

Analysis of Friction Utilization Within a Roadway Network Using Simulated Vehicle Trajectories *

Juliette Mitrovich¹, Srivenkata Satya Prasad Maddipati², Liming Gao³, Ilgin Guler⁴, Craig Beal⁵, and Sean Brennan⁶, *Member, IEEE*

Abstract— In a road network, differences in road geometry, traffic patterns, and traffic laws require a range of typical maneuvers, each of which strongly affects the vehicle's expected friction utilization. For example, very little tire force is required for driving straight on a low-speed road at a constant speed, performing no lane change maneuvers. Conversely, large tire forces may be required to navigate sharp highway curves, to stay in lane during sudden changes in lane offsets in a construction zone, or to stop abruptly from free-flow speed at a traffic light. Therefore, maps of this likely utilization are extremely valuable because they would reveal geolocations within a road network that require little relative friction utilization, and locations of large friction utilization. Such maps then may be useful to warn human drivers against lane changes on wet highway curves, to guide autonomous driving and/or driver assist algorithms toward geo-appropriate maneuver choices, or to modify as a function of weather the posted speed limits at road network locations whose normal maneuvering speed might violate friction limits.

The goal of this paper is to predict the areas of large friction utilization within a traffic network by using recorded vehicle trajectories. These trajectories are used as reference paths within a simulation of chassis dynamics along with a steering algorithm to predict the friction utilization as a function of road location. The friction utilization values are then mapped to geolocations to identify zones where friction utilization is largest. Knowing these locations allows the planning of maneuvers by both drivers and driving algorithms such that friction margins are maintained. The results show that, within a typical traffic network, there are significant and very highly localized areas where large friction utilization is typical.

I. INTRODUCTION

A fundamental tradeoff in vehicle operation is to balance the conflicting goals of reaching a destination as quickly as possible while minimizing speed to maximize safety. While the number of vehicle deaths decreased from 2017 to 2019 [1]–[3], the 43 thousand killed in 2021 mark a 10.5% increase in fatalities from 2020 [4]. Many factors have been found to increase traffic crash rates; but most notably, two of the largest factors include vehicle travelling speed and weather conditions. The balance between reasonable travel speeds and reasonable traveling weather affects the choice of speed limits, the design of road curvature, and even in the design of on-vehicle automated systems. Such operational and design decisions assume road conditions that, when violated, are often violated in specific locations and under specific conditions. If

the geo-location of these violation-prone areas are known, drivers can be warned when approaching to be more vigilant, automation systems can be tuned for robust operation in these zones, and/or transportation system operators can instrument these areas further with vehicle-to-infrastructure systems to obtain better data for operational decisions.

Vehicle speed and speed variation are well-known to affect road safety, specifically traffic crash rates [5]–[15]. In fact, Kloeden et al. [5] showed that by reducing the maximum free travelling speed on urban roads by just 5 km/h (3 mph), crash risk is reduced by 31%. Similarly, Elvik et al. [6] investigated the power model of the relationship between speed and road safety. Through a meta-analysis of 98 studies providing estimates of the relationship between speed and road safety, they showed that when speed decreases, the number of accidents or injured road users decreases in 95% of the cases. Similarly, Kloeden et al. [5] found that, when a vehicle travels faster than the average speed of the surrounding vehicles on urban roads, its risk of being involved in a car crash increases at a greater than exponential rate. Also, Qudus [16] found that a 1% increase in speed variation leads to a 0.3% increase in accident rates when all other conditions remain the same.

Weather conditions, including road surface conditions, also severely affect vehicle safety. Accident statistics show that 24% of all crashes are weather-related, with about 1% of those crashes leading to a fatality [17]. When adverse weather occurs, the road surface condition (RSC) is reduced [18]. The RSC is related to the friction supply (F_s) which is the amount of friction available at the tire-road interface. In this work, the term “friction” or “friction-coefficient” is assumed to be the sliding friction value of the tire-road averaged over the contact patch and is usually a normalized value that ranges from 0 to 1 for typical tire-road contact but can be much higher than 1 for high-performance tires or special road situations. The supply of friction must be greater than a maneuver's required utilization of friction (F_u). Specifically, it is an assumption of this work and of federal roadway design guides [19] that, to avoid skidding or spinning out in normal driving, the friction margin:

$$F_m = F_s - F_u \quad (1)$$

must be greater than zero. In other words, the friction utilization must be less than the friction supply. Examples where this assumption could be violated is in situations of tire blow-outs, collisions between vehicles, or very strong side-

*The work presented herein is supported by the National Science Foundation under grant numbers CNS-1932509, CNS-1931927, CNS-1932138 “CPS: Medium: Collaborative Research: Automated Discovery of Data Validity for Safety-Critical Feedback Control in a Population of Connected Vehicles”. The authors gratefully acknowledge this support.

Juliette Mitrovich¹, Satya Prasad², Liming Gao³, and Sean Brennan⁶ are with the College of Engineering, University Park, PA 16802 USA (e-mails: jfm5876, szm888, lug3582, and snb10@psu.edu).

Ilgin Guler⁴ is with the Department of Civil Engineering, University Park, PA 16802 USA (e-mail: iguler@engr.psu.edu).

Craig Beal⁵ is with the Department of Mechanical Engineering, Lewisburg, PA 17837 USA (e-mail: cbeal@bucknell.edu).

winds. To be clear, the friction utilization is the amount of friction that is used during a maneuver, which is not to be confused with friction demand, the friction required by a vehicle to maintain a hypothetical trajectory [20]. For example, for a vehicle to negotiate a tight turn at high-speed, the friction demand is very large – perhaps 2.0 or more to stay on the turn. However, if the vehicle starts to skid and not maintain the maneuver, its friction utilization will never be above the road friction value. For passenger vehicles, dry road friction is typically 0.7 to 0.9 [21].

The relationship between friction supply, friction demand, and friction utilization is thus an important physics-based surrogate safety metric for the ability or inability to control vehicle motion. As less friction is available to a vehicle, the more likely it is to spin out or skid unexpectedly [19]. The combination of excessive speeds and poor road conditions, particularly in adverse weather, can result in drivers overestimating their vehicle’s stopping distance [22], or ability to stop, slow down, or safely change lanes [22].

Roadway designers and operators must also consider how road geometry and rules-of-road affect the driver’s expected maneuvers and the resulting likely locations of skid events. Roadway design guides, for example the “Green Book” highway design guide in the United States [23], emphasizes analysis of friction supply and friction demand to set both roadway speed limits and road geometries such that the “design friction,” e.g., the friction a vehicle uses to navigate a roadway designed with particular speed limits and curvature, does not exceed the friction supply. However, these guides assume generically that a wet road condition would be the worst-case, and as well implicitly emphasize that certain areas of the roadways are likely to violate these assumptions first, for example roadways with high curvature. Unfortunately, a complete analysis of friction “hazard areas” is not possible in the design phase of a road, as some key factors may remain unknown, for example the as-built road surface friction. And even if known at the design stage, such hazard areas may not remain static as the road surface ages due to changing surfaces, operational conditions, and even vehicle technologies during the lifetime of each road segment.

A road’s friction supply is an extremely variable function of the road conditions (ice, snow, water), tire construction (smoothness, tread pattern, etc.), tire materials (tire compounds), and vehicle speed (hydroplaning) [20]. It is common to estimate a bulk friction supply value over the tire-patch to develop road rules, choose surface treatments, or design vehicle chassis control algorithms. And many technologies exist to estimate road friction experimentally or with model-based techniques [22] including friction-estimation [23] techniques such as classification systems, trained neural networks, and real-time model-based estimation algorithms [24]–[29].

Many prior research results emphasize that if friction is better known, Advanced Driver Assistant (ADAS) systems can provide increased road-safety benefits [31]. For example, intelligent speed adaptation (ISA), has proven benefits in road safety because it is a system that uses communication technology and vehicle information (e.g., road friction estimation data) to inform a vehicle of an appropriate speed limit [30]. ISA can be split into two classifications to define

the advisory speed of the vehicle: static or dynamic. Static ISA systems rely on fixed speed information, and dynamic ISA systems rely on changing speed and environmental information [31]. ISA can further be split into two modes that define the application of the advised speed: advisory or intervening [30]. Advisory ISA systems recommend a safe travelling speed to the driver through the vehicle’s dashboard and warn the driver if that speed is exceeded. Intervening ISA systems enforce the recommended speed by implementing a control action that is either overridable (voluntary) or non-overridable (mandatory). The results of this paper could augment such ISA functions by geo-locating areas of potentially hazardous friction conditions.

This paper presents an approach to map the locations of large friction utilization to its geolocation in a road network by analyzing simulated traffic trajectories. These results can then be used to determine how the trajectory of a vehicle – the speed of the vehicle, choice of lane change maneuvering, etc. – should change in adverse weather conditions. These mapped locations also could benefit friction estimation and the implementation of robust vehicle chassis controllers, and interested readers are referred to the National Academies report 774 by the authors analyzing low-friction effects on curve-keeping vehicle behaviors [19]. To present this process in further detail, the rest of the paper is organized as follows: section II documents the methods to complete this work, including running a calibrated traffic simulation of State College, Pennsylvania, post processing the traffic simulation data to obtain friction utilization and plotting the friction utilization to its geolocation within the State College Road network. Section III presents the figures and results of this work. Finally, the conclusion and future work of this study are discussed in Section IV.

II. METHODS

This section presents the methods used to obtain individual vehicle trajectories from traffic in the State College Road network, analyze the traffic trajectories to determine friction utilization, and map the locations of large friction utilization to its geolocation in the road network. Section A presents the State College OSM (Open Street Map) Road network used for a network traffic simulation, which is designed and managed by Dr. Ilgin Guler and her research team at the Pennsylvania State University; this section also discusses data management. Section B discusses post-processing and analysis of the traffic simulation data to determine friction utilization. Finally, Section C introduces the process to map friction demand to its geolocation.

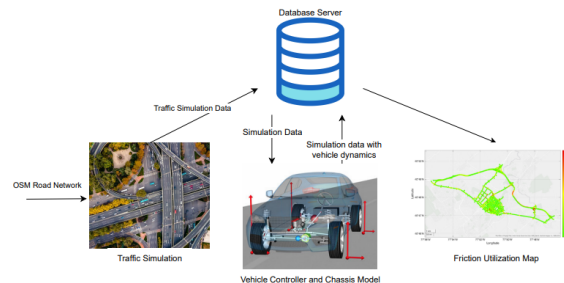


Figure 1. Illustration of the data flow strategy

A. Traffic Flow Modeling

While it is possible to simulate vehicles one-at-a-time traversing a roadway network using conventional chassis simulation tools (Simulink, for example), actual traffic involves interactions outside the ego-vehicle such as traffic signals, changing roadway rules as a function of road position, and interactions with surrounding vehicles which in turn, are interacting with each other and their own rule sets. A single vehicle simulation would have difficulty capturing many such factors, nor would it statistically predict where accelerations and decelerations occur resulting from such interactions. A microscopic traffic simulation was thus employed to capture the large-scale individual vehicle motions across the approximately 100km² traffic network, with approximately 15,000 interacting vehicles.

The first step of this work was to use an OSM representation of the State College Road network and implement it in a microscopic traffic simulation tool to produce reference vehicle trajectories throughout State College. The traffic modeling and simulation tool, AIMSUN (Advanced Interactive Microscopic Simulator for Urban and non-urban Networks) was employed. A traffic simulation was chosen because of its ability to create realistic traffic networks with ease, with traffic signals and volume levels calibrated by the work of Prof. Guler and her team using measured data for each signal and via traffic volume probes. With access to a calibrated road network, the traffic simulation provides reasonably accurate signal patterns, traffic volume, and traffic volume flow rate estimates.

To manage the data generated by the traffic simulation, a PostgreSQL 12.09 database server is used. The amount of data that is collected and analyzed in this work is not trivial, more than 100 GB of data for the 1-hour of peak traffic volume. And from this data, specific data points need to be found quickly and efficiently. An illustration of the data flow strategy is found in Figure 1.

B. Vehicle Model Simulation

Vehicle trajectories directly from the traffic simulation were not used to calculate friction utilization because that data is often inaccurate because traffic simulations do not account for lateral and rotational dynamics of a vehicle, and thus is especially incorrect in estimating the lateral (right and left in the vehicle frame) force component related to the tire's friction ellipse. Traffic simulations often allow vehicles to make perfect 90 degree turns which, in physical forces, would require near infinite tire forces; this disagrees with the gradual arced turns completed in real life, which are feasible with finite lateral tire forces. Further, the longitudinal dynamics (front to rear in the vehicle coordinate frame) are also grossly simplified in most traffic simulations. For example, when a vehicle comes to a stop within the traffic simulation, the deceleration profile may or may not meet physical constraints of the road.

After the traffic simulation data was collected, the data was corrected into representative chassis dynamics by using the traffic simulation vehicle trajectories as position reference requests in a vehicle chassis controller. This was an involved process that is described in detail in [32]. The vehicle controller then predicted a steering angle and wheel torque that gives feasible maneuvers, which were used as an input to a

vehicle chassis model, implemented as an ODE model in MATLAB, to "drive" the vehicle through the road network.

There are many different vehicle models to choose from, ranging in complexity, each with its own advantages. For example, the kinematic vehicle model [33] assumes the slip angles of both wheels is equal to zero, neglecting lateral forces generated by the tires. Because of this, the model is only feasible at longitudinal speeds that produce little to no lateral tire forces (less than 5m/s) [33]. As the longitudinal speed increases and/or the lateral tire forces and side slip angles increase, the accuracy of the kinematic model significantly decreases, and the use of a more complex vehicle model is necessary. The purpose of this paper is to discover areas of high friction utilization. At lower speeds, friction utilization is negligible, so it was determined a more complex model was needed.

In this study, a 7 degree-of-freedom (7DOF) vehicle model combined with a Pacejka brush tire model similar to the version presented by Rajamani in [33], was employed. As well, a 3DOF vehicle model was used. Both models yielded accurate results. However, due to the fast dynamics of the wheels in the 7DOF model, the time step to integrate the 7DOF model with a 4th order Runge-Kutta method had to be particularly small (0.001s) to ensure there would be no wheel lock up, which would lead to an inaccurate calculation of friction utilization. This choice of time step caused the code run time to be substantially high, with it taking about 80 hours to analyze 1 hour of traffic simulation data. In contrast, the 3DOF vehicle model generates longitudinal wheel forces by directly controlling longitudinal wheel slip. This allowed us to bypass the fast dynamics of the wheels to control the slower dynamics of the vehicle with a much larger time step of 0.05s. With the 3DOF model and larger time step, the 1-hour traffic simulation with nearly 15,000 vehicles can be analyzed in less than 3 hours with non-optimized MATLAB code.

In the model employed in this study, the 3 primary degrees of freedom on both the 3DOF and 7DOF models include: longitudinal motion, lateral motion, and yaw rate. The longitudinal and lateral velocities are represented by \dot{x} and \dot{y} , respectively, the yaw rate is represented by $\dot{\psi}$. The longitudinal motion is:

$$m\ddot{x} = (F_{xfl} + F_{xfr}) \cos(\delta) + F_{xrl} + F_{xrr} - (F_{yfl} + F_{yfr}) \sin(\delta) + m\dot{\psi}\dot{y}. \quad (2)$$

The lateral motion is:

$$m\ddot{y} = (F_{yfl} + F_{yfr}) \cos(\delta) + F_{yrl} + F_{yrr} + (F_{xfl} + F_{xfr}) \sin(\delta) - m\dot{\psi}\dot{x}. \quad (3)$$

The yaw motion is:

$$I_z\ddot{\psi} = a(F_{xfl} + F_{xfr}) \sin(\delta) + a(F_{yfl} + F_{yfr}) \cos(\delta) - b(F_{yrl} + F_{yrr}) + \frac{w}{2}(F_{xfr} - F_{xfl}) \cos(\delta) + \frac{w}{2}(F_{xrr} - F_{xrl}) + \frac{w}{2}(F_{yfl} - F_{yfr}) \sin(\delta). \quad (4)$$

The steering angle of the vehicle is denoted by δ . The longitudinal forces of the front left, front right, rear left, and rear right tires are given by F_{xfl} , F_{xfr} , F_{xrl} , and F_{xrr} , respectively. F_{yfl} , F_{yfr} , F_{yrl} , and F_{yrr} represent the lateral forces of the front left, front right, rear left, and rear right tires, respectively. Finally, a , b , and w represent the longitudinal

distance from the center of gravity (cg) of the vehicle to the front wheels, the longitudinal distance from the cg of the vehicle to the rear wheels, and the lateral distance between the right and left wheels, respectively.

For the 7DOF model, the four tires are represented in this work through the wheel rotational dynamics assuming drive torque and brake torque are known. The wheel rotational dynamics are given by:

$$J_w \dot{\omega} = T_d - T_b - r_{eff} F_x, \quad (5)$$

where T_d and T_b represent the drive torque and brake torque, respectively, with a proportional controller given in [32]. The effective wheel radius is symbolized by r_{eff} and J_w defines the rotational moment of inertia of each wheel. Equation (5) can be used for all four wheels. Of note: the wheel slip behavior such as Anti-Lock Braking or Traction Control should be included, but it is beyond the scope of this paper which is assuming typical driving behaviors to determine conditions, if any, in which typical driving may lead to requiring wheel-slip control.

Once the processed vehicle trajectories were created, the friction utilization was calculated based on the tire's friction ellipse. The friction ellipse represents the interaction between a vehicle's tire forces and the road. The ellipse is described by:

$$\left(\frac{F_x}{F_{x,max}} \right)^2 + \left(\frac{F_y}{F_{y,max}} \right)^2 \leq 1, \quad (6)$$

where F_x and F_y are the longitudinal and lateral tire force, respectively, and $F_{x,max}$ and $F_{y,max}$ represent the maximum achievable longitudinal and lateral tire force, respectively. The final friction utilization, F_u , is represented by the following equation used by Torbic et al. in [19]:

$$F_u = \sqrt{\left(\frac{F_x}{N} \right)^2 + \left(\frac{F_y}{N} \right)^2}, \quad (7)$$

where N represents the normal force of the tire. This equation is used for all four tires, where F_x , F_y , and N are the values of the tire of interest.

C. Mapping Friction Demand to its Geolocation

Once the friction utilization has been calculated for each vehicle trajectory, it is mapped to its associated geolocation on a map of State College. For the purposes of this paper, the "characteristic" behavior cases were analyzed. This "characteristic" behavior means that for every trajectory analyzed, the vehicles were assumed to be the same (length, weight, cornering stiffness, etc.), and only reference vehicle trajectories that had no lane changes were analyzed, as such maneuvers greatly change the friction demand (see [32] for details). Also, because multiple vehicles drive over each road segment in the traffic simulation, some sections included hundreds of vehicle traversals. The data analysis revealed that it is not necessary to simulate every single vehicle trajectory. Instead, if there were more than 75 vehicles traversing a section, only the first 75 vehicles were analyzed while keeping track of the minimum, maximum, and average friction utilization for each vehicle, stored relative to the vehicle's geolocation. In the traffic simulation, it was also assumed that the vehicles experience nominal road conditions. In other words, the road surface condition is representative of a dry road on a sunny day with the maximum amount of friction ($\mu = 1.0$) available to the vehicle.

1) Mapping Friction Utilization

A color bar using friction utilization values ranging from low of zero (green) to high of 1 (red) was chosen to represent the friction utilization values on the map of State College. In locations where the friction utilization is greater than 0.75, this indicates a location requiring a large amount of friction supply for vehicles to safely travel. In locations where the friction utilization ranges from 0.36 to 0.74, the map is generally in yellow, indicating a moderate amount of friction is used. Finally, locations where the friction utilization is lower than 0.35 are generally green, meaning minimal friction is used for a vehicle to safely travel. Figure 2 presents a map of the maximum friction utilization for the entire State College Road network. Spot locations of large friction utilization clearly show sharp curves, signalized intersections, highway off- and on-ramps, and stop sign areas.

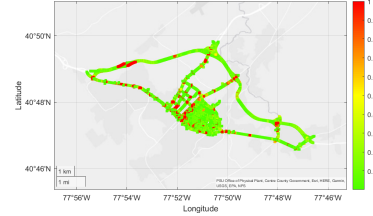


Figure 2. Maximum friction utilization map of the entire State College Road network

III. RESULTS



Figure 3. Maximum friction utilization map zoomed in on downtown State College

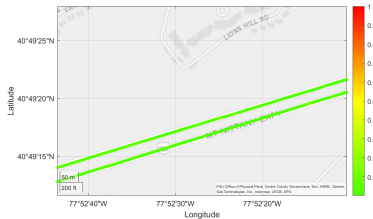


Figure 4. Maximum friction utilization map zoomed in on I99, representing straight-line, high-speed driving

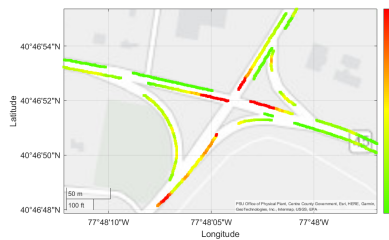


Figure 5. Maximum friction utilization map zoomed in on an intersection with on- and off-ramps, controlled by a stop light

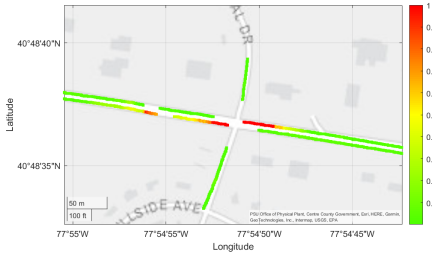


Figure 6. Maximum friction utilization map zoomed in on the intersection with a steep downhill on left-side approach.



Figure 7. Maximum friction utilization map of a particularly hazardous curve section on an interstate highway.

In urban driving situations, vehicles will either slow to a stop at an intersection or drive through it depending on the traffic signals. Depending on which maneuver is completed, the friction utilization is vastly different. This paper aims to identify areas of large friction utilization; therefore, we are more interested in the maximum friction used at an intersection to represent vehicles that must come to a stop. In contrast, the average friction utilization may occlude the stopping friction utilization. Therefore, the maximum friction utilization is mapped for figures that represent data from urban driving situations. Similarly in arterial driving situations, vehicles often perform braking and acceleration maneuvers to adjust to decreases and increases in speed limits, especially at on- and off-ramps. Therefore, it is necessary to map the maximum friction utilization in these situations.

Figure 3 presents a zoomed-in view of the maximum friction utilization of downtown State College, which is considered urban driving. The figure demonstrates that there is high friction utilization at locations within approximately 15 meters of a hard stop location (stop lights, stop signs). In these regions, friction utilization increases rapidly, with values ranging between 0.5 and 0.7 and occasionally reaching close to 1. This is consistent with the microscopic traffic dynamics used to generate the traffic simulation results; the simulation parameters are tuned from models of how vehicles are driven in the real world. It is outside the scope of this paper to assess the choice and tuning of these models.

Figure 4 presents a zoomed-in estimate of the maximum friction utilization of straight-line highway driving on Interstate 99 (I99). This location was specifically chosen to show the low friction utilization, as expected due to the minimal amount of hard braking or turning that occurs in typical straight-line highway driving.

Figure 5 presents a zoomed-in estimate of the maximum friction utilization of arterial driving at a traffic-light-

controlled intersection with both on- and off-ramps. At this location, an increase in friction utilization is seen due to braking, cornering, and a combination of the two. This information can be used to guide modifications to operational rules-of-road in response to road surface conditions to improve the safety of the road network. For example, Figure 6 presents a zoomed-in view of the friction utilization at a specific intersection on North Atherton Street. Common sense indicates that vehicles will slow down in the vicinity of a traffic light, but only the traffic simulator will easily indicate – based on queue length, road grade, rules of road, and surrounding intersections – where exactly that slow-down is most abrupt. At this location, it is seen that friction levels are approximately 0.3 for straight-line driving due to a steep downhill grade approaching the intersection and increasing to about 0.8 as vehicles decelerate near the signal light. And due to a combination of a queue length and steep grade, there is significant friction utilization on the left-hand approach to the intersection, something difficult to predict without the combination of traffic simulation data and chassis dynamic simulations. At this intersection in the real-world, the traffic signal light is switched off during icy/snowy conditions because the decrease in friction supply that occurs in such events causes vehicles to skid through the intersection.

In Figure 7, a section of Route 322 with particularly high maximum friction utilization is shown. This location is where many crashes have been reported or caused damage to the road infrastructure. Counterintuitively, the maximum friction utilization is on the south-bound area which is an uphill location which includes past fatalities at precisely the “red” locations; however, on this road segment, only the northbound (downhill) portion of the roadway has signage warning of a dangerous curve. This agreement between simulation and real-world observations, including insights beyond intuition, shows the potential of this work to improve road safety.

IV. CONCLUSIONS AND FUTURE WORK

This paper presented a method to analyze the friction utilization of a road network using simulated vehicle trajectories. A traffic simulation was chosen to create the reference trajectories due to its ability to create realistic traffic networks. A vehicle modeling simulation process was used subsequently to “drive” the network and create physically realistic trajectories to provide accurate friction utilization data. The resulting friction utilization values were mapped to their geolocation within the State College Road network.

The friction utilization map created in this work creates potential for the following improvements:

1. Designing vehicle maneuvers by both drivers and vehicle control algorithms such that friction margins are maintained within these zones by warning/choosing speeds and lane maneuvering accordingly.
2. Road policy, such as speed limits in the specific regions of high friction utilization, set to avoid vehicles demanding more friction than is available.

3. Engineering datasets illustrating where, in typical traffic patterns, dangerous geolocated areas may first emerge in adverse weather conditions.

REFERENCES

- [1] National Highway Traffic Safety Administration, "2017 Fatal Motor Vehicle Crashes: Overview," 2018.
- [2] National Highway Traffic Safety Administration, "2018 Fatal Motor Vehicle Crashes: Overview," 2019.
- [3] National Highway Traffic Safety Administration, "Overview of Motor Vehicle Crashes in 2019," 2020.
- [4] National Highway Traffic Safety Administration, "Early Estimates of Motor Vehicle Traffic Fatalities And Fatality Rate by Sub-Categories in 2021," May 2022.
- [5] C. Kloeden, G. Ponte, and A. McLean, "Traveling Speed and the Risk of Crash Involvement on Rural Roads," *Australian Transport Safety Bureau*, pp. 1–50, Jul. 2001.
- [6] R. Elvik, P. Christensen, and A. Amundsen, "Speed and Road Accidents: An Evaluation of the Power Model." Oslo, pp. 15, 2004.
- [7] L. Aarts and I. van Schagen, "Driving Speed and the Risk of Road Crashes: A Review," *Accid Anal Prev*, vol. 38, no. 2, pp. 215–224, Mar. 2006, doi: 10.1016/j.aap.2005.07.004.
- [8] J. Raiyn, "Real-time Intelligent Speed Adaptation in Heterogeneous Road Networks," in *2021 International Symposium on Networks, Computers and Communications (ISNCC)*, IEEE, Oct. 2021, pp. 1–7, doi: 10.1109/ISNCC52172.2021.9615840.
- [9] N. Garber and R. Gadirau, "Speed Variance and its Influence on Accidents," *AAA Foundation for Traffic Safety*. Washington, DC, pp. 1–69, 1988.
- [10] I. Galanis, I. Anagnostopoulos, P. Gurunathan, and D. Burkard, "Environmental-Based Speed Recommendation for Future Smart Cars," *Future Internet*, vol. 11, no. 3, pp. 78–96, Mar. 2019, doi: 10.3390/fi11030078.
- [11] L. S. Friedman, D. Hedeker, and E. D. Richter, "Long-Term Effects of Repealing the National Maximum Speed Limit in the United States," *Am J Public Health*, vol. 99, no. 9, pp. 1626–1631, Sep. 2009, doi: 10.2105/AJPH.2008.153726.
- [12] A. C. Wagenaar, F. M. Streff, and R. H. Schultz, "Effects of the 65 mph Speed Limit on Injury Morbidity and Mortality," *Accid Anal Prev*, vol. 22, no. 6, pp. 571–585, Dec. 1990, doi: 10.1016/0001-4575(90)90029-K.
- [13] A. A. M. Aljanahi, A. H. Rhodes, and A. V. Metcalfe, "Speed, Speed Limits and Road Traffic Accidents Under Free Flow Conditions," *Accid Anal Prev*, vol. 31, no. 1–2, pp. 161–168, Jan. 1999, doi: 10.1016/S0001-4575(98)00058-X.
- [14] A. Vadeby and Å. Forsman, "Speed Distribution and Traffic Safety Measures," in *Traffic Safety*, Hoboken, NJ, USA: John Wiley & Sons, Inc., 2016, pp. 161–176, doi: 10.1002/9781119307853.ch11.
- [15] N. J. Garber and R. Gadirau, "Factors Affecting Speed Variance and its Influence on Accidents," *Transportation Research Record 1213: Journal of the Transportation Research Board*. Transportation Research Board of the National Academies, Washington, DC, pp. 64–71, 1989.
- [16] M. Quddus, "Exploring the Relationship Between Average Speed, Speed Variation, and Accident Rates Using Spatial Statistical Models and GIS," *Journal of Transportation, Safety & Security*, vol. 5, no. 1, pp. 27–45, Jan. 2013.
- [17] P. A. Pisano, L. C. Goodwin, and M. A. Rossetti, "U.S Highway Crashes in Adverse Road Weather Conditions," in *24th Conference on International Interactive Information and Processing Systems for Meteorology, Oceanography and Hydrology*, New Orleans, LA, 2008, pp. 1–16.
- [18] T. Usman, L. Fu, and L. F. Miranda-Moreno, "Quantifying safety benefit of winter road maintenance: Accident frequency modeling," *Accid Anal Prev*, vol. 42, no. 6, pp. 1878–1887, Nov. 2010, doi: 10.1016/j.aap.2010.05.008.
- [19] D. Torbic *et al.*, "Superelevation Criteria for Sharp Horizontal Curves on Steep Grades." pp. 1–192, 2014.
- [20] R. Lutz, "A Comparison of Friction Supply, Friction Demand, and Maximum Design Friction on Sharp Horizontal Curves with Steep Grades," The Pennsylvania State University, 2013.
- [21] J. Y. Wong, *Theory of Ground Vehicles*, 3rd ed. 2001.
- [22] A. Abdi Kordani, O. Rahmani, A. S. Abdollahzadeh Nasiri, and S. M. Boroomandrad, "Effect of Adverse Weather Conditions on Vehicle Braking Distance of Highways," *Civil Engineering Journal*, vol. 4, no. 1, pp. 46–58, Feb. 2018, doi: 10.28991/cej-030967.
- [23] S. Khaleghian, A. Emami, and S. Taheri, "A technical survey on tire-road friction estimation," *Friction*, vol. 5, no. 2, pp. 123–146, Jun. 2017, doi: 10.1007/s40544-017-0151-0.
- [24] A Policy on Geometric Design of Highways and Streets (The Green Book), 7th ed. AASHTO, 2018.
- [25] J. Alonso *et al.*, "On-Board Wet Road Surface Identification Using Tyre/Road Noise and Support Vector Machines," *Applied Acoustics*, vol. 76, pp. 407–415, Feb. 2014, doi: 10.1016/j.apacoust.2013.09.011.
- [26] A. Howard and H. Seraji, "Vision-Based Terrain Characterization and Traversability Assessment," *J Robot Syst*, vol. 18, no. 10, pp. 577–587, Oct. 2001, doi: 10.1002/rob.1046.
- [27] A. Tuononen and L. Hartikainen, "Optical Position Detection Sensor to Measure Tyre Carcass Deflections in Aquaplaning," *International Journal of Vehicle Systems Modelling and Testing*, vol. 3, no. 3, pp. 189–199, 2008, doi: 10.1504/IJVSMT.2008.023837.
- [28] K. B. Singh and S. Taheri, "Estimation of Tire–Road Friction Coefficient and its Application in Chassis Control Systems," *Systems Science & Control Engineering*, vol. 3, no. 1, pp. 39–61, Jan. 2015, doi: 10.1080/21642583.2014.985804.
- [29] G. Baffet, A. Charara, D. Lechner, and D. Thomas, "Experimental Evaluation of Observers for Tire–Road Forces, Sideslip Angle and Wheel Cornering Stiffness," *Vehicle System Dynamics*, vol. 46, no. 6, pp. 501–520, Jun. 2008, doi: 10.1080/00423110701493963.
- [30] R. Rajamani, G. Phanomchoeng, D. Piyabongkarn, and J. Y. Lew, "Algorithms for Real-Time Estimation of Individual Wheel Tire-Road Friction Coefficients," *IEEE/ASME Transactions on Mechatronics*, vol. 17, no. 6, pp. 1183–1195, Dec. 2012, doi: 10.1109/TMECH.2011.2159240.
- [31] A. Shaout, D. Colella, and S. Awad, "Advanced Driver Assistance Systems - Past, Present and Future," in *2011 seventh International Computer Engineering Conference (ICENCO'2011)*, IEEE, Dec. 2011, pp. 72–82, doi: 10.1109/ICENCO.2011.6153935.
- [32] O. Carsten, F. Lai, K. Chorlton, P. Goodman, D. Carslaw, and S. Hess, "Speed Limit Adherence and its Effect on Road Safety and Climate Change." pp. 1–127, Oct. 2008.
- [33] R. H. Ordóñez-Hurtado, W. M. Griggs, K. Massow, and R. N. Shorten, "Intelligent Speed Advising Based on Cooperative Traffic Scenario Determination," in *Optimization and Optimal Control in Automotive Systems. Lecture Notes in Control and Information Sciences*, Springer, Cham, 2014, pp. 77–92, doi: 10.1007/978-3-319-05371-4_5.
- [34] F. Jiménez, F. Aparicio, and J. Páez, "Evaluation of In-Vehicle Dynamic Speed Assistance in Spain: Algorithm and Driver Behaviour," *IET Intelligent Transport Systems*, vol. 2, no. 2, pp. 132–143, 2008, doi: 10.1049/iet-its:20070019.
- [35] R. Gallen, N. Hautiere, A. Cord, and S. Glaser, "Supporting Drivers in Keeping Safe Speed in Adverse Weather Conditions by Mitigating the Risk Level," *IEEE Transactions on Intelligent Transportation Systems*, vol. 14, no. 4, pp. 1558–1571, Dec. 2013, doi: 10.1109/TITS.2013.2262523.
- [36] A. Hazaar, A. Lioi, and M. Bassani, "Development of a Novel Intelligent Speed Adaptation System Based on Available Sight Distance," *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2675, no. 9, pp. 1573–1584, Sep. 2021, doi: 10.1177/03611981211008885.
- [37] S. M. Savaresi, L. Piroddi, and M. Tanelli, "Real-Time Identification of Tire–Road Friction Conditions," *IET Control Theory & Applications*, vol. 3, no. 7, pp. 891–906, Jul. 2009, doi: 10.1049/iet-cta.2008.0287.
- [38] J. Mitrovich, "Database-Mediated Network Friction Utilization Analysis to Improve Road Safety," Masters, The Pennsylvania State University, 2023.
- [39] R. Rajamani, *Vehicle Dynamics and Control*. Boston, MA: Springer US, 2012, doi: 10.1007/978-1-4614-1433-9.