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# W3 Lesson 1



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## Vectors and Linear Transformations

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### Machine Learning motivation

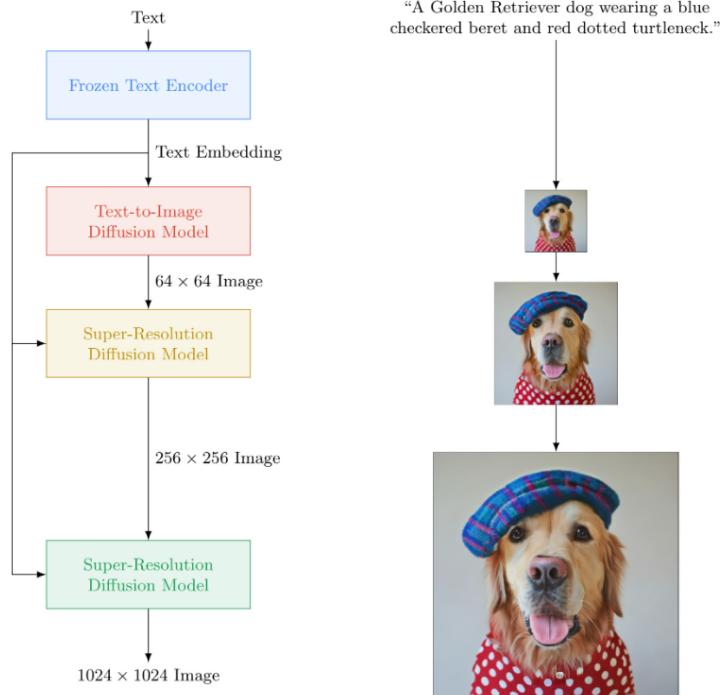
# Neural Networks - AI generated images



AI-generated human faces.

- Generative learning: Generating realistic looking images.

# Text-to-image and image-to-text generation



"A Golden Retriever dog wearing a blue checkered beret and red dotted turtleneck."



The screenshot shows an AI interface for generating text from images. On the left, there's a 'Model' section containing a large clock icon. On the right, there's an 'Output' section with a text input field containing the text "wall clock - wall clock." Above the input field, the number "3.0s" is displayed, likely indicating the time taken for the generation. There are also edit icons (pencil and X) at the top right of the output area.



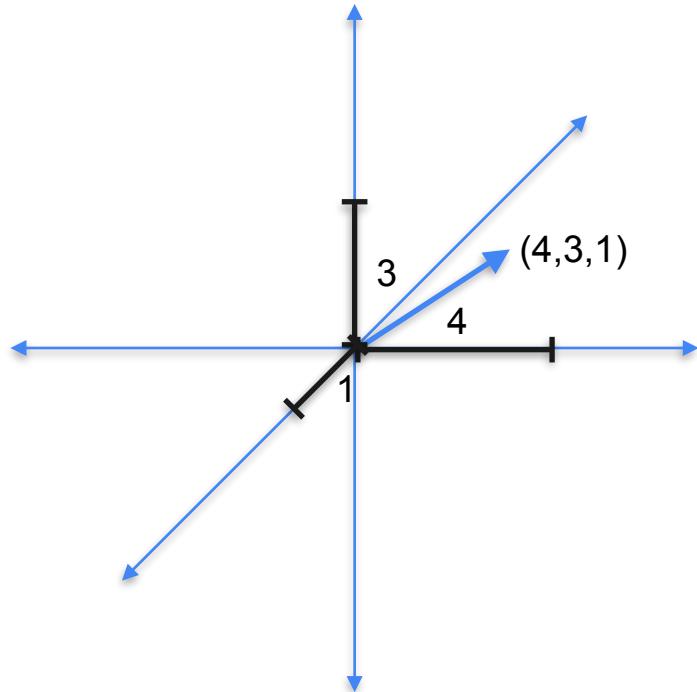
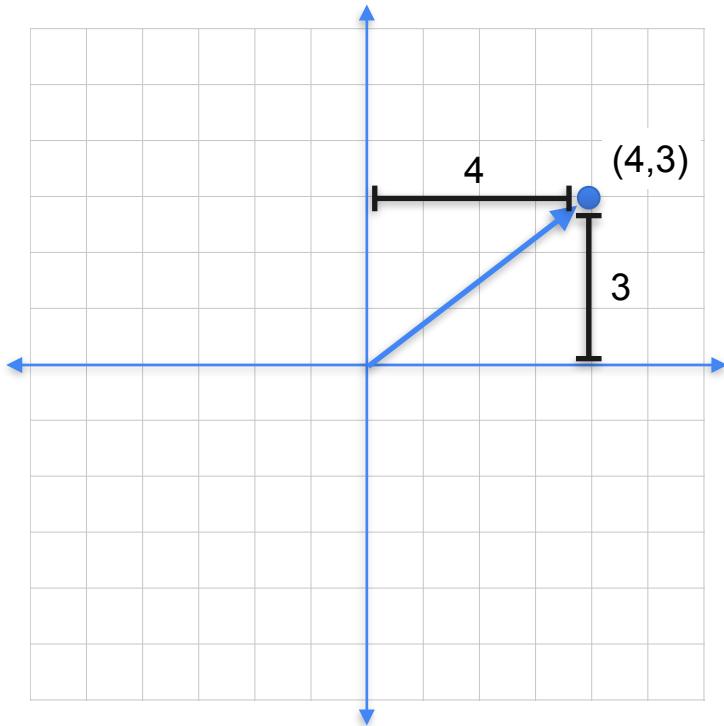
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# Vectors and Linear Transformations

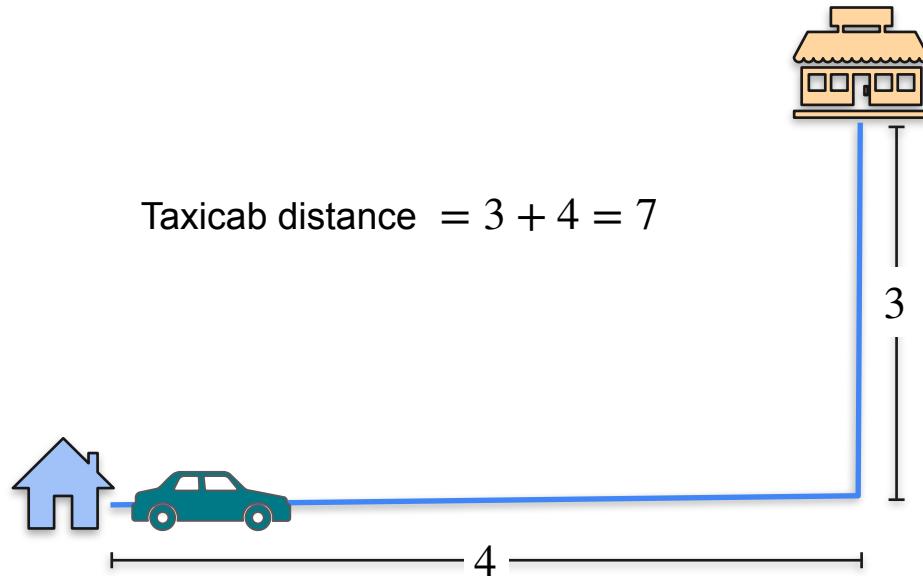
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## Vectors and their properties

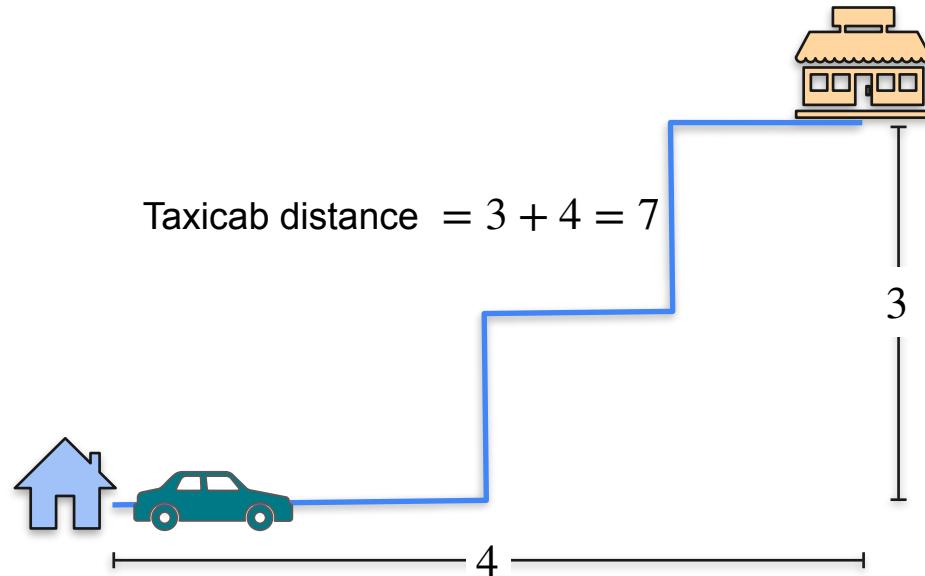
# Vectors



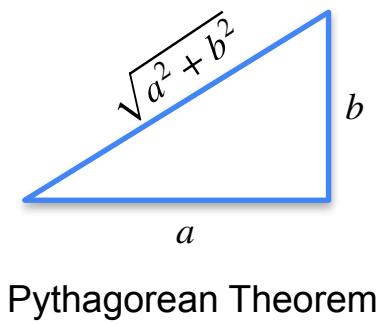
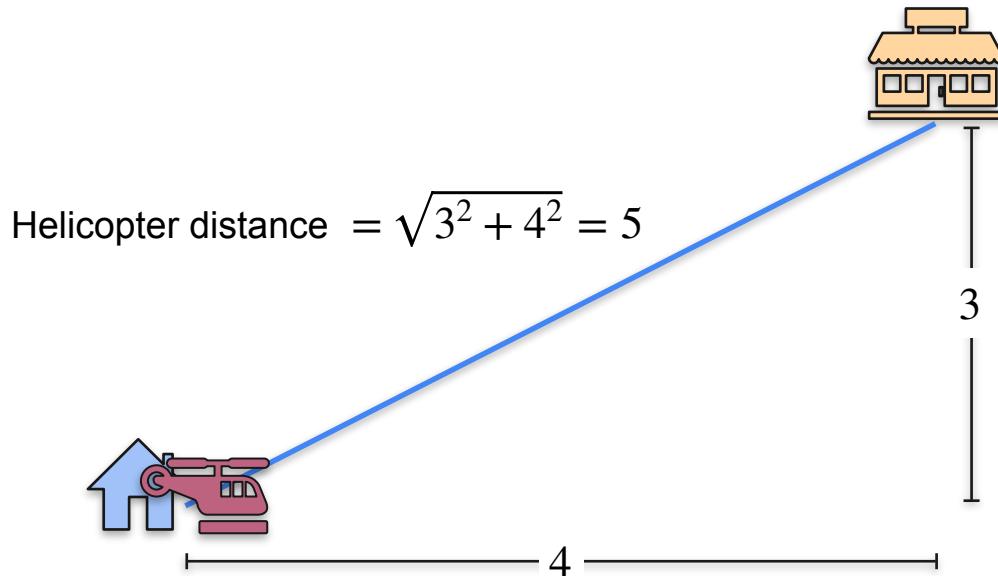
# How to get from point A to point B?



# How to get from point A to point B?

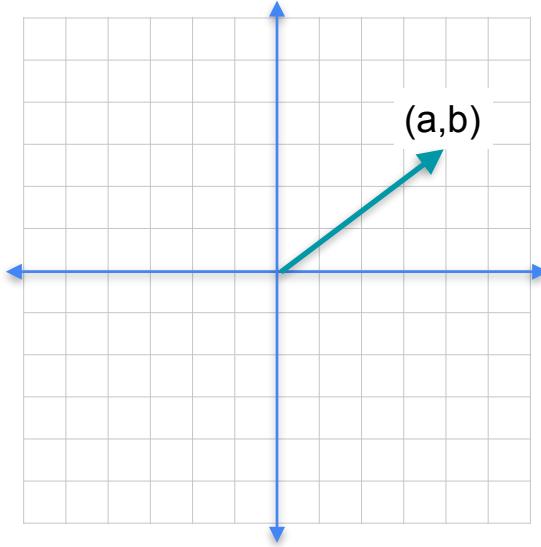


# How to get from point A to point B?

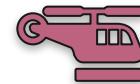


Pythagorean Theorem

# Norms

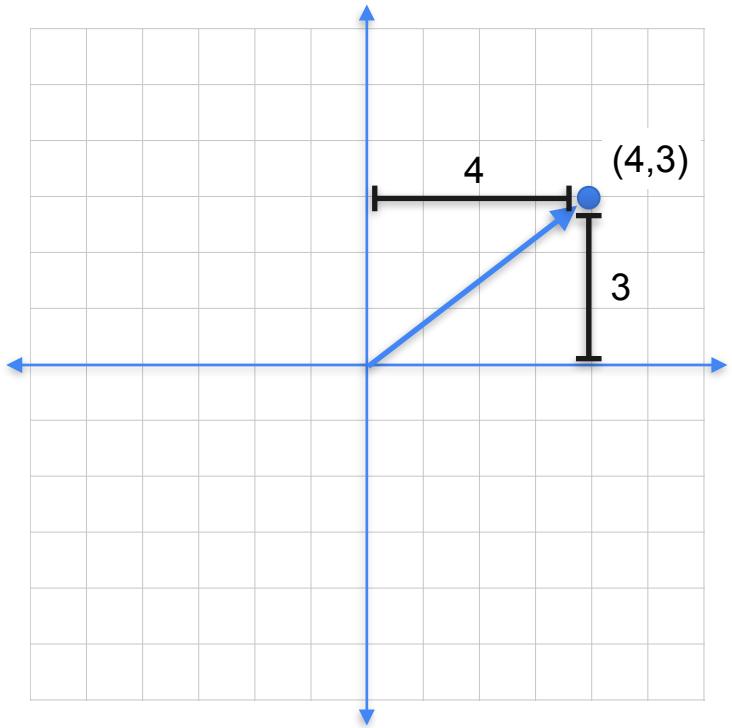


$$\text{L1-norm} = \|(a, b)\|_1 = |a| + |b|$$



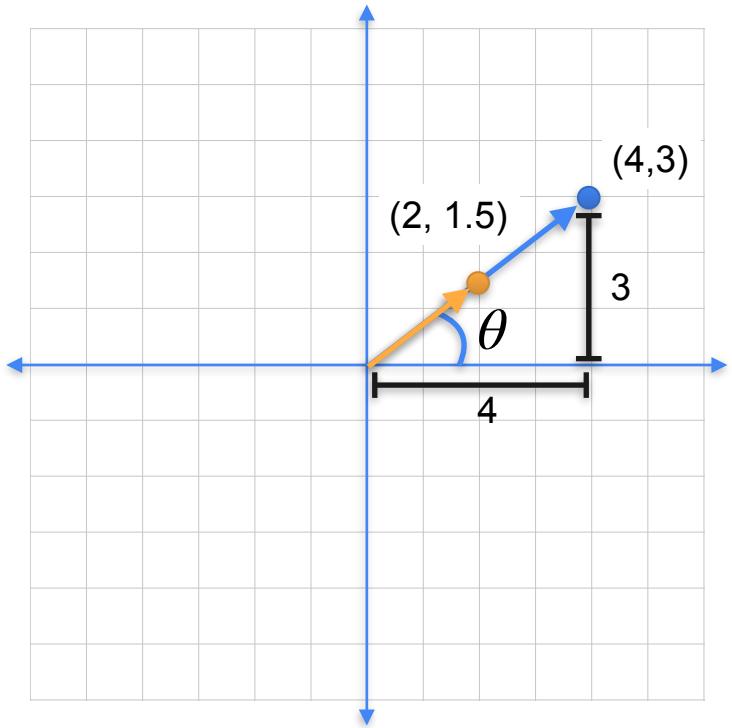
$$\text{L2-norm} = \|(a, b)\|_2 = \sqrt{a^2 + b^2}$$

# Norm of a vector



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

# Direction of a vector



$$\tan(\theta) = \frac{3}{4}$$

$$\theta = \arctan(3/4) = 0.64 = 36.87^\circ$$



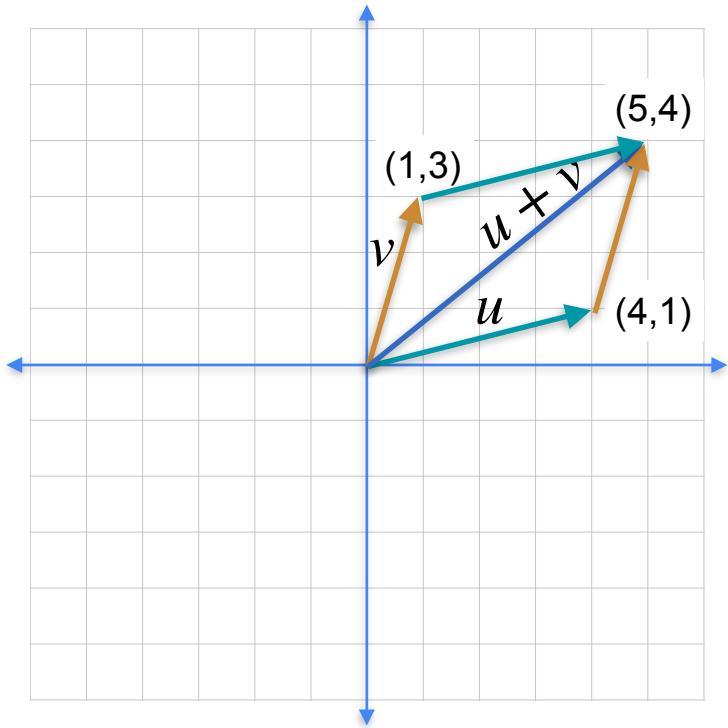
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# Vectors and Linear Transformations

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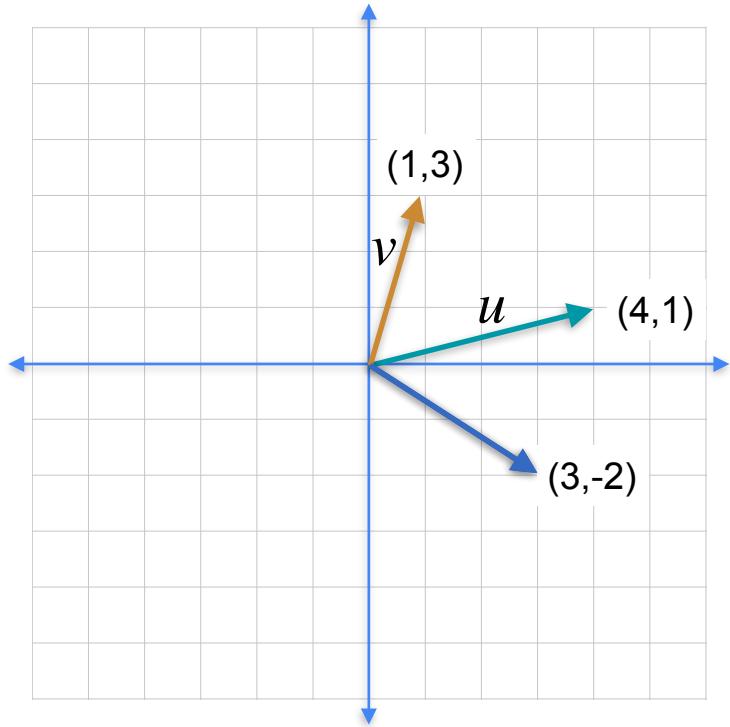
**Sum and difference of  
vectors**

# Sum of vectors



$$u + v = (4 + 1, 1 + 3) = (5, 4)$$

# Difference of vectors



$$u - v = (4 - 1, 1 - 3) = (3, -2)$$

# General definition: sum and difference

Same number  
of components

$$x = (x_1 \quad x_2 \quad \dots \quad x_n)$$
$$y = (y_1 \quad y_2 \quad \dots \quad y_n)$$

**Sum**

$$x + y = (x_1 + y_1 \quad x_2 + y_2 \quad \dots \quad x_n + y_n)$$

Sum component by component

**Difference**

$$x - y = (x_1 - y_1 \quad x_2 - y_2 \quad \dots \quad x_n - y_n)$$

Subtract component by component



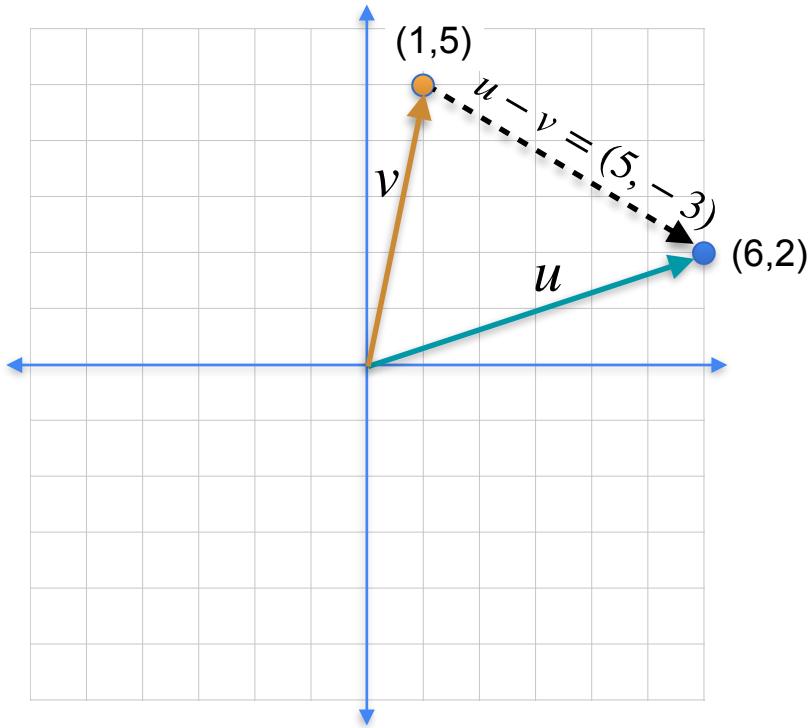
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# Vectors and Linear Transformations

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## Distance between vectors

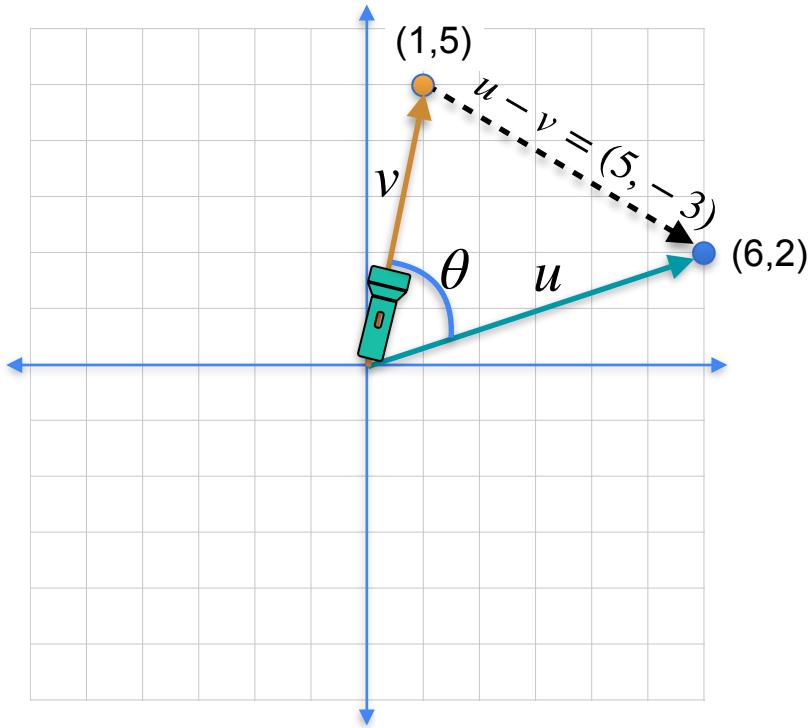
# Distances



L1-distance  $\|u - v\|_1 = |5| + |-3| = 8$

L2-distance  $\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$

# Distances



L1-distance  $\|u - v\|_1 = |5| + |-3| = 8$

L2-distance  $\|u - v\|_2 = \sqrt{5^2 + 3^2} = 5.83$

$\cos(\theta)$   
Cosine distance



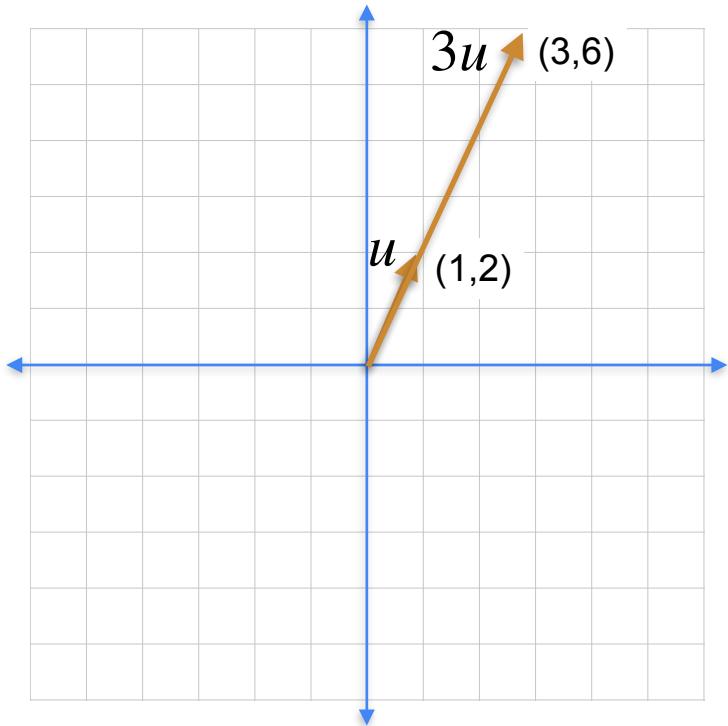
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# Vectors and Linear Transformations

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**Multiplying a vector by a scalar**

# Multiplying a vector by a scalar

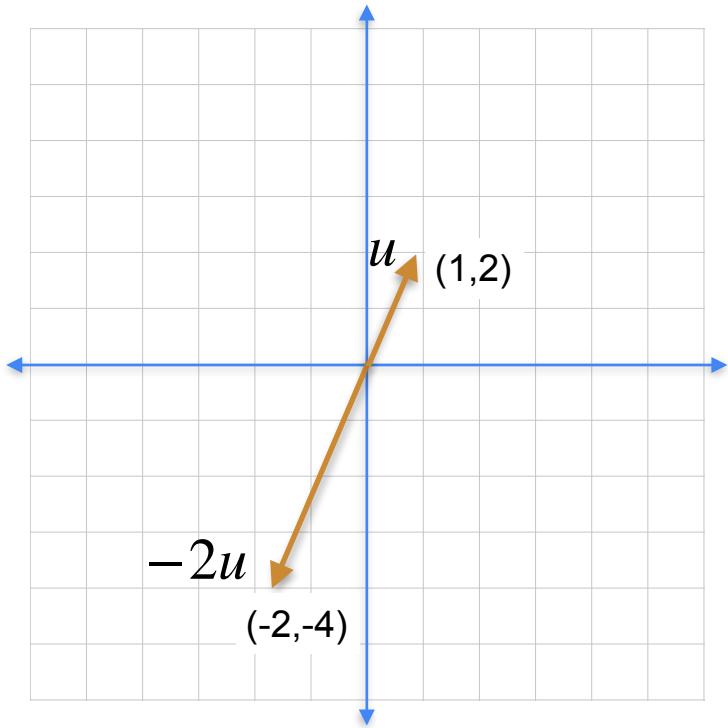


$$u = (1, 2)$$

$$\lambda = 3$$

$$\lambda u = (3, 6)$$

# If the scalar is negative



$$u = (1, 2)$$

$$\lambda = -2$$

$$\lambda u = (-2, -4)$$

# General definition

## Multiplication by a Scalar

$$x = (x_1 \quad x_2 \quad \dots \quad x_n)$$

$$\lambda x = (\lambda x_1 \quad \lambda x_2 \quad \dots \quad \lambda x_n)$$

# Video 6: The dot product



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# Vectors and Linear Transformations

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## The dot product

# A shortcut for linear operations

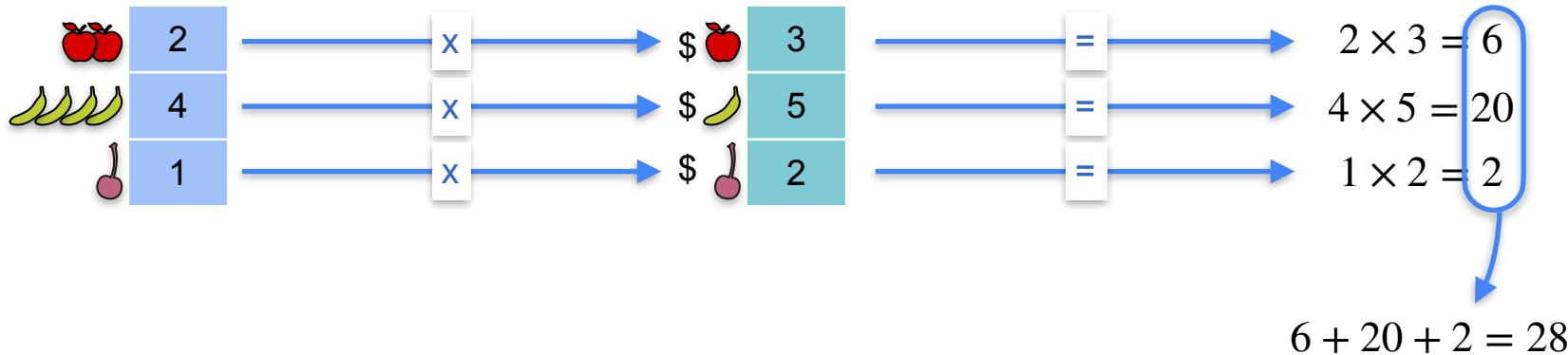
## Quantities

2 apples  
4 bananas  
1 cherry

## Prices

apples: \$3  
bananas: \$5  
cherries: \$2

Total price  
\$28



# The dot product

$$\begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} \cdot \begin{array}{c} \text{apple} \\ \text{banana} \\ \text{cherry} \end{array} = \$28$$

2	3
4	5
1	2

# The dot product

The diagram shows two vectors being multiplied. The first vector, on the left, is represented by a blue grid with three rows. The top row contains two red apples, the middle row contains four yellow bananas, and the bottom row contains one red cherry. To the right of this grid are its corresponding numerical values: 2, 4, and 1. A multiplication sign (·) is placed to the right of the grid. To the right of the multiplication sign is a teal grid with three rows. The top row contains one red apple with a dollar sign (\$), the middle row contains one yellow banana with a dollar sign (\$), and the bottom row contains one red cherry with a dollar sign (\$). To the right of this second grid are its corresponding numerical values: 3, 5, and 2. To the right of the second grid is an equals sign (=). To the right of the equals sign is the result: \$28.

$$\begin{matrix} \text{apple} \\ \text{banana} \\ \text{cherry} \end{matrix} \cdot \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} = \$28$$
$$\begin{matrix} \$\text{apple} \\ \$\text{banana} \\ \$\text{cherry} \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = \$28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

# The dot product

The diagram shows two vectors of fruits being multiplied. The first vector (left) has three elements: an apple (value 2), four bananas (value 4), and one cherry (value 1). The second vector (right) has three elements: an apple (\$3), a banana (\$5), and a cherry (\$2). The multiplication is shown as a dot product calculation:  $(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$ .

2	4	1
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.

\$ 3
\$ 5
\$ 2

= \$28

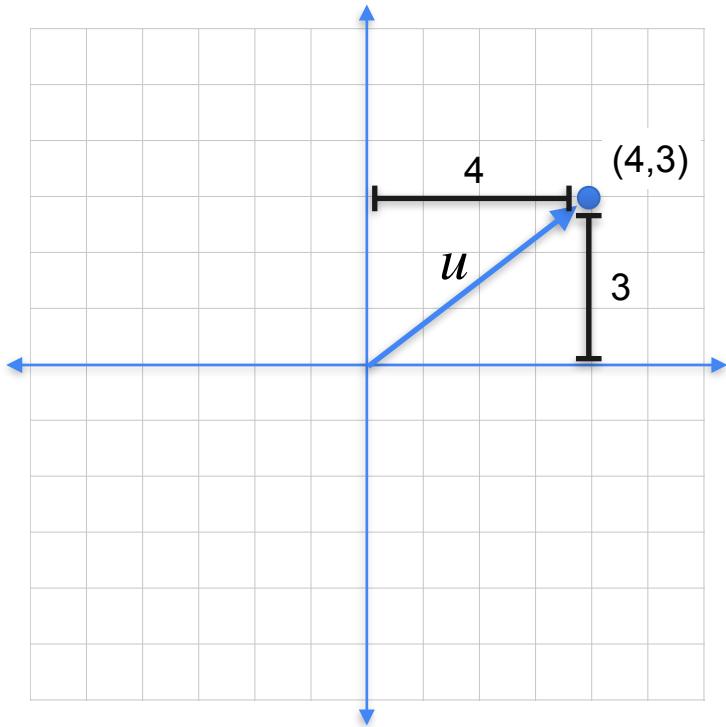
$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

# The dot product

$$\begin{matrix} 2 & 4 & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

# Norm of a vector using dot product



$$\sqrt{4^2 + 3^2} = \sqrt{25} = 5$$

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array} = \begin{array}{|c|c|} \hline 4 \\ \hline 3 \\ \hline \end{array} = 25$$

$$L2 - norm = \sqrt{dot\ product(u, u)}$$

$$\|u\|_2 = \sqrt{\langle u, u \rangle}$$

# Vector Transpose

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$(2 \times 3) + (4 \times 5) + (1 \times 2) = 28$$

# Vector Transpose

Transpose: convert columns to rows

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$\begin{matrix} 2 & 4 & 1 \end{matrix} \cdot \begin{matrix} 3 \\ 5 \\ 2 \end{matrix} = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

$$2 \cdot 3 + 4 \cdot 5 + 1 \cdot 2 = 28$$

# Vector Transpose

$$\begin{matrix} T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} \end{matrix} =$$

# Vector Transpose

$$\begin{matrix} & T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} & = & \begin{matrix} 2 & 4 & 1 \end{matrix} \end{matrix}$$

# Vector Transpose

$$\begin{matrix} & T \\ \begin{matrix} 2 \\ 4 \\ 1 \end{matrix} & = & \begin{matrix} 2 & 4 & 1 \end{matrix} & \begin{matrix} 2 & 4 & 1 \end{matrix} & = & T \end{matrix}$$

# Vector Transpose

$$\begin{matrix} 2 \\ 4 \\ 1 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \end{matrix}$$

$$\begin{matrix} 2 & 4 & 1 \end{matrix}^T = \begin{matrix} 2 \\ 4 \\ 1 \end{matrix}$$

# Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \text{Transpose the columns!}$$

# Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \end{matrix}$$

# Matrix Transpose

$$\begin{matrix} 2 & 5 \\ 4 & 7 \\ 1 & 3 \end{matrix}^T = \begin{matrix} 2 & 4 & 1 \\ 5 & 7 & 3 \end{matrix}$$

Columns → Rows

$3 \times 2$

$3 \times 2$

# General definition: dot product

Same number  
of components

$$x = (x_1 \quad x_2 \quad \dots \quad x_n) \qquad \qquad y = (y_1 \quad y_2 \quad \dots \quad y_n)$$

$$x \cdot y = (x_1 \times y_1) + (x_2 \times y_2) + \dots + (x_n \times y_n)$$

$\langle x, y \rangle$  is another notation for the dot product

$$x \cdot y^T = (x_1 \quad x_2 \quad \dots \quad x_n) \cdot \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$



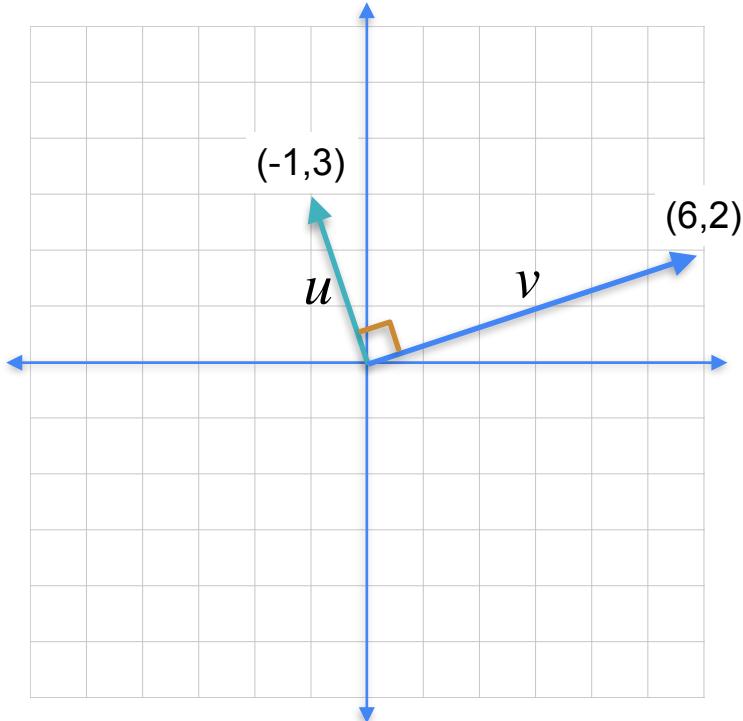
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# Vectors and Linear Transformations

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## Geometric dot product

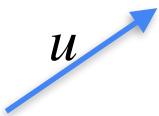
# Orthogonal vectors have dot product 0



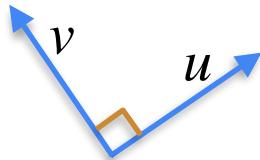
$$\begin{matrix} 6 & 2 \end{matrix} \times \begin{pmatrix} -1 \\ 3 \end{pmatrix} = 0$$

$$\langle u, v \rangle = 0$$

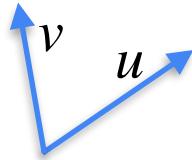
# The dot product



$$\langle u, u \rangle = \|u\|^2$$

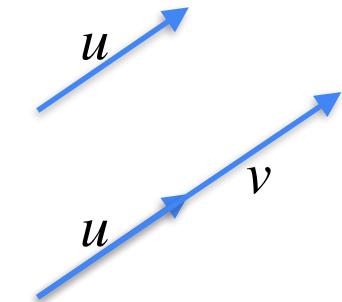


$$\langle u, v \rangle = 0$$

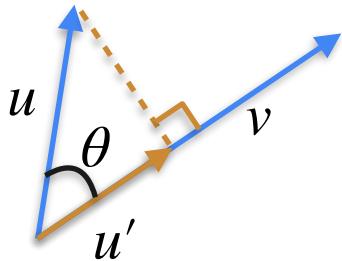


$$\langle u, v \rangle = ?$$

# The dot product



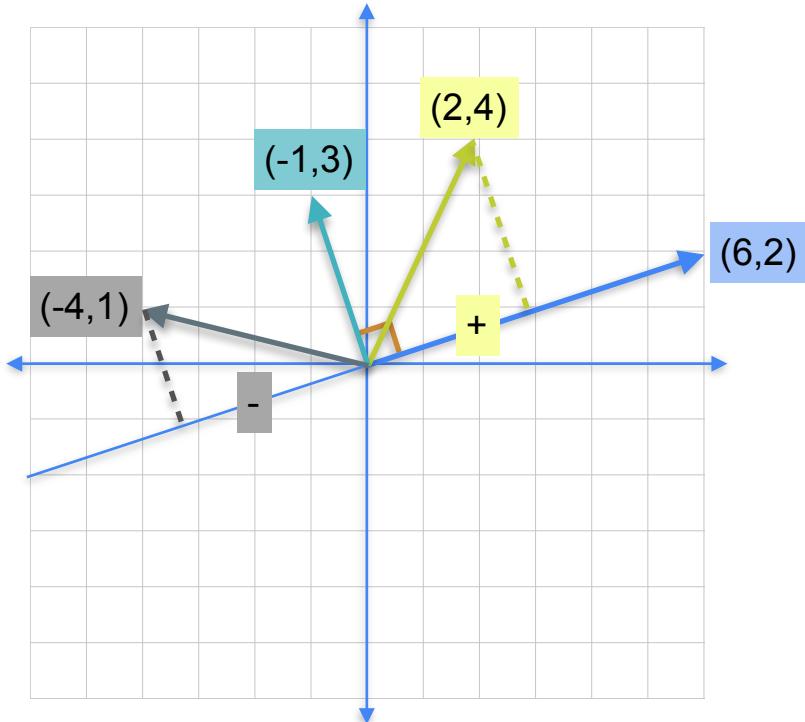
$$\langle u, u \rangle = \|u\|^2 = \|u\| \cdot \|u\|$$



$$\langle u, v \rangle = \|u\| \cdot \|v\|$$

$$\begin{aligned}\langle u, v \rangle &= \|u'\| \cdot \|v\| \\ &= \|u\| \|v\| \cos(\theta)\end{aligned}$$

# Geometric dot product

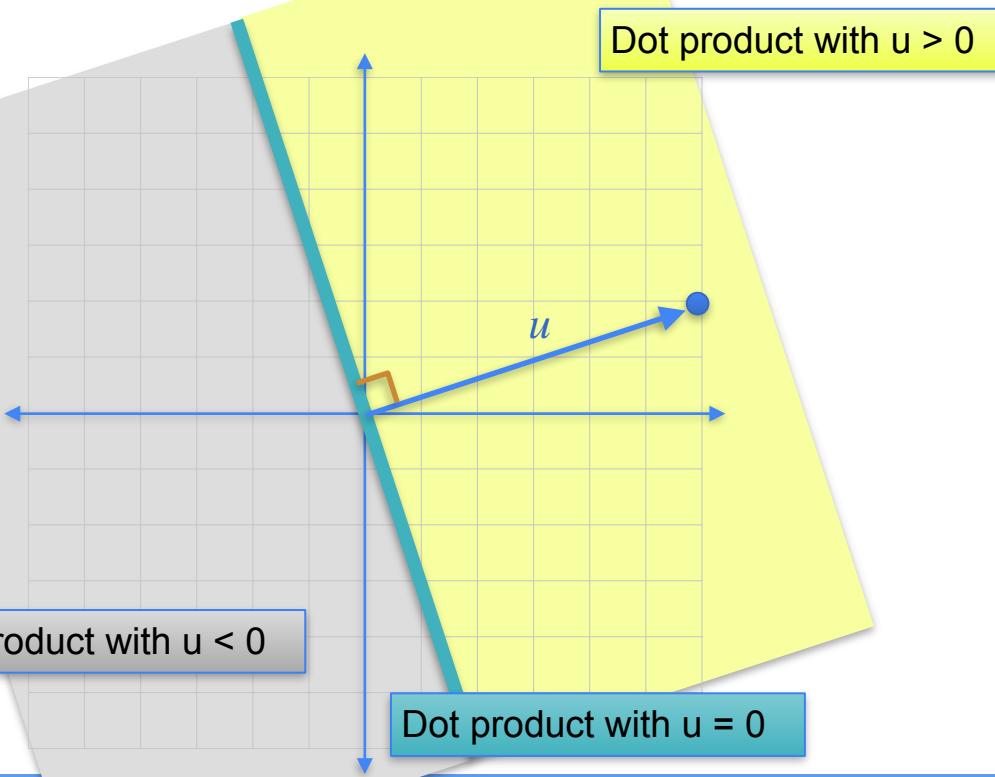


$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 2 \\ \hline 4 \\ \hline \end{array} = \begin{array}{|c|} \hline 20 \\ \hline \end{array} \quad \text{Positive}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -1 \\ \hline 3 \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 6 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline -4 \\ \hline 1 \\ \hline \end{array} = \begin{array}{|c|} \hline -22 \\ \hline \end{array} \quad \text{Negative}$$

# Geometric dot product



$$\langle u, v \rangle > 0$$

$$\langle u, v \rangle = 0$$

$$\langle u, v \rangle < 0$$



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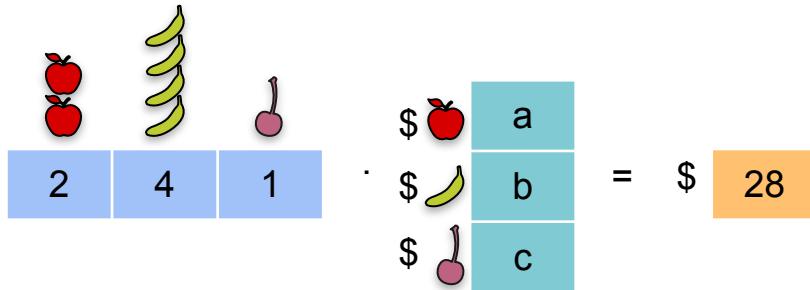
# Vectors and Linear Transformations

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**Multiplying a matrix by a  
vector**

# Equations as dot product

$$2a + 4b + c = 28$$



# Equations as dot product

$$a + b + c = 10$$

A diagram illustrating the equation  $a + b + c = 10$ . It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 1, and 1, each accompanied by a fruit icon: an apple, a banana, and a cherry respectively. The second vector is a column matrix with three rows, labeled  $a$ ,  $b$ , and  $c$  from top to bottom. The entries are \$1 for  $a$ , \$1 for  $b$ , and \$1 for  $c$ , each preceded by a dollar sign. The result of the multiplication is \$10, shown in an orange box.

$$a + 2b + c = 15$$

A diagram illustrating the equation  $a + 2b + c = 15$ . It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 2, and 1, each accompanied by a fruit icon: an apple, two bananas, and a cherry respectively. The second vector is a column matrix with three rows, labeled  $a$ ,  $b$ , and  $c$  from top to bottom. The entries are \$1 for  $a$ , \$2 for  $b$ , and \$1 for  $c$ , each preceded by a dollar sign. The result of the multiplication is \$15, shown in an orange box.

$$a + b + 2c = 12$$

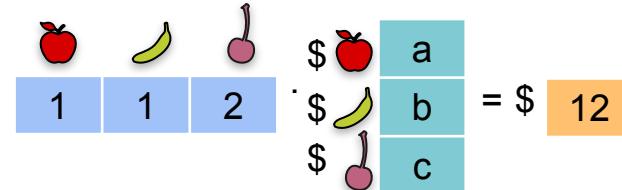
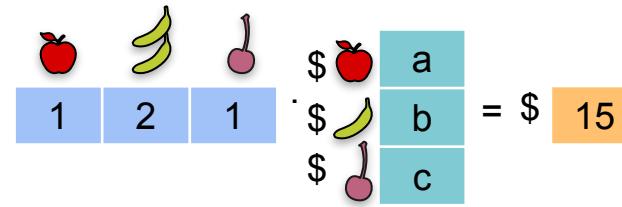
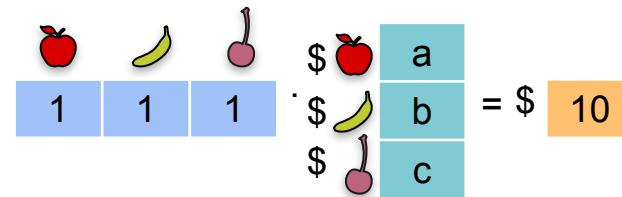
A diagram illustrating the equation  $a + b + 2c = 12$ . It shows two vectors being multiplied together. The first vector is composed of three blue boxes labeled 1, 1, and 2, each accompanied by a fruit icon: an apple, a banana, and two cherries respectively. The second vector is a column matrix with three rows, labeled  $a$ ,  $b$ , and  $c$  from top to bottom. The entries are \$1 for  $a$ , \$1 for  $b$ , and \$2 for  $c$ , each preceded by a dollar sign. The result of the multiplication is \$12, shown in an orange box.

# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

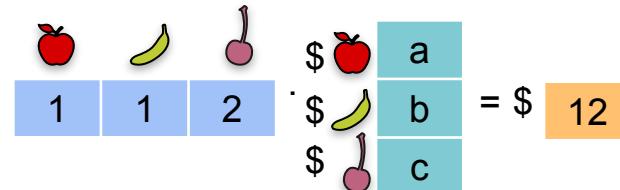
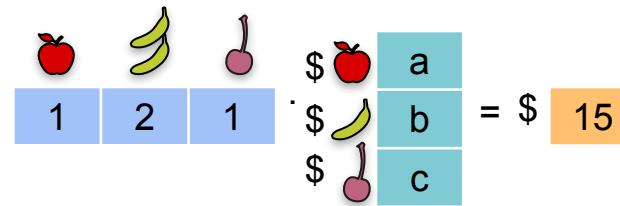
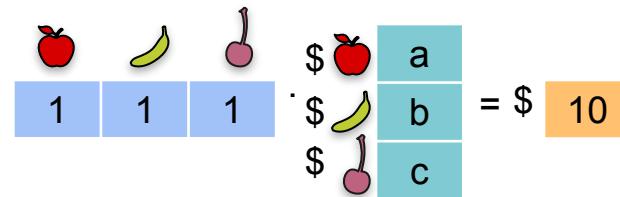


# Equations as dot product

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$



# Equations as dot product

## System of equations

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

## Matrix product

1	1	1
1	2	1
1	1	2

\$		a
\$		b
\$		c

= \$		
10		
15		
12		

# Equations as dot product

**System of equations**

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

**Matrix product**

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$

# Equations as dot product

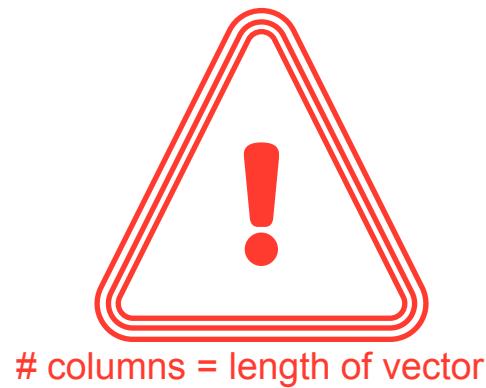
## Matrix product

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{matrix} \begin{matrix} a \\ b \\ c \end{matrix} = \begin{matrix} 10 \\ 15 \\ 12 \end{matrix}$$

$3 \times 3$    Length 3

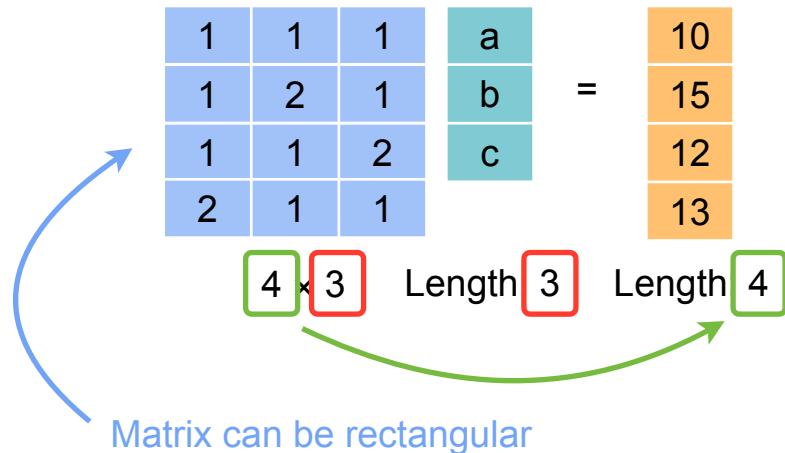


Matrix can be rectangular



# Equations as dot product

## Matrix product



# columns = length of vector

# W3 Lesson 2



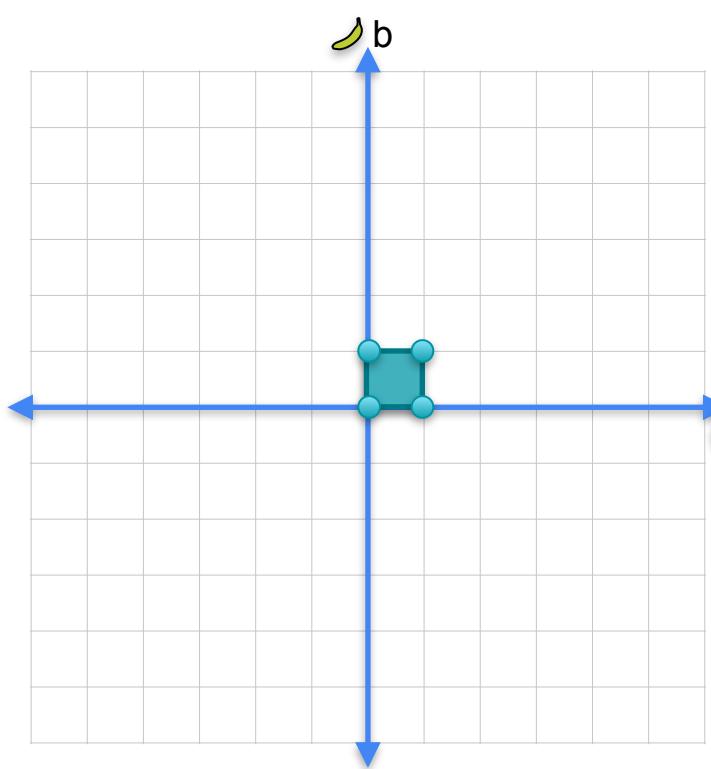
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# Vectors and Linear Transformations

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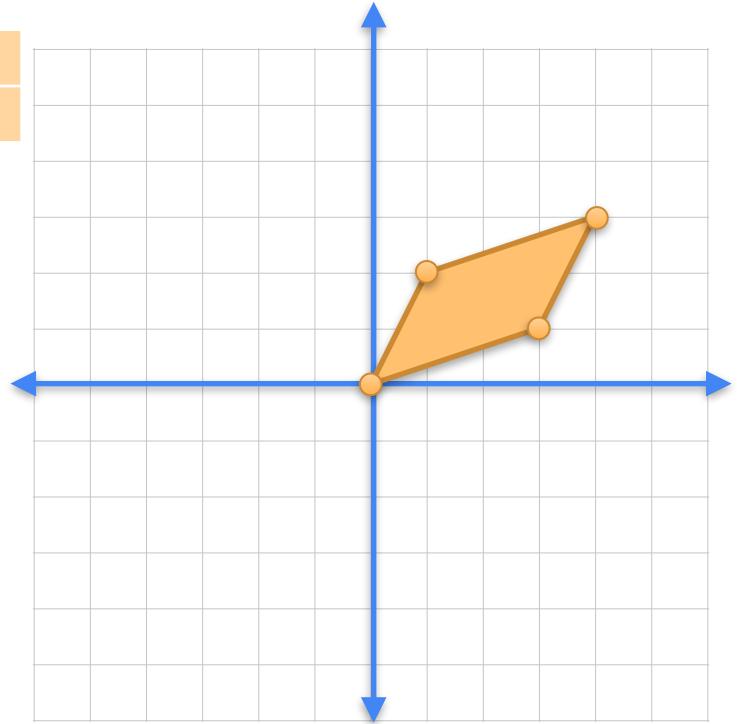
**Matrices as linear  
transformations**

# Matrices as linear transformations

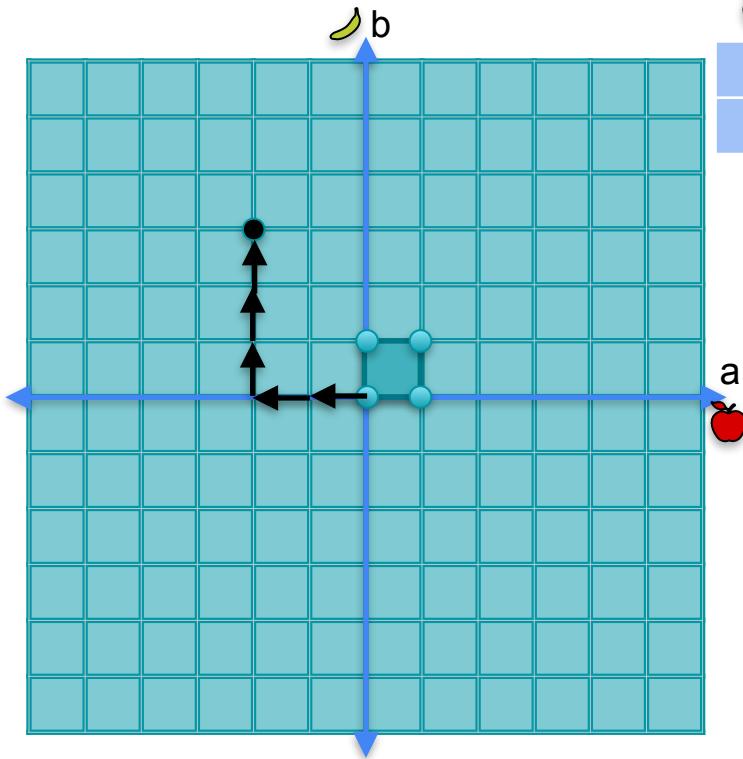


$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} & \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} 4 \\ 3 \end{matrix}$$

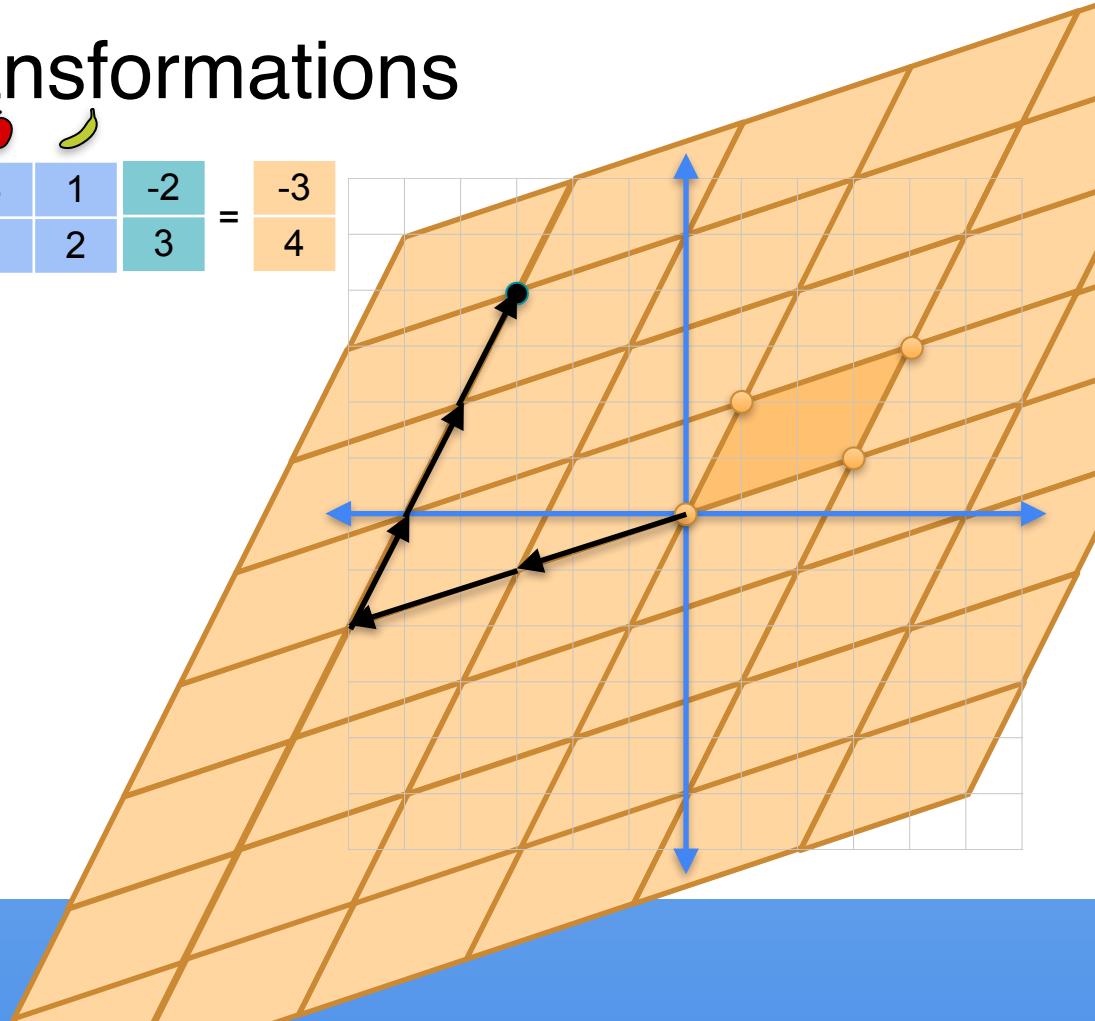
- $(0,0) \rightarrow (0,0)$
- $(1,0) \rightarrow (3,1)$
- $(0,1) \rightarrow (1,2)$
- $(1,1) \rightarrow (4,3)$



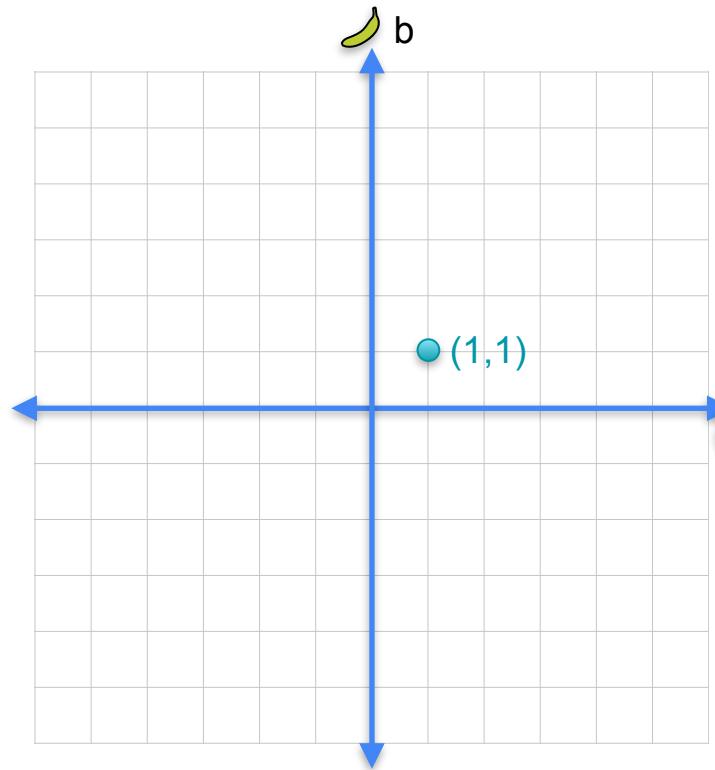
# Matrices as linear transformations



$$\begin{matrix} 3 & 1 & -2 \\ 1 & 2 & 3 \end{matrix} = \begin{matrix} -3 \\ 4 \end{matrix}$$



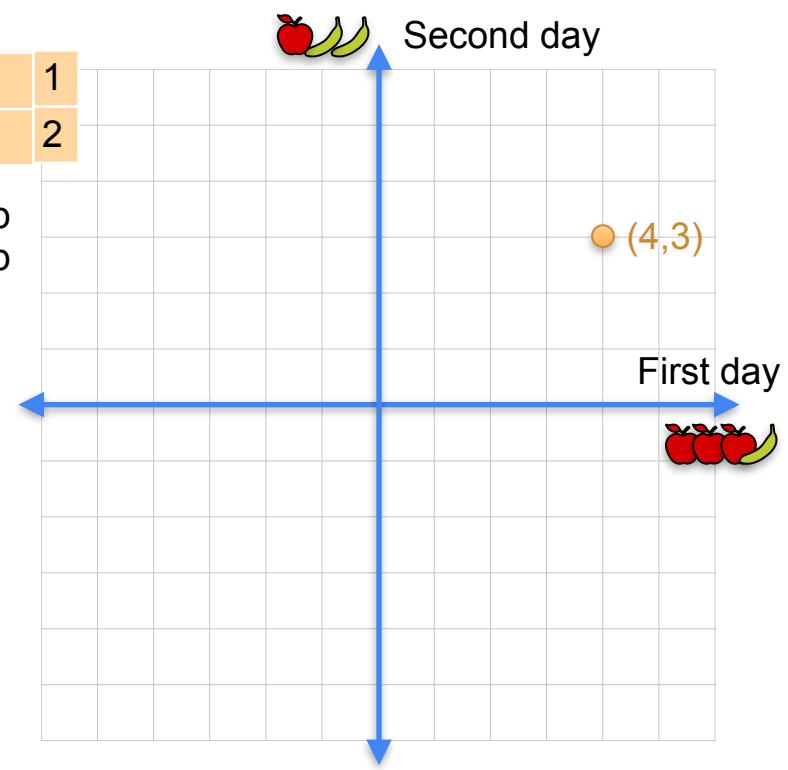
# Systems of equations as linear transformations



3	1	1	4	1
1	2	1	3	2

=

First day:  $3a + b$   
Second day:  $a + 2b$





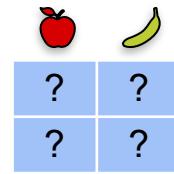
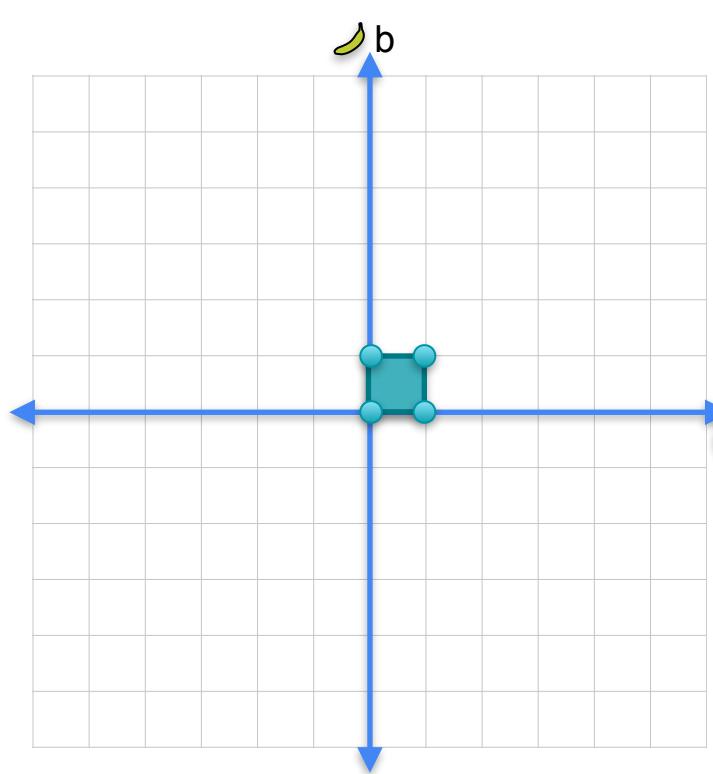
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# Vectors and Linear Transformations

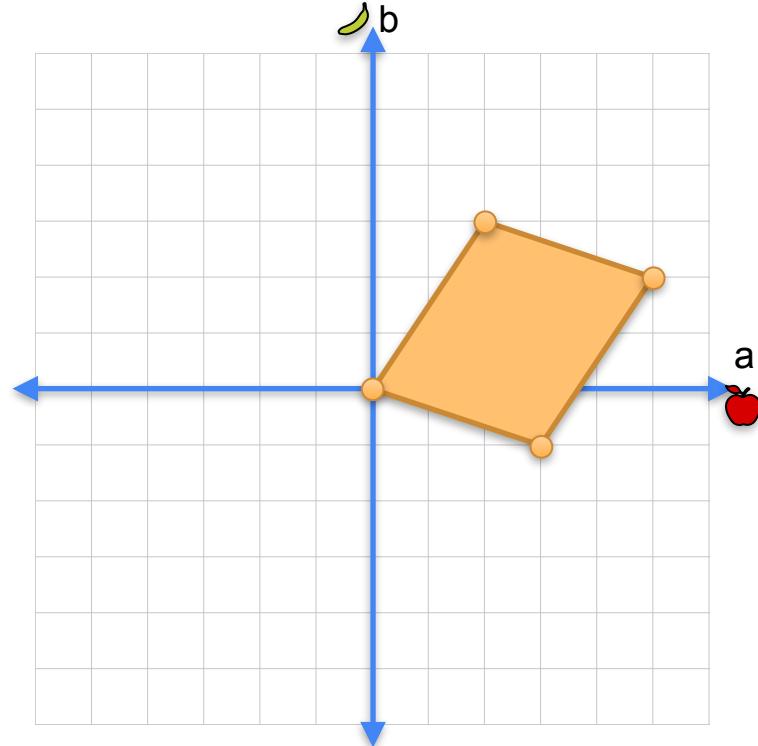
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**Linear transformations as  
matrices**

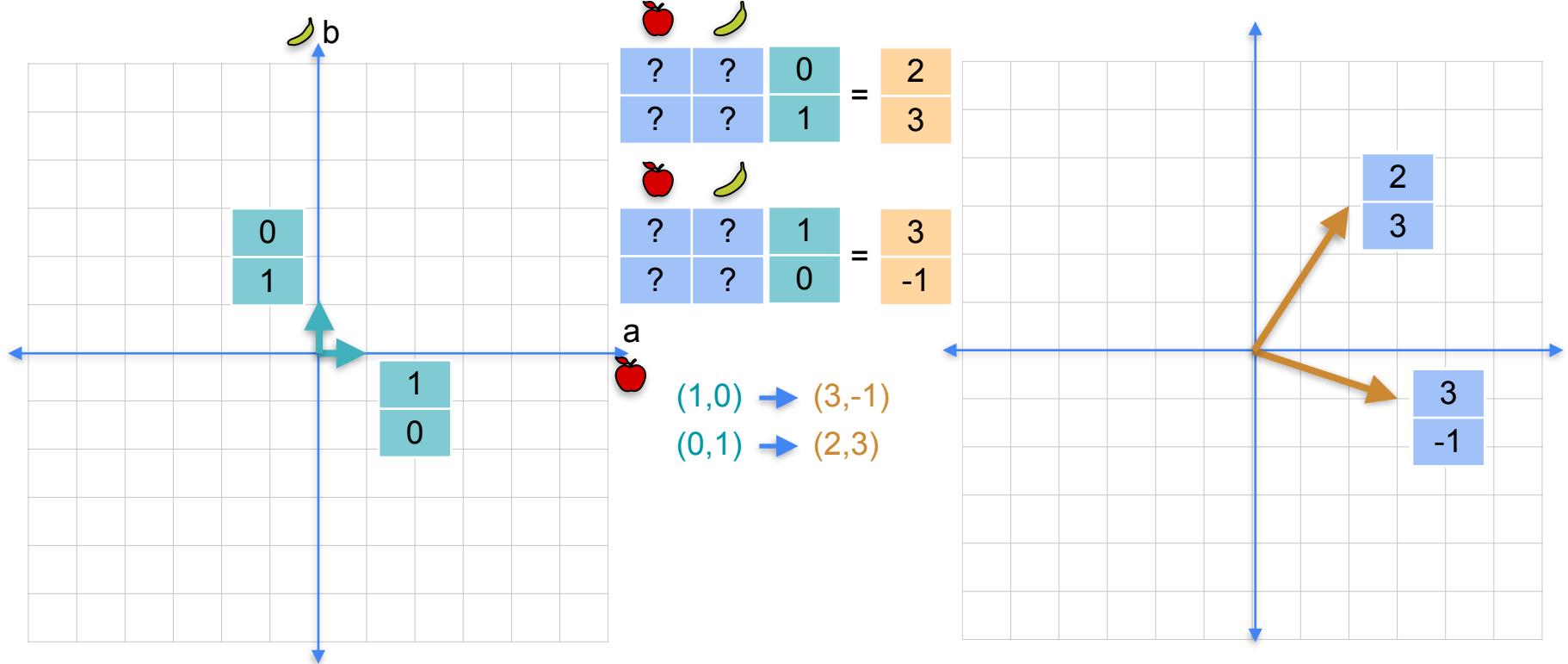
# Linear transformations as matrices



$(0,0) \rightarrow (0,0)$   
 $(1,0) \rightarrow (3,-1)$   
 $(0,1) \rightarrow (2,3)$   
 $(1,1) \rightarrow (5,2)$



# Linear transformations as matrices





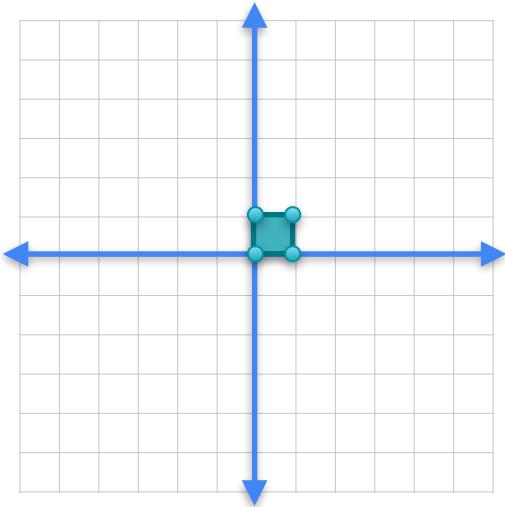
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# Vectors and Linear Transformations

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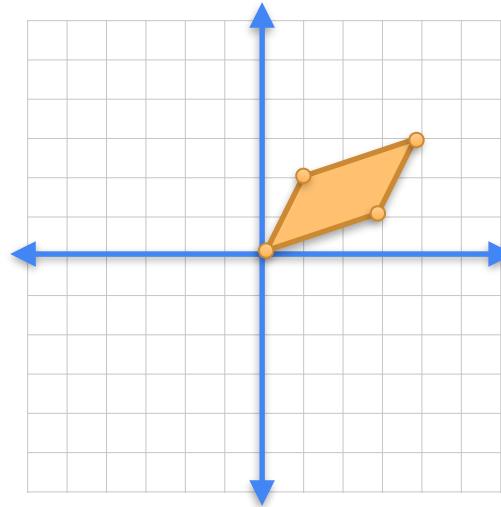
## Matrix multiplication

# Combining linear transformations

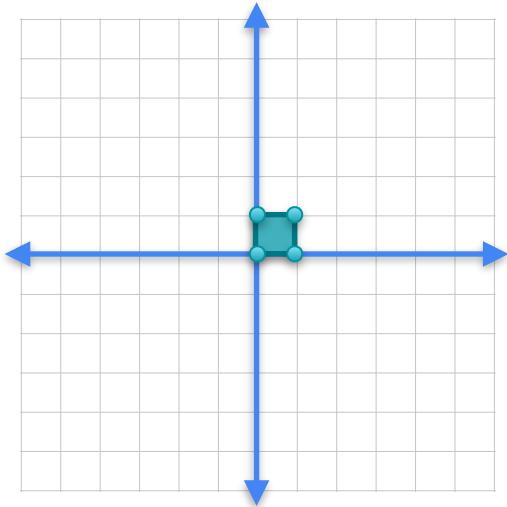


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

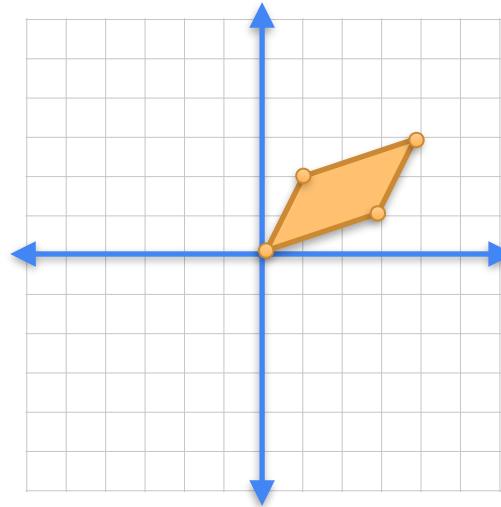


# Combining linear transformations

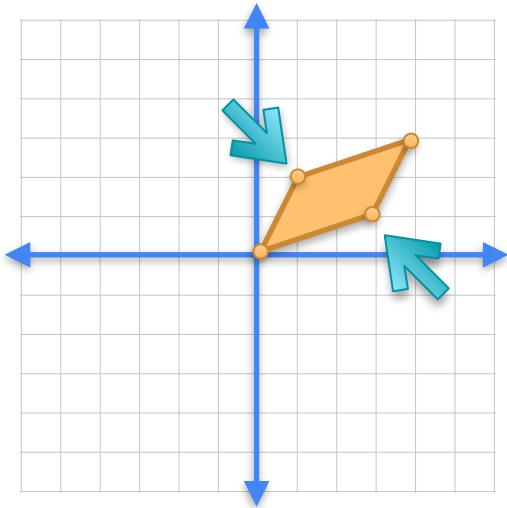


$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

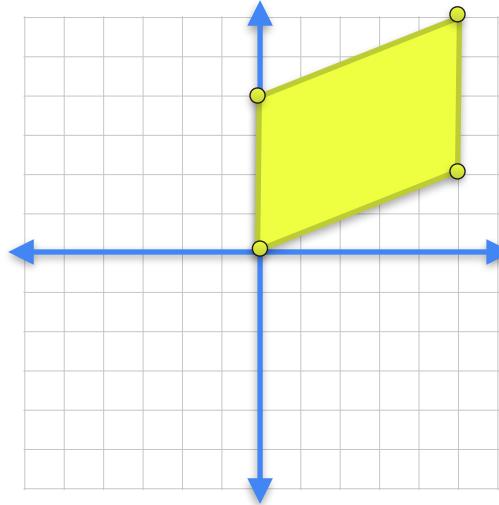


# Combining linear transformations

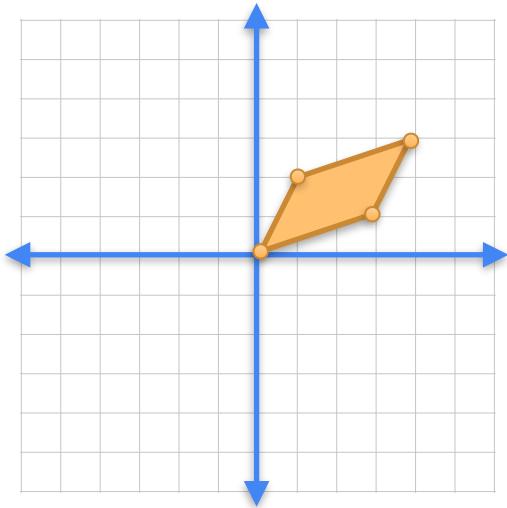


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$

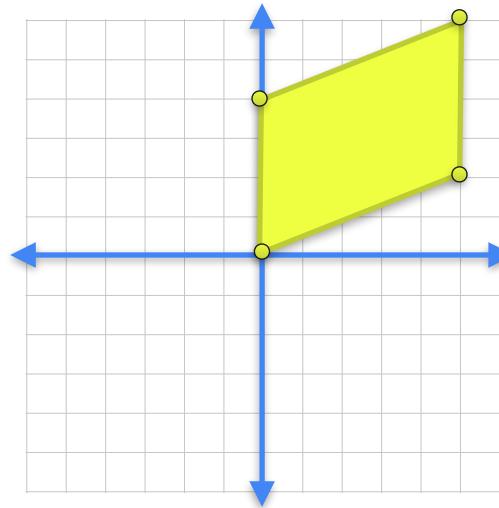


# Combining linear transformations

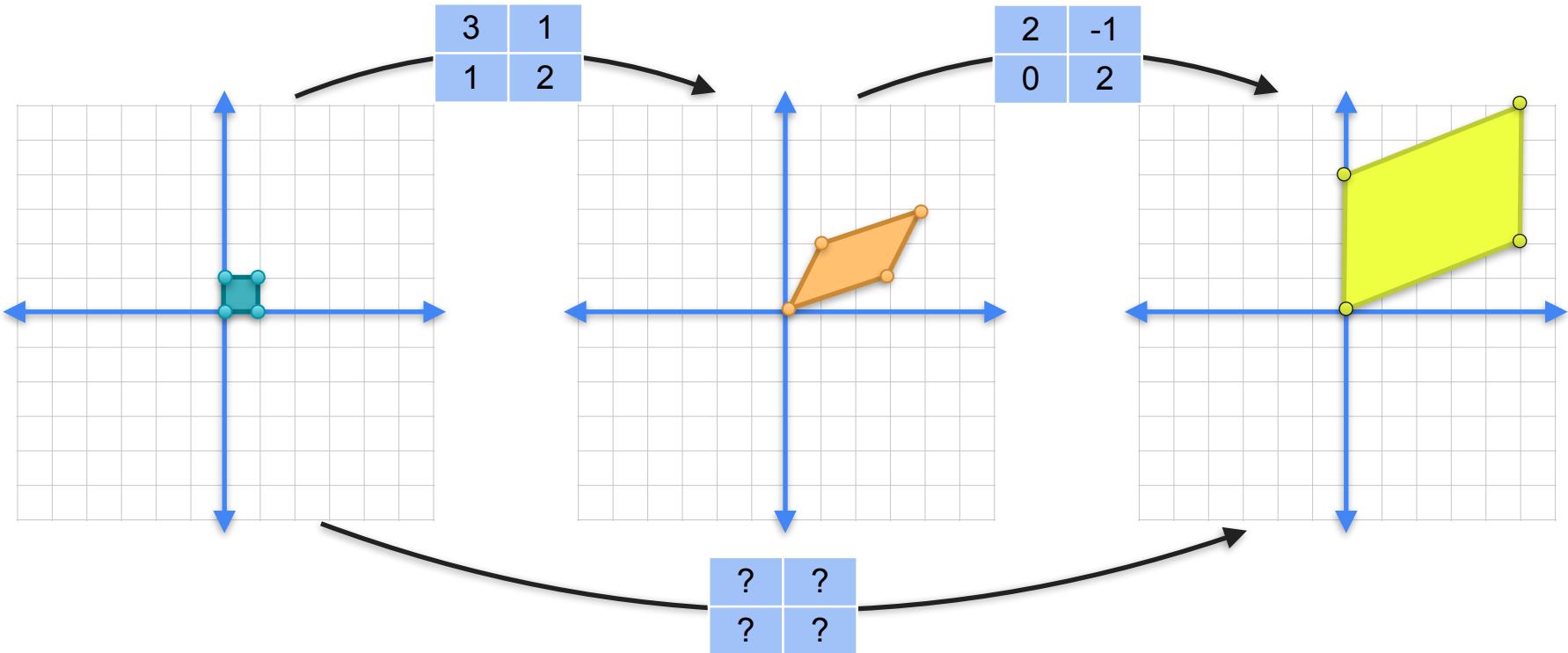


$$\begin{matrix} 2 & -1 & 3 \\ 0 & 2 & 1 \end{matrix} = \begin{matrix} 5 \\ 2 \end{matrix}$$

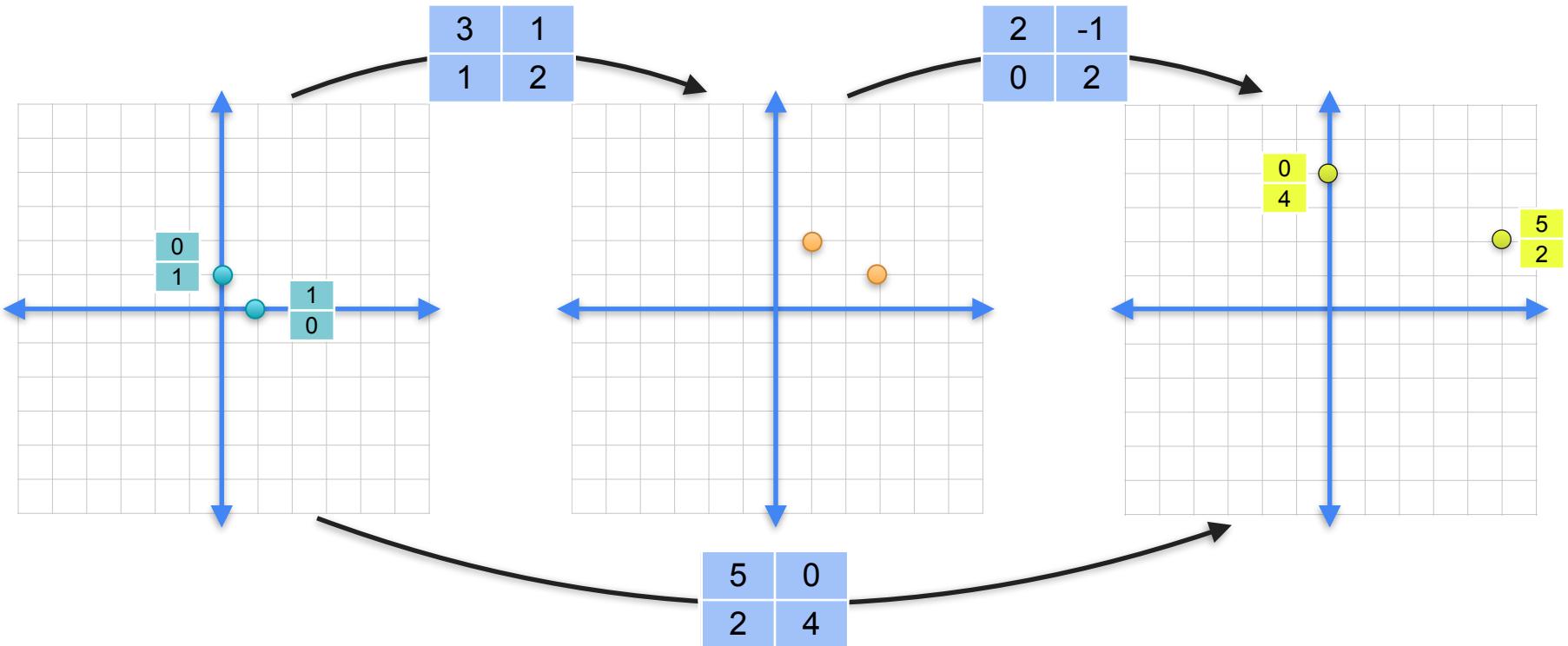
$$\begin{matrix} 2 & -1 & 1 \\ 0 & 2 & 2 \end{matrix} = \begin{matrix} 0 \\ 4 \end{matrix}$$



# Combining linear transformations



# Combining linear transformations



# Combining linear transformations

$$\begin{array}{c} \text{Second} \\ \downarrow \\ \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \end{array} \cdot \begin{array}{c} \text{First} \\ \downarrow \\ \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \end{array} = \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array}$$

# Dimension of the matrices

$$\begin{matrix} 2 & -1 \\ 0 & 2 \end{matrix} \cdot \begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} = \begin{matrix} 2 & -1 & 5 & 3 \\ 0 & 2 & 0 & 1 \\ 2 & -1 & 1 & 2 \\ 0 & 2 & 1 & 2 \end{matrix}$$

# Dimension of the matrices

$$\begin{array}{c} \begin{array}{|c|c|} \hline 2 & -1 \\ \hline 0 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} & = & \begin{array}{|c|c|} \hline 5 & 0 \\ \hline 2 & 4 \\ \hline \end{array} \\ 2 \times 2 & 2 \times 2 & 2 \times 2 \end{array}$$

# Dimension of the matrices

3	1	4
2	-1	2

$2 \times 3$

3	0	1	-2
1	5	2	0
-2	1	4	0

$3 \times 4$

=

?			

$2 \times 4$

# Dimension of the matrices

$$\begin{array}{c|c} \begin{matrix} 3 & 1 & 4 \\ 2 & -1 & 2 \end{matrix} & \cdot \begin{matrix} 3 & 0 & 1 & -2 \\ 1 & 5 & 2 & 0 \\ -2 & 1 & 4 & 0 \end{matrix} = \begin{matrix} 2 & 9 & 21 & -6 \\ ? & -3 & 8 & -4 \end{matrix} \\ \begin{matrix} 2 \times 3 \\ \text{red} \end{matrix} & \begin{matrix} 3 \times 4 \\ \text{red} \end{matrix} \end{array}$$

$\boxed{2 \times 4}$  (purple)

- Columns of first matrix must match rows of second (**numbers in red match**)
- Result takes number of rows from first matrix (**numbers in blue match**)
- Result takes number of columns from second matrix (**numbers in purple match**)



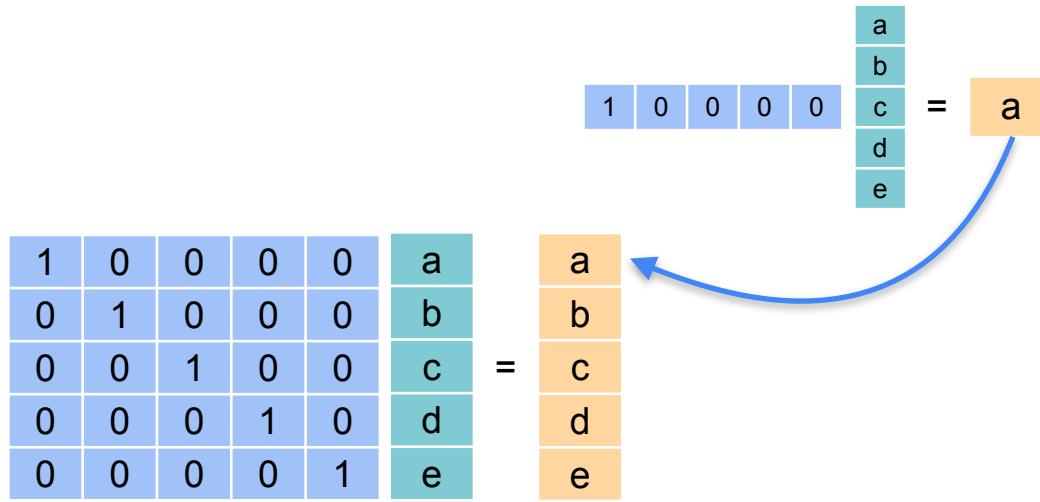
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# Vectors and Linear Transformations

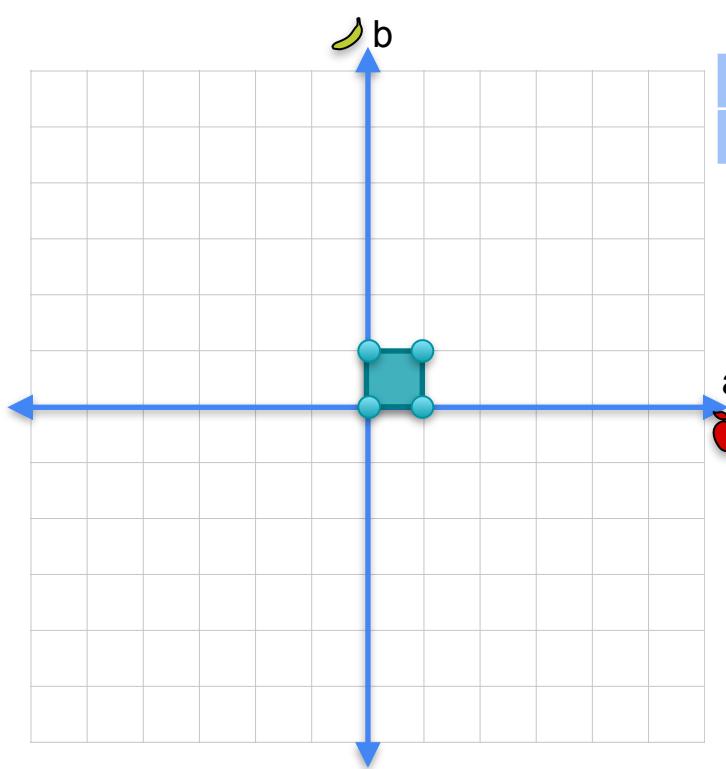
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## The identity matrix

# The identity matrix

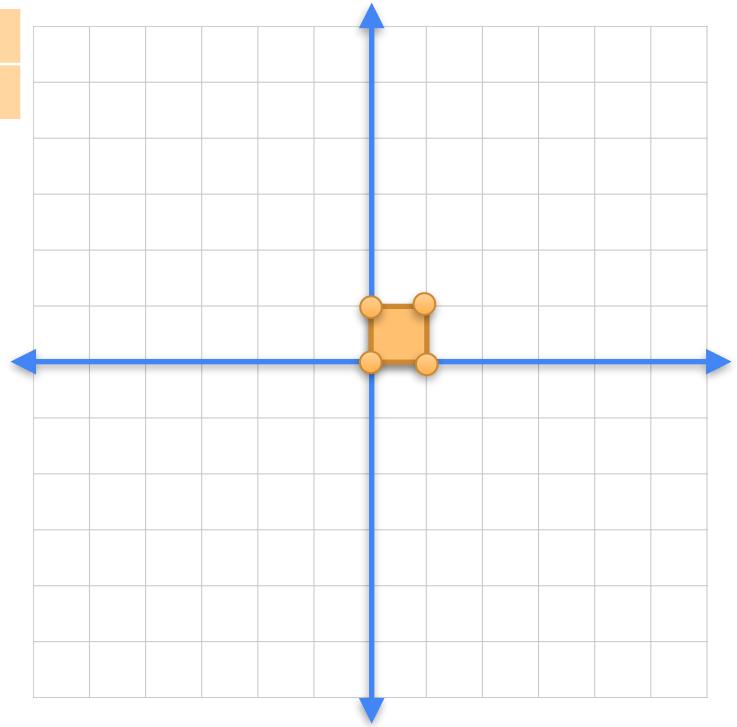


# The identity matrix



$$\begin{matrix} \text{apple} & \text{banana} \\ \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} x \\ y \end{matrix} = \begin{matrix} x \\ y \end{matrix} \end{matrix}$$

$(0,0) \rightarrow (0,0)$   
 $(1,0) \rightarrow (1,0)$   
 $(0,1) \rightarrow (0,1)$   
 $(1,1) \rightarrow (1,1)$





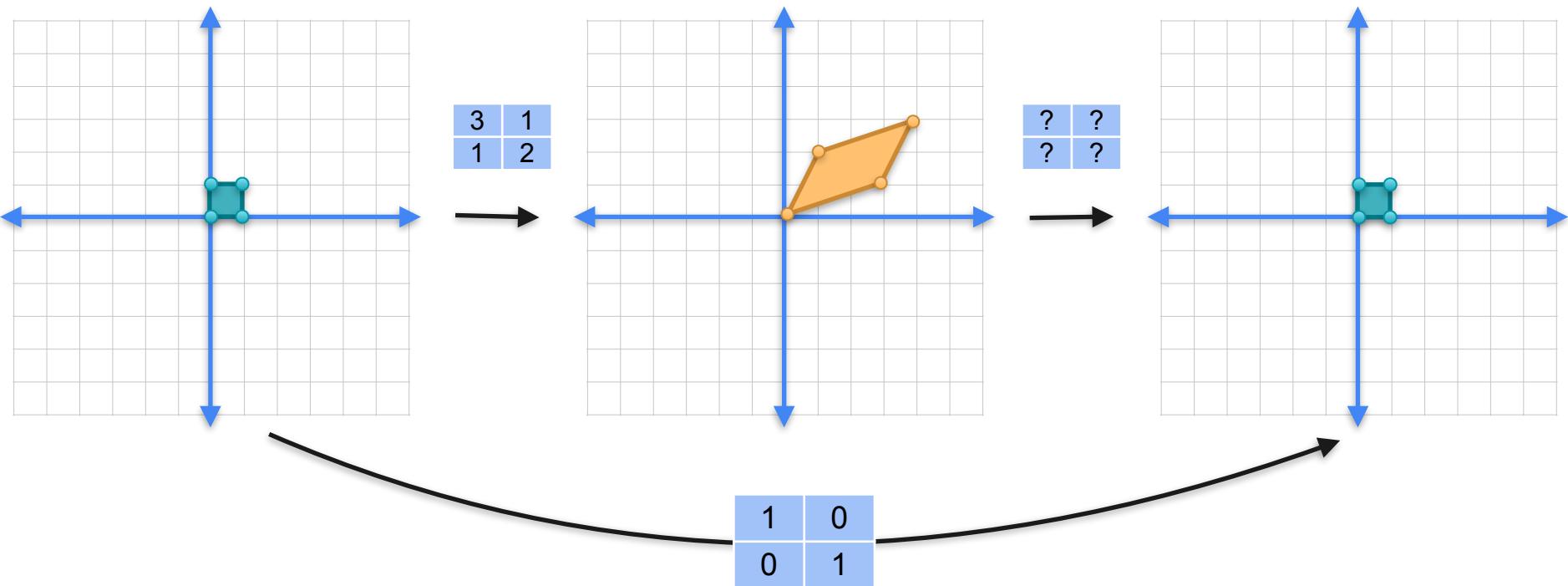
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# Vectors and Linear Transformations

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## Matrix inverse

# Matrix inverses



# Multiplying matrices

$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix} \cdot \begin{matrix} a & b \\ c & d \end{matrix} = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$$
$$\begin{matrix} 3 & 1 \\ 1 & 2 \end{matrix}^{-1} = \begin{matrix} 2/5 & -1/5 \\ -1/5 & 3/5 \end{matrix}$$


# How to find an inverse?

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 3a + 1c = 1 \quad a = \frac{2}{5}$$

$$\begin{array}{|c|c|} \hline 3 & 1 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 3b + 1d = 0 \quad b = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline a \\ \hline c \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 0 \\ \hline \end{array} \quad 1a + 2c = 0 \quad c = -\frac{1}{5}$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \left| \begin{array}{|c|c|} \hline b \\ \hline d \\ \hline \end{array} \right. = \begin{array}{|c|} \hline 1 \\ \hline \end{array} \quad 1b + 2d = 1 \quad d = \frac{3}{5}$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I couldn’t find it”

5	2
1	2

# Solution

- By solving the corresponding system of linear equations, we get the following.

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline 1 & 2 \\ \hline \end{array} \cdot \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} = \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet 5a + 2c = 1$$

$$\bullet a = 1/4$$

$$\begin{array}{|c|c|} \hline 5 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet 5b + 2d = 0$$

$$\bullet b = -1/4$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline a \\ \hline c \\ \hline \end{array} = \begin{array}{|c|} \hline 0 \\ \hline \end{array}$$

$$\bullet a + 2c = 0$$

$$\bullet c = -1/8$$

$$\begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array} \begin{array}{|c|} \hline b \\ \hline d \\ \hline \end{array} = \begin{array}{|c|} \hline 1 \\ \hline \end{array}$$

$$\bullet b + 2d = 1$$

$$\bullet d = 5/8$$

# Quiz

- Find the inverse of the following matrix. If you find that the task is impossible, feel free to click on “I’m reaching a dead end”

1	1
2	2

# Solutions

- The inverse doesn't exist!

We need to solve the following system of linear equations:

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline 2 & 2 \\ \hline \end{array} \quad \begin{array}{|c|c|} \hline a & b \\ \hline c & d \\ \hline \end{array} \quad = \quad \begin{array}{|c|c|} \hline 1 & 0 \\ \hline 0 & 1 \\ \hline \end{array}$$

$$a + c = 1$$

$$2b + 2d = 1$$

$$2a + 2c = 0$$

$$b + d = 0$$

This is clearly a contradiction, since equation 1 says  $a+c=1$ , and equation 3 says  $2a+2c=0$ .



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# Vectors and Linear Transformations

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**Which matrices have an inverse?**

# Which matrices have inverses?

$$5^{-1} = 0.2$$

$$8^{-1} = 0.125$$

$$0^{-1} = ???$$

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

Non-singular matrix  
Invertible

Non-singular matrix  
Invertible

Singular matrix  
Non-invertible

Det = 5 ← Det = 8 →

Non-zero determinants

Det = 0 ← Zero determinant →



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## Vectors and Linear Transformations

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**Neural networks and  
matrices**

# Quiz: Natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

## Scores:

Lottery: \_\_\_\_ points

Win: \_\_\_\_ points

## Examples

Lottery: 3 point

Win: 2 points

“Win, win the lottery!” : 7points

## Rule:

If the number of points of the sentence is bigger than \_\_\_\_,  
then the email is spam.

## Goal: Find the best points and threshold

Lottery: \_\_\_\_ point

Win: \_\_\_\_ point

Threshold: \_\_\_\_ points

# Quiz: Natural language processing

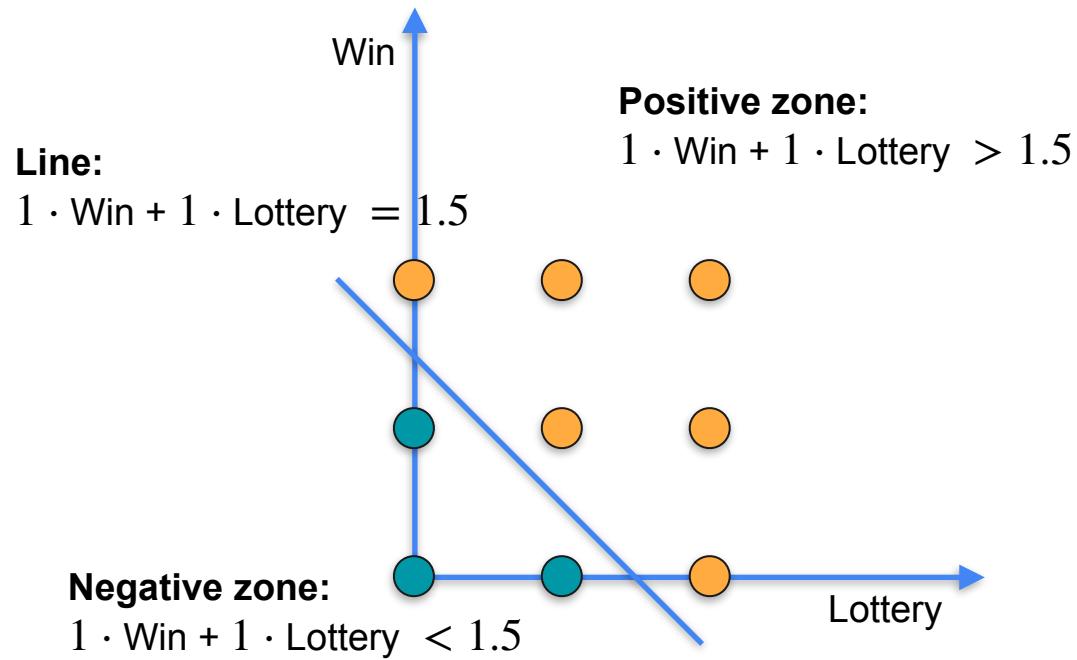
Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Score	> 1.5?
2	Yes
3	Yes
0	No
2	Yes
1	No
1	No
4	Yes
2	Yes
3	Yes

**Solution:**  
Lottery: 1 point  
Win: 1 point  
Threshold: 1.5 points

# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2



# Graphical natural language processing

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

= 3

Check: > 1.5?



Spam

# Dot product between vectors

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

$$= 1$$

Check:  $> 1.5?$



Not spam

# Matrix multiplication

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

$$\begin{matrix} \text{Model} \\ \hline 1 \\ 1 \end{matrix} = \begin{matrix} \text{Prod} \\ \hline 2 \\ 3 \\ 0 \\ 2 \\ 1 \\ 1 \\ 4 \\ 2 \\ 3 \end{matrix}$$

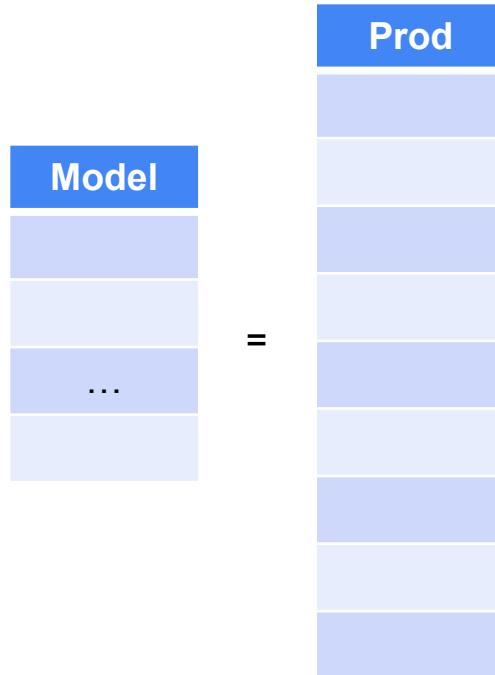
Check: >1.5?



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

# Perceptrons

Spam	Word1	Word2	...	WordN
Yes				
Yes				
No				
Yes				
No				
No				
Yes				
Yes				
Yes				



Check:



Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

# Threshold and bias

Spam	Lottery	Win	Bias
Yes	1	1	1
Yes	2	1	1
No	0	0	1
Yes	0	2	1
No	0	1	1
No	1	0	1
Yes	2	2	1
Yes	2	0	1
Yes	1	2	1

Check

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

$$1 \cdot \text{Win} + 1 \cdot \text{Lottery} - 1.5 > 0$$

Threshold

Bias

Check:  $> 0$ ?

Model
1
1
-1.5

Bias

# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1

$$\begin{matrix} \text{Model} \\ \begin{matrix} 1 \\ 1 \end{matrix} \end{matrix} = \begin{matrix} \text{Dot prod} \\ \begin{matrix} 0 \\ 1 \\ 1 \\ 2 \end{matrix} \end{matrix}$$

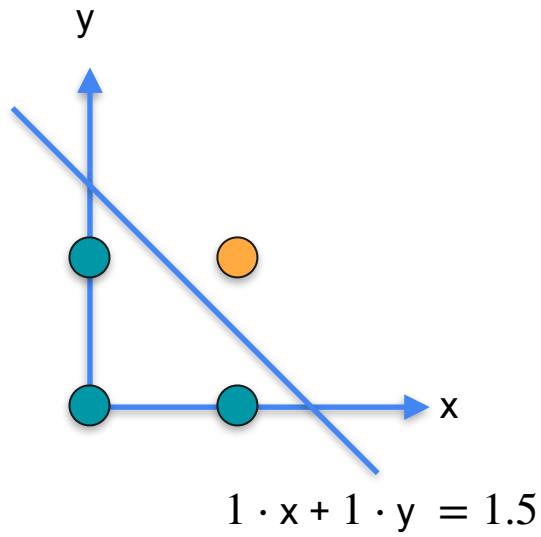
Check:  $>1.5?$



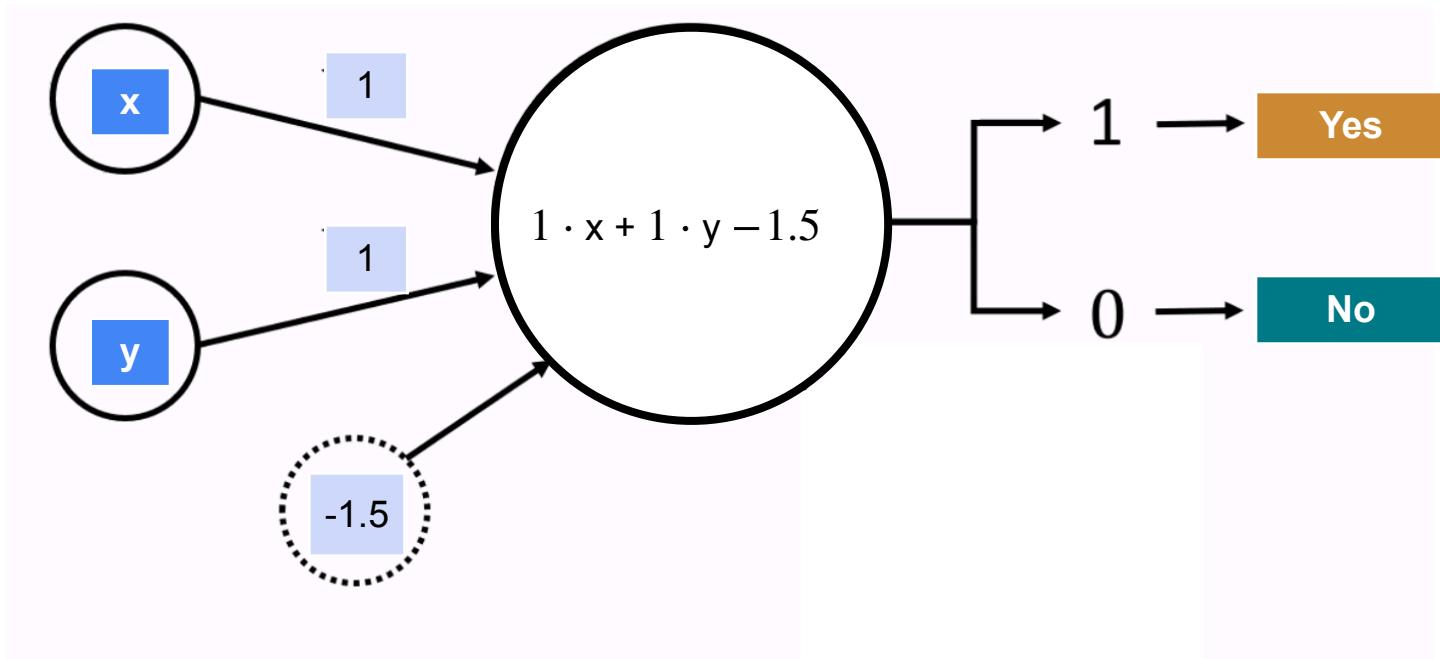
Check
No
No
No
Yes

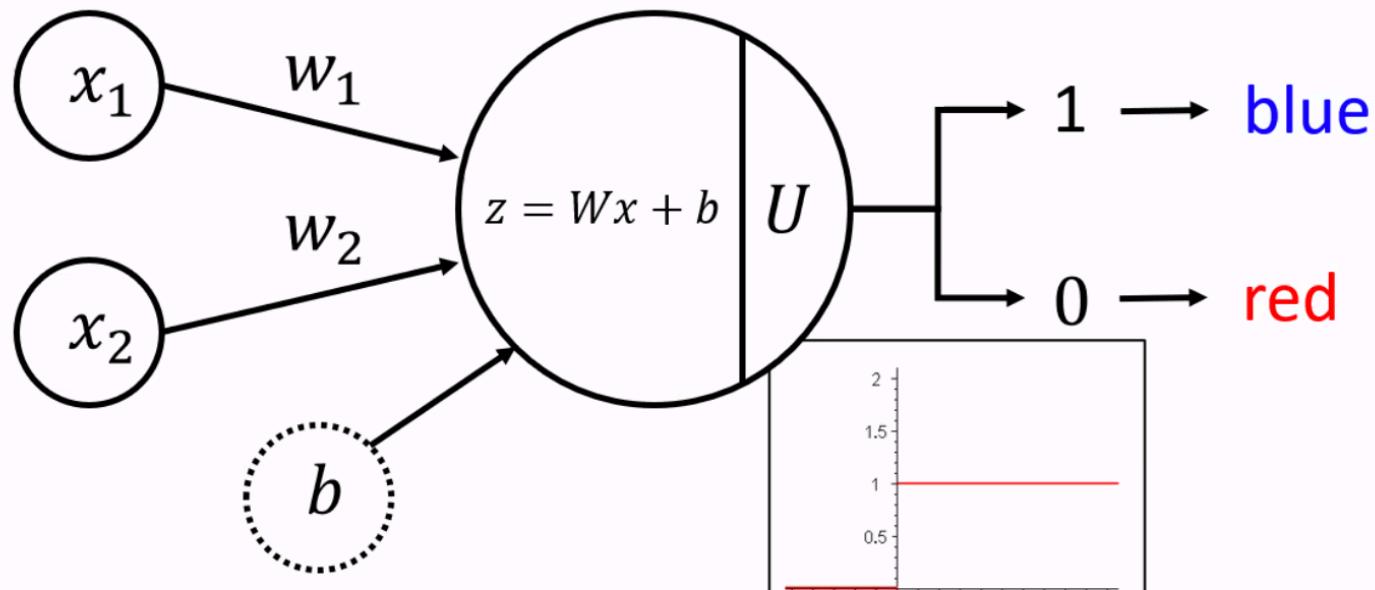
# The AND operator

AND	x	y
No	0	0
No	1	0
No	0	1
Yes	1	1



# The perceptron







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# Vectors and Linear Transformations

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## Conclusion