CECS 451 Assignment 4

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1. Imagine that one of the friends wants to avoid the other. The problem then becomes a **two-player pursuit–evasion game**. We assume now that the players take turns moving. The **game ends only when the players are on the same node**; the **terminal payoff to the pursuer is minus the total move taken**. An example is shown in Figure 1.

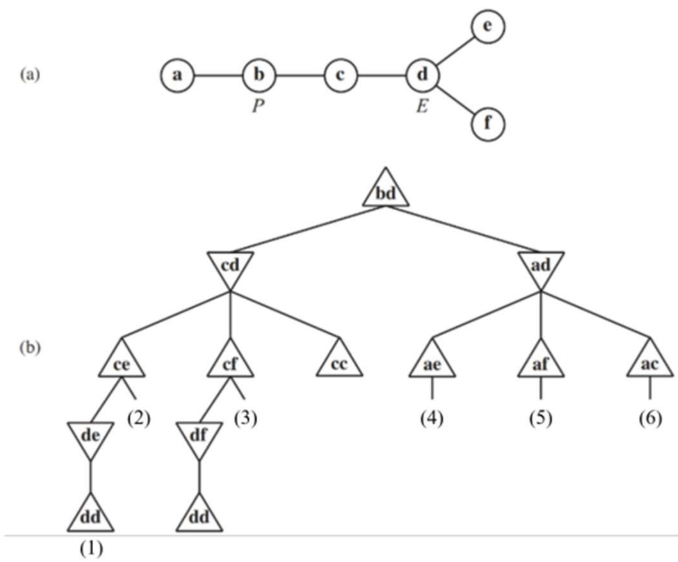

Figure 1: (a) A map where the cost of every edge is 1. Initially the pursuer P is at node b and the evader E is at node d. (b) A partial game tree for this map. Each node is labeled with the P, E positions. P moves first.

a. What is the terminal payoff at the node (1)?
   i. $\mathbf{-4}$

b. What are the positions of the two players at the node (2) and (2)'s children?
   i. Node (2): **be**
   ii. Node (2)'s children: **bd**

c. Can we assume the terminal payoff at the node (2) is less than -4? Answer yes or no, then explain your answers.
   i. **YES**. Node (2) only has a child, and it takes a total of 4 moves to get to that child. From (b) we have that node (2)'s child is bd, which means the two players are not on the same node. So, for the game to end, extra moves are needed. Therefore, we can assume that the terminal payoff at node (2) is less than -4.

d. Assume the terminal payoff at the node (4) is less than -4. Do we need expand the node (5) and (6)? Answer yes or no, then explain your answers.

i. **NO**.

The node cc's terminal payoff is -2. From (a), (b), and (c), we have that node ce's maximum value is -4. For a similar reason, node cf's maximum value is also -4. So, node cd's minimum value is -4.

Since node (4) is less than -4, node ad has a minimum value of less than -4. And we know that node cd on the LHS has a minimum value equal to -4. After comparing -4 and less than -4, the root node bd takes the maximum value, which is node cd's.

Therefore, with $\alpha - \beta$ pruning, we don't need to expand node (5) or node (6).

2. True or False?

a. $(A \land B) \models (A \Leftrightarrow B)$: **TRUE**

A	B	**A ∧ B**	**A ⇔ B**
True	True	**True**	**True**
True	False	**False**	**False**
False	True	**False**	**False**
False	False	**False**	**True**

b. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$: **TRUE**

$(C \lor \neg A) \land (C \lor \neg B) \equiv (A \Rightarrow C) \land (B \Rightarrow C)$

$(\neg A \lor C) \land (\neg B \lor C) \equiv (A \Rightarrow C) \land (B \Rightarrow C)$

$(A \Rightarrow C) \land (B \Rightarrow C) \equiv (A \Rightarrow C) \land (B \Rightarrow C)$

A	B	C	¬A ∧ ¬B	**C ∨ (¬A ∧ ¬B)**	A ⇒ C	B ⇒ C	**(A ⇒ C) ∧ (B ⇒ C)**
True	True	True	False	**True**	True	True	**True**
True	True	False	False	**False**	False	False	**False**
True	False	True	False	**True**	True	True	**True**
True	False	False	False	**False**	False	True	**False**
False	True	True	False	**True**	True	True	**True**
False	True	False	False	**False**	True	False	**False**
False	False	True	True	**True**	True	True	**True**
False	False	False	True	**True**	True	True	**True**

c. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$: **TRUE**

There is no row where the former is true but the latter isn't.

A	B	**A ∨ B**	C	D	E	¬C ∨ ¬D ∨ E	**(A ∨ B) ∧ (¬C ∨ ¬D ∨ E)**
True	True	**True**	True	True	True	True	**True**
True	True	**True**	True	True	False	False	**False**
True	True	**True**	True	False	True	True	**True**
True	True	**True**	True	False	False	True	**True**
True	True	**True**	False	True	True	True	**True**

True	True	**True**	False	True	False	True	**True**
True	True	**True**	False	False	True	True	**True**
True	True	**True**	False	False	False	True	**True**
True	False	**True**	True	True	True	True	**True**
True	False	**True**	True	True	False	False	**False**
True	False	**True**	True	False	True	True	**True**
True	False	**True**	True	False	False	True	**True**
True	False	**True**	False	True	True	True	**True**
True	False	**True**	False	True	False	True	**True**
True	False	**True**	False	False	True	True	**True**
True	False	**True**	False	False	False	True	**True**
False	True	**True**	True	True	True	True	**True**
False	True	**True**	True	True	False	False	**False**
False	True	**True**	True	False	True	True	**True**
False	True	**True**	True	False	False	True	**True**
False	True	**True**	False	True	True	True	**True**
False	True	**True**	False	True	False	True	**True**
False	True	**True**	False	False	True	True	**True**
False	True	**True**	False	False	False	True	**True**
False	False	**False**	True	True	True	True	**False**
False	False	**False**	True	True	False	False	**False**
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False	False	**False**	True	False	False	True	**False**
False	False	**False**	False	True	True	True	**False**
False	False	**False**	False	True	False	True	**False**
False	False	**False**	False	False	True	True	**False**
False	False	**False**	False	False	False	True	**False**

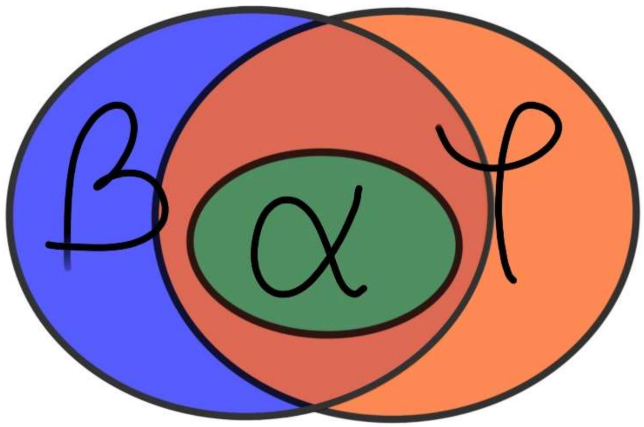
d. $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable: **TRUE**

A	B	$A \lor B$	$\neg(A \Longrightarrow B)$	$(\mathbf{A} \lor \mathbf{B}) \land \neg(\mathbf{A} \Rightarrow \mathbf{B})$
True	True	True	False	**False**
True	False	True	True	**True**
False	True	True	False	**False**
False	False	False	False	**False**

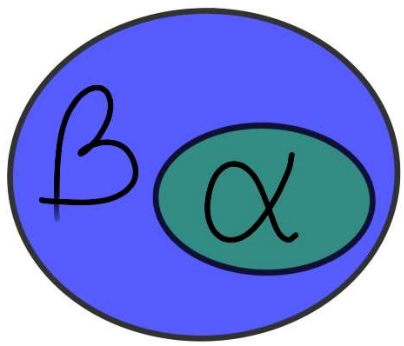
3. Prove using **Venn diagram**, or find a counterexample to the following assertion:

$\alpha| = (\beta \land \gamma)$ then $\alpha| = \beta$ and $\alpha| = \gamma$

This assertion is **TRUE**, because the Venn diagram for $\alpha| = (\beta \land \gamma)$ is



$\alpha| = \beta$ is



, and $\alpha| = \gamma$ is