# CECS 451
# Assignment 13

1. Consider the following data set comprised of three binary input attributes ($A_1$, $A_2$, and $A_3$) and one binary output:
   (a) (2 points) Compute $Gain(A_1)$.
   (b) (2 points) Compute $Gain(A_2)$.
   (c) (2 points) Compute $Gain(A_3)$.

| Example | $A_1$ | $A_2$ | $A_3$ | Output $y$ |
|---|---|---|---|---|
| $\mathbf{x}_1$ | 1 | 0 | 0 | 0 |
| $\mathbf{x}_2$ | 1 | 0 | 1 | 0 |
| $\mathbf{x}_3$ | 0 | 1 | 0 | 0 |
| $\mathbf{x}_4$ | 1 | 1 | 1 | 1 |
| $\mathbf{x}_5$ | 1 | 1 | 0 | 1 |
| | $p=\frac{4}{5}$ | $\frac{3}{5}$ | $\frac{2}{5}$ | |

Figure 1: Example data set

$$Gain(A) = B\left(\frac{p}{p+n}\right) - \sum_{k=1}^{d} \frac{p_k + n_k}{p+n} B\left(\frac{p_k}{p_k+n_k}\right)$$

positive example, negative example

$p$ = true cases
$n$ = false cases

$$\text{Entropy } B(q) = -(q\log_2 q + (1-q)\log_2(1-q))$$

a. $Gain(A_1)$

$$= B(y) - \left[\frac{p_{true}+n_{true}}{p+n} B\left(\frac{p_{true}}{p_{true}+n_{true}}\right) + \frac{p_{false}+n_{false}}{p+n} B\left(\frac{p_{false}}{p_{false}+n_{false}}\right)\right]$$

$$= B\left(\frac{2}{5}\right) - \left[\frac{4}{5} B\left(\frac{2}{4}\right) + \frac{1}{5} B\left(\frac{0}{1}\right)\right]$$

$$= -\left(\frac{2}{5}\log_2\frac{2}{5} + \frac{3}{5}\log_2\frac{3}{5}\right) - \left[\frac{4}{5}\cdot \underbrace{-\left(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{2}\log_2\frac{1}{2}\right)}_{=1 \text{ (lecture notes)}} + \frac{1}{5}\cdot \underbrace{-(0\log_2 0 + 1\log_2 1)}_{=0}\right]$$

$$= 0.97095 - \frac{4}{5}$$

$$= 0.17095$$

b. $Gain(A_2)$

$$= B\left(\frac{2}{5}\right) - \left[\frac{3}{5} B\left(\frac{2}{3}\right) + \overbrace{\frac{2}{5} B\left(\frac{0}{2}\right)}^{=0}\right]$$

$$= B\left(\frac{2}{5}\right) - \left[\frac{3}{5}\cdot -\left(\frac{2}{3}\log_2\frac{2}{3} + \frac{1}{3}\log_2\frac{1}{3}\right)\right]$$

$$= 0.97095 - 0.55098$$

$$= 0.41997$$

c. Gain($A_3$)

$$= B\left(\frac{2}{5}\right) - \left[\frac{2}{5}\overbrace{B\left(\frac{1}{2}\right)}^{1} + \frac{3}{5}B\left(\frac{1}{3}\right)\right]$$

$$= B\left(\frac{2}{5}\right) - \left[\frac{2}{5} + \frac{3}{5}\cdot -\left(\frac{1}{3}\log_2\frac{1}{3} + \frac{2}{3}\log_2\frac{2}{3}\right)\right]$$

$$= 0.97095 - \left[\frac{2}{5} + 0.55098\right]$$

$$= 0.97095 - 0.95097$$

$$= 0.01997$$

2. (6 points) Consider the XOR function of three binary input attributes ($A_1$, $A_2$, and $A_3$), which produces the value 1 if and only if an odd number of the three input attributes has value 1. Draw a minimal-sized decision tree for the three-input XOR function.

| $A_1$ | $A_2$ | $A_3$ | XOR |
|---|---|---|---|
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 0 | 0 | 0 |

DECISION TREE