CECS 451 Assignment 8
Ivy Nguyen 016618483

1. Given the full joint distribution shown in Figure 1, calculate the following (Write answers with a scale of 2, i.e., 0.12.)
   (a) (2 points) $P(toothache)$
   (b) (3 points) $\vec{P}(Cavity)$
   (c) (3 points) $\vec{P}(Toothache|cavity)$

	toothache		*¬toothache*	
	catch	*¬catch*	*catch*	*¬catch*
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

Figure 1: A full joint distribution for the Toothache, Cavity, Catch world.

a. $P(toothache) = 0.108 + 0.012 + 0.016 + 0.064 = 0.20$

b. $\vec{P}(Cavity) = \langle P(cavity), P(\neg cavity) \rangle$

$= \langle 0.108 + 0.012 + 0.072 + 0.008,\ 0.016 + 0.064 + 0.144 + 0.576 \rangle$

$= \langle 0.20, 0.80 \rangle$

c. $\vec{P}(Toothache \mid cavity) = \langle P(toothache \mid cavity), P(\neg toothache \mid cavity) \rangle$

$= \left\langle \frac{P(toothache, cavity)}{P(cavity)}, \frac{P(\neg toothache, cavity)}{P(cavity)} \right\rangle$

$= \left\langle \frac{0.108 + 0.012}{0.108 + 0.012 + 0.072 + 0.008}, \frac{0.072 + 0.008}{0.108 + 0.012 + 0.072 + 0.008} \right\rangle$

$= \langle 0.60, 0.40 \rangle$

2. Consider the Bayes net shown in Figure 2. Write answers with a scale of 2, i.e., 0.12.

(a) (3 points) Calculate the value of $P(b, i, \neg m, g, j)$.

(b) (4 points) Calculate the value of $\vec{P}(J|b, i, m)$.

(c) (5 points) Calculate the value of $\vec{P}(J|\neg b, \neg i, m)$.

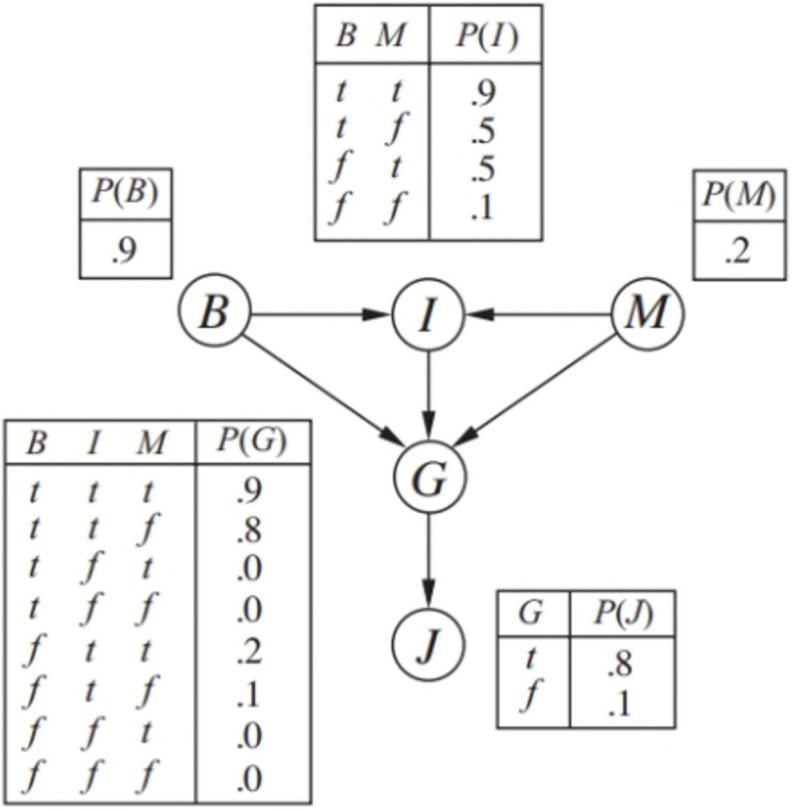

Figure 2: A simple Bayes net with Boolean variables.

a. $P(b, i, \neg m, g, j) = P(b)P(\neg m)P(i \mid b, \neg m)P(g \mid b, i, \neg m)P(j \mid g)$

$= 0.9 \cdot (1-0.2) \cdot 0.5 \cdot 0.8 \cdot 0.8$

≈ 0.23

b. $\vec{P}(J \mid b, i, m) = \alpha \vec{P}(J, b, i, m)$

$= \alpha \sum_g \vec{P}(J, b, i, m, g)$

$= \alpha \sum_g \vec{P}(J \mid g)P(b)P(i \mid b, m)P(m)P(g \mid b, i, m)$

$= \boxed{\alpha P(b)P(i \mid b, m)P(m)} \sum_g \vec{P}(J \mid g)P(g \mid b, i, m)$

$= 1$ (cancel out)

$= \sum_g \vec{P}(J \mid g)P(g \mid b, i, m)$

$= \vec{P}(J \mid g)P(g \mid b, i, m) + \vec{P}(J \mid \neg g)P(\neg g \mid b, i, m)$

$= \langle 0.8, (1-0.8) \rangle \cdot 0.9 + \langle 0.1, (1-0.1) \rangle \cdot (1-0.9)$

$= \langle 0.8, 0.2 \rangle \cdot 0.9 + \langle 0.1, 0.9 \rangle \cdot 0.1$

$= \left\langle \frac{73}{100}, \frac{27}{100} \right\rangle = \langle 0.73, 0.27 \rangle$

c. $\vec{P}(J \mid \neg b, \neg i, m) = \alpha \vec{P}(J, \neg b, \neg i, m)$

$= \alpha \sum_g \vec{P}(J, \neg b, \neg i, m, g)$

$= \alpha \sum_g \vec{P}(J \mid g)P(\neg b)P(\neg i \mid \neg b, m)P(m)P(g \mid \neg b, \neg i, m)$

$= \boxed{\alpha P(\neg b)P(\neg i \mid \neg b, m)P(m)} \sum_g \vec{P}(J \mid g)P(g \mid \neg b, \neg i, m)$

$= 1$ (cancel out)

$= \sum_g \vec{P}(J \mid g)P(g \mid \neg b, \neg i, m)$

$= \vec{P}(J \mid g)P(g \mid \neg b, \neg i, m) + \vec{P}(J \mid \neg g)P(\neg g \mid \neg b, \neg i, m)$

$= \langle 0.8, (1-0.8) \rangle \cdot 0 + \langle 0.1, (1-0.1) \rangle \cdot (1-0)$

$= \langle 0.10, 0.90 \rangle$