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Assignment 11

$X = R \quad e = u$

1. $\vec{P}(R_t | u_{1:t}) = \vec{P}(R_{t-1} | u_{1:t-1}) = \langle p, 1-p \rangle$

$$\vec{P}(R_t | u_{1:t}) = \alpha \vec{P}(u_t | \vec{R}_t) \sum_{r_{t-1}} \vec{P}(\vec{R}_t | r_{t-1}) P(r_{t-1} | u_{1:t-1})$$

$$\vec{P}(R_t | u_{1:t}) = \alpha \langle P(u_t | r_t), P(u_t | \neg r_t) \rangle \sum_{r_{t-1}} \langle P(r_t | r_{t-1}), P(\neg r_t | r_{t-1}) \rangle P(r_{t-1} | u_{1:t-1})$$

$$\vec{P}(R_t | u_{1:t}) = \alpha \langle P(u_t | r_t), P(u_t | \neg r_t) \rangle \left(\begin{array}{l} \langle P(r_t | r_{t-1}), P(\neg r_t | r_{t-1}) \rangle P(r_{t-1} | u_{1:t-1}) \\ + \\ \langle P(r_t | \neg r_{t-1}), P(\neg r_t | \neg r_{t-1}) \rangle P(\neg r_{t-1} | u_{1:t-1}) \end{array} \right)$$

$$\langle p, 1-p \rangle = \alpha \langle 0.7, 0.3 \rangle \left(\begin{array}{l} \langle 0.9, 0.1 \rangle \; p \\ + \\ \langle 0.2, 0.8 \rangle (1-p) \end{array} \right)$$

$$\langle p, 1-p \rangle = \alpha \langle 0.7, 0.3 \rangle \cdot \langle 0.9p - 0.2p + 0.2, 0.1p - 0.8p + 0.8 \rangle$$

$$\langle p, 1-p \rangle = \alpha \langle 0.7, 0.3 \rangle \cdot \langle 0.7p + 0.2, -0.7p + 0.8 \rangle$$

$$\langle p, 1-p \rangle = \alpha \langle 0.49p + 0.14, -0.21p + 0.24 \rangle$$

$$\langle p, 1-p \rangle = \frac{\langle 0.49p + 0.14, -0.21p + 0.24 \rangle}{0.28p + 0.38}$$

$$\langle 0.28p^2 + 0.38p, -0.28p^2 - 0.1p + 0.38 \rangle = \langle 0.49p + 0.14, -0.21p + 0.24 \rangle$$

$$0.28p^2 + 0.38p = 0.49p + 0.14$$

$$0.28p^2 - 0.11p - 0.14 = 0$$

$$p = \frac{0.11 \pm \sqrt{0.0121 + 0.1568}}{0.56}$$

$$= \frac{0.11 \pm \sqrt{0.1689}}{0.56}$$

$$= \frac{0.11 \pm 0.4110}{0.56}$$

$$= 0.9303, \; -0.\cancel{5375}$$

$p = 0.9303$

2. ES = getting enough sleep

e = red eyes

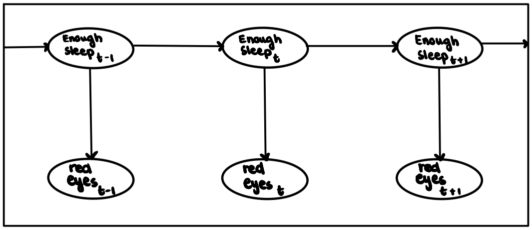
- $P(ES_0) = 0.7$
- $\vec{P}(\vec{ES}_t \mid es_{t-1}) = \langle 0.8, 0.2 \rangle$

  $\vec{P}(\vec{ES}_t \mid \neg es_{t-1}) = \langle 0.3, 0.7 \rangle$
- $\vec{P}(e_t \mid es_t) = \langle 0.2, 0.8 \rangle$

  $\vec{P}(e_t \mid \neg es_t) = \langle 0.7, 0.3 \rangle$

a.

$\vec{P}(ES_0) = \langle 0.7, 0.3 \rangle$



ES_t	$P(e_t)$	$P(\neg e_t)$
true	0.2	0.8
false	0.7	0.3

observation (sensor)

ES_{t-1}	$P(es_t)$	$P(\neg es_t)$
true	0.8	0.2
false	0.3	0.7

transition

b. $\vec{P}(ES_2 \mid e_{1:2})$

$$\vec{P}(ES_1 \mid e_1) = \alpha \vec{P}(e_1^t \mid \vec{ES}_1) \sum_{es_0} \vec{P}(ES_1 \mid es_0) P(es_0)$$

$$= \alpha \langle 0.2, 0.7 \rangle \begin{pmatrix} \langle 0.8, 0.2 \rangle\, 0.7 \\ + \\ \langle 0.3, 0.7 \rangle\, 0.3 \end{pmatrix}$$

$$= \alpha \langle 0.2, 0.7 \rangle \langle 0.56 + 0.09, 0.14 + 0.21 \rangle$$

$$= \alpha \langle 0.2, 0.7 \rangle \langle 0.65, 0.35 \rangle$$

$$= \alpha \langle 0.13, 0.245 \rangle$$

$$= \left\langle \frac{26}{75}, \frac{49}{75} \right\rangle$$

$$\vec{P}(ES_2 \mid e_{1:2}) = \alpha \vec{P}(e_2^f \mid \vec{ES}_2) \sum_{es_1} \vec{P}(ES_2 \mid es_1) P(es_1 \mid e_1)$$

$$= \alpha \langle 0.8, 0.3 \rangle \begin{pmatrix} \langle 0.8, 0.2 \rangle \cdot \frac{26}{75} \\ + \\ \langle 0.3, 0.7 \rangle \cdot \frac{49}{75} \end{pmatrix}$$

$$= \alpha \langle 0.8, 0.3 \rangle \left\langle 0.8 \cdot \frac{26}{75} + 0.3 \cdot \frac{49}{75}, 0.2 \cdot \frac{26}{75} + 0.7 \cdot \frac{49}{75} \right\rangle$$

$$= \alpha \langle 0.8, 0.3 \rangle \left\langle \frac{71}{150}, \frac{79}{150} \right\rangle$$

$$= \alpha \left\langle \frac{142}{375}, \frac{79}{500} \right\rangle$$

$$= \langle 0.7056, 0.2944 \rangle$$

enough sleep

not enough sleep

3. 101 × 3 world

start state : Up or Down

other states : Right

a. the utility of each action as a function of r

up action:

$$U([s_0=up, s_1=right, \dots, s_{101}=right]) = r^0 \cdot 0 + r^1 \cdot 50 + r^2(-1) + \dots + r^{100}(-1) + r^{101} \cdot 10$$

$$= 50r - \sum_{t=2}^{100} r^t + 10r^{101}$$

$$= \boxed{50r - r^2\left(\frac{1-r^{99}}{1-r}\right) + 10r^{101}}$$

$$\sum_{i=0}^{n-1} ar^i = \frac{a(1-r^n)}{1-r}$$

$r^2 + \dots + r^{100}$

$r^2(1 + \dots + r^{98})$

$n = 99 \quad a = r^2$

down action:

$$U([s_0=down, s_1=right, \dots, s_{101}=right]) = \boxed{-50r + r^2\left(\frac{1-r^{99}}{1-r}\right) - 10r^{101}}$$

c. $r = \frac{1}{2}$

$$U([s_0=up, s_1=right, \dots, s_{101}=right]) = 50\left(\tfrac{1}{2}\right) - \left(\tfrac{1}{2}\right)^2\left(\frac{1-\left(\frac{1}{2}\right)^{99}}{1-\frac{1}{2}}\right) + 10\left(\tfrac{1}{2}\right)^{101}$$

$$= 24.5$$

$$U([s_0=down, s_1=right, \dots, s_{101}=right]) = -50\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2\left(\frac{1-\left(\frac{1}{2}\right)^{99}}{1-\frac{1}{2}}\right) - 10\left(\tfrac{1}{2}\right)^{101}$$

$$= -24.5$$

Going up is recommended as the utility value of this action is larger than the utility value of going down

b.