Overview

These are notes and formulas from Understanding the Yield Curve: Part 1.

Definitions

- Par rate: Used to discount a set of cash flows to today.
- Spot rate: Used to discount a single future cash flow to today.
- Forward rate: Used to discount a single future cash flow to another future date; It is the interest rate for a loan between any two dates in the future, contracted today. [1]
- Par rates are averages of one or more spot rates
- Spot rates are geometric averages of one or more forward rates
- Forward rates measure the marginal reward for lengtening the maturity of the investment

General Notes

There are three main influences on the yield curve shape.

1. Market rate's expectations
2. Bond risk premia[2]
3. Convexity bias[3]

Forwards can be viewed as **break-even rates** for active portfolio managers.[4] If forward rates are realized, all positives earn the same return. Bearish bets are profitable when yields rise more than the forward rates. Bullish bets profit when rates rise less than what forwards imply – less than what is needed to offset positive carry.

Forwards are also **indicators of cheap maturity sectors**. High and low rates are more visible that the spot/par rates as the other rates are averages.

The one year forward rate $f_{n-1,n}$ is equal to n-year zero's rolling yield, such that it can quantify the rolling yield advantage.

[1] A multiyear spot can be decomposed into product of one year forward rates

[2] Premia tend to be high after poor economic conditions and low after strong economic conditions

[3] Positive convexity is desirable as it increases a bond's return regardless of how the yield moves. More convex bonds have lower yields. Short term bonds have little convexity.

[4] Recall that multiyear spots can be broken into one year forwards, considered the 'building block' for interest rates.

The Formulas

Relationship between n-year zero's price P_n and annualized n-year spot rate s_n.

$$P_n = \frac{100}{(1+s_n)^n}. \tag{1}$$

Computation of the annualized forward rate between maturities m and n.[5]

$$(1+f_{m,n})^{n-m} = \frac{(1+s_n)^n}{(1+s_m)^m} \tag{2}$$

Case of Eq(2) where m=1 and n=2

$$\frac{(1+s_2)^2}{1+f_{1,2}} = 1+s_1 \tag{3}$$

$$\frac{n!}{3!\,(n-3)!} = 74000 \tag{4}$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)!}}{3!\cancel{(n-3)!}} = 74000 \tag{5}$$

$$n(n-1)(n-2) = 444000 \tag{6}$$

$$n^3 - 3n^2 + 2n - 444000 = 0. \tag{7}$$

Solving for n using a computer[6] gives

$$n = 77.2932056177889. \tag{8}$$

So, there are probably around 77 panels available for the random comic.

[5] There is interest rate risk in this transaction that is not factored in here. Simply, this formula locks in a rate between m and n.

[6] `http://www.sympygamma.com/input/?i=solve%28n**3+-+3*n**2+%2B+2*n+-+444000%2C+n%29`