# Homework #7: Due Thursday, October 27th at 2:00 PM

- Submit a paper copy of your solutions before class on Thursday, October 27.
- All pages must be stapled (no folded corners please).
- No paper fringes on the edges please.
- Assignments MUST:
  - Be your own work. Though you may collaborate with others, everyone is responsible for their own work and plagiarism of any form is not tolerated.
  - Be complete. You must provide all work and/or explanations needed to find the solution. Answers with insufficient or incomplete supporting work may lose credit.
  - Adhere to the Code of Academic Honesty.
  - Be legible. Your solution to a problem must be clear, written in complete sentences. You may lose credit for work that is unclear or hard to follow.
- Assignments handed in during or after class will lose points for being late.
- Correct answers with missing or incomplete work may not earn full credit.
- Thanks!

1. Suppose a random sample with $X_1 = 4$, $X_2 = 7$, $X_3 = 9$, and $X_4 = 10$ is drawn from a distribution with pdf
$$f(x;\theta) = \left(\frac{\theta}{2\sqrt{x}}\right) e^{-\theta\sqrt{x}}$$
where $x > 0$. Find the maximum likelihood estimate for $\theta$. Be sure to show ALL of the steps of your work. Answers missing some steps or with incomplete work may lose credit. Round your answers to three decimal places if needed.

2. Let $X_1, X_2, \ldots X_n$ be a random sample from the distribution with pdf
$$f(x;\theta) = \frac{x^3 e^{-x/\theta}}{6\theta^4} \text{ for } x \geq 0.$$
Give the maximum likelihood estimate (MLE) of $\theta$. **Note your answer will depend on the values $\mathbf{x_1, x_2, \ldots, x_n}$.** Be sure to show ALL of the steps of your work. Answers missing some steps or with incomplete work may lose credit.

3. Let $X_1$, $X_2$, $X_3$ be a random sample from the distribution $f(x;\theta) = \theta x^{\theta-1}$ for $0 \leq x \leq 1$ where the parameter $\theta > 0$. Let $Y_1$, $Y_2$ be a random sample from the distribution $f(y;\theta) = 2\theta y^{2\theta-1}$ for $0 \leq y \leq 1$ where the parameter $\theta > 0$ is the same as with the $X$'s. Assume all $X$ and $Y$'s are independently chosen. If $X_1 = 1$,$X_2 = \frac{1}{4}$, $X_3 = \frac{1}{2}$, $Y_1 = \frac{1}{3}$ and $Y_2 = \frac{1}{5}$. Find the MLE of $\theta$. Be sure to show ALL of the steps of your work. Answers missing some steps or with incomplete work may lose credit. Round your answers to three decimal places if needed.

4. Let $X_1 = 0.5$, $X_2 = 0.6$, $X_3 = 0.25$, $X_4 = 0.9$, and $X_5 = 0.95$ be a random sample from a distribution with pdf
$$f(x;\theta) = \theta x^{\theta-1}\ ,\quad 0 \leq x \leq 1\ ,\quad \theta > 0.$$
   (a) Find the MLE of $\theta$.
   (b) Find the MoM estimate of $\theta$.

5. Suppose $Y_1, Y_2, \ldots, Y_n$ is a random sample from a distribution with pdf $f(y;\theta) = \theta e^{-\theta y}$ for $y > 0$ and where the parameter $\theta$. Find the method of moments estimator, $\hat{\theta}_{\text{MoM}}$, for the parameter $\theta$. **Note your answer will depend on the values $\mathbf{y_1, y_2, \ldots, y_n}$.** Be sure to show ALL of the steps of your work. Answers missing some steps or with incomplete work may lose credit.

6. Suppose $X_1$, $X_2$, and $X_3$ are a random sample of size 3 from a distribution with pdf $f(x;\theta)$. Let $\mu_X = E(X)$ and $\sigma_X^2 = \text{Var}(X)$ denote the expected value and variance of random variable $X$.

(a) Below are five different estimator for the parameter $\theta = \mu_X$. Determine whether the estimator is a biased or unbiased estimator for $\theta = \mu_X$. Answers without sufficient supporting work provided may lose credit.

i. $\theta_1 = X_1$

ii. $\theta_2 = X_2 + X_3$

iii. $\theta_3 = \dfrac{X_1 + X_3}{2}$

iv. $\theta_4 = \dfrac{2X_2 + 3X_3}{5}$

v. $\theta_5 = \dfrac{X_1 + 2X_2 + 2X_3}{5}$

(b) Which of the unbiased estimators for $\theta = \mu_X$ in part (a) is the most efficient (has the smallest variance)? Be sure to provide supporting that shows how you determined your answer. Answers without sufficient supporting work provided may lose credit. You do not need to conisder estimator(s) from part (a) that are biased. Only compare efficiencies of unbiased estimators.