# HW5

November 6, 2024

# 1 STOR 320 Homework 5: Visualization and Linear Regression

Please submit the solution to gradescope by 11:59 PM, Nov 7, Thursday.

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[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from wordcloud import WordCloud
import requests
from bs4 import BeautifulSoup
from sklearn.model_selection import train_test_split
import statsmodels.formula.api as smf
import statsmodels.api as sm
from statsmodels.stats.outliers_influence import variance_inflation_factor
```

## 1.1 Problem 1. (5 points).

A random walk is a mathematical concept that describes a path consisting of a series of random steps. A random walk progresses in discrete steps. In each step, the position either increases or decreases by a certain amount, typically +1 or -1.

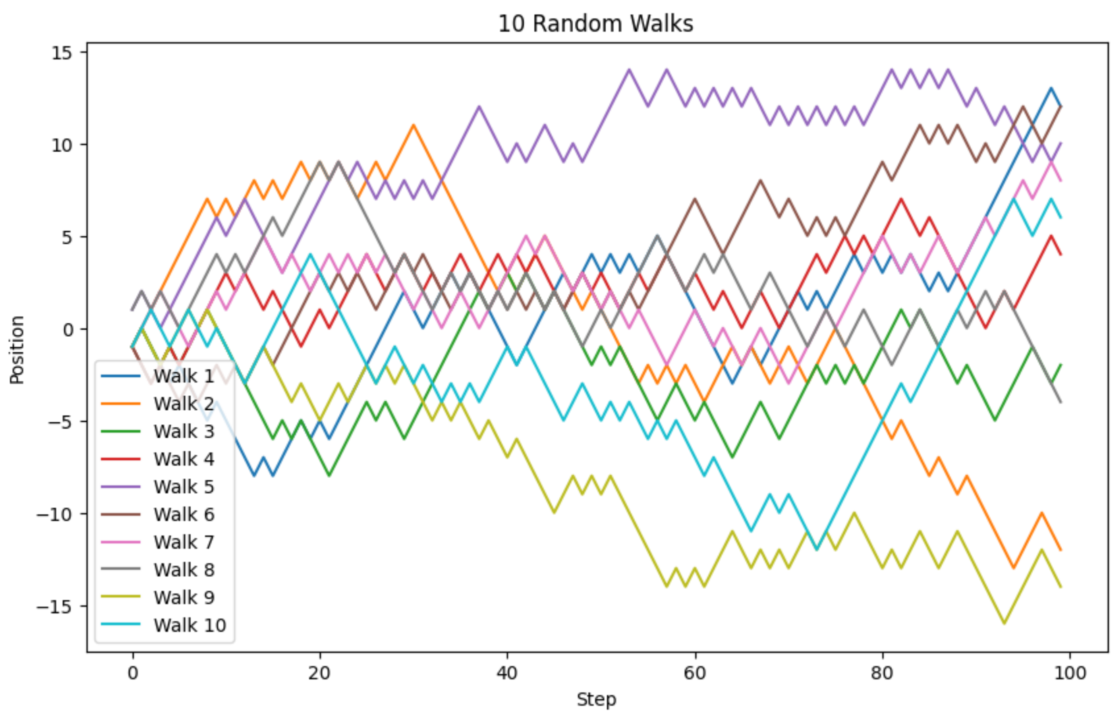
- 1.1 Generate 10 random walks, each starting at 0 and having 100 steps. Each step should be randomly either -1 or +1. Store each random walk as a separate line in a 2D array or list. Set random seed as 42. (3 points)
- 1.2 Visualize the 10 random walks in a single plot. Each random walk should have its own color and be labeled as Walk 1, Walk 2, …, Walk 10. (2 points)

[ ]:
```
#1.1
np.random.seed(42)

walks = np.cumsum(np.random.choice([-1, 1], size=(10, 100)), axis=1)
walks[0]
```

[ ]:
```
array([-1,  0, -1, -2, -3, -2, -3, -4, -5, -4, -5, -6, -7, -8, -7, -8, -7,
       -6, -5, -6, -5, -6, -5, -4, -3, -2, -1,  0,  1,  2,  1,  0,  1,  2,
```

```
 3,  2,  3,  2,  1,  0, -1, -2, -1,  0,  1,  2,  3,  2,  3,  4,  3,
 4,  3,  4,  3,  4,  5,  4,  3,  2,  1,  0, -1, -2, -3, -2, -1, -2,
-1,  0,  1,  2,  1,  2,  1,  2,  3,  4,  3,  4,  3,  4,  3,  4,  3,
 2,  3,  2,  3,  4,  5,  6,  7,  8,  9, 10, 11, 12, 13, 12])
```

[ ]:
```python
plt.figure(figsize=(10, 6))
for i in range(0,10):
    plt.plot(walks[i], label=f'Walk {i+1}')

plt.xlabel('Step')
plt.ylabel('Position')
plt.title('10 Random Walks')
plt.legend()
plt.show()
```




## 1.2 Problem 2. Recreate the `pd.get_dummies()` function from stratch. (20 points).

- 2.1 Write a function called `convert_to_binary` **from scratch** that takes a pandas DataFrame of categorical features and returns a DataFrame where each feature is converted into multiple binary columns. (8 points)
- Each binary column should represent a feature-category combination.
- The column names should follow the format feature_category. For example, if the feature

Color has values E, I, and J, the new columns should be Color_E, Color_I, and Color_J.

- 2.2 Apply the `convert_to_binary` to the `Color` and `Cut` columns in `diamonds-new.csv`. Print the converted dataframe. (4 points)
- 2.3 Apply the `pd.get_dummies()` function to the same dataset, and check if the output of `convert_to_binary` is the same as `pd.get_dummies()`. (8 points)

**Hint:** Use `np.allclose` to check if the outputs are the same.

[ ]:
```python
def convert_to_binary(df):
    binary_df = pd.DataFrame()
    for column in df.columns:
        if df[column].dtype == 'object':
            categories = sorted(df[column].unique())
            for category in categories:
                binary_column = f"{column}_{category}"
                binary_df[binary_column] = (df[column] == category)#.astype(int)
        else:
            binary_df[column] = df[column]
    return binary_df
```

[ ]:
```python
diamonds_new = pd.read_csv("diamonds-new.csv")
```

[ ]:
```python
diamonds_binary = convert_to_binary(diamonds_new)
diamonds_binary.head()
```

[ ]:
```
   Carat  Cut_Fair  Cut_Good  Cut_Ideal  Cut_Premium  Cut_Very Good  Color_D  \
0   0.23     False     False       True        False          False    False
1   0.21     False     False      False         True          False    False
2   0.23     False      True      False        False          False    False
3   0.29     False     False      False         True          False    False
4   0.31     False      True      False        False          False    False

   Color_E  Color_F  Color_G  …  Clarity_VS1  Clarity_VS2  Clarity_VVS1  \
0     True    False    False  …        False        False         False
1     True    False    False  …        False        False         False
2     True    False    False  …         True        False         False
3    False    False    False  …        False         True         False
4    False    False    False  …        False        False         False

   Clarity_VVS2  depth  table  Price     x     y     z
0         False   61.5   55.0  326.0  3.95  3.98  2.43
1         False   59.8   61.0  326.0  3.89  3.84  2.31
2         False   56.9   65.0  327.0  4.05  4.07  2.31
3         False   62.4   58.0  334.0  4.20  4.23  2.63
4         False   63.3   58.0  335.0  4.34  4.35  2.75

[5 rows x 27 columns]
```

[ ]:
```
dummies = pd.get_dummies(diamonds_new)
```

[ ]:
```
# ran into an error, checking boolean values
dummies_bool = dummies.select_dtypes(bool)
diamonds_binary_bool = diamonds_binary.select_dtypes(bool)

np.allclose(dummies_bool, diamonds_binary_bool)
```

[ ]: True

## 2 Problem 3: Simple linear regression with penguins dataset (15 points)

The dataset `penguins`contains measurements for different penguin species. The dataset includes the following relevant columns:

- bill_length_mm: Length of the penguin's bill in millimeters.
- bill_depth_mm: Depth of the penguin's bill in millimeters.
- species: Species of the penguin (e.g., Adelie, Gentoo, Chinstrap).
- **3.1** Create a scatter plot with a linear regression line for the entire dataset. X-axis is the `bill_length_mm` and y-axis is the `bill_depth_mm`. (3 points)
- **3.2** Based on your plot, describe the relationship between bill length and bill depth. What is the meaning of the slope of the line in the plot? (3 points)
- **3.3** Create a similar scatter plot with a linear regression line grouping by species. X-axis is the `bill_length_mm` and y-axis is the `bill_depth_mm`. (4 points)
- **3.4** Based on your plot, describe the relationship between bill length and bill depth. Describe the differences of plots between 3.1 and 3.3 (5 points)

[ ]:
```
#penguins = sns.load_dataset("penguins")
# had a hard time downloading penguins using sns, reading csv instead
penguins = pd.read_csv("penguins.csv")
```

[ ]:
```
all_columns = "+".join(penguins.columns.difference(["bill_depth_mm"]))
formula = "bill_depth_mm~" + all_columns +'-1'
mod = smf.ols(formula=formula,data=penguins).fit()
mod.summary()
```

[ ]:

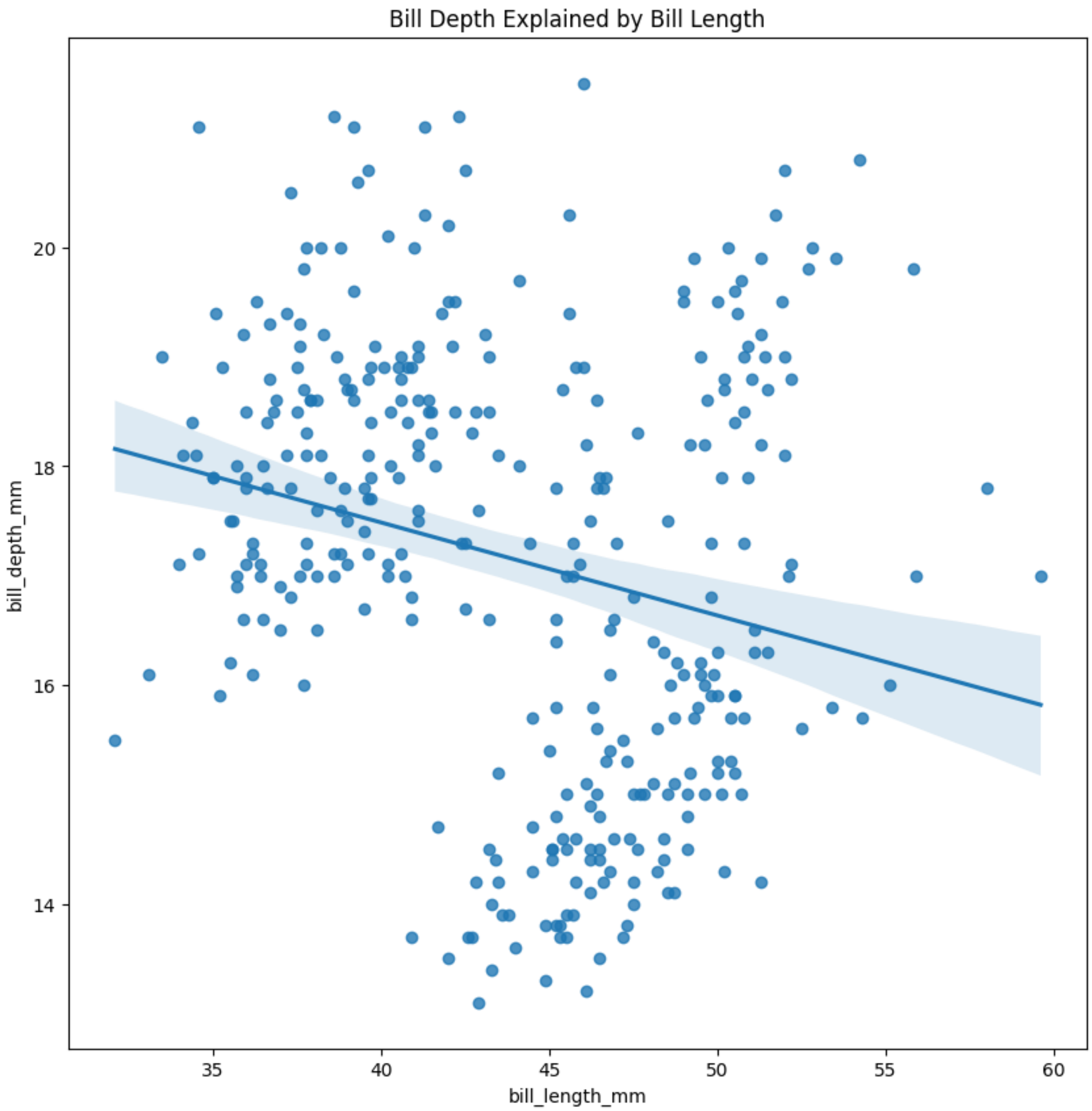
| | | | |
|---|---|---|---|
| **Dep. Variable:** | bill_depth_mm | **R-squared:** | 0.842 |
| **Model:** | OLS | **Adj. R-squared:** | 0.838 |
| **Method:** | Least Squares | **F-statistic:** | 215.5 |
| **Date:** | Tue, 05 Nov 2024 | **Prob (F-statistic):** | 8.35e-125 |
| **Time:** | 20:07:48 | **Log-Likelihood:** | -390.67 |
| **No. Observations:** | 333 | **AIC:** | 799.3 |
| **Df Residuals:** | 324 | **BIC:** | 833.6 |
| **Df Model:** | 8 | | |
| **Covariance Type:** | nonrobust | | |

| | **coef** | **std err** | **t** | **P> \|t\|** | **[0.025** | **0.975]** |
|---|---|---|---|---|---|---|
| **island[Biscoe]** | 10.9754 | 1.482 | 7.407 | 0.000 | 8.060 | 13.890 |
| **island[Dream]** | 10.8373 | 1.486 | 7.292 | 0.000 | 7.913 | 13.761 |
| **island[Torgersen]** | 11.0122 | 1.502 | 7.332 | 0.000 | 8.057 | 13.967 |
| **sex[T.MALE]** | 0.8932 | 0.136 | 6.546 | 0.000 | 0.625 | 1.162 |
| **species[T.Chinstrap]** | -0.3343 | 0.246 | -1.357 | 0.176 | -0.819 | 0.150 |
| **species[T.Gentoo]** | -4.9585 | 0.300 | -16.548 | 0.000 | -5.548 | -4.369 |
| **bill_length_mm** | 0.0384 | 0.020 | 1.945 | 0.053 | -0.000 | 0.077 |
| **body_mass_g** | 0.0005 | 0.000 | 3.409 | 0.001 | 0.000 | 0.001 |
| **flipper_length_mm** | 0.0188 | 0.008 | 2.239 | 0.026 | 0.002 | 0.035 |

| | | | |
|---|---|---|---|
| **Omnibus:** | 6.193 | **Durbin-Watson:** | 1.794 |
| **Prob(Omnibus):** | 0.045 | **Jarque-Bera (JB):** | 5.993 |
| **Skew:** | 0.288 | **Prob(JB):** | 0.0500 |
| **Kurtosis:** | 3.316 | **Cond. No.** | 2.55e+05 |

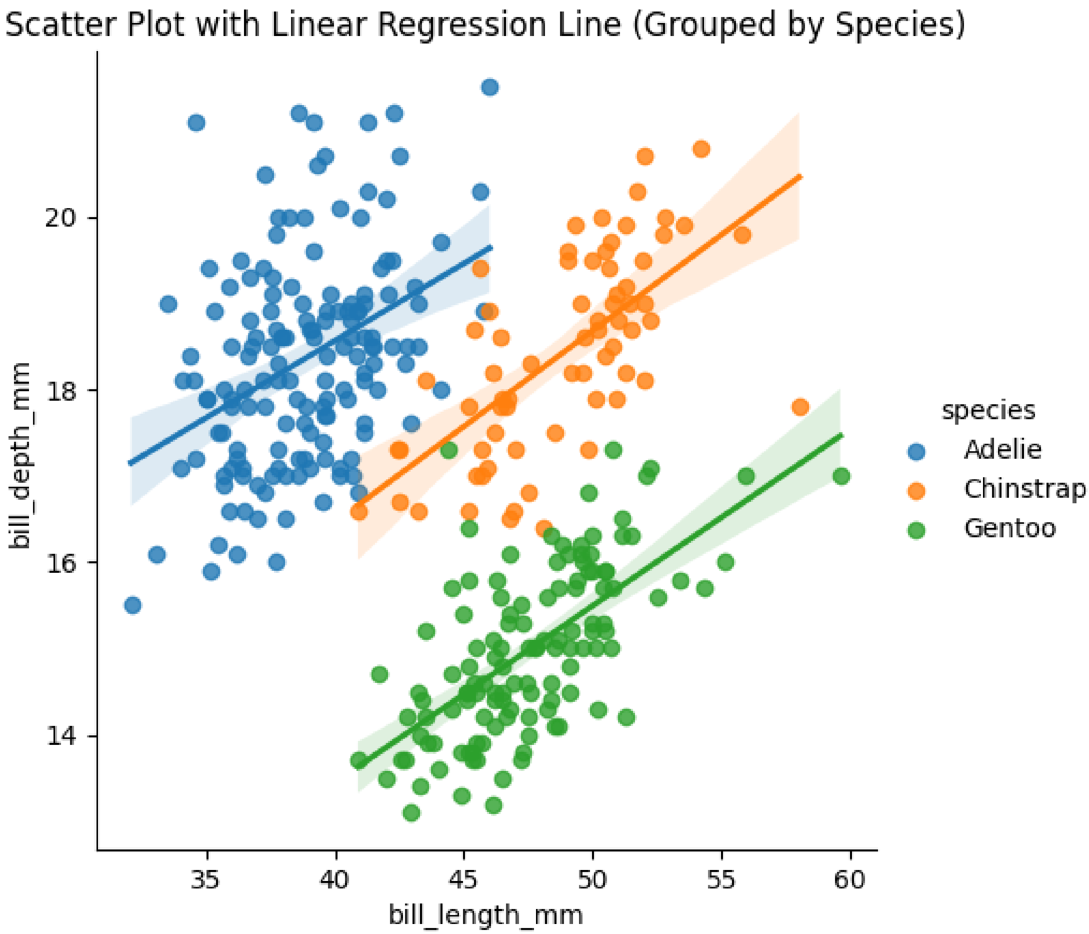
Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 2.55e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

[ ]:
```
#3.1
plt.figure(figsize=(10,10))
sns.regplot(x="bill_length_mm", y="bill_depth_mm", data=penguins)
plt.title("Bill Depth Explained by Bill Length")
plt.show()
```




Although there is a linear trend between bill length and depth, there does not appear to be a relationship between bill depth and length. The slope of the line indicates the line of “best fit”, as in, the average decrease in bill depth for every millimeter increase in bill length.

[ ]:
```
#3.3
sns.lmplot(x="bill_length_mm", y="bill_depth_mm", hue="species", data=penguins)
plt.title("Scatter Plot with Linear Regression Line (Grouped by Species)")
plt.show()
```




It seems that when grouping by species, the regression lines better fit the trends of the data. This conformity implies that bill depth and length are positively correlated for every species, but do not have the same correlation across species. Put another way, each species has its own regression line. The grouping caused different linear trends to emerge, specifically that bill depth tends to increase as bill length increases —- contrary to the findings in 3.1. The graph produced in 3.3 is much accurate and better at predicting bill depth than 3.1.

# 3 Problem 4: Linear regression with `diamonds-new.csv`. (60 points)

- 4.1 Read the `diamonds-new.csv`, check if there is any missing value. How many missing values are there? (3 points)
- 4.2 Remove the rows with missing values. Check if there is any missing value in the new table. (2 points)
- 4.3 Create a histgram of the price distribution. What do you observe? (5 points)
- 4.4 Create a scatter plot between `Price` and `Carat`, grouping by the level of `Cut`. What

difference do you observe for different `Cut` levels? Do you observe a linear trend? (5 points)

- 4.5 Create a scatter plot between `Price` and `depth`, grouping by the level of `Cut`. Do you observe a linear trend? (5 points)
- 4.6 Create a barplot between `Price` and `Color`, grouping by the level of `Cut`. Which type of `Color` and `Cut` combination has the largest average price? (5 points)
- 4.7 Use `train_test_split` to split the whole dataset into training set and testing set, according to the ratio 80% and 20%. How many rows in the training set? How many rows in the test set? (5 points)
- 4.8 Build a linear regression model to predict the prices based on all other columns. What is the value of in-sample R-squared? (5 points)
- 4.9 Calculate the VIF values for each column within `['Carat', 'depth', 'table', 'x', 'y', 'z']`. Is there any multicollinearity within these columns? (5 points)
- 4.10 Remove proper columns within `['Carat', 'depth', 'table', 'x', 'y', 'z']` based on the values of VIF. (5 points)
- 4.11 Build a new linear regression model using the selected columns in 4.10. What is the value of in-sample R-squared? Write down the math formulation for predicting the price of a diamonds based on the coefficient in this model. (5 points)
- 4.12 Calculate the out-of-sample R-squared for the model in 4.11. (5 points)
- 4.13 Visualize the prediction error as a function of the predicted prices. Add a horizontal line representing residual = 0. What do you observe? (5 points)

[ ]:
```python
diamonds = pd.read_csv("diamonds-new.csv")
```

[ ]:
```python
#4.1
missing_values = diamonds.isnull().sum()
total_missing = missing_values.sum()
print(f"Total missing values: {total_missing}")
```

```
Total missing values: 4
```

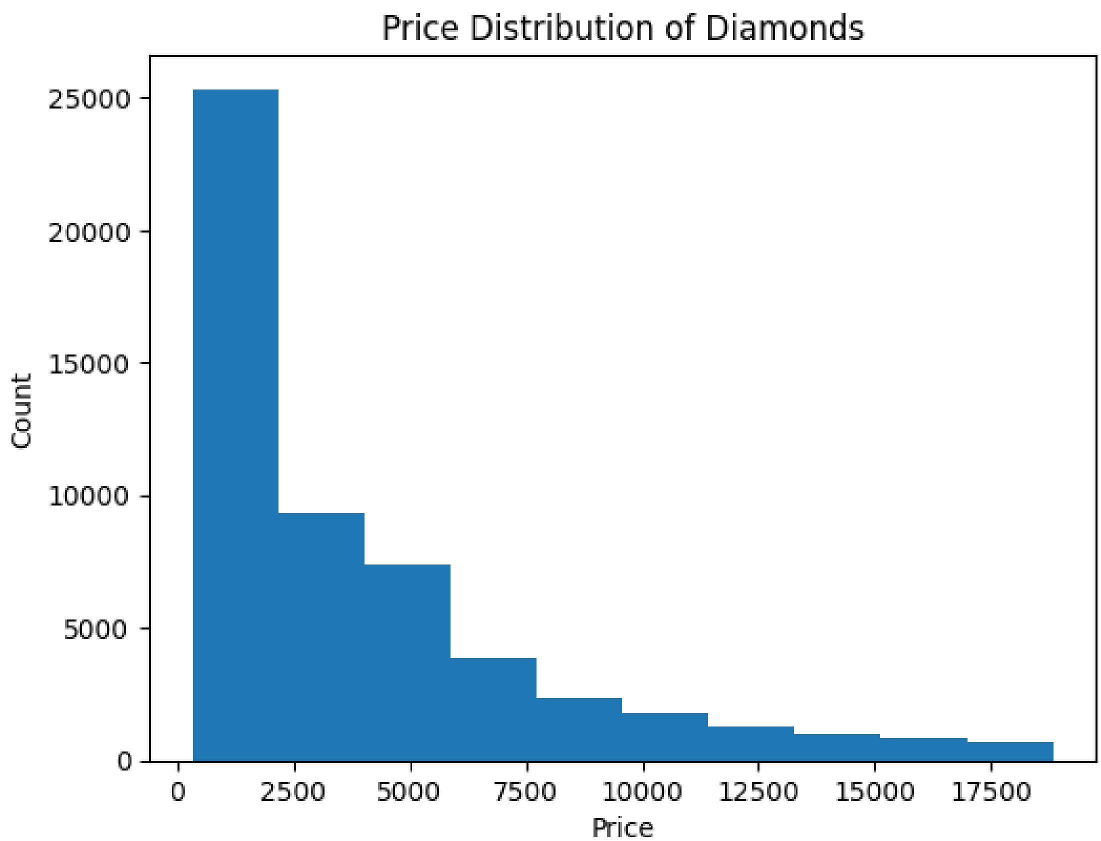
[ ]:
```python
#4.2
diamonds = diamonds.dropna()
```

[ ]:
```python
missing_values = diamonds.isnull().sum()
total_missing = missing_values.sum()
print(f"Total missing values: {total_missing}")
```

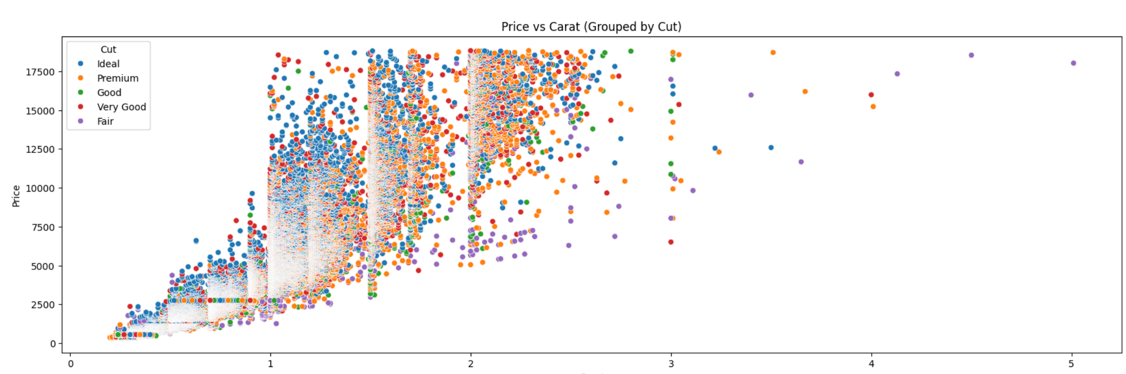
```
Total missing values: 0
```

[ ]:
```python
#4.3
plt.hist(diamonds["Price"])
plt.title("Price Distribution of Diamonds")
plt.ylabel("Count")
plt.xlabel("Price")
```

```
plt.show()
```

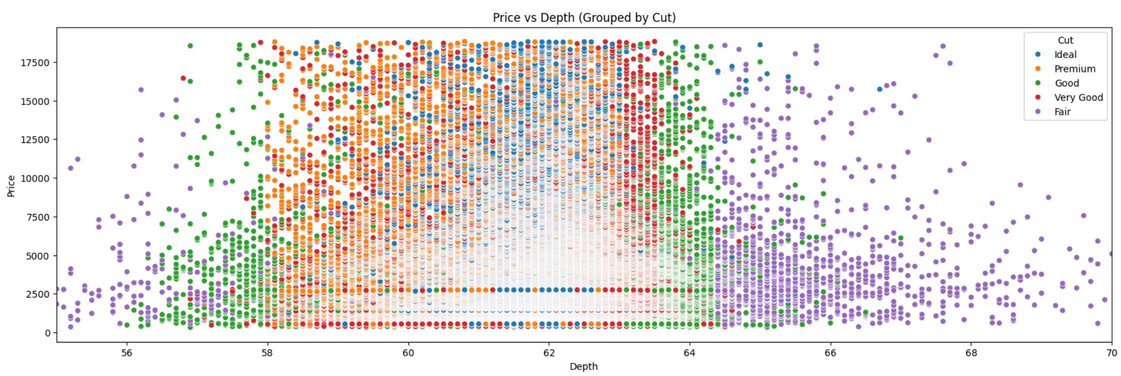



[ ]:
```
#4.4
plt.figure(figsize=(20, 6))
sns.scatterplot(x="Carat", y="Price", hue="Cut", data=diamonds)
plt.title("Price vs Carat (Grouped by Cut)")
plt.xlabel("Carat")
plt.ylabel("Price")
plt.show()
```



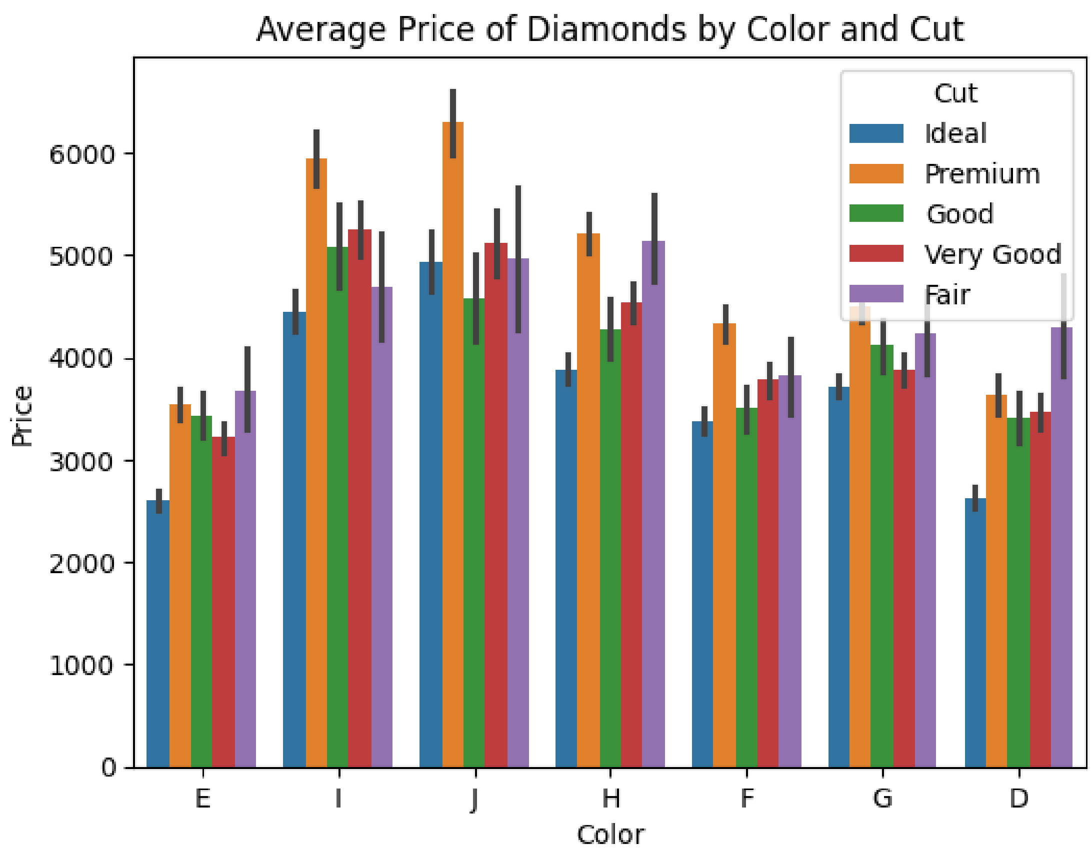
It seems like there's a linear trend between Price and Carats, without much influence from the cut. Perhaps it's easier to predict using the cut but generally it seems that as carats increase, price also increases, which makes sense intuitively. Larger diamonds should cost more.

[ ]:
```
#4.5
plt.figure(figsize=(20, 6))
sns.scatterplot(x="depth", y="Price", hue="Cut", data=diamonds)
plt.title("Price vs Depth (Grouped by Cut)")
plt.xlim(55,70) # visualizing the majority of the data
plt.xlabel("Depth")
plt.ylabel("Price")
plt.show()
```




There doesn't appear to be any linear trends with the data, it just seems normally distributed

[ ]:
```
#4.6
sns.barplot(x="Color", y="Price", hue="Cut", data=diamonds)
plt.title("Average Price of Diamonds by Color and Cut")
plt.show()
```




It seems that Premium diamonds of color J have the largest average price.

[ ]:
```
#4.7
diamonds_train, diamonds_test = train_test_split(diamonds, test_size=0.2,␣
  ↪random_state=42)
len(diamonds_train), len(diamonds_test)
```

[ ]: (43148, 10788)

[ ]:
```
#4.8
all_columns = "+".join(diamonds_train.columns.difference(["Price"]))
formula = "Price~" + all_columns +'-1'
mod = smf.ols(formula=formula,data=diamonds_train).fit()
print(f"R2 = {mod.rsquared}")
```

R2 = 0.917215815077376

[ ]:
```
#4.9
columns = ['Carat', 'depth', 'table', 'x', 'y', 'z']
for i in range(0, len(columns)):
```

```
    print(f"VIF of {columns[i]} =␣
↪{variance_inflation_factor(diamonds_train[columns], i)}")
```

```
VIF of Carat = 70.2972588045807
VIF of depth = 506.94872405185356
VIF of table = 494.5148862356101
VIF of x = 1267.0888081251753
VIF of y = 545.867416423977
VIF of z = 468.2185131382021
```

seems like there's a lot of multicollinearity, anything with a VIF>5 is quite high.

[ ]:
```
#4.10
diamonds_train = diamonds_train.drop("x", axis=1)
```

[ ]:
```
columns = ['Carat', 'depth', 'table', 'y', 'z']
for i in range(0, len(columns)):
    print(f"VIF of {columns[i]} =␣
↪{variance_inflation_factor(diamonds_train[columns], i)}")
```

```
VIF of Carat = 51.013874032155
VIF of depth = 506.76040501790834
VIF of table = 435.97651701813055
VIF of y = 350.6882481200804
VIF of z = 376.8530713427753
```

[ ]:
```
diamonds_train = diamonds_train.drop("depth", axis=1)
```

[ ]:
```
columns = ['Carat', 'table', 'y', 'z']
for i in range(0, len(columns)):
    print(f"VIF of {columns[i]} =␣
↪{variance_inflation_factor(diamonds_train[columns], i)}")
```

```
VIF of Carat = 41.92358071033758
VIF of table = 137.1411836227726
VIF of y = 350.64359471150425
VIF of z = 310.29158018199985
```

[ ]:
```
diamonds_train = diamonds_train.drop("y", axis=1)
```

[ ]:
```
columns = ['Carat', 'table', 'z']
for i in range(0, len(columns)):
    print(f"VIF of {columns[i]} =␣
↪{variance_inflation_factor(diamonds_train[columns], i)}")
```

```
VIF of Carat = 31.7223631361449
VIF of table = 100.59842427121707
VIF of z = 214.97552629167828
```

[ ]:
```
diamonds_train = diamonds_train.drop("z", axis=1)
```

[ ]:
```
columns = ['Carat', 'table']
for i in range(0, len(columns)):
    print(f"VIF of {columns[i]} =␣
 ↪{variance_inflation_factor(diamonds_train[columns], i)}")
```

```
VIF of Carat = 3.900079647930688
VIF of table = 3.900079647930639
```

[ ]:
```
#4.11
model = smf.ols("Price ~ Carat + table", data = diamonds_train).fit()
model.rsquared
```

[ ]: 0.8501611882175852

[ ]:
```
model.summary()
```

[ ]:

| | | | |
|---|---|---|---|
| **Dep. Variable:** | Price | **R-squared:** | 0.850 |
| **Model:** | OLS | **Adj. R-squared:** | 0.850 |
| **Method:** | Least Squares | **F-statistic:** | 1.224e+05 |
| **Date:** | Tue, 05 Nov 2024 | **Prob (F-statistic):** | 0.00 |
| **Time:** | 20:12:50 | **Log-Likelihood:** | -3.7818e+05 |
| **No. Observations:** | 43148 | **AIC:** | 7.564e+05 |
| **Df Residuals:** | 43145 | **BIC:** | 7.564e+05 |
| **Df Model:** | 2 | | |
| **Covariance Type:** | nonrobust | | |

| | **coef** | **std err** | **t** | **P>\|t\|** | **[0.025** | **0.975]** |
|---|---|---|---|---|---|---|
| **Intercept** | 1864.0999 | 193.237 | 9.647 | 0.000 | 1485.351 | 2242.849 |
| **Carat** | 7822.4531 | 15.967 | 489.921 | 0.000 | 7791.158 | 7853.748 |
| **table** | -72.5117 | 3.394 | -21.364 | 0.000 | -79.164 | -65.859 |

| | | | |
|---|---|---|---|
| **Omnibus:** | 11280.854 | **Durbin-Watson:** | 2.006 |
| **Prob(Omnibus):** | 0.000 | **Jarque-Bera (JB):** | 137267.090 |
| **Skew:** | 0.913 | **Prob(JB):** | 0.00 |
| **Kurtosis:** | 11.545 | **Cond. No.** | 1.49e+03 |

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.49e+03. This might indicate that there are strong multicollinearity or other numerical problems.

$$\hat{y} = 1864.0999 + 7822.4531\beta_1 - 72.5117\beta_2$$

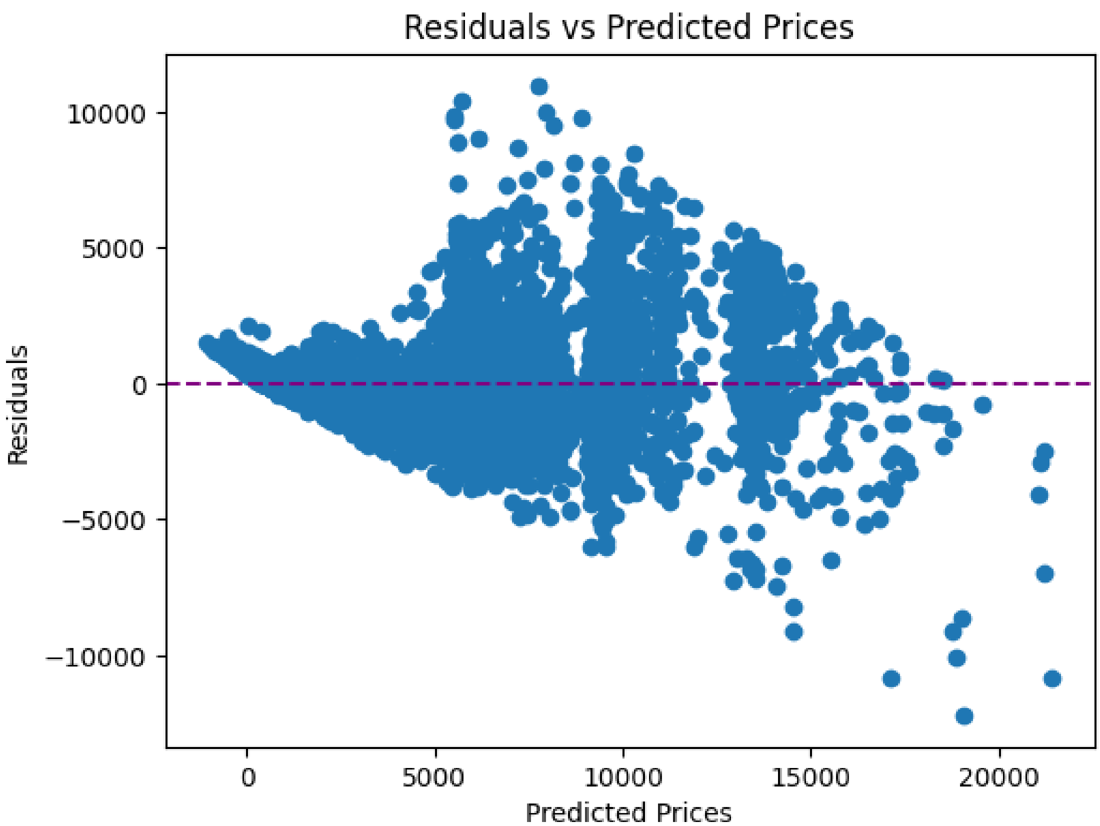
[ ]:
```
#4.12

def OSR2(y_train, y_test, y_pred):

    SSE = np.sum((y_test - y_pred)**2)
    SST = np.sum((y_test - np.mean(y_train))**2)
```

```
    return (1 - SSE/SST)
```

[ ]:
```
y_train = diamonds_train["Price"]
y_test = diamonds_test["Price"]
y_pred = model.predict(diamonds_test)

OSR2(y_train, y_test, y_pred)
```

[ ]: 0.8544465556805251

[ ]:
```
#4.13

residuals = y_test - y_pred

plt.scatter(y_pred, residuals)
plt.axhline(y=0, color="purple", linestyle="--")
plt.xlabel("Predicted Prices")
plt.ylabel("Residuals")
plt.title("Residuals vs Predicted Prices")
plt.show()
```



It seems that prices get a lot more variable as they increase, however it seems that the model tends to overprice more expensive diamonds.