Week13-1

November 25, 2024

1 Week 13-1 - RANDOM FORESTS AND BOOSTING

This lecture is comprised of 4 parts:

- 1. CART Baseline
- 2. Random Forests (basic model and cross-validation)
- 3. Gradient Boosted Trees (basic model and cross-validation)
- 4. Final Comparison

[41]:
```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

[42]:
```python
ctr = pd.read_csv("CTR.csv")
ctr.info()
ctr.head()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 6057 entries, 0 to 6056
Data columns (total 11 columns):
 #   Column         Non-Null Count  Dtype
---  ------         --------------  -----
 0   CTR            6057 non-null   float64
 1   titleWords     6057 non-null   int64
 2   adWords        6057 non-null   int64
 3   depth          6057 non-null   int64
 4   position       6057 non-null   int64
 5   advCTR         6057 non-null   float64
 6   advCTRInPos    6057 non-null   float64
 7   queryCTR       6057 non-null   float64
 8   queryCTRInPos  6057 non-null   float64
 9   gender         6057 non-null   object
 10  age            6057 non-null   object
dtypes: float64(5), int64(4), object(2)
memory usage: 520.7+ KB
```

[42]:
```
      CTR  titleWords  adWords  depth  position  advCTR  advCTRInPos  \
0  0.0000           8       17      1         1  0.0136       0.0153
1  0.0000           9       19      3         3  0.0199       0.0088
```

```
2  0.0675             6        30       2           1  0.0825        0.1002
3  0.0000             5        19       3           2  0.0116        0.0090
4  0.0000            10        22       1           1  0.0186        0.0284

   queryCTR  queryCTRInPos  gender    age
0    0.0000         0.0000    male   0-12
1    0.0394         0.0125    male  25-30
2    0.0200         0.0256  female  13-18
3    0.0042         0.0017  female  25-30
4    0.0294         0.0431  female   0-12
```

[43]:
```python
def OSR2(model, X_test, y_test, y_train):

    y_pred = model.predict(X_test)
    SSE = np.sum((y_test - y_pred)**2)
    SST = np.sum((y_test - np.mean(y_train))**2)

    return (1 - SSE/SST)
```

1.1 1. CART BASELINE

1.1.1 Decision Tree Regressor with CV

We will use a standard decision tree to establish a reference against which to evaluate the ensemble models

1.0 Train test split

[44]:
```python
from sklearn.model_selection import train_test_split

y = ctr['CTR']
X = pd.get_dummies(ctr.drop(['CTR'], axis=1))

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,␣
 ↪random_state=88)
X_train.shape, X_test.shape
```

[44]: ((4239, 18), (1818, 18))

1.1 Define the grid values and perform the Grid Search Cross-Validation

[61]:
```python
from sklearn.model_selection import GridSearchCV
from sklearn.tree import DecisionTreeRegressor
from sklearn.model_selection import KFold

grid_values = {'ccp_alpha': np.linspace(0, 0.001, 51)}

dtr = DecisionTreeRegressor(min_samples_leaf=5, min_samples_split=20,␣
 ↪random_state=88)
```

```
### Note that the line below is important. It ensures that the training data is␣
↪split into
### five folds randomly. Recall what we've seen in the discussion slides that␣
↪by default,
### GridSearchCV will split the training data without shuffling.
cv = KFold(n_splits=5,random_state=1,shuffle=True)
### by setting random_state as a fixed number, we ensure that each time the␣
↪GridSearchCV splits data, we get the
### same split.
dtr_cv = GridSearchCV(dtr, param_grid=grid_values, scoring='r2', cv=cv,␣
↪verbose=0)
dtr_cv.fit(X_train, y_train)
```

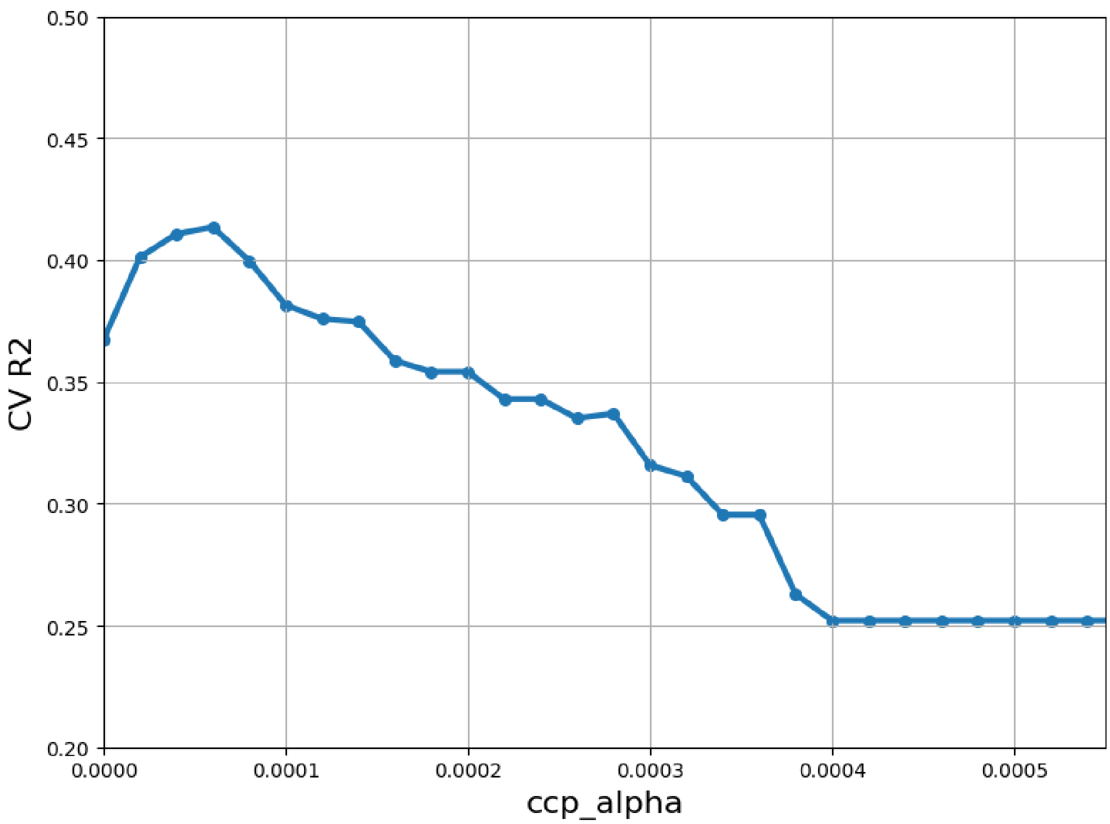
[61]:
```
GridSearchCV(cv=KFold(n_splits=5, random_state=1, shuffle=True),
             estimator=DecisionTreeRegressor(min_samples_leaf=5,
                                             min_samples_split=20,
                                             random_state=88),
             param_grid={'ccp_alpha': array([0.0e+00, 2.0e-05, 4.0e-05, 6.0e-05,
8.0e-05, 1.0e-04, 1.2e-04,
       1.4e-04, 1.6e-04, 1.8e-04, 2.0e-04, 2.2e-04, 2.4e-04, 2.6e-04,
       2.8e-04, 3.0e-04, 3.2e-04, 3.4e-04, 3.6e-04, 3.8e-04, 4.0e-04,
       4.2e-04, 4.4e-04, 4.6e-04, 4.8e-04, 5.0e-04, 5.2e-04, 5.4e-04,
       5.6e-04, 5.8e-04, 6.0e-04, 6.2e-04, 6.4e-04, 6.6e-04, 6.8e-04,
       7.0e-04, 7.2e-04, 7.4e-04, 7.6e-04, 7.8e-04, 8.0e-04, 8.2e-04,
       8.4e-04, 8.6e-04, 8.8e-04, 9.0e-04, 9.2e-04, 9.4e-04, 9.6e-04,
       9.8e-04, 1.0e-03])},
             scoring='r2')
```

1.2 Select the best hyperparameter

[7]:
```
ccp_alpha = dtr_cv.cv_results_['param_ccp_alpha'].data
R2_scores = dtr_cv.cv_results_['mean_test_score']

plt.figure(figsize=(8, 6))
plt.xlabel('ccp_alpha', fontsize=16)
plt.ylabel('CV R2', fontsize=16)
plt.scatter(ccp_alpha, R2_scores, s=30)
plt.plot(ccp_alpha, R2_scores, linewidth=3)
plt.grid(True, which='both')
plt.xlim([0, 0.00055])
plt.ylim([0.2, 0.5])

plt.tight_layout()
plt.show()
```



[8]:
```python
print('Best ccp_alpha', dtr_cv.best_params_)
```

```
Best ccp_alpha {'ccp_alpha': 6.000000000000001e-05}
```

1.3 Evaluate the model performance (trained on the entire training set)

[9]:
```python
# Model Evaluation
print('Cross-validated R2:', round(dtr_cv.best_score_, 5))
print('OSR2:', round(OSR2(dtr_cv, X_test, y_test, y_train), 5))
```

```
Cross-validated R2: 0.41349
OSR2: 0.4819
```

1.2 2. RANDOM FORESTS

1.2.1 2.1 Random Forest Regressor

[10]:
```python
from sklearn.ensemble import RandomForestRegressor

rf = RandomForestRegressor(max_features=5, min_samples_leaf=5,
                           n_estimators = 500, random_state=88, verbose=1)
# Note: you can change the verbose parameter to control how much training␣
↪progress is printed.
```

```
rf.fit(X_train, y_train)
```

```
[Parallel(n_jobs=1)]: Done  49 tasks      | elapsed:    0.2s
[Parallel(n_jobs=1)]: Done 199 tasks      | elapsed:    0.9s
[Parallel(n_jobs=1)]: Done 449 tasks      | elapsed:    2.0s
```

[10]:
```
RandomForestRegressor(max_features=5, min_samples_leaf=5, n_estimators=500,
                      random_state=88, verbose=1)
```

[11]:
```
rf.verbose = False

print('OSR2:', round(OSR2(rf, X_test, y_test, y_train), 5))
```

```
OSR2: 0.56467
```

Feature Importance

[12]:
```
pd.DataFrame({'Feature' : X_train.columns,
              'Importance score': 100*rf.feature_importances_}).round(1)
```

[12]:

	Feature	Importance score
0	titleWords	3.7
1	adWords	3.4
2	depth	2.3
3	position	2.2
4	advCTR	23.6
5	advCTRInPos	32.6
6	queryCTR	10.0
7	queryCTRInPos	17.9
8	gender_female	0.8
9	gender_male	0.5
10	gender_unknown	0.8
11	age_0-12	0.0
12	age_13-18	0.3
13	age_19-24	0.5
14	age_25-30	0.3
15	age_31-40	0.2
16	age_41+	0.0
17	age_unknown	0.8

1.2.2 2.2 Random Forest Regressor with CV

2.2.1 Define the grid values and perform the Grid Search Cross-Validation

[13]:
```
import time

grid_values = {'max_features': np.linspace(1,25,25, dtype='int32'),
               'min_samples_leaf': [5],
               'n_estimators': [40],
               'random_state': [88]}
```

```
tic = time.time()

rf2 = RandomForestRegressor()
# Note: here we set verbose=2 to keep track of the progress (the running time)␣
↪of the cross validation.
cv = KFold(n_splits=5,random_state=2333,shuffle=True)
rf_cv = GridSearchCV(rf2, param_grid=grid_values, scoring='r2', cv=cv,verbose=1)
rf_cv.fit(X_train, y_train)

toc = time.time()

print('time:', round(toc-tic, 2),'s')
```

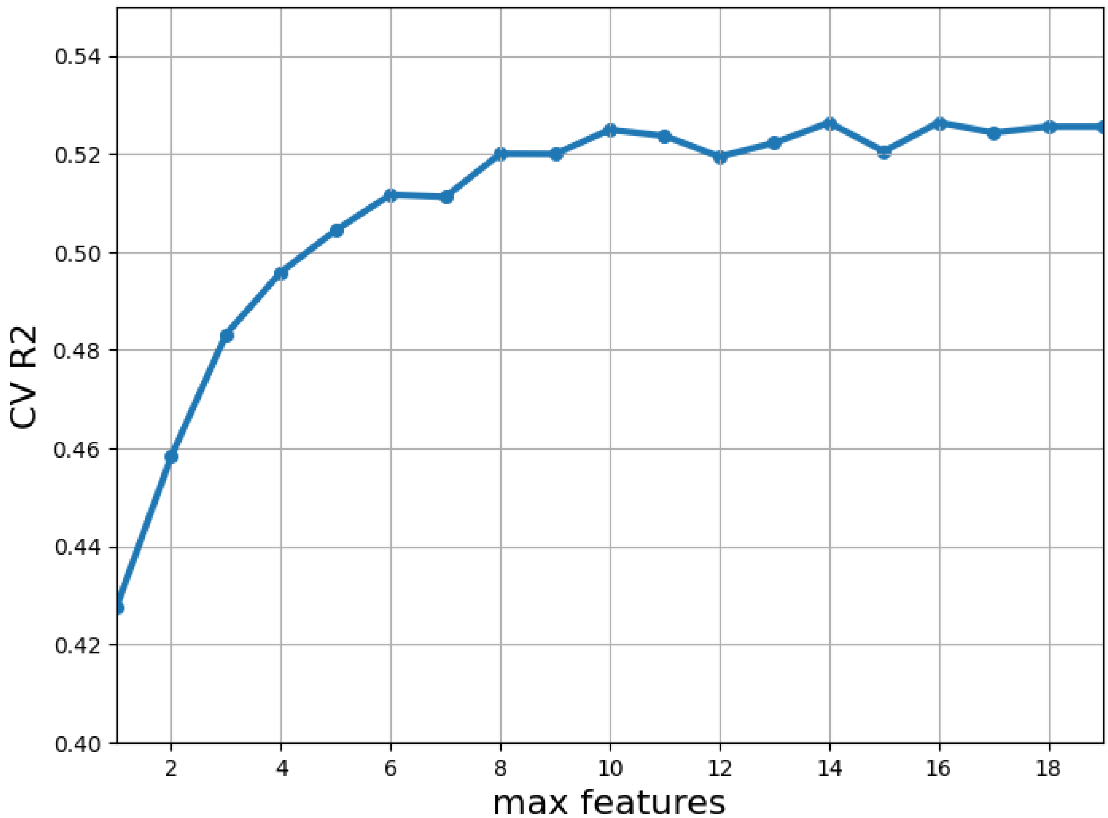
```
Fitting 5 folds for each of 25 candidates, totalling 125 fits
time: 37.07 s
```

2.2.2 Select the best hyperparameter

[14]:
```
max_features = rf_cv.cv_results_['param_max_features'].data
R2_scores = rf_cv.cv_results_['mean_test_score']

plt.figure(figsize=(8, 6))
plt.xlabel('max features', fontsize=16)
plt.ylabel('CV R2', fontsize=16)
plt.scatter(max_features, R2_scores, s=30)
plt.plot(max_features, R2_scores, linewidth=3)
plt.grid(True, which='both')
plt.xlim([1, 19])
plt.ylim([0.4, 0.55])
```

[14]:
```
(0.4, 0.55)
```



[15]:
```
print(rf_cv.best_params_)
```

```
{'max_features': 16, 'min_samples_leaf': 5, 'n_estimators': 40, 'random_state':
88}
```

2.2.3 Evaluate the model performance (trained on the entire training set)

[16]:
```
print('Cross-validated R2:', round(rf_cv.best_score_, 5))
print('OSR2:', round(OSR2(rf_cv, X_test, y_test, y_train), 5))
```

```
Cross-validated R2: 0.52633
OSR2: 0.55854
```

2.2.4 Feature Importance

[17]:
```
pd.DataFrame({'Feature' : X_train.columns,
              'Importance score': 100*rf_cv.best_estimator_.
 ↪feature_importances_}).round(1)
```

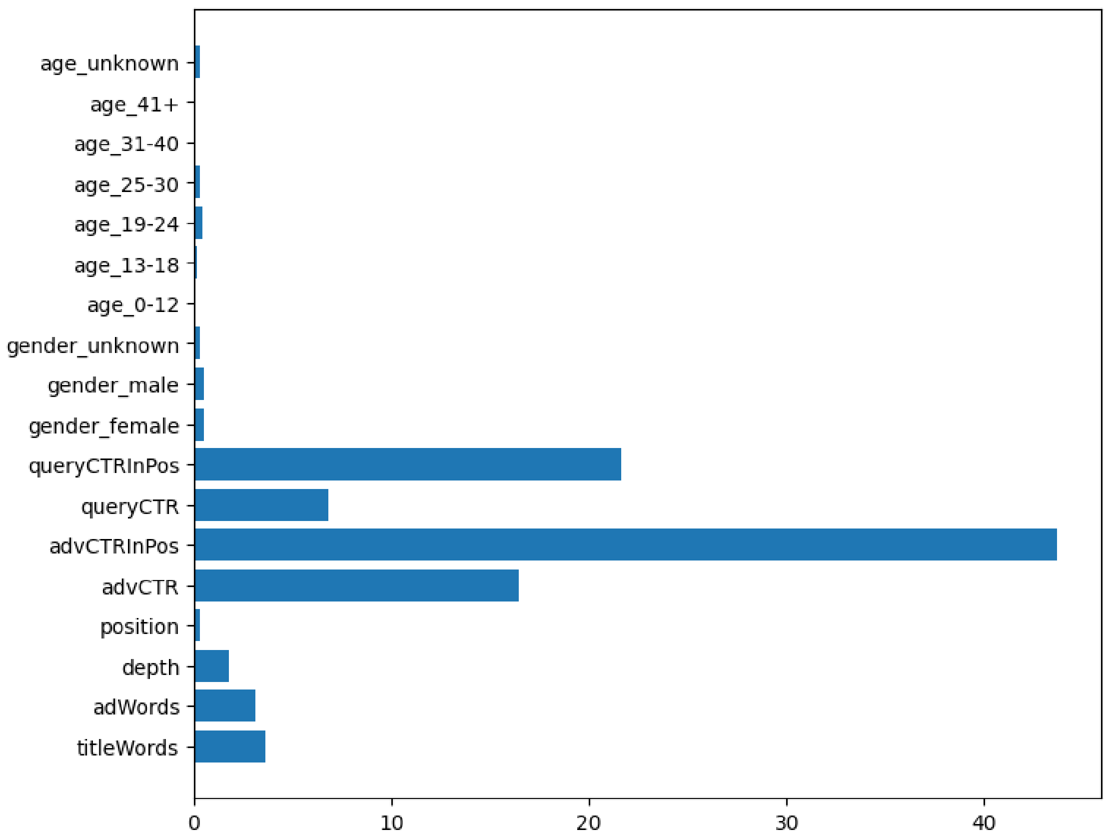
[17]:

	Feature	Importance score
0	titleWords	3.6
1	adWords	3.1
2	depth	1.8

```
3          position          0.3
4            advCTR         16.4
5       advCTRInPos         43.7
6          queryCTR          6.8
7     queryCTRInPos         21.6
8     gender_female          0.5
9       gender_male          0.5
10   gender_unknown          0.3
11          age_0-12         0.0
12         age_13-18         0.2
13         age_19-24         0.4
14         age_25-30         0.3
15         age_31-40         0.1
16           age_41+         0.0
17       age_unknown         0.3
```

[18]:

```
plt.figure(figsize=(8,7))
plt.barh(X_train.columns, 100*rf_cv.best_estimator_.feature_importances_)
plt.show()
```



1.3 3. GRADIENT BOOSTED TREES

1.3.1 3.1 Gradient Boosting Regressor

Controlling tree size using `max_leaf_nodes` vs. `max_depth`: https://scikit-learn.org/stable/modules/ensemble.html#controlling-the-tree-size

We choose use `max_leaf_nodes` as our primary parameter to control the size of the tree: we set `max_depth` to a large value so that it does not interfere with the construction of the trees.

[19]:
```
from sklearn.ensemble import GradientBoostingRegressor

gbr = GradientBoostingRegressor(n_estimators=2000, learning_rate= 0.001,␣
 ↪max_leaf_nodes=3,
                                max_depth=10, min_samples_leaf=10,␣
 ↪random_state=88, verbose=2)
gbr.fit(X_train, y_train)
```

```
      Iter       Train Loss   Remaining Time
         1           0.0060            2.21m
         2           0.0060            1.21m
         3           0.0060           51.93s
         4           0.0060           41.42s
         5           0.0060           35.11s
         6           0.0060           31.00s
         7           0.0060           27.97s
         8           0.0060           25.71s
         9           0.0060           23.96s
        10           0.0060           22.57s
        11           0.0060           21.45s
        12           0.0060           20.47s
        13           0.0060           19.65s
        14           0.0060           19.02s
        15           0.0060           18.42s
        16           0.0059           17.87s
        17           0.0059           17.41s
        18           0.0059           16.98s
        19           0.0059           16.64s
        20           0.0059           16.34s
        21           0.0059           16.04s
        22           0.0059           15.78s
        23           0.0059           15.56s
        24           0.0059           15.34s
        25           0.0059           15.15s
        26           0.0059           14.96s
        27           0.0059           14.78s
        28           0.0059           14.61s
        29           0.0059           14.46s
        30           0.0059           14.31s
```

31	0.0059	14.17s
32	0.0059	14.03s
33	0.0059	13.91s
34	0.0059	13.79s
35	0.0059	13.67s
36	0.0059	13.55s
37	0.0059	13.45s
38	0.0059	13.34s
39	0.0059	13.25s
40	0.0059	13.16s
41	0.0059	13.08s
42	0.0059	13.04s
43	0.0059	12.97s
44	0.0058	12.90s
45	0.0058	12.92s
46	0.0058	13.12s
47	0.0058	13.06s
48	0.0058	12.98s
49	0.0058	12.90s
50	0.0058	12.83s
51	0.0058	12.77s
52	0.0058	12.72s
53	0.0058	12.66s
54	0.0058	12.61s
55	0.0058	12.55s
56	0.0058	12.50s
57	0.0058	12.44s
58	0.0058	12.40s
59	0.0058	12.35s
60	0.0058	12.30s
61	0.0058	12.26s
62	0.0058	12.21s
63	0.0058	12.17s
64	0.0058	12.13s
65	0.0058	12.09s
66	0.0058	12.06s
67	0.0058	12.02s
68	0.0058	11.98s
69	0.0058	11.95s
70	0.0058	11.91s
71	0.0058	11.87s
72	0.0058	11.83s
73	0.0057	11.80s
74	0.0057	11.77s
75	0.0057	11.74s
76	0.0057	11.70s
77	0.0057	11.67s
78	0.0057	11.64s

79	0.0057	11.61s
80	0.0057	11.58s
81	0.0057	11.56s
82	0.0057	11.53s
83	0.0057	11.51s
84	0.0057	11.48s
85	0.0057	11.45s
86	0.0057	11.42s
87	0.0057	11.40s
88	0.0057	11.38s
89	0.0057	11.35s
90	0.0057	11.34s
91	0.0057	11.32s
92	0.0057	11.29s
93	0.0057	11.27s
94	0.0057	11.26s
95	0.0057	11.24s
96	0.0057	11.22s
97	0.0057	11.20s
98	0.0057	11.19s
99	0.0057	11.20s
100	0.0057	11.18s
101	0.0057	11.16s
102	0.0057	11.15s
103	0.0057	11.13s
104	0.0056	11.11s
105	0.0056	11.09s
106	0.0056	11.07s
107	0.0056	11.05s
108	0.0056	11.03s
109	0.0056	11.02s
110	0.0056	11.00s
111	0.0056	10.98s
112	0.0056	10.97s
113	0.0056	10.95s
114	0.0056	10.94s
115	0.0056	10.92s
116	0.0056	10.90s
117	0.0056	10.89s
118	0.0056	10.87s
119	0.0056	10.86s
120	0.0056	10.84s
121	0.0056	10.83s
122	0.0056	10.82s
123	0.0056	10.80s
124	0.0056	10.78s
125	0.0056	10.77s
126	0.0056	10.75s

127	0.0056	10.73s
128	0.0056	10.72s
129	0.0056	10.70s
130	0.0056	10.69s
131	0.0056	10.67s
132	0.0056	10.65s
133	0.0056	10.64s
134	0.0056	10.63s
135	0.0056	10.61s
136	0.0056	10.60s
137	0.0056	10.58s
138	0.0055	10.57s
139	0.0055	10.56s
140	0.0055	10.54s
141	0.0055	10.54s
142	0.0055	10.53s
143	0.0055	10.51s
144	0.0055	10.50s
145	0.0055	10.51s
146	0.0055	10.50s
147	0.0055	10.48s
148	0.0055	10.46s
149	0.0055	10.45s
150	0.0055	10.42s
151	0.0055	10.41s
152	0.0055	10.39s
153	0.0055	10.39s
154	0.0055	10.38s
155	0.0055	10.36s
156	0.0055	10.35s
157	0.0055	10.34s
158	0.0055	10.31s
159	0.0055	10.30s
160	0.0055	10.28s
161	0.0055	10.27s
162	0.0055	10.25s
163	0.0055	10.23s
164	0.0055	10.22s
165	0.0055	10.20s
166	0.0055	10.17s
167	0.0055	10.16s
168	0.0055	10.15s
169	0.0055	10.14s
170	0.0055	10.12s
171	0.0055	10.11s
172	0.0055	10.09s
173	0.0054	10.08s
174	0.0054	10.07s

175	0.0054	10.05s
176	0.0054	10.04s
177	0.0054	10.02s
178	0.0054	10.01s
179	0.0054	10.01s
180	0.0054	10.01s
181	0.0054	9.99s
182	0.0054	9.97s
183	0.0054	9.97s
184	0.0054	9.96s
185	0.0054	9.95s
186	0.0054	9.94s
187	0.0054	9.93s
188	0.0054	9.93s
189	0.0054	9.91s
190	0.0054	9.90s
191	0.0054	9.89s
192	0.0054	9.88s
193	0.0054	9.87s
194	0.0054	9.86s
195	0.0054	9.85s
196	0.0054	9.83s
197	0.0054	9.82s
198	0.0054	9.81s
199	0.0054	9.81s
200	0.0054	9.79s
201	0.0054	9.77s
202	0.0054	9.77s
203	0.0054	9.76s
204	0.0054	9.76s
205	0.0054	9.75s
206	0.0054	9.74s
207	0.0054	9.73s
208	0.0054	9.73s
209	0.0053	9.72s
210	0.0053	9.70s
211	0.0053	9.69s
212	0.0053	9.67s
213	0.0053	9.66s
214	0.0053	9.65s
215	0.0053	9.64s
216	0.0053	9.63s
217	0.0053	9.61s
218	0.0053	9.60s
219	0.0053	9.60s
220	0.0053	9.59s
221	0.0053	9.57s
222	0.0053	9.56s

223	0.0053	9.55s
224	0.0053	9.55s
225	0.0053	9.53s
226	0.0053	9.52s
227	0.0053	9.52s
228	0.0053	9.51s
229	0.0053	9.51s
230	0.0053	9.49s
231	0.0053	9.48s
232	0.0053	9.48s
233	0.0053	9.46s
234	0.0053	9.45s
235	0.0053	9.45s
236	0.0053	9.44s
237	0.0053	9.42s
238	0.0053	9.41s
239	0.0053	9.41s
240	0.0053	9.40s
241	0.0053	9.39s
242	0.0053	9.38s
243	0.0053	9.37s
244	0.0053	9.37s
245	0.0053	9.35s
246	0.0053	9.34s
247	0.0053	9.36s
248	0.0052	9.37s
249	0.0052	9.38s
250	0.0052	9.37s
251	0.0052	9.36s
252	0.0052	9.36s
253	0.0052	9.34s
254	0.0052	9.34s
255	0.0052	9.33s
256	0.0052	9.32s
257	0.0052	9.31s
258	0.0052	9.30s
259	0.0052	9.29s
260	0.0052	9.29s
261	0.0052	9.27s
262	0.0052	9.26s
263	0.0052	9.26s
264	0.0052	9.25s
265	0.0052	9.24s
266	0.0052	9.23s
267	0.0052	9.22s
268	0.0052	9.21s
269	0.0052	9.21s
270	0.0052	9.19s

271	0.0052	9.18s
272	0.0052	9.18s
273	0.0052	9.17s
274	0.0052	9.17s
275	0.0052	9.15s
276	0.0052	9.15s
277	0.0052	9.14s
278	0.0052	9.13s
279	0.0052	9.12s
280	0.0052	9.11s
281	0.0052	9.10s
282	0.0052	9.09s
283	0.0052	9.09s
284	0.0052	9.08s
285	0.0052	9.08s
286	0.0052	9.07s
287	0.0052	9.06s
288	0.0052	9.05s
289	0.0051	9.04s
290	0.0051	9.03s
291	0.0051	9.03s
292	0.0051	9.02s
293	0.0051	9.01s
294	0.0051	9.00s
295	0.0051	8.99s
296	0.0051	8.99s
297	0.0051	8.98s
298	0.0051	8.97s
299	0.0051	8.96s
300	0.0051	8.96s
301	0.0051	8.95s
302	0.0051	8.94s
303	0.0051	8.93s
304	0.0051	8.92s
305	0.0051	8.91s
306	0.0051	8.91s
307	0.0051	8.90s
308	0.0051	8.89s
309	0.0051	8.88s
310	0.0051	8.87s
311	0.0051	8.87s
312	0.0051	8.86s
313	0.0051	8.86s
314	0.0051	8.84s
315	0.0051	8.84s
316	0.0051	8.83s
317	0.0051	8.82s
318	0.0051	8.81s

319	0.0051	8.80s
320	0.0051	8.80s
321	0.0051	8.79s
322	0.0051	8.78s
323	0.0051	8.78s
324	0.0051	8.76s
325	0.0051	8.76s
326	0.0051	8.76s
327	0.0051	8.76s
328	0.0051	8.76s
329	0.0051	8.76s
330	0.0051	8.75s
331	0.0051	8.75s
332	0.0051	8.74s
333	0.0050	8.73s
334	0.0050	8.73s
335	0.0050	8.72s
336	0.0050	8.71s
337	0.0050	8.71s
338	0.0050	8.71s
339	0.0050	8.71s
340	0.0050	8.71s
341	0.0050	8.70s
342	0.0050	8.71s
343	0.0050	8.72s
344	0.0050	8.72s
345	0.0050	8.72s
346	0.0050	8.72s
347	0.0050	8.72s
348	0.0050	8.71s
349	0.0050	8.71s
350	0.0050	8.71s
351	0.0050	8.71s
352	0.0050	8.70s
353	0.0050	8.69s
354	0.0050	8.68s
355	0.0050	8.68s
356	0.0050	8.67s
357	0.0050	8.66s
358	0.0050	8.66s
359	0.0050	8.65s
360	0.0050	8.64s
361	0.0050	8.64s
362	0.0050	8.63s
363	0.0050	8.62s
364	0.0050	8.61s
365	0.0050	8.61s
366	0.0050	8.60s

367	0.0050	8.60s
368	0.0050	8.59s
369	0.0050	8.58s
370	0.0050	8.58s
371	0.0050	8.57s
372	0.0050	8.56s
373	0.0050	8.56s
374	0.0050	8.55s
375	0.0050	8.55s
376	0.0050	8.54s
377	0.0050	8.53s
378	0.0050	8.53s
379	0.0050	8.52s
380	0.0049	8.53s
381	0.0049	8.53s
382	0.0049	8.52s
383	0.0049	8.51s
384	0.0049	8.50s
385	0.0049	8.49s
386	0.0049	8.49s
387	0.0049	8.48s
388	0.0049	8.48s
389	0.0049	8.47s
390	0.0049	8.46s
391	0.0049	8.45s
392	0.0049	8.45s
393	0.0049	8.44s
394	0.0049	8.45s
395	0.0049	8.45s
396	0.0049	8.44s
397	0.0049	8.43s
398	0.0049	8.44s
399	0.0049	8.43s
400	0.0049	8.43s
401	0.0049	8.42s
402	0.0049	8.41s
403	0.0049	8.41s
404	0.0049	8.40s
405	0.0049	8.39s
406	0.0049	8.39s
407	0.0049	8.38s
408	0.0049	8.37s
409	0.0049	8.36s
410	0.0049	8.35s
411	0.0049	8.35s
412	0.0049	8.34s
413	0.0049	8.33s
414	0.0049	8.33s

415	0.0049	8.32s
416	0.0049	8.31s
417	0.0049	8.31s
418	0.0049	8.30s
419	0.0049	8.29s
420	0.0049	8.29s
421	0.0049	8.28s
422	0.0049	8.27s
423	0.0049	8.27s
424	0.0049	8.26s
425	0.0049	8.25s
426	0.0049	8.24s
427	0.0049	8.24s
428	0.0049	8.23s
429	0.0049	8.23s
430	0.0049	8.22s
431	0.0048	8.22s
432	0.0048	8.21s
433	0.0048	8.20s
434	0.0048	8.20s
435	0.0048	8.19s
436	0.0048	8.18s
437	0.0048	8.18s
438	0.0048	8.17s
439	0.0048	8.16s
440	0.0048	8.16s
441	0.0048	8.15s
442	0.0048	8.14s
443	0.0048	8.14s
444	0.0048	8.13s
445	0.0048	8.12s
446	0.0048	8.12s
447	0.0048	8.11s
448	0.0048	8.10s
449	0.0048	8.10s
450	0.0048	8.09s
451	0.0048	8.08s
452	0.0048	8.08s
453	0.0048	8.07s
454	0.0048	8.06s
455	0.0048	8.05s
456	0.0048	8.05s
457	0.0048	8.04s
458	0.0048	8.03s
459	0.0048	8.03s
460	0.0048	8.02s
461	0.0048	8.01s
462	0.0048	8.00s

463	0.0048	8.00s
464	0.0048	7.99s
465	0.0048	7.99s
466	0.0048	7.98s
467	0.0048	7.97s
468	0.0048	7.96s
469	0.0048	7.96s
470	0.0048	7.95s
471	0.0048	7.95s
472	0.0048	7.94s
473	0.0048	7.93s
474	0.0048	7.92s
475	0.0048	7.92s
476	0.0048	7.91s
477	0.0048	7.90s
478	0.0048	7.90s
479	0.0048	7.89s
480	0.0048	7.88s
481	0.0048	7.88s
482	0.0048	7.87s
483	0.0048	7.87s
484	0.0048	7.86s
485	0.0048	7.85s
486	0.0047	7.85s
487	0.0047	7.84s
488	0.0047	7.83s
489	0.0047	7.83s
490	0.0047	7.82s
491	0.0047	7.81s
492	0.0047	7.81s
493	0.0047	7.80s
494	0.0047	7.79s
495	0.0047	7.78s
496	0.0047	7.78s
497	0.0047	7.77s
498	0.0047	7.77s
499	0.0047	7.76s
500	0.0047	7.76s
501	0.0047	7.75s
502	0.0047	7.74s
503	0.0047	7.74s
504	0.0047	7.73s
505	0.0047	7.73s
506	0.0047	7.72s
507	0.0047	7.71s
508	0.0047	7.71s
509	0.0047	7.70s
510	0.0047	7.69s

511	0.0047	7.69s
512	0.0047	7.68s
513	0.0047	7.67s
514	0.0047	7.67s
515	0.0047	7.67s
516	0.0047	7.66s
517	0.0047	7.65s
518	0.0047	7.64s
519	0.0047	7.64s
520	0.0047	7.63s
521	0.0047	7.62s
522	0.0047	7.61s
523	0.0047	7.61s
524	0.0047	7.60s
525	0.0047	7.60s
526	0.0047	7.60s
527	0.0047	7.59s
528	0.0047	7.58s
529	0.0047	7.58s
530	0.0047	7.57s
531	0.0047	7.56s
532	0.0047	7.55s
533	0.0047	7.55s
534	0.0047	7.54s
535	0.0047	7.53s
536	0.0047	7.53s
537	0.0047	7.52s
538	0.0047	7.52s
539	0.0047	7.51s
540	0.0047	7.51s
541	0.0047	7.50s
542	0.0047	7.49s
543	0.0047	7.48s
544	0.0047	7.48s
545	0.0047	7.47s
546	0.0046	7.47s
547	0.0046	7.46s
548	0.0046	7.46s
549	0.0046	7.45s
550	0.0046	7.44s
551	0.0046	7.44s
552	0.0046	7.43s
553	0.0046	7.42s
554	0.0046	7.41s
555	0.0046	7.41s
556	0.0046	7.40s
557	0.0046	7.40s
558	0.0046	7.39s

559	0.0046	7.38s
560	0.0046	7.38s
561	0.0046	7.37s
562	0.0046	7.37s
563	0.0046	7.36s
564	0.0046	7.35s
565	0.0046	7.34s
566	0.0046	7.34s
567	0.0046	7.33s
568	0.0046	7.32s
569	0.0046	7.32s
570	0.0046	7.31s
571	0.0046	7.30s
572	0.0046	7.30s
573	0.0046	7.29s
574	0.0046	7.28s
575	0.0046	7.28s
576	0.0046	7.27s
577	0.0046	7.26s
578	0.0046	7.26s
579	0.0046	7.25s
580	0.0046	7.24s
581	0.0046	7.24s
582	0.0046	7.23s
583	0.0046	7.22s
584	0.0046	7.22s
585	0.0046	7.21s
586	0.0046	7.20s
587	0.0046	7.20s
588	0.0046	7.19s
589	0.0046	7.18s
590	0.0046	7.17s
591	0.0046	7.17s
592	0.0046	7.16s
593	0.0046	7.15s
594	0.0046	7.15s
595	0.0046	7.14s
596	0.0046	7.13s
597	0.0046	7.13s
598	0.0046	7.12s
599	0.0046	7.11s
600	0.0046	7.11s
601	0.0046	7.10s
602	0.0046	7.10s
603	0.0046	7.09s
604	0.0046	7.08s
605	0.0046	7.08s
606	0.0046	7.08s

607	0.0046	7.07s
608	0.0046	7.06s
609	0.0046	7.06s
610	0.0046	7.05s
611	0.0045	7.04s
612	0.0045	7.04s
613	0.0045	7.03s
614	0.0045	7.02s
615	0.0045	7.02s
616	0.0045	7.01s
617	0.0045	7.00s
618	0.0045	7.00s
619	0.0045	6.99s
620	0.0045	6.98s
621	0.0045	6.98s
622	0.0045	6.97s
623	0.0045	6.96s
624	0.0045	6.96s
625	0.0045	6.95s
626	0.0045	6.94s
627	0.0045	6.94s
628	0.0045	6.93s
629	0.0045	6.92s
630	0.0045	6.92s
631	0.0045	6.91s
632	0.0045	6.91s
633	0.0045	6.90s
634	0.0045	6.89s
635	0.0045	6.89s
636	0.0045	6.88s
637	0.0045	6.87s
638	0.0045	6.87s
639	0.0045	6.86s
640	0.0045	6.86s
641	0.0045	6.85s
642	0.0045	6.84s
643	0.0045	6.84s
644	0.0045	6.83s
645	0.0045	6.82s
646	0.0045	6.82s
647	0.0045	6.81s
648	0.0045	6.81s
649	0.0045	6.80s
650	0.0045	6.79s
651	0.0045	6.78s
652	0.0045	6.78s
653	0.0045	6.77s
654	0.0045	6.76s

655	0.0045	6.76s
656	0.0045	6.75s
657	0.0045	6.75s
658	0.0045	6.74s
659	0.0045	6.73s
660	0.0045	6.73s
661	0.0045	6.72s
662	0.0045	6.72s
663	0.0045	6.71s
664	0.0045	6.70s
665	0.0045	6.70s
666	0.0045	6.69s
667	0.0045	6.69s
668	0.0045	6.68s
669	0.0045	6.67s
670	0.0045	6.67s
671	0.0045	6.66s
672	0.0045	6.65s
673	0.0045	6.65s
674	0.0045	6.64s
675	0.0045	6.64s
676	0.0045	6.63s
677	0.0045	6.62s
678	0.0045	6.62s
679	0.0045	6.61s
680	0.0045	6.60s
681	0.0045	6.60s
682	0.0045	6.59s
683	0.0044	6.59s
684	0.0044	6.58s
685	0.0044	6.58s
686	0.0044	6.57s
687	0.0044	6.56s
688	0.0044	6.56s
689	0.0044	6.55s
690	0.0044	6.55s
691	0.0044	6.54s
692	0.0044	6.53s
693	0.0044	6.53s
694	0.0044	6.52s
695	0.0044	6.52s
696	0.0044	6.51s
697	0.0044	6.50s
698	0.0044	6.50s
699	0.0044	6.49s
700	0.0044	6.49s
701	0.0044	6.48s
702	0.0044	6.47s

703	0.0044	6.47s
704	0.0044	6.46s
705	0.0044	6.46s
706	0.0044	6.45s
707	0.0044	6.45s
708	0.0044	6.44s
709	0.0044	6.44s
710	0.0044	6.43s
711	0.0044	6.42s
712	0.0044	6.42s
713	0.0044	6.41s
714	0.0044	6.40s
715	0.0044	6.40s
716	0.0044	6.39s
717	0.0044	6.39s
718	0.0044	6.38s
719	0.0044	6.38s
720	0.0044	6.37s
721	0.0044	6.37s
722	0.0044	6.36s
723	0.0044	6.36s
724	0.0044	6.35s
725	0.0044	6.35s
726	0.0044	6.34s
727	0.0044	6.33s
728	0.0044	6.33s
729	0.0044	6.32s
730	0.0044	6.32s
731	0.0044	6.31s
732	0.0044	6.31s
733	0.0044	6.30s
734	0.0044	6.30s
735	0.0044	6.29s
736	0.0044	6.28s
737	0.0044	6.28s
738	0.0044	6.27s
739	0.0044	6.27s
740	0.0044	6.26s
741	0.0044	6.26s
742	0.0044	6.25s
743	0.0044	6.25s
744	0.0044	6.24s
745	0.0044	6.24s
746	0.0044	6.23s
747	0.0044	6.22s
748	0.0044	6.22s
749	0.0044	6.21s
750	0.0044	6.21s

751	0.0044	6.20s
752	0.0044	6.20s
753	0.0044	6.19s
754	0.0044	6.19s
755	0.0044	6.18s
756	0.0044	6.18s
757	0.0044	6.17s
758	0.0044	6.17s
759	0.0044	6.16s
760	0.0044	6.15s
761	0.0044	6.15s
762	0.0044	6.14s
763	0.0044	6.14s
764	0.0043	6.13s
765	0.0043	6.13s
766	0.0043	6.12s
767	0.0043	6.12s
768	0.0043	6.11s
769	0.0043	6.11s
770	0.0043	6.10s
771	0.0043	6.10s
772	0.0043	6.09s
773	0.0043	6.08s
774	0.0043	6.08s
775	0.0043	6.07s
776	0.0043	6.07s
777	0.0043	6.06s
778	0.0043	6.06s
779	0.0043	6.05s
780	0.0043	6.04s
781	0.0043	6.04s
782	0.0043	6.03s
783	0.0043	6.03s
784	0.0043	6.02s
785	0.0043	6.02s
786	0.0043	6.01s
787	0.0043	6.01s
788	0.0043	6.00s
789	0.0043	6.00s
790	0.0043	5.99s
791	0.0043	5.98s
792	0.0043	5.98s
793	0.0043	5.97s
794	0.0043	5.97s
795	0.0043	5.96s
796	0.0043	5.96s
797	0.0043	5.95s
798	0.0043	5.94s

799	0.0043	5.94s
800	0.0043	5.93s
801	0.0043	5.93s
802	0.0043	5.92s
803	0.0043	5.92s
804	0.0043	5.91s
805	0.0043	5.91s
806	0.0043	5.90s
807	0.0043	5.89s
808	0.0043	5.89s
809	0.0043	5.88s
810	0.0043	5.88s
811	0.0043	5.87s
812	0.0043	5.87s
813	0.0043	5.86s
814	0.0043	5.86s
815	0.0043	5.85s
816	0.0043	5.84s
817	0.0043	5.84s
818	0.0043	5.83s
819	0.0043	5.83s
820	0.0043	5.82s
821	0.0043	5.82s
822	0.0043	5.81s
823	0.0043	5.81s
824	0.0043	5.80s
825	0.0043	5.80s
826	0.0043	5.79s
827	0.0043	5.78s
828	0.0043	5.78s
829	0.0043	5.77s
830	0.0043	5.77s
831	0.0043	5.76s
832	0.0043	5.76s
833	0.0043	5.75s
834	0.0043	5.75s
835	0.0043	5.74s
836	0.0043	5.74s
837	0.0043	5.73s
838	0.0043	5.73s
839	0.0043	5.72s
840	0.0043	5.71s
841	0.0043	5.71s
842	0.0043	5.70s
843	0.0043	5.70s
844	0.0043	5.69s
845	0.0043	5.69s
846	0.0043	5.68s

847	0.0043	5.68s
848	0.0043	5.67s
849	0.0043	5.66s
850	0.0043	5.66s
851	0.0043	5.65s
852	0.0043	5.65s
853	0.0042	5.64s
854	0.0042	5.64s
855	0.0042	5.63s
856	0.0042	5.63s
857	0.0042	5.62s
858	0.0042	5.62s
859	0.0042	5.61s
860	0.0042	5.61s
861	0.0042	5.60s
862	0.0042	5.60s
863	0.0042	5.59s
864	0.0042	5.58s
865	0.0042	5.58s
866	0.0042	5.57s
867	0.0042	5.57s
868	0.0042	5.56s
869	0.0042	5.56s
870	0.0042	5.55s
871	0.0042	5.55s
872	0.0042	5.54s
873	0.0042	5.54s
874	0.0042	5.53s
875	0.0042	5.53s
876	0.0042	5.52s
877	0.0042	5.51s
878	0.0042	5.51s
879	0.0042	5.50s
880	0.0042	5.50s
881	0.0042	5.49s
882	0.0042	5.49s
883	0.0042	5.48s
884	0.0042	5.48s
885	0.0042	5.47s
886	0.0042	5.47s
887	0.0042	5.46s
888	0.0042	5.46s
889	0.0042	5.45s
890	0.0042	5.45s
891	0.0042	5.44s
892	0.0042	5.44s
893	0.0042	5.43s
894	0.0042	5.43s

895	0.0042	5.42s
896	0.0042	5.42s
897	0.0042	5.41s
898	0.0042	5.41s
899	0.0042	5.40s
900	0.0042	5.40s
901	0.0042	5.39s
902	0.0042	5.38s
903	0.0042	5.38s
904	0.0042	5.37s
905	0.0042	5.37s
906	0.0042	5.36s
907	0.0042	5.36s
908	0.0042	5.35s
909	0.0042	5.35s
910	0.0042	5.34s
911	0.0042	5.34s
912	0.0042	5.33s
913	0.0042	5.33s
914	0.0042	5.32s
915	0.0042	5.32s
916	0.0042	5.31s
917	0.0042	5.31s
918	0.0042	5.30s
919	0.0042	5.30s
920	0.0042	5.29s
921	0.0042	5.29s
922	0.0042	5.28s
923	0.0042	5.28s
924	0.0042	5.27s
925	0.0042	5.27s
926	0.0042	5.26s
927	0.0042	5.26s
928	0.0042	5.25s
929	0.0042	5.25s
930	0.0042	5.24s
931	0.0042	5.24s
932	0.0042	5.23s
933	0.0042	5.23s
934	0.0042	5.22s
935	0.0042	5.22s
936	0.0042	5.21s
937	0.0042	5.21s
938	0.0042	5.20s
939	0.0042	5.20s
940	0.0042	5.19s
941	0.0042	5.19s
942	0.0042	5.18s

943	0.0042	5.17s
944	0.0042	5.17s
945	0.0042	5.16s
946	0.0042	5.16s
947	0.0042	5.15s
948	0.0042	5.15s
949	0.0042	5.14s
950	0.0042	5.14s
951	0.0042	5.13s
952	0.0042	5.13s
953	0.0042	5.12s
954	0.0041	5.12s
955	0.0041	5.11s
956	0.0041	5.11s
957	0.0041	5.10s
958	0.0041	5.10s
959	0.0041	5.09s
960	0.0041	5.09s
961	0.0041	5.08s
962	0.0041	5.08s
963	0.0041	5.08s
964	0.0041	5.07s
965	0.0041	5.06s
966	0.0041	5.06s
967	0.0041	5.05s
968	0.0041	5.05s
969	0.0041	5.04s
970	0.0041	5.04s
971	0.0041	5.04s
972	0.0041	5.03s
973	0.0041	5.02s
974	0.0041	5.02s
975	0.0041	5.01s
976	0.0041	5.01s
977	0.0041	5.00s
978	0.0041	5.00s
979	0.0041	4.99s
980	0.0041	4.99s
981	0.0041	4.98s
982	0.0041	4.98s
983	0.0041	4.97s
984	0.0041	4.97s
985	0.0041	4.96s
986	0.0041	4.96s
987	0.0041	4.95s
988	0.0041	4.95s
989	0.0041	4.94s
990	0.0041	4.94s

991	0.0041	4.93s
992	0.0041	4.93s
993	0.0041	4.92s
994	0.0041	4.92s
995	0.0041	4.92s
996	0.0041	4.91s
997	0.0041	4.90s
998	0.0041	4.90s
999	0.0041	4.89s
1000	0.0041	4.89s
1001	0.0041	4.89s
1002	0.0041	4.88s
1003	0.0041	4.88s
1004	0.0041	4.87s
1005	0.0041	4.87s
1006	0.0041	4.86s
1007	0.0041	4.86s
1008	0.0041	4.85s
1009	0.0041	4.85s
1010	0.0041	4.84s
1011	0.0041	4.83s
1012	0.0041	4.83s
1013	0.0041	4.82s
1014	0.0041	4.82s
1015	0.0041	4.81s
1016	0.0041	4.81s
1017	0.0041	4.80s
1018	0.0041	4.80s
1019	0.0041	4.79s
1020	0.0041	4.79s
1021	0.0041	4.78s
1022	0.0041	4.78s
1023	0.0041	4.77s
1024	0.0041	4.77s
1025	0.0041	4.76s
1026	0.0041	4.76s
1027	0.0041	4.75s
1028	0.0041	4.75s
1029	0.0041	4.74s
1030	0.0041	4.74s
1031	0.0041	4.73s
1032	0.0041	4.73s
1033	0.0041	4.72s
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1037	0.0041	4.70s
1038	0.0041	4.70s

1039	0.0041	4.69s
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1047	0.0041	4.65s
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1051	0.0041	4.63s
1052	0.0041	4.63s
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1056	0.0041	4.61s
1057	0.0041	4.60s
1058	0.0041	4.60s
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1063	0.0041	4.57s
1064	0.0041	4.57s
1065	0.0040	4.56s
1066	0.0040	4.56s
1067	0.0040	4.55s
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1069	0.0040	4.54s
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1073	0.0040	4.52s
1074	0.0040	4.52s
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1076	0.0040	4.51s
1077	0.0040	4.50s
1078	0.0040	4.50s
1079	0.0040	4.49s
1080	0.0040	4.49s
1081	0.0040	4.48s
1082	0.0040	4.48s
1083	0.0040	4.47s
1084	0.0040	4.47s
1085	0.0040	4.46s
1086	0.0040	4.46s

1087	0.0040	4.45s
1088	0.0040	4.45s
1089	0.0040	4.44s
1090	0.0040	4.44s
1091	0.0040	4.43s
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1093	0.0040	4.42s
1094	0.0040	4.42s
1095	0.0040	4.41s
1096	0.0040	4.41s
1097	0.0040	4.40s
1098	0.0040	4.40s
1099	0.0040	4.39s
1100	0.0040	4.39s
1101	0.0040	4.38s
1102	0.0040	4.38s
1103	0.0040	4.37s
1104	0.0040	4.37s
1105	0.0040	4.36s
1106	0.0040	4.36s
1107	0.0040	4.35s
1108	0.0040	4.35s
1109	0.0040	4.34s
1110	0.0040	4.34s
1111	0.0040	4.33s
1112	0.0040	4.33s
1113	0.0040	4.32s
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1119	0.0040	4.29s
1120	0.0040	4.29s
1121	0.0040	4.28s
1122	0.0040	4.28s
1123	0.0040	4.27s
1124	0.0040	4.27s
1125	0.0040	4.26s
1126	0.0040	4.26s
1127	0.0040	4.25s
1128	0.0040	4.25s
1129	0.0040	4.24s
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1134	0.0040	4.22s

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1171	0.0040	4.04s
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1175	0.0040	4.02s
1176	0.0040	4.01s
1177	0.0040	4.01s
1178	0.0040	4.00s
1179	0.0040	4.00s
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1181	0.0040	3.99s
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1189	0.0040	3.95s
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1191	0.0039	3.94s
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1276	0.0039	3.53s
1277	0.0039	3.52s
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1298	0.0039	3.42s
1299	0.0039	3.42s
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1301	0.0039	3.41s
1302	0.0039	3.40s
1303	0.0039	3.40s
1304	0.0039	3.39s
1305	0.0039	3.39s
1306	0.0039	3.38s
1307	0.0039	3.38s
1308	0.0039	3.37s
1309	0.0039	3.37s
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1363	0.0038	3.11s
1364	0.0038	3.10s
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1367	0.0038	3.09s
1368	0.0038	3.08s
1369	0.0038	3.08s
1370	0.0038	3.07s
1371	0.0038	3.07s
1372	0.0038	3.06s
1373	0.0038	3.06s
1374	0.0038	3.05s

1375	0.0038	3.05s
1376	0.0038	3.04s
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1397	0.0038	2.94s
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1435	0.0038	2.76s
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1437	0.0038	2.75s
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1447	0.0038	2.70s
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1460	0.0038	2.64s
1461	0.0038	2.63s
1462	0.0038	2.63s
1463	0.0038	2.62s
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1469	0.0038	2.60s
1470	0.0038	2.59s

1471	0.0038	2.59s
1472	0.0038	2.58s
1473	0.0038	2.58s
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1499	0.0038	2.45s
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1501	0.0038	2.44s
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1542	0.0037	2.24s
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1544	0.0037	2.23s
1545	0.0037	2.22s
1546	0.0037	2.22s
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1989	0.0035	0.05s
1990	0.0035	0.05s
1991	0.0035	0.04s
1992	0.0035	0.04s
1993	0.0035	0.03s
1994	0.0035	0.03s
1995	0.0035	0.02s
1996	0.0035	0.02s
1997	0.0035	0.01s
1998	0.0035	0.01s

```
1999            0.0035             0.00s
2000            0.0035             0.00s
```

[19]:
```
GradientBoostingRegressor(learning_rate=0.001, max_depth=10, max_leaf_nodes=3,
                          min_samples_leaf=10, n_estimators=2000,
                          random_state=88, verbose=2)
```

[20]:
```python
print('OSR2:', round(OSR2(gbr, X_test, y_test, y_train), 5))
```

```
OSR2: 0.4388
```

Note that the OSR2 of the above gbr model is not very good. It is because the model is quite sensitive to hypterparameters. We will learn how to find the best parameters using cross-validation and see the improvement one could get from choosing the best hyperparameters.

1.3.2 3.2 Gradient Boosting Regressor with CV

Note that if you use the `GridSearchCV` function directly with the gbr as the classifier, the run-time is super long (e.g., more than 30 hours). The reason is that the naive implement of `GridSearchCV` does not take into account the special additive nature of the Boosting models.

For example, if you need to decide what is the best number of trees to include in your model and you would like to try values from 1 to 10000. The naive implementation of `GridSearchCV` would train 10000 different gbr models. However, a much more efficient way is to train a model with 10000 trees, and only subset a subset of them in your model when needed. This trick allows us to reduce the training time of gbr_cv to less than 2 hours.

If you use the `caret` package in `R`, they have implemented this efficient algorithm, but there is not a good counterpart in `Python sklearn`.

However, we change these values to a small subset to reduce the run-time.

1.1 Split the training data into 5 folds for cross validation

[21]:
```python
grid_values = {'n_estimators': np.arange(3000, 7000, 20)}

gbr = GradientBoostingRegressor(min_samples_leaf=5, n_estimators=120,␣
 ↪min_samples_split=20, random_state=88)

cv = KFold(n_splits=5,random_state=1,shuffle=True)

gbr_cv = GridSearchCV(gbr, param_grid=grid_values, scoring='r2', cv=cv,␣
 ↪verbose=1)
gbr_cv.fit(X_train, y_train)
```

```
Fitting 5 folds for each of 200 candidates, totalling 1000 fits
```

```
/Users/ivynangalia/Library/Python/3.12/lib/python/site-
packages/numpy/ma/core.py:2820: RuntimeWarning: invalid value encountered in
cast
  _data = np.array(data, dtype=dtype, copy=copy,
```

[21]:
```
GridSearchCV(cv=KFold(n_splits=5, random_state=1, shuffle=True),
             estimator=GradientBoostingRegressor(min_samples_leaf=5,
                                                 min_samples_split=20,
                                                 n_estimators=120,
                                                 random_state=88),
             param_grid={'n_estimators': array([3000, 3020, 3040, 3060, 3080,
3100, 3120, 3140, 3160, 3180, 3200,
       3220, 3240, 3260, 3280, 3300, 3320, 3340, 3360, 3380, 3400, 3420,
       3440, 3460, 3480, 3500, 3520, 3540, 3560…
       5640, 5660, 5680, 5700, 5720, 5740, 5760, 5780, 5800, 5820, 5840,
       5860, 5880, 5900, 5920, 5940, 5960, 5980, 6000, 6020, 6040, 6060,
       6080, 6100, 6120, 6140, 6160, 6180, 6200, 6220, 6240, 6260, 6280,
       6300, 6320, 6340, 6360, 6380, 6400, 6420, 6440, 6460, 6480, 6500,
       6520, 6540, 6560, 6580, 6600, 6620, 6640, 6660, 6680, 6700, 6720,
       6740, 6760, 6780, 6800, 6820, 6840, 6860, 6880, 6900, 6920, 6940,
       6960, 6980])},
             scoring='r2', verbose=1)
```

[ ]:
```

```

1.6 Train the model with the best hyperparameters on the full training data.

[58]:
```
gbr_cv = GradientBoostingRegressor(n_estimators = 5000, learning_rate = 0.005,␣
 ↪max_depth = 20,
                                   max_leaf_nodes=7,␣
 ↪min_samples_leaf=10,random_state=99,verbose=1)
gbr_cv.fit(X_train,y_train)
```

```
      Iter       Train Loss   Remaining Time
         1           0.0060            1.31m
         2           0.0060           54.35s
         3           0.0059           45.81s
         4           0.0059           41.34s
         5           0.0059           38.82s
         6           0.0059           36.53s
         7           0.0058           34.81s
         8           0.0058           33.55s
         9           0.0058           32.57s
        10           0.0058           31.54s
        20           0.0055           26.63s
        30           0.0053           24.98s
        40           0.0051           23.99s
        50           0.0050           24.07s
        60           0.0048           23.90s
        70           0.0047           23.70s
        80           0.0045           23.77s
        90           0.0044           23.73s
       100           0.0043           23.67s
```

```
     200           0.0035           24.50s
     300           0.0031           24.04s
     400           0.0029           23.41s
     500           0.0028           23.11s
     600           0.0026           22.95s
     700           0.0026           22.15s
     800           0.0025           21.45s
     900           0.0024           20.99s
    1000           0.0024           20.61s
    2000           0.0021           15.26s
    3000           0.0019           10.00s
    4000           0.0017            4.96s
    5000           0.0016            0.00s
```

[58]:
```
GradientBoostingRegressor(learning_rate=0.005, max_depth=20, max_leaf_nodes=7,
                          min_samples_leaf=10, n_estimators=5000,
                          random_state=99, verbose=1)
```

1.7 Evaluate the full model

[54]:
```python
print('OSR2:', round(OSR2(gbr_cv, X_test, y_test, y_train), 5))
```

```
OSR2: 0.53653
```

[55]:
```python
pd.DataFrame({'Feature' : X_train.columns,
              'Importance score': 100*gbr_cv.feature_importances_}).round(1)
```

[55]:

	Feature	Importance score
0	titleWords	1.7
1	adWords	2.0
2	depth	2.4
3	position	0.4
4	advCTR	13.2
5	advCTRInPos	52.6
6	queryCTR	4.8
7	queryCTRInPos	21.4
8	gender_female	0.5
9	gender_male	0.2
10	gender_unknown	0.2
11	age_0-12	0.0
12	age_13-18	0.0
13	age_19-24	0.1
14	age_25-30	0.2
15	age_31-40	0.0
16	age_41+	0.0
17	age_unknown	0.3

1.4 4. Final Comparison

1.4.1 4.1 Linear Regression Baseline

[56]:
```
from sklearn.linear_model import LinearRegression
import sklearn.metrics as metrics

lr = LinearRegression().fit(X_train, y_train)
```

1.4.2 4.2 Comparison Table

[62]:
```
comparison_data = {'Linear Regression': ['{:.3f}'.format(OSR2(lr, X_test,␣
 ↪y_test, y_train)),
                                         '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test, lr.predict(X_test))),
                                         '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test, lr.predict(X_test)))],
                   'Decision Tree Regressor': ['{:.3f}'.format(OSR2(dtr_cv,␣
 ↪X_test, y_test, y_train)),
                                               '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test, dtr_cv.predict(X_test))),
                                               '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test, dtr_cv.predict(X_test)))],
                   'Random Forest': ['{:.3f}'.format(OSR2(rf_cv, X_test,␣
 ↪y_test, y_train)),
                                     '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test, rf_cv.predict(X_test))),
                                     '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test, rf_cv.predict(X_test)))],
                   'Gradient Boosted Trees': ['{:.3f}'.format(OSR2(gbr_cv,␣
 ↪X_test, y_test, y_train)),
                                              '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test, gbr_cv.predict(X_test))),
                                              '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test, gbr_cv.predict(X_test)))]}

comparison_table = pd.DataFrame(data=comparison_data, index=['OSR2',␣
 ↪'Out-of-sample MSE', 'Out-of-sample MAE'])
comparison_table.style.set_properties(**{'font-size': '12pt',}).
 ↪set_table_styles([{'selector': 'th', 'props': [('font-size', '10pt')]}])
```

[62]:
```
<pandas.io.formats.style.Styler at 0x16ad2e9c0>
```

Let’s look at MAE restricted to CTR above and below 10%

1.5 In-class activity: Check if the prediction accuracy are different for low CTR and high CTR items. Use CTR <=0.1 or CTR > 0.1 as a threshold.

- Specifically, split the testing set to two subsets by CTR values. Evaluate the performance on each subset.

[59]:
```
X_test_low = X_test[y_test <= 0.1]
y_test_low = y_test[y_test <= 0.1]
```

[63]:
```
comparison_data = {'Linear Regression': ['{:.3f}'.format(OSR2(lr, X_test_low,␣
 ↪y_test_low, y_train)),
                                          '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test_low, lr.predict(X_test_low))),
                                          '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test_low, lr.predict(X_test_low)))],
                    'Decision Tree Regressor': ['{:.3f}'.format(OSR2(dtr_cv,␣
 ↪X_test_low, y_test_low, y_train)),
                                                '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test_low, dtr_cv.predict(X_test_low))),
                                                '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test_low, dtr_cv.predict(X_test_low)))],
                    'Random Forest': ['{:.3f}'.format(OSR2(rf_cv, X_test_low,␣
 ↪y_test_low, y_train)),
                                      '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test_low, rf_cv.predict(X_test_low))),
                                      '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test_low, rf_cv.predict(X_test_low)))],
                    'Gradient Boosted Trees': ['{:.3f}'.format(OSR2(gbr_cv,␣
 ↪X_test_low, y_test_low, y_train)),
                                               '{:.4f}'.format(metrics.
 ↪mean_squared_error(y_test_low, gbr_cv.predict(X_test_low))),
                                               '{:.3f}'.format(metrics.
 ↪mean_absolute_error(y_test_low, gbr_cv.predict(X_test_low)))]}

comparison_table = pd.DataFrame(data=comparison_data, index=['OSR2',␣
 ↪'Out-of-sample MSE', 'Out-of-sample MAE'])
comparison_table.style.set_properties(**{'font-size': '12pt',}).
 ↪set_table_styles([{'selector': 'th', 'props': [('font-size', '10pt')]}])
```

[63]:
```
<pandas.io.formats.style.Styler at 0x16ad2ede0>
```

[39]:
```
X_test_high = X_test[y_test >= 0.1]
y_test_high = y_test[y_test >= 0.1]
```

[64]:
```
comparison_data = {'Linear Regression': ['{:.3f}'.format(OSR2(lr, X_test_high,␣
 ↪y_test_high, y_train)),
```

```
                                         '{:.4f}'.format(metrics.
↪mean_squared_error(y_test_high, lr.predict(X_test_high))),
                                         '{:.3f}'.format(metrics.
↪mean_absolute_error(y_test_high, lr.predict(X_test_high)))],
                   'Decision Tree Regressor': ['{:.3f}'.format(OSR2(dtr_cv,␣
↪X_test_high, y_test_high, y_train)),
                                               '{:.4f}'.format(metrics.
↪mean_squared_error(y_test_high, dtr_cv.predict(X_test_high))),
                                               '{:.3f}'.format(metrics.
↪mean_absolute_error(y_test_high, dtr_cv.predict(X_test_high)))],
                   'Random Forest': ['{:.3f}'.format(OSR2(rf_cv, X_test_high,␣
↪y_test_high, y_train)),
                                     '{:.4f}'.format(metrics.
↪mean_squared_error(y_test_high, rf_cv.predict(X_test_high))),
                                     '{:.3f}'.format(metrics.
↪mean_absolute_error(y_test_high, rf_cv.predict(X_test_high)))],
                   'Gradient Boosted Trees': ['{:.3f}'.format(OSR2(gbr_cv,␣
↪X_test_high, y_test_high, y_train)),
                                              '{:.4f}'.format(metrics.
↪mean_squared_error(y_test_high, gbr_cv.predict(X_test_high))),
                                              '{:.3f}'.format(metrics.
↪mean_absolute_error(y_test_high, gbr_cv.predict(X_test_high)))]}

comparison_table = pd.DataFrame(data=comparison_data, index=['OSR2',␣
↪'Out-of-sample MSE', 'Out-of-sample MAE'])
comparison_table.style.set_properties(**{'font-size': '12pt',}).
↪set_table_styles([{'selector': 'th', 'props': [('font-size', '10pt')]}])
```

[64]: <pandas.io.formats.style.Styler at 0x16aef3320>