HW6

November 18, 2024

1 STOR 320 Homework 6 Cross Validation

Please submit the solution to gradescope by 11:59 PM, Nov 21, Thursday.

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[2]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split, GridSearchCV
import statsmodels.formula.api as smf
import statsmodels.api as sm
from sklearn.neighbors import KNeighborsRegressor
from sklearn.metrics import mean_squared_error, r2_score
from sklearn.preprocessing import StandardScaler
from sklearn.metrics import make_scorer
```

[3]:
```
ames = pd.read_feather('cleaned_ames.feather')
ames.head()
```

[3]:
```
   LogSalePrice MSSubClass MSZoning  LotFrontage  LotArea Street     Alley  \
0     12.278393         20       RL        141.0  31770.0   Pave  NoAccess
1     11.561716         20       RH         80.0  11622.0   Pave  NoAccess
2     12.055250         20       RL         81.0  14267.0   Pave  NoAccess
3     12.404924         20       RL         93.0  11160.0   Pave  NoAccess
4     12.154253         60       RL         74.0  13830.0   Pave  NoAccess

  LotShape LandContour Utilities  … PreCast Stone Stucco VinylSd WdSdng  \
0      IR1         Lvl    AllPub  …       0     0      0       0      0
1      Reg         Lvl    AllPub  …       0     0      0       1      0
2      IR1         Lvl    AllPub  …       0     0      0       0      1
3      Reg         Lvl    AllPub  …       0     0      0       0      0
4      IR1         Lvl    AllPub  …       0     0      0       1      0

  WdShing WdShng YearsSince1950Built YearsSince1950Remod  \
0       0      0                   10                  10
```

```
1        0       0                   11                    11
2        0       0                    8                     8
3        0       0                   18                    18
4        0       0                   47                    48

  YearsSince1950GarageBuilt
0                      10.0
1                      11.0
2                       8.0
3                      18.0
4                      47.0

[5 rows x 105 columns]
```

1. **Load the cleaned Ames dataset. Convert the categorical variables into dummy variables. Display the new ames table. How many columns are there? (5 points)**

[52]:
```python
ames_dummies = pd.get_dummies(ames)
ames_dummies.head()
```

[52]:
```
   LogSalePrice  LotFrontage  LotArea  MasVnrArea  BsmtFinSF1  BsmtFinSF2  \
0     12.278393        141.0  31770.0       112.0       639.0         0.0
1     11.561716         80.0  11622.0         0.0       468.0       144.0
2     12.055250         81.0  14267.0       108.0       923.0         0.0
3     12.404924         93.0  11160.0         0.0      1065.0         0.0
4     12.154253         74.0  13830.0         0.0       791.0         0.0

   BsmtUnfSF  TotalBsmtSF  X1stFlrSF  X2ndFlrSF  …  SaleType_New  \
0      441.0       1080.0     1656.0        0.0  …         False
1      270.0        882.0      896.0        0.0  …         False
2      406.0       1329.0     1329.0        0.0  …         False
3     1045.0       2110.0     2110.0        0.0  …         False
4      137.0        928.0      928.0      701.0  …         False

   SaleType_Oth  SaleType_VWD  SaleType_WD  SaleCondition_Abnorml  \
0         False         False         True                  False
1         False         False         True                  False
2         False         False         True                  False
3         False         False         True                  False
4         False         False         True                  False

   SaleCondition_AdjLand  SaleCondition_Alloca  SaleCondition_Family  \
0                  False                 False                 False
1                  False                 False                 False
2                  False                 False                 False
3                  False                 False                 False
4                  False                 False                 False
```

```
   SaleCondition_Normal  SaleCondition_Partial
0                  True                  False
1                  True                  False
2                  True                  False
3                  True                  False
4                  True                  False

[5 rows x 382 columns]
```

[49]:
```
print(f"There are {len(ames_dummies.columns)} columns.")
```

```
There are 382 columns.
```

2. **L1 Distance Calculation (5 points)**

- Define the L1 distance between two rows as the sum of the absolute values of the differences for all features. Calculate the L1 distance between the first and second rows of the dataset.
- For Boolean variables, you can treat `True` as 1 and `False` as 0.

[5]:
```
slice = ames_dummies[:2]
# convert boolean to int
slice = slice.astype(int)
L1 = np.sum(np.abs(slice.iloc[0] - slice.iloc[1]))
L1
```

[5]:
```
23031
```

3. **Feature Matrix and Target Vector Creation (10 points)**
   - Create a feature matrix `X` by dropping the `LogSalePrice` column from the dataset. Create a target vector `y` using the `LogSalePrice` column. (5 points)
   - Randomly split `X` and `y` into training and test sets with a 70/30 split. Name the training set `X_train` and `y_train`, and the test set `X_test` and `y_test`. Use random seed as `42`. (5 points)

[6]:
```
y = ames_dummies["LogSalePrice"]
X = ames_dummies.drop(columns=["LogSalePrice"])
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,␣
↪random_state=42)
```

4. **Rescaling the Feature Matrix (5 points)**
   - Rescale the values of `X_train` using a scaler (e.g., `StandardScaler`). Use the same scaling rule to transform `X_test`. Name the transformed sets `X_train_scaled` and `X_test_scaled`.

[7]:
```
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)
print(f"{X_train_scaled.shape}, {X_test_scaled.shape}")
```

```
(1935, 381), (830, 381)
```

5. **Simple L1 Prediction Model (10 points)**
    - Implement a simple prediction model: For each row in `X_test_scaled`, find the closest row in `X_train_scaled` in terms of L1 distance. Use the corresponding `y` value from `y_train` as the prediction.
    - Create predictions for all observations in `X_test_scaled` and name the prediction array `y_predict`.

[8]:
```
y_predict = []
for i in range(len(X_test_scaled)):
    L1_distances = np.sum(np.abs(X_train_scaled - X_test_scaled[i]), axis=1)
    closest_index = np.argmin(L1_distances)
    y_predict.append(y_train.iloc[closest_index])
y_predict = np.array(y_predict)
```

6. **Calculate Out-of-Sample R-squared (OSR2) (5 points)**
    - Calculate the OSR2 (out-of-sample R-squared) for the predictions from Problem 5.

[9]:
```
def OSR2(y_train, y_test, y_pred):

    SSE = np.sum((y_test - y_pred)**2)
    SST = np.sum((y_test - np.mean(y_train))**2)

    return (1 - SSE/SST)
```

[10]:
```
print(f"OSR2: {OSR2(y_train, y_test, y_predict)}")
```

```
OSR2: 0.7396347150902295
```

7. **Flexible L1 Prediction Model (10 points)**
    - Modify the prediction rule: For each row in `X_test_scaled`, find the five closest rows in `X_train_scaled` in terms of L1 distance and use the average `y` value of these five observations as the prediction.
    - Create predictions for all observations in `X_test_scaled` and name the prediction array `y_predict`.

[11]:
```
y_predict = []
for i in range(len(X_test_scaled)):
    L1_distances = np.sum(np.abs(X_train_scaled - X_test_scaled[i]), axis=1)
    closest_indices = np.argsort(L1_distances)[:5]
    y_predict.append(np.mean(y_train.iloc[closest_indices]))
y_predict = np.array(y_predict)
```

8. **Calculate OSR2 for the Modified Model (5 points)**
    - Calculate the OSR2 for the predictions from Problem 7.

[12]:
```
print(f"OSR2: {OSR2(y_train, y_test, y_predict)}")
```

```
OSR2: 0.7969887360628379
```

9. **KNN Model Using Sklearn (10 points)**

The above idea is called `K-Nearest Neighbors`. The K-Nearest Neighbors (KNN) algorithm is a non-parametric method used for regression and classification. It predicts the target for a new observation by averaging the target values of its k nearest training samples. This method is effective for capturing local structures in the data, which can lead to more flexible predictions compared to global linear models.

- Use the `KNeighborsRegressor(n_neighbors=k, metric='cityblock')` function in `sklearn` to build a KNN model with `k=5`. `metric='cityblock'` represents the L1 norm distance. (5 points)
- Print the OSR2 of the model. Is your OSR2 the same as the OSR2 in Problem 8? (5 points)

[13]:
```
KNN = KNeighborsRegressor(n_neighbors=5, metric="cityblock")
KNN.fit(X_train_scaled, y_train)
y_predict = KNN.predict(X_test_scaled)
print(f"OSR2: {OSR2(y_train, y_test, y_predict)}")
```

```
OSR2: 0.7969887360628379
```

The OSR2 is the same as the one in Problem 8.

10. **Effect of Large k (5 points)**

- Explain what happens when `k` equals the size of the training set. What is the predicted value in this case?

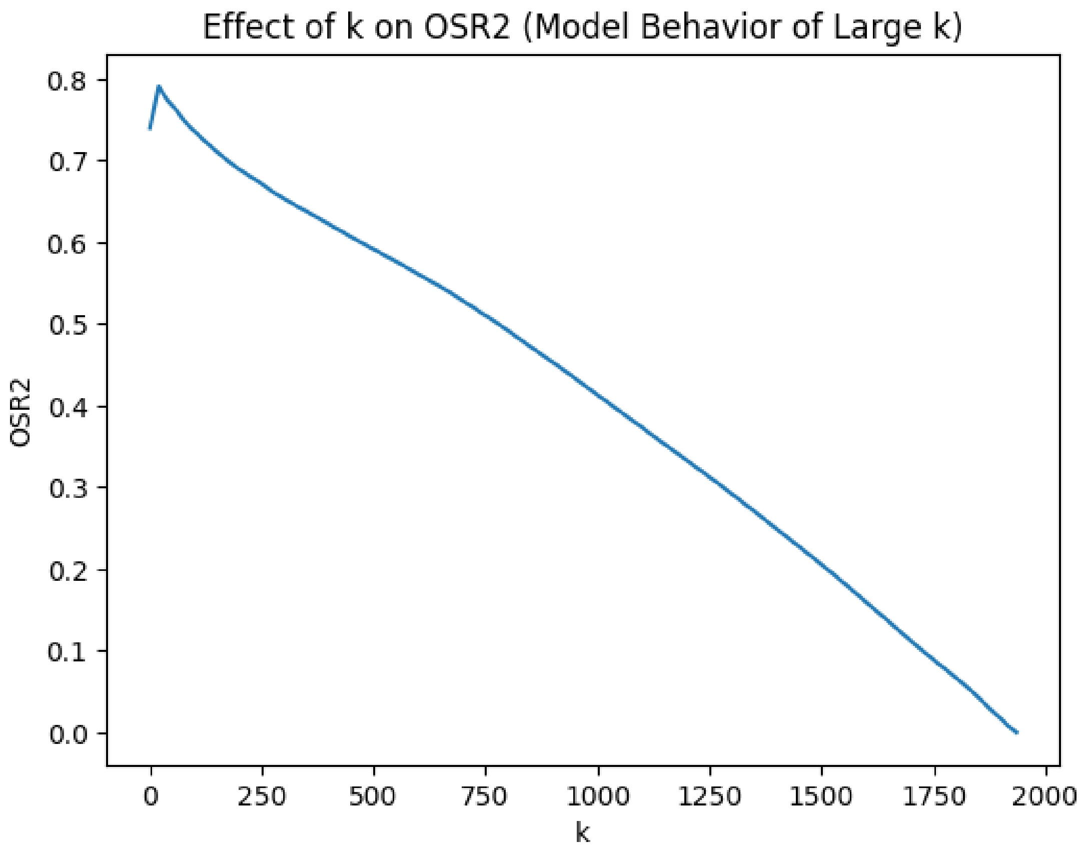
[30]:
```
k = len(X_train_scaled)
KNN = KNeighborsRegressor(n_neighbors=k, metric="cityblock")
KNN.fit(X_train_scaled, y_train)
y_predict = KNN.predict(X_test_scaled)
print(f"OSR2: {OSR2(y_train, y_test, y_predict)}")
print(f"y_predict is all the same value. ({y_predict.mean()})")
```

```
OSR2: 0.0
y_predict is all the same value. (12.047640175621291)
```

11. **Model Bias and Overfitting Discussion (5 points)**
    - Discuss how the model behaves as `k` increases. Does the model become more biased or less biased? Does it overfit more or less as `k` increases?

[23]:
```
k_range = np.linspace(1.0, len(X_train_scaled), 100)
OSR2_list = []

for k in k_range:
    KNN = KNeighborsRegressor(n_neighbors=int(k), metric="cityblock")
    KNN.fit(X_train_scaled, y_train)
    y_predict = KNN.predict(X_test_scaled)
    OSR2_list.append(OSR2(y_train, y_test, y_predict))
```

[22]:
```
plt.plot(k_range, OSR2_list)
plt.xlabel('k')
plt.ylabel('OSR2')
plt.title("Effect of k on OSR2 (Model Behavior of Large k)")
plt.show()
```



As k increases, the model becomes more biased towards the training data, making it worse at predicting the test data since it's overfitting the training data.

12. **Parameter Tuning with GridSearchCV (20 points)**
    - Create a custom function called `Mean_Absolute_Price_Error()` that converts the predicted log prices back to the original prices and calculates the Mean Absolute Error (MAE) between the predicted prices and true prices. (5 points)
    - Use `GridSearchCV` to tune the `k` parameter of the KNN model, using the custom `Mean_Absolute_Price_Error()` function as the metric for model selection. Use `5-fold` cross validation. (5 points)
    - Plot the `MAE` for different values of `k`. (5 points)
    - Identify the `k` value that results in the smallest `Mean_Absolute_Price_Error`. (5 points)

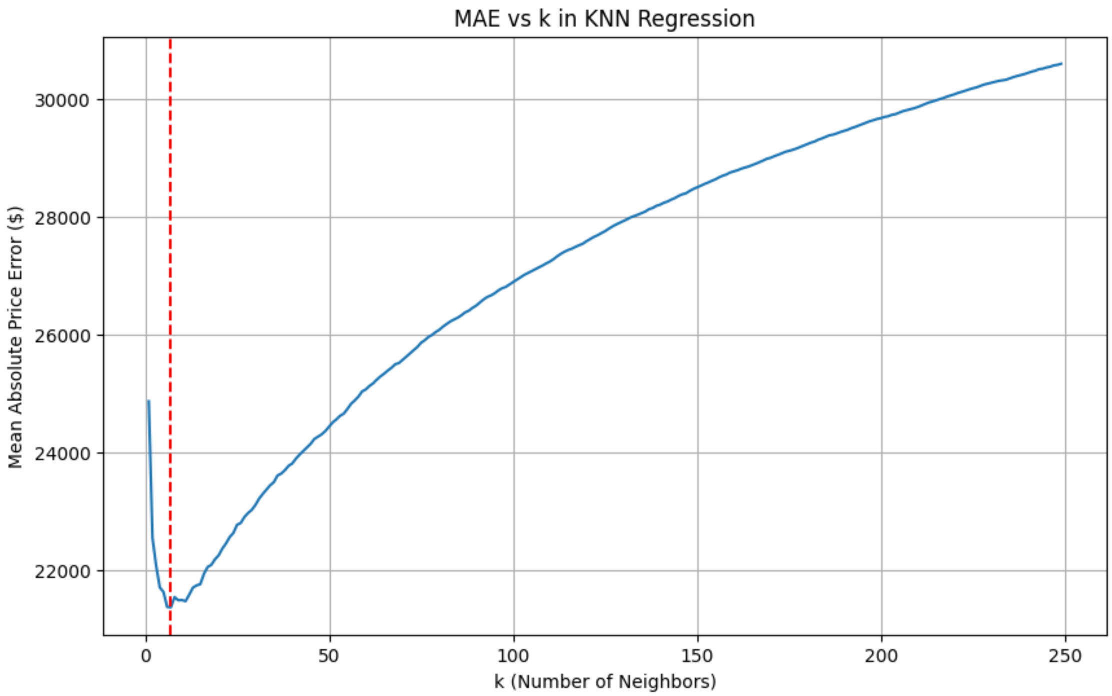
[31]:
```python
def Mean_Absolute_Price_Error(y_test, y_predict):
    return np.mean(np.abs(np.exp(y_test) - np.exp(y_predict)))
```

[35]:
```python
param_grid = {"n_neighbors": np.arange(1, 250)}
```

[36]:
```python
gs = GridSearchCV(KNN, param_grid, cv=5,␣
 ↪scoring=make_scorer(Mean_Absolute_Price_Error, greater_is_better=False))
gs.fit(X_train_scaled, y_train)

optimal_k = gs.best_params_["n_neighbors"]
```

[36]:
```
GridSearchCV(cv=5,
             estimator=KNeighborsRegressor(metric='cityblock',
                                           n_neighbors=1935),
             param_grid={'n_neighbors': array([  1,   2,   3,   4,   5,   6,
7,   8,   9,  10,  11,  12,  13,
        14,  15,  16,  17,  18,  19,  20,  21,  22,  23,  24,  25,  26,
        27,  28,  29,  30,  31,  32,  33,  34,  35,  36,  37,  38,  39,
        40,  41,  42,  43,  44,  45,  46,  47,  48,  49,  50,  51,  52,
        53,  54,  55,  56,  57,  58,  59,  60,  61,  62,  63,  64,  65,
        66,  67,  68,  69,  70,  71,  72,  73,  74,  75,  76,  77,  78,
        79,  80,  8…
       183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195,
       196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208,
       209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221,
       222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234,
       235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247,
       248, 249])},
             scoring=make_scorer(Mean_Absolute_Price_Error,
greater_is_better=False, response_method='predict'))
```

[48]:
```python
# Plot MAE vs k values
plt.figure(figsize=(10, 6))
plt.plot(param_grid["n_neighbors"], -gs.cv_results_["mean_test_score"],␣
 ↪label="Mean Absolute Price Error")
plt.xlabel("k (Number of Neighbors)")

plt.axvline(x=optimal_k, color='red', linestyle='--')

plt.ylabel("Mean Absolute Price Error ($)")
plt.title("MAE vs k in KNN Regression")
plt.grid(True)
plt.show()
```



[44]:
```
print(f"Best k value: {gs.best_params_["n_neighbors"]}")
print(f"Best MAE: {-gs.best_score_}")
```

```
Best k value: 7
Best MAE: 21371.824377847202
```

13. **Refit the Model (5 points)**
    - Refit the KNN model using the optimal `k` value from Problem 12. Print the out-of-sample performance (R2) of this final model.

[45]:
```
KNN = KNeighborsRegressor(n_neighbors=optimal_k, metric="cityblock")
KNN.fit(X_train_scaled, y_train)
y_predict = KNN.predict(X_test_scaled)
print(f"OSR2: {OSR2(y_train, y_test, y_predict)}")
```

```
OSR2: 0.7988402219884047
```