logistic_regression

November 17, 2024

1 STOR 320 Intro to Data Science: Logistic Regression

- 1. Logistic Regression with numerical variables
- 2. Logistic Regression with categorical variables
- 3. LDA

[ ]:
```
import numpy as np
import pandas as pd
```

As usual, we summon `numpy` and `pandas` for dataset representation and manipulation.

1.1 1. LOGISTIC REGRESSION (ONLY NUMERICAL VARIABLES)

1.1.1 1.1 Data loading

[ ]:
```
loans = pd.read_csv("loans.csv")
loans.info()
loans.head()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 9516 entries, 0 to 9515
Data columns (total 7 columns):
 #   Column          Non-Null Count  Dtype
---  ------          --------------  -----
 0   not.fully.paid  9516 non-null   int64
 1   installment     9516 non-null   float64
 2   log.annual.inc  9516 non-null   float64
 3   fico            9516 non-null   int64
 4   revol.bal       9516 non-null   float64
 5   inq.last.6mths  9516 non-null   int64
 6   pub.rec         9516 non-null   int64
dtypes: float64(3), int64(4)
memory usage: 520.5 KB
```

[ ]:
```
   not.fully.paid  installment  log.annual.inc  fico  revol.bal  \
0               0       829.10        4.929419   737     28.854
1               0       228.22        4.812913   707     33.623
2               0       366.86        4.505150   682      3.511
3               0       162.34        4.929419   712     33.667
```

```
4                 0       102.92         4.907411   667       4.740

   inq.last.6mths  pub.rec
0               0        0
1               0        0
2               1        0
3               1        0
4               0        0
```

1.1.2 1.2 Data cleaning and transformation

In Python, a convention is to name variables with underscores. This is slightly different from R.

Let us practice how to rename columns in Pandas.

```
[ ]: new_column_names = {'not.fully.paid':'not_fully_paid', 'log.annual.inc':
      ↪'log_annual_inc',
                         'revol.bal':'revol_bal', 'inq.last.6mths':'inq_last_6mths',␣
      ↪'pub.rec':'pub_rec'}
     loans.rename(columns = new_column_names, inplace = True)

     print(loans.columns)
```

```
Index(['not_fully_paid', 'installment', 'log_annual_inc', 'fico', 'revol_bal',
       'inq_last_6mths', 'pub_rec'],
      dtype='object')
```

Use `df.describe()`, you can have a quick overview of the data set. Here, observe that the first four variables ('installment', 'log_annual_inc', 'fico', and 'revol_bal') takes continuous numeric values and the last two variables ('inq_last_6mths', 'pub_rec') takes ingeter numeric values.

```
[ ]: loans.describe()
```

```
[ ]:        not_fully_paid  installment  log_annual_inc         fico     revol_bal  \
     count     9516.000000  9516.000000     9516.000000  9516.000000  9516.000000
     mean         0.159836   320.131185        4.748642   710.841950    16.988484
     std          0.366473   207.069870        0.265002    37.956246    33.721379
     min          0.000000    15.670000        3.277838   612.000000     0.000000
     25%          0.000000   164.020000        4.588821   682.000000     3.272750
     50%          0.000000   269.545000        4.748188   707.000000     8.687500
     75%          0.000000   435.405000        4.903323   737.000000    18.354250
     max          1.000000   940.140000        6.309584   827.000000  1207.359000

            inq_last_6mths      pub_rec
     count     9516.000000  9516.000000
     mean         1.572930     0.062211
     std          2.200329     0.262406
     min          0.000000     0.000000
     25%          0.000000     0.000000
```

```
50%          1.000000      0.000000
75%          2.000000      0.000000
max         33.000000      5.000000
```

1.1.3 1.3 Train-test split

[ ]:
```
from sklearn.model_selection import train_test_split

loans_train, loans_test = train_test_split(loans, test_size=0.3,␣
 ↪random_state=88)
loans_train.shape, loans_test.shape
```

[ ]:
```
((6661, 7), (2855, 7))
```

[ ]:
```
loans_train.head()
```

[ ]:
```
      not_fully_paid  installment  log_annual_inc  fico  revol_bal  \
6330               0       682.74        5.079181   697     72.682
3799               0       479.21        4.778151   677     14.180
4019               0       333.15        4.579784   702      5.520
7475               0       373.32        4.819544   782      6.449
8641               0        81.51        4.417638   662      0.558

      inq_last_6mths  pub_rec
6330               1        0
3799               1        0
4019               1        0
7475               2        0
8641               4        0
```

`test_size = 0.3` means that we will put 30% of the data in the test set, 70% in the training set.

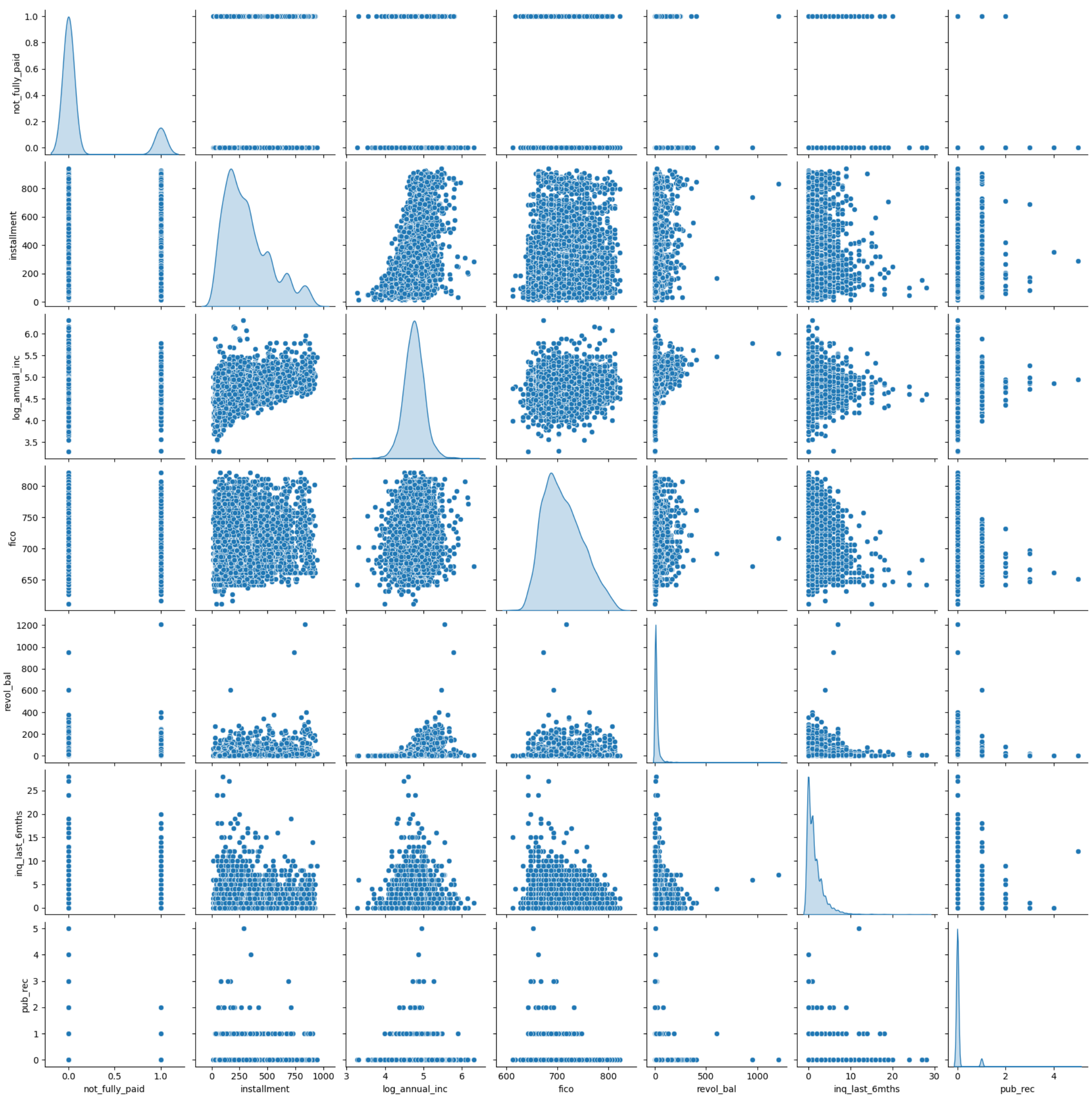
1.1.4 1.4 Baseline model

[ ]:
```
# How many loans have defaulted in the training set?

default_false = np.sum(loans_train['not_fully_paid'] == 0)
default_true = np.sum(loans_train['not_fully_paid'] == 1)

print(pd.Series({'0': default_false, '1': default_true}))
```

```
0    5585
1    1076
dtype: int64
```

[ ]:
```
#use seaborn.pairplot to plot scatter plot for continuous variables
import seaborn as sns
sns.pairplot(loans_train,diag_kind='kde')
```

[ ]: <seaborn.axisgrid.PairGrid at 0x14e53a450>



[ ]:
```
loans_train.corr()
```

[ ]:

	not_fully_paid	installment	log_annual_inc	fico	\
not_fully_paid	1.000000	0.029430	-0.039698	-0.144830	
installment	0.029430	1.000000	0.452113	0.094061	
log_annual_inc	-0.039698	0.452113	1.000000	0.124615	
fico	-0.144830	0.094061	0.124615	1.000000	
revol_bal	0.040423	0.224112	0.362875	-0.010072	
inq_last_6mths	0.141096	-0.005957	0.029720	-0.192888	
pub_rec	0.045955	-0.028412	0.022748	-0.142629	

```
                revol_bal  inq_last_6mths   pub_rec
not_fully_paid   0.040423        0.141096  0.045955
installment      0.224112       -0.005957 -0.028412
log_annual_inc   0.362875        0.029720  0.022748
fico            -0.010072       -0.192888 -0.142629
revol_bal        1.000000        0.026560 -0.027113
inq_last_6mths   0.026560        1.000000  0.066816
pub_rec         -0.027113        0.066816  1.000000
```

A baseline model can be a so-called "dummy" model, where the classifier predicts every new observation as the majority class. In our case, for a datapoint with any given features, the baseline model will always predict 'no default'.

[ ]:
```
# Accuracy of baseline model based on training data:
ACC = default_false/(default_false + default_true) #Accuracy = (TP+TN)/
 ↪(TP+TN+FP+FN)
ACC
```

[ ]: 0.8384626932892959

Note: we want to create models that performs better than the baseline model.

[ ]:
```
# EXERCISE: Compute accuracy of baseline on testing:
default_false_test = np.sum(loans_test['not_fully_paid'] == 0)
default_true_test = np.sum(loans_test['not_fully_paid'] == 1)
ACC_test = default_false_test/(default_false_test+default_true_test)
ACC_test
```

[ ]: 0.8441330998248686

[ ]:
```
# EXERCISE: What are the TPR and FPR rates of the baseline model?

# True positive: the proportion of actual positives that are correctly␣
 ↪identified as positive
# False positive: the proportion of actual negatives that are incorrectly␣
 ↪identified as positive

TPR = 0/default_true_test # TPR = TP/P = TP/(TP+FN)
FPR = 0/default_false_test # FPR = FP/N = FP/(FP+TN)
print(TPR,FPR)
```

```
0.0 0.0
```

[ ]:
```
# EXERCISE: What are the TNR and FNR rates of the baseline model?

# True negative: the proportion of actual negatives that are correctly␣
 ↪identified as negative
# False negative: the proportion of actual positives that are incorrectly␣
 ↪identified as negative
```

```
TNR = default_false_test/default_false_test # TNR = TN/N = TN/(TN+FP)
FNR = default_true_test/default_true_test # FNR = FN/P = FN/(TP+FN)
print(TNR,FNR)
```

```
1.0 1.0
```

1.1.5 1.5 Model Fitting (Logistic Regression)

Now we can use the statsmodels package to fit the training set to a logistic regression model

[ ]:
```
import statsmodels.formula.api as smf

?smf.logit
```

```
Signature:
smf.logit(formula,
data, subset=None,
drop_cols=None,
*args,
**kwargs)
Docstring:
Create a Model from a formula and dataframe.

Parameters
----------
formula : str or generic Formula object
    The formula specifying the model.
data : array_like
    The data for the model. See Notes.
subset : array_like
    An array-like object of booleans, integers, or index values that
    indicate the subset of df to use in the model. Assumes df is a
    `pandas.DataFrame`.
drop_cols : array_like
    Columns to drop from the design matrix.  Cannot be used to
    drop terms involving categoricals.
*args
    Additional positional argument that are passed to the model.
**kwargs
    These are passed to the model with one exception. The
    ``eval_env`` keyword is passed to patsy. It can be either a
    :class:`patsy:patsy.EvalEnvironment` object or an integer
    indicating the depth of the namespace to use. For example, the
    default ``eval_env=0`` uses the calling namespace. If you wish
    to use a "clean" environment set ``eval_env=-1``.

Returns
-------
```

```
model
    The model instance.

Notes
-----
data must define __getitem__ with the keys in the formula terms
args and kwargs are passed on to the model instantiation. E.g.,
a numpy structured or rec array, a dictionary, or a pandas DataFrame.
File:
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/statsmodels/base/model.py
Type:      method
```

[ ]:
```python
# Fit the logistic regression model

logreg = smf.logit(formula = 'not_fully_paid ~ installment + log_annual_inc +␣
↪fico + revol_bal + inq_last_6mths + pub_rec',
                   data = loans_train).fit(method = 'ncg')

print(logreg.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.437108
         Iterations: 4
         Function evaluations: 5
         Gradient evaluations: 5
         Hessian evaluations: 4
                           Logit Regression Results
==============================================================================
Dep. Variable:         not_fully_paid   No. Observations:                 6661
Model:                          Logit   Df Residuals:                     6654
Method:                           MLE   Df Model:                            6
Date:                Wed, 13 Nov 2024   Pseudo R-squ.:                 0.01154
Time:                        18:29:47   Log-Likelihood:                -2911.6
converged:                       True   LL-Null:                       -2945.6
Covariance Type:            nonrobust   LLR p-value:                 1.067e-12
================================================================================
==
                     coef    std err          z      P>|z|      [0.025
0.975]
--------------------------------------------------------------------------------
--
Intercept        -3.1e-06      0.901  -3.44e-06      1.000      -1.766
1.766
installment        0.0006      0.000      3.431      0.001       0.000
0.001
log_annual_inc -1.513e-05      0.151     -0.000      1.000      -0.295
0.295
```

```
fico              -0.0026      0.001     -2.807      0.005      -0.004
-0.001
revol_bal          0.0001      0.001      0.126      0.899      -0.002
0.002
inq_last_6mths  2.443e-05      0.016      0.002      0.999      -0.031
0.031
pub_rec         8.993e-07      0.125   7.19e-06      1.000      -0.245
0.245
================================================================================
==
```

1.1.6 1.6 Predictions

[ ]:
```
# Example of prediction for a new observation

new_obs = pd.DataFrame(data = {'installment' : [366], 'log_annual_inc' : [4.
 ↪51], 'fico' : [682],
                               'revol_bal' : [7.53], 'inq_last_6mths' : [1],␣
 ↪'pub_rec' : [0]})

logreg.predict(new_obs)
```

[ ]:
```
0    0.172312
dtype: float64
```

[ ]:
```
y_test = loans_test['not_fully_paid']

y_prob = logreg.predict(loans_test)
y_pred = pd.Series([1 if x > 0.5 else 0 for x in y_prob], index=y_prob.index)

# y_pred is the vector of probabilities as given by your model on the test set.␣
 ↪Values between 0 and 1.
# Remember, P(Yi = 1) = 1/(1 + e^(-(b0 + b1*x1 + b2*x2 +...)) )
```

1.1.7 1.7 Model Evaluation - Confusion Matrix

In order to evaluate the performance of our classification model, we can make use of confusion matrix to compute a variety of useful metrics

[ ]:
```
from sklearn.metrics import confusion_matrix

?confusion_matrix
```

```
Signature:
confusion_matrix(
    y_true,
    y_pred,
    *,
    labels=None,
```

```
sample_weight=None,

normalize=None,
)
Docstring:
Compute confusion matrix to evaluate the accuracy of a classification.

By definition a confusion matrix :math:`C` is such that :math:`C_{i, j}`
is equal to the number of observations known to be in group :math:`i` and
predicted to be in group :math:`j`.

Thus in binary classification, the count of true negatives is
:math:`C_{0,0}`, false negatives is :math:`C_{1,0}`, true positives is
:math:`C_{1,1}` and false positives is :math:`C_{0,1}`.

Read more in the :ref:`User Guide <confusion_matrix>`.

Parameters
----------
y_true : array-like of shape (n_samples,)
    Ground truth (correct) target values.

y_pred : array-like of shape (n_samples,)
    Estimated targets as returned by a classifier.

labels : array-like of shape (n_classes), default=None
    List of labels to index the matrix. This may be used to reorder
    or select a subset of labels.
    If ``None`` is given, those that appear at least once
    in ``y_true`` or ``y_pred`` are used in sorted order.

sample_weight : array-like of shape (n_samples,), default=None
    Sample weights.

    .. versionadded:: 0.18

normalize : {'true', 'pred', 'all'}, default=None
    Normalizes confusion matrix over the true (rows), predicted (columns)
    conditions or all the population. If None, confusion matrix will not be
    normalized.

Returns
-------
C : ndarray of shape (n_classes, n_classes)
    Confusion matrix whose i-th row and j-th
    column entry indicates the number of
    samples with true label being i-th class
```

```
    and predicted label being j-th class.

See Also
--------
ConfusionMatrixDisplay.from_estimator : Plot the confusion matrix
    given an estimator, the data, and the label.
ConfusionMatrixDisplay.from_predictions : Plot the confusion matrix
    given the true and predicted labels.
ConfusionMatrixDisplay : Confusion Matrix visualization.

References
----------
.. [1] `Wikipedia entry for the Confusion matrix
       <https://en.wikipedia.org/wiki/Confusion_matrix>`_
       (Wikipedia and other references may use a different
       convention for axes).

Examples
--------
>>> from sklearn.metrics import confusion_matrix
>>> y_true = [2, 0, 2, 2, 0, 1]
>>> y_pred = [0, 0, 2, 2, 0, 2]
>>> confusion_matrix(y_true, y_pred)
array([[2, 0, 0],
       [0, 0, 1],
       [1, 0, 2]])

>>> y_true = ["cat", "ant", "cat", "cat", "ant", "bird"]
>>> y_pred = ["ant", "ant", "cat", "cat", "ant", "cat"]
>>> confusion_matrix(y_true, y_pred, labels=["ant", "bird", "cat"])
array([[2, 0, 0],
       [0, 0, 1],
       [1, 0, 2]])

In the binary case, we can extract true positives, etc. as follows:

>>> tn, fp, fn, tp = confusion_matrix([0, 1, 0, 1], [1, 1, 1, 0]).ravel()
>>> (tn, fp, fn, tp)
(np.int64(0), np.int64(2), np.int64(1), np.int64(1))
File:
/Library/Frameworks/Python.framework/Versions/3.12/lib/python3.12/site-
packages/sklearn/metrics/_classification.py
Type:      function
```

To remind you of what each element of the confusion matrix represents:

TN FP

FN TP

[ ]:
```
cm = confusion_matrix(y_test, y_pred)
print ("Confusion Matrix : \n", cm)
```

```
Confusion Matrix :
 [[2410    0]
 [ 445    0]]
```

[ ]:
```
# compare the confusion matrix of the baseline model
baseline_model = [0]*loans_test.shape[0] # model of all zeros
confusion_matrix(loans_test['not_fully_paid'], baseline_model)
```

[ ]:
```
array([[2410,    0],
       [ 445,    0]])
```

[ ]:
```
print(cm.ravel())
```

```
[2410    0  445    0]
```

[ ]:
```
# Accuracy
(cm.ravel()[0]+cm.ravel()[3])/sum(cm.ravel())
```

[ ]:
```
0.8441330998248686
```

Be careful about the definitions of FPR, TPR, recall, precision, sensitivity, specificity etc: https://en.wikipedia.org/wiki/Sensitivity_and_specificity

[ ]:
```
# EXERCISE: What is the True Positive Rate ?
TPR_logit = cm.ravel()[3]/(cm.ravel()[3]+cm.ravel()[2])
print('TPR is: %.4f' % TPR_logit)

# EXERCISE: What is the False Positive rate ?
FPR_logit = cm.ravel()[1]/(cm.ravel()[1]+cm.ravel()[0])
print('FPR is: %.4f' % FPR_logit)
```

```
TPR is: 0.0000
FPR is: 0.0000
```

[ ]:
```
# Now, try threshold probability = 0.4
y_pred_40perc = pd.Series([1 if x > 0.4 else 0 for x in y_prob], index=y_prob.
 ↪index)
cm = confusion_matrix(y_test, y_pred_40perc)
print ("Confusion Matrix : \n", cm)
# EXERCISE: What is the Accuracy?
acc= (cm.ravel()[0]+cm.ravel()[3])/sum(cm.ravel())
print('Accuracy is: %.4f' %acc)

# EXERCISE: What is the True Positive Rate ?
TPR_logit = cm.ravel()[3]/(cm.ravel()[3]+cm.ravel()[2])
print('TPR is: %.4f' % TPR_logit)
```

```
# EXERCISE: What is the False Positive rate ?
FPR_logit = cm.ravel()[1]/(cm.ravel()[1]+cm.ravel()[0])
print('FPR is: %.4f' % FPR_logit)
```

```
Confusion Matrix :
 [[2410    0]
 [ 445    0]]
Accuracy is: 0.8441
TPR is: 0.0000
FPR is: 0.0000
```

Take away: After running logistic regression, one can change the threshold probability. This will affect the predicted label. A smaller threshold will result in more observations being predicted as positive.

1.2 In-class activity: print out the confusion matrix when the threshold is 0.2

[ ]:
```
y_pred_20perc = pd.Series([1 if x > 0.2 else 0 for x in y_prob], index=y_prob.
 ↪index)
cm = confusion_matrix(y_test, y_pred_20perc)
print ("Confusion Matrix : \n", cm)

acc= (cm.ravel()[0]+cm.ravel()[3])/sum(cm.ravel())
print('Accuracy is: %.4f' %acc)

TPR_logit = cm.ravel()[3]/(cm.ravel()[3]+cm.ravel()[2])
print('TPR is: %.4f' % TPR_logit)

FPR_logit = cm.ravel()[1]/(cm.ravel()[1]+cm.ravel()[0])
print('FPR is: %.4f' % FPR_logit)
```

```
Confusion Matrix :
 [[2313   97]
 [ 394   51]]
Accuracy is: 0.8280
TPR is: 0.1146
FPR is: 0.0402
```

1.3 2. Logistic Regression (Numerical + Categorical Variables)

1.3.1 1.1 Data Loading

[ ]:
```
churn = pd.read_csv("customerchurn.csv")
churn.info()
churn.head()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 7032 entries, 0 to 7031
```

```
Data columns (total 7 columns):
 #   Column           Non-Null Count  Dtype
---  ------           --------------  -----
 0   Churn            7032 non-null   int64
 1   MonthlyCharges   7032 non-null   float64
 2   SeniorCitizen    7032 non-null   int64
 3   PaymentMethod    7032 non-null   object
 4   InternetService  7032 non-null   object
 5   tenure           7032 non-null   int64
 6   Contract         7032 non-null   object
dtypes: float64(1), int64(3), object(3)
memory usage: 384.7+ KB
```

[ ]:
```
   Churn  MonthlyCharges  SeniorCitizen     PaymentMethod InternetService  \
0      0           29.85              0  Electronic check             DSL
1      0           56.95              0      Mailed check             DSL
2      1           53.85              0      Mailed check             DSL
3      0           42.30              0     Bank transfer             DSL
4      1           70.70              0  Electronic check     Fiber optic

   tenure        Contract
0       1  Month-to-month
1      34        One year
2       2  Month-to-month
3      45        One year
4       2  Month-to-month
```

1.3.2 1.2 Train Test Split

[ ]:
```
churn_train, churn_test = train_test_split(churn, test_size=0.3,␣
 ↪random_state=88)
churn_train.shape, churn_test.shape
```

[ ]:
```
((4922, 7), (2110, 7))
```

1.3.3 1.3 Model Fitting and Variable Selection

[ ]:
```
logreg = smf.logit(formula = 'Churn ~ MonthlyCharges + SeniorCitizen +␣
 ↪PaymentMethod + InternetService + tenure + Contract',
                       data = churn_train).fit()
print(logreg.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.421913
         Iterations 8
                           Logit Regression Results
==============================================================================
Dep. Variable:                  Churn   No. Observations:                 4922
```

```
Model:                          Logit   Df Residuals:                     4911
Method:                           MLE   Df Model:                           10
Date:                Wed, 13 Nov 2024   Pseudo R-squ.:                  0.2695
Time:                        18:29:48   Log-Likelihood:                -2076.7
converged:                       True   LL-Null:                       -2842.6
Covariance Type:            nonrobust   LLR p-value:                     0.000
================================================================================
=====================
                                       coef    std err          z      P>|z|
[0.025      0.975]
--------------------------------------------------------------------------------
---------------------
Intercept                           -0.8982      0.213     -4.217      0.000
-1.316      -0.481
PaymentMethod[T.Credit card]        -0.1066      0.134     -0.795      0.427
-0.369       0.156
PaymentMethod[T.Electronic check]    0.4394      0.110      3.984      0.000
0.223       0.656
PaymentMethod[T.Mailed check]       -0.0373      0.135     -0.277      0.782
-0.302       0.227
InternetService[T.Fiber optic]       0.8217      0.154      5.336      0.000
0.520       1.123
InternetService[T.No]               -0.7265      0.185     -3.925      0.000
-1.089      -0.364
Contract[T.One year]                -0.8444      0.124     -6.815      0.000
-1.087      -0.602
Contract[T.Two year]                -1.9124      0.226     -8.466      0.000
-2.355      -1.470
MonthlyCharges                       0.0067      0.004      1.857      0.063
-0.000       0.014
SeniorCitizen                        0.3852      0.097      3.981      0.000
0.196       0.575
tenure                              -0.0296      0.003    -11.559      0.000
-0.035      -0.025
================================================================================
=====================
```

[ ]:
```
# Let's remove MonthlyCharges
logreg2 = smf.logit(formula = 'Churn ~ SeniorCitizen + PaymentMethod +
 ↪InternetService + tenure + Contract',
                       data = churn_train).fit()
print(logreg2.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.422264
         Iterations 8
                           Logit Regression Results
==============================================================================
```

```
Dep. Variable:                  Churn   No. Observations:                 4922
Model:                          Logit   Df Residuals:                     4912
Method:                           MLE   Df Model:                            9
Date:                Wed, 13 Nov 2024   Pseudo R-squ.:                  0.2688
Time:                        18:29:48   Log-Likelihood:                -2078.4
converged:                       True   LL-Null:                       -2842.6
Covariance Type:            nonrobust   LLR p-value:                     0.000
================================================================================
=====================
                                        coef    std err          z      P>|z|
[0.025      0.975]
--------------------------------------------------------------------------------
---------------------
Intercept                            -0.5748      0.122     -4.724      0.000
-0.813      -0.336
PaymentMethod[T.Credit card]         -0.1121      0.134     -0.836      0.403
-0.375       0.151
PaymentMethod[T.Electronic check]     0.4389      0.110      3.982      0.000
0.223       0.655
PaymentMethod[T.Mailed check]        -0.0463      0.135     -0.344      0.731
-0.311       0.218
InternetService[T.Fiber optic]        1.0555      0.090     11.744      0.000
0.879       1.232
InternetService[T.No]                -0.9328      0.147     -6.331      0.000
-1.222      -0.644
Contract[T.One year]                 -0.8110      0.122     -6.622      0.000
-1.051      -0.571
Contract[T.Two year]                 -1.8682      0.225     -8.321      0.000
-2.308      -1.428
SeniorCitizen                         0.3818      0.097      3.948      0.000
0.192       0.571
tenure                               -0.0280      0.002    -11.650      0.000
-0.033      -0.023
================================================================================
=====================
```

[ ]:
```python
# Let's remove PaymentMethod Credit Card and PaymentMethodMailed check. How do
 ↪we do this?
# Create a new feature (dummy variable).

churn_train2 = churn_train.copy()
churn_train2['ElectronicCheck'] = (churn_train2['PaymentMethod'] == 'Electronic
 ↪check').astype('int64')
churn_train2.drop(columns=['PaymentMethod'], inplace=True)

# Let's do the same for test set as well
```

```
churn_test2 = churn_test.copy()
churn_test2['ElectronicCheck'] = (churn_test2['PaymentMethod'] == 'Electronic␣
↪check').astype('int64')
churn_test2.drop(columns=['PaymentMethod'], inplace=True)

churn_test2.head()
```

[ ]:

```
      Churn  MonthlyCharges  SeniorCitizen InternetService  tenure  \
2839      1           99.60              1     Fiber optic       4
4965      0           93.80              1     Fiber optic      13
6686      0           96.55              0     Fiber optic       5
4151      0           25.20              0              No      39
241       0          109.20              0     Fiber optic      72

            Contract  ElectronicCheck
2839  Month-to-month                1
4965  Month-to-month                0
6686  Month-to-month                1
4151        Two year                0
241         Two year                0
```

[ ]:

```
logreg3 = smf.logit(formula = 'Churn ~ SeniorCitizen + ElectronicCheck +␣
↪InternetService + tenure + Contract',
                       data = churn_train2).fit()
print(logreg3.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.422336
         Iterations 8
                           Logit Regression Results
==============================================================================
Dep. Variable:                  Churn   No. Observations:                 4922
Model:                          Logit   Df Residuals:                     4914
Method:                           MLE   Df Model:                            7
Date:                Wed, 13 Nov 2024   Pseudo R-squ.:                  0.2687
Time:                        18:29:49   Log-Likelihood:                -2078.7
converged:                       True   LL-Null:                       -2842.6
Covariance Type:            nonrobust   LLR p-value:                     0.000
================================================================================
==================
                                     coef    std err          z      P>|z|
[0.025      0.975]
--------------------------------------------------------------------------------
------------------
Intercept                         -0.6254      0.085     -7.363      0.000
-0.792      -0.459
InternetService[T.Fiber optic]     1.0549      0.089     11.817      0.000
0.880       1.230
```

```
InternetService[T.No]            -0.9313      0.146     -6.384      0.000
-1.217      -0.645
Contract[T.One year]             -0.8125      0.122     -6.635      0.000
-1.053      -0.573
Contract[T.Two year]             -1.8704      0.224     -8.332      0.000
-2.310      -1.430
SeniorCitizen                     0.3813      0.097      3.945      0.000
0.192       0.571
ElectronicCheck                   0.4901      0.080      6.098      0.000
0.333       0.648
tenure                           -0.0280      0.002    -11.928      0.000
-0.033      -0.023
================================================================================
==================
```

1.3.4 1.4 Predicting the Probability and Deciding the Threshold

[ ]:
```python
# 1. Predicting the probability of default
y_prob = logreg3.predict(churn_test2)

# 2. Determining the optimal threshold of the default probability
## price reduction = 800 and no price reduction = 1000
## 800 (1 - p/2) = 1000 (1 - p) -> p = 0.333 .
## The threshold of high churn-risk and low churn-risk is 0.333.

# 3. Predicting the label
y_pred = pd.Series([1 if x > 1/3 else 0 for x in y_prob], index=y_prob.index)
```

1.3.5 1.5 Confusion Matrix Given a Decision Threshold

[ ]:
```python
# Now we have our probabiltiy of default, we can construct a confusion matrix␣
 ↪based on decision tree threshold we have computed
# price reduction = 800 and no price reduction = 1000
# 800 (1 - p/2) = 1000 (1 - p) -> p = 0.333.
# High churn risk > 0.333.
# or high churn risk, the expected return of discount is higher than no␣
 ↪discount, b/c the prob of retention is lower for no discount

from sklearn.metrics import confusion_matrix
y_test = churn_test2['Churn']

cm = confusion_matrix(y_test, y_pred)
print ("Confusion Matrix : \n", cm)
```

```
Confusion Matrix :
 [[1213  329]
 [ 154  414]]
```

change threshold

```
[ ]: threshold = 1/4
y_pred = pd.Series([1 if x > threshold else 0 for x in y_prob], index=y_prob.
 ↪index)
cm = confusion_matrix(y_test, y_pred)
print ("Confusion Matrix : \n", cm)
```

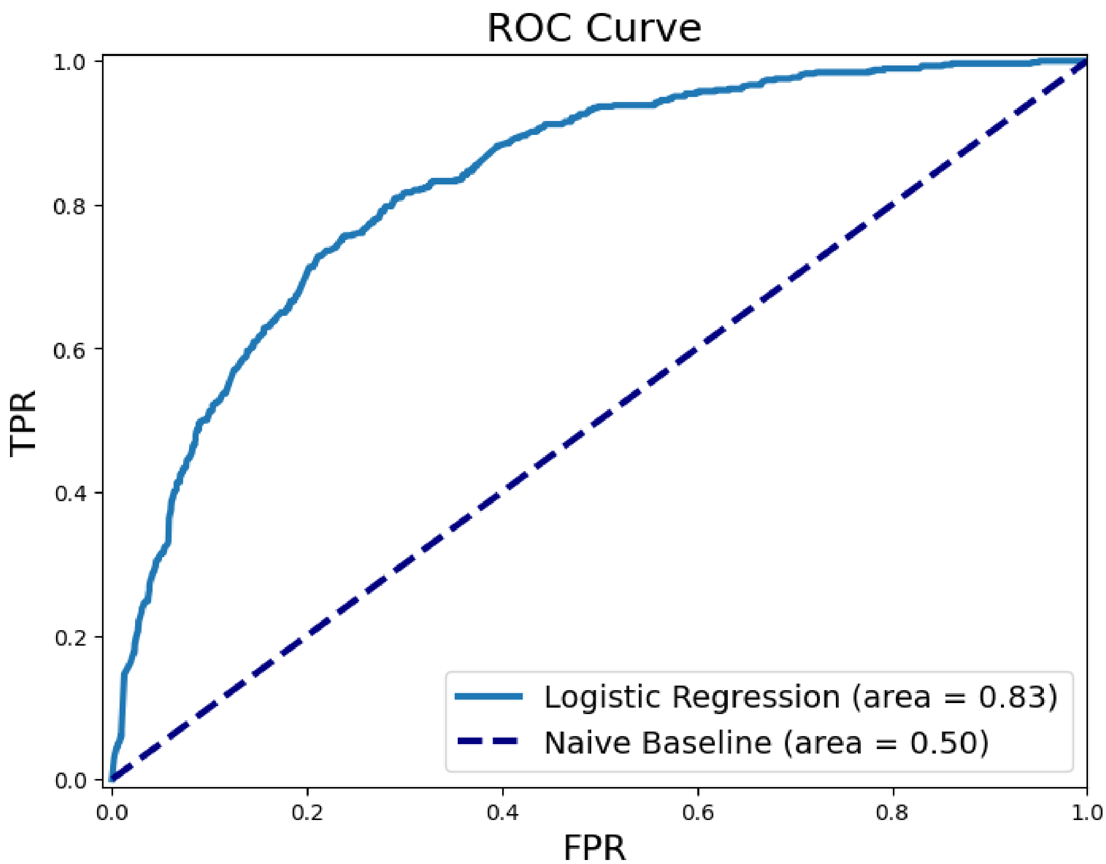
```
Confusion Matrix :
 [[1082  460]
 [ 105  463]]
```

1.3.6 1.7 ROC Curves

The ROC curve plots the TPR and FPR for every break-even threshold p between 0.0 and 1.0

```
[ ]: y_train = churn_train2['Churn']
X_train = churn_train2.drop(['Churn'], axis=1)

y_test = churn_test2['Churn']
X_test = churn_test2.drop(['Churn'], axis=1)
```

```
[ ]: import matplotlib.pyplot as plt
from sklearn.metrics import roc_curve, auc

fpr, tpr, _ = roc_curve(y_test, y_prob)
roc_auc = auc(fpr, tpr)

plt.figure(figsize=(8, 6))
plt.title('ROC Curve', fontsize=18)
plt.xlabel('FPR', fontsize=16)
plt.ylabel('TPR', fontsize=16)
plt.xlim([-0.01, 1.00])
plt.ylim([-0.01, 1.01])
plt.plot(fpr, tpr, lw=3, label='Logistic Regression (area = {:0.2f})'.
 ↪format(roc_auc))
plt.plot([0, 1], [0, 1], color='navy', lw=3, linestyle='--', label='Naive␣
 ↪Baseline (area = 0.50)')
plt.legend(loc='lower right', fontsize=14)
plt.show()
```



A good model has a large area under the ROC curve, want it to get as close to the top left corner as possible.

1.4 2. Linear Discriminant Analysis

1.4.1 2.1 Preliminary: one-hot encoding

[ ]:
```
from sklearn.preprocessing import OneHotEncoder
import pandas as pd

# Example data
df = pd.DataFrame({'Category': ['A', 'B', 'C', 'A']})

# OneHotEncoder = dummy variables
encoder = OneHotEncoder(sparse_output=False)
encoded_data = encoder.fit_transform(df[['Category']])
print(encoded_data)

# pd.get_dummies
dummies = pd.get_dummies(df, columns=['Category'])
```

```
print(dummies)

np.allclose(dummies, encoded_data)
```

```
[[1. 0. 0.]
 [0. 1. 0.]
 [0. 0. 1.]
 [1. 0. 0.]]
   Category_A  Category_B  Category_C
0        True       False       False
1       False        True       False
2       False       False        True
3        True       False       False
```

[ ]: True

[ ]:
```
from sklearn.preprocessing import OneHotEncoder
# initialize the OneHotEncoder
drop_enc = OneHotEncoder(drop='first').
 ↪fit(X_train[['InternetService','Contract']])
print(drop_enc.categories_)
```

```
[array(['DSL', 'Fiber optic', 'No'], dtype=object), array(['Month-to-month',
'One year', 'Two year'], dtype=object)]
```

[ ]:
```
# Perform the transformation for both the training and the test set.

X_train_categorical = drop_enc.
 ↪transform(X_train[['InternetService','Contract']]).toarray()
X_train_numerical = X_train[['MonthlyCharges','SeniorCitizen','tenure']].values
# combine the numerical variables and the one-hot encoded categorical variables
X_train_transformed = np.concatenate((X_train_numerical,X_train_categorical),␣
 ↪axis = 1)

X_test_categorical = drop_enc.transform(X_test[['InternetService','Contract']]).
 ↪toarray()
X_test_numerical = X_test[['MonthlyCharges','SeniorCitizen','tenure']].values
X_test_transformed = np.concatenate((X_test_numerical,X_test_categorical), axis␣
 ↪= 1)
```

1.4.2 2.2 Model Fitting and Prediction Making

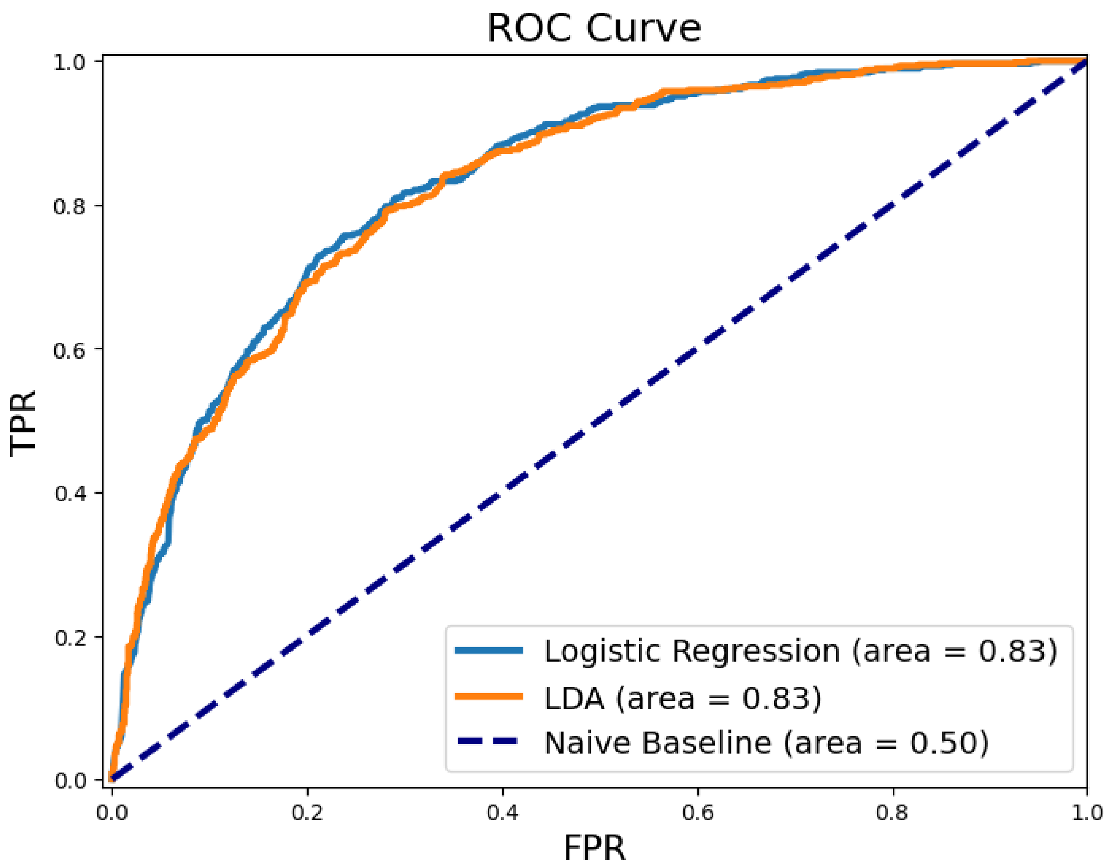
[ ]:
```
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.metrics import accuracy_score

lda = LinearDiscriminantAnalysis()
lda.fit(X_train_transformed, y_train)
```

```
y_prob_lda = lda.predict_proba(X_test_transformed)
y_pred_lda = pd.Series([1 if x > 1/3 else 0 for x in y_prob_lda[:,1]],␣
 ↪index=y_prob.index)

cm = confusion_matrix(y_test, y_pred_lda)
print ("Confusion Matrix: \n", cm)
print ("\nAccuracy:", accuracy_score(y_test, y_pred_lda))
```

```
Confusion Matrix:
 [[1206  336]
 [ 163  405]]

Accuracy: 0.7635071090047393
```

1.4.3 2.3 Plot the ROC Curve

[ ]:
```
fpr_lda, tpr_lda, _ = roc_curve(y_test, y_prob_lda[:,1])
roc_auc_lda = auc(fpr_lda, tpr_lda)

plt.figure(figsize=(8, 6))
plt.title('ROC Curve', fontsize=18)
plt.xlabel('FPR', fontsize=16)
plt.ylabel('TPR', fontsize=16)
plt.xlim([-0.01, 1.00])
plt.ylim([-0.01, 1.01])
plt.plot(fpr, tpr, lw=3, label='Logistic Regression (area = {:0.2f})'.
 ↪format(roc_auc))
plt.plot(fpr_lda, tpr_lda, lw=3, label='LDA (area = {:0.2f})'.
 ↪format(roc_auc_lda))
plt.plot([0, 1], [0, 1], color='navy', lw=3, linestyle='--', label='Naive␣
 ↪Baseline (area = 0.50)')
plt.legend(loc='lower right', fontsize=14)
plt.show()
```


ROC Curve
TPR
FPR
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Logistic Regression (area = 0.83)
LDA (area = 0.83)
Naive Baseline (area = 0.50)