Artificial Self-insurance for Heterogeneous Households

Using GQ-learning

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Contribution

Merge causal inference methods with reinforcement learning

This method could

- Relax assumptions on laws of motion while keeping casual interpretation.
- Use causal structural estimates to find decision rules.
- Use decision rules to simulate counterfactuals for policy analysis.
- Use sparse-on-time individual panel data with higher frequency and high dimension macro time series.

References

- HANK model: compute policy functions at a steady state to respond to shocks: Melcangi and Sterk (2024), Acharya et al. (2023), Luetticke (2021), Kaplan et al. (2018).
- Reinforcement learning applications: Cong et al. (2022), Rao and Jelvis (2023), or simply a policy function generator: Fatih and Paolo (2022), Moll (2024).
- Reduce the high-dimensional macro state space: Bayer et al.,(2024),Fernandez-Villaverde et al. (2021), Han et al. (2021), and Payne et al., 2024.

Based on the idea of

1. **Reinforcement learning:** Sutton and Barto (2013)
2. **G-estimation:** Robins et al. (1992a), Lewis and Syrgkanis (2021), Yang (2025).
3. **Adaptive learning:** Williams (2024), Evans and Honkapohja (2001), and Marcet and Sargent (1989)

Basic Algorithm

The basic algorithm includes:

- G-estimation, a method that predicts future micro-level states using past choices and current macro state.
- Q-learning, a reinforcement learning algorithm that derives policy functions through reward optimization.

GQ-learning combines the two, which is an algorithm for reinforcement learning.

G-estimation

Aims at estimating the non-linear law of motion $f(\cdot)$:

$$S_{i,t+1} = f(a_{i,t}, S_{i,t}, L_t | x_i) \tag{1}$$

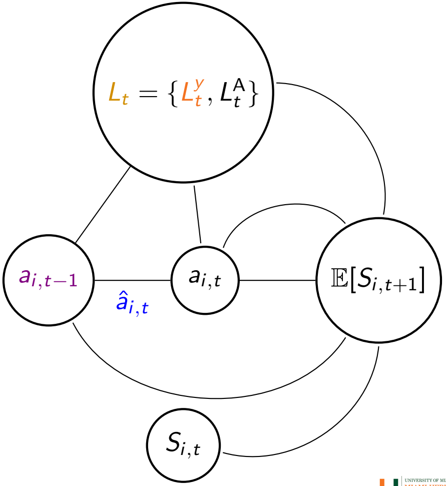
- $S_{i,t+1} = \{w_{i,t+1}, y_{i,t+1}\}$, micro-level state variables include wealth and income.
  - State variables.
- $a_{i,t} = \{\pi_{i,t}, c_{i,t}\}$, choice variables (action space) include portfolio choice $\pi_{i,t}$ and the consumption-to-wealth ratio $c_{i,t}$.
  - Action space.
- $L_t = \{L_t^y, L_t^A\}$, macro-level state includes the source of shock we are interested in L_t^y, and a component L_t^A.
  - Environment.
- x_i time invariant household specific variables.

Reference:Robins et al. 1992a,Robins, 1986.

G-estimation

G-estimation Stage 1:

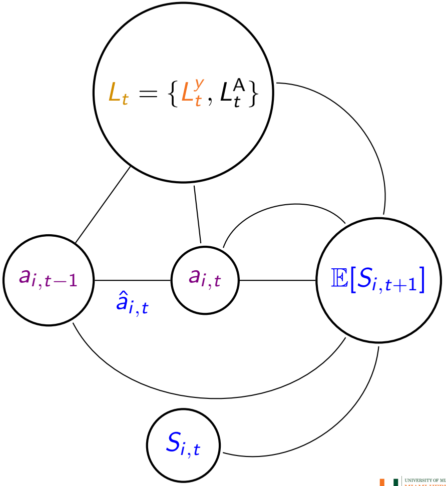
$$\mathbb{E}[\hat{a}_{i,t} | a_{i,t-1}, x_i, L_t^y, L_t^A)] \\ = \eta_0 + \eta_1 a_{i,t-1} + \eta_2 x_i + \eta_3 L_t^y + \eta_4 L_t^A$$



G-estimation

G-estimation Stage 2: Structural estimation for the future state

$$
\begin{aligned}
\mathbb{E}[S_{i,t+1} | a_{i,t}, a_{i,t-1}, x_i, S_{i,t}, L_t^y, L_t^A] \\
= & \beta_0 + \underbrace{\beta_1 a_{i,t-1} + \beta_2 S_{i,t} + \beta_3 x_i + \beta_4 [L_t^y + L_t^A]}_{\text{immediate effect}} \\
& + \beta_5 \underbrace{(1 + L_t^y + L_t^A)\hat{a}_{i,t}}_{\text{adjustment term of expectation}} \\
& + \psi \underbrace{(1 + L_t^y + L_t^A)a_{i,t}}_{\text{treatment effect}}
\end{aligned}
$$



Baseline model to Q-learning

The law of motion:

$$w_{i,t+1}(L_t) = \underbrace{[(1 + r^f) + \pi_{i,t}(r^s_{i,t}(L_t) - r^f)]}_{1+r_{i,t}}(w_{i,t}(L_t) - C_{i,t}) + y_{i,t}(L_t) \tag{2}$$

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Baseline model

$$V(S_{i,t}, L_t) = \max_{a_{i,t}}\{u(C_{i,t}) + \beta\mathbb{E}_t[V(S_{i,t+1}, L_{t+1})]\} \tag{3}$$

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The estimated law of motion:

$$\begin{aligned} S_{i,t+1} &= f(a_{i,t}, S_{i,t}, L_t|x_i) \\ L_{t+1} &= g(L_t, \epsilon_{t+1}) \end{aligned}$$

Given an arbitrary action $\bar{a}$, the Q-learning process is:

$$Q(\bar{a}, S_{i,t}, L_t) = u(\bar{c}) + \beta\mathbb{E}_t \max_a Q[a, f(\bar{a}, S_{i,t}, L_t|x_i), L_{t+1}]$$

Algorithm

A toy model - linear G-estimation with the temporal difference (TD) method of Q-learning

Set an initial Q-value for each agents $Q_{i,0}$

$\downarrow$

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With the current $a_{i,t}$, we could predict $w_{i,t+1}$ with the G-estimation, and update the Q-function:

$$\max_{c,\pi} Q_t(c, \pi, w_{t+1}(c_j, \pi_k)) = \max_{j,k} Q_t(j, k, n)$$

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Solve this to get the counterfactual optimal action $a^*_{i,t} = \{c^*_i, \pi^*_k\}$

Algorithm

The iteration method used here is the Temporal-Difference (TD). The Q-function is updating on a 3-D grid by ϵ-greedy methods

- **if exploitation** - choose actions according to the maximization.

$$Q_{t+1}\left(c_t,\pi_t,w_t\right)=Q_t\left(c_t,\pi_t,w_t\right)+\gamma*\left[u\left(c_t,w_t\right)+\beta Q_t\left(c^*,\pi^*,w_{t+1}\right)-Q_t\left(c_t,\pi_t,w_t\right)\right]$$

- **if exploration** - choose actions randomly from the grid.

$$Q_{t+1}\left(c_t,\pi_t,w_t\right)=Q_t\left(c_t,\pi_t,w_t\right)+\gamma*\left[u\left(c_t,w_t\right)+\beta Q_t\left(c_j,\pi_k,w_{t+1}\right)-Q_t\left(c_t,\pi_t,w_t\right)\right]$$

Stop when $\left|\left(Q_{n,t+1}-Q_{n,t}\right)/Q_{n,t}\right|<$ threshold - converges.

GQ-algorithm application to the real-world data

How do households make portfolio choices facing income and macro shocks?

Data:

Micro-level (PSID) 2009-2021 $S_{i,t}$, $a_{i,t}$, x_i

- Wealth and labor income $S_{i,t} = \{w_{i,t}, y_{i,t}\}$
- Stock value and consumption/wealth ratio $a_{i,t} = \{\pi_{i,t}, c_{i,t}\}$
- Baseline feature x_i

Macro-level (FRED) $L_t = \{L_t^y, L_t^s, r_t^f, L_t^A\}$

- High-dimensional macroeconomic variables L_t^A.
- Deflated S&P500 index L_t^s
- 3-Month Treasury Bill Secondary Market Rate r_t^f
- Aggregate income L_t^y
  "Real Rersonal Income Excluding Current Transfer Receipt".

G-estimation

Table: The second stage regression of G-estimation with PSID (2009-2021)

	$w_{i,t+1}$	$y_{i,t+1}$
$\pi_{i,t-1}$	$-4.252e+06$	$1.416e+04$
Last portfolio choice	$(4.04e+06)$	$(9.3e+04)$
$w_{i,t}$ or $y_{i,t}$	-0.10	0.69^{***}
Current states	(0.19)	(0.16)
x_i	71.78	-1.48
Initial inheritance	(82.37)	(1.85)

w is not well fitted: $R^2(w) = 0.711$ and adjusted $R^2(w) = 0.421$

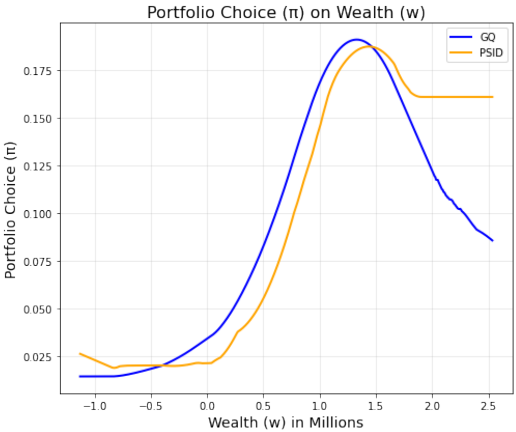
y is well fitted: $R^2(y) = 0.988$ and adjusted $R^2(y) = 0.976$

	$w_{i,t+1}$	$y_{i,t+1}$
L_t^y	-854.48	-10.38
Real personal income	(701.67)	(16.15)
L_t^s	-2428.33	-7.01
$S\&P500_t$	(2658.62)	(60.15)
r_t^f	$-3.212e+8^{*}$	$-5.06e+5$
risk free rate	$(1.36e+8)$	$(3.19e+6)$
L_t^{A1}	$6.269e+8^{*}$	$9.89e+5$
component 1	$(2.65e+05)$	$(6.22e+6)$
L_t^{A2}	$3.786e+06$	$2.101e+04$
Component 2	$(3.43e+05)$	$(7.78e+04)$
$(1+L_t)\times\hat{\pi}_{i,t}$	-3070.33	32.35
Effect of expectation	(3369.29)	(75.61)
$(1+L_t)\times\pi_{i,t}$	29.04	-3.94
Joint effect	(116.669)	(2.35)

Households' wealth levels are more affected by the macroeconomic factors, while labor income is more persistent.

Policy functions by GQ-learning compared with the real data

Reward function: $U(C) = \frac{C^{1-\gamma}}{1-\gamma}$, $\gamma = 0.85$, discount factor $\beta = 0.95$.



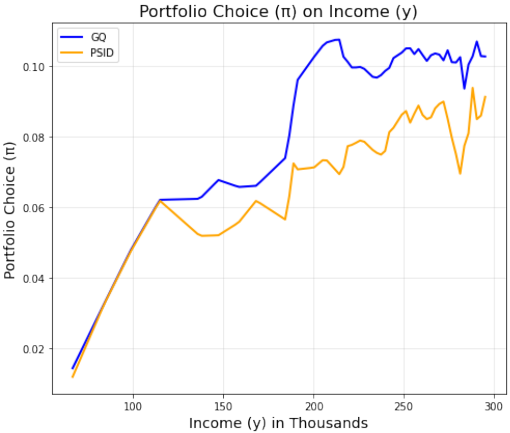


Figure: Recovery Policy Functions for Portfolio Choice.

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GQ-learning results: Consumption ratio $c_{i,t}$

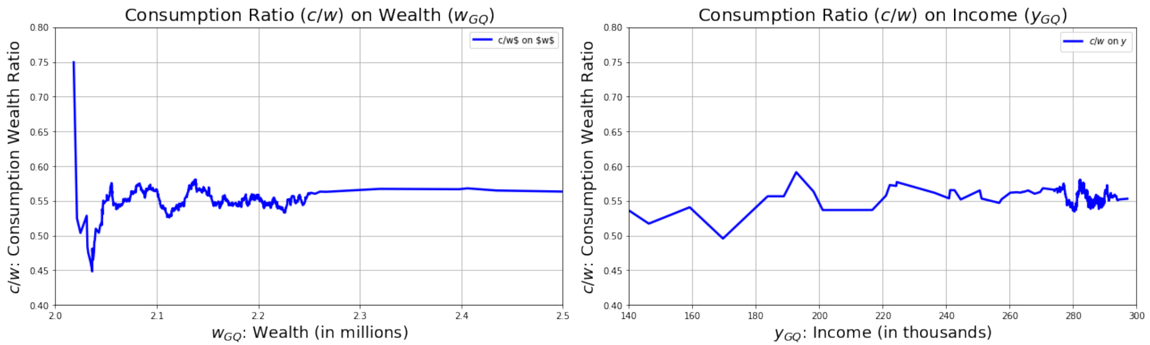
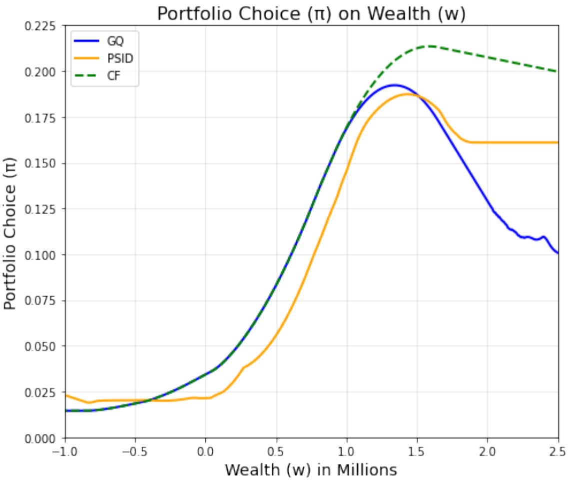


Figure: Recovery Policy Functions for Consumption Ratio.

This is not read from the data

π of a counterfactual low income level



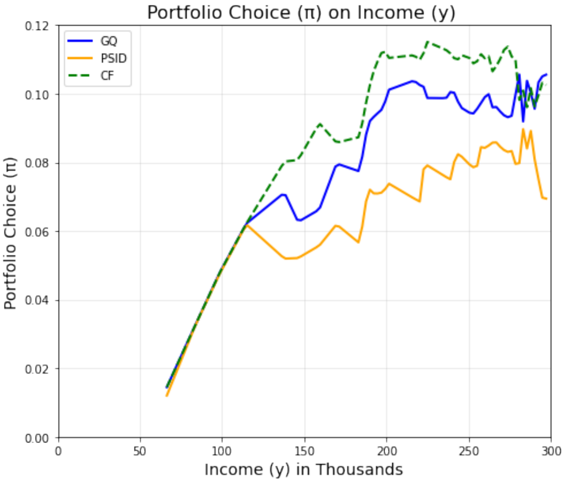
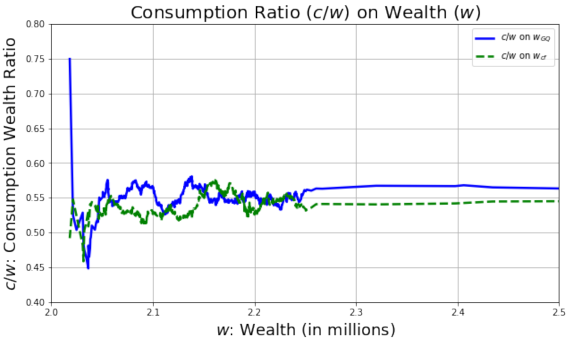
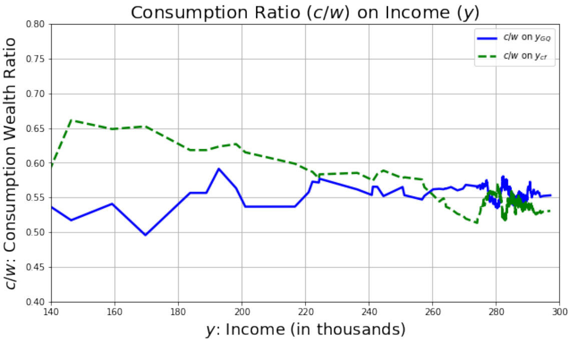


Figure: Aggregate income level is 8 trillion (16.2 trillion in Jan.2024)

c/w of a counterfactual low income level





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π of a counterfactual decrease real risk-free rate

Figure: If cut r_f by 1% (the mean of r_f is 1.18%)

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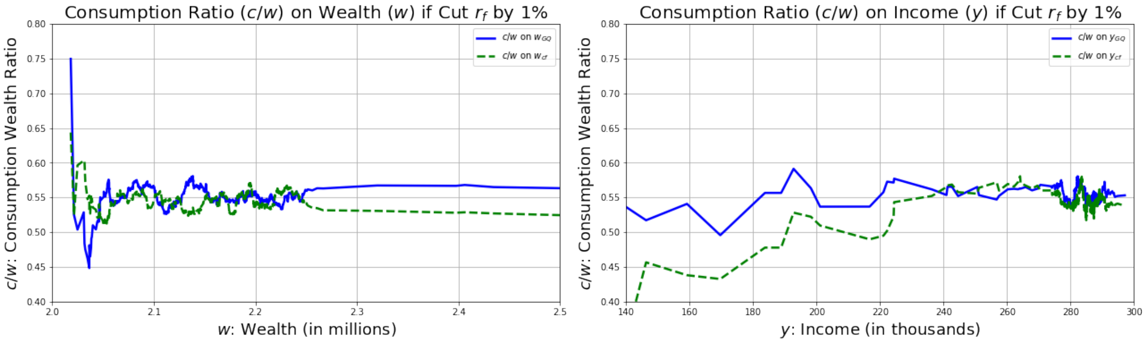

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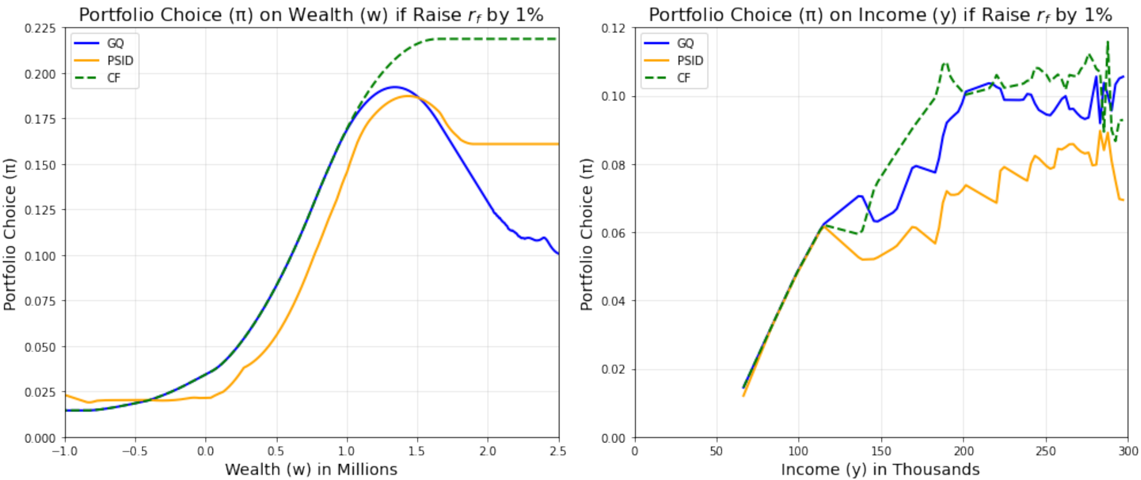
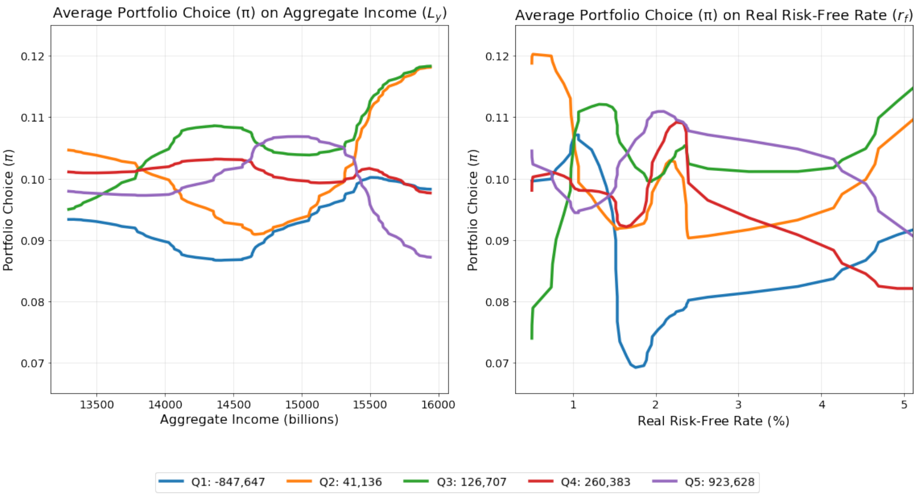


Figure: If raise r_f by 1% (the mean of r_f is 1.18%)

Observe choices at finer granular



Households are in quintile based on their wealth in 2009.

The End