Lab 2: Parking Guidance System

Due: Friday 2/26 at 11:59pm

Background

The University of California, Berkeley is going to develop a Parking Guidance and Information (PGI) system based on smart-phone Apps. This PGI system is designed to aid in the search for vacant parking spaces by directing drivers to parking lots where occupancy levels are low. The objective is to reduce search time, which in turn reduces congestion on the surrounding roads with related benefits to air pollution. Thus improves the parking experience.

A simple case is illustrated in Figure 1. A driver will check their smartphone app before commuting and receive an instruction of where to park. Suppose this driver will strictly follow the parking guidance and park on the instructed parking lot. The optimization problem is to assign N drivers into M parking lots while minimizing the total time cost of parking. The search time in parking lot i follows the congestion function developed by the Bureau of Public Roads (BPR), which we will term $S_i(q)$.

$$S_i(q_i) = t_{0i} e^{\alpha_i (\frac{q_i}{C_i})^{\beta_i}}$$

where
t_{0i} = search time when the lot is empty.
C_i = operational capacity of lot i.
q_i = number of occupied spaces in lot i
β_i and α_i are parameters associated to this function. You will be asked to interpret these two parameters later.

The total time cost of parking in lot i, denoted as $T_i(q_i)$, also includes the time to walk into campus, t_{wi}.

$$T_i(q_i) = S_i(q_i) + t_{wi}$$

Assume all drivers have the same destination which is the center of the campus. Then t_{wi} will be the same for all drivers parking in lot i.

There are over 40 parking lots near the Berkeley campus. In this lab assignment we only consider 6 large ones, e.g. Lower Hearst, Upper Hearst, Foothill, Bancroft, Underhill and the Berkeley Way Lot. Parameters associated to these parking lots are in table 1.

Figure 1: Visualization of smarphone based PGI system.

Table 1: Parameters for the parking time function

Parking Lots	α	β	C	t_0	t_w
Lower Hearst(q_1)	0.92	1.91	600	2	3
Upper Hearst(q_2)	0.94	1.84	280	2	4
Foothill(q_3)	0.16	2.02	183	1	13
Bancroft(q_4)	0.4	4.19	232	1	5
Underhill(q_5)	1.07	1.93	920	2	7
Berkeley Way(q_6)	0.51	4.23	100	1	7

Questions

1. **The BPR function** Plot the parking search time functions $S_i(q_i)$ with respect to the ratio of occupancy, $\frac{q_i}{C_i}$, for our 6 parking lots. Let α, β, t_0 be the values in table 1 and $\frac{q_i}{C_i}$ vary from 0 to 1.5. (When q_i is greater than C_i, drivers are parked in the corridors of the lot. In this case, parking search time is much longer.)

   For the following problems, assume that once a driver decides to park the car into parking lot i, he or she will stay at i.

2. **Parking Information System**. The parking guidance system starts to send instructions 8 am after everyday. Before 8 am in the morning the PGI system shows all available parking spaces in every parking lot. In this case drivers have perfect knowledge about parking costs and choose the parking lot with minimal time cost. This behavioural assumption leads to deterministic user equilibrium. [1]

[1] Further readings about traffic assignment and deterministic user equilibrium Click Here

This problem is equivalent to the following nonlinear optimization program, denoted as NLP_U:

$$NLP_U : \min_{q_i} Z^U = \sum_{i=1}^{6} \int_0^{q_i} T_i(x)dx$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{6} q_i = Q_0 \\ & q_i \geqslant 0 : \ \forall i \end{aligned}$$

where Q_0 is the total number of drivers arriving before 8 am.

(a) In this part, let Q_0 be 3000 and ignore the inequality constraint on q_i. Rewrite the equality constraint into the objective function, where $q_6 = Q_0 - \sum_{i=1}^{5} q_i$. Therefore NLP_U becomes an unconstrained nonlinear problem. Apply gradient descent method with constant step size to find the solution for NLP_U. (Stop at k^{th} iteration if $\sum_{i=1}^{5} |q_i^{k+1} - q_i^k| < 10^{-6}$). Create a table to report your solution, including q_i*, $S(q_i*)$ and $T(q_i*)$. Briefly discuss your findings. [2]

(b) Claim:
Under deterministic user equilibrium, if the solutions q_i* are all strictly positive, the parking times in each parking lot $T(q_i*)$ will be equal.
Prove this claim by using Lagrangian multiplier λ for the equality constraint. What does the Lagrangian multiplier at the optimal solution λ^* mean in practice? Could you solve λ^* from part 2(b)?

(c) Write down the KKT condition for NLP_U. Prove that if $T_i(0) > -\lambda^*$, no one will park on lot i ($q_T* = 0$).
Hint: for a general programming problem:

$$\begin{aligned} &\min : f(x) \\ &st : h_i(x) = 0 \\ &g_j(x) \leq 0 \end{aligned}$$

We write the Lagrangian as:

$$L(x, \lambda_i, \mu_j) = f(x) + \sum_i \lambda_i h_i(x) + \sum_j \mu_j g_i(x)$$

[2]
Think before you code.
Design your math equations before coding.
Design your algorithm and variables before coding.
Be careful about the correspondence of parameters and variables.

(d) In this part, let Q_0 be 2000. Use part 2(d) to prove that:
The inequality constraint on q_i will be active for Foothill lot, while the q_i* are strictly positive for other five lots.

(e) In this part, let Q_0 vary from 2000 to 3000 with step size 100. Solve for q_i* with respect to different values of Q_0. Create a plot to report your q_i*. Briefly discuss your findings. (Hint: q_i* are functions of Q_0. Figure 2 shows an example of a part of these functions with $Q_0 \in [0, 2000]$. You should plot the 2000 to 3000 part.)

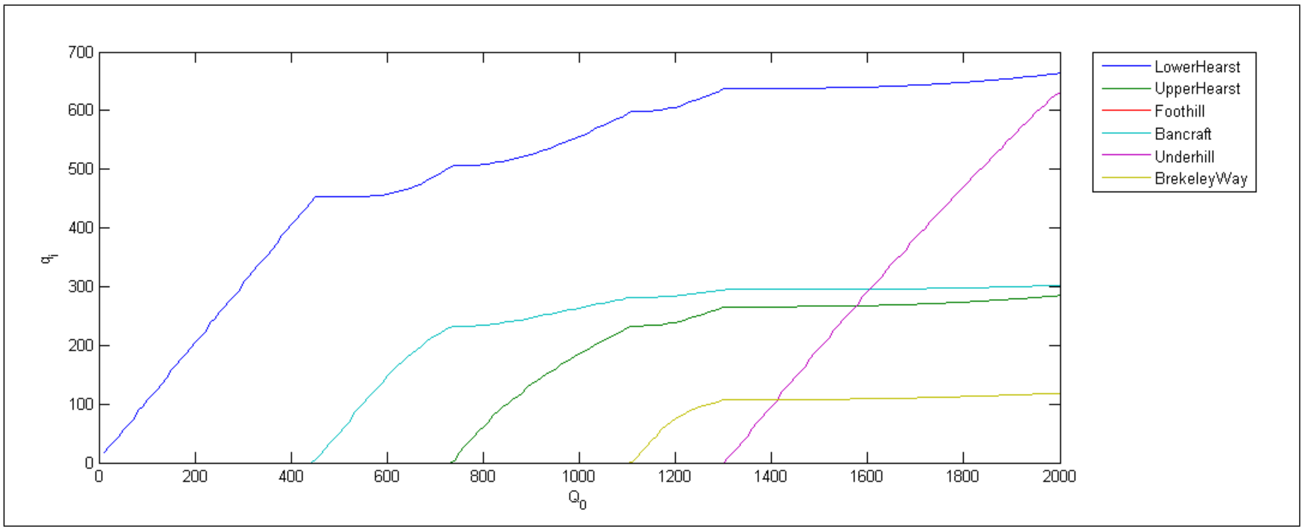

Figure 2: q_i* are functions of Q_0.

3. **Parking Guidance System** Assume at 8:01 am, Q_1 drivers arrive at the campus lots and search for parking space at the same time. The PGI system also starts to send instructions to them. The purposes of the Parking Guidance system is to minimize total system travel time. This assignment can be thought of as a model in which congestion is minimized when drivers are told which parking lots to use. (This may not a behaviorally realistic model, but it can be useful to transport planners and engineers, trying to manage the traffic to minimize travel costs and therefore achieve an optimum social equilibrium.) This problem is equivalent to the following nonlinear optimization program, denoted as NLP_S:

$$NLP_S : \min_{q_i} Z^S = \sum_{i=1}^{6} T(q_i + q_i*)q_i$$

$$\begin{aligned} s.t. \quad & \sum_{i=1}^{6} q_i = Q_1 \\ & q_i \geqslant 0 : \ \forall i \end{aligned}$$

where q_i^* is the solution to question 2(a).

(a) Let Q_1 be 500 and solve for the NLP_S with the method in part 2(b). However, instead of a constant step size, apply backtracking line search to find the optimal solution. You can ignore inequality constraint on q_i. Create a table to report your solution, including q_i^{*S}, $S(q_i^{*S})$ and $T(q_i^{*S})$. What is the total parking time for these $Q1$ drivers (What is $Z^S(q_i^{*S})$)?

(b) Let Q_1 be 500. Assume the parking guidance system does not exist and drivers behave as the same as question 2. Solve for the new user equilibrium parking assignment q_i^{*U} using method in part 2(b). What is the total parking time for these $Q1$ drivers (What is $Z^S(q_i^{*U})$)? Is a parking guidance system necessary?.

1 Sample Code

1.1 Backtracking Line Search

```
function alphak = linesearch(f,d,x,rho,c)
%function alphak = linesearch(f,d,x,rho,c)
%Backtracking line search
%See Algorithm 3.1 on page 37 of Nocedal and Wright
%Input Parameters :
%f: MATLAB file that returns function value
%d: The search direction
%x: previous iterate
%rho :- The backtrack step between (0,1) usually 1/2
%c: parameter between 0 and 1 , usually 10^-2
%Output : alphak: step length calculated by algorithm

 alphak = 1;
[fk, gk] = feval(f,x);% gk is the gradient at x, fk is the fucntion value at x
xx = x;
x = x - alphak*d;
fk1 = feval(f,x);
while fk1 > fk - c*alphak*(gk'*d)
alphak = alphak*rho;
x = xx - alphak*d;
fk1 = feval(f,x);
end
```