

## **Business Report for DHL Supply Chain**

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# **I. Executive Summary**

## **INCENTIVES**

DHL Supply Chain is a transportation company that transports commercial products to different places in the world. It is also a part of the Go Green environmental protection program initiated by its parent firm, Deutsche Post DHL. As the 2009 World Economic Forum publication had declared, the annual greenhouse gas emissions have reached around 50,000 mega-tonnes of CO<sub>2</sub> per year and around 5.5% are contributed by the logistics and transport sector, which is a firm that DHL Supply Chain is majoring in. The company itself aims to be environmentally sustainable. Therefore, this report is going to demonstrate how the company is going to use its budget (3.3 billion as proposed by the problem) efficiently while at the same time minimizing carbon dioxide (CO<sub>2</sub>) emission in the transportation of LCD TVs from their manufacturing bases to the respective distribution centre.

## **PRELIMINARY SUGGESTIONS**

- *Choose More Environmentally-Friendly Transportation Methods:* In general, water shipments have the lowest CO<sub>2</sub> emission, road shipments have moderate CO<sub>2</sub> emission, and air shipments have the highest CO<sub>2</sub> emission. Interestingly, air shipments are also the most expensive, and water shipments are the cheapest. Hence, we suggest assigning as many shipping units into water transportation as possible to minimize the emission of CO<sub>2</sub> and the shipping cost.
- *Minimize Unused Shipments:* Assign the units shipped by Air, Road, and Rail transportation to the exact minimum amount of required units in accordance to the Exhibit 1 data sheet to utilize water transportation as much as possible.

- *Choose Regular Air and Road Network as air and road transportation:* We have two different modes of air transportation (i.e. Regular Air and Air Express) and three different modes of road transportation (i.e. Road, Road LTL, and Road Network). To satisfy the minimum shipment required for air and road transportation, we suggest choosing Regular Air and Road Network as primary air and road transportation for their lower CO<sub>2</sub> emission per Ton-Km.
- *Increasing Overall Budget:* As suggested by the problem, increasing the budget might be beneficial for reducing the CO<sub>2</sub> emission for the budget can be used to produce more LCD TVs in ODMs that are closer to DC. Though these ODMs generally have higher production/shipping cost, they will produce lower CO<sub>2</sub> emission during transportation.

### **QUESTIONS TO BE ADDRESSED**

- The given table suggests that shipment costs are higher for those ODMs that are further away from the DC. However, these ODMs usually also have lower production/shipping costs. In this report, we will seek ways to balance shipping costs and production/shipping costs while minimizing CO<sub>2</sub> emission.
- If increasing the budget can lead to lower CO<sub>2</sub> emissions, to what extent can we increase the budget while still making meaningful reductions in CO<sub>2</sub> emission?

### **SUGGESTED PLANS AFTER DATA ANALYSIS**

With a budget of 3.3 billion (after 10% increase from 3 billion), we proposed the following production/shipping plan to satisfy all the demands and constraints while minimizing the CO<sub>2</sub> emissions = 3406.42 ton (Following table 9.2 in Appendix):

- ❑ produce/ship 228000 units LCD 42" from ODM 4 by water

- ❑ produce/ship 92000 units of LCD 42” from ODM 5 by road network
- ❑ produce 600000 units of LCD 42” from ODM 7, and ship 46000 units by regular air, and 138000 units by rail, and 416000 units by water
- ❑ produce/ship 259062 units of LCD 32” from ODM 1 by water
- ❑ Produce 270938 units of LCD 32” from ODM 2, and ship 53000 units by regular air, 79500 units by road network, 79500 units by rail, and 58938 units by water

### **INTERPRETATIONS OF THE GIVEN PLAN**

ODM 4, ODM 5, and ODM 7 are chosen to be the manufacturing ODMs for LCD 42”, and ODM 1 and ODM 2 are chosen to be the manufacturing ODMs for LCD 32”.

Based on Table 9.1, there are several different reasons as to why ODM 4, 5 and 7 are chosen to be the manufacturing ODMs for LCD 42”. ODM 4 has the lowest production cost for LCD 42”. Therefore, despite the relatively higher shipping cost and distance to DC, it is still efficient cost wise. ODM 7 has the second lowest water transportation cost and is the second closest ODM to DC. Given water shipments produce the least CO<sub>2</sub> emission per Ton-Km, ODM 7 is one of the most efficient ODMs that should be utilized. Even though ODM 2 has the lowest cost of water transportation, it is less efficient compared to ODM 7 for it’s more than twice as far to DC as ODM 7. Therefore ODM 2 is not efficient for minimizing CO<sub>2</sub> emissions. ODM 5 is also utilized in the proposed plan for it is the closest ODM to DC, which makes it advantageous for reducing the CO<sub>2</sub> emissions.

To satisfy the minimum number of LCD 32” to be shipped, it is better to utilize ODM 2 to satisfy minimum shipments of air/road/rail transportation for it’s closer to DC compared to ODM 1. However, ODM 1 can become advantageous for its lower production and shipment cost

under certain circumstances (i.e. when the number of production units is large). Hence, both ODM 1 and ODM 2 are utilized for satisfying the demand/constraints of LCD 32”.

In conclusion, the given plan mainly addresses three factors of ODMs: distance to DC, production cost and water transportation cost. The chosen ODMs might not have the most optimal values for all of the factors, but they are at least advantageous for some of the factors.

### **FURTHER INVESTIGATIONS TO REDUCE CO<sub>2</sub> EMISSION**

To satisfy all the constraints and demands, a budget of at least 2.999 billion is needed.

The following table shows the optimal CO<sub>2</sub> emission at different budgets:

Budget	Manufacturing ODMs	Optimal CO <sub>2</sub> Emission
3 billion	1,4	7401.25 tons
3.3 billion	4,5,7	3406.42 tons
3.6 billion	5,7	3306.15 tons
3.9 billion	5,7	3293.33 tons

As the given budget increases from 3 billion to 3.3 billion, the optimal CO<sub>2</sub> emission decreases drastically by 53%. However, starting from 3.3 billion, every 0.3 billion raise in budget can only reduce the optimal CO<sub>2</sub> emission by about 3%. The optimal CO<sub>2</sub> emission won't decrease further even if we increase the budget to be greater than 3.9 billion.

Given a budget of 3 billion, ODM 5 and ODM 7 are not chosen to be manufacturing ODMs though they are two of the closest ODMs to DC. ODM 1 and ODM 4 are chosen mainly because of the low production cost and shipping cost. CO<sub>2</sub> emission depends on the weight of products, the distance between ODMs and DC, and the CO<sub>2</sub> emission per Ton-Km unit. It is not realistic to change the last two factors, but the company might be able to reduce the weight of TVs. However, according to sensitivity analysis of ODM 5, mass reduction will not be an

effective way to make ODM 5 more advantageous in reducing CO<sub>2</sub> emission because ODM 5 (the closest ODM to DC) is not chosen at a 3.0 billion budget not because of the weight of the product but mainly because of an insufficient budget. Hence, when at a 3.0 billion budget, budget increase will be extremely crucial for reducing CO<sub>2</sub> emission.

**The following discussion will be based on a budget of 3.3 billion:**

Transportation by water should receive the most attention for it creates the least amount of CO<sub>2</sub> emission per Ton-Km unit shipment. Hence, ODMs with the relatively low water transportation cost such as ODM 2, ODM 4, and ODM 7 should be prioritized. ODM 7 is particularly important because it has low water transportation cost, and it is the one of the closest ODMs to DC. Sensitivity analysis indicates that increasing the maximum production capacity of ODM 7 will be beneficial for reducing CO<sub>2</sub> emission. Every unit increase in the maximum production capacity of ODM 7 will result in 0.134 kg decrease in optimal CO<sub>2</sub> emission when all other conditions are kept constant.

In this problem, the company needs to satisfy the minimum shipment requirements for different transportation methods. Decreasing the minimum shipment constraints for air/road/rail transportation will allow more shipments by water transportation, which is more environmentally friendly. Every unit of decrease in the required minimum shipments by regular air/air express of LCD 42"/LCD 32" will result in 23.04/37.91 kg decrease in the optimal CO<sub>2</sub> emission. Decreasing the minimum shipping requirements by road/road LTL/road network can also reduce the optimal CO<sub>2</sub> emission, but their effects are not as prominent. Every unit of decrease in the minimum shipments by road/road LTL/road network of LCD 42"/LCD 32" will reduce the optimal CO<sub>2</sub> emission by 0.259/1.422 kg. For rail transportation, every unit of decrease in the minimum shipments by rail of LCD 42"/LCD 32" will reduce the optimal CO<sub>2</sub> emission by

0.368/0.584 kg. Hence, if possible, more attention should be given to reducing the minimum requirement of air shipments. However, air transportation is oftentimes used for urgent shipments. Hence, stocking the TV via water transportation during seasons with less demand can help the company to reduce CO<sub>2</sub> emission while having more freedom to keep the cost low.

## **CONCLUSION & RECOMMENDATIONS: FUTURE PLANS BY SENSITIVITY**

### **ANALYSIS**

In addition to the preliminary suggestion:

- Choose more environmentally-friendly transportation methods
- Minimize unused shipments
- Choose regular air and road network as air and road transportation
- Increasing overall budget

After looking at the sensitivity analysis, some of the best ways to reduce CO<sub>2</sub> emission are:

- Raise the budget as much as possible up to 3.9 billion.
- Expand the maximum production capacity of ODM 7 when at a 3.3 billion budget, and the capacity of ODM 4 when at a 3.0 billion budget.
- If possible, the company can choose to further expand the inventory amount. Ship TVs by water transportation and store them at DC during periods with low demands so that regular air/air express can be used less during periods with higher demands.

### 1. TABLE 9.1: DATA SHEET IN DETERMINING CONSTRAINTS

## DATA SHEET

Supply Chain Production and Shipping Constraints	
Minimum production of LCD42* (ditto for LCD32*) at any manufacturing ODM	200,000 units
Maximum production of LCD42* at manufacturing OEM (ditto for LCD32*) at manufacturing ODM	600,000 units
Minimum number of units of LCD42* (LCD32*) to be shipped by Regular Air or Air Express	46,000 (53,000)
Minimum number of units of LCD42* (LCD32*) to be shipped by Road or Road LTL or Road Network	92,000 (79,500)
Minimum number of units of LCD42* (LCD32*) to be shipped by Rail	138,000 (79,500)

<sup>1</sup> Arithmetically, a TV weighing say 20 kgs and shipped by road over 800 kms will emit  $(20 / 1000) \times 0.0613 \times 800$  or about 0.98 kg of CO<sub>2</sub>.

**2. TABLE 9.2: TABLE SOLUTION FOR EACH UNITS SHIPPED BY DIFFERENT**

ODM[illegible]



LCD 32" ODM 2	53000	0	0	0	79500	79500	58938	270938
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### 3. LINEAR PROGRAMING FORMULATION

- *Variable X*

Variables No.	Explanations
1,9,17,25,33,41,49	Units of LCD42" shipped by regular air from ODM1-7
57,65	Units of LCD32" shipped by regular air from ODM1-2
2,10,18,26,34,42,50	Units of LCD42" shipped by air express from ODM1-7
58,66	Units of LCD32" shipped by air express from ODM1-2
3,11,19,27,35,43,51	Units of LCD42" shipped by road from ODM1-7
59,67	Units of LCD32" shipped by road from ODM1-2
4,12,20,28,36,44,52	Units of LCD42" shipped by road LTL from ODM1-7
60,68	Units of LCD32" shipped by road LTL from ODM1-2
5,13,21,29,37,45,53	Units of LCD42" shipped by road-network from ODM1-7
61,69	Units of LCD32" shipped by road-network from ODM1-2
6,14,22,30,38,46,54	Units of LCD42" shipped by rail from ODM1-7
62,70	Units of LCD32" shipped by rail from ODM1-2
7,15,23,31,39,47,55	Units of LCD42" shipped by water from ODM1-7
63,71	Units of LCD32" shipped by water from ODM1-2
8,16,24,32,40,48,56	Units of LCD42" produced by not shipped from ODM1-7
64,72	Units of LCD32" produced by not shipped from ODM1-2

- *Param B: Constraints Value*

Constraints	Explanations
1-9=600000	The maximum units of production for each ODM is 600000
10=46000	Minimum production of LCD42" to be shipped by Regular Air or Air Express
11=53000	Minimum production of LCD32" to be shipped by Regular Air or Air Express
12=92000	Minimum production of LCD42" to be shipped by Road or Road LTL or Road Network
13=79500	Minimum production of LCD32" to be shipped by Road or Road LTL or Road Network
14=138000	Minimum production of LCD42" to be shipped by Rail
15=79500	Minimum production of LCD32" to be shipped by Rail
16-31=0	ODM 5 does not have access to Regular Air/Air Express/Rail/Water ODM 6 does not have access to Road/Road LTL/Road-Network/Rail
32=920000	Units of LCD42" to be shipped
33=530000	Units of LCD32" to be shipped
34=330000	Budget=3.3 billion (units is million in this constrain)

- *Param A: Matrix for Linear Programing*

- Matrix A's dimension is  $34 \times 72$  for 72 variables and 34 constraints.
- Formulations for each constraints are as follows:

- $$\begin{bmatrix} -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & \dots & 0 & 0 & 0 & 0 & 0 \\ 0 & .. & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & \dots & 0 & 0 & 0 \\ \vdots & & & & & & & & & & & & & & \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \end{bmatrix}$$

- $(k \times 8 + 1)th, (k \times 8 + 2)th, k \in \{0, \dots, 6\}$  variables are 1. Otherwise, coefficients are 0.
- Constraints/Row 11: Only the coefficients corresponding to  $(k \times 8 + 1)th, (k \times 8 + 2)th, k \in \{7, 8\}$  variables are 1. Otherwise, coefficients are 0.
- Constraints/Row 12: Only the coefficients corresponding to  $(k \times 8 + 3)th, (k \times 8 + 4)th, (k \times 8 + 5)th k \in \{0, \dots, 6\}$  variables are 1. Otherwise, coefficients are 0.
- Constraints/Row 13: Only the coefficients corresponding to  $(k \times 8 + 3)th, (k \times 8 + 4)th, (k \times 8 + 5)th k \in \{7, 8\}$  variables are 1. Otherwise, coefficients are 0.
- Constraints/Row 14: Only the coefficients corresponding to  $(k \times 8 + 6)th k \in \{0, \dots, 6\}$  variables are 1. Otherwise, coefficients are 0.
- Constraints/Row 15: Only the coefficients corresponding to  $(k \times 8 + 6)th k \in \{7, 8\}$  variables are 1. Otherwise, coefficients are 0.

- Constraints/Row 16/18/20/22/24/26/28/30: Coefficients for Variable 33/34/38/39/43/44/45/46 is 1, 0 otherwise.
- Constraints/Row 19/21/23/25/27/29/31: Coefficients for Variable 33/34/38/39/43/44/45/46 is 1, 0 otherwise.
- Constraints/Row 32: Coefficients for Variable 1-56 are 1, 0 otherwise.
- Constraints/Row 33: Coefficients for Variable 57-72 are 1, 0 otherwise.
- Constraints/Row 34: Coefficient for Variable  $i \in \{1...56\}$  is calculated as

$$\text{production cost of LCD 42" at corresponding ODM} + 0.022 \times \text{shipping cost per metric ton for corresponding shipping method}$$

- Constraints/Row 34: Coefficient for Variable  $i \in \{57...72\}$  is calculated:  $\text{production cost of LCD 32" at corresponding ODM} + 0.0165 \times \text{shipping cost per metric ton for corresponding shipping method}$

- *Param C: Coefficients for Each Variables*

- Matrix C's dimension is  $1 \times 72$ , with the  $i$ th entry of matrix C being the  $\text{CO}_2$  emission per unit of the  $i$ th variable.
- For example, variable 1 represents the number of units of LCD 42" produced by ODM1 and shipped by regular air. The 1st entry of matrix C is:  

$$0.022(\text{LCD 42" mass in ton}) \times 2508(\text{distance from ODM1 to DC}) \times 1.44(\text{CO}_2 \text{ emission per Ton-Km of Regular Air})$$

The rest of the coefficients can be calculated in a similar way.

- *Formula Used*

- For AMPL input, the model is  $A \times C \geq B$

#### 4. **SENSITIVITY ANALYSIS (done with 3.3 billion budget unless specified otherwise)**

- *Constraints 1-9: Maximum production of Each ODM*

Constraints	1	2	3	4	5	6	7
Shadow Price	0	0	0	0	0	0	0.134
Slack	600000	60000	60000	372000	508000	600000	0

According to the sensitivity analysis, the shadow prices for constraints 1-6 are 0, which means that increasing/decreasing the maximum production of ODM 1-6 for LCD42” won’t change the optimal objective function when keeping the same ODMs chosen. The shadow price for constraint 7 is 0.134, which means that the CO<sub>2</sub> emission will decrease by 0.134 kg for each unit increase in the maximum production of ODM 7. The sensitivity analysis also indicates that we can change the maximum capacity of ODM 7 from 287505 units to 671095 units without changing the optimal basis. Hence, given all other conditions to be the same, increasing the maximum production capacity of ODM 7 to be 671095 will return the minimum CO<sub>2</sub> emission.

*At a budget of 3.0 billion, the sensitivity analysis results for Constraints 1-7 are:*

Constraints	1	2	3	4	5	6	7
Shadow Price	0	0	0	0.4223	0	0	0
Slack	28000	600000	600000	0	600000	600000	600000

The sensitivity analysis indicates that when at a budget of 3.0 billion, expanding the maximum production capacity of ODM 4 is more important. For each unit expansion of maximum production capacity of ODM 4 up to 705160 units, the optimal CO<sub>2</sub> emission will decrease by 0.422 kg.

- *Constraints 10-15: Minimum shipment requirements for air/road/rail transportation*

Constraints	10	11	12	13	14	15
Shadow Price	23.041	37.918	0.260	1.422	0.368	0.584

The shadow price for constraints 10/11 is 23.041/37.918, which means that if the minimum required units of LCD 42"/LCD 32" shipped by air transportation increases 1 unit, the optimal CO<sub>2</sub> emission will increase 23.041/37.918 kg. Similarly, we can conclude that increasing the minimum required shipments of LCD 42" by road transportation will result in the least increase in optimal CO<sub>2</sub> emission, and that increasing the minimum required shipments of LCD 32" by rail transportation will result in the least increase in optimal CO<sub>2</sub> emission. This result is consistent with the fact that air transportation creates the highest CO<sub>2</sub> emission per Ton-Km unit. Hence, if it is possible to reduce the minimum required shipments, we should pay more attention to reduce the requirement of air transportation.

- *Constraints 32-33: Units to be shipped of LCD42", LCD 32"*

Constraints	32	33
Shadow Price	1.642	1.488

The table above indicates that if the required shipping unit for LCD42"/LCD32" increases by 1 unit, the optimal CO<sub>2</sub> emission will increase by 1.64/1.48 kg when keeping all other conditions to be the same. The shadow price for Constraint 32 is greater than that of Constraint 33 because the weight of LCD42" is greater than that of LCD32". However, since the demands for TVs are usually fixed, it is not likely for the company to change these constraints.

- *Constraint 34: Budget (Unit=\$1000 in this part)*

Budget	3.0 billion	3.3 billion	3.6 billion	3.9 billion
ODMs Chosen for LCD 42”	ODM 1,4	ODM 4,5,7	ODM 5, 7	ODM 5, 7
Optimal CO <sub>2</sub> Emission	7401.25	3406.42 tons	3306.15 tons	3293.33 tons
Shadow Price	50.309	0.641	0.0911	0

By examining the shadow price for constraint 34 at different budgets, we can conclude as budget increases, the shadow price for the constraint decreases. At a budget of 3.0 billion, increasing \$1000 in budget will reduce the optimal CO<sub>2</sub> emission by 50.309 kg. However, as the budgets increase, the amount of optimal CO<sub>2</sub> emission that can be reduced by budget increase decreases drastically. Eventually, increasing the budget to be higher than 3.9 billion won't reduce the optimal CO<sub>2</sub> emission, which means the budget increase will not be meaningful up to that point.

From the table above, we can conclude that as the budget increases, it is more likely that the optimal solution will choose ODM 5 and ODM 7 to be manufacturing ODMs. This result makes sense for ODM 5 and 7 are the top two closest ODMs to DC. When the mass of products are kept constant, distance between ODMs and DC will be the only determining factor for CO<sub>2</sub> emission. Hence, when provided enough budget, the optimal plan will automatically favor ODMs with the shortest distance to DC though their production costs might be high.

- *Coefficient for Variables 49-55 (corresponding to ODM 7 at a budget of 3 billion)*

Constraints	49	50	51	52	53	54	55
Reduced Cost	3.46585	14.158	7.171	5.746	5.461	7.943	6.527
CO <sub>2</sub> emission per Ton-Km unit	1.44 (air)	1.44 (air)	0.0613 (road)	0.0613 (road)	0.0613 (road)	0.0285 (rail)	0.007 (water)

R	2.407	9.832	116.98	93.736	89.086	278.70	932.429
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- *Coefficient for Variables 49-55 (corresponding to ODM 5 at a budget of 3 billion)*

Constraints	33	34	35	36	37	38	39
Reduced Cost	0	0	38.624	37.911	37.555	30.531	32.817
CO <sub>2</sub> emission per Ton-Km unit	1.44 (air)	1.44 (air)	0.0613 (road)	0.0613 (road)	0.0613 (road)	0.0285 (rail)	0.007 (water)
R	0	0	630.08	618.45	612.64	1071.3	4688.14
Minimum Mass Reduction	$\frac{30.513 \text{ Kg}}{30 \text{ Km} \times 0.0285 \text{ Kg/Ton} \times \text{Km}} = 35.7 \text{ ton}$ <p>(which is greater than the original mass)</p>						

By examining the reduced cost for variables 49-55, we can see that the CO<sub>2</sub> emission for each unit product produced by ODM 7 and shipped by regular air needs to be reduced by about 3.47 kg before this variable can be considered (nonzero) in the optimal solution. To reduce the CO<sub>2</sub> emission per unit product, we can choose to either reduce the weight of the product, the distance of each ODM to DC or the CO<sub>2</sub> emission per Ton-Km unit. It is only realistic to consider reducing the weight of the product since we can't easily move ODMs or alter the CO<sub>2</sub> emission per Ton-Km unit. Considering the table above, the reduced cost for air transportation is the greatest, the reduced cost for road and rail transportations are moderate, and the reduced cost for water transportation is the smallest. If ODM 7 were to be chosen as a manufacturing ODM by reducing the weight of LCD 42", the variable with the smallest reduced cost to CO<sub>2</sub> emission per Ton-Km unit ratio (R) will give the least and the most realistic weight reduction. The table above indicates that variable 49 will have the lowest R, which gives

$\frac{2.407 \text{ Kg}}{686 \text{ Km} \times 1.44 \text{ Kg/Ton} \times \text{Km}} = 0.00243 \text{ ton}$ , which is 11% of the original weight of LCD 42". Hence,

ODM 7 can be included as one of the manufacturing ODMs if the company can reduce 10% of the original weight of LCD 42". However, by utilizing the air transportation, more CO<sub>2</sub> emission can be created. Hence, we will not include mass reduction in our recommendations.

If we repeat similar calculations for ODM 5 (shown in the table with Constraints 33-39), we obtain the result that the mass reduction is greater than the original mass of LCD 42", which is unrealistic. Hence there is no mass reduction can be done to include ODM 5 as a manufacturing ODM. ODM 5 is not chosen mainly because of insufficient budget, and mass reduction will not be an efficient plan to reduce CO<sub>2</sub> emissions.



## III.AMPL SCREENSHOTS

- Dataset

```
param n := 72;
param m := 34;

param C :=
1 79.453439999999999
2 79.453439999999999
3 3.382288800000000
4 3.382288800000000
5 3.382288800000000
6 1.572516000000000
7 0.386232000000000
8 0
9 49.199040000000000
10 49.199040000000000
11 2.094375800000000
12 2.094375800000000
13 2.094375800000000
14 0.973731000000000
15 0.239162000000000
16 0
17 43.718399999999995
18 43.718399999999995
19 1.861068000000000
20 1.861068000000000
21 1.861068000000000
22 0.865260000000000
23 0.212520000000000
24 0
25 68.112000000000000
26 68.112000000000000
27 2.899490000000000
28 2.899490000000000
29 2.899490000000000
30 1.348050000000000
```





## • Model

```
param n;  
param m;  
  
set J:= {1..n};  
set I:= {1..m};  
  
param C {J}>=0;  
param B {I};  
param A {I,J};  
  
var X {J}>=0;  
  
minimize z: sum {j in J} C[j]* X[j];  
  
s.t. Constraint {i in I}:  
    sum {j in J} A[i,j]*X[j] >= B[i];
```

## • Running Code

```
reset;  
  
model IEOR162project.mod;  
  
data IEOR32.dat;  
  
expand z, Constraint;  
  
option solver cplex;  
option cplex_options 'sensitivity';  
  
solve;  
  
display X,z, _varname, _var, _var.rc,_var.down,_var.current, _var.up, _conname, _con, _con.slack, _con.up, _con.current, _con.down;|
```

- Output at 3 Billion Budget

```

suffix current OUT;
X [*] :=
1  0      16      0      31 414306      46      0      61 79500
2  0      17      0      32      0      47      0      62 79500
3  0      18      0      33      0      48      0      63 318000
4  0      19      0      34      0      49      0      64      0
5  90305.7 20      0      35      0      50      0      65      0
6  0      21      0      36      0      51      0      66      0
7  229694 22      0      37      0      52      0      67      0
8  0      23      0      38      0      53      0      68      0
9  0      24      0      39      0      54      0      69      0
10 0      25 46000 40      0      55      0      70      0
11 0      26      0      41      0      56      0      71      0
12 0      27      0      42      0      57 53000 72      0
13 0      28      0      43      0      58      0
14 0      29 1694.3 44      0      59      0
15 0      30 138000 45      0      60      0
;

z = 7401250

:  _varname  _var  _var.rc  _var.down  _var.current  :=
1  'X[1]'    0      11.2863  68.1671  79.4534
2  'X[2]'    0      18.4141  61.0394  79.4534
3  'X[3]'    0      1.49683  1.88546  3.38229
4  'X[4]'    0      0.427667 2.95462  3.38229
5  'X[5]'    90305.7 0      3.35259  3.38229
6  'X[6]'    0      0.169334 1.40318  1.57252
7  'X[7]'    229694 1.42109e-14 -2.60397 0.386232
8  'X[8]'    0      99.7829  -99.7829 0
9  'X[9]'    0      51.6678  -2.46873 49.199
10 'X[10]'   0      64.4978  -15.2987 49.199
11 'X[11]'   0      14.8204  -12.7261 2.09438
12 'X[12]'   0      13.3949  -11.3005 2.09438

```

12	'X[12]'	0	13.3949	-11.3005	2.09438
13	'X[13]'	0	13.2523	-11.1579	2.09438
14	'X[14]'	0	14.8948	-13.9211	0.973731
15	'X[15]'	0	13.0389	-12.7997	0.239162
16	'X[16]'	0	113.396	-113.396	0
17	'X[17]'	0	48.453	-4.73464	43.7184
18	'X[18]'	0	59.8575	-16.1391	43.7184
19	'X[19]'	0	31.1086	-29.2475	1.86107
20	'X[20]'	0	29.683	-27.822	1.86107
21	'X[21]'	0	29.5405	-27.6794	1.86107
22	'X[22]'	0	31.3078	-30.4426	0.86526
23	'X[23]'	0	30.1752	-29.9627	0.21252
24	'X[24]'	0	129.918	-129.918	0
25	'X[25]'	46000	2.84217e-14	-67.5254	68.112
26	'X[26]'	0	7.12778	60.9842	68.112
27	'X[27]'	0	1.06917	1.83032	2.89949
28	'X[28]'	0	0.0712778	2.82821	2.89949
29	'X[29]'	1694.3	0	-1e+20	2.89949
30	'X[30]'	138000	0	-0.9519	1.34805
31	'X[31]'	414306	1.42109e-14	0.301649	0.3311
32	'X[32]'	0	99.838	-99.838	0
33	'X[33]'	0	0	0	0
34	'X[34]'	0	0	0	0
35	'X[35]'	0	38.6239	-38.5835	0.040458
36	'X[36]'	0	37.9112	-37.8707	0.040458
37	'X[37]'	0	37.5548	-37.5143	0.040458
38	'X[38]'	0	30.5312	0	0
39	'X[39]'	0	32.817	0	0
40	'X[40]'	0	136.403	-136.403	0
41	'X[41]'	0	68.3906	-46.5314	21.8592
42	'X[42]'	0	82.6461	0	0
43	'X[43]'	0	28.4953	0	0
44	'X[44]'	0	28.4953	0	0
45	'X[45]'	0	28.4953	0	0



```

45 'X[45]'      0      28.4953      0      0
46 'X[46]'      0      30.618      -30.1854      0.43263
47 'X[47]'      0      36.3693      -36.2631      0.10626
48 'X[48]'      0      136.076      -136.076      0
49 'X[49]'      0      3.46585      18.2666      21.7325
50 'X[50]'      0      14.1575      7.57496      21.7325
51 'X[51]'      0      7.17139      -6.24625      0.92514
52 'X[52]'      0      5.74583      -4.8207      0.92514
53 'X[53]'      0      5.46072      -4.53558      0.92514
54 'X[54]'      0      7.9427      -7.51258      0.430122
55 'X[55]'      0      6.52672      -6.42108      0.105644
56 'X[56]'      0      106.917      -106.917      0
57 'X[57]'      53000      0      -50.6027      59.5901
58 'X[58]'      0      5.34583      54.2442      59.5901
59 'X[59]'      0      1.12263      1.41409      2.53672
60 'X[60]'      0      0.32075      2.21597      2.53672
61 'X[61]'      79500      0      -1.1537      2.53672
62 'X[62]'      79500      0      -0.672576      1.17939
63 'X[63]'      318000      0      -94.0278      0.289674
64 'X[64]'      0      91.4618      -91.4618      0
65 'X[65]'      0      29.051      7.84827      36.8993
66 'X[66]'      0      38.6735      -1.77423      36.8993
67 'X[67]'      0      9.88023      -8.30945      1.57078
68 'X[68]'      0      8.81107      -7.24029      1.57078
69 'X[69]'      0      8.70415      -7.13337      1.57078
70 'X[70]'      0      9.80904      -9.07874      0.730298
71 'X[71]'      0      8.54407      -8.3647      0.179371
72 'X[72]'      0      100.437      -100.437      0
;
:      _var.up      _conname      _con      _con.slack      :=
1      1e+20      'Constraint[1]'      0      280000
2      1e+20      'Constraint[2]'      0      6e+05
3      1e+20      'Constraint[3]'      0      6e+05
4      1e+20      'Constraint[4]'      0.422388      0

```

4	1e+20	'Constraint[4]'	0.422388	0
5	1e+20	'Constraint[5]'	0	6e+05
6	1e+20	'Constraint[6]'	0	6e+05
7	0.415683	'Constraint[7]'	0	6e+05
8	1e+20	'Constraint[8]'	0	70000
9	1e+20	'Constraint[9]'	0	6e+05
10	1e+20	'Constraint[10]'	135.637	0
11	1e+20	'Constraint[11]'	110.193	0
12	1e+20	'Constraint[12]'	4.92056	0
13	1e+20	'Constraint[13]'	3.69042	0
14	1e+20	'Constraint[14]'	2.29995	0
15	1e+20	'Constraint[15]'	1.85196	0
16	1e+20	'Constraint[16]'	0	0
17	1e+20	'Constraint[17]'	101.875	0
18	1e+20	'Constraint[18]'	0	0
19	1e+20	'Constraint[19]'	101.875	0
20	1e+20	'Constraint[20]'	0	0
21	1e+20	'Constraint[21]'	0	0
22	1e+20	'Constraint[22]'	0	0
23	1e+20	'Constraint[23]'	0	0
24	1e+20	'Constraint[24]'	0	0
25	71.5778	'Constraint[25]'	0	0
26	1e+20	'Constraint[26]'	0	0
27	1e+20	'Constraint[27]'	0	0
28	1e+20	'Constraint[28]'	0	0
29	2.92919	'Constraint[29]'	0	0
30	1.51738	'Constraint[30]'	0	0
31	3.3213	'Constraint[31]'	0	0
32	1e+20	'Constraint[32]'	103.59	-1.16415e-10
33	0	'Constraint[33]'	94.3174	0
34	0	'Constraint[34]'	50.309	4.65661e-10
35	1e+20	.	.	.
36	1e+20	.	.	.
37	1e+20	.	.	.



38	0	.	.	.
39	0	.	.	.
40	1e+20	.	.	.
41	1e+20	.	.	.
42	0	.	.	.
43	0	.	.	.
44	0	.	.	.
45	0	.	.	.
46	1e+20	.	.	.
47	1e+20	.	.	.
48	1e+20	.	.	.
49	1e+20	.	.	.
50	1e+20	.	.	.
51	1e+20	.	.	.
52	1e+20	.	.	.
53	1e+20	.	.	.
54	1e+20	.	.	.
55	1e+20	.	.	.
56	1e+20	.	.	.
57	64.9359	.	.	.
58	1e+20	.	.	.
59	1e+20	.	.	.
60	1e+20	.	.	.
61	2.85747	.	.	.
62	10.9884	.	.	.
63	2.14164	.	.	.
64	1e+20	.	.	.
65	1e+20	.	.	.
66	1e+20	.	.	.
67	1e+20	.	.	.
68	1e+20	.	.	.
69	1e+20	.	.	.
70	1e+20	.	.	.
71	1e+20	.	.	.

```

71 1e+20 . . .
72 1e+20 . . .
;
: _con.up _con.current _con.down :=
1 -320000 -6e+05 -1e+20
2 0 -6e+05 -1e+20
3 0 -6e+05 -1e+20
4 -598027 -6e+05 -705160
5 0 -6e+05 -1e+20
6 0 -6e+05 -1e+20
7 0 -6e+05 -1e+20
8 -530000 -6e+05 -1e+20
9 0 -6e+05 -1e+20
10 46010.7 46000 45430.8
11 53014.2 53000 52241.1
12 92376.5 92000 75580.8
13 80002 79500 52742.8
14 138565 138000 107898
15 80253 79500 39364.1
16 0 0 0
17 0 0 0
18 0 0 0
19 0 0 0
20 0 0 0
21 0 0 0
22 0 0 0
23 0 0 0
24 0 0 0
25 0 0 0
26 0 0 0
27 0 0 0
28 0 0 0
29 0 0 0
30 0 0 0

```

```

31 0 0 0
32 920007 920000 919626
33 530008 530000 529589
34 -2999990 -3e+06 -3000770
.

```

- Output for 3.3 Billion Budget

```

suffix up OUT;
suffix down OUT;
suffix current OUT;
X [*] :=
1   0   16   0   31 228000   46   0   61   0
2   0   17   0   32   0   47   0   62   0
3   0   18   0   33   0   48   0   63 259062
4   0   19   0   34   0   49 46000   64   0
5   0   20   0   35   0   50   0   65 53000
6   0   21   0   36   0   51   0   66   0
7   0   22   0   37 92000   52   0   67   0
8   0   23   0   38   0   53   0   68   0
9   0   24   0   39   0   54 138000   69 79500
10  0   25   0   40   0   55 416000   70 79500
11  0   26   0   41   0   56   0   71 58938.1
12  0   27   0   42   0   57   0   72   0
13  0   28   0   43   0   58   0
14  0   29   0   44   0   59   0
15  0   30   0   45   0   60   0
;

z = 3406420

# $5 = _var.current
:  _varname      _var      _var.rc      _var.down      $5      _var.up
:=
1  'X[1]'        0        56.9509      22.5025      79.4534      1e+20
2  'X[2]'        0        57.0418      22.4116      79.4534      1e+20
3  'X[3]'        0        2.83967      0.542619     3.38229     1e+20
4  'X[4]'        0        2.82604      0.556246     3.38229     1e+20
5  'X[5]'        0        2.82059      0.561697     3.38229     1e+20
6  'X[6]'        0        0.894746     0.67777      1.57252     1e+20
7  'X[7]'        0        0.0598128    0.326419     0.386232    1e+20
8  'X[8]'        0        1.27176     -1.27176     0           1e+20

```

9	'X[9]'	0	27.5968	21.6022	49.199	1e+20
10	'X[10]'	0	27.7603	21.4387	49.199	1e+20
11	'X[11]'	0	1.73798	0.356391	2.09438	1e+20
12	'X[12]'	0	1.71982	0.37456	2.09438	1e+20
13	'X[13]'	0	1.718	0.376377	2.09438	1e+20
14	'X[14]'	0	0.491273	0.482458	0.973731	1e+20
15	'X[15]'	0	0.0808015	0.158361	0.239162	1e+20
16	'X[16]'	0	1.44527	-1.44527	0	1e+20
17	'X[17]'	0	22.1451	21.5733	43.7184	1e+20
18	'X[18]'	0	22.2904	21.428	43.7184	1e+20
19	'X[19]'	0	1.71525	0.14582	1.86107	1e+20
20	'X[20]'	0	1.69708	0.16399	1.86107	1e+20
21	'X[21]'	0	1.69526	0.165806	1.86107	1e+20
22	'X[22]'	0	0.593373	0.271887	0.86526	1e+20
23	'X[23]'	0	0.272907	-0.0603865	0.21252	1e+20
24	'X[24]'	0	1.65584	-1.65584	0	1e+20
25	'X[25]'	0	45.6048	22.5072	68.112	1e+20
26	'X[26]'	0	45.6957	22.4163	68.112	1e+20
27	'X[27]'	0	2.35219	0.5473	2.89949	1e+20
28	'X[28]'	0	2.33947	0.560018	2.89949	1e+20
29	'X[29]'	0	2.33856	0.560927	2.89949	1e+20
30	'X[30]'	0	0.665599	0.682451	1.34805	1e+20
31	'X[31]'	228000	0	0.197086	0.3311	0.390913
32	'X[32]'	0	1.26708	-1.26708	0	1e+20
33	'X[33]'	0	0	0	0	0
34	'X[34]'	0	0	0	0	0
35	'X[35]'	0	0.0136268	0.0268312	0.040458	1e+20
36	'X[36]'	0	0.00454228	0.0359157	0.040458	1e+20
37	'X[37]'	92000	0	-0.219348	0.040458	0.0450003
38	'X[38]'	0	0	0	0	0
39	'X[39]'	0	0.101326	0	0	0
40	'X[40]'	0	1.73849	-1.73849	0	1e+20
41	'X[41]'	0	0.818575	21.0406	21.8592	1e+20
42	'X[42]'	0	1.00027	0	0	0

```

42 'X[42]'      0      1.00027      0      0      0
43 'X[43]'      0      0.763267     0      0      0
44 'X[44]'      0      0.763267     0      0      0
45 'X[45]'      0      0.763267     0      0      0
46 'X[46]'      0      0.157465     0.275165   0.43263    1e+20
47 'X[47]'      0      0.246947    -0.140687   0.10626    1e+20
48 'X[48]'      0      1.73432     -1.73432    0      1e+20
49 'X[49]'      46000   -8.88178e-16 -1.30844    21.7325    21.8687
50 'X[50]'      0      0.136268    21.5962    21.7325    1e+20
51 'X[51]'      0      0.620175     0.304964    0.92514    1e+20
52 'X[52]'      0      0.602006     0.323133    0.92514    1e+20
53 'X[53]'      0      0.598372     0.326767    0.92514    1e+20
54 'X[54]'      138000   0      0.062419    0.430122    0.587587
55 'X[55]'      416000   0     -0.0518215   0.105644    0.239658
56 'X[56]'      0      1.4967     -1.4967      0      1e+20
57 'X[57]'      0      22.0313     37.5587     59.5901    1e+20
58 'X[58]'      0      22.0995     37.4906     59.5901    1e+20
59 'X[59]'      0      0.856995     1.67972     2.53672    1e+20
60 'X[60]'      0      0.846775     1.68994     2.53672    1e+20
61 'X[61]'      0      0.842687     1.69403     2.53672    1e+20
62 'X[62]'      0      0.318346     0.861041     1.17939    1e+20
63 'X[63]'      259062   -2.22045e-16 0.244016     0.289674    0.418643
64 'X[64]'      0      1.16571     -1.16571     0      1e+20
65 'X[65]'      53000   3.9968e-15   -1.01843     36.8993     37.0219
66 'X[66]'      0      0.122642     36.7766     36.8993     1e+20
67 'X[67]'      0      0.0149895    1.55579     1.57078     1e+20
68 'X[68]'      0      0.00136268   1.56942     1.57078     1e+20
69 'X[69]'      79500   0      0.14803     1.57078     1.57214
70 'X[70]'      79500   0      0.146667     0.730298     1.04864
71 'X[71]'      58938.1 -2.22045e-16 0.0504022    0.179371     0.225029
72 'X[72]'      0      1.2801     -1.2801      0      1e+20
;
# $3 = _con.slack
# $5 = _con.current

```



	_conname	_con	\$3	_con.up	\$5	_con.down
:						
:=						
1	'Constraint[1]'	0	6e+05	0	-6e+05	-1e+20
2	'Constraint[2]'	0	6e+05	0	-6e+05	-1e+20
3	'Constraint[3]'	0	6e+05	0	-6e+05	-1e+20
4	'Constraint[4]'	0	372000	-228000	-6e+05	-1e+20
5	'Constraint[5]'	0	508000	-92000	-6e+05	-1e+20
6	'Constraint[6]'	0	6e+05	0	-6e+05	-1e+20
7	'Constraint[7]'	0.134014	0	-287505	-6e+05	-671095
8	'Constraint[8]'	0	340938	-259062	-6e+05	-1e+20
9	'Constraint[9]'	0	329062	-270938	-6e+05	-1e+20
10	'Constraint[10]'	23.0409	0	50597.3	46000	25792.5
11	'Constraint[11]'	37.9177	0	57969.8	53000	29143.6
12	'Constraint[12]'	0.259806	0	103810	92000	40087.4
13	'Constraint[13]'	1.42275	0	125397	79500	0
14	'Constraint[14]'	0.367703	0	288399	138000	0
15	'Constraint[15]'	0.583631	0	124959	79500	0
16	'Constraint[16]'	0	0	0	0	0
17	'Constraint[17]'	21.9938	0	0	0	0
18	'Constraint[18]'	0	0	0	0	0
19	'Constraint[19]'	21.9938	0	0	0	0
20	'Constraint[20]'	0	0	0	0	0
21	'Constraint[21]'	0.252187	0	0	0	0
22	'Constraint[22]'	0	0	0	0	0
23	'Constraint[23]'	0	0	0	0	0
24	'Constraint[24]'	0	0	0	0	0
25	'Constraint[25]'	0	0	0	0	0
26	'Constraint[26]'	0	0	0	0	0
27	'Constraint[27]'	0	0	0	0	0
28	'Constraint[28]'	0	0	0	0	0
29	'Constraint[29]'	0	0	0	0	0
30	'Constraint[30]'	0	0	0	0	0
31	'Constraint[31]'	0	0	0	0	0
32	'Constraint[32]'	1.64179	0	924960	920000	898198

33	'Constraint[33]'	1.48808	0	535425	530000	508165
34	'Constraint[34]'	0.641203	0	-3289860	-3300000	-3344560
;						

- Output for 3.6 Billion Budget

```

suffix up OUT;
suffix down OUT;
suffix current OUT;
X [*] :=
1      0      13      0      25      0      37 403405      49      46000      61      0
2      0      14      0      26      0      38      0      50      0      62      0
3      0      15      0      27      0      39      0      51      0      63      0
4      0      16      0      28      0      40      0      52      0      64      0
5      0      17      0      29      0      41      0      53      0      65 53000
6      0      18      0      30      0      42      0      54 138000      66      0
7      0      19      0      31      0      43      0      55 332595      67      0
8      0      20      0      32      0      44      0      56      0      68      0
9      0      21      0      33      0      45      0      57      0      69 79500
10     0      22      0      34      0      46      0      58      0      70 79500
11     0      23      0      35      0      47      0      59      0      71 318000
12     0      24      0      36      0      48      0      60      0      72      0
;

z = 3306150

# $5 = _var.current
:  _varname      _var      _var.rc      _var.down      $5      _var.up
:=
1      'X[1]'      0      57.6306      21.8228      79.4534      1e+20
2      'X[2]'      0      57.6435      21.8099      79.4534      1e+20
3      'X[3]'      0      3.27052      0.111773      3.38229      1e+20
4      'X[4]'      0      3.26858      0.113708      3.38229      1e+20
5      'X[5]'      0      3.26781      0.114482      3.38229      1e+20
6      'X[6]'      0      1.12626      0.44626      1.57252      1e+20
7      'X[7]'      0      0.268267      0.117965      0.386232      1e+20
8      'X[8]'      0      0.18061      -0.18061      0      1e+20
9      'X[9]'      0      27.5041      21.6949      49.199      1e+20
10     'X[10]'     0      27.5273      21.6717      49.199      1e+20

```

12	'X[12]'	0	2.00647	0.0879058	2.09438	1e+20
13	'X[13]'	0	2.00621	0.0881638	2.09438	1e+20
14	'X[14]'	0	0.555209	0.418522	0.973731	1e+20
15	'X[15]'	0	0.145064	0.0940985	0.239162	1e+20
16	'X[16]'	0	0.205251	-0.205251	0	1e+20
17	'X[17]'	0	22.0276	21.6908	43.7184	1e+20
18	'X[18]'	0	22.0482	21.6702	43.7184	1e+20
19	'X[19]'	0	1.80565	0.0554211	1.86107	1e+20
20	'X[20]'	0	1.80307	0.0580014	1.86107	1e+20
21	'X[21]'	0	1.80281	0.0582594	1.86107	1e+20
22	'X[22]'	0	0.476642	0.388618	0.86526	1e+20
23	'X[23]'	0	0.149487	0.063033	0.21252	1e+20
24	'X[24]'	0	0.235155	-0.235155	0	1e+20
25	'X[25]'	0	46.2885	21.8235	68.112	1e+20
26	'X[26]'	0	46.3014	21.8106	68.112	1e+20
27	'X[27]'	0	2.78705	0.112437	2.89949	1e+20
28	'X[28]'	0	2.78525	0.114244	2.89949	1e+20
29	'X[29]'	0	2.78512	0.114373	2.89949	1e+20
30	'X[30]'	0	0.901125	0.446925	1.34805	1e+20
31	'X[31]'	0	0.21247	0.11863	0.3311	1e+20
32	'X[32]'	0	0.179945	-0.179945	0	1e+20
33	'X[33]'	0	0	0	0	0
34	'X[34]'	0	0	0	0	0
35	'X[35]'	0	0.00193522	0.0385228	0.040458	1e+20
36	'X[36]'	0	0.000645075	0.0398129	0.040458	1e+20
37	'X[37]'	403405	0	-0.353362	0.040458	0.0410968
38	'X[38]'	0	0	0	0	0
39	'X[39]'	0	0	0	0	0
40	'X[40]'	0	0.246893	-0.246893	0	1e+20
41	'X[41]'	0	0.244006	21.6152	21.8592	1e+20
42	'X[42]'	0	0.269809	0	0	0
43	'X[43]'	0	0.872067	0	0	0
44	'X[44]'	0	0.872067	0	0	0
45	'X[45]'	0	0.872067	0	0	0



```

45 'X[45]'      0      0.872067      0      0      0
46 'X[46]'      0      0.0435465     0.389084     0.43263     1e+20
47 'X[47]'      0      0.0546309     0.0516291    0.10626     1e+20
48 'X[48]'      0      0.246301     -0.246301     0      1e+20
49 'X[49]'      46000    1.60982e-15   -0.0951781    21.7325     21.7518
50 'X[50]'      0      0.0193522     21.7131     21.7325     1e+20
51 'X[51]'      0      0.828086     0.0970541    0.92514     1e+20
52 'X[52]'      0      0.825505     0.0996344    0.92514     1e+20
53 'X[53]'      0      0.824989     0.10015      0.92514     1e+20
54 'X[54]'      138000    0      0.0995054    0.430122     0.473668
55 'X[55]'      332595    2.77556e-17   0.040458     0.105644     0.281931
56 'X[56]'      0      0.193522     -0.193522     0      1e+20
57 'X[57]'      0      22.5971      36.9929     59.5901     1e+20
58 'X[58]'      0      22.6068      36.9833     59.5901     1e+20
59 'X[59]'      0      0.950464     1.58625      2.53672     1e+20
60 'X[60]'      0      0.949012     1.5877       2.53672     1e+20
61 'X[61]'      0      0.948432     1.58828      2.53672     1e+20
62 'X[62]'      0      0.430521     0.748866     1.17939     1e+20
63 'X[63]'      0      0.0946378     0.195036     0.289674     1e+20
64 'X[64]'      0      0.165548     -0.165548     0      1e+20
65 'X[65]'      53000    -2.16493e-15   0.00926535    36.8993     36.9167
66 'X[66]'      0      0.017417     36.8819     36.8993     1e+20
67 'X[67]'      0      0.00212875    1.56865      1.57078     1e+20
68 'X[68]'      0      0.000193522    1.57059      1.57078     1e+20
69 'X[69]'      79500    0      0.17492      1.57078     1.57098
70 'X[70]'      79500    5.55112e-17   0.174727     0.730298     1.16082
71 'X[71]'      318000    2.77556e-17   -0.185858     0.179371     0.274009
72 'X[72]'      0      0.181794     -0.181794     0      1e+20
;
# $5 = _con.current
:      _conname      _con      _con.slack      _con.up      $5
:=
1      'Constraint[1]'      0      6e+05      0      -6e+05
2      'Constraint[2]'      0      6e+05      0      -6e+05

```

```

3   'Constraint[3]'      0          6e+05          0          -6e+05
4   'Constraint[4]'      0          6e+05          0          -6e+05
5   'Constraint[5]'      0          196595         -403405         -6e+05
6   'Constraint[6]'      0          6e+05          0          -6e+05
7   'Constraint[7]'      0          83405.3         -516595         -6e+05
8   'Constraint[8]'      0          6e+05          0          -6e+05
9   'Constraint[9]'      0          70000          -530000         -6e+05
10  'Constraint[10]'     21.8277          0          73073          46000
11  'Constraint[11]'     36.89            0          84961.5         53000
12  'Constraint[12]'      0          311405         403405          92000
13  'Constraint[13]'      1.39586          0          397500          79500
14  'Constraint[14]'      0.330617          0          505172         138000
15  'Constraint[15]'      0.555571          0          397500          79500
16  'Constraint[16]'      0            0            0            0
17  'Constraint[17]'     20.9351          0            0            0
18  'Constraint[18]'      0            0            0            0
19  'Constraint[19]'     20.9351          0            0            0
20  'Constraint[20]'      0            0            0            0
21  'Constraint[21]'      0.369682          0            0            0
22  'Constraint[22]'      0            0            0            0
23  'Constraint[23]'      0.053255          0            0            0
24  'Constraint[24]'      0            0            0            0
25  'Constraint[25]'      0            0            0            0
26  'Constraint[26]'      0            0            0            0
27  'Constraint[27]'      0            0            0            0
28  'Constraint[28]'      0            0            0            0
29  'Constraint[29]'      0            0            0            0
30  'Constraint[30]'      0            0            0            0
31  'Constraint[31]'      0            0            0            0
32  'Constraint[32]'      0.304768         1.16415e-10     940570          920000
33  'Constraint[33]'      0.365229          0          559253          530000
34  'Constraint[34]'      0.0910608        -4.65661e-10    -3540290        -3600000
;
:   _con.down      :=

```

```
: _con.down :=
1  -1e+20
2  -1e+20
3  -1e+20
4  -1e+20
5  -1e+20
6  -1e+20
7  -1e+20
8  -1e+20
9  -1e+20
10 0
11 0
12 -1e+20
13 0
14 0
15 0
16 0
17 0
18 0
19 0
20 0
21 0
22 0
23 0
24 0
25 0
26 0
27 0
28 0
29 0
30 0
31 0
32 855642
33 461048
```

```
32 855642
33 461048
34 -3740730
;
```

- Output for 3.9 Billion Budget

```

suffix up OUT;
suffix down OUT;
suffix current OUT;
X [*] :=
1      0   13      0   25      0   37      0   49  46000   61      0
2      0   14      0   26      0   38      0   50      0   62      0
3      0   15      0   27      0   39      0   51      0   63      0
4      0   16      0   28      0   40      0   52      0   64      0
5      0   17      0   29      0   41      0   53      0   65  53000
6      0   18      0   30      0   42      0   54  138000   66      0
7      0   19      0   31      0   43      0   55  136000   67  79500
8      0   20      0   32      0   44      0   56      0   68      0
9      0   21      0   33      0   45      0   57      0   69      0
10     0   22      0   34      0   46      0   58      0   70  79500
11     0   23      0   35  6e+05  47      0   59      0   71  318000
12     0   24      0   36      0   48      0   60      0   72      0
;

z = 3293330

# $5 = _var.current
:  _varname  _var      _var.rc      _var.down      $5      _var.up
:=
1  'X[1]'      0  57.721      21.7325      79.4534      1e+20
2  'X[2]'      0  57.721      21.7325      79.4534      1e+20
3  'X[3]'      0  3.27664      0.105644      3.38229      1e+20
4  'X[4]'      0  3.27664      0.105644      3.38229      1e+20
5  'X[5]'      0  3.27664      0.105644      3.38229      1e+20
6  'X[6]'      0  1.14239      0.430122      1.57252      1e+20
7  'X[7]'      0  0.280588      0.105644      0.386232      1e+20
8  'X[8]'      0  0      0      0      1e+20
9  'X[9]'      0  27.4666      21.7325      49.199      1e+20
10 'X[10]'     0  27.4666      21.7325      49.199      1e+20
11 'X[11]'     0  1.98873      0.105644      0.09438      1e+20

```

12	'X[12]'	0	1.98873	0.105644	2.09438	1e+20
13	'X[13]'	0	1.98873	0.105644	2.09438	1e+20
14	'X[14]'	0	0.543609	0.430122	0.973731	1e+20
15	'X[15]'	0	0.133518	0.105644	0.239162	1e+20
16	'X[16]'	0	0	0	0	1e+20
17	'X[17]'	0	21.9859	21.7325	43.7184	1e+20
18	'X[18]'	0	21.9859	21.7325	43.7184	1e+20
19	'X[19]'	0	1.75542	0.105644	1.86107	1e+20
20	'X[20]'	0	1.75542	0.105644	1.86107	1e+20
21	'X[21]'	0	1.75542	0.105644	1.86107	1e+20
22	'X[22]'	0	0.435138	0.430122	0.86526	1e+20
23	'X[23]'	0	0.106876	0.105644	0.21252	1e+20
24	'X[24]'	0	0	0	0	1e+20
25	'X[25]'	0	46.3795	21.7325	68.112	1e+20
26	'X[26]'	0	46.3795	21.7325	68.112	1e+20
27	'X[27]'	0	2.79385	0.105644	2.89949	1e+20
28	'X[28]'	0	2.79385	0.105644	2.89949	1e+20
29	'X[29]'	0	2.79385	0.105644	2.89949	1e+20
30	'X[30]'	0	0.917928	0.430122	1.34805	1e+20
31	'X[31]'	0	0.225456	0.105644	0.3311	1e+20
32	'X[32]'	0	0	0	0	1e+20
33	'X[33]'	0	0	0	0	0
34	'X[34]'	0	0	0	0	0
35	'X[35]'	6e+05	-1.38778e-17	-1e+20	0.040458	0.040458
36	'X[36]'	0	-1.38778e-17	0.040458	0.040458	1e+20
37	'X[37]'	0	-1.38778e-17	0.040458	0.040458	1e+20
38	'X[38]'	0	0	0	0	0
39	'X[39]'	0	0	0	0	0
40	'X[40]'	0	0.065186	-0.065186	0	1e+20
41	'X[41]'	0	0.12672	21.7325	21.8592	1e+20
42	'X[42]'	0	0.12672	0	0	0
43	'X[43]'	0	0.82489	0	0	0
44	'X[44]'	0	0.82489	0	0	0
45	'X[45]'	0	0.82489	0	0	0



```

46 'X[46]'      0      0.002508      0.430122      0.43263      1e+20
47 'X[47]'      0      0.000616      0.105644      0.10626      1e+20
48 'X[48]'      0      0      0      0      1e+20
49 'X[49]'      46000  1.6237e-15      0.105644      21.7325      21.7325
50 'X[50]'      0      1.6237e-15      21.7325      21.7325      1e+20
51 'X[51]'      0      0.819496      0.105644      0.92514      1e+20
52 'X[52]'      0      0.819496      0.105644      0.92514      1e+20
53 'X[53]'      0      0.819496      0.105644      0.92514      1e+20
54 'X[54]'      138000  1.38778e-17      0.105644      0.430122      0.43263
55 'X[55]'      136000  0      0.040458      0.105644      0.10626
56 'X[56]'      0      0      0      0      1e+20
57 'X[57]'      0      22.6908      36.8993      59.5901      1e+20
58 'X[58]'      0      22.6908      36.8993      59.5901      1e+20
59 'X[59]'      0      0.965935      1.57078      2.53672      1e+20
60 'X[60]'      0      0.965935      1.57078      2.53672      1e+20
61 'X[61]'      0      0.965935      1.57078      2.53672      1e+20
62 'X[62]'      0      0.449089      0.730298      1.17939      1e+20
63 'X[63]'      0      0.110302      0.179371      0.289674      1e+20
64 'X[64]'      0      0      0      0      1e+20
65 'X[65]'      53000  2.02616e-15      0.179372      36.8993      36.8993
66 'X[66]'      0      2.02616e-15      36.8993      36.8993      1e+20
67 'X[67]'      79500  2.77556e-17      0.179372      1.57078      1.57078
68 'X[68]'      0      2.77556e-17      1.57078      1.57078      1e+20
69 'X[69]'      0      2.77556e-17      1.57078      1.57078      1e+20
70 'X[70]'      79500  2.77556e-17      0.179372      0.730298      1.17939
71 'X[71]'      318000  0      0      0.179371      0.289674
72 'X[72]'      0      0      0      0      1e+20
;
# $5 = _con.current
:      _conname      _con      _con.slack      _con.up      $5      _con.down
:=
1      'Constraint[1]'      0      6e+05      0      -6e+05      -1e+20
2      'Constraint[2]'      0      6e+05      0      -6e+05      -1e+20
3      'Constraint[3]'      0      6e+05      0      -6e+05      -1e+20

```

4	'Constraint[4]'	0	6e+05	0	-6e+05	-1e+20
5	'Constraint[5]'	0.065186	0	-320000	-6e+05	-736000
6	'Constraint[6]'	0	6e+05	0	-6e+05	-1e+20
7	'Constraint[7]'	0	280000	-320000	-6e+05	-1e+20
8	'Constraint[8]'	0	6e+05	0	-6e+05	-1e+20
9	'Constraint[9]'	0	70000	-530000	-6e+05	-1e+20
10	'Constraint[10]'	21.6268	0	111594	46000	0
11	'Constraint[11]'	36.7199	0	130438	53000	0
12	'Constraint[12]'	0	508000	6e+05	92000	-1e+20
13	'Constraint[13]'	1.39141	0	397500	79500	0
14	'Constraint[14]'	0.324478	0	274000	138000	0
15	'Constraint[15]'	0.550927	0	397500	79500	0
16	'Constraint[16]'	0	0	0	0	0
17	'Constraint[17]'	20.7169	0	0	0	0
18	'Constraint[18]'	0	0	0	0	0
19	'Constraint[19]'	20.7169	0	0	0	0
20	'Constraint[20]'	0	0	0	0	0
21	'Constraint[21]'	0.346126	0	0	0	0
22	'Constraint[22]'	0	0	0	0	0
23	'Constraint[23]'	0.035838	0	0	0	0
24	'Constraint[24]'	0	0	0	0	0
25	'Constraint[25]'	0	0	0	0	0
26	'Constraint[26]'	0	0	0	0	0
27	'Constraint[27]'	0	0	0	0	0
28	'Constraint[28]'	0	0	0	0	0
29	'Constraint[29]'	0	0	0	0	0
30	'Constraint[30]'	0	0	0	0	0
31	'Constraint[31]'	0	0	0	0	0
32	'Constraint[32]'	0.105644	0	986153	920000	784000
33	'Constraint[33]'	0.179371	0	6e+05	530000	212000
34	'Constraint[34]'	0	144658	-3755340	-3900000	-1e+20

ampl: