Chapter IV: OLAP

Knowledge Discovery in Databases

Luciano Melodia M.A.
Evolutionary Data Management, Friedrich-Alexander University Erlangen-Nürnberg

Summer semester 2021

Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

What is a data warehouse?

Defined in many different ways, but not rigorously:

- A **decision-support** database that is **maintained separately** from the organization's operational database.
- Supports information processing by providing a solid platform of **consolidated, historical data** for analysis.

Famous:

A data warehouse is a subject-oriented, integrated, time-variant, and nonvolatile collection of data in support of management's decision-making process.

– W. H. Inmon.

Data warehousing: The process of constructing and using data warehouses.

Data warehouse – subject-oriented

Organized around major subjects.

Such as customer, product, sales.

Focusing on the modeling and analysis of data for decision makers.

Not on daily operations or transaction processing.

Provide a simple and concise view around particular subject issues.

By excluding data that are not useful in the decision-support process.

Data warehouse – integrated

Constructed by integrating multiple heterogeneous data sources.

- Relational databases, flat files, online transaction records, . . .

Data-cleaning and data-integration techniques are applied.

- Ensure consistency in naming conventions, encoding structures, attribute measures, etc. among different data sources.
- E.g., hotel price: currency, tax, breakfast covered, etc.
- When data is moved to the warehouse, it is converted.
- ETL – Extraction, Transformation, Loading, see below.

Data warehouse – time variant

The time horizon for a data warehouse is significantly longer than that of operational systems.

Operational database: current-value data.

Data warehouse: provide information from a historical perspective, e.g. past 5 — 10 years.

**Every key structure in the data warehouse contains
an element of time, explicitly or implicitly.**

The key of operational data may or may not contain a "time element."

Data warehouse – nonvolatile

A physically separate store of data.

- Transformed from the operational environment.
- By copying.

No operational update of data:

- Hence, does not require transaction processing, i.e. no logging, recovery, concurrency control, etc.
- Requires only three operations:
    - Initial loading of data.
    - Refresh (update, often periodically, e.g. over night).
    - Access of data.

OLTP vs. OLAP

	OLTP	OLAP
users	clerk, IT professional	knowledge worker
function	day-to-day operations	decision support
DB design	application-oriented	decision support
data	current, up-to-date; detailed, flat relational; isolated	historical; summarized, multidimensional, integrated, consolidated
usage	repetitive	ad-hoc
access	read/write; index/hash on primary key	lots of scans
unit of work	short, simple transaction	complex query
#-records accessed	10	10^6
#-users	1000	100
DB size	100 MB to GB	100 GB to TB
quantification	transaction throughput	query throughput, response

Why a separate data warehouse?

High performance for both systems:

DBMS: tuned for OLTP; Access methods, indexing concurreny control, recovery.

Warehouse: tuned for OLAP; Complex OLAP queries, multidimensional view, consolidation.

Different functions and different data:

Missing data:

Decision support (DS) requires **historical data**
which operational DBs do not typically maintain.

Data consolidation:

DS requires **consolidation** (aggregation, summarization)
of data from heterogeneous sources.

Data quality:

Different sources typically use inconsistent data representations,
codes and formats which have to be reconciled.

Note: There are more and more systems which perform OLAP analysis directly on relational databases.

Three Data warehouse models

Enterprise Warehouse:

Collects all of the information about subjects spanning the entire organization.

Data mart:

A **subset** of corporate-wide data that is of value to a **specific group of users.**

Its scope is confined to specific, selected groups, such as marketing data mart.

Independent vs. dependent (directly from warehouse) data mart.

Virtual warehouse:

A set of **views** over operational databases.

Only some of the possible summary views may be materialized.

Extraction, transformation, and loading (ETL)

Extraction:

Get data from multiple, heterogeneous, and external sources.

Cleaning:

Detect errors in the data and rectify them if possible.

Transformation:

Convert data from legacy or host format to warehouse format.

Loading:

Sort, summarize, consolidate, compute views, check integrity, and build indexes and partitions.

Refresh:

Propagate only the updates from the data sources to the warehouse.

Metadata repository

Metadata: the data defining Data warehouse objects.

Description of the structure of the data warehouse:

- Schema, view, dimensions, hierarchies, derived-data definition, data-mart locations and contents.

Operational metadata:

- Data lineage (history of migrated data and transformation path).
- Currency of data (active, archived, or purged).
- Monitoring information (warehouse-usage statistics, error reports, audit trails).

Algorithms used for summarization.

Mapping from operational environment to data warehouse.

Data related to system performance:

- Warehouse schema, view and derived-data definitions.

Business data:

- Business terms and definitions, ownership of data, charging policies.

Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

From tables and spreadsheets to data cubes

Data warehouse: basic concepts.

- Based on a **multidimensional data model** which views data in the form of a **data cube**.

Data cube.

- Allows data (here: sales) to be modeled and viewed in multiple dimensions.
  - **Dimension tables:** such as: `item (item_name, brand, type)`, or: `time (day, week, month, quarter, year)`.
  - **Fact table:** Contains **measures** (such as `dollars_sold`) and references (`foreign keys`) to each of the related dimension tables.

***n*-dimensional base cube.**

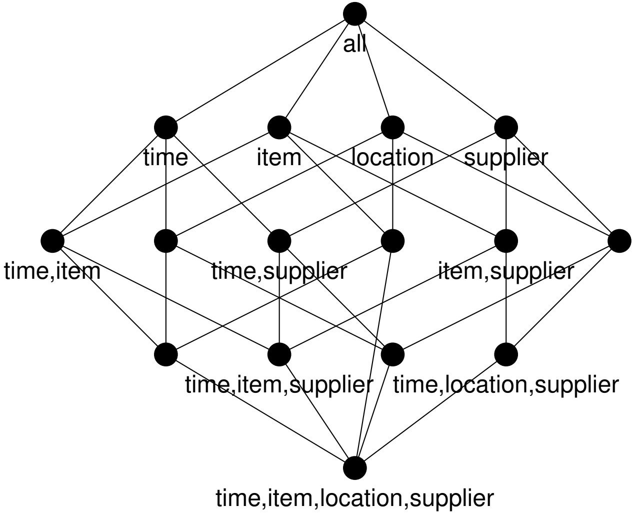
- Called a base cuboid in Data warehousing literature.

Top most 0-dimensional cuboid.

- Holds the highest-level of summarization.
- Called the apex cuboid.

Lattice of cuboids. (Forms a data cube)

Cube: a lattice of cuboids



0-dimensional (apex) cuboid

1-dimensional cuboid

2-dimensional cuboid

3-dimensional cuboid

4-dimensional (base) cuboid

Conceptual modeling of data warehouses

Star schema:.

A fact table in the middle connected to a set of dimension tables.

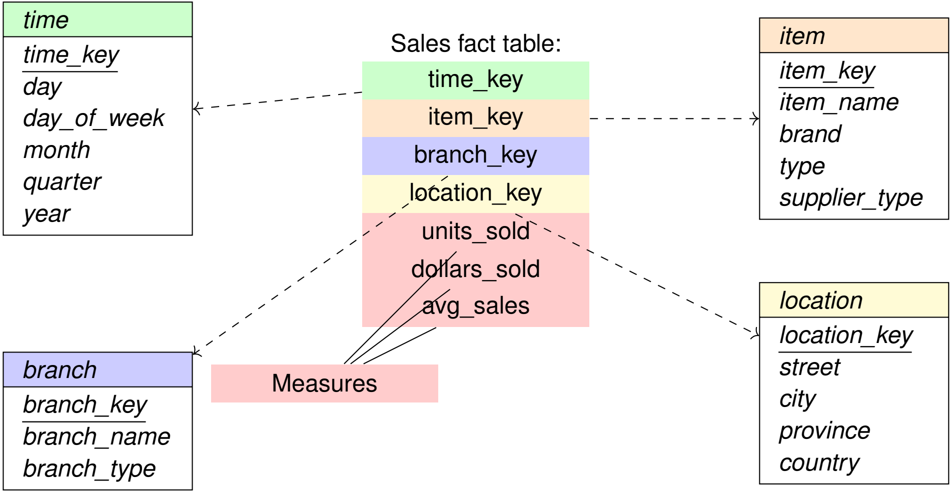
Snowflake schema:.

A refinement of the star schema where some dimensional hierarchy
is **normalized** into a set of smaller dimension tables,
forming a shape similar to a snowflake.

Fact constellations:.

Multiple fact tables sharing dimension tables,
viewed as a collection of stars, therefore called
galaxy schema or fact constellation.

Example of star schema



time
time_key
day
day_of_week
month
quarter
year

Sales fact table:

time_key
item_key
branch_key
location_key
units_sold
dollars_sold
avg_sales

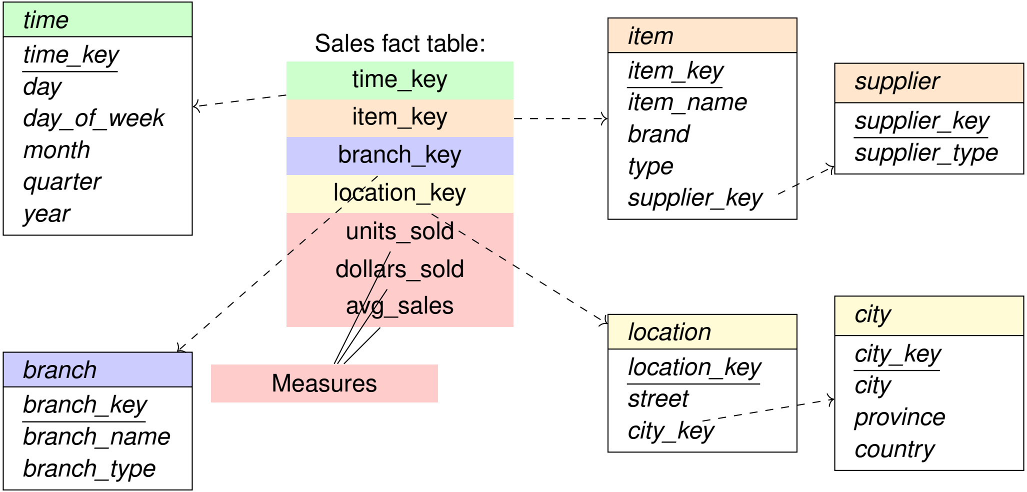
item
item_key
item_name
brand
type
supplier_type

branch
branch_key
branch_name
branch_type

Measures

location
location_key
street
city
province
country

Example of snowflake schema



time

time_key
day
day_of_week
month
quarter
year

Sales fact table:

time_key
item_key
branch_key
location_key
units_sold
dollars_sold
avg_sales

Measures

item

item_key
item_name
brand
type
supplier_key

supplier

supplier_key
supplier_type

branch

branch_key
branch_name
branch_type

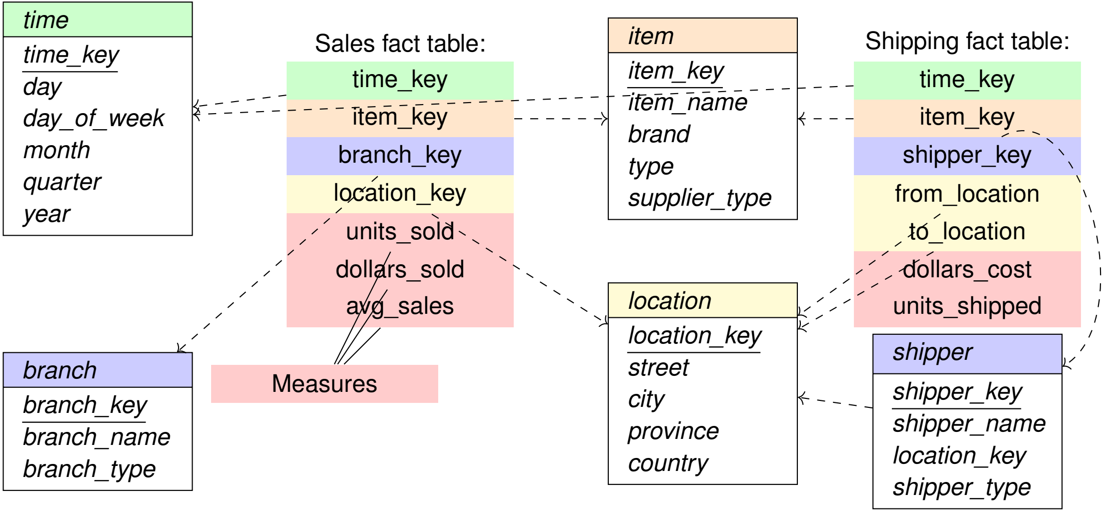
location

location_key
street
city_key

city

city_key
city
province
country

Example of fact constellation



time

time_key
day
day_of_week
month
quarter
year

Sales fact table:

time_key
item_key
branch_key
location_key
units_sold
dollars_sold
avg_sales

Measures

branch

branch_key
branch_name
branch_type

item

item_key
item_name
brand
type
supplier_type

location

location_key
street
city
province
country

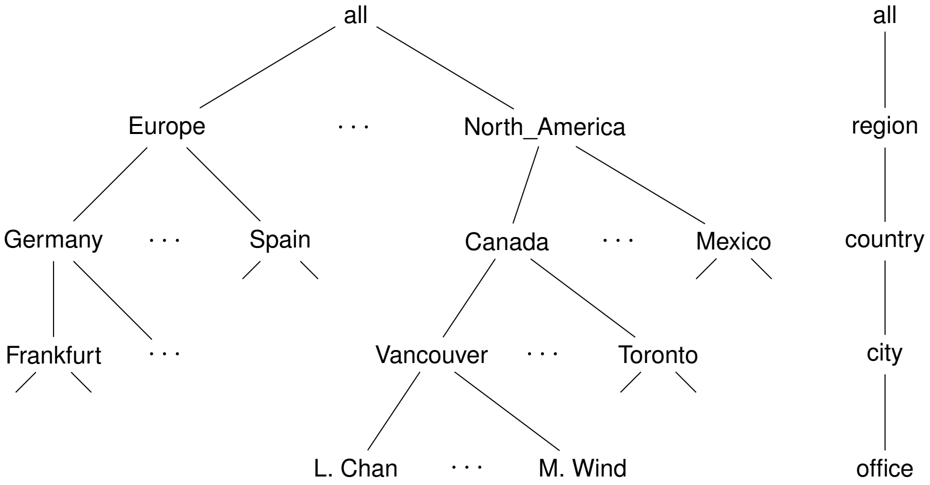
Shipping fact table:

time_key
item_key
shipper_key
from_location
to_location
dollars_cost
units_shipped

shipper

shipper_key
shipper_name
location_key
shipper_type

A concept hierarchy: dimension (location)



Data-cube measures: three categories

Distributive:

If the result derived by applying the function to the *n* aggregate values obtained for *n* partitions of the dataset is the same as that derived by applying the function on all the data without partitioning.

E.g. `COUNT`, `SUM`, `MIN`, `MAX`.

Functional:

If it can be computed by an algebraic function with *M* arguments, each of which is obtained by applying a distributive aggregate function.

E.g. `AVG`, $\texttt{MIN}_N$, `STD`.

Holistic:

If there is no constant bound on the storage size needed to describe a subaggregate.

E.g. `MEDIAN`, `MODE`, `RANK`.

Aggregation type

Non-trivial property.

Next to name and value range.

Defines the set of aggregation operations that can be executed on a measure (a fact).

FLOW:

Any aggregation.

E.g. sales turnover.

STOCK:

No temporal aggregation.

E.g. stock, inventory.

VPU (Value per Unit:

No summarization.

E.g. price, tax, in general factors.

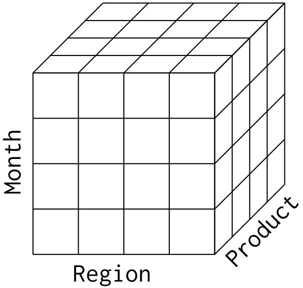
(Always applicable: `MIN`, `MAX` and `AVG`).

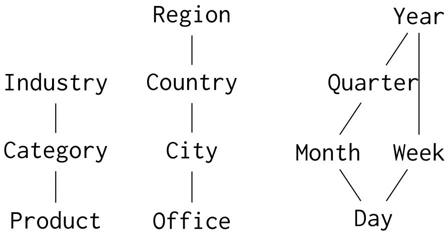
Aggregation type

Sales volume as a function of product, month, and region.

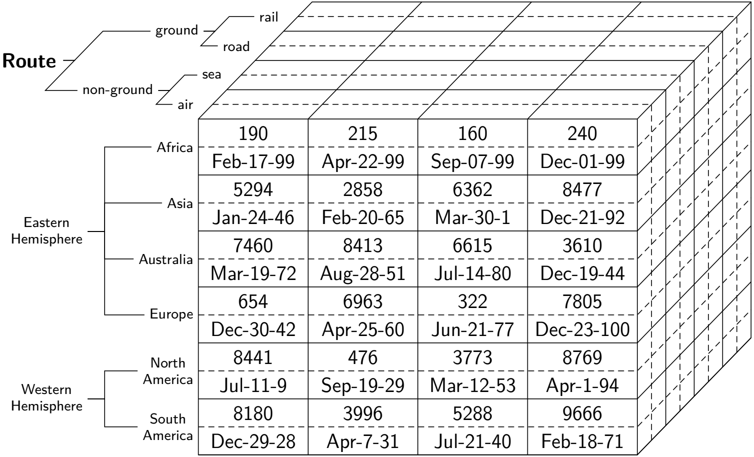
Dimensions: `Product`, `Location`, `Time`.

Hierarchical summarization paths.



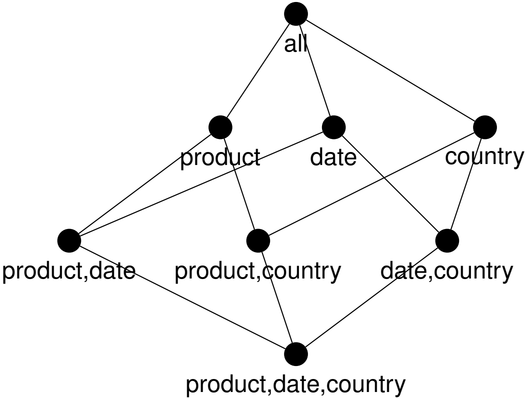


Data cube sample



Eastern Hemisphere	Africa	190 Feb-17-99	215 Apr-22-99	160 Sep-07-99	240 Dec-01-99
	Asia	5294 Jan-24-46	2858 Feb-20-65	6362 Mar-30-1	8477 Dec-21-92
	Australia	7460 Mar-19-72	8413 Aug-28-51	6615 Jul-14-80	3610 Dec-19-44
	Europe	654 Dec-30-42	6963 Apr-25-60	322 Jun-21-77	7805 Dec-23-100
Western Hemisphere	North America	8441 Jul-11-9	476 Sep-19-29	3773 Mar-12-53	8769 Apr-1-94
	South America	8180 Dec-29-28	3996 Apr-7-31	5288 Jul-21-40	9666 Feb-18-71

Cuboids corresponding to the cube



0-dimensional (apex) cuboid

1-dimensional cuboid

2-dimensional cuboid

3-dimensional (base) cuboid

Typical OLAP operations

Roll up (drill up): summarize data.

By climbing up hierarchy or by dimension reduction.

Drill down (roll down): reverse of roll up.

From higher-level summary to lower-level summary or detailed data, or introducing new dimensions.

Slice and dice: project and select.

Pivot (rotate):

Reorient the cube, visualization, 3D to series of 2D planes.

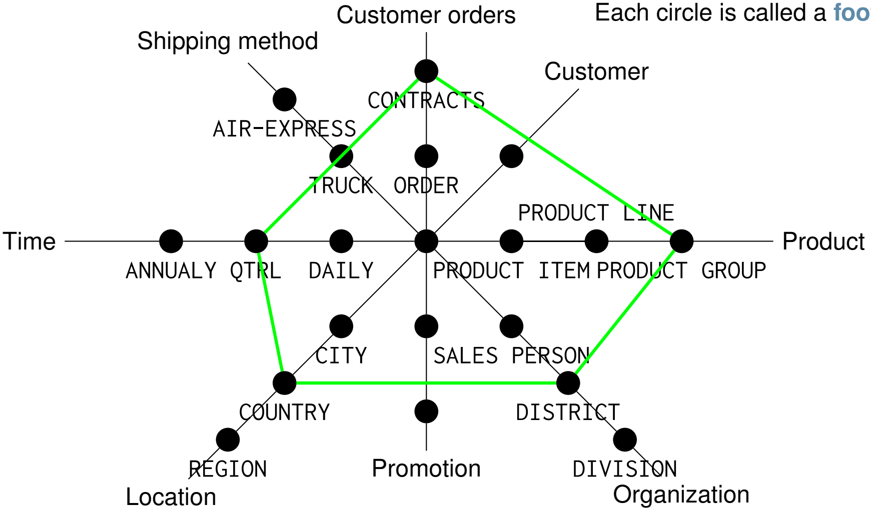
Other operations:

Drill across: involving (across) more than one fact table.

Drill through: through the bottom level of the cube to its back-end relational tables (using SQL).

A star-net query model

Each circle is called a **footprint**.



Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

Design of data warehouse: a business-analysis framework

Four views regarding the design of a data warehouse:

Top-down view:

Allows selection of the relevant information necessary for the data warehouse.

Data-source view:

Exposes the information being captured, stored, and managed by operational systems.

Data warehouse view:

Consists of fact tables and dimension tables.

Business-query view:

Sees the perspectives of data in the warehouse from the view of the end-user.

Data warehouse design process

Top-down, bottom-up approaches or a combination of both:

- **Top-down:** starts with overall design and planning (mature).
- **Bottom-up:** starts with experiments and prototypes (rapid).

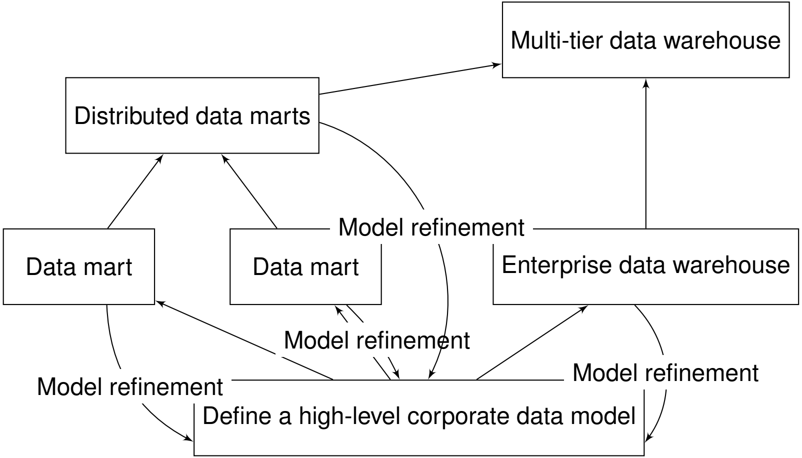
From software-engineering point of view:

- **Waterfall:** structured and systematic analysis at each step before proceeding to the next.
- **Spiral:** rapid generation of increasingly functional systems, short turn-around time, quick turn-around.

Typical Data warehouse design process:

- Choose a **business process** to model, e.g., orders, invoices, etc.
- Choose a **grain** (atomic level of data) of the business process.
- Choose a **dimensions** that will apply to each fact-table record.
- Choose a **measure** that will populate each fact-table record.

Data warehouse development: a recommended approach



Data warehouse usage

Three kinds of Data warehouse applications.

Information processing.

Supports querying, basic statistical analysis, and reporting using crosstabs, tables, charts and graphs.

Analytical processing.

Multidimensional analysis of data warehouse data.

Supports basic OLAP operations, slice-dice, drilling, pivoting.

Data mining.

Knowledge discovery from hidden patterns.

Supports associations, constructing analytical models, performing classification and prediction, and presenting the mining results using visualization tools.

From online analytical processing (OLAP) to online analytical mining (OLAM)

Why online analytical mining?

- DW contains integrated, consistent, cleaned data.

Available information-processing structure surrounding data warehouses.

- ODBC, OLEDB, Web access, service facilities, reporting, and OLAP tools.

OLAP-based exploratory data analysis.

- Mining with drilling, dicing, pivoting, etc.

Online selection of data-mining functions.

- Integration and swapping of multiple mining functions, algorithms, and tasks.

Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

Efficient data-cube computation

Data cube can be viewed as a lattice of cuboids.

The bottom-most cuboid is the base cuboid.

The top-most cuboid (apex) contains only one cell.

How many cuboids in an *n*-dimensional cube with L_i levels associated with dimension *i*?

$$T = \prod_{i=1}^{n} (L_i + 1). \tag{1}$$

Materialization of data cube.

Materialize each (cuboid) (full materialization), none (no materialization), or some (partial materialization).

Selection of cuboids to materialize based on size, sharing, access frequency, etc.

The "compute cube" operator

Cube definition and computation in DMQL:

```
DEFINE CUBE sales [item, city, year]:
SUM (sales_in_dollars);
COMPUTE CUBE sales;
```

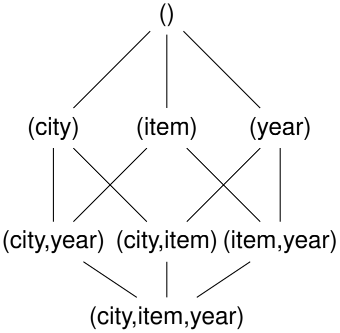
Transform it into an SQL-like language:

with a new operator `CUBE BY` (Gray et al. 9́6).

```
SELECT item, city, year, SUM (amount)
FROM sales
CUBE BY item, city, year;
```

Need to compute the following `Group bys`:

```
(date, product, customer),
(date, product), (date, customer),
(product, customer),
(date), (product), (customer)
()
```



()

(city) (item) (year)

(city,year) (city,item) (item,year)

(city,item,year)

Indexing OLAP data: bitmap index

Index on a particular column.

Each value in the column has a bit vector: bit-op is fast.

Length of bit vector: # of records in base table.

***i*-th bit set, if *i*-th row of base table has value of bit vector.**

Not suitable for high-cardinality domains:

A bit compression technique called Word-Aligned Hybrid (WAH) makes it work for high-cardinality domain as well [Wu et al., TODS'06].

Base table

Cust	Region	Type
$C1$	Asia	Retail
$C2$	Europe	Dealer
$C3$	Asia	Dealer
$C4$	America	Retail
$C5$	Europe	Dealer

Index on region

RecID	Asia	Europe	America
1	1	0	0
2	0	1	0
3	1	0	0
4	0	0	1
5	0	1	0

Index on type

RecID	Retail	Dealer
1	1	0
2	0	1
3	0	1
4	1	0
5	0	1

Indexing OLAP data: join indices

Join index:

$$\text{JI}(\texttt{R-id}, \texttt{S-id}) \quad \text{where} \quad \text{R}(\texttt{R-id}, \ldots) \bowtie \text{S}(\texttt{S-id}, \ldots). \tag{2}$$

Traditional indices map the values to a list of record ids.

Materializes rel. join in JI file and speeds up relational join.

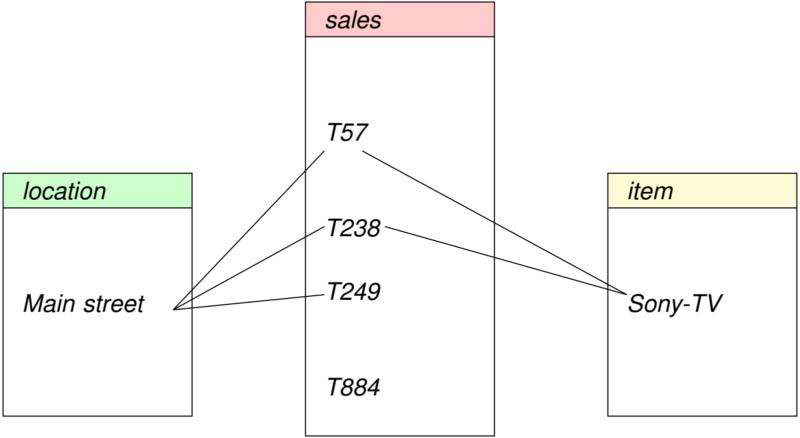
In data warehouses, join index relates the values of the dimensions of a star schema to rows in the fact table.

E.g. fact table: Sales and two dimensions location and item.

A join index on location maintains for each distinct location a list of `R-ids` of the tuples recording the Sales in that location.

Join indices can span multiple dimensions.

Indexing OLAP data: join indices (example)



Efficient processing of OLAP queries

Determine which operations should be performed on the available cuboids.

Transform drill, roll, etc. into corresponding SQL and/or OLAP operations.
E.g. `dice` = `selection` + `projection`.

Determine which materialized cuboid(s) should be selected for OLAP operation.

Let the query to be processed be on {`brand, province_or_state`} with the condition "`year = 2004`", and there are 4 materialized cuboids available:

1) `year, item_name, city`
2) `year, brand, country`
3) `year, brand, province_or_state`
4) `item_name, province_or_state where year = 2004`

Which should be selected to process the query?

Explore indexing structures and compressed vs. dense-array structures in MOLAP.

OLAP server architectures

Relational OLAP (ROLAP).

Use relational or extended-relational DBMS to store and manage warehouse data and OLAP middleware.

Include optimization of DBMS backend, implementation of aggregation navigation logic, and additional tools and services.

Greater scalability.

Multidimensional OLAP (MOLAP).

Sparse array-based multidimensional storage engine.

Fast indexing to pre-computed summarized data.

Hybrid OLAP (HOLAP) (e.g., Microsoft SQL-Server).

Flexibility, e.g., low level: relational, high-level: array.

Specialized SQL servers (e.g., Redbricks).

Specialized support for SQL queries over star/snowflake schemas.

Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

Data generalization

Summarize data:

- **By replacing relatively low-level values**
  e.g. numerical values for the attribute `age`
  with higher-level concepts
  e.g. `young`, `middle-aged` and `senior`.
- **By reducing the number of dimensions**
  e.g. removing `birth_date` and `telephone_number`
  when summarizing the behavior of a group of students.
- Describe concepts in concise and succinct terms at generalized (rather than low) levels of abstractions:
  - Facilitates users in examining the general behavior of the data.
  - Makes dimensions of a data cube easier to grasp.

Attribute-oriented induction

Proposed in 1989 (KDD'89 workshop).

Not confined to categorical data nor to particular measures.

How is it done?

- Collect the **task-relevant data** (initial relation) using a relational database query.
- Perform **generalization** by attribute removal or attribute generalization.
- Apply **aggregation** by merging identical, generalized tuples and accumulating their respective counts.
- Interaction with users for knowledge presentation.

Attribute-oriented induction: an example

Example: Describe general characteristics of graduate students in a University database.

Step 1: Fetch relevant set of data using an SQL statement, e.g.

```
SELECT name, gender, major, birth_place, birth_date, residence, phone#, gpa)
FROM student
WHERE student_status IN ”Msc”, ”MBA”, ”PhD”;
```

Step 2: Perform attribute-oriented induction.

Step 3: Present results in generalized-relation, cross-tab, or rule forms.

Class characterization: an initial relation (I)

Name	Gender	Major	Birth place	Birth date	Residence	Phone number	GPA
Jim	M	CS	Vancouver, BC, Canada	08-21-76	3511 Main St., Richmond	687-4598	3.67
Scott Lachance	M	CS	Montreal, Que, Canada	28-07-75	345 1st Ave., Richmond	253-9106	3.70
Laura Lee	F	Physics	Seattle, WA, USA	25-08-70	125 Austin Ave., Burnaby	420-5232	3.83
Removed	Retained	Sci, Eng, Bus	Country	Age range	City	Removed	Excl, Vg,. . .

Class characterization: prime generalized relation (II)

Gender	**Major**	**Birth region**	**Age range**	**Residence**	**GPA**	**Count**
M	Science	Canada	20-35	Richmond	Very good	16
F	Science	Foreign	25-30	Burnaby	Excellent	22
...	...	...	...	...	...	...

Class characterization: an example (III)

Cross-table of birth region and gender:

	Canada	Foreign	Total
M	16	14	30
F	10	22	32
Total	26	36	62

Basic principles of attribute-oriented induction

Data focusing:

Task-relevant data, including dimensions
The result is the **initial relation**.

Attribute removal:

Remove attribute A, if there is a large set of distinct values for A,
but (1) there is no generalization operator on A,
or (2) A's higher-level concepts are expressed in terms of other attributes.

Attribute generalization:

If there is a large set of distinct values for A,
and there exists a **set of generalization operators** on A,
then select an operator and generalize A.

Attribute-threshold control:

Typical 2-8, specified/default.

Generalized-relation-threshold control:

Control the final relation/rule size.

Attribute-oriented induction: basic algorithm

InitialRel:

Query processing of task-relevant data, deriving the initial relation.

PreGen:

Based on the analysis of the number of distinct values in each attribute, determine generalization plan for each attribute: removal? Or how high to generalize?

PrimeGen:

Based on the PreGen plan, perform generalization to the right level to derive a "prime generalized relation", accumulating the counts.

Presentation:

User interaction:

1. Adjust levels by drilling.
2. Pivoting.
3. Mapping into rules, cross tabs, visualization presentations.

Presentation of generalized results

Generalized relation:

Relations where some or all attributes are generalized, with counts or other aggregation values accumulated.

Cross tabulation:

Mapping results into cross-tabulation form (similar to contingency tables).
Visualization techniques: pie charts, bar charts, curves, cubes, and other visual forms.

Quantitative characteristic rules:

Mapping generalized result into characteristic rules with quantitative information associated with it, e.g.

$$\begin{aligned} \text{grad}(x) \wedge \text{male}(x) &\implies \text{birth\_region}(x) && (3)\\ &= \text{"Canada"}[t : 53\%] \vee \text{birth\_region}(x) && (4)\\ &= \text{"foreign"}[t : 47\%]. && (5) \end{aligned}$$

Mining-class comparisons

Comparison: Comparing two or more classes.

Method:

- Partition the set of relevant data into the **target class** and the **contrasting class(es)**.
- Generalize both classes to the same high-level concepts (i.e. AOI).
  - Including aggregation.
- Compare tuples with the same high-level concepts.
- Present for each tuple its description and two measures.
  - Support – distribution within single class (counts, percentage).
  - Comparison – distribution between classes.
- Highlight the tuples with strong discriminant features.

Relevance Analysis:

- Find attributes (features) which best distinguish different classes.

Concept description vs. cube-based OLAP

Similarity:

Data generalization.

Presentation of data summarization at multiple levels of abstraction.

Interactive drilling, pivoting, slicing and dicing.

Differences:

OLAP has systematic preprocessing, query independent, and can drill down to rather low level.

AOI has automated desired-level allocation and may perform dimension-relevance analysis/ranking when there are many relevant dimensions.

AOI works on data which are not in relational forms.

Chapter IV: Data warehousing and online analytical processing

Data warehouse: basic concepts.

Data warehouse modeling: data cube and OLAP.

Data warehouse design and usage.

Data warehouse Implementation.

Data generalization by attribute-oriented induction.

Summary.

Summary

Data warehousing: multi-dimensional model of data.

- A data cube consists of dimensions and measures.
- Star schema, snowflake schema, fact constellations.
- OLAP operations: drilling, rolling, slicing, dicing and pivoting.

Data warehouse architecture, design, and usage.

- Multi-tiered architecture.
- Business-analysis design framework.
- Information processing, analytical processing, data mining, OLAM (Online Analytical Mining).

Implementation: efficient computation of data cubes.

- Partial vs. full vs. no materialization.
- Indexing OALP data: Bitmap index and join index.
- OLAP query processing.
- OLAP servers: ROLAP, MOLAP, HOLAP.

Data generalization: attribute-oriented induction.

References (I)

S. Agarwal, R. Agrawal, P. M. Deshpande, A. Gupta, J. F. Naughton, R. Ramakrishnan, and S. Sarawagi: On the computation of multidimensional aggregates. VLDB'96.

D. Agrawal, A. E. Abbadi, A. Singh, and T. Yurek: Efficient view maintenance in data warehouses. SIGMOD'97.

R. Agrawal, A. Gupta, and S. Sarawagi: Modeling multidimensional databases. ICDE'97.

S. Chaudhuri and U. Dayal: An overview of data warehousing and OLAP technology. ACM SIGMOD Record, 26:65-74, 1997-

E. F. Codd, S. B. Codd, and C. T. Salley: Beyond decision support. Computer World, 27, July 1993.

J. Gray, et al. Data cube: A relational aggregation operator generalizing group-by, cross-tab and sub-totals. Data Mining and Knowledge Discovery, 1:29-54, 1997.

A. Gupta and I. S. Mumick. Materialized Views: Techniques, Implementations, and Applications. MIT Press, 1999.

J. Han: Towards on-line analytical mining in large databases. ACM SIGMOD Record, 27:97-107, 1998.

References (II)

V. Harinarayan, A. Rajaraman, and J. D. Ullman: Implementing data cubes efficiently. SIGMOD'96.

J. Hellerstein, P. Haas, and H. Wang: Online aggregation. SIGMOD'97.

C. Imhoff, N. Galemmo, and J. G. Geiger: Mastering Data Warehouse Design: Relational and Dimensional Techniques. John Wiley, 2003.

W. H. Inmon: Building the Data Warehouse. John Wiley, 1996.

R. Kimball and M. Ross: The Data Warehouse Toolkit: The Complete Guide to Dimensional Modeling. 2ed. John Wiley, 2002.

P. O'Neil and G. Graefe: Multi-table joins through bitmapped join indices. ACM SIGMOD Record, 24:8–11, Sept. 1995.

P. O'Neil and D. Quass: Improved query performance with variant indexes. SIGMOD'97.

Microsoft. OLEDB for OLAP programmer's reference version 1.0. In http://www.microsoft.com/data/oledb/olap, 1998.

Thank you for your attention.
Any questions about the fourth chapter?

Ask them now, or again, drop me a line:
✉ `luciano.melodia@fau.de`.