Chapter VI: Classification

Knowledge Discovery in Databases

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Chapter VI: Classification

Classification: basic concepts.

Decision-tree induction.

Bayes classification methods.

Rule-based classification.

Model evaluation and selection.

Techniques to improve classification accuracy: ensemble methods.

Summary.

Supervised vs. unsupervised learning

Supervised learning (classification).

- Supervision:
  - The **training data** (observations, measurements, etc.) are accompanied by **labels** indicating the **class** of the observations.
  - New data is classified based on a **model** created from the training data.

Unsupervised learning (clustering).

- The class labels of training data are unknown.
  - Or rather, there are no training data.
- Given a set of measurements, observations, etc., the goal is to find classes or clusters in the data.
  - See next chapter.

Prediction problems: classification vs. numerical prediction

Classification:

- Predicts **categorical class labels** (discrete, nominal).
- Constructs a model based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data.

Numerical prediction:

- Models **continuous-valued functions**.
- I.e. predicts missing or unknown (future) values.

Typical applications of classification:

- Credit/loan approval: Will it be paid back?
- Medical diagnosis: Is a tumor cancerous or benign?
- Fraud detection: Is a transaction fraudulent or not?
- Web-page categorization: Which category is it?

Classification – a two-step process

Model construction: describing a set of predetermined classes:

- Each tuple/sample is assumed to belong to a predefined class, as determined by the **class-label attribute**.
- The set of tuples used for model construction is the **training set**.
- The **model** is represented as classification rules, decision trees, or mathematical formulae.

Model usage, for classifying future or unknown objects:

- Estimate **accuracy** of the model:
  - The known label of **test samples** is compared with the result from the model.
  - **Accuracy rate** is the percentage of test-set samples that are correctly classified by the model.
  - Test set is independent of training set (otherwise overfitting).
- If the accuracy is acceptable, **use the model** to classify data tuples whose class labels are not known.

Classification – a two-step process

Training data

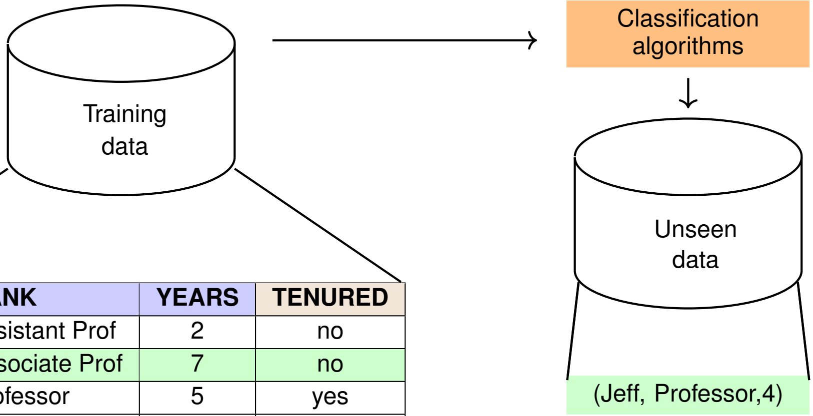
Classification algorithms

Classifier (model)

NAME	RANK	YEARS	TENURED
Mike	Assistant Prof	3	no
Mary	Assistant Prof	7	yes
Bill	Professor	2	yes
Jim	Associate Prof	7	yes
Dave	Assistant Prof	6	no
Anne	Associate Prof	3	no

```
if RANK ='Professor'
or YEARS >6
then TENURED ='yes'
```

Process (II): using the model in prediction



NAME	RANK	YEARS	TENURED
Tom	Assistant Prof	2	no
Merlisa	Associate Prof	7	no
George	Professor	5	yes
Joseph	Assistant Prof	7	yes

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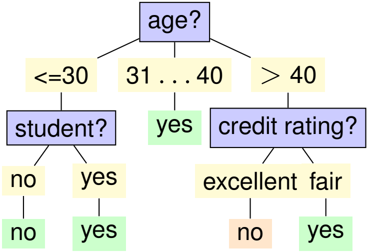
Decision-tree induction: an example

Training dataset: buys_computer.

The dataset follows an example of Quinlan's ID3 (playing tennis).

Resulting tree:



age	income	student	credit_rating	buys_coputer
$\leq$ 30	high	no	fair	no
$\leq$ 30	high	no	excellent	no
31 . . . 40	high	no	fair	yes
$>$ 40	medium	no	fair	yes
$>$ 40	low	yes	fair	yes
$>$ 40	low	yes	excellent	no
31 . . . 40	low	yes	excellent	yes
$\leq$ 30	medium	no	fair	no
$\leq$ 30	low	no	fair	yes
$>$ 40	medium	yes	fair	yes
$\leq$ 30	medium	yes	excellent	yes
31 . . . 40	medium	no	excellent	yes
31 . . . 40	high	yes	fair	yes
$>$ 40	medium	no	excellent	no

Algorithm for decision-tree induction

Basic algorithm (a greedy algorithm):

- Tree is constructed in a **top-down recursive divide-and-conquer manner.**
- Attributes are categorical.
  - If not: discretize in advance.
- At start, all the training examples are at the root.
- Examples are **partitioned recursively** based on selected attributes.
- Test attributes are selected on the basis of a heuristic or statistical measure.
  - E.g. information gain – see on the next slide.

Conditions for stopping partitioning:

- All samples for a given node belong to the same class.
- There are no remaining attributes for further partitioning.
  - Majority voting is employed for classifying the leaf.
- There are no samples left (i.e. partition for particular value is empty).

Attribute-selection measure: information gain (ID3/C4.5)

Select the attribute with the highest information gain.

Let p_i be the probability that an arbitrary tuple in D belings to class C_i, estimated by $\frac{|C_i|}{|D|}$, such that $1 \leq i \leq m$.

Expected information (entropy)needed to classify a tuple in D:

$$\text{Info}(D) = -\sum_{i=1}^{m} p_i \log_2(p_i). \tag{1}$$

Information needed (after using attribute A to split D into v partitions) to classify D:

$$\text{Info}_A(D) = \sum_{j=1}^{v} \left(\frac{|D_j|}{|D|} \text{Info}(D_j) \right). \tag{2}$$

Information gained by branching on A:

$$\text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D). \tag{3}$$

Attribute selection: information gain

Class P: buys_computer = "yes"

Class N: buys_computer = "no"

$$\text{Info}(D) = I(9,5) = -\frac{9}{14}\log_2(\frac{9}{14}) - \frac{5}{14}\log_2(\frac{5}{14}) = 0.94$$

age	p	n	$I(p,n)$
≤ 30	2	3	0.971
$31\dots40$	4	0	0
> 40	3	2	0.971

Similarly,

$$\text{Gain}(\texttt{income}) = 0.029,$$
$$\text{Gain}(\texttt{student}) = 0.151,$$
$$\text{Gain}(\texttt{credit\_rating}) = 0.048.$$

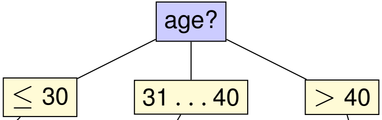
$$\text{Info}_{\text{age}}(D) = \frac{5}{14}I(2,3) + \frac{4}{14}I(4,0) + \frac{5}{14}I(3,2) = 0.694.$$

$\frac{5}{14}I(2,3)$ means "age ≤ 30" has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence,

$$\text{Gain}(\text{age}) = \text{Info}(D) - \text{Info}_{\text{age}}(D) = 0.246.$$

age	income	student	credit_rating	buys_computer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
$31\dots40$	high	no	fair	yes
> 40	medium	no	fair	yes
> 40	low	yes	fair	yes
> 40	low	yes	excellent	no
$31\dots40$	low	yes	excellent	yes
≤ 30	medium	no	fair	no
> 40	medium	yes	fair	yes
≤ 30	medium	yes	excellent	yes
$31\dots40$	medium	no	fair	yes
$31\dots40$	high	yes	fair	yes
> 40	medium	no	excellent	no

Partitioning in the example



income	student	credit_rating	buys_computer
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

income	student	credit_rating	
high	no	fair	
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

income	student	credit_rating	buys_computer
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

Computing information gain for continuous-valued attributes

Let attribute A be a continuous-valued attribute.

Must determine the best split point for A.

- Sort the values of A in increasing order.
- Typically, the midpoint between each pair of adjacent values is considered as a possible split point.
  - $\frac{a_i+a_{i+1}}{2}$ is the midpoint between the values of a_i and a_{i+1}.
- The point with the minimum expected information requirement for A is selected as the split point for A.

Split:

- D_1 is the set of tuples in D satisfying $A \leq$ split point, and D_2 is the set of tuples in D satisfying $A >$ split point.

So to say: Discretization as you go along.

- For this particular purpose.

Gain ratio for attribute selection (C4.5)

Information-gain measure is biased towards attributes with a large number of values. C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain):

$$\mathrm{SplitInfo}_A(D) = -\sum_{j=1}^{v} \frac{|D_j|}{|D|} \log_2\left(\frac{|D_j|}{|D|}\right), \tag{4}$$

$$\mathrm{GainRatio}(A) = \frac{\mathrm{Gain}(A)}{\mathrm{SplitInfo}_A(D)}. \tag{5}$$

Example:

$$\mathrm{SplitInfo}_{\texttt{income}}(D) = -\frac{4}{14}\log_2\left(\frac{4}{14}\right) - \frac{6}{14}\log_2\left(\frac{6}{14}\right) - \frac{4}{14}\log_2\left(\frac{4}{14}\right) = 1.557, \tag{6}$$

$$\mathrm{GainRatio}(\texttt{income}) = \frac{0.029}{1.557} = 0.019. \tag{7}$$

The attribute with the maximum gain ratio is selected as the splitting attribute.

Gini index

Corrado Gini (1884 – 1965).

Italian statistician and sociologist.

Also called Gini coefficient.

Measures statistical dispersion.

Zero expresses perfect equality where all values are the same.

One expresses maximal inequality among values.

Based on the Lorenz curve.

Plots the proportion of the total sum of values (y-axis) that is cumulatively assigned to the bottom $x\%$ of the population.

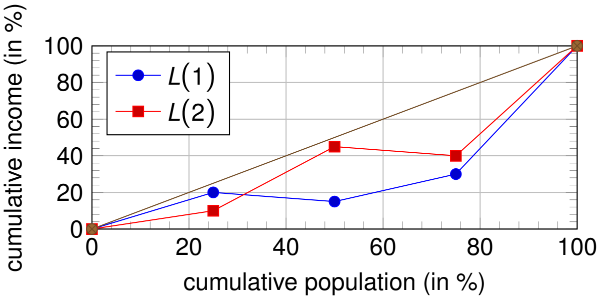
Line at 45 degrees thus represents perfect equality of value distribution.

Gini coefficient then is . . .

. . . the ratio of the area that lies between the line of equality and the Lorenz curve over the total area under the line of equality.

Gini index (II)

Example: Distribution of incomes.



Gini index (CART, IBM IntelligentMiner)

If a dataset D contains examples from n classes, Gini index gini(D) is defined as:
gini(D) = 1-$\sum_{j=1}^{n} p_j^2$, where p_j is the relative frequency of class j in D.

If a dataset D is split on A into two subsets D_1 and D_2, the Gini index gini(D) is defined as

$$\text{gini}_A(D) = \frac{|D_1|}{|D|}\text{gini}(D_1) + \frac{|D_2|}{|D|}\text{gini}(D_2). \tag{8}$$

Reduction in impurity:

$$\Delta\text{gini}_A(A) = \text{gini}(D) - \text{gini}_A(D). \tag{9}$$

The attribute A provides the smallest gini$_A(D)$ (or the largest reduction in impurity) is chosen to split the node.

Need to enumerate all the possible splitting points for each attribute.

Computation of Gini index

Example:

D has 9 tuples in buys_computer = "yes" and 5 in "no", thus

$$\text{gini}(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right) = 0.459. \tag{10}$$

Suppose the attribute `income` partitions D into 10 in D_1 : $\{\texttt{low},\texttt{medium}\}$ and 4 in D_2 : $\{\texttt{high}\}$:

$$\text{gini}(D|_{D[\texttt{income}]=''medium'',''low''}) \tag{11}$$

$$= \left(\frac{10}{14}\right)\text{gini}(D_1) + \frac{4}{14}\text{gini}(D_2) \tag{12}$$

$$= \frac{10}{14}\left(1 - \left(\frac{7}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) + \frac{4}{14}\left(1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2\right) = \tag{13}$$

$$= 0.443 = \text{gini}(D|_{D[\texttt{income}]=''high''}). \tag{14}$$

Computation of Gini index (II)

Example (cont.):

$\text{gini}\left(D|_{D[\texttt{income}]=''low'',''high''}\right) = 0.458,$
$\text{gini}\left(D|_{D[\texttt{income}]=''medium'',''high''}\right) = 0.450.$
Thus, splot on the {"low","medium"} and {"high"}, since it has the lowest gini index.

All attributes are assumed continuous-valued.

May need other tools, e.g. clustering, to get the possible split values.

Can be modified for categorical attributes.

Computation of Gini index (II)

The three measures, in general, return good results, but

- **Information gain:**
  - Biased towards multi-valued attributes.
- **Gain ratio:**
  - Tends to prefer unbalanced splits in which one partition is much smaller than the others.
- **Gini index:**
  - Biased to multi-valued attributes.
  - Has difficulty when number of classes is large.
  - Tends to favor tests that result in equal-sized partitions and purity in both partitions.

Other attribute-selection measures

CHAID:

A popular decision-tree algorithm, measure based on χ^2 test for independence.

C-SEP:

Performs better than information gain and Gini index in certain cases.

G-statistic:

Has a close approximation to χ^2 distribution.

MDL (Minimal Description Length) principle:

I.e. the simplest solution is preferred.

The best tree is the one that requires the fewest number of bits to both (1) encode the tree and (2) encode the exceptions to the tree.

Multivariate splits:

Partitioning based on multiple variable combinations.

CART: finds multivariate splits based on a linear combination of attributes.

Which attribute-selection measure is the best?

Most give good results, none is significantly superior to others.

Thank you for your attention.

Any questions about the sixt chapter?

Ask them now, or again, drop me a line:

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