

Chapter VI: Classification

Knowledge Discovery in Databases

Luciano Melodia M.A.

Evolutionary Data Management, Friedrich-Alexander University Erlangen-Nürnberg

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Chapter VI: Classification

Classification: basic concepts.

Decision-tree induction.

Bayes classification methods.

Rule-based classification.

Model evaluation and selection.

Techniques to improve classification accuracy: ensemble methods.

Summary.

Supervised vs. unsupervised learning

Supervised learning (classification).

Supervision:

The **training data** (observations, measurements, etc.) are accompanied by **labels** indicating the **class** of the observations.

New data is classified based on a **model** created from the training data.

Unsupervised learning (clustering).

The class labels of training data are unknown.

Or rather, there are no training data.

Given a set of measurements, observations, etc., the goal is to find classes or clusters in the data.

See next chapter.

Prediction problems: classification vs. numerical prediction

Classification:

Predicts **categorical class labels** (discrete, nominal).

Constructs a model based on the training set and the values (class labels) in a classifying attribute and uses it in classifying new data.

Numerical prediction:

Models **continuous-valued functions**.

I.e. predicts missing or unknown (future) values.

Typical applications of classification:

Credit/loan approval: Will it be paid back?

Medical diagnosis: Is a tumor cancerous or benign?

Fraud detection: Is a transaction fraudulent or not?

Web-page categorization: Which category is it?

Classification – a two-step process

Model construction: describing a set of predetermined classes:

Each tuple/sample is assumed to belong to a predefined class, as determined by the **class-label attribute**.

The set of tuples used for model construction is the **training set**.

The **model** is represented as classification rules, decision trees, or mathematical formulae.

Model usage, for classifying future or unknown objects:

Estimate **accuracy** of the model:

The known label of **test samples** is compared with the result from the model.

Accuracy rate is the percentage of test-set samples that are correctly classified by the model.

Test set is independent of training set (otherwise overfitting).

If the accuracy is acceptable, **use the model** to classify data tuples whose class labels are not known.

Classification – a two-step process



Process (II): using the model in prediction



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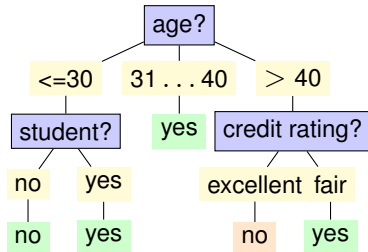
Summary.

Decision-tree induction: an example

Training dataset: buys_computer.

The dataset follows an example of Quinlan's ID3 (playing tennis).

Resulting tree:



age	income	student	credit_rating	buys_coputer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
31 ... 40	high	no	fair	yes
> 40	medium	no	fair	yes
> 40	low	yes	fair	yes
> 40	low	yes	excellent	no
31 ... 40	low	yes	excellent	yes
≤ 30	medium	no	fair	no
≤ 30	low	no	fair	yes
> 40	medium	yes	fair	yes
≤ 30	medium	yes	excellent	yes
31 ... 40	medium	no	excellent	yes
31 ... 40	high	yes	fair	yes
> 40	medium	no	excellent	no

Algorithm for decision-tree induction

Basic algorithm (a greedy algorithm):

Tree is constructed in a **top-down recursive divide-and-conquer manner**.

Attributes are categorical.

If not: discretize in advance.

At start, all the training examples are at the root.

Examples are **partitioned recursively** based on selected attributes.

Test attributes are selected on the basis of a heuristic or statistical measure.

E.g. information gain – see on the next slide.

Conditions for stopping partitioning:

All samples for a given node belong to the same class.

There are no remaining attributes for further partitioning.

Majority voting is employed for classifying the leaf.

There are no samples left (i.e. partition for particular value is empty).

Attribute-selection measure: information gain (ID3/C4.5)

Select the attribute with the highest information gain.

Let p_i be the probability that an arbitrary tuple in D belongs to class C_i , estimated by $\frac{|C_i|}{|D|}$, such that $1 \leq i \leq m$.

Expected information (entropy) needed to classify a tuple in D :

$$\text{Info}(D) = - \sum_{i=1}^m p_i \log_2(p_i). \quad (1)$$

Information needed (after using attribute A to split D into v partitions) to classify D :

$$\text{Info}_A(D) = \sum_{j=1}^v \left(\frac{|D_j|}{|D|} \text{Info}(D_j) \right). \quad (2)$$

Information gained by branching on A :

$$\text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D). \quad (3)$$

Attribute selection: information gain

Class P: buys_computer = "yes"

Class N: buys_computer = "no"

$$\text{Info}(D) = I(9, 5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.94$$

age	p	n	$I(p, n)$
≤ 30	2	3	0.971
31 ... 40	4	0	0
> 40	3	2	0.971

Similarly,

$$\text{Gain}(\text{income}) = 0.029,$$

$$\text{Gain}(\text{student}) = 0.151,$$

$$\text{Gain}(\text{credit_rating}) = 0.048.$$

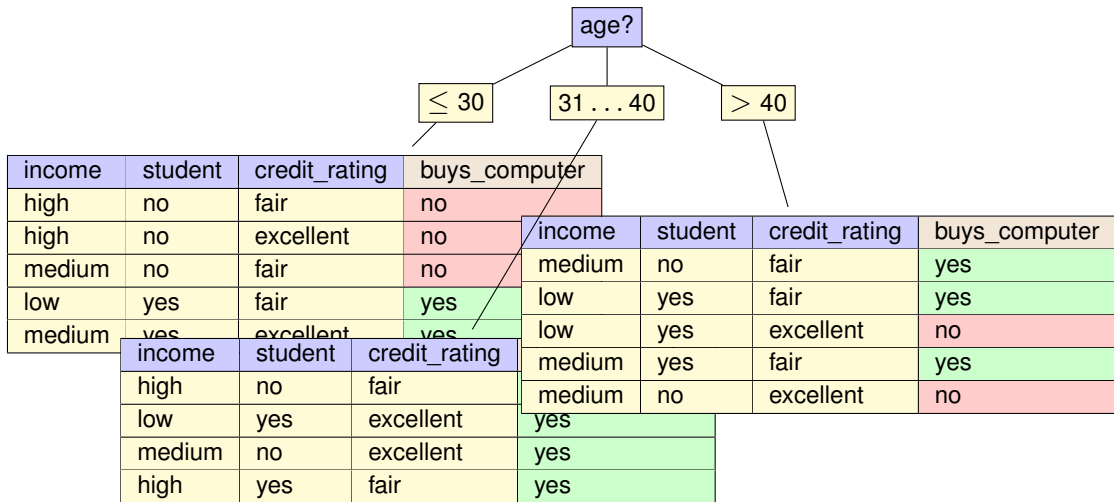
$$\text{Info}_{\text{age}}(D) = \frac{5}{14} I(2, 3) + \frac{4}{14} I(4, 0) + \frac{5}{14} I(3, 2) = 0.694.$$

$\frac{5}{14} I(2, 3)$ means "age ≤ 30 " has 5 out of 14 samples, with 2 yes'es and 3 no's. Hence,

$$\text{Gain}(\text{age}) = \text{Info}(D) - \text{Info}_{\text{age}}(D) = 0.246.$$

age	income	student	credit_rating	buys_computer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
31 ... 40	high	no	fair	yes
> 40	medium	no	fair	yes
> 40	low	yes	fair	yes
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31 ... 40	low	yes	excellent	yes
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≤ 30	medium	yes	excellent	yes
31 ... 40	medium	no	fair	yes
31 ... 40	high	yes	fair	yes
> 40	medium	no	excellent	no

Partitioning in the example



Computing information gain for continuous-valued attributes

Let attribute A be a continuous-valued attribute.

Must determine the best split point for A.

Sort the values of A in increasing order.

Typically, the midpoint between each pair of adjacent values is considered as a possible split point.

$\frac{a_i + a_{i+1}}{2}$ is the midpoint between the values of a_i and a_{i+1} .

The point with the minimum expected information requirement for A is selected as the split point for A.

Split:

D_1 is the set of tuples in D satisfying $A \leq \text{split point}$,
and D_2 is the set of tuples in D satisfying $A > \text{split point}$.

So to say: Discretization as you go along.

For this particular purpose.

Gain ratio for attribute selection (C4.5)

Information-gain measure is biased towards attributes with a large number of values. C4.5 (a successor of ID3) uses gain ratio to overcome the problem (normalization to information gain):

$$\text{SplitInfo}_A(D) = - \sum_{j=1}^v \frac{|D_j|}{|D|} \log_2 \left(\frac{|D_j|}{|D|} \right), \quad (4)$$

$$\text{GainRatio}(A) = \frac{\text{Gain}(A)}{\text{SplitInfo}_A(D)}. \quad (5)$$

Example:

$$\text{SplitInfo}_{\text{income}}(D) = -\frac{4}{14} \log_2 \left(\frac{4}{14} \right) - \frac{6}{14} \log_2 \left(\frac{6}{14} \right) - \frac{4}{14} \log_2 \left(\frac{4}{14} \right) = 1.557, \quad (6)$$

$$\text{GainRatio}(\text{income}) = \frac{0.029}{1.557} = 0.019. \quad (7)$$

The attribute with the maximum gain ratio is selected as the splitting attribute.

Gini index

Corrado Gini (1884 – 1965).

Italian statistician and sociologist.

Also called Gini coefficient.

Measures statistical dispersion.

Zero expresses perfect equality where all values are the same.

One expresses maximal inequality among values.

Based on the Lorenz curve.

Plots the proportion of the total sum of values (y -axis) that is cumulatively assigned to the bottom $x\%$ of the population.

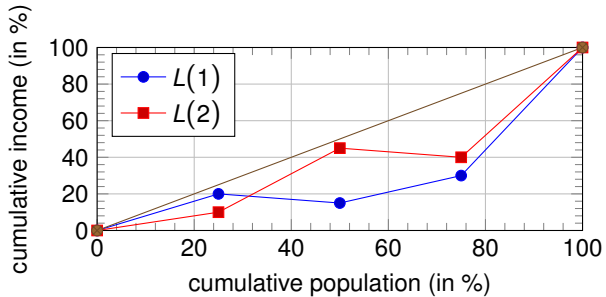
Line at 45 degrees thus represents perfect equality of value distribution.

Gini coefficient then is . . .

. . . the ratio of the area that lies between the line of equality and the Lorenz curve over the total area under the line of equality.

Gini index (II)

Example: Distribution of incomes.



Gini index (CART, IBM IntelligentMiner)

If a dataset D contains examples from n classes, Gini index $\text{gini}(D)$ is defined as:

$$\text{gini}(D) = 1 - \sum_{j=1}^n p_j^2, \quad (8)$$

where p_j is the relative frequency of class j in D .

If a dataset D is split on A into two subsets D_1 and D_2 , the Gini index $\text{gini}(D)$ is defined as

$$\text{gini}_A(D) = \frac{|D_1|}{|D|} \text{gini}(D_1) + \frac{|D_2|}{|D|} \text{gini}(D_2). \quad (9)$$

Reduction in impurity:

$$\Delta \text{gini}_A(A) = \text{gini}(D) - \text{gini}_A(D). \quad (10)$$

The attribute A provides the smallest $\text{gini}_A(D)$ (or the largest reduction in impurity) is chosen to split the node.

Need to enumerate all the possible splitting points for each attribute.

Computation of Gini index

Example:

D has 9 tuples in $\text{buys_computer} = \text{"yes"}$ and 5 in "no" , thus

$$\text{gini}(D) = 1 - \left(\frac{9}{14}\right)^2 - \left(\frac{5}{14}\right)^2 = 0.459. \quad (11)$$

Suppose the attribute income partitions D into 10 in $D_1 : \{\text{low}, \text{medium}\}$ and 4 in $D_2 : \{\text{high}\}$:

$$\text{gini}(D|_{D[\text{income}] = \text{"medium"}, \text{"low"}}) \quad (12)$$

$$= \left(\frac{10}{14}\right) \text{gini}(D_1) + \frac{4}{14} \text{gini}(D_2) \quad (13)$$

$$= \frac{10}{14} \left(1 - \left(\frac{7}{10}\right)^2 - \left(\frac{3}{10}\right)^2\right) + \frac{4}{14} \left(1 - \left(\frac{2}{4}\right)^2 - \left(\frac{2}{4}\right)^2\right) = \quad (14)$$

$$= 0.443 = \text{gini}(D|_{D[\text{income}] = \text{"high"}}). \quad (15)$$

Computation of Gini index (II)

Example (cont.):

$$\text{gini}(D|_{D[\text{income}] = \text{"low"}, \text{"high"}}) = 0.458,$$

$$\text{gini}(D|_{D[\text{income}] = \text{"medium"}, \text{"high"}}) = 0.450.$$

Thus, split on the {"low", "medium"} and {"high"}, since it has the lowest gini index.

All attributes are assumed continuous-valued.

May need other tools, e.g. clustering, to get the possible split values.

Can be modified for categorical attributes.

Computation of Gini index (II)

The three measures, in general, return good results, but

Information gain:

Biased towards multi-valued attributes.

Gain ratio:

Tends to prefer unbalanced splits in which one partition is much smaller than the others.

Gini index:

Biased to multi-valued attributes.

Has difficulty when number of classes is large.

Tends to favor tests that result in equal-sized partitions and purity in both partitions.

Other attribute-selection measures

CHAID:

A popular decision-tree algorithm, measure based on χ^2 test for independence.

C-SEP:

Performs better than information gain and Gini index in certain cases.

G-statistic:

Has a close approximation to χ^2 distribution.

MDL (Minimal Description Length) principle:

I.e. the simplest solution is preferred.

The best tree is the one that requires the fewest number of bits to both (1) encode the tree and (2) encode the exceptions to the tree.

Multivariate splits:

Partitioning based on multiple variable combinations.

CART: finds multivariate splits based on a linear combination of attributes.

Which attribute-selection measure is the best?

Most give good results, none is significantly superior to others.

Overfitting and tree pruning

Overfitting: An induced tree may overfit the training data.

Too many branches, some may reflect anomalies due to noise or outliers.

Poor accuracy for unseen samples.

Two approaches to avoid overfitting:

Prepruning:

Halt tree construction early.

Do not split a node, if this would result in the goodness measure falling below a threshold.

Difficult to choose an appropriate threshold.

Postpruning:

Remove branches from a "fully grown" tree.

Get a sequence of progressively pruned trees.

Use a set of data different from the training data to decide which is the "best pruned tree."

Enhancements to Basic Decision-Tree Induction

Allow for continuous-valued attributes.

Dynamically define new discrete-valued attributes that partition the values of continuous-valued attributes into a discrete set of intervals.

Handle missing attribute values.

Assign the most common value of the attribute.

Assign probability to each of the possible values.

Attribute construction.

Create new attributes based on existing ones that are sparsely represented.

This reduces fragmentation, repetition, and replication.

Classification in large databases

Classification – a classical problem extensively studied by statisticians and machine-learning researchers.

Scalability:

Classifying datasets with millions of examples and hundreds of attributes with reasonable speed.

Why is decision-tree induction popular?

Relatively fast learning speed (compared to other classification methods).

Convertible to simple and easy-to-understand classification rules.

Can use SQL queries for accessing databases.

Classification accuracy comparable with other methods.

RainForest (Gehrke, Ramakrishnan & Ganti, VLDB'98).

Builds an AVC-list (attribute, value, class label).

Scalability framework for RainForest

Separates the scalability aspects from the criteria that determine the quality of the tree.

Builds an AVC-list:

AVC (Attribute, Value, Class_label).

AVC-set (of an attribute X):

Projection of training dataset onto the attribute X and class label where counts of individual class label are aggregated.

AVC-group (of a node n):

Set of AVC-sets of all predictor attributes at the node n .

RainForest: training set and its AVC-sets

age	income	student	credit_rating	buys_computer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
31 ... 40	high	no	fair	yes
> 40	medium	no	fair	yes
> 40	low	yes	fair	yes
> 40	low	yes	excellent	no
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≤ 30	medium	no	fair	no
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> 40	medium	yes	fair	yes
≤ 30	medium	yes	excellent	yes
31 ... 40	medium	no	excellent	yes
31 ... 40	high	yes	fair	yes
> 40	medium	no	excellent	no

AVC-set on age:

age	yes	no
≤ 30	2	3
31 ... 40	4	0
> 40	3	2

AVC-set on income:

income	yes	no
high	2	2
medium	4	2
low	3	1

RainForest: training set and its AVC-sets (II)

age	income	student	credit_rating	buys_computer
≤ 30	high	no	fair	no
≤ 30	high	no	excellent	no
31 ... 40	high	no	fair	yes
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31 ... 40	low	yes	excellent	yes
≤ 30	medium	no	fair	no
≤ 30	low	yes	fair	yes
> 40	medium	yes	fair	yes
≤ 30	medium	yes	excellent	yes
31 ... 40	medium	no	excellent	yes
31 ... 40	high	yes	fair	yes
> 40	medium	no	excellent	no

AVC-set on student:

student	yes	no
yes	6	1
no	3	4

AVC-set on credit_rating:

credit_rating	yes	no
fair	6	2
excellent	3	3

BOAT (Bootstrapped optimistic algorithm for tree construction)

Use a statistical technique called bootstrapping to create several smaller samples (subsets), each fitting in memory.

See on the subsequent slides.

Each subset is used to create a tree, resulting in several trees.

These trees are examined and used to construct a new tree T' .

It turns out that T' is very close to the tree that would be generated using the whole data set together.

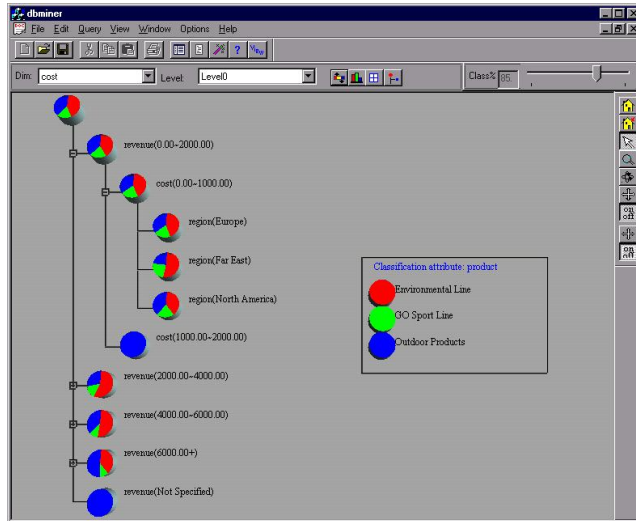
Advantages:

Requires only two scans of DB.

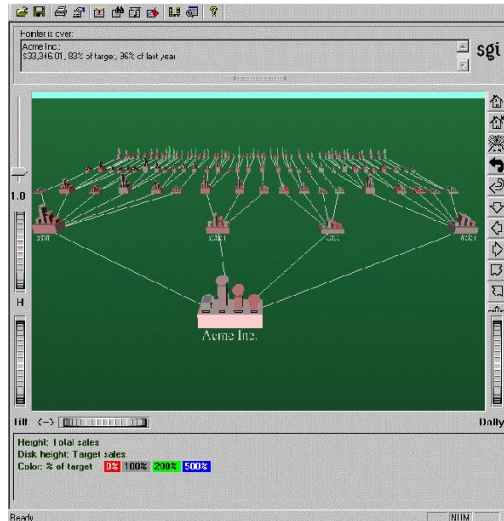
An incremental algorithm:

Take insertions and deletions of training data and update the decision tree.

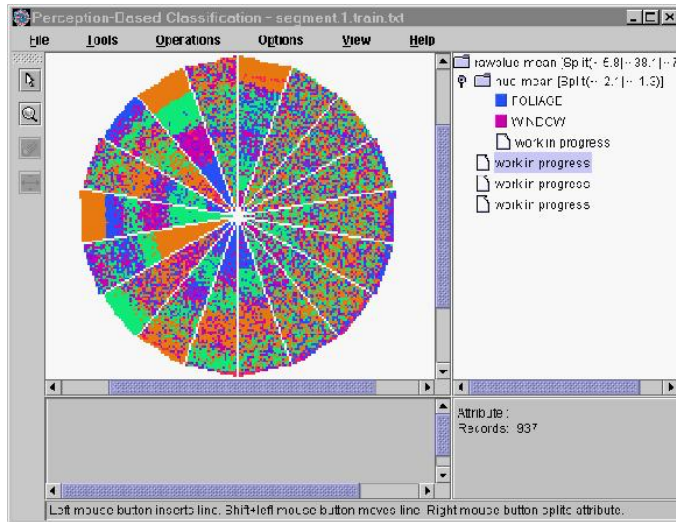
Presentation of classification results



Visualization of a decision tree in SGI/MineSet 3.0



Interactive visual mining by perception-based classification (PBC)



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Summary.

Bayesian classification: why?

A statistical classifier:

Performs probabilistic prediction, i.e. predicts class-membership probabilities.

Foundation: Bayes' Theorem.

Performance:

A simple Bayesian classifier (naïve Bayesian classifier) has performance comparable with decision tree and selected neural-network classifiers.

Incremental:

Each training example can incrementally increase/decrease the probability that a hypothesis is correct—prior knowledge can be combined with observed data.

Standard:

Even when Bayesian methods are computationally intractable, they can provide a standard of optimal decision making against which other methods can be measured.

Bayes' Theorem: basics

Let X be a data sample ("evidence").

The class label shall be unknown.

Let C_i be the hypothesis that X belongs to class i .

Classification is to determine $P(C_i|X)$:

Posteriori probability: the probability that the hypothesis holds given the observed data sample X .

$P(C_i)$:

Prior probability: the initial probability.

E.g. X will buy computer, regardless of age, income, . . .

$P(X)$:

Probability that sample data is observed.

$P(X|C_j)$:

Likelihood: the probability of observing the sample X given that the hypothesis holds.

E.g. given that X buys computer, the probability that X is 31 . . . 40, medium income.

Bayes' Theorem (II)

Given training data X , the posteriori probability $P(C_i|X)$ of a hypothesis C_i follows from the Bayes' Theorem:

$$P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}. \quad (16)$$

Predicts that X belongs to C_i iff the probability $P(C_i|X)$ is **the highest** among all the $P(C_k|X)$ for all k classes.

Practical difficulty:

Requires initial knowledge of many probabilities.

Significant computational cost.

Towards naïve Bayesian classifier

Let D be a training set of tuples and their associated class labels.

Each tuple is represented by an n -dimensional attribute $X = (x_1, x_2, \dots, x_n)$.

Suppose there are m classes C_1, C_2, \dots, C_m .

Classification is to derive the maximum posteriori probability.

i.e. the maximal $P(C_i|X)$.

This can be derived from Bayes' Theorem:

$$P(C_i|X) = \frac{P(X|C_i)P(C_i)}{P(X)}. \quad (17)$$

Since $P(X)$ is constant for all classes, we must maximize only:

$$P(X|C_i)P(C_i). \quad (18)$$

Derivation of naïve Bayes classifier

A simplifying assumption: attributes are conditionally independent.

I.e. no dependence relation between attributes (which is "naïve").

$$P(X|C_i) = \prod_{k=1}^n P(x_k|C_i) = P(x_1|C_i)P(x_2|C_i) \cdots P(x_k|C_i).$$

This greatly reduces the computation cost:

Only count the class distribution.

If A_k is categorical,

$P(x_k|C_i)$ is the number of tuples in C_i having value x_k for A_k
divided by $|C_{i,D}|$ (the number of tuples of C_i in D).

If A_k is continuous-valued,

$P(x_k|C_i)$ is usually computed based on Gaussian distribution with a mean μ and standard deviation σ :

$$G(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

and $P(x_k|C_i)$ is $P(x_k|C_i) = G(x_k, \mu_{C_i}, \sigma_{C_i})$.

Naïve Bayesian classifier: training dataset

Classes:

C_1 : buys_computer = "yes".

C_2 : buys_computer = "no".

Data sample:

$X = (\text{age} \leq 30,$
 $\text{income} = \text{"medium"},$
 $\text{student} = \text{"yes"},$
 $\text{credit_rating} = \text{"fair"}).$

age	income	student	credit_rating	buys_computer
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≤ 30	medium	yes	excellent	yes
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31 ... 40	high	yes	fair	yes
> 40	medium	no	excellent	no

Naïve Bayesian classifier: an example

$P(C_i)$:

$$P(\text{buys_computer} = \text{"yes"}) = \frac{9}{14} = 0.643.$$

$$P(\text{buys_computer} = \text{"no"}) = \frac{5}{14} = 0.357.$$

$X = (\text{age} \leq 30, \text{income} = \text{"medium"}, \text{student} = \text{"yes"}, \text{credit_rating} = \text{"fair"}).$

Compute $P(X|C_i)$ for each class:

$$P(\text{age} \leq 30 | \text{buys_computer} = \text{"yes"}) = \frac{2}{9} = 0.222.$$

$$P(\text{age} \leq 30 | \text{buys_computer} = \text{"no"}) = \frac{3}{5} = 0.6.$$

$$P(\text{income} = \text{"medium"} | \text{buys_computer} = \text{"yes"}) = \frac{4}{9} = 0.444.$$

$$P(\text{income} = \text{"medium"} | \text{buys_computer} = \text{"no"}) = \frac{2}{5} = 0.4.$$

$$P(\text{student} = \text{"yes"} | \text{buys_computer} = \text{"yes"}) = \frac{6}{9} = 0.667.$$

$$P(\text{student} = \text{"yes"} | \text{buys_computer} = \text{"no"}) = \frac{1}{5} = 0.2.$$

$$P(\text{credit_rating} = \text{"fair"} | \text{buys_computer} = \text{"yes"}) = \frac{6}{9} = 0.667.$$

$$P(\text{credit_rating} = \text{"fair"} | \text{buys_computer} = \text{"no"}) = \frac{2}{5} = 0.4.$$

Naïve Bayesian classifier: an example (II)

$P(C_i)$:

$$P(X|\text{buys_computer} = \text{"yes"}) = 0.222 \cdot 0.444 \cdot 0.667 \cdot 0.667 = 0.044.$$

$$P(X|\text{buys_computer} = \text{"no"}) = 0.6 \cdot 0.4 \cdot 0.2 \cdot 0.4 = 0.019.$$


$P(X|C_i) \cdot P(C_i)$:

$$P(X|\text{buys_computer} = \text{"yes"}) \cdot P(\text{buys_computer} = \text{"yes"}) = 0.028.$$

$$P(X|\text{buys_computer} = \text{"no"}) \cdot P(\text{buys_computer} = \text{"no"}) = 0.007.$$

Therefore, X belongs to class C_1 ($\text{buys_computer} = \text{"yes"}$).

Thank you for your attention.
Any questions about the sixth chapter?

Ask them now, or again, drop me a line:
 `luciano.melodia@fau.de`.