3D Incompressible fluid simulation

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Abstract

Incompressible fluid simulation using MaC grid
Based on the paper by Cline[1]

[1] D. Cline, D. Cardon, and P. K. Egbert, "Fluid flow for the rest of us: Tutorial of the marker and cell method in computer graphics," *Brigham Young University*, 2013

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Theoretical

Reduced Navier-Stokes equation

$$\nabla \cdot u = 0$$

$$\frac{\partial u}{\partial t} = -(\nabla \cdot u)u - \nabla p + f_{ext}$$

Solve the navier-stokes in steps to make the problem easier

MaC grid

Staggered grid: velocities live on mid face
Pressure lives in cell centers
Used to increase the stability of the simulation

Advection

Firstly advection of velocity through the fluid

$$\frac{\partial u^*}{\partial t} = -(\nabla \cdot u)u$$

Use backwards particle trace
Trace particle backwards with a time of $-dt$

$$u_x^* = u_x.interpolate(pos(u_x) - dt * u(pos(u_x)))$$

External forces

Fairly straightforward

Simply add acceleration to the velocity components

Calculate Pressure

Pressure at different points dependent upon the others

Thus linear system of equations

We solve this iteratively

Interpolation

Simulation uses RK2 trilinear interpolation
Trace particle for half the time and determine velocity again
Then trace using this velocity for full time

$$v = u(p);$$
$$v2 = u(p + 0.5 * dt * v);$$
$$p = p + dt * v2;$$

Implementation

Framework

Used practicals as framework
Using libigl as a renderer
Polyvox for creating meshes from fluid grid (marching cubes)

Structure

Using dense grid for small implementation
- Easy to upgrade to hashmap
Functions map to the steps described in [1]

[1] D. Cline, D. Cardon, and P. K. Egbert, “Fluid flow for the rest of us: Tutorial of the marker and cell method in computer graphics,” *Brigham Young University*, 2013

Rendering

Using voxelization: suboptimal

Demo

Questions?