CHAPTER 16

16.1 The area and volume can be computed as

$$A = \pi r^2 + 2\pi rh \qquad\qquad V = \pi r^2 h$$

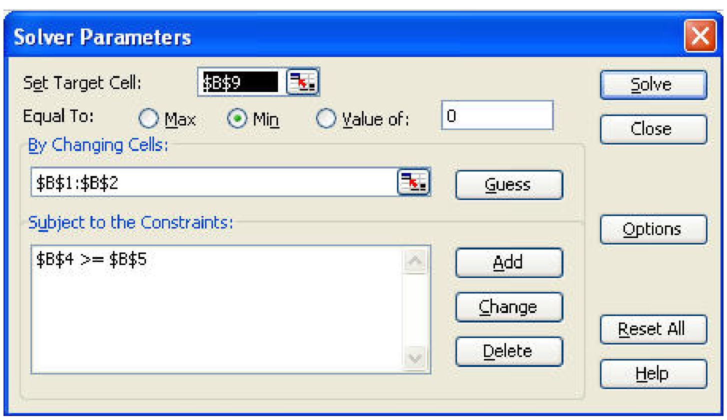
An Excel spreadsheet can be set up to solve the problem as

	A	B
1	radius	1
2	height	1
3		
4	volume	3.141593
5	desired volume	0.2
6		
7	side area	6.283185
8	bottom area	3.141593
9	total area	9.424778

The formulas are

	A	B
1	radius	1
2	height	1
3		
4	volume	=PI()*B1^2*B2
5	desired volume	0.2
6		
7	side area	=2*PI()*B1*B2
8	bottom area	=PI()*B1^2
9	total area	=SUM(B7:B8)

The Solver can be called and set up as



The resulting solution is

	A	B
1	radius	0.399491
2	height	0.398901
3		
4	volume	0.199999
5	desired volume	0.2
6		
7	side area	1.001272
8	bottom area	0.501376
9	total area	1.502647

Thus, the Solver says that the optimal cylindrical container is one where the radius equals the height. For the case of the desired $V = 0.2\ \text{m}^3$, the dimensions are $r = h = 0.399491$ m.

The general result of $r = h$ can be verified using calculus as follows. First, we can solve the volume equation for h as

$$h = \frac{V}{\pi r^2} \tag{3}$$

This can be substituted into the area equation to give

$$A = \pi r^2 + 2\pi r \frac{V}{\pi r^2} = \pi r^2 + \frac{2V}{r}$$

We can differentiate this equation with respect to r to yield

$$\frac{dA}{dr} = 2\pi r - \frac{2V}{r^2}$$

which can be set equal to zero and solved for

$$r = \sqrt[3]{\frac{V}{\pi}}$$

This result can then be substituted into Eq. 3 which can be solved for

$$h = \sqrt[3]{\frac{V}{\pi}}$$

Thus, we prove that the optimal container has $r = h = (V/\pi)^{1/3}$. For our desired volume of 0.5 m^3, this means that $r = h = (0.2/\pi)^{1/3} = 0.399295$ m, which confirms the result obtained numerically with the Excel Solver.

16.2 (a) The area and volume can be computed as

$$A = \pi r^2 + \pi r\sqrt{r^2 + h^2} \qquad\qquad V = \frac{\pi r^2 h}{3}$$

An Excel spreadsheet can be set up to solve the problem as

	A	B
1	radius	1
2	height	1
3		
4	volume	1.047198
5	desired volume	0.2
6		
7	top area	3.141593
8	side area	4.442883
9	total area	7.584476

The formulas are

	A	B	C	D
1	radius	1		
2	height	1		
3				
4	volume	=PI()*B1^2*B2/3		
5	desired volume	0.2		
6				
7	top area	=PI()*B1^2		
8	side area	=PI()*B1*SQRT(B1^2+B2^2)		
9	total area	=B7+B8		

The Solver can be called and set up as

The resulting solution is

	A	B
1	radius	0.40705
2	height	1.152665
3		
4	volume	0.199999
5	desired volume	0.2
6		
7	top area	0.52053
8	side area	1.563222
9	total area	2.083752

(b) For this case, the area and volume can be computed as

$$A = \pi r\sqrt{r^2 + h^2} \qquad\qquad V = \frac{\pi r^2 h}{3}$$

An Excel spreadsheet can be set up to solve the problem in a similar fashion to part **(a)** with the result: $r = 0.513334$ m and $h = 0.72477$ m.

16.3 First printing errata: the formula for volume should be: $V = \dfrac{\pi h(h^2 + 3r^2)}{6}$

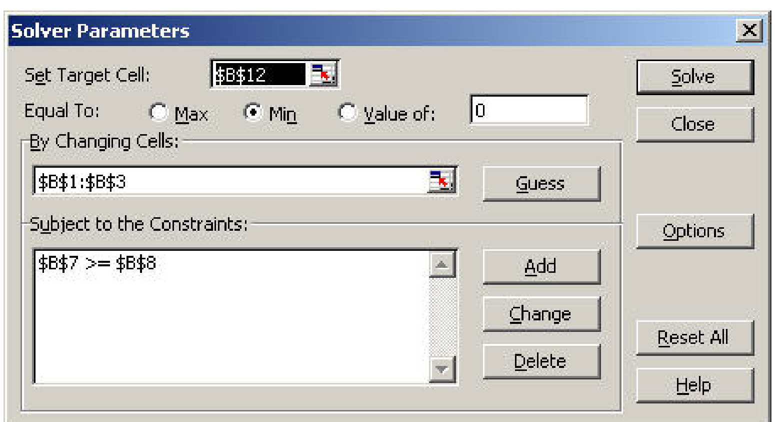
The area and volume can be computed as

$$A = 2\pi rL + 2\pi(h^2 + r^2) \qquad\qquad V = \pi r^2 L + 2\frac{\pi h(h^2 + 3r^2)}{6}$$

(a) An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D
1	r	0.1		
2	h	0.1		
3	L	0.1		
4				
5	cylinder volume	=PI()*B1^2*B3		
6	end volumes	=2*PI()*B2*(B2^2+3*B1^2)/6		
7	volume	=B5+B6		
8	desired volume	0.2		
9				
10	cylinder area	0.062832		
11	end areas	=2*PI()*(B2^2+B1^2)		
12	total area	=B10+B11		

The Solver can be called and set up as

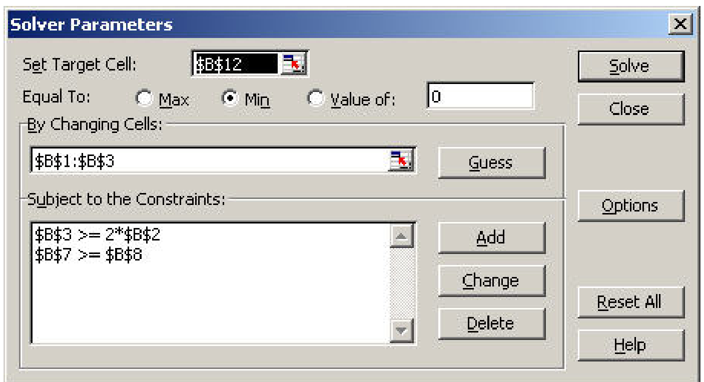


Note that you should also select: Options → Assume non-negative. The resulting solution is

	A	B
1	r	0.36278
2	h	0.362787
3	L	0
4		
5	cylinder volume	0
6	end volumes	0.2
7	volume	0.2
8	desired volume	0.2
9		
10	cylinder area	0
11	end areas	1.653882
12	total area	1.653882

Thus, the result indicates that the tank with the minimum area is spherical.

(b) The constraint can be entered as



The resulting solution is

	A	B
1	r	0.334278
2	h	0.183739
3	L	0.367479
4		
5	cylinder volume	0.129003
6	end volumes	0.070997
7	volume	0.2
8	desired volume	0.2
9		
10	cylinder area	0.771827
11	end areas	0.914217
12	total area	1.686044

16.4 This problem can be solved in a number of different ways. For example, using the golden section search, the result is

i	c_l	$g(c_l)$	c_2	$g(c_2)$	c_1	$g(c_1)$	c_u	$g(c_u)$	d	c_{opt}	ε_a
1	0.0000	0.0000	3.8197	0.2330	6.1803	0.1310	10.0000	0.0641	6.1803	3.8197	100.00%
2	0.0000	0.0000	2.3607	0.3350	3.8197	0.2330	6.1803	0.1310	3.8197	2.3607	100.00%
3	0.0000	0.0000	1.4590	0.3686	2.3607	0.3350	3.8197	0.2330	2.3607	1.4590	100.00%
4	0.0000	0.0000	0.9017	0.3174	1.4590	0.3686	2.3607	0.3350	1.4590	1.4590	61.80%
5	0.9017	0.3174	1.4590	0.3686	1.8034	0.3655	2.3607	0.3350	0.9017	1.4590	38.20%
6	0.9017	0.3174	1.2461	0.3593	1.4590	0.3686	1.8034	0.3655	0.5573	1.4590	23.61%
7	1.2461	0.3593	1.4590	0.3686	1.5905	0.3696	1.8034	0.3655	0.3444	1.5905	13.38%

8	1.4590	0.3686	1.5905	0.3696	1.6718	0.3688	1.8034	0.3655	0.2129	1.5905	8.27%
9	1.4590	0.3686	1.5403	0.3696	1.5905	0.3696	1.6718	0.3688	0.1316	1.5905	5.11%
10	1.5403	0.3696	1.5905	0.3696	1.6216	0.3694	1.6718	0.3688	0.0813	1.5905	3.16%
11	1.5403	0.3696	1.5713	0.3696	1.5905	0.3696	1.6216	0.3694	0.0502	1.5713	1.98%
12	1.5403	0.3696	1.5595	0.3696	1.5713	0.3696	1.5905	0.3696	0.0311	1.5713	1.22%
13	1.5595	0.3696	1.5713	0.3696	1.5787	0.3696	1.5905	0.3696	0.0192	1.5713	0.75%

Thus, after 13 iterations, the method is converging on the true value of c = 1.5679 which corresponds to a maximum specific growth rate of g = 0.36963.

16.5 (a) The LP formulation is given by

Maximize $C = 30X + 30Y + 35Z$ {Maximize profit}

subject to

$6X + 4Y + 12Z \le 2500$ {Raw chemical constraint}
$0.05X + 0.1Y + 0.2Z \le 55$ {Time constraint}
$X + Y + Z \le 450$ {Storage constraint}
$X, Y, Z \ge 0$ {Positivity constraints}

(b) The simplex tableau for the problem can be set up and solved as

Basis	C	X	Y	Z	S1	S2	S3	Solution	Intercept
P	1	-30	-30	-35	0	0	0	0	
S1	0	6	4	12	1	0	0	2500	208.3333
S2	0	0.05	0.1	0.2	0	1	0	55	275
S3	0	1	1	1	0	0	1	450	450

Basis	C	X	Y	Z	S1	S2	S3	Solution	Intercept
P	1	-12.5	-18.3333	0	2.91667	0	0	7291.667	
Z	0	0.5	0.33333	1	0.08333	0	0	208.3333	625
S2	0	-0.05	0.03333	0	-0.0167	1	0	13.33333	400
S3	0	0.5	0.66667	0	-0.0833	0	1	241.6667	362.5

Basis	C	X	Y	Z	S1	S2	S3	Solution	Intercept
P	1	1.25	0	0	0.625	0	27.5	13937.5	
Z	0	0.25	0	1	0.125	0	-0.5	87.5	700
S2	0	-0.075	0	0	-0.0125	1	-0.05	1.25	-100
Y	0	0.75	1	0	-0.125	0	1.5	362.5	-2900

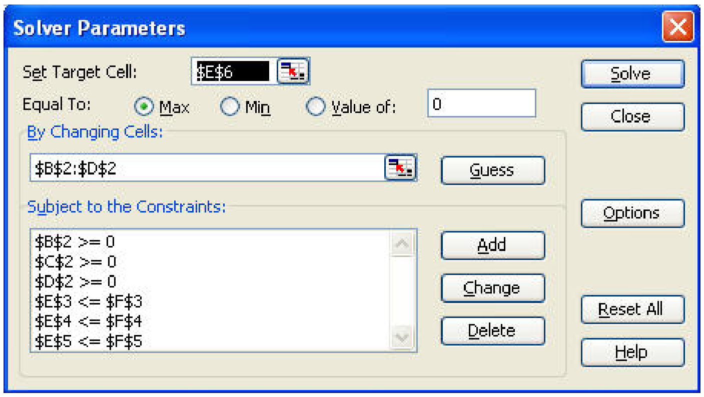
(c) An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D	E	F
1		Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	0	0		
3	material	6	4	12	0	2500
4	time	0.05	0.1	0.2	0	55
5	storage	1	1	1	0	450
6	profit	30	30	35	0	

The formulas are

	A	B	C	D	E	F
1		Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	0	0		
3	material	6	4	12	=B3*B$2+C3*C$2+D3*D$2	2500
4	time	0.05	0.1	0.2	=B4*B$2+C4*C$2+D4*D$2	55
5	storage	1	1	1	=B5*B$2+C5*C$2+D5*D$2	450
6	profit	30	30	35	=B6*B$2+C6*C$2+D6*D$2	

The Solver can be called and set up as



The resulting solution is

	A	B	C	D	E	F
1		Product 1	Product 2	Product 3	Total	Constraint
2	amount	0	362.5	87.5		
3	material	6	4	12	2500	2500
4	time	0.05	0.1	0.2	53.75	55
5	storage	1	1	1	450	450
6	profit	30	30	35	13937.5	

In addition, a sensitivity report can be generated as

	A	B	C	D	E	F	G	H
1	**Microsoft Excel 11.0 Sensitivity Report**							
2	**Worksheet: [Prob1604.xls]Sheet1**							
3	**Report Created: 6/29/2005 8:38:14 AM**							
4								
5								
6	Adjustable Cells							
7				Final	Reduced	Objective	Allowable	Allowable
8		Cell	Name	Value	Cost	Coefficient	Increase	Decrease
9		B2	amount Product 1	0	-1.25	30	1.25	1E+30
10		C2	amount Product 2	362.5	0	30	5	1.666666667
11		D2	amount Product 3	87.5	0	35	55	5
12								
13	Constraints							
14				Final	Shadow	Constraint	Allowable	Allowable
15		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
16		E3	material Total	2500	0.625	2500	100	700
17		E4	time Total	53.75	0	55	1E+30	1.25
18		E5	storage Total	450	27.5	450	25	241.6666667

(d) The high shadow price for storage from the sensitivity analysis from **(c)** suggests that increasing storage will result in the best increase in profit.

16.6 An LP formulation for this problem can be set up as

$$\text{Maximize } P = 2000Z_1 - 75Z_2 + 250Z_3 - 300W \quad \{\text{Maximize profit}\}$$

subject to

$$Z_1 + Z_2 \le 7500 \quad \{X \text{ material constraint}\}$$
$$2.5Z_1 + Z_3 \le 12{,}500 \quad \{Y \text{ material constraint}\}$$
$$Z_1 - Z_2 - Z_3 - W = 0 \quad \{\text{Waste constraint}\}$$

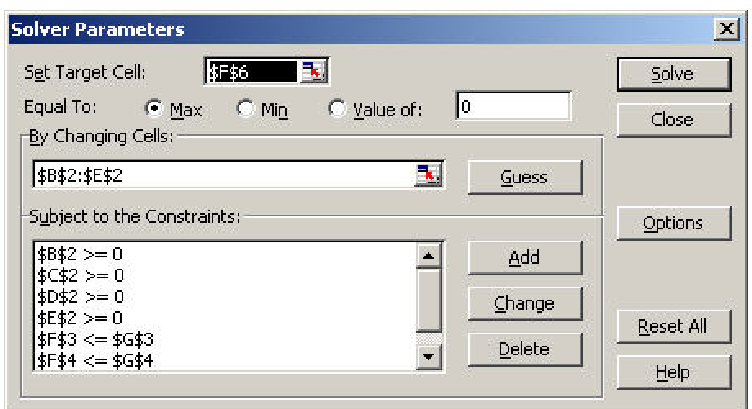
An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D	E	F	G
1		Z1	Z2	Z3	W	total	constraint
2	amount	0	0	0	0		
3	amount X	1	1	0	0	0	7500
4	amount Y	2.5	0	1	0	0	10000
5	amount W	1	-1	-1	-1	0	
6	profit	2500	-50	-200	-300	0	

The formulas are

	A	B	C	D	E	F	G
1		Z1	Z2	Z3	W	total	constraint
2	amount	0	0	0	0		
3	amount X	1	1	0	0	=B3*B$2+C3*C$2+D3*D$2+E3*E$2	7500
4	amount Y	2.5	0	1	0	=B4*B$2+C4*C$2+D4*D$2+E4*E$2	10000
5	amount W	1	-1	-1	-1	=B5*B$2+C5*C$2+D5*D$2+E5*E$2	
6	profit	2500	-50	-200	-300	0	

The Solver can be called and set up as



The resulting solution is

	A	B	C	D	E	F	G
1		Z1	Z2	Z3	W	total	constraint
2	amount	4000	3500	0	500		
3	amount X	1	1	0	0	7500	7500
4	amount Y	2.5	0	1	0	10000	10000
5	amount W	1	-1	-1	-1	0	
6	profit	2500	-50	-200	-300	9675000	

This is an interesting result which might seem counterintuitive at first. Notice that we create some of the unprofitable Z_2 while producing none of the profitable Z_3. This occurred because we used up all of Y in producing the highly profitable Z_1. Thus, there was none left to produce Z_3.

16.7 Substitute $x_B = 1 - x_T$ into the pressure equation,

$$(1 - x_T)P_{sat_B} + x_T P_{sat_T} = P$$

and solve for x_T,

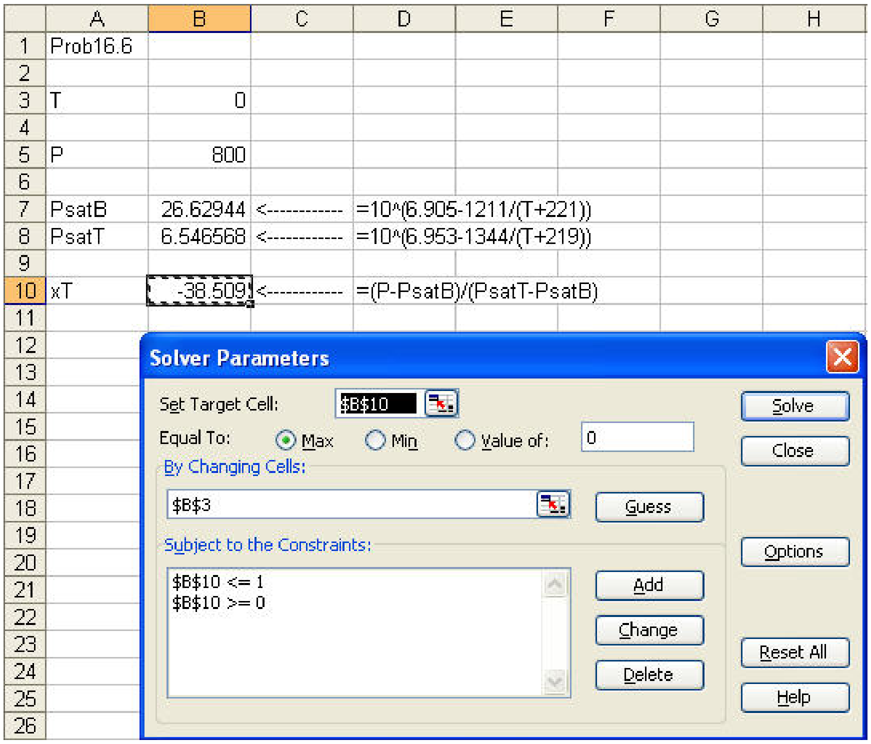
$$x_T = \frac{P - P_{sat_B}}{P_{sat_T} - P_{sat_B}} \qquad (1)$$

where the partial pressures are computed as

$$P_{sat_B} = 10^{\left(6.905 - \frac{1211}{T+221}\right)}$$

$$P_{sat_T} = 10^{\left(6.953 - \frac{1344}{T+219}\right)}$$

The solution then consists of maximizing Eq. 1 by varying T subject to the constraint that $0 \le x_T \le 1$. The Excel Solver can be used to obtain the solution. Here is how the worksheet can be set up:



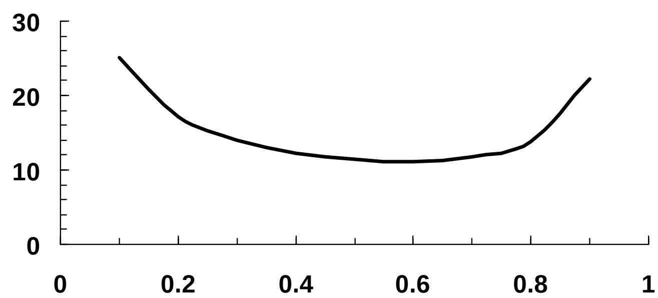
The result is $T = 112.8592$ as shown below:

	A	B	C	D	E	F
1	Prob16.6					
2						
3	T	112.8592				
4						
5	P	800				
6						
7	PsatB	1895.496	<------------	=10^(6.905-1211/(T+221))		
8	PsatT	800.0004	<------------	=10^(6.953-1344/(T+219))		
9						
10	xT	1	<------------	=(P-PsatB)/(PsatT-PsatB)		

16.8 This is a straightforward problem of varying x_A in order to minimize

$$f(x_A) = \left(\frac{1}{(1-x_A)^2}\right)^{0.6} + 6\left(\frac{1}{x_A}\right)^{0.6}$$

First, the function can be plotted versus x_A



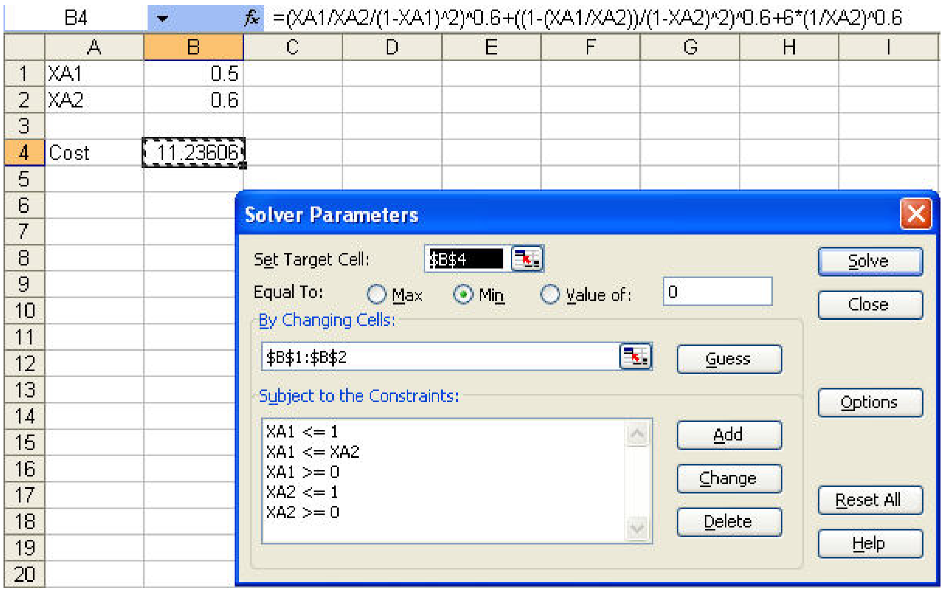
The result indicates a minimum between 0.5 and 0.6. Using golden section search or a package like Excel or MATLAB yields a minimum of 0.587683.

16.9 This is a case of constrained nonlinear optimization. The conversion factors range between 0 and 1. In addition, the cost function can not be evaluated for certain combinations of X_{A1} and X_{A2}. The problem is the second term,

$$\left(\frac{1-\frac{x_{A1}}{x_{A2}}}{(1-x_{A2})^2}\right)^{0.6}$$

If $x_{A1} > x_{A2}$, the numerator will be negative and the term cannot be evaluated.

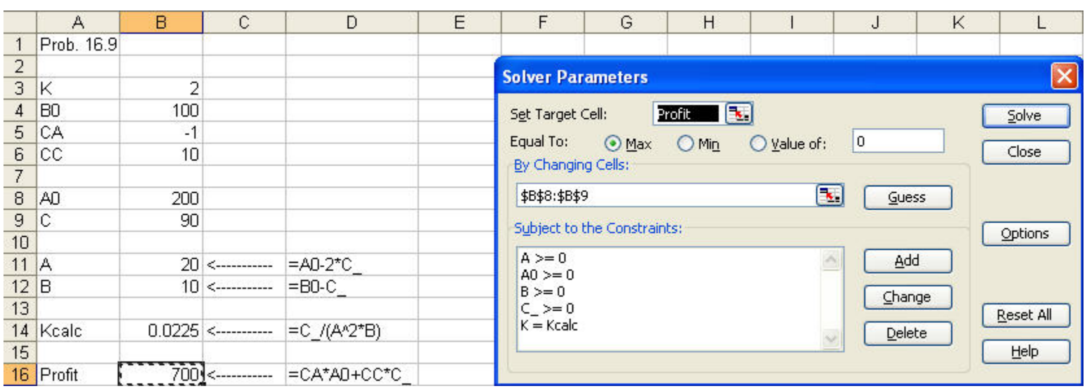
Excel Solver can be used to solve the problem:



The result is

	A	B
1	XA1	0.368949
2	XA2	0.627265
3		
4	Cost	11.12039

16.10 This problem can be set up on Excel and the answer generated with Solver. Note that we have named the cells with the labels in the adjacent left columns.

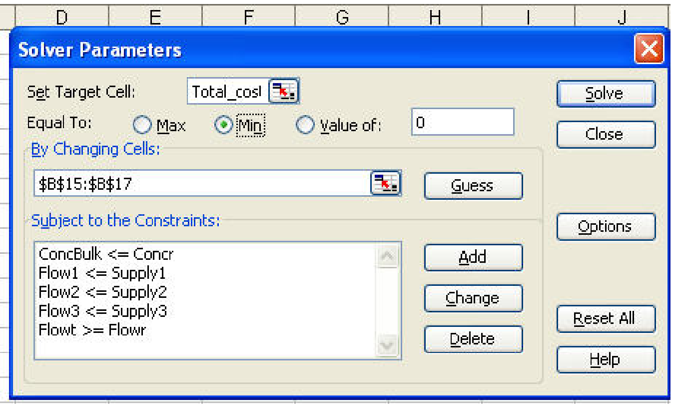


The solution is

	A	B	C	D
1	Prob. 16.9			
2				
3	K	2		
4	B0	100		
5	CA	-1		
6	CC	10		
7				
8	A0	208.085		
9	C	99.41872		
10				
11	A	9.247574	<-----------	=A0-2*C_
12	B	0.581276	<-----------	=B0-C_
13				
14	Kcalc	2	<-----------	=C_/(A^2*B)
15				
16	Profit	786.1022	<-----------	=CA*A0+CC*C_

16.11 The problem can be set up in Excel Solver. Note that we have named the cells with the labels in the adjacent left columns.

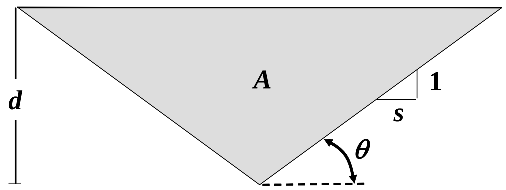
	A	B	C	D
1	Prob. 16.10			
2				
3	Unitcost1	$ 0.50	$/L	
4	Unitcost2	$ 1.00	$/L	
5	Unitcost3	$ 1.20	$/L	
6				
7	conc1	135	mg/L	
8	conc2	100	mg/L	
9	conc3	75	mg/L	
10				
11	Supply1	2000000	L/d	
12	Supply2	1000000	L/d	
13	Supply3	500000	L/d	
14				
15	Flow1	300000	L/d	
16	Flow2	300000	L/d	
17	Flow3	300000	L/d	
18				
19	Flowt	900000	L/d	=Flow1+Flow2+Flow3
20				
21	Flowr	1000000	L/d	
22				
23	ConcBulk	103.3333333	mg/L	=(Flow1*conc1+Flow2*conc2+Flow3*conc3)/Flowt
24				
25	Concr	100	mg/L	
26				
27	Total cost	$ 810,000.00		=Unitcost1*Flow1+Unitcost2*Flow2+Unitcost3*Flow3



The solution is

15	Flow1	357143	L/d
16	Flow2	142857	L/d
17	Flow3	500000	L/d
18			
19	Flowt	1000000	L/d
20			
21	Flowr	1000000	L/d
22			
23	ConcBulk	100.0000001	mg/L
24			
25	Concr	100	mg/L
26			
27	Total cost	$ 921,428.57	

16.12 Here is a diagram for this problem:



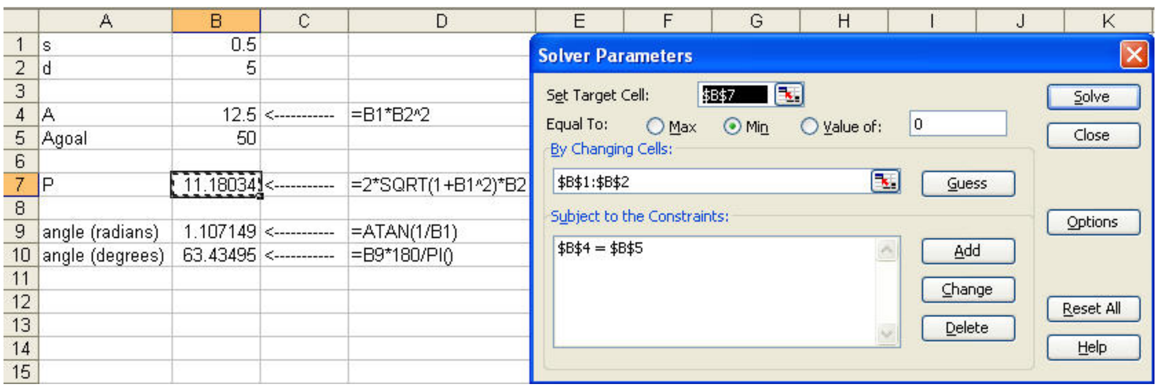
The following formulas can be developed:

$$\theta = \tan^{-1}\frac{1}{s} \qquad (1)$$

$$P = 2d\sqrt{1+s^2} \qquad (2)$$

$$A = sd^2 \qquad (3)$$

Then the following Excel worksheet and Solver application can be set up:



	A	B	C	D
1	s	0.5		
2	d	5		
3				
4	A	12.5	<----------	=B1*B2^2
5	Agoal	50		
6				
7	P	11.18034	<----------	=2*SQRT(1+B1^2)*B2
8				
9	angle (radians)	1.107149	<----------	=ATAN(1/B1)
10	angle (degrees)	63.43495	<----------	=B9*180/PI()
11				
12				
13				
14				
15				

Our goal is to minimize the wetted perimeter by varying the side slope and the depth. We apply the constraint that the computed area must equal the desired area. The result is

	A	B	C	D
1	s	1.000088		
2	d	7.070756		
3				
4	A	50	<----------	=B1*B2^2
5	Agoal	50		
6				
7	P	20	<----------	=2*SQRT(1+B1^2)*B2
8				
9	angle (radians)	0.785354	<----------	=ATAN(1/B1)
10	angle (degrees)	44.99747	<----------	=B9*180/PI()

Thus, this specific application indicates that a 45° angle yields the minimum wetted perimeter.

The verification that this result is universal can be attained inductively or deductively. The inductive approach involves trying several different desired areas in conjunction with our solver solution. As long as the desired area is greater than 0, the result for the optimal design will be 45°.

The deductive verification involves calculus. First, Eq. 3 can be solved for *d* and the result substituted into Eq. 2 to give

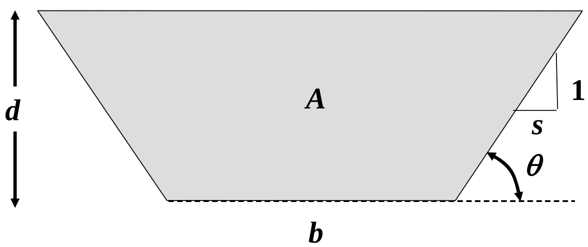
$$P = 2\sqrt{A\left(s+\frac{1}{s}\right)} \tag{4}$$

The minimum wetted perimeter should occur when the derivative of the perimeter with respect to s flattens out. That is, the slope is zero. Setting the derivative of Eq. 4 to zero yields,

$$\frac{dP}{ds} = \frac{1-\frac{1}{s^2}}{\sqrt{s+\frac{1}{s}}} = 0 \tag{5}$$

We can see that the derivative is zero if $s = 1$. According to Eq. 1, this corresponds to $\theta = 45^\circ$. Thus, the result obtained numerically is shown to be universal.

16.13 Here is a diagram for this problem:



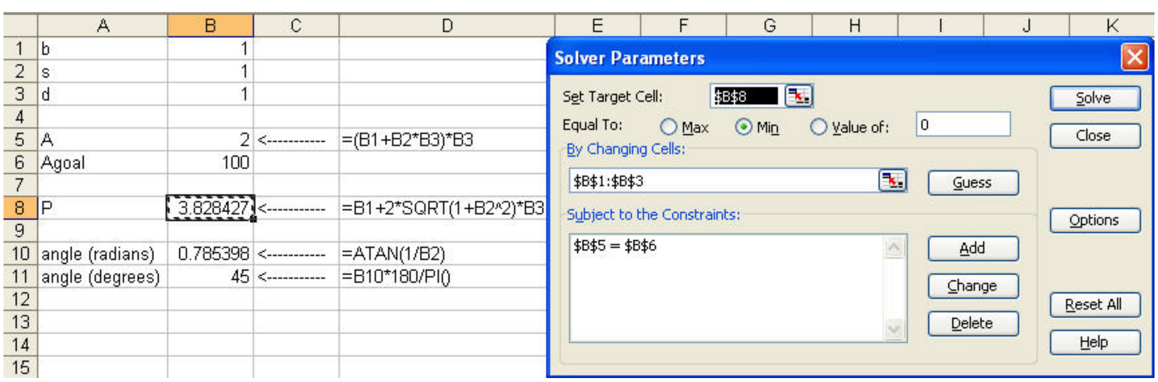
The following formulas can be developed:

$$\theta = \tan^{-1}\frac{1}{s} \tag{1}$$

$$P = b + 2d\sqrt{1+s^2} \tag{2}$$

$$A = (b+sd)d \tag{3}$$

Then the following Excel worksheet and Solver application can be set up:



	A	B	C	D
1	b	1		
2	s	1		
3	d	1		
4				
5	A	2	<-----------	=(B1+B2*B3)*B3
6	Agoal	100		
7				
8	P	3.828427	<-----------	=B1+2*SQRT(1+B2^2)*B3
9				
10	angle (radians)	0.785398	<-----------	=ATAN(1/B2)
11	angle (degrees)	45	<-----------	=B10*180/PI()
12				
13				
14				
15				

Our goal is to minimize the wetted perimeter by varying the depth, side slope and bottom width. We apply the constraint that the computed area must equal the desired area. The result is

	A	B	C	D
1	b	8.773829		
2	s	0.577343		
3	d	7.598379		
4				
5	A	100	<-----------	=(B1+B2*B3)*B3
6	Agoal	100		
7				
8	P	26.32148	<-----------	=B1+2*SQRT(1+B2^2)*B3
9				
10	angle (radians)	1.047203	<-----------	=ATAN(1/B2)
11	angle (degrees)	60.0003	<-----------	=B10*180/PI()

Thus, this specific application indicates that a 60° angle yields the minimum wetted perimeter.

The verification of whether this result is universal can be attained inductively or deductively. The inductive approach involves trying several different desired areas in conjunction with our solver solution. As long as the desired area is greater than 0, the result for the optimal design will be 60°.

The deductive verification involves calculus. First, we can solve Eq. 3 for *b* and substitute the result into Eq. 2 to give,

$$P = \frac{A}{d} + d\left(2\sqrt{1+s^2} - s\right) \tag{4}$$

If both *A* and *d* are constants and *s* is a variable, the condition for the minimum perimeter is $dP/ds = 0$. Differentiating Eq. 4 with respect to *s* and setting the resulting equation to zero,

$$\frac{dP}{ds} = d\left(\frac{2s}{\sqrt{1+s^2}} - 1\right) = 0 \tag{4}$$

Therefore, we obtain $s = 1/\sqrt{3}$. Using Eq. 1, this corresponds to $\theta = 60°$.

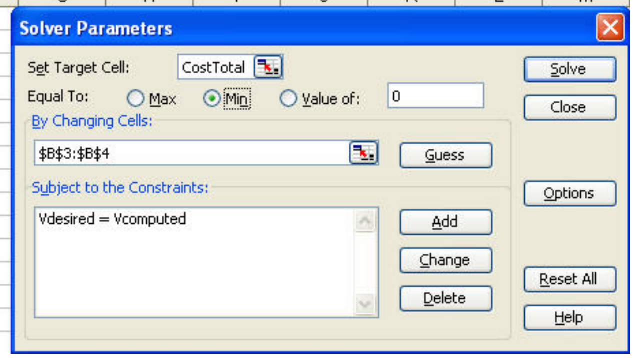
16.14

$$A_{ends} = 2\pi r^2$$
$$A_{side} = 2\pi rh$$
$$A_{total} = A_{ends} + A_{side}$$
$$V_{computed} = \pi r^2 h$$
$$Cost = F_{ends}A_{ends} + F_{side}A_{side} + F_{coating}A_{coating}$$

Then the following Excel worksheet and Solver application can be set up:

CostTotal f_x =SUM(CostEnd:CostOp)

	A	B	C	D	E	F
1	Prob. 16.13					
2						
3	hside	1	m	Vdesired	10	m3
4	dend	1	m	Vcomputed	0.785398163	m3
5						
6	dend:hside	1		Aend	1.570796327	m2
7				Aside	3.141592654	m2
8	rend	0.5	m	Atotal	4.71238898	
9						
10	FEnd	$ 200.00	$/m2	CostEnd	$ 314.16	
11	FSide	$ 100.00	$/m2	CostSide	$ 314.16	
12	FCoat	$ 50.00	$/m2	CostCoat	$ 235.62	
13						
14				CostTotal	$ 863.94	
15						

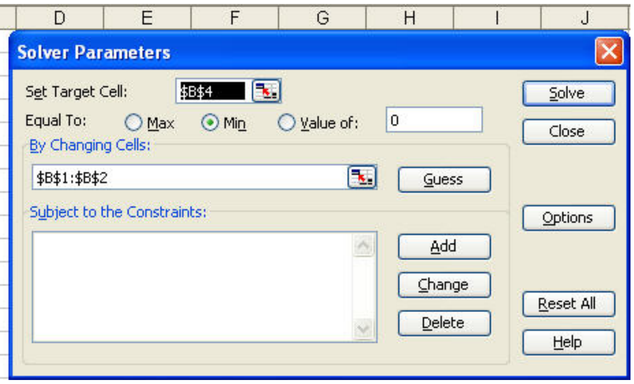


which results in the following solution:

	A	B	C	D	E	F
1	Prob. 16.13					
2						
3	hside	3.282282	m	Vdesired	10	m3
4	dend	1.96955	m	Vcomputed	10	m3
5						
6	dend:hside	0.600055		Aend	6.093321721	m2
7				Aside	20.30920276	m2
8	rend	0.984775	m	Atotal	26.40252448	
9						
10	FEnd	$ 200.00	$/m2	CostEnd	$ 1,218.66	
11	FSide	$ 100.00	$/m2	CostSide	$ 2,030.92	
12	FCoat	$ 50.00	$/m2	CostCoat	$ 1,320.13	
13						
14				CostTotal	$ 4,569.71	

16.15 As shown below, Excel Solver gives: $x = 0.5$, $y = 0.8$ and $f_{min} = -0.85$.

	A	B	C
1	x	1	
2	y	1	
3			
4	f(x,y)	0	
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			



	A	B
1	x	0.5
2	y	0.8
3		
4	f(x,y)	-0.85

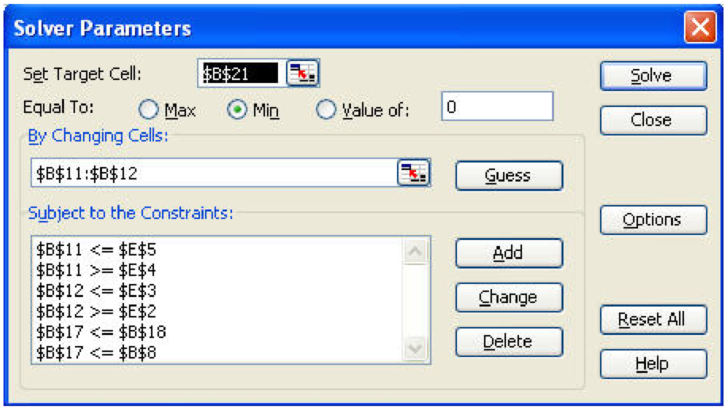
16.16 An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D	E
1	Parameters				
2	c1	4		d1	1
3	c2	2		d2	10
4	H	275		t1	0.1
5	P	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9					
10	Decision variables				
11	t	0.5			
12	d	10			
13					
14	Computed quantities			goals:	
15	W	10.79922			
16	I	196.8404			
17	sigma	127.324	<	550	
18	sigmab	1471.876			
19					
20	Objective function				
21	C	63.1969			

The formulas are

	A	B	C	D	E
1	Parameters				
2	c1	4		d1	1
3	c2	2		d2	10
4	H	275		t1	0.1
5	P	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9					
10	Decision va				
11	t	0.5			
12	d	10			
13					
14	Computed c			goals:	
15	W	=PI()*B12*B11*B4*B7			
16	I	=PI()/8*B12*B11*(B12^2+B11^2)			
17	sigma	=B5/PI()/B12/B11	<	550	
18	sigmab	=PI()*B6*B16/B4^2/B12/B11			
19					
20	Objective fu				
21	C	=B2*B15+B3*B12			

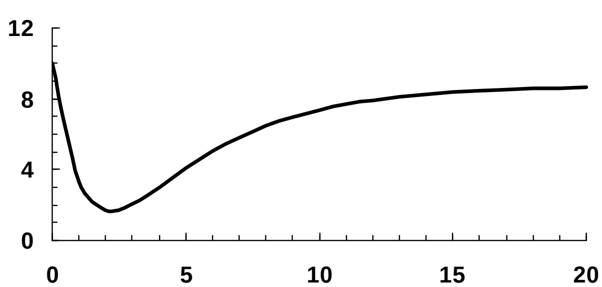
The Solver can be called and set up as



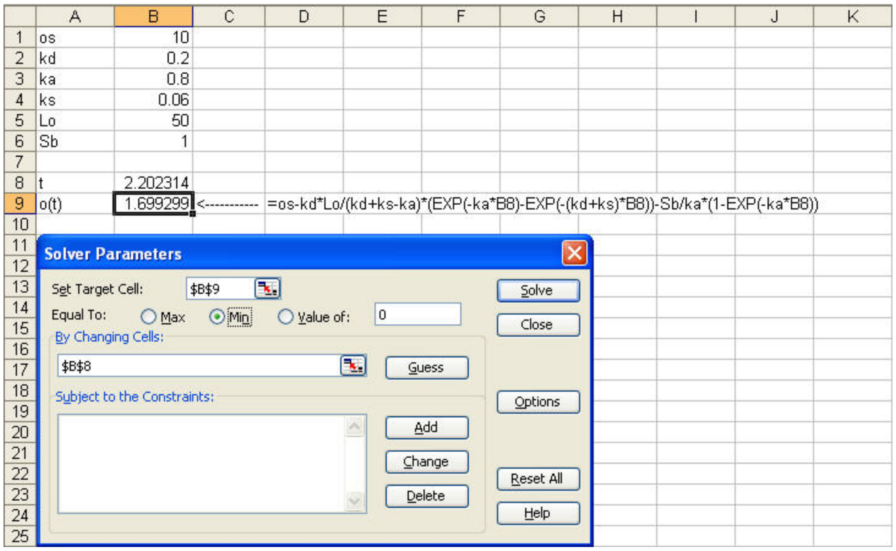
The resulting solution is

	A	B	C	D	E
1	Parameters				
2	c1	4		d1	1
3	c2	2		d2	10
4	H	275		t1	0.1
5	P	2000		t2	1
6	E	900000			
7	rho	0.0025			
8	sigmamax	550			
9					
10	Decision variables				
11	t	0.189207			
12	d	6.117589			
13					
14	Computed quantities			goals:	
15	W	2.5			
16	I	17.02759			
17	sigma	550	<	550	
18	sigmab	550			
19					
20	Objective function				
21	C	22.23518			
22					

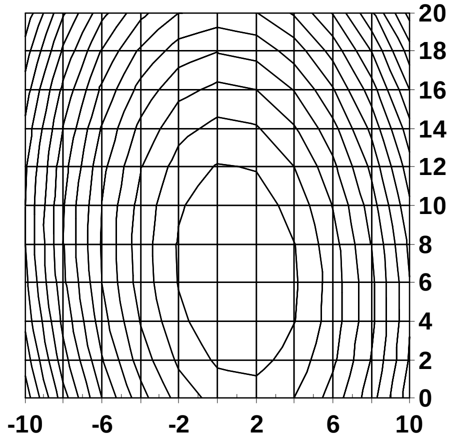
16.17 A plot of the function indicates a minimum at about $t = 2.2$.



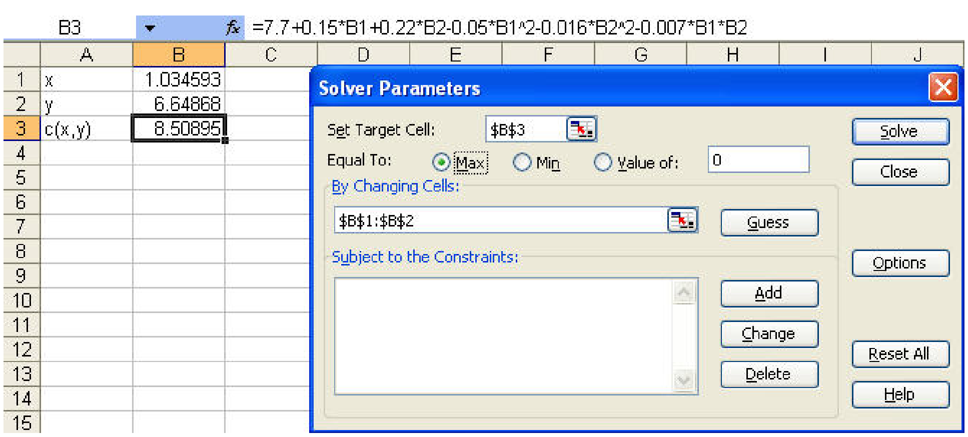
The Excel Solver can be used to determine that a minimum of $o = 1.699$ occurs at a value of t = 2.2023.



16.18 This problem can be solved graphically by using a software package to generate a contour plot of the function. For example, the following plot can be developed with Excel. As can be seen, a minimum occurs at approximately $x = 1$ and $y = 7$.



We can use a software package like Excel to determine the maximum precisely as $x = 1.034593$ and $y = 6.64868$.



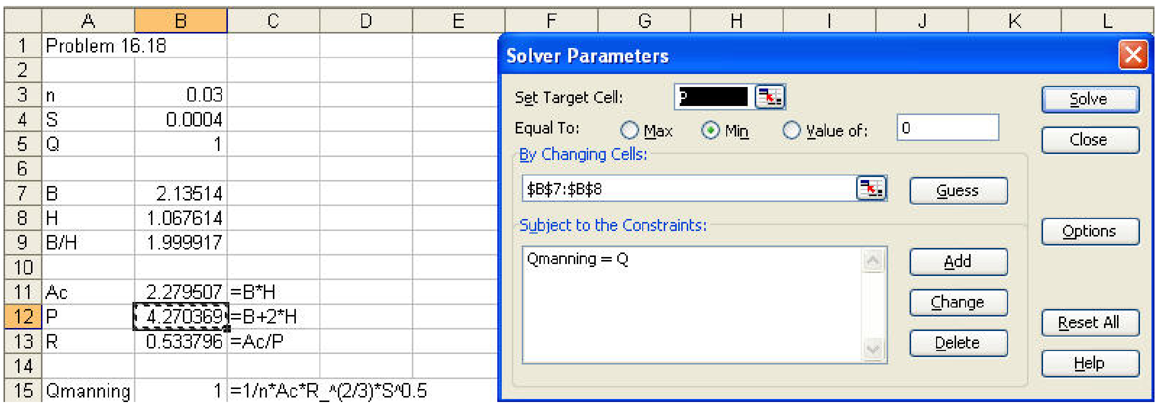
16.19 (a) The problem consists of

$$\min\ P = B + 2 * H$$

Subject to

$$\frac{1}{n} BH \left(\frac{BH}{B + 2H} \right)^{2/3} S^{1/2} = Q$$

The problem can be set up and solved with the Excel Solver as in



As can be seen, the result shows that the dimensions for the minimum wetted perimeter correspond to having the bottom width that is twice the length of each vertical side.

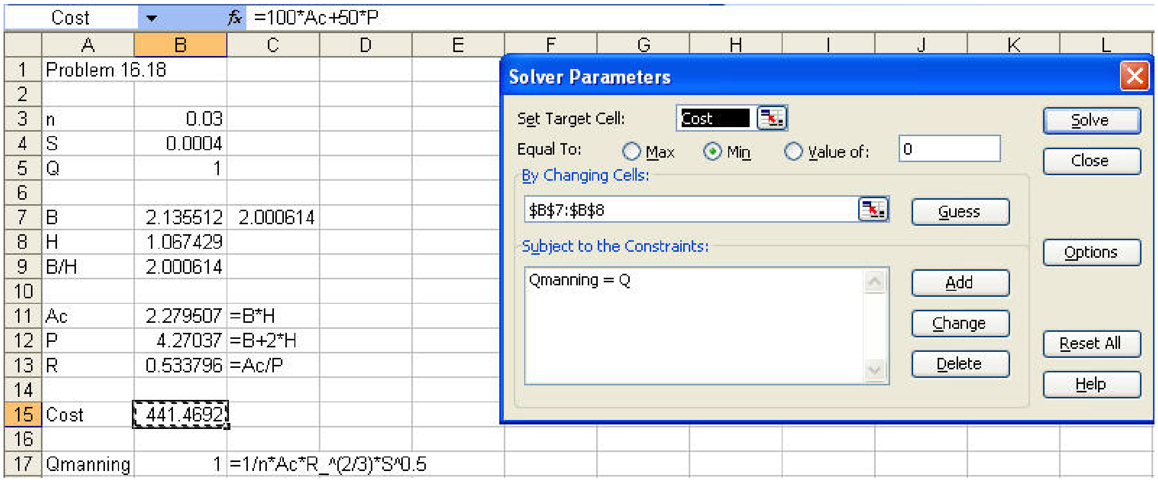
(b) Now we can redo the problem as a cost minimization:

$$\min\ C = 100A_c + 50P$$

Subject to

$$\frac{1}{n} BH \left(\frac{BH}{B + 2H} \right)^{2/3} S^{1/2} = Q$$

The problem can be set up and solved with the Excel Solver as in



Very interestingly, the result is identical to that obtained when cost was not an issue!!!

(c) The constraint can be rewritten as

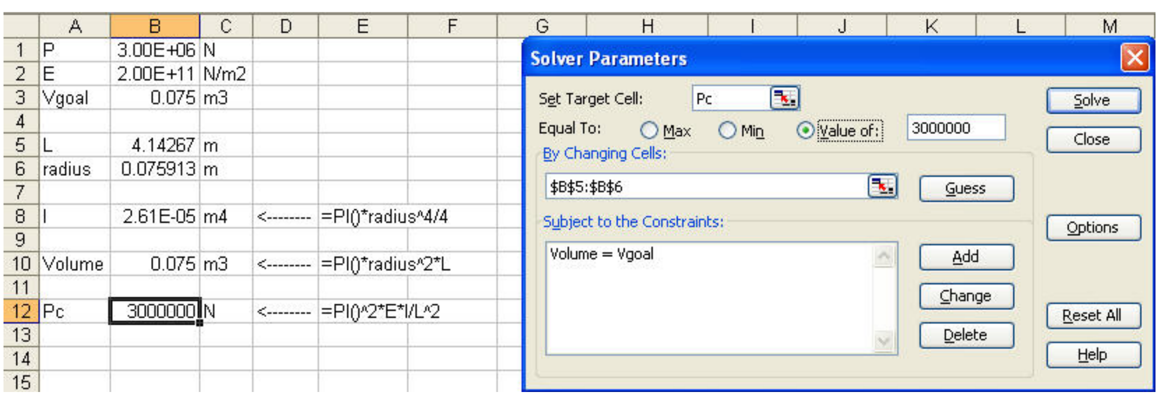
$$\frac{(BH)^{5/2}}{B+2H}=\left(\frac{nQ}{S^{1/2}}\right)^{3/2} = \text{constant}$$

or

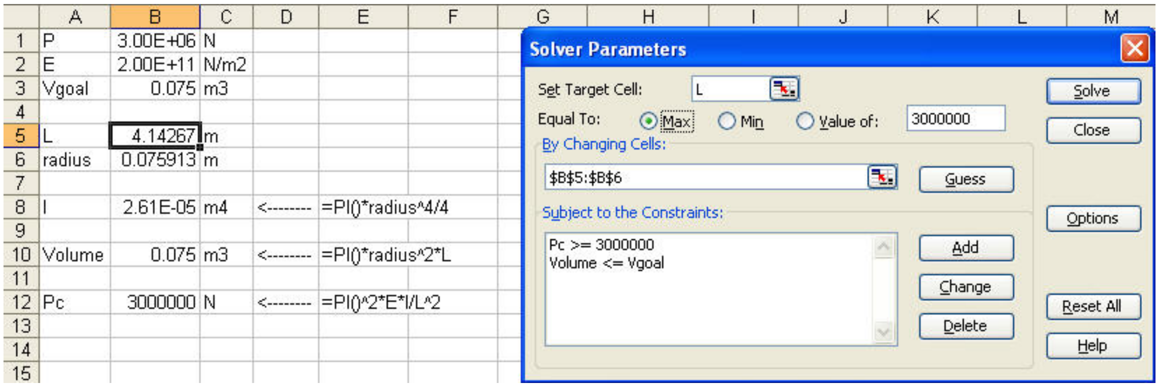
$$BH = \text{constant} \times (B+2H)^{2/5}$$

Therefore, both A_c and P are minimized simultaneously. This is great, because the excavation costs will be proportional to the cross-sectional area. Hence, by having the bottom width twice the length of each vertical side, we will minimize both excavation and lining costs simultaneously!!!

16.20 Using Excel Solver,



An alternative solution can be developed by maximizing L subject to Volume $\le 0.075\ \text{m}^3$ and $P_c \ge$ 3,000,000 N,



16.21 The total flow in the river: $F = 2\times10^6$ m^3/d.

The flow into the channels:

$$f_1 + f_2 \le 0.7F = 1.4\times10^6 \ \text{m}^3/\text{d}$$

Minimum channel flows for navigation:

$$f_1 \ge 0.3\times10^6 \ \text{m}^3/\text{d}$$
$$f_2 \ge 0.2\times10^6 \ \text{m}^3/\text{d}$$

Political constraints:

$$\frac{|f_1 - f_2|}{f_1 + f_2} \le 0.4$$

leads to

$$f_2 \ge \frac{3}{7} f_1$$

$$f_2 \le \frac{7}{3} f_1$$

Maintenance cost per year, $C \le \$1.8\times10^6$

Channel 1: $C_1 = 1.1f_1$
Channel 2: $C_2 = 1.4f_2$

leads to

$$1.1f_1 + 1.4f_2 \le 1.8\times10^6$$

Power revenue (revenue per year):

Channel 1: $r_{p1} = 4f_1$
Channel 2: $r_{p2} = 3f_2$

Irrigation revenue (revenue per year):

Channel 1: loss, $\alpha_1 = 0.3$
value/yr: $i_1 = 3.2(1 - \alpha) f_1 = 2.24 f_1$
Channel 2: loss, $\alpha_2 = 0.2$
value/yr: $i_2 = 3.2(1 - \alpha) f_2 = 2.56 f_2$

Net revenue = Revenue – losses

$P = 4f_1 + 3f_2 + 2.24f_1 + 2.56f_2 - 1.1f_1 - 1.4f_2$
$P = 5.14f_1 + 4.16f_2$

Therefore, the problem is formulated as

Decision variables:
f_1: flow in channel 1
f_2: flow in channel 2

Maximize: $P = 5.14f_1 + 4.16f_2$

Subject to

$f_1 + f_2 \le 1.4\times10^6$	channel flow
$1.1f_1 + 1.4f_2 \le 1.8\times10^6$	maintenance
$0.43f_1 - f_2 \le 0$	political constraint 1
$-2.33f_1 + f_2 \le 0$	political constraint 2
$f_1 \ge 0.3\times10^6$	minimum channel flow 1
$f_2 \ge 0.2\times10^6$	minimum channel flow 2

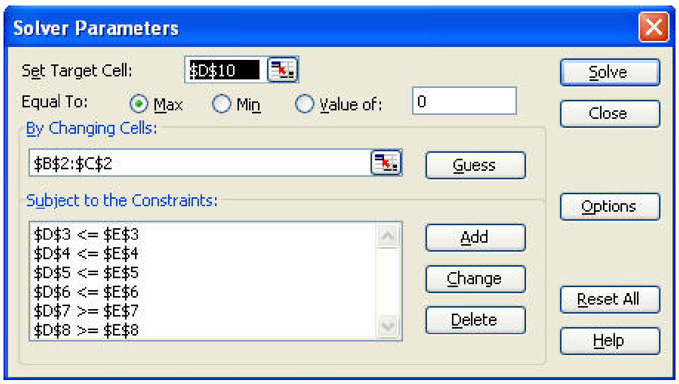
The problem can then be set up and solved with a tool such as Excel:

	A	B	C	D	E
1		Channel 1	Channel 2	total	constraint
2	Flow	0	0		
3		1	1	0	1.40E+06
4		1.1	1.4	0	1.80E+06
5		0.43	-1	0	0
6		-2.33	1	0	0
7		1		0	3.00E+05
8			1	0	2.00E+05
9					
10	Profit	5.14	4.16	0	

The cell formulas are

	A	B	C	D	E
1		Channel 1	Channel 2	total	constraint
2	Flow	0	0		
3		1	1	=B3*B$2+C3*C$2	1400000
4		1.1	1.4	=B4*B$2+C4*C$2	1800000
5		0.43	-1	=B5*B$2+C5*C$2	0
6		-2.33	1	=B6*B$2+C6*C$2	0
7		1		=B7*B$2+C7*C$2	300000
8			1	=B8*B$2+C8*C$2	200000
9					
10	Profit	5.14	4.16	=B10*B$2+C10*C$2	

The Excel Solver can be invoked as



The resulting solution is

	A	B	C	D	E
1		Channel 1	Channel 2	total	constraint
2	Flow	979021	420979		
3		1	1	1400000	1.40E+06
4		1.1	1.4	1666294	1.80E+06
5		0.43	-1	0	0
6		-2.33	1	-1860140	0
7		1		979021	3.00E+05
8			1	420979	2.00E+05
9					
10	Profit	5.14	4.16	6783441	

16.22 The weight of the truss is equal to

$$W = \rho(L_1 A_c + L_2 A_t + L_3 A_c)$$

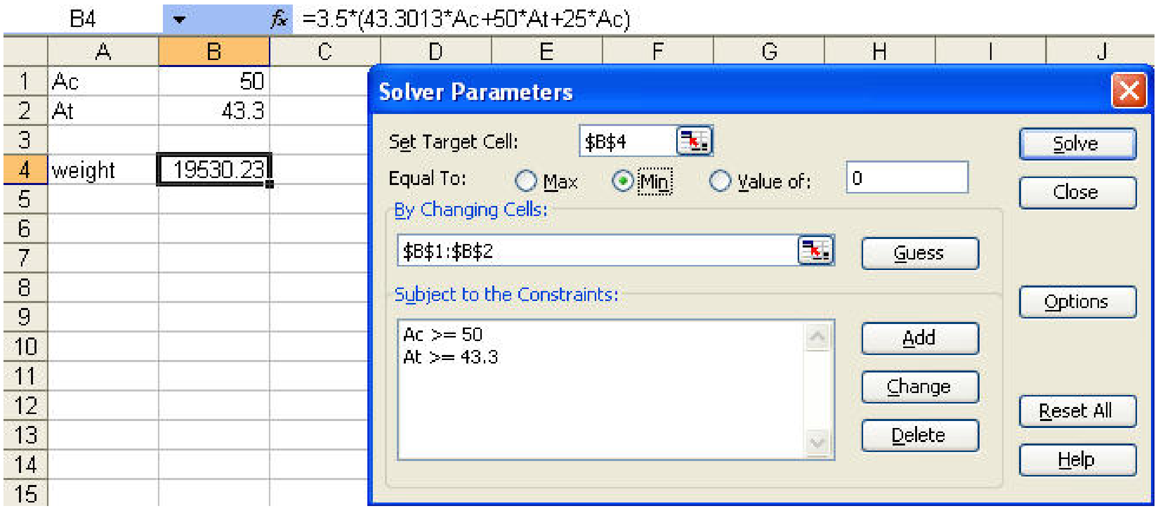
where ρ = density, L_i = length of member i, A_c = cross-sectional area of compression member, and A_t = cross-sectional area of tension member. The lengths of the 3 members can be determined as L_1 =43.3013, L_2 = 50, and L_3 = 25. Therefore, the solution can be formulated as a linear programming problem as

Minimize: $W = 3.5(43.3013A_c + 50A_t + 25A_c)$

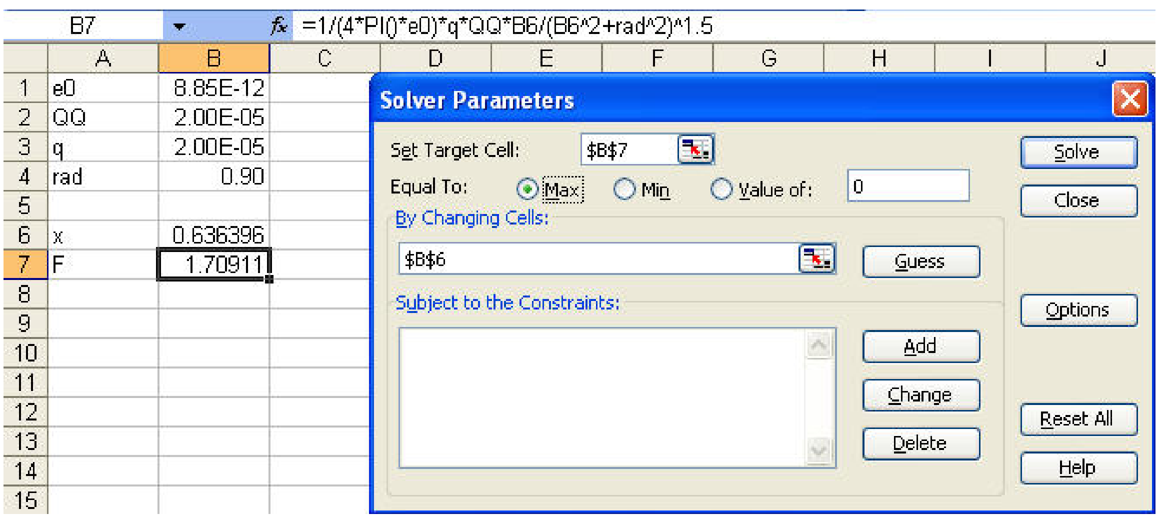
subject to

$$A_c \geq 50$$
$$A_t \geq 43.3$$

The solution can be developed in Excel using the Solver tool,



16.23 The solution can be developed in Excel using the Solver tool,



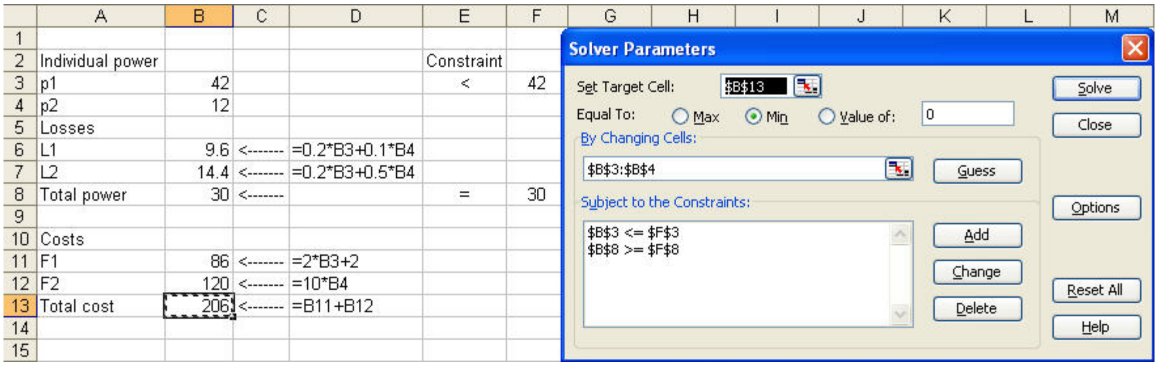
16.24 The problem can be formulated as

Minimize $\quad C = 2p_1 + 10p_2 + 2$

subject to $\quad 0.6p_1 + 0.4p_2 \geq 30$

$\quad p_1 \leq 42$

Using the Excel Solver:



	A	B	C	D	E	F
1						
2	Individual power				Constraint	
3	p1	42			<	42
4	p2	12				
5	Losses					
6	L1	9.6	<-------	=0.2*B3+0.1*B4		
7	L2	14.4	<-------	=0.2*B3+0.5*B4		
8	Total power	30	<-------		=	30
9						
10	Costs					
11	F1	86	<-------	=2*B3+2		
12	F2	120	<-------	=10*B4		
13	Total cost	206	<-------	=B11+B12		
14						
15						

16.25 This is a trick question. Because of the presence of (1 – s) in the denominator, the function will experience a division by zero at the maximum. This can be rectified by merely canceling the (1 – s) terms in the numerator and denominator to give

$$T = \frac{15s}{4s^2 - 3s + 4}$$

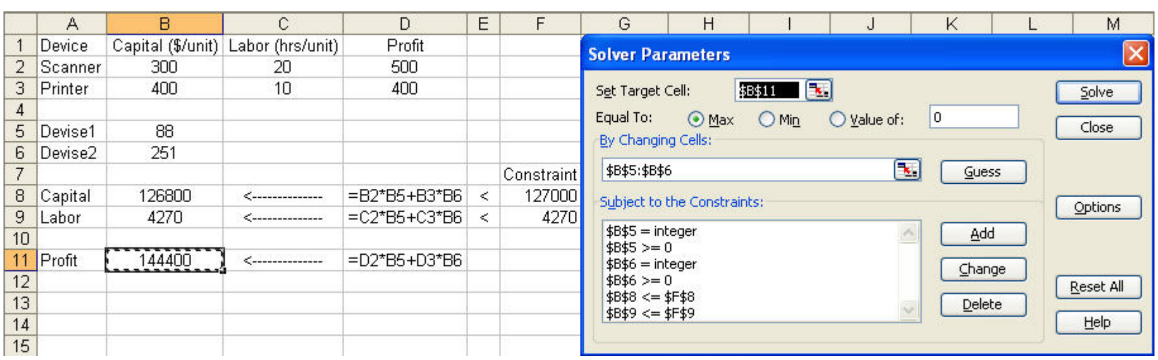
Any of the optimizers described in this section can then be used to determine that the maximum of T = 3 occurs at s = 1.

16.26 (a) An LP formulation for this problem can be set up as

$$\text{Maximize} \quad P = 500x_1 + 400x_2$$

$$\text{subject to} \quad \begin{aligned} &300x_1 + 400x_2 \le 127{,}000 \\ &20x_1 + 10x_2 \le 4{,}270 \\ &x_1, x_2 \ge 0 \end{aligned}$$

An Excel spreadsheet can be set up to solve the problem as



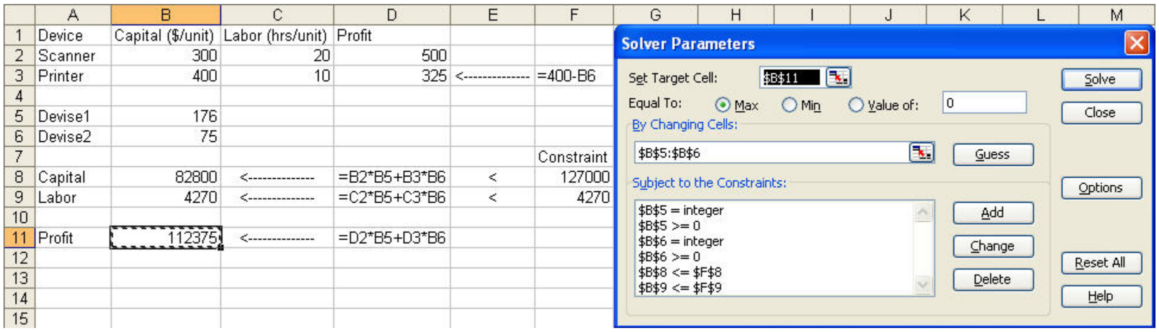
	A	B	C	D	E	F
1	Device	Capital ($/unit)	Labor (hrs/unit)	Profit		
2	Scanner	300	20	500		
3	Printer	400	10	400		
4						
5	Devise1	88				
6	Devise2	251				
7						Constraint
8	Capital	126800	<--------------	=B2*B5+B3*B6	<	127000
9	Labor	4270	<--------------	=C2*B5+C3*B6	<	4270
10						
11	Profit	144400	<--------------	=D2*B5+D3*B6		
12						
13						
14						
15						

(b) This problem can be formulated as

$$\text{Maximize} \quad P = 500x_1 + (400 - x_2)x_2$$

$$\text{subject to} \quad \begin{aligned} &300x_1 + 400x_2 \le 127{,}000 \\ &20x_1 + 10x_2 \le 4{,}270 \\ &x_1, x_2 \ge 0 \end{aligned}$$

An Excel spreadsheet can be set up to solve the problem as



	A	B	C	D	E	F
1	Device	Capital ($/unit)	Labor (hrs/unit)	Profit		
2	Scanner	300	20	500		
3	Printer	400	10	325	<--------------	=400-B6
4						
5	Devise1	176				
6	Devise2	75				
7						Constraint
8	Capital	82800	<--------------	=B2*B5+B3*B6	<	127000
9	Labor	4270	<--------------	=C2*B5+C3*B6	<	4270
10						
11	Profit	112375	<--------------	=D2*B5+D3*B6		
12						
13						
14						
15						

16.27 An LP formulation for this problem can be set up as

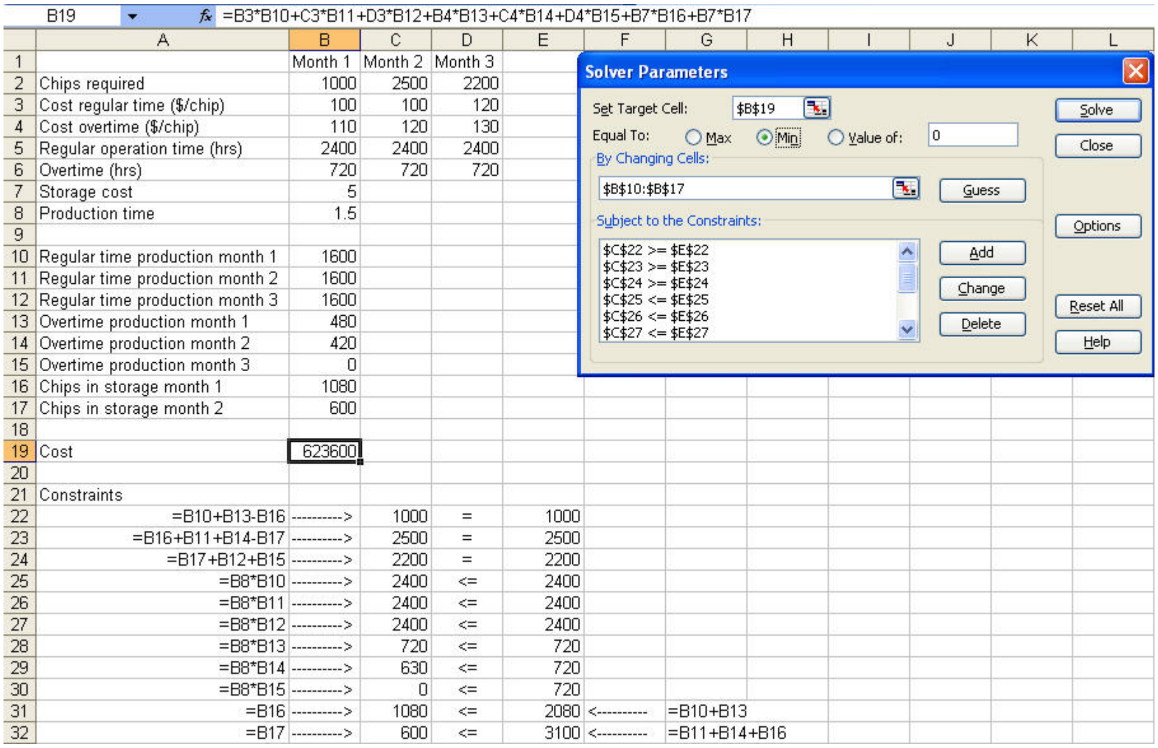
Decision variables: x_{ri} = chips produced in regular time for month i
x_{oi} = chips produced in overtime for month i
x_{si} = chips stored for month i

Minimize $C = 100x_{r1} + 100x_{r2} + 120x_{r3} + 110x_{o1} + 120x_{o2} + 130x_{o3} + 5x_{s1} + 5x_{s2}$

subject to

$$x_{r1} + x_{o1} - x_{s1} \geq 1,000$$
$$x_{s1} + x_{r2} + x_{o2} - x_{s2} \geq 2,500$$
$$x_{s2} + x_{r3} + x_{o3} \geq 2,200$$
$$1.5x_{r1} \leq 2,400$$
$$1.5x_{r2} \leq 2,400$$
$$1.5x_{r3} \leq 2,400$$
$$1.5x_{o1} \leq 720$$
$$1.5x_{o2} \leq 720$$
$$1.5x_{o3} \leq 720$$
$$x_{r1}, x_{r2}, x_{r3}, x_{o1}, x_{o2}, x_{o3}, x_{s1}, x_{s2} \geq 0$$

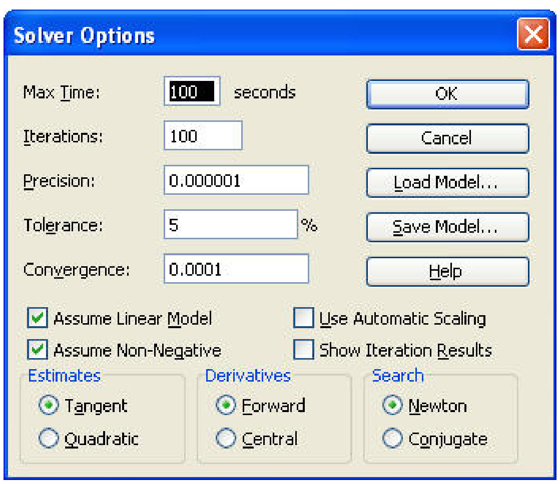
An Excel spreadsheet can be set up to solve the problem as



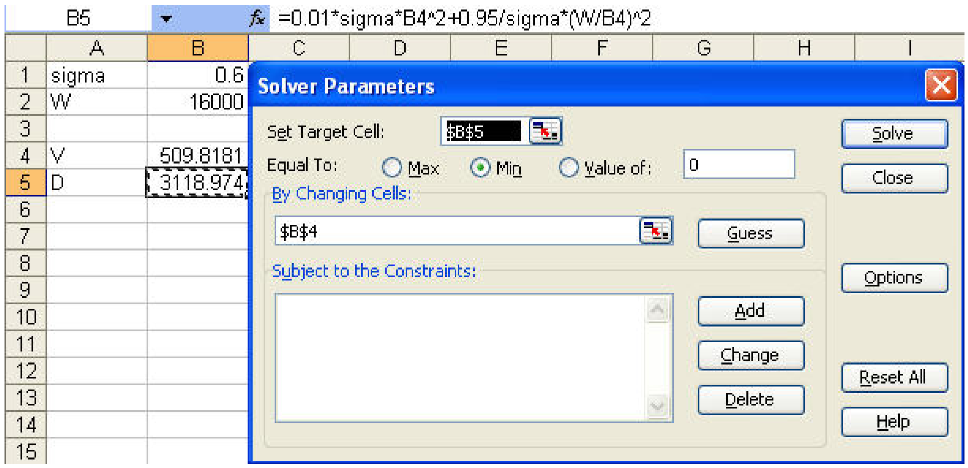
B19 f_x =B3*B10+C3*B11+D3*B12+B4*B13+C4*B14+D4*B15+B7*B16+B7*B17

	A	B	C	D	E	F	G
1		Month 1	Month 2	Month 3			
2	Chips required	1000	2500	2200			
3	Cost regular time ($/chip)	100	100	120			
4	Cost overtime ($/chip)	110	120	130			
5	Regular operation time (hrs)	2400	2400	2400			
6	Overtime (hrs)	720	720	720			
7	Storage cost	5					
8	Production time	1.5					
9							
10	Regular time production month 1	1600					
11	Regular time production month 2	1600					
12	Regular time production month 3	1600					
13	Overtime production month 1	480					
14	Overtime production month 2	420					
15	Overtime production month 3	0					
16	Chips in storage month 1	1080					
17	Chips in storage month 2	600					
18							
19	Cost	623600					
20							
21	Constraints						
22	=B10+B13-B16	---------->	1000	=	1000		
23	=B16+B11+B14-B17	---------->	2500	=	2500		
24	=B17+B12+B15	---------->	2200	=	2200		
25	=B8*B10	---------->	2400	<=	2400		
26	=B8*B11	---------->	2400	<=	2400		
27	=B8*B12	---------->	2400	<=	2400		
28	=B8*B13	---------->	720	<=	720		
29	=B8*B14	---------->	630	<=	720		
30	=B8*B15	---------->	0	<=	720		
31	=B16	---------->	1080	<=	2080	<----------	=B10+B13
32	=B17	---------->	600	<=	3100	<----------	=B11+B14+B16

Note that before depressing the Solve button, the Options button should be depressed and the following boxes should be selected: “Assume Linear Model” and “Assume Non-Negative.”



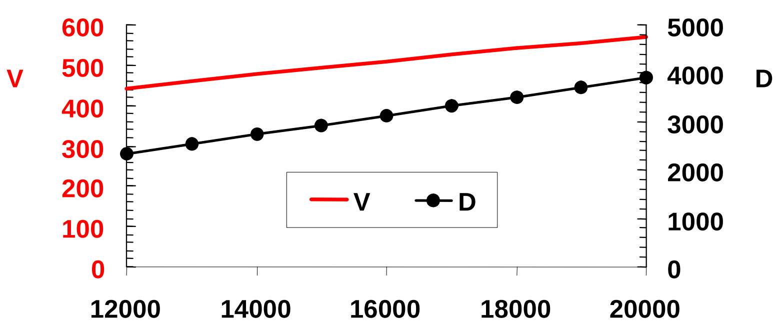
16.28 A tool such as the Excel Solver can be used to determine the solution as



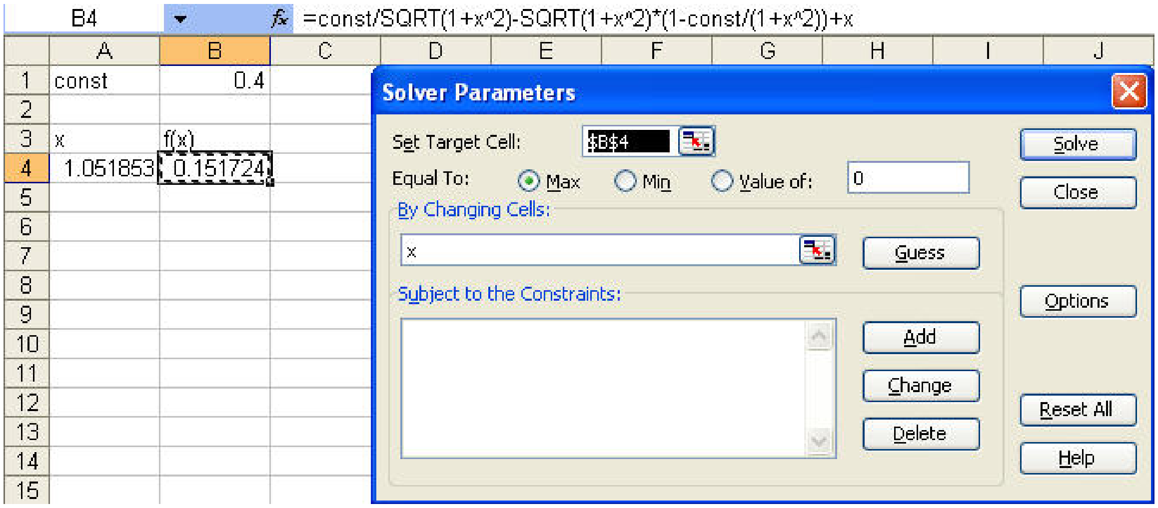
The approach can be implemented to evaluate other values of W with a constant σ to yield the following results:

W	*V*	*D*
12000	441.5154	2339.231
13000	459.5438	2534.167
14000	476.8912	2729.102
15000	493.6293	2924.038
16000	509.8181	3118.974
17000	525.5085	3313.910
18000	540.7438	3508.846
19000	555.5614	3703.782
20000	569.9940	3898.718

The optimal velocity along with the minimal drag can be plotted versus weight. As shown below, the relationship is fairly linear for the specified range.



16.29 A tool such as the Excel Solver can be used to determine the solution as



16.30 An LP formulation for this problem can be set up as

Minimize $C = 0.05X + 0.025Y + 0.15Z$ {Minimize cost}

subject to
$X + Y + Z \geq 6$ {Performance constraint}
$X + Y < 2.5$ {Safety constraint}
$X - Y \geq 0$ {*X*-*Y* Relationship constraint}
$Z - 0.5Y \geq 0$ {*Y*-*Z* Relationship constraint}

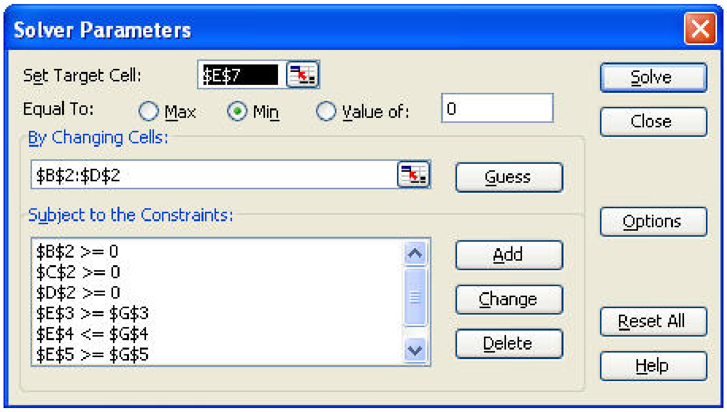
An Excel spreadsheet can be set up to solve the problem as

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	0	0	0			
3	Performance	1	1	1	0	>=	6
4	Safety	1	1	0	0	<=	2.5
5	X-Y	1	-1	0	0	>=	0
6	Z-0.5*Y	0	-0.5	1	0	>=	0
7	Cost	0.05	0.025	0.15	0		

The formulas are

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	0	0	0			
3	Performance	1	1	1	=B3*B$2+C3*C$2+D3*D$2	>=	6
4	Safety	1	1	0	=B4*B$2+C4*C$2+D4*D$2	<=	2.5
5	X-Y	1	-1	0	=B5*B$2+C5*C$2+D5*D$2	>=	0
6	Z-0.5*Y	0	-0.5	1	=B6*B$2+C6*C$2+D6*D$2	>=	0
7	Cost	0.05	0.025	0.15	=B7*B$2+C7*C$2+D7*D$2		

The Solver can be called and set up as



The resulting solution is

	A	B	C	D	E	F	G
1		X	Y	Z	Total		Constraint
2	Amount	1.25	1.25	3.5			
3	Performance	1	1	1	6	>=	6
4	Safety	1	1	0	2.5	<=	2.5
5	X-Y	1	-1	0	0	>=	0
6	Z-0.5*Y	0	-0.5	1	2.875	>=	0
7	Cost	0.05	0.025	0.15	0.61875		

16.31 An LP formulation for this problem can be set up as

Decision variables: x_i = quantity of part i

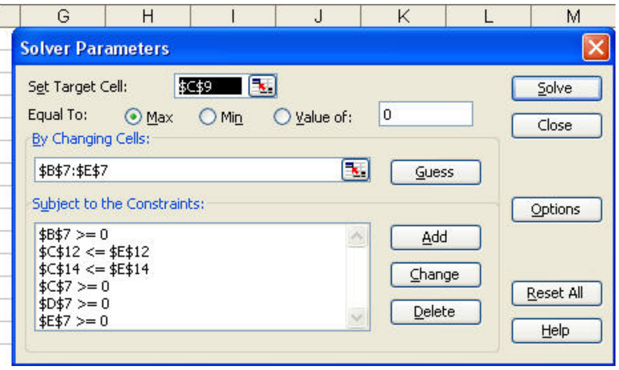
$$\text{Minimize} \quad P = 375x_A + 275x_B + 475x_C + 325x_D$$

$$\text{subject to} \quad 2.5x_A + 1.5x_B + 2.75x_C + 2x_D \le 640$$

$$3.5x_A + 3x_B + 3x_C + 2x_D \le 960$$

A tool such as the Excel Solver can be used to determine the solution as

	A	B	C	D	E	F
1		Part				
2		A	B	C	D	
3	Fabrication time (hr/100 units)	2.5	1.5	2.75	2	
4	Finishing time (hr/100 units)	3.5	3	3	2	
5	Profit ($/100 units)	375	275	475	325	
6						
7	Quantity	0	192	128	0	
8	Profit					
9	=B5*B7+C5*C7+D5*D7+E5*E7	------->	113600			
10	Constraints:					
11	Fabr Time					
12	=B3*B$7+C3*C$7+D3*D$7+E3*E$7	------->	640	<=	640	
13	Finish time					
14	=B4*B$7+C4*C$7+D4*D$7+E4*E$7	------->	960	<=	960	
15						



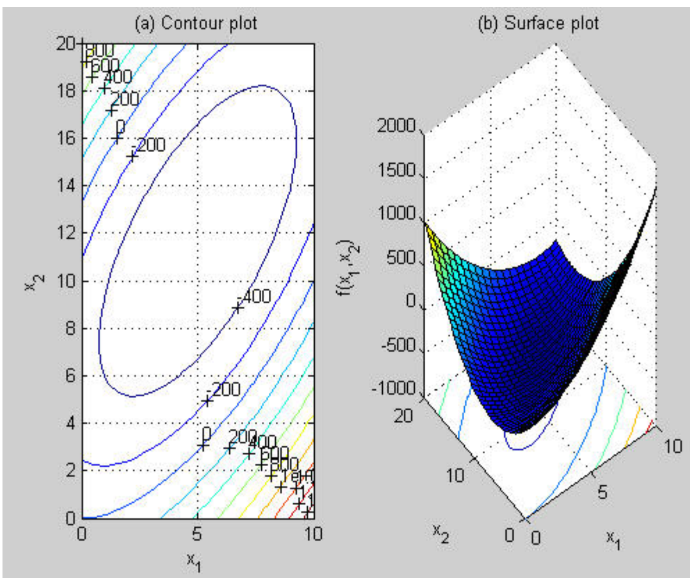
Thus, the results indicate that if we produce none of parts A and D and 192 and 128 of B and C, respectively, we will generate a maximum profit of $113,600.

16.32 The potential energy function can be written as

$$PE = 0.5k_a x_1^2 + 0.5k_b(x_1 - x_2)^2 - Fx_2$$

Contour and surface plots can be generated with the following MATLAB script,

```
ka=20;kb=15;F=100;
x=linspace(0,10,20);y=linspace(0,20,40);
[x1,x2] = meshgrid(x,y);
Z=0.5*ka*x1.^2+0.5*kb*(x2-x1).^2-F*x2;
subplot(1,2,1);
cs=contour(x1,x2,Z);clabel(cs);
xlabel('x_1');ylabel('x_2');
title('(a) Contour plot');grid;
subplot(1,2,2);
cs=surfc(x1,x2,Z);
zmin=floor(min(Z));
zmax=ceil(max(Z));
xlabel('x_1');ylabel('x_2');zlabel('f(x_1,x_2)');
title('(b) Mesh plot');
```



The values of x_1 and x_2 that minimize the PE function can be determined as

```
>> PE=@(x) 0.5*ka*x(1).^2+0.5*kb*(x(2)-x(1)).^2-F*x(2);
>> [xmin,PEmin]=fminsearch(PE,[5,5])
xmin =
   4.99998098312971  11.66668727728067
PEmin =
   -5.833333333179395e+002
```

16.33 This problem can be approached by developing the following equation for the potential energy of the bracketing system,

$$V(x,y)=\frac{EA}{\ell}\left(\frac{w}{2\ell}\right)^2 x^2+\frac{EA}{\ell}\left(\frac{h}{\ell}\right)^2 y^2 - Fx\cos\theta - Fy\sin\theta$$

Solving for an individual angle is straightforward. For $\theta = 30^{\circ}$, the given parameter values can be substituted to yield

$V(x, y) = 5{,}512{,}026x^2 + 28{,}471{,}210y^2 - 5000x - 8660y$

The minimum of this function can be determined in a number of ways. For example, using the Excel Solver, the minimum potential energy is −3.62 with deflections of $x = 0.000786$ and $y = 0.0000878$ m.

Of course, the Excel Solver could be implemented repeatedly for different values of θ to assess how the solution changes as the angle changes. Alternatively, a macro or a program can be written so that multiple optimizations can be implemented simultaneously. Of course, for this case a multidimensional search algorithm would have to be implemented. In any event, the results are displayed in the following figure:

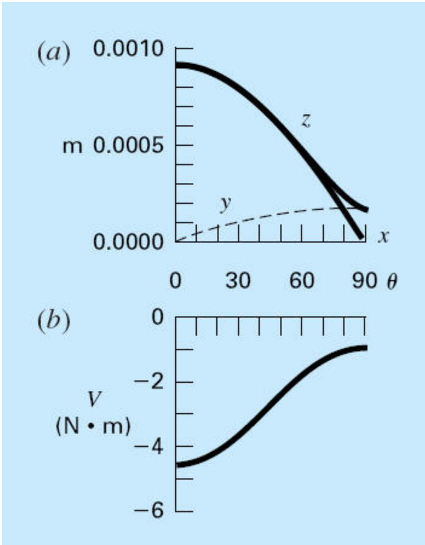


Figure P16.33 (*a*) The impact of different angles on the deflections (note that *z* is the resultant of the *x* and *y* components) and (*b*) potential energy of a part of the frame of a mountain bike subjected to a constant force.

As might be expected (Fig. P16.33*a*), the *x* deflection is at a maximum when the load is pointed in the *x* direction ($\theta = 0^o$) and the y deflection is at a maximum when the load is pointed in the *y* direction ($\theta = 90^o$). However, notice that the x deflection is much more pronounced than that in the *y* direction. This is also manifest Fig. P16.33*b*, where the potential energy is higher at low angles. Both results are due to the geometry of the strut. If *w* were made bigger, the deflections would be more uniform.