CHAPTER 21

21.1 (a) Analytical solution:

$$\int_0^{\pi/2} (8+4\cos x)\ dx = \left[8x+4\sin x\right]_0^{\pi/2} = 8(\pi/2)+4\sin(\pi/2)-0 = 16.56637$$

(b) Trapezoidal rule (n = 1):

$$I = (1.570796-0)\frac{12+8}{2} = 15.70796 \qquad \varepsilon_t = \left|\frac{16.56637-15.70796}{16.56637}\right| \times 100\% = 5.182\%$$

(c) Trapezoidal rule (n = 2):

$$I = (1.570796-0)\frac{12+2(10.82843)+8}{4} = 16.35861 \quad \varepsilon_t = 1.254\%$$

Trapezoidal rule (n = 4):

$$I = (1.570796-0)\frac{12+2(11.69552+10.82843+9.530734)+8}{8} = 16.51483 \quad \varepsilon_t = 0.311\%$$

(d) Simpson's 1/3 rule:

$$I = (1.570796-0)\frac{12+4(10.82843)+8}{6} = 16.57549 \quad \varepsilon_t = 0.055\%$$

(e) Simpson's rule (n = 4):

$$I = (1.570796-0)\frac{12+4(11.69552+9.530734)+2(10.82843)+8}{12} = 16.56691 \quad \varepsilon_t = 0.0032\%$$

(f) Simpson's 3/8 rule:

$$I = (1.570796-0)\frac{12+3(11.4641+10)+8}{8} = 16.57039 \quad \varepsilon_t = 0.024\%$$

(g) Simpson's rules (n = 5):

$$\begin{aligned} I &= (0.628319-0)\frac{12+4(11.80423+11.23607}{6} \\ &\quad + (1.570796-0.628319)\frac{11.23607+3(10.35114+9.236068)+8}{8} \\ &= 7.377818+9.188887 = 16.5667 \quad \varepsilon_t = 0.002\% \end{aligned}$$

21.2 (a) Analytical solution:

$$\int_0^4 \left(1-e^{-x}\right) dx = \left[x+e^{-x}\right]_0^3 = 3+e^{-3}-0-e^0 = 2.049787$$

(b) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{0+0.950213}{2} = 1.425319$$

$$\varepsilon_t = \left|\frac{2.049787-1.425319}{2.049787}\right| \times 100\% = 30.47\%$$

(c) Trapezoidal rule (n = 2):

$$I = (3-0)\frac{0+2(0.77687)+0.950213}{4} = 1.877964 \qquad \varepsilon_t = 8.38\%$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0+2(0.527633+0.77687+0.894601)+0.950213}{8} = 2.005658 \qquad \varepsilon_t = 2.15\%$$

(d) Simpson's 1/3 rule:

$$I = (3-0)\frac{0+4(0.77687)+0.950213}{6} = 2.028846 \qquad \varepsilon_t = 1.02\%$$

(e) Simpson's rule (n = 4):

$$I = (3-0)\frac{0+4(0.527633+0.894601)+2(0.77687)+0.950213}{12} = 2.048222 \qquad \varepsilon_t = 0.08\%$$

(f) Simpson's 3/8 rule:

$$I = (3-0)\frac{0+3(0.632121+0.864665)+0.950213}{8} = 2.040213 \qquad \varepsilon_t = 0.47\%$$

(g) Simpson's rules (n = 5):

$$I = (1.2-0)\frac{0+4(0.451188)+0.698806}{6}$$
$$+(3-1.2)\frac{0.698806+3(0.834701+0.909282)+0.950213}{8}$$
$$= 0.500712 + 1.548218 = 2.04893 \qquad \varepsilon_t = 0.04\%$$

21.3 (a) Analytical solution:

$$\int_{-2}^{4} (1 - x - 4x^3 + 2x^5)\, dx = \left[x - \frac{x^2}{2} - x^4 + \frac{x^6}{3} \right]_{-2}^{4} = 1104$$

(b) Trapezoidal rule (n = 1):

$$I = (4-(-2))\frac{-29+1789}{2} = 5280 \qquad \varepsilon_t = \left|\frac{1104-5280}{1104}\right| \times 100\% = 378.26\%$$

(c) Trapezoidal rule (n = 2):

$$I = (4-(-2))\frac{-29+2(-2)+1789}{4} = 2634 \quad \varepsilon_t = 138.59\%$$

Trapezoidal rule (n = 4):

$$I = (4-(-2))\frac{-29+2(1.9375-2+131.3125)+1789}{8} = 1516.875 \quad \varepsilon_t = 37.398\%$$

(d) Simpson's 1/3 rule (n = 2):

$$I = (4-(-2))\frac{-29+4(-2)+1789}{6} = 1752 \quad \varepsilon_t = 58.7\%$$

(e) Simpson's 3/8 rule:

$$I = (4-(-2))\frac{-29+3(1+31)+1789}{8} = 1392 \quad \varepsilon_t = 26.087\%$$

(f) Boole's rule (n = 5):

$$I = (4-(-2))\frac{7(-29)+32(1.9375)+12(-2)+32(131.3125)+7(1789)}{90} = 1104 \quad \varepsilon_t = 0\%$$

21.4 Analytical solution:

$$\int_1^2 (x+1/x)^2\, dx = \left[\frac{x^3}{3}+2x-\frac{1}{x}\right]_1^2 = 4.33333$$

Trapezoidal rule (n = 1):

$$(2-1)\frac{4+6.25}{2} = 5.125$$

The results are summarized below:

n	**Integral**	ε_t
1	5.125000	6.034%
2	4.909722	1.580%
3	4.867685	0.711%
4	4.852744	0.402%

21.5 Analytical solution:

$$\int_{-3}^{5} (4x-3)^3 \, dx = \left[\frac{1}{16}(4x-3)^4\right]_{-3}^{5} = 2056$$

Simpson's rule (n = 4):

$$I = (5-(-3))\frac{-3375+4(-343+729)+2(1)+4913}{12} = 2056 \qquad \varepsilon_t = 0\%$$

Simpson's rules (n = 5):

$$I = (0.2-(-3))\frac{-3375+4(-636.056)-10.648}{6}$$

$$+(5-0.2)\frac{-10.648+3(74.088+1191.016)+4913}{8} = -3162.6+5218.598 = 2056 \qquad \varepsilon_t = 0\%$$

Because Simpson's rules are perfect for cubics, both versions yield the exact result for this cubic polynomial.

21.6 Analytical solution:

$$\int_{0}^{3} x^2 e^x \, dx = \left[(x^2-2x+2)e^x\right]_{0}^{3} = 98.42768$$

Trapezoidal rule (n = 4):

$$I = (3-0)\frac{0+2(1.190813+10.0838+48.03166)+180.7698}{8} = 112.2684 \qquad \varepsilon_t = 14.062\%$$

Simpson's rule (n = 4):

$$I = (3-0)\frac{0+4(1.190813+48.03166)+2(10.0838)+180.7698}{12} = 99.45683 \qquad \varepsilon_t = 1.046\%$$

21.7 Analytical solution:

$$\int_{0}^{1} 15^{2x} \, dx = \left[\frac{1}{2\ln 15}15^{2x}\right]_{0}^{1} = 41.35817$$

(a) Trapezoidal rule (n = 1):

$$I = (1-0)\frac{1+225}{2} = 113 \qquad \varepsilon_t = \left|\frac{41.35817-113}{41.35817}\right| \times 100\% = 173.223\%$$

(b) Simpson's 1/3 rule (n = 2):

$$I = (1-0)\frac{1+4(15)+225}{6} = 47.6667 \qquad \varepsilon_t = 15.253\%$$

(c) Simpson's 3/8 rule:

$$I = (1-0)\frac{1+3(6.082202+36.99318)+225}{8} = 44.40327 \quad \varepsilon_t = 7.363\%$$

(d) Boole's rule:

$$I = (1-0)\frac{7(1)+32(3.872983)+12(15)+32(58.09475)+7(225)}{90} = 41.61075 \quad \varepsilon_t = 0.6107\%$$

(e) Midpoint method:

$$I = (1-0)15 = 15 \qquad \varepsilon_t = 63.731\%$$

(f) 3-segment-2-point open integration formula:

$$I = (1-0)\frac{6.082202+36.99318}{2} = 21.53769 \qquad \varepsilon_t = 47.924\%$$

(g) 4-segment-3-point open integration formula:

$$I = (1-0)\frac{2(3.872983)-15+2(58.09475)}{3} = 36.31182 \qquad \varepsilon_t = 12.202\%$$

21.8 Analytical solution:

$$\int_0^3 \left(5+3\cos x\right) dx = \left[5x+3\sin x\right]_0^3 = 15.42336$$

(a) Trapezoidal rule (n = 1):

$$I = (3-0)\frac{8+2.030023}{2} = 15.04503 \qquad \varepsilon_t = \left|\frac{15.42336-15.04503}{15.42336}\right| \times 100\% = 2.453\%$$

(b) Simpson's 1/3 rule (n = 2):

$$I = (3-0)\frac{8+4(5.212212)+2.030023}{6} = 15.43943 \quad \varepsilon_t = 0.104\%$$

(c) Simpson's 3/8 rule:

$$I = (3-0)\frac{8+3(6.620907+3.751559)+2.030023}{8} = 15.43028 \quad \varepsilon_t = 0.045\%$$

(d) Simpson's rules (n = 5):

$$\begin{aligned} I &= (1.2-0)\frac{8+4(7.476007)+6.087073}{6} \\ &\quad + (3-1.2)\frac{6.087073+3(4.318394+2.787819)+2.030023}{8} \\ &= 8.79822+6.62304 = 15.42126 \qquad \varepsilon_t = 0.014\% \end{aligned}$$

(e) Boole's rule:

$$I = (3-0)\frac{7(8)+32(7.195067)+12(5.212212)+32(3.115479)+7(2.030023)}{90} = 15.42314$$

$$\varepsilon_t = 0.0014\%$$

(f) Midpoint method:

$$I = (3-0)5.212212 = 15.63663 \qquad \varepsilon_t = 1.383\%$$

(g) 3-segment-2-point open integration formula:

$$I = (3-0)\frac{6.620907+3.751559}{2} = 15.5587 \qquad \varepsilon_t = 0.877\%$$

(h) 4-segment-3-point open integration formula:

$$I = (3-0)\frac{2(7.195067)-5.212212+2(3.115479)}{3} = 15.40888 \qquad \varepsilon_t = 0.094\%$$

21.9 Analytical solution:

$$z(t) = \int_0^t \sqrt{\frac{gm}{c_d}} \tanh\left(\sqrt{\frac{gc_d}{m}}t\right) dt = \left[\frac{m}{c_d}\ln\left[\cosh\left(\sqrt{\frac{gc_d}{m}}t\right)\right]\right]_0^t$$

$$z(10) = \left[\frac{68.1}{0.25}\ln\left[\cosh\left(\sqrt{\frac{9.8(0.25)}{68.1}}(10)\right)\right]\right]_0^{10} = 333.9262$$

Thus, the result to 3 significant digits is 334. Here are results for various multiple-segment trapezoidal rules:

n	***I***
1	246.9593
2	314.4026
3	325.4835
4	329.216
5	330.9225
6	331.8443
7	332.3984
8	332.7573
9	333.0031
10	333.1788
11	333.3087
12	333.4074
13	333.4842
14	333.5452

Thus, a 14-segment application gives the result to 4 significant digits.

21.10
(a) Trapezoidal rule (n = 5):

$$I = (0.5-0)\frac{1+2(8+4+3.5+5)+1}{10} = 2.15$$

(b) Simpson's rules (n = 5):

$$I = (0.2-0)\frac{1+4(8)+4}{6} + (0.5-0.2)\frac{4+3(3.5+5)+1}{8} = 1.233333 + 1.14375 = 2.377083$$

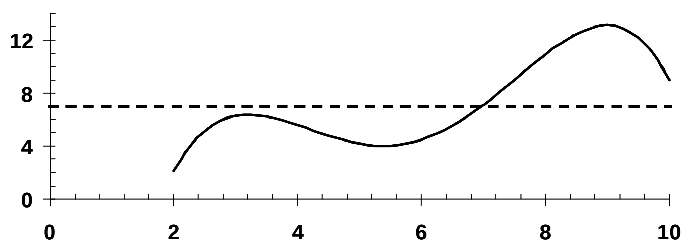
21.11
(a) Trapezoidal rule (n = 6):

$$I = (10-(-2))\frac{35+2(5-10+2+5+3)+20}{12} = 65$$

(b) Simpson's rules (n = 6):

$$I = (10-(-2))\frac{35+4(5+2+3)+2(-10+5)+20}{18} = 56.66667$$

21.12 (a) A graph suggests that the mean value is about 7.



(b) Analytical solution:

$$\frac{\int_2^{10} -46+45.4x-13.8x^2+1.71x^3-0.0729x^4 dx}{10-2}$$

$$= \frac{\left[-46x+22.7x^2-4.6x^3+0.4275x^4-0.01458x^5\right]_2^{10}}{10-2} = \frac{58.62656}{8} = 7.32832$$

(c) Numerical solution:

$$I = (5.2-2)\frac{2.1136+4(6.129359)+4.065999}{6}$$

$$+(10-5.2)\frac{4.065999+3(6.416489+12.20762)+9}{8}$$

$$= 16.37175 + 41.36299 = 57.73475$$

$$\text{Average} = \frac{57.73475}{10-2} = 7.216843$$

$$\varepsilon_t = \left|\frac{7.32832 - 7.216843}{7.32832}\right| \times 100\% = 1.521\%$$

21.13 (a) Analytical solution:

$$\int_0^{0.6} 2e^{-1.5x} dx = \left[-1.33333e^{-1.5x}\right]_0^{0.6} = -0.54209 - (-1.33333) = 0.79124$$

(b) Trapezoidal rule

$$I = (0.05-0)\frac{2+1.8555}{2} + (0.15-0.05)\frac{1.8555+1.597}{2} + (0.25-0.15)\frac{1.597+1.3746}{2}$$
$$+(0.35-0.25)\frac{1.3746+1.1831}{2} + (0.475-0.35)\frac{1.1831+0.9808}{2}$$
$$+(0.6-0.475)\frac{0.9808+0.8131}{2} = 0.79284$$

$$\varepsilon_t = \left|\frac{0.79124 - 0.79284}{0.79124}\right| \times 100\% = 0.2022\%$$

(c) Trapezoidal/Simpson’s rules

$$I = (0.05-0)\frac{2+1.8555}{2} + (0.35-0.05)\frac{1.8555+3(1.597+1.3746)+1.1831}{8}$$
$$+(0.6-0.35)\frac{1.1831+4(0.9808)+0.8131}{6} = 0.791282$$

$$\varepsilon_t = \left|\frac{0.79124 - 0.791282}{0.79124}\right| \times 100\% = 0.0052\%$$

21.14 (a) Analytical solution:

$$\int_{-2}^{2}\int_0^4 (x^2 - 3y^2 + xy^3)\ dx\ dy = \int_{-2}^{2} \left[\frac{x^3}{3} - 3y^2 x + y^3\frac{x^2}{2}\right]_0^4\ dy$$

$$= \int_{-2}^{2} \frac{64}{3} - 12y^2 + 8y^3\ dy = \left[\frac{64}{3}y - 4y^3 + 2y^4\right]_{-2}^{2} = 21.33333$$

(b) Sweeping across the *x* dimension:

<u>$y = -1$:</u>

$$I = (4-0)\frac{-12+2(-24)-28}{4} = -88$$

<u>$y = 0$:</u>

$$I = (4-0)\frac{0+2(4)+16}{4} = 24$$

<u>$y = 1$:</u>

$$I = (4-0)\frac{-12+2(8)+36}{4} = 40$$

These values can then be integrated along the *y* dimension:

$$I = (2-(-2))\frac{-88+2(24)+40}{4} = 0$$

$$\varepsilon_t = \left|\frac{21.33333-0}{21.33333}\right| \times 100\% = 100\%$$

(c) Sweeping across the *x* dimension:

$\underline{y = -1:}$

$$I = (4-0)\frac{-12+4(-24)-28}{6} = -90.66667$$

$\underline{y = 0:}$

$$I = (4-0)\frac{0+4(4)+16}{6} = 21.333333$$

$\underline{y = 1:}$

$$I = (4-0)\frac{-12+4(8)+36}{6} = 37.33333$$

These values can then be integrated along the *y* dimension:

$$I = (2-(-2))\frac{-90.66667+4(21.33333)+37.33333}{6} = 21.33333 \qquad \varepsilon_t = 0\%$$

21.15 (a) Analytical solution:

$$\int_{-2}^{2}\int_{0}^{2}\int_{-3}^{1}(x^3-3yz)\;dx\;dy\;dz = \int_{-2}^{2}\int_{0}^{2}\left[\frac{x^4}{4}-3yzx\right]_{-3}^{1}\;dy\;dz$$

$$= \int_{-2}^{2}\int_{0}^{2} -20-12yz\;\;dy\;dz = \int_{-2}^{2}\left[-20y-6y^2z\right]_{0}^{2}\;dz = \int_{-2}^{2} -40-24z\;\;dz$$

$$= \left[-40z-12z^2\right]_{-2}^{2} = -160$$

(b) For z = –2, sweeping across the *x* dimension:

$\underline{z = -2;\; y = 0:}$

$$I = (1-(-3))\frac{-27+4(-1)+1}{6} = -20$$

$\underline{z = -2;\; y = 1:}$

$$I = (1-(-3))\frac{-21+4(5)+7}{6} = 4$$

$\underline{z = -2;\; y = 2:}$

$$I = (1-(-3))\frac{-15+4(11)+13}{6} = 28$$

These values can then be integrated along the *y* dimension:

$$I = (2-0)\frac{-20+4(4)+28}{6} = 8$$

For *z* = 0, similar calculations yield

$z = 0; y = 0$: $I = -20$
$z = 0; y = 1$: $I = -20$
$z = 0; y = 2$: $I = -20$

These values can then be integrated along the *y* dimension:

$$I = (2-0)\frac{-20+4(-20)-20}{6} = -40$$

For *z* = 2, similar calculations yield

$z = 2; y = 0$: $I = -20$
$z = 2; y = 1$: $I = -44$
$z = 2; y = 2$: $I = -68$

These values can then be integrated along the *y* dimension:

$$I = (2-0)\frac{-20+4(-44)-68}{6} = -88$$

Finally, these results can be integrated along the *z* dimension,

$$I = (2-(-2))\frac{8+4(-40)-88}{6} = -160 \qquad \varepsilon_t = 0\%$$

21.16 Here is a VBA program that is set up to duplicate the computation performed in Example 21.2.

```
Option Explicit
Sub TestTrapm()
Dim n As Integer, i As Integer
Dim a As Double, b As Double, h As Double
Dim x(100) As Double, f(100) As Double
'Enter data and integration parameters
a = 0
b = 0.8
n = 2
h = (b - a) / n
'generate data from function
x(0) = a
f(0) = fx(a)
For i = 1 To n
  x(i) = x(i - 1) + h
  f(i) = fx(x(i))
Next i
'invoke function to determine and display integral
MsgBox "The integral is " & Trapm(h, n, f)
End Sub
Function Trapm(h, n, f)
```

```
Dim i As Integer, sum As Double
sum = f(0)
For i = 1 To n - 1
  sum = sum + 2 * f(i)
Next i
sum = sum + f(n)
Trapm = h * sum / 2
End Function
Function fx(x)
fx = 0.2 + 25 * x - 200 * x ^ 2 + 675 * x ^ 3 - 900 * x ^ 4 + 400 * x ^ 5
End Function
```

21.17 Here is a VBA program that is set up to duplicate the computation performed in Example 21.5.

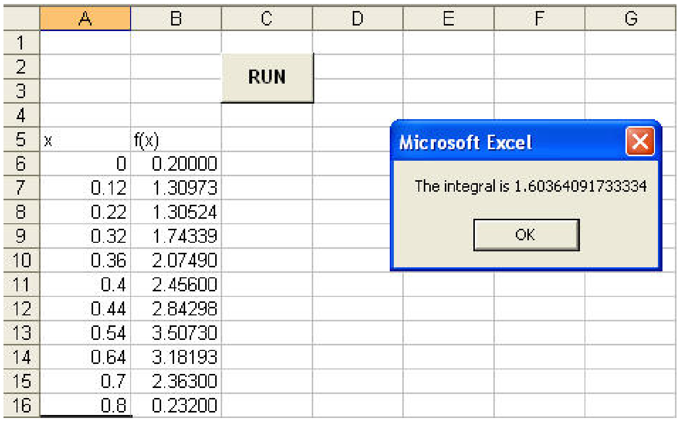
```
Option Explicit
Sub TestSimpm()
Dim n As Integer, i As Integer
Dim a As Double, b As Double, h As Double
Dim x(100) As Double, f(100) As Double
'Enter data and integration parameters
a = 0
b = 0.8
n = 4
h = (b - a) / n
'generate data from function
x(0) = a
f(0) = fx(a)
For i = 1 To n
  x(i) = x(i - 1) + h
  f(i) = fx(x(i))
Next i
'invoke function to determine and display integral
MsgBox "The integral is " & Simp13m(h, n, f)
End Sub
Function Simp13m(h, n, f)
Dim i As Integer
Dim sum As Double
sum = f(0)
For i = 1 To n - 2 Step 2
  sum = sum + 4 * f(i) + 2 * f(i + 1)
Next i
sum = sum + 4 * f(n - 1) + f(n)
Simp13m = h * sum / 3
End Function
Function fx(x)
fx = 0.2 + 25 * x - 200 * x ^ 2 + 675 * x ^ 3 - 900 * x ^ 4 + 400 * x ^ 5
End Function
```

21.18 Here is a VBA program that is set up to duplicate the computation performed in Example 21.8.

```
Option Explicit
Sub TestUneven()
Dim n As Integer, i As Integer
Dim label As String
Dim a As Double, b As Double, h As Double
Dim x(100) As Double, f(100) As Double
'Enter data
Sheets("Sheet1").Select
Range("a6").Select
n = ActiveCell.Row
Selection.End(xlDown).Select
```

```
n = ActiveCell.Row - n
'Input data from sheet
Range("a6").Select
For i = 0 To n
  x(i) = ActiveCell.Value
  ActiveCell.Offset(0, 1).Select
  f(i) = ActiveCell.Value
  ActiveCell.Offset(1, -1).Select
Next i
'invoke function to determine and display integral
MsgBox "The integral is " & Uneven(n, x, f)
End Sub
Function Uneven(n, x, f)
Dim k As Integer, j As Integer
Dim h As Double, sum As Double, hf As Double
h = x(1) - x(0)
k = 1
sum = 0#
For j = 1 To n
  hf = x(j + 1) - x(j)
  If Abs(h - hf) < 0.000001 Then
    If k = 3 Then
      sum = sum + Simp13(h, f(j - 3), f(j - 2), f(j - 1))
      k = k - 1
    Else
      k = k + 1
     End If
  Else
    If k = 1 Then
      sum = sum + Trap(h, f(j - 1), f(j))
    Else
      If k = 2 Then
        sum = sum + Simp13(h, f(j - 2), f(j - 1), f(j))
      Else
        sum = sum + Simp38(h, f(j - 3), f(j - 2), f(j - 1), f(j))
      End If
      k = 1
    End If
  End If
  h = hf
Next j
Uneven = sum
End Function
Function Trap(h, f0, f1)
Trap = h * (f0 + f1) / 2
End Function
Function Simp13(h, f0, f1, f2)
Simp13 = 2 * h * (f0 + 4 * f1 + f2) / 6
End Function
Function Simp38(h, f0, f1, f2, f3)
Simp38 = 3 * h * (f0 + 3 * (f1 + f2) + f3) / 8
End Function
Function fx(x)
fx = 0.2 + 25 * x - 200 * x ^ 2 + 675 * x ^ 3 - 900 * x ^ 4 + 400 * x ^ 5
End Function
```

To run the program, the data is entered in columns A and B on a worksheet labeled Sheet1. After the macro is run, a message box displays the integral estimate as shown.



x	f(x)
0	0.20000
0.12	1.30973
0.22	1.30524
0.32	1.74339
0.36	2.07490
0.4	2.45600
0.44	2.84298
0.54	3.50730
0.64	3.18193
0.7	2.36300
0.8	0.23200

21.19 The required quantities can be computed at the sample points and tabulated as

y, m	*H*, m	*U*, m/s	*HU* (m^2/s)
0	0	0	0
1	1	0.1	0.1
3	1.5	0.12	0.18
5	3	0.2	0.6
7	3.5	0.25	0.875
8	3.2	0.3	0.96
9	2	0.15	0.3
10	0	0	0

The cross-sectional area can be computed with a combination of the trapezoidal rule and two applications of Simpson's 3/8 rule,

$$A_c = \int_0^y H(y)\,dy = (1-0)\frac{0+1}{2} + (7-1)\frac{1+3(1.5+3)+3.5}{8} + (10-7)\frac{3.5+3(3.2+2)+0}{8}$$
$$= 0.5 + 13.5 + 7.1625 = 21.1625 \text{ m}^2$$

The average depth is

$$H = \frac{A_c}{y} = \frac{21.1625 \text{ m}^2}{10 \text{ m}} = 2.11625 \text{ m}$$

The flow is

$$Q = \int_0^y H(y)U(y)\,dy = (1-0)\frac{0+0.1}{2} + (7-1)\frac{0.1+3(0.18+0.6)+0.875}{8} + (10-7)\frac{0.875+3(0.96+0.3)+0}{8}$$
$$= 0.05 + 2.48625 + 1.745625 = 4.281875 \ \frac{\text{m}^2}{\text{s}}$$

The average velocity is

$$U = \frac{Q}{A_c} = \frac{4.281875 \text{ m}^3/\text{s}}{21.1625 \text{ m}^2} = 0.202333 \ \frac{\text{m}}{\text{s}}$$

21.20 The following table lays out the solution:

t, hr	*c*, mg/L	*Q*	*Qc*		*Q*	*Qc*
0	1	15	15			
1	1.5	12.92893	19.3934	Trap	13.96447	17.1967
5.5	2.3	10.7612	24.75077			
10	2.1	20	42	Simp 1/3	113.9606	240.5947
12	4	25	100			
14	5	28.66025	143.3013	Simp 1/3	99.10684	390.2008
16	5.5	30	165			
18	5	28.66025	143.3013			
20	3	25	75	Simp 3/8	172.2308	857.4038
24	1.2	15	18		80	186
				Integrals→	479.2627	1691.396
					Ratio→	3.529163

Therefore, the average concentration leaving the reactor is 3.529 mg/L.

21.21 (a) Analytical solution:

$$M = \int_0^{11} 5 + 0.25x^2 \; dx = \left[5x + 0.083333x^3\right]_0^{11} = 165.9167$$

(b) Trapezoidal rule:

$$I = (1-0)\frac{5+5.25}{2} + (2-1)\frac{5.25+6}{2} + \bullet\bullet\bullet = 166.375$$

(c) Simpson's rule:

$$I = (2-0)\frac{5+4(5.25)+6}{6} + (4-2)\frac{6+4(7.25)+9}{6} + \bullet\bullet\bullet = 165.9167$$

21.22 Trapezoidal/Simpsons rules

$$I = (2-0.5)\frac{336+294.4}{2} + (4-2)\frac{294.4+4(266.4)+260.8}{6}$$
$$+ (10-4)\frac{260.8+3(260.5+249.6)+193.6}{8} + (11-10)\frac{193.6+165.6}{2} = 2681.192$$

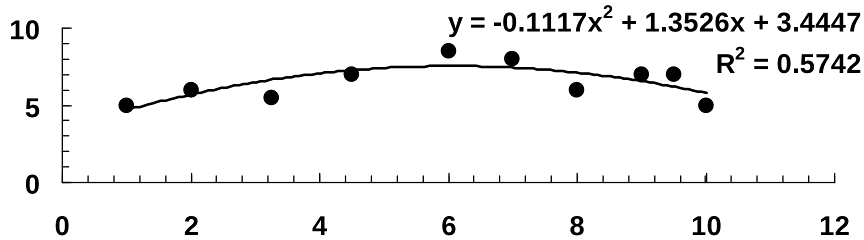
21.23 (a) Trapezoidal rule

$$I = (2-1)\frac{5+6}{2} + (3.25-2)\frac{6+5.5}{2} + \bullet\bullet\bullet = 60.375\frac{\text{m}\cdot\text{min}}{\text{s}} \times \frac{60\text{ s}}{\text{min}} = 3{,}622.5\text{ m}$$

(b) Trapezoidal/Simpsons rules

$$I = (2-1)\frac{5+6}{2} + (4.5-2)\frac{6+4(5.5)+7}{6} + (6-4.5)\frac{7+8.5}{2}$$
$$+ (9-6)\frac{8.5+3(8+6)+7}{8} + (10-9)\frac{7+4(7)+5}{6} = 59.9375\frac{\text{m}\cdot\text{min}}{\text{s}} \times \frac{60\text{ s}}{\text{min}} = 3{,}596.25\text{ m}$$

(c) We can use regression to fit a quadratic equation to the data

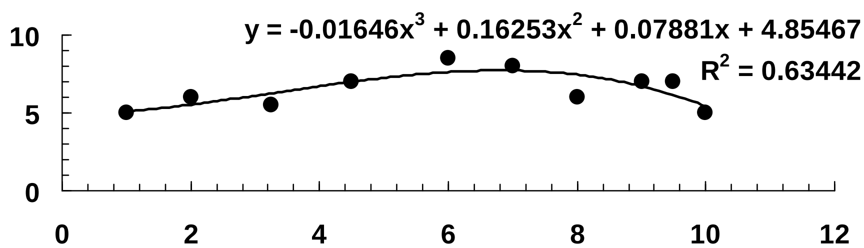


This equation can be integrated to yield

$$M = \int_1^{10} -0.1117x^2 + 1.3526x + 3.4447\, dx = \left[-0.03723x^3 + 0.6763x^2 + 3.4447x\right]_1^{10}$$

$$= 60.7599 \frac{\text{m} \cdot \text{min}}{\text{s}} \times \frac{60\ \text{s}}{\text{min}} = 3,645.594\ \text{m}$$

We can use regression to fit a cubic equation to the data



This equation can be integrated to yield

$$M = \int_1^{10} -0.01646x^3 + 0.16253x^2 + 0.07881\text{x} + 4.85467\, dx$$

$$= \left[-0.00412x^4 + 0.054177x^3 + 0.039405x^2 + 4.85467x\right]_1^{10}$$

$$= 60.56973 \frac{\text{m} \cdot \text{min}}{\text{s}} \times \frac{60\ \text{s}}{\text{min}} = 3,634.184\ \text{m}$$

21.24 We can set up a table that contains the values comprising the integrand

x, cm	ρ, g/cm^3	A_c, cm^2	$\rho \times A_c$, g/cm
0	4	100	400
200	3.95	103	406.85
300	3.89	106	412.34
400	3.8	110	418
600	3.6	120	432
800	3.41	133	453.53
1000	3.3	150	495

We can integrate this data using a combination of the trapezoidal and Simpson's rules,

$$I = (200-0)\frac{400+406.85}{2} + (400-200)\frac{406.85+4(412.34)+418}{6}$$
$$+(1000-400)\frac{418+3(432+453.53)+495}{8} = 430{,}877.9 \text{ g} = 430.8779 \text{ kg}$$

21.25 We can set up a table that contains the values comprising the integrand

***t*, hr**	***t*, d**	**rate (cars/4 min)**	**rate (cars/d)**
7:30	0.312500	18	6480
7:45	0.322917	24	8640
8:00	0.333333	26	9360
8:15	0.343750	20	7200
8:45	0.364583	18	6480
9:15	0.385417	9	3240

We can integrate this data using a combination of Simpson's 3/8 and 1/3 rules. This yields the number of cars that go through the intersection between 7:30 and 9:15 (1.75 hrs),

$$I = (0.34375-0.3125)\frac{6480+3(8640+9360)+7200}{8}$$
$$+(0.385417-0.34375)\frac{7200+4(6480)+3240}{6}$$
$$= 262.375+252.5 = 516.875 \text{ cars}$$

The number of cars going through the intersection per minute can be computed as

$$\frac{516.875 \text{ cars}}{1.75 \text{ hr}}\frac{\text{hr}}{60 \text{ min}} = 4.9226\frac{\text{cars}}{\text{min}}$$

21.26 We can use Simpson's 1/3 rule to integrate across the *y* dimension,

<u>$x = 0$:</u>

$$I = (4-0)\frac{-2+4(-4)-8}{6} = -17.3333$$

<u>$x = 4$:</u>

$$I = (4-0)\frac{-1+4(-3)-8}{6} = -14$$

<u>$x = 8$:</u>

$$I = (4-0)\frac{4+4(1)-6}{6} = 1.3333$$

<u>$x = 12$:</u>

$$I = (4-0)\frac{10+4(7)+4}{6} = 28$$

These values can then be integrated along the *x* dimension with Simpson's 3/8 rule:

$$I = (12-0)\frac{-17.3333+3(-14+1.3333)+28}{8} = -41$$