CHAPTER 24

24.1 Simpson's rules can be implemented as

$$I = (-50-(-150))\frac{0.11454+4(0.11904)+0.12486}{6}$$
$$+(100-(-50))\frac{0.12486+3(0.132+0.14046)+0.15024}{8}$$
$$=11.926+20.484=32.41$$

The amount of heat required can be computed as

$$\Delta H = 32.41(1200) = 38892$$

24.2 The first iteration involves computing 1 and 2 segment trapezoidal rules and combining them as

$$I = \frac{4(32.581875)-33.0975}{3} = 32.41$$

and computing the approximate error as

$$\varepsilon_a = \left|\frac{32.41-32.581875}{32.41}\right| \times 100\% = 0.5303\%$$

The computation can be continues as in the following tableau until $\varepsilon_a < 0.01\%$.

	1	**2**	**3**
n	**ε_a →**	**0.5303%**	**0.0000%**
1	33.09750000	32.41000000	32.41000000
2	32.58187500	32.41000000	
4	32.45296875		

24.3 Change of variable:

$$x = \frac{100-150}{2}+\frac{100-(-150)}{2}x_d = -25+125x_d$$
$$dx = \frac{100-(-150)}{2}dx_d = 125dx_d$$
$$I = \int_{-1}^{1} (0.132+1.56\times10^{-4}(-25+125x_d)+2.64\times10^{-7}(-25+125x_d)^2)\ 125\ dx_d$$

Therefore, the transformed function is

$$f(x_d) = 125\ (0.132+1.56\times10^{-4}(-25+125x_d)+2.64\times10^{-7}(-25+125x_d)^2)$$

Two-point formula:

$$I = f\left(\frac{-1}{\sqrt{3}}\right)+f\left(\frac{1}{\sqrt{3}}\right)=14.91679+17.49321=32.41$$

Three-point formula:

$$\begin{aligned} I &= 0.5555556\, f(-0.7745967) + 0.8888889\, f(0) + 0.5555556\, f(0.7745967) \\ &= 0.5555556(14.61418) + 0.8888889(16.03313) + 0.5555556(18.07082) \\ &= 8.11899 + 14.25167 + 10.03934 = 32.41 \end{aligned}$$

Thus, both give exact results because the errors are

2 point: $E_t \propto f^{(4)}(\xi)$
3 point: $E_t \propto f^{(6)}(\xi)$

which are zero for a second-order polynomial.

24.4 Simpson's rules can be implemented as

$$I = (30-0)\frac{10+3(35+55)+52}{8} + (40-30)\frac{52+4(40)+37}{6} + (50-40)\frac{37+4(32)+34}{6}$$
$$= 1245 + 415 + 331.6667 = 1991.6667$$

The amount of mass can be computed as

$$M = 4\frac{\text{m}^3}{\text{min}}\left(1991.6667\frac{\text{mg min}}{\text{m}^3}\right) = 7{,}966.667 \text{ mg}$$

24.5 The solution to this problem amounts to evaluating the integral,

$$M = \int_{t_1}^{t_2} Qc\; dt$$

MATLAB can be used to develop a solution:

```
>> t=[0 10 20 30 35 40 45 50];
>> Q=[4 4.8 5.2 5.0 4.6 4.3 4.3 5.0];
>> c=[10 35 55 52 40 37 32 34];
>> Qc=Q.*c;
>> M=trapz(t,Qc)
M =
  9.5185e+003
```

The problem can also be solved with a combination of Simpson's 3/8 and 1/3 rules:

```
>> M=(30-0)*(Qc(1)+3*(Qc(2)+Qc(3))+Qc(4))/8;
>> M=M+(50-30)*(Qc(4)+4*(Qc(5)+Qc(7))+2*Qc(6)+Qc(8))/12
M =
  9.6235e+003
```

Thus, the answers are 9.5185 g (trapezoidal rule) and 9.6235 g (Simpson's rules).

24.6 Equation (23.9) can be used to compute the derivative as

$x_0 = 0$ $\quad f(x_0) = 0.06$
$x_1 = 1$ $\quad f(x_1) = 0.32$

$x_2 = 3$ $f(x_2) = 0.6$

$$f'(0) = 0.06\frac{2(0)-1-3}{(0-1)(0-3)} + 0.32\frac{2(0)-0-3}{(1-0)(1-3)} + 0.6\frac{2(0)-0-1}{(3-0)(3-1)} = -0.08 + 0.48 - 0.1 = 0.3$$

The mass flux can be computed as

$$\text{Mass flux} = -1.52\times10^{-6}\ \frac{\text{cm}^2}{\text{s}} 0.3\times10^{-6}\ \frac{\text{g}}{\text{cm}^4} = -4.56\times10^{-13}\ \frac{\text{g}}{\text{cm}^2\text{s}}$$

where the negative sign connotes transport from the sediments into the lake. The amount of mass transported into the lake can be computed as

$$\text{Mass transport} = 4.56\times10^{-13}\ \frac{\text{g}}{\text{cm}^2\text{s}}(3.6\times10^6\ \text{m}^2)\frac{365\text{ d}}{\text{yr}}\frac{86{,}400\text{ s}}{\text{d}}\frac{10{,}000\text{ cm}^2}{\text{m}^2}\frac{\text{kg}}{1000\text{ g}} = 517.695\text{ kg}$$

24.7 For the equally-spaced points, we can use the second-order formulas from Figs. 23.1 through 23.3. For example, for the first point ($t = 0$), we can use

$$f'(0) = \frac{-0.77 + 4(0.7) - 3(0.4)}{2(10)} = 0.0415\frac{\text{barrels}}{\text{min}}$$

However, the points around $t = 30$ are unequally spaced so we must use Eq. (23.9) to compute the derivative as

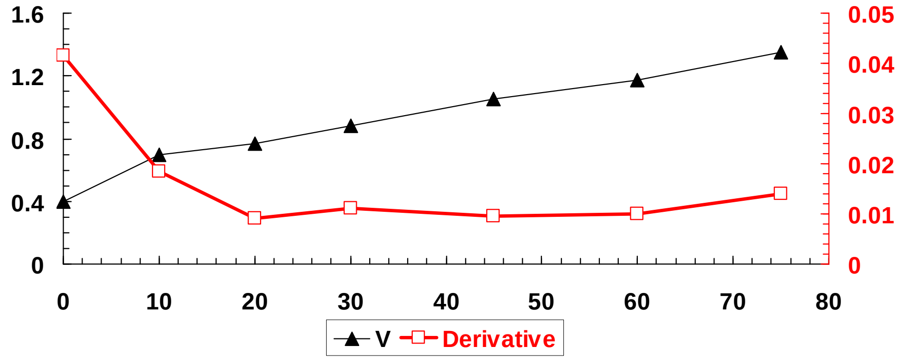
$x_0 = 20$ $f(x_0) = 0.77$
$x_1 = 30$ $f(x_1) = 0.88$
$x_2 = 45$ $f(x_2) = 1.05$

$$f'(30) = 0.77\frac{2(30)-30-45}{(20-30)(20-45)} + 0.88\frac{2(30)-20-45}{(30-20)(30-45)} + 1.05\frac{2(30)-20-30}{(45-20)(45-30)} = 0.011133$$

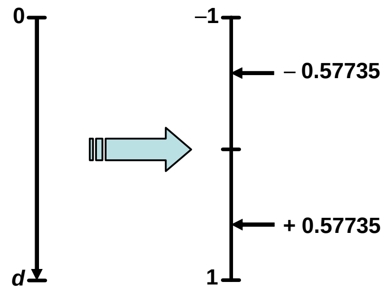
All the results can be summarized as

t	*V*	Derivative	Method
0	0.4	0.0415	Forward
10	0.7	0.0185	Centered
20	0.77	0.009	Centered
30	0.88	0.011133	Unequal
45	1.05	0.009667	Centered
60	1.17	0.01	Centered
75	1.35	0.014	Backward

The derivatives can be plotted versus time as



24.8 If we use 2-point Gauss quadrature the depth domain can be mapped onto the [–1, 1] domain as depicted here



This spacing corresponds to 21.1%*d* and 78.9%*d*. Thus, if we wanted to get a good estimate, we would average the two samples taken at about 20% and 80% of the depth. It should be noted that the U.S. Geological Survey recommends this procedure for making flow measurements in rivers.

24.9 Because the data is equispaced, we can use the second-order finite divided difference formulas from Figs. 23.1 through 23.3. For the first point, we can use

$$\frac{d\sigma}{d\varepsilon} = \frac{-176 + 4(96.6) - 3(87.8)}{255 - 153} = -0.5196$$

For the intermediate points, we can use centered differences. For example, for the second point

$$\frac{d\sigma}{d\varepsilon} = \frac{176 - 87.8}{255 - 153} = 0.8647$$

For the last point, we can use a backward difference

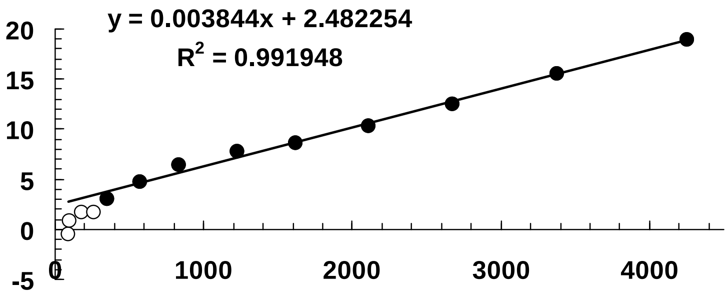
$$\frac{d\sigma}{d\varepsilon} = \frac{3(4258) - 4(3380) + 2678}{765 - 663} = 18.94118$$

All the values can be tabulated as

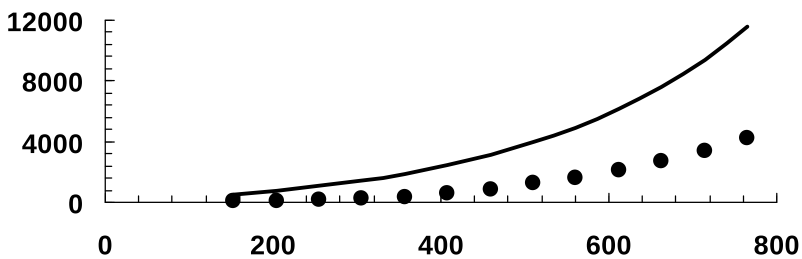
σ	ε	$d\sigma/d\varepsilon$
87.8	153	–0.5196
96.6	204	0.8647
176	255	1.6314

263	306	1.7157
351	357	3.0196
571	408	4.7353
834	459	6.4510
1229	510	7.7451
1624	561	8.6078
2107	612	10.3333
2678	663	12.4804
3380	714	15.4902
4258	765	18.9412

We can plot these results and after discarding the first few points, we can fit a straight line as shown (note that the discarded points are displayed as open circles).



Therefore, the parameter estimates are E_o = 2.48225 and a = 0.003844. The data along with the first equation can be plotted as



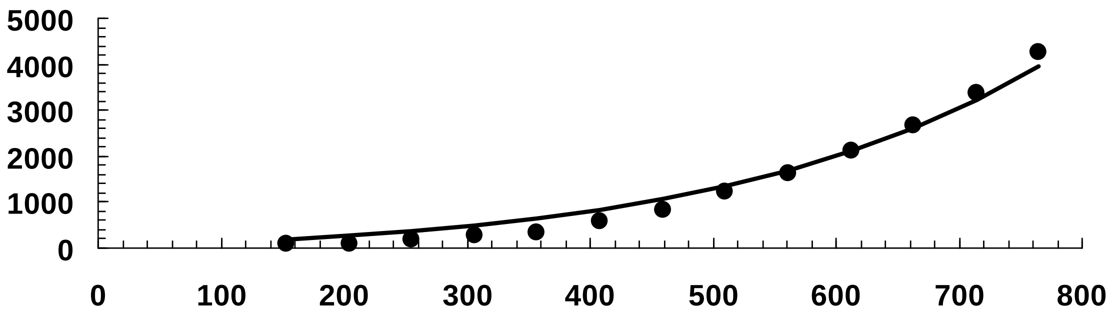
As described in the problem statement, the fit is not very good. We therefore pick a point near the midpoint of the data interval

$$(\bar{\varepsilon} = 612, \bar{\sigma} = 2107)$$

We can then plot the second model

$$\sigma = \left(\frac{2107}{e^{0.003844(612)} - 1} \right) (e^{0.003844\varepsilon} - 1) = 221.5719(e^{0.003844\varepsilon} - 1)$$

The result is a much better fit



24.10 An exponential model can be fit to the four points from t = 17 to 23. The result is

$$c = 2132.9e^{-0.3277t}$$

This equation can then be used to generate data from t = 25 through 35. This synthetic data along with the previous measured data can then be integrated with the trapezoidal rule,

$$I = (7-5)\frac{0+0.1}{2} + (9-7)\frac{0.1+0.11}{2} + \cdots + (33-31)\frac{0.0826+0.0429}{2} + (35-33)\frac{0.0429+0.0223}{2} = 61.2049$$

All the evaluations are summarized in the following table,

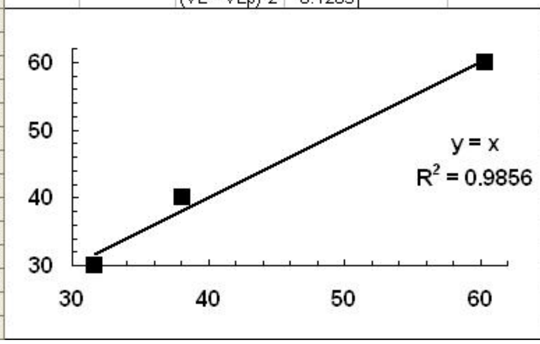
t	*c*, original	*c*, synthetic	Trap rule
5	0	0	
7	0.1	0.1	0.1
9	0.11	0.11	0.21
11	0.4	0.4	0.51
13	4.1	4.1	4.5
15	9.1	9.1	13.2
17	8	8	17.1
19	4.2	4.2	12.2
21	2.3	2.3	6.5
23	1.1	1.1	3.4
25	0.9	**0.5902**	1.6902
27	1.75	**0.3065**	0.8967
29	2.06	**0.1591**	0.4656
31	2.25	**0.0826**	0.2417
33	2.32	**0.0429**	0.1255
35	2.43	**0.0223**	0.0652

The cardiac output can then be computed as

$$C = \frac{5.6}{61.2049}60 = 5.48975\frac{\text{L}}{\text{min}}$$

24.11 The following Excel Solver application can be used to estimate: k = 0.09916 and A = 6.98301.

	A	B	C	D	E	F	G	H	I	J	K	L
1	**k**	**0.009916**										
2	**A**	**6.983009**										
3												
4	Patient	A			Patient	B			Patient	C		
5	Age	65				43				80		
6	VL	60				40				30		
7	age(yrs)	P (mmHG)	P - 13	Trap	Age (yrsr)	P (mmHG)	P - 13	Trap	Age (yrsr)	P (mmHG)	P - 13	Trap
8	25	13	0		25	11	-2		25	13	0	
9	40	15	2	15	40	30	17	112.5	40	14	1	7.5
10	50	22	9	55	41	32	19	18	50	15	2	15
11	60	23	10	95	42	33	20	19.5	60	17	4	30
12	65	24	11	52.5	43	35	22	21	80	19	6	100
13												
14			Integral	217.5			Integral	171			Integral	152.5
15												
16			VLp	60.3469			VLp	38.0552			VLp	31.6773
17												
18			(VL - VLp)^2	0.1203			(VL - VLp)^2	3.7823			(VL - VLp)^2	2.8133
19												
20											**SumSq**	**6.7159**



24.12 The following Excel spreadsheet is set up to use (left) a combination of the trapezoidal and Simpsons rules and (right) just the trapezoidal rule:

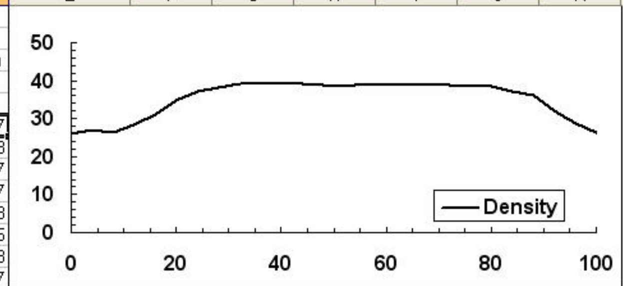
C22 =C20*24*Area

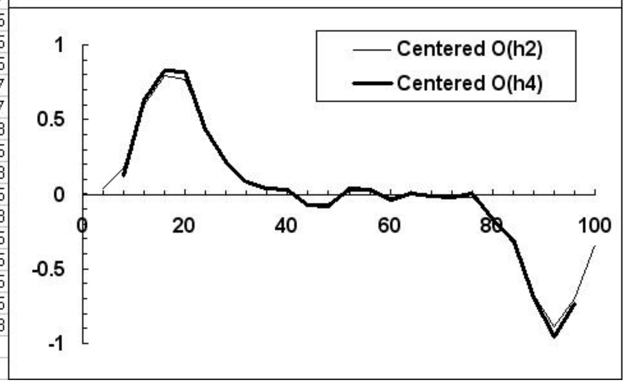
	A	B	C	D	E	F	G	H
1	**Prob 24.12**							
2								
3	Area	12	cm2					
4								
5	Simp1/3-3/8-Trap					Trap		
6	Time	Flux	Integral	Rule		Time	Flux	Trapezoidal
7	0	15				0	15	
8	1	14				1	14	14.5
9	2	12	27.66667	1/3 rule		2	12	13
10	3	11				3	11	11.5
11	4	9				4	9	10
12	5	8	30	3/8 rule		5	8	8.5
13	10	5				10	5	32.5
14	15	2.5				15	2.5	18.75
15	20	2	60.9375	3/8 rule		20	2	11.25
16	24	1	6	Trap rule		24	1	6
17								
18		Integral	124.6042					111.5
19								
20		Average	5.19184				Average	4.64583333
21								
22	Mass delivered		**1495.25**					**1338**

24.13

D6 =(-B8+8*B7-8*B5+B4)/12/(A6-A5)

	A	B	C	D
1	Prob. 24.13			
2				
3	Distance	Density	Centered O(h2)	Centered O(h4)
4	0	26.013		
5	4	26.955	0.04225	
6	8	26.351	0.1735	0.125354167
7	12	28.343	0.593625	0.630833333
8	16	31.1	0.7905	0.826916667
9	20	34.667	0.768875	0.818291667
10	24	37.251	0.45075	0.434270833
11	28	38.273	0.2315	0.217875
12	32	39.103	0.094	0.079895833
13	36	39.025	0.041125	0.036291667
14	40	39.432	0.01725	0.0268125
15	44	39.163	-0.064	-0.077125
16	48	38.92	-0.0665	-0.08225
17	52	38.631	0.0255	0.041291667
18	56	39.124	0.02275	0.030229167
19	60	38.813	-0.024875	-0.036770833
20	64	38.925	-0.001125	0.005125
21	68	38.804	-0.014875	-0.016770833
22	72	38.806	-0.01725	-0.0174375
23	76	38.666	-0.0185	0.006020833
24	80	38.658	-0.166875	-0.163625
25	84	37.331	-0.33475	-0.306125
26	88	35.98	-0.674375	-0.6946875
27	92	31.936	-0.892125	-0.959875
28	96	28.843	-0.703375	-0.732958333
29	100	26.309	-0.337125	
30	104	26.146		





The extremes for both cases are at 16 and 92

24.14

$$I = \int_0^{30} 200z\left[\frac{z}{5+z}\right]e^{-z/15}\ dx$$

	1	2	3	4
n	$\varepsilon_a \rightarrow$	11.9724%	0.1646%	0.01412%
1	10440.1504	20034.6250	19520.5626	19345.7378
2	17636.0064	19552.6915	19348.4694	
4	19073.5202	19361.2333		
8	19289.3051			

$$I = \int_0^{30} 200\left[\frac{z}{5+z}\right]e^{-z/15}\ dx$$

	1	2	3	4
n	$\varepsilon_a \rightarrow$	17.8666%	0.9589%	0.03823%
1	348.0050	1219.6400	1440.6846	1476.7973
2	1001.7312	1426.8693	1476.2330	
4	1320.5848	1473.1478		
8	1435.0070			

$$d = \frac{19345.74}{1476.797} = 13.0998$$

$$V = \frac{1476.797(13.0998)}{3} = 6448.58$$

$$T = \frac{6448.58}{0.995} = 6480.985$$
$$H = 1476.797 - 6480.985(0.0995) = 831.94$$

24.15 Change of variable:

$$x = \frac{30+0}{2} + \frac{30-0}{2} x_d = 15 + 15x_d$$
$$dx = \frac{30-0}{2} dx_d = 15 dx_d$$
$$I = \int_0^{30} 200z\left[\frac{z}{5+z}\right] e^{-z/15}\ dz$$
$$I = \int_{-1}^{1} 200(15+15x_d)\left[\frac{15+15x_d}{5+(15+15x_d)}\right] e^{-(15+15x_d)/15}\ 15\ dx_d$$

Therefore, the transformed function is

$$f(x_d) = 3000\frac{(15+15x_d)^2}{20+15x_d} e^{-(1+x_d)}$$

Five-point formula:

$$\begin{aligned} I &= 0.236927f(-0.90618) + 0.478629f(-0.53847) + 0.568889f(0) \\ &\qquad + 0.478629f(0.53847) + 0.236927f(0.90618) \\ &= 0.236927(844.2582) + 0.478629(7601.173) + 0.568889(12415.93) \\ &\qquad + 0.478629(12217.47) + 0.236927(10852.84) = 19,320.41 \end{aligned}$$

$$I = \int_0^{30} 200\left[\frac{z}{5+z}\right] e^{-z/15}\ dz$$
$$I = \int_{-1}^{1} 200\left[\frac{15+15x_d}{5+(15+15x_d)}\right] e^{-(15+15x_d)/15}\ 15\ dx_d$$

Therefore, the transformed function is

$$f(x_d) = 3000\frac{15+15x_d}{20+15x_d} e^{-(1+x_d)}$$

Five-point formula:

$$\begin{aligned} I &= 0.236927f(-0.90618) + 0.478629f(-0.53847) + 0.568889f(0) \\ &\qquad + 0.478629f(0.53847) + 0.236927f(0.90618) \\ &= 0.236927(599.9124) + 0.478629(1097.966) + 0.568889(827.7287) \\ &\qquad + 0.478629(529.4212) + 0.236927(379.5667) = 1,481.865 \end{aligned}$$

$$d = \frac{19320.41}{1481.865} = 13.038$$

$$V = \frac{1481.865(13.038)}{3} = 6440.185$$

$$T = \frac{6440.185}{0.995} = 6472.548$$

$$H = 1481.865 - 6472.548(0.0995) = 837.846$$

24.16 Trapezoidal rule:

$$I = (30-0)\frac{0 + 2(68.924 + 57.481 + 39.845 + 26.026 + 16.549) + 10.372}{2(6)} = 1070.057$$

Simpson's 1/3 rule:

$$I = (30-0)\frac{0 + 4(68.924 + 39.845 + 16.549) + 2(57.481 + 26.026) + 10.372}{3(6)} = 1131.098$$

Simpson's 3/8 rule:

$$I = (15-0)\frac{0 + 3(68.924 + 57.481) + 39.845}{8}$$
$$+ (30-15)\frac{39.845 + 3(26.026 + 16.549) + 10.372}{8} = 785.7375 + 333.6435 = 1119.381$$

24.17 Trapezoidal rule (h = 4):

$$I = (20-0)\frac{0 + 2(2 + 4 + 4 + 3.4) + 0}{10} = 53.6 \text{ m}^2$$

Trapezoidal rule (h = 2):

$$I = (20-0)\frac{0 + 2(1.8 + 2 + 4 + 4 + 6 + 4 + 3.6 + 3.4 + 2.8) + 0}{20} = 63.2 \text{ m}^2$$

Simpson's 1/3 rule:

$$I = (20-0)\frac{0 + 4(1.8 + 4 + 6 + 3.6 + 2.8) + 2(2 + 4 + 4 + 3.4) + 0}{30} = 66.4 \text{ m}^2$$

24.18 A table can be set up to hold the values that are to be integrated:

***y*, m**	***H*, m**	***U*, m/s**	***UH*, m^2/s**
0	0.5	0.03	0.015
2	1.3	0.06	0.078
4	1.25	0.05	0.0625
5	1.7	0.12	0.204
6	1	0.11	0.11
9	0.25	0.02	0.005

The cross-sectional area can be evaluated using a combination of Simpson's 1/3 rule and the trapezoidal rule:

$$A_c = (4-0)\frac{0.5+4(1.3)+1.25}{6} + (6-4)\frac{1.25+4(1.7)+1}{6} + (9-6)\frac{1+0.25}{2}$$
$$= 4.633333 + 3.016667 + 1.875 = 9.525 \text{ m}^2$$

The flow can be evaluated in a similar fashion:

$$Q = (4-0)\frac{0.015+4(0.078)+0.0625}{6} + (6-4)\frac{0.0625+4(0.204)+0.11}{6} + (9-6)\frac{0.11+0.005}{2}$$
$$= 0.259667 + 0.3295 + 0.1725 = 0.761667 \ \frac{\text{m}^3}{\text{s}}$$

24.19 (*a*) The deflection is computed by the following integration,

$$y(x) = \int_0^x \theta(x)\ dx$$

Here is the numerical evaluation of this integral with the trapezoidal rule.

$$y(x_n) = \sum_{i=1}^{n}\left[(x_i - x_{i-1})\frac{\theta(x_{i-1})+\theta(x_i)}{2}\right]$$

In addition, an exact, closed form solution can be developed by integrating Eq. P24.19 to give

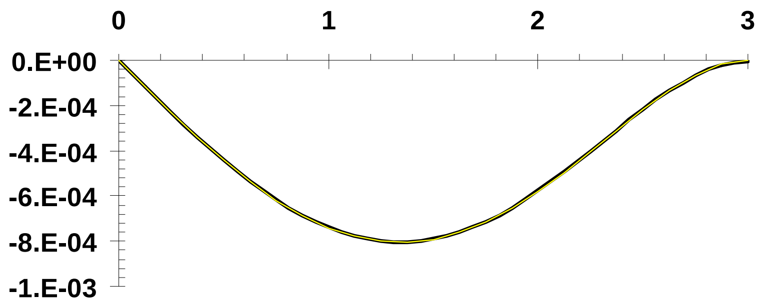
$$y(x) = \frac{w_0}{120EIL}\int_0^x -5x^4 + 6L^2x^2 - L^4\ dx = \frac{w_0}{120EIL}\left(-x^5 + 2L^2x^3 - L^4x\right)$$

The following table implements this equation. The first two columns hold the x and θ values (Eq. P24.19) and the third column holds the trapezoidal rule evaluation for each interval. The fourth column then holds the summation that represents the deflection,

x	***θ(x)***	**Trap rule**	***y(x)***	***y*true**
0	-9.375E-04	0.000E+00	0.000E+00	0.000E+00
0.125	-9.277E-04	-1.166E-04	-1.166E-04	-1.168E-04
0.25	-8.987E-04	-1.142E-04	-2.307E-04	-2.311E-04
0.375	-8.508E-04	-1.093E-04	-3.401E-04	-3.407E-04
0.5	-7.849E-04	-1.022E-04	-4.423E-04	-4.431E-04
0.625	-7.022E-04	-9.294E-05	-5.352E-04	-5.362E-04
0.75	-6.042E-04	-8.165E-05	-6.169E-04	-6.180E-04
0.875	-4.929E-04	-6.857E-05	-6.855E-04	-6.867E-04
1	-3.704E-04	-5.395E-05	-7.394E-04	-7.407E-04
1.125	-2.392E-04	-3.810E-05	-7.775E-04	-7.789E-04
1.25	-1.022E-04	-2.134E-05	-7.988E-04	-8.003E-04
1.375	3.729E-05	-4.059E-06	-8.029E-04	-8.044E-04
1.5	1.758E-04	1.332E-05	-7.896E-04	-7.910E-04
1.625	3.094E-04	3.032E-05	-7.593E-04	-7.606E-04
1.75	4.338E-04	4.645E-05	-7.128E-04	-7.141E-04
1.875	5.445E-04	6.114E-05	-6.517E-04	-6.527E-04
2	6.366E-04	7.382E-05	-5.779E-04	-5.787E-04
2.125	7.047E-04	8.383E-05	-4.940E-04	-4.946E-04
2.25	7.434E-04	9.051E-05	-4.035E-04	-4.037E-04
2.375	7.466E-04	9.313E-05	-3.104E-04	-3.102E-04
2.5	7.082E-04	9.093E-05	-2.195E-04	-2.188E-04

2.625	6.214E-04	8.310E-05	-1.364E-04	-1.352E-04
2.75	4.794E-04	6.880E-05	-6.756E-05	-6.577E-05
2.875	2.748E-04	4.713E-05	-2.043E-05	-1.795E-05
3	0.000E+00	1.717E-05	-3.254E-06	0.000E+00

The numerical and exact results can be plotted below where the results are in such good agreement that the graphs are indistinguishable.



(b) The moment can be computed as

$$M(x) = EI\frac{d}{dx}\theta(x)$$

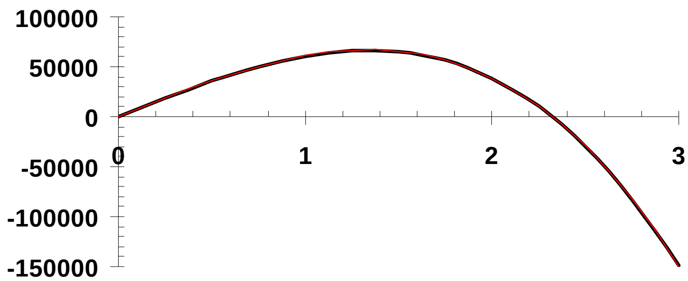
For comparison purposes, an exact, closed form solution can be developed by differentiating Eq. P24.19 to give

$$M(x) = EI\frac{d}{dx}\left[\frac{w_0}{120EIL}(-5x^4 + 6L^2x^2 - L^4)\right] = \frac{w_0}{120L}(-20x^3 + 12L^2x)$$

The following table uses divided differences of order $O(h^2)$ to evaluate the derivative. The fourth and fifth columns hold the numerical moment and the exact moment, respectively. Note that centered differences are used for the intermediate intervals and forward and backward differences are used for the first and last intervals, respectively. The numerical and exact results can be plotted below where the results are in such good agreement that the graphs are indistinguishable.

x	$\theta(x)$	$d\theta(x)/dx$	$M(x)$	M_{true}
0	-9.375E-04	6.782E-07	40.7	0.0
0.125	-9.277E-04	1.553E-04	9320.7	9347.9
0.25	-8.987E-04	3.080E-04	18478.7	18533.0
0.375	-8.508E-04	4.552E-04	27311.2	27392.6
0.5	-7.849E-04	5.943E-04	35655.4	35763.9
0.625	-7.022E-04	7.225E-04	43348.5	43484.2
0.75	-6.042E-04	8.371E-04	50227.9	50390.6
0.875	-4.929E-04	9.355E-04	56130.6	56320.5
1	-3.704E-04	1.015E-03	60894.1	61111.1
1.125	-2.392E-04	1.073E-03	64355.5	64599.6
1.25	-1.022E-04	1.106E-03	66352.0	66623.3
1.375	3.729E-05	1.112E-03	66720.9	67019.3
1.5	1.758E-04	1.088E-03	65299.5	65625.0
1.625	3.094E-04	1.032E-03	61924.9	62277.6
1.75	4.338E-04	9.406E-04	56434.5	56814.2
1.875	5.445E-04	8.111E-04	48665.4	49072.3
2	6.366E-04	6.409E-04	38454.9	38888.9
2.125	7.047E-04	4.273E-04	25640.2	26101.3
2.25	7.434E-04	1.676E-04	10058.6	10546.9

2.375	7.466E-04	-1.409E-04	-8452.7	-7937.3
2.5	7.082E-04	-5.009E-04	-30056.4	-29513.9
2.625	6.214E-04	-9.153E-04	-54915.4	-54345.7
2.75	4.794E-04	-1.387E-03	-83192.3	-82595.5
2.875	2.748E-04	-1.917E-03	-115049.9	-114426.0
3	0.000E+00	-2.479E-03	-148738.6	-150000.0



The shear can be computed as the derivative of the moment. For centered, finite divided differences, this can be represented as

$$V(x_i) = \frac{d}{dx} M(x_i) \cong \frac{M(x_{i+1}) - M(x_{i-1})}{x_{i+1} - x_{i-1}} \qquad (1)$$

or alternatively as the product of *EI* times the second derivative of the slope. For centered , finite divided differences, this can be represented as,

$$V(x_i) = EI \frac{d^2}{dx^2} \theta(x_i) \cong EI \frac{\theta(x_{i+1}) - 2\theta(x_i) + \theta(x_{i-1})}{(x_{i+1} - x_i)^2} \qquad (2)$$

Note that for the last interval, the backward $O(h^2)$ formula from Table 23.2 is used as in

$$V(x_n) = EI \frac{d^2}{dx^2} \theta(x_n) \cong EI \frac{2\theta(x_n) - 5\theta(x_{n-1}) + 4\theta(x_{n-2}) - \theta(x_{n-3})}{(x_n - x_{n-1})^2}$$

For comparison purposes, an exact, closed form solution can be developed by differentiating the moment solution to give

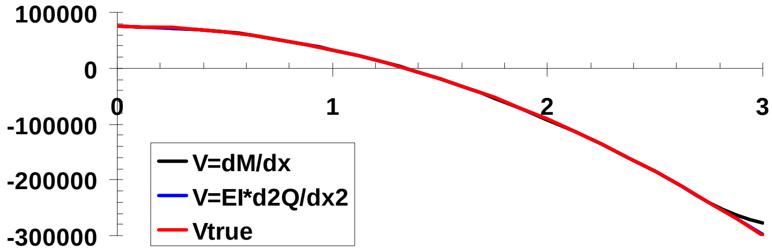
$$V(x) = \frac{w_0}{120L} \frac{d}{dx}(-20x^3 + 12L^2 x) = \frac{w_0}{120L}(-60x^2 + 12L^2)$$

The following table displays the results. The fourth and sixth columns hold the resulting moment for Eqs. (1) and (2), respectively. The exact moment is displayed in column seven. Note that centered differences are used for the intermediate intervals and forward and backward differences are used for the first and last intervals, respectively.

x	***θ(x)***	***M(x)***	***V(x) = dM/dx***	***$d^2\theta(x)/dx^2$***	***V(x)***	***Vtrue***
0	-9.375E-04	40.7	74728.7	1.270E-03	76193.6	75000.0
0.125	-9.277E-04	9320.7	73752.2	1.237E-03	74240.5	74349.0
0.25	-8.987E-04	18478.7	71961.8	1.205E-03	72287.3	72395.8
0.375	-8.508E-04	27311.2	68706.6	1.151E-03	69032.1	69140.6
0.5	-7.849E-04	35655.4	64149.3	1.075E-03	64474.8	64583.3
0.625	-7.022E-04	43348.5	58289.9	9.769E-04	58615.5	58724.0
0.75	-6.042E-04	50227.9	51128.5	8.576E-04	51454.0	51562.5

0.875	-4.929E-04	56130.6	42664.9	7.165E-04	42990.5	43099.0
1	-3.704E-04	60894.1	32899.3	5.537E-04	33224.8	33333.3
1.125	-2.392E-04	64355.5	21831.6	3.693E-04	22157.1	22265.6
1.25	-1.022E-04	66352.0	9461.8	1.631E-04	9787.3	9895.8
1.375	3.729E-05	66720.9	-4210.1	-6.474E-05	-3884.5	-3776.0
1.5	1.758E-04	65299.5	-19184.0	-3.143E-04	-18858.5	-18750.0
1.625	3.094E-04	61924.9	-35460.1	-5.856E-04	-35134.5	-35026.0
1.75	4.338E-04	56434.5	-53038.2	-8.785E-04	-52712.7	-52604.2
1.875	5.445E-04	48665.4	-71918.4	-1.193E-03	-71592.9	-71484.4
2	6.366E-04	38454.9	-92100.7	-1.530E-03	-91775.2	-91666.7
2.125	7.047E-04	25640.2	-113585.1	-1.888E-03	-113259.5	-113151.0
2.25	7.434E-04	10058.6	-136371.5	-2.267E-03	-136046.0	-135937.5
2.375	7.466E-04	-8452.7	-160460.1	-2.669E-03	-160134.5	-160026.0
2.5	7.082E-04	-30056.4	-185850.7	-3.092E-03	-185525.2	-185416.7
2.625	6.214E-04	-54915.4	-212543.4	-3.537E-03	-212217.9	-212109.4
2.75	4.794E-04	-83192.3	-240538.2	-4.004E-03	-240212.7	-240104.2
2.875	2.748E-04	-115049.9	-262185.3	-4.492E-03	-269509.5	-269401.0
3	0.000E+00	-148738.6	-276833.8	-4.980E-03	-298806.4	-300000.0

The numerical and exact results can be plotted below where the results are in such good agreement for Eq. (2) that the graphs are indistinguishable. In contrast, Eq. (1) does not do as well for the last point.



24.20 The slope is equal to the derivative of the deflection:

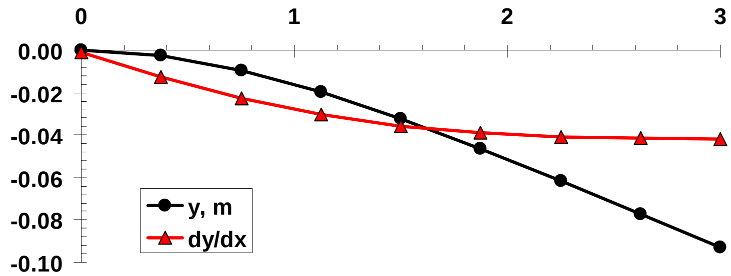
$$\frac{dy}{dx} = \theta(x) \tag{1}$$

After converting the deflections to meters, finite-divided differences of order $O(h^2)$ can be used to numerically estimated the slope. Note that forward, centered and backward differences are used for the first, the intermediate and the last points, respectively. For example, for the first point,

$$\theta(0) = \frac{d}{dx}y(0) \cong \frac{-0.009484 + 4(0.002571) - 3(0)}{0.75 - 0} = -0.00107$$

The solution is displayed in the following table and graph. The graph also shows the deflection.

***x*, m**	***y*, m**	**$\theta = dy/dx$**
0	0.000000	-0.00107
0.375	-0.002571	-0.01265
0.75	-0.009484	-0.02282
1.125	-0.019689	-0.03037
1.5	-0.032262	-0.03563
1.875	-0.046414	-0.03899
2.25	-0.061503	-0.04085
2.625	-0.077051	-0.04166
3	-0.092750	-0.04207



The moment can be computed in two ways,

$$M(x) = EI\frac{d\theta}{dx} \tag{2}$$

$$M(x) = EI\frac{d^2y}{dx^2} \tag{3}$$

Using the result for slope computed previously, Eq. (2) can be implemented by estimating the derivative of the slope with finite-divided differences of order $O(h^2)$ in a similar fashion to how we estimated the slope itself previously. For example, for the first point,

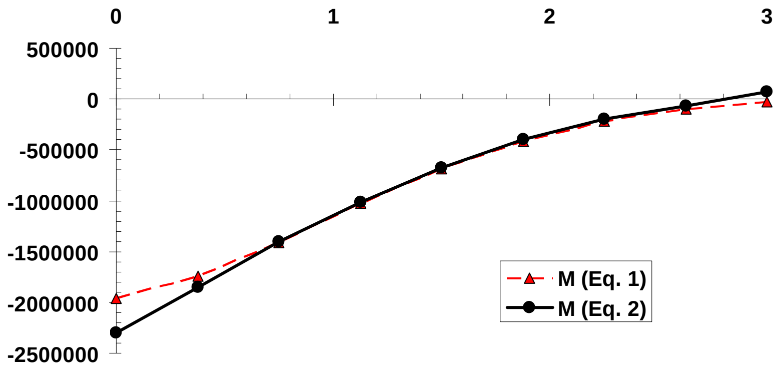
$$M(0) = EI\frac{d}{dx}\theta(0) \cong 200\times10^9(0.0003)\frac{-0.02282+4(-0.01265)-3(-0.00107)}{0.75-0}$$
$$= 60\times10^6(-0.0327431) = -1{,}964{,}587 \text{ N m}$$

For Eq. (3), the second derivative is estimated in a similar fashion. Again, for the first point,

$$M(0) = EI\frac{d^2}{dx^2}y(0) \cong 200\times10^9(0.0003)\frac{-0.019689+4(-0.009484)-5(-0.002571)+2(0)}{(0.375-0)^2}$$
$$= 60\times10^6(-0.03834) = -2{,}300{,}587 \text{ N m}$$

Thus, we can see that the two approaches yield different results. The reason for this discrepancy is that Eq. (2) is based on an estimated value of the slope (which is approximate) whereas Eq. (3) is based on the original data. Therefore, the use of Eq. (3) is preferable. Both solutions are displayed in the following table and graph.

x, m	*y*, m	θ	$d\theta/dx$	*M* (Eq. 2)	d^2y/dx^2	*M* (Eq. 3)
0	0.000000	-0.001067	-0.032743	-1964587	-0.038343	-2300587
0.375	-0.002571	-0.012645	-0.029010	-1740587	-0.030876	-1852587
0.75	-0.009484	-0.022824	-0.023634	-1418027	-0.023410	-1404587
1.125	-0.019689	-0.030371	-0.017079	-1024747	-0.016839	-1010347
1.5	-0.032262	-0.035633	-0.011490	-689387	-0.011228	-673707
1.875	-0.046414	-0.038988	-0.006955	-417280	-0.006663	-399787
2.25	-0.061503	-0.040849	-0.003566	-213973	-0.003264	-195840
2.625	-0.077051	-0.041663	-0.001621	-97280	-0.001074	-64427
3	-0.092750	-0.042065	-0.000526	-31573	0.001116	66987



Using the same reasoning as for the slope and moment, the shear can be computed in a number of ways

$$V(x) = \frac{dM}{dx} \tag{4}$$

$$V(x) = EI\frac{d^2\theta}{dx^2} \tag{5}$$

$$V(x) = EI\frac{d^3y}{dx^3} \tag{6}$$

Because it deals directly with the original data, we choose to use Eq. (6). However, this represents a problem as the third derivative estimators from Chap. 23 require 4 points for $O(h)$ accuracy and 5 points for $O(h^2)$ accuracy. Here is the best that we can do with these formulas,

At x = 0: $O(h^2)$

$$V(0) = 60\times10^6\,\frac{-3(-0.032262)+14(-0.019689)-24(-0.009484)+18(-0.002571)-5(0)}{2(0.375)^3}$$

$$= 60\times10^6(0.023495) = 1{,}409{,}707 \text{ N}$$

At x = 0.375 (not possible with formulas from Chap. 23)

At x = 0.75: $O(h)$

$$V(0.75) = 60\times10^6\,\frac{-0.032262-2(-0.019689)+2(-0.002571)-0}{2(0.375)^3}$$

$$= 60\times10^6(0.018716) = 1{,}122{,}987 \text{ N}$$

At x = 1.125: $O(h^2)$

$$V(1.125) = 60\times10^6\,\frac{-0.061503+8(0.046414)-13(0.032262)+13(-0.009484)-8(-0.002571)+0}{8(0.375)^3}$$

$$= 60\times10^6(0.016291556) = 977{,}493 \text{ N}$$

At x = 1.5: $O(h^2)$ can be used

V(1.5) = 818,204 N

At x = 1.875: $O(h^2)$ can be used

V(1.875) = 640,427 N

At x = 2.25: $O(h)$ can be used

V(2.25) = 447,147 N

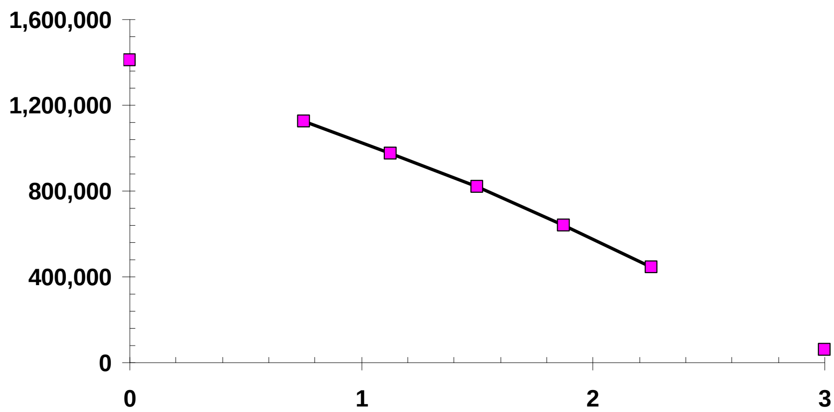
At x = 2.625 (not possible with formulas from Chap. 23)

At x = 3: $O(h^2)$

$$V(3)=60\times10^{6}\,\frac{5(-0.092750)-18(-0.077051)+24(-0.061503)-14(-0.046414)+3(-0.032262)}{2(0.375)^{3}}$$

$$=60\times10^{6}(0.001005037)=60,302\text{ N}$$

The results are plotted below:

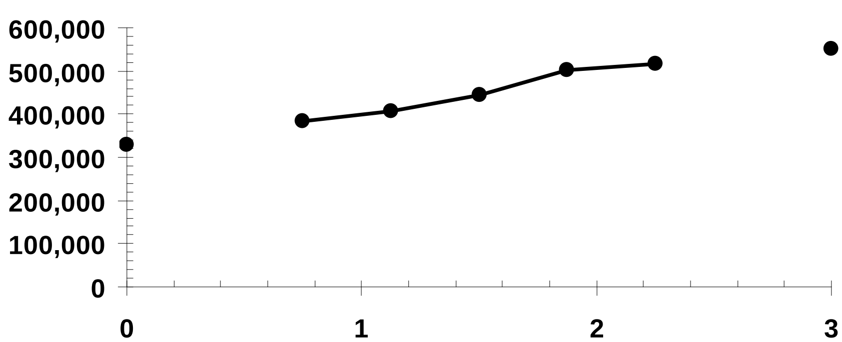


The distributed load, $w(x)$, can be computed as

$$w(x)=-EI\,\frac{d^{4}y}{dx^{4}}$$

As was the case with the shear, the 4th derivative cannot be estimated at all the points. The results are shown below:

***x*, m**	***y*, m**	***d⁴y/dx⁴***	***w*(x), N/m**
0	0.000000	-0.00546	327,680
0.375	-0.002571		
0.75	-0.009484	-0.00637	382,293
1.125	-0.019689	-0.00680	408,083
1.5	-0.032262	-0.00739	443,480
1.875	-0.046414	-0.00839	503,151
2.25	-0.061503	-0.00860	515,793
2.625	-0.077051		
3	-0.092750	-0.00920	552,201

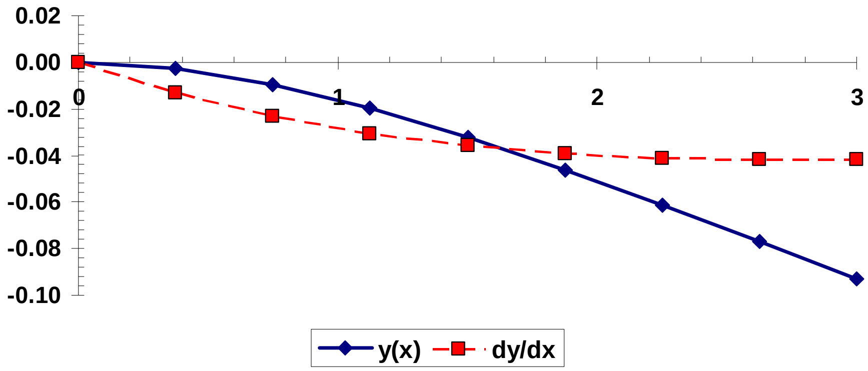


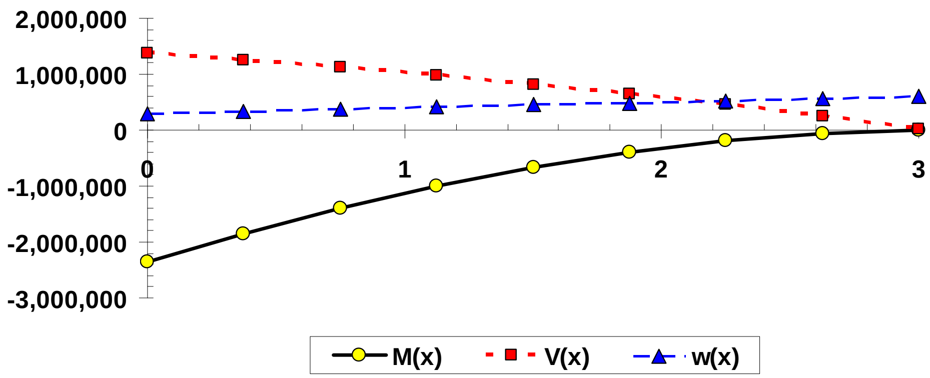
Although the formulas in Chap. 23 are inadequate for the shear and the moment, an alternative approach would be to fit the data with a fifth-order polynomial. If this is done, the result is

$$y(x) = 2.097902\times10^{-8} - 5.9457964\times10^{-5}x - 1.9532391\times10^{-2}x^2 - 3.8330795\times10^{-3}x^3$$
$$- 2.0866626\times10^{-4}x^4 - 1.3873758\times10^{-5}x^5$$

This polynomial can then be differentiated analytically to generate the slope, moment, shear and load. The results are tabulated and graphed below.

x	*y*(*x*)	*q*(*x*)	*M*(*x*)	*V*(*x*)	*w*(*x*)
0	0.000000	-0.000059	-2,343,887	1,379,909	300,479
0.375	-0.002571	-0.013137	-1,848,427	1,260,205	337,939
0.75	-0.009484	-0.023264	-1,400,489	1,126,455	375,398
1.125	-0.019689	-0.030753	-1,005,341	978,657	412,857
1.5	-0.032262	-0.035952	-668,252	816,812	450,316
1.875	-0.046414	-0.039238	-394,488	640,920	487,775
2.25	-0.061503	-0.041026	-189,318	450,981	525,234
2.625	-0.077051	-0.041759	-58,009	246,994	562,693
3	-0.092750	-0.041915	-5,828	28,961	600,153





24.21 In order to generate the average rate in units of car/min, the integral to be evaluated along with the units is

$$\text{average rate} = \frac{\int_0^{24\text{ hr}} \text{rate(car / min) } dt}{24\text{ hr}}$$

The integral can be evaluated with the trapezoidal and Simpson's rules as tabulated below:

Time (hr)	Rate (car/min)	Method	Integral
0	2		
2	2		
4	0	Simp 1/3	6.666667
5	2		
6	6	Simp 1/3	4.666667
7	7		
8	23		
9	11	Simp 3/8	40.125
10.5	4	Trap	11.25
11.5	11		
12.5	12	Simp 1/3	20
14	8	Trap	15
16	7	Trap	15
17	26		
18	20	Simp 1/3	43.66667
19	10		
20	8	Simp 1/3	22.66667
21	10		
22	8	Simp 1/3	18.66667
23	7		
24	3	Simp 1/3	13

The summation of the last column yields the integral estimate of 210.7083 cars·hr/min. Dividing by 24 hrs yields the average rate of 210.7083/24 = 8.779514 cars/min. This can then be multiplied by 1,440 min/d to yield 12,642.5 cars/d.

24.22 The values needed to perform the evaluation can be tabulated:

Height *l*, m	Force, *F*(*l*), N/m	*l*×*F*(*l*)
0	0	0
30	340	10200
60	1200	72000
90	1600	144000
120	2700	324000
150	3100	465000
180	3200	576000
210	3500	735000
240	3800	912000

Because there are an even number of equally-spaced segments, we can evaluate the integrals with the multi-segment Simpson's 1/3 rule.

$$F = (240-0)\frac{0+4(340+1600+3100+3500)+2(1200+2700+3200)+3800}{24} = 521,600$$

$$I = (240-0)\frac{0+4(10200+144000+465000+735000)+2(72000+324000+576000)+912000}{24}$$

$$= 82,728,000$$

The line of action can therefore be computed as

$$d = \frac{82,728,000}{521,600} = 158.6043$$

24.23 The values needed to perform the evaluation can be tabulated:

z	*w*(*z*)	$\rho gw(z)(60-z)$	$z\rho gw(z)(60-z)$
60	200	0	0
50	190	1.862E+07	9.310E+08
40	175	3.430E+07	1.372E+09
30	160	4.704E+07	1.411E+09
20	135	5.292E+07	1.058E+09
10	130	6.370E+07	6.370E+08
0	122	7.174E+07	0

Because there are an even number of equally-spaced segments, we can evaluate the required integrals with the multi-segment Simpson's 1/3 rule.

$$f_t = \int_0^D \rho gw(z)(D-z)\, dz$$

$$= (60-0)\frac{7.174 + 4(6.37 + 4.704 + 1.862) + 2(5.292 + 3.43) + 0}{18} \times 10^7 = 2.54539 \times 10^9$$

$$\int_0^D \rho gzw(z)(D-z)\, dz = (60-0)\frac{0 + 4(6.371 + 4.112 + 9.31) + 2(10.584 + 13.72) + 0}{18} \times 10^8$$

$$= 5.59253 \times 10^{10}$$

The line of action can therefore be computed as

$$d = \frac{5.59253 \times 10^{10}}{2.54539 \times 10^9} = 21.97 \text{ m}$$

24.24 One way to determine the volume is to integrate the data from mid-Jan through mid-Jan by duplicating the mid-Jan value at the end of the record. Since we are dealing with long-term averages, this is valid. Further, for simplicity, we can assume that every month is 30 days long. We can then integrate the data. As shown in the following table, we have used the a combination of the trapezoidal and Simpson's rules.

day	Flow	Trap/Simp	Method
15	30		
45	38		
75	82		
105	125	5793.75	Simp 3/8
165	95	6600	Trap
255	20	5175	Trap
285	22		
315	24	1320	Simp 1/3
345	35		
375	30	1940	Simp 1/3

The summation of the integral estimates is 20,828.75 $m^3 \cdot d/s$. We can divide this quantity by the integration interval (360 d) to yield an average flow of 57.8576 m^3/s. This quantity can then be multiplied by the number of seconds per year to arrive at the volume of water that flows down the river over a typical year.

$$\text{Annual Flow} = 57.8576 \times 60\frac{\text{s}}{\text{min}} \times 60\frac{\text{min}}{\text{hr}} \times 24\frac{\text{hr}}{\text{d}} \times 365\frac{\text{d}}{\text{yr}} = 1.8246 \times 10^9 \text{ m}^3$$

24.25 The integral to be evaluated is

$$h = e_{ab} A \int_0^t q \; dt$$

Simpson's rules can be used to evaluate the integral,

$$I = (8-0)\frac{0.1+4(5.32+8)+2(7.8)+8.03}{12} + (14-8)\frac{8.03+3(6.27+3.54)+0.2}{8}$$
$$= 51.34 + 28.245 = 79.585$$

We can then multiply this result by the area and the efficiency to give

$$h = 0.45(100,000 \text{ cm}^2)79.585 \text{ cal/cm}^2 = 5,371,988 \text{ cal}$$

24.26 Because the values are equally spaced, we can use the second-order forward difference from Fig. 23.1 to compute the derivative as

$x_0 = 0$	$f(x_0) = 20$
$x_1 = 0.08$	$f(x_1) = 17$
$x_2 = 0.16$	$f(x_2) = 15$

$$f\,'(0) = \frac{-15+4(17)-3(20)}{0.16} = -43.75\frac{°\text{C}}{\text{m}}$$

The coefficient of thermal conductivity can then be estimated as $k = -60$ W/m^2/(-43.75 $^{\text{o}}$C/m) = 1.37143 W/($^{\text{o}}$C·m).

24.27 First, we can estimate the areas by numerically differentiating the volume data. Because the values are equally spaced, we can use the second-order difference formulas from Fig. 23.1 to compute the derivatives at each depth. For example, at the first depth, we can use the forward difference to compute

$$A_s(0) = -\frac{dV}{dz}(0) = -\frac{-1,963,500+4(5,105,100)-3(9,817,500)}{8} = 1,374,450 \text{ m}^2$$

For the interior points, second-order centered differences can be used. For example, at the second point at (z = 4 m),

$$A_s(4) = -\frac{dV}{dz}(4) = -\frac{1,963,500-9,817,500}{8} = 981,750 \text{ m}^2$$

The other interior points can be determined in a similar fashion

$$A_s(8) = -\frac{dV}{dz}(8) = -\frac{392,700-5,105,100}{8} = 589,050 \text{ m}^2$$
$$A_s(12) = -\frac{dV}{dz}(12) = -\frac{0-1,963,500}{8} = 245,437.5 \text{ m}^2$$

For the last point, the second-order backward formula yields

$$A_s(16) = -\frac{dV}{dz}(16) = -\frac{3(0)-4(392,700)+1,963,500}{8} = -49,087.5 \text{ m}^2$$

Since this is clearly a physically unrealistic result, we will assume that the bottom area is 0. The results are summarized in the following table along with the other quantities needed to determine the average concentration.

z, m	V, m^3	c, g/m^3	A_s, m^2	$c \times A_s$
0	9817500	10.2	1374450.0	14019390
4	5105100	8.5	981750.0	8344875
8	1963500	7.4	589050.0	4358970
12	392700	5.2	245437.5	1276275
16	0	4.1	0	0

The necessary integrals can then be evaluated with the multi-segment Simpson's 1/3 rule,

$$\int_0^z A_s(z)\,dz = (16-0)\frac{1,374,450+4(981,750+245,437.5)+2(589,050)+0}{12} = 9,948,400 \text{ m}^3$$

$$\int_0^z c(z)A_s(z)\,dz = (16-0)\frac{14,019,390+4(8,344,875+1,276,275)+2(4,358,970)+0}{12} = 81,629,240 \text{ g}$$

The average concentration can then be computed as

$$\bar{c} = \frac{\int_0^Z c(z)A_s(z)\,dz}{\int_0^Z A_s(z)\,dz} = \frac{81,629,240}{9,948,400} = 8.205263\frac{\text{g}}{\text{m}^3}$$

24.28 The integral to be determined is

$$I = \int_0^{1/2} (5e^{-1.25t}\sin 2\pi t)^2\,dt$$

Change of variable:

$$x = \frac{0.5+0}{2} + \frac{0.5-0}{2}x_d = 0.25+0.25x_d$$

$$dx = \frac{0.5-0}{2}dx_d = 0.25dx_d$$

$$I = \int_{-1}^{1} (5e^{-1.25(0.25+0.25x_d)}\sin 2\pi(0.25+0.25x_d))^2\ 0.25\,dx_d$$

Therefore, the transformed function is

$$f(x_d) = 0.25\ (5e^{-1.25(0.25+0.25x_d)}\sin 2\pi(0.25+0.25x_d))^2$$

Five-point formula:

$$\begin{aligned} I &= 0.236927f(-0.90618)+0.478629f(-0.53847)+0.568889f(0) \\ &\qquad +0.478629f(0.53847)+0.236927f(0.90618) \\ &= 0.236927(0.127087)+0.478629(2.059589)+0.568889(3.345384) \\ &\qquad +0.478629(1.050662)+0.236927(0.040941) = 3.431617 \end{aligned}$$

Therefore, the RMS current can be computed as

$$I_{\text{RMS}} = \sqrt{3.431617} = 1.852463$$

24.29 The integral to be determined is

$$I = \int_0^{1/2} (5e^{-1.25t}\sin 2\pi t)^2 \ dt$$

Five applications of Simpson's 1/3 rule can be applied with h = 0.05 as tabulated below:

t	*i*²	Simpsons 1/3 rule
0	0	
0.05	2.106774	
0.1	6.726726	0.252563698
0.15	11.24592	
0.2	13.7153	1.090428284
0.25	13.38154	
0.3	10.68149	1.298715588
0.35	6.820993	
0.4	3.177481	0.685715716
0.45	0.775039	
0.5	0	0.104627262
	Sum →	3.432050547

Therefore, the RMS current can be computed as

$$I_{\text{RMS}} = \sqrt{3.432051} = 1.852579$$

24.30

$$I = \int_0^{1/2} (5e^{-1.25t}\sin 2\pi t)^2 \ dt$$

	1	2	3	4
n	ε_a →	25.0000%	1.9781%	0.0198933%
1	0.00000000	4.46051190	3.38814962	3.43184278
2	3.34538393	3.45517226	3.43116008	
4	3.42772518	3.43266084		
8	3.43142692			

Therefore, the RMS current can be computed as

$$I_{\text{RMS}} = \sqrt{3.431843} = 1.852523$$

24.31 For the equispaced data, we can use the second-order finite divided difference formulas from Figs. 23.1 through 23.3. For example, for the first point, we can use

$$\frac{di}{dt} = \frac{-0.32 + 4(0.16) - 3(0)}{0.2 - 0} = 1.6$$

For the intermediate equispaced points, we can use centered differences. For example, for the second point

$$\frac{di}{dt} = \frac{0.32 - 0}{0.2 - 0} = 1.6$$

For the last point, we can use a backward difference

$$\frac{di}{dt} = \frac{3(2) - 4(0.84) + 0.56}{0.7 - 0.3} = 8$$

One of the points (t = 0.3) has unequally spaced neighbors. For this point, we can use Eq. (23.9) to compute the derivative as

$x_0 = 0.2$ $\quad f(x_0) = 0.32$
$x_1 = 0.3$ $\quad f(x_1) = 0.56$
$x_2 = 0.5$ $\quad f(x_2) = 0.84$

$$f\,'(0) = 0.32\frac{2(0.3) - 0.3 - 0.5}{(0.2 - 0.3)(0.2 - 0.5)} + 0.56\frac{2(0.3) - 0.2 - 0.5}{(0.3 - 0.2)(0.3 - 0.5)} + 0.84\frac{2(0.3) - 0.2 - 0.3}{(0.5 - 0.2)(0.5 - 0.3)} = 2.066667$$

We can then multiply these derivative estimates by the inductance. All the results are summarized below:

t	*i*	*di/dt*	V_L
0	0	1.6	6.4
0.1	0.16	1.6	6.4
0.2	0.32	2	8
0.3	0.56	2.066667	8.266667
0.5	0.84	3.6	14.4
0.7	2	8	32

24.32 We can solve Faraday's law for inductance as

$$L = \frac{\int_0^t V_L\ dt}{i}$$

We can evaluate the integral using a combination of the trapezoidal and Simpson's rules,

$$\begin{aligned} I &= (20-0)\frac{0+4(18)+29}{6} + (80-20)\frac{29+3(44+49)+46}{8} + (120-80)\frac{46+35}{2} \\ &\quad + (180-120)\frac{35+26}{2} + (280-180)\frac{26+15}{2} + (400-280)\frac{15+7}{2} \\ &= 336.6667 + 2655 + 1620 + 1830 + 2050 + 1320 = 9811.667 \end{aligned}$$

The inductance can then be computed as

$$L = \frac{9811.667}{2} = 4905.833\frac{\text{volts ms}}{\text{A}}\frac{\text{s}}{1000\text{ ms}} = 4.905833\text{ H}$$

24.33 The average voltage can be computed as

$$\bar{V} = \frac{\int_0^{60} i(t)R(i)\ dt}{60}$$

We can use the formulas to generate values of $i(t)$ and $R(i)$ and their product for various equally-spaced times over the integration interval as summarized in the table below. The last column shows the integral of the product as calculated with Simpson's 1/3 rule.

t	***i*(*t*)**	***R*(*i*)**	***i*(*t*) × *R*(*i*)**	**Simpson's 1/3**
0	3600.000	36469.784	131291223	
6	2950.461	29916.029	88266063	1075071847
12	2288.787	23235.215	53180447	
18	1726.549	17553.332	30306694	381186392
24	1260.625	12839.641	16185971	
30	878.355	8966.984	7876196	102019847
36	569.294	5830.320	3319166	
42	327.533	3370.360	1103904	15944048
48	151.215	1568.912	237242	
54	41.250	436.372	18000	618486
60	0.000	0.000	0	
			Sum →	**1,574,840,619**

The average voltage can therefore be computed as

$$\bar{V} = \frac{1{,}574{,}840{,}619}{60} = 2.6247 \times 10^7$$

24.34 We can use the trapezoidal rule to integrate the data. For example at $t = 0.2$, the voltage is computed as

$$V(0.2) = \frac{1}{10^{-5}}(0.2 - 0)\frac{0.0002 + 0.0003683}{2} = 5.683$$

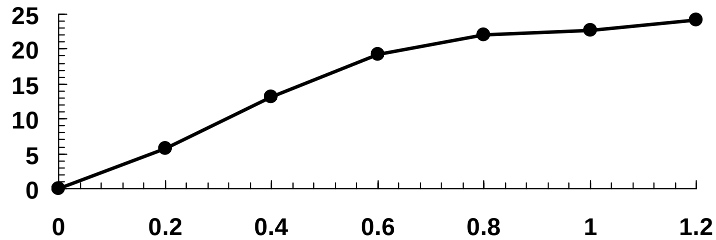
At $t = 0.4$, we add this value to the integral from $t = 0.2$ to 0.4,

$$V(0.4) = 5.683 + \frac{1}{10^{-5}}(0.4 - 0.2)\frac{0.0003683 + 0.0003819}{2} = 13.185$$

The remainder of the values can be computed in a similar fashion,

***t*, s**	***i*, A**	**Trap**	***V*(*t*)**
0	0.0002000	0	0
0.2	0.0003683	5.683	5.683
0.4	0.0003819	7.502	13.185
0.6	0.0002282	6.101	19.286
0.8	0.0000486	2.768	22.054
1	0.0000082	0.568	22.622
1.2	0.0001441	1.523	24.145

The plot of voltage versus time can be developed as



24.35 The work can be computed as

$$W = \int_0^{30} (1.6x - 0.045x^2)\cos(0.8 + 0.125x - 0.009x^2 + 0.0002x^3)\,dx$$

For the 4-segment trapezoidal rule, we can compute values of the integrand at equally-spaced values of x with h = 7.5. The results are summarized in the following table,

x	$F(x)$	$\theta(x)$	$F(x)\cos\theta(x)$	Trap Rule
0	0	0.8	0	
7.5	9.46875	1.315625	2.390018	8.962569
15	13.875	1.325	3.376187	21.623271
22.5	13.21875	1.334375	3.096162	24.271308
30	7.5	1.85	-2.066927	3.859631
			Sum →	**58.716779**

The finer-segment versions can be generated in a similar fashion. The results are summarized below:

Segments	W
4	58.71678
8	64.89955
16	66.41926

24.36 The work can be computed as

$$W = \int_0^{30} (1.6x - 0.045x^2)\cos(0.8 + 0.125x - 0.009x^2 + 0.0002x^3)\,dx$$

(a) For the 4-segment Simpson's 1/3 rule, we can compute values of the integrand at equally-spaced values of x with h = 7.5. The results are summarized in the following table,

x	$F(x)$	$\theta(x)$	$F(x)\cos\theta(x)$	Simpson's 1/3 Rule
0	0	0.8	0	
7.5	9.46875	1.315625	2.390018	32.34065
15	13.875	1.325	3.376187	
22.5	13.21875	1.334375	3.096162	34.23477
30	7.5	1.85	-2.066927	
			Sum →	66.57542

The finer-segment versions can be generated in a similar fashion. The results are summarized below:

Segments	W
4	66.57542
8	66.96047
16	66.92583

(b)

n	1	2	3	4
	$\varepsilon_a \rightarrow$	38.5532%	0.9312%	0.0051%
1	-31.003903	57.189106	67.201176	66.982729
2	35.140854	66.575421	66.986143	
4	58.716779	66.960473		
8	64.899549			

(c) Change of variable:

$$x = \frac{30+0}{2} + \frac{30-0}{2} x_d = 15 + 15x_d$$

$$dx = \frac{30-0}{2} dx_d = 15 dx_d$$

Therefore, the transformed function is

$$f(x_d) = 15(1.6(15+15x_d) - 0.045(15+15x_d)^2)$$
$$\times \cos(0.8 + 0.125(15+15x_d) - 0.009(15+15x_d)^2 + 0.0002(15+15x_d)^3)$$

Two-point formula:

$$I = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = 35.64278 + 38.20683 = 73.84961$$

Three-point formula:

$$I = 0.5555556 f(-0.7745967) + 0.8888889 f(0) + 0.5555556 f(0.7745967)$$
$$= 0.5555556(31.49657) + 0.8888889(50.64281) + 0.5555556(7.748426)$$
$$= 17.4981 + 45.01583 + 4.304681 = 66.8186$$

The results for the 2- through 6-point formulas are summarized below:

points	*W*
2	73.8496
3	66.8186
4	66.7734
5	66.9145
6	66.9225

24.37 The work can be computed as

$$W = \int_0^{30} (1.6x - 0.045x^2) \cos(-0.00055x^3 + 0.0123x^2 + 0.13x)\, dx$$

For the 4-segment trapezoidal rule, we can compute values of the integrand at equally-spaced values of x with h = 7.5. The results are summarized in the following table,

x	$F(x)$	$\theta(x)$	$F(x)\cos\theta(x)$	Trap Rule
0	0	0	0	

7.5	9.46875	1.434844	1.283339	4.812522
15	13.875	2.86125	-13.333330	-45.187464
22.5	13.21875	2.887031	-12.792760	-97.972837
30	7.5	0.12	7.446065	-20.050109
			Sum →	-158.39789

The finer-segment versions can be generated in a similar fashion. The results are summarized below:

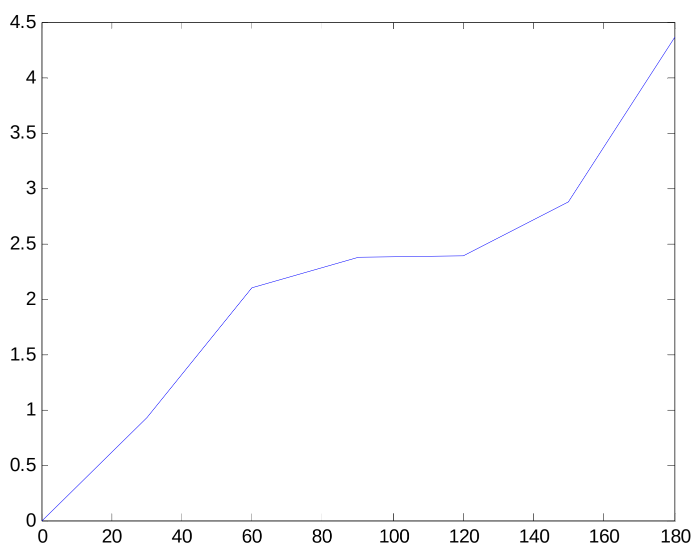
Segments	*W*
4	-158.398
8	-159.472
16	-157.713

The computation can be also implemented with a tool like MATLAB,

```
>> F=@(x) (1.6*x-0.045*x.^2).*cos(-0.00055*x.^3+0.0123*x.^2+0.13*x);
>> W=quad(F,0,30)
W =
 -157.0871
```

24.38 First printing errata: Note that the angle is in degrees.

```
>> plot(theta,W)
>> x=[0 1 2.7 3.8 3.7 3 1.4];
>> theta=[0 30 60 90 120 150 180];
>> F=cos(theta*pi/180);
>> W=cumtrapz(x,F)
W =
         0    0.9330    2.0941    2.3691    2.3941    2.8722    4.3651
>> plot(theta,W)
```



24.39 The work is computed as the product of the force times the distance, where the latter can be determined by integrating the velocity data (recall Eq. PT6.5),

$$W = F\int_0^t v(t)\, dt$$

A table can be set up holding the velocities at evenly spaced times over the integration interval. The Simpson's 1/3 rule can then be used to integrate this data as shown in the last column of the table

t	*v*	Simp 1/3 rule
0	0	
1	4	8
2	8	
3	12	24
4	16	
5	17	34.6667
6	20	
7	25	50.6667
8	32	
9	41	82.6667
10	52	
11	65	130.6667
12	80	
13	97	194.6667
14	116	
	Sum →	**525.3333**

Thus, the work can be computed as

$$W = 200\text{ N}(525.3333\text{ m}) = 105{,}066.7\text{ N}\cdot\text{m}$$

24.40 Because the data is equispaced, we can use the second-order finite divided difference formulas from Figs. 23.1 through 23.3. For the first point, we can use

$$\frac{dT}{dt} = \frac{-30 + 4(44.5) - 3(80)}{10 - 0} = -9.2$$

For the intermediate points, we can use centered differences. For example, for the second point

$$\frac{dT}{dt} = \frac{30 - 80}{10 - 0} = -5$$

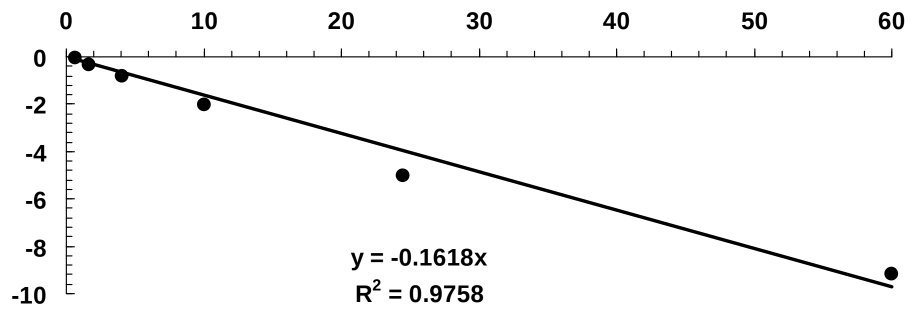
We can analyze the other interior points in a similar fashion. For the last point, we can use a backward difference

$$\frac{dT}{dt} = \frac{3(20.7) - 4(21.7) + 24.1}{25 - 15} = -0.06$$

All the values can be tabulated as

t	*T*	$T - T_a$	*dT/dt*
0	80	60	-9.2
5	44.5	24.5	-5
10	30	10	-2.04
15	24.1	4.1	-0.83
20	21.7	1.7	-0.34
25	20.7	0.7	-0.06

If Newton's law of cooling holds, we can plot dT/dt versus $T - T_a$ and the points can be fit with a linear regression with zero intercept to estimate the cooling rate. As in the following plot, the result is k = 0.1618/min.



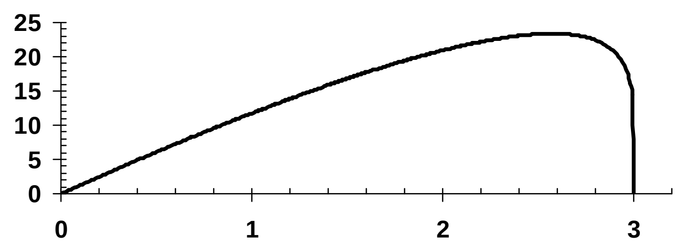
24.41 As in the plot, the initial point is assumed to be $e = 0$, $s = 40$. We can then use a combination of the trapezoidal and Simpsons rules to integrate the data as

$$I = (0.02-0)\frac{40+40}{2} + (0.05-0.02)\frac{40+37.5}{2} + (0.25-0.05)\frac{37.5+4(43+60)+2(52)+60}{12} =$$
$$0.8 + 1.1625 + 4.358333 + 5.783333 = 12.10417$$

24.42 The function to be integrated is

$$Q = \int_0^3 2\left(1-\frac{r}{r_0}\right)^{1/6} (2\pi r)\, dr$$

A plot of the integrand can be developed as



As can be seen, the shape of the function indicates that we must use fine segmentation to attain good accuracy. Here are the results of using a variety of segments.

n	*Q*
2	25.1896
4	36.1635
8	40.9621
16	43.0705
32	44.0009
64	44.4127
128	44.5955
256	44.6767
512	44.7128
1024	44.7289
2048	44.7361
4096	44.7392
8192	44.7407
16384	44.7413
32768	44.7416
65536	44.7417

131072	44.7418
262144	44.7418

Therefore, the result to 4 significant figures appears to be 44.7418.

24.43 The work is computed as

$$W = \int_0^x F \, dx$$

A table can be set up holding the forces at the various displacements. A combination of Simpson's 1/3 and 3/8 rules can be used to determine the integral as shown in the last two columns,

***x*, m**	***F*, N**	**Integral**	**Method**
0	0		
0.05	10		
0.1	28	1.133333	Simp 1/3
0.15	46		
0.2	63	4.583333	Simp 1/3
0.25	82		
0.3	110		
0.35	130	14.41875	Simp 3/8
	Sum→	**20.13542**	

24.44 Although the first three points are equally spaced, the remaining values are unequally spaced. Therefore, a good approach is to use Eq. 23.9 to perform the differentiation for all points. The results are summarized below:

t	***x***	***v***	***a***
0	153	70.19231	-47.09922
0.52	185	52.88462	-19.46883
1.04	208	49.94473	-10.82145
1.75	249	37.25169	-26.58748
2.37	261	16.05181	-24.50300
3.25	271	6.59273	-10.80561
3.83	273	0.30382	-10.88031

24.45 The distance traveled is equal to the integral of velocity

$$y = \int_{t_1}^{t_2} v(t) \, dt$$

A table can be set up holding the velocities at evenly spaced times (h = 1) over the integration interval. The Simpson's 1/3 rule can then be used to integrate this data as shown in the last column of the table

t	***v***	**Simp 1/3 rule**
0	0	
1	6	19.33333
2	34	
3	84	175.3333
4	156	
5	250	507.3333
6	366	
7	504	1015.333
8	664	

9	846	1699.333
10	1050	
11	1045	2090
12	1040	
13	1035	2070
14	1030	
15	1025	2050
16	1020	
17	1015	2030
18	1010	
19	1005	2010
20	1000	
21	1052	2105.333
22	1108	
23	1168	2337.333
24	1232	
25	1300	2601.333
26	1372	
27	1448	2897.333
28	1528	
29	1612	3225.333
30	1700	
	Sum →	**26833.33**

Since the underlying functions are second order or less, this result should be exact. We can verify this by evaluating the integrals analytically,

$$y = \int_0^{10} 11t^2 - 5t \;\; dt = \left[3.66667t^3 - 2.5t^2\right]_0^{10} = 3416.667$$

$$y = \int_{10}^{20} 1100 - 5t \;\; dt = \left[1100t - 2.5t^2\right]_{10}^{20} = 10,250$$

$$y = \int_{20}^{30} 50t + 2(t-20)^2 \;\; dt = \left[\frac{2}{3}t^3 - 15t^2 + 800t\right]_{20}^{30} = 13,166.67$$

The total distance traveled is therefore 3416.667 + 10,250 + 13,166.67 = 26,833.33.

24.46 6-segment trapezoidal rule:

$$y = (30-0)\frac{0 + 2(97.422 + 207.818 + 333.713 + 478.448 + 646.579) + 844.541}{2(6)} = 10,931.25$$

6-segment Simpson's 1/3 rule:

$$y = (30-0)\frac{0 + 4(97.422 + 333.713 + 646.579) + 2(207.818 + 478.448) + 844.541}{3(6)} = 10,879.88$$

6-point Gauss quadrature:

$$y = 10,879.61914$$

Romberg integration:

	1	2	3	4
n	ε_a →	4.0634%	0.0098%	0.0000291%
1	12668.10909	10896.96330	10879.82315	10879.62077

2	11339.74974	10880.89441	10879.62393
4	10995.60824	10879.70333	
8	10908.67956		

Numerical differentiation can be used to compute acceleration as a function of time by first generating a table of times and velocities. Then $O(h^2)$ finite-divided differences can be used to compute the accelerations. Using h = 2, the value for the initial time can be computed with a forward divided difference,

$$a(0) = \frac{dv(0)}{dt} \cong \frac{-3(76.9693) + 4(37.5477) - 0}{2(2)} = 18.3053$$

For the intermediate values, centered differences can be employed. For example, for the acceleration at t = 2,

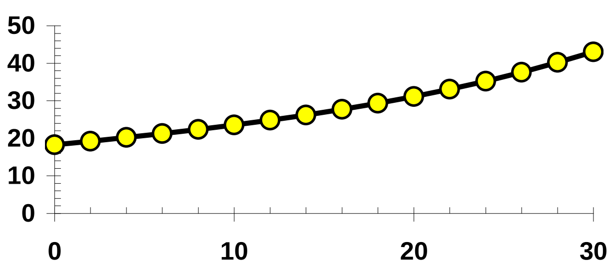
$$a(2) = \frac{dv(2)}{dt} \cong \frac{76.9693 - 0}{2(2)} = 19.2423$$

For the last value at t = 30 s, we can use a backward difference

$$a(0) = \frac{dv(0)}{dt} \cong \frac{3(844.5406) - 4(761.2555) + 683.5345}{2(2)} = 43.0336$$

All of the results are displayed in the following table and graph:

t	*v*	*a*
0	0.0000	18.3053
2	37.5477	19.2423
4	76.9693	20.2111
6	118.3921	21.2468
8	161.9565	22.3565
10	207.8183	23.5486
12	256.1509	24.8325
14	307.1481	26.2192
16	361.0277	27.7217
18	418.0350	29.3551
20	478.4482	31.1373
22	542.5842	33.0896
24	610.8065	35.2376
26	683.5345	37.6122
28	761.2555	40.2515
30	844.5406	43.0336



24.47 We can use the second-order difference formulas from Fig. 23.1 to compute the derivatives at each depth. For example, at the first point, we can use the forward difference to compute

$$\frac{d\varepsilon}{dt}(0.085) = -\frac{-0.16 + 4(0.13) - 3(0.1)}{8} = 0.05994$$

For the interior points, second-order centered differences can be used. For example, at the second point at (t = 0.586),

$$\frac{d\varepsilon}{dt}(0.586) = -\frac{0.16 - 0.1}{8} = 0.05994$$

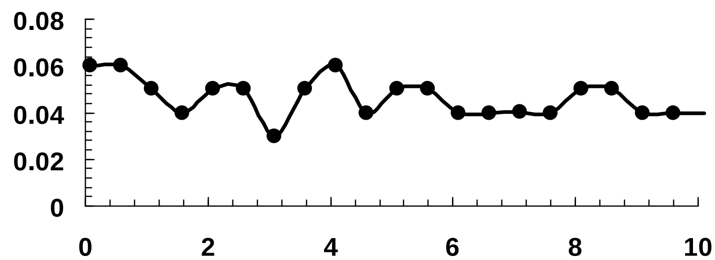
The other interior points can be determined in a similar fashion. For the last point, the second-order backward formula yields

$$\frac{d\varepsilon}{dt}(10.097) = -\frac{3(0.56) - 4(0.54) + 0.52}{8} = 0.03988$$

All the results are summarized in the following table.

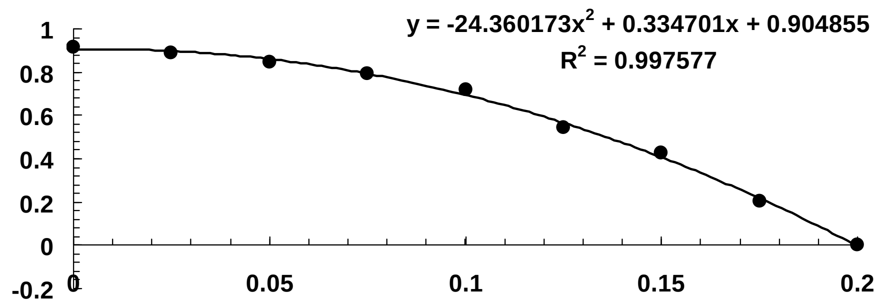
t	ε	***dε/dt***	***t***	ε	***dε/dt***
0.085	0.10	0.05994	5.591	0.37	0.04995
0.586	0.13	0.05994	6.091	0.39	0.03996
1.086	0.16	0.04995	6.592	0.41	0.03996
1.587	0.18	0.03996	7.092	0.43	0.04
2.087	0.20	0.04995	7.592	0.45	0.03996
2.588	0.23	0.04995	8.093	0.47	0.04995
3.088	0.25	0.02997	8.593	0.50	0.04995
3.589	0.26	0.04995	9.094	0.52	0.03996
4.089	0.30	0.05994	9.594	0.54	0.03988
4.590	0.32	0.03996	10.097	0.56	0.03988
5.090	0.34	0.04995			

The derivatives can be plotted:



After about nine points, the derivatives have settled down and the mean and standard deviation can be computed as 0.04328 and 0.004926, respectively.

24.48 (a) After converting the radii to units of meters, a polynomial can be fit to the velocity data



The flow can then be evaluated analytically as

$$Q = \int_0^{0.2} 2\pi(-24.360173r^3 + 0.334701r^2 + 0.904855r)\,dr$$

$$= 2\pi\left[-6.09004325r^4 + 0.111567r^3 + 0.4524275r^2\right]_0^{0.2} = 0.05809161\frac{\text{m}^3}{\text{s}}$$

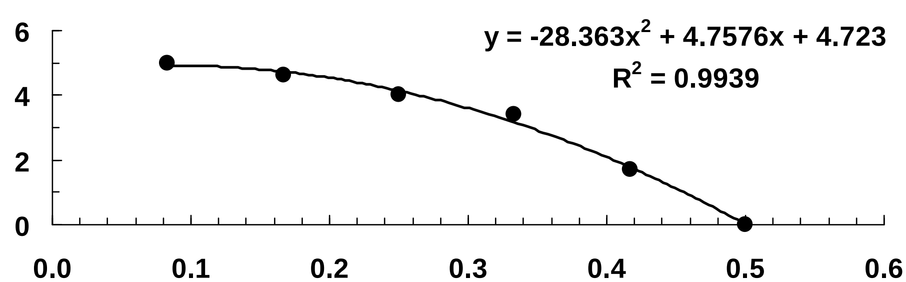
(b) The data and the integrand can be tabulated as shown below. Simpson's 1/3 rule can then be computed and summed in the last column.

r, m	*v*, m/s	2π*rv*	Simp 1/3
0	0.914	0	
0.025	0.89	0.139801	0.006877
0.05	0.847	0.266093	
0.075	0.795	0.374635	0.018470
0.1	0.719	0.451761	
0.125	0.543	0.426471	0.021334
0.15	0.427	0.402438	
0.175	0.204	0.22431	0.010831
0.2	0	0	
		Sum→	**0.057512**

(c)

$$\varepsilon_t = \left|\frac{0.05809161 - 0.057512}{0.05809161}\right| \times 100\% = 0.998\%$$

24.49 (a) After converting the radii to units of feet, a polynomial can be fit to the non-core velocity data



The noncore flow can then be evaluated analytically as

$$Q_{\text{noncore}} = \int_{0.08333}^{0.5} 2\pi(-28.3629r^3 + 4.75757r^2 + 4.723r)\, dr$$

$$= 2\pi\left[-7.09072r^4 + 1.585857r^3 + 2.3615r^2\right]_{0.08333}^{0.5} = 2.06379\frac{\text{ft}^3}{\text{s}}$$

This can be added to the core flow to determine the total flow

$$Q = Q_{\text{noncore}} + Q_{\text{core}} = 2.170447\frac{\text{ft}^3}{\text{s}} + 5\frac{\text{ft}}{\text{s}}\pi(0.083333\text{ ft})^2 = 2.06379 + 0.109083 = 2.172873\frac{\text{ft}^3}{\text{s}}$$

(b) The data and the integrand can be tabulated as shown below. Simpson's rules can then be computed and summed in the last column. Note that because we have an odd number of segments, we must use a combination of Simpson's 1/3 and 3/8 rules.

***r*, ft**	***v*, ft/s**	**2π*rv***	**Integral**	**Method**
0.083333	5	2.617994		
0.166667	4.62	4.838053		
0.250000	4.01	6.298893	0.785253	Simp 1/3
0.333333	3.42	7.162831		
0.416667	1.69	4.42441		
0.500000	0	0	1.283144	Simp 3/8
		Sum→	**2.068397**	

This numerical estimate can be added to the core flow to determine the total flow

$$Q = Q_{\text{noncore}} + Q_{\text{core}} = 2.068397\frac{\text{ft}^3}{\text{s}} + 5\frac{\text{ft}}{\text{s}}\pi(0.083333\text{ ft})^2 = 2.068397 + 0.109083 = 2.17748\frac{\text{ft}^3}{\text{s}}$$

(c)

$$\varepsilon_t = \left|\frac{2.172873 - 2.17748}{2.172873}\right| \times 100\% = 0.21\%$$

24.50 The following Excel worksheet solves the problem. Note that the derivative is calculated with a centered difference,

$$\frac{dV}{dT} = \frac{V_{450K} - V_{350K}}{100K}$$

F4 =C4-400*E4

	A	B	C	D	E	F	G	H	I
1	Prob. 24.50								
2									
3	P,atm	T=350K	T=400K	T=450K	dVdT	(V - T (dV/dT)p)	Integral		
4	0.1	220	250	282.5	0.625	0			
5	5	4.1	4.7	5.23	0.0113	0.18	0.441	Trap	4.9
6	10	2.2	2.5	2.7	0.005	0.5	1.7	Trap	5
7	20	1.35	1.49	1.55	0.002	0.69	5.95	Trap	10
8	25	1.1	1.2	1.24	0.0014	0.64			5
9	30	0.9	0.99	1.03	0.0013	0.47	6.2	Simp1/3	5
10	40	0.68	0.75	0.78	0.001	0.35	4.1	Trap	10
11	45	0.61	0.675	0.7	0.0009	0.315			5
12	50	0.54	0.6	0.62	0.0008	0.28	3.15	Simp1/3	5
13									
14						Total Integral =	**21.541**		

24.51 A single application of the trapezoidal rule yields:

$$I = (23-3)\frac{12.5+1.2}{2} = 137$$

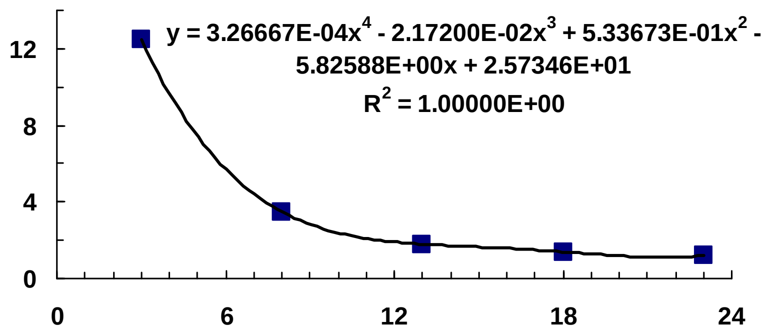
A 2-segment trapezoidal rule gives

$$I = (23-3)\frac{12.5+2(1.8)+1.2}{4} = 86.5$$

A 4-segment trapezoidal rule gives

$$I = (23-3)\frac{12.5+2(3.5+1.8+1.4)+1.2}{8} = 67.75$$

Because we do not know the true value, it it would seem impossible to estimate the error. However, we can try to fit different order polynomials to see if we can get a decent fit to the data. This yields the surprising result that a 4th-order polynomial results in almost a perfect fit. For example, using the Excel trend line gives:



This can be integrated analytically to give 60.955. Note that the same result would result from using Boole's rule, Romberg integration or Gauss quadrature.

Therefore, we can estimate the errors as

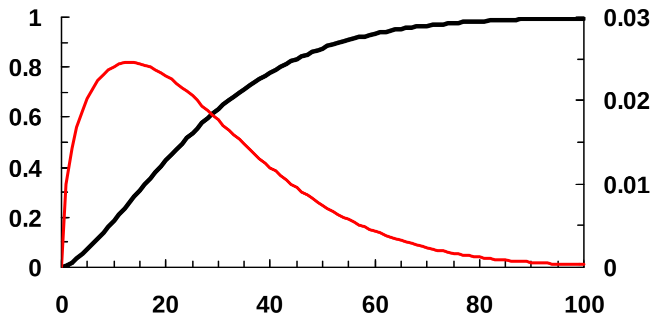
$$I = \left|\frac{60.955-137}{60.955}\right| \times 100\% = 124.76\%$$

$$I = \left|\frac{60.955-86.5}{60.955}\right| \times 100\% = 41.91\%$$

$$I = \left|\frac{60.955-67.75}{60.955}\right| \times 100\% = 11.15\%$$

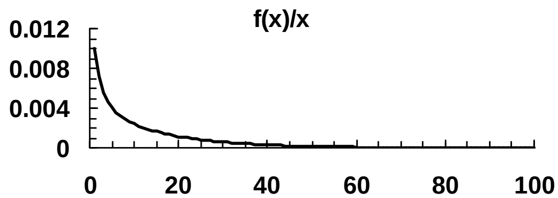
The ratio of these is 124.76:41.91:11.15 = 11.2:3.8:1. Thus, the result approximates the quartering of the error that we would expect according to Eq. 21.13.

24.52 (b) This problem can be solved in a number of ways. One approach is to set up Excel with a series of equally-spaced x values from 0 to 100. Then one of the formulas described in this Part of the book can be used to numerically compute the derivative. For example, x values with an interval of 1 and Eq. 23.9 can be used. The resulting plot of the function and its derivative is



(b) Inspection of this plot indicates that the maximum derivative occurs at about a diameter of 13.3.

(c) The function to be integrated looks like



This can be integrated from 1 to a high number using any of the methods provided in this book. For example, the trapezoidal rule can be used to integrate from 1 to 100, 1 to 200 and 1 to 300 using $h = 1$. The results are:

h	***I***
100	0.073599883
200	0.073632607
300	0.073632609

Thus, the integral seems to be converging to a value of 0.073633. S_m can be computed as 6×0.073633 = 0.4418.

24.53 (a) First, the distance can be converted to meters. Then, Eq. (23.9) can be used to compute the derivative at the surface as

$x_0 = 0$ $\quad f(x_0) = 900$
$x_1 = 0.01$ $\quad f(x_1) = 480$
$x_2 = 0.03$ $\quad f(x_2) = 270$

$$f'(0) = 900\frac{2(0)-0.01-0.03}{(0-0.01)(0-0.03)} + 480\frac{2(0)-0-0.03}{(0.01-0)(0.01-0.03)} + 270\frac{2(0)-0-0.01}{(0.03-0)(0.03-0.01)} = -52{,}500\frac{\text{K}}{\text{m}}$$

The heat flux can be computed as

$$\text{Heat flux} = -0.028\frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{K}}\left(-52{,}500\frac{\text{K}}{\text{m}}\right) = 1{,}470\frac{\text{W}}{\text{m}^2}$$

(b) The heat transfer can be computed by multiplying the flux by the area

$$\text{Heat transfer} = 1,470 \frac{\text{W}}{\text{m}^2}(200 \text{ cm} \times 50 \text{ cm}) \frac{\text{m}^2}{10,000 \text{ cm}^2} = 1,470 \text{ W}$$

24.54 (a) The pressure drop can be determined by integrating the pressure gradient

$$p = \int_{x_1}^{x_2} -\frac{8\mu Q}{\pi r(x)^4}\, dx$$

After converting the units to meters, a table can be set up holding the data and the integrand. The trapezoidal and Simpson's rules can then be used to integrate this data as shown in the last column of the table.

***x*, m**	***r*, m**	**integrand**	**integral**	**method**
0	0.002	-7957.75		
0.02	0.00135	-38333.2		
0.04	0.00134	-39490.3	-1338.5392	Simp 1/3
0.05	0.0016	-19428.1		
0.06	0.00158	-20430.6		
0.07	0.00142	-31315.3	-713.9319	Simp 3/8
0.1	0.002	-7957.75	-589.0959	Trap
		Sum→	**-2641.5670**	

Therefore, the pressure drop is computed as –2,641.567 N/m^2.

(b) The average radius can also be computed by integration as

$$\bar{r} = \frac{\int_{x_1}^{x_2} r(x)\, dx}{x_2 - x_1}$$

The numerical evaluations can again be determined by a combination of the trapezoidal and Simpson's rules.

***x*, m**	***r*, m**	**integral**	**method**
0	0.00200		
0.02	0.00135		
0.04	0.00134	5.83E-05	Simp 1/3
0.05	0.00160		
0.06	0.00158		
0.07	0.00142	4.61E-05	Simp 3/8
0.1	0.00200	5.13E-05	Trap
	Sum→	**0.0001557**	

Therefore, the average radius is 0.0001557/0.1 = 0.001557 m. This value can be used to compute a pressure drop of

$$\Delta p = \frac{dp}{dx}\Delta x = -\frac{8\mu Q}{\pi r^4}\Delta x = -\frac{8(0.005)0.00001}{\pi(0.001557)^4} 0.1 = -2166.95 \frac{\text{N}}{\text{m}^2}$$

Thus, there is less pressure drop if the radius is at the constant mean value.

(c) The average Reynolds number can be computed by first determining the average velocity as

$$v = \frac{Q}{A_c} = \frac{0.00001}{\pi(0.001557)^2} = \frac{0.00001}{7.6152 \times 10^{-6}} = 1.31317 \frac{\text{m}}{\text{s}}$$

Then the Reynolds number can be computed as

$$\text{Re} = \frac{1 \times 10^3 (1.31317) 0.0031138}{0.005} = 817.796$$

24.55 After converting the units to meters, a table can be set up holding the data and the integrand. The trapezoidal and Simpson's rules can then be used to integrate this data as shown in the last column of the table.

***r*, m**	***v*, m/s**	**integrand**	**integral**	**method**
0	10	0		
0.016	9.69	1.168974	0.036905	Simp 1/3
0.032	9.3	2.243851		
0.048	8.77	3.173964	0.100138	Simp 1/3
0.064	7.95	3.836262		
0.0747	6.79	3.824296	0.040984	Trap
0.0787	5.57	3.305149	0.014259	Trap
0.0795	4.89	2.931144	0.002495	Trap
0.08	0	0	0.000733	Trap
		Sum→	0.195514	

Therefore, the mass flow rate is 0.195514 kg/s.