CHAPTER 4

4.1 Use the stopping criterion: $\varepsilon_s = 0.5\times10^{2-2}\% = 0.5\%$

True value: $\cos(\pi/3) = 0.5$

<u>zero order:</u>

$$\cos\left(\frac{\pi}{3}\right)=1$$

$$\varepsilon_t=\left|\frac{0.5-1}{0.5}\right|\times100\%=100\%$$

<u>first order:</u>

$$\cos\left(\frac{\pi}{3}\right)=1-\frac{(\pi/3)^2}{2}=0.451689$$

$$\varepsilon_t=9.66\% \qquad \varepsilon_a=\left|\frac{0.451689-1}{0.451689}\right|\times100\%=121.4\%$$

<u>second order:</u>

$$\cos\left(\frac{\pi}{3}\right)=0.451689+\frac{(\pi/3)^4}{24}=0.501796$$

$$\varepsilon_t=0.359\% \qquad \varepsilon_a=\left|\frac{0.501796-0.451689}{0.501796}\right|\times100\%=9.986\%$$

<u>third order:</u>

$$\cos\left(\frac{\pi}{3}\right)=0.501796-\frac{(\pi/3)^6}{720}=0.499965$$

$$\varepsilon_t=0.00709\% \qquad \varepsilon_a=\left|\frac{0.499965-0.501796}{0.499965}\right|\times100\%=0.366\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.2 Use the stopping criterion: $\varepsilon_s = 0.5\times10^{2-2}\% = 0.5\%$

True value: $\sin(\pi/3) = 0.866025\ldots$

<u>zero order:</u>

$$\sin\left(\frac{\pi}{3}\right)=\frac{\pi}{3}=1.047198$$

$$\varepsilon_t=\left|\frac{0.866025-1.047198}{0.866025}\right|\times100\%=20.92\%$$

<u>first order:</u>

$$\sin\left(\frac{\pi}{3}\right)=1.047198-\frac{(\pi/3)^3}{6}=0.855801$$

$$\varepsilon_t=1.18\% \qquad \varepsilon_a=\left|\frac{0.855801-1.047198}{0.855801}\right|\times100\%=22.36\%$$

second order:

$$\sin\left(\frac{\pi}{3}\right) = 0.855801 + \frac{(\pi/3)^5}{120} = 0.866295$$

$$\varepsilon_t = 0.031\% \qquad \varepsilon_a = \left|\frac{0.866295 - 0.855801}{0.866295}\right| \times 100\% = 1.211\%$$

third order:

$$\sin\left(\frac{\pi}{3}\right) = 0.866295 - \frac{(\pi/3)^7}{5040} = 0.866021$$

$$\varepsilon_t = 0.000477\% \qquad \varepsilon_a = \left|\frac{0.866021 - 0.866295}{0.866021}\right| \times 100\% = 0.0316\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.3 (a) For this case $x_i = 0$ and $h = x$. Thus,

$$f(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2} + \cdots$$

$$f(0) = f'(0) = f''(0) = e^0 = 1$$

$$f(x) = 1 + x + \frac{x^2}{2} + \cdots$$

(b)

$$f(x_{i+1}) = e^{-x_i} - e^{-x_i}h + e^{-x_i}\frac{h^2}{2} - e^{-x_i}\frac{h^3}{6} + \cdots$$

for $x_i = 0.2$, $x_{i+1} = 1$ and $h = 0.8$. True value = $e^{-1} = 0.367879$.

zero order:

$$f(1) = e^{-0.2} = 0.818731 \qquad \varepsilon_t = \left|\frac{0.367879 - 0.818731}{0.367879}\right| \times 100\% = 122.55\%$$

first order:

$$f(1) = 0.818731 - 0.818731(0.8) = 0.163746 \qquad \varepsilon_t = \left|\frac{0.367879 - 0.163746}{0.367879}\right| \times 100\% = 55.49\%$$

second order:

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731\frac{0.8^2}{2} = 0.42574 \quad \varepsilon_t = \left|\frac{0.367879 - 0.42574}{0.367879}\right| \times 100\% = 15.73\%$$

third order:

$$f(1) = 0.818731 - 0.818731(0.8) + 0.818731\frac{0.8^2}{2} - 0.818731\frac{0.8^3}{6} = 0.355875$$

$$\varepsilon_t = \left|\frac{0.367879 - 0.355875}{0.367879}\right| \times 100\% = 3.26\%$$

4.4 True value: $f(2.5) = \ln(2.5) = 0.916291...$

zero order:

$$f(2.5) = f(1) = 0 \qquad \varepsilon_t = \left|\frac{0.916291 - 0}{0.916291}\right| \times 100\% = 100\%$$

first order:

$$f(2.5) = f(1) + f'(1)(2.5-1) = 0 + 1(1.5) = 1.5 \qquad \varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$$

second order:

$$f(2.5) = 1.5 + \frac{f''(1)}{2}(2.5-1)^2 = 1.5 + \frac{-1}{2}1.5^2 = 0.375 \qquad \varepsilon_t = \left|\frac{0.916291 - 0.375}{0.916291}\right| \times 100\% = 59.074\%$$

third order:

$$f(2.5) = 0.375 + \frac{f^{(3)}(1)}{6}(2.5-1)^3 = 0.375 + \frac{2}{6}1.5^3 = 1.5 \qquad \varepsilon_t = \left|\frac{0.916291 - 1.5}{0.916291}\right| \times 100\% = 63.704\%$$

fourth order:

$$f(2.5) = 1.5 + \frac{f^{(4)}(1)}{24}(2.5-1)^4 = 1.5 + \frac{-6}{24}1.5^4 = 0.234375 \quad \varepsilon_t = \left|\frac{0.916291 - 0.234375}{0.916291}\right| \times 100\% = 74.421\%$$

Thus, the process seems to be diverging suggesting that a smaller step would be required for convergence.

4.5 True value: $f(3) = 554$.

zero order:

$$f(3) = f(1) = -62 \qquad \varepsilon_t = \left|\frac{554 - (-62)}{554}\right| \times 100\% = 111.191\%$$

first order:

$$f(3) = -62 + f'(1)(3-1) = -62 + 70(2) = 78 \qquad \varepsilon_t = 85.921\%$$

second order:

$$f(3) = 78 + \frac{f''(1)}{2}(3-1)^2 = 78 + \frac{138}{2}4 = 354 \qquad \varepsilon_t = 36.101\%$$

third order:

$$f(3) = 354 + \frac{f^{(3)}(1)}{6}(3-1)^3 = 354 + \frac{150}{6}8 = 554 \qquad \varepsilon_t = 0\%$$

Thus, the third-order result is perfect because the original function is a third-order polynomial.

4.6 True value:

$$f'(x) = 75x^2 - 12x + 7$$

$$f'(2) = 75(2)^2 - 12(2) + 7 = 283$$

function values:
$x_{i-1} = 1.8$ $\quad f(x_{i-1}) = 50.96$
$x_i = 2$ $\quad f(x_i) = 102$
$x_{i+1} = 2.2$ $\quad f(x_{i+1}) = 164.56$

forward:

$$f'(2) = \frac{164.56 - 102}{0.2} = 312.8 \qquad \varepsilon_t = \left|\frac{283 - 312.8}{283}\right| \times 100\% = 10.53\%$$

backward:

$$f'(2) = \frac{102 - 50.96}{0.2} = 255.2 \qquad \varepsilon_t = \left|\frac{283 - 255.2}{283}\right| \times 100\% = 9.823\%$$

centered:

$$f'(2) = \frac{164.56 - 50.96}{2(0.2)} = 284 \qquad \varepsilon_t = \left|\frac{283 - 284}{283}\right| \times 100\% = 0.353\%$$

Both the forward and backward have errors that can be approximated by (recall Eq. 4.15),

$$|E_t| \approx \frac{f''(x_i)}{2} h$$

$$f''(2) = 150x - 12 = 150(2) - 12 = 288$$

$$|E_t| \approx \frac{288}{2} 0.2 = 28.8$$

This is very close to the actual error that occurred in the approximations

forward: $\quad |E_t| \approx |283 - 312.8| = 29.8$
backward: $\quad |E_t| \approx |283 - 255.2| = 27.8$

The centered approximation has an error that can be approximated by,

$$E_t \approx -\frac{f^{(3)}(x_i)}{6} h^2 = -\frac{150}{6} 0.2^2 = -1$$

which is exact: $E_t = 283 - 284 = -1$. This result occurs because the original function is a cubic equation which has zero fourth and higher derivatives.

4.7 True value:

$$f''(x) = 150x - 12$$

$$f''(2) = 150(2) - 12 = 288$$

$h = 0.25$:

$$f''(2) = \frac{f(2.25) - 2f(2) + f(1.75)}{0.25^2} = \frac{182.1406 - 2(102) + 39.85938}{0.25^2} = 288$$

$h = 0.125$:

$$f''(2)=\frac{f(2.125)-2f(2)+f(1.875)}{0.125^2}=\frac{139.6738-2(102)+68.82617}{0.125^2}=288$$

Both results are exact because the errors are a function of 4th and higher derivatives which are zero for a 3rd-order polynomial.

4.8 For $\Delta\tilde{T}$ =20,

$$\Delta H(\tilde{T})=\left|\frac{\partial H}{\partial T}\right|\Delta\tilde{T}$$

$$\frac{\partial H}{\partial T}=4Ae\sigma T^3=4(0.15)0.9(5.67\times10^{-8})650^3=8.408$$

$$\Delta H(\tilde{T})=8.408(20)=168.169$$

Exact error:

$$\Delta H_{\text{true}}=\frac{H(670)-H(630)}{2}=\frac{1542.468-1205.81}{2}=168.3286$$

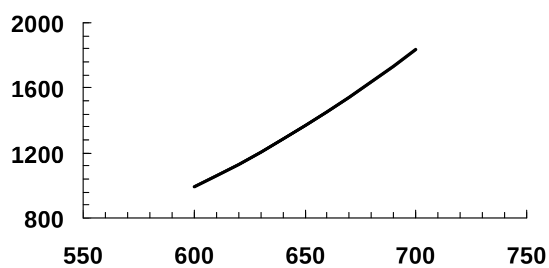
Thus, the first-order approximation is close to the exact result.

For $\Delta\tilde{T}$ =40, $\Delta H(\tilde{T})=8.408(40)=336.3387$

Exact error:

$$\Delta H_{\text{true}}=\frac{H(690)-H(610)}{2}=\frac{1735.055-1059.83}{2}=337.6124$$

Again, the first-order approximation is close to the exact result. The results are good because *H*(*T*) is nearly linear over the ranges we are examining. This is illustrated by the following plot.



4.9 For a sphere, $A = 4\pi r^2$. Therefore,

$$H=4\pi r^2 e\sigma T^4$$

At the mean values of the parameters,

$$H(0.15,0.9,550)=4\pi(0.15)^2 0.90(5.67\times10^{-8})(550)^4=1320.288$$

$$\Delta H=\left|\frac{\partial H}{\partial r}\right|\Delta\tilde{r}+\left|\frac{\partial H}{\partial e}\right|\Delta\tilde{e}+\left|\frac{\partial H}{\partial T}\right|\Delta\tilde{T}$$

$$\frac{\partial H}{\partial r} = 8\pi r e\sigma T^4 = 17{,}603.84$$
$$\frac{\partial H}{\partial e} = 4\pi r^2\sigma T^4 = 1466.987$$
$$\frac{\partial H}{\partial T} = 16\pi r^2 e\sigma T^3 = 9.6021$$
$$\Delta H = 17603.84(0.01) + 1466.987(0.05) + 9.6021(20) = 441.4297$$

To check this result, we can compute

$$H(0.14, 0.85, 530) = 4\pi(0.14)^2 0.85(5.67\times10^{-8})(530)^4 = 936.6372$$
$$H(0.16, 0.95, 570) = 4\pi(0.16)^2 0.95(5.67\times10^{-8})(570)^4 = 1829.178$$
$$\Delta H_{\text{true}} = \frac{1829.178 - 936.6372}{2} = 446.2703$$

4.10

$$\frac{\partial v}{\partial c} = \frac{cgte^{-(c/m)t} - gm\left(1 - e^{-(c/m)t}\right)}{c^2} = -1.38666$$
$$\Delta v(\tilde{c}) = \left|\frac{\partial v}{\partial c}\right|\Delta\tilde{c} = 1.38666(1.5) = 2.079989$$
$$v(12.5) = \frac{9.8(50)}{12.5}\left(1 - e^{-12.5(6)/50}\right) = 30.4533$$
$$v = 30.4533 \pm 2.079989$$

Thus, the bounds computed with the first-order analysis range from 28.3733 to 32.5333. This result can be verified by computing the exact values as

$$v(c - \Delta c) = \frac{9.8(50)}{11}\left(1 - e^{-(11/50)6}\right) = 32.6458$$
$$v(c + \Delta c) = \frac{9.8(50)}{14}\left(1 - e^{-(14/50)6}\right) = 28.4769$$

Thus, the range of ±2.0844 is close to the first-order estimate.

4.11

$$v(12.5) = \frac{9.8(50)}{12.5}\left(1 - e^{-12.5(6)/50}\right) = 30.4533$$
$$\Delta v(\tilde{c}, \tilde{m}) = \left|\frac{\partial v}{\partial c}\right|\Delta\tilde{c} + \left|\frac{\partial v}{\partial m}\right|\Delta\tilde{m}$$
$$\frac{\partial v}{\partial c} = \frac{cgte^{-(c/m)t} - gm\left(1 - e^{-(c/m)t}\right)}{c^2} = -1.38666$$
$$\frac{\partial v}{\partial m} = -\frac{gt}{m}e^{-(c/m)t} + \frac{g}{c}\left(1 - e^{-(c/m)t}\right) = 0.346665$$
$$\Delta v(\tilde{c}, \tilde{m}) = \left|-1.38666\right|(1.5) + \left|0.346665\right|(2) = 2.079989 + 0.69333 = 2.773319$$
$$v = 30.4533 \pm 2.773319$$

4.12 The condition number is computed as

$$CN = \frac{\tilde{x} f'(\tilde{x})}{f(\tilde{x})}$$

(a) $$CN = \frac{1.00001\left[\dfrac{1}{2\sqrt{1.00001-1}}\right]}{\sqrt{1.00001-1}+1} = \frac{1.00001(158.1139)}{1.003162} = 157.617$$

The result is ill-conditioned because the derivative is large near $x = 1$.

(b) $$CN = \frac{10(-e^{-10})}{e^{-10}} = \frac{10(-4.54\times10^{-5})}{4.54\times10^{-5}} = -10$$

The result is ill-conditioned because x is large.

(c) $$CN = \frac{300\left[\dfrac{300}{\sqrt{300^2+1}}-1\right]}{\sqrt{300^2+1}-300} = \frac{300(-5.555556\times10^{-6})}{0.0016667} = -0.99999444$$

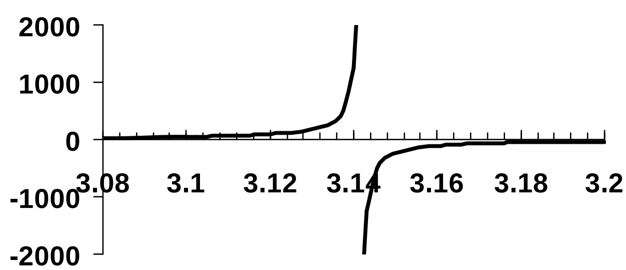
The result is well-conditioned.

(d) $$CN = \frac{x\dfrac{-xe^{-x}-e^{-x}+1}{x^2}}{\left(\dfrac{e^{-x}-1}{x}\right)} = \frac{0.001(0.499667)}{-0.9995} = -0.0005$$

The result is well-conditioned.

(e) $$CN = \frac{x\dfrac{(1+\cos x)\cos x+\sin x(\sin x)}{(1+\cos x)^2}}{\dfrac{\sin x}{1+\cos x}} = \frac{3.141907(20{,}264{,}237)}{-6366.2} = -10{,}001$$

The result is ill-conditioned because, as in the following plot, the function has a singularity at $x = \pi$.



4.13 Addition and subtraction:

$$f(u,v) = u + v$$
$$\Delta f = \left|\frac{\partial f}{\partial u}\right|\Delta\tilde{u} + \left|\frac{\partial f}{\partial v}\right|\Delta\tilde{v}$$
$$\left|\frac{\partial f}{\partial u}\right| = 1 \qquad \left|\frac{\partial f}{\partial v}\right| = 1$$
$$f(\tilde{u},\tilde{v}) = \Delta\tilde{u} + \Delta\tilde{v}$$

<u>Multiplication:</u>

$$f(u,v) = u \cdot v$$
$$\left|\frac{\partial f}{\partial u}\right| = v \qquad \left|\frac{\partial f}{\partial v}\right| = u$$
$$f(\tilde{u},\tilde{v}) = |\tilde{v}|\Delta\tilde{u} + |\tilde{u}|\Delta\tilde{v}$$

<u>Division:</u>

$$f(u,v) = u / v$$
$$\left|\frac{\partial f}{\partial u}\right| = \frac{1}{v} \qquad \left|\frac{\partial f}{\partial v}\right| = \frac{u}{v^2}$$
$$f(\tilde{u},\tilde{v}) = \left|\frac{1}{v}\right|\Delta\tilde{u} + \left|\frac{u}{v^2}\right|\Delta\tilde{v}$$
$$f(\tilde{u},\tilde{v}) = \frac{|v|\Delta\tilde{u} + |u|\Delta\tilde{v}}{|v^2|}$$

4.14

$$f(x) = ax^2 + bx + c$$
$$f'(x) = 2ax + b$$
$$f''(x) = 2a$$

Substitute these relationships into Eq. (4.4),

$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + bx_i + c + (2ax_i + b)(x_{i+1} - x_i) + \frac{2a}{2!}(x_{i+1}^2 - 2x_{i+1}x_i + x_i^2)$$

Collect terms

$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + 2ax_i(x_{i+1} - x_i) + a(x_{i+1}^2 - 2x_{i+1}x_i + x_i^2) + bx_i + b(x_{i+1} - x_i) + c$$
$$ax_{i+1}^2 + bx_{i+1} + c = ax_i^2 + 2ax_ix_{i+1} - 2ax_i^2 + ax_{i+1}^2 - 2ax_{i+1}x_i + ax_i^2 + bx_i + bx_{i+1} - bx_i + c$$
$$ax_{i+1}^2 + bx_{i+1} + c = (ax_i^2 - 2ax_i^2 + ax_i^2) + ax_{i+1}^2 + (2ax_ix_{i+1} - 2ax_{i+1}x_i) + (bx_i - bx_i) + bx_{i+1} + c$$
$$ax_{i+1}^2 + bx_{i+1} + c = ax_{i+1}^2 + bx_{i+1} + c$$

4.15 The first-order error analysis can be written as

$$\Delta Q = \left|\frac{\partial Q}{\partial n}\right|\Delta n + \left|\frac{\partial Q}{\partial S}\right|\Delta S$$

$$\frac{\partial Q}{\partial n} = -\frac{1}{n^2}\frac{(BH)^{5/3}}{(B+2H)^{2/3}}S^{0.5} = -50.74 \qquad \frac{\partial Q}{\partial S} = \frac{1}{n}\frac{(BH)^{5/3}}{(B+2H)^{2/3}}\frac{1}{2S^{0.5}} = 2536.9$$

$$\Delta Q = \left|-50.74\right|0.003 + \left|2536.9\right|0.00003 = 0.152 + 0.076 = 0.228$$

The error from the roughness is about 2 times the error caused by the uncertainty in the slope. Thus, improving the precision of the roughness measurement would be the best strategy.

4.16 Use the stopping criterion: $\varepsilon_s = 0.5\times10^{2-2}\% = 0.5\%$

True value: 1/(1 – 0.1) = 1.111111….

<u>zero order:</u>

$$\frac{1}{1-x} = 1 \qquad \varepsilon_t = \left|\frac{1.11111-1}{1.11111}\right|\times100\% = 10\%$$

<u>first order:</u>

$$\frac{1}{1-x} = 1+0.1 = 1.1 \qquad \varepsilon_t = 1\% \qquad \varepsilon_a = \left|\frac{1.1-1}{1.1}\right|\times100\% = 9.0909\%$$

<u>second order:</u>

$$\frac{1}{1-x} = 1+0.1+0.01 = 1.11 \qquad \varepsilon_t = 0.1\% \qquad \varepsilon_a = \left|\frac{1.11-1.1}{1.11}\right|\times100\% = 0.9009009\%$$

<u>third order:</u>

$$\frac{1}{1-x} = 1+0.1+0.01+0.001 = 1.111 \qquad \varepsilon_t = 0.01\% \qquad \varepsilon_a = \left|\frac{1.111-1.11}{1.111}\right|\times100\% = 0.090009\%$$

Since the approximate error is below 0.5%, the computation can be terminated.

4.17

$$\Delta(\sin\phi_0) = \left|\frac{d\sin\phi_0}{d\alpha}\right|\Delta\alpha$$

$$\frac{d\sin\phi_0}{d\alpha} = \frac{-\beta}{2\sqrt{(1+\alpha)(1+\alpha-\alpha\beta)}} + \sqrt{1-\frac{\alpha\beta}{1+\alpha}}$$

where $\beta = (v_e/v_0)^2 = 4$ and $\alpha = 0.25$ to give,

$$\frac{d\sin\phi_0}{d\alpha} = \frac{-4}{2\sqrt{(1+0.25)(1+0.25-0.25(4))}} + \sqrt{1-\frac{0.25(4)}{1+0.25}} = -3.1305$$

$$\Delta(\sin\phi_0) = 3.1305\Delta\alpha$$

For $\Delta\alpha = 0.25(0.02) = 0.005$,

$$\Delta(\sin\phi_0) = 3.1305(0.005) = 0.015652$$

$$\sin\phi_0 = (1+0.25)\sqrt{1-\frac{0.25}{1+0.25}4} = 0.559017$$

Therefore,

$$\max \sin\phi_0 = 0.559017 + 0.015652 = 0.574669$$
$$\min \sin\phi_0 = 0.559017 - 0.015652 = 0.543365$$
$$\max \phi_0 = \arcsin(0.574669)\times\frac{180}{\pi} = 35.076°$$
$$\min \phi_0 = \arcsin(0.543365)\times\frac{180}{\pi} = 32.913°$$

4.18 The derivatives can be computed as

$$f(x) = x - 1 - 0.5\sin x$$
$$f'(x) = 1 - 0.5\cos x$$
$$f''(x) = 0.5\sin x$$
$$f^{(3)}(x) = 0.5\cos x$$
$$f^{(4)}(x) = -0.5\sin x$$

The first through fourth-order Taylor series expansions can be computed based on Eq. 4.5 as

<u>First-order:</u>
$$f_1(x) = f(a) + f'(a)(x-a)$$
$$f_1(x) = \frac{\pi}{2} - 1 - 0.5\sin\frac{\pi}{2} + \left[1 - 0.5\cos\frac{\pi}{2}\right]\left(x - \frac{\pi}{2}\right) = x - 1.5$$

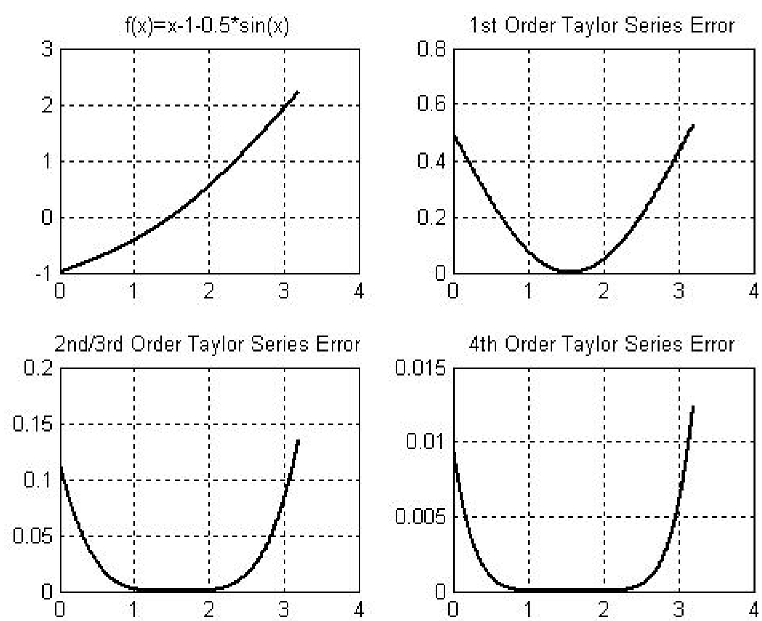
<u>Second-order:</u>
$$f_2(x) = f_1(x) + \frac{f''(a)}{2}(x-a)^2$$
$$f_2(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2$$

<u>Third-order:</u>
$$f_3(x) = f_2(x) + \frac{f^{(3)}(a)}{6}(x-a)^3$$
$$f_3(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 + \frac{0.5\cos(\pi/2)}{6}(x-a)^3$$
$$f_3(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2$$

<u>Fourth-order:</u>
$$f_4(x) = f_3(x) + \frac{f^{(4)}(a)}{24}(x-a)^4$$
$$f_4(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 - \frac{0.5\sin(\pi/2)}{24}(x - \pi/2)^4$$
$$f_4(x) = x - 1.5 + 0.25\sin(\pi/2)(x - \pi/2)^2 - \frac{1}{48}(x - \pi/2)^4$$

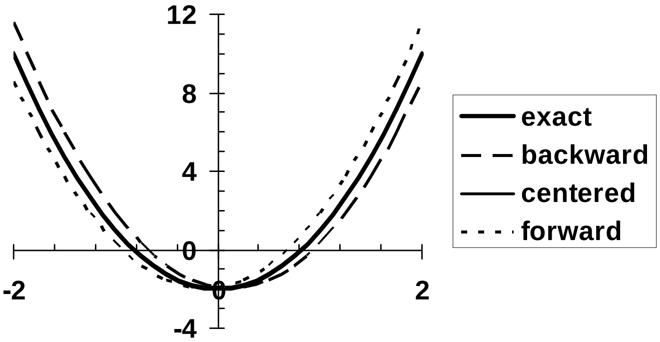
Note the 2nd and 3rd Order Taylor Series functions are the same. The following MATLAB script file which implements and plots each of the functions indicates that the 4th-order expansion satisfies the problem requirements.

```
x=0:0.001:3.2;
f=x-1-0.5*sin(x);
subplot(2,2,1);
plot(x,f);grid;title('f(x)=x-1-0.5*sin(x)');hold on

f1=x-1.5;
e1=abs(f-f1);    %Calculates the absolute value of the difference/error
subplot(2,2,2);
plot(x,e1);grid;title('1st Order Taylor Series Error');

f2=x-1.5+0.25.*((x-0.5*pi).^2);
e2=abs(f-f2);
subplot(2,2,3);
plot(x,e2);grid;title('2nd/3rd Order Taylor Series Error');

f4=x-1.5+0.25.*((x-0.5*pi).^2)-(1/48)*((x-0.5*pi).^4);
e4=abs(f4-f);
subplot(2,2,4);
plot(x,e4);grid;title('4th Order Taylor Series Error');hold off
```

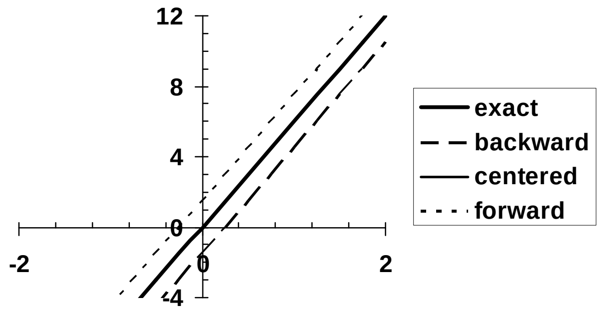


4.19 Here are Excel worksheets and charts that have been set up to solve this problem:

	A	B	C	D	E	F	G	H
1	dx	0.25						
2								
3	First	f(x)	f(x-dx)	f(x+dx)	f'(x)-exact	f'(x)-back	f'(x)-cent	f'(x)-forw
4	-2	0	-2.89063	2.140625	10	11.5625	10.0625	8.5625
5	-1.75	2.140625	0	3.625	7.1875	8.5625	7.25	5.9375
6	-1.5	3.625	2.140625	4.546875	4.75	5.9375	4.8125	3.6875
7	-1.25	4.546875	3.625	5	2.6875	3.6875	2.75	1.8125
8	-1	5	4.546875	5.078125	1	1.8125	1.0625	0.3125
9	-0.75	5.078125	5	4.875	-0.3125	0.3125	-0.25	-0.8125
10	-0.5	4.875	5.078125	4.484375	-1.25	-0.8125	-1.1875	-1.5625
11	-0.25	4.484375	4.875	4	-1.8125	-1.5625	-1.75	-1.9375
12	0	4	4.484375	3.515625	-2	-1.9375	-1.9375	-1.9375
13	0.25	3.515625	4	3.125	-1.8125	-1.9375	-1.75	-1.5625
14	0.5	3.125	3.515625	2.921875	-1.25	-1.5625	-1.1875	-0.8125
15	0.75	2.921875	3.125	3	-0.3125	-0.8125	-0.25	0.3125
16	1	3	2.921875	3.453125	1	0.3125	1.0625	1.8125
17	1.25	3.453125	3	4.375	2.6875	1.8125	2.75	3.6875
18	1.5	4.375	3.453125	5.859375	4.75	3.6875	4.8125	5.9375
19	1.75	5.859375	4.375	8	7.1875	5.9375	7.25	8.5625
20	2	8	5.859375	10.89063	10	8.5625	10.0625	11.5625



	A	B	C	D	E	F	G	H	I	J
1	dx	0.25								
2										
3	x	f(x)	f(x-dx)	f(x+dx)	f(x-2dx)	f(x+2dx)	f''(x)-exact	f''(x)-back	f''(x)-cent	f''(x)-forw
4	-2	0	-2.89063	2.140625	-6.625	3.625	-12	-13.5	-12	-10.5
5	-1.75	2.140625	0	3.625	-2.89063	4.546875	-10.5	-12	-10.5	-9
6	-1.5	3.625	2.140625	4.546875	0	5	-9	-10.5	-9	-7.5
7	-1.25	4.546875	3.625	5	2.140625	5.078125	-7.5	-9	-7.5	-6
8	-1	5	4.546875	5.078125	3.625	4.875	-6	-7.5	-6	-4.5
9	-0.75	5.078125	5	4.875	4.546875	4.484375	-4.5	-6	-4.5	-3
10	-0.5	4.875	5.078125	4.484375	5	4	-3	-4.5	-3	-1.5
11	-0.25	4.484375	4.875	4	5.078125	3.515625	-1.5	-3	-1.5	0
12	0	4	4.484375	3.515625	4.875	3.125	0	-1.5	0	1.5
13	0.25	3.515625	4	3.125	4.484375	2.921875	1.5	0	1.5	3
14	0.5	3.125	3.515625	2.921875	4	3	3	1.5	3	4.5
15	0.75	2.921875	3.125	3	3.515625	3.453125	4.5	3	4.5	6
16	1	3	2.921875	3.453125	3.125	4.375	6	4.5	6	7.5
17	1.25	3.453125	3	4.375	2.921875	5.859375	7.5	6	7.5	9
18	1.5	4.375	3.453125	5.859375	3	8	9	7.5	9	10.5
19	1.75	5.859375	4.375	8	3.453125	10.89063	10.5	9	10.5	12
20	2	8	5.859375	10.89063	4.375	14.625	12	10.5	12	13.5



4.20 We want to find the value of h that results in an optimum of

$$E = \frac{\varepsilon}{h} + \frac{h^2 M}{6}$$

Differentiating gives

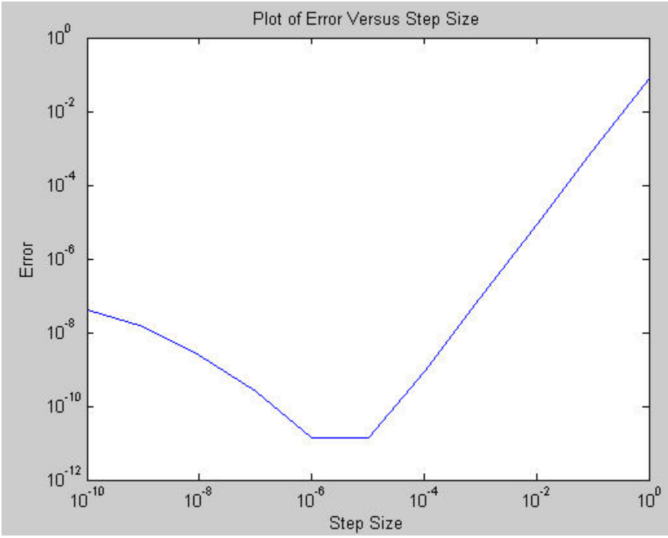
$$\frac{dE}{dh} = -\frac{\varepsilon}{h^2} + \frac{M}{3} h$$

This result can be set to zero and solved for $h^3 = 3\varepsilon/M$. Taking the cube root gives

$$h_{opt} = \sqrt[3]{\frac{3\varepsilon}{M}}$$

4.21 Using the same function as in Example 4.8:

```
>> ff=@(x) cos(x);
>> df=@(x) -sin(x);
>> diffex(ff,df,pi/6,11)

   step size   finite difference    true error
  1.0000000000 -0.42073549240395  0.0792645075961
  0.1000000000 -0.49916708323414  0.0008329167659
  0.0100000000 -0.49999166670833  0.0000083332917
  0.0010000000 -0.49999991666672  0.0000000833333
  0.0001000000 -0.49999999916672  0.0000000008333
  0.0000100000 -0.49999999998662  0.0000000000134
  0.0000010000 -0.50000000001438  0.0000000000144
  0.0000001000 -0.49999999973682  0.0000000002632
  0.0000000100 -0.50000000251238  0.0000000025124
  0.0000000010 -0.49999998585903  0.0000000141410
  0.0000000001 -0.50000004137019  0.0000000413702
```

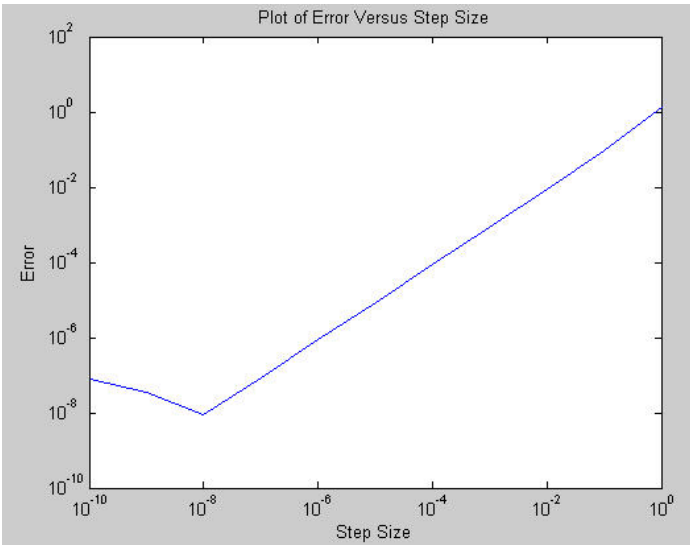


4.22 First, we must develop a function like the one in Example 4.8, but to evaluate a forward difference:

```
function prob0422(func,dfunc,x,n)
format long
dftrue=dfunc(x);
h=1;
H(1)=h;
D(1)=(func(x+h)-func(x))/h;
E(1)=abs(dftrue-D(1));
for i=2:n
  h=h/10;
  H(i)=h;
  D(i)=(func(x+h)-func(x))/h;
  E(i)=abs(dftrue-D(i));
end
L=[H' D' E']';
fprintf('   step size   finite difference    true error\n');
fprintf('%14.10f %16.14f %16.13f\n',L);
loglog(H,E),xlabel('Step Size'),ylabel('Error')
title('Plot of Error Versus Step Size')
format short
```

We can then use it to evaluate the same case as in Example 4.8:

```
>> ff=@(x) -0.1*x^4-0.15*x^3-0.5*x^2-0.25*x+1.2;
>> df=@(x) -0.4*x^3-0.45*x^2-x-0.25;
>> prob0422(ff,df,0.5,11)
   step size   finite difference    true error
  1.0000000000 -2.23750000000000  1.3250000000000
  0.1000000000 -1.00360000000000  0.0911000000000
  0.0100000000 -0.92128509999999  0.0087851000000
  0.0010000000 -0.91337535009994  0.0008753500999
  0.0001000000 -0.91258750349987  0.0000875034999
  0.0000100000 -0.91250875002835  0.0000087500284
  0.0000010000 -0.91250087497219  0.0000008749722
  0.0000001000 -0.91250008660282  0.0000000866028
  0.0000000100 -0.91250000888721  0.0000000088872
  0.0000000010 -0.91249996447829  0.0000000355217
  0.0000000001 -0.91250007550059  0.0000000755006
```



4.23 A MATLAB M-file can be written as

```
function [ser, ea, i, et] = coscomp(x, es, maxit)
if nargin<1,error('at least 1 input argument required'), end
if nargin<2|isempty(es)|es<=0, es=0.000001; end
if nargin<3|isempty(maxit)|maxit<0, maxit=100; end
i = 0; tru = cos(x);
ser = 0;
while (1)
  serold = ser;
  ser = ser + (-1)^i * x^(2*(i+1)-2) / factorial(2*(i+1)-2);
  if ser ~= 0, ea=abs((ser - serold)/ser)*100; end
  if ea <= es | i >= maxit, break, end
  i = i + 1;
end
et = abs((tru - ser)/tru)*100;
```

This function can be used to evaluate the case from Prob. 4.1,

```
>> [ser, ea, i, et] = coscomp(pi/3)

ser =
          0.5
ea =
  7.2617e-007
i =
     6
et =
  4.3555e-009
```