statemech: liouville-von Neumann proof that entropy in constant in time

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1 $i\hbar\frac{\partial\rho}{\partial t} = [H, \rho]$

using this to show that $\frac{\partial S}{\partial t} = 0$

$$S = -k\,\mathrm{tr}(\rho\ln(\rho)))$$

$$\frac{\partial S}{\partial t} = -k\,\mathrm{tr}\left(\frac{\partial\rho}{\partial t}\ln(\rho)\right) + \rho\frac{1}{\rho}\frac{\partial\rho}{\partial t}$$

for a system to have a constant energy H commutes with ρ which means $\frac{\partial\rho}{\partial t} = 0$

$$\therefore \frac{\partial S}{\partial t} = 0 \tag{1}$$