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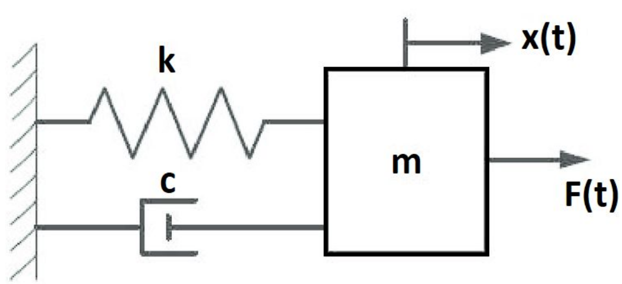
Final Project: Mass-Spring-Damper System

1.1 Introduction

The goal in this problem is to develop a script that takes in mass (m), damping ratio (c), spring constant (k), and driving force (F) to find the reactionary motion of the mass over time. Essentially, the script should be able to accomplish the following task using the forward Euler, 2nd order runge-kutta, and 4th order runge-kutta numerical schemes. The script should first solve the response of the mass-spring-damper and plot the position of the mas vs time for all three numeric schemes above. Then the script should derive the transfer function and describe the relationship between normalized frequency, absolute gain, and phase shift through plots. Finally, the script should generate a video of the mass position as it evolves over time for overdamped, critically damped, and under damped systems.

1.2 Model and Theory



ma+cv+kx = F(t)

F(t) = $a_0\sin(t/2\pi)$

Forward Euler numeric scheme

$df/dt = [f(t_{k+1},y) - f(t_k,y)]/ \Delta t$

$d\varphi/dt = f(t)$

$dx(t)/dt = \varphi(t)$

2nd Order Runge-Kutta

$c_1 = \Delta t\, f(t_k,y_k)$

$c_2 = \Delta t\, f(t_k + ½ \Delta t, y_k\ ½\, c_1)$

$y_{k+1} = y_k + c_2$

4th Order Runge-Kutta

$c_1 = \Delta t\, f(t_k,y_k)$

$c_2 = \Delta t\, f(t_k + 1/2\, \Delta t, y_k + ½\, c_1)$

$c_3 = \Delta t\, f(t_k + 1/2\, \Delta t, y_k + ½\, c_2)$

$c_4 = \Delta t\, f(t_k + \Delta t, y_k + c_3)$

$y_{k+1} = y_k + c_1/6 + c_2/3 + c_3/3 + c_4/6$

Frequency Response

$\omega_n = sqrt(k/m)$

$\varepsilon = c/2 * sqrt(1/(m*k))$

$\lambda = \omega/\omega_n$

$B/A = G(j\omega) = \omega^2/[(\omega_n^2 - \omega^2) + j(2\varepsilon\omega*\omega_n)]$

$\Phi_G = -arctan(2\varepsilon\lambda/(1-\lambda^2))$

Where…

$a_0 = 2$

t= time

c_1, c_2, c_3, c_4 are all constants

Δt = change in time

x_0 = Current position

m = mass

k = spring constant

c = damping constant

f - forcing Function

dt - Timestep

type - Type of differentiation (Forward Euler or Runge Kutta)

$\varepsilon > 1$ (Over-damped)

$\varepsilon = 1$ (Critically-damped)

$\varepsilon < 1$ (Under-damped)

1.3 Methods and Pseudo-code

The flow of calculation should be as follows:

1.Utilize the forward euler numeric scheme to solve for the homogeneous and in-homogenous response of the mass-spring-damper ODE's and PDE's.
2.Then plot position of the mass over time
3.Repeat previous steps for both the 2nd Order Runge-Kutta and the 4th Order Runge-Kutta
4.Derive the transfer function
5.Plot the frequency responses bode plots (1)normalized frequency vs absolute gain and (2) normalized frequency vs phase shift
6.Create an animation of the mass damper system in action for the 4th Order Runge-Kutta so that it will run for over-damped, critically-damped, and under-damped systems.
7.Make (3) clips that are approximately 10 seconds long

1.4 Calculations and Results

Derivation of Transfer function:

$mx'' + cx' + kx = f(t)$

$x'' + 2\varepsilon\omega_n x' + \omega_n^2 x = f(t)$

$x(t) = Be^{(\sigma + jw)t}$

$x'(t) = (\sigma + j\omega)Be^{(\sigma + jw)t}$

$x''(t) = (\sigma^2 - \omega^2)Be^{(\sigma + jw)t}$

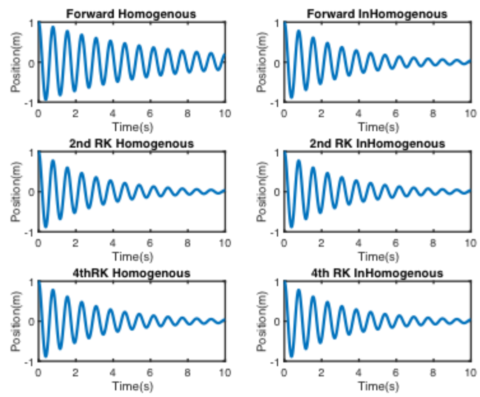
$$Asin(\omega t) = (\sigma^2 - \omega^2)Be^{(\sigma + jw)t} + 2\varepsilon\omega_n(\sigma + j\omega)Be^{(\sigma + jw)t} + \omega_n^2 Be^{(\sigma + jw)t}$$

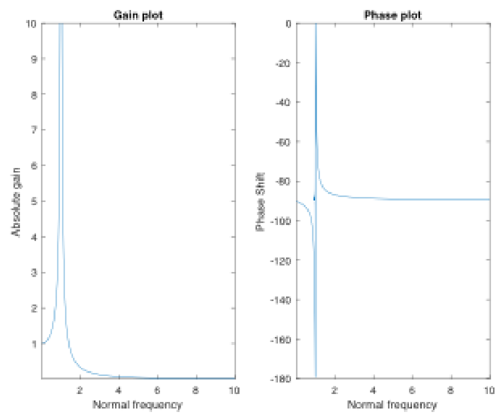
$$B/A = e^{jwt} / ((\sigma^2 - \omega^2)Be^{(\sigma + jw)t} + 2\varepsilon\omega_n(\sigma + j\omega)Be^{(\sigma + jw)t} + \omega_n^2 Be^{(\sigma + jw)t})$$

$$= 1/ ((\sigma^2 - \omega^2)Be^{(\sigma + jw)t} + 2\varepsilon\omega_n(\sigma + j\omega)Be^{(\sigma + jw)t} + \omega_n^2 Be^{(\sigma + jw)t})$$

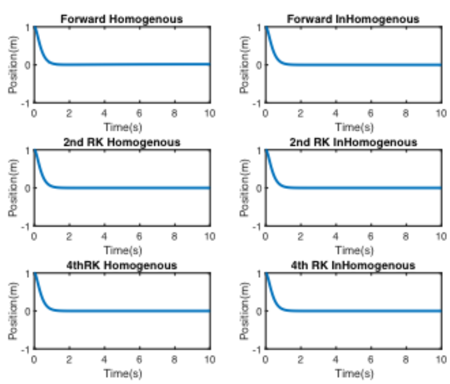
$$= 1/(-\omega^2 + \omega_n^2 + 2\varepsilon\omega_n\omega j)$$

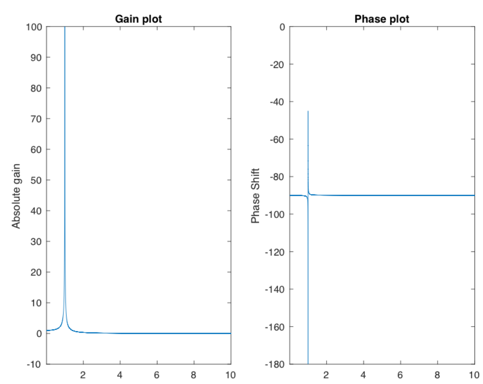
The resulting output from the script for <u>case 1</u> where a0 = 2 , m = 3 , k = 200 , c = 2 is:



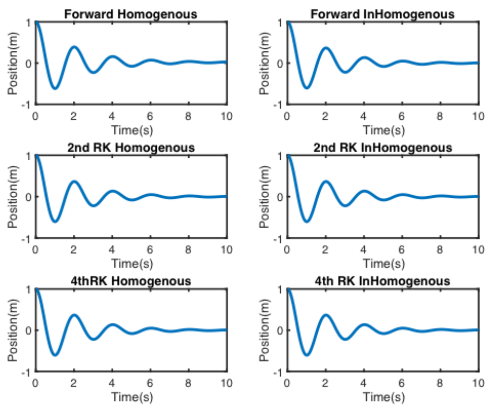


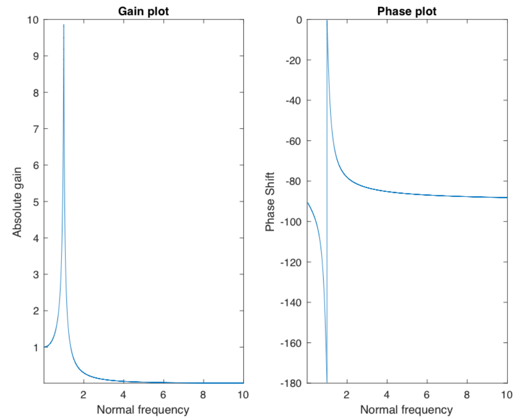
The resulting output from the script for <u>case 3</u> where a0 = 2 , m = 5 , k = 125 , c = 50 is:



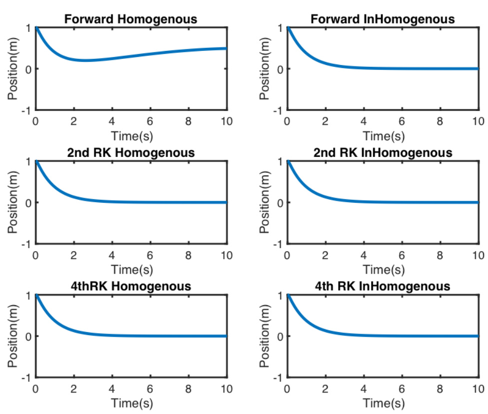


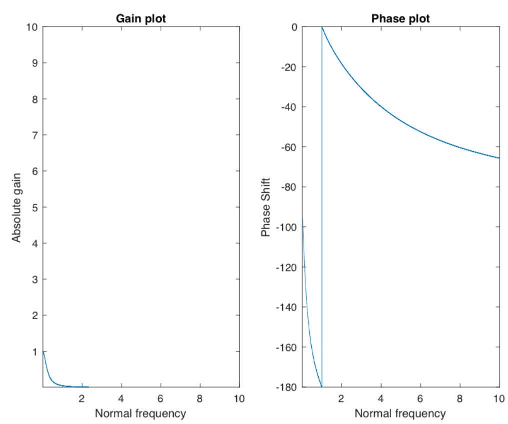
The resulting output from the script for <u>case 5</u> where a0 = 2 , m = 10 , k = 100 , c = 10 is:

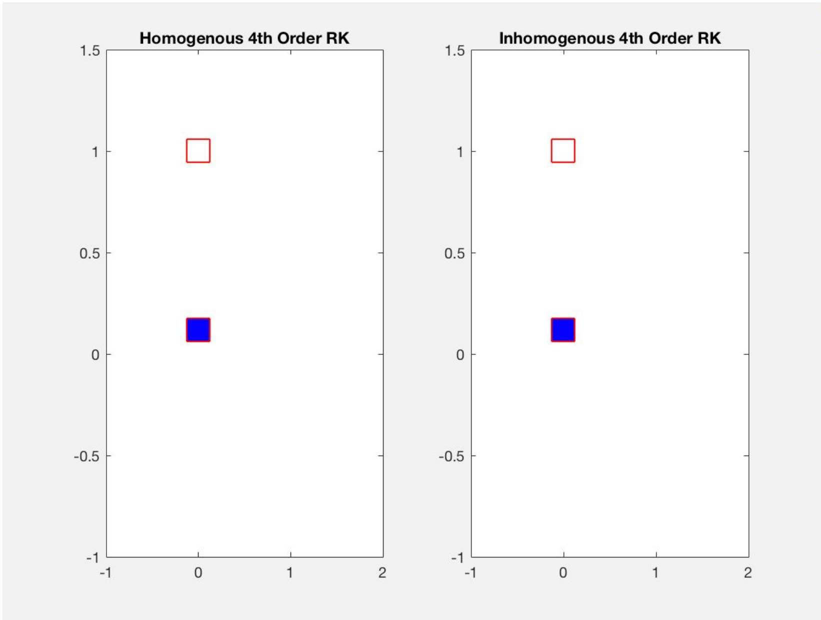




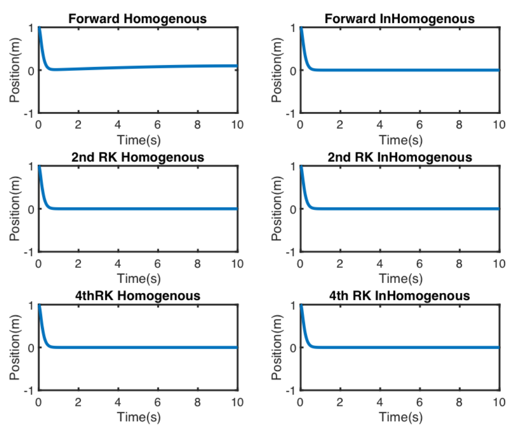
The resulting output from the script for <u>case 6</u> where a0 = 10 , m = 1 , k = 20 , c = 20 is:

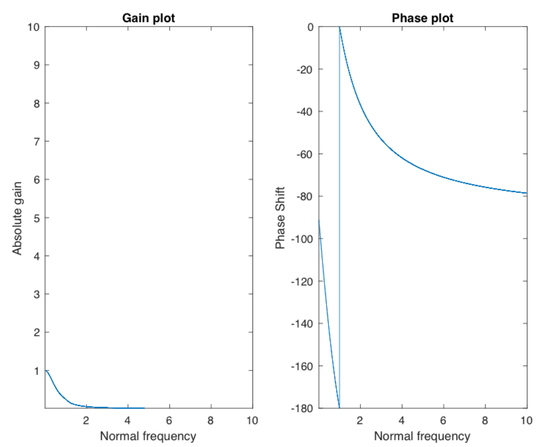


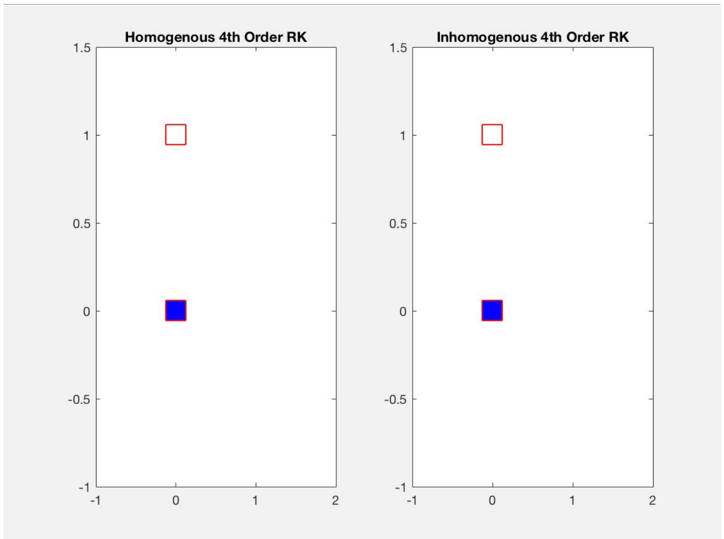




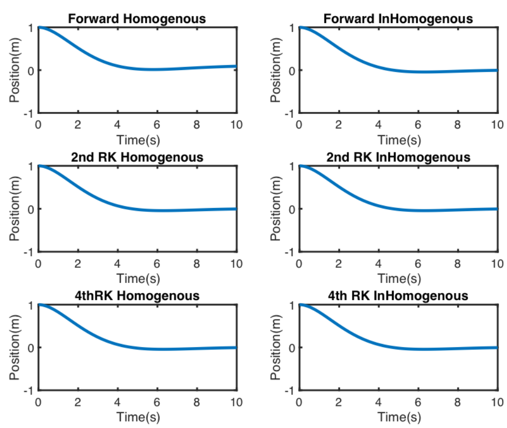
The resulting output from the script for <u>case 7</u> where a0 = 10 , m = 1 , k = 100 , c = 20 is:

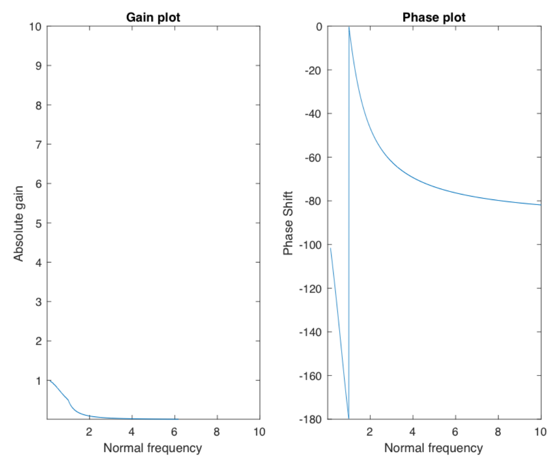


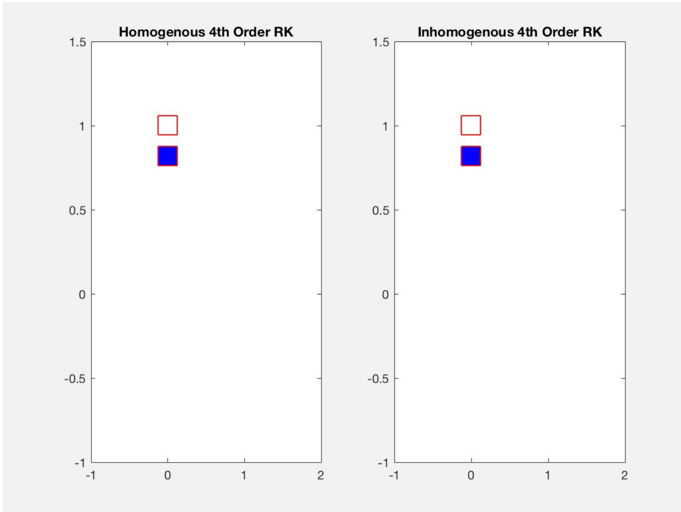




The resulting output from the script for case 8 where a0 = 1 , m = 20 , k = 10 , c = 20 is:







1.5 Discussions and Conclusions

Clearly, from the graphs above, cases 1 and 5 have the most oscillation. Furthermore, the 4th Order Runge-Kutta is the most accurate because it uses the first four derivatives to approximate the motion of the mass spring system compared to the Forward Euler, which uses the first derivative or the 2nd Order Runge-Kutta, which uses the first two derivatives.

Discuss the differences between all three methods of differentiation.

All three methods of differentiation ultimately solve the homogeneous and in-homogenous response of the mass-spring-damper. The forward euler uses the first derivative, the 2nd Order Runge-Kutta uses the first two derivatives, and the 4th Order Runge-Kutta uses the first four derivatives to solve ordinary and partial differential equations.

Discuss the role of the damping ratio (Ɛ) to the overall change in gain and phase shift with respect to normalized frequency. Why do we call a damping ratio over, critically, or under damped?

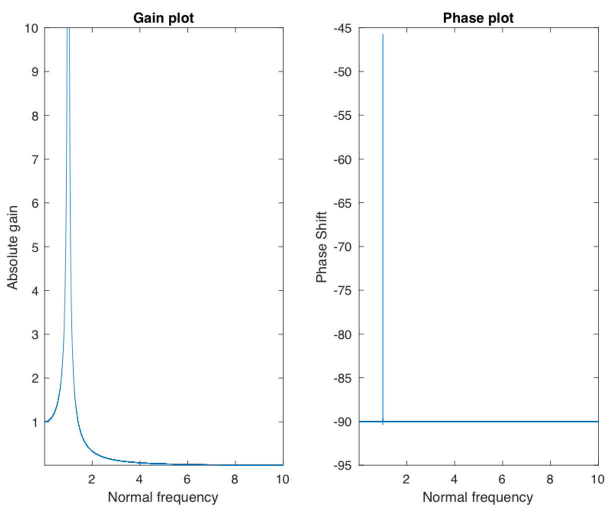
As you increase the damping ratio, the limit as x→ infinity is 0 for the phase shift. For the gain, as you increase the damping ratio, the gain is transposed to the left. We call a damping ratio over,critically, or under damped because of the force that is exerted on the system makes the return to 0 at different rates. When the damping ratio > 1, it is overdamped; when the damping ratio is = 1, it is critically damped; when the damping ratio < 1 , it is underdamped

1.6 Extra Credit

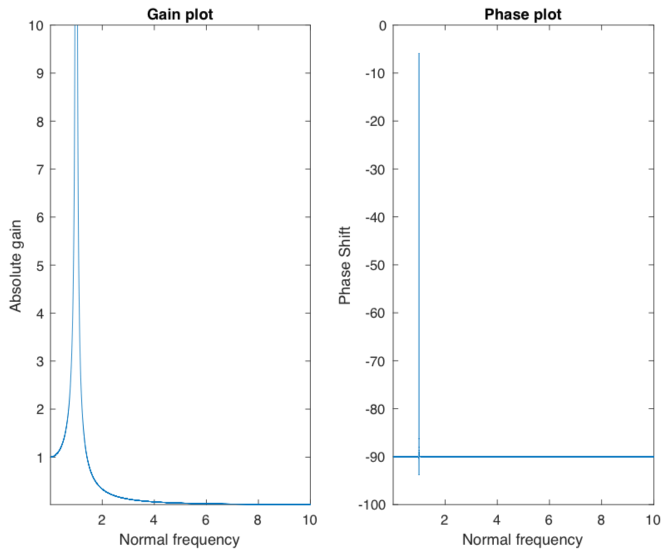
Discuss the effect of aliasing on data acquisition by finding differences in frequency response using the standard 60 Hz method, and 30 Hz, 20 Hz, 10 Hz, and 5 Hz.

Obviously from aliasing data acquisition of the frequency response using 60 Hz, 30 Hz, 20 Hz, 10 Hz, and 5 Hz, these results revealed that the peak increases gradually as you decrease the Hz frequency; this is an inversely proportional relationship.

Frequency at 60Hz



Frequency at 40 Hz



Frequency at 30 Hz