IWOAT SUMMER SCHOOL 2024:
MOTIVIC STABLE HOMOTOPY THEORY

Day 1: 4 lectures: Introduction to motivic categories.

Lectures 1-2. A quick overview, presentable infinity categories and localizations.

Lecture 3. The construction of the unstable $\mathbb{A}^1$-homotopy category over a base. A good primary source is Hoyois' article on the equivariant setting, taking $G = \{id\}$, supplemented by Morel-Voevodsky.
Functorialities, f_*, f^*. Thom spaces. Homotopy purity.

Lecture 4. The stable $\mathbb{A}^1$-homotopy category, as $\mathbb{P}^1$-spectra. Suspension functors, "ambidexterity", purity. 6 functors (just construction and properties).

Day 2: 4 Lectures: connectivity, homotopy t-structure and Morel's theorem on stable π_0.

Lectures 1-2. Morel's S^1-$\mathbb{A}^1$-connectivity theorem. Introduce S^1-spectra, follow Morel's paper "The stable $\mathbb{A}^1$-connectivity theorems".

Lecture 3. Promote to the connectivity theorem for $\mathbb{P}^1$-spectra. Introduce the S^1 and $\mathbb{P}^1$ homotopy t-structures. Discuss the heart of the homotopy t-structure as homotopy modules, following Morel's ICTP notes.

Lecture 4. Introduce the Milnor-Witt K-sheaves. State the main properties without much proofs, following the ICTP notes, state Morel's theorems on stable $\pi_{n,n}$, and give a sketch of the proof, following the ICTP notes.

Day 3: 2 Lectures. Motivic cohomology.
5pt] Lecture 1. Motivic cohomology via the Voevodsky motivic complexes

Lecture 2. Representing motivic cohomology in SH(k), following Röndigs-Østvær.

Day 4. 4 Lectures. K-theory, algebraic cobordism, and the slice tower.

Lecture 1. BGL and the unstable representation theorem of Morel-Voevodsky. The construction of KGL.

Lecture 2. The stable representation theorem for K-theory. The construction of MGL, the formal group law for MGL and the computation of motivic cohomology of MGL.

Lecture 3. Introduction to Voevodsky's slice tower.

Lecture 4. The slice spectral sequence. Slices of KGL and $H\mathbb{Z}$ and the spectrral sequence from motivic cohomology to K-theory.

Day 5. 4 Lectures. Hopkins-Morel-Hoyois theorem, slices of KGL and MGL.

Lecture 1: Statement of the HMH theorem: Introduce the Steenrod algebra and the motivic version as blackbox.
Lecture 2. Properties of $H\mathbb{Q}$ following Cisinski-Deglise, especially the idempotent property. Proof of the $\mathbb{Q}$-version of HMH

Lecture 3. Finish the proof of the HMH theorem

Lecture 4. Compute the slices of MGL.