IWOAT SUMMER SCHOOL 2024: MOTIVIC STABLE HOMOTOPY THEORY

Day 1: 4 Lectures: Introduction to motivic categories.

Lectures 1-2. Definition of infinity-categories as quasi-categories, existence of mapping spaces, interpretation. Presentable infinity categories. Bousfield localization. All of this will have to be presented very tersely, mostly as a reminder.
Lecture 3. The construction of the unstable $\mathbb{A}^1$-homotopy category over a base. Functorialities, f_*, f^*. Thom spaces. Homotopy purity.

Lecture 4. The stable $\mathbb{A}^1$-homotopy category. As $\mathbb{P}^1$-spectra? Suspension functors, "ambidexterity", purity. 6 functors (just construction and properties).

Day 2: 4 Lectures: connectivity, homotopy t-structure and Morel's theorem on stable π_0.

Lecture 1. Morel's S^1-$\mathbb{A}^1$-connectivity theorem. Introduce S^1-spectra, follow Morel's paper "The stable $\mathbb{A}^1$-connectivity theorems".
Lecture 2. Promote to the connectivity theorem for $\mathbb{P}^1$-spectra. Introduce the S^1 and $\mathbb{P}^1$ homotopy t-structures.
Lecture 3. The heart of the homotopy t-structure as homotopy modules, following Morel's ICTP notes.
Lecture 4. Introduce the Milnor-Witt K-sheaves. State the main properties without much proofs, following the ICTP notes, state Morel's theorems on stable $\pi_{n,n}$, and give a sketch of the proof, following the ICTP notes.

Day 3: 2 Lectures. Motivic cohomology.

Lecture 1. Motivic cohomology via the Voevodsky motivic complexes

Lecture 2. Representing motivic cohomology in $\mathrm{SH}(k)$, following Röndigs-Østvær.

Day 4. 4 Lectures. K-theory, algebraic cobordism, and the slice tower.

Lecture 1. BGL and the unstable representation theorem of Morel-Voevodsky.

Lecture 2. KGL and the stable representation theorem.

Lecture 3. The construction of MGL. The formal group law. Computation of motivic cohomology of MGL.

Lecture 4. Introduction to Voevodsky's slice tower.

Day 5. 4 Lectures. Hopkins-Morel-Hoyois theorem, slices of KGL and MGL.

Lecture 1: Statement of the HMH theorem: Introduce the Steenrod algebra and the motivic version as blackbox.
Lecture 2. Computation of the homology (rational and mod ℓ) of regular quotients of MGL.
Lecture 3. Finish the proof of the HMH theorem.
Lecture 4. Compute the slices of KGL and MGL.