$\Rightarrow X^{S^1} = X^C$ is $\mathbb{F}_p$-acyclic.

use the same trick in case 1

$\Rightarrow (X/X^{S^1})/S^1$ is $\mathbb{F}_p$-acyclic.

$\Rightarrow X^{S^1}$ is $\mathbb{F}_p$-acyclic.

Case 6 $\quad G = S^1 \quad \mathbb{Q}$ coefficient.

Apply a rational Vietoris–Smith theorem,

we can prove this case similarly as case 5'

ex. Check this case.

Case 5+6 $\quad G = S^1$ for all abelian grp. coefficient.

Case 7. $\quad G = T^n = \underbrace{S^1 \times \cdots \times S^1}_{n \text{ copy}}$

$S^1 = T \triangleleft \cdots T^{n-1} \triangleleft T^n$

induction reduce to case 6+5.

Case 8. $\quad G$ cpt Lie

Fact: G connected cpt Lie grp. with maximal torus T^n

Then. $\chi(G/N_G(T^n)) = 1$

Consider.

$$H^*(X/N_G(T^n)) \xrightarrow{\pi^*} H^*(X/G) \to H^*(X/N_G(T^n))$$

iso.

But: X acyclic $\Rightarrow X/T^n$ acyclic (case 7)

$\Rightarrow X/N_G(T^n)$ acyclic (case 3+4)

+ the fact $N_G(T^n)/T^n$ is a finite.

in a cpt Lie grp G with maximal torus T^n.