Problem Set 3
Intertemporal Choice

1. **The Time Aggregation Problem.**

   Suppose that consumption data C are collected annually (as if the government asks everybody "how much did you spend last year?" and adds up the answers), with years indexed by t, but agents make their consumption decisions once every six months ('semiannually'), in periods indexed by τ. Assume quadratic utility and $\mathsf{R}\beta = 1$ so that, at the semiannual frequency (6 months), consumption follows a random walk:

   $$C_{\tau+1} = C_\tau + \epsilon_{\tau+1} \tag{1}$$

   where $\mathbb{E}_t[\epsilon_{t+n}] = 0 \ \forall \ n > 0$.

   Suppose that for year $t = 0$ we refer to the first six-month period as $\tau = 0$ and the second six-month period as $\tau + 1$.

   a) Denote the measured change in annual consumption $\Delta\bar{C}_t = C_\tau + C_{\tau+1} - C_{\tau-1} - C_{\tau-2}$. Show that it is related to the underlying consumption changes by

   $$\Delta\bar{C}_t = \Delta C_{\tau+1} + 2\Delta C_\tau + \Delta C_{\tau-1} \tag{2}$$

   b) Is the change in measured annual consumption uncorrelated with the previous value of the change in measured annual consumption? In light of this, is measured annual consumption a random walk?

   c) Show that the previous period's measured income change can be written as

   $$\Delta\bar{Y}_{t-1} = \Delta Y_{\tau-1} + 2\Delta Y_{\tau-2} + \Delta Y_{\tau-3}, \tag{3}$$

   Is the previous period's measured income change correlated with recorded change in annual consumption? Is $\Delta\bar{Y}_{t-2}$ correlated with recorded change in annual consumption?

   d) Suppose we are now testing the consumption random walk hypothesis along the lines of Flavin (1981). We use time aggregated consumption and income data (this is all we have) to estimate

   $$\Delta\bar{C}_t = \alpha_0 + \alpha_1\Delta\bar{Y}_{t-1}, \tag{4}$$

   Suppose our disaggregated consumption data (if available) follows a random walk, what would we expect to find for the value of the coefficient α_1 in our estimates?

   e) Use a computer program to create an artificial dataset that has the characteristics of the model outlined above. That is, the computer program will create artificial semiannual consumption data that follows a random walk (you will need to use your software package's built-in "random number generator"), starting from some initial period when (say) consumption starts at a level of 100. (Suppose, for simplicity, that in each semiannual period consumption either rises by 1 unit, or falls by 1 unit, randomly). Simulate 1000 semiannual consumption observations. Given this simulated semiannual data, construct "measured annual consumption" by aggregating, and then perform a regression of $\Delta\bar{C}_t$ on $\Delta\bar{Y}_{t-1}$. Confirm that the results you get correspond to what would be expected from the mathematical derivations performed above.

References

Flavin, Marjorie B. (1981): "The Adjustment of Consumption to Changing Expectations About Future Income," *Journal of Political Economy*, 89, 974–1009, http://www.jstor.org/stable/1830816.