# Problem Set 1b

## Intertemporal Choice

1. **The Income, Substitution, and Human Wealth Effects in a Two Period Lifetime (Fisher (1930)).** For a consumer who solves the following maximization problem,

$$\begin{aligned} \mathrm{v}(y_1) &= \max_{c_1} \ \mathrm{u}(c_1) + \beta \mathrm{u}(c_2) \\ &\text{s.t.} \\ c_2 &= (y_1 - c_1)\mathsf{R} + y_2 \\ \mathrm{u}(c) &= \frac{c^{1-\rho}}{1-\rho}. \end{aligned}$$

   a) Solve for consumption and saving in the first period as a function of $\mathsf{R}, y_1, y_2, \rho$, and $\beta$.

   b) Suppose that all noncapital income is earned in the first period of life, i.e. $y_2 = 0$. How does the sign of the derivative of saving with respect to the interest rate depend on the value of $\rho$ (consider the cases of $\rho \to 0, 0 < \rho < 1, \rho = 1, \rho > 1$, and $\rho \to \infty$)? Give a verbal explanation.

   c) Now suppose that all noncapital income is earned in the second period of life, i.e. $y_1 = 0, y_2 > 0$. For a given value of $\rho$, how does the responsiveness of consumption and saving to the interest rate compare to the case when all noncapital income was earned in the first period? What is the name of the additional effect on saving? (Hint: There is a particular value of $\rho$ which makes this question easy.)

   d) Now suppose that noncapital income is earned equally in both periods, $y_1 = y_2$. How does the response of consumption and saving to interest rates depend on $\rho$? Describe and explain the special results obtained as $\rho$ approaches infinity and and as it approaches zero.

2. **Income Growth Over the Lifetime Versus Between Generations (Modigliani (1986), Carroll and Summers (1991)).** This question concerns the effects on aggregate saving of income growth over the lifetime versus income growth between generations. Consider an overlapping generations economy in which each individual lives for two periods. Population is constant, so that the population growth factor between generations is $\Xi = 1$; normalize population itself to $\boldsymbol{N} = 1$ per generation. The individuals' noncapital incomes in each period are exogenous. The first period noncapital income of an individual born at time $t$ is $y_{t,1}$, and the second-period noncapital income of the same individual is $y_{t+1,2} = \mathsf{X} y_{t,1}$ where $\mathsf{X}$ can be greater than or less than one. The consumer solves the optimization problem:

$$\max \quad \log(c_{t,1}) + \beta \log(c_{t+1,2}) \tag{1}$$

$$\text{s.t.} \tag{2}$$

$$c_{t+1,2} = (y_{t,1} - c_{t,1})\mathsf{R} + y_{t+1,2}. \tag{3}$$

Finally, between generations the first period noncapital incomes grow by a factor $\mathsf{G} = (1 + \mathsf{g})$ so that:

$$y_{t+1,1} = \mathsf{G} y_{t,1}$$

For the purposes of this question, consider this to be an open economy so that the aggregate interest rate $\mathsf{R}$ and noncapital incomes $y$ are fixed (that is, don't try to derive their values from an aggregate production function).

a) How does an increase in the growth rate of noncapital income over the lifetime, $\mathsf{X}$, affect the saving of young households? Explain.

b) Calculate the level of aggregate saving $S_t = K_{t+1} - K_t$ as a function of $y_{t,1}$, and then calculate the aggregate saving rate out of noncapital income $\sigma_t = S_t/Y_t = S_t/(y_{t,1} + y_{t,2})$. How is the aggregate saving rate related to the growth rate of income between generations, $\mathsf{G}$? How and why does the answer depend on the relationship between $\beta$ and $\mathsf{X}/\mathsf{R}$?

c) Thus far in the problem, we have assumed that $\mathsf{X}$ is independent of $\mathsf{G}$; that is, we have assumed that the rate at which income grows during your lifetime is unrelated to the rate at which each generation's income exceeds the income of the previous generation. Now assume that $\mathsf{X} = \gamma \mathsf{G}$ for some constant $\gamma$. Also assume (for simplicity) that $\beta = 1/\mathsf{R}$ (qualitative results are the same when $\beta \neq 1/\mathsf{R}$, but the analysis is messier). Now when does an increase in $\mathsf{G}$ increase or reduce the aggregate saving rate?

d) Empirical evidence shows that the ratio of the income of the old (people aged 55-85) to income of the young (people aged 25-55) is about 0.7 in both the US and in Japan. From the late 1940s to the late 1980s, Japan's economic growth rate was about 4 percent per year in per-capita terms. Over the same period income growth in the US was about 1 percent per capita. Japan's aggregate saving rate was also much higher than the US saving rate during this period. Discuss whether the overlapping generations model can explain Japan's high saving rate as being the result of its rapid growth rate (continue to assume $\beta = 1/\mathsf{R}$). (Hint: start by figuring out the OLG model's implications for the ratio of the income of the old to income of the young.)

# References


Carroll, Christopher D., and Lawrence H. Summers (1991): "Consumption Growth Parallels Income Growth: Some New Evidence," in *National Saving and Economic Performance*, ed. by B. Douglas Bernheim, and John B. Shoven. Chicago University Press, Chicago, http://econ.jhu.edu/people/ccarroll/papers/CParallelsY.pdf.

Fisher, Irving (1930): *The Theory of Interest*. MacMillan, New York.

Modigliani, Franco (1986): "Life Cycle, Individual Thrift, and the Wealth of Nations," *American Economic Review*, 3(76), 297–313, Available at http://www.jstor.org/stable/1813352.