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# Wealth distribution in life-cycle economies

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#### Abstract

This paper compares the age—wealth distribution produced in life-cycle economies to the corresponding distribution in the US economy. The idea is to calibrate the model economies to match features of the US earnings distribution and then examine the wealth distribution implications of the model economies. The findings are that the calibrated model economies with earnings and lifetime uncertainty can replicate measures of both aggregate wealth and transfer wealth in the US. Furthermore, the model economies produce the US wealth Gini and a significant fraction of the wealth inequality within age groups. However, the model economies produce less than half the fraction of wealth held by the top 1 percent of US households.

Key words: Wealth distribution

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#### 1. Introduction

This paper investigates wealth distribution at a quantitative level by pursuing the research program put forward by Atkinson (1971). The program is to first describe a parametric class of economies where people differ only insofar as they

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are at different stages of the life-cycle. The model economies are then calibrated and compared to actual age—wealth distributions. New features are to be added to this 'basic' life-cycle framework to improve the match between theory and observation. A list of features to be investigated includes (1) earnings, health, and longevity uncertainty, (2) household structure, (3) institutional features such as social security, income taxation, and social insurance, and (4) market features such as borrowing constraints and the absence of some insurance markets.

The basic life-cycle model has a number of problems as a model of wealth distribution. First, White (1978) argues that aggregate savings tend to be too low in calibrated versions of the basic life-cycle model to explain US savings rates. Thus, in the model that she considers the capital stock tends to be too low to explain aggregate wealth. Second, Kotlikoff and Summers (1981) calculate that the vast majority of the US capital stock can be attributed to intergenerational transfers rather than to accumulation out of earnings that are the emphasis of the basic life-cycle model of capital accumulation. Third, wealth holding is much more concentrated in the upper tail of the wealth distribution than the basic model predicts. This point is developed in Atkinson (1971) and extended by Oulton (1976), Davies and Shorrocks (1978), and others. Fourth, wealth is as unequally distributed within an age group in the US as it is in the overall wealth distribution. However, in the basic model wealth only differs across agents in different age groups.

These observations motivate asking whether a few realistic modifications of the basic life-cycle framework might produce an age-wealth distribution that more closely resembles the US age-wealth distribution. The modifications of the basic model investigated here are the presence of earnings and lifetime uncertainty and the absence of markets for insuring this uncertainty. There are a number of reasons why these modifications might be important. First, people will now save for precautionary as well as life-cycle (retirement) reasons. Therefore, with appropriate restrictions on preferences (see Skinner, 1988; Caballero, 1991), aggregate wealth will be higher than in the basic model. Second, earnings uncertainty will be added in a way that matches the magnitudes of individual earnings variation and the inequality in the US earnings distribution. Thus, a significant part of observed wealth inequality may be due to earnings inequality. Part of observed wealth inequality may reflect the fact that some people have permanently higher earnings than others and therefore carry higher wealth levels into retirement. In addition, part of wealth inequality could be due to the lack of insurance markets for insuring earnings uncertainty. In the absence of these insurance markets, individual luck will lead even ex-ante identical people to realize differing wealth levels over time. People experiencing high earnings shocks will accumulate wealth and the opposite will occur for people experiencing low earnings shocks.

Lifetime uncertainty could also be important for several reasons. First, in the presence of lifetime uncertainty and in the absence of markets to insure this uncertainty, there will be accidental bequests. The passing of these bequests

could have substantial effects on both aggregate wealth and the distribution of wealth. Second, Kotlikoff and Summers (1981) have calculated that intergenerational transfers account for the vast majority of the US capital stock. This seems to rule out pure life-cycle models as models of how capital is actually accumulated. It is therefore interesting to investigate models with lifetime uncertainty and an absence of perfect annuity markets. These models follow a logic that is very similar to pure life-cycle models (see Davies, 1981) and yet allow the possibility that a substantial part of capital could be attributed to intergenerational transfers arising from accidental bequests.

At a formal level, the model economies that are investigated are generalizations of the Diamond (1965) growth model. The agents populating these economies live for realistic life spans. They experience variations in earnings for both deterministic and idiosyncratic reasons. They save for retirement, for precautionary reasons and in case of a long lifetime. The assets that agents can hold are either riskless debt or physical capital. In addition, agents are required to participate in a social security system. The stationary equilibrium age—wealth distributions of the model economies are characterized using computational methods.

The paper is organized as follows. Section 2 reviews in detail several features of the US wealth distribution. Section 3 describes the economies to be investigated. Section 4 describes the calibration of the economies. Section 5 presents the results. Section 6 concludes.

### 2. Wealth observations

This section presents measures of aggregate wealth, transfer wealth, and wealth distribution in the US. In a later section these observations will be compared to the corresponding facts from the model economies.

Table 1 presents the observations discussed in this section. First, consider the measurement of aggregate wealth. The measure of aggregate wealth used in this paper is the capital—output ratio. In the US this ratio is anywhere between 2 and 3. When capital is defined as residential structures, plant and equipment, inventories, land, and consumer durables, the average capital—output ratio from

Table 1				
Wealth	in	the	US	economy

Capital–	Transfer wealth	Wealth	Perc	entage	Percent with				
output ratio	ratio	Gini	1%	5%	20%	40%	60%	•	zero or negative wealth (%)
3.0	0.78-1.32	0.72	28	49	75	89	96	99	5.815.0

1959–92 is 3.0 (see Auerbach and Kotlikoff, 1995). When capital is defined to exclude land, consumer durables, and residential structures owned by the government, the capital-output ratio is below 2 (see Stokey and Rebelo, 1995).

Transfer wealth as a fraction of total wealth in the US economy is estimated by Kotlikoff and Summers (1981). They separate total wealth into a transfer and a life-cycle wealth component. Transfer wealth for a given person alive at a point in time is defined as the current value of all current and past nongovernmental transfers received by that person, where current values are calculated using realized after-tax rates of return on wealth holdings. The remaining component of wealth is life-cycle wealth. Kotlikoff and Summers (1981) present four direct estimates of aggregate life-cycle wealth. The estimates range from -32 to 19 percent of aggregate wealth. Thus, their indirect estimate of transfer wealth ranges from 81 to 132 percent of total wealth.

The magnitude and even the definition of life-cycle wealth and transfer wealth have been the subject of debate. Modigliani (1988) and Kotlikoff (1988) discuss the issues. Modigliani argues that the correct treatment of consumer durables lowers the above estimates of transfer wealth. Kotlikoff (1988) states that after correcting for consumer durables the lowest measure of transfer wealth is reduced from 81 to 78 percent of total wealth. These estimates of transfer wealth are listed in Table 1.

Table 1 also presents measures of wealth distribution. Two standard ways of describing wealth distribution are the Gini coefficient and the fraction of wealth held by the wealthiest households. The estimates of the Gini coefficient and the fraction of wealth held by the top wealth-holders come from Wolff (1987). The table states that the top 1 percent own 28 percent, the top 20 percent own 75 percent, and the top 80 percent own 99 percent of total wealth. The wealth Gini coefficient is 0.72.

Wolff's estimates are based on data from the 1983 Survey of Consumer Finances (SCF). The SCF was specifically designed as a wealth survey and is therefore widely believed to offer one of the most accurate descriptions of wealth distribution. The survey defines wealth as owner-occupied housing, other real estate, cash, financial securities, unincorporated business equity, insurance and pension cash surrender value, miscellaneous assets less mortgage, and other debt. Wolff adjusts the survey data to account for consumer durables and household inventories not measured in the survey data and to account for under reporting of financial assets and equities. The first adjustment reduces the wealth Gini, as durables and household inventories are relatively evenly distributed,

<sup>&</sup>lt;sup>1</sup>Recall that the Gini coefficient is the area between the Lorenz curve and the 45 degree line as a percentage of the area below the 45 degree line. The Lorenz curve for the wealth distribution is the graph generated by plotting the fraction of total wealth held by a group as a function of the group's fraction of the population. The groups considered are the households having a wealth level below a specified level.

and the second adjustment increases the wealth Gini. The overall effect of both adjustments is to increase the wealth Gini from 0.66 to 0.72. When the concept of wealth measured is financial wealth, then the wealth Gini and the fraction of wealth held by the top wealth-holders are even larger than the figures in Table 1. This is because financial wealth tends to be more unequally distributed than other forms of wealth. When wealth distribution is inferred from estate tax data rather than surveys, the fraction of wealth held by top wealth-holders is substantially smaller. Using the estate tax method, Avery et al. (1988) report that in 1983 the top 1 percent held 19.7 percent of the wealth rather than the 28 percent estimated from the survey data.

The measurement of wealth inequality discussed above is also sensitive to the addition of social security benefits. For example, Feldstein (1976) calculates that the introduction of social security wealth reduces wealth concentration significantly. Using the 1962 Survey of Financial Characteristics of Consumers (SFCC), he finds that the wealth Gini declined from 0.72 to 0.51 after accounting for social security. Wolff (1992) finds that the Gini coefficient for the 1983 SCF falls to 0.64 with the addition of social security wealth. The fact that the measurement of wealth concentration is sensitive to the definition of wealth suggests that it would be useful to design theoretical models with this measurement problem in mind. For this reason I directly model the social security system to facilitate matching the US wealth distribution data to the distributions in model economies.

Table 1 also reports the fraction of households holding either zero or negative wealth. The raw data on which these facts are based is again the 1983 SCF. A total of 15.0 percent of the households hold either zero or negative wealth. This figure decreases to 5.8 percent when consumer durables are included in the definition of wealth.<sup>2</sup> Projector and Weiss (1966) report that 16 percent of households in the 1962 SFCC hold zero or negative wealth.

## 3. The economies investigated

The economies that are investigated are generalizations of the Diamond (1965) growth model. The particular modeling framework is similar to that used by Imrohoroglu, Imrohoroglu, and Joines (1995) to investigate the welfare benefits of alternative social security replacement rates.<sup>3</sup> The framework is rich

<sup>&</sup>lt;sup>2</sup>These facts are from unpublished research described to me by Edward Wolff in a phone conversation on February 18, 1994.

<sup>&</sup>lt;sup>3</sup>Imrohoroglu et al. (1995) build on a tradition that incorporates social security into the neoclassical growth model. This literature includes the work of Kotlikoff (1979), Auerbach and Kotlikoff (1987), and Hubbard and Judd (1987) among others.

enough for agents to have retirement, precautionary, and lifetime uncertainty savings motives. All the economies investigated are either special cases or simple modifications of the economy described in this section.

### 3.1. The environment

I consider an overlapping generations economy. Each period a continuum of agents are born. Agents live a maximum of N periods and face a probability  $s_t$  of surviving up to age t conditional on surviving up to age t-1. The population grows at rate n. These demographic patterns are stable so that age t agents make up a constant fraction  $\mu_t$  of the population at any point in time. All age 1 agents have identical preferences for consumption:

$$\mathbf{E}\left[\sum_{t=1}^{N} \beta^{t} \left(\prod_{j=1}^{t} s_{j}\right) u(c_{t})\right].$$

The period utility function u(c) is of the constant relative-risk aversion class, where  $\sigma$  is the coefficient of relative risk aversion.

$$u(c) = c^{(1-\sigma)}/(1-\sigma).$$

An agent's labor endowment is given by a function e(z, t) that depends on the agent's age t and on an idiosyncratic labor productivity shock z. The shock z takes on a finite number of possible values in the set Z and follows a Markov process. Labor productivity shocks are independent across agents. This implies that there is no uncertainty over the aggregate labor endowment even though there is uncertainty at the individual level. The function e(z, t) is described in detail in Section 4.

There is a constant returns to scale production technology that converts capital K and labor L into output Y. Each period capital depreciates at rate  $\delta$ .

$$Y = F(K, L) = AK^{\alpha}L^{1-\alpha}.$$

## 3.2. The arrangement

I consider an arrangement where each period an age t agent chooses consumption c and risk-free asset holdings a'. An agent's decision problem is described below in the language of dynamic programming. An agent's individual state at a point in time is denoted x = (a, z), where a is asset holdings carried into the period and z is the labor endowment shock. Optimal decision rules are functions for consumption c(x, t) and asset holdings a(x,t) that solve the

<sup>&</sup>lt;sup>4</sup>The weights  $\mu_t$  are normalized to sum to 1, where  $\mu_{t+1} = (s_{t+1}/(1+n)) \mu_t$ .

dynamic programming problem, given that after the terminal period N the value function is set to zero, V(x, N + 1) = 0.

$$V(x, t) = \max_{(c, a')} u(c) + \beta s_{t+1} E[V(a', z', t+1)|x]$$

subject to

$$(1) \quad c + a' \le a(1 + r(1 - \tau)) + (1 - \theta - \tau)e(z, t)w + T + b_t,$$

(2) 
$$c \ge 0$$
,  $a' \ge a$  and  $a' \ge 0$  if  $t = N$ .

At a point in time an agent's resources are derived from asset holdings a, labor endowment e(z,t), a lump-sum transfer T, and an age-dependent social security benefit  $b_t$ . Asset holdings pay a risk-free interest rate r and labor receives a real wage w. Capital and labor income are taxed at rate  $\tau$ . In addition, there is a social security tax  $\theta$  on labor earnings. The social security benefit  $b_t$  is zero before the retirement age R and equals a fixed benefit level b after retirement. Since the benefit level is the same for all agents, there is no linkage between an individual agent's earnings and future social security benefits. This assumption can be viewed as a rough first approximation to the highly redistributive nature of the actual link between earnings and benefits; see Musgrave and Musgrave (1976, Ch. 31) or Stiglitz (1988, Ch. 13). The assumption eases the computational burden significantly as a variable capturing an agent's earnings history need not be included as part of an individual agent's state.

Social security is explicitly modeled as social security redistributes a significant fraction of income and previous research suggests that this redistribution has a large effect on the capital—output ratio. Furthermore, the concept of wealth that is commonly measured in survey data is one that excludes the value of future social security benefits. Thus, modeling social security facilitates matching US wealth distribution data to the distribution in model economies.

Agents are allowed to borrow up to a credit limit  $\underline{a}$ . The only additional restriction is that if an agent survives to the terminal age N, then asset holdings must be nonnegative,  $a' \ge 0$ . Of course, the credit limit can be set sufficiently low so that the only binding requirement is that in the last period of life the agent holds no debt.

## 3.3. Equilibria

To state the equilibrium concept, some way of describing heterogeneity in the economy at a point in time is needed. At a point in time agents are heterogeneous in their age t and in their individual state x. A probability measure  $\psi_t$  defined on subsets of the state space is a natural way of describing the distribution of individual states across age t agents. So let  $(X, B(X), \psi_t)$  be a probability space, where  $X = [a, \infty) \times Z$  is the state space and B(X) is the

Borel  $\sigma$ -algebra on X. Thus, for each set  $B \in B(X)$ ,  $\psi_t(B)$  is the fraction of age t agents whose individual states lie in B as a proportion of all age t agents. These agents then make up a fraction  $\mu_t \psi_t(B)$  of all agents in the economy, where  $\mu_t$  is the fraction of age t agents in the economy. The distribution of individual states across age 1 agents is determined by the exogenous initial distribution of labor productivity since all agents start out with no assets. The distribution of individual states across agents age  $t = 2, 3, \ldots, N$  is then given recursively as follows:

$$\psi_t(B) = \int_X P(x, t-1, B) \, \mathrm{d}\psi_{t-1} \quad \text{for all} \quad B \in B(X).$$

The function P(x, t, B) is a transition function which gives the probability that an age t agent transits to the set B next period given that the agent's current state is x. The transition function is determined by the optimal decision rule on asset holding and by the exogenous transition probabilities on the labor productivity shock z.<sup>5</sup>

I focus on an equilibrium concept where factor prices are constant over time and where capital, labor, transfers, and government consumption are constant in per capita terms. In addition, the age-wealth distribution is stationary or unchanged over time. Equilibria are described as follows.

Definition. A stationary equilibrium is  $(c(x, t), a(x, t), r, w, K, L, T, G, \tau, \theta, b)$  and distributions  $(\psi_1, \psi_2, \dots, \psi_N)$  such that:

- 1. c(x, t) and a(x, t) are optimal decision rules.
- 2. Competitive input markets:

$$w = F_2(K, L)$$
 and  $r = F_1(K, L) - \delta$ .

3. Markets clear:

i) 
$$\sum_{t} \mu_{t} \int_{X} (c(x, t) + a(x, t)) d\psi_{t} + G = F(K, L) + (1 - \delta)K$$
,

ii) 
$$\sum_{t} \mu_{t} \int_{X} a(x, t) d\psi_{t} = (1 + n)K,$$

iii) 
$$\sum_{t} \mu_{t} \int_{X} e(z, t) d\psi_{t} = L.$$

<sup>&</sup>lt;sup>5</sup>The transition function is  $P(x, t, B) = \text{Prob}(\{z' \in Z: (a(x, t), z') \in B\}|z)$ , where the relevant probability is the conditional probability that describes the behaviour of the Markov process z.

4. Distributions are consistent with individual behavior:

$$\psi_{t+1}(B) = \int_X P(x, t, B) d\psi_t \quad \text{for } t = 1, \dots, N-1 \text{ and for all } B \in B(X).$$

- 5. Government budget constraint:  $G = \tau(rK + wL)$ .
- 6. Social security benefits equal taxes:  $\theta wL = b \left( \sum_{t=R}^{N} \mu_t \right)$ .
- 7. Transfers equal accidental bequests:

$$T = \left[ \sum_{t} \mu_{t} (1 - s_{t+1}) \int_{X} a(x, t) (1 + r(1 - \tau)) d\psi_{t} \right] / (1 - n).$$

A brief discussion of the equilibrium concept is in order. Equilibrium condition 1 says that agents optimize. Condition 2 says that factor prices are equal to marginal products. The first market clearing condition is that aggregate consumption plus asset holdings plus government consumption equals the current output plus the capital stock after depreciation. The second is that asset holdings are sufficient to keep the capital stock per capita constant over time. The last market clearing condition is just that the aggregate labor input per capita equals the labor input summed over the population. Equilibrium condition 4 says that the distribution of individual states over the population is consistent with the optimal decision rules implicit in the transition function. Equilibrium conditions 5 and 6 say that income taxes are sufficient to pay for government consumption and that social security taxes are sufficient to cover the benefits paid to agents past the retirement age. Thus, the social security system works on a pay-as-you-go basis. The remaining equilibrium condition is that lump-sum transfers equal accidental bequests. Thus, accidental bequests are fully taxed by the government and redistributed in equal amounts to all living agents each period. In a later section I will allow accidental bequests to be passed directly to living agents without taxation.

## 4. Calibration

## 4.1. Parameters of the model economies

The preference parameters  $(\beta, \sigma)$  are set using a model period of one year. The value of the discount factor is Hurd's (1989) estimate in economies where mortality risk is accounted for separately. In models without mortality uncertainty the discount factor is set equal to 0.994. This makes the discount factor in the certain lifetimes model equal to the average discount factor (including mortality risk) in the uncertain lifetimes model. The value of the coefficient of

Table 2	2
Model	parameters

β	σ	A	α	δ	N	R	St	n	τ	0	<u>a</u>
1.011	1.5	0.895944	0.36	0.06	56, 79	46	*	0.012	*	0.10	0, -w

<sup>\*</sup>See the discussion below.

relative risk aversion follows the estimates of the microeconomic studies reviewed by Auerbach and Kotlikoff (1987) and Prescott (1986).

The technology parameters  $(A, \alpha, \delta)$  are set as follows. The technology level A is normalized so that the wage equals 1.0 when the capital-output ratio equals 3.0 and the labor input per capita is normalized at 1.0. Capital's share of output  $\alpha$  is set following the discussion in Prescott (1986). The depreciation rate  $\delta$  is set to match the US depreciation-output ratio following the estimate of Stokey and Rebelo (1995).

The demographic parameters  $(N, R, s_t, n)$  are set using a model period of one year. Thus, agents are born at a real-life age of 20 (model period 1) and live up to a maximum real-life age of 98 (model period 79). Agents retire at a real-life age of 65 (model period 46). The survival probabilities are set according to the actuarial estimates in Jordan (1975). The population growth rate n is set to equal the average growth rate in the US from 1950–92 as reported in the Economic Report of the President (1994, Table B32).

Tax rates  $(\tau, \theta)$  are set as follows. The income tax  $\tau$  is set to match the average share of government consumption in output. The measure of government consumption is federal, state, and local government consumption as reported in the Economic Report of the President (1994, Table B1). As the average ratio was 0.195 from 1959–93, the tax rate is set at  $0.195/(1 - \delta K/Y)$ . The tax rate is greater than 0.195 as capital income is taxed only after subtracting depreciation. The social security tax rate equals the average for the 1980's of the contribution to social security programs as a fraction of labor income. The data on contributions come from Table M-3 of the Social Security Bulletin and exclude unemployment insurance contributions.

The credit limit (a) is set at 0 and for comparison purposes at -w. A credit limit of 0 means that agents are not allowed to hold net debt. A credit limit of

<sup>&</sup>lt;sup>6</sup>In the business cycle literature it is common to set the annual depreciation rate between 0.08 and 0.10. This allows the model to be calibrated to match the capital—output ratio and the average return to capital. These models are then used to investigate business cycle fluctuations. The idea here is to see if the model matches both the aggregate quantity and the distribution of wealth. Therefore, the depreciation rate is calibrated to match the depreciation—output ratio. Using this method, Stokey and Rebelo (1995) calculate that the depreciation rate is 0.06.

-w means that agents are allowed to borrow up to one year's average earnings in the economy.

## 4.2. Calibration of the labor endowment process

I investigate a labor endowment process with regression towards the mean in log labor endowment. This labor endowment or earnings process has been estimated in a number of studies. See Atkinson et al. (1992) for a review of this literature. In this earnings process  $y_t$  and  $\bar{y}_t$  are respectively the log labor endowment of an age t agent and the mean log endowment of age t agents and  $\varepsilon_t$  is a labor endowment shock that is distributed N(0,  $\sigma_{\varepsilon}^2$ ) and independently over time.<sup>7</sup>

$$y_t - \bar{y}_t = \gamma(y_{t-1} - \bar{y}_{t-1}) + \varepsilon_t.$$

There are a number of nice features of this earnings process. First,  $\bar{y}_t$  can be chosen to match the US age-earnings profile. Second, the process generates a log normal earnings distribution within each cohort. More precisely, if the log endowment of the initial cohort of agents is normally distributed (i.e.,  $y_1$  is distributed  $N(\bar{y}_1, \sigma_{y_1}^2)$ ), then the log endowment for the cohort will continue to be normally distributed over time. This is useful as the log normal distribution has long been used to describe the distribution of earnings. Third, time series evidence on individual earnings has been used to estimate the regression towards the mean parameter,  $\gamma$ . Finally, the variances  $(\sigma_{\varepsilon}^2, \sigma_{y_1}^2)$  can be selected to match some properties of the US earnings Gini.

I use data from two sources to calibrate the age-earnings profile  $(\bar{y}_1, \ldots, \bar{y}_N)$ . First, I use data on the median earnings of males in cross section from the Social Security Bulletin (1981).<sup>8</sup> The median earnings data are then multiplied by the labor force participation rates of males in each age group. Participation rates are taken from the Handbook of Labor Statistics (1985). The resulting earnings profile is shown in Fig. 1.

A number of studies have estimated the magnitude of idiosyncratic uncertainty at the household level. For example, Lillard and Willis (1978) estimate using data from the Panel Study of Income Dynamics the component of log earnings variance that is not explained by education, experience, race, fixed individual effects, and a host of other variables. They estimate that this variance

<sup>&</sup>lt;sup>7</sup>I originally considered a process with serially correlated shocks. Creedy (1985) shows that these shocks exihibit moderate negative serial correlation. One reason for negative serial correlation is the presence of a purely temporary component of earnings uncertainty. This paper abstracts from such temporary shocks. The work of Huggett and Ventura (1995) suggests that abstracting from these shocks is not critical for issues of wealth concentration.

<sup>&</sup>lt;sup>8</sup>If earnings are log normally distributed, then mean log earnings and median earnings are related as follows: median earnings = exp{mean log earnings}.

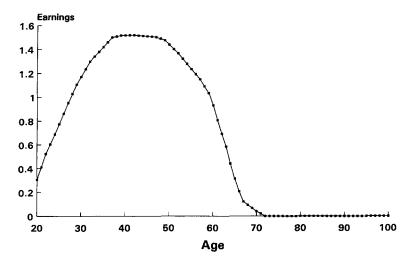


Fig. 1. Earnings profile (ratio to overall mean).

is 0.071. Carroll (1992) reports similar results and argues that the majority of the variance is due to true stochastic variation rather than measurement error. Therefore, in the baseline model I choose  $\sigma_{\varepsilon}^2 = 0.045$  and in Huggett (1995) investigate the sensitivity of the results to higher and lower values. This value implies that a one standard deviation shock increases or decreases earnings by about 20 percent.

The remaining parameters of the earnings process are the variance of log earnings of age 1 agents and the regression towards the mean parameter. I use the following considerations to set these two parameters. First, Henle and Ryscavage (1980) calculate that the US earnings Gini for men averaged 0.42 in the period 1958–77. Second, a number of studies document that measures of earnings inequality for cohorts of agents increase over time; see Creedy and Hart (1979) and Shorrocks (1980). Third, Lillard (1977) and Shorrocks (1980) have estimated the earnings Gini for young agents at 0.254 and 0.268 respectively. I take these estimates as lower bounds as they are based on relatively small samples and only include agents with nonzero earnings in the sample period. Fourth, Atkinson et al. (1992) report that estimates of the regression towards the mean parameter  $\gamma$  vary from 0.65 to 0.95 in annual data.

Based on these considerations, I choose  $\sigma_{y_1}^2 = 0.38$  and set the regression towards the mean parameter to match the overall earnings Gini in the US. Given that the earnings variance is set at  $\sigma_{\varepsilon}^2 = 0.045$ , this implies that the regression parameter is  $\gamma = 0.96$  which is slightly above the estimates in the literature. With these choices the earnings Gini is 0.33 for 20-year-olds and increases monotonically to 0.41 for 65-year-olds. The overall earnings Gini in

the model is then 0.42 as cohorts differ in their average earnings levels at a point in time. The top 1, 5, 10, and 20 percent of wage earners then earn respectively 6.0, 19.2, 30.6, and 47.4 percent of total earnings. 10

For computational reasons, the earnings process just specified must be approximated with a finite number of states. The procedure is as follows. I define a Markov process z, where  $z_t = (y_t - \bar{y}_t)$ , and then approximate this process using 18 possible states for z:

$$z_t = \gamma z_{t-1} + \varepsilon_t.$$

The states z are equally spaced and range from  $-4\sigma_{y_1}$  to  $4\sigma_{y_2}$ . In addition, to take account of extreme earnings shocks, the shock can also take on the value  $6\sigma_{y_1}$ . Agents receiving the highest earnings shock receive 40 times the median earnings of agents in their cohort. The transition probabilities between states are calculated by integrating the area under the normal distribution conditional on the current value of the state. In summary, the labor endowment process is given by  $e(z, t) = e^{(z_1 + \hat{y}_2)}$ , where  $z_t$  is a finite Markov chain.

#### 5. Results

This section examines the degree to which calibrated life-cycle economies match the US wealth distribution observations reviewed in Section 2. The notion of wealth used in the model economies is net asset holdings, a. This choice reflects the fact that the concept of wealth typically measured in the US data is one that capitalizes the value of physical capital but not human capital.<sup>12</sup> All the details of how the results reported here are computed are described in the Appendix.

## 5.1. Capital-output ratios

Tables 3 and 4 compare the US economy and the model economies along a number of dimensions. Focus first on the capital—output ratio. There is

<sup>&</sup>lt;sup>9</sup>This value was calculated for the working age population in the economy where the population grows at 1.2 percent and where agents have lifetime uncertainty. The working ages in the model economy are ages 1–46 which corresponds to ages 20–65 in the US economy.

<sup>&</sup>lt;sup>10</sup>These figures are in line with the US earnings distribution. See Wolff (1983).

<sup>&</sup>lt;sup>11</sup>The 18 state approximation puts a little more weight in the upper tail than the theoretical distribution. In particular, the top 1, 5, 10, and 20 percent of earners hold 6.5, 21.1, 33.2, and 47.5 percent of earnings.

<sup>&</sup>lt;sup>12</sup>Clearly, the concept of wealth is highly dependent on market structure and institutions. As an additional example, note that the current value of future net social security benefits is also not counted as wealth.

Table 3 Wealth distribution (risk aversion coefficient  $\sigma = 1.5$ )

Credit Earnings limit shock $\underline{a}$ $\sigma_{\varepsilon}^2$	_	ıgs	Transfer	XX7 1.1	Percer the to	ntage wea	Zero or	
	K/Y	wealth ratio	Wealth Gìni	1%	5%	20%	negative wealth (%)	
US economy		3.0	0.78-1.32	0.72	28	49	75	5.8-15.0
Certain	lifetimes							
0.0	0.00	2.9	0.0	0.47	2.4	11.6	42.8	14.0
- w	0.00	2.8	0.0	0.54	2.7	12.7	46.6	25.0
0.0	0.045	3.2	0.0	0.70	10.8	32.4	68.9	19.0
-w	0.045	3.1	0.0	0.74	11.1	33.8	72.3	24.0
Uncerta	in lifetimes							
0.0	0.00	3.1	1.03	0.46	2.5	11.7	42.8	11.0
- w	0.00	3.0	1.07	0.49	2.6	12.1	44.3	12.0
0.0	0.045	3.4	0.84	0.69	10.9	32.9	70.0	17.0
w	0.045	3.2	0.89	0.76	11.8	35.6	75.5	24.0

Table 4 Wealth distribution (risk aversion coefficient  $\sigma = 3.0$ )

Credit Earnings limit shock $\underline{a}$ $\sigma_{\varepsilon}^2$		Transfer	Wealth Gini	Percer the to	itage wea	Zero or		
	K/Y	wealth ratio		1%	5%	20%	negative wealth (%)	
US econ	iomy	3.0	0.78-1.32	0.72	28	49	75	5.8-15.0
Certain	lifetimes							
0.0	0.00	2.3	0.0	0.51	2.7	13.0	46.1	21.0
w	0.00	2.0	0.0	0.62	3.3	14.7	52.5	29.0
0.0	0.045	2.9	0.0	0.66	10.5	32.0	66.6	3.0
- w	0.045	2.8	0.0	0.73	11.4	34.0	73.1	23.0
Uncertai	in lifetimes							
0.0	0.00	2.5	2.54	0.50	2.6	12.6	45.3	21.0
- w	0.00	2.3	4.30	0.61	3.1	14.3	51.1	29.0
0.0	0.045	3.0	1.28	0.72	12.1	35.7	71.7	19.0
- w	0.045	2.8	1.75	0.84	13.8	40.4	80.2	40.0

considerable variation in this ratio. Part of the variation occurs as a result of changes in the credit limit. When the credit limit is relaxed by one year's average earnings, the capital—output ratio declines as agents can hold savings in some form other than physical capital. The effect of lowering the credit limit is to

decrease the capital—output ratio by 3 to 13 percent. These magnitudes could be interpreted as the effect on steady state capital arising from improvements in financial intermediation. There would be an even greater fall in capital—output ratios in most of the economies if the credit limit were eliminated while still maintaining the terminal wealth constraint. For example, eliminating the credit limit in the certain lifetimes economy in Table 4 with no earnings uncertainty further reduces the ratio from 2.0 to 1.9.

A large part of the variation in the capital-output ratio in Tables 3 and 4 is due to earnings uncertainty. Earnings uncertainty adds a precautionary savings motive to the model. The partial equilibrium literature on precautionary savings shows (see Leland, 1968; Sandmo, 1970) that savings increase with added uncertainty when marginal utility is a convex function. In steady state the gross savings rate (S/Y) in the model economy is related to the capital-output ratio as follows:  $S/Y = (n + \delta)K/Y$ . Thus, the general equilibrium effect of adding earnings uncertainty is to increase the gross savings rate by 1-2 percent of output when risk aversion is  $\sigma = 1.5$ . Savings increase by 4-5 percent of output when risk aversion is  $\sigma = 3.0$ . Clearly, the savings effect is much stronger when agents are more risk-averse. The magnitude of the general equilibrium effects of earnings uncertainty calculated here are within the range calculated by Aiyagari (1994) using the infinitely-lived agent abstraction.

## 5.2. Transfer wealth

Recall from the discussion in Section 2 that aggregate wealth can be divided into a transfer and a life-cycle wealth component. Within the model economies, this can also be done as follows. First, consider the period budget constraint for an age t agent that is provided below. The budget constraint can then be rewritten by recursively substituting the budget constraint into itself. This produces the second equation below. The first term in the equation is the Kotlikoff-Summers definition of life-cycle wealth for an individual agent, whereas the second term is transfer wealth. At a point in time the aggregate transfer wealth in the economy is then given by the third equation below. This is simply the sum of the transfer wealth of each agent alive at a point in time in the economy.

$$a_{t+1} = a_t(1 + r(1 - \tau)) + (1 - \theta - \tau)e(z, t)w + b_t - c_t + T,$$

$$a_{t+1} = \sum_{j=0}^{t-1} \left\{ (1 - \theta - \tau)e(z, t - j)w + b_{t-j} - c_{t-j} \right\} (1 + r(1 - \tau))^j + \sum_{j=0}^{t-1} T(1 + r(1 - \tau))^j,$$

Aggregate transfer wealth =  $\sum_{t} \mu_{t} \sum_{j=0}^{t-1} T(1 + r(1-\tau))^{j}$ .

Tables 3 and 4 report the aggregate transfer wealth in the economy as a fraction of total wealth. It is clear that the model economies with certain lifetimes have no transfer wealth as there are no intergenerational transfers aside from social security transfers. In contrast, the model economies with uncertain lifetimes have no difficulty producing magnitudes of transfer wealth that equal or exceed the estimates for the US economy listed in Tables 3 and 4.

Transfer wealth in the model economy is determined by demographics, returns to capital, and the pattern of transfers received at different stages of the life cycle. The magnitudes of transfer wealth in the model economies do not arise because of counterfactual assumptions on demographics. The reader should recall that population growth rates and mortality probabilities are chosen to match US data. Neither do the magnitudes arise from a systematic bias in the rate of return to capital. All of the uncertain lifetime economies in Table 3 have an after-tax return that is at or below the 4.5 percent average after-tax return calculated for the US economy by Kotlikoff and Summers (1981, p. 715). At the same time, two of the economies in Table 4 do have very large amounts of transfer wealth arising from the fact that the rate of return to capital is well above the US level. The final determinate of transfer wealth is the path of transfers received over the life cycle. A number of assumptions could be important here. One of these is that there is no growth in output per person in the economy. Given that this is a counterfactual assumption, it would clearly be interesting to conduct a sensitivity analysis along this dimension. Another is the assumption of equal division of accidental bequests. It is possible that other ways of passing these bequests could dramatically affect the size of transfer wealth. I leave these topics for future research.

#### 5.3. Wealth distribution

Features of the wealth distribution in the model economies are also described in Tables 3 and 4. The tables show that the model economies are capable of generating the US wealth Gini coefficient. However, it is clear that the model economies generate the US Gini by generating a high fraction of zero and negative wealth-holders and not by concentrating enough wealth in the extreme upper tail of the wealth distribution. I now review these results in detail.

The results show that the top 1 percent of wealth-holders hold 2–3 percent of total wealth in the economies without earnings uncertainty. The results in Atkinson's (1971) work correspond to the first two rows in Tables 3 and 4. The basic life-cycle model does not generate the 28 percent of wealth held by the top 1 percent in the US economy. The basic model also fails to generate the 75 percent held by the top 20 percent in the US economy. The intuition for this result is simply that there are no multi-millionaires in a model in which all agents within an age group hold the same wealth. The addition of lifetime

uncertainty does not help to concentrate wealth in the upper tail of the wealth distribution for exactly the same reason.

In contrast, adding earnings inequality does improve the match between theory and observation. The top 1 percent now hold 10–14 percent of total wealth. The top 20 percent now hold 67–80 percent of total wealth as compared to the 75 percent that is held by the top 20 percent in the US. In summary, the model economies with earnings uncertainty can match the US facts on the fraction of wealth held by the top 20 percent while generating a little less than half the wealth held by the top 1 percent in the US.

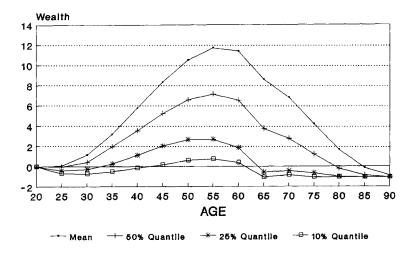
The model economies generate a high fraction of agents with zero or negative wealth holdings. As will be seen in the next section, the low wealth-holders are mainly young and old agents, although there are agents of all ages at low wealth levels (see Fig. 2 in the next section). Young agents tend to hold little wealth because they start out with zero wealth and because they expect to have much higher earnings in the future. Therefore, consumption smoothing dictates that most young agents hold little wealth. The presence of earnings uncertainty does little to change this. The very old also tend to hold little wealth. There are several reasons for this. One of these reasons is that old agents discount the future at a high rate as their survival probabilities are decreasing in age. This implies that they eventually have a declining consumption profile and hence need little wealth to finance their consumption. Other reasons for low wealth holding among the aged are described in the next subsection.

The fraction of agents with zero or negative wealth is at or above US levels even when agents are not allowed to go into debt. This does not seem to be a property of the infinitely-lived agent model. In the infinitely-lived agent model relatively few agents tend to be exactly at the corner of the borrowing constraint.<sup>13</sup> Thus, life-cycle considerations seem to be important for generating low wealth levels.

## 5.4. Wealth profiles

Fig. 2 shows both the mean and various quantiles of the wealth distribution within age groups. The figure corresponds to the model economy with earnings and lifetime uncertainty, where  $\sigma=1.5$  and the credit limit is set at  $\underline{a}=-w$ . In Fig. 2 many agents past the retirement age hold negative wealth levels. This is possible because they receive a social security annuity that they can use to pay back their debt by the terminal period. The pattern of asset holdings for agents past the retirement age has a prominent feature. As agents age an increasing fraction are at the borrowing constraint. This occurs for three reasons. First, old agents discount their last few periods of life with a relatively low effective

<sup>&</sup>lt;sup>13</sup>Huggett (1993) and Aiyagari (1994) generate this result.



## **Uncertain Lifetimes**

Fig. 2. Wealth profiles.

discount factor due to their decreased survival probability. This means that agents eventually prefer a decreasing consumption profile and therefore run their assets down to low levels. <sup>14</sup> Second, this effect is strengthened further because agents receive a social security annuity that cannot be sold in the market. This means that agents reduce their nonsocial security wealth first. Finally, these agents no longer have a precautionary savings motive as they do not receive labor income and are not subject to health uncertainty or other shocks that could motivate precautionary asset holdings in old age.

The age—wealth distribution in the model economy can be compared to the cross-sectional distribution in the US economy. The data for the US economy is presented in Fig. 3. The data is from Radner (1989) and is based on the 1984 Survey of Income and Program Participation (SIPP). Figs. 2 and 3 are similar in a number of respects. First, the fact that the median lies below the mean indicates that the wealth distribution within each age group is skewed to the right in both the model economy and the US economy. Second, a high fraction of young agents hold zero and negative wealth in both economies. Finally, a high fraction of agents in all age groups hold either very little or zero wealth in both economies.

Diamond and Hausman (1984) describe the low wealth-holding of households in their prime earnings years. They calculate that 7 percent of their sample of

<sup>&</sup>lt;sup>14</sup>Leung (1994) argues that in continuous time models agents will run down assets to zero before the terminal period.

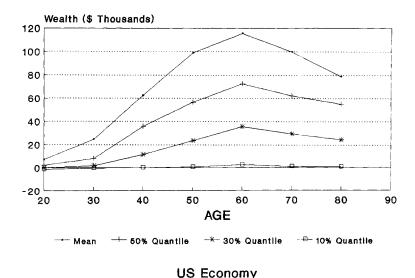


Fig. 3. Wealth profiles.

men aged 45–59 held negative net wealth. Diamond and Hausman (1980, p. 84) state: 'The presence of so little wealth accumulation is, itself, a reflection on the limitations of at least the strongest versions of the life-cycle theory'. It is therefore interesting to note that the life-cycle economies considered here introduce earnings variation as the sole source of heterogeneity within an age group. Nevertheless, the model economies generate a surprising amount of low wealth-holdings even among agents aged 45–59. In Fig. 2 the peak wealth level for the 10 percent quantile occurs at age 55 at a wealth level of 1.2. Since the output per person in the model economy is 1.63, this level corresponds to a maximum wealth level of about 70 percent of average annual income in the economy. Thus, it seems that even relatively simple modifications of the basic life-cycle model can come close to these low wealth-holding observations.

One of the main reasons why agents aged 45–59 hold so little wealth in this model is that social security benefits are independent of earnings history. Thus, agents with low earnings are anticipating very generous benefits and therefore carry low asset levels into retirement. The opposite occurs for agents with very high earnings. They realize that social security benefits will be a small fraction of current earnings and therefore carry high asset levels into retirement. It would be interesting to see how sensitive the low asset holding results of this paper are to over estimating the redistribution that goes on within an age group through the social security system. This could be done by modeling more carefully the

link between an agent's earnings history and the level of social security benefits received.<sup>15</sup>

## 5.5. Wealth concentration within age groups

A notable feature of the basic life-cycle model is that all agents within an age group hold the same level of wealth. This is not a feature of the US data. In fact, in the US the Gini coefficient within an age group is similar to the level of the overall wealth Gini. This fact is documented for the United States by Projector and Weiss (1966) and Greenwood (1987). Atkinson (1971) obtains a similar result for Great Britain.

The patterns in the model economies are compared to the US data in Figs. 4 and 5. In these figures the US data comes from Projector and Weiss (1966). First, note that there is a slight U-shape in the age—Gini profile in that wealth tends to be more concentrated among the youngest and oldest age groups than among the middle-age groups. Even though the U-shape may not prove to be a robust fact of US wealth distribution, it is interesting to consider the shape of the age—Gini profile in the model economies. Figs. 4 and 5 show that the model

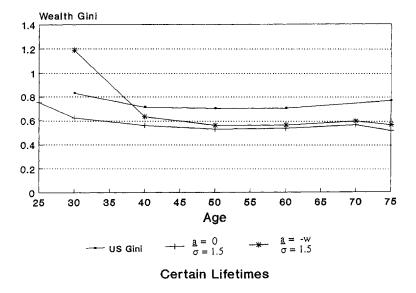


Fig. 4. Gini coefficients within age groups.

<sup>&</sup>lt;sup>15</sup>Hubbard et al. (1995) argue that low levels of wealth holding can also arise in life-cycle models when the receipt of social insurance payments is conditioned on the level of wealth holding.

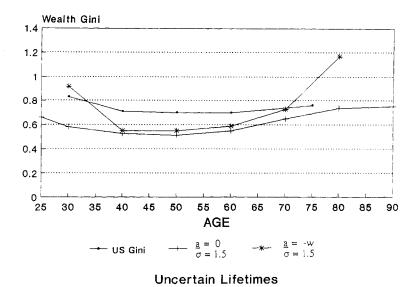


Fig. 5. Gini coefficients within age groups.

economies with uncertain lifetimes generate a U-shape, whereas the economies with certain lifetimes do not. The explanation for the U-shape in the economies with uncertain lifetimes seems to be related to the fact that the very young and the very old age groups both have a high fraction of agents holding zero or negative wealth. The basic reasons for why these groups tend to hold little wealth were discussed in the previous two subsections.

Consider now the level of the Gini coefficient within age groups. Figs. 4 and 5 show that the model economies generate a substantial amount of heterogeneity within age groups. This heterogeneity is due solely to differences in earnings over time between agents in the same age group. This is true as all agents start out with no assets and as bequests are taxed away and returned in equal amounts to all living agents.

The fact that the model economies generate substantial heterogeneity is interesting as the previous focus in the literature was on the importance of inheritance for explaining wealth differences within age groups. Clearly, the message is not that inheritance is unimportant but rather that the inequality in the earnings distribution is capable of explaining a substantial portion of observed wealth inequality within age groups. In fact, one could conjecture that adding inheritance in the form of a bequest at the time of death could generate even more within-age-group heterogeneity. One relatively simple way of adding such an accidental bequest to the model is simply to allow estates to be passed directly to living agents instead of taxing them fully and then redistributing them

to agents each period as an annuity. I have experimented with this type of bequest and discovered that the degree of wealth inequality within an age group and in the overall distribution does not increase. This is true even though bequests are received once in a lifetime and in very unequal amounts.

There are two main reasons why these accidental bequests don't matter very much for wealth inequality. The first is that fewer agents actually hold low wealth levels when estate taxation is eliminated. Thus, agents lose a source of annuity income and replace this with increased asset holding. The second is that the increase in wealth levels held by the wealthiest households is not enough to counteract the overall increase in wealth holding. Perhaps the key reason why wealth is not more concentrated is that in the model described above a large bequest is not any more likely to be received by a wealthy agent than by a poor agent in the same age group. This result suggests that deeper models of the family may be needed if bequest behavior is to matter for wealth concentration.

#### 6. Conclusion

This paper investigates the degree to which two modifications of the basic life-cycle model produce a wealth distribution that more closely resembles features of the US wealth distribution. The two modifications considered are the presence of earnings and lifetime uncertainty and the absence of markets for insuring this uncertainty. The main findings of this investigation are as follows. First, model economies with these features are able to replicate measures of both the aggregate wealth and the transfer wealth in the US economy. Second, the model economies produce a number of the features of the distribution of wealth in the US. In particular, the models can match the US wealth Gini and the fraction of wealth held by the top 20 percent of US households. However, there is still a gap between theory and observation. The model economies examined do not generate all of the concentration of wealth in the upper tail of the distribution. In particular, the model economies generate only about half of the wealth held by the top 1 percent in the US. In addition, although the models produce a significant fraction of the wealth inequality within age groups, they do not explain all of the within-age-group inequality.

In the future it would be interesting to investigate the following issues. First, there is the question of whether the concentration of wealth in the extreme upper tail of the wealth distribution is sensitive to alternative specifications of the

<sup>&</sup>lt;sup>16</sup>Bequests were received once in a lifetime. The date of receipt was random and the conditional probability of receiving a bequest given that one had not yet been received was the probability that agents 30 years older had died. Finally, the distribution from which bequests were drawn was the same for all agents and was the actual distribution of estates left by agents exiting the economy.

earnings process. This may be important as the model economies nearly match the fraction of wealth held by the top 20 percent but fail to divide it correctly within the extreme upper tail. I have examined (Huggett, 1995) versions of the earnings process considered in this paper with both more and less regression to the mean, while at the same time adjusting the variance of the shocks to remain consistent with the inequality in the US earnings distribution. These economies do not produce substantially more wealth concentration in the upper tail. However, alternative specifications may show otherwise. Perhaps the more likely case is that wealth concentration in the upper tail is due to features that were abstracted from in this paper. Thus, one could investigate other ways of getting rich. Among the possibilities to consider are deeper models of the family (see Laitner, 1992) and modeling entrepreneurs (see Quadrini, 1995).

Second, it would be interesting to investigate some of the other implications of the model structure investigated here. One direction to pursue is to see if the model can account for some of the stylized facts of the distribution of savings. One interesting stylized fact is that high-income households save on average a much higher fraction of income than do low-income households in US cross-section data. This particular fact is addressed by Huggett and Ventura (1995) who find that models of the type considered here are capable of replicating some of the magnitudes of average savings rates among different income groups observed in US cross-section data.

## 7. Appendix

The algorithm for computing equilibria in economies with certain lifetimes is as follows:<sup>17</sup>

- 1. Choose K.
- 2. Set w and r according to equilibrium condition 2.
- 3. Given w and r, find a(x, t) by solving the dynamic programming problem.
- 4. Calculate the wealth distribution and the new capital stock K'.
- 5. If K is approximately equal to K' stop. Otherwise adjust K and repeat step  $2.^{18}$

The algorithm for solving the dynamic programming problem is identical to the algorithm described in Huggett (1993). The basic idea is to approximate the true decision rules with piecewise-linear functions. The approximate decision

 $<sup>^{17}</sup>$ In economies with lifetime uncertainty the algorithm remains the same except that iterations are on both capital (K) and transfers (T) rather than just capital.

<sup>&</sup>lt;sup>18</sup>To make the algorithm stable the value of the capital stock  $(K_{t+1})$  used on iteration t+1 was partially adjusted (i.e.,  $K_{t+1} = aK_t + (1-a)K'_t$  where 0 < a < 1).

rules satisfy the Euler equation exactly on an arbitrarily specified set of grid points. Between grid points the decision rules are given by a linear interpolation. This is reasonable as the true decision rules are increasing in the level of the risk-free asset. The Euler equation to the dynamic programming problem is given by

$$u'(a(1 + r(1 - \tau)) + (1 - \theta - \tau)e(z, t)w + T + b_t - a')$$

$$\geq \beta s_{t+1} E[V_1(a', z', t+1)|x]$$

$$= \beta s_{t+1} E[V_1(a', z', t+1)|x] \quad \text{if} \quad a' > a.$$

Therefore, at a grid point (a, z) the value of the decision rule a(a, z, t) is given by the value of a' that solves the above Euler equation. It can be shown that the derivative of the value function at age t is given by

$$V_1(a, z, t) = u'(a(1 + r(1 - \tau)) + (1 - \theta - \tau)e(z, t)w + T$$
$$+ b_t - a(a, z, t))(1 + r(1 - \tau)).$$

Thus, these two equations form a recursive algorithm for generating the optimal decision rule at each age, given that an agent's asset holdings are zero at age N, a(x, N) = 0.

To solve the dynamic programming problem I put a uniform grid on the space of asset holdings. The number of grid points varies between as little as 41 for economies without earnings uncertainty to as many as 301 for the economies with earnings uncertainty. The distance between grid points is 0.25 units of output in economies without earnings uncertainty and 0.40 units of output in economies with earnings uncertainty. This tends to be about 25 to 40 percent of average earnings per person in the model economies.

The wealth distributions  $(\psi_1, \psi_2, \dots, \psi_N)$  can be calculated in a number of ways. One method is to simulate draws of the uncertainty in the economy for individual agents starting from birth and to use the optimal decision rule a(x, t) to determine how the individual state x varies over time. With a large enough number of draws the distributions could be approximated. The method used here is different. It is simply to iterate on equilibrium condition 4. This condition is the law of motion for the age—wealth distribution. Since the initial distribution  $\psi_1$  is known, the law of motion can be used to generate the subsequent distributions. The details for carrying this out are described in Huggett (1993).

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