MACROECONOMIC FLUCTUATIONS WITH HANK & SAM: AN ANALYTICAL APPROACH

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Abstract
Recently developed HANK models, which combine Heterogeneous Agents and New Keynesian frictions, have had a considerable impact on macroeconomics. However, due to the complexity of such models, the literature has focused on numerically solved models and therefore little is known about their general properties. This paper presents a tractable HANK model which integrates Search and Matching (SAM) frictions in the labor market. The model features endogenous idiosyncratic earnings risk which may be procyclical or countercyclical. When this risk is countercyclical, which we argue is the empirically plausible case, there is downward pressure on real interest rates in recessions due to a precautionary saving motive. We show that in this setting (a) the economy may get stuck in a high-unemployment steady state, (b) the Taylor principle is insufficient to eliminate local indeterminacy of the intended steady state, and (c) nominal rigidities and incomplete markets are complementary in terms of amplifying the impact of shocks on the macro economy. (JEL: E10, E21, E24, E30, E52)

Keywords: Sticky prices, incomplete asset markets, matching frictions, multiple equilibria, amplification.

1. Introduction

The recently developed HANK literature, which merges Heterogeneous Agents (HA) models and New Keynesian (NK) models, has already had a considerable impact. Its results speak to key issues in macroeconomics, such as the transmission of conventional monetary policy, the impact of forward guidance and credit policies, and the design of automatic stabilizers and fiscal policy. Moreover, this new class of models has been applied to understand the financial crisis and the Great Recession.[1]

However, so far the HANK literature has relied mostly on insights from calibrated models which are solved numerically. Therefore, little is known about the generality of the results and how they relate to the deeper structure of the economy. This paper addresses this issue. We formulate a HANK model and show that its properties can be characterized analytically when making simplifying assumptions. We exploit this tractability to provide insights into the local and global determinacy of equilibria, the conduct of monetary policy, and the impact of interactions between heterogeneity and market frictions on short- and long-run macroeconomic outcomes.

The model features frictions in goods, labor and financial markets. In the goods market, firms are monopolistically competitive and are subject to nominal rigidities in price setting as in the NK tradition. As a result, macroeconomic outcomes are affected by changes in the nominal interest rate set by the central bank. The labor market is characterized by Search and Matching (SAM) frictions in the Diamond-Mortensen-Pissarides tradition.[2] Firms hire by posting vacancies, whereas households are either employed or unemployed. In the former case, a household receives wage income determined via Nash bargaining. When unemployed, a household receives an endowment of goods and may choose to search for employment or exit the labor market. Because of the SAM frictions, job prospects are uncertain which exposes households to idiosyncratic income risk. Finally, financial markets are incomplete, in the sense that households cannot fully insure against this income risk. Instead, they attempt to self insure through savings in risk free bonds or in equity. Due to the combination of heterogeneous agents, nominal rigidities and search and matching frictions we dub this model "HANK&SAM."

Our aim is to characterize key aspects of the equilibrium explicitly.[3] We make three assumptions which allow us to accomplish this aim. First, we assume that

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1. See e.g. Auclert (2019), Bayer, Pham-Dao, Luetticke and Tjaden (2019), Beaudry, Galizia and Portier (2017), Berger, Dew-Becker, Schmidt and Takahasi (2016), Braun and Nakajima (2012), Challe, Matheron, Ragot and Rubio-Ramirez (2017), den Haan, Rendahl and Riegler (2018), Heathcote and Perri (2018), Gornemann, Kuester and Nakajima (2016), Guerrieri and Lorenzoni (2017), Kaplan, Moll and Violante (2018), Kekre (2016), Luetticke (2015), McKay, Nakamura and Steinsson (2016), McKay and Reis (2016ab), and Ravn and Sterk (2017).

2. Due to the search and matching frictions, our model includes involuntary unemployment similar to recent contributions to the NK literature such as Gertler and Trigari (2009), Blanchard and Galí (2010), Ravenna and Walsh (2011), and Christiano, Eichenbaum and Trabandt (2016).

3. In this respect, the paper complements Clarida, Galí and Gertler (1999) who characterize equilibrium properties of the standard NK model based on a highly intuitive, three-equation version.

households cannot go short on equity and that they face a borrowing constraint on bonds which allows only employed households to borrow. Secondly, we assume that there are differences across households in labor market productivity and in the returns obtained from investment in firm equity. Third, we assume that there are two types of households. The first type has zero productivity if working on the market but can participate in the equity market, whereas the second type instead is unable to obtain returns from equity investment but has positive productivity when taking a job. In combination, these assumptions imply that, in equilibrium, firms are owned by capitalists who drop out of bond and labor markets, while workers hold no equity and are either employed or unemployed. Moreover, in equilibrium all households end up consuming their income period-by-period and the real interest rate has to satisfy the Euler equation of employed workers.[4] Consequently, the equilibrium features inequality in outcomes but the wealth distribution is degenerate, which enables an analytical characterization of the properties of the model.

We derive four main sets of results. First, we show that the HANK&SAM model features an *endogenous earnings risk wedge,* which derives from the combination of a precautionary saving motive on the part of workers and endogeneity of labor market transitions due to the matching framework. This wedge appears in the Euler equation of the employed workers, and is a source of fluctuations in aggregate goods demand and in the real interest rate. Werning (2015) similarly highlights such a wedge in an analytical "aggregated" Euler equation, but does not model explicitly how it is determined in equilibrium.

We demonstrate how, with SAM frictions, the endogenous earnings risk wedge is pinned down by the tightness of the labor market, which determines both labor market transition rates and real wages. Because of these two forces, the precautionary saving motive may be either procyclical (because wages rise in booms), or countercyclical (because unemployment risk is higher in recessions). We argue that countercyclicality of the wedge is the empirically plausible case and that this leads to amplification of shocks. Intuitively, when the countercyclical force dominates, firms post more vacancies in booms thereby lowering the risk of unemployment, which weakens the precautionary saving motive, and thus stimulates goods demand inducing firms to post more vacancies etc. (and vice versa in a recession).[5] When earnings risk is acyclical, either because the two forces exactly cancel out or because earnings risk is exogenous, the model's implications for aggregate fluctuations are similar to those of two-agent New Keynesian models (see Debortoli and Galí, 2017).

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4. Our setup extends earlier work deriving tractability from assumptions on the borrowing limit, see Krusell, Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), McKay and Reis (2016b), Bilbiie (2017) and Ravn and Sterk (2017). In these models, agents are unable to borrow. In our analysis, the employed households – who end up pricing the bonds – are in principle able to borrow but choose voluntarily not to do so.

5. On the other hand, a tight labor market pushes up wages, which means that employed workers suffer a higher income drop when losing their job, strengthening the precautionary saving motive. This is the procyclical force referred to above.

Secondly, we study which steady states can arise. Similarly to the NK model, the economy may hover around an intended steady state where the central bank's target for inflation (and other variables) is satisfied, or around a low-activity liquidity trap where the interest rate is constrained by a lower bound (as in Benhabib, Schmitt-Grohé and Uribe (2001, 2002)). However, the equilibrium real interest rate in the intended steady state depends on policy parameters and inflation may be positive in the liquidity trap steady state. These implications both contrast with the standard NK model, in which the long-run real interest rate is determined by preferences and the liquidity trap has to be deflationary. Moreover, both implications follow from the precautionary saving motive through which the steady-state real interest rate depends on labor market tightness.

Moreover, the interactions between HANK and SAM may give rise to a third steady state: *an unemployment trap.* This steady state arises when endogenous earnings risk is sufficiently countercyclical and it features high unemployment and low (but positive) inflation. It arises because of the amplification mechanism described above, by which expectations of weak labor and goods demand reinforce each other, to the point that firms may entirely stop hiring if stabilization policy fails to respond sufficiently aggressively.

Third, we study determinacy properties of the steady states. We show that that when the endogenous earning risk wedge is countercyclical, the "Taylor principle" (see e.g. Woodford, 2003, Chapter 2) may fail to guarantee local determinacy of the intended steady state. In this case, policy must be even more aggressive than prescribed by the Taylor principle in order to rule out self-fulfilling fluctuations. Intuitively, monetary policy should not only rule out local indeterminacy due to nominal rigidities, but also address the amplification mechanism described above. Importantly, this mechanism derives from the risk channel that is absent in the representative-agent New Keynesian models and in two-agent versions of that model, as in for example Galí, López-Salido and Vallés (2003), Bilbiie (2008), and Broer et al. (2019). Additionally, we show that the unemployment trap is locally determinate under a standard interest rate rule which responds more than one-for-one to inflation. Around this steady state, the monetary policy rule determines the rate of inflation, but has no grip on unemployment.

Fourth, we demonstrate that, in the presence of endogenous risk, nominal rigidities and market incompleteness are complementary in their influence on how the economy responds to shocks in the vicinity of the intended steady-state: Stronger nominal rigidities increase the impact of market incompleteness and vice versa. This interaction between the frictions is important for understanding why this new literature may have fundamentally new implications. We also show that under some circumstances the interactions may become sufficiently strong that technology shocks become inflationary (because they expand demand) and have larger real effects when nominal rigidities are stronger, results that both are in contrast to the standard NK model.

While our analysis rests on the analytical convenience produced by the simplifying assumptions that we make, we believe that the insights are general and apply

to models with a non-degenerate wealth distribution and with more complicated asset structures, with fiscal policy etc. It goes beyond the purpose of the paper to demonstrate robustness in a fully fledged setting but we do show how the introduction of capital accumulation and other extensions do not materially change our results.

Acharya and Dogra (2019) instead achieve tractability by assuming CARA utility and, importantly, by abstracting from occasionally binding borrowing constraints. An important difference is that in their setting, the Marginal Propensities to Consume (MPCs) are the same across households, whereas in ours they differ strongly. This aspect of our analysis is important: MPC heterogeneity is often emphasized as a central property that differentiates HANK models from representative-agent NK models, see e.g. Kaplan Moll and Violante (2018). Specifically, when MPCs differ across households, redistribution across households can have large aggregate effects. Another key difference is that they postulate a reduced-form relation between income risk and aggregate output, whereas we model this relation structurally, via SAM frictions. That said, focusing on the importance of the cyclicality of income risk and equilibrium determinacy, Acharya and Dogra (2019) do confirm our results. We go further in also considering the impact on long-run equilibria and on the amplification of shocks.

The remainder of this paper is organized as follows. Section 2 presents the model. In Sections 3 and 4 we study, respectively, steady states and local fluctuations. Section 5 focuses on empirics and extensions.

2. The Model

The economy consists of households who consume, save and work, firms which hire labor, produce output and set prices, and a monetary authority which sets the short term nominal interest rate. Firms are monopolistically competitive and face nominal price rigidities as in the NK tradition, the labor market is frictional as in the Diamond-Mortensen-Pissarides tradition, and households face incomplete asset markets as in the Aiyagari-Bewley-Huggett tradition.

2.1. Environment

Preferences: A continuum of mass 1 of infinitely lived single-member households, indexed by $i \in (0,1)$, derive utility from a basket of non-durable goods, $c_{i,s}$, and maximize the expected discounted present value of their utility streams:

$$\mathcal{V}_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{c_{i,s}^{1-\mu} - 1}{1 - \mu} - \zeta n_{i,s} \right), \tag{1}$$

where $c_{i,s}$ is a constant elasticity of substitution aggregator of a basket of consumption goods, c_i^j:

$$c_{i,s} = \left(\int_j \left(c_{i,s}^j \right)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}, \tag{2}$$

$\gamma > 1$ is the elasticity of substitution between goods varieties, $\mathbb{E}_t x_s$ is the date t conditional expectation of some variable x_s, $\beta \in (0,1)$ is the subjective discount factor, $\mu > 0$ is the measure of relative risk aversion, $n_{i,s}$ denotes the household's labor market status, and $\zeta > 0$ is a utility cost of working.

Households either work full time as employees, or do not work in the market at all:

$$n_{i,s} = \begin{cases} 1 & \text{if employed at date } s \\ 0 & \text{if not employed at date } s \end{cases}. \tag{3}$$

We let z_i denote the individual-specific productivity level of household i when a job is held at a firm. This level is determined ex ante and is constant over time. For simplicity, we assume that $z_i \in \{0,1\}$, i.e. a household has either full productivity in a job ($z_i = 1$) or none at all ($z_i = 0$). Households of the latter type opt to be out the labor force and never search for a job. We denote their share in the overall population by $\xi \in (0,1)$.

Production technology: A continuum of final goods producers, indexed by $j \in (0, M)$, each produce a differentiated good using labor. Their technologies are:

$$y_{j,s} = \exp(A_s) n_{j,s}, \tag{4}$$

where $y_{j,s}$ is firms j's output and $n_{j,s}$ its employment. A_s is an *aggregate* stochastic productivity shock which follows a first-order autoregressive process:

$$A_s = \rho_A A_{s-1} + \sigma_A \varepsilon_s^A, \tag{5}$$

where $\rho_A \in (-1,1)$, $\sigma_A > 0$ and $\varepsilon_s^A \sim \mathcal{N}(0,1)$.

Employment Dynamics and Matching technology: Firm-worker relationships are formed in a frictional matching market. In order to hire, firms post vacancies, $v_{j,s}$, at cost $\kappa > 0$ per unit. Each vacancy is filled with probability $q_s \in [0,1]$ and firms are assumed to be sufficiently large that q_s is also the fraction of vacancies that are filled.[6] The job separation probability is constant and denoted by $\omega \in [0,1]$. The law of motion of employment in firm j is given by:

$$n_{j,s} = (1-\omega) n_{j,s-1} + v_{j,s} q_s, \tag{6}$$

and we impose that vacancies cannot be negative:

$$v_{j,s} \geq 0, \tag{7}$$

see also Petrosky-Nadeau, Zhang and Kuehn (2018).

Workers who are currently without a job, and willing to work, search for jobs and are matched with a firm with probability $\eta_s \in [0,1]$. The job finding and the

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6. This is useful because we will later assume symmetry across firms and the large firm assumption avoids having to consider that the measure of vacancies filled by individual firms is stochastic.

vacancy filling rates are taken as given by agents but are endogenously determined in equilibrium by a matching function which relates the measure of new matches, m_s, to the measures aggregate vacancies and workers searching for a jobs. We assume a Cobb-Douglas matching function:

$$m_s = e_s^\alpha v_s^{1-\alpha}, \tag{8}$$

where $\alpha \in (0,1)$, $e_s \geq 0$ is the measure of job searchers, and $v_s = \int_j v_{j,s} dj$ is the measure of aggregate vacancies.[7] It follows that the job finding and vacancy filling probabilities are determined by labor market tightness, $\theta_s \equiv \frac{v_s}{e_s}$, respectively as:

$$\eta_s = \frac{m_s}{e_s} = \theta_s^{1-\alpha}, \tag{9}$$

$$q_s = \frac{m_s}{v_s} = \theta_s^{-\alpha} = \eta_s^{\frac{\alpha}{\alpha-1}}. \tag{10}$$

Timing: We assume that agents receive information about aggregate productivity shocks at the beginning of each period. Existing worker-firm relationships are resolved at the end of the period and new ones are formed at the beginning of each period. Households take their consumption/saving decisions after new matches are formed. Job separations are exogenous and affect existing hires randomly, so that employees perceive ω to be the risk that they lose their current job. Aggregate employment and the number of job searchers evolve, respectively, as:

$$n_s = (1-\omega) n_{s-1} + \eta_s e_s, \tag{11}$$

$$e_s = 1 - n_{s-1} - \xi + \omega n_{s-1}. \tag{12}$$

Prices Setting: Firms are monopolistically competitive and set the prices of their products, $P_{j,s}$, subject to a quadratic price adjustment cost as in Rotemberg (1982). We anticipate that in the symmetric equilibria we study, wages do not vary across firms. Let $w_{i,s}$ denote the real wage the firm pays to a type i worker, $n_{j,s}^i$ firm j's employment of type i workers, $y_s = \int_j y_{j,s} dj$ aggregate output, and P_s be the aggregate price level. Firms maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \Lambda_{j,t,s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - \int_i w_{i,s} n_{j,s}^i di - \kappa v_{j,s} - \frac{\varphi}{2} \left(\frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right)^2 y_s \right], \tag{13}$$

subject to (4), (6), (7) and a goods demand function induced by the CES aggregator in (2):

$$y_{j,s} = \left(\frac{P_{j,s}}{P_s} \right)^{-\gamma} y_s. \tag{14}$$

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7. It is trivial to include a match efficiency parameter in the matching function. We have normalized this parameter to one because it impacts on the results in a symmetric (inverse) manner to κ, the vacancy posting cost.

$\Lambda_{j,t,s}$ is the discount factor of the firm's owners (discussed in Section 2.3), and $\varphi \geq 0$ parameterizes the extent of nominal rigidities. The price index associated with Equation (14) is given by $P_s = \left(\int_j P_{j,s}^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}$.

Wage Determination: Real wages are determined in bilateral Nash bargaining games between workers and firms and are renegotiated period by period. Risk aversion on the part of households and lack of insurance against job loss imply that a worker's surplus from holding a job generally depends on her wealth, see Krusell, Mukoyama and Sahin (2010). We therefore index the households' value and surplus functions by i. Firms are assumed to be symmetric and hence we do not include a firm index in the bargaining equations.

The wage solves the following maximization problem:

$$\max \left(S_{i,s}^n \right)^{\upsilon} \left(S_s^f \right)^{1-\upsilon}, \tag{15}$$

where $S_{i,s}^n$ is the worker's match surplus, S_s^f is the firm's surplus and $\upsilon \in (0,1)$ is the worker's bargaining weight. We assume that were negotiations to fall through, the worker becomes unemployed for at least one period while the firm can attempt to hire a new worker in the same period. $S_{i,s}^n$ is determined as the difference between the value of being employed ($V_{i,s}^n$), and the value of being unemployed ($V_{i,s}^u$), to be defined below:

$$S_{i,s}^n = V_{i,s}^n - V_{i,s}^u,$$

Since the firm posts vacancies to hire a replacement worker should the current negotiations fail, the surplus of the match to the firm satisfies:

$$S_s^f = \frac{\kappa}{q_s}. \tag{16}$$

Monetary Policy: The monetary authority sets the short-term nominal interest rate and responds to inflation, given by $\Pi_s \equiv \frac{P_s}{P_{s-1}}$, and to labor market tightness. The latter variable naturally captures, inversely, the degree of labor market slack. The interest rate rule is given by:

$$R_s = \max \left\{ \overline{R} \left(\frac{\Pi_s}{\overline{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\overline{\theta}} \right)^{\delta_\theta}, 1 \right\}, \tag{17}$$

where $\overline{R}, \overline{\Pi}, \overline{\theta}, \delta_\pi, \delta_\theta \geq 0$ are policy parameters and the 'max' operator captures the Zero Lower Bound (ZLB) on the net nominal interest rate, $R_s - 1$. We will later allow for shocks to the monetary policy rule.

Financial Markets and Budget Constraints: We assume lack of insurance contracts against job loss. However, households without a job produce $\vartheta > 0$ units

of the aggregate consumption good at home and may be able to self-insure through savings.[8]

Households have access to two financial assets. The first is a zero coupon one-period nominal bond which can be purchased at price $1/R_s$ units of currency at date s. Let the household's purchases of bonds at date s be given by $b_{i,s}$ (in real terms). We impose the following borrowing constraint:

$$b_{i,s} \geq -\psi z_i w_{i,s} n_{i,s}, \tag{18}$$

which allows a household to borrow up to a multiple $\psi > 0$ of its current labor income. The economy is closed and there is no government debt. Hence, the aggregate net supply of bonds is equal to zero, i.e. $\int b_{i,s} di = 0$.

A second asset available to households is firm equity, i.e. claims to the dividend streams of the firms. Let $h_{i,s}$ denote household i's purchases of equity in period s. We rule out short selling of equity, $h_{i,s} \geq 0$. The total mass of firms is unity, and hence clearing in the market for equity implies that $\int h_{i,s} di = 1$.

The budget constraint of a household is given by:

$$c_{i,s} + \frac{b_{i,s}}{R_s} + p_{h,s} h_{i,s} \leq z_i w_{i,s} n_{i,s} + \vartheta \left(1 - n_{i,s}\right) + \frac{b_{i,s-1}}{\Pi_s} + \left(1 - \tau_i\right) \left(p_{h,s} + d_{h,s}\right) h_{i,s-1}, \tag{19}$$

where $p_{h,s}$ denotes the equity price, $d_{h,s}$ is the dividend. $\tau_i \in [0,1]$ is an individual-specific cost of holding equity, which introduces ex-ante differences across households in net asset returns, see Fagereng et al. (2016) for empirical evidence.

2.2. Household Dynamic Optimization Problems

We now state the households' dynamic optimization problems. For ease of presentation, we remove time indices and let a prime denote the next period. We also suppress the agent index i. We further let a superscript n (u) denote an employed (unemployed) household and denote the individual asset state by $x = [b_{-1}, h_{-1}]$, and the aggregate state by X.

Consider first the households with $z = 1$, i.e. those who are productive if they hold a job at a firm. The Bellman equation for an employed household is:

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\begin{aligned}
\mathbf{V}^{z=1,n}\left(x, X\right) = \max_{c, x'} \quad & \frac{c^{1-\mu} - 1}{1 - \mu} - \zeta \\
& + \beta \mathbb{E}\left[\left(1 - \omega\left(1 - \eta'\right)\right) \mathbf{V}^{z=1,n}\left(x', X'\right) + \omega\left(1 - \eta'\right) \mathbf{V}^{z=1,u}\left(x', X'\right)\right]
\end{aligned}
$$

subject to (18)-(19), setting $n = 1$. A currently employed worker experiences a job separation with probability ω and, in that case, finds a new job with probability

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8. The model would be equivalent if we assume that unemployed workers receive a constant real benefit financed by lump-sum taxation of the entrepreneurs. Assuming a constant replacement ratio instead would rule out the possibility of procyclical earnings risk. In Appendix A9 we consider a version of the model where unemployed agents receive benefits that are sticky in nominal terms which under some conditions can provide an automatic stabilizer.

η' at the beginning of the next period. $\omega\left(1-\eta'\right)$ is therefore the probability that a currently employed worker is without a job next period and $1-\omega\left(1-\eta'\right)$ is the complement probability. A household who is unemployed, and thus searching for a job, will be in employment at the beginning of next period with probability η'. Hence, the Bellman equation for an unemployed worker is:

$$\mathbf{V}^{z=1,u}\left(x,X\right)=\max_{c,x'}\frac{c^{1-\mu}-1}{1-\mu}+\beta\mathbb{E}\left[\eta'\mathbf{V}^{z=1,n}\left(x',X'\right)+\left(1-\eta'\right)\mathbf{V}^{z=1,u}\left(x',X'\right)\right],$$

subject to (18)-(19), setting $n=0$.

Next, consider the households who are out of the labor force (i.e. those with $z=0$). Their dynamic optimization problem is given by:

$$\mathbf{V}^{z=0}\left(x,X\right)=\max_{c,x'}\frac{c^{1-\mu}-1}{1-\mu}+\beta\mathbb{E}\mathbf{V}^{z=0}\left(x',X'\right),$$

subject to (18)-(19), setting $n=0$.

2.3. Equilibrium and tractability

We focus upon symmetric equilibria in which the intermediate goods producers choose the same actions. Formally,

Definition 1: *A recursive equilibrium is a set of policy functions for quantities, (c,b',h',v), prices (P,R,w,Π), labor market variables (η,q,e,n), value functions $\left(\mathbf{V}^{z=1,n},\mathbf{V}^{z=1,u},\mathbf{V}^{z=0}\right)$, and a distribution of agents over assets, productivity and labor market states such that: (i) The policy functions (c,b',h') solve the households' problems stated above; (ii) (v,P) solve the firms' problems; (iii) The goods and asset markets clear; (iv) Aggregate labor market variables evolve according to (8)-(11); (v) The wage solves the Nash bargaining problem; (vi) The central bank implements the policy rule; (vii) Actual and perceived laws of motion of x and X coincide.*

We aim at obtaining an analytical solution of the model in order to bring out some of its key properties in a transparent manner. For that reason, we make the following simplifying assumption:

Assumption 1: *There are two ex-ante types of households who differ with respect to z_i and τ_i. The first type, "capitalists", obtain full returns from equity investment ($\tau_i=0$), but have zero productivity as workers ($z_i=0$). Their initial holdings of equity are the same. The second type, "workers", have a positive productivity in a job at a firm ($z_i=1$), but cannot obtain returns from equity ($\tau_i=1$).*

In other words, workers can participate in the labor market but not in the firm equity market, whereas the precise opposite is true for the capitalists. We now make the following conjecture:

Conjecture: *Given assumption 1, the following two sets of conditions hold in any steady-state equilibrium and in a neighborhood around the steady state:*

$$b_{i,s} = 0 \; \forall \; i, \tag{20}$$

$$1 = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega \left(1 - \eta_{s+1}\right) \left(\frac{\vartheta}{w_s}\right)^{-\mu} + \left(1 - \omega \left(1 - \eta_{s+1}\right)\right) \left(\frac{w_{s+1}}{w_s}\right)^{-\mu} \right) \tag{21}$$

Equation (20) states that *all* individual households hold exactly zero bonds. Equation (21) is the Euler equation of the employed workers. Importantly, the equation contains only aggregate variables, which will allow us to solve for the equilibrium real interest rate referring only to aggregate variables.

We now verify that (20) and (21) are indeed consistent with utility maximization of all households, and with market clearing. First, recall that the capitalists are out of the labor force and therefore they are not exposed to idiosyncratic risk.[9] Given symmetry in initial asset holdings, this implies that capitalists make identical decisions. In particular, the capitalists all set $h_{i,s} = 1/\xi$, the population share of the capitalists. This outcome and condition (20) imply that all households consume their current income. For that reason, wages equalize across all employed households. Given (20) and (21), we can therefore distinguish between three distinct groups of households, with consumption levels given by:

$$c_{i,s}^{z=0} = \frac{1}{\xi} \left(y_s - \kappa v_s - w_s n_s - \frac{\varphi}{2} \left(\Pi_s - 1\right)^2 y_s \right) + \vartheta, \tag{22}$$

$$c_{i,s}^{z=1,u} = \vartheta, \tag{23}$$

$$c_{i,s}^{z=1,n} = w_s. \tag{24}$$

Here, $c_{i,s}^{z=0}$ is the consumption level of a capitalist. The term between large brackets on the right hand side of equation (22) is the dividend income received by the firm owners which is non-negative in the steady-state due to monopolistic competition. The second term is the home production. Workers hold no equity and equations (23)-(24) are the equilibrium consumption levels of unemployed and employed workers, respectively, under Assumption 1. Since there is no consumption heterogeneity across households conditional on their type and employment status, we drop the i-subscript and denote consumption levels as $c_{c,s} = c_{i,s}^{z=0}$, $c_{u,s} = c_{i,s}^{z=1,u}$, and $c_{n,s} = c_{i,s}^{z=1,n}$, where subscript c, u, and n denote, respectively, the capitalists, the unemployed workers, and the employed workers. We also note that firms discount profits at a common discount rate of the capitalists, $\Lambda_{t,s} = \beta \left(c_{c,t}/c_{c,s}\right)^{\mu}$, who own the firms.

Equations (20) and (22)-(24) are trivially consistent with clearing of the goods, bond, and equity markets. To see why they also satisfy optimality of households

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9. Even if these households were to participate in the labor market, they would be relatively well insured against idiosyncratic risk due to wealth (equity holdings).

decisions, consider the Euler equations for bonds for the three types of agents:

$$c_{c,s}^{-\mu} \;\geq\; \beta\mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{c,s+1}^{-\mu}, \tag{25}$$

$$c_{u,s}^{-\mu} \;\geq\; \beta\mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left((1-\eta_{s+1}) c_{u,s+1}^{-\mu} + \eta_{s+1} c_{n,s+1}^{-\mu} \right), \tag{26}$$

$$c_{n,s}^{-\mu} \;\geq\; \beta\mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega (1-\eta_{s+1}) c_{u,s+1}^{-\mu} + (1 - \omega (1-\eta_{s+1})) c_{n,s+1}^{-\mu} \right). \tag{27}$$

Each of these conditions holds with strict inequality if the household is liquidity constrained and otherwise with equality. The conjecture implies that in any steady state, the real interest rate lies below the subjective discount rate, i.e. $\frac{R}{\Pi} < \frac{1}{\beta}$. Both (25) and (26) are consistent with this, provided that they do not hold with equality.[10] The Euler equation of the employed households is also consistent with the conjecture. This can be seen by combining (27) with (24) and (23), which induces (21) to hold with equality. The latter is necessary since otherwise the employed households would face a binding borrowing constraint. This, however, would mean that they hold positive amounts of debt, which would violate bond market clearing given that the asset-rich and the unemployed hold zero bonds.[11]

Figure 1 is an illustration of the steady-state bond demand schedules of the three groups of households as functions of the real interest rate. These functions are upward sloping under fairly general conditions. To the far right is the demand schedule of the unemployed workers who have a strong intertemporal incentive to borrow, realizing that in the future they might find a job and receive more income. In the middle is the bond demand function of capitalists who face no idiosyncratic risk and therefore will only hold bonds in the steady state if the real interest rate equals at least $1/\beta$. At real rates lower than $1/\beta$, equity dominates bonds in return and hence the equity holders would like to go short on bonds but are prevented from doing so by the borrowing constraint. The left schedule represents the employed workers, who are most eager to save, due to their precautionary savings motive. They end up holding zero bonds, but not because they are forced by a constraint since they could in principle save or borrow. Rather, the equilibrium real interest rate adjusts downward to a point at

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10. To see this note that in a steady state (25) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}$, whereas (27) reduces to $\frac{R_s}{\Pi} \leq \frac{1}{\beta}((1-\eta) + \eta \left(\frac{\vartheta}{w} \right)^{-\mu})^{-1} < \frac{1}{\beta}$.

11. One might wonder if there are related steady-state equilibria in which the capitalists and the unemployed are at the constraint but there is heterogeneity in wealth and consumption within the group of employed households. This is not the case. To understand why, first note that on average the employed would have to hold zero bonds. Second, note that any employed household just coming out of unemployment would start with zero wealth. At the conjectured interest rate, they choose to buy exactly zero bonds. However, if the real interest rate were higher than the one conjectured, households would gradually accumulate bonds during the employment spell, monotonically converging to a certain target level of wealth. This, however, would mean that average bond holdings are positive, violating bond market clearing. Conversely, if the interest rate were lower than the one conjectured, aggregate bond holdings would be negative.

FIGURE 1. Illustration of steady-state bond demand schedules.
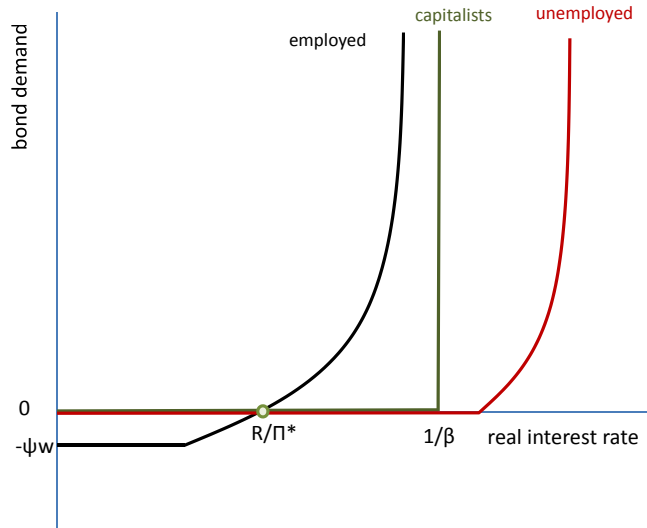
which they voluntarily hold zero bonds (which is required for clearing of the bond market).

We thus arrive at a tractable characterization of the household block in the model. All households hold zero bonds and the employed workers determine the real interest rate, whereas the remaining households face a binding liquidity constraint. The fact that we allow those who price the bond to borrow sets us apart from earlier literature achieving tractability in incomplete-markets models, which typically assume that no one can borrow, see Krusell, Mukoyama and Smith (2011), Werning (2015), McKay, Nakamura and Steinsson (2017), McKay and Reis (2017), Ravn and Sterk (2017) and Bilbiie (2019).[12]

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12. In those studies, there is typically a continuum of equilibrium real interest rates, which is not the case in our analysis.

The set of conditions that characterize the resulting equilibrium of the model are then given by:

$$w_s^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} w_{s+1}^{-\mu} \left[1 + \omega \left(1 - \eta_{s+1} \right) \left(\left(\vartheta / w_{s+1} \right)^{-\mu} - 1 \right) \right], \tag{28}$$

$$\gamma mc_s = \varphi \left(\Pi_s - 1 \right) \Pi_s - \varphi \beta \mathbb{E}_s \left(\frac{c_{c,s+1}}{c_{c,s}} \right)^{-\mu} \left(\Pi_{s+1} - 1 \right) \Pi_{s+1} y_{s+1} / y_s + \gamma - 1, \tag{29}$$

$$mc_s = \frac{1}{\exp(A_s)} \left(w_s + \frac{\kappa}{q_s} - \lambda_{v,s} - (1 - \omega) \beta \mathbb{E}_s \left(\frac{c_{c,s+1}}{c_{c,s}} \right)^{-\mu} \left\{ \frac{\kappa}{q_{s+1}} - \lambda_{v,s+1} \right\} \right) \tag{30}$$

$$R_s = \max \left\{ \overline{R} \left(\frac{\Pi_s}{\overline{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\overline{\theta}} \right)^{\delta_\theta}, 1 \right\}, \tag{31}$$

$$c_{c,s} = \frac{1}{\xi} \left(y_s - \kappa v_s - w_s n_s - \frac{\varphi}{2} \left(\Pi_s - 1 \right)^2 y_s \right) + \vartheta, \tag{32}$$

$$q_s = \theta_s^{-\alpha} = \eta_s^{\frac{\alpha}{\alpha-1}}, \tag{33}$$

$$y_s = \exp(A_s) n_s, \tag{34}$$

$$n_s = (1 - \omega) n_{s-1} + \eta_s e_s, \tag{35}$$

$$e_s = 1 - n_{s-1} - \xi + \omega n_{s-1}, \tag{36}$$

$$w_s = w(\eta_s). \tag{37}$$

where mc_s denotes real marginal costs and $\lambda_{v,s}$ is the Kuhn-Tucker multiplier on the vacancy non-negativity constraint (7). In addition, the stochastic process for productivity is given by (5).

The first equation is the employed workers' Euler equation. The second condition is the standard optimal price-setting equation well-known from the NK literature which links inflation dynamics and real marginal costs. The third equation defines real marginal costs, mc_s. To produce an additional unit of output, firms must employ $1/\exp(A_s)$ workers who are paid w_s and requires posting $1/q_s$ vacancies at a cost of κ each. The last term in this expression captures the fact that current hires survive until the next period with probability $(1 - \omega)$ thereby reducing future hiring costs. (31) is the interest rate rule. Equations (32)-(36) define capitalists' consumption, the relationship between the vacancy filling rate, labor market tightness and the job finding, equilibrium output, and the laws of motion for employment and unemployment. Finally, (37) defines the real wage (the solution to the Nash bargaining game) to be an implicit (non-decreasing) function of the job finding rate, see Appendix A1.1.

We will consider the properties of the steady state of the model and local equilibria around the steady state allowing for fluctuations in response to aggregate shocks. Provided that shocks are not too large, in the equilibria we examine capitalists and unemployed workers are liquidity constrained, whereas the employed workers are not. Since there is no aggregate supply of bonds in which the employed can save, the

employed perpetually choose to hold zero bonds. Thus, (20) and (21) continue to hold in the neighborhoods of steady states.[13]

2.4. The Endogenous Earnings Risk Channel

In this incomplete-markets model, apart from in very special circumstances, employed agents will be on their Euler equation. Hence, the savings choices of employed agents and the equilibrium real interest rate have to satisfy these agents' Euler equation, which can be expressed as:

$$c_{n,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{n,s+1}^{-\mu} \left[1 + \omega \left(1 - \eta_{s+1} \right) \left((\vartheta/w_{s+1})^{-\mu} - 1 \right) \right]. \qquad (38)$$

This Euler equation sets the marginal utility of current consumption equal to the discounted expected marginal utility next period times the real interest rate. It differs from the Euler equation that guides choices in standard complete-markets model due the precautionary savings term in square brackets which derives from lack of insurance against unemployment risk. We will now discuss how this modification of the Euler equation combines with frictions in labor and goods markets to create an *endogenous* earnings risk wedge which is important for understanding the properties of the model. Not only is this wedge endogenous, it is also cyclical and it introduces a channel through which shocks are amplified (or stabilized). This wedge is the key new ingredient through which the model adds new insights relative to the earlier NK literature.

To understand this, we consider simplified versions of the model. Consider first a case in which income risk is exogenous and constant. In particular, let us abstract from unemployment risk ($\omega = 0$) and assume that changes in wages (and hence consumption) are log-normally distributed with a constant conditional variance. Also, the gross real interest rate is constant and equal to R/Π. Given CRRA preferences and denoting the constant (conditional) variance of employed workers' consumption by σ^2, in this case the Euler equation can be expressed as:

$$\ln c_{n,s} = \mathbb{E}_s \ln c_{n,s+1} - \left[\ln \frac{\beta R}{\mu \Pi} + \mu \sigma^2/2 \right],$$

Hence, earnings risk introduces a precautionary saving term which is proportional to the risk aversion parameter. As the precautionary motive enters as a constant term, it does not have an important effect on macroeconomic outcomes, apart from the steady-state real interest rate falling below the pure rate of time-preference.

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13. It is worth stressing that the property that consumption equals income for the households is an equilibrium feature. Since we allow for debt, the Marginal Propensity to Consume (MPC) out of an idiosyncratic temporary income shock **does not equal one** for the employed agents and the capitalists in the model. The MPC equals one only for the borrowing-constrained agents (the unemployed).

Storesletten, Telmer and Yaron (2004) argue that idiosyncratic risk is strongly countercyclical. Allowing for such countercyclical changes in risk and assuming it happens exogenously implies (under the same assumptions as above regarding log normality of wages and constant real interest rates) the following Euler equation:

$$\ln c_{n,s} \;\;=\;\; \mathbb{E}_s \ln c_{n,s+1} - \left[\ln \frac{\beta R}{\mu \Pi} + \mu \sigma_s^2/2\right],$$

Hence, due to countercyclical earnings risk, demand contracts in recessions relative to booms because of the increase in the precautionary saving motive. In the HANK setting, lower goods demand reduces labor demand but if the earnings risk is exogenous, there is no further feedback to the demand side.

Now introduce unemployment risk which naturally implies countercyclicality of earnings risk. In particular, assume that a currently employed worker remains employed next period with exogenous probability p_{s+1}^{nn} and faces unemployment with the complement probability. Assume also that wages are exogenous. In this case, employed workers' Euler equation is given as:

$$c_{n,s}^{-\mu} = \beta E_s \frac{R_{s+1}}{\Pi_{s+1}} c_{n,s+1}^{-\mu} \left[p_{s+1}^{nn} + \left(1 - p_{s+1}^{nn}\right) \left(\frac{c_{u,s+1}}{c_{n,s+1}}\right)^{-\mu}\right].$$

The term in the square bracket captures the impact of precautionary saving against job loss. This wedge will fluctuate over time but in an entirely exogenous way. How the wedge moves over time will depend on the joint probability distribution of unemployment risk and the income loss due to unemployment. When either unemployment risk or the loss due to unemployment rise, employed workers increase their precautionary saving propensity which puts downward pressure on the real interest rate and on current consumption (at the given real interest rate). As in the previous example, an increase in precautionary saving, gives rise to a demand contraction, which impacts on the supply side in a sticky price setting. However, if wages and job transition rates are exogenous, there is no further feedback from this. One new aspect of this model relative to the case outlined above with countercyclical wage risk is that agents face "discrete" income risk due to unemployment which implies that employed workers face left skewed earnings distributions.

Now let us return to the baseline model. This corresponds to the case in which the labor market transition rates are endogenous and determined by labor market tightness, $p_{s+1}^{nn} = \left(1 - \omega \left(1 - \eta_{s+1}\right)\right),$ and in which wages are determined endogenously through Nash bargaining. The precautionary wedge in Equation (38) now moves endogenously with the risk of unemployment and the loss of earnings given job loss. This has fundamental consequences because it introduces a feedback mechanism. As above, demand contracts at the current real interest rate when either the risk of unemployment or the costs thereof rise. A contraction in demand will now be reflected in lower vacancy postings which impact on the job finding rate and on wages and thereby induces a feedback mechanism which sets the HANK&SAM model apart from

both models with complete markets, models with exogenous earnings uncertainty (as above), and models with flexible prices.

The feedback mechanism may move pro- or countercyclically. Suppose the economy enters a recession during which the job finding rate drops. Worsening labor market conditions stimulate precautionary saving amongst employed workers because they perceive a higher risk of unemployment. However, as the labor market deteriorates, wages are also likely to fall which implies a smaller income loss in case of unemployment which accordingly implies less precautionary saving. When the first of these effects dominates, a case we refer to as *countercyclical* endogenous earnings risk, the employed workers' increased propensity to save in a recession implies lower goods demand which reduces firms' labor demand which in turn induces further risk of unemployment creating a feedback mechanism which amplifies the impact of shocks relative to models with exogenous risk (or complete insurance).[14] Conversely, when the wage effect dominates, the procyclical endogenous earnings risk case, the wedge has a stabilizing effect because the demand for precautionary savings increases in booms and declines in recessions. Hence, the properties of the endogenous risk that arise in the model will be key for the model's implications.

Because the cyclicality of risk is central to the discussion that follows, it is useful to consider whether the endogenous risk wedge is likely to be procyclical or countercyclical. Storesletten, Telmer and Yaron (2004) study PSID household income data and conclude that idiosyncratic labor market income risk is strongly countercyclical. Studying a 10 percent sample of all U.S. working-age males, Guvenen, Ozkan and Song (2014) show that the countercyclicality of earnings risk derives from increased left-skewness in recessions (i.e. higher likelihood of large earnings losses and lower likelihood of large gains) rather than from a countercylical variance.

These findings indicate that sources of countercyclical risk are likely to dominate. Furthermore, our emphasis on unemployment risk is consistent with the left-skewness of earnings changes in recessions as emphasized by Guvenen et al. (2014). In Section 7, we will use the model to further argue that countercyclical risk is the empirically relevant case, once one takes a view on plausible values of the parameters entering the wedge. For these reasons, we concentrate most of our discussion on the case where the endogenous earnings risk wedge is countercyclical, although we will point out how this contrasts with the procyclical case.

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14. The link from the Euler equation to the labor demand happens as follows. Higher precautionary saving puts downward pressure on the real interest rate. As long as the central bank operates an active rule for the nominal interest rate, the downward pressure on the real interest rate induces a drop in the inflation rate. According to the condition for optimal price setting, lower inflation requires real marginal costs to fall. Lower marginal costs, in turn, requires either real wages to drop or hiring costs to decline. In general, both real wages and hiring costs decline. The latter, in turn, requires firms to hire less, which induces a further decline in the job finding rate.

3. Steady-state Equilibria

This section discusses the set of steady-state equilibria that can arise absent aggregate shocks.

3.1. Global Determinacy

We first turn attention to the the properties of the economy in the absence of aggregate shocks. Hence, we now impose that $\sigma_A = 0$ and assume away other sources of fluctuations such as stochastic sunspots and transitional dynamics. Formally:

Definition 2: *A steady-state equilibrium of the model, is a set of time-invariant solutions to equations (28)-(37).*

An important difference vis-à-vis the extant complete-markets NK literature is that although aggregate variables are constant in the steady state, labor market participants still face idiosyncratic risk due to lack of insurance against earnings risk. We will now show that this aspect matters for the long-run properties of the model.

We indicate steady-state values by removing time subscripts from variables. Define for convenience $R^* \equiv \overline{R} \left(\overline{\Pi} \right)^{-\delta_\pi} \left(\overline{\theta} \right)^{-\delta_\theta}$. Combining Equations (28)-(37), we can characterize the steady-state equilibria of the model as the solutions to two non-linear equations in inflation and in the job finding rate:

$$\varphi \left(1 - \beta \right) \left(\Pi - 1 \right) \Pi = 1 - \gamma + \gamma \left(w \left(\eta \right) + \left(\kappa \eta^{\alpha/(1-\alpha)} - \lambda_v \right) \left(1 - \beta \left(1 - \omega \right) \right) \right), \tag{PC}$$

$$1 = \beta \frac{\max\{ R^* \Pi^{\delta_\pi} \eta^{\delta_\theta/(1-\alpha)}, 1 \}}{\Pi} \boldsymbol{\Theta}^{SS} \left(\eta \right), \tag{EE}$$

where $\boldsymbol{\Theta}^{SS} \left(\eta \right)$ is the steady-state endogenous risk wedge, which can be expressed as a function of the job finding rate:

$$\boldsymbol{\Theta}^{SS} \left(\eta \right) \equiv 1 + \omega \left(1 - \eta \right) \left[\left(\vartheta / w \left(\eta \right) \right)^{-\mu} - 1 \right] \geq 1.$$

The (**PC**) and (**EE**) schedules can both be considered as defining a two-dimensional relationship between the job finding rate η and the inflation rate Π; the steady-state equilibria relate to the intersections of these relationship. The steady-state values of all other variables can be found as functions of inflation and the job finding rate.

Equation (**PC**) is the steady-state version of the optimality condition for price setting ("the NK Phillips Curve") into which we have substituted the steady-state condition for marginal costs and used the implicit real wage solution from the Nash bargaining game. Both real wages and the cost of hiring depend positively on the job finding rate. As long as firms post vacancies, it follows from the implicit function theorem that the slope of equation (**PC**) is given as:

$$\frac{d\Pi}{d\eta} \mid_{PC, v > 0} = \frac{\gamma \left[w' \left(\eta \right) + \left(1 - \beta \left(1 - \omega \right) \right) \kappa \alpha / \left(1 - \alpha \right) \eta^{\frac{2\alpha - 1}{1 - \alpha}} \right]}{\varphi \left(1 - \beta \right) \left(2\Pi - 1 \right)}$$

In a (η, Π)-space, the **PC** relationship is (i) vertical if prices are flexible ($\varphi = 0$), (ii) positive sloped if prices are sticky, $w' > 0$ and $\Pi > 1/2$, and (iii) vertical if $\lambda_v > 0$ (i.e. if firms are constrained by non-negativity of vacancies).[15] As regards case (ii), in the appendix we show that wages are increasing in the job finding rate; the restriction on the inflation rate simply requires less than 50 percent deflation, a rather mild condition.

Equation (**EE**) is the steady-state version of the employed households' Euler equation (21), where we have substituted in the policy rule and used that employed workers' consumption equals the real wage. This equation also defines a relationship between Π and η (consistent with utility maximization by the employed agents) which depends on the job finding rate because of (a) idiosyncratic unemployment risk households, and (b) because of the central bank response to the state of the labor market. The slope of this schedule depends on whether the Zero Lower Bound (ZLB) on the net interest rate is binding or not, and is given as:

$$
\begin{aligned}
\frac{d\Pi}{d\eta}\,|_{EE,R>1} &= \frac{\delta_\theta/\left(1-\alpha\right) + \frac{\partial \boldsymbol{\Theta}^{SS}(\eta)/\partial\eta}{\boldsymbol{\Theta}^{SS}(\eta)/\eta}}{1-\delta_\pi}\frac{\Pi}{\eta} \\
\frac{d\Pi}{d\eta}\,|_{EE,R=1} &= \beta\partial\boldsymbol{\Theta}^{SS}\left(\eta\right)/\partial\eta \\
\partial\boldsymbol{\Theta}^{SS}\left(\eta\right)/\partial\eta &= -\omega[(\vartheta/w)^{-\mu}-1] + \mu\omega\left(1-\eta\right)/\eta\left(\vartheta/w\right)^{-\mu}\chi
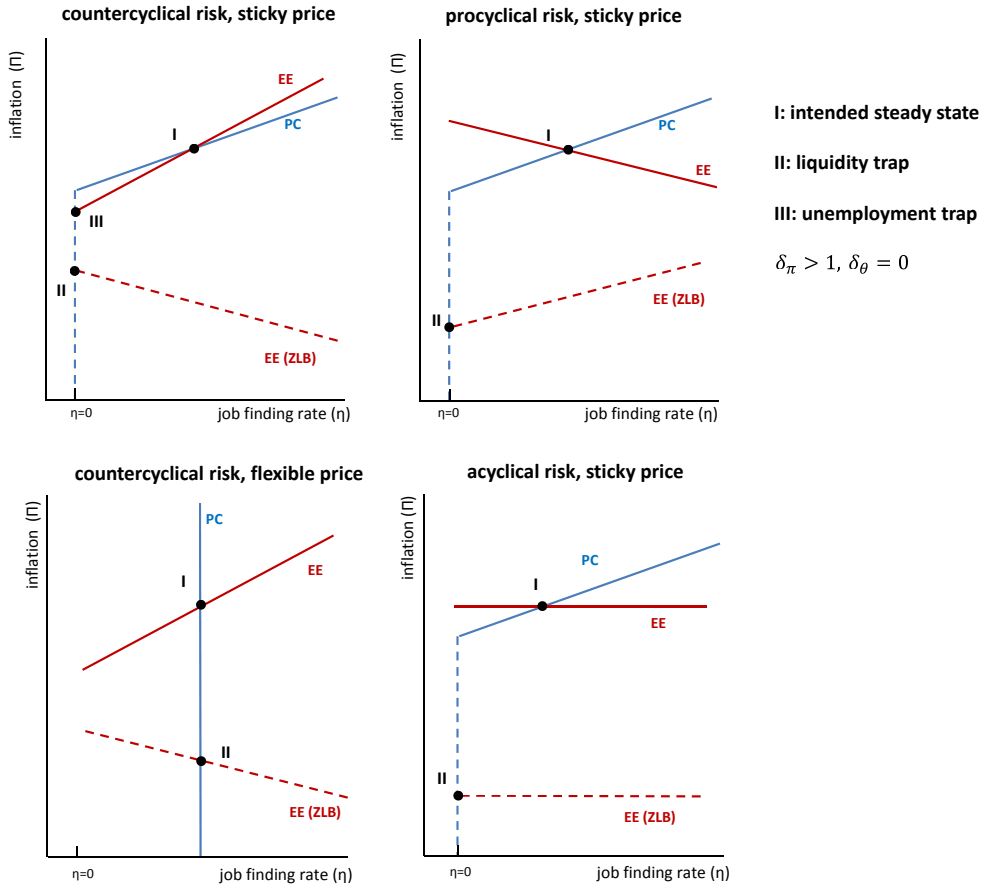\end{aligned}
$$

where $\chi = (\partial w/\partial\eta)\left(\eta/w\right)$ is the elasticity of the real wage to the job finding rate. When the ZLB is not binding, the slope of the (**EE**) relationship depends on the elasticity of the risk wedge and on the policy responses to deviations of tightness and inflation from their targets. Under the assumptions that $\delta_\pi > 1$ (the Taylor principle) and $\delta_\theta = 0$, the cyclicality of earnings risk determines the slope of of this relationship.

When wages are unresponsive to the job finding rate, $w'(\eta) \simeq 0$, earnings risk is countercyclical and $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta < 0$ because unemployment risk completely determines the overall earnings risk. In this case, the (**EE**) relationship defines a positive relationship between steady-state inflation and job finding prospects because households' precautionary saving motive declines as the job finding rate rises which puts upward pressure on real interest rates and therefore on inflation. When the wage elasticity is sufficiently large, i.e. when $w'(\eta)$ is large, overall earnings risk becomes procyclical, so that $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta \geq 0$. In this case, wages increase sharply with the job finding rate and precautionary saving now rises when the job finding rate increases.

Second, the slope of the **EE** schedule depends on whether or not the ZLB on the nominal interest rate binds. For simplicity, consider initially a case in which the interest rate rule only reacts to inflation (i.e. $\delta_\theta = 0$) and satisfies the Taylor principle, so that away from the ZLB the real interest rate is increasing in the inflation rate. While the slope of **EE** schedule depends on $\boldsymbol{\Theta}^{SS}(\eta)$, the sign of the slope reverses under a binding ZLB as can be seen from the expressions above.

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15. Note that in case (iii) it is the case that $\eta = 0$, regardless the precise level of inflation.

Figure 2. Illustration of steady-state equilibria.

coun(title area: "countercyclical risk, sticky price"; "procyclical risk, sticky price"; "countercyclical risk, flexible price"; "acyclical risk, sticky price")

I: intended steady state

II: liquidity trap

III: unemployment trap

$\delta_\pi > 1,\ \delta_\theta = 0$

Before stating our results formally, it is useful to consider the steady-state properties of the model from a graphical representation of the (**PC**) and (**EE**) relationships. Figure 2 illustrates the two relationships for four different cases relating to whether earnings risk is pro-, counter- or acyclical and whether prices are sticky or not. We assume that $\delta_\pi > 1$ and $\delta_\theta = 0$. Under countercyclical risk, the **EE** schedule is positively sloped when the ZLB does not bind because higher job finding rates stimulates current consumption unless real interest rates rise (which requires higher inflation when $\delta_\pi > 1$). The exact opposite is true under procyclical risk, i.e. **EE** slopes downward away from the ZLB whereas **EE**(ZLB) slopes upward. Under acyclical risk the **EE** schedule is horizontal, both at the ZLB and away from it. We can now state the following result:

Proposition 1: *Suppose that the Taylor principle holds and that $\delta_\theta \geq 0$. The economy may have at least three different steady states*:

I **Intended steady state** $\left(\eta^I, \Pi^I\right)$ *where* $\eta^I, \Pi^I > 0$. *This steady state occurs at an intersection of the* **PC** *and the* **EE** *schedule at* $\eta > 0$.

II **Liquidity trap** $\left(\eta^{LT}, \Pi^{LT}\right)$ *where* $\eta^{LT} < \eta^I$ *and* $\Pi^{LT} < \Pi^I$. *This steady state arises at an intersection of the* **PC** *and the* **EE**(**ZLB**) *schedule*.

III **Unemployment trap** $\left(\eta^U, \Pi^U\right)$ *where* $\eta^U = 0$ *and* $\Pi^{LT} < \Pi^U < \Pi^I$. *This steady state occurs if there is an intersection of the* **PC** *the* **EE** *schedule at* $\eta = 0$.

Under the assumption that the **PC** and the **EE** relationships are monotonic, a necessary condition for the existence of the unemployment trap is that $\frac{d\Pi}{d\eta}\mid_{EE,R>1} \left(\eta^I, \Pi^I\right) > \frac{d\Pi}{d\eta}\mid_{PC,v>0} \left(\eta^I, \Pi^I\right)$ which in turn requires $\partial\boldsymbol{\Theta}^{SS}(\eta)/\partial\eta < 0$.

The intended steady state: This equilibrium corresponds to the steady state usually considered in the NK literature. There are, however, important differences between the properties of the equilibria under complete and incomplete markets. Under complete markets, the steady state real interest rate needs to equal $1/\beta$ in order to be consistent with constant consumption. Without full insurance, and regardless of the slope of $\boldsymbol{\Theta}^{SS}(\eta)$, the wedge in (**EE**) exceeds unity, which reduces the equilibrium real interest rate below the inverse of the discount factor. In particular, the steady-state real interest rate is determined as:

$$\left(\frac{R}{\Pi}\right)^I = \frac{1}{\beta\Theta^{SS}(\eta^I)} < \frac{1}{\beta} = \left(\frac{R}{\Pi}\right)^{CM} \tag{39}$$

where $(R/\Pi)^{CM}$ denotes the deterministic steady-state real interest rate under complete markets.

The lower steady-state real interest rate under incomplete markets derives from the precautionary saving motive induced by idiosyncratic unemployment risk. Since the job finding rate in the intended steady state depends on the interest rate rule, economic policy is a co-determinant of the long-run real interest rate. As long as equilibrium wages do not depend on market incompleteness, however, the central bank can replicate the steady-state levels of unemployment and inflation that would prevail under complete markets. Suppose for example that the central bank targets price stability, $\overline{\overline{\Pi}} = 1$, and let η^{CM} denote the steady-state job finding rate under complete markets. Then the central bank can implement the same outcome under incomplete markets by setting $\overline{\theta} = \left(\eta^{CM}\right)^{\frac{1}{1-\alpha}}$ and $\overline{R} = 1/\left(\beta\boldsymbol{\Theta}^{SS}\left(\eta^{CM}\right)\right)$. This is not possible, however, without the use of fiscal policy if wages depend on market incompleteness, as they will in general.[16]

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16. The reason is that wages impact on the Phillips curve as well. In this case, the intended steady state can replicated by taxing labor income and by adjusting $\overline{R}$.

The liquidity trap: As in the standard NK model, the model features a liquidity trap induced by the ZLB. This is the type of equilibrium examined by Benhabib, Schmitt-Grohé and Uribe (2001, 2002) and by Mertens and Ravn (2014). The properties of the liquidity trap in the incomplete-markets setting, however, may be quite different from the standard complete-markets model. In the standard NK model, the liquidity trap steady state is characterized by deflation because $(R/\Pi)^{CM} = 1/\beta$ regardless of whether the ZLB is binding or not (implying that $\Pi^{LT} = 1/\beta$ under complete markets). However, although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation. Temporary episodes at the ZLB will be even more deflationary than this in the standard model, since the stochastic Euler equation in that case will only be satisfied as long as $\Pi < \beta$ during the ZLB regime.[17] It is important to notice that these implications are independent of the arguments that enter the interest rate rule.

The incomplete-markets model has different implications and, in particular, it may be consistent with positive inflation in the liquidity trap. From Equation (39) it follows that the steady-state real interest rate under incomplete markets depends the job finding rate through its impact on the endogenous risk wedge and this may induce steady-states at the ZLB with positive inflation. To see this, note that the steady-state Euler equation and the policy rule for the interest rate imply that the following two conditions must be satisfied in a liquidity trap (LT):

$$
\begin{aligned}
\Pi^{LT} &= \beta \mathbf{\Theta}^{SS}\left(\eta^{LT}\right) > \beta, \\
\Pi^{LT} &< \overline{\Pi\theta}^{\delta_\theta/\delta_\pi} \overline{R}^{-1/\delta_\pi} \left(\eta^{LT}\right)^{-(\delta_\theta/\delta_\pi)/(1-\alpha)}.
\end{aligned}
$$

If $\delta_\theta = 0$ and the policy maker is targeting price stability (or positive inflation), the policy rule implies that $\Pi^{LT} < \overline{\Pi R}^{-1/\delta_\pi} < 1$, so that the liquidity trap is deflationary, given that in the intended steady state $\Pi^I = \overline{\Pi} = 1$ and $R^I = \overline{R} > 1$. When $\delta_\theta > 0$, however, inflation may be positive or negative in the liquidity trap. In particular, steady-state inflation is likely to be positive if $\left(\vartheta/w\left(\eta^{LT}\right)\right)^{-\mu} \gg 1$ and wages are not too responsive to the job finding rate, i.e. when the endogenous risk wedge is sufficiently countercyclical. Intuitively, under these circumstances, deteriorating labor market conditions induce a stronger precautionary saving motive, which puts downward pressure on real and nominal interest rates. Such high demand for savings reduces goods demand which in turn implies a further decline in tightness and in nominal rates the end-product of which may be that the ZLB may be reached at a *positive* inflation rate.

This result is interesting since it implies that purely expectational liquidity traps can arise even in inflationary environments. The incomplete markets model has other

---

17. Suppose that the ZLB regime persists with probability p while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\Pi^{LT} = \beta\left(p + (1-p)\left(c^I/c^{LT}\right)^{-\mu}\right)$ where Π^{LT} is the inflation rate during the liquidity trap, c^I is consumption in the intended steady-state and c^{LT} is consumption in the liquidity trap. This condition implies $\Pi^{LT} < \beta$ as long as $c^I > c^{LT}$.

interesting implications for liquidity traps. We develop some further implications for zero lower bound equilibria in Appendix A10 where we show that the current model paves the way for labor market uncertainty triggering liquidity traps and that it might reverse the so-called supply side paradox.

The Unemployment Trap: The possible emergence of a third steady state is perhaps the most interesting consequence of introducing incomplete markets. In this equilibrium, firms stop posting vacancies, unemployed workers see no prospects of finding jobs, and the economy may fail at escaping the equilibrium because of a very severe demand contraction induced by precautionary saving. Note that the inflation rate in this equilibrium lies in between the inflation rates of the intended steady-state and the liquidity trap. Under the assumptions that the **PC** and the **EE** relationships are monotonic, that the Taylor principle holds and $\delta_\theta \geq 0$, this steady-state *cannot* exist if prices are flexible, if markets are complete, or, if prices are sticky, when the endogenous earnings risk is either acyclical or procyclical.

The unemployment trap steady state can instead arise when markets are incomplete, prices are sticky, and the endogenous risk wedge is sufficiently countercyclical. In this case, expectations of poor labor market conditions may trigger such an increase in desired savings (and therefore lower goods demand) that firms' reductions in hiring sends the economy on a downward spiral towards an outcome where firms no longer want to hire because of lack of demand for their goods. For this to be possible, endogenous risk must be sufficiently countercyclical that the Euler equation schedule becomes steeper than the Phillips curve schedule. This is the necessary condition that is stated in Proposition 1 (under the assumption of monotonicity of the **EE** and **PC** relationships). Sufficient conditions are harder to state but can easily be checked in applications to a given model by examining its properties in the limit as the economy approaches the state where firms choose not to post vacancies.[18]

The likelihood of the existence of the unemployment trap is higher when monetary policy reacts little to inflation and/or labor market tightness, and when hiring costs are limited.[19] If the central bank implements sufficiently aggressive policies (policy rules with sufficiently large values of δ_π and/or δ_θ), the unemployment trap can be ruled out because the central bank neutralizes the impact of deteriorating labor market conditions through interest rate cuts. We discuss the details of this in Appendix A5.

In the unemployment trap, agents survive in the limit through the availability of home production. One might consider this outcome too extreme but it is

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18. In particular, let η_{min} denote the job finding rate when firms post no vacancies. Then the necessary condition for existence of an unemployment trap is that $lim_{\eta \to \eta_{min}} \Pi^{PC}(\eta_{min}) > lim_{\eta \to \eta_{min}} \Pi^{EE}(\eta_{min}) > 0$. The last inequality here assures that the ZLB is not binding.

19. Note that the unemployment trap can be ruled out if the government allows the real interest rate to depend negatively on inflation. Such a policy, however, would make the intended steady-state locally indeterminate.

straightforward to extend the model so that the unemployment trap displays high but finite unemployment.[20] In Appendix A1.2 we show an example of a model with positive liquidity (solved numerically) in which the unemployment trap occurs at a positive level of unemployment, as in that model the **EE** curve slopes downward for very low values of the job finding rate, but upward sloping for higher values.

Alternatively, one could consider the following specification of the matching function:

$$\widetilde{m}_s = e_s^{\alpha} v_s^{1-\alpha}, \tag{40}$$

where $v_s = \int_j (v_{j,s} + \widetilde{v})\, dj$ is the measure of aggregate vacancies. Here $v_{j,s}$ denotes the measure of formal vacancies posted by firm j (at the flow cost κ per vacancy) while $\widetilde{v} \geq 0$ denotes a fixed amount of "informal" vacancies, which are costless. The idea here is that some job matches are formed even when firms do not actively post any vacancies. This could happen through informal channels such as "word of mouth." Along these lines, Davis, Faberman and Haltiwanger (2013) find that a substantial fraction of establishment-level hiring takes place without formal vacancies having been posted.

For this specification of the matching function, the availability of "informal vacancies," implies that market activity does not get wiped out in the unemployment trap and that the job finding rate in this equilibrium remains strictly positive:

$$\widetilde{\eta} = \frac{\widetilde{e}^{\alpha} v^{1-\alpha}}{\widetilde{e}},$$

where $\widetilde{e}$ satisfies the condition:

$$\widetilde{e} = (1 - \xi) \frac{\omega}{\omega (1 - \widetilde{\eta}) + \widetilde{\eta}}.$$

In Appendix A6 we illustrate graphically the determination of the unemployment trap in this version of the model. The outcome is equivalent to the situation in Figure 2, apart from the limit point displaying a low but positive job finding rate.

The unemployment trap is an intriguing outcome. The slow recovery after the Great Recession and the very protracted nature of the surge in unemployment observed in the U.S. (and many other OECD economies) have spurred a renewed interest in "secular stagnation": equilibrium outcomes consistent with long periods of low activity and high unemployment. Hansen (1939) argued that such outcomes (with negative natural real interest rates) were most likely produced by a combination of a low rate of technological progress and population aging, implying high savings rates and low investment rates. Recently, Eggertsson and Mehrotra (2014) have argued that deleveraging may lead to secular stagnation and exacerbate the problems that follow from an aging population and falling investment goods prices.

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20. Interestingly, when the central bank's policy rule for the short term interest rate depends on labor market tightness, the unemployment trap, if it exists, displays low but strictly positive job finding rates. We discuss this in Appendix A5.

The unemployment trap that can arise in our model offers an alternative perspective of secular stagnation, which ties together low real interest rates, high unemployment and low activity. Importantly, the unemployment trap can occur in our model purely because of expectations and thus does not rely on sudden changes in population growth, technological progress or financial tightening. Furthermore, while the nominal interest rate may be low in the unemployment trap, its root cause does not derive from the ZLB on nominal interest rates. Therefore, the ongoing discussions about re-design of monetary policy to prevent secular stagnation by avoiding the ZLB may be in vain.

3.2. Local Determinacy

Next we consider the local determinacy properties of the steady states of the model. This is an important question in macroeconomics as much attention is often focused on characterizing fluctuations in the vicinity of the steady state in which case it is of first-order importance whether equilibria are locally unique. To investigate local determinacy, we log-linearize the model around the steady states.

The log-linearized model: Let a hat denote a log deviation from the intended steady state, i.e. $\widehat{x}_s = \ln x_s - \ln \overline{x}^I$, where $\overline{x}^I$ denotes the value of x_s in the intended steady state (discussed above). We assume that monetary policy parameters are such that $\overline{R}$, $\overline{\theta}$ and $\overline{\Pi}$ correspond to the levels of, respectively, R, θ and Π, in the intended steady state. In order to simplify the expressions marginally, we assume that the intended steady-state displays price stability, $\overline{\Pi}^I = 1$. Moreover, since we want to focus on stability properties we assume for the moment that there are no productivity shocks.

The log-linearized model can be represented as:

$$\widehat{c}_{n,s} - \beta \overline{R} \mathbb{E}_s \widehat{c}_{n,s+1} = -\frac{1}{\mu} \left(\widehat{R}_s - \mathbb{E}_s \widehat{\Pi}_{s+1} - \beta \overline{R} \Theta^F \mathbb{E}_s \widehat{\eta}_{s+1} \right), \tag{41}$$

$$\frac{\varphi}{\gamma} \left(\widehat{\Pi}_s - \beta \mathbb{E}_s \widehat{\Pi}_{s+1} \right) = w \widehat{w}_s + \frac{1-\gamma}{\gamma} A_s + \frac{\kappa}{q} \frac{\alpha}{1-\alpha} \widehat{\eta}_s$$
$$- (1-\omega) \beta \frac{\kappa}{q} \left(\frac{\alpha}{1-\alpha} \mathbb{E}_s \widehat{\eta}_{s+1} + \mathbb{E}_s \widehat{\Lambda}_{c,s,s+1} \right), \tag{42}$$

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s + \delta_\theta \widehat{\theta}_s, \tag{43}$$

$$\widehat{w}_s = \chi \widehat{\eta}_s, \tag{44}$$

$$\widehat{c}_{n,s} = \widehat{w}_s, \tag{45}$$

$$\widehat{\Lambda}_{c,s-1,s} = \widetilde{\mu} \left(\widehat{c}_{c,s} - \widehat{c}_{c,s-1} \right), \tag{46}$$

$$c_c \widehat{c}_{c,s} = \frac{1}{\xi} \left[y \widehat{y}_s - \kappa v \widehat{v}_s - w n \left(\widehat{w}_s + \widehat{n}_s \right) \right], \tag{47}$$

$$\widehat{y}_s = \widehat{n}_s, \tag{48}$$

$$\widehat{n}_s = (1-\omega) \widehat{n}_{s-1} + \frac{\eta e}{n} \left(\widehat{\eta}_s + \widehat{e}_s \right), \tag{49}$$

$$e \widehat{e}_s = (\omega - 1) n \widehat{n}_{s-1}, \tag{50}$$

where $\widetilde{\mu}$ denotes the risk aversion of the entrepreneurs and Θ^F is given as:

$$\Theta^F \equiv \omega\eta\left((\vartheta/w)^{-\mu}-1\right)-\chi\mu\omega\left(1-\eta\right),$$

and the elasticity $\chi > 0$ is a convolution of the model's deep parameters, which captures how the wage responds to fluctuations in the job finding rate and depends critically on the bargaining parameter υ.

Equation (41) is the log-linearized Euler equation of the employed workers. As already discussed, the key differences relative to complete markets are (i) stronger discounting, $\beta\overline{R} < 1$, and (ii) the presence of the endogenous risk wedge which fluctuates proportionally with the expected job finding rate and captures the precautionary saving motive.[21] Its strength and cyclicality is determined by Θ^F, which depends on structural parameters. The first part, $\omega\eta\left((\vartheta/w)^{-\mu}-1\right) > 0$, captures the impact of earnings risk (due to fluctuations unemployment risk) on precautionary savings.[22] The second part, $-\chi\mu\omega\left(1-\eta\right) < 0$, relates to changes in earnings risk which derive from wage fluctuations which is a procyclical risk channel as long $\chi > 0$. We refer to $\Theta^F > 0$ as countercyclical endogenous earning risk and $\Theta^F < 0$ as procyclical earnings risk. If consumption losses upon unemployment are large, e.g. in the face of little insurance, risk will countercyclical. On the other hand, if wages are elastic and procyclical (high χ), risk will be procyclical. When $\Theta^F = 0$, the endogenous risk wedge vanishes and the above equation reduces to the log-linearized Euler equation obtained in standard representative-agent models, aside from the constant $\beta\overline{R} < 1$.

Equation (42) is the log-linearization of the firms' optimal price-setting condition into which we have substituted the log-linearized expression for real marginal costs and exploited that $q_s = \eta_s^{-\frac{\alpha}{1-\alpha}}$. The left-hand side of the above equation is the sticky-price wedge, which vanishes in the absence of nominal rigidities ($\varphi = 0$) or in the limit with perfect competition ($\gamma \to \infty$). The right-hand side is the log-linearized marginal cost, which is standard given the presence of search and matching frictions. Note that the Kuhn-Tucker multiplier on the vacancy non-negativity constraint disappears because we log-linearize around the intended steady-state.

Equation (43) is the log-linearized interest rate rule. Since the interest rate is positive in the intended steady-state, this relationship depends on the central bank's response to deviations of inflation and labor market tightness from their targets. (44) is the log-linearized real wage solution to the Nash bargaining game while (45) exploits the fact that employed workers in equilibrium consume their income. Finally,

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21. The incomplete markets wedge that occurs in the log-linearized Euler equation differs from its steady state version because of a different normalization and because of the impact of wage fluctuations on savings, see below.

22. The first part of Θ^F becomes zero if the steady-state job finding rate, η, equals zero. The reason for this is technical, however. Note that $\mathbb{E}_s\widehat{\eta}_{s+1}$ is the *percentage* deviation in the expected job finding rate from its steady-state value. If the steady-state value is zero, no percentage deviation represents any actual change.

equations (46)-(50) are the log-linearized expressions for the capitalists' intertemporal marginal rate of substitution, capitalists equilibrium consumption, the production function, and the laws of motion of employment and unemployment, respectively.

Characterizing the stability conditions of this set of equations is in general feasible only numerically. However, under two simplifying assumptions, we obtain a very simple characterization.

Assumption 2: *Assume that capitalists are risk neutral ($\widetilde{\mu} = 0$) and that $\varphi_\pi = 1/\beta > 1$.*

Risk neutrality of the capitalists implies that the local stability properties can be determined from the first five equations only and that there are no endogenous state variables. In Appendix A3 we relax this assumption and show that it has no material consequences for our results. The second equation is inconsequential, since the coefficient on tightness, δ_θ, is left unrestricted.[23]

We can now state the main result regarding local determinacy of the intended steady-state:

Proposition 2: *Under Assumptions 1 and 2, local determinacy of the intended steady-state with price stability requires that:*

$$\frac{\varphi}{\gamma} \left(\beta^2 \overline{R} \Theta^F - \frac{\beta \delta_\theta}{1-\alpha} \right) - w\chi - \frac{\varphi}{\gamma} \mu \beta \left(1 - \beta \overline{R} \right) \chi < \frac{\kappa}{q} \frac{\alpha}{1-\alpha} \left(1 - \beta \left(1 - \omega \right) \right).$$

Proof: Under Assumption 2, the model can be reduced down to a single stochastic forward looking difference equation for the job finding rate (see Appendix A2 for the details):

$$E_s \widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s, \tag{51}$$

$$\Psi \equiv \frac{\varphi \gamma^{-1} \mu \chi \beta + \varphi \gamma^{-1} \frac{\beta \delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q} \frac{\alpha}{1-\alpha}}{\frac{\kappa}{q} \frac{\alpha \beta (1-\omega)}{1-\alpha} + \varphi \gamma^{-1} \mu \beta^2 \overline{R} \chi + \varphi \gamma^{-1} \beta^2 \overline{R} \Theta^F} = \frac{\Psi^N}{\Psi^D}.$$

Both the numerator, Ψ^N, and the denominator, Ψ^D, are positive, and local determinacy requires $\Psi > 1$, which holds under the condition stated in Proposition 2. QED.

In order to understand the consequences of this, consider initially the local determinacy condition when real wages are inelastic, i.e. constant.

Determinacy around the intended steady state under inelastic real wages: Imposing that $\chi = 0$, so that real wages are constant, simplifies the local determinacy

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23. The log-linearized model contains no endogenous state variables and hence for any desired pair of values δ_π and δ_θ there exists a value δ_θ^* such that the same solution is obtained under the restriction that $\delta_\pi = \frac{1}{\beta}$.

condition to:

$$\underbrace{\frac{\varphi}{\gamma}}_{\textbf{(i), (ii)}} \underbrace{(\beta^2 \overline{R} \Theta^F}_{\textbf{(iii)}} \underbrace{-\frac{\beta \delta_\theta}{1-\alpha})}_{\textbf{(iv)}} < \underbrace{\frac{\kappa}{q} \frac{\alpha}{1-\alpha} \left(1 - \beta \left(1 - \omega\right)\right)}_{\textbf{(v)}}$$

This condition clarifies the importance of the various market frictions and their interaction. In particular, we can separate the roles of the four types of market frictions present in the model (incomplete markets, sticky prices, imperfect competition and labor adjustment costs), as well as on monetary policy:

(i) *Price rigidity.* The stickier are prices (higher φ), the more likely is the possibility that the equilibrium is locally indeterminate. If prices are fully flexible ($\varphi = 0$) the equilibrium is always determinate since the left-hand side collapses to zero and the right-hand side is strictly positive.

(ii) *Imperfect competition.* Less substitutability across goods (lower γ) impacts on the determinacy condition symmetrically to larger nominal rigidities. Under perfect competition ($\gamma \to \infty$) the equilibrium is always determinate.

(iii) *Endogenous earnings risk.* When $\chi = 0$, the endogenous risk wedge is countercyclical or acyclical, $\Theta^F \equiv \omega \eta \left((\vartheta/w)^{-\mu} - 1\right) \geq 0$. A larger endogenous risk wedge unambiguously demands more aggressive monetary policy to ensure local determinacy of the intended equilibrium. When $\Theta^F = 0$, the equilibrium is always locally determinate.

(v) *Monetary policy.* The more aggressively monetary policy responds to tightness, i.e. the higher δ_θ, the less likely is local indeterminacy.

(iv) *Labor adjustment cost.* The term $\frac{\kappa}{q} \frac{\alpha}{1-\alpha} \left(1 - \beta \left(1 - \omega\right)\right)$ denotes the steady-state marginal cost of hiring a worker today rather than tomorrow, so we can think of it as a labor adjustment cost, i.e. a real labor rigidity. Note that this cost is proportional to the steady-state hiring cost $\frac{\kappa}{q}$. The larger the labor adjustment costs, the more likely is local determinacy of the equilibrium.

The introduction of incomplete markets has fundamental consequences for local determinacy of the intended steady-state. First, in HANK&SAM, when risk is countercyclical, the Taylor principle, $\delta_\pi > 1$, no longer suffices to guarantee local determinacy. The reason for this is that, when endogenous earnings risk is countercyclical, expectations of higher inflation may be self-fulfilling even if the central were to stabilize the direct impact of inflation on the real interest rate since demand (and thus inflation) is also stimulated by a decline in unemployment risk. Thus, monetary policy needs to be even more aggressive to rule out expectational equilibria.

Secondly, the wedges interact in important ways in the HANK&SAM model. As long as monetary policy dominates the endogenous risk wedge, $\Theta^F < \beta \delta_\theta$, the sticky-price wedge and the labor market wedge are irrelevant and the intended equilibrium is locally determinate. However, once the endogenous risk wedge dominates the monetary policy effect, $\Theta^F > \beta \delta_\theta$, the three wedges *all* matter and market incompleteness, nominal rigidities and risk aversion become complements, making local indeterminacy increasingly likely in combination.

An intriguing insight regards the impact of labor market frictions since the higher is the labor adjustment cost, the *less* likely it is for indeterminacy to happen. The reason for this is that when it is costly for firms to adjust on the labor margin, they are more likely to adjust prices which neutralizes the feedback mechanism and makes local indeterminacy less likely.

The above analysis complements a literature which has studied local determinacy in New Keynesian Models with both forward-looking and "rule-of-thumb households", see for example Galí, López-Salido and Vallès (2003) and Bilbiie (2008). A crucial difference with our environment, however, is that in these models there is no idiosyncratic risk and hence no precautionary saving motive. In our model, the precautionary motive, coupled with endogenous risk, is the key source behind the breakdown of the Taylor principle, as demonstrated above.

Determinacy around the intended steady state under flexible real wages: Consider again the equation in Proposition 2. Elastic wages ($\chi > 0$) moderates the insights above in three ways. First, more elastic wages impacts on the endogenous risk channel since $\Theta^F = \omega \eta \left((\vartheta/w)^{-\mu} - 1 \right) - \chi \mu \omega (1 - \eta)$ is decreasing in χ. For that reason, higher wage flexibility makes it more likely that the intended equilibrium is locally determinate. If wage flexibility is sufficiently high that $\Theta^F < 0$, local determinacy is guaranteed as long as $\delta_\theta \geq 0$.

Second, wage flexibility creates a *marginal cost channel* via the term $-w\chi$, as it pushes down wage costs during times of low market tightness. This also makes local determinacy more likely. Finally, wage flexibility generates a *discounting channel*, which enters via the term $-\varphi\gamma^{-1}\mu\beta \left(1 - \beta\overline{R} \right) \chi$ which also increases the likelihood of local determinacy of the intended steady-state. This term arises only under incomplete markets, but does not require job risk to be endogenous. It emerges due to the Euler equation "discount" on future income (consumption), $\beta\overline{R} < 1$. See McKay, Nakamura and Steinsson (2017) for a discussion of this discount in relation to the "forward guidance puzzle".

Hence, all three channels of wage flexibility are stabilizing in the vicinity of the intended steady state and push the model towards the determinacy region of the parameter space.

Determinacy around the unemployment trap: We now consider local determinacy around the unemployment trap. To this end, we exploit that the non-negativity constraint on vacancies binds. Hence, we can drop Equation (42) and set η_s equal to 0 (or equal to a lower bound if some frictionless hiring is introduced). Thus, the job finding rate is trivially determined. The Euler equation, log-linearized around the unemployment trap, is given by:

$$0 = \delta_\pi \widehat{\Pi}_s - \mathbb{E}_s \widehat{\Pi}_{s+1}.$$

It follows immediately that the steady state is locally determinate if and only if $\delta_\pi > 1$, i.e. the interest rate elasticity with respect to inflation exceeds unity. Thus,

the unemployment trap is determinate under a standard Taylor rule which responds more than one-for-one to inflation.

4. Fluctuations

We now solve for the local dynamics in the vicinity of the intended steady state in response to aggregate shocks. We focus on the impact of productivity shocks and monetary policy shocks, but it is not difficult to derive the implications for other shocks, such as mark-up shocks or non-fundamental "belief shocks."[24]

4.1. Productivity shocks

Proposition 3: *Under Assumptions 1 and 2, and assuming local determinacy, the solutions to the job finding rate and the inflation rate in response to productivity shocks are given as*:

$$\widehat{\eta}_s = \Gamma_\eta^A A_s, \tag{52}$$

$$\widehat{\Pi}_s = \Gamma_\Pi^A A_s, \tag{53}$$

$$\Gamma_\eta^A = \frac{\gamma - 1}{\varphi\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho_A\beta\overline{R}\Theta^F\right) + \gamma\frac{\kappa}{q}\frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha} + \left(\gamma w + \varphi\mu\beta\left(1 - \rho_A\beta\overline{R}\right)\right)\chi} \geq 0,$$

$$\Gamma_\Pi^A = \frac{\beta^2\overline{R}\Theta^F\rho_A - \frac{\beta\delta_\theta}{1-\alpha} - \mu\chi\beta\left(1 - \rho_A\beta\overline{R}\right)}{1 - \beta\rho_A}\Gamma_\eta^A.$$

Proof: Imposing Assumptions 1 and 2, the log-linearized solution for the job finding rate when there are productivity shocks can be written as (see Appendix A2 for a derivation):

$$\mathbb{E}_s\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s - \Omega A_s,$$

$$A_s = \rho_A A_{s-1} + \sigma_A\varepsilon_s^A,$$

$$\Omega = \frac{(\gamma - 1)/\gamma}{\Psi^D},$$

where $\Psi > 1$ under local determinacy and $\Omega > 0$ since we established above that $\Psi^D > 0$ and $\gamma > 1$. Applying the method of undetermined coefficients to a guess of the form $\widehat{\eta}_s = \Gamma_\eta^A A_s$ gives us:

$$\Gamma_\eta^A = \frac{\Omega}{\Psi - \rho_A}.$$

Inserting the expressions for Ω and Ψ delivers the solution for Γ_η^A in Proposition 3. Next, guess that the solution for the inflation rate of the form $\widehat{\Pi}_s = \Gamma_\Pi^A A_s$ and insert

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24. We outline the implications for belief shocks in Appendix A2. For an analysis of technology shocks and monetary policy shocks in the standard New Keynesian model, see Galí (1999).

into the log-linearized Euler equation yields the solution for inflation including Γ_Π^A. QED.

These solutions for the dynamics of the job finding rate and inflation can be used to derive the solutions for all other relevant variables. The policy functions have interesting properties. Consider first the job finding rate. Higher productivity lowers marginal costs, induces firms to post more vacancies and therefore improves job finding prospects for the unemployed, $\Gamma_\eta^A > 0$. From Proposition 3 it also follows that the impact of technology shocks on the job finding rate depends positively on the endogenous risk wedge, $\frac{\partial \Gamma_\eta^A}{\partial \Theta^F} \geq 0$. Hence, countercyclical endogenous earnings risk , $\Theta^F > 0$, amplifies the impact of productivity shocks on the job finding rate relative to the acyclical risk case. When the earnings risk is countercyclical, lower productivity implies worse job finding prospects which stimulates employed workers' desired savings, leading firms to post fewer vacancies which induces worse job finding prospects etc. Procylical endogenous earnings risk, $\Theta^F < 0$, instead stabilizes the impact of productivity shocks on the job finding rate for symmetric reasons.

It also follows from Proposition 3 that the incomplete markets model may have the – perhaps counter-intuitive – implication that higher price stickiness can amplify the impact productivity shocks on the job finding rate. In particular, this happens when:

$$\rho_A \beta \overline{R} \Theta^F > \frac{\delta_\theta}{1-\alpha} + \mu\beta \left(1 - \rho_A \beta \overline{R}\right) \chi, \tag{54}$$

i.e. when endogenous earnings risk is sufficiently countercyclical relative to the stabilizing effects of monetary policy responses to labor market slack and wage flexibility. In this case, there is in this case complementarity between the endogenous risk wedge and sticky prices. This complementarity arises because, the costlier it is to adjust prices, the more hiring responds to variations in demand, and variations in hiring is the core of the amplification mechanism. When condition (54) is instead violated, higher price stickiness is stabilizing. Note also that when (54) holds and prices are sticky, more aggressive monetary policy dampens the response, since $\frac{\partial \Gamma_\eta^A}{\partial \delta_\theta} \leq 0$.

The impact of productivity shocks on the job finding rate is muted by elastic real wages, $\frac{\partial \Gamma_\eta^A}{\partial \chi} < 0$, since real wage flexibility both moderates the countercyclical movements in precautionary saving and implies a less elastic response of marginal costs to productivity shocks.[25] Finally, high vacancy posting costs are also stabilizing, $\frac{\partial \Gamma_\eta^A}{\partial \kappa} < 0$, because they lead firms to load more of the adjustment on prices than on hiring.

The properties of the response of inflation to technology shocks are more complex than in the standard NK model, due to the endogenous risk channel. In the NK

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25. The facts that $\frac{\partial \Gamma_\eta^A}{\partial \chi} < 0$ follows from differentiation and using that $\beta \overline{R} \leq 1$, $\rho_A \in (-1, 1)$ and $\frac{\partial \Theta^F}{\partial \chi} < 0$.

FIGURE 3. Response of CPI inflation to a positive TFP shock.
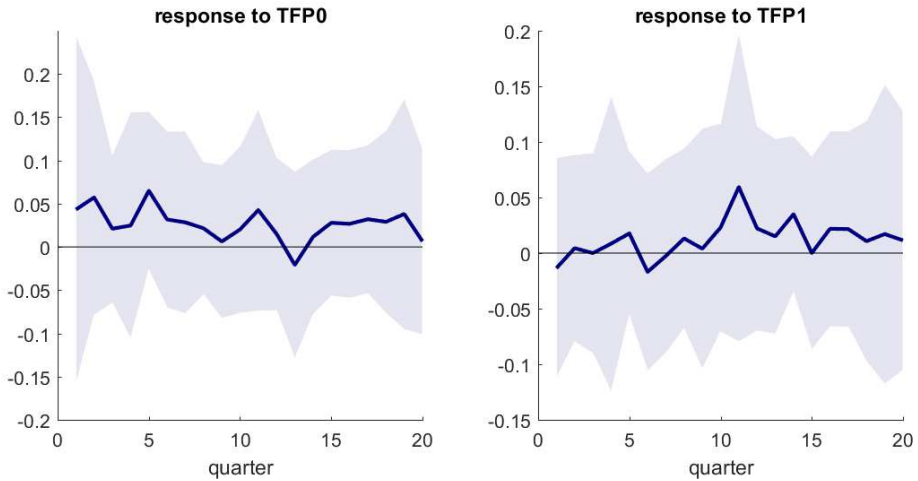
Notes: We plot Local projection IRFs with fours lags of the dependent variable as controls. Confidence bands, shown by the shaded areas, are 90 percent using Newey-West errors. Inflation is measured as 400*ln(CPI(t)/CPI(t-1)). TFP is 400*ln(TFP(t)/TFP(t-1)) with and with controls for factor utilization. Sample period is 1980 – 2019:4, quarterly.

model, when the productivity process is stationary, higher productivity implies lower marginal costs which means that inflation falls. In the incomplete markets model, inflation may increase or decrease even when productivity shocks are stationary. A positive relationship between inflation and productivity (i.e. $\Gamma_\Pi^A > 0$) arises when condition (54) holds. When the inequality is satisfied, the demand stimulus from higher productivity is sufficiently strong that inflation actually *rises*. Otherwise, i.e. when (54) is violated, the marginal cost effect dominates. Thus, a strongly countercyclical risk wedge will tend to imply a small decline or even an increase in inflation in response to a positive productivity shock.

The possibility that increased productivity is inflationary is not a mere theoretical curiosity. In Figure 3, we show the impulse response of CPI inflation to TFP shocks where the latter correspond to those estimated by Fernald and Wang (2016). Using local projection, we regressed (400 times) quarterly (log) changes in the CPI on TFP (log) growth (times 100) for a sample that starts in 1980. Depending on whether one controls for movements in factor utilization or not, higher TFP either leaves inflation unchanged or gives rise to higher inflation. While the empirical results come with a

fair amount of uncertainty, they do suggest that a positive inflation response is not simply an odd feature of our model.[26]

4.2. Monetary policy shocks

We now consider the effects of monetary policy shocks. We introduce an exogenous shock e_t^R to the interest rate rule:

$$R_s = \max \left\{ \overline{R} \left(\frac{\Pi_s}{\overline{\overline{\Pi}}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\overline{\theta}} \right)^{\delta_\theta} \exp \left(z_t^R \right), 1 \right\}.$$

z_t^R follows an AR(1) process with a persistence parameter given by ρ_R.

Proposition 4. Under Assumptions 1 and 2, and assuming local determinacy, the

solutions to the job finding rate and the inflation rate in response to productivity shocks are given as:

$$\widehat{\eta}_s = \Gamma_\eta^R z_s^R, \tag{55}$$

$$\widehat{\Pi}_s = \Gamma_\Pi^R z_s^R, \tag{56}$$

$$\Gamma_\eta^R = \frac{-\varphi}{\varphi\beta \left(\frac{\delta_\theta}{1-\alpha} - \rho_R \overline{R} \Theta^F \right) + \frac{\gamma}{\beta} \frac{\kappa}{q} \frac{\alpha(1-\beta\rho_R(1-\omega))}{1-\alpha} + \left(\frac{\gamma}{\beta} w + \varphi\mu \left(1 - \rho_R \beta \overline{R} \right) \right) \chi} < 0,$$

$$\Gamma_\Pi^R = \frac{\beta \left(\beta \overline{R} \Theta^F \rho_R - \frac{\delta_\theta}{1-\alpha} - \chi \left(1 - \rho_R \beta \overline{R} \right) \right) \Gamma_\eta^R - \beta}{1 - \beta\rho_R}.$$

Proof: Guessing that the solutions are of the form $\widehat{\eta}_s = \Gamma_\eta^R z_s^R$ and $\widehat{\Pi}_s = \Gamma_\Pi^R z_s^R$, inserting the guesses in the Euler equation and the optimal price setting condition, and using the method of undetermined coefficients deliver the solutions stated in the proposition. QED.

Following the same logic as above, we can verify that $\Gamma_\eta^R < 0$. That is, a contractionary monetary policy shock triggers a decline in the job finding rate and hence in output, as in the standard NK model. This decline is amplified by the presence of countercyclical risk, since $\frac{\partial \Gamma_\eta^R}{\partial \Theta^F} > 0$ provided that there is some persistence in the monetary policy shocks ($\rho_R > 0$). Intuitively, the boom in demand created by a monetary expansion reduces idiosyncratic risk which triggers a further boom in demand. Hence, market incompleteness will tend to provide a more powerful role for monetary policy shocks in the case where the endogenous earnings risk is countercyclical.

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26. The result holds also for the core PCE and here the positive response holds regardless of the TFP measure. The results also hold true for a sample period that starts in 1984, the sample split that Fernald and Wang (2016) focus upon.

5. An Empirical Perspective and Extensions

In this section we discuss various extensions of the model and provide an empirical perspective. The latter is not meant as a formal test of the model, but rather as a cursory, indicative comparison of the model to the data.

### 5.1. Is Earnings Risk Pro- or Countercyclical?

We have shown that many properties of the model depend on the cyclical nature of the endogenous earnings risk. When this risk is countercyclical (dominated by unemployment risk), the model implies that monetary policy must be more aggressive to rule out local indeterminacy than in the standard NK model, that productivity shocks may be inflationary and have larger real effects, and that the economy may be susceptible to "unemployment traps." Procyclical endogenous earnings risk instead implies stabilization of the economy relative to the standard NK representative agent model.

One indirect check on which of these cases is likely to be more relevant comes from the relationship between variations in job finding rates and the real interest rate. When markets are complete, agents save purely for reasons related to intertemporal smoothing. Hence, the propensity to save will be high in booms and low recessions. Therefore, when jobs are easy to find because the economy is doing well, real interest rates will tend to be low because of high desired savings and vice versa. Similarly, in an incomplete-markets model with acyclical earnings risk, there is a precautionary saving motive, but it is constant over time. Procyclical earnings risk induces an even stronger negative covariance between job finding rates and the real interest rate because booms not only stimulate saving through the intertemporal channel but also through precautionary saving. By contrast, when earnings risk is strongly countercyclical, (employed) agents have an incentive to save for precautionary reasons in recessions, which can induce a positive comovement between job finding rates and the real interest rate. Formally, log-linearizing the employed workers' Euler equation, we obtain the following expression for the real interest rate,

$$\widehat{R}_s^r = -\left(1-\alpha\right)\left(\mu\chi - \beta\overline{R}\rho_A\left(\mu\chi + \Theta^F\right)\right)\widehat{\theta}_s$$

where $\widehat{R}_s^r = \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$ is the ex-ante real interest rate (in percentage deviation from its steady state). Under complete markets, acyclical or procyclical earnings risk, real interest rates and labor market tightness (and therefore the job finding rate) comove negatively. When endogenous earnings risk is countercyclical and $\Theta^F > \left(1 - \beta\overline{R}\rho_A\right)/\beta\overline{R}\rho_A\mu\chi > 0$, however, tightness and real interest rates become positively related. This happens as the precautionary saving motive strengthens in recessions, dominating the change in the intertemporal motive.

Figure 4 illustrates the relationship between real interest rates and labor market tightness (the ratio of job vacancies to unemployment) in U.S. data. These two variables comove positively, indicating that real interest rates are low when jobs are

FIGURE 4. Real interest rate (R^r) and labor market tightness (v/u) in the data.
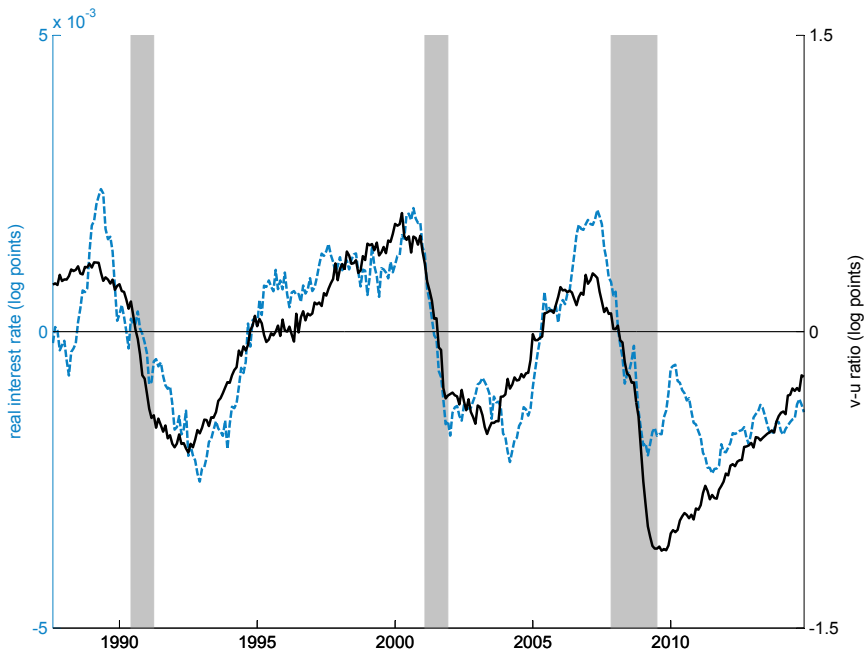
Notes: Real interest rate and labor market tightness (vacancy-unemployment ratio) in the United States; deviations from trend. The real interest rate is expressed on a monthly basis and is computed as the Federal Funds rate minus a six-month moving average of CPI inflation. Vacancies are measured as the composite Help Wanted index from Barnichon (2010). Data series were logged and de-trended using a linear trend estimated over the period up to the end of 2007.

hard to find and vice versa. This points towards dominance of the countercyclical endogenous earnings risk channels.

An alternative approach is to evaluate Θ^F directly. While the precise expression for this parameter depends on assumptions we have made, confronting the analytical model with the data is an insightful exercise, which at least indicates the likely sign of this key parameter. We showed above that

$$\Theta^F \equiv \omega\eta\left((\vartheta/w)^{-\mu} - 1\right) - \chi\mu\omega\left(1 - \eta\right),$$

To evaluate the parameters entering in this expression, we consider a time period of one month. We calibrate the steady-state job finding rate η and the job loss rate ω to the average of their counterparts in the Current Population Survey (CPS) (we look at data from January 1990 until August 2019).[27] The average monthly job finding

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27. In particular, we measure η_t as the unemployment-to-employment transition rate, $\omega_t = \frac{u_t - u_{t-1}(1-\eta_t)}{(1-u_{t-1})(1-\eta_t)}$, given a series for the unemployment rate u_t, consistent with the timing assumptions in our model.

rate is 25.2 percent while the job loss rate is estimated to be two percent per month. Recall further that $1 - \vartheta/w = 1 - c_u/c_n$ is the decline in consumption upon job loss. Following Karabarbounis and Chodorow-Reich (2017), we assume that consumption drops 20 percent upon job loss. However, we also consider a much smaller drop of only 5 percent. For the risk aversion parameter μ we consider both $\mu = 0.5$ and $\mu = 2$, as estimates of this parameters vary across studies in the literature.

The final parameter that matters is the wage flexibility parameter, $\chi = \frac{\partial \ln w_s}{\partial e_s} \frac{\partial e_s}{\partial \ln \eta_s}$. We can obtain the second term by differentiating the transition equation for unemployment (the number of searchers) with respect to the (log of the) job finding rate. Evaluated at the steady state this gives $\frac{\partial e_s}{\partial \ln \eta_s} = -\eta e - \omega \eta (1 - e)$, which we evaluate using CPS data. The semi-elasticity of the wage with respect to unemployment, $\frac{\partial \ln w_s}{\partial e_s}$, has been estimated in several studies. Gertler, Huckfeldt and Trigari (2016) estimate this elasticity to be $\frac{\partial \ln w_s}{\partial e_s} = -0.16$ for job stayers (see their Table 2, fourth column). We take this number as our baseline, since Θ^F captures the expected wage of those currently employed, in the event they remain employed. However, we also consider a much larger elasticity of -1.5, which is in the ballpark of the estimates which Gertler et al. estimate specifically for new hires from unemployment (-0.164) and job switchers (-2.085).[28]

Table 1 shows the results. The countercyclical effect of unemployment risk clearly dominates the procyclical effect of wage risk (i.e. $\Theta^F > 0$). Only when we assume *both* a small consumption drop (5 percent) and a very elastic wage ($\frac{\partial \ln w_s}{\partial e_s} = -1.5$), do we find that the effect of wage risk slightly dominates. Given that these values are relatively unlikely in the light of most studies in the literature, we conclude that countercyclical earnings risk is probably the more relevant case.

5.2. Extensions

One may conjecture that the introduction of capital accumulation mutes the amplification mechanism we have discussed in this paper. In particular, when the model implies amplification, the downward pressure on real interest rates in a recession reduces the cost of capital therefore stimulating capital investment. Hence, while precautionary savings reduces goods demand in recessions, this force might be neutralized by increasing demand for capital from the corporate sector. Going against this conjecture, however, depressed goods demand also induces lower *return* on capital investment which magnifies the amplification. Hence, it is ex ante unclear whether introducing capital into the model mutes the results we have discussed above or whether it introduces even further amplification.

It is not possible to solve the model in closed form when we introduce capital. We therefore investigate this issue by calibrating a numerically solved model and

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28. We also estimated χ directly by running a regression of w_s on the job finding rate η_s, and a time trend. Here, we measured w_s as average hourly earnings of production and nonsupervisory employees, deflated by the CPI. While results vary across specifications, the largest wage elasticity we found was $\widehat{\chi} = 0.03$ which corresponds to about $\frac{\partial \ln w_s}{\partial u_s} = -1.5$.

examining its implications for the impact of productivity shocks, see Appendix A7. We assume that capital is owned by the capitalists and that the production function is Cobb-Douglas. Capitalists are assumed to be risk averse with preferences given in (1). For simplicity, we set the real wage constant, an outcome that corresponds to the Nash bargaining solution when the value of leisure is sufficiently high and firms' bargaining power is close to one.

Figure 9 in Appendix A7 illustrates the impact of a one percent negative productivity shock on unemployment in the economy with and without capital accumulation, comparing the outcome of the model with sticky prices and countercyclical endogenous risk with versions of the model that assume either flexible prices or exogenous risk. We find that the introduction of capital accumulation preserves the amplification mechanism, or even makes it slightly stronger. In other words, it is the demand effect that dominates in our simplified setting.

While above we have focused at the effects of shocks when the economy is away from the Zero Lower Bound (ZLB), in Appendix A10 we discuss some dynamic properties at the ZLB. In particular, we show that with endogenous earnings risk negative supply shocks may bring the economy to the ZLB, unlike in the standard NK model. Moreover, we show that the endogenous risk channel may overturn the property of the standard NK model that positive supply shocks tend to be contractionary at the ZLB.

Other interesting extensions include positive supply of nominal bonds and/or a positive borrowing limit for the unemployed. These extensions, however, require a full-blown numerical approach, which we deliberately avoided in this paper in order to gain insights from analytical formulas.

6. Conclusion

In this paper we have provided insights into the global and local properties of a HANK model with Search And Matching (SAM) frictions in the labor market. Our analysis was aimed at providing analytically-based insights and we accomplished this by making assumptions that, while retaining the essential ingredients of incomplete markets, imply a degenerate wealth distribution. The assumptions that we made might be thought to be strong, but they enable us to generate a series of results that we believe have a more general nature and should impact on future developments in this new literature.

Our key results are that (i) the HANK&SAM model features endogenous earnings risk, which derives from an interaction between goods demand and labor demand. This interaction is missing in NK models and in HANK models without unemployment risk or other sources of endogenous idiosyncratic and uninsurable income risk; (ii) The endogenous earnings risk is countercylical when the downside risk due to unemployment rising in recessions dominates the upside risk due to real wages rising in booms; (iii) When earnings risk is countercyclical, the impact of productivity shocks and monetary policy shocks on job finding rates and other real variables

tends to be amplified, relative to the standard NK model, or to models with procyclical endogenous earnings risk; Moreover, in this case, nominal rigidities and incomplete markets are complements; (iv) The Taylor principle may fail to deliver local determinacy of the intended steady state when earnings risk is countercyclical; And finally, (v) the economy may get stuck in a high unemployment - low inflation steady state, the unemployment trap, when endogenous earnings risk is sufficiently countercyclical unless monetary policy is very aggressive. We have also shown that the HANK model with countercyclical earnings risk can potentially resolve a number of puzzles that have arisen in the macroeconomic literature. These puzzles pertain to the existence of persistent low growth equilibria with low but positive inflation, the impact of supply shocks on inflation dynamics, and the presence of inflation at the ZLB. The model may also provide a coherent framework for understanding the positive relationship between real interest rates and labor market tightness which can be observed in the US.

These results add to the literature that has studied HANK models with labor market frictions such as Challe (2019), Challe and Ragot (2016), den Haan et al (2018), McKay and Reis (2016b) as well as our own earlier work, Ravn and Sterk (2017). These papers all stress the importance of labor market frictions for aggregate outcomes (or optimal policies) in a HANK setting but the current paper stands out in providing analytical insights in the pinpointing the mechanisms just summarized. The results are also important for the rapidly growing HANK literature. There is currently some discussion about whether these models bring much new to the table regarding aggregate fluctuations relative to two-agent versions of NK models. We have shown that a key feature that should be introduced into HANK models is endogenous and cyclical earnings risk channels. Search And Matching models provide a natural way of accomplishing this task, but there are alternatives.

Given our focus on analytical insights, we ignored issues related to accumulation of assets which would be important when confronting the model with micro data. We have also ignored fiscal policy issues and we have kept the analysis positive, thereby ignoring normative issues. Firms were kept symmetric and we did not introduce heterogeneous firm aspect which could matter for aggregate outcomes too. Each of these aspects would be very interesting avenues for future research. Further, in Appendix A9, we demonstrate that under incomplete markets the NK model becomes useful to analyze the link between monetary policy and financial asset prices. While we limit the analysis to simple analytical exercises, it would be interesting to evaluate the extent to which a full-scale heterogeneous-agents NK can explain observed asset prices. Vice versa, financial markets data may be useful to impose empirical discipline on the new generation of NK models.

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