

CONSUMPTION PUZZLES AND PRECAUTIONARY SAVINGS

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When marginal utility is convex, agents accumulate savings as a precautionary measure against labor-income eventualities. This paper shows that precautionary savings can go a long way in making the *excess-growth*, *excess-smoothness*, and *excess-sensitivity* features of consumption consistent with the stochastic processes of labor income observed in the U.S. at the microeconomic level.

1. Introduction

Most of the *empirical* research on the Life-Cycle/Permanent-Income hypothesis (LCH/PIH), which deals with the relation between income and consumption disturbances, uses certainty-equivalence assumptions. One possible explanation for the repeated use of this specification is the degree of difficulty involved in obtaining closed-form solutions in the multiperiod optimization problem of a consumer who faces a random sequence of (uninsurable) labor income and whose utility function is not quadratic.¹

Unfortunately, by using a quadratic utility function important aspects of the problem are ignored. Theoretical studies have shown that whenever the utility function is separable and has a positive third derivative ($U''' > 0$) – a property of utility functions that exhibit nonincreasing absolute risk aversion – an increase in labor-income uncertainty, when insurance markets are not complete, will reduce current consumption and alter the slope of the consumption path [e.g., Leland (1968), Sandmo (1970), Dreze and Modigliani (1972), Miller (1974, 1976), Caballero (1987a), Kimball (1988), and Skinner (1988)]. These results are confirmed, for the case of a CRRA (constant relative risk aversion) utility function and i.i.d. (independently identically distributed) labor income

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¹Some *theoretical* exceptions are Merton (1971), Sibley (1975), Schechtman and Escudero (1977), Levhari, Mirman, and Zilcha (1980).

by the numerical simulations performed by Zeldes (1989a).² This paper, on the other hand, studies the interactions between precautionary-savings motives and different stochastic processes of labor income, including processes in which the variance of labor-income innovations is stochastic. This represents a step in the direction of bridging the gap between the theoretical precautionary-savings literature and the ongoing empirical consumption literature. As a result, several empirical puzzles can be (potentially) simultaneously explained.

Section 2 sets up the basic model and assumptions and discusses some of the limitations of the approach followed. Section 3 studies the interactions between riskiness, convexity of marginal utility, persistence of labor-income shocks, and the behavior of the expected and actual trajectory of consumption. The results obtained show that precautionary-savings behavior can account – under reasonable parameter assumptions – for the ‘persistent growth of consumption, even when the real interest has been negative’ [Deaton (1986)]. This extends previous theoretical literature (*op. cit.*) by allowing for income processes consistent with panel-data and time-series work. For example, using MaCurdy’s (1982) or Hall and Mishkin’s (1982) income-processes estimates together with a coefficient of relative risk aversion of three, yields an ‘excess’ consumption growth of about 2% (per year). Alternatively, given the consumption path, precautionary savings can explain the relatively low real interest rate observed in the postwar U.S. data. For example, an infinite-horizon model, with no population growth and the same parametric assumptions made above, yields a steady-state real interest approximately 6% lower than the discount rate. This section also shows that the difference in the path of consumption of two households (or economies) that face different interest rates, is attenuated (exacerbated) when labor income is nonstationary (stationary) and precautionary motives exist. Very loosely interpreted – and with all the limitations of comparative-statics experiments – this can partly rationalize some zero, or even negative, intertemporal substitution-parameter estimates [Hall (1988), Runkle (1986)].

Without exceptions, the theoretical literature mentioned above takes labor-income uncertainty as a given constant. Experiments are then performed through a once and for all unanticipated change in the level of labor-income uncertainty. Section 4 extends the existent literature beyond comparative statics by providing a simple model in which the variance of labor income is truly stochastic. Again a closed form for consumption is obtained. **An important corollary of this section is that, in the presence of precautionary motives, labor-income conditional heteroskedasticity affects the marginal propensity to**

²Zeldes’s (1989a) recent revision of his thesis paper also includes a log-random-walk process for labor income. However, numerical problems only allow him to deal with short horizons in this case.

consume even when the predisposition to risk does not change with the level of wealth, as is the case with the exponential utility function used in this paper. Under reasonable assumptions, the marriage between precautionary-savings motives and conditional heteroskedasticity can account, simultaneously, for two of the most important puzzles in the macro-consumption literature: The excess sensitivity [Flavin (1981), Hayashi (1982)] and the excess smoothness [Deaton (1986), Campbell and Deaton (1989)] of consumption to anticipated and unanticipated labor-income changes, respectively.³ Section 5 presents concluding remarks.

2. The problem setup and main assumptions

The problem to be solved is that of an infinitely-lived representative consumer.⁴ This consumer takes decisions in discrete time and has a time-separable utility function. He works a fixed number of hours (per year) for which he receives 'labor income'. The mean and variance of labor income are the only sources of uncertainty considered.⁵ He is also allowed to buy and sell a riskless bond, but there is no insurance market for labor income.⁶

Formally the problem can be stated as

$$\max_{c_{t+i}} E_t \left[\sum_{i=0}^{\infty} (1 + \delta)^{-i} U(c_{t+i}) \right], \quad (1)$$

subject to

$$c_{t+i} = y_{t+i} + (1 + r) A_{t+i-1} - A_{t+i},$$

$$\lim_{i \rightarrow \infty} A_{t+i} (1 + r)^{-i} = 0,$$

$$A_{t-1} \text{ given,}$$

where

E_t = conditional expectations operator,

δ = discount rate,

³It can also account for the excess smoothness of savings to unanticipated changes on income [Campbell (1988)]. In fact the latter is another dimension of the consumption-excess smoothness puzzle.

⁴See Caballero (1987b) for the case of finite and random horizon in the context of a multiperiod OLG model.

⁵Adding other sources of uncertainty, like tastes, does not convey major additional complications [e.g., Caballero (1988)].

⁶Incomplete markets is enough to get the results of this paper. Although in this case the concept of riskiness becomes more delicate since only the uninsurable part of income interacts with the precautionary-savings motive.

U = instantaneous utility function (felicity),

c = consumption,

y = labor income,

A = nonhuman wealth,

r = riskless return on the bond.

The simplest form to introduce precautionary savings is through an exponential utility function. This paper specializes to this type of preferences in spite of some of its unpleasant features like the possibility of negative consumption. (Unfortunately, explicitly imposing nonnegativity constraints impedes finding a tractable solution.⁷) Also undesirable is the fact that absolute risk aversion is constant, hence the elasticity of risky investments with respect to wealth is zero.⁸ Nevertheless, these assumptions allow us to isolate the precautionary-savings motive as the difference between the results in this paper and the results in standard certainty-equivalence empirical models⁹ used in the empirical literature. Section 4 briefly discusses the robustness of the main results of the paper to departures in the direction of a more appealing decreasing absolute risk-aversion utility function.

3. The slope of the consumption path

This section restricts labor income to processes with i.i.d. (independently identically distributed) innovations (hence the variance of the innovations is assumed, momentarily, constant) and concentrates on the interaction between different labor-income processes and the slope of the consumption path. After

⁷See Zeldes (1989b) for an example of the effects of binding liquidity and nonnegativity constraints.

⁸The CES or CRRA utility function does not exhibit these problems. However, finding a closed-form solution in this case seems to be much harder. The main problem arises from (i) the dependence of the precautionary-savings motive on the level of wealth and (ii) the effect of any labor-income shock on wealth. This combination produces a drift term (in the logs) that is not constant, complicating the solution for $\{v_{t+j}\}$, the innovation in the consumption process (see section 3 below).

⁹Dreze and Modigliani (1972) showed that the effect of an increase in uncertainty on current consumption can be decomposed into an 'income' and a 'substitution' effect. The income effect is the change in consumption due to the new expected utility level resulting from a change in the degree of uncertainty. This effect is always negative in the presence of risk aversion. The substitution effect, on the other hand, is the change in consumption due to the change in the desired optimal wealth at the time of receiving the uncertain income. When the utility function is exponential, absolute risk aversion does not depend on the level of wealth, hence the substitution effect is zero. This has important implications for the sensitivity of current consumption to income shocks. On the other hand, if the utility function exhibits decreasing absolute risk aversion (e.g., CES), the substitution effect is negative since people care less about future uncertainty if they have more wealth at that time. Whenever uncertainty rises, it is optimal to shift to the future more resources than those indicated by the income effect. The opposite happens when the utility function exhibits increasing absolute risk aversion (e.g., quadratic utility). Kimball (1988) provides an alternative interpretation in terms of the relative size of absolute prudence and risk aversion.

presenting the general framework, it argues that both micro and macro U.S. data are consistent with an important gap between the rate of consumption growth and the difference between the real interest rate and the discount rate (multiplied by the coefficient of intertemporal substitution).

Proposition 1 below summarizes the main implications of precautionary savings – in the context of the model presented here – for the consumption path. Perhaps of independent relevance is the simple approach followed in obtaining a closed-form solution for the consumption function: (i) given the set-up of the problem ‘guess’ the stochastic process for consumption¹⁰ in such a way that the Euler equation is satisfied, and (ii) use the (*ex post*) intertemporal budget constraint to infer, first, the characteristics of the stochastic component of the consumption process and, second, the consumption function itself.¹¹ The main results of the proposition are highlighted at the end of the section through several examples.

Proposition 1. If (a) the instantaneous utility function is exponential:

$$U(c_t) = -(1/\theta)e^{-\theta c_t},$$

with θ the coefficient of risk aversion, (b) income follows any ARMA process (with possibly a unit root), and (c) the return on assets is certain; then

- (i) *the stochastic process of consumption is a martingale with drift,*
- (ii) *the drift is increasing on riskiness in the Rothschild–Stiglitz (1970) sense¹² and on the persistence of labor-income shocks,*
- (iii) *the disturbance of the stochastic process of consumption is equal to the annuity value of the contemporaneous innovation in income, and*
- (iv) *when $r = \delta$, the consumption function can be decomposed, additively, in a term analogous to that of the certainty-equivalence case and a precautionary savings term.¹³*

¹⁰This is the main difference with (informal) dynamic programming techniques, where the guess is on the value function or on the consumption function itself. Certainly formal procedures like the symmetries approach presented in Boyd (1989) are more general, but at the cost of additional complexity.

¹¹This approach is closely related, in spirit, to the Martingale approach [Cox and Huang (1985)]. It could also be considered as a stochastic version of the approach followed under certainty by Modigliani and Brumberg (1954). Finally, Kotlikoff and Pakes (1984) use a somewhat similar technique.

¹²See Silbey (1975) and Miller (1976) for analogous results in the i.i.d. labor-income case.

¹³If r is different from δ , it is still possible to decompose the consumption function into a precautionary-savings term and a second term. However, the latter is no longer identical to consumption under a quadratic utility function.

Proof. For notational convenience, it is useful to assume that the interest rate is constant and equal to the discount rate.¹⁴

If a solution to (1) exists, it must satisfy the Euler equation

$$e^{-\theta c_t} = E_t[e^{\theta c_{t+1}}]. \quad (2)$$

The first step in finding a feedback solution is to make a guess on the form of the stochastic process followed by consumption. The functional form of this process is easily chosen by using the Euler equation. In this particular case – as well as in the quadratic case – the best guess is to assume that the process is linear in levels:¹⁵

$$c_{t+i} = \Gamma_{t+i-1} + \phi_{t+i-1}c_{t+i-1} + v_{t+i}, \quad (3)$$

with Γ_{t+i-1} and ϕ_{t+i-1} , as well as the distribution function of the innovation v_{t+i} , to be determined during the derivation of the solution.

Plugging (3) into (2) yields

$$e^{(-\theta c_t(1-\phi_t)+\theta \Gamma_t)} = E_t[e^{-\theta v_{t+1}}].$$

It is apparent that $\phi_t = 1$ for all t , otherwise consumption would be determined by the Euler equation, regardless of the budget constraint. After this condition is imposed, the Euler equation can be used to find the functional form of Γ :

$$\Gamma_t = (1/\theta) \ln E_t[e^{-\theta v_{t+1}}]. \quad (4)$$

The value of Γ_t depends on the distribution of v_{t+1} , hence it cannot be determined yet. Nonetheless, Jensen's inequality and (4) are enough to show that even when $r = \delta$ the slope of the consumption path is positive. If $\theta > 0$,

$$\theta \Gamma_t = \ln E_t[e^{-\theta v_{t+1}}] > E_t[-\theta v_{t+1}] = 0.$$

The next step is to find the distribution of v_{t+1} . For this, the source of uncertainty has to be made explicit. Assume that the only source of uncertainty is labor income and that this has a moving-average representation with

¹⁴If the interest rate is constant but different from the discount rate, the only change is that the slope term, Γ (to be discussed later), has an extra term equal to $(r - \delta)/\theta$. On the other hand, if the interest rate is nonconstant (but deterministic), in addition to the extra term in the slope the notation becomes slightly more cumbersome.

¹⁵If the utility function is CES the natural guess is a linear function in the logarithms.

ψ_i representing the i th MA coefficient, hence

$$E_t[y_{t+i}] - E_{t-1}[y_{t+i}] = \psi_i w_t,$$

with $\psi_0 \equiv 1$, $|\sum_{i=0}^{\infty} \alpha^i \psi_i| < \infty$, $\alpha \equiv (1+r)^{-1}$, and w_t an i.i.d. innovation disturbance.

The intertemporal budget constraint, $\sum_{i=0}^{\infty} \alpha^i (c_{t+i} - y_{t+i}) = A_t$, can now be written as follows:

$$\sum_{i=0}^{\infty} \alpha^i c_{t+i} - \sum_{i=1}^{\infty} \alpha^i (y_{t+i} - E_t[y_{t+i}]) - \sum_{i=0}^{\infty} \alpha^i E_t[y_{t+i}] = A_t. \quad (5)$$

But $y_{t+i} - E_t[y_{t+i}] = \psi_0 w_{t+i} + \psi_1 w_{t+i-1} + \dots + \psi_{i-1} w_{t+1}$. Replacing this and the expression for c_{t+i} given in (3) into (5) yields

$$\begin{aligned} & \sum_{i=0}^{\infty} \alpha^i c_t + \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i \Gamma_{t+j-1} - \sum_{i=0}^{\infty} \alpha^i E_t[y_{t+i}] \\ & + \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i v_{t+i} - \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i \psi_{i-j} w_{t+j} = A_t. \end{aligned} \quad (6)$$

Taking expectations conditional on the information available at time t yields the consumption function

$$c_t = y_t^p - (1-\alpha) \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i \Gamma_{t+j-1},$$

with $y_t^p \equiv (1-\alpha)(A_t + \sum_{i=0}^{\infty} \alpha^i E_t[y_{t+i}])$, the definition of permanent income.

This has proved part (iv) of Proposition 1.

Next, the stochastic sequence $\{v_{t+i}\}$ can be identified by replacing c_t back in (6) (the *ex post* budget constraint), obtaining the condition

$$\sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i (v_{t+j} - \psi_{i-j} w_{t+j}) = 0.$$

But this is satisfied for all t if and only if the following condition holds:¹⁶

$$\sum_{i=1}^{\infty} \alpha^i (v_h - \psi_{i-1} w_h) = 0 \quad \text{for all } h.$$

¹⁶An alternative way to derive the same condition is by noticing that both w_t and v_t are serially independent.

Eq. (7) yields the solution for the v_t 's,

$$v_t = \Psi w_t, \quad (8)$$

with $\Psi \equiv (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \psi_i$, proving part (iii) of the proposition.

Eq. (8) establishes that the consumption innovation is equal to the marginal propensity to consume out of wealth, $(1 - \alpha)$, times the revision in human wealth due to an income innovation, $\Psi w_t / (1 - \alpha)$. For example, if labor income follows a white-noise process $\Psi = (1 - \alpha)$, whereas if labor income can be better described by a random walk, $\Psi = 1$.

Replacing (8) back into expressions (3) and (4) gives a full characterization of the consumption process:

$$c_{t+i} = \Gamma_{t+i-1} + c_{t+i-1} + \Psi w_{t+i},$$

and of the slope of the consumption path:

$$\Gamma_{t+i-1} = (1/\theta) \ln E_{t+i-1} [e^{-\theta \Psi w_{t+i}}].$$

Furthermore, if the w_{t+i} 's are i.i.d. (hence so are the v_{t+i} 's), Γ is constant and the consumption function simplifies to

$$c_t = y_t^p - [\alpha/(1 - \alpha)] \Gamma,$$

i.e., for any given level of wealth consumption is equal to consumption under certainty equivalence minus a term related to precautionary-savings behavior.

The last step of this proof is to show that Γ is increasing on riskiness and persistence of labor-income shocks.

Rothschild and Stiglitz (1970) definition of riskiness implies that if v_{t+1} is riskier than v_{t+1}^* :

$$-(1/\theta) E_t [e^{-\theta v_{t+1}^*}] > -(1/\theta) E_t [e^{-\theta v_{t+1}}]. \quad (9)$$

Multiplying by $-\theta$ both sides of (9) yields the proof of the first part of (ii):

$$E_t [e^{-\theta v_{t+1}}] > E_t [e^{-\theta v_{t+1}^*}],$$

therefore

$$(1/\theta) \ln E_t [e^{-\theta v_{t+1}}] > (1/\theta) \ln E_t [e^{-\theta v_{t+1}^*}].$$

Finally, for a given distribution of the labor income shock, w , an increase in its persistence means that Ψ , and therefore the variance of v , rises. But in this

Table 1
Some examples of Γ , $\Psi = 1$ (percentage of average consumption).^a

Distribution of w	Γ expression	$(\sigma_w/y, \theta c)$		
		(0.1, 1)	(0.2, 1)	(0.1, 9)
N	$(\theta/2)\sigma_w^2$	1.000	2.000	4.500
Be	$\theta^{-1} \ln[(e^{\theta\sigma_w} + e^{-\theta\sigma_w})/2]$	0.499	1.987	3.998
Be^b	$\theta^{-1} \ln[0.1e^{3\theta\sigma_w} + 0.9e^{-\theta\sigma_w/3}]$	0.547	2.388	8.530
Be^g	$\theta^{-1} \ln[0.9e^{\theta\sigma_w/3} + 0.1e^{-3\theta\sigma_w}]$	0.458	1.679	2.224
U	$\theta^{-1} \{\ln[e^{\theta\sigma_w 3^{1/2}} - e^{-\theta\sigma_w 3^{1/2}}] - \ln[2\theta\sigma_w 3^{1/2}]\}$	0.500	1.992	4.183

^a $N(0, \sigma_w^2)$ = Normal,

$Be(0, \sigma_w^2)$ = Bernoulli taking values σ_w and $-\sigma_w$ with probability 0.5 each (w.p. 0.5),

$Be^b(0, \sigma_w^2)$ = Bernoulli (bad) taking values $\sigma_w/3$ (w.p. 0.9) and $-3\sigma_w$ (w.p. 0.1),

$Be^g(0, \sigma_w^2)$ = Bernoulli (good) taking values $3\sigma_w$ (w.p. 0.1) and $-\sigma_w/3$ (w.p. 0.9),

$U(0, \sigma_w^2)$ = Uniform with support $[-\sigma_w 3^{1/2}, \sigma_w 3^{1/2}]$,

Ψ = Annuity value of a unitary income innovation,

θc = Average relative risk aversion,

σ_w/y = Annual uncertainty (rate),

Γ = Consumption path's slope (for the case in which interest and discount rates are equal).

case (i.e., when only a location parameter of the distribution has changed),¹⁷ there is a one to one relation between an increase in the variance and more risk. As a result, the proof of the previous paragraph also applies to this case. Q.E.D.

The purpose of the previous proposition was to summarize the results of the multiperiod problem posed in section 2, under i.i.d. labor-income innovations (not to be confused with i.i.d. labor income). Most of the qualitative results shown above were known in the context of two-period models or multiperiod models with very restrictive income-processes. The main reason to go into the trouble of solving the multiperiod general-income-process case is to be able to assess (quantify) the potential importance of precautionary savings for the time-series behavior of consumption. Using the results of the proposition it is possible to do so [see Skinner (1988) and Zeldes (1989a) for alternative sets of estimates derived under different assumptions but leading to very similar conclusions]. Tables 1 and 2 present suggestive preliminary numbers. Table 1 assumes that labor income follows a random walk (i.e., $\psi_i = \Psi = 1$ for all i) and that $r = \delta$. The first two columns of this table show the distributional assumptions for labor-income innovations and the corresponding expression for Γ , respectively. Columns three to five present the values of Γ for different combinations of standard deviations of labor income (in rates) and the average

¹⁷See Rothschild and Stiglitz (1970, p. 241).

Table 2
Some examples of Γ , N distribution, $r = 4\%$ ^a (percentage of average annual consumption).^b

AR coefficient ^c	ψ	$(\sigma_w/y, \theta_c)$		
		(0.1, 1)	(0.2, 1)	(0.1, 9)
0.80	0.167	0.014	0.056	0.125
0.90	0.286	0.041	0.164	0.367
0.95	0.444	0.099	0.395	0.884
1.00	1.000	0.500	2.000	4.500
0.10 ^d	1.106	0.612	2.448	5.508
0.50 ^d	1.926	1.855	7.419	16.691

^aAnnual real interest rate.

^b ψ = Annuity value of a unitary-income innovation,

θ_c = Average relative risk aversion,

σ_w/y = Annual uncertainty (rate),

Γ = Consumption path's slope (for the case in which interest and discount rates are equal).

^cAutoregressive coefficient of an AR(1) model for labor income.

^dAutoregressive coefficient of the first-differences process.

coefficient of relative risk aversion. It is apparent from here that precautionary savings can account for substantial growth in consumption. The values of Γ are uniformly large for realistic parametric assumptions (see section 3.1 below). The role of uncertainty is important, doubling the standard deviation of labor income more than triples Γ in all the examples.

Another interesting issue highlighted by table 1 is the important effect that asymmetries may have. The Bernoulli examples clearly show that as the bad state deteriorates (preserving the expected value) Γ becomes larger. If one considers that one of the most important sources of big shocks must be unemployment, bad asymmetries are likely to occur, enhancing the cause for precautionary savings.

Table 1 shows that an increase in the convexity of marginal utility, here indexed by the coefficient of absolute risk aversion, raises Γ . This is a result that does not hold for every distribution function and parameter sets, however it holds for reasonable ones.

Table 2, on the other hand, highlights the role of labor income's persistence in determining precautionary savings. Labor income is described by a first-order autoregressive process, either in levels or first differences, with normal innovations. The first column presents the autoregressive coefficient of labor income, whereas the second column shows the annuity value of a unitary-income innovation. Columns three to five show the value of Γ for different combinations of standard deviations of labor income (in rates) and the average coefficient of relative risk aversion; the distinction between (labor-income) processes that have persistent effects and those that only have transitory effects

is dramatic.¹⁸ Furthermore, if the effects of a labor-income shock are not expected to remain for a very long time (for a given variance of labor-income innovations), precautionary savings are uninteresting as determinants of the slope of the consumption path.

The next subsection reviews empirical evidence on parameters shown to be crucial for precautionary savings in the previous proposition. These parameters are then used to assess the potential magnitude of precautionary-savings effects for the U.S.

3.1. U.S. data and consumption growth

The results derived above are particularly useful for panel-data and time-series studies of consumption behavior. Here I present some examples based on U.S. data, illustrating the potential importance of precautionary-savings arguments.

Determining the proper measure of the degree of labor-income uncertainty is a difficult task. Aggregate measures of income uncertainty are easily attainable, however they are not likely to provide a good proxy for the uncertainty faced by individuals unless idiosyncratic risk is fully insurable. On the other hand, disaggregate studies usually involve short time-series observations. The latter impedes a good understanding of the degree of persistence of labor-income shocks, a crucial issue in establishing the links between income and human-wealth uncertainty.

MaCurdy (1982) presents a very careful panel-data study on real earnings and wages. He designs a random sample from the Michigan Panel Study of Income and Dynamics for the years 1968–77. His preferred estimates for the (normalized) real wages and earnings,¹⁹ respectively, are

$$\Delta x_{1t} = e_{1t} - 0.484e_{1t-1} - 0.066e_{1t-2}$$

and

$$\Delta x_{2t} = e_{2t} - 0.411e_{2t-1} - 0.106e_{2t-2},$$

with $\sigma_{e_1}^2 = 0.061$, $\sigma_{e_2}^2 = 0.054$, and x_1 and x_2 real wages and earnings, respectively.

¹⁸This comment applies to persistent processes which have first differences that have positive (i.e., the annuity value of human-wealth increases by more than current income), or at least not too negative, autocovariances.

¹⁹Here I have approximated the logarithm of the ratio x_{it}/x_{it-1} by the normalized difference $(x_{it} - x_{it-1})/\bar{x}_t$. Except for the conditional heteroskedasticity issue (that makes table 3 estimates slightly upward biased) extensively discussed in section 4, this is a fairly good approximation.

These estimates have to be taken with caution. They reflect total income faced by individuals. Hence, on one hand, they are an upper bound for the uninsurable component for labor income. For instance, the possibility of self-insuring within the family [Kotlikoff and Spivak (1981)] may reduce nondiversifiable risk. On the other, some of MaCurdy's procedures bias downward the estimate of total labor-income uncertainty faced by individuals. Only white males between the age of 25 and 46 that have been continuously married to the same spouse are considered. All these characteristics tend to smooth large and unexpected changes in income. Also, to avoid outliers, observations with large changes in wages and/or earnings were excluded. Finally, a dummy for each year was included. This rules out aggregate uncertainty, perhaps the most uninsurable of all the risks faced by an individual.

An independent set of estimates comes from Hall and Mishkin (1982). They measure income uncertainty through the residual of a regression of PSID data on demographic and life-cycle variables. They then decompose this disturbance into a random-walk component and a transitory moving-average term. Here I normalize their measures by the median family income for the period and sum permanent and transitory components [see Granger and Morris (1976)] in order to obtain estimates comparable to those presented above. The result of such procedure is

$$\Delta x_{3t} = e_{3t} - 0.360e_{3t-1} - 0.080e_{3t-2} - 0.060e_{3t-3},$$

with $\sigma_{e_3}^2 = 0.041$ and x_3 is Hall and Mishkin's measure of uncertain income. These estimates should also be taken with caution since they assume that demographic and life-cycle elements are fully predictable.

Finally, evidence on aggregate data suggests even larger persistence than what is indicated by MaCurdy's and Hall and Mishkin's panel estimates [e.g., Campbell and Deaton (1989)], therefore it seems safe to conclude that the estimates of precautionary savings based on MaCurdy's and Hall and Mishkin's data are a lower bound for a representative-agent model.²⁰

Another parameter of primary importance is the coefficient of risk aversion. If one is willing to restrict this coefficient to be equal to the inverse of the coefficient of intertemporal substitution, it is possible to use the wealth of results found in the 'Euler equation' approach literature. Recent work in this area [Hall (1988), Caballero (1988)] suggests that the intertemporal substitution parameter is unlikely to be larger than 0.3. This implies, under some

²⁰This 'representative-agent' model, nonetheless, does not allow individual agents to fully diversify away 'idiosyncratic' uncertainty.

Table 3
Estimates of Γ , $r = 4\%$ (annual) (in percentages).^a

Income process ^b	$\theta_c = 1$	$\theta_c = 3$	$\theta_c = 9$
Wage un. (Ma)	0.65	1.95	5.85
Earnings un. (Ma)	0.70	2.10	6.30
Income un. (H-M)	0.57	1.71	5.12

^a Ψ = Annuity value of a unitary-income innovation,

θ_c = Average relative risk aversion,

σ_w = Annual uncertainty (rate),

Γ = Consumption path's slope (for the case in which interest and discount rates are equal).

^bWage un. (Ma) = $\Psi^2 \sigma_w^2 = 0.013$, according to MaCurdy's wage equation.

Earnings un. (Ma) = $\Psi^2 \sigma_w^2 = 0.014$, according to MaCurdy's earnings equation.

Income un. (H-M) = $\Psi^2 \sigma_w^2 = 0.011$, according to Hall and Mishkin's income equation.

standard preference restrictions, a coefficient of relative²¹ risk aversion above three. Friend and Blume (1975), on the other hand, obtained an independent measure of the degree of risk aversion using cross-sectional data on households asset holdings. Their comprehensive analysis of the data led them to the conclusion that the coefficient of relative risk aversion of U.S. asset holders exceeds two.

Table 3 combines these estimates to speculate on the possible importance of precautionary motives for U.S. wealth accumulation, under the assumption that labor-income innovations are normally distributed. The numbers there shown are potentially able to account for the 'excess of growth' puzzle first noticed by Deaton (1986).²² In this table the rows represent the different income processes reviewed above and the columns different levels of average relative risk aversion. For example, using MaCurdy's (1982) earnings-process estimates (row 2), together with a coefficient of relative risk aversion of three (column 2) and a real interest rate of 4% per year, yields an 'excess' consumption growth of about 2% per year. This result is not significantly affected by the income process used. On the other hand, lowering the average relative risk aversion to one (column 1) reduces the excess consumption growth to about 0.7%, whereas raising this coefficient to nine brings excess growth up to 6%. The importance of precautionary savings highlighted here is consistent with

²¹Notice that in the model developed in the paper risk aversion is characterized by a constant coefficient of absolute risk aversion. In the experiments performed below, the coefficient of absolute risk aversion times the average consumption level is set equal to the coefficient of relative risk aversion estimates.

²²The traditional warning applies. In the context of an OLG model, this rate of growth is filtered by demographic factors.

the results obtained in Zeldes's (1989) numerical solutions and Skinner's (1988) Taylor approximation for the case of the CRRA utility function.

3.2. Other results

Another result worth noticing is shown in Corollary 1 below.

Corollary 1. In the presence of precautionary-savings motives, the difference in the slope of the consumption path of two households (or economies) that face different interest rates but are otherwise equal, is dampened (exacerbated) when an increase in today's income conveys a weakly monotonic increase (decrease) of future's labor income.

Proof.

$$\partial \Psi / \partial \alpha = - \sum_{i=1}^{\infty} \alpha^i \psi_{i-1} + (1 - \alpha) \sum_{i=1}^{\infty} i \alpha^{i-1} \psi_{i-1}$$

or

$$\partial \Psi / \partial \alpha = \sum_{i=1}^{\infty} \alpha^i \psi_{i-1} [i(1 - \alpha) / \alpha - 1].$$

It is clear that for any constant ψ_{i-1} this expression is equal to zero. But $\psi_0 \equiv 1$, therefore this result only includes the case when labor income follows a random walk. If ψ_{i-1} is decreasing on i , this expression is negative since the weights of large i 's are smaller, and it is precisely for large i 's that the first (positive) term in square brackets is relatively more important. The opposite happens when ψ_{i-1} is increasing. Finally,

$$\text{sign}(\partial \Gamma / \partial r - \partial \Gamma^{pi} / \partial r) = \text{sign}(\partial \Psi / \partial r) = - \text{sign}(\partial \Psi / \partial \alpha),$$

where $\Gamma^{pi} = (r - \delta) / \theta$, proving the corollary. Q.E.D.

Loosely interpreted, and with all the limitations of comparative-statics experiments, the case of increasing ψ_{i-1} could explain the extremely small, or even negative, intertemporal-substitution parameters estimates found in the literature [e.g., Hall (1988)]. However, this channel could not explain why intertemporal-substitution parameter estimates are small, but only why they may show up as negative once they are already small. To illustrate this, consider the labor-income persistence estimates obtained by Campbell and Deaton (1989), an average annual interest rate equal to 4%, and an estimate of labor-income uncertainty equal to 10% per year. In this case if the true coefficient of intertemporal substitution is 1.0, the estimated parameter would

be equal to 0.92. On the other hand, if the true coefficient is 0.05, the estimated parameter would be equal to -0.1 .

Finally, before moving to the next section, there are some remarks to be made. First, when the horizon and/or retirement are finite, Ψ is no longer constant. If the ψ_i 's are increasing (decreasing), Γ_t decreases (increases) as time passes. This results from the change in the annuity value of an income shock when the horizon is shortened.

Second, contrary to what happens when marginal utility is linear, any change in the underlying process of income (as a result of policy perhaps) changes not only the conditional mean and variance of income but also the slope of the consumption path.

Third, given the expected present value of lifetime consumption, a positive Γ implies less consumption than under certainty equivalence during the early years, and more during the late years. In the context of a finite-horizon model, if the assumption of a flat expected income path is added, precautionary savings produce a 'humped savings' model [Harrod (1948)] in the same way as retirement does in the Modigliani and Brumberg (1954) model.²³

And fourth, it is always possible to perform the inverse experiment. That is, to take the consumption path as given and ask what happens to the interest rate. In this case, all the results used in this section to explain high consumption growth can be used to explain low riskless interest rates (relative to the complete-markets case). For example, an economy with no population growth no longer has a steady-state interest rate equal to the discount rate. Instead, $r = \delta - \theta^2 \sigma^2 / 2$ (in the case of Normal labor-income innovations). If MaCurdy's or Hall and Mishkin's income processes are taken to be the actual processes and the coefficient of relative risk aversion is equal to three, the steady-state real interest rate would be approximately 6% lower than the discount rate.

4. Beyond comparative statics

The specification of section 3 is useful to isolate the effect of precautionary savings on average consumption growth. A natural question, addressed by table 1 and most of the papers previously cited, is how does the consumption path change as labor-income uncertainty rises. Answers are provided by performing comparative-statics experiments in models derived under the assumption that second and higher moments of the labor-income process are known and constant. Conversely, this section studies the behavior of consumption in a model with truly stochastic higher moments. Important implications for the marginal propensity to consume obtain from higher-moments uncertainty. In fact, it is in general no longer the case that the marginal propensity to consume implied by a CARA utility function is equivalent to that of

²³See Caballero (1987b).

certainty-equivalent models. For example, once higher-moments uncertainty is considered, reasonable parametric assumptions make the data consistent with the excess-smoothness and excess-sensitivity puzzles.

Proposition 2 below derives a closed-form consumption function under very general conditions about the slope of the consumption path. The stochastic variance case is just a corollary of this proposition (Corollary 2).

Proposition 2. If the assumptions of Proposition 1 hold, although now labor-income innovations are independently nonidentically distributed so the slope of the consumption path is stochastic and (assumed) representable by

$$E_t[\Gamma_{t+j}] - E_{t-1}[\Gamma_{t+j}] = \phi_j z_t \quad \text{with} \quad \sum_{i=0}^{\infty} \alpha^i \phi_i < \infty,$$

then the consumption function and processes are

$$c_t = y_t^p - [\alpha/(1-\alpha)]\Gamma_t - (1-\alpha) \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i \sum_{h=0}^t (\phi_{h+j-1} - \phi_h) z_{t-h}$$

and

$$c_{t+1} = \Gamma_t + c_t + v_{t+1}, \tag{10}$$

with

$$v_{t+j} = \Psi w_{t+j} - [\alpha\Phi/(1-\alpha)]z_{t+j},$$

$$\Phi \equiv (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \phi_i,$$

$$\Gamma_t = (1/\theta) \ln E_t[e^{-\theta v_{t+1}}].$$

Proof. First notice that the form of the consumption process is identical to the process shown in Proposition 1 since $E_t[f(\Gamma_t)] = f(\Gamma_t)$, for any function $f(\cdot)$. If this is the case, the same steps of Proposition 1 can be used to determine the expression for Γ_t . Furthermore, each Γ_{t+j} can be described by

$$\Gamma_{t+j} = \Gamma_t + \sum_{h=1}^j \phi_{j-h} z_{t+h} + \sum_{h=0}^t (\phi_{h+j} - \phi_h) z_{t-h}.$$

This, plus the income process, and recalling that by integrating (10) consumption can be written as

$$c_{t+i} = c_t + \sum_{j=1}^i v_{t+j} + \sum_{j=1}^i \Gamma_{t+j-1},$$

are enough to find the solution.

Replacing these expressions into the budget constraint, and using the information structure of the problem, yields the consumption function shown in the proposition.

As before, plugging back the consumption function in the budget constraint yields the following restrictions on the unknown terms:

$$\sum_{i=1}^{\infty} \alpha^i \left[\sum_{j=1}^i (v_{t+j} - \psi_{i-j} w_{t+j}) + [i > 1] \sum_{j=2}^i \sum_{h=1}^{j-1} \phi_{j-1-h} z_{t+h} \right] = 0 \quad (11)$$

for all t .

As in Proposition 1, (11) is satisfied (period-by-period) almost surely if and only if

$$v_{t+j} = \Psi w_{t+j} - [\alpha \Phi / (1 - \alpha)] z_{t+j},$$

with $\Phi \equiv (1 - \alpha) \sum_{i=0}^{\infty} \alpha^i \phi_i$, so the consumption process (including its slope) is fully determined. Q.E.D.

Corollary 2. If the variance of labor income follows the process

$$E_t [\sigma_{w,t+j}^2] - E_{t-1} [\sigma_{w,t+j}^2] = \phi_j v_t \quad \text{with} \quad \sum_{i=0}^{\infty} \alpha^i \phi_i < \infty,$$

and the conditional (on higher moments) distribution of the consumption innovation can be well approximated by a normal distribution, then:

$$\begin{aligned} c_t = & y_t^p - (\theta/2) / [\alpha / (1 - \alpha)] E_t [\sigma_{v,t+1}^2] \\ & - (\theta/2) (1 - \alpha) \sum_{i=1}^{\infty} \alpha^i \sum_{j=1}^i \sum_{h=0}^j (\phi_{h+j-1} - \phi_h) v_{t-h}, \end{aligned}$$

and the consumption process is

$$c_{t+1} = (\theta/2) E_t [\sigma_{v,t+1}^2] + c_t + v_{t+1},$$

with

$$v_{t+j} = \psi w_{t+j} - (\theta/2) [\alpha \Phi / (1 - \alpha)] v_{t+j},$$

Proof. Direct from Proposition 2. Q.E.D.

The first two terms in the consumption function are similar to those obtained when higher moments were assumed constant (Proposition 1), al-

though now $E_t[\sigma_{vt+1}^2]$ replaces the constant consumption variance attained before. The third term in the consumption function incorporates the fact that the expected variance of consumption innovations for periods beyond $t+1$ may differ from $E_t[\sigma_{vt+1}^2]$. After a positive variance shock, $E_t[\sigma_{vt+1}^2]$ overestimates the relevant variance forecasts to determine the optimal precautionary-savings amount, if labor income's variance follows a mean-reverting process. Conversely, if the labor-income variance process has long-run responses larger than short-run responses, $E_t[\sigma_{vt+1}^2]$ underestimates the proper variance forecasts to determine precautionary savings. Finally, if labor-income variance follows a random walk, the last term of the consumption function disappears.

Another important difference is that now the consumption disturbance includes a term that takes into account revisions in variance forecasts. When the variance forecast rises, current consumption must fall to accommodate a larger slope of the consumption path, without violating the budget constraint [Caballero (1988)]. It is clear, nonetheless, that if income and variance innovations are uncorrelated, the variance of consumption exceeds the variance implied by the permanent-income hypothesis ($\Psi^2\sigma_w^2$). This is counterfactual in the light of Campbell and Deaton's (1989) excess-smoothness finding. However, as long as the correlation between income and variance innovations is allowed to be positive, excess smoothness is easily attainable. In fact, the condition

$$[2\Psi\sigma_{wt}/(\alpha\theta\Phi\sigma_v/(1-\alpha))]\rho_{wz} > \frac{1}{2}$$

yields excess smoothness in the sense described by Campbell and Deaton.²⁴

It is perhaps of more interest to study expected changes in consumption *conditional* on a change in income, i.e., the marginal propensity to consume (out of labor income) taking into account the role of income in signaling a change in labor income's level and variance. Notice that this is a somewhat unconventional definition of the marginal propensity to consume, however, it is consistent with the concept used in Zeldes (1989a) and is a natural extension of Flavin (1981)'s definition of marginal propensity to consume out of current

²⁴Strictly speaking, the excess-smoothness result found by Campbell and Deaton (1989) refers to the unconditional variance of Δc , whereas this paper refers to the conditional (on information available at time t) variance of Δc_{t+1} . Adding the term $\sum_{i=0}^{\infty} \phi_i^2 \sigma_v^2$ to the conditional variance of consumption growth,

$$\sigma_c^2 = \Psi^2\sigma_w^2 + \frac{\alpha^2\Phi^2}{(1-\alpha)^2}\sigma_v^2 - \frac{2\alpha\Psi\Phi}{1-\alpha}\sigma_w,$$

permits us to calculate the unconditional variance of Δc . The extra term is negligible when compared to $\alpha^2\Phi^2\sigma_v^2/(1-\alpha)^2$ for any long-lasting but stationary process of slope shocks. On the other hand, if slope shocks are nonstationary the extra term blows up. However, it is still the case that the sample estimate (e.g., for 200 observations) of this extra term is orders of magnitude smaller than the sample estimate of the term involving σ_v^2 in the expression for σ_c^2 , therefore the conditional statements made in this paper are empirically relevant to address Campbell and Deaton's puzzle.

income taking into account the role of the latter in signaling future income changes.

For this assume that both shocks are joint-normally distributed. In this case the conditional expectation of v given w is

$$E[v|w] = \Psi w - (\theta/2)(\alpha\Phi\sigma_v\rho_{wv}/(1-\alpha)\sigma_w)w, \quad (12)$$

where ρ_{wv} is the correlation between income and variance shocks. Eq. (12) shows that even in the case of the CARA utility function the marginal propensity to consume is reduced (relative to the certainty-equivalence model) when labor income and its variance innovations are positively correlated. A simple example shows the potential importance of this phenomenon. Suppose that $\Phi = \Psi$, $\theta_c = 1$ (the coefficient of relative risk aversion), the quarterly real interest rate is one percent, and the correlation between labor-income and variance innovations is one. Then, even if the variance (standard deviation) of labor income is 39,204 (198) times larger than the variance (standard deviation) of the variance of labor income, the sensitivity of consumption to income innovations, i.e., the marginal propensity to consume, is just one half of what is implied by the certainty-equivalence model.

It turns out that this simple example is not very far from what could be happening in U.S. aggregate data. In their paper, Campbell and Deaton show the excess-smoothness result in the framework of what they call ‘a logarithmic version of the permanent-income income model’. This logarithmic version intends to take into account the fact that labor-income innovations are retrieved from a logarithmic regression (notice that MaCurdy’s equations are also estimated in logarithms). Their conclusion is that consumption’s response to income shocks is only 58% of what is implied by the theory. But an homoskedastic labor-income process in logarithms implies conditional heteroskedasticity on levels, yielding a positive correlation between labor-income and variance innovations.

If full idiosyncratic insurance existed, aggregate uncertainty would be the right measure of individual uncertainty. In this case, precautionary savings would be negligible and certainly not an explanation for excess smoothness. However, incomplete markets are likely to be present. In order to generate the excess-smoothness parameters implied by the model presented in this paper, table 4 uses Campbell and Deaton’s point estimates of persistence but lets individual uncertainty be larger than aggregate uncertainty, and more in line with MaCurdy’s and Hall and Mishkin’s microeconomic estimates. The first column shows a range of possible values of the standard deviation of labor-income innovations, whereas the remaining columns present the excess-smoothness parameters for different levels of risk aversion. These show that the excess-smoothness puzzle could be rationalized by the marriage between precautionary savings and the conditional heteroskedasticity of the labor-

Table 4
Excess smoothness: Λ ,^a $\Psi = 1.8$,^b $r = 4\%$ (annual).^c

σ_w/y	$\theta_c = 1$	$\theta_c = 3$	$\theta_c = 9$
0.05	0.952	0.869	0.689
0.10	0.833	0.624	0.357
0.15	0.689	0.425	0.198
0.20	0.555	0.294	0.122
0.30	0.357	0.156	0.058
0.50	0.166	0.062	0.022

^a $\Lambda = 1$ corresponds to the certainty equivalence case.

^bCampbell and Deaton's estimate.

^c Ψ = Annuity value of a unitary-income innovation,

θ_c = Average relative risk aversion,

σ_w/y = Annual uncertainty (rate).

income level implied by a log-linear income process. For example, this table shows that, if the annual real interest rate is 4%, $\Psi = 1.8$ [Campbell and Deaton (1989)], $\theta_c = 3$, and $\sigma_w/y = 10\%$, consumption responses to labor-income changes are only about 60% of responses to similar income changes under certainty equivalence.²⁵ Larger risk aversion and/or uninsurable income risk yield an even milder response of consumption to income news. **On the other hand, excess sensitivity²⁶ can be easily explained by the positive correlation of lagged income changes with $E_t[\sigma_{vt+1}^2]$.**

It is important to realize that even though the CARA utility-function assumption is necessary to obtain the closed-form solutions derived above, the principle underlying the interaction between the marginal propensity to consume and the relationship between labor-income and its variance innovations is much more general. In fact this principle only depends on the fact that if income changes are a signal for variance shifts and agents exhibit precautionary-savings motives, consumption responses to income changes will be damp-

²⁵ Notice that as the interest rate lowers, excess smoothness becomes more pronounced. This results from the reduced discounting of future changes in the slope (increase in the present value of slope shocks). For example, if r is equal to 2% per year, the same set of parameters used in the previous example delivers an excess-smoothness coefficient on the order of 45% (of the certainty-equivalence response).

²⁶ Campbell and Deaton made clear that the apparently contradictory findings of excess sensitivity and excess smoothness are in fact consistent. The former refers to the reaction of consumption to *anticipated* changes in income, whereas the latter refers to the response of consumption to *unanticipated* changes in income. Moreover, if savings Granger-cause income (they do!), then both excess sensitivity and smoothness reflect the violation of the same orthogonality condition. Therefore, when an explanation for excess smoothness is given in the paper, it is also an explanation for excess sensitivity (if consumption responds too little today, then it must respond too much in the future in order to satisfy the budget constraint). Needless to say, the same explanation can be given to the excess smoothness of savings [Campbell (1988)], another reflection of the same 'failure' of the certainty-equivalence/permanent-income model.

ened relative to the case in which income innovations do not convey any information about variance shifts. For example, a CRRA utility function can be locally approximated²⁷ by a CARA utility function in which the coefficient of absolute risk aversion changes (inversely) with income innovations.²⁸ In this case, consumption (local) responses to positive income innovations convey the same two effects described for the CARA, i.e., the direct wealth effect (certainty-equivalence response) and the offsetting precautionary-savings response to an increase in forecasted variance, plus a third effect due to the reduction in the coefficient of absolute risk aversion that tends to raise current consumption. Caballero's (1987a) approximation as well as Zeldes's (1989a) numerical simulations suggest that when income follows a geometric random walk and the utility function is CRRA, the second effect, i.e., the precautionary-savings/conditional-heteroskedasticity effect described above, dominates the third effect.²⁹ Hence, consumption exhibits excess smoothness to unanticipated income changes as in the case of the CARA utility function. Furthermore, Zeldes's (1989a) numerical simulations show that in the case of the CRRA utility function the excess-smoothness effect is more important when the level of wealth of consumers is lower. This is entirely consistent with eq. (12) in this paper, since in the case of the CRRA a lower level of wealth (hence of consumption) implies – *ceteris paribus* – a larger coefficient of *absolute* risk aversion.

5. Conclusions

The aim of this paper has been to bridge the gap between the theoretical work on precautionary savings due to the presence of uninsurable labor-income uncertainty and the empirical consumption literature. The first part of the paper obtained a closed-form solution for consumption under very general ARIMA labor-income processes and showed that as long as precautionary savings are taken into account, the *excess of consumption growth puzzle* [Deaton (1986)] is consistent with the stochastic processes of labor income estimated for the U.S. For example, using MaCurdy's (1982) or Hall and Mishkin's (1982) income-processes estimates together with a coefficient of

²⁷Up to my knowledge no exact closed-form solution has been found for the CRRA case with incomplete insurance for labor income.

²⁸One way to see the additional complexity of the CRRA utility function is to notice that the absolute risk-aversion coefficient depends on the consumption level (an endogenous variable) as opposed to the income level (an exogenous variable).

²⁹Strictly speaking, once the local approximation mentioned above is abandoned, there is a fourth effect due to the fact that consumers can reduce the relative importance of future labor-income riskiness by saving more today in order to lower the coefficient of absolute risk aversion faced in the future (by consuming more). This effect can certainly be responsible for the fact that consumption exhibits excess smoothness but it does not hamper the fact that the precautionary-savings/conditional-heteroskedasticity effect is present.

relative risk aversion of three, yields an 'excess' consumption growth of about 2%. Alternatively, given the consumption path, precautionary savings can explain the relatively low real interest rate observed in the postwar U.S. data. For example, an infinite-horizon model with no population growth and the same parametric assumptions made above, yields a steady-state real interest rate approximately 6% lower than the discount rate. The order of magnitude of these results is consistent with the results obtained through numerical procedures by Zeldes (1989a) for the case of a CRRA utility function.

The second part of the paper goes beyond comparative-statics experiments by allowing for truly stochastic variance processes. Again, the consumption function was obtained in closed form (feed-back). An important result of this section was to show that once higher moments are stochastic and possibly correlated with income innovations, it is no longer true that the marginal propensity to consume out of current income obtained under a CARA utility function equals that of a certainty-equivalent model. In particular, if the labor income and its variance innovations are positively correlated, the marginal propensity to consume implied by a CARA utility function is lower than that of the certainty-equivalent model. A corollary of this is that the marriage of precautionary savings and conditional heteroskedasticity of labor income is potentially able to provide simultaneous explanations for the *excess smoothness* [Deaton (1986), Campbell and Deaton (1989)] and the *excess sensitivity* [Flavin (1981)] of consumption to unanticipated and anticipated labor-income changes, respectively.

Many arguments have emerged to explain some of these puzzles (e.g., general equilibrium considerations, myopia, liquidity constraints, and different assumptions about the labor-income process), but none of these seems to give as many simultaneous answers as precautionary savings. This might be one of the most important sources of identification of the origins of the puzzles.

Appendix

This appendix shows the derivation underlying the excess-smoothness results presented in table 4.

Campbell and Deaton found an AR(1) in first differences of the logs to be the most appropriate description of the income process. In order to simplify, however, it is useful to approximate their result by, first, reducing the process to a random walk in logs (although preserving the unconditional variance of the percentage income changes). And second, by approximating the logarithmic process by

$$y_{t+i} = y_{t+i-1}(1 + \varepsilon_{t+i}),$$

with ε a normal i.i.d. disturbance.

In this context,³⁰

$$v_{t+j} = (\theta/2) \Delta \sigma_{v_{t+j}}^2, \quad (\text{A.1})$$

and $\Phi = 1$.

Even though there is only one source of uncertainty, ϵ , the correlation between w_{t+j} ($= y_{t+j-1} \epsilon_{t+j}$) and v_{t+j} needs not be equal to one since their relation is not linear. Nevertheless, $\rho_{wv} = 1$ is a good approximation for small income changes. In this case,

$$\sigma_{v_{t+j}}^2 = (\sigma_{w_{t+j}} - \alpha \sigma_{v_{t+j}} / (1 - \alpha))^2. \quad (\text{A.2})$$

Neglecting the changes in the variance of the variance of labor income and other third-order terms, yields the following expression for the change in the variance of consumption innovations:

$$\Delta \sigma_{v_{t+j}}^2 = 2 \sigma_{v_{t+j}} (\Delta \sigma_{w_{t+j}}),$$

but $\Delta \sigma_{w_{t+j}} = y_{t+j-1} \epsilon_{t+j} \sigma_\epsilon$, and therefore

$$\Delta \sigma_{v_{t+j}}^2 = 2 \sigma_{v_{t+j}} y_{t+j-1} \epsilon_{t+j} \sigma_\epsilon.$$

Replacing this expression in (A.1) allows the computation of the standard deviation of v_{t+j} :

$$\sigma_{v_{t+j}} = \sigma_{v_{t+j}} y_{t+j-1} \sigma_\epsilon^2 \theta.$$

Finally, substituting this in (A.2) and solving for the standard deviation of v_{t+j} yields

$$\sigma_{v_{t+j}} = \Lambda \sigma_{w_{t+j}} \quad \text{with} \quad \Lambda = \left(1 + \left[\alpha (\theta y_{t+j-1}) \sigma_\epsilon^2 / (1 - \alpha) \right] \right)^{-1},$$

the excess-smoothness parameter (computed at average income) reported in table 4.

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³⁰And assuming that the distribution of v can be approximated by a normal distribution.

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