



# How large is the stock component of human capital? ☆



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## ARTICLE INFO

### Article history:

Received 14 July 2014

Received in revised form 26 May 2016

Available online 15 June 2016

### JEL classification:

D91

E21

G12

J24

### Keywords:

Value of human capital

Idiosyncratic and aggregate risk

Incomplete markets

## ABSTRACT

This paper examines the value of an individual's human capital and the associated return on human capital using U.S. data on male earnings and financial asset returns. We measure the size of the stock component of human capital and assess the implications for life-cycle portfolio decisions. We find that (1) the value of human capital is far below the value implied by discounting earnings at the risk-free rate and (2) the stock component of the value of human capital is smaller than the bond component at all ages and typically averages less than 35 percent of the value of human capital. Data properties that increase the stock component of the value of human capital also act to lower the stock share held in financial wealth.

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## 1. Introduction

A common view is that by far the most valuable asset that most people own is their human capital. We provide a detailed characterization of the value and return to human capital, and their implications for portfolio choice over the lifecycle. We estimate a statistical model for male earnings and stock returns to describe how earnings move with age, education and a rich structure of aggregate and idiosyncratic shocks. We then embed this statistical model into a decision problem of the type analyzed in the literature on the income-fluctuation problem. We use the stochastic discount factor produced by a solution to this decision problem to value future earnings after taxes and transfers.

We highlight two main findings. First, the value of human capital is far below the value that would be implied by discounting net earnings at the risk-free interest rate. The most important reason for this is the large amount of idiosyncratic earnings risk that we estimate from U.S. data. An agent's stochastic discount factor covaries negatively with this component of earnings risk.

This finding is particularly relevant with respect to the view that various legal impediments, including personal bankruptcy laws, hinder greater skill investment and greater risk sharing in an individual's future earnings. The finding that individual human capital valuations are far below the value implied by discounting earnings at the risk-free rate suggests that individual valuations are well below market valuations. If so, then absent these impediments there is ample scope for alternative financial arrangements to arise to share some of this idiosyncratic risk.<sup>1</sup>

☆ This work was previously entitled "The Money Value of a Man".

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<sup>1</sup> Nerlove (1975) discusses the early literature on the potential for market arrangements to arise to better share future earnings risk.

Our second main finding involves decomposing the value of human capital at each age into a stock, bond and orthogonal value component. We find that the stock component is typically below 35 percent of the value of human capital. This holds for two different educational groups (high school or college educated males) under a wide range of attitudes towards risk-aversion. We determine the stock share by projecting the sum of next period's earnings and human capital value onto next period's bond and stock returns. We then value these components using the individual's stochastic discount factor.

This finding is relevant for the portfolio allocation literature. Much of this literature tries to understand two findings: low participation rates in the stock market and the age profile for the average stock share of the financial wealth portfolio, conditional on participation. Explanations are often framed in terms of deviations from a benchmark model (e.g. Samuelson, 1969), with one safe and one risky financial asset and constant-relative-risk-aversion preferences, that implies constant portfolio shares.

There are two main views in this literature. One view holds that adding a realistic labor income and financial asset returns process to the benchmark model does not produce these two findings. Thus, an explanation lies elsewhere. For example, adding fixed costs of entering equity markets, a housing choice or imperfect information and learning may help to reduce participation rates, especially at young ages, to be closer to those in data.<sup>2</sup> An alternative view is that realistic labor income and financial asset returns alone go a long ways towards producing both findings. Benzoni et al. (2007) and Lynch and Tan (2011) argue for this view.

There are three curious things about this literature. First, the former papers believe that the stock share implicit in the value of future earnings is small, whereas the latter papers believe it is large. Second, Benzoni et al. (2007) is the only paper to calculate this stock share even though it is central in all papers. They calculate that the stock share is 50 percent at age 20 and remains at 50 percent for the first half of the working lifetime. This leads to non-participation in equity markets early in life with sufficiently high relative risk aversion. Third, none of these papers go very far towards estimating the relationship between aggregate earnings risk and stock returns or estimating how the variance or skewness of idiosyncratic earnings shocks covaries with aggregate risk. Thus, differences in model portfolio choice implications are partly due to differences in assumptions rather than the weight of evidence viewed through estimated statistical models.

We find that the average stock share of the value of human capital is positive, but is robustly below the 50 percent value calculated by Benzoni et al. (2007). A number of model features lead to a positive stock share. For example, social security retirement benefits that are positively linked to the level of average earnings, a left-skewed distribution of idiosyncratic shocks, cyclical variation in the variance and skewness of idiosyncratic shocks and a positive conditional correlation between stock returns and the aggregate component of individual earnings all contribute towards a positive stock share. They also have support in US data. We do not find much empirical support for the claim that allowing cointegration between the aggregate component of earnings and stock returns is key to producing a large stock share. Benzoni et al. (2007) calculate the stock share after roughly calibrating such a cointegrated process. In contrast, all our work is based on estimating the relationship between earnings and stock returns.

Our work is most closely related to two literatures. First, a long line of work values human capital by discounting future earnings using a deterministic interest rate or discount factor.<sup>3</sup> Our work differs as discounting is done using an individual's stochastic discount factor, which produces an individual-specific value of human capital. Huggett and Kaplan (2011) is more closely related. They put bounds on individual human capital values using knowledge of the earnings and asset returns process and Euler equation restrictions. Second, there is a vast literature on financial asset allocation decisions. While our work relates to this literature, it differs by its focus on decomposing the value of human capital based on an earnings-stock-returns process estimated from micro data.

The remainder of the paper is organized as follows. Section 2 presents the theoretical framework. Sections 3 to 5 present our main findings. Section 6 explores the robustness and the key drivers of these findings. Section 7 concludes.

## 2. Theoretical framework

This section presents the framework, defines and decomposes the value of human capital and illustrates the value and return concepts with a simple example.

### 2.1. Decision Problem

An agent solves Problem P1. Lifetime utility  $U(c)$  is determined by a consumption plan  $c = (c_1, \dots, c_J)$ . Consumption at age  $j$  is given by a function  $c_j : Z^j \rightarrow R_+^1$  that maps shock histories  $z^j = (z_1, \dots, z_j) \in Z^j$  into consumption. All the variables that we analyze are functions of these shocks.

Problem P1:  $\max U(c)$  subject to

$$(1) \ c_j + \sum_{i \in \mathcal{I}} a_{j+1}^i = \sum_{i \in \mathcal{I}} a_j^i R_j^i + e_j \text{ and } c_j \geq 0, \forall j$$

$$(3) \ a_{J+1}^i = 0, \forall i \in \mathcal{I}$$

<sup>2</sup> See Coco et al. (2005), Gomes and Michaelides (2005), Coco (2005) and Chang et al. (2015).

<sup>3</sup> See Farr (1853), Dublin and Lotka (1930), Weisbrod (1961), Becker (1975), Graham and Webb (1979), Jorgenson and Fraumeni (1989), Haveman et al. (2003). Some of this work calculates an aggregate value of human capital.

The budget constraint says that period resources are divided between consumption  $c_j$  and savings  $\sum_{i \in \mathcal{I}} a_{j+1}^i$ . Period resources are determined by an exogenous earnings process  $e_j$  and by the value of financial assets brought into the period  $\sum_{i \in \mathcal{I}} a_j^i R_j^i$ . The value of financial assets is determined by the amount  $a_j^i$  of savings allocated to each financial asset  $i \in \mathcal{I} = \{1, \dots, I\}$  and by the gross return  $R_j^i > 0$  to each asset  $i$ .

## 2.2. Value and return concepts

The value of human capital  $v_j$  is defined to equal expected discounted dividends (i.e. net earnings) at a solution  $(c^*, a^*) = ((c_1^*, \dots, c_J^*), \{(a_1^{*,i}, \dots, a_{J+1}^{*,i})\}_{i \in \mathcal{I}})$  to Problem P1. Discounting is done using the agent's stochastic discount factor  $m_{j,k}$  from the solution to Problem P1. It captures the agent's marginal valuation of an extra period  $k$  consumption good in terms of the period  $j$  consumption good. It contains a conditional probability term  $P(z^k|z^j)$  because human capital values are stated using the mathematical expectations operator  $E$ .<sup>4</sup>

$$v_j(z^j) \equiv E\left[\sum_{k=j+1}^J m_{j,k} e_k | z^j\right] \text{ and } m_{j,k}(z^k) \equiv \frac{\partial U(c^*) / \partial c_k(z^k)}{\partial U(c^*) / \partial c_j(z^j)} \frac{1}{P(z^k|z^j)}$$

Given the value concept, we define the gross return  $R_{j+1}^h$  to human capital to be next period's value and dividend divided by this period's value:  $R_{j+1}^h = \frac{v_{j+1} + e_{j+1}}{v_j}$ . The return to human capital is then well integrated into standard asset pricing theory. Off corners, all returns  $R_{j+1}$  satisfy the same type of restriction:  $E[m_{j,j+1} R_{j+1} | z^j] = 1$ .<sup>5</sup>

## 2.3. An interpretation

We provide an interpretation for  $v_j$ . The value  $v_j$  is the price at which an agent would be willing to sell a marginal share of a claim to their future earnings stream. It is thus the value of all the shares (total shares are normalized to 1) in the future earnings stream. The price process  $\{v_j\}_{j=1}^J$  has the property that if the agent were allowed to change share holdings in this earnings stream at any age at these prices, then the agent would optimally decide not to change share holdings and would make exactly the same consumption and asset choices  $(c^*, a^*)$  that were optimal in Problem P1.<sup>6</sup> The value  $v_j$  is not the market or social value of future earnings.

## 2.4. A decomposition

We decompose human capital values into financial asset components and a residual-value component. To do so, we project next period's human capital payout  $v_{j+1} + e_{j+1}$  onto the space of conditional asset returns. The decomposition is carried out in the two equations below. The human capital payout contains a component  $(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i)$  spanned by gross asset returns and a component  $(\epsilon_{j+1})$  orthogonal to asset returns, where  $\alpha_j^i$  are the projection coefficients.<sup>7</sup> The orthogonal component  $\epsilon_{j+1}$  will be mean zero when one of the financial assets is riskless.

$$v_j = E[m_{j,j+1}(v_{j+1} + e_{j+1}) | z^j] = E[m_{j,j+1}(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i + \epsilon_{j+1}) | z^j]$$

$$v_j = \sum_{i \in \mathcal{I}} \alpha_j^i E[m_{j,j+1} R_{j+1}^i | z^j] + E[m_{j,j+1} \epsilon_{j+1} | z^j]$$

## 2.5. A simple example

A simple example illustrates the value and return concepts. An agent's preferences are given by a constant relative risk aversion utility function. Earnings follow an exogenous Markov process. There is a single, risk-free financial asset.<sup>8</sup>

<sup>4</sup> The expectations operator *integrates* the relevant age  $k$  functions with respect to the distribution  $P(z^k|z^j)$ . In all of our applied work, the set of partial shock histories is finite and, thus, integration is straightforward summation.

<sup>5</sup> This holds for the return to human capital because  $v_j = E[\sum_{k=j+1}^J m_{j,k} e_k | z^j]$  implies  $E[m_{j,j+1}(\frac{v_{j+1} + e_{j+1}}{v_j}) | z^j] = 1$ .

<sup>6</sup> Broadly, the pricing of human capital generalizes the method of pricing a non-traded asset in Lucas (1978). One proposes a second economy where trade in the non-traded asset is allowed and then finds prices that persuade the agent not to do so. The result claimed in the text holds for general utility functions under a concavity assumption and is not sensitive to the nature of the earnings process or the financial asset returns process. It extends to economies with valued leisure and endogenous earnings. See Huggett and Kaplan (2012, Theorem 1) for a proof. It also holds under a variety of borrowing constraints on financial asset holdings.

<sup>7</sup> When the agent is off corners in the holding of asset  $i$  then the Euler equation  $E[m_{j,j+1} R_{j+1}^i | z^j] = 1$  holds. In our application, we allow corner solutions in which case  $E[m_{j,j+1} R_{j+1}^i | z^j] \leq 1$ .

<sup>8</sup> The model is a finite-lifetime version of the permanent-shock model analyzed by Constantinides and Duffie (1996).

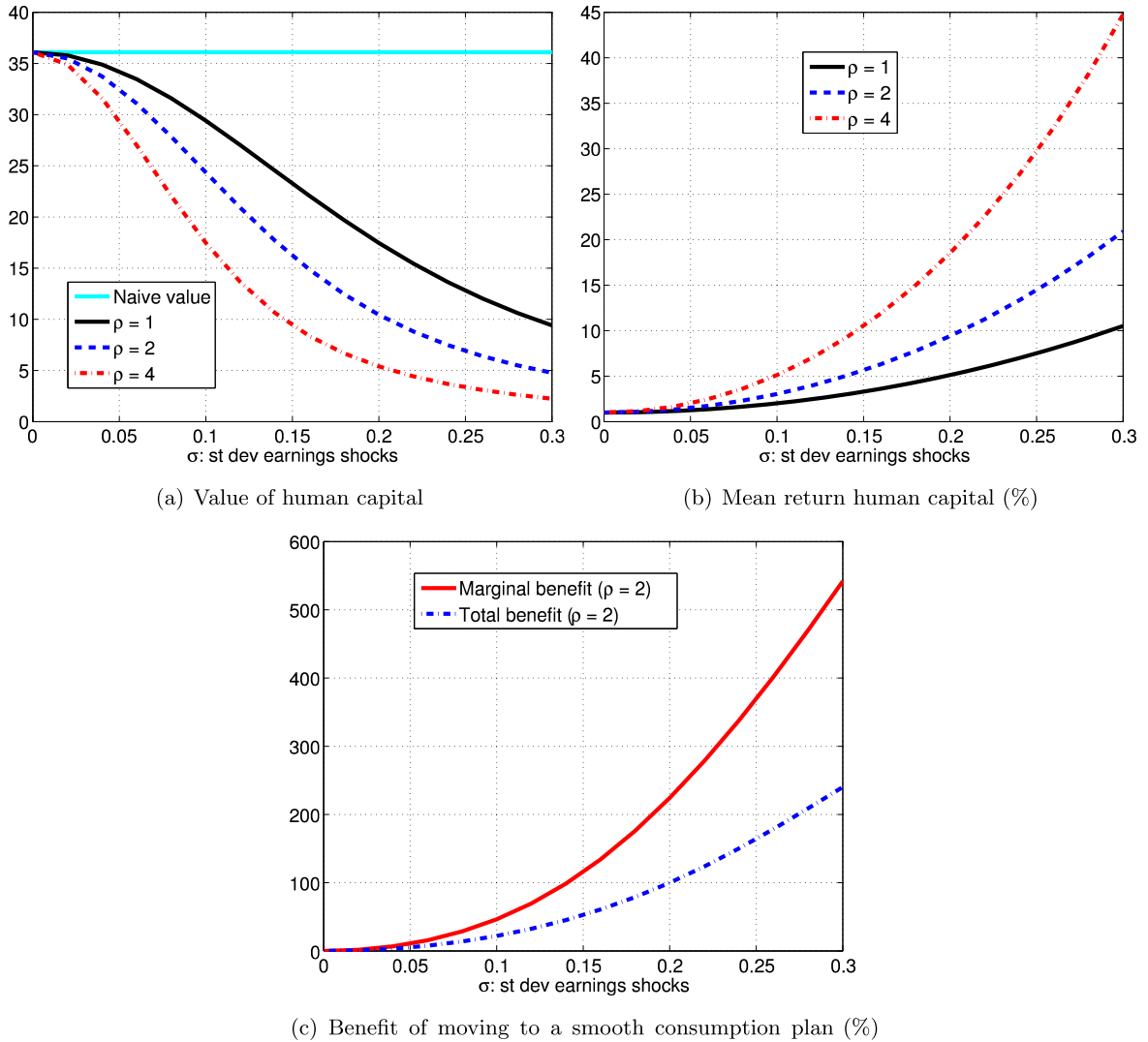


Fig. 1. Human capital values and returns: a simple example.

$$\text{Utility: } U(c) = E[\sum_{j=1}^J \beta^{j-1} u(c_j) | z^1], \text{ where } u(c_j) = \begin{cases} \frac{c_j^{1-\rho}}{(1-\rho)} & : \quad \rho > 0, \rho \neq 1 \\ \log(c_j) & : \quad \rho = 1 \end{cases}$$

Earnings:  $e_j = \prod_{k=1}^j z_k$ , where  $\ln z_k \sim N(\mu, \sigma^2)$  is i.i.d.

Decision Problem:  $\max U(c)$  subject to

$$(1) c_j + a_{j+1} \leq a_j(1+r) + e_j, (2) c_j \geq 0, a_{j+1} \geq 0$$

When  $1+r = \frac{1}{\beta} \exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$  and initial financial assets are zero, then setting consumption equal to earnings each period is optimal. The stochastic discount factor equals  $m_{j,k}(z^k) \equiv \frac{\partial U(c^*)/\partial c_k(z^k)}{\partial U(c^*)/\partial c_j(z^j)} \frac{1}{P(z^k|z^j)} = \frac{\beta^{k-j} u'(e_k(z^k))}{u'(e_j(z^j))}$ . This example leads to a closed-form formula where  $v_j$  is proportional to earnings  $e_j$  and where  $R_j^h$  is a time-invariant function of the period shock  $z_j$ .

Fig. 1 illustrates some quantitative properties. The parameter  $\sigma$ , governing the standard deviation of earnings shocks, varies over the interval  $[0, 0.3]$  and  $\mu = -\sigma^2/2$ . As all agents start with earnings equal to 1, the expected earnings profile over the lifetime is flat and equals 1 in all periods. The lifetime is  $J = 46$  periods which can be viewed as covering real-life ages 20–65. The interest rate is fixed at  $r = .01$ . Thus, the discount factor  $\beta$  is adjusted to be consistent with this interest rate given the remaining parameters:  $1+r = \frac{1}{\beta} \exp(\rho\mu - \frac{\rho^2\sigma^2}{2})$ .

Fig. 1 shows that the value  $v_1$  of an age 1 agent's human capital falls and that the mean return in any period rises as the shock standard deviation increases. Thus, a high mean return on human capital is the flip side of a low value attached to future earnings. These patterns are amplified as the preference parameter  $\rho$  increases.

Fig. 1 also plots the “naive value”. The naive value equals earnings discounted at a constant interest rate  $r$  that we set equal to the risk-free rate (i.e.  $v_1^{\text{naive}} = E[\sum_{j=2}^J \frac{e_j}{(1+r)^{j-1}} | z^1]$ ). This follows a traditional empirical procedure that is employed in the literature as was mentioned in the introduction. The naive value is exactly the same in each economy in Fig. 1 because the risk-free interest rate and the mean earnings profile are unchanged across economies. Our notion of value  $v_1$  differs from  $v_1^{\text{naive}}$  because the agent's stochastic discount factor covaries with earnings. Fig. 1 shows that negative covariation can be substantial.

Fig. 1 plots the total benefit and the marginal benefit of moving from the model consumption plan  $c$  to a smooth consumption plan where  $c_j^{\text{smooth}} = E_1[c_j] = E_1[e_j] = 1$ . The benefit function  $\Omega$  is defined, following Alvarez and Jermann (2004), by the first equation below. The total benefit is  $\Omega(1)$  and the marginal benefit is  $\Omega'(0)$ . The marginal benefit in Fig. 1 increases as the standard deviation of the period earnings shock increases.

$$U((1 + \Omega(\gamma))c) = U((1 - \gamma)c + \gamma c^{\text{smooth}})$$

$$\Omega'(0) = \frac{\sum_{j=1}^J \sum_{z^j} \frac{\partial U(c)}{\partial c_j(z^j)} (c_j^{\text{smooth}}(z^j) - c_j(z^j))}{\sum_{j=1}^J \sum_{z^j} \frac{\partial U(c)}{\partial c_j(z^j)} c_j(z^j)} = \frac{E[\sum_{j=1}^J m_{1,j} c_j^{\text{smooth}} | z^1]}{v_1(z^1) + e_1(z^1) + a_1(z^1)(1+r)} - 1$$

The marginal benefit is tightly connected to the value  $v_1$  of human capital. To see this, differentiate the first equation above with respect to  $\gamma$ . This implies the leftmost equality in the second equation above. The rightmost equality holds by rearrangement because the individual solves Problem P1.<sup>9</sup> The numerator term in the second equation is pinned down by asset prices so that  $E[\sum_{j=1}^J m_{1,j} c_j^{\text{smooth}} | z^1] = \sum_{j=1}^J (\frac{1}{1+r})^{j-1}$ , whereas the denominator is determined by the value of human capital plus initial earnings and initial wealth. The only unobservable is the value of human capital. The theory then implies that a high marginal benefit of moving towards perfect consumption smoothing coincides with a low value of human capital. This straightforward point has not, to the best of our knowledge, been noted in the literature on the value of human capital.

### 3. Empirics: earnings and asset returns

We outline an empirical framework for idiosyncratic earnings shocks, aggregate earnings shocks and stochastic stock returns. Let  $e_{i,j,t}$  denote real pretax annual earnings for individual  $i$  of age  $j$  in year  $t$ . We assume that the natural logarithm of earnings consists of an aggregate component ( $u^1$ ) and an idiosyncratic component ( $u^2$ ) and

$$\log e_{i,j,t} = u_t^1 + u_{i,j,t}^2. \quad (1)$$

In Section 3.1 we describe the structure of the idiosyncratic component of earnings, our estimation procedure and the fit of the estimated model. In Section 3.2 we describe the structure and estimation of the joint process for the aggregate component of earnings and stock returns.

#### 3.1. Idiosyncratic component of earnings

The idiosyncratic component of earnings is the sum of four orthogonal components: a common age effect  $\kappa_j$ , an individual-specific fixed effect  $\xi_i$ , a persistent component  $\zeta$  and a transitory component  $v$ .

$$u_{i,j,t}^2 = \kappa_j + \xi_i + \zeta_{i,j,t} + v_{i,j,t} \quad (2)$$

$$\zeta_{i,j,t} = \rho \zeta_{i,j-1,t-1} + \eta_{i,j,t}$$

$$\zeta_{i,0,t} = 0.$$

The common age effect is modeled as a quartic polynomial. The individual fixed effects are assumed to be normally distributed with a constant variance  $\sigma_\xi^2$ . The transitory idiosyncratic shocks are assumed to be distributed according to a distribution with zero mean, variance  $\sigma_{v,j}^2$ , and third central moment  $\mu_{3,v,j}$ . In order to capture life-cycle properties of the variance and skewness of earnings we allow the moments of the transitory component to be age-dependent and model this as a quartic polynomial.

Persistent idiosyncratic shocks are assumed to be distributed according to a distribution with zero mean, variance  $\sigma_{\eta,t}^2(X_t)$  and third central moment  $\mu_{3,\eta,t}(X_t)$ . The variance and skewness have a linear trend, in order to capture low

<sup>9</sup> More specifically, convert the period budget constraints in Problem P1 into an age-1 budget constraint, using the fact that the Euler equation holds at a solution to Problem P1. Then the age-1 value of the consumption plan equals the value of human capital, earnings at age 1 and initial wealth.

frequency trends over the sample period, and are state dependent via the variable  $X_t$ .<sup>10</sup> We model  $X_t$  as a two-state process. Specifically, we set  $X_t = 1_{\Delta u_t^1 > 0}$  so that  $X_t$  is an indicator function taking on the value 1 in booms and 0 in recessions. Thus, aside from a trend term, the variance and skewness of the persistent innovations take on different values in expansions and contractions.

We estimate the idiosyncratic earnings process using data on male annual labor earnings from the Panel Study of Income Dynamics (PSID) from 1967 to 1996.<sup>11</sup> We focus on male heads of households between ages 22 and 60 with real annual earnings of at least \$1,000. Our measure of annual gross labor earnings includes pre-tax wages and salaries from all jobs, plus commission, tips, bonuses and overtime, as well as the labor part of income from self-employment. Labor earnings are inflated to 2008 dollars using the CPI All Urban series. We also consider two sub-samples. Individuals with 12 or fewer years of education are included in the High School sub-sample, while those with at least 16 years or a Bachelor's degree are included in the College sub-sample.

Estimation is done in two stages. In the first stage we estimate  $\kappa_j$  by regressing log real annual earnings on a quartic polynomial in age and a full set of year dummies. This is done separately for the three education samples. On the basis of the first-stage results for the PSID, and related results for the Current Population Survey data and NIPA data, we set the contraction years over the time interval 1967–1996 to be 1970, 1974–1975, 1979–1982, 1989–1991 and 1993.

Residuals from this first-stage regression are then used to estimate the remaining parameters of the individual earnings equation:  $(\rho, \sigma_\xi^2, \sigma_{\eta,t}^2(X_t), \mu_{3,\eta,t}(X_t), \sigma_{v,j}^2, \mu_{3,v,j})$ . A GMM estimator is then used to estimate the parameters, using the full set of second and third-order autocovariances as moments.<sup>12</sup>

The estimated process delivers a good fit to the variance and third central moment of the earnings distribution as a function of both age and time. The fit of these and other moments for the full sample is displayed in Fig. 2. Corresponding results for the College and High School samples are contained in the Appendix.

We highlight three findings from Table 1. First, transitory shocks are left skewed for the full sample and the college sample. Left skewness is needed to match the left skewness of the first-stage residuals as documented in Fig. 2. Guvenen et al. (2014) document that male earnings growth rates are left skewed in administrative data. Second, the variance and left skewness of persistent shocks is higher in recessions than in booms. Consistent with the findings in Storesletten et al. (2004), there is evidence for counter-cyclical variance even when the framework is generalized to account for skewness and a time trend. However, the cyclical variation that we estimate is less dramatic than their findings. Third, the autoregression parameter  $\rho$  is higher for the full sample and the college sample compared to the high school sample. Thus, persistent innovations of a given magnitude will be of greater proportional importance for those with a college than a high school education.

The parameter estimates are broadly consistent with those from related specifications (that do not account for skewness), that have been estimated elsewhere in the literature and summarized in Meghir and Pistaferri (2010).<sup>13</sup> We note that our estimate of the variance of the transitory component is approximately 0.1 larger than what has been estimated by others (see for example, Guvenen, 2009). The source of this difference is due entirely to our broader sample selection. Since it is likely that a substantial fraction of this variance is due to measurement error, we make an adjustment when using these estimates as parameters in the structural model.<sup>14</sup>

### 3.2. Aggregate component of earnings and stock returns

Our benchmark model is a standard, first-order VAR for  $\Delta y_t = (\Delta u_t^1 \quad \log R_t^s)'$ . The error term  $\varepsilon_t$  is a vector of zero mean IID random variables with covariance matrix  $\Sigma$ .

$$\Delta y_t = \gamma + \Gamma \Delta y_{t-1} + \varepsilon_t \quad (3)$$

<sup>10</sup> Allowing for a trend in the shock variances is important for accurately estimating cyclical variation in the variance and skewness. This is because of the well-documented increase in the variance of idiosyncratic earnings shocks over this period. See for example Heathcote et al. (2010).

<sup>11</sup> After 1996 the PSID was converted into a biannual survey.

<sup>12</sup> When computing the auto covariance function, individuals are grouped into 5-year age cells so that when calculating covariances at age  $j$ , individuals aged  $j \in [j-2, j+2]$  are used. Only cells with at least 30 observations are retained. The moments are weighted by  $n_{j,t,l}^{0.5}$  where  $n_{j,t,l}$  is the number of observations used to calculate the covariance at lag  $l$  in year  $t$  for age  $j$ . The maximum lag length is  $l = 10$ . Individuals aged 22 to 60 are used to construct the empirical auto-covariance functions. This means that variances, covariance and third moments from ages 24 to 58 are effectively used in the estimation. Standard errors are calculated by bootstrap with 39 repetitions, thus accounting for estimation error induced by the first-stage estimation.

<sup>13</sup> Our model is estimated using data on log earnings levels. Estimation using data on log earnings growth rates would yield larger estimates of the persistent or permanent shocks. See Heathcote et al. (2010) for a discussion of this issue. We favor the estimation in levels since it allows us to accurately capture the age profile of the cross-sectional variance of earnings.

<sup>14</sup> Using indirect inference on a structural model of consumption and savings behavior, Guvenen and Smith (2011) estimate that the variance of measurement error in male log annual earnings is around 0.02–0.025. Using a validation study of the PSID, French (2004) concludes that the variance of measurement error in the PSID is around 0.01. However, both of their samples are substantially more selected than ours, with a cross-sectional variance about 0.1 lower. Assuming that half of this additional variance is due to measurement error, would suggest that around 0.05–0.06 of the estimated transitory variance is measurement error. Accordingly, we adjust our estimates of the variance  $\sigma_{v,j}^2$  down by 1/3 at all ages. We adjust the third central moment at each age so that the coefficient of skewness,  $\mu_{3,v,j}/\sigma_{v,j}^3$  remains unchanged after adjusting the denominator.

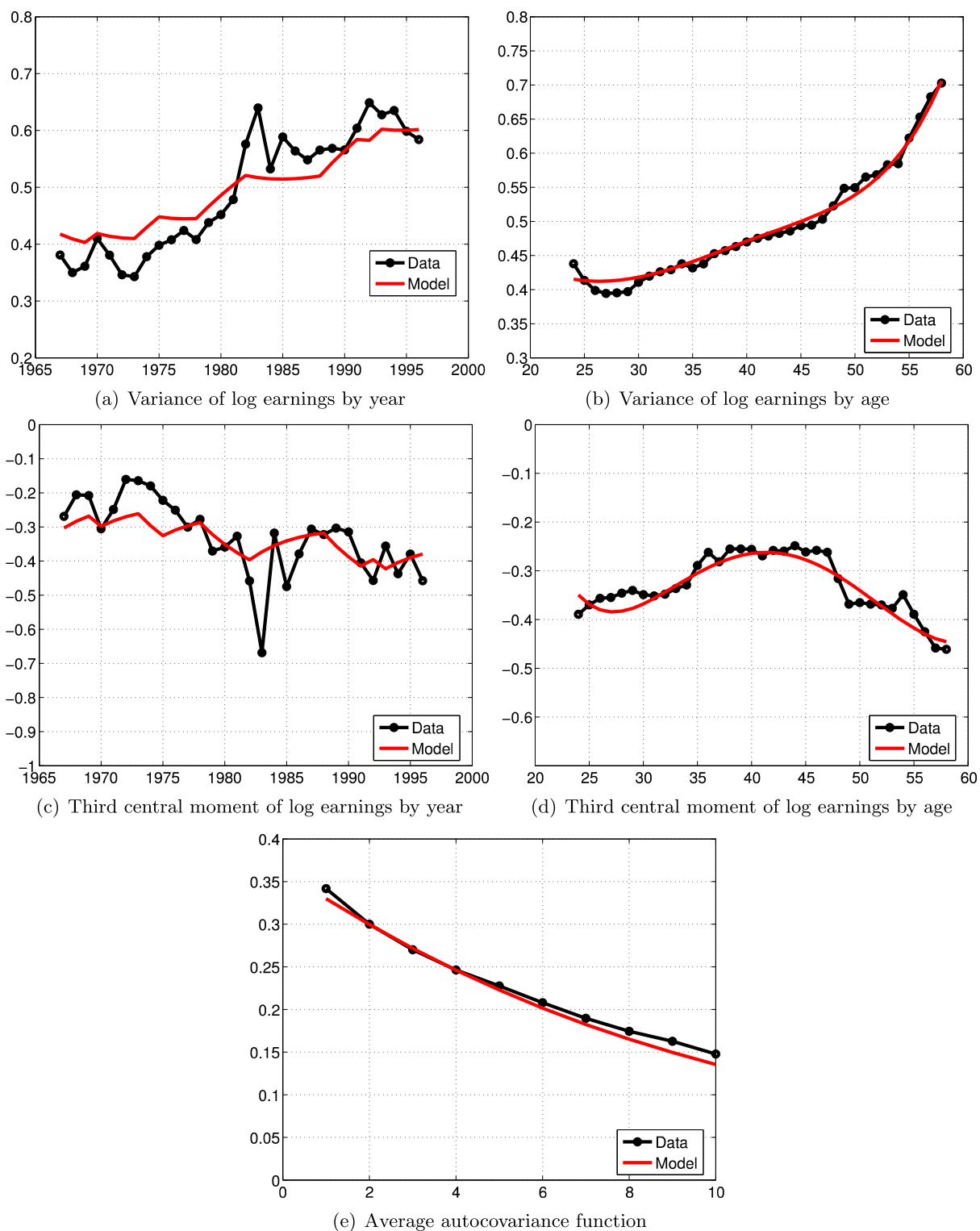


Fig. 2. Fit of estimated idiosyncratic earnings model for the full sample.



**Table 1**  
Parameter estimates for the idiosyncratic earnings process.

	Full sample	College sample	High School sample
<u>Fixed effect</u>			
$\sigma_{\xi}^2$	0.082 (0.013)	0.064 (0.015)	0.137 (0.032)
<u>Persistent component</u>			
$\rho$	0.942 (0.012)	0.951 (0.015)	0.843 (0.097)
$\sigma_{\eta}^2$ : boom	0.038 (0.005)	0.037 (0.006)	0.047 (0.023)
$\sigma_{\eta}^2$ : recession	0.058 (0.005)	0.048 (0.008)	0.072 (0.020)
$\sigma_{\eta}^2$ : linear trend	0.001 (0.000)	0.001 (0.000)	0.002 (0.002)
$\mu_{3,\eta}$ : boom	−0.020 (0.014)	−0.006 (0.015)	−0.149 (0.181)
$\mu_{3,\eta}$ : recession	−0.061 (0.013)	−0.040 (0.019)	−0.190 (0.170)
$\mu_{3,\eta}$ : linear trend	−0.001 (0.001)	−0.001 (0.001)	−0.003 (0.002)
<u>Transitory component</u>			
$\sigma_v^2$	0.132 (0.005)	0.139 (0.006)	0.128 (0.023)
$\mu_{3,v}$	−0.161 (0.027)	−0.162 (0.029)	−0.002 (0.172)

**Notes:** Models of the moments of the transitory shock include a fourth-order polynomial in age. The reported moments for transitory shocks are averages over the age range. Standard errors are computed by block bootstrap with 39 repetitions.

We estimate (3) using data on male annual labor earnings from the Current Population Survey (CPS) from 1967 to 2008. Our sample selection criteria and definition of earnings are the same as those used for the PSID, described in section 3.1. We construct an empirical counterpart to  $u_t^1$  by estimating a median regression (Least Absolute Deviations) of earnings on a full set of age and time dummies. We use a median regression rather than OLS since it is more robust to the effects of changes in top coding in the CPS over our sample period. We use the estimates of  $\hat{u}_t^1$  from our CPS sample, rather than corresponding estimates from the PSID, as input to the estimation because CPS data has both a longer time dimension and a larger cross-section sample each year compared to PSID data.

Data on equity and bond returns are annual returns. Equity returns are based on a value-weighted portfolio of all NYSE, AMEX and NASDAQ stocks including dividends. Bond returns are based on 1-month Treasury bill returns.<sup>15</sup> Real returns are calculated by adjusting for realized inflation using the CPI All Urban series.

Table 2 reports the estimation results. The parameter estimates reveal a moderate degree of persistence in aggregate earnings growth. The covariance between innovations to earnings growth and innovations to stock returns is positive in all models. Thus, the estimated models imply a positive conditional correlation between earnings growth and stock returns. This is one feature, among others, that will later produce a positive conditional correlation between stock and human capital returns and a positive stock component of the value of human capital.

The implied steady-state dynamics are reported in Table 3. The estimated model matches the observed correlation structure well. When we input the estimated process into our economic model, we adjust the constants  $(\gamma_1, \gamma_2)$  estimated in Table 2 so that all models produce in steady state  $E[\log R^S] = .041$  and  $E[\Delta u^1] = 0$ . This facilitates comparisons of human capital value and return properties across models.

In Appendix A.3 we consider a generalization of (3). The generalization allows us to address the possibility that earnings  $u_t^1$  and a process generating log stock returns are cointegrated. We assume that  $y_t = (u_t^1 \ P_t)'$  follows a  $p$ -th order VAR, where  $P_t$  is a process generating stock returns (i.e.  $\Delta P_t = \log R_t^S$ ). We show that this VAR can be written as a Vector Error Correction Mechanism (VECM), a useful tool in the cointegration literature. We present lag order selection tests that suggest the presence of two lags, i.e.  $p = 2$ . We also present tests of the cointegrating rank of this system. We interpret these test findings as providing only weak evidence for cointegration. This leads us to adopt (3) as the benchmark process. However, since these tests may all have relatively little power given the short time series, we also estimate the model with cointegration and assess their implications within the portfolio choice model.

<sup>15</sup> All returns come from Kenneth French's data archive. We restrict attention to the period 1967–2008 since this is the period that the CPS earnings data covers.



**Table 2**

Parameter estimates for the aggregate stochastic process.

		Full sample	College sample	High School sample
<b>Equation (1): <math>\Delta u_t^1</math></b>				
$\Delta u_{t-1}^1$	$\Gamma_{11}$	0.383 (0.14)	0.260 (0.15)	0.348 (0.14)
$\log R_{t-1}^s$	$\Gamma_{12}$	0.044 (0.02)	0.04 (0.02)	0.057 (0.02)
Constant	$\gamma_1$	−0.004 (0.00)	−0.003 (0.00)	−0.009 (0.00)
<b>Equation (2): <math>\log R_t^s</math></b>				
$\Delta u_{t-1}^1$	$\Gamma_{21}$	−2.149 (1.15)	−2.203 (1.29)	−1.731 (0.97)
$\log R_{t-1}^s$	$\Gamma_{22}$	0.106 (0.17)	0.153 (0.18)	0.101 (0.17)
Constant	$\gamma_2$	0.032 (0.03)	0.031 (0.03)	0.024 (0.03)
<b>Var-Cov matrix</b>				
$\text{var}(\varepsilon_{1,t}) \times 10^{-4}$		4.42	4.24	6.49
$\text{var}(\varepsilon_{2,t}) \times 10^{-2}$		3.2	3.24	3.23
$\text{cov}(\varepsilon_{1,t}, \varepsilon_{2,t}) \times 10^{-3}$		1.23	1.24	1.52

**Notes:** Standard errors in parentheses.**Table 3**

Implied steady-state statistics for the aggregate stochastic process.

	Full		College		High School	
	Data	Model	Data	Model	Data	Model
$E(\log R_t^b)$	0.012	0.012	0.012	0.012	0.012	0.012
$E(\log R_t^s)$	0.041	0.045	0.041	0.040	0.041	0.045
$E(\Delta u_t^1)$	−0.002	−0.004	0.000	−0.001	−0.007	−0.010
$sd(\Delta u_t^1)$	0.025	0.025	0.023	0.023	0.030	0.030
$sd(\log R_t^s)$	0.187	0.187	0.187	0.187	0.187	0.187
$\text{corr}(\Delta u_t^1, \log R_t^s)$	0.184	0.177	0.248	0.251	0.207	0.194
$\text{corr}(\Delta u_t^1, \Delta u_{t-1}^1)$	0.425	0.441	0.346	0.341	0.386	0.416
$\text{corr}(\log R_t^s, \log R_{t-1}^s)$	0.057	0.055	0.057	0.084	0.057	0.047
$\text{corr}(\Delta u_t^1 \log R_{t-1}^s)$	0.372	0.398	0.377	0.387	0.387	0.420
$\text{corr}(\log R_t^s, \Delta u_{t-1}^1)$	−0.292	−0.270	−0.225	−0.235	−0.289	−0.261

**Notes:** Table shows average moments in the data, together with implied steady-state statistics from the corresponding estimated model. Data cover the period 1967–2008. When implementing the estimated processes in the structural model, we adjust the constants  $(\gamma_1, \gamma_2)$  estimated in Table 2 so that all models have  $E[\log R_t^s] = 0.041$  and  $E[\Delta u_t^1] = 0$ .

#### 4. The benchmark model

We now use the theoretical framework and the empirical results to quantify the value and return to human capital. The benchmark model has two financial assets. Asset  $i = 1$  is riskless and asset  $i = 2$  is risky. The agent cannot go short on either financial asset. Initial resources at age  $j = 1$  are denoted by  $x$  which describes the sum of initial earnings and financial wealth.

Benchmark model:  $\max U(c)$  subject to  $c \in \Gamma_1(x, z^1)$

$$\Gamma_1(x, z^1) = \{c = (c_1, \dots, c_J) : \exists (a^1, a^2) \text{ s.t. } 1 - 2 \text{ holds } \forall j\}$$

1.  $c_j + \sum_{i \in \mathcal{I}} a_{j+1}^i \leq x$  for  $j = 1$  and  $c_j + \sum_{i \in \mathcal{I}} a_{j+1}^i \leq \sum_{i \in \mathcal{I}} a_j^i R_j^i + e_j$  for  $j > 1$
2.  $c_j \geq 0$  and  $a_{j+1}^1, a_{j+1}^2 \geq 0, \forall j$

The utility function  $U(c) = U^1(c_1, \dots, c_J)$  is of the type employed by [Epstein and Zin \(1991\)](#). It is defined recursively by applying an aggregator  $W$  and a certainty equivalent  $F$ . The certainty equivalent encodes attitudes towards risk with  $\alpha$  governing risk-aversion. The aggregator encodes attitudes towards intertemporal substitution where  $\gamma$  is the inverse of the intertemporal elasticity of substitution. We allow for mortality risk via the one-period-ahead survival probability  $\psi_{j+1}$ .

**Table 4**  
Parameter values for the benchmark model.

Category	Symbol	Parameter value
Demographics	$J, Ret$ $\psi_{j+1}$ Survival probability	$(J, Ret) = (69, 40)$ U.S. Life Table
Preferences	$\alpha$ Risk aversion $1/\gamma$ Intertemporal substitution $\beta$ Discount factor	$\alpha \in \{4, 6, 10\}$ $1/\gamma = 1.17$ see Notes
Returns	$R^s, R^b$	Tables 2–3
Earnings	$e_j(z) = \begin{cases} z_1 \exp(\kappa_j + \xi + \zeta + \nu)(1 - \tau) & \text{if } j < Ret \\ z_1 b(\xi)(1 - \tau) & \text{if } j \geq Ret \end{cases}$ $\zeta' = \rho\zeta + \eta'$ and $\eta' \sim GN(0, \sigma_\eta^2(X), \mu_{3,\eta}(X))$ $\xi \sim N(0, \sigma_\xi^2)$ and $\nu \sim GN(0, \sigma_{\nu,j}^2, \mu_{3,\nu,j})$	$\tau = .27$ $b(\cdot)$ see text Tables 1–2
Initial wealth	$\sum_{i \in \mathcal{I}} a_1^i R_1^i$	$\sum_{i \in \mathcal{I}} a_1^i R_1^i = 0.3E[e_1]$

**Notes:**  $\beta$  is calibrated to generate a steady-state ratio of wealth to income equal to 4.1. All sensitivity analyses are performed by re-calibrating  $\beta$  to generate the same ratio. Survival probabilities are smoothed versions of male values from the 1989–1991 US Decennial Life Tables in [NCHS \(1992\)](#). Smoothing is done using a nine point moving average.  $E[e_1]$  denotes mean earnings at age 1 in the model.

$$U^j(c_j, \dots, c_J) = W(c_j, F(U^{j+1}(c_{j+1}, \dots, c_J)), j)$$

$$W(a, b, j) = [(1 - \beta)a^{1-\gamma} + \beta\psi_{j+1}b^{1-\gamma}]^{1/(1-\gamma)} \text{ and } F(x) = (E[x^{1-\alpha}])^{1/(1-\alpha)}$$

Table 4 summarizes the parameters in the benchmark model.

**Demographics:** Agents start economic life at real-life age 22, retire at age 61 and live at most up to age 90. Thus, we set  $J = 69$  and  $Ret = 40$ . Agents face a conditional probability  $\psi_{j+1}$  of surviving from period  $j$  to period  $j + 1$  that is set to estimates for males from the 1989–1991 US Decennial Life Tables in [NCHS \(1992\)](#).

**Preferences:** We set the preference parameters to values estimated from Euler equation restrictions. [Vissing-Jorgensen and Attanasio \(2003\)](#) estimate  $1/\gamma = 1.17$  for a preferred specification and conclude that the risk aversion parameter  $\alpha$  in the interval  $[5, 10]$  can be obtained under realistic assumptions, based on household-level data. Thus, the special case of constant-relative-risk-aversion (CRRA) preferences, where  $\gamma = \alpha$ , is not the parameter configuration that best fits the Euler equation restrictions. We examine model implications for  $1/\gamma = 1.17$  and  $\alpha \in \{4, 6, 10\}$ . We later investigate CRRA preferences to determine whether our main conclusions are robust across many parameter configurations. We set the discount factor  $\beta$  so that, given all other model parameters, the model estimated based on the full sample produces an average wealth-income ratio equal to 4.1. This is the 2010 US value documented by [Piketty and Zucman \(2014\)](#).

**Initial wealth:** Initial wealth is set to equal 30 percent of mean earnings at age 22.

**Earnings and asset returns:** Earnings and asset returns in the benchmark model are based on the estimates in [Tables 1–3](#) for the case of no cointegration. The earnings that enter the model are earnings after taxes and transfers. Before the retirement age, model earnings  $e_j(z)$  are the process estimated in [Tables 1–3](#) times  $(1 - \tau)$ , where  $\tau = .27$ . After the retirement age, model earnings  $e_j(z)$  equal model social security benefits after taxes.

$$e_j(z) = z_1 g_j(z_2) = \begin{cases} \exp(u^1) \exp(\kappa_j + \xi + \zeta + \nu)(1 - \tau) & j < \text{Retire} \\ \exp(u^1) b(\xi)(1 - \tau) & \text{otherwise} \end{cases}$$

We group the variables from the statistical model into a state variable  $z = (z_1, z_2)$ , where  $z_1 = \exp(u^1)$  captures the aggregate component of earnings and  $z_2 = (\xi, \zeta, \nu, \Delta u^1, \log R^s)$  captures the idiosyncratic components  $(\xi, \zeta, \nu)$ , the growth in the aggregate component of earnings and the stock return.<sup>16</sup> The aggregate innovations governing earnings and stock returns are assumed to be jointly normally distributed with the covariance matrix estimated in [Table 2](#). Shocks are discretized following the method described in [Appendix A.2](#). Since shock innovations are jointly normal, they are not drawn from a fat-tailed or time varying distribution.

The fixed effect  $\xi$  is normally distributed with the variance given in [Table 1](#). The transitory shock  $\nu$  and the persistent shock innovations  $\eta$  follow a Generalized Normal distribution, determined by the first three central moments. The second and third central moments of the persistent shock innovations are state dependent as described in [Table 1](#). The age-dependent second and third moments of the transitory shock distribution are scaled as discussed in footnote 14. See [Hosking and Wallis \(1997, Appendix A.8\)](#) for a discussion of the Generalized Normal distribution.

**Social security:** The nature of social security benefits is potentially of great importance for how people value future earnings flows after taxes and transfers. Social security wealth is by some calculations the single most important asset type

<sup>16</sup> [Appendix A.2](#) discusses how we compute model solutions. [Appendix A.3](#) discusses the statistical model with cointegration. The state variable in the model with cointegration is  $z = (z_1, z_2)$ , where  $z_2 = (\xi, \zeta, \nu, \Delta u^1, \log R^s, w)$ .

for many older households.<sup>17</sup> Social security benefits in the model are an annuity payment which is determined by the aggregate earnings level  $z_1$  when the agent reaches the retirement age and by a concave benefit function  $b$ . We adopt the benefit function employed by Huggett and Parra (2010) which captures the bend-point structure of old-age benefits in the U.S. social security system. We employ the computationally-useful assumption that the benefit function applies only to an agent's idiosyncratic fixed effect rather than to an average of the agent's past earnings as in the U.S. system. Thus, the model benefit is risky after entering the labor market only because the aggregate component of earnings at the time of retirement is risky. Old-age benefit payments in the U.S. system are indexed to average economy-wide earnings when an individual hits age 60.<sup>18</sup> This is captured within the model by the fact that benefits are proportional to  $z_1 \equiv \exp(u^1)$ .

Properties of the benchmark model are displayed in Fig. 3. It is constructed by simulating many shock histories, calculating decisions along these histories and then taking averages at each age. Fig. 3 shows that mean consumption, wealth and earnings net of taxes and transfers are hump shaped. Financial wealth is exhausted before the end of life whereupon agents live off social security.

Equity participation rates in Fig. 3 are above US values at all ages for all the risk aversion values we analyze. Participation rates are calculated among agents with strictly positive financial assets as is done in the literature. Chang et al. (2015) provide a useful review of findings in the literature and document properties in data from the Survey of Consumer Finances (SCF). They find an average participation rate of 55 percent in the SCF 1998–2007 for risky financial assets that include stock, trusts, mutual funds and retirement accounts with exposure to risky assets. They find a hump-shaped participation rate with age where the rate is roughly 30 percent within the 21–25 age group. Equity participation rates in the model are just as high or higher at all ages as the patterns in Fig. 3 when we analyze constant-relative-risk-aversion (CRRA) preferences for CRRA values ranging from 1 to 6.

The results summarized above lead us to conclude that the benchmark model is far away from producing the low equity participation rates early in the working lifetime found in US data. This holds for a wide range of preference parameter configurations. Thus, we provide support for one view within the portfolio literature: the explanation of these portfolio facts is likely to rest on model properties beyond those encompassed by enriching the Samuelson (1969) model with empirically realistic earnings and financial asset return properties. This negative view is based on the robust result that the stock share of the value of human capital is not particularly large when earnings and asset returns are driven by a process estimated from US data.

## 5. Human capital values and returns: benchmark model

We report properties of the benchmark model based on the high school and the college subsamples. Results for the full sample are typically between the results for these education groups.

### 5.1. Human capital values

Fig. 4 plots the value of human capital in the benchmark model and a decomposition of this value. The mean value of human capital over the lifetime is hump shaped and is lower for higher values of the risk aversion coefficient.<sup>19</sup> For comparison purposes, we also plot the value of human capital that would be implied by discounting future earnings at the risk-free rate. We label this the naive value.

Our notion of value lies far below the naive value. This occurs because of negative covariation between an agent's stochastic discount factor and earnings and because agents are sometimes on the corner of the risk-free asset choice. Corner solutions occur more frequently early in life for college agents than for high school agents due to differences in the mean earnings profile. When an agent is on the corner, then the agent discounts certain future earnings at more than the risk-free rate.

We now decompose human capital values into a bond, a stock and a residual-value component. This follows the method discussed in section 2. To do so, we project next period's human capital payout  $v_{j+1} + e_{j+1}$  onto the space of conditional gross bond and stock returns.<sup>20</sup>

$$v_j = E[m_{j,j+1}(v_{j+1} + e_{j+1})|z^j] = E[m_{j,j+1}(\sum_{i \in \mathcal{I}} \alpha_j^i R_{j+1}^i + \epsilon_{j+1})|z^j]$$

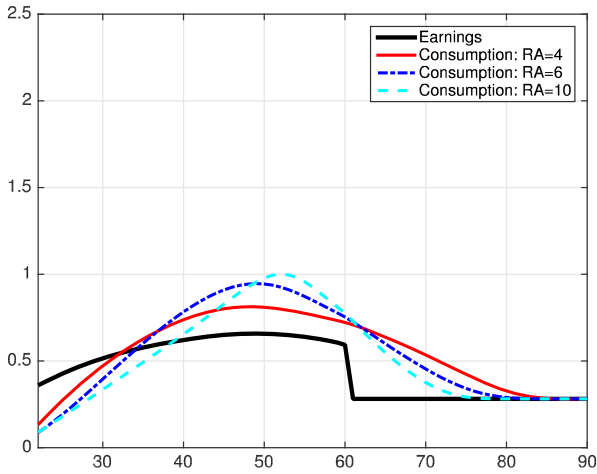
Fig. 4 calculates the value of the bond and stock components  $\alpha_j^b E[m_{j,j+1} R_{j+1}^b | z^j]$  and  $\alpha_j^s E[m_{j,j+1} R_{j+1}^s | z^j]$  as well as the value of the orthogonal component  $E[m_{j,j+1} \epsilon_{j+1} | z^j]$ . It then expresses these values as a fraction of the value of human

<sup>17</sup> Poterba et al. (2011) calculate that the capitalized wealth implicit in social security retirement annuities is approximately 33 percent of all wealth for households aged 65–69 and is a much larger percentage of individual wealth for households with low wealth.

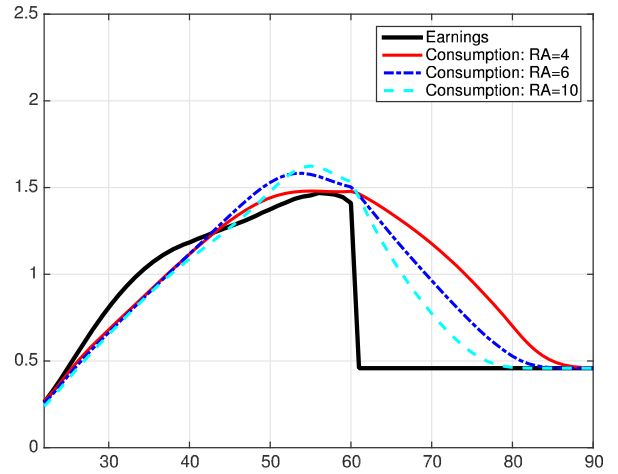
<sup>18</sup> See the Social Security Handbook (2012, Ch. 7).

<sup>19</sup> Figs. 3(a)–(b) are constructed by first computing human capital values at each age as a function of the state. We then calculate the sample average of the value at each age, conditional on survival, by simulating many realizations of the state variable over the lifetime. Computational methods are described in the Appendix.

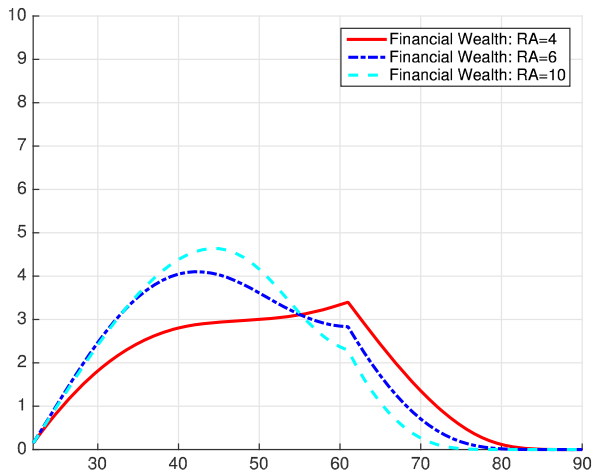
<sup>20</sup> The Appendix describes our methods for computing the projection coefficients  $\alpha^i$  in the value decomposition.



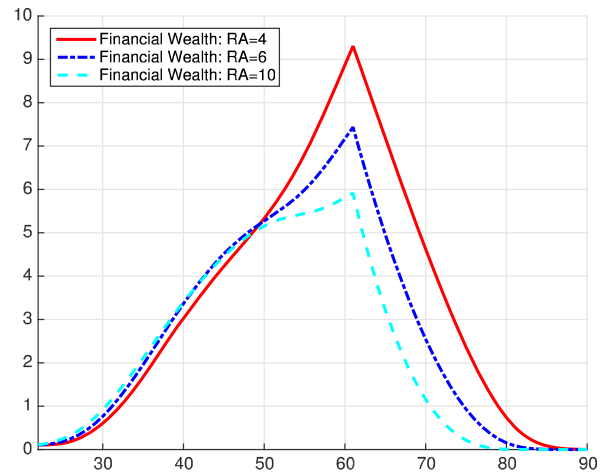
(a) Mean earnings, consumption



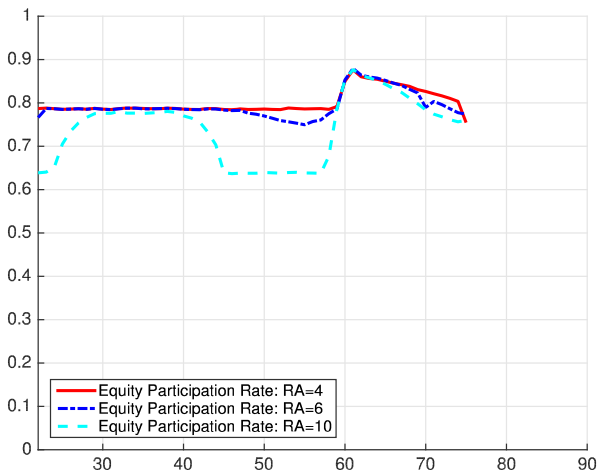
(b) Mean earnings, consumption



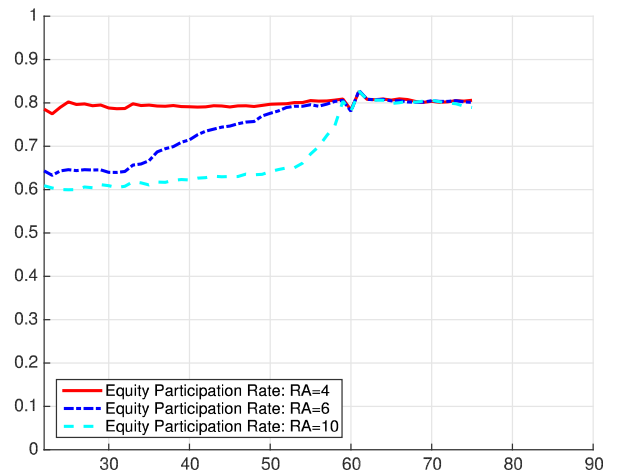
(c) Mean financial wealth



(d) Mean financial wealth



(e) Equity participation rate



(f) Equity participation rate

**Notes:** Left panels refer to high-school sub-sample, right panels refer to college sub-sample. The vertical scale in (a)–(b) is in units of 100,000 dollars in year 2008. Participation rates are conditional on having positive financial wealth. Equity shares are conditional on having positive equity.

**Fig. 3.** Life-cycle profiles in the benchmark model.

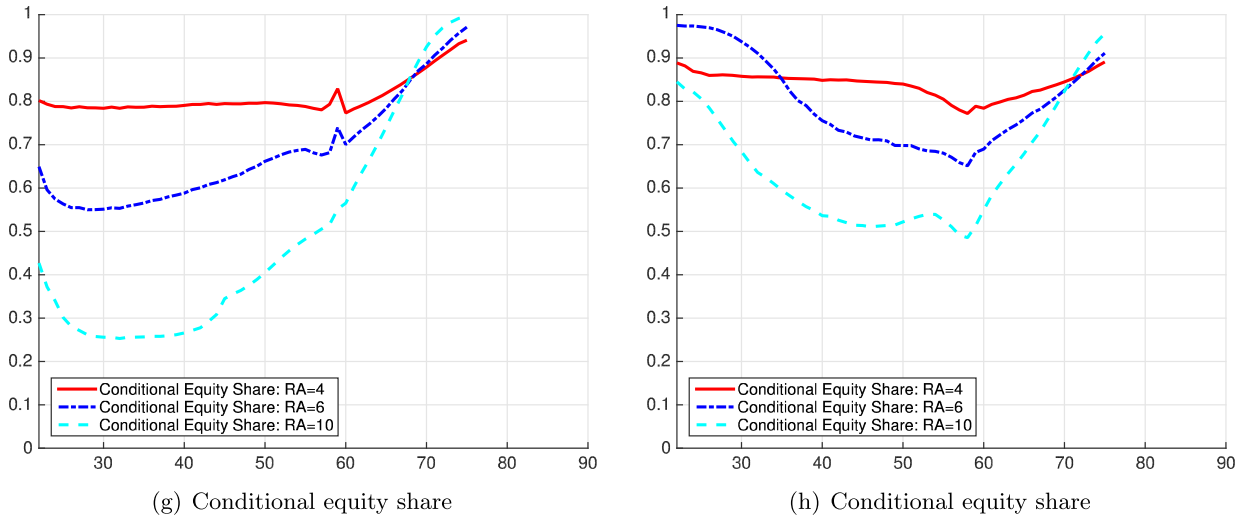


Fig. 3. (continued)

capital at each age and then averages these ratios across the states that occur at each age. Appendix A.2 describes our methods for computing the projection coefficients in the value decomposition and each asset's share in the value of human capital. The bond component averages more than 80 percent of the value of human capital. This holds for both education groups and for a range of risk-aversion parameters.

Fig. 4 shows that the stock component of the value of human capital is positive on average. It averages below 35 percent at all ages for high school agents and below 20 percent for college agents for a wide range of risk-aversion parameters. The stock component is positive, at a given age and state, provided that the sum of next period's earnings and human capital value covaries positively with the return to stock, conditional on this period's state. Our empirical work, as summarized in Table 2, directly relates to the conditional comovement of earnings and stock returns since  $cov(\varepsilon_{1,t}, \varepsilon_{2,t}) > 0$  in all the estimated models.

The orthogonal component has a large negative value early in the lifetime. Given that the orthogonal component has a zero mean, this is due to strong negative covariation between the orthogonal component and the stochastic discount factor early in life. This occurs, for example, because consumption and future utility are increasing in the realization of the persistent idiosyncratic earnings shock, other things equal. Section 6 will show that the persistent shock component is particularly important early in life in lowering human capital values. Such a shock has many periods over which it impacts future earnings.

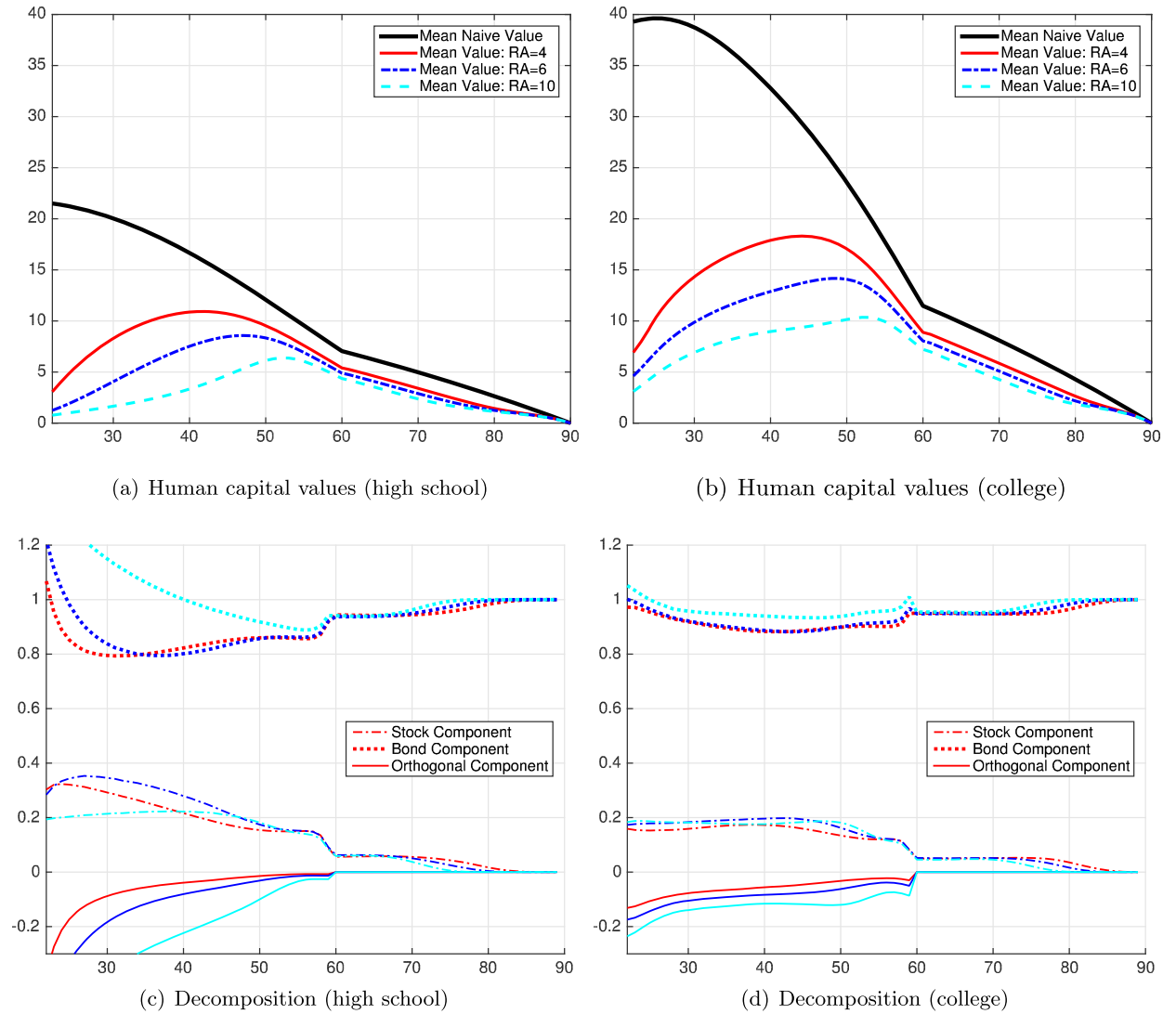
While it may seem plausible that the value of human capital is largely bond-like during retirement, it is useful to understand why it is not always 100 percent bond-like. If a retired agent will in all future date-events end up holding positive bonds, then the decomposition will indeed calculate that this agent's human capital in retirement is 100 percent bond-like as social security annuity payments in the model are riskless after retirement. However, if an agent hits the corner of the bond decision in the future under some sequence of risky stock returns, then this is not true. The mean of the agent's stochastic discount factor will be less than the inverse of the gross risk-free rate in such an event. Thus, the agent discounts future social security transfers at greater than the risk-free rate beyond this date. The value of these transfers at earlier dates takes on a positive stock component provided that a corner solution is induced by low stock return realizations. In summary, while the value of human capital is mostly bond-like in retirement, it is not 100 percent bonds because agents run down financial assets, hit a corner solution on the holdings of the risk-free asset and live off social security transfers.

## 5.2. Human capital returns

Fig. 5 plots properties of human capital returns. Mean returns are very large early in the working lifetime. To understand what drives the mean human capital returns, it is useful to return to the main ideas used in the value decomposition. The first equation below decomposes gross returns by decomposing the future payout into a bond, a stock and an orthogonal component. The second equation shows that the conditional mean human capital return always equals the weighted sum of the conditional mean of the bond and stock return.<sup>21</sup>

$$R_{j+1}^h \equiv \frac{v_{j+1} + e_{j+1}}{v_j} = \frac{\alpha_j^b R_{j+1}^b + \alpha_j^s R_{j+1}^s + \epsilon}{v_j}$$

<sup>21</sup> The orthogonal component drops out as, with a risk-free asset, the mean of the orthogonal component is zero.



Notes: The vertical scale in Fig. 3(a)–(b) is in units of 100,000 dollars in year 2008.

Fig. 4. Human capital values and a decomposition.

$$E[R_{j+1}^h|z^j] = \frac{\alpha_j^b}{v_j} E[R_{j+1}^b|z^j] + \frac{\alpha_j^s}{v_j} E[R_{j+1}^s|z^j]$$

The weights on the bond and stock return do not always sum to one. When the agent's Euler equation for both stock and bonds hold with equality, then these weights will sum to more than one exactly when the value of the orthogonal component is negative.<sup>22</sup> The value of the orthogonal component of human capital payouts is negative early in the working lifetime. Human capital returns can vastly exceed a convex combination of stock and bond returns when the weights sum to more than one.

The mean return to human capital is near the risk-free rate immediately after retirement but subsequently increases. The high return towards the end of the lifetime might at first seem odd. This should not be surprising, however, as in the penultimate period  $v_{j-1} = E[m_{j-1,j}e_j]$  and  $1 = E[m_{j-1,j}e_j/v_{j-1}]$ . As the payment  $e_j$ , conditional on surviving to the last period, is certain, the return is  $R_j^h = e_j/v_{j-1} = 1/E[m_{j-1,j}]$ . Thus, the return equals the risk-free bond rate when the agent is off the corner (i.e.  $R_j^h = 1/E[m_{j-1,j}] = R^b$ ) but can exceed the risk-free rate when the agent is on the corner (i.e.

<sup>22</sup> In this case,  $v_j = E[m_{j+1}(v_{j+1} + e_{j+1})] = \alpha_j^b + \alpha_j^s + E[m_{j+1}e_{j+1}]$  and  $E[m_{j+1}e_{j+1}] < 0$  imply  $\alpha_j^b/v_j + \alpha_j^s/v_j > 1$ . Of course, the weights for decomposing returns can and do sum to more than one even when Euler equations do not hold with equality.

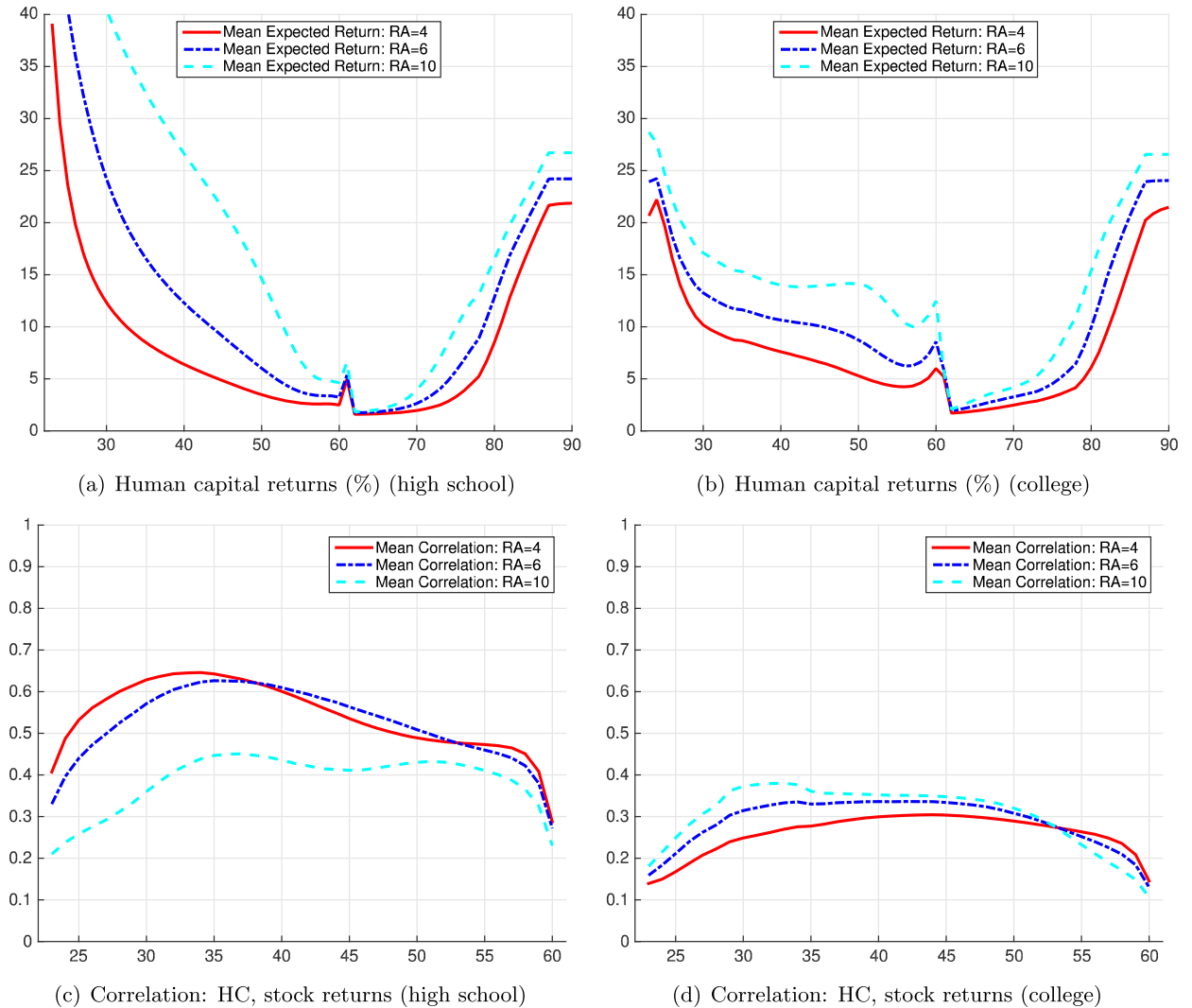


Fig. 5. Properties of human capital returns.

$R_J^h = 1/E[m_{J-1,J}] \geq R^b$ ). Towards the end of the lifetime an increasing fraction of agents in the model are on this corner consistent with the wealth profile in Fig. 3.

The positive correlation between human capital returns and stock returns in Fig. 5 is based in part on two properties. First, innovations to the aggregate component of earnings growth and to stock returns are positively correlated. This implies that the component of human capital returns related directly to the earnings payout next period covaries positively with stock returns. Second, the old-age transfer benefit formula in the benchmark model is proportional to the aggregate component of earnings at the retirement age. The U.S. social security system has a similar feature as old-age benefits are proportional to a measure of average earnings in the economy when the worker turns age 60, other things equal.

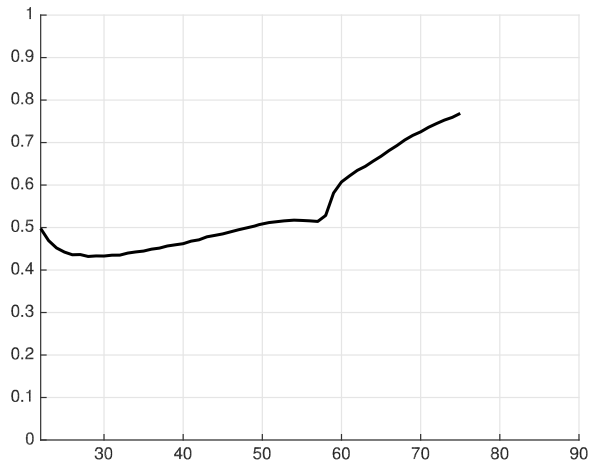
### 5.3. Portfolio allocation

Fig. 6 describes portfolio allocation based on financial wealth and on overall wealth (i.e. financial plus human wealth). Fig. 6(a)–(b) plots the average share of financial wealth in stock among those holding positive assets. The high-school agents have a lower average stock share than the college agents over most of the working lifetime.

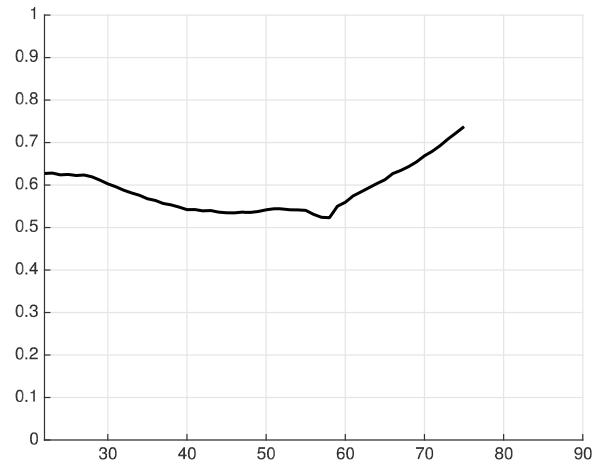
Fig. 6 provides two descriptions of portfolio allocation based on overall wealth. First, consider how overall wealth is split between stock, bonds and the value of human capital. For both the high school and college groups, human capital is the largest component of wealth on average at each age. This holds despite the fact that the value of human capital to the individual is far below the naive value over the working lifetime.

Fig. 6 also divides overall wealth into a stock, bond and an orthogonal value component. The stock component is then the sum of stock directly held in the financial wealth portfolio and the stock position embodied in the value of human capital

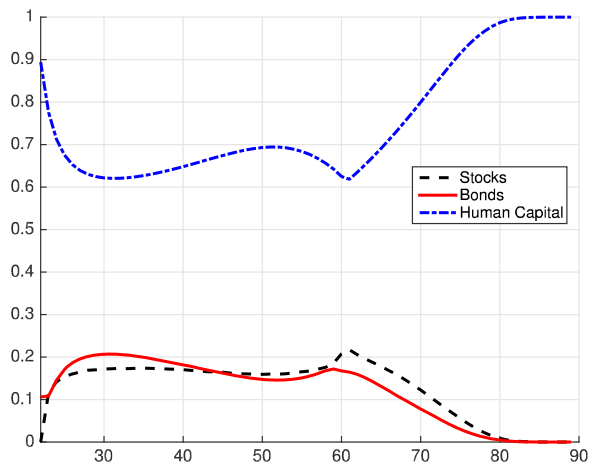




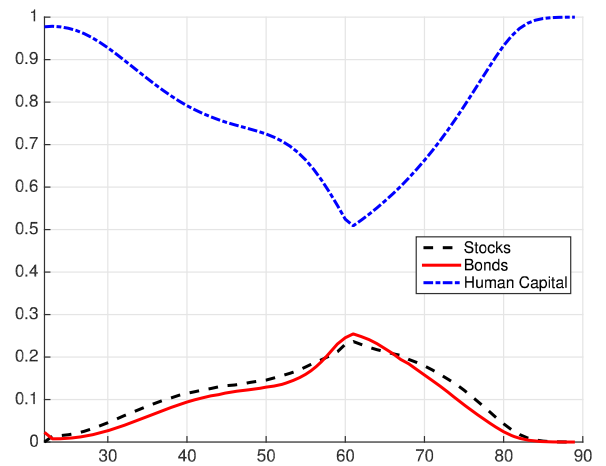
(a) Stock share of financial wealth (high school)



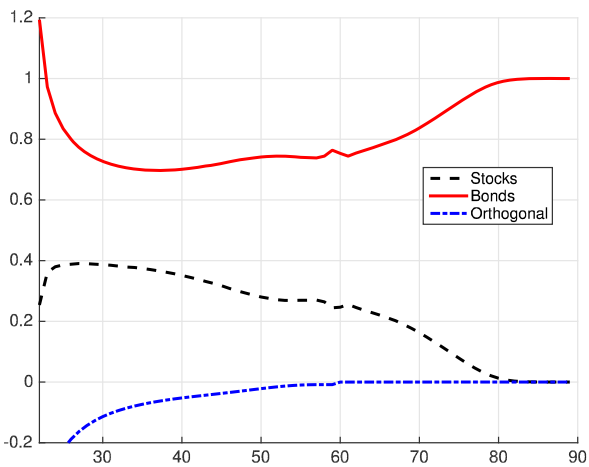
(b) Stock share of financial wealth (college)



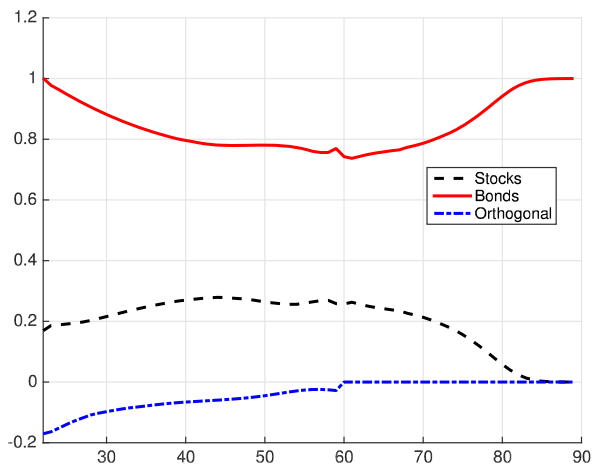
(c) Portfolio shares (high school)



(d) Portfolio shares (college)



(e) Overall portfolio shares (high school)



(f) Overall portfolio shares (college)

**Notes:** Financial portfolio shares in panels (a)–(b) are averages over the sub-population with positive asset holdings. All results are for risk aversion equal to 6.

**Fig. 6.** Portfolio shares in the benchmark model.

based on the analysis in section 5.1. We find that the bond share of overall wealth exceeds the stock share at all ages. The stock share for the college group averages between 20 to 30 percent over the working lifetime, whereas for high school it average between 20 to 40 percent over the working lifetime. The overall stock share early in life is largely determined by the decomposition analysis presented earlier in Fig. 4. This is because financial assets are small in value compared to the value of human capital and negative positions in either financial asset are not allowed.

It is perhaps useful to view the average stock shares in Fig. 6(e)–(f) from the perspective of the stock share formula  $\frac{1}{\alpha} \frac{E[R^*]}{\text{Var}(R^*)} \doteq \frac{1}{6} \frac{.05}{.04} \doteq .21$  holding in the Samuelson (1969) model. In this formula  $\alpha$  is the relative-risk-aversion coefficient and we plug in the mean and the variance of the excess return  $R^*$  on equity.<sup>23</sup> While background risk, correlated returns, multiple-period horizons and corner solutions are present in the benchmark model underlying Fig. 6 and invalidate this simple formula (see Gollier, 2002), it is interesting to see that average overall stock shares in Fig. 6 are not wildly at odds with this formula.

## 6. Discussion: robustness and drivers

This section determines the robustness and drivers of two findings: (1) the value of human capital is far below the naive value and (2) the stock component of human capital averages less than 35 percent of the value of human capital at all ages.

### 6.1. Value of human capital

What drives the value of human capital to be substantially below the naive value? To answer this question, we consider a number of perturbations of the benchmark model. For each perturbation, we recalculate human capital values and then plot the results. The benchmark model is the model in Table 4 that sets risk aversion to  $\alpha = 6$  and that sets earnings to the process without cointegration estimated for the full sample.

We first consider perturbations that help agents to better smooth consumption. Intuitively, such perturbations lessen the negative covariation between the stochastic discount factor and earnings. One perturbation starts agents off with an initial wealth of 1 times mean earnings at age 22 rather than the benchmark value of 30 percent of mean earnings. The other perturbations allow agents to hold negative balances in the risk-free asset up to 1.0 times mean earnings or up to the natural borrowing limit. Fig. 7 shows that while all perturbations increase the human capital value early in life, values remain well below the naive value.

Next we examine the extent to which transitory or persistent idiosyncratic shocks are key drivers. First, we eliminate transitory or persistent shocks. Fig. 7 shows that eliminating transitory shocks increases human capital values. However, the quantitative effects are small compared to the massive impact of eliminating persistent idiosyncratic shocks. Eliminating persistent shocks produces more than a tripling of the value of human capital early in life. Lastly, we impose that persistent shocks have no skewness and no cyclical variation in variance or skewness. Fig. 7 shows that this increases human capital values and that its effect is more powerful than eliminating transitory shocks. This foreshadows the importance of skewness as a driver of the stock component of human capital values that we find in the next subsection. We note that for each of these three changes we re-estimate the model with the new restrictions imposed.

We examine the effect of altering the preference parameter  $\gamma$ , while keeping relative risk aversion fixed. The value of  $1/\gamma$  controls the intertemporal elasticity of substitution (IES). Fig. 7 shows that increasing the IES to 2 increases the human capital value slightly, whereas decreasing the IES to 0.5 reduces the value of human capital. Neither change alters the finding that human capital values are substantially below naive values.

### 6.2. Stock share of human capital values

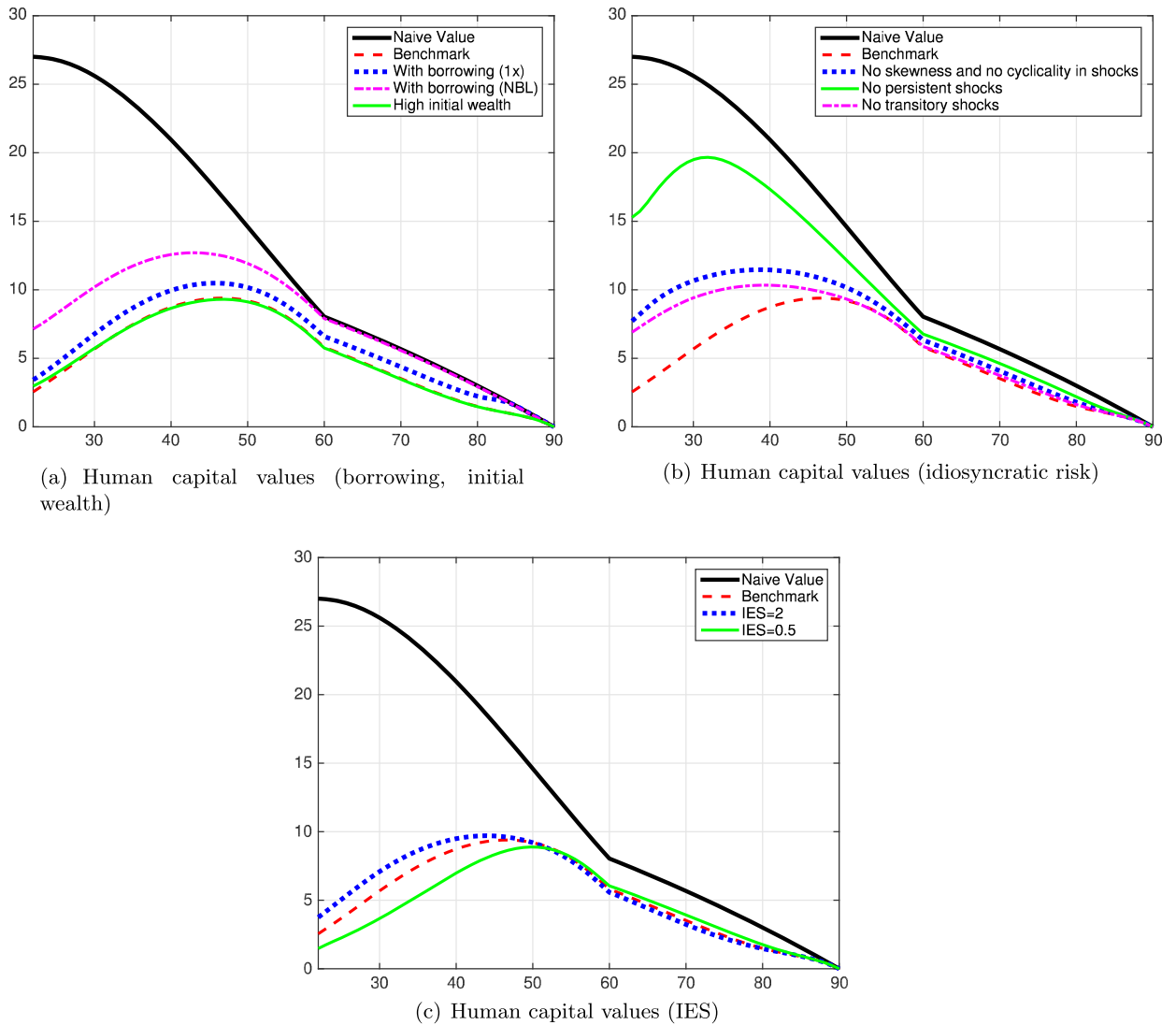
What drives the magnitude of the stock share of human capital values? We answer this question by analyzing four perturbations of the benchmark model. We (i) vary the borrowing constraint, (ii) vary the IES and/or risk aversion, (iii) eliminate the cyclical changes in persistent idiosyncratic shocks and/or eliminate the left-skewness in these shocks and (iv) allow cointegration.

Fig. 8 shows the results. Raising the IES to 2 lowers the stock component slightly whereas lowering the IES to 0.5 raises the stock share slightly. Allowing borrowing up to one times mean earnings has almost no visible impact on the stock share and we therefore do not display this result. Fig. 8(b) analyzes what happens when we increase risk aversion  $\alpha$  and reduce the IES measured by  $1/\gamma$  at the same time by setting  $\gamma = \alpha$ , effectively imposing CRRA preferences. The stock share rises as  $\alpha$  increases.

Next we eliminate skewness and/or eliminate the cyclical variation in shock distributions. The case of no skewness means that the generalized normal distribution analyzed in the benchmark model is replaced by the normal distribution. In all cases, we re-estimate the idiosyncratic shock process under the new restrictions to best match data moments.

Fig. 8 shows that eliminating left-skewness and eliminating cyclicity decreases the stock share of human capital values. This could be stated more positively if one took the benchmark model to be the model with no skewness and no cyclical

<sup>23</sup> The mean and variance are calculated, based on a lognormal distribution, using the mean and variance of log returns from US data in Table 3.



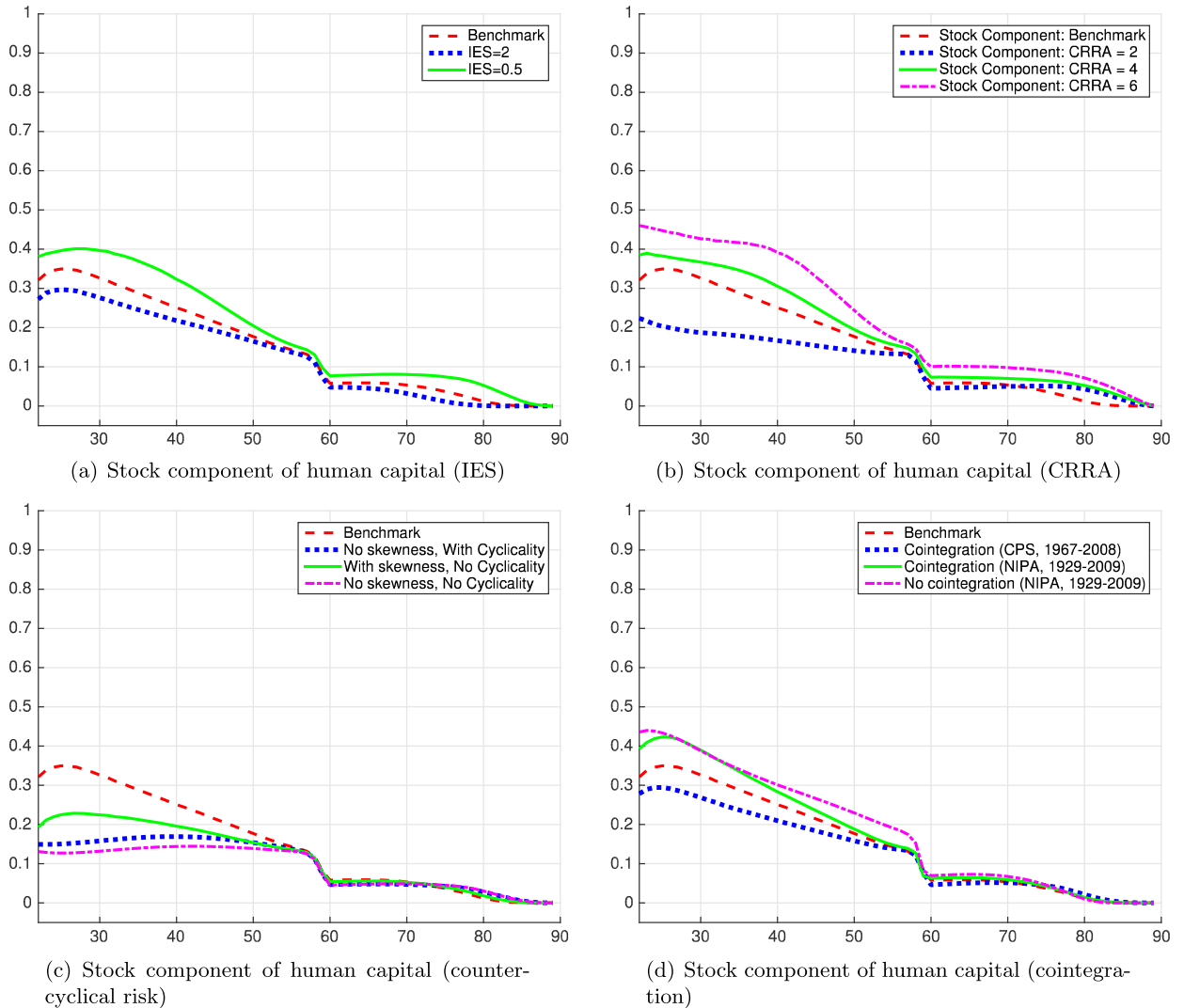
**Notes:** In panel (a) “With borrowing (1×)” refers to the model that allows borrowing up to 1 times average annual earnings, and “With borrowing (NBL)” refers to model that allows borrowing up to the “Natural Borrowing Limits” i.e. limits that impose only that the agent must be able to repay his debt in all states of the world.

**Fig. 7.** What drives the value of human capital?

changes in idiosyncratic shocks. Using that model as a benchmark, implies that allowing left-skewed shocks and allowing the distribution of such shocks to vary cyclically increases the stock share of human capital. The incremental effect of adding left-skewness is substantially larger than adding cyclical variation in a distribution displaying no skewness.<sup>24</sup> Thus, we find that skewness is a quantitatively important factor that increases the stock share of human capital values and lowers human capital values.

We now consider statistical models that allow the aggregate component of earnings to be cointegrated with stock returns. Appendix A.3 presents a time series model that allows for cointegration and that nests the benchmark time series model from section 3.2 as a special case. Table A.4 and Table A.5 present parameter estimates and steady state properties of the estimated model. Some intuition for why cointegration may matter is that over long horizons aggregate shock histories with high realizations for the aggregate component of earnings will be associated with histories of high stock returns. Correspondingly, low earnings histories will be associated with histories of low stock returns. Cointegration implies that two series do not drift apart. Such a process could lead an individual's earnings far into the future to take on stock-like features.

<sup>24</sup> Storesletten et al. (2007) and Lynch and Tan (2011) argue that adding counter-cyclical variation in the variance of idiosyncratic shocks, in a model without skewness, reduces the stock share of the financial asset portfolio.



**Notes:** When comparing different models estimated on the same data set or the same model estimated on different data sets, the constants in all models are reset so that  $E[\Delta u_t^c] = 0$  and  $E[\log R_t^c] = 0.041$  as previously noted in Table 3. The benchmark model is the model from Table 4 with  $\alpha = 6$  and without cointegration.

**Fig. 8.** What drives the stock component of human capital?

Fig. 8 addresses whether cointegration is important for the magnitude of the stock share of human wealth. First, we compare the benchmark model and the model allowing cointegration when both are estimated using CPS 1967–2008 data for the full sample. Allowing cointegration slightly decreases the stock share of human capital values. One might be skeptical that a cointegrated relationship can be precisely estimated over a short time period. For this reason, we repeat the analysis using an aggregate measure of earnings growth to proxy male earnings growth. We use NIPA 1929–2009 data on aggregate wages and salaries and divide this by the labor force to get average earnings. We re-estimate both models using data over the period 1929–2009.<sup>25</sup> Fig. 8 shows that the stock share increases somewhat using NIPA data when these estimated models are incorporated into the economic model. However, allowing cointegration does not significantly alter the stock share compared to the case of no cointegration, given the new data.

In summary, Fig. 8 shows that two changes to the benchmark model produce a larger stock share. These are to decrease the IES to values below 1 and to re-estimate the model using NIPA data over a longer time period. We also find that skewness is an important driver of the stock share. Abstracting from skewness implies a much smaller stock share of

<sup>25</sup> Table A.5 in Appendix A.3 presents steady state properties of these different models.

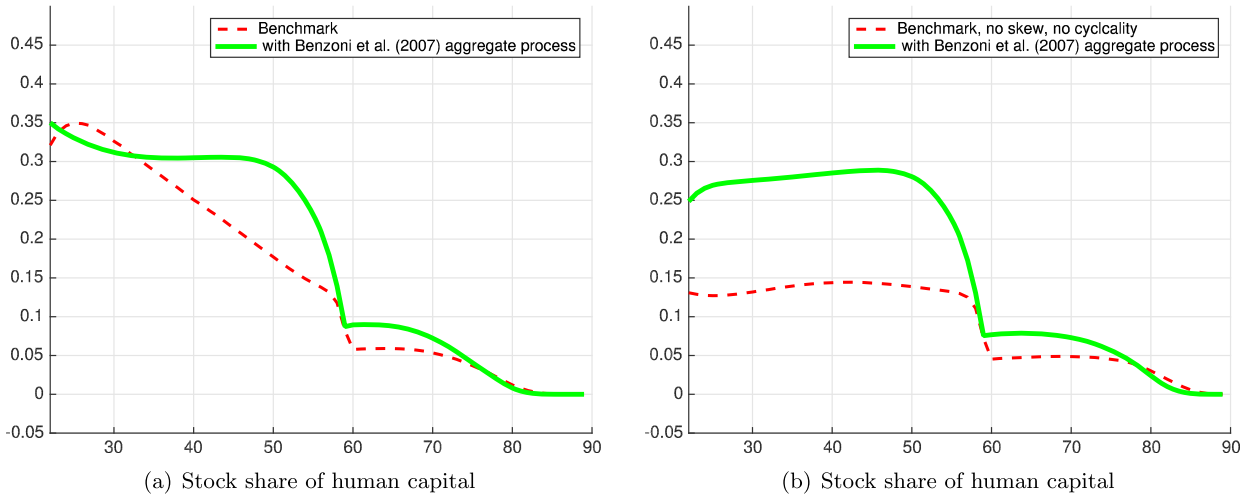


Fig. 9. Analysis of Benzoni et al. (2007).

human capital.<sup>26</sup> While one may conjecture that allowing cointegration may substantially increase the stock share of the value of human capital, we do not find support for this when the models are estimated using the same data set.

This last finding may seem surprising in light of the work of Benzoni et al. (2007). The main focus of their work is to examine how cointegration affects portfolio choice as a single parameter  $\kappa$  that controls the strength of adjustment in the cointegrating relationship increases. Stock holding in the financial asset portfolio is zero early in life in their model for a sufficiently large value of  $\kappa$ , when the relative-risk aversion coefficient is set to equal 5. For the parameter configuration that they highlight, the stock component of the value of human capital is 50 percent at age 20 and remains at 50 percent throughout the first half of the working lifetime.

Our model differs from their work in a number of dimensions. For example, we model social security benefits, allow idiosyncratic shocks to be drawn from a skewed distribution and allow for cyclical changes in idiosyncratic shocks. They abstract from all of these features. In addition, the methodology differs. While we estimate the idiosyncratic and aggregate shock structure, they roughly calibrate a cointegrated aggregate process governing stock returns and the aggregate component of earnings.

To make contact with Benzoni et al. (2007), we simply take the calibrated aggregate process from their work and substitute it into our model. Their calibrated process has the discrete-time approximation of the form indicated below. We take as given their parameter value for  $\kappa = 0.15$  and their values governing the variance-covariance structure of the shock terms  $(\epsilon_1, \epsilon_2, \epsilon_3)$ . We adjust the constant terms  $(\gamma_1, \gamma_2)$  so that  $E[\log R] = .041$  and  $E[\Delta u] = 0$  in all models.<sup>27</sup>

$$\begin{pmatrix} \Delta u_{t+1} \\ \log R_{t+1} \\ w_{t+1} \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -\kappa \\ 0 & 0 & 0 \\ 0 & 0 & 1 - \kappa \end{pmatrix} \begin{pmatrix} \Delta u_t \\ \log R_t \\ w_t \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t+1} \\ \epsilon_{2,t+1} \\ \epsilon_{3,t+1} \end{pmatrix} \quad (4)$$

Fig. 9(a) compares the benchmark model (without cointegration) from this section to the model that results from replacing the aggregate dynamics with the Benzoni et al. (2007) process featuring cointegration. When inserted into our framework, the Benzoni et al. (2007) process produces a stock share that is well below the 50 percent value that they highlight.

Fig. 9(b) considers a different comparison. The benchmark model is now modified to exclude skewness and cyclical variation in idiosyncratic risk, but is still estimated to best match data. We then insert the Benzoni et al. (2007) process into this model. Fig. 9(b) shows that once again both models produce a stock share that is well below 50 percent over the lifetime. The results in Fig. 9 are essentially unchanged if the mean log earnings growth rate is increased to equal 1 percent in all the models rather than the benchmark value of zero.

<sup>26</sup> One might conjecture that increasing the mean growth rate  $E[\Delta u_t^*]$  from the sample average of zero might matter. Increasing this mean from zero to 1 or 2 percent substantially increases individual human capital values while leaving the stock share essentially unchanged in the benchmark model.

<sup>27</sup> Appendix A.3 shows how we go from their continuous-time formulation to the discrete-time model and computes steady-state properties. The Benzoni et al. (2007) process produces an unconditional standard deviation of  $SD(\Delta u_t) = 0.069$  which is more than twice the values that we calculate in US earnings data as documented in Table A.5 and Table A.6.

## 7. Conclusion

Our analysis highlights two main properties of human capital values based on an analysis of US data on males earnings and financial asset returns: (1) the value of human capital is far below the value implied by discounting future earnings at the risk-free rate and (2) the stock component of the value of human capital averages less than 35 percent at each age over the working lifetime. These properties hold for (i) different educational groups, (ii) a wide range of parameters characterizing risk aversion and intertemporal substitution, (iii) a range of assumptions on borrowing constraints and (iv) two different statistical models for earnings estimated using male earnings data.

We investigate the main drivers of these two findings. Persistent idiosyncratic shocks and the left skewness of idiosyncratic shocks are two key drivers of low human capital values. In our model framework, an agent's stochastic discount factor falls for larger realizations of idiosyncratic shocks, other things equal. A number of model features lead to a positive stock component of the value of human capital including (i) social security benefits linked to average earnings, (ii) positive conditional correlation between the aggregate component of earnings and stock returns and (iii) left-skewed idiosyncratic earnings shocks. We provide support for these features in US data. We do not find much support in US data for the idea that cointegration between the aggregate component of earnings and stock returns is a key factor driving the size of the stock share of the value of human capital.

## Appendix A

### A.1. Model fit

The fit of the earnings model for the high school and college samples is provided in Figs. 10 and 11 respectively.

### A.2. Computation

This section describes our methods to compute solutions to the benchmark model and to compute values and returns.

#### A.2.1. Value function and decision rules

To compute the optimal value function  $V_j^*$  and optimal decision rules to the model in section 4, we employ the method of dynamic programming. This involves computing functions  $V_j$  solving the Bellman equation (BE). Of course, the idea is that  $V_j = V_j^*$ . In stating  $\hat{\Gamma}_j(x, z)$  in Bellman's equation, we impose the restrictions from the original budget constraint  $\Gamma_j(x, z)$ . We also use the fact that shocks are Markovian so that the current shock, denoted  $z$ , rather than partial histories  $z^j$  contain all relevant information. We model the shock  $z = (z_1, z_2)$  as stated in section 4.

$$\begin{aligned} V_j^*(x, z) &\equiv \max W(c_j, F(U(c_{j+1}, \dots, c_j)), j) \text{ s.t. } c \in \Gamma_j(x, z) \\ (\text{BE}) \quad V_j(x, z) &= \max W(c_j, F(V_{j+1}(x', z')), j) \text{ s.t. } (c, a^1, a^2) \in \hat{\Gamma}_j(x, z) \\ \hat{\Gamma}_j(x, z) &= \{(c, a^1, a^2) : c + \sum_{i \in \mathcal{I}} a^i \leq x, c \geq 0, a^1, a^2 \geq 0\} \end{aligned}$$

We compute solutions to Bellman's equation only when the first component of the shock  $z = (z_1, z_2)$  takes the value  $z_1 = 1$ . This is indicated below. To do so requires knowledge of  $V_{j+1}(x', z'_1, z'_2)$  at all values of  $z'_1$ . Lemma 1 below shows that  $V_j^*(\lambda x, \lambda z_1, z_2) = \lambda V_j^*(x, z_1, z_2)$ ,  $\forall \lambda > 0$  and therefore  $V_j^*(x, z_1, z_2) = z_1 V_j^*(\frac{x}{z_1}, 1, z_2)$ . In the Algorithm described below, we make use of this key property. In Lemma 1,  $\Gamma(x, z)$  is homogeneous provided  $c \in \Gamma(x, z) \Rightarrow \lambda c \in \Gamma(\lambda x, \lambda z)$ ,  $\forall \lambda > 0$ .

$$V_j(x, 1, z_2) = \max_{(c, a^1, a^2) \in \hat{\Gamma}_j(x, 1, z_2)} W(c_j, F(V_{j+1}(x', z'_1, z'_2)), j)$$

#### Lemma 1.

- (i) Assume  $U$  is homothetic and  $\Gamma(x, z)$  is homogeneous.  $c^* \in \operatorname{argmax} \{U(c) : c \in \Gamma(x, z)\}$  implies  $\lambda c^* \in \operatorname{argmax} \{U(c) : c \in \Gamma(\lambda x, \lambda z)\}$ ,  $\forall \lambda > 0$ .
- (ii) In the benchmark model  $V_j^*(\lambda x, \lambda z_1, z_2) = \lambda V_j^*(x, z_1, z_2)$ ,  $\forall \lambda > 0$

#### Proof. (i) Obvious

(ii) Follows from Lemma 1(i) after noting two things. First, EZ preferences are homothetic and, in fact, homogeneous of degree 1. Second,  $\Gamma_j(x, z)$  is homogeneous in  $(x, z_1)$  for any fixed  $z_2$ . This is implied because the earnings function from the benchmark model is  $e_j = z_1 g_j(z_2)$  and  $z'_1 = z_1 f_{j+1}(z'_2)$ , where  $z_2$  is Markov and primes denote next period values. These two properties hold both for the model with and without cointegration.  $\square$

The Lagrange function corresponding to (BE) is stated below along with first-order conditions.

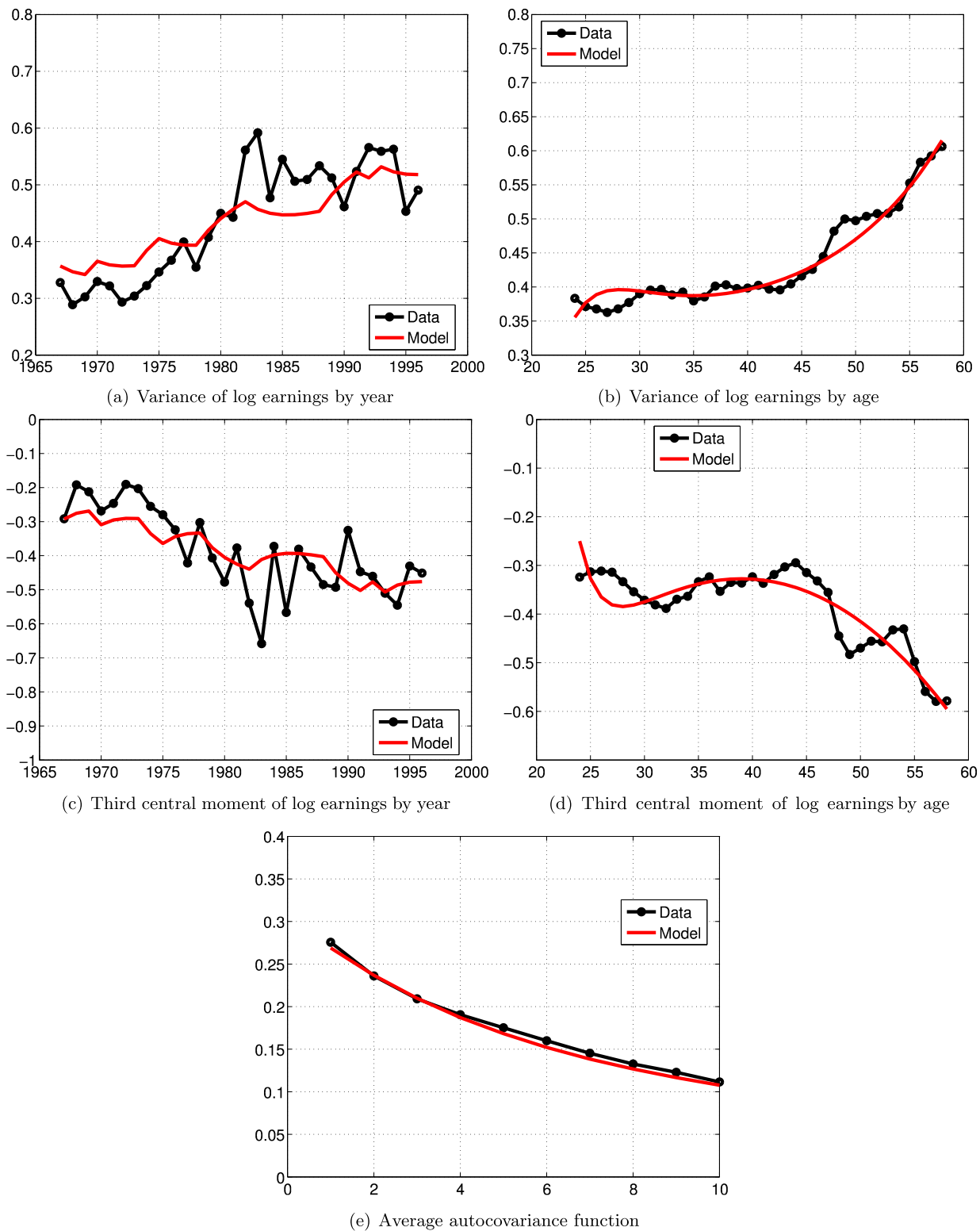


Fig. 10. Fit of estimated idiosyncratic earnings model for High School sample.



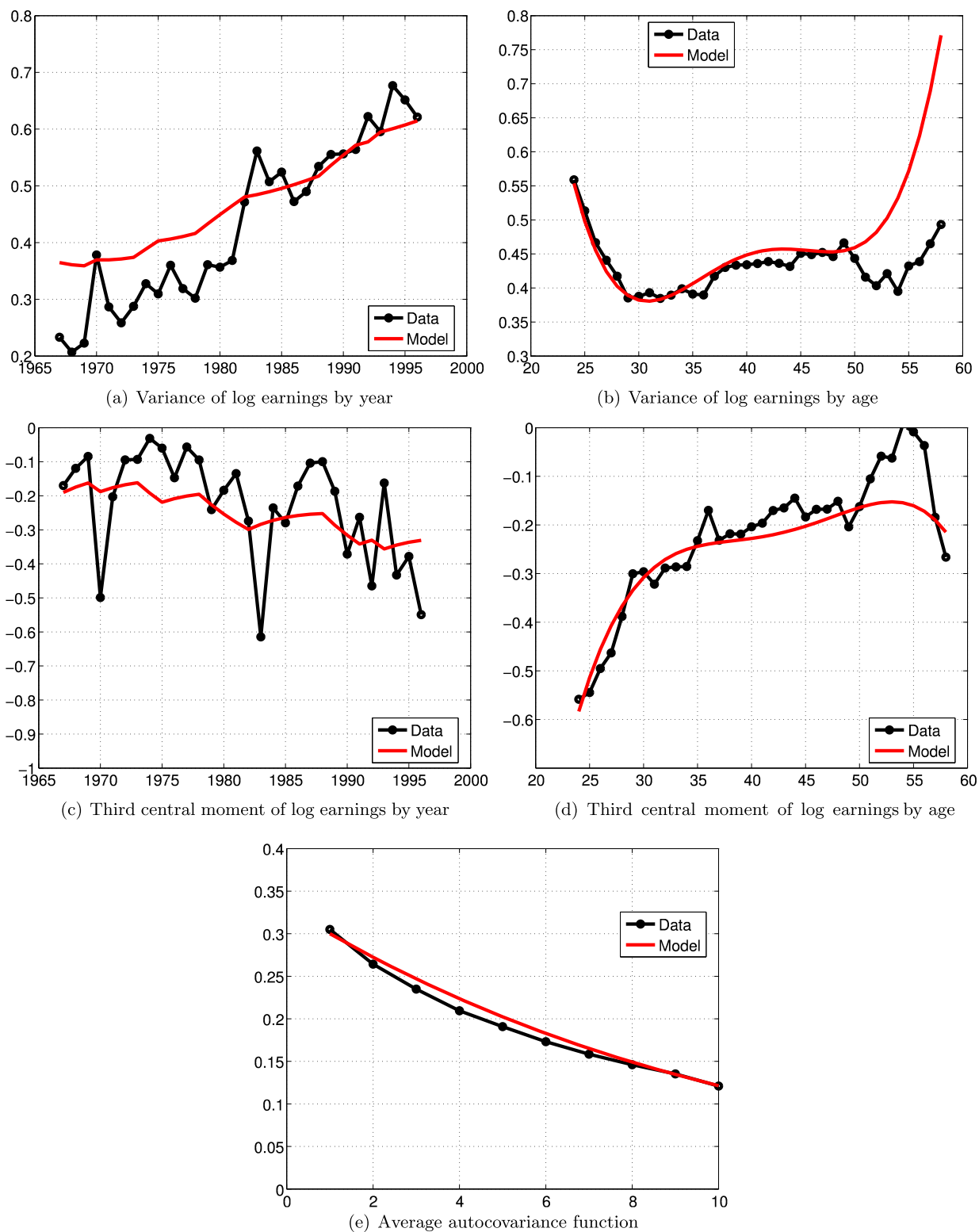


Fig. 11. Fit of estimated idiosyncratic earnings model for College sample.

$$\mathcal{L} = W(x - \sum_{i=1}^2 a^i, F(V_{j+1}(x', z'_1, z'_2)), j) + \lambda_1[a^1 - 0] + \lambda_2[a^2 - 0]$$

- (1)  $-W_1 + W_2 dF/da^1 + \lambda_1 = 0$
- (2)  $-W_1 + W_2 dF/da^2 + \lambda_2 = 0$
- (3) *constraints + complementary slackness*

We rewrite equations (1)–(2) below after imposing the functional forms from section 4. The Algorithm is then based on repeatedly solving these Euler equations.

$$(1') -1 + \beta \psi_{j+1} E[(\frac{c_{j+1}}{c_j})^{-\gamma} (\frac{V_{j+1}}{F(V_{j+1})})^{\gamma-\alpha} R^1(z') | x, z] + \lambda'_1 = 0$$

$$(2') -1 + \beta \psi_{j+1} E[(\frac{c_{j+1}}{c_j})^{-\gamma} (\frac{V_{j+1}}{F(V_{j+1})})^{\gamma-\alpha} R^2(z') | x, z] + \lambda'_2 = 0$$

#### Algorithm:

1. Set  $V_j(x, 1, z_2) = W(x, 0)$  and  $c_j(x, 1, z_2) = x$  at grid points  $(x, z_2)$ .
2. Given  $(V_{j+1}(x, 1, z_2), c_{j+1}(x, 1, z_2))$ , compute  $(a_{j+1}^1(x, 1, z_2), a_{j+1}^2(x, 1, z_2))$  at grid points  $(x, 1, z_2)$  by solving (1')–(2') and (3).
3. Set  $c_j(x, 1, z_2) = x - \sum_i a_{j+1}^i(x, 1, z_2)$  and  $V_j(x, 1, z_2) = W(c_j(x, 1, z_2), F(V_{j+1}), j)$  at grid points.
4. Repeat 2–3 for successive lower ages.

To carry out this Algorithm we mention two points. First, evaluating (1')–(2') involves an interpolation of the first component of the functions  $(V_{j+1}, c_{j+1})$ . Second, evaluating (1')–(2') also involves knowledge of  $(V_{j+1}, c_{j+1})$  when the second component of these functions differs from  $z_1 = 1$ . This is accomplished by using Lemma 1 as indicated below.

$$V_{j+1}(x', z'_1, z'_2) = z'_1 V_{j+1}(\frac{x'}{z'_1}, 1, z'_2) \text{ and } c_{j+1}(x', z'_1, z'_2) = z'_1 c_{j+1}(\frac{x'}{z'_1}, 1, z'_2)$$

$$x' = \sum_i a_{j+1}^i(x, 1, z_2) R^i(z') + e_{j+1}(z')$$

#### A.2.2. Human capital values and returns

We describe how to compute human capital values and returns. Let  $(v_j(x, z), R_{j+1}(x, z, z'))$  denote the value and the return to human capital. These functions are recursive versions of the values and returns defined in section 2. Human capital values  $v_j(x, z)$  follow the recursion (\*\*), given  $v_j(x, z) = 0$ :

$$(**) v_j(x, z) = E[m_{j+1}(x, z, z')(v_{j+1}(x', z') + e_{j+1}(z')) | z]$$

$$R_{j+1}(x, z, z') = \frac{v_{j+1}(x', z') + e_{j+1}(z')}{v_j(x, z)}$$

$$m_{j+1}(x, z, z') = \beta \psi_{j+1} (\frac{c_{j+1}(x', z')}{c_j(x, z)})^{-\gamma} (\frac{V_{j+1}(x', z')}{F(V_{j+1}(x', z'))})^{\gamma-\alpha}$$

$$x' = \sum_i a_{j+1}^i(x, z) R^i(z') + e_{j+1}(z')$$

Although the recursive structure above is a step in the right direction, it is not practical to implement because the aggregate component of earnings  $z_1$  “fans out” over time in the benchmark model. Instead, we compute the functions  $(\hat{v}_j, \hat{m}_{j+1})$  defined below and then use Lemma 2 to compute values and returns.  $\hat{v}_j$  is defined recursively, given  $\hat{v}_j = 0$ . To compute  $(\hat{v}_j, \hat{m}_{j+1})$ , we require as inputs the functions  $(c_j, a_{j+1}^1, a_{j+1}^2, V_j)$  from the previous sections computed on the restricted domain. In what follows, we write earnings as  $e_j(z) = z_1 g_j(z_2)$  and use the fact that  $z'_1 = z_1 f_{j+1}(z'_2)$  which is consistent with the model from section 4. In retirement  $z'_1 = z_1 f_{j+1}(z'_2) = z_1$ .

$$\hat{v}_j(\hat{x}, z_2) = E[\hat{m}_{j+1}(\hat{x}, z_2, z'_2) f_{j+1}(z'_2) (\hat{v}_{j+1}(\hat{x}', z'_2) + g_{j+1}(z'_2)) | z_2]$$

$$\hat{m}_{j+1}(\hat{x}, z_2, z'_2) \equiv \beta \psi_{j+1} (\frac{f_{j+1}(z'_2) c_{j+1}(\hat{x}', 1, z'_2)}{c_j(\hat{x}, 1, z_2)})^{-\gamma} (\frac{f_{j+1}(z'_2) V_{j+1}(\hat{x}', 1, z'_2)}{F(f_{j+1}(z'_2) V_{j+1}(\hat{x}', 1, z'_2))})^{\gamma-\alpha}$$

$$\hat{x}' \equiv \frac{\sum_i a_{j+1}^i(\hat{x}, 1, z_2) R^i(z'_2) + f_{j+1}(z'_2) g_{j+1}(z'_2)}{f_{j+1}(z'_2)}$$

**Lemma 2** says that the value of human capital is proportional to  $z_1$  other things equal and after correcting for financial asset holdings. It also says that the stochastic discount factor and the return to human capital are independent of the level of  $z_1$ , after correcting for financial asset holdings. **Lemma 2** and the associated formulas allow the computation of statistics of  $(v_j, R_j)$  over the lifetime by means of simulating lifetime draws of  $z_2$  shocks and using  $\hat{v}_j$  and the computed decision rules.

**Lemma 2.** In the benchmark model the following hold when  $\hat{x} = x/z_1$ :

- (i)  $m_{j+1}(x, z, z') = \hat{m}_{j+1}(\hat{x}, z_2, z'_2)$
- (ii)  $v_j(x, z) = z_1 v_j(\hat{x}, 1, z_2) = z_1 \hat{v}_j(\hat{x}, z_2)$
- (iii)  $R_{j+1}(x, z, z') = \frac{f_{j+1}(z'_2)(\hat{v}_{j+1}(\hat{x}', z'_2) + g_{j+1}(z'_2))}{\hat{v}_j(\hat{x}, z_2)}$ .

**Proof.** (i) The result follows from direct substitution of the consumption and the value function into the definition of  $m_{j+1}(x, z, z')$ . Here we use **Lemma 1** so that  $c_j(x, z) = z_1 c_j(\frac{x}{z_1}, 1, z_2)$  and  $V_{j+1}(x', z') = z'_1 V_{j+1}(\frac{x'}{z'_1}, 1, z'_2)$ . We also use the fact that  $z'_1 = z_1 f_{j+1}(z'_2)$  and that  $F$  is homogeneous of degree 1.

(ii) **Lemma 2**(ii) holds trivially for  $j = J$ . We show it holds for  $j$  given it holds for  $j + 1$ . The first line below uses the definition and the induction hypothesis. The leftmost equality in the second line follows from the first line, **Lemma 2**(i) and the induction hypothesis. The rightmost equality follows from the definition of  $\hat{v}_j$ .

$$\begin{aligned} v_j(x, z) &= E[m_{j+1}(x, z, z')(z'_1 v_{j+1}(\hat{x}', 1, z'_2) + z'_1 g_{j+1}(z'_2)) | z] \\ v_j(x, z) &= E[\hat{m}_{j+1}(\hat{x}, z_2, z'_2)(z'_1 \hat{v}_{j+1}(\hat{x}', z'_2) + z'_1 g_{j+1}(z'_2)) | z] = z_1 \hat{v}_j(\hat{x}, z_2) \end{aligned}$$

(iii) The first line follows from the definition, **Lemma 2**(ii) and the structure of earnings. The second line follows from the first and  $z'_1 = z_1 f_{j+1}(z'_2)$ .

$$\begin{aligned} R_{j+1}(x, z, z') &= \frac{v_{j+1}(x', z') + e_{j+1}(z')}{v_j(x, z)} = \frac{z'_1 \hat{v}_{j+1}(\hat{x}', z'_2) + z'_1 g_{j+1}(z'_2)}{z_1 \hat{v}_j(\hat{x}, z_2)} \\ R_{j+1}(x, z, z') &= \frac{f_{j+1}(z'_2)(\hat{v}_{j+1}(\hat{x}', z'_2) + g_{j+1}(z'_2))}{\hat{v}_j(\hat{x}, z_2)} \quad \square \end{aligned}$$

An algorithm to compute the naive value  $v_j^n(z)$  is provided. First, we list some useful points from theory, where  $z = (z_1, z_2)$ ,  $e_j(z) = z_1 g_j(z_2)$  and  $z'_1 = z_1 f_{j+1}(z'_2)$ . The first two equations are Bellman equations. The third equation is an implication of theory. It follows from the first equation by backwards induction and substituting in for earnings.

$$\begin{aligned} v_j^n(z) &\equiv E[\frac{1}{1+r}(v_{j+1}^n(z') + e_{j+1}(z')) | z] \\ \hat{v}_j^n(z_2) &\equiv E[\frac{f_{j+1}(z'_2)}{1+r}(\hat{v}_{j+1}^n(z'_2) + g_{j+1}(z'_2)) | z_2] \\ v_j^n(z) &= z_1 \hat{v}_j^n(z_2) \end{aligned}$$

The algorithm is as follows. Step 1: compute the functions  $\hat{v}_j^n(z_2)$  by iterating on Bellman's equation. Step 2: simulate histories of  $z_2$  shocks. Step 3: compute  $z_1$  histories using step 2 and  $z'_1 = z_1 f_{j+1}(z'_2)$ . Step 4: compute histories  $v_j^n(z)$  using (i)  $v_j^n(z) = z_1 \hat{v}_j^n(z_2)$ , (ii)  $\hat{v}_j^n(z_2)$  from step 1, (iii) shock histories from steps 2–3.

### A.2.3. Decomposing human capital values

We decompose the value of human capital into a bond, a stock and a residual value component. We then calculate the bond and stock shares of human capital at different ages and states. To do so, apply the Projection Theorem to the payout  $y = v_{j+1}(x', z') + e_{j+1}(z')$ . By construction, the residual  $\epsilon \equiv y - \alpha^b R^b + \alpha^s R^s$  is orthogonal to each gross asset return. We calculate  $(\alpha^s, \alpha^b)$  by solving the system  $E[\epsilon R^s] = 0$  and  $E[\epsilon R^b] = 0$ . The expectations operator  $E$  is conditional on  $(x, z)$ .

$$\begin{aligned} v_j(x, z) &= E[m_{j+1} y] = E[m_{j+1}(\alpha^b R^b + \alpha^s R^s + \epsilon)] \\ v_j(x, z) &= \alpha^b E[m_{j+1} R^b] + \alpha^s E[m_{j+1} R^s] + E[m_{j+1} \epsilon] \\ \text{share}_j^i(x, z) &\equiv \frac{\alpha_j^i(x, z) E[m_{j+1} R^i]}{v_j(x, z)} \text{ for } i = s, b \end{aligned}$$

**Lemma 3** below is useful in theory and computation. It says that  $\text{share}_j^i(x, z)$  is invariant to scaling up or down  $(x, z_1)$ . Thus, shares can be computed for a single value  $z_1 = 1$  to determine the share decomposition for all  $z_1$  values.

**Table A.1**  
Model moments.

	Full sample	College sample	High School sample
<b>Idiosyncratic component</b>			
Variance persistent component: true	0.340	0.339	0.193
Variance persistent component: discretized	0.324	0.329	0.184
Skewness persistent component: true	−1.14	−0.738	−4.73
Skewness persistent component: discretized	−1.20	−0.815	−4.43
<b>Aggregate component</b>			
Variance aggregate earnings growth: true	0.00632	0.00539	0.00918
Variance aggregate earnings growth: discretized	0.00657	0.00560	0.00953
Variance stock returns: true	0.0349	0.0351	0.0350
Variance stock returns: discretized	0.0333	0.0336	0.0334

**Lemma 3.** In the benchmark model the following holds for  $i = s, b$ :

$$share_j^i(\lambda x, \lambda z_1, z_2) = share_j^i(x, z_1, z_2), \forall \lambda > 0$$

**Proof.** The first line is the definition of the share. The second line uses Lemma 1 and the fact that the solution  $(\alpha^b, \alpha^s)$  to the linear system scales linearly in  $(x, z_1)$ . This latter fact holds as the payout scales linearly in  $(x, z_1)$ . To show this, write the payoff:  $y = v_{j+1}(x', z_1 f_{j+1}(z_2'), z_2') + z_1 f_{j+1}(z_2') g_{j+1}(z_2')$ . The payoff scales in  $(x, z_1)$  because  $v_{j+1}$  scales in its first two components (Lemma 2(ii)) and  $x'$  scales in  $(x, z_1)$ . Lemma 3 then follows if  $E[m_{j+1} R^i | \lambda x, \lambda z_1, z_2]$  is constant in  $\lambda$ . This holds because financial asset returns  $R^i$  depend only on  $z_2'$ ,  $z_2$  is Markov and  $m_{j+1}$  is homogeneous of degree zero in  $(x, z_1)$  by Lemma 2(i).

$$\begin{aligned} share_j^i(\lambda x, \lambda z_1, z_2) &= \frac{\alpha_j^i(\lambda x, \lambda z_1, z_2) E[m_{j+1} R^i | \lambda x, \lambda z_1, z_2]}{v_j(\lambda x, \lambda z_1, z_2)} \\ share_j^i(\lambda x, \lambda z_1, z_2) &= \frac{\lambda \alpha_j^i(x, z_1, z_2) E[m_{j+1} R^i | \lambda x, \lambda z_1, z_2]}{\lambda v_j(x, z)} \\ share_j^i(\lambda x, \lambda z_1, z_2) &= \frac{\alpha_j^i(x, z_1, z_2) E[m_{j+1} R^i | \lambda x, \lambda z_1, z_2]}{v_j(x, z)} \quad \square \end{aligned}$$

#### A.2.4. Discretization of stochastic processes

For computation, we discretize the idiosyncratic component of earnings, the aggregate component of earnings and stock returns. We construct our discrete approximations to the estimated stochastic processes as follows.

For the idiosyncratic earnings process, both the persistent and transitory innovations are drawn from a three-parameter Generalized Normal distribution. The (inverse) map from the mean, variance and third central moment to the parameters of the Generalized Normal is unique. Thus, we set model parameters consistent with the values in Table 1 as indicated in Table 4. For a given number of grid points, we choose the upper and lower bounds and the (non-linear) spacing of the grid points for each component to minimize the distance between the average variance and average skewness over the lifecycle of the discretized process, and the corresponding moments from the estimated continuous process, excluding the estimated trend. We fill in transition probabilities by assigning to each grid point the mass implied by the continuous Generalized Normal distribution between the mid-points of adjacent grid points. We construct separate discretizations for booms and recessions. We use 7 grid points for the transitory component, 11 grid points for the persistent component and 3 grid points for the fixed effect. Table A.1 reports moments from the continuous and discretized persistent processes for the baseline model. The moments for the transitory process are matched exactly, for each age, by construction.

For the aggregate earnings process, we assume that innovations are drawn from a joint normal distribution. We use equally spaced grid points in each dimension where the upper and lower bounds are a constant multiple of the unconditional variance in the respective dimensions. We choose this multiple to minimize the distance between the second moments of the underlying estimated continuous process and the discretized process. We use 4 grid points for aggregate earnings growth, 5 grid points for equity returns and in the models that include cointegration we include 3 grid points for the cointegrating vector. Table A.1 reports moments from the continuous and discretized processes for the baseline model.

We impose a constant lower bound on the possible realizations of the combined process for total earnings, in order to minimize numerical inaccuracies that result from very low probability low earnings realizations. We set this minimum at 5% of average earnings. The minimum does not bind except for the high school sample at young ages where there is large amount of negative skewness in persistent shocks.

### A.3. Time series model

#### A.3.1. Full description of stochastic model for aggregate variables

We assume the following general VAR model for  $y_t = (u_t^1 \ P_t)$ :

$$y_t = v(t) + \sum_{i=1}^p A_i y_{t-i} + \varepsilon_t$$

where  $\varepsilon_t$  is a vector of mean zero IID random variables with covariance matrix  $\Sigma$ .  $v(t)$  is a quadratic time trend which is parameterized below. We restrict attention to values of  $p \leq 2$  to keep the state space manageable. This model has a general VECM form given by

$$\Delta y_t = v + \delta t + \alpha \beta' y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t$$

where the vector  $\beta$  is known as the cointegrating vector. We can split the constant and trend terms into components as follows

$$v = \alpha \mu + \gamma$$

$$\delta t = \alpha \rho t + \tau t$$

where  $\gamma' \alpha \mu = 0$  and  $\tau' \alpha \rho = 0$ . In this case the VECM model can be written as

$$\Delta y_t = \gamma + \tau t + \alpha (\beta' y_{t-1} + \mu + \rho t) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \varepsilon_t$$

It is useful to define the one-dimensional object  $w_t = \beta' y_t + \mu + \rho(t+1)$  which, in the case of cointegration ( $\alpha \neq 0$ ), is a stationary random variable. By construction,  $w_t$  evolves as

$$w_t = w_{t-1} + \beta' \Delta y_t + \rho$$

Since we would like a system where  $w_t$  evolves based on variables at  $t-1$  or earlier, we can re-write this as

$$\begin{aligned} w_t &= w_{t-1} + \beta' \gamma + \beta' \tau t + \beta' \alpha w_{t-1} + \sum_{i=1}^{p-1} \beta' \Gamma_i \Delta y_{t-i} + \beta' \varepsilon_t \\ &= (\beta' \gamma + \rho) + \beta' \tau t + (1 + \beta' \alpha) w_{t-1} + \sum_{i=1}^{p-1} \beta' \Gamma_i \Delta y_{t-i} + \beta' \varepsilon_t \end{aligned}$$

In all of our analyses we assume that  $\tau = 0$ . The model for  $p = 2$  can then be written as

$$\begin{pmatrix} \Delta y_t \\ w_t \end{pmatrix} = \begin{pmatrix} \gamma \\ \beta' \gamma + \rho \end{pmatrix} + \begin{pmatrix} \Gamma & \alpha \\ \beta' \Gamma & 1 + \beta' \alpha \end{pmatrix} \begin{pmatrix} \Delta y_{t-1} \\ w_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t \\ \beta' \varepsilon_t \end{pmatrix}$$

We adopt the [Johansen \(1995\)](#) normalization for  $\beta$ , which implies for the two variable case that we can write

$$\Delta w_t = \Delta u_t^1 + \beta_2 \log R_t^s + \rho$$

The model with  $p = 2$  becomes

$$\begin{pmatrix} \Delta u_t^1 \\ \log R_t^s \\ w_t \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_1 + \beta_2 \gamma_2 + \rho \end{pmatrix} + \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \alpha_1 \\ \Gamma_{21} & \Gamma_{22} & \alpha_2 \\ \Gamma_{11} + \beta_2 \Gamma_{21} & \Gamma_{12} + \beta_2 \Gamma_{22} & 1 + \alpha_1 + \beta_2 \alpha_2 \end{pmatrix} \begin{pmatrix} \Delta u_{t-1}^1 \\ \log R_{t-1}^s \\ w_{t-1} \end{pmatrix} \quad (5)$$

$$+ \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \\ \varepsilon_{1t} + \beta_2 \varepsilon_{2t} \end{pmatrix} \quad (6)$$

When cointegration is not allowed (i.e.  $\alpha_1 = \alpha_2 = 0$  is imposed), then the model reduces to the benchmark model from section 2.3.

**Table A.2**

Lag-order selection tests for VAR.

Lag	P-value	FPE	AIC	HQIC	SBIC
<u>Full sample</u>					
0		.001762	−0.666	−0.635	−0.579
1	0.000	.000020	−5.169	−5.077	−4.911
2	0.004*	.000016*	−5.370*	−5.217*	−4.939*
3	0.160	.000018	−5.333	−5.118	−4.729
<u>College sub-sample</u>					
0		.001882	−0.600	−0.569	−0.514
1	0.000	.000015	−5.422	−5.330*	−5.163*
2	0.078*	.000015*	−5.432*	−5.279	−5.001
3	0.374	.000018	−5.248	−4.973	−4.744
<u>High School sub-sample</u>					
0		.003469	0.012	0.042	0.098
1	0.000	.000028	−4.801	−4.709	−4.543
2	0.014*	.000025*	−4.919*	−4.766*	−4.488*
3	0.132	.000028	−4.837	−4.561	−4.257

Notes: Lag order selected by each criteria is denoted by \*. P-values are from likelihood ratio tests of the null that true lag length is  $p - 1$  or less.

**Table A.3**

Cointegration rank selection tests.

Maximum rank	Trace statistic	5% critical value	Eigenvalue	SBIC	HQIC
<u>Full sample</u>					
0	9.95*	15.41	0.220*	−5.05*	−5.21
1	0.00	3.76	0.000	−5.02	−5.26*
<u>College sub-sample</u>					
0	9.49*	15.41	0.200*	−5.08*	−5.247
1	0.55	3.76	0.014	−5.03	−5.274*
<u>High School sub-sample</u>					
0	9.91*	15.41	0.218*	−4.66*	−4.824
1	0.07	3.76	0.002	−4.63	−4.875*

Notes: Rank of cointegration by each criteria is denoted by \*. Trace statistic criteria is obtained by selecting the lowest rank that cannot be rejected.

### A.3.2. Estimation of aggregate stochastic process

We test for the lag order of the underlying VAR. Table A.2 reports results from likelihood ratio tests and a number of commonly used statistical information criteria. All criteria suggest a lag length of  $p = 2$ . The college and high-school sub-samples, and alternative measures of aggregate earnings and alternative sample periods all indicate the presence of two lags.

We also test for the presence of cointegration. Table A.3 reports results from tests of the cointegrating rank based on the methods in Johansen (1995). Our results suggest only very weak evidence for cointegration. Alternative variable definitions, specifications and time periods lead to similar results.

Table A.4 presents the parameter estimates of the models with three education groups that allow cointegration. Table A.5 presents the average moments in the data together with the implied steady-state statistics from the model for three different data samples: the full sample from the CPS 1967–2008, NIPA 1967–2008 and NIPA 1929–2009. The NIPA measure of earnings growth is the change in the log of total wages and salaries per member of the labor force.

### A.3.3. Benzoni et al. (2007)

We describe the construction of the system of equations underlying the results in Fig. 9 from section 6. The three equations below are equations 2, 8 and 14 from Benzoni et al. (2007), where  $y_t$  is log dividends,  $R_t$  is the gross stock return and  $u_t$  is the log of the common component of earnings. The parameter  $\kappa$  is the key adjustment parameter controlling the strength of cointegration.

$$dy_t = (g - \sigma^2/2)dt + \sigma dz_3$$

$$R_t - 1 = \mu dt + \sigma dz_3$$

$$d(u_t - y_t - \bar{u}\bar{y}) = -\kappa(u_t - y_t - \bar{u}\bar{y})dt + v_1 dz_1 - v_3 dz_3$$

**Table A.4**

Parameter estimates for the aggregate stochastic process with cointegration.

		Full sample	College sample	High School sample
<b>Equation (1): <math>\Delta u_t^1</math></b>				
$\Delta u_{t-1}^1$	$\Gamma_{11}$	0.364 (0.19)	0.12 (0.15)	0.295 (0.18)
$\log R_{t-1}^s$	$\Gamma_{12}$	0.045 (0.02)	0.016 (0.02)	0.058 (0.03)
Constant	$\gamma_1$	−0.004 (0.00)	−0.005 (0.00)	−0.008 (0.00)
<b>Equation (2): <math>\log R_t^s</math></b>				
$\Delta u_{t-1}^1$	$\Gamma_{21}$	0.473 (1.42)	−2.248 (1.45)	0.236 (1.18)
$\log R_{t-1}^s$	$\Gamma_{22}$	0.054 (0.16)	0.145 (0.21)	0.072 (0.17)
Constant	$\gamma_2$	0.00 (0.03)	0.029 (0.04)	0.00 (0.03)
<b>Var-cov matrix</b>				
$\text{var}(\varepsilon_{1,t}) \times 10^{-4}$		4.42	3.37	6.44
$\text{var}(\varepsilon_{2,t}) \times 10^{-2}$		2.57	3.24	2.92
$\text{cov}(\varepsilon_{1,t}, \varepsilon_{2,t}) \times 10^{-3}$		1.28	1.21	2.00
<b>Cointegrating vector</b>				
$\log R_t^s$	$\beta_2$	0.309 (0.10)	−0.211 (0.06)	0.469 (0.15)
Trend	$\rho$	−0.019 (0.01)	0.016 (0.00)	−0.026 (0.01)
Constant	$\mu$	−0.67	0.343	−0.976
<b>Adjustment parameters</b>				
$\Delta u_t^1$	$\alpha_1$	0.007 (0.05)	−0.196 (0.07)	0.017 (0.04)
$\log R_t^s$	$\alpha_2$	−1.04 (0.36)	−0.063 (0.64)	−0.651 (0.23)

Notes: Standard errors in parentheses.

**Table A.5**

Implied steady-state statistics: alternative data sources and sample periods.

	No cointegration					
	CPS: 1967–2008		NIPA: 1967–2008		NIPA: 1929–2009	
	Data	Model	Data	Model	Data	Model
$E(\log R_t^s)$	0.041	0.045	0.041	0.044	0.068	0.069
$E(\Delta u_t^1)$	−0.002	−0.004	0.004	0.001	0.012	0.012
$sd(\Delta u_t^1)$	0.025	0.025	0.024	0.025	0.029	0.029
$sd(\log R_t^s)$	0.187	0.187	0.187	0.187	0.178	0.178
$\text{corr}(\Delta u_t^1, \log R_t^s)$	0.184	0.177	0.234	0.216	0.070	0.071
$\text{corr}(\Delta u_t^1, \Delta u_{t-1}^1)$	0.425	0.441	0.429	0.460	0.398	0.384
$\text{corr}(\log R_t^s, \log R_{t-1}^s)$	0.057	0.055	0.058	0.057	−0.054	−0.052
$\text{corr}(\Delta u_t^1 \log R_{t-1}^s)$	0.372	0.398	0.640	0.685	0.680	0.675
$\text{corr}(\log R_t^s, \Delta u_{t-1}^1)$	−0.292	−0.270	−0.189	−0.194	−0.096	−0.096
	With cointegration					
	CPS: 1967–2008		NIPA: 1967–2008		NIPA: 1929–2009	
	Data	Model	Data	Model	Data	Model
$E(\log R_t^s)$	0.041	0.070	0.041	0.070	0.068	0.045
$E(\Delta u_t^1)$	−0.002	−0.002	0.004	0.005	0.012	0.004
$sd(\Delta u_t^1)$	0.025	0.025	0.024	0.024	0.029	0.026
$sd(\log R_t^s)$	0.187	0.187	0.187	0.182	0.178	0.172
$\text{corr}(\Delta u_t^1, \log R_t^s)$	0.184	0.155	0.234	0.178	0.070	−0.020
$\text{corr}(\Delta u_t^1, \Delta u_{t-1}^1)$	0.425	0.435	0.429	0.435	0.398	0.260
$\text{corr}(\log R_t^s, \log R_{t-1}^s)$	0.057	0.005	0.058	−0.007	−0.054	−0.096
$\text{corr}(\Delta u_t^1 \log R_{t-1}^s)$	0.372	0.394	0.640	0.664	0.680	0.660
$\text{corr}(\log R_t^s, \Delta u_{t-1}^1)$	−0.292	−0.283	−0.189	−0.213	−0.096	−0.176

Notes: Table shows average moments in the data, together with implied steady-state statistics from the corresponding estimated model. NIPA data is total wage and salaries per member of the labor force. When implementing the estimated processes in the structural model, we adjust the constants ( $\gamma_1, \gamma_2$ ) so that all models have  $E[\log R_t^s] = 0.041$  and  $E[\Delta u_t^1] = 0$ .



**Table A.6**  
Implied steady-state statistics: two models.

	Benchmark model	Benzoni process
$sd(\Delta u_t^1)$	0.025	0.069
$sd(\log R_t^S)$	0.187	0.160
$corr(\Delta u_t^1, \log R_t^S)$	0.177	0.000
$corr(\Delta u_t^1, \Delta u_{t-1}^1)$	0.441	0.327
$corr(\log R_t^S, \log R_{t-1}^S)$	0.055	0.000
$corr(\Delta u_t^1 \log R_{t-1}^S)$	0.398	0.347
$corr(\log R_t^S, \Delta u_{t-1}^1)$	−0.270	0.000

**Notes:** Table shows implied steady-state statistics. When implementing the estimated processes in the structural model, we adjust the constant terms so that all models have  $E[\log R_t^S] = 0.041$  and  $E[\Delta u_t^1] = 0$ .

The three equations below are a discrete-time approximation of this continuous-time process, where  $(z_{1,t}, z_{3,t})$  are independent standard normal random variables. We rewrite this system of equations as system (7) below, using  $\Delta u_t \equiv u_t - u_{t-1}$  and  $w_t \equiv (u_t - y_t - \bar{u}y)$ .

$$y_{t+1} - y_t = g - \sigma^2/2 + \sigma z_{3,t+1}$$

$$\log R_{t+1} = \mu + \sigma dz_{3,t+1}$$

$$(u_{t+1} - y_{t+1} - \bar{u}y) - (u_t - y_t - \bar{u}y) = -\kappa(u_t - y_t - \bar{u}y) + \nu_1 dz_{1,t+1} - \nu_3 dz_{3,t+1}$$

$$\begin{pmatrix} \Delta u_{t+1} \\ \log R_{t+1} \\ w_{t+1} \end{pmatrix} = \begin{pmatrix} g - \sigma^2/2 \\ \mu \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & -\kappa \\ 0 & 0 & 0 \\ 0 & 0 & 1 - \kappa \end{pmatrix} \begin{pmatrix} \Delta u_t \\ \log R_t \\ w_t \end{pmatrix} + \begin{pmatrix} \nu_1 z_{1,t+1} + (\sigma - \nu_3) z_{3,t+1} \\ \sigma z_{3,t+1} \\ \nu_1 z_{1,t+1} - \nu_3 z_{3,t+1} \end{pmatrix} \quad (7)$$

System (7) produces the steady-state statistics listed in Table A.6. This occurs when we set  $(\kappa, g, \nu_1, \nu_3, \sigma, \mu) = (.15, .018, .05, .16, .07)$ , which are the benchmark parameter values used by Benzoni et al. (2007), and when we alter the constant terms in (7) to produce the same steady-state mean values ( $E[\log R]$ ,  $E[\Delta u]$ ) as in both of our benchmark models. This leaves the variance-covariance properties of the model unchanged.

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