How Do Agents Form Macroeconomic Expectations?

Evidence from Inflation Uncertainty

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Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Macroeconomic expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...

Macroeconomic expectation formation

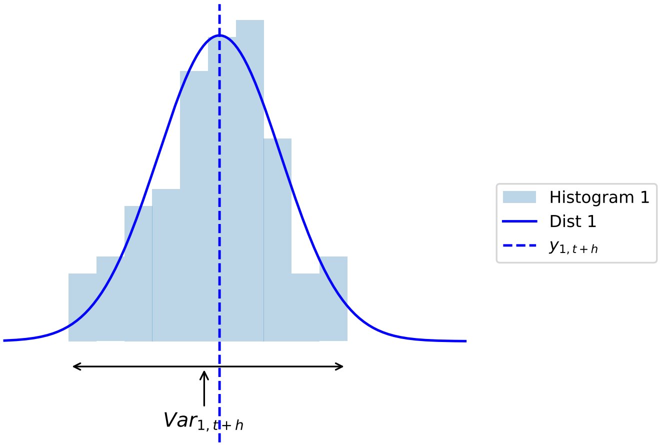
- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...
- Testing these models using survey expectations
  e.g. (Coibion and Gorodnichenko, 2012)
  - Forecast errors (FE)
  - Disagreement (Disg)
  - This paper: +Uncertainty (Var)

Why uncertainty?

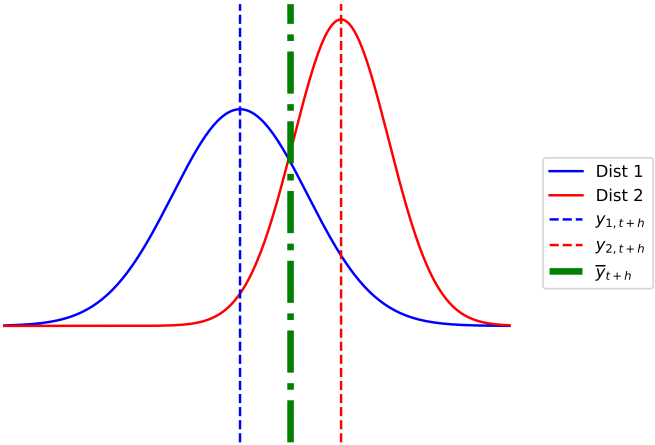
Uncertainty (or higher moments) matters for both

- individual economic decisions
  - precautionary saving motives
  - portfolio investments
  - mortgage choices
  - wage bargaining
- and aggregate outcomes
  - inflation dynamics
  - asset prices

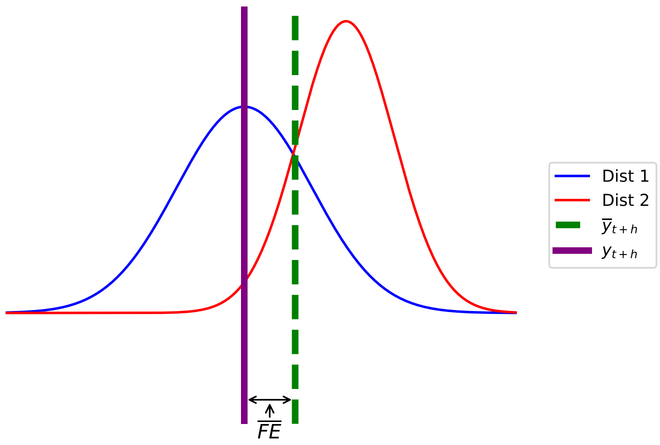
Density forecasts: an example



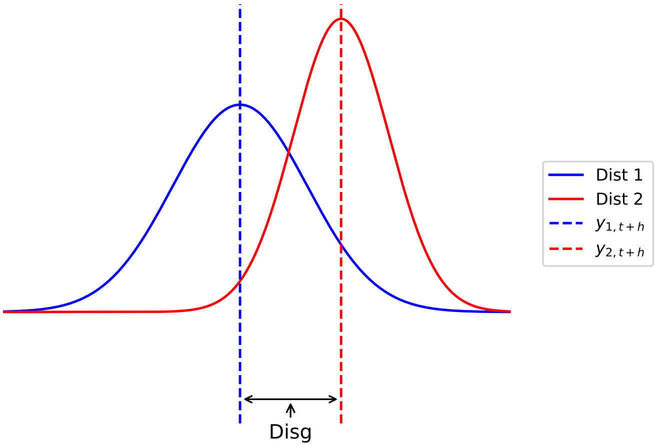
Average expectation



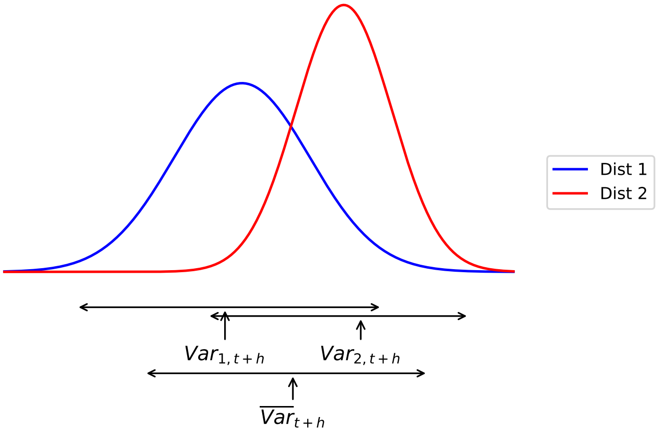
Average forecast errors (FE)



Disagreement (Disg)



Average uncertainty (Var)



Preview of the findings

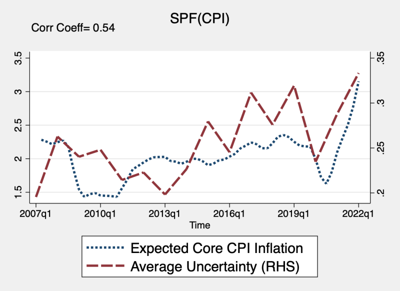
- Competing theories have distinctive predictions about Var
  - Information rigidity $\rightarrow$ ex-ante $\overline{Var} >$ ex-post $\overline{FE}^2$
- Additional evidence
  - Uncertainty revision is inefficient
  - SE more robust than NI
  - State-dependence: inflation $\uparrow$ rigidity $\downarrow$
  - Coexisting with overreaction at the individual level
- Inflation contains persistent and transitory components

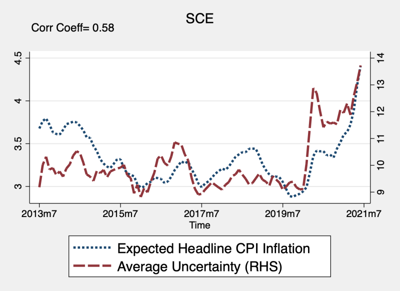
Data

Density forecast of inflation

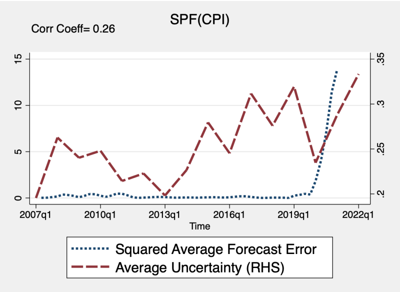
	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1 and 3-yr-ahead inflation	current-year and 1-yr-ahead q4/q4 Core CPI and Core PCE inflation
Survey Structure	fix-event	fix-horizon
Panel Structure	unbalanced, stay up to 12 months	unbalanced, average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

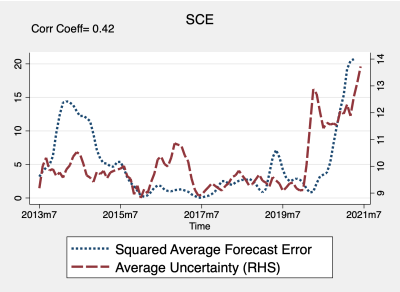
Expected inflation and uncertainty



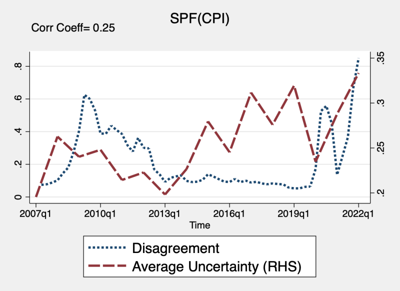


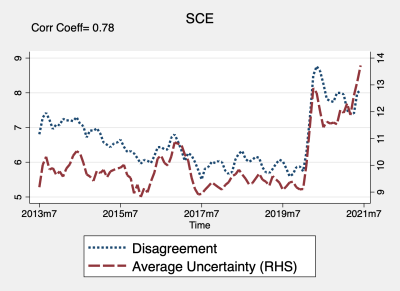
Forecast error and uncertainty





Disagreement and uncertainty





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FIRE predictions

Inflation process (AR1)

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

FIRE

$$\overline{FE}^*_{t+1|t} = -\omega_{t+1} \rightarrow \quad \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{\text{Var}}^*_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{Disg}^*_{\bullet+1|\bullet} = 0$$

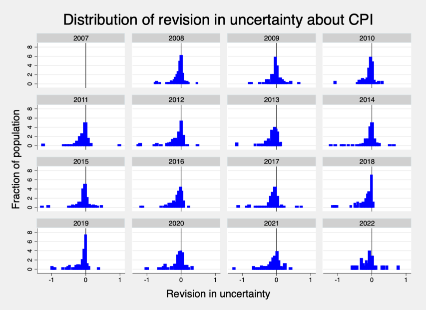
A first look at the data

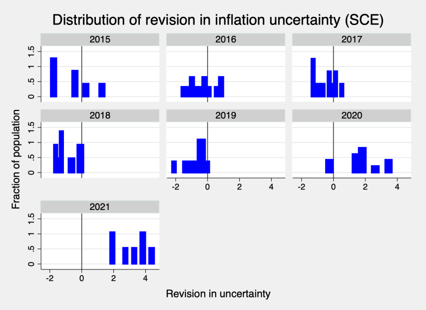
	SPF	SCE
InfAV	0	0
InfVar	0.219	1.282
InfATV	0.194	1.206
FE	0.125	1.812
FEVar	0.136	0.935
Disg	0.161	2.805
Var	0.213	1.749

- Demeaned realized inflation and inflation expectations.
- Household fixed effects controlled.
- Before 2020.

Uncertainty revision

More certain when getting closer to realization?





Efficiency tests with uncertainty

Do revisions reflect only common resolution of uncertainty?

$$\text{Var}_{i,t|t} - \text{Var}_{i,t|t-1} = \alpha^{\text{var}} + \beta^{\text{Var}}(\text{Var}_{i,t-1|t-1} - \text{Var}_{i,t-1|t-2}) + \psi_t^{var} + \zeta_{i,t}^{var}$$

- $\beta^{var} = 0$ under FIRE
- $\alpha^{var} < 0$ time-invariant uncertainty reduction
- ψ_t^{var}: time-varying innovations

Efficiency tests: professionals

	Mean revision	4q before	4q before	5q before
L4.InfExp_Var_rv		0.448***	0.456***	
		(0.056)	(0.058)	
L5.InfExp_Var_rv				0.440***
				(0.053)
Constant	-0.091***	-0.049***	-0.048***	-0.049***
	(0.000)	(0.008)	(0.005)	(0.005)
R2	0.047	0.196	0.248	0.249
N	1529	1157	1157	1021
Time FE	Yes	No	Yes	Yes

Taking stock

- Evidence rejecting FIRE
  - Inefficient revisions in Var
  - Disg > 0
  - FE^2 $<$ Var

Taking stock

- Evidence rejecting FIRE
  - Inefficient revisions in Var
  - Disg > 0
  - FE^2 $<$ Var
- Also, observed rankings help identify theories
  - SE$>$ NI, DE, DENI:
    - FE^2 $<$ Var
    - Sticky updating implies inefficient revisions

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Sticky expectations (SE)

[Mankiw and Reis, 2002, Carroll, 2003, etc]

With an updating rate of λ (FIRE when $\lambda = 1$)

$$\overline{FE}^{se}_{t+1|t} = (1-\lambda)\rho\overline{FE}^{se}_{t|t-1} - \lambda\omega_{t+1}$$

$$\rightarrow \overline{FE}^{se2}_{\bullet+1|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+t|\bullet} = \sigma^2_\omega$$

$$\overline{\text{Var}}^{se}_{\bullet+1|\bullet} = \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\overline{\text{Var}}^{*}_{t+1|t-\tau} = \frac{1}{1-(1-\lambda)\rho^2}\sigma^2_\omega \geq \overline{\text{Var}}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{se}_{\bullet+1|\bullet} \geq 0$$

Noisy information (NI)

[Lucas, 1972, Woodford, 2001, Sims, 2003 and Maćkowiak and Wiederholt, 2009, etc]

With noisiness of public and private signals σ_{pb}^2 and σ_{pr}^2

$$\overline{FE}^{ni}_{t+1|t} = (1 - PH)\rho\overline{FE}^{ni}_{t|t-1} + \rho P_\epsilon \epsilon_t + \overline{FE}^{*}_{t+1|t}$$

$$\rightarrow \overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^2 P_\epsilon^2 \sigma_{pb}^2 + \sigma_\omega^2}{(PH)^2} \geq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\mathrm{Var}^{ni}_{\bullet+1|\bullet} = \rho^2 \mathrm{Var}^{ni}_{\bullet|\bullet} + \sigma_\omega^2 \geq \mathrm{Var}^{*}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{Disg}^{ni}_{\bullet+1|\bullet} = \frac{\rho^2 P_\xi^2}{1 - (1 - PH)^2\rho^2}\sigma_{pr}^2 \geq 0$$

Kalman gain: $P = [P_\epsilon, P_\xi] = \overline{\mathrm{Var}}^{ni}_{\bullet|\bullet-1} H (H' \overline{\mathrm{Var}}^{ni}_{\bullet|\bullet-1} H + \Sigma^v)^{-1}$

Diagnostic expectations (DE)

[Bordalo, Gennaioli, and Shleifer, 2018, Bordalo, Gennaioli, Ma, et al., 2020, etc]

With overreaction parameter $\hat{\theta}(>0)$ and dispersion σ_θ^2

$$\overline{FE}^{de}_{t+1|t} = \overline{FE}^{*}_{t+1|t} - \hat{\theta}\rho \mathrm{FE}^{de}_{t|t-1}$$

$$\rightarrow \overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{1}{1+\hat{\theta}^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{\mathrm{Var}}^{de}_{\bullet+1|\bullet} = \overline{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{de}_{\bullet+1|\bullet} \geq 0$$

Comparing theories

Table: Model-implied ranking of moments

Model	Predictions
FIRE	$\overline{Var}^{*} = \overline{FE}^{*2} = \sigma_{\omega}^{2}; \overline{Disg}^{*} = 0$

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SE	$\boxed{\overline{FE}^{2} < \overline{FE}^{*2} = \overline{Var}^{*} = \sigma_{\omega}^{2} < \overline{Var}; \overline{Disg} > 0}$

Comparing theories

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NI	$\overline{FE}^2 > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^*; \overline{Disg} > 0$

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Comparing theories

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NI	$\overline{FE}^{2} > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^{*}; \overline{Disg} > 0$
DE	$\overline{FE}^{2} < \overline{FE}^{*2} = \overline{Var}^{*} = \overline{Var}; \overline{Disg} > 0$
DENI	$\overline{FE}^{2} >< \overline{FE}^{*2}, \overline{Var} > \overline{Var}^{*}, \overline{Disg} > 0$

Structural Estimation: SMM

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

- $o \in \{se, ni, de, deni\} \times \{ar, sv\}$
- Γ^o: parameter space
- H: real-time historical realizations
- W: weighting matrix

Model estimates for professionals

SE as an example

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$	ρ	σ_ω
FE	0.36	0.99	0.23
FE+Disg	0.28	0.99	0.23
FE+Disg+Var	0.26	0.99	0.23

Evidence for subjective models

[Jain, 2019, Macaulay and Moberly, 2022, Farmer, Nakamura, and Steinsson, 2021]

SE						
Moments Used	2-Step Estimate			Joint Estimate		
	$\hat{\lambda}$	ρ	σ_ω	$\hat{\lambda}$	ρ	σ_ω
FE	0.36	0.99	0.23	0.18	0.97	0.11
FE+Disg	0.28	0.99	0.23	0.22	0.95	0.14
FE+Disg+Var	0.26	0.99	0.23	0.32	0.9	0.22

NI requires highly noisy signals

NI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	0.87	0.99	0.23	0	0.15	0.97	0.11
FE+Disg	1.5	2.26	0.99	0.23	1.48	2.33	0.97	0.11
FE+Disg+Var	2.64	3	0.99	0.23	3	3	0.94	0.16

Patterns of households

Sticky, underreactive and widely dispersed

SE				
Moments Used	2-Step Estimate			
	$\hat{\lambda}$	ρ	σ_ω	
FE	0.36	0.98	0.45	
FE+Disg	0.36	0.98	0.45	
FE+Disg+Var	0.36	0.98	0.45	
NI				
Moments Used	2-Step Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	1	0.98	0.45
FE+Disg	3	1.18	0.98	0.45
FE+Disg+Var	2.06	3	0.98	0.45
DENI				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	N/A	N/A	0.98	0.45
FE+Disg	-0.54	3	0.98	0.45
FE+Disg+Var	-0.35	2.43	0.98	0.45

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Stochastic volatility (SV)

[Stock and Watson, 2007]

Process of inflation

$$y_t = \overbrace{\zeta_t}^{\text{Permanent component}} + \overbrace{\eta_t}^{\text{Transitory}}$$

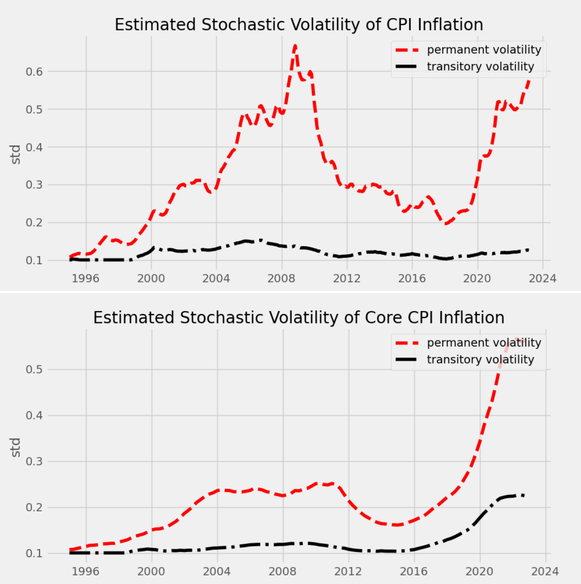
$$\zeta_t = \zeta_{t-1} + z_t$$

$$z_t = \sigma_{z,t}\xi_{z,t}, \quad \eta_t = \sigma_{\eta,t}\xi_{\eta,t}, \quad \xi_t = [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I)$$

$$\log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t}, \qquad \log \sigma^2_{z,t} = \log \sigma^2_{z,t-1} + \mu_{z,t}$$

$$\mu_t = [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)$$

Estimated SV



More sensible est of NI for professionals

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.2		0.3	
FE+Disg	0.25		0.36	
FE+Disg+Var	0.36		0.36	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.68	0.24	2.3	3
FE+Disg	0.67	0.24	2.3	3
FE+Disg+Var	0.64	0.21	2.3	3

NI remains a poor fit of households

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.27		0.36	
FE+Disg	0.2		0.27	
FE+Disg+Var	0.26		0.26	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_{\epsilon}$	$\hat{\sigma}_{\xi}$	$\hat{\sigma}_{\epsilon}$	$\hat{\sigma}_{\xi}$
FE	N/A	N/A	N/A	N/A
FE+Disg	N/A	N/A	N/A	N/A
FE+Disg+Var	N/A	N/A	N/A	N/A

Higher inflation, less rigidity

[Coibion and Gorodnichenko, 2015, Weber et al., 2023]

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.27		0.36	
FE+Disg	0.2		0.27	
FE+Disg+Var	0.26		0.26	
DENI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	$\hat{\sigma}_{\xi}$	$\hat{\theta}$	$\hat{\sigma}_{\xi}$
FE	-0.48	0.64	0.43	0.26
FE+Disg	-0.48	0.64	0.43	0.26
FE+Disg+Var	-0.48	0.64	0.43	0.26

Scoring card

Table: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	Yes	Yes	No
Sensitive to the assumed inflation process?	No	Yes	Yes	No
Sensitive to two-step or joint estimate?	No	No	No	Yes
Sensitive to the type of agents?	No	Yes	Yes	Yes

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