How Do Agents Form Macroeconomic Expectations?

Evidence from Inflation Uncertainty

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Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...
- Testing these models using survey expectations
  e.g. (Coibion and Gorodnichenko, 2012)
  - Forecast errors (FE)
  - Disagreement (Disg)
  - This paper: +Uncertainty (Var)

Why (inflation) uncertainty?

Uncertainty (or higher moments) matters for both

- individual economic decisions
  - through precautionary saving motives
  - through portfolio investments
- and aggregate outcomes
  - inflation dynamics
  - asset prices

Preview of the findings

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- Sticky expectation (SE) best jointly explains FE, Disg and Var,
- ... and more robust to
    1. type of agents: households or professionals
    2. moments used: FE, Disg, Var, etc.
    3. separate or joint estimation of inflation + expectation formation
    4. various inflation processes: AR or stochastic volatility (SV)

Preview of the findings

- Competing theories have distinctive predictions about uncertainty
- Sticky expectation (SE) best jointly explains FE, Disg and Var,
- ... and more robust to
    1. type of agents: households or professionals
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    3. separate or joint estimation of inflation + expectation formation
    4. various inflation processes: AR or stochastic volatility (SV)
- Additional evidence rejecting FIRE
    1. Uncertainty is widely dispersed
    2. Revision is inefficient

Data

Density forecast of inflation

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1-yr-ahead inflation	1-yr-ahead Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

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FIRE predictions

Inflation process (AR1)

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

FIRE

$$\overline{FE}^*_{t+1|t} = -\omega_{t+1} \rightarrow \quad \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{\text{Var}}^*_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{Disg}^*_{\bullet+1|\bullet} = 0$$

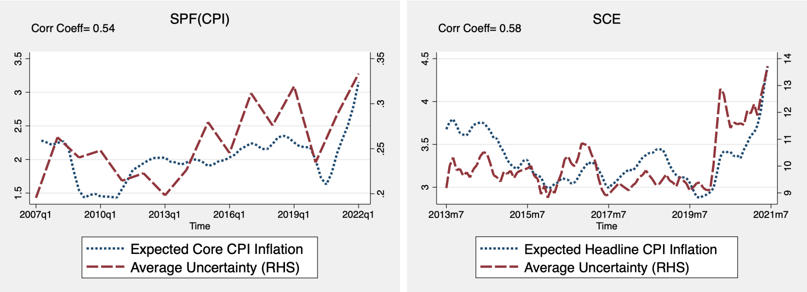
FIRE predictions v.s. data

	SPF	SCE
InfAV	0	0
InfVar	0.219	1.282
InfATV	0.194	1.206
FE	0.125	1.812
FEVar	0.136	0.935
Disg	0.161	2.805
Var	0.213	1.749

- Demeaned realized inflation and inflation expectations.
- Household fixed effects controlled.

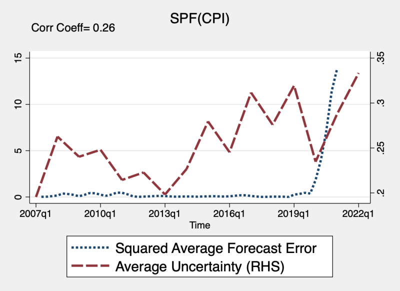
FIRE predictions v.s. data

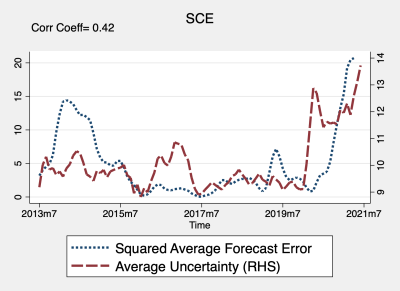
Expected inflation and uncertainty



FIRE predictions v.s. data, continued

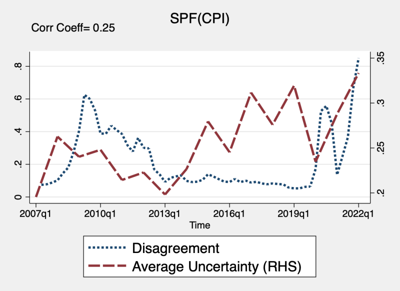
Forecast error and uncertainty

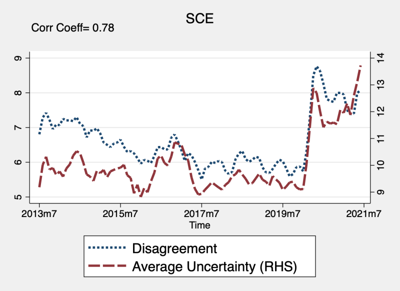




FIRE predictions v.s. data, continued

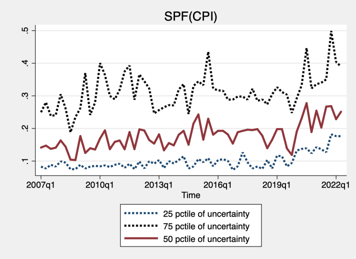
Disagreement and uncertainty

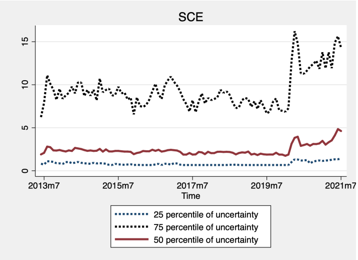




Other evidence rejecting FIRE

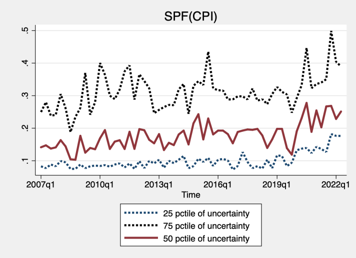
Dispersion in uncertainty

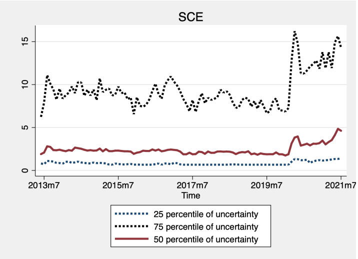




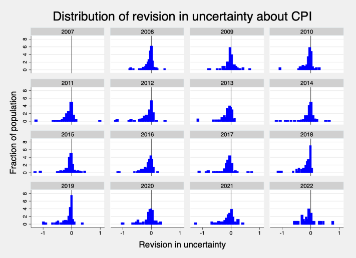
Other evidence rejecting FIRE

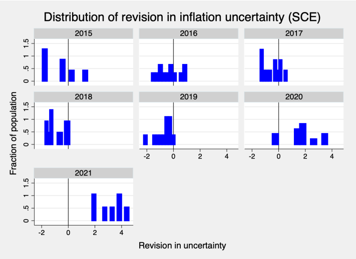
Dispersion in uncertainty





Inefficient revisions in uncertainty





Efficiency tests of uncertainty revision

$$\mathrm{Var}_{i,t|t} - \mathrm{Var}_{i,t|t-1} = \alpha^{\mathrm{var}} + \beta^{\mathrm{Var}}(\mathrm{Var}_{i,t-1|t-1} - \mathrm{Var}_{i,t-1|t-2}) + \psi_t^{var} + \zeta_{i,t}^{var}$$

- $\beta^{var} = 0$ under FIRE
- $\alpha^{var} < 0$ time-invariant uncertainty reduction
- ψ_t^{var}: time-varying innovations

Efficiency tests: professionals

	Mean revision	4q before	4q before	5q before
L4.InfExp_Var_rv		0.448***	0.456***	
		(0.056)	(0.058)	
L5.InfExp_Var_rv				0.440***
				(0.053)
Constant	-0.091***	-0.049***	-0.048***	-0.049***
	(0.000)	(0.008)	(0.005)	(0.005)
R2	0.047	0.196	0.248	0.249
N	1529	1157	1157	1021
Time FE	Yes	No	Yes	Yes

Efficiency tests: households

	Mean revision	24m before	25m before	26m before
L24.InfExp_Var_rv		-0.666***		
		(0.151)		
L25.InfExp_Var_rv			-0.501*	
			(0.208)	
L26.InfExp_Var_rv				-0.376
				(0.219)
L12.InfExp_FE2		0.357***	0.328***	0.306***
		(0.039)	(0.058)	(0.056)
Constant	0.778***	-0.623*	-0.426	-0.275
	(0.225)	(0.272)	(0.337)	(0.345)
R2	0.000	0.527	0.498	0.478
N	88	64	63	62
Time FE	No	No	No	No

Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - FE^2 $<$ Var

Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - FE$^2 <$ Var
- Also, these patterns help identify competing theories
  - SE$>$ NI, DE, DENI:
    - FE$^2 <$ Var
    - Sticky updating implies inefficient revisions

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The role of stochastic volatility

Sticky expectations (SE)

[Mankiw and Reis, 2002, Carroll, 2003, etc]

With an updating rate of λ (FIRE when $\lambda = 1$)

$$\overline{FE}^{se}_{t+1|t} = (1-\lambda)\rho\overline{FE}^{se}_{t|t-1} - \lambda\omega_{t+1}$$

$$\rightarrow \overline{FE}^{se2}_{\bullet+1|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+t|\bullet} = \sigma^2_\omega$$

$$\overline{\text{Var}}^{se}_{\bullet+1|\bullet} = \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\overline{\text{Var}}^{*}_{t+1|t-\tau} = \frac{1}{1-(1-\lambda)\rho^2}\sigma^2_\omega \geq \overline{\text{Var}}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{se}_{\bullet+1|\bullet} \geq 0$$

Noisy information (NI)

[Lucas, 1972, Woodford, 2001, Sims, 2003 and Maćkowiak and Wiederholt, 2009, etc]

With noisiness of public and private signals σ^2_{pb} and σ^2_{pr}

$$\overline{FE}^{ni}_{t+1|t} = (1 - PH)\rho \overline{FE}^{ni}_{t|t-1} + \rho P_\epsilon \epsilon_t + \overline{FE}^{*}_{t+1|t}$$

$$\rightarrow \overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_\epsilon \sigma^2_{pb} + \sigma^2_\omega}{(PH)^2} \geq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\text{Var}^{ni}_{\bullet+1|\bullet} = \rho^2 \text{Var}^{ni}_{\bullet|\bullet} + \sigma^2_\omega \geq \text{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{ni}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_\xi}{1 - (1 - PH)^2 \rho^2} \sigma^2_{pr} \geq 0$$

Kalman gain: $P = [P_\epsilon, P_\xi] = \overline{\text{Var}}^{ni}_{\bullet|\bullet-1} H (H' \overline{\text{Var}}^{ni}_{\bullet|\bullet-1} H + \Sigma^v)^{-1}$

Diagnostic expectations (DE)

[Bordalo, Gennaioli, and Shleifer, 2018, Bordalo, Gennaioli, Ma, et al., 2020, etc]

With overreaction parameter $\hat{\theta}(>0)$ and dispersion σ_θ^2

$$\overline{FE}^{de}_{t+1|t} = \overline{FE}^{*}_{t+1|t} - \hat{\theta}\rho \mathrm{FE}^{de}_{t|t-1}$$

$$\rightarrow \overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{1}{1+\hat{\theta}^2\rho^2}\sigma_\omega^2 \leq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{\mathrm{Var}}^{de}_{\bullet+1|\bullet} = \overline{Var}^{*}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{Disg}^{de}_{\bullet+1|\bullet} \geq 0$$

Comparing theories

Table: Model-implied ranking of moments

Model	Predictions
FIRE	$\overline{Var}^* = \overline{FE}^{*2} = \sigma^2_\omega; \overline{Disg}^* = 0$

Comparing theories

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FIRE	$\overline{Var}^* = \overline{FE}^{*2} = \sigma^2_\omega; \overline{Disg}^* = 0$
SE	$\overline{FE}^2 < \overline{FE}^{*2} = \overline{Var}^* = \sigma^2_\omega < \overline{Var}; \overline{Disg} > 0$

Comparing theories

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SE	$\overline{FE}^{2} < \overline{FE}^{*2} = \overline{Var}^{*} = \sigma^2_\omega < \overline{Var}; \overline{Disg} > 0$
NI	$\overline{FE}^{2} > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^{*}; \overline{Disg} > 0$

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NI	$\overline{FE}^{2} > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^{*}; \overline{Disg} > 0$
DE	$\overline{FE}^{2} < \overline{FE}^{*2} = \overline{Var}^{*} = \overline{Var}; \overline{Disg} > 0$

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NI	$\overline{FE}^2 > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^*; \overline{Disg} > 0$
DE	$\overline{FE}^2 < \overline{FE}^{*2} = \overline{Var}^* = \overline{Var}; \overline{Disg} > 0$
DENI	$\overline{FE}^2 >< \overline{FE}^{*2}, \overline{Var} > \overline{Var}^*, \overline{Disg} > 0$

Structural Estimation: SMM

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

- $o \in \{se, ni, de, deni\} \times \{ar, sv\}$
- Γ^o: parameter space
- H: real-time historical realizations
- W: weighting matrix

Structural Estimation: Professionals

SE								
Moments Used	2-Step Estimate			Joint Estimate				
	$\hat{\lambda}$	ρ	σ_ω	$\hat{\lambda}$	ρ	σ_ω		
FE	0.36	0.99	0.23	0.18	0.97	0.11		
FE+Disg	0.28	0.99	0.23	0.22	0.95	0.14		
FE+Disg+Var	0.26	0.99	0.23	0.32	0.9	0.22		
NI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	0.87	0.99	0.23	0	0.15	0.97	0.11
FE+Disg	1.5	2.26	0.99	0.23	1.48	2.33	0.97	0.11
FE+Disg+Var	2.64	3	0.99	0.23	3	3	0.94	0.16
DE								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\theta}$	σ_θ	ρ	σ_ω	$\hat{\theta}$	σ_θ	ρ	σ_ω
FE	0.64	0.58	0.99	0.23	0.81	1.68	0.97	0.11
FE+Disg	0.27	2.2	0.99	0.23	0.38	2.1	0.9	0.2
FE+Disg+Var	0.42	2.1	0.99	0.23	0.33	2.1	0.9	0.23
DENI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\theta}$	$\hat{\sigma}_\xi$	ρ	σ_ω	$\hat{\theta}$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0.76	0	0.99	0.23	0.82	0	0.97	0.11
FE+Disg	0.85	0.14	0.99	0.23	N/A	N/A	N/A	N/A
FE+Disg+Var	0.85	0.16	0.99	0.23	N/A	N/A	N/A	N/A

Structural Estimation: Households

SE				
Moments Used	2-Step Estimate			
	$\hat{\lambda}$	ρ	σ_ω	
FE	0.36	0.98	0.45	
FE+Disg	0.36	0.98	0.45	
FE+Disg+Var	0.36	0.98	0.45	
NI				
Moments Used	2-Step Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	1	0.98	0.45
FE+Disg	3	1.18	0.98	0.45
FE+Disg+Var	2.06	3	0.98	0.45
DE				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	σ_θ	ρ	σ_ω
FE	0.49	0.5	0.98	0.45
FE+Disg	1.91	5	0.98	0.45
FE+Disg+Var	1.03	5	0.98	0.45
DENI				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	N/A	N/A	0.98	0.45
FE+Disg	-0.54	3	0.98	0.45
FE+Disg+Var	-0.35	2.43	0.98	0.45

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The role of stochastic volatility

Stochastic volatility (SV)

[Stock and Watson, 2007]

Process of inflation

$$y_t = \overbrace{\zeta_t}^{\text{Permanent component}} + \overbrace{\eta_t}^{\text{Transitory}}$$

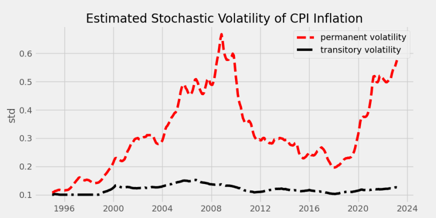
$$\zeta_t = \zeta_{t-1} + z_t$$

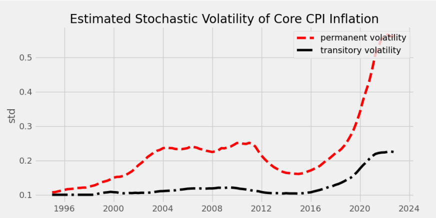
$$z_t = \sigma_{z,t}\xi_{z,t}, \quad \eta_t = \sigma_{\eta,t}\xi_{\eta,t}, \quad \xi_t = [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I)$$

$$\log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t}, \qquad \log \sigma^2_{z,t} = \log \sigma^2_{z,t-1} + \mu_{z,t}$$

$$\mu_t = [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)$$

Estimated SV





Structural Estimation with SV: Professionals

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.2		0.3	
FE+Disg	0.25		0.36	
FE+Disg+Var	0.36		0.36	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.68	0.24	2.3	3
FE+Disg	0.67	0.24	2.3	3
FE+Disg+Var	0.64	0.21	2.3	3
DE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	σ_{θ}	$\hat{\theta}$	σ_{θ}
FE	-0.03	0.54	0.31	0.41
FE+Disg	-0.03	0.16	0.28	0.19
FE+Disg+Var	-0.04	0.16	0.31	0.19
DENI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	$\hat{\sigma}_{pr}$	$\hat{\theta}$	$\hat{\sigma}_{pr}$
FE	0.64	0.47	-0.25	0.93
FE+Disg	0.82	0.26	-0.26	0.93
FE+Disg+Var	0.82	0.24	-0.26	0.93

Structural Estimation with SV: Households

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.27		0.36	
FE+Disg	0.2		0.27	
FE+Disg+Var	0.26		0.26	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$
FE	N/A	N/A	N/A	N/A
FE+Disg	N/A	N/A	N/A	N/A
FE+Disg+Var	N/A	N/A	N/A	N/A
DE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	σ_θ	$\hat{\theta}$	σ_θ
FE	-0.09	0.58	-0.07	0.57
FE+Disg	0.29	0.57	0.47	1.07
FE+Disg+Var	0.29	0.57	0.28	1.07
DENI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	$\hat{\sigma}_\xi$	$\hat{\theta}$	$\hat{\sigma}_\xi$
FE	-0.48	0.64	0.43	0.26
FE+Disg	-0.48	0.64	0.43	0.26
FE+Disg+Var	-0.48	0.64	0.43	0.26

Scoring card

Table: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	Yes	Yes	No
Sensitive to the assumed inflation process?	No	Yes	Yes	No
Sensitive to two-step or joint estimate?	No	No	No	Yes
Sensitive to the type of agents?	No	Yes	Yes	Yes

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