How Do Agents Form Inflation Expectations?

Evidence from the Forecast Uncertainty

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Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...
- Testing these models using survey expectations e.g. (Coibion and Gorodnichenko, 2012)
  - Forecast errors (FE)
  - Disagreement (Disg)
  - This paper: +Uncertainty (Var)

Why inflation uncertainty?

Uncertainty (or higher moments) matters for both

- individual economic decisions
  - through precautionary saving motives
  - through portfolio investment
- and aggregate outcomes
  - inflation dynamics
  - asset prices

Preview of the findings

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- Sticky expectation (SE) best jointly explains FE, Disg and Var,
- ... and more robust to
  1. type of agents: households or professionals
  2. moments used: FE, Disg, Var, etc.
  3. separate or joint estimation of inflation + expectation formation
  4. various inflation processes: AR or stochastic volatility (SV)

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- Sticky expectation (SE) best jointly explains FE, Disg and Var,
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  3. separate or joint estimation of inflation + expectation formation
  4. various inflation processes: AR or stochastic volatility (SV)
- Additional evidence rejecting FIRE
  1. Uncertainty is widely dispersed
  2. Revision is inefficient

Data

Density forecast of inflation

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1-yr-ahead inflation	1-yr-ahead Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

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FIRE predictions

Inflation process (AR1)

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

FIRE

$$\overline{FE}^*_{t+1|t} = -\omega_{t+1} \rightarrow \quad \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{\text{Var}}^*_{\bullet+1|\bullet} = \sigma_\omega^2$$

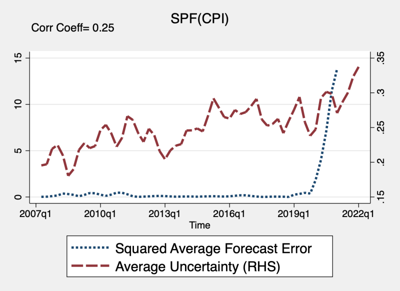
$$\overline{Disg}^*_{\bullet+1|\bullet} = 0$$

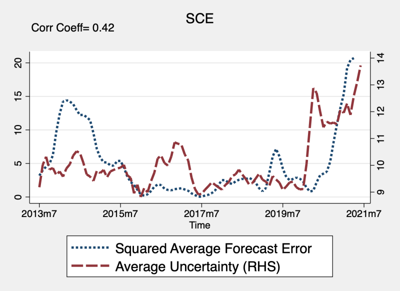
FIRE predictions v.s. data

	SPF	SCE	FIRE+AR	FIRE+SV
InfAV	0	0	0	0
InfVar	0.159	0.653	$\sigma_\omega^2/(1-\rho^2)$	N/A
InfATV	0.125	0.621	$\rho\sigma_\omega^2/(1-\rho^2)$	N/A
FE	0.136	1.772	0	0
FEVar	0.133	0.923	σ_ω^2	$\bar{\sigma}_\eta^2+\bar{\sigma}_\epsilon^2$
FEATV	0.097	0.89	0	0
Disg	0.183	2.585	0	0
DisgVar	0.028	0.057	0	0
DisgATV	0.021	0.025	0	0
Var	0.242	1.75	σ_ω^2	$\bar{\sigma}_\eta^2+\bar{\sigma}_\epsilon^2$
VarVar	0.001	0.023	0	>0
VarATV	0.001	0.004	0	>0

FIRE predictions v.s. data, continued

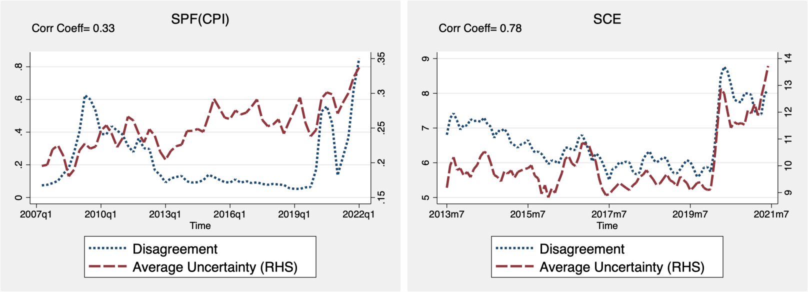
Forecast error and uncertainty





FIRE predictions v.s. data, continued

Disagreement and uncertainty



Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - $\text{FE}^2 \neq$ Var
  - $\text{FE}^2 >$ InfVar and Var $>$ InfVar

Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - FE$^2 \neq$ Var
  - FE$^2 >$ InfVar and Var $>$ InfVar
- Also, these patterns help identify competing theories
  - SE$>$ NI, DE, DENI:
    - Disg and dispersion in Var because of different updating
    - Sticky updating implies inefficient revisions

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Sticky expectations (SE)

[Mankiw and Reis, 2002, Carroll, 2003, etc]

With an updating rate of λ (FIRE when $\lambda = 1$)

$$\overline{FE}^{se}_{t+1|t} = (1-\lambda)\rho\overline{FE}^{se}_{t|t-1} - \lambda\omega_{t+1}$$

$$\rightarrow \overline{FE}^{se2}_{\bullet+1|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+t|\bullet} = \sigma^2_\omega$$

$$\overline{\text{Var}}^{se}_{\bullet+1|\bullet} = \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\overline{\text{Var}}^{*}_{t+1|t-\tau} = \frac{1}{1-(1-\lambda)\rho^2}\sigma^2_\omega \geq \overline{\text{Var}}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{se}_{\bullet+1|\bullet} \geq 0$$

Noisy information (NI)

[Lucas, 1972, Woodford, 2001, Sims, 2003 and Maćkowiak and Wiederholt, 2009, etc]

With noisiness of public and private signals σ^2_{pb} and σ^2_{pr}

$$\overline{FE}^{ni}_{t+1|t} = (1 - PH)\rho\overline{FE}^{ni}_{t|t-1} + \rho P_{\epsilon}\epsilon_t + \overline{FE}^{*}_{t+1|t}$$

$$\rightarrow \overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_{\epsilon}\sigma^2_{pb} + \sigma^2_{\omega}}{(PH)^2} \geq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_{\omega}$$

$$\text{Var}^{ni}_{\bullet+1|\bullet} = \rho^2\text{Var}^{ni}_{\bullet|\bullet} + \sigma^2_{\omega} \geq \text{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_{\omega}$$

$$\overline{Disg}^{ni}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_{\xi}}{1 - (1 - PH)^2\rho^2}\sigma^2_{pr} \geq 0$$

Kalman gain: $P = [P_{\epsilon}, P_{\xi}] = \overline{\text{Var}}^{ni}_{\bullet|\bullet-1}H(H'\overline{\text{Var}}^{ni}_{\bullet|\bullet-1}H + \Sigma^{v})^{-1}$

Diagnostic expectations (DE)

[Bordalo, Gennaioli, and Shleifer, 2018, Bordalo, Gennaioli, Ma, et al., 2020, etc]

With overreaction parameter $\hat{\theta}(> 0)$ and dispersion σ_θ^2

$$\overline{FE}^{de}_{t+1|t} = \overline{FE}^{*}_{t+1|t} - \hat{\theta}\rho \mathrm{FE}^{de}_{t|t-1}$$

$$\rightarrow \overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{1}{1+\hat{\theta}^2\rho^2}\sigma_\omega^2 \leq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{\mathrm{Var}}^{de}_{\bullet+1|\bullet} = \overline{Var}^{*}_{\bullet+1|\bullet} = \sigma_\omega^2$$

$$\overline{Disg}^{de}_{\bullet+1|\bullet} \geq 0$$

Structural Estimation: SMM

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

- $o \in \{se, ni, de, deni\} \times \{ar, sv\}$
- Γ^o: parameter space
- H: real-time historical realizations
- W: weighting matrix

Structural Estimation: Professionals

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.99	0.23	0.18	1	0.01				
FE+Disg	0.3	0.99	0.23	0.18	1	0				
FE+Disg+Var	0.32	0.99	0.23	0.21	1	0.02				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	2.73	3	0.99	0.23	3	3	1	0.02		
FE+Disg	3	2.95	0.99	0.23	3	3	1	0.02		
FE+Disg+Var	3	3	0.99	0.23	1.97	1.17	1	0		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	0.25	1.57	0.99	0.23	1.39	2.83	0.9	0.17		
FE+Disg	1.12	1.62	0.99	0.23	0.05	1.73	1	0.01		
FE+Disg+Var	0.29	1.81	0.99	0.23	1.19	1.68	0.91	0.16		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	0.3	0.02	2.56	0.99	0.23	-1.14	2.75	0.12	1	0
FE+Disg	-1.21	3	0.09	0.99	0.23	-0.82	2.45	2.83	1	0.01
FE+Disg+Var	-0.8	3	0.09	0.99	0.23	1.67	3	3	1	0.03

Structural Estimation: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.98	0.41	0.2	0.98	0.16				
FE+Disg	0.35	0.98	0.41	0.2	0.98	0.1				
FE+Disg+Var	0.35	0.98	0.41	0.2	0.98	0.1				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	3	0.45	0.98	0.41	2.98	0.28	0.95	0.24		
FE+Disg	1.55	0.36	0.98	0.41	3	0.28	0.95	0.24		
FE+Disg+Var	2.58	0.97	0.98	0.41	2.9	1.02	0.96	0.24		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_{θ}	ρ	σ	$\hat{\theta}$	σ_{θ}	ρ	σ		
FE	-0.47	0.81	0.98	0.41	-0.61	4.15	0.95	0.25		
FE+Disg	-0.3	2.08	0.98	0.41	-0.08	2.13	0.95	0.25		
FE+Disg+Var	-0.35	2.08	0.98	0.41	0.36	2.07	0.9	0.36		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	N/A	N/A	N/A	0.98	0.41	-0.39	1.86	0.03	0.95	0.24
FE+Disg	N/A	N/A	N/A	0.98	0.41	N/A	N/A	N/A	N/A	N/A
FE+Disg+Var	N/A	N/A	N/A	0.98	0.41	0.35	0	1.32	0.96	0.22

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Stochastic volatility (SV)

[Stock and Watson, 2007]

Process of inflation

$$y_t = \overbrace{\zeta_t}^{\text{Permanent component}} + \overbrace{\eta_t}^{\text{Transitory}}$$

$$\zeta_t = \zeta_{t-1} + z_t$$

$$z_t = \sigma_{z,t}\xi_{z,t}, \quad \eta_t = \sigma_{\eta,t}\xi_{\eta,t}, \quad \xi_t = [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I)$$

$$\log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t}, \qquad \log \sigma^2_{z,t} = \log \sigma^2_{z,t-1} + \mu_{z,t}$$

$$\mu_t = [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)$$

Structural Estimation with SV: Professionals

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.34		
FE+Disg+Var	0.34		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0.27	0.04	
FE+Disg	0.27	0.04	
FE+Disg+Var	0.27	0.05	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.39	0.69	
FE+Disg	-0.39	0.7	
FE+Disg+Var	-0.39	0.7	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	1.38	3	1.72
FE+Disg	3	3	2.03
FE+Disg+Var	3	3	2.28

Structural Estimation with SV: Households

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0.44	2.37	
FE+Disg	0.44	2.61	
FE+Disg+Var	0.44	2.9	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.05	0.58	
FE+Disg	-0.05	0.56	
FE+Disg+Var	-0.05	0.56	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	N/A	N/A	N/A
FE+Disg	0.71	0.01	0.95
FE+Disg+Var	N/A	N/A	N/A

Scoring card

Table: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	No	Yes	Yes
Sensitive to the assumed inflation process?	No	Yes	Yes	Yes
Sensitive to two-step or joint estimate?	Yes	Yes	Yes	Yes
Sensitive to the type of agents?	Yes	Yes	Yes	Yes

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