# How Do Agents Form Macroeconomic Expectations? Evidence from Inflation Uncertainty

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**Abstract**

This paper explores the behaviors of *uncertainty* through the lens of several popular models of expectation formation. Full-information-rational-expectation (FIRE) predicts the *ex-ante* uncertainty and the variance of *ex-post* forecast errors are both equal to the conditional volatility. Incomplete-information models such as Sticky Expectation (SE) and Noisy Information (NI) and non-rational models such as Diagnostic Expectations (DE) predict distinctive rankings of these moments. It is also shown that uncertainty provides additional parametric restrictions to distinguish models that predict similar aggregate patterns of forecast errors and disagreement.

**Keywords: Inflation, Expectation Formation, Rigidity, Overreaction, Uncertainty, Density Forecast**

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# 1 Introduction

Macroeconomists developed different models of expectation formation to account for observed patterns of survey expectations which are inconsistent with the benchmark of full-information rational expectations (FIRE). Some of the models, such as Sticky Expectations (SE)[1] and Noisy Information (NI)[2] feature incomplete information and information rigidity in the form of sluggish responsiveness to aggregate shocks, as documented by Coibion and Gorodnichenko (2012, 2015). Other models such as Diagnostic Expectation (DE) (Bordalo et al., 2018, 2020) implies the overreaction to the news at the individual level.[3]

The development of these models[4] has been primarily focused on the patterns of forecast errors, revision, and disagreement in the survey data. What remains underexplored in this literature is the implications of expectation formation about *uncertainty*, which, here, strictly refers to the dispersion of density forecasts. This paper does a cross-moment estimation of various selective theories by jointly accounting for its predictions about different forecast moments: forecast errors (FE), cross-sectional disagreements (Disg), and uncertainty (Var). Uncertainty is shown to provide additional information to identify different models of expectation. [5]

A more important motivation for bringing uncertainty into the studies on expectation formation is the increasing emphasis on higher-order moments, in addition to the average mean expectations, in driving business cycle fluctuations. [6] Such macroeconomic effects may take into effects through a wide range of channels of households and firms, such as precautionary saving motives[7], and investment [8]and hiring decisions. Therefore, macroeconomic models intended to capture these channels need to make sure the dynamics of uncertainty are consistent with the selected model of expectation formation.

Consider FIRE to get some intuition on why various models of expectation formation have distinctive predictions about the rankings and parameter restrictions regarding various forecasting moments. With complete information and an identical model as assumed, forecast uncertainty only reflects the conditional variance of the unforecastable component of the inflation shock. Therefore, the ex-ante uncertainty would be exactly equal to the variance of ex-post forecast errors, and to the size of the conditional volatility of inflation.

In contrast, the two models featuring incomplete information/information rigidity,

[1]Mankiw and Reis (2002); Carroll (2003); Reis (2006).

[2]Lucas (1972), Woodford (2001),Maćkowiak and Wiederholt (2009)

[3]Kohlhas and Walther (2021) proposes an extended model of NI allowing for multiple unobserved components to reconcile the coexistence of under- and overreaction.

[4]For instance, Coibion and Gorodnichenko (2012, 2015); Bordalo et al. (2020)

[5]Some other contemporaneous papers also structurally estimate theories on expectation formation based on single or multiple moments of surveyed expectations, such as Giacomini et al. (2020); Xie (2019); Bordalo et al. (2020); Farmer et al. (2021). However, none of these studies explore the behaviors of uncertainty.

[6]See Bloom (2009), Jurado et al. (2015), Leduc and Liu (2016), and Dietrich et al. (2022) for empirical evidence that uncertainty, measured differently, have significantly negative aggregate demand effects.

[7]See, for instance, Binder (2017); Bayer et al. (2019); Coibion et al. (2021).

[8]Basu and Bundick (2017).

Sticky Expectations (SE), and Noisy Information (NI) predict that ex-ante uncertainty contains additional components in addition to the conditional volatility given a perfectly updated information set. In SE, the extra uncertainty arises compared to in FIRE because of lagged updating of the most recent realized information, hence a slower resolution of uncertainty. It results in a higher uncertainty than the volatility of the unrealized shock. Meanwhile, the variance of ex-post forecast errors is reduced compared to FIRE as information rigidity attenuates the average forecast responses to shocks to inflation. These patterns are actually observed in both SCE and SPF.

In NI, the uncertainty endogenously depends on the noisiness of signals and determines agents' degree of reaction to the news in the Kalman filtering problem. The model predicts both uncertainty and variance of forecast error to be greater than the conditional volatility of the inflation due to the presence of noises. However, it accommodates a flexible relative size between the two depending on the nosiness of signals. The model-consistent noisiness of signals hence becomes a quantitative question.

Different from models of information rigidity, the canonical model of Diagnostic Expectation (DE), although assuming that agents extrapolatively revise their mean forecast of the variable, keeps the uncertainty equal to that in FIRE. At the same time, DE predicts an attenuated variance of forecast errors due to the mean-reverting nature of the overreaction to persistent shocks. A hybrid of Diagnostic Expectations and Noisy Information (DENI) proposed by Bordalo et al. (2020) entails both overreaction mechanisms and dispersed noisy information. Uncertainty is unambiguously larger than the conditional volatility of the variable due to the presence of noisy information. Meanwhile, the variation of forecast error could be either attenuated or amplified depending on the parameter values of the model.

These predictions allow for a structural estimation that jointly uses various moments from survey data. The headline finding from doing such is that using additional information from uncertainty not only confirms the previous empirical finding of information rigidity using low-order moments in the literature (Coibion and Gorodnichenko (2012, 2015); Coibion et al. (2018)), but also favors the SE over NI mechanism as a source of information rigidity.

In particular, SE consistently generates a sensible updating rate of around one-third per period for both types of agents, consistent with the many estimates in the literature. In contrast, the estimated nosiness of public and private signals $\sigma_\epsilon$ and $\sigma_\xi$ in NI are unrealistic high, and unstable, i.e. at a ballpark value of 3 percentage points or higher. These are arguably too high given an unconditional standard deviation of inflation of 0.8 (for headline CPI) or 0.4 for (Core PCE) in the sample period. The intuition behind the poor fit of canonical NI is that Kalman filtering requires agents to efficiently decide their responsiveness to new information based on the prior uncertainty and nosiness of information. Therefore, effectively, only extremely imprecise signals could result in persistent information rigidity as we see in the data.

In addition to the evidence for rigidity, the estimates of DE and DENI do suggest a coexistent overreacting mechanism at the individual level, i.e. a non-zero fraction of agents in the economy has a positive degree of overreaction $\hat{\theta}$. This is consistent with the finding in the literature showing the coexistence of rigidity and overreaction (Angeletos et al. (2021); Kohlhas and Walther (2021)).

The baseline estimation assumes a quite stylized data-generating process of AR(1), this was not very ill-suited for characterizing the baseline sample period in this paper, the two-decade low-inflation era till 2020. But to better capture the time-varying dynamics of inflation and expectation moments over the period of rising inflation, I turn to a more realistic inflation process featuring both permanent and transitory components of time-varying volatility, as estimated in Stock and Watson (2007) and many following studies. Such a formulation of multi-component and time-varying volatility significantly improves the within-model consistency of all models under consideration. This is consistent with the mounting evidence that inflation is in fact subject to shocks with different persistence, and the perceived relative importance between various components is often one source of disagreement among macroeconomists, not to mention ordinary households.

With the SV formulation, I also alter the estimation across low and high inflation periods to examine the possible state dependence of expectation formation. This follows from a long list of studies that show that the information rigidity and degree of overreaction may just ultimately hinge on the underlying macroeconomic conditions. Through the lens of SE, both households and professionals have increased the updating rate of new information in the recent period. The estimates of DENI, however, seem to suggest divergent patterns of the two types of agents. In particular, professionals have shifted from overreaction on average and relatively precise individual information [9] to underreaction and more dispersed information in the high inflation episode. Households, in contrast, have become overreactive as inflation has elevated in recent years, consistent with the intuitive pattern that news coverage and social interactions have significantly resulted in household attentiveness to inflation news.

In addition to the structural estimation, the paper also shows two empirical patterns and reduced-form regression results involving uncertainty that are in violation of FIRE. First, the persistent cross-sectional dispersion of uncertainty across individual forecasters (Figure 2), like that in the average forecasts, is inconsistent with the FIRE as the latter assumes that agents agree on the data generating process and have the common knowledge of available information. Second, dynamically, across different forecast vintages, the uncertainty revision is a measure of information gain or the degree of forecasting efficiency. FIRE implies that information gain is sufficiently large to match the volatility of realized shocks, but information rigidity implies inefficient revisions in uncertainty. But, empirically, this paper shows that the uncertainty revisions have a serial correlation that is not consistent with the level of forecast efficiency predicted by rational expectation.

## Related Literature

This paper is related to four strands of literature. First, it is related to a series of empirical studies directly testing and evaluating various theories on expectation formation using survey data. For instance, Mankiw and Reis (2002), Mankiw et al. (2003), Carroll (2003), Branch (2004), etc., were early examples of such work. More recent examples include Coibion and Gorodnichenko (2012, 2015); Coibion et al. (2018) that test common implications of various theories with different micro-foundations. In addition to testing particular sets of theories, there are also a number of papers that show people's

[9] Consistent with the findings of Bordalo et al. (2018).

expectations are driven by individual heterogeneity such as socioeconomic characteristics, cognitive abilities, experiences of macroeconomic histories (Malmendier and Nagel (2015), Das et al. (2017) and D'Acunto et al. (2019)[10]). In terms of the methodology, this paper is closest to Giacomini et al. (2020), which estimates theories of expectation formation using cross-moment restrictions. However, all of these studies simply rely upon point forecasts instead of density forecasts or surveyed uncertainty. This is one theme on which this paper differs from the existing literature.

Second, this paper is related to the macroeconomic literature on measuring uncertainty, especially those using survey data.[11] Various proxies of uncertainty that have often been used include ex-ante cross-sectional disagreement (Bachmann et al., 2013), *approximated* conditional volatility based on time-series forecasting (e.g. Jurado et al. (2015)), and ex-post forecast errors (Bachmann et al., 2013; Rossi and Sekhposyan, 2015). Some studies empirically evaluated the correlation between the aforementioned proxies and the uncertainty measured by the dispersion of density forecasts. Zarnowitz and Lambros (1987) made a clear conceptual distinction between disagreement and uncertainty, and found a very low correlation between the two in early sample of SPF. Follow-up studies (Rich and Tracy, 2010; D'Amico and Orphanides, 2008; Abel et al., 2016; Glas, 2020; Rich and Tracy, 2021) echoed such a finding, mostly based on SPF data, although Bomberger (1996); Giordani and Söderlind (2003); Lahiri and Sheng (2010) arrive at different conclusions. One key point often overlooked or taken for granted by this literature is that the relationship between various ex-ante uncertainty, ex-post forecast errors and disagreement depends on the mechanisms of expectation formation. My paper makes such assumptions explicitly when evaluating the relationships.

Third, Manski (2004), Delavande et al. (2011), Manski (2018) and many other papers have long advocated for eliciting probabilistic questions measuring subjective uncertainty in economic surveys. Although the initial suspicion concerning people's ability in understanding, using, and answering probabilistic questions is understandable, Bertrand and Mullainathan (2001) and other work have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) have a thorough discussion on designing, experimenting, and implementing the consumer expectation surveys to ensure the quality of the responses.[12] Broadly speaking, the literature has argued that going beyond the revealed preference approach, availability of survey data provides economists with direct information on agents' expectations and helps avoid imposing arbitrary assumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only

[10]See D'Acunto et al. (2023) for a thorough survey of the empirical evidence of heterogeneous inflation expectations and their drivers.

[11]Survey-based uncertainty measures are among various methods seen in the literature such as using news texts (Bloom, 2009), econometric methods (Jurado et al. (2015)), and market derivatives (e.g. VIX index), as summarized in Cascaldi-Garcia et al. (2023). Besides, Binder (2017) creates a novel measure using *household* survey data based on the insight from cognitive science that people tend to round numbers when facing with higher uncertainty.

[12]Others include Van der Klaauw et al. (2008) and Delavande (2014), etc. See Bassetti et al. (2023) for a complete survey on methods of extracting information from density forecasts and their macroeconomic applications.

the expectation but also the perceived risks matter a great deal.

Finally, the literature that has been originally developed under the theme of forecast efficiency (Nordhaus, 1987; Davies and Lahiri, 1995; Clements, 1997; Faust and Wright, 2008; Patton and Timmermann, 2012) provides a framework analyzing the dynamics of uncertainty useful for the purpose of this paper. The focus of the forecasting efficiency literature is evaluating forecasters' performance and improving forecasting methodology, but it can be adapted to test the theories of expectation formation of different types of agents. This is especially relevant to this paper as I focus on uncertainty.

The paper is organized as followed. Section 2 shows the stylized patterns of different forecasting moments of professional forecasts and households. Section 4 first sets up a unified framework in which testable predictions of different theories can be compared. Also, I derive various moment conditions from these theories. Section 3 undertakes reduced-form time-series regressions that test the null hypothesis of FIRE and the implications of different theories. Section 5 includes results from estimating the theory-specific parameters using the simulated method of moments. It also evaluates the sensitivity of the model specification. Section 7 concludes the paper and discusses the future research directions.

# 2 Theoretical Benchmark and Basic Facts

## 2.1 Full-information rational expectation (FIRE)

Assume the underlying data generating process of $y_t$ is $AR(1)$ with a persistence parameter $0 < \rho < 1$ and i.i.d. shock $\omega_t$ whose time-invariant volatility is $\sigma_\omega$.

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma^2_\omega) \tag{1}$$

FIRE benchmark assumes that all agents perfectly observe $y_t$ at time $t$ and understand the true process of $y$. Therefore, the individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to the average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ and $t+h$.

$$\overline{FE}^*_{t+h|t} = -\sum_{s=1}^{h} \rho^{s-1}\omega_{t+h+1-s} \tag{2}$$

I use the superscript of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time $t$. This provides a well-known null hypothesis of FIRE.[13]

[13] Another well-known prediction of FIRE is that forecast errors of non-overlapping horizon are not correlated, namely, $Cov(\overline{FE}^*_{t+h|t}, \overline{FE}^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h$. This is not the case within $h$ periods, as the realized shocks in overlapping periods enter both forecast errors.

The unconditional variance of h-period-ahead FE, or equivalently, the expected value of its square (due to zero unconditional mean), is equal to the following. ($\bullet$ indicates that it is unconditional on $t$.)

$$\overline{FE}^{*2}_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)} \sigma^2_\omega \tag{3}$$

The uncertainty about future $y$ simply comes from uncertainty about unrealized shocks between $t$ and $t+h$. With the same model in mind (Equation 1) and the same information $y_t$, everyone's uncertainty is equal to the weighted sum of the future volatility before its realization (Equation 4), which is exactly equal to the variance of forecast errors, $\overline{FE}^{*2}_{\bullet+h|\bullet}$.

$$\overline{\mathrm{Var}}^{*}_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)} \sigma^2_\omega \tag{4}$$

Another testable implication of rationality lies in the revision of uncertainty. Moving from $t$ to $t+1$, for instance, the revision in uncertainty is simply a negative constant independent of the time. (Equation 5) There is an unambiguous reduction in uncertainty (or information gain in the forecasting literature) as more and more shocks have realized. In the most intuitive case, from the one-step-ahead forecast at $t-1$ to that of $t$, i.e. $h = 1$, the variance drops exactly by the resolution of the uncertainty of $\omega_t$, which is $\sigma^2_\omega$, to zero uncertainty.

$$\overline{\mathrm{Var}}^{*}_{t+h|t+1} - \overline{\mathrm{Var}}^{*}_{t+h|t} = -\rho^{2(h-1)} \sigma^2_\omega \tag{5}$$

Lastly, FIRE predicts a zero disagreement, and it is so regardless of the behaviors of forecast errors and uncertainty.

$$\overline{Disg}^{*}_{\bullet+h|\bullet} = 0 \quad \forall t \tag{6}$$

## 2.2 Density survey of inflation

This paper uses density forecasts of inflation by professionals and households, where respondents are asked to assign probabilities to various ranges of values of future inflation.

*Survey of Professional Forecasters* (SPF) collects professionals' individual density forecasts of core CPI and core PCE inflation since 2007.[14] In each quarter, density forecasts of fourth-quarter-to-forth-quarter inflation in the current year and next year are elicited. As a result, both quarterly revisions and annual revisions can be calculated. This makes it possible to directly test the implications of the revisions in

[14]Previous studies used an extended sample of the density forecast of GDP deflator starting from 1968 in the predecessor of SPF, or the NBER-ASA Economic Outlook Survey.

uncertainty. In addition, because the forecasts are reported in all quarters of a year regarding Q4/Q4 inflation, the forecast horizons change within a year.

The New York Fed's *Survey of Consumer Expectations* (SCE), which started in 2013, also asked households to report their distribution forecasts of 1-year- and 3-year-ahead inflation for various ranges of values each month.[15] This allows for comparing the 3-year-ahead forecast at time $t-3$ with the 1-year-ahead forecast at $t-1$. Since the maximum duration for households to stay in the panel is 12 months (for about one-third of the households), forecast revision can only be examined at the population level. One major difference between SCE and SPF is that the former elicits fix-horizon expectations, instead of fix-event.[16]

Converting expressed probability forecasts based on externally divided bins into an underlying subjective distribution requires a density estimation. I closely follow Engelberg et al. (2009)'s method with a small modification to estimate the density distribution of each individual respondent in SPF.[17] For SCE, I directly use the estimates by the New York Fed (Armantier et al., 2017), following the same method.

To address potential biases from extreme answers or data errors, I remove outliers from both the top and bottom one percent of the SPF sample's mean forecast and uncertainty estimates and three percent of the SCE's sample, respectively.[18]

## 2.3 A first look at the data

Table 1 reports the empirically computed moments of professionals and households. For the benchmark assumption of AR(1) process of inflation, I restrict the sample to be before March 2020, in which inflation remained stably low. This was also the period for which most of the empirical results showing the deviations from FIRE in inflation expectations were established.

Regardless of the realized inflation volatility, the survey moments alone exhibit patterns inconsistent with the predictions of FIRE: *ex-ante* uncertainty is sizably larger than the average squared *ex-post* forecast error in both SPF and SCE, in addition to a positive disagreement.

Additional patterns emerge when an estimated inflation process is considered. The quarterly core CPI inflation during the sample period had persistence of $\hat{\rho}=0.99$ and volatility $\hat{\sigma}_\omega=0.23$. The conditional volatility of 4-quarter-ahead annualized inflation, $\sum_{s=1}^{4}\rho^{2(s-1)}\sigma^2_\omega$, is therefore, approximately equal to 0.162. This should be equal to the size of Var and variance of FE under FIRE. In the data, however, quarterly professional forecasts of the core CPI inflation have Var of 0.213, which is larger than conditional

[15] The survey respondents are guaranteed to assign probabilities to all bins that sum up to one, as a feature of the online survey design.

[16] See Clements et al. (2023) for a detailed discussion of these differences in survey structure.

[17] Answers with at least 3 bins with positive probabilities or 2 bins but open-ended from either left or right are fit with a generalized beta distribution. Depending on if there is an open-ended bin on either side with positive probability, a 2-parameter or 4-parameter beta distribution is estimated, respectively. Those with only two bins with positive probabilities and adjacent are fit with a triangular distribution. Answers with only one bin of positive probability are fit by a uniform distribution. See the Python program with detailed steps of estimation.

[18] This entails dropping 6,528 observations for mean forecasts and 5,096 observations for uncertainty, out of a total of 68,887 observations.

volatility. Meanwhile, $FEVar = 0.136$, is smaller than the unconditional volatility. In addition, Disg is 0.161, significantly positive instead of zero like in FIRE.

Household expectations exhibit different rankings of these moments. During the sample period of 2013-2020, the monthly headline CPI inflation is estimated to have a persistence parameter of 0.98 and volatility of 0.45.[19] This, when translated into a 12-month-ahead annualized inflation forecast, implies a conditional volatility of 1.972. Meanwhile, household forecasts had a variance of FE of 0.935, a Disg of 2.805, and Var of 1.75. Both the variance of FE and Var are smaller than the conditional volatility. Note that these are computed based on residuals by first controlling for the individual fixed effects. This follows from the rich empirical evidence[20] that individual-specific effects such as demographics and experience result in persistent differences in expectations.

We will revisit these stylized patterns through the lens of alternative theories of expectation formation discussed in Section 4.

Table 1: Moments of Inflation and Expectations

| | SPF | SCE |
|---|---|---|
| InfAV | 0 | 0 |
| InfVar | 0.219 | 1.282 |
| InfATV | 0.194 | 1.206 |
| FE | 0.125 | 1.812 |
| FEVar | 0.136 | 0.935 |
| Disg | 0.161 | 2.805 |
| Var | 0.213 | 1.749 |

This table reports the moments of demeaned realized inflation and inflation expectations, respectively, for the SPF's core CPI forecast and SCE's headline CPI forecast. The sample period is 2007M1-2020M3 for SPF and 2013M1-2020M3 for SCE. For SCE moments, both disagreement (Disg) and uncertainty (Var) are computed using the regression residuals of individual mean forecast and uncertainty after controlling for individual fixed effects.

## 2.4 Time variations of the FE, Disg, and Var

Despite the stark differences in magnitudes between professionals' and households' forecasting moments, both types of agents share common patterns in terms of the relationship across various moments. Figure 1a, 1b, and 1c plot the population uncertainty against expected inflation, squared forecast errors, and disagreements, respectively.

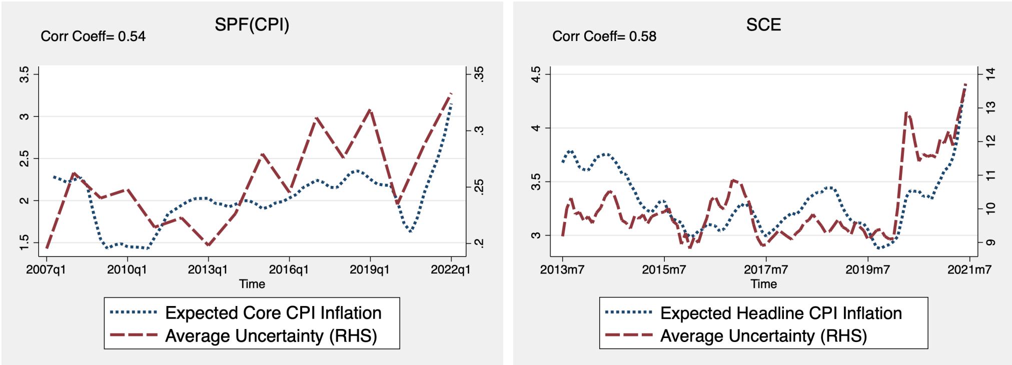
Both academic research (Friedman, 1977; Ball et al., 1990; Ball, 1992) and anecdotal evidence indicate that higher inflation or expected inflation is generally associated with greater inflation uncertainty. This relationship is partially reflected in the positive correlation observed between expected inflation and directly measured forecasting

[19] The higher volatility compared to that of core CPI is due to a higher frequency and inclusion of more volatile items in the CPI basket.

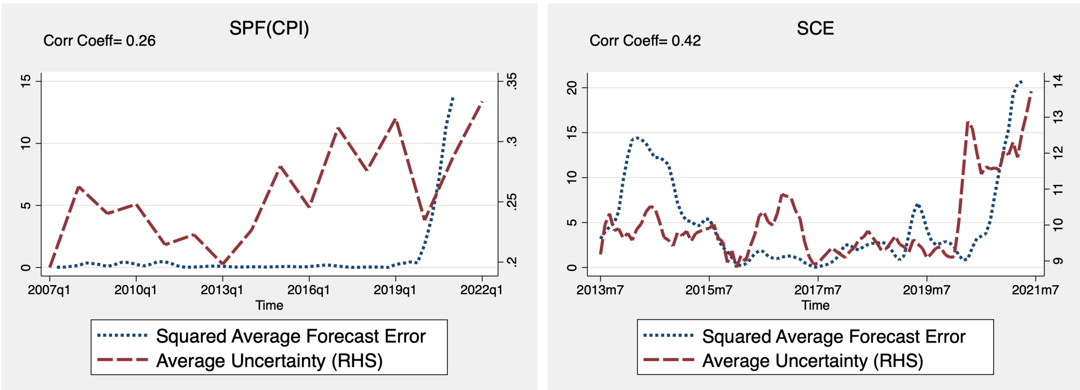
[20] Malmendier and Nagel (2015), Das et al. (2017), D'Acunto et al. (2019).

Figure 1: Uncertainty and Other Moments

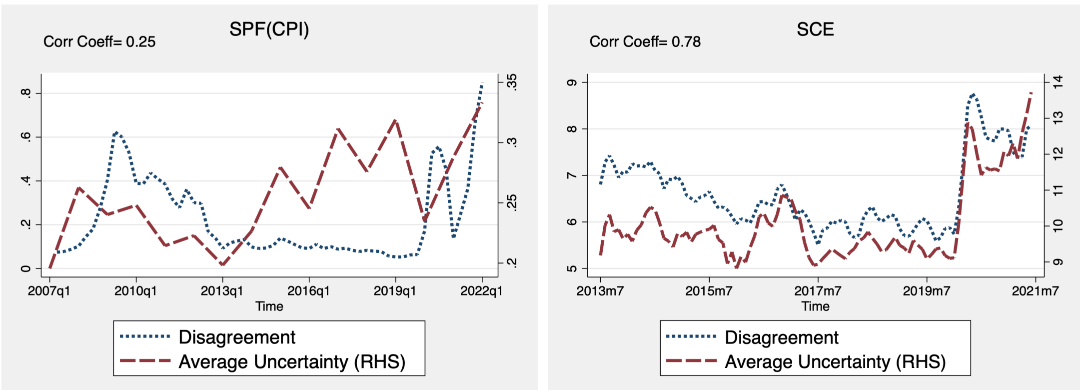
(a) Expected Inflation and Uncertainty




(b) Squared Forecast Errors and Uncertainty




(c) Disagreement and Uncertainty




Note: From the left to right: SPF's forecasts of core CPI and SCE's household forecast of headline CPI. From the top to the bottom: uncertainty (dash line) versus expected inflation (dot) with a correlation coefficient of 0.54, and 0.58, respectively; uncertainty (dash) versus square of the realized forecast errors (dot) with a correlation coefficient of 0.26, 0.42, respectively; uncertainty (dash) versus disagreements (dot) with a correlation coefficient of 0.25, and 0.78, respectively.

uncertainty among both households and professional forecasters, with correlation coefficients of 0.54 for SPF and 0.58 for SCE between 2007-2021. However, it is important to note that this correlation is primarily driven by recent dynamics since 2020, with the onset of higher inflation. Prior to 2020, the correlation coefficients were notably smaller (0.15 for SPF and -0.24 for SCE).

Figure 1b inspects the relationship between the size of the forecast error and uncertainty. According to the benchmark prediction under FIRE, the ex-ante forecast uncertainty is equal to the variance of ex-post forecast errors on average, as shown in Table 1. In the data, the correlation coefficients of the two are 0.26, and 0.42 for SPF Core CPI forecasts and SCE's forecasts, respectively. Excluding the post-2020 sample yields even smaller correlation coefficients. The fact that the two are at most weakly correlated is inconsistent with the FIRE benchmark prediction.

Figure 1c plots the relationship between disagreement and uncertainty. A large body of empirical literature in macroeconomics uses disagreement, which is often more available than uncertainty, as a proxy of the latter. This practice implicitly assumes some form of deviation from the benchmark FIRE, as in FIRE, regardless of the inflation process, disagreement should be always zero, and it is therefore not correlated with the average uncertainty.[21] The empirical correlation between disagreement and uncertainty is indeed weakly positive, which is 0.25 for professionals, and 0.78 for households. The positive correlation between the two was primarily driven by the post-2020 dynamics, which saw rapidly rising inflation.

To summarize, compared to the FIRE prediction of a perfect correlation between ex-ante uncertainty and the size of ex-post forecast error in the benchmark FIRE, the empirical patterns of professionals and household expectations exhibit divergent and time-varying behaviors of the two. In particular, the uncertainty is more correlated with expected inflation, disagreement, and size of forecast errors when the level of inflation is already high.

## 2.5 Cross-sectional dispersion of forecast uncertainty

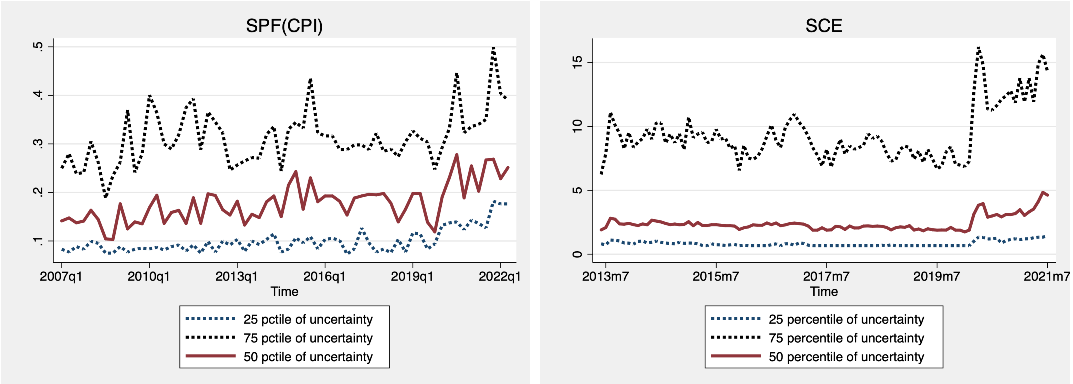
Persistent dispersion in expectations has been among the most commonly cited evidence that is inconsistent with the assumption of identical expectations predicted by FIRE (Mankiw et al. (2003)). A similar argument can be made with the dispersion in forecasting uncertainty, as FIRE predicts individuals share an equal degree of uncertainty.[22]

[21] Several studies since Zarnowitz and Lambros (1987) found conflicting evidence about the correlation between disagreement and uncertainty. (Bomberger, 1996; Giordani and Söderlind, 2003; Rich and Tracy, 2010; Lahiri and Sheng, 2010; D'Amico and Orphanides, 2008; Binder, 2017; Glas, 2020; Rich and Tracy, 2021). Most of the comparisons are based on professional forecasters, with the exception being Binder (2017), which, by measuring the uncertainty based on rounding, found a high correlation between the two measures. More recently, Manski (2018) points out that much empirical work has confused dispersion with uncertainty.

[22] In contrast, SE predicts that the uncertainty of individuals differs in that agents are not equally updated at a point in time. NI generates a homogeneous degree of uncertainty only under the stringent conditions of equal precision of signals and the same prior for uncertainty (Equation 24). DE predicts an equal degree of uncertainty across agents (Equation 30). Therefore, taken by the face value, the presence of dispersion of uncertainty across agents is not consistent with predictions of FIRE, or the canonical version of NI and DE.

Figure 2 plots the median uncertainty along with its 25/75 percentiles in both SCE and SPF. There is persistent dispersion in uncertainty across agents. The dispersion in uncertainty of households is much greater than that of the professionals. The IQR of the uncertainty of households is around 150-200 times(12–14 times in standard deviation terms) of that of professional forecasters.

Figure 2: Dispersion of Uncertainty




One difference in the distribution of uncertainty between households and professionals is that the distribution of the former is more skewed toward the right (higher uncertainty), while professional forecasters disagree in uncertainty more symmetric around its cross-sectional mean.

Another pattern worth discussing in Figure 2 is that there is a notable rise in the dispersion of uncertainty along the rapid rise in inflation in 2020, which was primarily driven by an increase in uncertainty reported in the upper end of the forecasts.

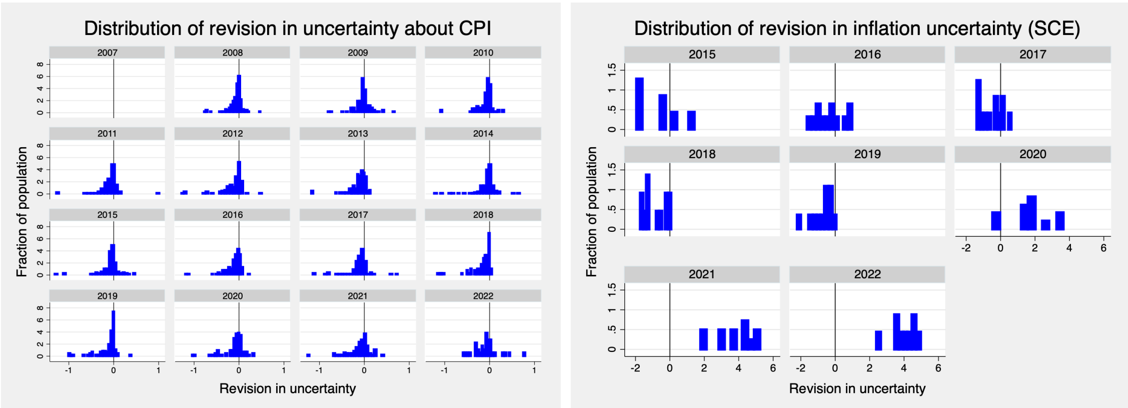
## 2.6 Revision in uncertainty

Under AR(1) process of inflation, FIRE predicts an unambiguous reduction in uncertainty as one approaches the date of realization, where the drop is exactly equal to the conditional volatility of the realized shocks $\sigma^2_\omega$.

Figure 3 plots the average revision in uncertainty in SPF and SCE. Since the individual-specific revisions are not available in SCE, I instead calculate the revisions in the average uncertainty across all respondents, or, more specifically, the difference between 1-year-ahead uncertainty and 3-year-ahead forecasts made two years ago. In most of the years, both histograms suggest that revisions are left-skewed relative to zero. This implies, on average, forecasters feel more certain about their nowcasts relative to their forecasts made before. However, over the entire sample, there is always a positive fraction of forecasters who revise uncertainty upward, which is inconsistent with the benchmark prediction with the AR(1) process of inflation. Positive uncertainty revisions were particularly common in the sample after 2020, a period with rapidly rising inflation due to a combination of various demand and supply shocks.

It is worth noting that the existence of positive uncertainty revisions does not necessarily reject the FIRE assumption in more general conditions. The pattern could also be reconciled by alternative models where the inflation volatility is stochastic instead of deterministic. With the former scenario, newly arrived information may cause an upward revision in the conditional perceived uncertainty of inflation, even though uncertainty in the period elapsed has resolved. Therefore, in the later part of this paper, I explore the patterns of uncertainty in conjunction with the alternative assumption of stochastic volatility.

Figure 3: Distribution of Uncertainty Revision




Note: the revisions in SPF (left) are calculated at the individual level and is the difference between forecast uncertainty in quarter $q$ about current-year q4/q4 inflation and that of the next-year q4/q4 inflation in quarter $q-4$. The revisions in SCE (right) are calculated at the population level, and it is the difference of the average 1-year-ahead uncertainty at month $m$ and the 3-year-ahead uncertainty at $m-24$.

# 3 Reduced-form Tests of FIRE Using Uncertainty

This section presents additional evidence for information rigidity based on uncertainty, in the spirit of forecasting efficiency by Nordhaus (1987). It is an extension of revision tests on mean forecasts by Fuhrer (2018) to uncertainty.[23]

In plain words, the revision is efficient if the following two criteria are satisfied: (1) forecast revision does not depend on past information, including the past revisions; (2) the drop in uncertainty of all individual forecasters are identical and sufficiently rapid to reflect the volatility of the realized shocks.

The mean revision test takes a similar form of that used in Fuhrer (2018). To convey the intuition the most easily, we first use 1-step forecast-to-nowcast revision as an example.

$$y_{i,t|t} - y_{i,t|t-1} = \alpha + \beta(y_{i,t-1|t-1} - y_{i,t-1|t-2}) + \psi_t + \zeta_{i,t} \tag{7}$$

[23]As a validation, I first reproduce a number of reduced-form statistical tests of FIRE only using information from forecast errors primarily following Mankiw et al. (2003), and the results are reported in Table 9 in Appendix 7. Consistent with the existing findings, the results reject the null hypothesis of unbiasedness in forecasts, non-serial correlation of non-overlapping forecast errors, and efficient use of information in forecasting.

In the above equation, $\beta = 0$ according to FIRE, because rational forecast revision only responds to newly realized shocks, thus it is not predictable by past revisions. Time-fixed effect $\phi_t$ is included to capture any innovation in time $t$ to the common information set of all forecasters that induce rational revisions.

The test based on uncertainty simply replaces the revision of forecast with the revision in uncertainty, as shown below.

$$\mathrm{Var}_{i,t|t} - \mathrm{Var}_{i,t|t-1} = \alpha^{\mathrm{var}} + \beta^{\mathrm{Var}}(\mathrm{Var}_{i,t-1|t-1} - \mathrm{Var}_{i,t-1|t-2}) + \psi_t^{var} + \zeta_{i,t}^{var} \tag{8}$$

Equation 5 predicts that under FIRE, individual uncertainty revisions are all identical and equal to the innovation of the conditional volatility of inflation. This means that under FIRE the size of revisions in uncertainty is the same by all forecasters and, hence should be fully absorbed by either the time-invariant constant $\alpha^{var}$ or the time-varying fixed effect $\psi_t^{var}$. Meanwhile, the auto-correlation coefficient $\beta^{\mathrm{var}}$ takes the value of zero under FIRE. A higher value of $\beta^{\mathrm{var}}$ indicates a slower speed of the drop in uncertainty, or forecast inefficiency, possibly due to information rigidity.[24]

The two aforementioned regressions need to be adapted to be strictly consistent with the specific data structure in SPF and SCE. In particular, the revision in SPF is computed between the forecasts of the current-year q4/q4 inflation and the forecasts made 4 quarters before regarding the next-year q4/q4 inflation. The lagged revision, a measure of past information, was made 4 quarter before. For SCE, revisions and lagged revisions are regarding forecasts made 24 months before. This is critical as revisions are expected to be correlated within the forecast horizon even under the assumption of FIRE. Furthermore, since individual revisions are not observed in SCE, I can only run regressions using average expectations and uncertainty. Hence, I cannot control for time-fixed effects.

The top panel of Table 2 reports the results for mean revisions (InfExp_Mean_rv). The first column of each panel regards the regression on a constant. Average revisions of forecasts of CPI and PCE by SPF inflation are both negative and significant, indicating an average downward revision in over the sample period. The second to fourth columns of each panel in the upper panel examine the dependence of revisions on past information beyond forecast horizons. In particular, revisions are negatively correlated with the median SPF forecasts made 4 quarters before (InfExp_Mean_ct50), and are also serially correlated 4 or 5 quarters apart, and the coefficients are positive and significant. This implies that individual revisions in forecasts react to lagged information, some evidence against the null hypothesis of FIRE.

Similar to professionals, average household revisions in SCE also positively depend on past revisions made 2 years before. Since individual revisions are not available, time-fixed effect cannot be controlled to absorb all common contemporary innovations. Instead, I control for the average forecast error (InfExp_FE) that has just realized in the same period, meant to capture innovations in the information set common to all forecasters. It is found to be negatively correlated with average revisions, implying information rigidity.[25]

[24] This interpretation corresponds to Equation 13 for SE and Equation 25 for NI.

[25] Coibion and Gorodnichenko (2012) shows that information rigidity implies that past forecast

The bottom panel reports the new results of this paper using revision in uncertainty. (InfExp_Var_rv) The first column tests the mean revision against the null hypothesis of zero. For professional forecasters, the mean revisions in uncertainty are negative and statistically significant, confirming our observation from Figure 3 that forecasters are more certain about current inflation compared to her previous year forecast.

The second to fourth column in bottom panel of Table 2 show that revision in uncertainty of non-overlapping forecasts are serially correlated in both SPF and SCE. SPF forecasters' uncertainty revision from one year ago positively predict current revisions in uncertainty. With the aggregate information innovation to be absorbed by time-fixed effects and the constant, the coefficients of individual past revisions remain significant.

Households in SCE exhibit similar patterns of rigidity despite some differences with SPF professionals. In particular, the average revision in uncertainty between 3-year-ahead to 1-year-ahead forecasts is significantly negative, when past revisions and squared realized forecast errors are controlled (InfExp_FE2). This implies that on average, households are also more certain in their 1-year forecast than in 3-year-ahead forecast. In addition, past uncertainty revisions are significantly correlated with current revisions, although negatively. Upward revisions 1 year ago are usually followed by downward revisions later. The coefficients remain significant even though I control for the size of realized average forecast errors over the past year. Higher realized squared forecast errors predict a larger uncertainty revision. This is different from the prediction of FIRE, according to which the two, on average, should be negatively correlated one by one, i.e. the resolution of forecast errors is equal to the reduction in uncertainty over the same period.

In summary, the empirical tests in this section use the uncertainty revision to show additional evidence for the deviation from FIRE in expectation formation. In particular, information rigidity of incorporating new information implies inefficiency of revisions in forecasts and a drop in uncertainty. The reduced-form results do confirm this pattern. The next section proceeds to compare a variety of candidate theories of expectation formation that may lead to such patterns.

# 4 Alternative Theories of Expectation Formation

## 4.1 Sticky expectation (SE)

The theory of sticky expectation (Mankiw and Reis (2002), Carroll (2003)), regardless of its micro-foundation[26], builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One tractable assumption is that agents update their information with a homogeneous and time-independent probability, denoted by $\lambda$. Specifically, at any point of time $t$, each agent learns about the up-to-date realization of $y_t$ with the probability of $\lambda$; otherwise, they form the expectation based

errors and current revisions are positively correlated, where forecast errors are defined as the opposite sign as in this paper.

[26]For instance, Mankiw and Reis (2002) models SE a result of individual attention choice subject to information cost, while Carroll (2003) models SE as a gradual diffusion of information among the population.

Table 2: Tests of Revision Efficiency Using Mean Revision and Uncertainty

| | SPF CPI | | | | SPF PCE | | | | | SCE | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Test 1. Revision efficiency of mean forecast | | | | | | | | | | | | | |
| | Mean revision | Past info | 4q before | 5q before | Mean revision | Past info | 4q before | 4-5 q before | | Mean revision | 24m before | 25m before | 26m before |
| L4.InfExp_Mean_ct50 | | -0.512* | | | | -0.503 | | | L24.InfExp_Mean_rv | | 0.233* | | |
| | | (0.227) | | | | (0.280) | | | | | (0.100) | | |
| L4.InfExp_Mean_rv | | | 0.211*** | | | | 0.265*** | | L25.InfExp_Mean_rv | | | 0.225* | |
| | | | (0.047) | | | | (0.052) | | | | | (0.103) | |
| L5.InfExp_Mean_rv | | | | 0.218*** | | | | 0.265*** | L26.InfExp_Mean_rv | | | | 0.254* |
| | | | | (0.046) | | | | (0.054) | | | | | (0.098) |
| | | | | | | | | | InfExp_FE | | -0.330*** | -0.327*** | -0.331*** |
| | | | | | | | | | | | (0.046) | (0.047) | (0.047) |
| Constant | -0.075*** | 0.994* | -0.028*** | -0.016 | -0.036*** | 0.888 | 0.001 | 0.026*** | Constant | -0.106 | 0.504*** | 0.510*** | 0.530*** |
| | (0.000) | (0.487) | (0.007) | (0.007) | (0.000) | (0.528) | (0.007) | (0.007) | | (0.054) | (0.074) | (0.080) | (0.080) |
| R2 | 0.376 | 0.030 | 0.439 | 0.441 | 0.393 | 0.027 | 0.461 | 0.461 | R2 | 0.000 | 0.595 | 0.600 | 0.616 |
| N | 1697 | 1697 | 1290 | 1149.000 | 1362 | 1362 | 1033 | 924.000 | N | 74 | 50 | 49 | 48 |
| Time FE | Yes | No | Yes | Yes | Yes | No | Yes | Yes | Time FE | No | No | No | No |
| Test 2. Revision efficiency of uncertainty | | | | | | | | | | | | | |
| | Mean revision | 4q before | 4q before | 5q before | Mean revision | 4q before | 4q before | 4-5 q before | | Mean revision | 24m before | 25m before | 26m before |
| L4.InfExp_Var_rv | | 0.448*** | 0.456*** | | | 0.384*** | 0.395*** | | L24.InfExp_Var_rv | | -0.679** | | |
| | | (0.056) | (0.058) | | | (0.044) | (0.044) | | | | (0.202) | | |
| L5.InfExp_Var_rv | | | | 0.440*** | | | | 0.406*** | L25.InfExp_Var_rv | | | -0.740** | |
| | | | | (0.053) | | | | (0.042) | | | | (0.216) | |
| | | | | | | | | | L26.InfExp_Var_rv | | | | -0.874*** |
| | | | | | | | | | | | | | (0.201) |
| | | | | | | | | | InfExp_FE2 | | 0.510*** | 0.568*** | 0.600*** |
| | | | | | | | | | | | (0.101) | (0.092) | (0.091) |
| Constant | -0.091*** | -0.049*** | -0.048*** | -0.049*** | -0.079*** | -0.051*** | -0.050*** | -0.053*** | Constant | 0.114 | -1.121*** | -1.237*** | -1.366*** |
| | (0.000) | (0.008) | (0.005) | (0.005) | (0.000) | (0.007) | (0.003) | (0.003) | | (0.180) | (0.247) | (0.204) | (0.237) |
| R2 | 0.047 | 0.196 | 0.248 | 0.249 | 0.054 | 0.145 | 0.215 | 0.204 | R2 | 0.000 | 0.372 | 0.402 | 0.448 |
| N | 1529 | 1157 | 1157 | 1021 | 1439 | 1091 | 1091 | 960 | N | 74 | 50 | 49 | 48 |
| Time FE | Yes | No | Yes | Yes | Yes | No | Yes | Yes | Time FE | No | No | No | No |

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

on the most recent up-to-date realization of $y_{t-\tau}$, where $\tau$ is the elapsed time since the last update.

The average forecast under SE is a weighted average of up-to-date rational expectation and lagged average expectation, as reproduced below.[27] It can also be expressed as a weighted average of all the past rational forecasts of $y$. Setting $\lambda = 1$, then the SE collapses to FIRE, and the average forecast is equal to $y$'s long-run mean of zero.

$$\bar{y}^{se}_{t+h|t} = \lambda \underbrace{y^{*}_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}^{se}_{t+h|t-1}}_{\text{average forecast at } t-1} \tag{9}$$

It follows that the average forecast errors are serially correlated, as described in Equation 10.

$$\overline{FE}^{se}_{t+h|t} = (1-\lambda)\rho\overline{FE}^{se}_{t+h-1|t-1} + \lambda FE^{*}_{t+h|t} \tag{10}$$

The unconditional variance of the h-period-ahead forecast error (or its square) is proportional to that of the FIRE. It is also easy to confirm the former is always smaller than the latter as long as there is stickiness ($\lambda < 1$). Intuitively speaking, stickiness in expectation implies attenuated responses to inflation shocks than in FIRE, hence a lower variation in forecast errors across time.

$$\overline{FE}^{se2}_{\bullet+h|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2} FE^{*2}_{\bullet+h|\bullet} \leq FE^{*2}_{\bullet+h|\bullet} \tag{11}$$

Like average forecasts, average uncertainty at time $t$ is also a weighted average of uncertainty to agents whose last updates took place in different periods of the past: $t-\tau \quad \forall \quad \tau = 0, 1...\infty$. (Equation 12) Since at any point in time, there are agents who have not updated the recent realization of the shocks, thus with higher uncertainty, the population uncertainty is unambiguously higher than in the case of FIRE. (See Appendix 7 for detailed derivations.)

$$\begin{aligned}
\overline{\text{Var}}^{se}_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction of non-updater until } t-\tau} \underbrace{\text{Var}^{*}_{t+h|t-\tau}}_{\text{uncertainty based on updating by } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma^2_{\omega} \\
&= (1 - \frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2}) \frac{1}{1-\rho^2}\sigma^2_{\omega} \\
&\geq \overline{\text{Var}}^{*}_{t+h|t}
\end{aligned} \tag{12}$$

For example, the average uncertainty regarding 1-period-ahead inflation ($h = 1$), is equal to $\frac{\sigma^2_{\omega}}{1-(1-\lambda)\rho^2}$, which collapses to $\sigma^2_{\omega}$ when $\lambda = 1$ under FIRE, and takes a larger value for any $0 < \lambda < 1$.

[27]See Coibion and Gorodnichenko (2012) for detailed steps.

With respect to revision, the inefficiency of reducing uncertainty in SE takes the following form at the aggregate level. Since not all agents incorporate the recently realized shocks, the revision in average uncertainty exhibits a serial correlation, as described in Equation 13. It is a weighted average of the resolution of uncertainty from the most recent shocks and its lagged counterpart.

$$\overline{\text{Var}}^{se}_{t+h|t+1} - \overline{\text{Var}}^{se}_{t+h|t} = (1-\lambda)(\overline{\text{Var}}^{se}_{t+h|t} - \overline{\text{Var}}^{se}_{t+h|t-1}) - \lambda\rho^{2(h-1)}\sigma^2_\omega \tag{13}$$

In particular, the second component is the information gain from the most recent realization of the shock, underweighted by $\lambda < 1$. The first component is the inefficiency sourced from the stickiness of updating. The higher rigidity (lower $\lambda$), the smaller the efficiency gain or uncertainty reduction compared to FIRE.

Lastly, SE also predicts non-zero disagreements and sluggish adjustment compared to FIRE. This is because of different lags in updating across populations.

$$\overline{Disg}^{se}_{t+h|t} = \lambda \sum_{\tau=0}^{\infty}(1-\lambda)^\tau (y_{t+h|t-\tau} - \bar{y}_{t+h|t})^2 \tag{14}$$

In summary, SE predicts a higher average uncertainty and a lower expected forecast error square than their counterparts in FIRE, both of which should be identical to the conditional volatility of inflation under FIRE. In addition, SE predicts positive disagreements. These patterns are indeed observed in survey data, as reported in Table 1. Next, we move to other theories to see if such patterns are distinctive predictions by SE.

### 4.2 Noisy information (NI)

A class of models (Lucas (1972), Sims (2003), Woodford (2001), and Maćkowiak and Wiederholt (2009), etc), noisy information (NI hereafter), describes the expectation formation as a process of extracting the underlying variable $y_t$ from a sequence of realized signals.

I assume the same signal structure as in Coibion and Gorodnichenko (2015) by assuming that an agent $i$ observes two signals, $s^{pb}$ being a public signal common to all agents, and $s^{pr}_i$ being a private signal specific to the agent $i$. The generating process of two signals is assumed to be the following.

$$\begin{aligned} s^{pb}_t &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \\ s^{pr}_{i,t} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma^2_\xi) \end{aligned} \tag{15}$$

Stacking the two signals into one vector $s_{i,t} = [s^{pb}_t, s^{pr}_{i,t}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$, the equations above can be rewritten as

$$\begin{aligned} s_{i,t} &= Hy_t + v_{i,t} \\ &\text{where } H = [1,1]' \end{aligned} \tag{16}$$

Now any agent trying to forecast the future $y$ has to form her expectation of the contemporaneous $y$. Denote it as $y^{ni}_{i,t|t}$, which needs to be inferred from the signals particular to the agent $i$. The agent's best h-period ahead forecast is simply iterated $h$ periods forward based on the AR(1) process, and it is equal to $\rho^h y^{ni}_{i,t|t}$.

So the crucial difference between NI from FIRE lies in the nowcasting. In particular, the agent makes her best guess of $y_t$ using Kalman filtering at the time $t$. The nowcast of individual $i$ is the posterior mean based on her prior and realized signals $s_{i,t}$.

$$\begin{aligned} y^{ni}_{i,t|t} &= \underbrace{y^{ni}_{i,t|t-1}}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\ &= (1-PH)y^{ni}_{t|t-1} + PHy_t + Pv_{i,t} \end{aligned} \tag{17}$$

where the Kalman gain $P$ is a vector of size of two that determines the degrees of reaction to signals.

$$P = [P_\epsilon, P_\xi] = \text{Var}^{ni}_{i,t|t-1} H'(H'\text{Var}^{ni}_{i,t|t-1}H + \Sigma^v)^{-1} \tag{18}$$

where $\text{Var}^{ni}_{i,t|t-1}$ is the forecast uncertainty of $y_t$ based on prior beliefs up to $t-1$ and $\Sigma^v$ is a 2-by-2 matrix indicating the noisiness of the two signals.

$$\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix} \tag{19}$$

Therefore, the Kalman gain $P$ is a function of prior uncertainty, $\text{Var}_{t|t-1}$, and the noisiness of the signals $\Sigma^v$. Note that beyond the steady state, $P$ is time-varying as the variance is updated by the agent each period (as in Equation 23). With constant volatility in AR(1), we can focus on the Kalman gain in the steady state, corresponding to a constant variance. Therefore, I drop time $t$ from $P$.

Individual forecast partially responds to new signals as $PH < 1$. $PH = 1$ is a special case when either signal is perfectly precise, thus $\Sigma^v$ takes zero in one of the diagonal entries, then the formula collapses to FIRE. This makes $PH$ a comparable parameter with $\lambda$ in SE that governs information rigidity.

What differentiates average forecasts from individual's is the role played by private signals. On average, private signals cancel out across agents, therefore, only public signals enter the average forecast, hence, average forecast errors (Equation 21).

$$\begin{aligned} \bar{y}^{ni}_{t+h|t} &= \rho^h[(1-PH) \underbrace{\bar{y}^{ni}_{t|t-1}}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average signals}} ] \\ &= (1-PH)\bar{y}^{ni}_{t+h|t-1} + \rho^h PHy_t + \rho^h P_\epsilon \epsilon_t \end{aligned} \tag{20}$$

The population average forecast error under NI takes a similarly recursive form as in SE.

$$\overline{FE}^{ni}_{t+h|t} = (1-PH)\rho\overline{FE}^{ni}_{t+h-1|t-1} + \rho^h P_\epsilon \epsilon_t + FE^*_{t+h|t} \tag{21}$$

The average square (or the unconditional variance) of the forecast errors is unambiguously greater than $FE^{*2}_{t+h|t}$ in FIRE, as shown in Equation 22. The simple reason for this is that the forecast errors under NI always come from not only the realized shocks to inflation but also nowcasting noises.

$$\overline{FE}^{ni2}_{\bullet+h|\bullet} = \frac{\rho^{2h}P^2_\epsilon\sigma^2_\epsilon + FE^{*2}_{\bullet+h|\bullet}}{(PH)^2} \geq FE^{*2}_{\bullet+h|\bullet} \tag{22}$$

Kalman filtering also updates the variance recursively according to the following rule. The posterior uncertainty at time $t$ is a linear function of prior uncertainty and noisiness of signals.

$$\begin{aligned} \text{Var}^{ni}_{i,t|t} &= (1-PH)\text{Var}^{ni}_{i,t|t-1} \\ &= (1-PH)(\rho^2\text{Var}^{ni}_{i,t-1|t-1} + \sigma^2_\omega) \end{aligned} \tag{23}$$

The unconditional nowcasting variance can be solved as the steady-state value in Equation 23. In the steady-state, there is no heterogeneity across agents in forecasting uncertainty and the nowcasting uncertainty becomes a constant. Thus, we can drop the subscript $i$. Note that the average uncertainty non-linearly depends on the noisiness of the two signals $\sigma^2_\epsilon$ and $\sigma^2_\xi$, as well as the volatility of inflation itself.

The $h$-period-ahead forecasting uncertainty comes from both nowcasting uncertainty and volatility of unrealized shocks in the future. (Equation 24)

$$\text{Var}^{ni}_{i,t+h|t} = \rho^{2h}\text{Var}^{ni}_{i,t|t} + \sum_{s=1}^{h}\rho^{2(s-1)}\sigma^2_\omega \geq \text{Var}^*_{t+h|t} \tag{24}$$

Equation 23 also directly gives the revision in uncertainty from time $t-1$ to $t$. The newly arrived information, albeit noisy, still brings about information gains, thus leading to an unambiguous drop in uncertainty. But because the signal is not perfect, i.e. $\Sigma^v$'s diagonal is non-zero, there is inefficiency in reducing uncertainty compared to in FIRE.

$$\text{Var}^{ni}_{i,t|t} - \text{Var}^{ni}_{i,t|t-1} = -PH\text{Var}_{i,t|t-1} < 0 \tag{25}$$

As a result of NI mechanism, the revision in h-period-ahead uncertainty from $t-1$ to $t$ only partially reacts to the resolution of uncertainty from newly realized shock $\omega_t$ in the past period.

NI also predicts non-zero disagreement in the presence of private signals. The size of the disagreement depends on, but is not a strictly increasing function of, the noisiness of the private signals. It is so because if the noisiness of private signals $\sigma_\xi$ is much larger, say infinity, than that of the public signal $\sigma_\epsilon$, agents will optimally not at all react to private signals. In this scenario, the disagreement will no longer increase with $\sigma_\xi$.

$$\overline{Disg}^{ni}_{t+h|t} > 0 \tag{26}$$

In summary, both the average squared forecast error and uncertainty are greater than conditional volatility of the inflation because of the presence of noisy signals. Meanwhile, their relative size are ambiguous. Disagreement is positive as long as there is dispersed information in the form of private signals, and they are not too noisy. In addition, all three moments contain parametric restrictions about the noisiness of public and private signals. It is possible that, under a range of parameter values, NI generates moments that are consistent with the observed data from the survey. We leave this task for the structural estimation in Section 5.

## 4.3 Diagnostic expectations (DE)

Different from the previous two theories featuring informational rigidity, diagnostic expectation (Bordalo et al. (2018)) introduces an extrapolation mechanism in expectation formation that results in overreactions to the news (Bordalo et al. (2020)). Both SE and NI deviate from FIRE in terms of the information set available to the agents (the "FI" assumption), while DE deviates from FIRE in terms of the processing of an otherwise fully updated information set (the "RE" assumption).

Skipping over its micro foundation, Equation 27 captures the essence of DE mechanism. Each individual $i$'s h-period-ahead forecast consists of two components. The first component can be considered as a rational forecast based on the fully updated $y_t$. The second component corresponds to the potential overreaction to the unexpected surprises from $t-1$ to $t$. The degree of overreaction is governed by the parameter $\theta$. The premise of DE model is that $\theta > 0$, which captures the fact that the agent overly responds to the realized forecast errors. The model collapses to the FIRE when $\theta = 0$. Meanwhile, as argued in Bordalo et al. (2020), a negative $\theta$ is not incompatible with an interpretation of underreaction if we treat DE as a more generalized model of expectation formation.

$$\bar{y}^{de}_{i,t+h|t} = \rho^h y_t + \theta_i(\rho^h y_t - \bar{y}^{de}_{i,t+h|t-1}) \tag{27}$$

There is no room for disagreement with a homogeneous degree of overreaction. To account for the possibility of a positive disagreement, I assume $\theta$ to be different across different agents. Therefore, I add the subscript $i$ to the parameter. Since agents are equally informed about the realizations of the variable, the only room for disagreement to be positive is heterogeneous degrees of overreaction. To capture this, I assume $\theta_i$ to follow a normal distribution across the population, $N(\hat{\theta}, \sigma^2_\theta)$. So the DE model has two parameters. Disagreement increases with the dispersion of overreaction, $\sigma_\theta$.

The average forecast takes exactly the same form, with the individual-specific $\theta_i$ replaced by the population average $\hat{\theta}$. Therefore, we focus on average forecast errors directly. The average forecast error under DE evolves as the following (See Appendix 7 for derivations)

$$\overline{FE}^{de}_{t+h|t} - FE^*_{t+h|t} = -\hat{\theta}\rho(\overline{FE}^{de}_{t+h-1|t-1} - FE^*_{t+h-1|t-1}) + \rho^h\hat{\theta}\omega_t \tag{28}$$

Intuitively, moving from $t-1$ to $t$, the h-period-ahead FE exceeds that of FIRE by two components, one is the mean-reverting change from the FE deviation in t-1; the other is the overreaction to the the newly realized shock. Combined, they can be interpreted as the overreaction to the surprise to the expectation formed at $t-1$.

The square or the unconditional variance of h-period-ahead forecast errors is the most straightforward in the special case $h=1$, which is equal to the following. It is smaller than the variance of FE in FIRE benchmark and the conditional volatility of the inflation, $\sigma^2_\omega$.

$$\overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{\sigma^2_\omega}{1+\hat{\theta}^2\rho^2} \tag{29}$$

Finally, as to the uncertainty, since the mechanism of extrapolation in DE does not change the agent's perceived distribution of future shocks, the benchmark DE model the forecast uncertainty to remain the same as in FIRE.

$$\overline{Var}^{de}_{t+h|t} = \overline{Var}^{*}_{t+h|t} \tag{30}$$

In summary, under DE, the ex-ante uncertainty, which is identical to the conditional volatility of inflation, is greater than the square of ex-post forecast error. The variability of average forecast errors are smaller than that in FIRE because of its mean-reversion.

## 4.4 Diagnostic expectation (DE) augmented with heterogeneous information (DENI)

Bordalo et al. (2020) embeds heterogeneous information in a standard DE model. Their motivations are to generate cross-sectional disagreement in forecasts and coexistence of under-reaction in consensus forecasts and overreaction at individual levels. The framework is essentially a hybrid of the NI and DE.[28] It maintains the assumption regarding how agents overreact to new information at individual levels, but the information is no longer the real-time realization of the variable $y_t$ itself, but noisy signals of it, which we denote as $s_{i,t}$. I assume a more general signal structure than in Bordalo et al. (2020) to include both public and private signals, as assumed in Section 4.2.

Then the h-period-ahead forecast takes a recursive form as follows.

$$y^{deni}_{i,t+h|t} = y^{deni}_{i,t+h|t-1} + (1+\theta)P^{deni}H(\rho^h s_{i,t} - y^{deni}_{i,t+h|t-1}) \tag{31}$$

$P^{deni} = [P_\epsilon, P_\xi]$, is the vector of Kalman gain as a function of nowcasting uncertainty $\text{Var}^{deni}_{t|t}$ and nosiness of signals $\sigma_\epsilon, \sigma_\xi$. With $\theta = 0$, Equation 31 becomes identical to that in NI with one private signal, where forecast is a Kalman-gain-weighted average

[28] It is called "the Diagnostic Kalman Filter" in Bordalo et al. (2020).

of new information and the prior. Any $\theta > 0$ implies overreaction beyond the optimal Kalman gain.

With such a mechanism, the average FE evolves as the following. (See Appendix 7 for derivations.) At any $t$, the deviation of FE from FIRE consists of its mean-reversion from $t-1$, the overreaction to the Kalman-gain weighted inflation shock, and the public noise.

$$\begin{aligned}\overline{FE}^{deni}_{t+h|t} - \overline{FE}^{*}_{t+h|t} = & -\theta\rho(\overline{FE}^{deni}_{t+h-1|t-1} - FE^{*}_{t+h-1|t-1}) \\ & + \rho^h((1+\theta)P_\epsilon - 1)\omega_t + \rho^h(1+\theta)P_\epsilon\epsilon_t\end{aligned} \tag{32}$$

Taking $h = 1$ as an example, the unconditional variance of FE is equal to the following.

$$\overline{FE}^{deni2}_{\bullet+1|\bullet} = \frac{\sigma^2_\omega + \rho^2(1+\theta)^2(1-P_\epsilon)^2\sigma^2_\omega + \rho^2(1+\theta)^2P^2_\epsilon\sigma^2_\epsilon}{1+\theta^2\rho^2} \tag{33}$$

The above equation collapses to that in FIRE when the public information is perfectly precise ($P_\epsilon = 1$, $\sigma_\epsilon = 0$) and there is no overreaction ($\theta = 0$). It collapses to DE when $\theta$ remains positive and $P_\epsilon = 1$, $\sigma_\epsilon = 0$. Compared to DE model in which the variation of average forecast error is attenuated, NI model introduces a counteracting force that makes variation of FE possibly bigger than FIRE due to the existence of the noisy signals. Therefore, the relative size between the variance of FE in DE and FIRE is ambiguous.

Forecast uncertainty under DENI is identical to that in NI, because only the NI mechanism affects the behaviors of uncertainty.

$$\overline{Var}^{deni}_{t+h|t} = \overline{Var}^{ni}_{t+h|t} \tag{34}$$

Finally, the DENI also predicts positive Disg for any noisy private information: $\sigma_\xi > 0$.

### 4.5 Comparing theories

Table 3 summarizes the predictions by different theories.

Table 3: Model-implied ranking of moments

| Model | Predictions |
|---|---|
| FIRE | $\overline{Var}^* = \overline{FE}^{*2} = \sigma^2_\omega; \overline{Disg}^* = 0$ |
| SE | $\overline{FE}^2 < \overline{FE}^{*2} = \overline{Var}^* = \sigma^2_\omega < \overline{Var}; \overline{Disg} > 0$ |
| NI | $\overline{FE}^2 > \overline{FE}^{*2}; \overline{Var} > \overline{Var}^*; \overline{Disg} > 0$ |
| DE | $\overline{FE}^2 < \overline{FE}^{*2} = \overline{Var}^* = \overline{Var}; \overline{Disg} > 0$ |
| DENI | $\overline{FE}^2 >< \overline{FE}^{*2}, \overline{Var} > \overline{Var}^*, \overline{Disg} > 0$ |

In addition, for each theory, not only forecast error but also higher moments, disagreement, and uncertainty contain restrictions to identify the model parameters within each theory. I will use these moment conditions to estimate each theory in Section 5.

# 5 Model Estimation and Sensitivity Analysis

## 5.1 SMM Estimation

The reduced-form tests in Section 3 provide additional evidence for rejecting the null hypothesis of FIRE. But there are two limitations with such an approach in terms of identifying differences among non-FIRE theories. First, the coefficient estimates from the reduced-form regression cannot always be mapped into a structural parameter of the particular model, especially when reported expectations and forecast horizons are at different time frequencies. Second, even if it does so, the tests fall short of simultaneously utilizing all the restrictions across moments implied by a particular non-FIRE theory, as discussed in great detail in Section 4. In this section, I undertake a structural estimation that jointly accounts for cross-moment restrictions.

Since many of the moment conditions cannot be easily derived as a closed-form function of parameters, I adopt the simulated method of moment (SMM). In a nutshell, the estimation chooses the best set of model parameters by minimizing the weighted distances between the data moments and the model-simulated moments. For a given process of inflation, and a particular theory of expectation formation, the vector of the parameters estimates is defined as the minimizer of the following objective function.

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

where $\Omega^o$ stands for the parameters of the particular pair of theories of expectation and inflation process, i.e. $o \in \{se, ni, de, deni\} \times \{ar, sv\}$. $\Gamma^o$ represents the corresponding parameter space respecting the model-specific restrictions. $M_{data}$ is a vector of the unconditional moments that is computed from data on expectations and inflation. $F^o$ is the simulated model moments under the theory pair $o$. $W$ is the weighting matrix used for the SMM estimation. I report estimation results using the 2-step feasible SMM approach, in which the inverse of the variance-covariance matrix from the 1st-step estimation using identity matrix is used as the $W$ in the second step, which has been shown to give asymptotically efficient estimates of the model parameters.

Crucially, notice that the model-implied moments $F^o$ are not just a function of model parameters $\Omega^o$, but also a function of the corresponding information set available to the forecasters. I use $H$ to represent the historical realizations of the variables in the agents' information set that are used as the inputs for forecasts.

It is also important to mimic the information set that was truly available to the agents at each point in time in history.[29] Therefore, I use the real-time data on historical inflation that was publicly available at each point in time instead of the historical data series released later, since the latter usually experiences many rounds

[29]For the importance of using real-time data to study survey forecasts, see Faust and Wright (2008), Faust and Wright (2009), and so on.

of revisions over time. I obtained the data from the Real-Time Data Research Center hosted by the Philadelphia Fed. [30]

The estimation is also specific to the choice of moments to be matched. I focus on the unconditional population moments across time in the sample. In particular, they include the mean ($FE$), variance($FEVar$) and auto-covariance($FEATV$) of population forecast error, the mean disagreement ($Disg$), and the mean uncertainty ($Var$). For joint estimation, two unconditional moments of inflation are used, the variance($InfVar$) and auto-covariance($InfATV$). Table 1 reports the size of these moments computed for both SPF and SCE.

The model-implied moment conditions also implicitly depend on the parameters of the inflation process for a given model. This point is illustrated well in Bordalo et al. (2020). For instance, the observed overreaction in DE is lower for an AR(1) process with higher persistence. In recovering the model parameters associated with expectation formation, it is important to take into account the information contained in expectation data regarding the process of inflation itself. To account for this, I undertake both 2-step and joint estimation. The former is to first externally estimate the inflation process and then estimate expectation formation, separately, treating the inflation parameter as the *true* data generating process of inflation. The latter refers to jointly estimating parameters of inflation and expectation.

These alternative specifications of the estimation also serve as a model sensitivity analysis with respect to the following criteria: (1) different choices of moments; (2) AR(1) and SV for the process of inflation. (3) two-step and joint estimation. (4) for both professionals and households. A reasonable theory of expectation formation ought to be relatively robust to these criteria. I discuss the findings in greater detail along these four dimensions the next.

## 5.2 Moments matching and parameter estimates

Table 4 presents the SMM estimates for professionals, as a benchmark. For each theory, I estimate the theory both in 2 steps or jointly using expectations and inflation moments. Different rows within each panel report the estimates depending on various choices of moments used for estimation: forecast errors only (FE), forecast error and disagreement (FE+Disg), and the two plus uncertainty (FE+Disg+Var).

### 5.2.1 Cross-moment consistency of each theory

Among the four models under consideration, SE and DENI outperform others in terms of the within-model robustness against targeted moments, as shown in the estimation of professional forecasts in Table 4.

For SE, the estimated quarterly updating rate $\lambda$ is between 0.22- 0.36 across different combinations of moments. The estimate is 0.36 when only FE moments are targeted. It is smaller, 0.28 and 0.26, respectively, when Disg and Var are sequentially included. These imply that the information rigidity is revealed through both low and high-order moments.

[30] https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/.

Table 4: SMM Estimates of Different Models: Professionals

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SE | | | | | | | | |
| Moments Used | 2-Step Estimate | | | Joint Estimate | | | | |
| | $\hat{\lambda}$ | $\rho$ | $\sigma_\omega$ | $\hat{\lambda}$ | $\rho$ | $\sigma_\omega$ | | |
| FE | 0.36 | 0.99 | 0.23 | 0.18 | 0.97 | 0.11 | | |
| FE+Disg | 0.28 | 0.99 | 0.23 | 0.22 | 0.95 | 0.14 | | |
| FE+Disg+Var | 0.26 | 0.99 | 0.23 | 0.32 | 0.9 | 0.22 | | |
| NI | | | | | | | | |
| Moments Used | 2-Step Estimate | | | | Joint Estimate | | | |
| | $\hat{\sigma}_\epsilon$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ | $\hat{\sigma}_\epsilon$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ |
| FE | 0 | 0.87 | 0.99 | 0.23 | 0 | 0.15 | 0.97 | 0.11 |
| FE+Disg | 1.5 | 2.26 | 0.99 | 0.23 | 1.48 | 2.33 | 0.97 | 0.11 |
| FE+Disg+Var | 2.64 | 3 | 0.99 | 0.23 | 3 | 3 | 0.94 | 0.16 |
| DE | | | | | | | | |
| Moments Used | 2-Step Estimate | | | | Joint Estimate | | | |
| | $\hat{\theta}$ | $\sigma_\theta$ | $\rho$ | $\sigma_\omega$ | $\hat{\theta}$ | $\sigma_\theta$ | $\rho$ | $\sigma_\omega$ |
| FE | 0.64 | 0.58 | 0.99 | 0.23 | 0.81 | 1.68 | 0.97 | 0.11 |
| FE+Disg | 0.27 | 2.2 | 0.99 | 0.23 | 0.38 | 2.1 | 0.9 | 0.2 |
| FE+Disg+Var | 0.42 | 2.1 | 0.99 | 0.23 | 0.33 | 2.1 | 0.9 | 0.23 |
| DENI | | | | | | | | |
| Moments Used | 2-Step Estimate | | | | Joint Estimate | | | |
| | $\hat{\theta}$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ | $\hat{\theta}$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ |
| FE | 0.76 | 0 | 0.99 | 0.23 | 0.82 | 0 | 0.97 | 0.11 |
| FE+Disg | 0.85 | 0.14 | 0.99 | 0.23 | N/A | N/A | N/A | N/A |
| FE+Disg+Var | 0.85 | 0.16 | 0.99 | 0.23 | N/A | N/A | N/A | N/A |

For DENI, the implied overreaction parameter $\theta$ is in a range of 0.76-0.85, suggesting the existence of overreaction mechanisms in the population. Meanwhile, the noisiness of the private signals $\sigma_\xi$ is around 0.14-0.16 in percentage points.

In contrast, the NI estimates of $\sigma_\epsilon$ and $\sigma_\xi$ are rather volatile across targeted moments. When only FE is targeted, the estimation points to a highly precise public signal and mildly noisy private signals. But when Disg and Var are included, the estimated nosiness of both signals significantly increases. They are often so large in magnitudes that they hit the externally set upper bound of 3. These are highly noisy signals compared to the conditional standard deviation of inflation shocks $\sigma_\omega = 0.22$. Although qualitatively, NI mechanisms accommodate patterns of information rigidity similar to SE, quantitatively the required noisiness of the signals is less sensible to interpret.

DE estimates are also sensitive to moment restrictions, although all the estimates confirm the existence of a positive mass of overreacting agents. $\hat{\theta}$ is estimated to range from 0.27 to 0.81 depending on the estimation specification. Using disagreement helps identify the population dispersion in the degree of overreaction $\sigma_\theta$, which is estimated to be 2.2. This suggests a significant amount of heterogeneity in the degree of reaction to the news in the lens of DE.

#### 5.2.2 Interactions between expectation formation and inflation process

In all four models, the estimated parameters vary when one jointly estimates expectation and inflation process parameters. With the benchmark AR(1) process, both the persistence of the shock to inflation $\rho$ and the overall volatility of the inflation shock $\sigma_\omega$ determine the value of the corresponding moments under a particular model of expectation formation.[31] The differences between 2-step estimation and joint estimation reveal such inter-dependence.

An alternative interpretation of the joint estimates is that they reveal possibly subjective models of inflation as perceived by the forecasters which may be different from the one estimated from historical data retrospectively. The joint estimation results seem to support such an interpretation. The estimates of SE, NI, and DE all produce very similar parameters of expectation formation, yet rather different inflation persistence and volatility. The survey-implied persistence of inflation and conditional volatility are both smaller than those estimated solely based on inflation data. This implies that in addition to information rigidity and overreaction mechanisms in the canonical versions of these models, allowing for the possibility of a subjective model is necessary to fit the joint dynamics and inflation and expectation better.

#### 5.2.3 Professionals versus households

Table 5 reports the estimates for households. The updating rate in SE is 0.36, implying around one-third chance of updating per month. This is a slightly lower degree of stickiness than professionals. It is well documented in the literature that household expectations tend to be more inattentive to economic news than professionals.[32] But the SE results of our estimates show that the major differences are not simply due to the differences in updating rates of information.

NI estimates of households, when all moments are targeted, reveal extremely noisy signals. The public signals need to have a noisiness of 2 to 3 percentage points while private signals need to be between 1 to 3 percentage points.

DE estimates of households suggest a consistently positive and greater degree of overreaction, together with an extremely large dispersion of 5.0, which gives also a positive mass of underreactive agents.

Compared to the previous three models, DENI estimates of the household are significantly different from that of professionals. The average degree of overreaction $\hat{\theta}$ becomes negative, taking the value of -0.35 to -0.54, implying underreaction on average. In addition, the nosiness of private signals is estimated to be still extremely large, taking values of 2.43 to 3.

[31] Afrouzi et al. (2023) emphasizes this point in the context of DE. They further address such non-identification challenges in lab experiments by exogenously altering the persistence of the data-generating process.

[32] See Cornand and Hubert (2022) for a recent discussion on this point.

Table 5: SMM Estimates of Different Models: Households

| | | | | |
|---|---|---|---|---|
| SE | | | | |
| Moments Used | 2-Step Estimate | | | |
| | $\hat{\lambda}$ | $\rho$ | $\sigma_\omega$ | |
| FE | 0.36 | 0.98 | 0.45 | |
| FE+Disg | 0.36 | 0.98 | 0.45 | |
| FE+Disg+Var | 0.36 | 0.98 | 0.45 | |
| NI | | | | |
| Moments Used | 2-Step Estimate | | | |
| | $\hat{\sigma}_\epsilon$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ |
| FE | 0 | 1 | 0.98 | 0.45 |
| FE+Disg | 3 | 1.18 | 0.98 | 0.45 |
| FE+Disg+Var | 2.06 | 3 | 0.98 | 0.45 |
| DE | | | | |
| Moments Used | 2-Step Estimate | | | |
| | $\hat{\theta}$ | $\sigma_\theta$ | $\rho$ | $\sigma_\omega$ |
| FE | 0.49 | 0.5 | 0.98 | 0.45 |
| FE+Disg | 1.91 | 5 | 0.98 | 0.45 |
| FE+Disg+Var | 1.03 | 5 | 0.98 | 0.45 |
| DENI | | | | |
| Moments Used | 2-Step Estimate | | | |
| | $\hat{\theta}$ | $\hat{\sigma}_\xi$ | $\rho$ | $\sigma_\omega$ |
| FE | N/A | N/A | 0.98 | 0.45 |
| FE+Disg | -0.54 | 3 | 0.98 | 0.45 |
| FE+Disg+Var | -0.35 | 2.43 | 0.98 | 0.45 |

# 6 Inflation with stochastic volatility (SV)

This section considers an alternative data-generating process of inflation using the Unobservable Component/Stochastic Volatility (UCSV, or SV) model proposed by Stock and Watson (2007), which is arguably a more realistic stochastic process of inflation given the presence of macroeconomic shocks of varying persistence.

The SV extension achieves two objectives. Firstly, a basic inflation process with constant volatility does not account for the observed time-varying pattern of forecast uncertainty, nor its correlation with other moments such as disagreement and forecast error size, as shown in Section 2.4. This is particularly relevant to account for the rapid rise of Disg, FE, and Var over the inflationary period since 2020.

Second, allowing for SV in the inflation process serves as a robustness test of various theories of expectation formation, as it captures the sensitivity of these theories to the assumed underlying generating process of inflation. This extension provides a more comprehensive analysis of the relationship between inflation dynamics and expectation formation.

In particular, UCSV assumes that inflation consists of a permanent $\zeta$ and transitory component $\eta$.[33] Time variations in the relative size of the volatility of two components $\sigma^2_\zeta$ and $\sigma^2_\eta$ drive time variations of the persistence of inflation shocks. The logged volatility of the two components themselves follows a random walk subject to shocks $\mu_\zeta$ and $\mu_\eta$.

$$
\begin{aligned}
y_t &= \zeta_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\nu_{\eta,t} \\
\zeta_t &= \zeta_{t-1} + z_t, \quad \text{where } z_t = \sigma_{z,t}\nu_{\epsilon,t} \\
\log \sigma^2_{\eta,t} &= \log \sigma^2_{\eta,t-1} + \mu_{\eta,t} \\
\log \sigma^2_{z,t} &= \log \sigma^2_{z,t-1} + \mu_{\epsilon,t}
\end{aligned}
\tag{35}
$$

The shocks to the level of the two components $\eta_t$, and $z_t$, and those to their volatility, $\mu_{\eta,t}$ and $\mu_{z,t}$, are drawn from the following normal distributions, respectively. The only parameter of the model is $\gamma$, which determines the smoothness of the time-varying volatility.

$$
\begin{aligned}
\nu_t &= [\nu_{\eta,t}, \nu_{z,t}] \sim N(0, I) \\
\mu_t &= [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)
\end{aligned}
\tag{36}
$$

I reproduce the estimates of Stock and Watson (2007) using the Markov Chain Monte Carlo (MCMC) algorithm covering the extended period till March 2023. The estimated time-varying permanent and transitory volatility of both Core CPI and headline CPI is shown in Figure 4. The figure depicts intuitive patterns of higher permanent volatility of inflation around the 2008 Great Recession and the Covid era since 2020.

## 6.1 Model predictions under SV

The information set necessary for forecasting is different in SV from that in an AR(1) process. Consider first the benchmark case of FIRE. At the time $t$, the FIRE agent sees the most recent and past realization of all stochastic variables as of $t$, including $y_t$, $\zeta_t$, $\eta_t$, $\sigma_{\eta,t}$, $\sigma_{z,t}$. Using the superscript $*sv$ to denote the FIRE benchmark prediction under the stochastic volatility, and suppressing the individual subscript $i$ (because there is no disagreement in FIRE), the h-period-ahead forecast of inflation is equal to the contemporaneous realization of the permanent component, $\epsilon_t \equiv \zeta_t$.

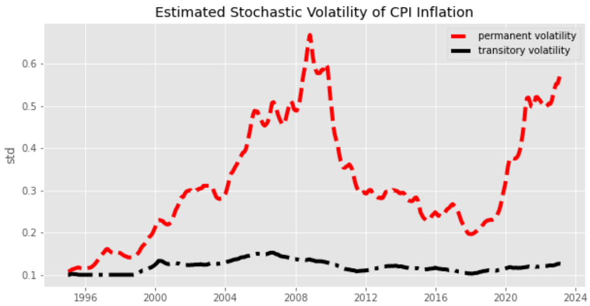
$$
\overline{y}^{*sv}_{t+h|t} = \zeta_t \tag{37}
$$

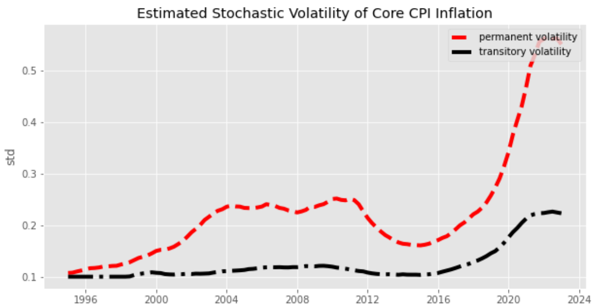
Under FIRE, forecast error is simply the cumulative sum of unrealized permanent and transitory shocks from $t$ to $t+h$, which is equal to the following. And, disagreement is zero across agents in FIRE.

$$
\overline{FE}^{*sv}_{t+h|t} = -\sum_{s=1}^{h} (\eta_{t+s} + z_{t+s}) \tag{38}
$$

[33]For such a multi-component formulation of the inflation process, see Kohlhas and Walther (2021); Farmer et al. (2021).

Figure 4: Stochastic Volatility of Inflation




Note: This figure plots the estimated stochastic volatility of permanent and transitory components of monthly headline CPI (top) and quarterly core CPI inflation (bottom) using the same approach as in Stock and Watson (2007).

The h-step-ahead conditional variance, or the forecast uncertainty is time-varying, as the volatility is stochastic now. It is essentially the conditional expectation of the cumulative sum of future volatility given the current realizations of the component-specific volatility at $t$.

$$\begin{aligned}\overline{Var}^{*sv}_{t+h|t} &= \sum_{s=1}^{h} E_t(\sigma^2_{\eta,t+s}) + E_t(\sigma^2_{z,t+s}) \\ &= \sigma^2_{\eta,t}\sum_{s=1}^{h} exp^{-0.5s\gamma} + \sigma^2_{z,t} exp^{-0.5h\gamma}\end{aligned} \tag{39}$$

**SESV** Under the sticky expectation (SE), an agent whose most recent up-to-date update happened in $t-\tau$ only has seen the realizations of $y$, $\zeta$, $\eta$, $\sigma_\eta$, $\sigma_z$ till $t-\tau$. The average forecast is hence the weighted average of all past realizations of the permanent component up to $t$.

$$y^{sesv}_{t+h|t-\tau} = \sum_{\tau=0}^{\infty} \lambda(1-\lambda)^{\tau}\zeta_{t-\tau} \tag{40}$$

The distribution of lagged updating is also reflected in the average forecast uncertainty. The population average uncertainty is a weighted average of FIRE uncertainty at $t, t-1 ... t-\tau ... t-\infty$. (Equation 41) The key difference in SV from AR(1) is that the average uncertainty exhibits a positive serial correlation under SV. Expectations being sticky further increase the positive serial correlation compared to that in FIRE due to the lag in updating the shocks to the volatility. The predictions regarding both forecast errors and disagreements under SV are the same as under the AR(1) model.

$$Var^{sesv}_{t+h|t} = \sum_{\tau=0}^{\infty} \lambda(1-\lambda)^{\tau} Var^{*sv}_{t+h|t-\tau} \tag{41}$$

**NISV** Under noisy information (NI), in order to forecast future $y$, the agent at time $t$ needs to form her best nowcast of the permanent component $\zeta_t$, denoted as $\bar{\zeta}_{t|t}$, using noisy signals and Kalman filtering. We assume again that the noisy signals of $\zeta_t$ consist of a public signal $s^{pb}_t$ and a private signal $s^{pr}_{i,t}$ containing noises around the true realization of $\zeta_t$. Following a long tradition of modeling the signaling-extraction problem in this two-component context, we further assume the public signal $s^{pb}_t = y_t$, meaning the inflation realization itself is the public signal of the permanent component. Accordingly, the transitory shock $\eta_t$ is equivalent to the realized noise of the public signal $\epsilon_t$ in the benchmark NI model with AR(1) process.

Then the average forecast is a Kalman-gain-weighted average of prior belief and new information.

$$y^{nisv}_{t+h|t} = \bar{\zeta}_{t|t} = (1 - P^{sv}_t H) y^{nisv}_{t+h-1|t-1} + P^{sv}_t H \zeta_t + P^{sv}_{\eta,t} \eta_t \tag{42}$$

In the above equation, Kalman gain $P^{sv}_t = [P^{sv}_{\eta,t}, P^{sv}_{\xi,t}]$ is a function of forecasting uncertainty $Var^{svni}_{t|t-1}$, the constant noisiness of private signal $\sigma_\xi$ and that of public signal, $\sigma_{\eta,t}$, which is also the time-varying volatility of the transitory component of the inflation.

What is different under time-varying volatility is that there is no steady-state Kalman gain and uncertainty that are independent of time because the underlying volatility of the variable is time-varying. This also implies that the rigidity induced by the noisiness of information is state-dependent. In each period, agents in the economy will update their forecasts based on the realized volatility. In periods with high (low) fundamental volatility, the Kalman gain from noisy signals is larger (smaller) thus the agents will be more (less) responsive to the new information. There is no such state-dependence of rigidity in the canonical SE.

The mechanisms of DE and DENI exactly mimic that under AR(1) except that the average volatility is time-varying now.

## 6.2 The role of stochastic volatility

Table 6 and 7 report the estimates under SV, respectively, for the low-inflation pre-pandemic period and the extended sample covering the high-inflation era between 2020-2023. I juxtapose the two episodes to explore possible state-dependence of expectation formation, especially given the rapid rise in inflation in the post-2020 era.

The major finding from the estimates is that SV process significantly improves the within-model consistency across targeted moments for both types of agents and both sample periods. This is probably not surprising given the two-component formulation proves to be a more realistic foundation of the inflation dynamics as the previous literature established.

The improvement in model consistency is the most obvious in NI estimates of professionals in which the benchmark estimates under AR(1) process produce unrealistically imprecise signals. With SV, the estimated noisiness of private signals falls into a more reasonable range of values, i.e. 0.21-0.68. It is more reasonable to assume that forecasters imperfectly observe the permanent component instead of inflation itself.

Despite this improvement in the cross-moment consistency for the low-inflation sample, however, NI's estimates remain extremely large once the sample includes the recent inflation episodes. In addition, the estimation of NI for households fails to converge in all specifications. This implies that the model has a rather poor fit to household expectations even if a more realistic inflation process is assumed.

Among all theories, SE gives the closest parameter estimates to that of the benchmark AR(1). The updating rate is estimated to remain in the range of $0.2 - 0.36$ for both households and professionals. This suggests that SE has a very good consistency against the assumed inflation process in capturing the overall patterns of the survey expectations. This is consistent with the preliminary diagnosis simply based on the empirical rankings of moments that are the most consistent with SE predictions.

Estimates of DENI under SV also point to a very similar degree of overreaction of professionals ($\hat{\theta}$ around 0.6-0.82) and underreaction of households ($\hat{\theta}$ around -0.48) as in the benchmark AR(1) estimates. The estimated dispersion of private information also remains similar to SV assumption.

## 6.3 Expectation formation when inflation is high

The benchmark estimation is based on the low-inflation sample period before 2020 where our assumption of stationary AR(1) process of inflation is a reasonable one. But SV formulation naturally fits better the dynamics of inflation once we want to examine if the estimates change when the sample covers the episode of high inflation and volatility, as shown in Figure 4.

For most models, the parameter estimates change significantly between two sample periods. Such changes do not necessarily invalidate the model mechanisms but instead reflect possible state-dependence of expectation formation. The difference in estimates between the two sample periods does suggest both professionals and households have altered their responsiveness to the inflation news.

In particular, for professionals, the SE estimates imply an on average higher updating rate $\lambda = 0.36$. Households' updating rate is estimated to be higher than in the pre-2020 sample. Both exhibit less information rigidity in the form of SE. The estimates of the DE model corroborate this pattern. Both professionals and households exhibit an average higher degree of overreaction. $\hat{\theta}$ changes from $-0.03$ to 0.3 for professionals and from 0.29 to 0.47 for households. This echoes the findings of Coibion and Gorodnichenko (2015) that the information rigidity is state-dependent. Goldstein (2023); Pfäuti (2023) found that inflation expectations exhibit less rigidity when inflation is elevated.

Different from the finding that information rigidity is lessened in the high inflation episode for both types of agents, the DENI estimates depict a more divergent pattern between the two types. Professionals have on average turned to underreaction with more dispersed information: $\hat{\theta}$ changes from 0.82 to -0.26, and $\sigma_\xi$ changes from 0.24 to 0.93. In contrast, households have turned to overreaction with less dispersed information: $\hat{\theta}$ changes from -0.48 to 0.43, and $\sigma_\xi$ changes from 0.64 to 0.26.

Table 6: SMM Estimates of Different Models under Stochastic Volatility: Professionals

| Before March 2020 | | | Till March 2023 | |
|---|---|---|---|---|
| SE | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\lambda}$ | | $\hat{\lambda}$ | |
| FE | 0.2 | | 0.3 | |
| FE+Disg | 0.25 | | 0.36 | |
| FE+Disg+Var | 0.36 | | 0.36 | |
| NI | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\sigma}_{pb}$ | $\hat{\sigma}_{pr}$ | $\hat{\sigma}_{pb}$ | $\hat{\sigma}_{pr}$ |
| FE | 0.68 | 0.24 | 2.3 | 3 |
| FE+Disg | 0.67 | 0.24 | 2.3 | 3 |
| FE+Disg+Var | 0.64 | 0.21 | 2.3 | 3 |
| DE | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\theta}$ | $\sigma_\theta$ | $\hat{\theta}$ | $\sigma_\theta$ |
| FE | -0.03 | 0.54 | 0.31 | 0.41 |
| FE+Disg | -0.03 | 0.16 | 0.28 | 0.19 |
| FE+Disg+Var | -0.04 | 0.16 | 0.31 | 0.19 |
| DENI | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\theta}$ | $\hat{\sigma}_{pr}$ | $\hat{\theta}$ | $\hat{\sigma}_{pr}$ |
| FE | 0.64 | 0.47 | -0.25 | 0.93 |
| FE+Disg | 0.82 | 0.26 | -0.26 | 0.93 |
| FE+Disg+Var | 0.82 | 0.24 | -0.26 | 0.93 |

## 6.4 The final assessment of different models

To summarize, Table 8 reports my evaluation of the four theories under consideration based on four sensitivity criteria laid out in the previous section. According to this evaluation, SE seems to capture the average behavior of expectations better than the other three theories, as it constantly produces a stable range of estimates of updating rate around 0.2 to 0.3 across all specifications.

NI, another theory that also features information rigidity and captures similar qualitative patterns as SE, exhibits less cross-moment consistency. The major weakness

Table 7: SMM Estimates of Different Models under Stochastic Volatility: Households

| Before March 2020 | | | Till March 2023 | |
|---|---|---|---|---|
| SE | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\lambda}$ | | $\hat{\lambda}$ | |
| FE | 0.27 | | 0.36 | |
| FE+Disg | 0.2 | | 0.27 | |
| FE+Disg+Var | 0.26 | | 0.26 | |
| NI | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\sigma}_\epsilon$ | $\hat{\sigma}_\xi$ | $\hat{\sigma}_\epsilon$ | $\hat{\sigma}_\xi$ |
| FE | N/A | N/A | N/A | N/A |
| FE+Disg | N/A | N/A | N/A | N/A |
| FE+Disg+Var | N/A | N/A | N/A | N/A |
| DE | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\theta}$ | $\sigma_\theta$ | $\hat{\theta}$ | $\sigma_\theta$ |
| FE | -0.09 | 0.58 | -0.07 | 0.57 |
| FE+Disg | 0.29 | 0.57 | 0.47 | 1.07 |
| FE+Disg+Var | 0.29 | 0.57 | 0.28 | 1.07 |
| DENI | | | | |
| Moments Used | 2-Step Estimate | | 2-Step Estimate | |
| | $\hat{\theta}$ | $\hat{\sigma}_\xi$ | $\hat{\theta}$ | $\hat{\sigma}_\xi$ |
| FE | -0.48 | 0.64 | 0.43 | 0.26 |
| FE+Disg | -0.48 | 0.64 | 0.43 | 0.26 |
| FE+Disg+Var | -0.48 | 0.64 | 0.43 | 0.26 |

of the model is that it produces unrealistically large sizes of the parameters to match the rigidity of the data. This is per se not a rejection of the theory. It is indeed found that once a more realistic inflation process of SV is used, NI estimation produces much more consistent and sensible values of parameters for professionals. Despite of this improvement for professionals, however, NI proves to be a poor model to fit the patterns of household expectations. Although the previous literature (Coibion and Gorodnichenko (2012, 2015)) treat SE and NI as two indistinguishable theories that both produce information rigidity, this paper shows that using information from uncertainty significantly disciplines the parameter choices and allows me to distinguish the two theories by their model sensitivity.

Compared to the two rigidity models, a modified canonical DE that allows for heterogeneous degrees of over/underreaction, are estimated to reveal a large degree of heterogeneity ranging from overreaction to underreaction across individuals. In addition, the model estimates are rather sensitive along many dimensions. The estimates with the high inflation episode do suggest a shift toward an average degree of overreaction of both professionals and households.

A hybrid of DE and NI, as in Bordalo et al. (2020), which accommodates the coexistent overreacting mechanisms and dispersed noisy information, does improve the fit of the model and its robustness compared to DE. The estimates feature a reasonable degree of underreaction (overreaction) of households with dispersed information in the low-inflation environment but an opposite pattern once the high-inflation episode is included in the estimation.

Table 8: Evaluation of different model

| Criteria | SE | NI | DE | DENI |
|---|---|---|---|---|
| Sensitive to moments used for estimation? | No | Yes | Yes | No |
| Sensitive to the assumed inflation process? | No | Yes | Yes | No |
| Sensitive to two-step or joint estimate? | No | No | No | Yes |
| Sensitive to the type of agents? | No | Yes | Yes | Yes |

# 7 Conclusion

Most studies on expectation formation that document how it deviates from the FIRE benchmark have focused on the first moment, namely the mean forecasts and the cross-sectional dispersion of the forecasts. However, this paper has shown that the surveyed forecasting uncertainty by professionals and households provides useful information for understanding the exact mechanisms of expectation formation. It not only provides additional reduced-form testing results of rejecting FIRE, such as persistent disagreements in forecasting uncertainty and its inefficient revisions but also provides additional moment restrictions to any particular model of expectation formation, which helps identify differences across theories.

At least three lines of questions remain unresolved in this paper and require future research. First, this paper focuses on a selective list of models of expectation formation and inevitably omits various others likewise proven to match certain aspects of surveyed inflation expectations, such as adaptive learning (Marcet and Sargent, 1989; Evans and Honkapohja, 2012), experience-based learning (Malmendier and Nagel, 2015), heterogeneous models (Patton and Timmermann, 2010; Farmer et al., 2021), and asymmetric attention (Kohlhas and Walther, 2021). It would be a fruitful attempt to explore the corresponding predictions of these models about uncertainty. Second, throughout the analysis, we maintained the normality/symmetric assumptions of the shocks and ignored beliefs in tail events or even higher moments. It would be natural to explore how different theories of expectation formation may contain different predictions on tail beliefs. Finally, although this paper focuses only on macroeconomic expectations regarding inflation, it is worth asking if the belief formation regarding individual variables such as income bears similar mechanisms and matches the observed empirical patterns of surveyed expectations and risks.[34]

[34] A few recent studies on income/wage/unemployment/job-search expectations: Mueller et al. (2021); Wang (2022); Koşar and Van der Klaauw (2022); Jäger et al. (2022); Caplin et al. (2023).

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# Appendix

## Detailed derivation

### SE

$$
\begin{aligned}
\overline{Var}_t^{se}(y_{t+h}) &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction who does not update until } t-\tau} \underbrace{Var^*_{t|t-\tau}(y_{t+h})}_{\text{uncertainty of most recent update at } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma^2_{\omega} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} (1+\rho^2+...+\rho^{2(h+\tau-1)})\sigma^2_{\omega} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{\rho^{2(h+\tau)}-1}{\rho^2-1}\sigma^2_{\omega} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{\rho^{2(h+\tau)}}{\rho^2-1}\sigma^2_{\omega} - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma^2_{\omega} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \rho^{2\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma^2_{\omega} - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma^2_{\omega} \\
&= \sum_{\tau=0}^{+\infty} \lambda((1-\lambda)\rho^2)^{\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma^2_{\omega} - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma^2_{\omega} \\
&= \frac{\lambda}{(1-\rho^2+\lambda\rho^2)} \sum_{\tau=0}^{+\infty} (1-\rho^2+\lambda\rho^2)(1-(1-\rho^2+\lambda\rho^2))^{\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma^2_{\omega} \\
&- \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma^2_{\omega} \\
&= (\frac{\lambda\rho^{2h}}{(1-\rho^2+\lambda\rho^2)(\rho^2-1)} - \frac{1}{\rho^2-1})\sigma^2_{\omega} \\
&= (\frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2} - 1)\frac{\sigma^2_{\omega}}{\rho^2-1} \\
&= (\frac{\lambda\rho^{2h}-1+\rho^2-\lambda\rho^2}{1-\rho^2+\lambda\rho^2})\frac{\sigma^2_{\omega}}{\rho^2-1}
\end{aligned}
\tag{43}
$$

### NI

The steady-state nowcasting uncertainty $\text{Var}^{ni}_{ss}$ is solved using the updating equation (Equation 23).

$$
\begin{aligned}
&\overline{\mathrm{Var}}^{ni}_{t|t} = \overline{\mathrm{Var}}^{ni}_{t|t-1} - \mathrm{Var}^{ni}_{t|t-1} H'(H\mathrm{Var}^{ni}_{t|t-1}H' + \Sigma^v)^{-1} H\overline{\mathrm{Var}}^{ni}_{t|t-1} \\
&\rightarrow \overline{\mathrm{Var}}^{ni}_{t|t} = \rho^2(\overline{\mathrm{Var}}^{ni}_{t-1|t-1} + \sigma^2) \\
&- \rho^2(\overline{\mathrm{Var}}^{ni}_{ss} + \sigma^2)H'(H\rho^2(\overline{\mathrm{Var}}^{ni}_{ss} + \sigma^2)H' + \Sigma^v)^{-1} H\overline{\mathrm{Var}}^{ni}_{ss} \\
&\rightarrow \overline{\mathrm{Var}}^{ni}_{ss} = \rho^2(\overline{\mathrm{Var}}^{ni}_{ss} + \sigma^2_\omega) \\
&- \rho^2(\overline{\mathrm{Var}}^{ni}_{ss} + \sigma^2_\omega)H'(H\rho^2(\overline{\mathrm{Var}}^{ni}_{ss} + \sigma^2_\omega)H' + \Sigma^v)^{-1} H\overline{\mathrm{Var}}^{ni}_{ss}
\end{aligned} \tag{44}
$$

**DE**

$$
\begin{aligned}
FE^{de}_{i,t+h|t} &= y^{de}_{i,t+h|t} - y_{t+h} \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y^{de}_{i,t+h|t-1}) \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= FE^*_{t+h|t} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i FE^{de}_{i,t+h|t-1} \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i(\rho FE^{de}_{i,t+h-1|t-1} - \omega_{t+h}) \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} + \theta_i\omega_{t+h} \\
&= (1+\theta_i)FE^*_{t+h|t} + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^*_{t+h-1|t} + (1+\theta_i)(-\omega_{t+h}) + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^*_{t+h-1|t} - \omega_{t+h} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^*_{t+h|t} + \theta_i FE^*_{t+h-1|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^*_{t+h|t} + \theta_i(FE^*_{t+h-1|t} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= FE^*_{t+h|t} + \theta_i(\rho FE^*_{t+h-1|t-1} + \rho^h\omega_t - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= FE^*_{t+h|t} - \theta_i\rho(FE^{de}_{i,t+h-1|t-1} - FE^*_{t+h-1|t-1}) + \theta_i\rho^h\omega_t
\end{aligned} \tag{45}
$$

**DENI**

Current forecast error is

$$
\begin{aligned}
\overline{FE}^{deni}_{t|t} &= \rho y^{deni}_{t-1|t-1} + (1+\theta)P_\epsilon(s^{pb}_t - \rho y^{deni}_{t-1|t-1}) - y_t \\
&= \rho(\overline{FE}^{deni}_{t-1|t-1} + y_{t-1}) + (1+\theta)P_\epsilon(s^{pb}_t - \rho y^{deni}_{t-1|t-1}) - y_t \\
&= \rho(\overline{FE}^{deni}_{t-1|t-1} + y_{t-1}) + (1+\theta)P_\epsilon(y_t + \epsilon_t - \rho(\overline{FE}^{deni}_{t-1|t-1} + y_{t-1})) - \rho y_{t-1} - \omega_t \\
&= \rho\overline{FE}^{deni}_{t-1|t-1} + (1+\theta)P_\epsilon(\rho y_{t-1} + \omega_t + \epsilon_t - \rho(\overline{FE}^{deni}_{t-1|t-1} + y_{t-1}) - \omega_t \\
&= \rho\overline{FE}^{deni}_{t-1|t-1} + (1+\theta)P_\epsilon(\omega_t + \epsilon_t - \rho\overline{FE}^{deni}_{t-1|t-1}) - \omega_t \\
&= \rho\overline{FE}^{deni}_{t-1|t-1} - (1+\theta)\rho\overline{FE}^{deni}_{t-1|t-1} + (1+\theta)P_\epsilon(\omega_t + \epsilon_t) \\
&= -\theta\rho\overline{FE}^{deni}_{t-1|t-1} + ((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t
\end{aligned} \tag{46}
$$

Furthermore, we know

$$\begin{aligned}\overline{FE}^{deni}_{t+h|t} &= \rho^h \overline{FE}^{deni}_{t|t} + FE^*_{t+h|t} \\ \overline{FE}^{deni}_{t+h-1|t-1} &= \rho^h \overline{FE}^{deni}_{t-1|t-1} + FE^*_{t+h-1|t-1}\end{aligned} \tag{47}$$

So,

$$\begin{aligned}\overline{FE}^{deni}_{t+h|t} &= \rho^h \overline{FE}^{deni}_{t|t} + FE^*_{t+h|t} \\ &= \rho^h(-\theta\rho\overline{FE}^{deni}_{t-1|t-1} + ((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^*_{t+h|t} \\ &= -\theta\rho(\overline{FE}^{deni}_{t+h-1|t-1} - FE^*_{t+h-1|t-1}) + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^*_{t+h|t} \\ &= -\theta\rho\overline{FE}^{deni}_{t+h-1|t-1} + \theta\rho FE^*_{t+h-1|t-1} + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^*_{t+h|t} \\ &= \theta\rho(FE^*_{t+h-1|t-1} - \overline{FE}^{deni}_{t+h-1|t-1}) + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^*_{t+h|t}\end{aligned} \tag{48}$$

Rearranging it, we get

$$\overline{FE}^{deni}_{t+h|t} - FE^*_{t+h|t} = -\theta\rho(\overline{FE}^{deni}_{t+h-1|t-1} - FE^*_{t+h-1|t-1}) + \rho^h((1+\theta)P_\epsilon - 1)\omega_t + \rho^h(1+\theta)P_\epsilon\epsilon_t \tag{49}$$

Set h=1, we get

$$\overline{FE}^{deni}_{t+1|t} - FE^*_{t+1|t} = -\theta\rho(\overline{FE}^{deni}_{t|t-1} - FE^*_{t|t-1}) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \tag{50}$$

When $\theta = 0$, $P_\epsilon = 1$ and $\epsilon_t = 0$, the equation collapses to FIRE.
Which is equivalent to the following.

$$\begin{aligned}\overline{FE}^{deni}_{t+1|t} + \omega_{t+1} &= -\theta\rho(\overline{FE}^{deni}_{t|t-1} + \omega_t) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \\ \rightarrow \overline{FE}^{deni}_{t+1|t} &= -\theta\rho(\overline{FE}^{deni}_{t|t-1} + \omega_t) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \\ \rightarrow \overline{FE}^{deni}_{t+1|t} &= -\theta\rho\overline{FE}^{deni}_{t|t-1} - \theta\rho\omega_t + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho\overline{FE}^{deni}_{t|t-1} - \theta\rho\omega_t + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho\overline{FE}^{deni}_{t|t-1} - (\rho(1+\theta)P_\epsilon - \rho - \theta\rho)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho\overline{FE}^{deni}_{t|t-1} - (\rho P_\epsilon + \rho\theta P_\epsilon - \rho - \theta\rho)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho\overline{FE}^{deni}_{t|t-1} - \rho(P_\epsilon + \theta P_\epsilon - 1 - \theta)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho\overline{FE}^{deni}_{t|t-1} + \rho((1+\theta)(1-P_\epsilon))\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1}\end{aligned} \tag{51}$$

This means

$$\overline{FE}^{deni2}_{\bullet+1|\bullet} = \frac{\sigma^2_\omega + \rho^2(1+\theta)^2(1-P_\epsilon)^2\sigma^2_\omega + \rho^2(1+\theta)^2P^2_\epsilon\sigma^2_\epsilon}{1+\theta^2\rho^2} \tag{52}$$

## Reduced-form tests with forecast errors

The FE-based null-hypothesis of FIRE utilize the moment restrictions on forecast errors. In plain words, the null hypotheses of the three tests are the following. First, since the forecasts are on average unbiased according to FIRE, forecast errors across agents should converge to zero in a large sample. Second, forecast errors of non-overlapping forecasting horizon are not serially correlated. Third, forecast errors cannot be predicted by any information available at the time of the forecast, including the mean forecast itself and other variables that are in the agent's information set. This follows from Equation 2. In addition, I include what is called a weak version of the FE-based test which explores the serial correlation of forecast errors in overlapping periods, i.e. 1-year-ahead forecasts within one year. The forecast errors are correlated to the extent of the realized shocks in the overlapping periods. So the positive serial correlation does not directly violate FIRE. But the correlation of overlapping forecast errors still contains useful information about the size of the realized shocks.

Individual-level data are used whenever possible, utilizing the panel structure of both surveys. Since test 2 and 3 requires individual forecasts in vintages that are more than one year apart while SCE only surveys each household for 12 months, the two tests are done with the population average expectations for SCE. Also, the regressions are adjusted accordingly depending on the quarterly and monthly frequency of SPF and SCE. Since these regressions are based on 1-year inflation in overlapping periods, Newy-West standard error is computed for hypothesis testing.

First, all three forecast series easily reject the null hypothesis of unbiasedness at the significance level of 0.1%. There are upward biases in both professional forecasts of core PCE inflation and households' forecast of headline inflation [35], while at the same time professionals underpredicted core CPI inflation over the entire sample period. This was primarily driven by the under-prediction of the inflation over the recent two years since the Pandemic.

Second, the average point forecast one year ago predicts the forecast errors of both groups at the significance level of 0.1%. For headline CPI inflation, for instance, one percentage point inflation forecast corresponds to 0.35 percentage points of the forecast errors one year later. Thus, test 2 in Table 9 easily rejects the second hypothesis test of FIRE that past information does not predict future forecast errors. This suggests that both types of agents inefficiently utilize all information when making the forecasts.

Third, forecast errors are positively correlated with the forecast errors one year ago, with a significant coefficient ranging from 0.35 to 0.572. A higher positive autocorrelation coefficient of forecast errors by households is consistent with the common finding that households are subject to more information rigidity than attentive professionals.

Lastly, test 4 in Table 9 presents a higher serial correlation of forecast errors produced within a year. For SPF forecasts, the serial correlation does not exist beyond 2 quarters, implying the relative efficiency of professional forecasts. For the households, the forecast errors are more persistent over the entire year, in that current forecast errors are correlated with all past forecast errors over the past three quarters. Although the persistence of 1-year forecast errors within one year does not directly violate FIRE,

[35] Coibion et al. (2018) finds the same upward bias for firms' managers.

the fact that households' forecast errors are more persistent than professionals' indicates that the former group is subject to a higher degree of rigidity than the latter one.

Table 9: Tests of Rationality and Efficiency Using Forecast Errors

| | SPF CPI | SPF PCE | SCE |
|---|---|---|---|
| Test 1: Bias | | | |
| Constant | -3.021*** | 0.460*** | 1.673*** |
| | (0.242) | (0.047) | (0.008) |
| N | 5510 | 1610 | 112668 |
| Test2: FE Depends on past information | | | |
| Forecast 1-yr before | 0.350*** | 0.460*** | 4.190*** |
| | (0.035) | (0.047) | (0.659) |
| Constant | -3.452*** | -2.333*** | -12.92*** |
| | (0.386) | (0.192) | (2.213) |
| N | 3945 | 1610 | 84 |
| $R^2$ | 0.828 | 0.826 | 0.311 |
| Test3: FE of non-overllaping forecast horizons are serially correlated | | | |
| Forecast Error 1-year before | 0.350*** | 0.460*** | 0.572** |
| | (0.035) | (0.047) | (0.195) |
| Constant | 0.314 | -1.351*** | -0.149 |
| | (0.231) | (0.156) | (0.445) |
| N | 3945 | 1610 | 84 |
| $R^2$ | 0.828 | 0.826 | 0.0957 |
| Time FE | Yes | Yes | No |
| Test4: Overlapping FE are serially correlated | | | |
| Forecast Error 1-q before | 0.502*** | 0.551*** | 0.327*** |
| | (0.060) | (0.075) | (0.010) |
| Forecast Error 2-q before | 0.0901 | 0.231*** | 0.341*** |
| | (0.064) | (0.060) | (0.024) |
| Forecast Error 3-q before | 0.146* | 0.0693 | 0.333*** |
| | (0.065) | (0.052) | (0.023) |
| Constant | 1.147*** | -0.356*** | 0.509*** |
| | (0.224) | (0.058) | (0.035) |
| N | 2971 | 1338 | 4432 |
| $R^2$ | 0.890 | 0.903 | 0.243 |

Note: white standard errors reported in the parentheses of estimations. *** p<0.001, ** p<0.01 and * p<0.05.