How Do Agents Form Inflation Expectations? Evidence from the Forecast Uncertainty

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Abstract

Existing empirical tests of different models of expectation formation using survey expectations have been primarily based on patterns of forecasts errors, revisions and disagreements. This paper explores the implications of expectation formation for *uncertainty*, which is available only in density forecasts. Full-information rational expectations benchmark (FIRE) implies that ex-ante uncertainty, the size of ex-post forecast errors are both identical to the conditional volatility of inflation, in addition to a zero disagreement. This paper uses empirically observed deviations from such predictions as additional moment restrictions for identifying various workhorse models of expectation formation. It is shown that information from uncertainty helps differentiate theories with seemingly similar aggregate patterns: e.g. sticky expectations versus noisy information. In addition, some models (sticky expectation) are found to be more robust than other models (noisy information and diagnostic expectations) to incorporating information from uncertainty.

Keywords: Inflation, Expectation Formation, Rigidity, Overreaction, Uncertainty, Density Forecast

JEL Codes: D83, E31, E70

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1 Introduction

Theories on how agents form expectations in ways deviating from rational expectations (RE) have been proliferating over the past decade. On one hand, various theories built upon different micro-foundations produce somewhat similar macro patterns. For instance, the information rigidity documented by Coibion and Gorodnichenko (2012, 2015), i.e. the sluggish response in aggregate expectations to new information, can be micro-founded by both Sticky Expectations (SE)[1] and Noisy Information (NI)[2]. On the other hand, there are important and subtle differences in testable predictions from various theories regarding both individual forecasts and aggregate moments. For instance, in contrast with the models featuring information rigidity at the aggregate level, the theory of Diagnostic Expectation (DE) (Bordalo et al., 2018, 2020) implies overreaction to the news at the individual level. [3]

One of the crucial steps ahead in advancing this literature is to test these theories better using expectation surveys. This paper does a cross-moment estimation of each theory by jointly accounting for its predictions about different moments of the forecasts such as forecast errors (FE), cross-sectional disagreements (Disg), and *uncertainty* (Var). Although reduced-form tests focused on first moments have been sufficient to reject the null of the full-information rational expectations (FIRE), identifying the differences among these non-FIRE theories requires more information from second or higher-order moments and needs to rely on more restrictions across moments. Such an idea builds on the seminal work Coibion and Gorodnichenko (2012), which compares different theories by examining if the impulse responses of inflation expectations to externally estimated shocks align with the qualitative predictions of these models. This paper adopts a similar approach, [4] with the key novelty being utilizing an additional moment of forecasts in the estimation, i.e. the uncertainty.

Existing work that studies expectation formation based on survey data mostly focuses on the individual patterns of the mean forecast, forecast errors, and, most, cross-sectional disagreements. But there are additional insights from the forecast uncertainty, which is only available if each survey respondent is asked to assign their own perceived probabilities to a range of values of the variable. Meanwhile, in addition to identifying theories of expectation formation, understanding the drivers of uncertainty per se is important since inflation uncertainty has both microeconomic implications via precautionary saving motives and portfolio investments, as well as macroeconomic implications on inflation dynamics and asset prices.

Two patterns of the observed uncertainty are in violation of benchmark full-information rational expectation (FIRE). First, the persistent cross-sectional dispersion of uncertainty across individual forecasters (Figure 2), like that in the average forecasts, is inconsistent with the FIRE as the latter assumes that agents agree on the data generating process and have the common knowledge of available information. Second,

[1] Mankiw and Reis (2002); Carroll (2003); Reis (2006).

[2] Lucas (1972), Woodford (2001).

[3] Recently, Kohlhas and Walther (2021) proposes an extended model of NI allowing for multiple unobserved components to reconcile the coexistence of under- and overreaction.

[4] Some other contemporaneous papers also structurally estimate theories on expectation formation based on single or multiple moments of surveyed expectations, such as Giacomini et al. (2020); Xie (2019); Bordalo et al. (2020); Farmer et al. (2021).

dynamically, across different vintages of the forecast, the revision in uncertainty is a measure of information gain or the degree of forecasting efficiency (Table 4). FIRE implies that information gain is sufficiently large to match the volatility of realized shocks, but information rigidity implies inefficient revisions in uncertainty.

A deeper insight in this paper is that various models of expectation formation have distinctive parameter restrictions about the uncertainty and its relationship with other moments such as forecast errors and disagreement. Utilizing information from uncertainty helps identify model parameters, especially when the commonly used moments are not sufficient. In addition, even for competing theories that have similar qualitative patterns, a cross-moment structural estimation could be useful in evaluating if the indirectly implied model parameters from the survey data are empirically realistic.

On the theoretical front, this paper contributes to the literature on expectation formation by explicitly deriving predictions of the uncertainty of three non-FIRE workhorse models of expectation formation and one hybrid version seen in the literature: (1) Sticky Expectations (SE); (2) Noisy Information (NI); (3) Diagnostic Expectations (DE); (4) Diagnostic Expectations/Noisy Information (DENI).[5]

FIRE benchmark predicts the ex-ante uncertainty to be exactly equal to the variance of ex-post forecast errors, and to the size of the conditional volatility of inflation. But other models imply different patterns. For instance, in SE, the extra uncertainty arises compared to in FIRE because of lagged updating of the most recent information, hence a slower resolution of uncertainty. It also predicts a greater uncertainty than the size of ex-post forecast errors (a pattern observed in both SCE and SPF). In NI, the uncertainty endogenously depends on the noisiness of signals and determines agents' degree of reaction to the news in the Kalman filtering problem. The model accommodates a flexible relative size of the ex-ante uncertainty and ex-post forecast errors depending on the size of the nosiness of signals. Different from both, the canonical model of DE assumes the uncertainty to be equal to that in FIRE.

These predictions allow for empirical parameter identification of each model using survey data. The headline finding from doing such is that using additional information from surveyed inflation uncertainty further confirms the previous empirical finding of information rigidity using low-order moments in the literature (Coibion and Gorodnichenko (2012, 2015); Coibion et al. (2018)). This means qualitatively, SE and NI outperform the DE and DENI in capturing the rigidity in the aggregate forecasting moments, including that in uncertainty. Note, however, that the estimates of DE do suggest a coexistent overreacting mechanism at the individual level, i.e. a non-zero fraction of agents in the economy has a positive degree of overreaction $\hat{\theta}$. This is consistent with the finding in the literature showing the coexistence of rigidity and overreaction (Angeletos et al. (2021); Kohlhas and Walther (2021)).

The key real-value-added by surveyed uncertainty turns out to be differentiating two rigidity models, SE and NI. Despite the seemingly similar qualitative patterns, SE outperforms NI in yielding more sensible and consistent model parameters. In particular, the estimated updating rates λ are consistently around one-third (meaning one-third fraction of the population update each quarter for both households and professionals), while in contrast, the estimated nosiness of public and private signals

[5] For instance, Bordalo et al. (2020) embeds DE in a NI model to jointly account for the heterogeneous and extrapolating expectations.

σ_ϵ and σ_ξ in NI are unrealistic high and unstable, i.e. at a ballpark value of 3 percentage points or higher. These are arguably too high given an unconditional standard deviation of inflation of 0.8 (for headline CPI) or 0.4 for (Core PCE) in the sample period.

With comparable estimates of different theories using cross-moments restrictions, one can evaluate the model sensitivity of each model of expectation formation in four dimensions: (1) the moments used for estimation, i.e. only the forecast error or higher moments such as disagreement and uncertainty. (2) the specification of the underlying process of the inflation, i.e. an AR(1) with constant volatility or one with different components of the time-varying volatility as that in Stock and Watson (2007). (3) if estimating the underlying process and expectations separately or jointly. The former basically recover the inflation process only based on inflation data, while the latter lets the expectations provide information for estimating the inflation process. (4) whether it accounts for different agents such as households and forecasters' expectations equally well.[6]

Some key findings from the model sensitivity evaluation are as follows, which are also summarized as a scoring card in Table 10.

- **Overall robustness** Sticky expectations (SE) produces the most stable parameter estimate in accounting for the observed patterns of *aggregate* expectations and inflation among competing theories, for both households and professionals, regardless of targeted moments, and for both AR(1) and SV model of inflation.

- **Targeted moments** The parameter estimates of both SE and NI are insensitive to the use of moments in estimation, while in contrast, the estimates of DE and DENI vary greatly depending on the moments used.

- **Mutual-dependence of expectation formation and inflation process** All the theories are somewhat sensitive to whether jointly estimate the inflation process and the survey moments, or separately estimate the two. This reflects the mutual dependence of the underlying inflation process and the degree of deviation from FIRE postulated by a particular theory. For instance, a higher jointly estimated persistence of inflation ρ always comes with a lower estimate of updating rate λ (higher rigidity) in SE, and a smaller degree of overreaction $\hat{\theta}$ in DE. [7] Intuitively speaking, a great degree of reaction in inflation expectations to shocks may come from either a FIRE response to a highly persistent shock, or a lack of information rigidity (or, on the flip side, overreaction).

- **The role of stochastic volatility** SE estimate is most robust to an alternative process of SV. In contrast, DE and DENI are particularly sensitive to alternating the inflation process. Across all theories, however, allowing for two unobserved components and SV significantly improves the within-model consistency of each theory. Most notably, modeling NI as a signal-extraction problem of unobserved components as in SV produces much more sensible and stable estimates of the noisiness of signals.

[6] Cornand and Hubert (2022) documents systematic differences in forecasts of various types of agents in terms of revision frequency and size of cross-sectional disagreement.

[7] Such interdependence is also discussed in Afrouzi et al. (2020) in the context of DE models.

- **The type of agents** Within each theory, households' expectations exhibit less consistency across moments compared to professionals. This evidence of inconsistency adds to the evidence rejecting full-information-rationality based on reduced-form tests.

Finally, in addition to the structural estimation with survey data, I also provide additional reduced-form evidence in violation of the benchmark FIRE predictions using the revision of uncertainty. Like auto-regression tests of revisions in forecasts, revision in uncertainty can also be used to test the efficiency in expectation formation. The new result from this paper is that the revision in uncertainty has a serial correlation that is not consistent with the level of forecast efficiency predicted by rational expectation. This provides an additional result that rejects the null of the rational expectation hypothesis.

1.1 Related Literature

This paper is related to four strands of literature. First, it is related to a series of empirical studies directly testing and evaluating various theories on expectation formation using survey data. For instance, Mankiw et al. (2003), Carroll (2003), Branch (2004). More recent examples include Coibion and Gorodnichenko (2012, 2015); Coibion et al. (2018) that test common implications of various theories with different micro-foundations. In addition to testing particular sets of theories, there are also a number of papers that show people's expectations are driven by individual heterogeneity such as socioeconomic characteristics, cognitive abilities, experiences of macroeconomic histories (Malmendier and Nagel (2015), Das et al. (2017) and D'Acunto et al. (2019)[8]). In terms of the methodology, this paper is closest to Giacomini et al. (2020), which estimates theories of expectation formation using cross-moment restrictions. However, all of these studies simply rely upon point forecasts instead of density forecasts or surveyed uncertainty. This is one theme on which this paper differs from the existing literature.

Second, Manski (2004), Delavande et al. (2011), Manski (2018) and many other papers have advocated long for eliciting probabilistic questions measuring subjective uncertainty in economic surveys. Although the initial suspicion concerning people's ability in understanding, use, and answering probabilistic questions is understandable, Bertrand and Mullainathan (2001) and other work have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) have a thorough discussion on designing, experimenting, and implementing the consumer expectation surveys to ensure the quality of the responses.[9] Broadly speaking, the literature has argued that going beyond the revealed preference approach, availability of survey data provides economists with direct information on agents' expectations and helps avoids imposing arbitrary assumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only the expectation but also the perceived risks matter a great deal.

[8] See D'Acunto et al. (2023) for a thorough survey of the empirical evidence of heterogeneous inflation expectations and their drivers.

[9] Other literature includes Van der Klaauw et al. (2008) and Delavande (2014), etc.

Third, by approximating subjective uncertainty directly from density responses, this paper contributes to the literature that develops and uses a variety of measures of uncertainty, especially in the macroeconomic context. There is a long tradition of approximating uncertainty by measures that are more directly available in survey data or that can be estimated by econometric methods. For instance, Bachmann et al. (2013) use ex-ante disagreement and ex-post forecast errors computed from forecasters' surveys as proxies of uncertainty. Jurado et al. (2015) define the time-varying uncertainty as conditional volatility of the non-forecastable component of a variable and estimate it using multiple macroeconomic series. Binder (2017) approximate uncertainty from rounding in survey data based on the insights from cognitive literature. Besides, the text-based approach such as Bloom (2009) constructs indices of policy uncertainty based on texts of newspaper reporting. Although these proxies are all meant to capture the notion of uncertainty, as shown in Section 2.4, cross-validation seems to suggest they are statistically uncorrelated or even negatively correlated.

Fourth, the literature that has been originally developed under the theme of forecast efficiency[10] provides a framework analyzing the dynamics of uncertainty useful for the purpose of this paper. The focus of the forecasting efficiency literature is evaluating forecasters' performance and improving forecasting methodology, but it can be adapted to test the theories of expectation formation of different types of agents. This is especially relevant to this paper as I focus on uncertainty.

The paper is organized as followed. Section 2 shows the stylized patterns of different forecasting moments of professional forecasts and households. Section 3 first sets up a unified framework in which testable predictions of different theories can be compared. Also, I derive various moment conditions from these theories. Section 4 undertakes reduced-form time-series regressions that test the null hypothesis of FIRE and the implications of different theories. Section 5 includes results from estimating the theory-specific parameters using the simulated method of moments. It also evaluates the sensitivity of the model specification. Section 6 concludes the paper and discusses the future research directions.

2 Data and Facts

2.1 Definition and notation

An agent i is forming expectations about a stochastic macroeconomic variable y_{t+h} [11], the inflation in this paper. Denote $f_{i,t+h|t}$ as agent i's h-period-ahead density forecast. $f_{i,t+h|t}$ is the conditional density of y_{t+h} given the information set $I_{i,t}$ available at time t.

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

[10] Nordhaus (1987), Davies and Lahiri (1995), Clements (1997), Faust and Wright (2008), Patton and Timmermann (2012).

[11] Only in the context of aggregate variable, it makes sense to study the population moments such as average expectations and disagreements. Studying expectations of idiosyncratic variables requires individual panel data, as well as the idiosyncratic realizations of the variable.

The information set could be agent-specific, thus it has subscript i. The specific content contained in I_t varies from different models of expectation. For instance, sticky expectation (SE) and rational inattention[12] literature all assume that agents are not able to update new information instantaneously. So the information set may not contain the most recent realization of the variable of forecast y_t. In contrast, NI assumes that the information set only contains noisy signals of the underlying variables. Different theories may also differ in terms of the mapping from information to conditional density forecasts. [13] For instance, DE deviates from Bayesian learning by allowing agents to overweight new information that is particularly salient.

Accordingly, h-period-ahead mean forecast at t, denoted as $y_{i,t+h|t}$, is the conditional expectation of y_{t+h} by the agent i.

$$y_{i,t+h|t} \equiv E_{i,t}(y_{t+h}) = \int y_{t+h} f_{i,t+h|t} dy_{t+h}$$

Similarly, individual forecasting variance $Var_{i,t+h|t}$, hereafter termed as individual uncertainty in this paper, is the conditional variance corresponding to the forecast density distribution.

Individual forecast error $FE_{i,t+h|t}$ is the difference of individual forecast at time t and ex-post realized value of y_{t+h}. By definition. positive (negative) forecast errors mean overpredicting (underpredicting) the variables.

$$FE_{i,t+h|t} = y_{i,t+h|t} - y_{t+h}$$

The population analogs of the individual mean forecast, uncertainty, and forecast errors are simply the average of the individual moments taken across agents. Denote them as $\bar{y}_{t+h|t}$, $\overline{Var}_{t+h|t}$, and $\overline{FE}_{t+h|t}$, respectively. Hereafter, they are referred to as the population mean forecast, population uncertainty, and population forecast error, respectively. In addition, disagreement is defined as the cross-sectional variance of mean forecasts of individual agents, denoted as $\overline{Disg}_{t+h|t}$.

I refer to the 3 individual indicators and 4 population indicators defined above as moments, and they are listed in Table 1.

Table 1: Definition and Notation of Moments

Individual Moments	Population Moments
Mean forecast: $y_{i,t+h|t}$	Average forecast: $\bar{y}_{t+h|t}$
Forecast error: $FE_{i,t+h|t}$	Average forecast error: $\overline{FE}_{t+h|t}$
Uncertainty: $\mathrm{Var}_{i,t+h|t}$	Average uncertainty: $\overline{\mathrm{Var}}_{t+h|t}$
	Disagreements: $\overline{Disg}_{t+h|t}$

[12] Sims (2003).

[13] There are other classes of models that fall into this category, which assume alternative mapping from the agent's information set to the conditional density. For instance, Patton and Timmermann (2010); Farmer et al. (2021) find that the disagreements are driven by not only the difference in information but also heterogeneity in prior and models. Macaulay and Moberly (2022) shows survey evidence for heterogeneity in the perceived persistence of inflation shocks. More theoretical work includes multi-prior or model uncertainty such as Hansen and Sargent (2001), Hansen and Sargent (2008), etc.

2.2 Benchmark predictions from full-information rational expectation (FIRE)

We start by assuming an underlying data generating process of inflation. In the benchmark scenario, we assume that y_t follows $AR(1)$ with persistence parameter $0 < \rho < 1$ and i.i.d. shock ω_t with a time-invariant volatility of σ_ω.

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma^2_\omega) \tag{1}$$

The assumption of i.i.d. shock with constant volatility is inadequate to match the time-varying pattern of uncertainty. Therefore, as an extension, I also consider an alternative assumption that inflation consists of two unobservable components with stochastic volatility. I will specify this explicitly later to focus on comparing different models of expectation formation under a simple AR(1) process.

In the FIRE benchmark, it is assumed that all agents perfectly observe y_t at time t and understand the true process of y. Therefore, the individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to the average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ and $t+h$.

$$FE^*_{t+h|t} = -\sum_{s=1}^{h} \rho^{s-1}\omega_{t+h-s} \tag{2}$$

I use the superscript of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time t. This provides a well-known null hypothesis of FIRE. [14]

The unconditional variance of h-period-ahead FE, or equivalently, its square (due to its zero unconditional mean), is equal to the following.

$$FE^{*2}_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)}\sigma^2_\omega \tag{3}$$

The uncertainty about future y simply comes from uncertainty about unrealized shocks between t and $t+h$. With the same model in mind (Equation 1) and the same information y_t, everyone's uncertainty is equal to the weighted sum of the future volatility before its realization (Equation 4), which is exactly equal to the variance of forecast errors, $FE^{*2}_{\bullet+h|\bullet}$. In FIRE, there is neither disagreement about the mean, nor disagreement about the uncertainty.[15]

$$\text{Var}^*_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)}\sigma^2_\omega \tag{4}$$

[14] Another well-known prediction of FIRE is that forecast errors of non-overlapping horizon are not correlated, namely, $Cov(FE^*_{t+h|t}, FE^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h$. This is not the case within h periods, as the realized shocks in overlapping periods enter both forecast errors.

[15] This is the same to Jurado et al. (2015)'s terminology.

The time-series behavior of h-period-ahead uncertainty, i.e. $\text{Var}_{t+h|t}$, $\text{Var}_{t+h+1|t+1}$, etc., depends on the true process of y. Specifically, it depends on whether σ^2_ω is time-varying. If time-invariant, h-period-ahead uncertainty is simply a constant. I assume this in the baseline case. But, generally, it may not be true. In the extension, I make alternative assumptions of the inflation process allowing for stochastic volatility.[16]

Another testable implication of rationality lies in the revision of uncertainty. Hereafter, we refer to revision (instead of change) as the difference of moments across vintages of the forecast with the fixed terminal date of realization. For instance, the uncertainty revision for from $t-1$ to t the h-period-ahead forecast is the difference between the uncertainty about y_{t+h} at time t and the uncertainty about y_{t+h} at time $t-1$.

Moving from t to $t+1$, for instance, the revision in uncertainty is simply a negative constant independent of the time. There is an unambiguous reduction in uncertainty (or information gain in the forecasting literature) as more and more shocks have been realized. In the most intuitive case, from the one-step-ahead forecast at $t-1$ to that of t, i.e. $h=1$, the variance drops exactly by the resolution of the uncertainty of ω_t, which is σ^2_ω, to zero uncertainty.

$$\text{Var}^*_{t+h|t+1} - \text{Var}^*_{t+h|t} = -\rho^{2(h-1)}\sigma^2_\omega \tag{5}$$

Lastly, FIRE has predictions about the disagreement. As agents perfectly update the same information, there is no disagreement at any point of time.

$$Disg^*_{t+h|t} = 0 \quad \forall t \tag{6}$$

Table 2 summarizes all other expectation moments of 1-period-ahead inflation ($h=1$) as predicted by FIRE and a process of AR1. Both variances of forecast errors ($FEVar$) and average uncertainty Var are equal to the size of the shock to inflation σ^2_ω, while in contrast, the disagreement is always zero, hence has zero correlation with $FEVar$ and Var. In Section 2.4, I discuss in greater detail how the data counterparts of the forecast moments are inconsistent with these predictions.

2.3 Data

This paper uses one special survey feature of professionals' and households' inflation expectations, which is the density forecast. In particular, respondents are asked to report their perceived distribution, instead of a point forecast, of future inflation.

Survey of Professional Forecasters (SPF) has elicited density forecasts of core CPI and core PCE inflation since 2007. In addition, for these two inflation measures, both forecasts of current-year inflation, basically nowcast, and one-year-ahead forecasts are elicited via density surveys. This makes it possible to directly test the implications of the revisions in uncertainty.

[16] For example, Justiniano and Primiceri (2008), Vavra (2013) on time-varying volatility of inflation.

Table 2: Moments of Inflation and Expectations

	SPF	SCE	FIRE+AR	FIRE+SV
InfAV	0	0	0	0
InfVar	0.159	0.653	$\sigma^2_\omega/(1-\rho^2)$	N/A
InfATV	0.125	0.621	$\rho\sigma^2_\omega/(1-\rho^2)$	N/A
FE	0.136	1.772	0	0
FEVar	0.133	0.923	σ^2_ω	$\bar{\sigma}^2_\eta + \bar{\sigma}^2_z$
FEATV	0.097	0.89	0	0
Disg	0.183	2.585	0	0
DisgVar	0.028	0.057	0	0
DisgATV	0.021	0.025	0	0
Var	0.242	1.75	σ^2_ω	$\bar{\sigma}^2_\eta + \bar{\sigma}^2_z$
VarVar	0.001	0.023	0	>0
VarATV	0.001	0.004	0	>0

This table reports the moments of demeaned inflation and inflation expectations of both SPF and SCE used in the model estimation. Core CPI inflation is used for SPF, and headline inflation is used for SCE. The sample period is 2007M1-2020M3 for SPF and 2013M1-2020M3 for SCE. For SCE moments, both disagreement (Disg) and uncertainty (Var) are computed using the regression residuals of individual mean forecast and uncertainty after controlling for individual fixed effects.

The New York Fed's *Survey of Consumer Expectations* (SCE), which started in 2013, also asked households to report their distribution forecasts of 1-year- and 3-year-ahead inflation for various ranges of values each month.[17] This allows for comparing the 3-year-ahead forecast at time $t-3$ with the 1-year-ahead forecast at $t-1$. Since the maximum duration for households to stay in the panel is 12 months (for about one-third of the households), forecast revision can be only examined at the population level. The advantage of SCE compared to SPF is its monthly frequency. This provides an invaluable chance to explore the dynamics of uncertainty. A summary of the two surveys is in Table 3.

Converting expressed probability forecasts based on externally divided bins into an underlying subjective distribution requires a density distribution. I closely follow Engelberg et al. (2009)'s method with a small modification to estimate the density distribution of each individual respondent in SPF. [18] For SCE, I directly use the estimates by the New York Fed researchers Armantier et al. (2017), following the same method.

To avoid the biases introduced by extreme answers or data errors, I drop the outliers

[17]The survey respondents are guaranteed to assign probabilities to all bins that sum up to one, as a feature of the online survey design.

[18]Answers with at least 3 bins with positive probabilities or 2 bins but open-ended from either left or right are fit with a generalized beta distribution. Depending on if there is an open-ended bin on either side with positive probability, a 2-parameter or 4-parameter beta distribution is estimated, respectively. Those with only two bins with positive probabilities and adjacent are fit with a triangular distribution. Answers with only one bin of positive probability are fit by a uniform distribution. See the Python program with detailed steps of estimation.

of mean forecast and uncertainty estimates at both the top and bottom one percentile for the SPF sample and at top and bottom three percent. [19] All the results in this paper are robust to different thresholds.

Table 3: Information about the Survey Data

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1-yr-ahead inflation	1-yr-ahead Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

In this paper, I primarily focus on two measures of inflation: the headline CPI for households and core CPI for professional forecasters.

2.4 Stylized facts about uncertainty

2.4.1 Relationship between the size of forecast errors, disagreement, and uncertainty

Despite the stark differences in magnitudes between professionals' and households' forecasting moments, both types of agents share common patterns in terms of the relationship across various moments. Figure 1a, 1b, and 1c plot the population uncertainty against expected inflation, squared forecast errors, and disagreements in the first, second, and third rows, respectively.

Both academic research[20] and popular narratives suggest that higher inflation or expected inflation is generally associated with greater inflation uncertainty. This relationship is partially reflected in the positive correlation observed between expected inflation and directly measured forecasting uncertainty among both households and professional forecasters, with correlation coefficients of 0.28 for SPF and 0.58 for SCE from 2007-2021. However, it is important to note that this correlation is primarily driven by recent dynamics since 2020, with the onset of higher inflation. Prior to 2020, the correlation coefficients were much lower, at 0.12 and -0.24 for SPF and SCE, respectively.

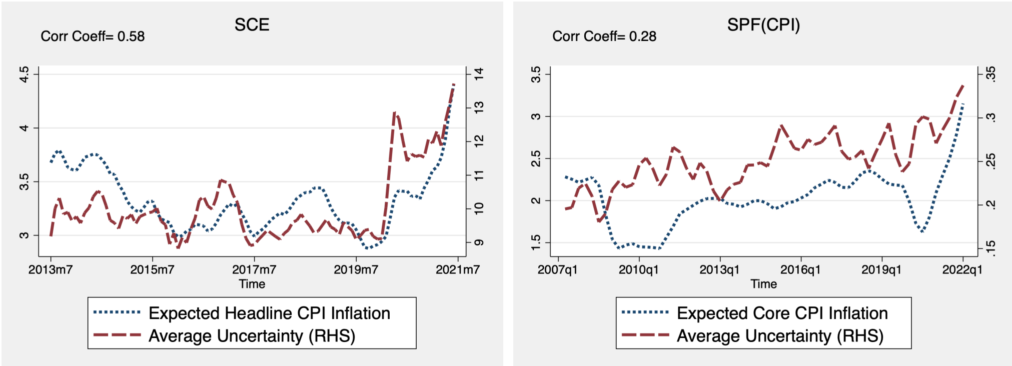
Figure 1b inspects the relationship between the size of the forecast error and uncertainty. According to the benchmark prediction under FIRE, the ex-ante forecast uncertainty is equal to the variance of ex-post forecast errors on average, as shown in Table 2. In the data, the correlation coefficients of the two are 0.25, and 0.42 for SPF Core CPI forecasts and SCE's forecasts, respectively. Excluding the post-2020 sample

[19] For mean forecasts and uncertainty, respectively, this means dropping 6528 and 5096 observations, out of 68887 observations in total.

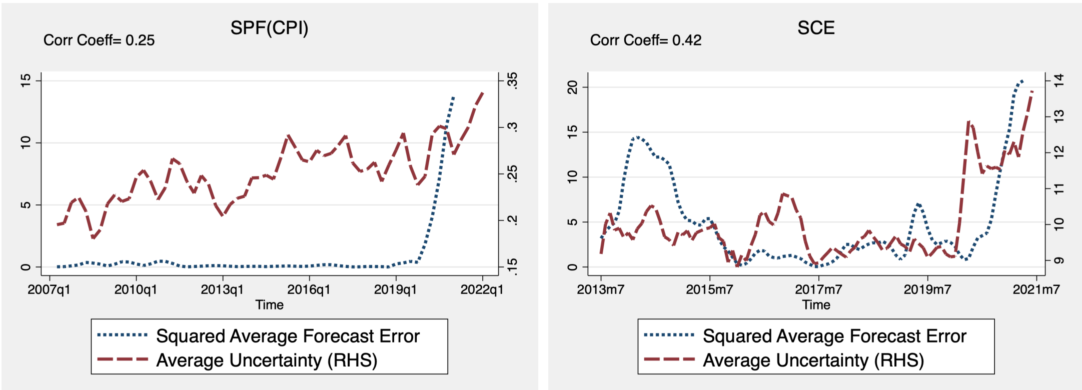
[20] For example Ball et al. (1990); Ball (1992).

Figure 1: Uncertainty and Other Moments

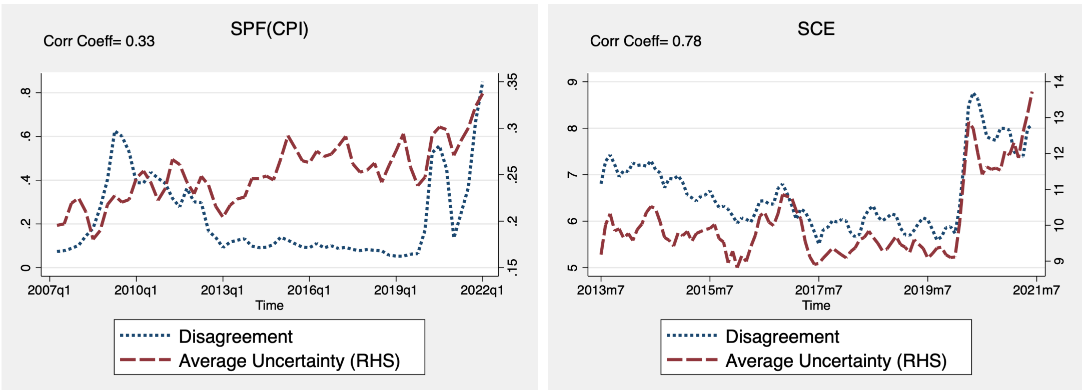
(a) Expected Inflation and Uncertainty



(b) Squared Forecast Errors and Uncertainty



(c) Disagreement and Uncertainty



Note: From the left to right: SPF's forecasts of core CPI and SCE's household forecast of headline CPI. From the top to the bottom: uncertainty (dash line) versus expected inflation (dot) with a correlation coefficient of 0.58, and 0.28, respectively; uncertainty (dash) versus square of the realized forecast errors (dot) with a correlation coefficient of 0.25, 0.42, respectively; uncertainty (dash) versus disagreements (dot) with a correlation coefficient of 0.33, and 0.78, respectively.

yields even smaller correlation coefficients. The fact that the two are at most weakly correlated is inconsistent with the FIRE benchmark prediction.

Figure 1c examines the relationship between disagreement and uncertainty. A large body of empirical literature in macroeconomics uses disagreement, which is often more available than uncertainty, as a proxy of the latter.[21] This practice implicitly assumes some form of deviation from the benchmark FIRE, as in FIRE, regardless of the inflation process, disagreement should be always zero, and it is therefore not correlated with the average uncertainty.[22] The empirical correlation between disagreement and uncertainty is indeed weakly positive, which is 0.33 for professionals, and 0.78 for households. The positive correlation between the two was primarily driven by the post-2020 dynamics, which saw rapidly rising inflation.

To summarize, compared to the FIRE prediction of a perfect correlation between ex-ante uncertainty and the size of ex-post forecast error in the benchmark FIRE, the empirical patterns of professionals and household expectations exhibit divergent and time-varying behaviors of the two.

2.4.2 Dispersion in forecast uncertainty

A persistent disagreement in expectations has been used as important evidence inconsistent with the assumption of identical expectations predicted by FIRE (Mankiw et al. (2003)). A similar argument can be made with the dispersion in forecasting uncertainty. FIRE predicts individuals share an equal degree of uncertainty. [23]

Figure 2 plots the median uncertainty along with its 25/75 percentiles in both SCE and SPF. There is persistent dispersion in uncertainty across agents. The dispersion in uncertainty of households is much greater than that of the professionals. The IQR of the uncertainty of households is around 150-200 times(12–14 times in standard deviation terms) of that of professional forecasters.

One difference in the distribution of uncertainty between households and professionals is that the distribution of former is more skewed toward the right (higher uncertainty), while professional forecasters disagree in uncertainty more symmetrically around its cross-sectional mean.[24].

Another pattern worth discussing in Figure 2 is that there is a notable rise in the dispersion of professional forecasts in the post-2020 period with a rapid rise in inflation, primarily driven by an increase of the upper end of the forecasts (i.e. 75 percentile

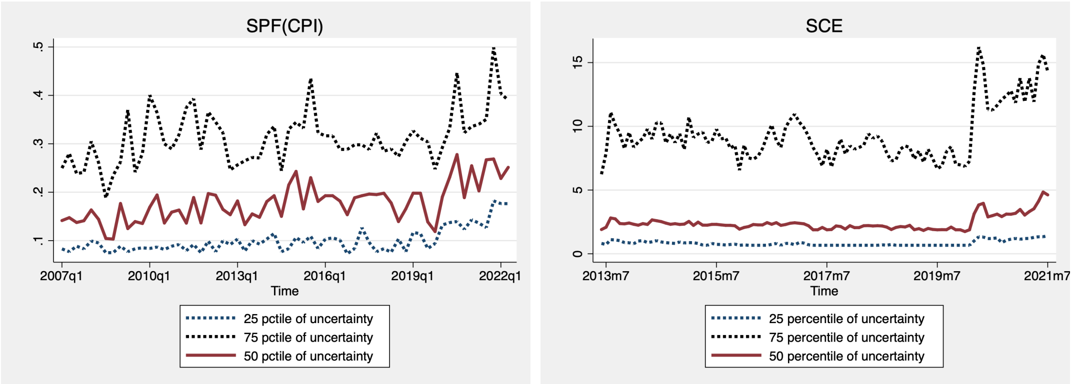
[21] For instance, Bachmann et al. (2013) used ex-ante disagreement and ex-post forecast errors as two measures of uncertainty and find that both uncertainty indicators lead to a reduction in real economic activity.

[22] This point was made very clearly by Zarnowitz and Lambros (1987). Manski (2018) also points out that much empirical work has confused dispersion with uncertainty.

[23] In contrast, SE predicts that the uncertainty of individuals differs in that agents are not equally updated at a point in time. NI generates a homogeneous degree of uncertainty only under the stringent conditions of an equal precision of signals and the same prior for uncertainty (Equation 23). DE predicts an equal degree of uncertainty across agents (Equation 28). Therefore, taken by the face value, the presence of dispersion of uncertainty across agents is not consistent with predictions of FIRE, the canonical version of NI and DE.

[24] Kumar et al. (2015) also presents the dispersion in uncertainty using a shorter period of sample for SCE.

Figure 2: Dispersion of Uncertainty



forecast increases from 1% to 2%). [25]

2.4.3 Revision in uncertainty

FIRE also predicts an unambiguous reduction in uncertainty as one approaches the date of realization, where the drop is exactly equal to the volatility of the realized shocks. I only focus on SPF forecasts here, as the individual-specific revisions are not available in SCE. Figure 3 plots the average revision in mean forecasts and uncertainty from 1-year-ahead forecasts in year $t-1$ to the current-year nowcast in year t. The more negative range in which the revision lies, the more "rational" the forecast. The histogram suggests that uncertainty revisions are left-skewed relative to zero. This implies, on average, forecasters feel more certain about their nowcasts relative to their forecast made one year before. However, over the entire sample, there is always a positive fraction of forecasters who revise uncertainty upward, which is inconsistent with the benchmark prediction. A formal test of revision equal to zero or negative will be carried out in Section 4.

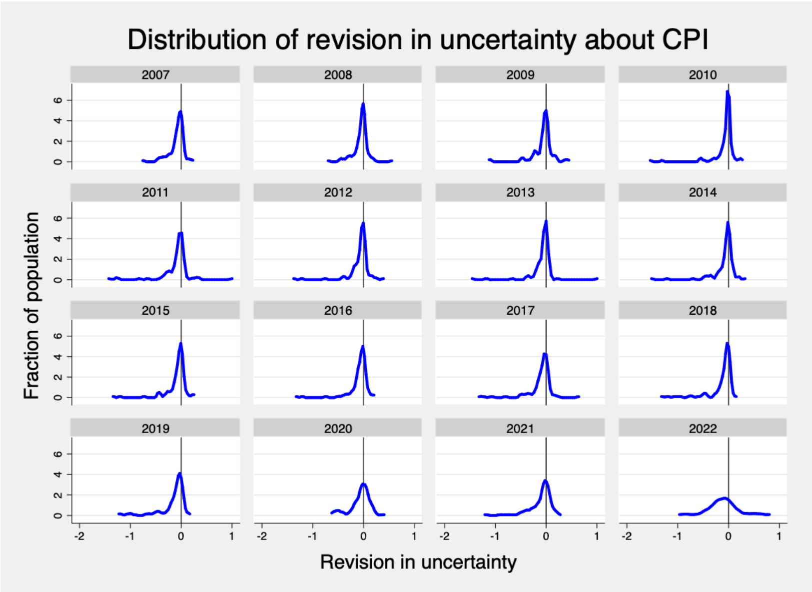
3 Theories of Expectation Formation

3.1 Sticky expectation (SE)

The theory of sticky expectation (Mankiw and Reis (2002), Carroll (2003) and others.), regardless of various micro-foundations, builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One tractable assumption is that agents update their information with a homogeneous and time-independent probability, denoted by λ. Specifically, at any point of time t, each agent learns about the up-to-date realization of y_t with the probability of λ; otherwise, they form the expectation based on the most recent up-to-date realization of $y_{t-\tau}$, where τ is the elapsed time since the last update.

[25] This should be interpreted with caution, since the disagreements of SPF forecasts shown in Figure 1c actually exhibit a gradual decline.

Figure 3: Distribution of Uncertainty Revisions



The average forecast under SE is a weighted average of update-to-date rational expectation and lagged average expectation, as reproduced below.[26] It can also be expressed as a weighted average of all the past realizations of y. Setting $\lambda = 1$, then the SE collapses to FIRE, and the average forecast is equal to y's long-run mean of zero.

$$\bar{y}^{se}_{t+h|t} = \lambda \underbrace{y^*_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}^{se}_{t+h|t-1}}_{\text{average forecast at } t-1} \tag{7}$$

It follows that the average forecast errors are serially correlated, as described in Equation 8.

$$\overline{FE}^{se}_{t+h|t} = (1-\lambda)\rho \overline{FE}^{se}_{t+h-1|t-1} + \lambda FE^*_{t+h|t} \tag{8}$$

The unconditional variance of the h-period-ahead forecast error (or its square) is proportional to that of the FIRE. It is also easy to confirm the former is always smaller than the latter as long as there is stickiness ($\lambda < 1$). Intuitively speaking, stickiness in expectation implies attenuated responses to shocks than in FIRE, hence a lower variation in forecast errors across time.

$$\overline{FE}^{se2}_{\bullet+h|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2} FE^{*2}_{\bullet+h|\bullet} \leq FE^{*2}_{\bullet+h|\bullet} \tag{9}$$

Like average forecasts, average uncertainty at time t is also a weighted average of uncertainty to agents whose last updates took place in different periods of the past:

[26]See Coibion and Gorodnichenko (2012) for detailed steps.

$t-\tau \quad \forall \quad \tau = 0, 1...\infty$. (Equation 10) Since at any point in time, there are agents who have not updated the recent realization of the shocks, thus with higher uncertainty, the population uncertainty is unambiguously higher than in the case of FIRE. (See Appendix 6 for detailed derivations.)

$$
\begin{aligned}
\overline{\text{Var}}^{se}_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction of non-updater until } t-\tau} \underbrace{\text{Var}^{*}_{t+h|t-\tau}}_{\text{uncertainty based on updating by } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma^2_{\omega} \\
&= (\frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2} - 1)\frac{1}{\rho^2-1}\sigma^2_{\omega} \\
&\geq \overline{\text{Var}}^{*}_{t+h|t}
\end{aligned} \tag{10}
$$

With respect to revision, the inefficiency of reducing uncertainty in SE takes the following form at the aggregate level. Since not all agents incorporate the recently realized shocks, the revision in average uncertainty exhibits a serial correlation, as described in Equation 11. It is a weighted average of the resolution of uncertainty from the most recent shocks and its lagged counterpart.

$$
\overline{\text{Var}}^{se}_{t+h|t+1} - \overline{\text{Var}}^{se}_{t+h|t} = (1-\lambda)(\overline{\text{Var}}^{se}_{t+h|t} - \overline{\text{Var}}^{se}_{t+h|t-1}) - \lambda\rho^{2(h-1)}\sigma^2_{\omega} \tag{11}
$$

In particular, the second component is the information gain from the most recent realization of the shock, underweighted by $\lambda < 1$. The first component is the inefficiency sourced from the stickiness of updating. The higher rigidity (lower λ), the smaller the efficiency gain or uncertainty reduction compared to FIRE.

Lastly, SE also predicts non-zero disagreements and sluggish adjustment compared to FIRE. This is because of different lags in updating across populations.

$$
\overline{Disg}^{se}_{t+h|t} = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (y_{t+h|t-\tau} - \bar{y}_{t+h|t})^2 \tag{12}
$$

In summary, SE predicts a higher average uncertainty and a lower forecast error square than their counterparts in FIRE, both of which should be identical to the conditional volatility of inflation under FIRE. In addition, SE predicts positive disagreements. These patterns are indeed observed in survey data, as reported in Table 2. Next, we move to other theories to see if such patterns are distinctive predictions by SE.

3.2 Noisy information (NI)

A class of models (Lucas (1972), Sims (2003), Woodford (2001), and Maćkowiak and Wiederholt (2009), etc), noisy information (NI hereafter), describes the expectation formation as a process of extracting the underlying variable y_t from a sequence of realized signals.

I adopt the signal structure laid out in Coibion and Gorodnichenko (2015) by assuming that an agent i observes two signals, s^{pb} being a public signal common to all agents, and s^{pr}_i, being a private signal specific to the agent i. The generating process of two signals is assumed to be the following.

$$\begin{aligned} s^{pb}_t &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2_\epsilon) \\ s^{pr}_{i,t} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0, \sigma^2_\xi) \end{aligned} \tag{13}$$

Stacking the two signals into one vector $s_{i,t} = [s^{pb}_t, s^{pr}_{i,t}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$, the equations above can be rewritten as

$$\begin{aligned} s_{i,t} &= Hy_t + v_{i,t} \\ &\text{where } H = [1, 1]' \end{aligned} \tag{14}$$

Now any agent trying to forecast the future y has to form her expectation of the contemporaneous y. Denote it as $y^{ni}_{i,t|t}$, which needs to be inferred from the signals particular to the agent i. The agent's best h-period ahead forecast is simply iterated h periods forward based on the AR(1) process, and it is equal to $\rho^h y^{ni}_{i,t|t}$.

So the crucial difference between NI from FIRE lies in the nowcasting. In particular, the agent makes her best guess of y_t using Kalman filtering at the time t. The nowcast of individual i is the posterior mean based on her prior and realized signals $s_{i,t}$.

$$\begin{aligned} y^{ni}_{i,t|t} &= \underbrace{y^{ni}_{i,t|t-1}}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\ &= (1 - PH)y^{ni}_{t|t-1} + PHy_t + Pv_{i,t} \end{aligned} \tag{15}$$

where the Kalman gain P is a vector of size of two that determines the degrees of reaction to signals.

$$P = [P_\epsilon, P_\xi] = \text{Var}^{ni}_{i,t|t-1} H(H' \text{Var}^{ni}_{i,t|t-1} H + \Sigma^v)^{-1} \tag{16}$$

$\text{Var}^{ni}_{i,t|t-1}$ is the forecast uncertainty of y_t based on prior beliefs up to $t-1$. And Σ^v is a 2-by-2 matrix indicating the noisiness of the two signals.

$$\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix} \tag{17}$$

Individual forecast partially responds to new signals as $PH < 1$. $PH = 1$ is a special case when both signals are perfect, thus $\Sigma^v = 0$, then the formula collapses to FIRE.

A comparable parameter with $1-\lambda$ in SE that governs rigidity in NI is $1-PH$. It is a function of prior uncertainty, $\text{Var}_{t-1|t-1}$, and the noisiness of the signals determined by Σ^v. Note that beyond the steady state, P is time-varying as the variance is updated by the agent each period (as in Equation 21). With constant volatility in AR(1), we can focus on the Kalman gain in the steady state, corresponding to a constant variance. Therefore, I drop time t from P.

What differentiates average forecasts from individual's is the role played by private signals. On average, private signals cancel out across agents, therefore, only public signals enter the average forecast, hence, average forecast errors (Equation 19).

$$\begin{aligned}\bar{y}^{ni}_{t+h|t} &= \rho^h[(1-PH)\underbrace{\bar{y}^{ni}_{t|t-1}}_{\text{Average prior}} + P\underbrace{\bar{s}_t}_{\text{Average signals}}] \\ &= (1-PH)\bar{y}^{ni}_{t+h|t-1} + \rho^h PHy_t + \rho^h P_\epsilon \epsilon_t\end{aligned} \tag{18}$$

The population average forecast error under NI takes a similarly recursive form as in SE.

$$\overline{FE}^{ni}_{t+h|t} = (1-PH)\rho\overline{FE}^{ni}_{t+h-1|t-1} + \rho^h P_\epsilon \epsilon_t + FE^*_{t+h|t} \tag{19}$$

The average square (or the unconditional variance) of the forecast errors is unambiguously greater than $FE^{*2}_{t+h|t}$ in FIRE, as shown in Equation 20. The simple reason for this is that the forecast errors under NI always come from not only the realized shocks to inflation but also nowcasting noises.

$$\overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^{2h}P^2_\epsilon\sigma^2_\epsilon + FE^{*2}_{\bullet+h|\bullet}}{(PH)^2} \tag{20}$$

Kalman filtering also updates the variance recursively according to following rule. The posterior uncertainty at time t is a linear function of prior uncertainty and noisiness of signals.

$$\text{Var}^{ni}_{i,t|t} = \text{Var}^{ni}_{i,t|t-1} - \text{Var}^{ni}_{i,t|t-1}H'(H\text{Var}^{ni}_{i,t|t-1}H' + \Sigma^v)^{-1}H\text{Var}^{ni}_{i,t|t-1} \tag{21}$$

The unconditional nowcasting variance can be solved as the steady-state value of Equation 21. In the steady-state, there is no heterogeneity across agents in forecasting uncertainty and the nowcasting uncertainty becomes a constant. Thus, we can drop the subscript i. Note that the average uncertainty non-linearly depends on the noisiness of the two signals σ^2_ϵ and σ^2_ξ, as well as the volatility of inflation itself.

Equation 21 also directly gives the revision in uncertainty from time $t-1$ to t. The newly arrived information, albeit noisy, still brings about information gains, thus leading to an unambiguous drop in uncertainty. But due to the signal is not perfect, i.e. $\Sigma^v \neq 0$, there is inefficiency in reducing uncertainty compared to in FIRE.

$$\text{Var}^{ni}_{i,t|t} - \text{Var}^{ni}_{i,t|t-1} = -\text{Var}^{ni}_{i,t|t-1}H'(H\text{Var}^{ni}_{i,t|t-1}H' + \Sigma^v)^{-1}H\text{Var}_{i,t|t-1} < 0 \tag{22}$$

The h-period-ahead forecasting uncertainty comes from both nowcasting uncertainty and volatility of unrealized shocks in the future. (Equation 23)

$$\text{Var}^{ni}_{i,t+h|t} = \rho^{2h}\text{Var}^{ni}_{i,t|t} + \sum_{s=1}^{h} \rho^{2(s-1)}\sigma^2_{\omega} \geq \text{Var}^{*}_{t+h|t} \tag{23}$$

As a result of NI mechanism, the revision in h-period-ahead uncertainty from $t-1$ to t only partially reacts to the resolution of uncertainty from newly realized shock ω_t in the past period.

NI also predicts non-zero disagreement in the presence of private signals. The size of the disagreement depends on, but is not a strictly increasing function of, the noisiness of the private signals. It is so because if the noisiness of private signals σ_ξ is extremely larger than that of the public signal σ_ϵ, agents will optimally choose not at all react to private signals. In this scenario, the disagreement will no longer increase with σ_ξ.

$$\overline{Disg}^{ni}_{t+h|t} > 0 \tag{24}$$

In summary, the relative size of the square of forecast error and uncertainty is ambiguous under NI, in contrast with that in SE. Disagreement is possibly positive as long as there is dispersed information in the form of private signals, and they are not too noisy. Although there is no clear-cut prediction about the relative size, all three moments contain parametric restrictions about the noisiness of public and private signals. It is possible that, under a range of parameter values, NI generates moments that are consistent with the observed data from the survey. We leave this task for the structural estimation in Section 5.

3.3 Diagnostic expectations (DE)

Different from the previous two theories featuring informational rigidity, diagnostic expectation (Bordalo et al. (2018)) introduces an extrapolation mechanism in expectation formation that results in overreactions to the news (Bordalo et al. (2020)). Both SE and NI deviate from FIRE in terms of the information set available to the agents, while DE deviates from FIRE in terms of the processing of an otherwise fully updated information set.

Skipping over its micro foundation, Equation 25 captures the essence of DE mechanism. Each individual i's h-period-ahead forecast consists of two components. The first component can be considered as a rational forecast based on the fully updated y_t. The second component corresponds to the potential overreaction to the unexpected surprises from $t-1$ to t. The degree of overreaction is governed by the parameter θ. The premise of DE model is that $\theta > 0$, which captures the fact that the agent overly responds to the realized forecast errors. The model collapses to the FIRE when $\theta = 0$. Meanwhile, as argued in Bordalo et al. (2020), a negative θ is not incompatible with an interpretation of underreaction if we treat DE as a more generalized model of expectation formation.

$$\bar{y}^{de}_{i,t+h|t} = \rho^h y_t + \theta_i(\rho^h y_t - \bar{y}^{de}_{i,t+h|t-1}) \tag{25}$$

There is no room for disagreement with a homogeneous degree of overreaction. To account for the possibility of a positive disagreement, I assume θ to be different across different agents. Therefore, I add the subscript i to the parameter. Since agents are equally informed about the realizations of the variable, the only room for disagreement to be positive is heterogeneous degrees of overreaction. To capture this, I assume θ_i to follow a normal distribution across the population, $N(\hat{\theta}, \sigma^2_\theta)$. So the DE model has two parameters. Disagreement increases with the dispersion of overreaction, σ_θ.

The average forecast takes exactly the same form, with the individual-specific θ_i replaced by the population average $\hat{\theta}$. Therefore, we focus on average forecast errors directly. The average forecast error under DE evolves as the following (See Appendix 6 for derivations)

$$\overline{FE}^{de}_{t+h|t} - FE^*_{t+h|t} = -\hat{\theta}\rho(\overline{FE}^{de}_{t+h-1|t-1} - FE^*_{t+h-1|t-1}) + \rho^h\hat{\theta}\omega_t \tag{26}$$

The formula contains the intuition behind the extrapolation mechanism in DE: moving from $t-1$ to t, the h-period-ahead forecast error exceeds that of FIRE forecast error by exactly the surprise to the expectation formed at $t-1$ with a degree of overreaction $\hat{\theta}$.

The square or the unconditional variance of h-period-ahead forecast errors is the most straightforward in the special case $h = 1$, which is equal to the following. It is smaller than the variance of FE in FIRE benchmark and the conditional volatility of the inflation, σ^2_ω.

$$\overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{\sigma^2_\omega}{1 + \hat{\theta}^2\rho^2} \tag{27}$$

Finally, as to the uncertainty, since the mechanism of extrapolation in DE does not change the agent's perceived distribution of future shocks, the benchmark DE model the forecast uncertainty to remain the same as in FIRE.

$$\overline{Var}^{de}_{t+h|t} = \overline{Var}^*_{t+h|t} \tag{28}$$

In summary, under DE, the ex-ante uncertainty, which is identical to the conditional volatility of inflation, is also greater than the square of ex-post forecast error. This is a prediction not directly differentiable from that of SE.

3.4 Diagnostic Expectation (DE) augmented with heterogeneous information (DENI)

Bordalo et al. (2020) embeds heterogeneous information in a standard DE model. Their motivation is primarily to generate cross-sectional disagreement in forecasts because the baseline version predicts zero dispersion unless heterogeneity in the degree of overreaction is introduced in the first place, as we did in the previous section. The framework is essentially a hybrid of the NI and DE. It maintains the assumption regarding how agents overreact to new information at individual levels, but the information is no longer the real-time realization of the variable y_t but one public and one private noisy signal of it: $s_{i,t} = [s_t^{pb}, s_{i,t}^{pr}]'$, the same as in NI.

Then the h-period-ahead forecast is as follows.

$$y_{i,t+h|t}^{deni} = y_{i,t+h|t-1}^{deni} + (1+\theta)PH(\rho^h s_{i,t} - y_{i,t+h|t-1}^{deni}) \tag{29}$$

P is the Kalman gain as defined in NI as a function of nowcasting uncertainty $\text{Var}_{t|t}^{deni}$ and nosiness of signals σ_ϵ and σ_ξ. This implies the average forecast errors evolves as the following. (See Appendix 6 for derivations.)

$$\overline{FE}_{t+h|t}^{deni} - \overline{FE}_{t+h|t}^{*} = -\theta\rho(\overline{FE}_{t+h-1|t-1}^{deni} - FE_{t+h-1|t-1}^{*}) + \rho^h((1+\theta)P_\epsilon - 1)\omega_t + \rho^h(1+\theta)P_\epsilon\epsilon_t \tag{30}$$

The unconditional variance of 1-period-ahead forecast error is equal to the following. It collapses to FIRE when the private information is perfectly precise ($P_\epsilon = 1$, $\sigma_\epsilon = 0$) and there is no overreaction ($\theta = 0$).

$$\overline{FE}_{\bullet+1|\bullet}^{deni2} = \frac{\sigma_\omega^2 + \rho^2(1+\theta)^2(1-P_\epsilon)^2\sigma_\omega^2 + \rho^2(1+\theta)^2P_\epsilon^2\sigma_\epsilon^2}{1+\theta^2\rho^2} \tag{31}$$

Forecast uncertainty under DENI is identical to that in NI, because only the NI mechanism affects the behaviors of uncertainty.

$$\overline{Var}_{t+h|t}^{deni} = \overline{Var}_{t+h|t}^{ni} \tag{32}$$

3.5 An alternative process of inflation: stochastic volatility (SV)

This section considers an alternative data generating process of the inflation using the Unobservable Components/Stochastic Volatility (UCSV, or SV) model proposed by Stock and Watson (2007), which is found to produce a better fit to the time series dynamics of inflation.

In this paper, the use of stochastic volatility (SV) has two main objectives. Firstly, a basic inflation process with constant volatility does not account for the observed time-varying pattern of forecast uncertainty, nor its correlation with other moments such as disagreement and forecast error size, as illustrated in Figure 2.4. The incorporation of stochastic volatility could possibly accommodate these stylized facts.

Secondly, allowing for stochastic volatility in the inflation process serves as a robustness test of various theories of expectation formation, as it captures the sensitivity of these theories to the assumed underlying generating process of inflation. This extension provides a more comprehensive analysis of the relationship between inflation dynamics and expectation formation.

In particular, UCSV assumes that inflation consists of a permanent ζ and transitory component η. Time variations in the relative size of the volatility of two components

σ^2_ζ and σ^2_η drive time variations of the persistence of inflation shocks. The logged volatility of the two components themselves follow a random walk subject to shocks μ_ζ and μ_η.

$$\begin{aligned}
y_t &= \zeta_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\nu_{\eta,t} \\
\zeta_t &= \zeta_{t-1} + z_t, \quad \text{where } z_t = \sigma_{z,t}\nu_{\epsilon,t} \\
\log\sigma^2_{\eta,t} &= \log\sigma^2_{\eta,t-1} + \mu_{\eta,t} \\
\log\sigma^2_{z,t} &= \log\sigma^2_{z,t-1} + \mu_{\epsilon,t}
\end{aligned} \tag{33}$$

The shocks to the level of the two components η_t, and z_t, and those to their volatility, $\mu_{\eta,t}$ and $\mu_{z,t}$, are drawn from the following Normal distributions, respectively. The only parameter of the model is γ, which determines the smoothness of the time-varying volatility.

$$\begin{aligned}
\nu_t &= [\nu_{\eta,t}, \nu_{z,t}] \sim N(0, I) \\
\mu_t &= [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)
\end{aligned} \tag{34}$$

The information set necessary for forecasting is different in SV from that in an AR(1) process. Consider first the benchmark case of FIRE. At time t, the FIRE agent sees the most recent and past realization of all stochastic variables as of t, including y_t, ζ_t, η_t, $\sigma_{\eta,t}$, $\sigma_{z,t}$. Using the superscript $*sv$ to denote the FIRE benchmark prediction under the stochastic volatility, and suppressing the individual subscript i (because there is no disagreement in FIRE), the h-period-ahead forecast of inflation is equal to the contemporaneous realization of the permanent component, $\epsilon_t \equiv \zeta_t$.

$$\overline{y}^{*sv}_{t+h|t} = \zeta_t \tag{35}$$

Under FIRE, forecast error is simply the cumulative sum of unrealized permanent and transitory shocks from t to $t+h$, which is equal to the following. And, disagreement is zero across agents in FIRE.

$$\overline{FE}^{*sv}_{t+h|t} = -\sum_{s=1}^{h}(\eta_{t+s} + z_{t+s}) \tag{36}$$

The h-step-ahead conditional variance, or the forecast uncertainty is time-varying, as the volatility is stochastic now. It is essentially the conditional expectation of the cumulative sum of future volatility given the current realizations of the component-specific volatility at t.

$$\begin{aligned}
\overline{Var}^{*sv}_{t+h|t} &= \sum_{s=1}^{h} E_t(\sigma^2_{\eta,t+s}) + E_t(\sigma^2_{z,t+s}) \\
&= \sigma^2_{\eta,t}\sum_{s=1}^{h} exp^{-0.5s\gamma} + \sigma^2_{z,t} exp^{-0.5h\gamma}
\end{aligned} \tag{37}$$

SESV Under the sticky expectation (SE), an agent whose most recent up-to-date update happened in $t-\tau$ only has seen the realizations of y, ζ, η, σ_η, σ_z till $t-\tau$. The average forecast is hence the weighted average of all past realizations of the permanent component up to t.

$$y^{sesv}_{t+h|t-\tau} = \sum_{\tau=0}^{\infty} \lambda(1-\lambda)^{\tau}\zeta_{t-\tau} \tag{38}$$

The distribution of lagged updating is also reflected in the average forecast uncertainty. The population average uncertainty is a weighted average of FIRE uncertainty at $t, t-1...t-\tau...t-\infty$. (Equation 39) The key difference in SV from AR(1) is that the average uncertainty exhibits a positive serial correlation under SV. Expectations being sticky further increases the positive serial correlation compared to that in FIRE due to the lag in updating the shocks to the volatility. The predictions regarding both forecast errors and disagreements under SV are the same as under the AR(1) model.

$$Var^{sesv}_{t+h|t} = \sum_{\tau=0}^{\infty} \lambda(1-\lambda)^{\tau} Var^{*sv}_{t+h|t-\tau} \tag{39}$$

NISV Under noisy information (NI), in order to forecast future y, the agent at time t needs to form her best nowcast of the permanent component ζ_t, denoted as $\bar{\zeta}_{t|t}$, using noisy signals and Kalman filtering. We assume again that the noisy signals of ζ_t consist of a public signal s^{pb}_t and a private signal $s^{pr}_{i,t}$ containing noises around the true realization of ζ_t. Following a long tradition of modeling the signaling-extraction problem in this two-component context, we further assume the public signal $s^{pb}_t = y_t$, meaning the inflation realization itself is the public signal of the permanent component. Accordingly, the transitory shock η_t is equivalent to the realized noise of the public signal ϵ_t in the benchmark NI model with AR(1) process.

$$y^{nisv}_{t+h|t} = \bar{\zeta}_{t|t} = (1 - P^{sv}_t H) y^{nisv}_{t+h-1|t-1} + P^{sv}_t H \zeta_t + P^{sv}_{\eta,t}\eta_t \tag{40}$$

In the above equation, Kalman gain $P^{sv}_t = [P^{sv}_{\eta,t}, P^{sv}_{\xi,t}]$ is a function of forecasting uncertainty $Var^{svni}_{t|t-1}$, the constant noisiness of private signal σ_ξ and that of public signal, $\sigma_{\eta,t}$, which is also the time-varying volatility of the transitory component of the inflation.

What is different under time-varying volatility is that there is no steady-state Kalman gain and uncertainty that are independent of time because the underlying volatility of the variable is time-varying. This also implies that the rigidity induced by the noisiness of information is state-dependent. At each period, the agents in the economy will update their forecasts based on the realized volatility. In periods with high (low) fundamental volatility, the Kalman gain from noisy signals is larger (smaller) thus the agents will be more (less) responsive to the new information. There is no such state-dependence of rigidity in SE.

The mechanisms of DE and DENI exactly mimic that under AR(1) except that the average volatility is time-varying now.

3.6 Comparing theories

We summarize the predictions by different theories here.

- In contrast with the FIRE prediction that ex-ante forecast uncertainty ($\overline{Var}$) is equal to the square of ex-post forecast errors $\overline{FE}^2$, SE and DE both predict the former to be greater than the latter. NI does not have such a clear-cut prediction, and the relative size of the two depend on the parameter configurations.
- The canonical SE and NI predict positive disagreement ($\overline{Disg}$). Only a modified version of DE or its hybrid with NI predict positive disagreement.

The general takeaway is that not only the first moment such as forecast error but also higher moments, disagreement, and uncertainty contain restrictions to identify the model parameters within each theory. We will utilize these moment conditions to estimate each theory in Section 5.

4 Reduced-form Tests of FIRE

As a credible starting benchmark, I first reproduce a number of reduced-form statistical tests of FIRE only using information from forecast errors primarily following Mankiw et al. (2003), and the results are reported in Table 11 in Appendix 6. Consistent with the existing findings, the results reject the null hypothesis of unbiasedness in forecasts, non-serial correlation of non-overlapping forecast errors, and efficient use of information in forecasting.

This section presents a number of new tests relying on uncertainty in Table 4 and 5, in the spirit of forecasting efficiency by Nordhaus (1987). It is an extension of revision tests on mean forecasts by Fuhrer (2018) to the forecast uncertainty.

Table 4 focuses on estimating forecasting efficiency using revisions of mean forecasts and uncertainty, hereafter referred to as revision-based tests. In plain words, the revision from 1-year-ahead forecasts to nowcasts of current-year inflation is efficient if the following two criteria are satisfied: (1) forecast revision does not depend on past information, including the past revisions; (2) the drop in uncertainty is sufficiently rapid to reflect the uncertainty of all realized shocks.

The mean revision test by Fuhrer (2018) takes the following form (using 1 period as an example):

$$y_{i,t+1|t+1} - y_{i,t+1|t} = \alpha + \beta(y_{i,t+1|t} - y_{i,t+1|t-1}) + \epsilon_{i,t+1} \tag{41}$$

In the above equation $\beta = 0$ according to FIRE, because rational forecast revision only responds to newly realized shocks, thus it is not predictable by past revisions.[27]. Since we have four vintages of the forecasts from SPF, the above specification can include lagged revisions up to 4 quarters.

[27]Adding $y_{t+1|t}$ to both sides of Equation 41 gives an equivalent null hypothesis used by Fuhrer (2018): coefficient of regression of $y_{t+1|t+1}$ on $y_{t+1|t}$ is $1 - \beta = 1$.

The test with uncertainty simply replaces the revision of forecast with the revision in uncertainty, as shown below.

$$\text{var}_{i,t+1|t+1} - \text{var}_{i,t+1|t} = \alpha^{\text{var}} + \beta^{\text{var}}(\text{var}_{i,t+1|t} - \text{var}_{i,t+1|t-1}) + \zeta_{i,t+1} \tag{42}$$

This regression follows from Equation 11 for SE and Equation 22 for NI. The auto-correlation coefficient β^{var} speaks to the speed of the drop in uncertainty, which takes a zero value of zero under FIRE and one with perfect rigidity. Depending on the model, one can interpret it as the particular structural parameter of rigidity.

The top panel in Table 4 presents the results for the mean forecast. Following Fuhrer (2018), I include the median forecast available at time t and $t-1$ as an indicator of past information for the revision regression. In the first column of each panel, I report the regression on a constant.

The mean revision in the forecast is mildly negative and significant for CPI forecast. The second to fourth columns of each panel in Table 4 examine the autocorrelation of revisions, including different lags. Revisions of forecasts are serially correlated over 4 quarters, and the coefficients are all positive and significant. Also, the median forecasts as the past information always predict a negative revision with significant coefficients. This is evidence against the null hypothesis of FIRE, and my estimates are comparable with those by Fuhrer (2018).

The bottom panel reports auto-regression results for revision in uncertainty. Again, the first column first tests the mean revision against the null being zero. For professional forecasters, the mean revisions in uncertainty are negative (0.25-0.3 percentage points equivalence in the standard deviation of uncertainty) and statistically significant, confirming our observation from Figure 3 that forecasters are more certain about current inflation compared to her previous year forecast. However, for households at the population level, moving from 3-year-ahead inflation to 1-year-ahead inflation 2 years later, the drop in uncertainty is not significantly different from zero, suggesting inefficient forecasting and inconsistency with the FIRE.

The second to fourth column shows a positive serial correlation of revision in uncertainty for both CPI and PCE forecasts. The revision to CPI seems more efficient as serial correlation is with only a one-quarter lag. For PCE, the revisions in uncertainty are serially correlated with all past three quarters.

Table 5 presents the results with the revision replaced with the change in mean forecasts and uncertainty, i.e. from $y_{t|t-1}$ to $y_{t+1|t}$. As we have discussed in Section 3, the auto-correlation of change in mean forecast and uncertainty do not bear testable predictions from FIRE. But if the forecasts and uncertainty are persistent in their first difference, it may imply that the agent does not react to the news and newly realized shocks sufficiently. In addition, auto-correlation regressions of this kind are a useful characterization of the time series dynamics of forecasts. With the variable being the first difference, the panel structure of SCE and SPF allows for calculating changes in individual levels for a greater sample size, especially for households. Besides auto-regression, I also report the constant estimate of the changes in the first column of each sub-panel.

The most noticeable pattern for both professionals and households and for both mean forecast and uncertainty is that past changes predict future changes with univer-

Table 4: Tests of Revision Efficiency Using Mean Revision and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 1. Revision efficiency of mean forecast													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_rv		0.482***	0.406***	0.400***		0.569***	0.473***	0.456***	L.InfExp_Mean_rv		0.986***	0.844***	0.861***
		(0.037)	(0.040)	(0.044)		(0.050)	(0.069)	(0.080)			(0.048)	(0.111)	(0.114)
L2.InfExp_Mean_rv			0.153***	0.121**			0.164**	0.133*	L2.InfExp_Mean_rv			0.130	0.119
			(0.036)	(0.039)			(0.051)	(0.058)				(0.115)	(0.125)
L3.InfExp_Mean_rv				0.054				0.080	L3.InfExp_Mean_rv			0.047	0.250
				(0.045)				(0.040)				(0.116)	(0.157)
SPFCPI_ct50		0.393**	0.370*	0.354*					L4.InfExp_Mean_rv				-0.195
		(0.120)	(0.142)	(0.149)									(0.156)
SPFPCE_ct50						0.380**	0.341*	0.340*	L5.InfExp_Mean_rv				0.091
						(0.120)	(0.134)	(0.143)					(0.141)
									L6.InfExp_Mean_rv				-0.155
													(0.114)
Const	-0.079*	-0.858**	-0.809**	-0.773*	-0.056	-0.732**	-0.654**	-0.644*	Const	-0.032	0.015	0.025	0.014
	(0.036)	(0.249)	(0.290)	(0.307)	(0.033)	(0.222)	(0.244)	(0.263)		(0.045)	(0.018)	(0.017)	(0.018)
N	1765	1501	1295	1136	1513	1275	1086	945	N	86	85	83	80
R2	0.000	0.281	0.302	0.296	0.000	0.373	0.381	0.375	R2	0.000	0.858	0.876	0.884
Test 2. Revision efficiency of uncertainty													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_rv		0.339***	0.166	0.185*		0.395***	0.352***	0.285***	L.InfExp_Var_rv		0.749***	0.770***	0.844***
		(0.074)	(0.093)	(0.089)		(0.057)	(0.066)	(0.070)			(0.074)	(0.151)	(0.167)
L2.InfExp_Var_rv			0.162*	0.217**			0.206***	0.114	L2.InfExp_Var_rv			0.014	-0.097
			(0.078)	(0.073)			(0.058)	(0.058)				(0.174)	(0.203)
L3.InfExp_Var_rv				0.231**				0.271***	L3.InfExp_Var_rv			-0.005	-0.218
				(0.079)				(0.069)				(0.123)	(0.175)
Constant	-0.087***	-0.053***	-0.053***	-0.031**	-0.076***	-0.048***	-0.035***	-0.026***	L4.InfExp_Var_rv				0.378*
	(0.008)	(0.007)	(0.007)	(0.009)	(0.006)	(0.006)	(0.006)	(0.005)					(0.153)
									L5.InfExp_Var_rv				-0.134
													(0.231)
									L6.InfExp_Var_rv				0.066
													(0.144)
									Const	0.020	-0.006	0.037	0.037
										(0.157)	(0.103)	(0.103)	(0.106)
N	1696	1435	1237	1082	1622	1370	1171	1013	N	86	85	83	80
R2	0.000	0.132	0.092	0.210	0.000	0.146	0.211	0.263	R2	0.000	0.567	0.602	0.639

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

sally negative coefficients across different horizons. Most of the negative coefficients are statistically significant at the 1% level. For instance, each percentage point increase in SPF's CPI forecast in the previous quarter predicts around 0.29 percentage points decrease in the next quarter. This negative correlation is smaller but remains significant into further past, i.e. -0.24 for two-quarters lag and -0.1 for three-quarters lag. Data on SCE is monthly, so lags are included for up to six months. The negative correlation between past and current changes is all negative and significant for households. The sizes of the correlation coefficients are comparable with professional forecasts when the monthly coefficients are converted to their quarterly average, i.e. -0.3 to -0.4.

Such an auto-correlation of change in mean forecast is very much reflected in the same regression for uncertainty, as reported in the bottom panel of Table 5. For SPF of CPI and PCE, respectively, one unit increase in uncertainty about 1-year-ahead inflation in the previous quarter predicts around 0.39 and 0.44 units of drop in the next quarter. The effect holds up to two quarters for professionals and 5 months for households.

This evidence suggests that both the mean forecast and uncertainty of individuals are mean-reverting. An essentially equivalent explanation is that both series are realizations of noisy signals around their respective long-run mean. This will lead to the exact negative correlation of the first differences we have seen.

The second noticeable result lies in the constant regressions reported in the first column of each sub-panel in Table 5. It implies that households constantly lower their mean forecasts as well as uncertainty from month to month, while professional forecasts do not behave in such a pattern. In particular, the constant regression of the change in the mean forecast for SCE gives an estimated coefficient of -0.05 which is significant in the 5% level. Individual households' 1-year-ahead inflation expectations keep being downward adjusted each month compared to their previous answer. What is more interesting is that their uncertainty about 1-year-ahead inflation also decreases each month. The size of the downward adjustment is -1.39 unit and statistically significant at the level of 0.1%. This negative significant and constant-coefficient remains throughout all auto-regressions, implying it is not driven by time-varying changes.

The most natural explanation for this, is that repeatedly surveyed households have become more informative about inflation over time. Given the unconditional forecast errors of inflation by households are positive, a downward adjustment of inflation stands for a less-biased forecast. [28]

In summary, the major additional insights that arise from the empirical tests of this section is that the rigidity of incorporating new information in forming expectations implies noticeable inefficiency of revisions in forecasts and a drop in uncertainty.

[28] The possibility that the surveys' information set is influenced by the survey itself, or "learning through surveys"(Kim and Binder, 2020), is a double-edged sword. On one hand, this poses a methodological challenge to the survey designers of expectations as to if surveys can objectively elicit the "true" expectations held by the respondents. On the other hand, researchers can use the survey as a meaningful intervention tool to identify the effect of factors such as information provision and attention. Recent examples of this type of research include: Coibion et al. (2018) for firms and Coibion et al. (2019) for households.

Table 5: Weak Tests of Revision Efficiency Using Change in Forecasts and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 3. Weak. Efficiency of change in forecast													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_ch		-0.249***	-0.308***	-0.311***		-0.289***	-0.324***	-0.324***	L.InfExp_Var_ch		-0.408***	-0.569***	-0.612***
		(0.062)	(0.054)	(0.060)		(0.060)	(0.080)	(0.089)			(0.005)	(0.008)	(0.012)
L2.InfExp_Mean_ch			-0.194**	-0.177**			-0.100	-0.100	L2.InfExp_Var_ch			-0.305***	-0.385***
			(0.064)	(0.060)			(0.081)	(0.096)				(0.007)	(0.012)
L3.InfExp_Mean_ch				-0.075*				0.014	L3.InfExp_Var_ch			-0.126***	-0.241***
				(0.036)				(0.064)				(0.007)	(0.012)
									L4.InfExp_Var_ch				-0.142***
													(0.012)
									L5.InfExp_Var_ch				-0.078***
													(0.011)
									L6.InfExp_Var_ch				-0.038***
													(0.008)
Constant	-0.019	-0.015	-0.019	-0.021	0.017	0.026	0.019	0.017	Constant	-0.034**	-0.018	0.002	0.020
	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.024)	(0.026)	(0.028)		(0.013)	(0.014)	(0.017)	(0.019)
N	4286	3519	2971	2581	1791	1538	1338	1189	N	85166	67555	43489	20894
R2	0.000	0.072	0.103	0.089	0.000	0.077	0.088	0.088	R2	0.000	0.187	0.259	0.276
Test 4. Weak. Efficiency of change in uncertainty													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean change	t-1	t-1- t-2	t-1-t-3		Mean change	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_ch		-0.381***	-0.542***	-0.601***		-0.352***	-0.428***	-0.513***	L.InfExp_Var_ch		-0.396***	-0.572***	-0.653***
		(0.071)	(0.075)	(0.069)		(0.043)	(0.053)	(0.054)			(0.008)	(0.010)	(0.016)
L2.InfExp_Var_ch			-0.280***	-0.428***			-0.237***	-0.430***	L2.InfExp_Var_ch			-0.307***	-0.430***
			(0.059)	(0.061)			(0.044)	(0.044)				(0.011)	(0.018)
L3.InfExp_Var_ch				-0.293***				-0.382***	L3.InfExp_Var_ch			-0.124***	-0.269***
				(0.078)				(0.059)				(0.006)	(0.016)
									L4.InfExp_Var_ch				-0.155***
													(0.015)
									L5.InfExp_Var_ch				-0.079***
													(0.012)
									L6.InfExp_Var_ch				-0.029**
													(0.009)
Constant	-0.002	-0.001	0.005	0.006	0.003	0.004	0.006	0.008		-0.710***	-0.717***	-0.593***	-0.448***
	(0.005)	(0.006)	(0.005)	(0.005)	(0.004)	(0.005)	(0.005)	(0.004)		(0.072)	(0.064)	(0.072)	(0.076)
N	1685	1439	1251	1104	1629	1406	1225	1079		88052	69979	45003	21476
R2	0.000	0.130	0.240	0.286	0.000	0.125	0.161	0.286		0.000	0.187	0.271	0.311

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

5 Model Estimation and Sensitivity Analysis

5.1 SMM Estimation

The reduced-form tests in Section 4 are sufficient in rejecting the null hypothesis of FIRE. But there are two limitations with these tests in terms of identifying differences among non-FIRE theories. First, the coefficient estimates from the reduced-form regression cannot always be mapped into a structural parameter of the particular model. Second, even if it does so, the tests fall short of simultaneously utilizing all the restrictions across moments implied by a particular non-FIRE theory, as discussed in great detail in Section 3. In this section, I undertake a structural estimation that jointly accounts for cross-moment restrictions.

Since many of the moment conditions cannot be easily derived as a closed-form function of parameters, I adopt the simulated method of moment (SMM). In a nutshell, the estimation chooses the best set of model parameters by minimizing the weighted distances between the data moments and the model-simulated moments. For a given process of inflation, and a particular theory of expectation formation, the vector of the parameters estimates is defined as the minimizer of the following objective function.

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin}(M_{\text{data}} - F^o(\Omega^o, H))W(M_{\text{data}} - F^o(\Omega^o, H))'$$

where Ω^o stands for the parameters of the particular pair of theory of expectation and inflation process, i.e. $o \in \{se, ni, de, deni\} \times \{ar, sv\}$. Γ^o represents the corresponding parameter space respecting the model-specific restrictions. M_{data} is a vector of the unconditional moments that is computed from data on expectations and inflation. F^o is the simulated model moments under the theory pair o. W is the weighting matrix used for the SMM estimation. I report estimation results using the 2-step feasible SMM approach, in which the inverse of the variance-covariance matrix from the 1st-step estimation using identity matrix is used as the W in the second step, which has been shown to give asymptotically efficient estimates of the model parameters.

Crucially, notice that the model-implied moments F^o are not just a function of model parameters Ω^o, but also a function of the corresponding information set available to the forecasters. I use H to represent the historical realizations of the variables in the agents' information set that are used as the inputs for forecasts. Although the real-time history is the same across models, the mapping between the history to data moments depends on model specifics. For instance, although real-time inflation is the only variable in the information set for AR(1) process, the information set in SV contains both the permanent component of the inflation and the realized levels of volatility. Since the different components are not directly observed from historical data, I estimate it using the Markov Chain Monte Carlo (MCMC) procedure developed by Stock and Watson (2007) in this context. The estimated time-varying permanent and transitory volatility of both Core CPI and headline CPI is shown in Appendix 4.

It is also important to mimic the information set that was truly available to the agents at each point in time in history.[29] Therefore, I use the real-time data on

[29] For the importance of using real-time data to study survey forecasts, see Faust and Wright (2008), Faust and Wright (2009) and so on.

historical inflation that was publicly available at each point of the time instead of the historical data released later, since it is well known among macroeconomists that the latter typically incorporates many rounds of revisions over time. I obtain the data from the Real-Time Data Research Center hosted by Philadelphia Fed[30].

The estimation is also specific to the choices of moments included in computing the distances. I focus on the unconditional moments of (independent of the time) at the population level, defined in Table 1. In particular, they include the mean (FE), variance($FEVar$) and auto-covariance($FEATV$) of population forecast error, the mean($Disg$), variance($DisgVar$) and auto-covariance($DisgATV$) of disagreement, and the mean (Var), variance($VarVar$) and auto-covariance($VarATV$) of uncertainty. When the joint estimation is done, two unconditional moments of the inflation are used, the variance($InfVar$) and auto-covariance($InfATV$). Table 2 reports the size of these moments computed for both SPF and SCE.

The model-implied moment conditions also implicitly depend on the parameters of the inflation process for a given model. This point is illustrated well in Bordalo et al. (2020). For instance, the observed overreaction in DE is lower for an AR(1) process with higher persistence. In recovering the model parameters associated with expectation formation, it is important to take into account the information contained in expectation data regarding the process of inflation per se. To handle this, I undertake both 2-step and joint estimation. The former refers to the first externally estimating the inflation process and then estimating expectation formation separately treating the inflation parameter as the *true* data generating process of inflation. The latter refers to jointly estimating parameters of inflation and expectation.

These alternative specifications of the estimation also serve as a model sensitivity analysis with respect to the following criteria: (1) different choices of moments; (2) AR(1) and SV for the process of inflation. (3) two-step and joint estimation. (4) for both professionals and households. A reasonable theory of expectation formation ought to be relatively robust to these criteria. I discuss the findings in greater detail along these four dimensions the next.

5.2 Moments matching and parameter estimates

We first focus on the professionals' expectations as a benchmark, since professionals may exhibit less severe deviation from FIRE than households do, according to much previous empirical evidence. A first look at the data moments of SPF, as reported in Table 2, already provides clues as to the degree of the deviations from FIRE benchmark predictions laid out in Section 3. When separately estimated, the quarterly core CPI inflation during 1995-2020 followed an AR(1) process with the persistence parameter $\hat{\rho} = 0.98$ and volatility $\hat{\sigma}_\omega = 0.22$. The FIRE benchmark predicts zero forecast error, zero disagreements, an uncertainty level identical to the conditional volatility of inflation shocks, and zero time-variation and zero auto-correlation of these population forecasting moments. But in the data, quarterly professional forecasts of the core CPI inflation have a mildly positive average forecast error of 0.136 percentage points, a degree of Disg of 0.183, and an Var of 0.242, both of which are larger than the inflation volatility of $\sigma_\omega^2 = 0.04 \approx 0.22^2$. The ex-ante uncertainty is greater than

[30] https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/.

the variance of ex-post forecast error, or equivalently, its variance $FEVar = 0.133$, which is inconsistent with the FIRE prediction.

Table 6 presents the SMM estimates for professionals. For each theory, I estimate the theory both in 2 steps or jointly using expectations and inflation moments. Different rows within each panel report the estimates depending on various choices of moments used for estimation: forecast errors only (FE), forecast error and disagreement (FE+Disg), and the two plus uncertainty (FE+Disg+Var).

Table 6: SMM Estimates of Different Models: Professionals

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.99	0.23	0.18	1	0.01				
FE+Disg	0.3	0.99	0.23	0.18	1	0				
FE+Disg+Var	0.32	0.99	0.23	0.21	1	0.02				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	2.73	3	0.99	0.23	3	3	1	0.02		
FE+Disg	3	2.95	0.99	0.23	3	3	1	0.02		
FE+Disg+Var	3	3	0.99	0.23	1.97	1.17	1	0		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	0.25	1.57	0.99	0.23	1.39	2.83	0.9	0.17		
FE+Disg	1.12	1.62	0.99	0.23	0.05	1.73	1	0.01		
FE+Disg+Var	0.29	1.81	0.99	0.23	1.19	1.68	0.91	0.16		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	0.3	0.02	2.56	0.99	0.23	-1.14	2.75	0.12	1	0
FE+Disg	-1.21	3	0.09	0.99	0.23	-0.82	2.45	2.83	1	0.01
FE+Disg+Var	-0.8	3	0.09	0.99	0.23	1.67	3	3	1	0.03

5.2.1 Cross-moment consistency within a theory of expectation

Among the four models under consideration, SE and NI outperform DE and DENI in terms of their within-model robustness against the targeted moments, as shown in the estimation of professional forecasts in Table 6.

For SE, the estimated quarterly updating rate is between 0.3-0.35 across different combinations of moments. Such a medium degree of information rigidity seems to reflect the counteracting evidence both for and against information rigidity. On one hand, average forecast errors of a much higher variance (FEVar = 0.133) than the inflation volatility $0.22^2 \approx 0.04$, and a positive disagreement of 0.183 and higher uncertainty of 0.242 than the conditional volatility are all in line with the patterns of the information rigidity in the form of SE. On the other hand, only a mild serial correlation in average forecast errors (with an auto-covariance FEATV = 0.097) and a small-sized disagreement (Disg = 0.183) suggests the degree of the rigidity is not extremely large.

The NI estimates of the noisiness of both public and private signals, despite their cross-moment consistency, turn out to be so large in magnitudes that they often hit the externally set upper bounds of 3 in estimation. These are highly noisy signals compared to the conditional standard deviation of inflation shocks $\sigma_\omega = 0.22$. This suggests that dispersed and noisy information does play a role in generating rigidity seen both in forecast errors and uncertainty, as well as the observed disagreement, but the implied nosiness required to match such moments appears to be less sensible in sizes. Targeting uncertainty in addition to FE and Disg does put some "discipline" on the parameters, generating smaller, albeit still large, noisiness of both signals ($\sigma_{pb} = 1.97$ and $\sigma_{pr} = 1.17$).

Compared to the two rigidity models, DE estimates are more sensitive to moment restrictions, although all the estimates confirm the existence of a positive mass of overreacting agents. With only information from FE, the estimated overreaction parameter of DE θ is around 0.25 on average for all forecasters. Using disagreement helps identify the population dispersion in the degree of overreaction σ_θ, which is estimated to be 1.62, but doing so also leads to a significantly larger estimate of average overreaction $\hat{\theta} = 1.12$. This is so because disagreements depend on not only the dispersion of overreaction parameter σ_θ, but also the average degree to which agents overreact. When information from uncertainty is incorporated, the estimated $\hat{\theta}$ and σ_θ reverse to resemble those based on only FE only.

DENI seems to have even less cross-moment consistency. The implied overreaction parameter spans positive to negative values for professionals, depending on the specific moments that are used. This partly reflects the fact DENI has the most parameters among all theories, and hence a larger degree of freedom. The problem also manifests itself in non-convergence in the estimation of households although additional information from uncertainty, in theory, could help identify parameters.

5.2.2 Interactions between expectation formation and inflation process

In all four models, estimated parameters of expectation formation vary when one jointly estimates expectation and inflation process parameters. With the benchmark AR(1) process, both the persistence of the shock to inflation ρ and the overall volatility of the inflation shock σ_ω determine what the FIRE forecasts moments should be. Therefore, model-specific forecasting moments are not only a function of the model parameters but also the process parameters of inflation. The differences between 2-step estimation and joint estimation reveal such inter-dependence.

For SE, letting professional forecasts reveal information about the inflation process leads to a more persistent (close to the unit root) and lower conditional volatility: the persistence parameter ρ becomes $1,0$ and the inflation volatility becomes $0.0 - 0.02$, compared to 0.99 and 0.23 when estimated separately. Simultaneously, the joint estimation produces a lower updating rate λ (a higher rigidity) around $0.18 - 0.21$.

This is consistent with the underlying mechanisms of SE. Moving to the household estimates in Table 7, the joint estimation produces an identical degree of persistence, but smaller conditional volatility ($\sigma_\omega = 0.1$ from 0.41). A lower conditional volatility means it requires a higher degree of rigidity (λ becomes 0.2 from 0.35) to allow the model to match the observed level of FE, Disg, and Var.

NI is also found to be sensitive toward the estimation procedure, especially when uncertainty is included for targeted moments. Again, the joint estimation reveals a more persistent inflation process, $\rho = 1.0$, and a smaller conditional volatility close to zero. The jointly implied nosiness of both signals becomes smaller in the joint estimation (σ_{pb} becomes 1.97 from 3 and σ_{pr} becomes 1.12 from 3). The mutual dependence between the noisiness of signals and the volatility of inflation shocks is not surprising because, in NI, both shocks to inflation itself or to the signals contribute to forecast errors and forecast uncertainty. Although the two are clearly distinguishable from the point of view of the modeler, the distinction between the two is indistinguishable for the agents who make forecasts.

Very similar to that for SE, DE estimates in joint estimation demonstrate substitution between the persistence ρ and overreaction $\hat{\theta}$. A higher persistence in joint estimation comes with a lower degree of overreaction and vice versa. This echoes the finding in Afrouzi et al. (2020) about the interdependence between the implied persistence in forecasting and the underlying degree of overreaction.

DENI performs the worst in terms of its sensitivity against the estimation procedure, implying it is very dependent on the persistence and volatility of the inflation process. Taking the most likely credible estimate that utilizes information from all moments, the joint estimation implies a significantly higher degree of overreaction $\hat{\theta} = 1.67$, much noisier private signals (σ_{pr} becomes 3 from 0.09, together with a more persistent inflation process with smaller conditional volatility.

5.2.3 Professionals versus households

In contrast with professionals, raw household forecasts see a more substantial deviation from FIRE in every dimension. During the sample period of 2013-2020, the monthly headline CPI inflation is estimated to have a persistence parameter of 0.98 and volatility of 0.41.[31] The household forecasts had an average forecast error of 1.77, a disagreement of 2.585, and an average uncertainty of 1.75. Note that these are computed based on residuals by first controlling for the individual fixed effects. This follows from the emerging evidence[32] that individual-specific effects such as demographics and experience play important roles in driving systematic differences in expectations. After controlling for this ex-ante heterogeneity in beliefs, households' expectations exhibit similar yet still more distorted expectations than that of professionals.

Table 7 presents the estimates for households after excluding the fixed effects on forecasts, forecast errors, and uncertainty. Overall, the estimates are surprisingly similar to that for professionals, and each model, in some sense, exhibits even more model consistency across moment conditions and estimation procedures. Most noticeably, the

[31] The higher volatility compared to that of core CPI is due to a higher frequency and inclusion of more volatile items in the CPI basket.

[32] Malmendier and Nagel (2015), Das et al. (2017), D'Acunto et al. (2019).

Table 7: SMM Estimates of Different Models: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.98	0.41	0.2	0.98	0.16				
FE+Disg	0.35	0.98	0.41	0.2	0.98	0.1				
FE+Disg+Var	0.35	0.98	0.41	0.2	0.98	0.1				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	3	0.45	0.98	0.41	2.98	0.28	0.95	0.24		
FE+Disg	1.55	0.36	0.98	0.41	3	0.28	0.95	0.24		
FE+Disg+Var	2.58	0.97	0.98	0.41	2.9	1.02	0.96	0.24		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_{θ}	ρ	σ	$\hat{\theta}$	σ_{θ}	ρ	σ		
FE	-0.47	0.81	0.98	0.41	-0.61	4.15	0.95	0.25		
FE+Disg	-0.3	2.08	0.98	0.41	-0.08	2.13	0.95	0.25		
FE+Disg+Var	-0.35	2.08	0.98	0.41	0.36	2.07	0.9	0.36		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	N/A	N/A	N/A	0.98	0.41	-0.39	1.86	0.03	0.95	0.24
FE+Disg	N/A	N/A	N/A	0.98	0.41	N/A	N/A	N/A	N/A	N/A
FE+Disg+Var	N/A	N/A	N/A	0.98	0.41	0.35	0	1.32	0.96	0.22

updating rate in SE falls into a very similar range of values of $0.31 - 0.35$ per month. This is comparable to the estimates for professionals. It is well documented in the literature that household expectations have more severe deviations from FIRE than professionals.[33] But the SE results of our estimates show that the major differences are not simply due to the differences in updating rates of information.

NI estimates of households also reveal intuitive patterns. Compared to those of professionals, the NI estimates of households show equally noisy public signals ($1.5 - 3.0$) and more precise private signals ($0.36 - 0.97$) depending on the targeted moments, according to both 2-step and join estimations. By the NI mechanism, more precise private signals lead households to primarily react to private signals, giving a bigger room for disagreement. Conditional on primarily reacting to private signals, the sizable nosiness also implies a higher steady-state uncertainty that is closer to the data.

Unlike professionals, the DE estimates of households suggest a consistently negative, instead of positive, value of the average overreaction parameter, in conjunction with a large dispersion that gives a positive mass of overreacting agents. On average, households underreact instead of overreact to the news. But at the individual level,

[33]See Cornand and Hubert (2022) for a detailed discussion on this point.

overrreacton exists.

As for professionals, the household estimates of DENI are equally sensitive across targeted moments. The most informative estimates using all moments seem to suggest average underreaction, extremely noisy public signals, and very precise noisy signals.

5.2.4 Alternative inflation process with stochastic volatility

Table 8 and 9 show the estimation of the model-specific parameters allowing the alternative inflation process featuring stochastic volatility and two separate unobserved components of different persistence. Compared to the benchmark AR(1) process, there are three crucial implications for expectation formation. The first is that now the SV model admits time-varying volatility, which has more potential to be consistent with the time-varying pattern of the forecast uncertainty, primarily in the expectations of the household as shown in Section 2.4.1. The second is that given the permanent component is a random walk, the shock to the permanent component is permanent instead of persistent. This seems to be more consistent with the finding previously that joint estimation oftentimes implies a unit-root process of inflation instead of an AR(1). Finally, the alternative process seems to lead to a more interpretable version of NI compared to that in AR(1) in which the agents learn about the permanent component using the realized inflation to make forecasts.

An overarching finding is that SV process of inflation significantly improves the within-model consistency of every model in consideration and for both types of agents.

Among all theories, SE gives the closest parameter estimates to that of the benchmark AR(1). Surprisingly enough, admitting stochastic volatility reveals an almost identical information rigidity in SE, i.e. an updating rate of $0.34-0.35$ for both households and professionals. This is not a mechanical coincidence, since the SVSE model assumes agents do not only infrequently update realized shocks but also the shocks to the volatility, as explicitly discussed in the section 3.5. Therefore, the dynamics of uncertainty seen from data do provide useful additional information now to identify information rigidity than the benchmark model. This suggests that SE has a very good consistency against the assumed inflation process.

Such information rigidity is also indirectly confirmed by the revealed underreaction according to DESV estimates. On average, both households and professionals underreact (a negative $\hat{\theta}$) instead of overreact to inflation news under the SV model of inflation. More specifically, professionals show significantly more underreaction with SV than the benchmark estimates, and households show less with the modified process. It is worth noting that again that the dispersion in overreaction does accommodate a positive mass of overreacting agents as that in SV. Therefore, the finding the there are individual overreaction is robust to SV process of inflation.

The estimates for NI augmented with SV are a lot different from that of benchmark estimates but appear to be more sensible, benefiting from the modified inflation process. This ultimately comes from a more sensible assumption in NISV that what agents do not perfectly observe is the permanent component instead of the inflation itself. The estimated noisiness of public signals is close to the size of the average conditional volatility of inflation, by the assumption of the NISV model. The private signals of households remain extremely large (2.37-2.9), and significantly larger than that of professionals (0.04).

Table 8: SMM Estimates of Different Models under Stochastic Volatility: Professionals

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.34		
FE+Disg+Var	0.34		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0.27	0.04	
FE+Disg	0.27	0.04	
FE+Disg+Var	0.27	0.05	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.39	0.69	
FE+Disg	-0.39	0.7	
FE+Disg+Var	-0.39	0.7	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	1.38	3	1.72
FE+Disg	3	3	2.03
FE+Disg+Var	3	3	2.28

5.3 The scoring card of different theories

To summarize, Table 10 reports my evaluation of the four theories under consideration based on four sensitivity criteria laid out in the previous section. According to this evaluation, SE seems to capture the average behavior of expectations better than the other three theories.

NI, another theory that also features information rigidity and captures similar qualitative patterns as SE, does show some cross-moment consistency. But the major weakness of the model is that they produce unrealistically large sizes of the parameters to match the rigidity of the data. This is per se not a rejection of the theory. It is indeed found that once a more realistic inflation process of SV is used, NI estimation produces significantly more consistent and sensible values of parameters for both types of agents. Although the previous literature (Coibion and Gorodnichenko (2012, 2015)) treat SE and NI as two indistinguishable theories that both produce information rigidity, this paper shows that using information from uncertainty significantly disciplines

Table 9: SMM Estimates of Different Models under Stochastic Volatility: Households

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0.44	2.37	
FE+Disg	0.44	2.61	
FE+Disg+Var	0.44	2.9	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.05	0.58	
FE+Disg	-0.05	0.56	
FE+Disg+Var	-0.05	0.56	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	N/A	N/A	N/A
FE+Disg	0.71	0.01	0.95
FE+Disg+Var	N/A	N/A	N/A

the parameter choices and allows me to distinguish the two theories by their model sensitivity.

Compared to the two rigidity models, a modified canonical DE that allows for heterogeneous degrees of over/underreaction, although producing a sensible finding of the coexistence of average underreaction and individual overreaction, turns out to be sensitive along all four dimensions. Only with a modified inflation process, do the estimates all become more stable across moments and estimation procedures.

A hybrid of DE and NI, which accommodates the coexistent overreacting mechanism and dispersed noise information, proves to be equally sensitive along all dimensions. One diagnosis of the problem might be that such sensitivity comes from too much degree of freedom in choosing parameters in DENI.

A final point worth making here is that the model sensitivity discussed above shall not be interpreted as an entire rejection of the model being considered. Instead, it just cautions against using that particular model for accounting for the surveyed inflation expectation in this particular sample period (2007-2020). As one piece of evidence that such model evaluation could be sample- and domain-specific, I report the structural estimates with a sample extended to include 2020-2022, a period with significantly higher realized inflation, in the Appendix 6. It is perhaps not surprising to find that both DE and DENI models exhibit more consistency in estimates. This may suggest that the overreaction mechanism as featured in DE and DENI has been probably more relevant in the recent sample of higher and more salient inflation news.

Table 10: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	No	Yes	Yes
Sensitive to the assumed inflation process?	No	Yes	Yes	Yes
Sensitive to two-step or joint estimate?	Yes	Yes	Yes	Yes
Sensitive to the type of agents?	Yes	Yes	Yes	Yes

6 Conclusion

Most studies on expectation formation that document how it deviates from the FIRE benchmark have focused on the first moment, namely the mean forecasts and the cross-sectional dispersion of the forecasts. However, this paper has shown that the surveyed forecasting uncertainty by professionals and households provides useful information for understanding the exact mechanisms of expectation formation. It not only provides additional reduced-form testing results of rejecting FIRE, such as persistent disagreements in forecasting uncertainties and its inefficient revisions but also provides additional moment restrictions to any particular model of expectation formation, which helps identify differences across theories.

At least three lines of questions remain unresolved in this paper and require future research. First, this paper focuses on a selective list of models of expectation formation and inevitably omits various others likewise proven to match certain aspects of surveyed inflation expectations, such as adaptive learning (Marcet and Sargent, 1989; Evans and Honkapohja, 2012), experience-based learning (Malmendier and Nagel, 2015), heterogeneous models (Patton and Timmermann, 2010; Farmer et al., 2021), and asymmetric attention (Kohlhas and Walther, 2021). It would be a fruitful attempt to explore the corresponding predictions of these models about uncertainty. Second, throughout the analysis, we maintained the normality/symmetric assumptions of the shocks and ignored beliefs in tail events or even higher moments. It would be natural to explore how different theories of expectation formation may contain different predictions on tail beliefs. Finally, although this paper focuses only on macroeconomic expectations regarding inflation, it is worth asking if the belief formation regarding individual variables such as income bears similar mechanisms and matches the observed empirical patterns of surveyed expectations and risks.[34]

[34] A few recent studies on income/wage/unemployment/job-search expectations: Mueller et al. (2021); Wang (2022); Koşar and Van der Klaauw (2022); Jäger et al. (2022); Caplin et al. (2023).

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Appendix

Detailed derivation

SE

$$
\begin{aligned}
\bar{V}ar_t(y_{t+h}) &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction who does not update until } t-\tau} \underbrace{Var_{t|t-\tau}(y_{t+h})}_{\text{Variance of most recent update at } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}(1+\rho^2+...+\rho^{2(h+\tau-1)})\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{\rho^{2(h+\tau)}-1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{\rho^{2(h+\tau)}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\rho^{2\tau}\frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda((1-\lambda)\rho^2)^{\tau}\frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \frac{\lambda}{(1-\rho^2+\lambda\rho^2)} \sum_{\tau=0}^{+\infty}(1-\rho^2+\lambda\rho^2)(1-(1-\rho^2+\lambda\rho^2))^{\tau}\frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 \\
&- \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau}\frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= (\frac{\lambda\rho^{2h}}{(1-\rho^2+\lambda\rho^2)(\rho^2-1)} - \frac{1}{\rho^2-1})\sigma_{\omega}^2 \\
&= (\frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2} - 1)\frac{\sigma_{\omega}^2}{\rho^2-1} \\
&= (\frac{\lambda\rho^{2h}-1+\rho^2-\lambda\rho^2}{1-\rho^2+\lambda\rho^2})\frac{\sigma_{\omega}^2}{\rho^2-1}
\end{aligned}
\tag{43}
$$

NI

The steady-state nowcasting uncertainty Var_{ss}^{ni} is solved using the updating equation (Equation 21).

$$
\begin{aligned}
&\mathrm{Var}^{ni}_{t|t} = \mathrm{Var}^{ni}_{t|t-1} - \mathrm{Var}^{ni}_{t|t-1}H'(H\mathrm{Var}^{ni}_{t|t-1}H' + \Sigma^{v})^{-1}H\mathrm{Var}^{ni}_{t|t-1} \\
&\rightarrow \mathrm{Var}^{ni}_{t|t} = \rho^2(\mathrm{Var}^{ni}_{t-1|t-1} + \sigma^2) \\
&- \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2)H'(H\rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2)H' + \Sigma^{v})^{-1}H\mathrm{Var}^{ni}_{ss} \\
&\rightarrow \mathrm{Var}^{ni}_{ss} = \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_{\omega}) \\
&- \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_{\omega})H'(H\rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_{\omega})H' + \Sigma^{v})^{-1}H\mathrm{Var}^{ni}_{ss}
\end{aligned}
\tag{44}
$$

DE

$$
\begin{aligned}
FE^{de}_{i,t+h|t} &= y^{de}_{i,t+h|t} - y_{t+h} \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y^{de}_{i,t+h|t-1}) \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= FE^{*}_{t+h|t} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= (1+\theta_i)FE^{*}_{t+h|t} - \theta_i FE^{de}_{i,t+h|t-1} \\
&= (1+\theta_i)FE^{*}_{t+h|t} - \theta_i(\rho FE^{de}_{i,t+h-1|t-1} - \omega_{t+h}) \\
&= (1+\theta_i)FE^{*}_{t+h|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} + \theta_i\omega_{t+h} \\
&= (1+\theta_i)FE^{*}_{t+h|t} + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^{*}_{t+h-1|t} + (1+\theta_i)(-\omega_{t+h}) + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^{*}_{t+h-1|t} - \omega_{t+h} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^{*}_{t+h|t} + \theta_i FE^{*}_{t+h-1|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^{*}_{t+h|t} + \theta_i(FE^{*}_{t+h-1|t} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= FE^{*}_{t+h|t} + \theta_i(\rho FE^{*}_{t+h-1|t-1} + \rho^h\omega_t - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= FE^{*}_{t+h|t} - \theta_i\rho(FE^{de}_{i,t+h-1|t-1} - FE^{*}_{t+h-1|t-1}) + \theta_i\rho^h\omega_t
\end{aligned}
\tag{45}
$$

DENI

Current forecast error is

$$
\begin{aligned}
FE^{deni}_{t|t} &= \rho y^{deni}_{t-1|t-1} + (1+\theta)P_{\epsilon}(s^{pb}_t - \rho y^{deni}_{t-1|t-1}) - y_t \\
&= \rho(FE^{deni}_{t-1|t-1} + y_{t-1}) + (1+\theta)P_{\epsilon}(s^{pb}_t - \rho y^{deni}_{t-1|t-1}) - y_t \\
&= \rho(FE^{deni}_{t-1|t-1} + y_{t-1}) + (1+\theta)P_{\epsilon}(y_t + \epsilon_t - \rho(FE^{deni}_{t-1|t-1} + y_{t-1})) - \rho y_{t-1} - \omega_t \\
&= \rho FE^{deni}_{t-1|t-1} + (1+\theta)P_{\epsilon}(\rho y_{t-1} + \omega_t + \epsilon_t - \rho(FE^{deni}_{t-1|t-1} + y_{t-1}) - \omega_t \\
&= \rho FE^{deni}_{t-1|t-1} + (1+\theta)P_{\epsilon}(\omega_t + \epsilon_t - \rho FE^{deni}_{t-1|t-1}) - \omega_t \\
&= \rho FE^{deni}_{t-1|t-1} - (1+\theta)\rho FE^{deni}_{t-1|t-1} + (1+\theta)P_{\epsilon}(\omega_t + \epsilon_t) \\
&= -\theta\rho FE^{deni}_{t-1|t-1} + ((1+\theta)P_{\epsilon} - 1)\omega_t + (1+\theta)P_{\epsilon}\epsilon_t
\end{aligned}
\tag{46}
$$

Furthermore, we know

$$\begin{aligned} FE^{deni}_{t+h|t} &= \rho^h FE^{deni}_{t|t} + FE^{*}_{t+h|t} \\ FE^{deni}_{t+h-1|t-1} &= \rho^h FE^{deni}_{t-1|t-1} + FE^{*}_{t+h-1|t-1} \end{aligned} \tag{47}$$

So,

$$\begin{aligned} FE^{deni}_{t+h|t} &= \rho^h FE^{deni}_{t|t} + FE^{*}_{t+h|t} \\ &= \rho^h(-\theta\rho FE^{deni}_{t-1|t-1} + ((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^{*}_{t+h|t} \\ &= -\theta\rho(FE^{deni}_{t+h-1|t-1} - FE^{*}_{t+h-1|t-1}) + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^{*}_{t+h|t} \\ &= -\theta\rho FE^{deni}_{t+h-1|t-1} + \theta\rho FE^{*}_{t+h-1|t-1} + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^{*}_{t+h|t} \\ &= \theta\rho(FE^{*}_{t+h-1|t-1} - FE^{deni}_{t+h-1|t-1}) + \rho^h(((1+\theta)P_\epsilon - 1)\omega_t + (1+\theta)P_\epsilon\epsilon_t) + FE^{*}_{t+h|t} \end{aligned} \tag{48}$$

Rearranging it, we get

$$FE^{deni}_{t+h|t} - FE^{*}_{t+h|t} = -\theta\rho(FE^{deni}_{t+h-1|t-1} - FE^{*}_{t+h-1|t-1}) + \rho^h((1+\theta)P_\epsilon - 1)\omega_t + \rho^h(1+\theta)P_\epsilon\epsilon_t \tag{49}$$

Set h=1, we get

$$FE^{deni}_{t+1|t} - FE^{*}_{t+1|t} = -\theta\rho(FE^{deni}_{t|t-1} - FE^{*}_{t|t-1}) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \tag{50}$$

When $\theta = 0$, $P_\epsilon = 1$ and $\epsilon_t = 0$, the equation collapses to FIRE.
Which is equivalent to the following.

$$\begin{aligned} FE^{deni}_{t+1|t} + \omega_{t+1} &= -\theta\rho(FE^{deni}_{t|t-1} + \omega_t) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \\ \rightarrow FE^{deni}_{t+1|t} &= -\theta\rho(FE^{deni}_{t|t-1} + \omega_t) + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t \\ \rightarrow FE^{deni}_{t+1|t} &= -\theta\rho FE^{deni}_{t|t-1} - \theta\rho\omega_t + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho FE^{deni}_{t|t-1} - \theta\rho\omega_t + \rho((1+\theta)P_\epsilon - 1)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho FE^{deni}_{t|t-1} - (\rho(1+\theta)P_\epsilon - \rho - \theta\rho)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho FE^{deni}_{t|t-1} - (\rho P_\epsilon + \rho\theta P_\epsilon - \rho - \theta\rho)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho FE^{deni}_{t|t-1} - \rho(P_\epsilon + \theta P_\epsilon - 1 - \theta)\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \\ &= -\theta\rho FE^{deni}_{t|t-1} + \rho((1+\theta)(1-P_\epsilon))\omega_t + \rho(1+\theta)P_\epsilon\epsilon_t - \omega_{t+1} \end{aligned} \tag{51}$$

This means

$$FE^{deni2}_{\bullet+1|\bullet} = \frac{\sigma^2_\omega + \rho^2(1+\theta)^2(1-P_\epsilon)^2\sigma^2_\omega + \rho^2(1+\theta)^2P^2_\epsilon\sigma^2_\epsilon}{1+\theta^2\rho^2} \tag{52}$$

Reduced-form tests with forecast errors

The FE-based null-hypothesis of FIRE utilize the moment restrictions on forecast errors. In plain words, the null hypotheses of the three tests are the following. First, since the forecasts are on average unbiased according to FIRE, forecast errors across agents should converge to zero in a large sample. Second, forecast errors of non-overlapping forecasting horizon are not serially correlated. Third, forecast errors cannot be predicted by any information available at the time of the forecast, including the mean forecast itself and other variables that are in the agent's information set. This follows from Equation 2. In addition, I include what is called a weak version of the FE-based test which explores the serial correlation of forecast errors in overlapping periods, i.e. 1-year-ahead forecasts within one year. The forecast errors are correlated to the extent of the realized shocks in the overlapping periods. So the positive serial correlation does not directly violate FIRE. But the correlation of overlapping forecast errors still contains useful information about the size of the realized shocks.

Individual-level data are used whenever possible, utilizing the panel structure of both surveys. Since test 2 and 3 requires individual forecasts in vintages that are more than one year apart while SCE only surveys each household for 12 months, the two tests are done with the population average expectations for SCE. Also, the regressions are adjusted accordingly depending on the quarterly and monthly frequency of SPF and SCE. Since these regressions are based on 1-year inflation in overlapping periods, Newy-West standard error is computed for hypothesis testing.

First, all three forecast series easily reject the null hypothesis of unbiasedness at the significance level of 0.1%. There are upward biases in both professional forecasts of core PCE inflation and households' forecast of headline inflation [35], while at the same time professionals underpredicted core CPI inflation over the entire sample period. This was primarily driven by the under-prediction of the inflation over the recent two years since the Pandemic.

Second, the average point forecast one year ago predicts the forecast errors of both groups at the significance level of 0.1%. For headline CPI inflation, for instance, one percentage point inflation forecast corresponds to 0.35 percentage points of the forecast errors one year later. Thus, test 2 in Table 11 easily rejects the second hypothesis test of FIRE that past information does not predict future forecast errors. This suggests that both types of agents inefficiently utilize all information when making the forecasts.

Third, forecast errors are positively correlated with the forecast errors one year ago, with a significant coefficient ranging from 0.35 to 0.572. A higher positive autocorrelation coefficient of forecast errors by households is consistent with the common finding that households are subject to more information rigidity than attentive professionals.

Lastly, test 4 in Table 11 presents a higher serial correlation of forecast errors produced within a year. For SPF forecasts, the serial correlation does not exist beyond 2 quarters, implying the relative efficiency of professional forecasts. For the households, the forecast errors are more persistent over the entire year, in that current forecast errors are correlated with all past forecast errors over the past three quarters. Although the persistence of 1-year forecast errors within one year does not directly violate FIRE,

[35] Coibion et al. (2018) finds the same upward bias for firms' managers.

the fact that households' forecast errors are more persistent than professionals' indicates that the former group is subject to a higher degree of rigidity than the latter one.

Table 11: Tests of Rationality and Efficiency Using Forecast Errors

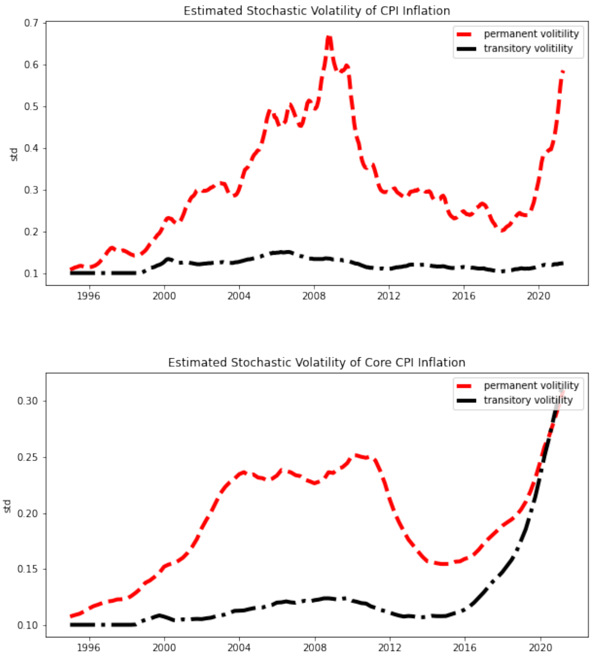
	SPF CPI	SPF PCE	SCE
Test 1: Bias			
Constant	-3.021***	0.460***	1.673***
	(0.242)	(0.047)	(0.008)
N	5510	1610	112668
Test2: FE Depends on past information			
Forecast 1-yr before	0.350***	0.460***	4.190***
	(0.035)	(0.047)	(0.659)
Constant	-3.452***	-2.333***	-12.92***
	(0.386)	(0.192)	(2.213)
N	3945	1610	84
R^2	0.828	0.826	0.311
Test3: FE of non-overllaping forecast horizons are serially correlated			
Forecast Error 1-year before	0.350***	0.460***	0.572**
	(0.035)	(0.047)	(0.195)
Constant	0.314	-1.351***	-0.149
	(0.231)	(0.156)	(0.445)
N	3945	1610	84
R^2	0.828	0.826	0.0957
Time FE	Yes	Yes	No
Test4: Overlapping FE are serially correlated			
Forecast Error 1-q before	0.502***	0.551***	0.327***
	(0.060)	(0.075)	(0.010)
Forecast Error 2-q before	0.0901	0.231***	0.341***
	(0.064)	(0.060)	(0.024)
Forecast Error 3-q before	0.146*	0.0693	0.333***
	(0.065)	(0.052)	(0.023)
Constant	1.147***	-0.356***	0.509***
	(0.224)	(0.058)	(0.035)
N	2971	1338	4432
R^2	0.890	0.903	0.243

Note: white standard errors reported in the parentheses of estimations. *** p<0.001, ** p<0.01 and * p<0.05.

Estimates of stochastic volatility model of inflation

Figure 4 plots the estimated stochastic volatility of permanent and transitory component of inflation, respectively, as specified in Equation 33, using the same estimation method of Stock and Watson (2007).

Figure 4: Stochastic Volatility of Inflation



Note: this figure plots the estimated stochastic volatility of permanent and transitory components of monthly headline CPI (top) and quarterly core CPI inflation (bottom) using the same approach as in Stock and Watson (2007).

Structural estimation covering Pandemic sample (2020-2023)

Table 13, 12 report the estimates for households and professionals, respectively, using only sample till March 2022 for AR(1) inflation process. Table 15, 14 report the estimates based on the alternative UCSV model of inflation process.

Table 12: SMM Estimates of Different Models: Professionals

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.99	0.23	0.2	1	0.06				
FE+Disg	0.34	0.99	0.23	N/A	N/A	N/A				
FE+Disg+Var	0.35	0.99	0.23	N/A	N/A	N/A				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	2.49	3	0.99	0.23	1.46	3	1	0.09		
FE+Disg	1.89	3	0.99	0.23	0	1.45	1	0.03		
FE+Disg+Var	0.39	3	0.99	0.23	0	0.28	0.9	0.45		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	-0.07	0.61	0.99	0.23	-0.43	1.25	1	0.06		
FE+Disg	-0.29	1.87	0.99	0.23	0.97	1.67	0.91	0.44		
FE+Disg+Var	0.67	1.67	0.99	0.23	0.73	1.63	0.9	0.46		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	0.05	0.61	0.31	0.99	0.23	0.77	2.74	3	0.99	0.11
FE+Disg	1.09	3	2.74	0.99	0.23	1.09	3	3	0.99	0.12
FE+Disg+Var	1.09	3	3	0.99	0.23	1.25	3	3	0.99	0.13

Table 13: SMM Estimates of Different Models: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.98	0.41	0.21	0.95	0.64				
FE+Disg	0.34	0.98	0.41	N/A	N/A	N/A				
FE+Disg+Var	0.35	0.98	0.41	N/A	N/A	N/A				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	0.01	2.89	0.98	0.41	0.12	0	0.95	0.65		
FE+Disg	0.02	2.91	0.98	0.41	0.02	2.05	0.95	0.63		
FE+Disg+Var	3	3	0.98	0.41	3	3	0.92	0.81		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_{θ}	ρ	σ	$\hat{\theta}$	σ_{θ}	ρ	σ		
FE	0.95	0.72	0.98	0.41	1.05	0.86	0.95	0.62		
FE+Disg	1.04	1.16	0.98	0.41	1.05	1.15	0.95	0.62		
FE+Disg+Var	1.04	1.16	0.98	0.41	1.12	1.14	0.93	0.75		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	0.79	0.02	2.98	0.98	0.41	0.78	0.02	0.74	0.95	0.62
FE+Disg	N/A	N/A	N/A	0.98	0.41	0.74	0.01	1.89	0.9	0.86
FE+Disg+Var	1.62	3	0.65	0.98	0.41	2.42	3	1.68	0.94	0.67

Table 14: SMM Estimates of Different Models under Stochastic Volatility: Professionals

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	N/A	N/A	
FE+Disg	0.02	0	
FE+Disg+Var	N/A	N/A	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.29	0.67	
FE+Disg	-0.29	0.75	
FE+Disg+Var	-0.29	0.75	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	3	0.27	0.27
FE+Disg	3	2.84	0.24
FE+Disg+Var	3	3	0.25

Table 15: SMM Estimates of Different Models under Stochastic Volatility: Households

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0.05	2.96	
FE+Disg	0.05	2.99	
FE+Disg+Var	0.05	2.82	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	1.28	0.84	
FE+Disg	1.28	1.2	
FE+Disg+Var	1.28	1.2	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	N/A	N/A	N/A
FE+Disg	0.71	0.01	0.95
FE+Disg+Var	N/A	N/A	N/A