How Do Agents Form Macroeconomic Expectations?

Evidence from Inflation Uncertainty

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Roadmap

Motivations

Differentiating non-FIRE models

The role of stochastic volatility

Macroeconomic expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...

Macroeconomic expectation formation

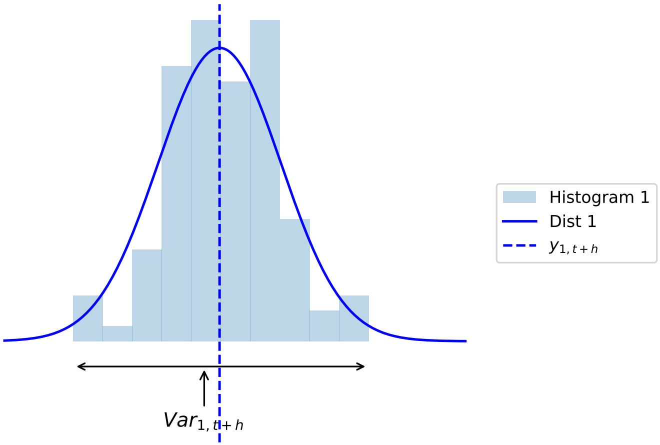
- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...
- Testing these models using survey expectations
  e.g. (Coibion and Gorodnichenko, 2012)
  - Forecast errors (FE)
  - Disagreement (Disg)
  - This paper: +Uncertainty (Var)

Why uncertainty?

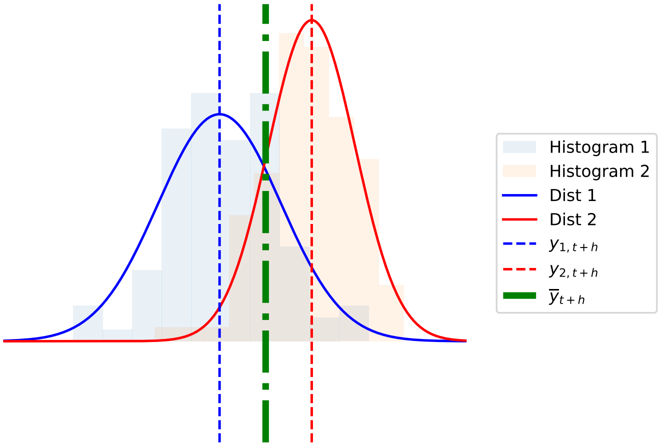
Uncertainty (or higher moments) matters for both

- individual economic decisions
  - precautionary saving motives
  - portfolio investments
  - mortgage choices
  - wage bargaining
- and aggregate outcomes
  - inflation dynamics
  - asset prices

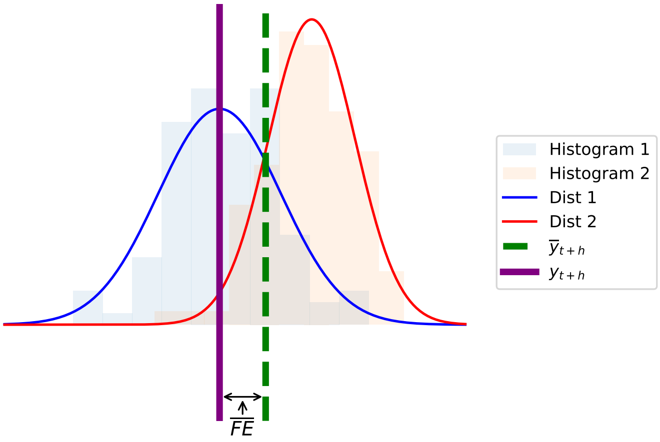
Density forecasts: an example



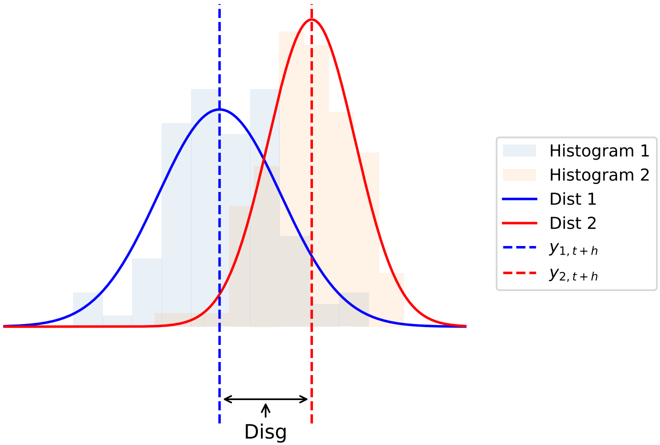
Average expectation



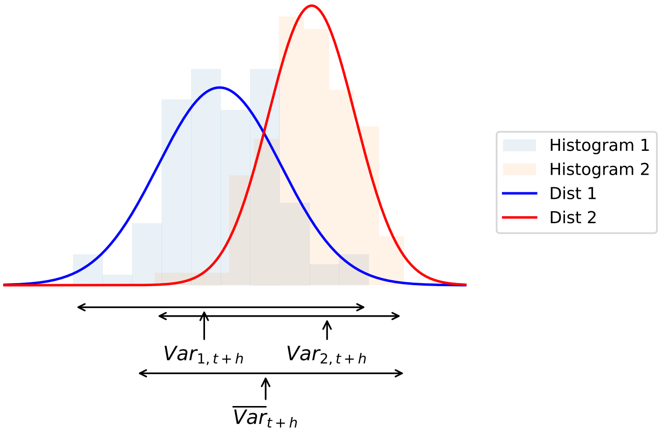
Average forecast errors (FE)



Disagreement (Disg)



Average uncertainty (Var)



Preview of the findings

- Competing theories have distinctive predictions about Var
  - Information rigidity $\rightarrow$ ex-ante $\overline{Var}$ > ex-post $\overline{FE}^2$
- Additional evidence
  - SE more robust than NI
    - Var more due to lagged updating, than noisy information
  - State-dependence: inflation $\uparrow$ rigidity $\downarrow$
  - Coexisting with overreaction at the individual level
- Inflation contains persistent and transitory components

Literature

Studies on expectation formation using survey data

- Structural estimation: [Giacomini, Skreta, and Turen, 2020; Xie, 2023; Bordalo, Gennaioli, Ma, et al., 2020; Farmer, Nakamura, and Steinsson, 2021; Ryngaert, 2017]
- Others: [Mankiw, Reis, and Wolfers (2003), Carroll (2003), Branch (2004), Coibion and Gorodnichenko (2015)]

Measures of uncertainty: [Bachmann, Elstner, and E. R. Sims (2013), Jurado, Ludvigson, and Ng (2015), Rossi and Sekhposyan (2015), Binder (2017),Cascaldi-Garcia et al. (2023)]

- Differentiating Disg and Var: [Rich and Tracy (2010), D'Amico and Orphanides (2008), Abel et al. (2016), Glas (2020), and Rich and Tracy (2021)]

Eliciting probabilistic/density expectations [Manski (2004), Delavande, Giné, and McKenzie (2011), Manski (2018)]

Data

Density forecast of inflation

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1 and 3-yr-ahead inflation	current-year and 1-yr-ahead q4/q4 Core CPI and Core PCE inflation
Survey Structure	fix-horizon	fix-event
Panel Structure	unbalanced, stay up to 12 months	unbalanced, average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

FIRE predictions

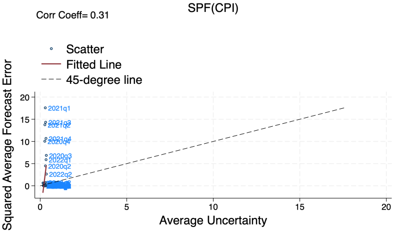
Inflation process (AR1)

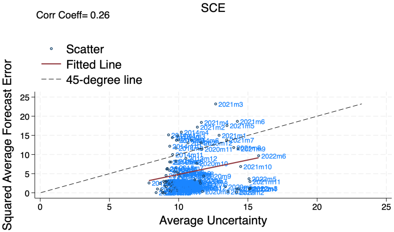
$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

FIRE

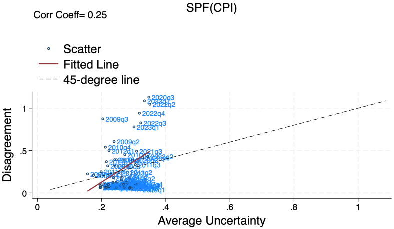
$$\overline{FE}^*_{t+1|t} = -\omega_{t+1} \rightarrow \quad \begin{aligned} \overline{FE}^{*2}_{\bullet+1|\bullet} &= \sigma_\omega^2 \\ \overline{\text{Var}}^*_{\bullet+1|\bullet} &= \sigma_\omega^2 \\ \overline{Disg}^*_{\bullet+1|\bullet} &= 0 \end{aligned}$$

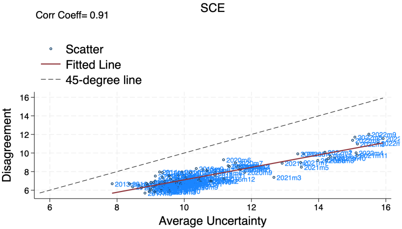
Squared forecast errors and uncertainty



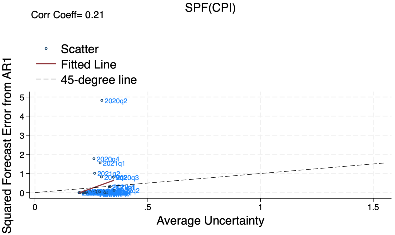


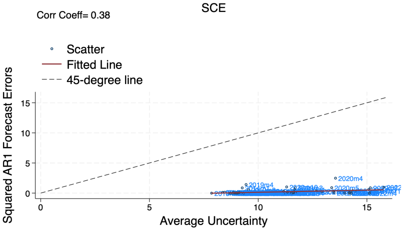
Disagreement and uncertainty





Conditional volatility and uncertainty





Roadmap

Motivations

Differentiating non-FIRE models

The role of stochastic volatility

Sticky expectations (SE)

[Mankiw and Reis, 2002, Carroll, 2003, etc]

With an updating rate of λ (FIRE when $\lambda = 1$)

$$\overline{FE}^{se}_{t+1|t} = (1-\lambda)\rho\overline{FE}^{se}_{t|t-1} - \lambda\omega_{t+1}$$

$$\rightarrow \overline{FE}^{se2}_{\bullet+1|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+t|\bullet} = \sigma^2_\omega$$

$$\overline{\text{Var}}^{se}_{\bullet+1|\bullet} = \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\overline{\text{Var}}^{*}_{t+1|t-\tau} = \frac{1}{1-(1-\lambda)\rho^2}\sigma^2_\omega \geq \overline{\text{Var}}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{se}_{\bullet+1|\bullet} \geq 0$$

Noisy information (NI)

[Lucas, 1972, Woodford, 2001, C. A. Sims, 2003 and Maćkowiak and Wiederholt, 2009, etc]

With noisiness of public and private signals σ^2_{pb} and σ^2_{pr}

$$\overline{FE}^{ni}_{t+1|t} = (1 - PH)\rho\overline{FE}^{ni}_{t|t-1} + \rho P_\epsilon \epsilon_t + \overline{FE}^{*}_{t+1|t}$$

$$\rightarrow \overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_\epsilon \sigma^2_{pb} + \sigma^2_\omega}{(PH)^2} \geq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\text{Var}^{ni}_{\bullet+1|\bullet} = \rho^2 \text{Var}^{ni}_{\bullet|\bullet} + \sigma^2_\omega \geq \text{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{ni}_{\bullet+1|\bullet} = \frac{\rho^2 P^2_\xi}{1 - (1 - PH)^2\rho^2}\sigma^2_{pr} \geq 0$$

Kalman gain: $P = [P_\epsilon, P_\xi] = \overline{\text{Var}}^{ni}_{\bullet|\bullet-1} H (H' \overline{\text{Var}}^{ni}_{\bullet|\bullet-1} H + \Sigma^v)^{-1}$

Diagnostic expectations (DE)

[Bordalo, Gennaioli, and Shleifer, 2018, Bordalo, Gennaioli, Ma, et al., 2020, etc]

With overreaction parameter $\hat{\theta}(> 0)$ and dispersion σ^2_θ

$$\overline{FE}^{de}_{t+1|t} = \overline{FE}^{*}_{t+1|t} - \hat{\theta}\rho \mathrm{FE}^{de}_{t|t-1}$$

$$\rightarrow \overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{1}{1+\hat{\theta}^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{\mathrm{Var}}^{de}_{\bullet+1|\bullet} = \overline{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{de}_{\bullet+1|\bullet} \geq 0$$

Comparing theories

Table: Model Predictions

	FIRE	SE	NI	DE	DENI
Fact 1: Uncertainty greater than forecast error variance	No	Yes	?	Yes	?

Comparing theories

Table: Model Predictions

	FIRE	SE	NI	DE	DENI
Fact 1: Uncertainty greater than forecast error variance	No	Yes	?	Yes	?
Fact 2: Uncertainty greater than conditional volatility	No	Yes	Yes	No	Yes

Comparing theories

Table: Model Predictions

	FIRE	SE	NI	DE	DENI
Fact 1: Uncertainty greater than forecast error variance	No	Yes	?	Yes	?
Fact 2: Uncertainty greater than conditional volatility	No	Yes	Yes	No	Yes
Fact 3: Positive disagreement	No	Yes	Yes	Yes	Yes

Structural Estimation: SMM

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

- $o \in \{se, ni, de, deni\} \times \{ar, sv\}$
- Γ^o: parameter space
- H: real-time historical realizations
- W: weighting matrix

Model estimates for professionals

SE as an example

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$	ρ	σ_ω
FE	0.36	0.99	0.23
FE+Disg	0.28	0.99	0.23
FE+Disg+Var	0.26	0.99	0.23

Evidence for subjective models

[Jain, 2019, Macaulay and Moberly, 2022, Farmer, Nakamura, and Steinsson, 2021]

SE						
Moments Used	2-Step Estimate			Joint Estimate		
	$\hat{\lambda}$	ρ	σ_ω	$\hat{\lambda}$	ρ	σ_ω
FE	0.36	0.99	0.23	0.18	0.97	0.11
FE+Disg	0.28	0.99	0.23	0.22	0.95	0.14
FE+Disg+Var	0.26	0.99	0.23	0.32	0.9	0.22

NI requires highly noisy signals

NI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	0.87	0.99	0.23	0	0.15	0.97	0.11
FE+Disg	1.5	2.26	0.99	0.23	1.48	2.33	0.97	0.11
FE+Disg+Var	2.64	3	0.99	0.23	3	3	0.94	0.16

Patterns of households

Sticky, underreactive and widely dispersed

SE				
Moments Used	2-Step Estimate			
	$\hat{\lambda}$	ρ	σ_ω	
FE	0.36	0.98	0.45	
FE+Disg	0.36	0.98	0.45	
FE+Disg+Var	0.36	0.98	0.45	
NI				
Moments Used	2-Step Estimate			
	$\hat{\sigma}_\epsilon$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	0	1	0.98	0.45
FE+Disg	3	1.18	0.98	0.45
FE+Disg+Var	2.06	3	0.98	0.45
DENI				
Moments Used	2-Step Estimate			
	$\hat{\theta}$	$\hat{\sigma}_\xi$	ρ	σ_ω
FE	N/A	N/A	0.98	0.45
FE+Disg	-0.54	3	0.98	0.45
FE+Disg+Var	-0.35	2.43	0.98	0.45

Roadmap

Motivations

Differentiating non-FIRE models

The role of stochastic volatility

Stochastic volatility (SV)

[Stock and Watson, 2007]

Process of inflation

$$y_t = \overbrace{\zeta_t}^{\text{Permanent component}} + \overbrace{\eta_t}^{\text{Transitory}}$$

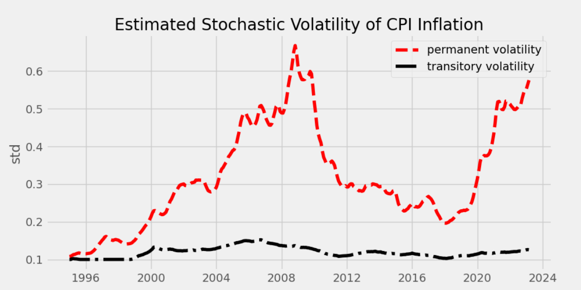
$$\zeta_t = \zeta_{t-1} + z_t$$

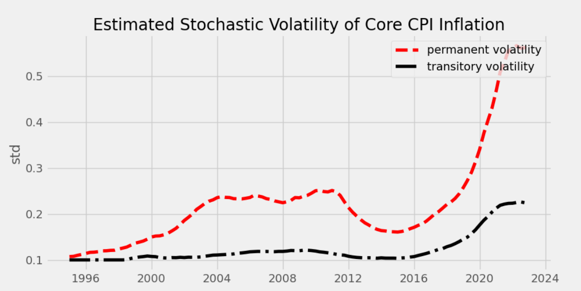
$$z_t = \sigma_{z,t}\xi_{z,t}, \quad \eta_t = \sigma_{\eta,t}\xi_{\eta,t}, \quad \xi_t = [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I)$$

$$\log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t}, \qquad \log \sigma^2_{z,t} = \log \sigma^2_{z,t-1} + \mu_{z,t}$$

$$\mu_t = [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)$$

Estimated SV





More sensible est of NI for professionals

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.2		0.3	
FE+Disg	0.25		0.36	
FE+Disg+Var	0.36		0.36	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.68	0.24	2.3	3
FE+Disg	0.67	0.24	2.3	3
FE+Disg+Var	0.64	0.21	2.3	3

NI remains a poor fit of households

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.27		0.36	
FE+Disg	0.2		0.27	
FE+Disg+Var	0.26		0.26	
NI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\sigma}_{\epsilon}$	$\hat{\sigma}_{\xi}$	$\hat{\sigma}_{\epsilon}$	$\hat{\sigma}_{\xi}$
FE	N/A	N/A	N/A	N/A
FE+Disg	N/A	N/A	N/A	N/A
FE+Disg+Var	N/A	N/A	N/A	N/A

Higher inflation, less rigidity

[Coibion and Gorodnichenko, 2015, Weber et al., 2023]

Before March 2020			Till March 2023	
SE				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\lambda}$		$\hat{\lambda}$	
FE	0.27		0.36	
FE+Disg	0.2		0.27	
FE+Disg+Var	0.26		0.26	
DENI				
Moments Used	2-Step Estimate		2-Step Estimate	
	$\hat{\theta}$	$\hat{\sigma}_\xi$	$\hat{\theta}$	$\hat{\sigma}_\xi$
FE	-0.48	0.64	0.43	0.26
FE+Disg	-0.48	0.64	0.43	0.26
FE+Disg+Var	-0.48	0.64	0.43	0.26

Scoring card of model robustness

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	Yes	Yes	No
Sensitive to the assumed process?	No	Yes	Yes	No
Sensitive to two-step or joint estimate?	No	No	No	Yes
Sensitive to the type of agents?	No	Yes	Yes	Yes

- But no single model explains all aspects of survey expectations

Conclusion

- Belief is not just expectations, but also second or higher moments

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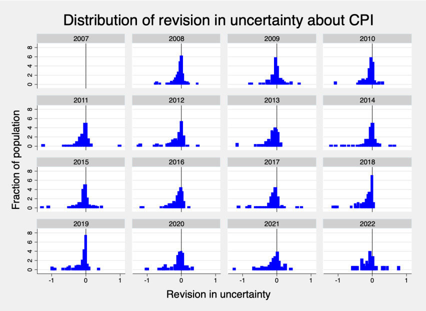
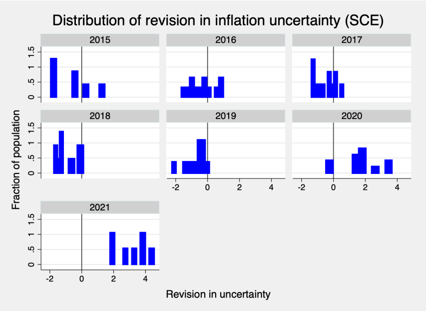
Uncertainty revision

More certain when getting closer to realization?





Efficiency tests with uncertainty

Do revisions reflect only common resolution of uncertainty?

$$\text{Var}_{i,t|t} - \text{Var}_{i,t|t-1} = \alpha^{\text{var}} + \beta^{\text{Var}}(\text{Var}_{i,t-1|t-1} - \text{Var}_{i,t-1|t-2}) + \psi_t^{var} + \zeta_{i,t}^{var}$$

- $\beta^{var} = 0$ under FIRE
- $\alpha^{var} < 0$ time-invariant uncertainty reduction
- ψ_t^{var}: time-varying innovations

Efficiency tests: professionals

	Mean revision	4q before	4q before	5q before
L4.InfExp_Var_rv		0.448***	0.456***	
		(0.056)	(0.058)	
L5.InfExp_Var_rv				0.440***
				(0.053)
Constant	-0.091***	-0.049***	-0.048***	-0.049***
	(0.000)	(0.008)	(0.005)	(0.005)
R2	0.047	0.196	0.248	0.249
N	1529	1157	1157	1021
Time FE	Yes	No	Yes	Yes