How Do Agents Form Inflation Expectations? Evidence from the Forecast Uncertainty

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Abstract

Density forecasts of macroeconomic variables provide one additional moment restriction, uncertainty, for testing various models of expectation formation deviating from the full-information rational expectations (FIRE). This paper first documents the persistent dispersion and inefficient revisions in inflation uncertainty of professionals and households, and how the uncertainty conveys different information from the widely used proxies to uncertainty such as cross-sectional disagreement and forecast errors. Second, I provide additional reduced-form test results and structural estimates for three models deviating from FIRE: sticky expectations (SE), noisy information (NI), and diagnostic expectations (DE) by jointly accounting for its predictions for different moments. The estimates suggest the SE model with an inflation process featuring stochastic volatility accounts for the joint dynamics of inflation and forecast moments better than other competing theories.

Keywords: Inflation, Expectation Formation, Rigidity, Overreaction, Uncertainty, Density Forecast

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1 Introduction

Theories on how agents form expectations in ways deviating from rational expectations (RE) have proliferated over the past decade. On one hand, various theories built upon different micro-foundations produce somewhat similar macro patterns. For instance, the expectation rigidity documented by Coibion and Gorodnichenko (2012, 2015), i.e. the sluggish response in aggregate expectations to new information, can be micro-founded by both Sticky Expectations (SE)[1] and Noisy Information (NI)[2]. On the other hand, there are important and subtle differences in testable predictions from various theories regarding both individual forecasts and aggregate moments. For instance, in contrast with the models featuring expectation rigidity at the aggregate level, the theory of Diagnostic Expectation (DE) (Bordalo et al., 2018, 2020) predicts overreaction to news at the individual level. [3]

One of the crucial steps ahead in advancing this literature is to test these theories better using expectation surveys. This paper does a cross-moment estimation of each theory by jointly accounting for its predictions about different moments of the forecasts such as forecast errors, cross-sectional disagreements, and uncertainty. Although reduced-form tests focused on first moments have been powerful in rejecting the null of the full-information rational expectations (FIRE), identifying the differences among these non-FIRE theories requires more information from second or higher moments and needs to rely on more restrictions across moments. Such an idea builds on the seminal work Coibion and Gorodnichenko (2012), which compares different theories by examining if the impulse responses of inflation expectations to externally estimated shocks align with the qualitative predictions of these models. This paper adopts a similar approach[4], with the key novelty being utilizing an additional moment of forecasts in the estimation, i.e. the uncertainty.

Existing work that studies expectation formation utilizing survey data mostly focuses on the individual and cross-sectional patterns of the mean forecast, forecast errors and, at the most, disagreement. But there are additional insights from the forecast uncertainty, which is only available if each survey respondents is asked to assign their own perceived probabilities to a range of values of the variable. For instance, the dispersion of uncertainty across individual forecasters, as that of the average forecasts, is inconsistent with the benchmark full-information rational expectation (FIRE) as the latter assumes that agents agree on the data generating process and have the common knowledge of available information. Dynamically, across different vintages of the forecast, the revision in uncertainty is a measure of information gain or the degree of forecasting efficiency. Besides, the relationship between uncertainty with other moments such as forecast errors and disagreement provides a way to check if the surveyed expectations are consistent with certain theories of expectation formation. In addition, although some theories have similar qualitative predictions about the forecasting

[1] Mankiw and Reis (2002); Carroll (2003); Reis (2006).

[2] Lucas (1972), Woodford (2001).

[3] Alternatively, Kohlhas and Walther (2021) reconciles the coexistence of underreaction to news and extrapolation relying upon a variant of the mechanism of rational inattention.

[4] This is not the only paper that estimates theories on expectation formation using cross-moment restrictions. Recent examples pursuing similar path includes, for instance, Giacomini et al. (2020); Farmer et al. (2021).

moments, the configurations of the model parameters consistent with the theory may not be empirically realistic.

In the theoretical front, this papers shows that various workhorse theories of expectation formation have distinctive predictions for forecast uncertainty in addition to its implications for commonly explored moments such as forecast errors and disagreement. In particular, in SE, the extra uncertainty in addition to the uncertainty associated with rational forecasting arises from non-updating of the most recent information. While in NI model, the uncertainty endogenously depends on the noisiness of signals and determines agents' degree of reaction to the news in the Kalman filtering problem. Different from either, the canonical model of DE maintains the constant variance assumption. It poses a question if the extrapolation is also relevant in the second moment. Moreover, different models featuring information rigidity in incorporating new information predict inefficiency in forecast revisions. This paper shows that such inefficiency can not only be seen in the average forecast, but also in the forecast uncertainty. Because of these reasons, uncertainty seen in the data imposes additional restrictions on parametric configurations that produce the degree of under/overreaction seen in the data.

Empirically, with comparable estimates of different theories using cross-moments restrictions, I can evaluate to what extent each theory fits the observed expectation data and inflation dynamics. In addition, I can evaluate each model of expectations formation in terms of its sensitivity with respect to four criteria (1) the moments used for estimation, i.e. only the forecast error or higher moments such as disagreement and uncertainty. (2) the specification of the underlying process of the inflation, i.e. an AR(1) with constant volatility or one with different components of the time-varying volatility. (3) if estimating the underlying process and expectations separately or jointly. The former basically recovers the inflation process only based on inflation data, while the latter let the expectations provide information for estimating the inflation process. (4) whether it accounts for different agents such as households and forecasters' expectations equally well.

In particular, this paper evaluates three workhorse theories on expectation formation and some hybrid versions seen in the literature. (1) sticky expectations (SE). (2) noisy information (NI). (3) diagnostic expectations (DE). (4) diagnostic expectations/noisy information (DENI).[5] One major distinction between the first two and DE is that the former predict underreaction due to rigidity, and the latter predicts overreaction. The distinction between SE and NI is, to a large extent, quantitative. The test of the two models relies upon which model can generate patterns consistent with the data with a realistic configuration of parameters. In particular, the estimation in this paper suggests that an unrealistic high degree of noisiness of the signals is needed for the rigidity in NI while a more realistic updating frequency is sufficient for the SE.

Other findings from the paper are the following.

- Overall, sticky expectations (SE) augmented with inflation with stochastic volatility (SV) fits the joint dynamics of inflation expectations and inflation better than other theories, for both households and professionals.

[5]For instance, Bordalo et al. (2020) embeds DE in a NI model, which is also called a "a diagnostic Kalman filtering problem".

- Three theories perform indistinguishably well in matching the dynamics of forecast errors, but differ in matching the moment conditions of disagreement and uncertainty.

- None of the three theories' parameter estimates are particularly sensitive to the use of moments in estimation, although the estimates of DE and DENI are not always consistent with the premises of model assumptions.

- Theories differ in their sensitivity to whether jointly estimating the inflation process and the survey moments or separately estimating the two. On both fronts, SE and DE have similar performances, but the NI coefficient estimate changes substantially.

- Within each theory, households' expectations exhibit less consistency across moments compared to professionals. This evidence of inconsistency adds to the evidence rejecting full-information-rationality based on reduced-form tests.

In addition to structural estimation, I also provide additional reduced-form evidence in violation of the benchmark FIRE predictions using the revision of uncertainty. As autoregression tests of revisions in forecasts, revision in uncertainty can be also used to test the efficiency in expectation formation. The new result from this paper is that the revision in uncertainty has a serial correlation that is not consistent with the level of forecast efficiency predicted by rational expectation. This provides an additional result that rejects the null of the rational expectation hypothesis.

1.1 Related Literature

This paper is related to four strands of literature. First, it is related to a series of empirical work directly testing and evaluating various theories on expectations formation using survey data. For instance, Mankiw et al. (2003), Carroll (2003), Branch (2004). More recent examples include Coibion et al. (2018) on firms' managers. In addition to testing particular sets of theories, there is also a number of papers that show people's expectations are driven by idiosyncratic demographics, cognitive abilities and macroeconomic histories experienced (Malmendier and Nagel (2015), Das et al. (2017) and D'Acunto et al. (2019), etc.). In terms of the methodology, this paper is closest to Giacomini et al. (2020), which estimates theories of expectation formation using cross-moments restrictions. However, all of these studies simply rely upon point forecasts instead of density forecast or surveyed uncertainty. This is one theme on which this paper differs from the existing literature.

Second, Manski (2004), Delavande et al. (2011), Manski (2018) and many other papers have advocated long for eliciting probabilistic questions measuring subjective uncertainty in economic surveys. Although the initial suspicion concerning to people's ability in understanding, using and answering probabilistic questions is understandable, Bertrand and Mullainathan (2001) and other work have shown respondents have the consistent ability and willingness to assign a probability (or "percent chance") to future events. Armantier et al. (2017) have a thorough discussion on designing, experimenting and implementing the consumer expectation surveys to ensure the quality of the responses[6]. Broadly speaking, the literature has argued that going beyond the revealed preference approach, availability of survey data provides economists with direct information on agents' expectations and helps avoids imposing arbitrary assumptions. This insight holds for not only point forecast but also and even more importantly, for uncertainty, because for any economic decision made by a risk-averse agent, not only the expectation but also the perceived risks matter a great deal.

Third, by approximating subjective uncertainty directly from density responses, this paper contributes to the literature that develops and uses a variety of measures of uncertainty, especially in the macroeconomic context. There is a long tradition of approximating uncertainty by measures that are more directly available in survey data or that can be estimated by econometric methods. For instance, Bachmann et al. (2013) use ex-ante disagreement and ex-post forecast errors computed from forecasters' surveys as proxies of uncertainty. Jurado et al. (2015) define the time-varying uncertainty as conditional volatility of the non-forecastable component of a variable, and estimate it using multiple macroeconomic series. Binder (2017) approximate uncertainty from rounding in survey data based on the insights from cognitive literature. Besides, the text-based approach such as Bloom (2009) constructs indices of policy uncertainty based on texts of newspaper reporting. Although these proxies are all meant to capture the notion of uncertainty, as shown in Section 2.4, cross-validation seems to suggest they are statistically uncorrelated or even negatively correlated.

Fourth, the literature that has been originally developed under the theme of forecast efficiency[7] provides a framework analyzing the dynamics of uncertainty useful for the purpose of this paper. The focus of the forecasting efficiency literature is evaluating forecasters' performance and improving forecasting methodology, but it can be adapted to test the theories of expectation formation of different types of agents. This is especially relevant to this paper as I focus on the uncertainty.

The paper is organized as followed. Section 2 shows the stylized patterns of different forecasting moments of professional forecasts and households. Section 3 first sets up a unified framework in which testable predictions of different theories can be compared. Also, I derive a various moment conditions from these theories. Section 4 undertakes reduced-form time-series regressions that test the null hypothesis of FIRE and implications of different theories. Section 5 includes results from estimating the theory-specific parameters using simulated method of moments. It also evaluates the sensitivity of the model specification. Section 6 concludes the paper and discusses the future directions of the research.

[6]Other literature includes Van der Klaauw et al. (2008) and Delavande (2014), etc.

[7]Nordhaus (1987), Davies and Lahiri (1995), Clements (1997), Faust and Wright (2008), Patton and Timmermann (2012).

2 Data and Facts

2.1 Definition and notation

An agent i is forming expectations about a stochastic macroeconomic variable y_{t+h} [8], the inflation in this paper. Denote $f_{i,t+h|t}$ as agent i's h-period-ahead density forecast. $f_{i,t+h|t}$ is the conditional density of y_{t+h} given the information set $I_{i,t}$ available at time t.

$$f_{i,t+h|t} \equiv f_{i,t}(y_{t+h}|I_{i,t})$$

The information set could be agent-specific, thus it has subscript i. The specific content contained in I_t varies from different models of expectation. For instance, sticky expectation (SE) and rational inattention[9] literature all assume that agents are not able to update new information instantaneously. So the information set may not contain the most recent realization of the variable of forecast y_t. In contrast, NI assumes that the information set only contains noisy signals of the underlying variables. Different theories may also differ in terms of the mapping from information to conditional density forecasts. [10] For instance, DE deviates from Bayesian learning by allowing agents to overweight new information that is particularly salient.

Accordingly, h-period-ahead mean forecast at t, denoted as $y_{i,t+h|t}$, is the conditional expectation of y_{t+h} by the agent i.

$$y_{i,t+h|t} \equiv E_{i,t}(y_{t+h}) = \int y_{t+h} f_{i,t+h|t} dy_{t+h}$$

Similarly, individual forecasting variance $Var_{i,t+h|t}$, hereafter termed as individual uncertainty in this paper, is the conditional variance corresponding to the forecast density distribution.

Individual forecast error $FE_{i,t+h|t}$ is the difference of individual forecast at time t and ex post realized value of y_{t+h}. By definition. positive (negative) forecast errors mean overpredict (underpredict) the variables.

$$FE_{i,t+h|t} = y_{i,t+h|t} - y_{t+h}$$

The population analogs of individual mean forecast, uncertainty and forecast errors are simply the average of the individual moments taken across agents. Denote them as $\bar{y}_{t+h|t}$, $\overline{Var}_{t+h|t}$, and $\overline{FE}_{t+h|t}$, respectively. Hereafter, they are termed as the population mean forecast, population uncertainty and population forecast error, respectively. In

[8]Only in the context of aggregate variable, it makes sense to study the population moments such as average expectations and disagreements. Studying expectations of idiosyncratic variables requires individual panel data, as well as the idiosyncratic realizations of the variable.

[9]Sims (2003).

[10]There are other classes of models that fall into this category, which assume alternative mapping from the agent's information set to the conditional density. For instance, Patton and Timmermann (2010); Farmer et al. (2021) finds that the disagreements are driven by not the only difference in information but also heterogeneity in prior and models. Macaulay and Moberly (2022) shows survey evidence for heterogeneity in perceived persistence of inflation shocks. More theoretical work includes multi-prior or model uncertainty such as Hansen and Sargent (2001), Hansen and Sargent (2008), etc.

addition, disagreement is defined as the cross-sectional variance of mean forecasts of individual agents, denoted as $\overline{Disg}_{t+h|t}$.

I refer to the 3 individual indicators and 4 population indicators defined above as moments, and they are listed in Table 1.

Table 1: Definition and Notation of Moments

Individual Moments	Population Moments
Mean forecast: $y_{i,t+h\|t}$	Average forecast: $\bar{y}_{t+h\|t}$
Forecast error: $FE_{i,t+h\|t}$	Average forecast error: $\overline{FE}_{t+h\|t}$
Uncertainty: $\text{Var}_{i,t+h\|t}$	Average uncertainty: $\overline{\text{Var}}_{t+h\|t}$
	Disagreements: $\overline{Disg}_{t+h\|t}$

2.2 Benchmark predictions from full-information rational expectation (FIRE)

We start by assuming an underlying process of inflation. In the benchmark scenario, we assume the underlying true process of y_t is $AR(1)$ with persistence parameter $0 < \rho < 1$ and i.i.d. shock ω_t.

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma^2_\omega) \tag{1}$$

The assumption of i.i.d. shock with constant volatility turns out insufficient to match the time-varying pattern of uncertainty. Therefore, as an extension, I also consider an alternative assumption that inflation consists of two unobservable components with stochastic volatility. I will specify this explicitly later to focus on comparing different models of expectation formation under a simple AR(1) process.

In the FIRE benchmark, it is assumed that all agents perfectly observe y_t at time t and understand the true process of y. Therefore, the individual forecast is $\rho^h y_t$, which is shared by all agents. Therefore, it is also equal to the average forecast.

Both individual and population forecast errors are simply the realized shocks between $t+1$ and $t+h$.

$$FE^*_{t+h|t} = -\sum_{s=1}^{h} \rho^(s-1)\omega_{t+h-s} \tag{2}$$

I use superscript of $*$ to denote all the moments according to FIRE. It is easy to see that the forecast error is orthogonal to information available till time t. This provides a well known null hypothesis of FIRE. [11] These FE-based restrictions of FIRE provide the foundations for the tests used in Section 6.

[11] Another well-known prediction of FIRE is that forecast errors of non-overlapping horizon are not correlated, namely, $Cov(FE^*_{t+h|t}, FE^*_{t+s+h|t+s}) = 0 \quad \forall s \geq h$. This is not the case within h periods, as the realized shocks in overlapping periods enter both forecast errors.

The unconditional variance of the FE, or equivalently, its square (due to its zero unconditional mean), is equal to the following.

$$FE^{*2}_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)} \sigma^2_{\omega} \tag{3}$$

The uncertainty about future y simply comes from uncertainty about unrealized shocks between t and $t+h$. With the same model in mind (Equation 1) and the same information y_t, everyone's uncertainty is equal to the weighted sum of the future volatility before its realization (Equation 4). In FIRE, there is neither disagreement about the mean, nor disagreement about the uncertainty.[12] Furthermore, it is exactly equal to the variance of forecast errors, $FE^{*2}_{\bullet+h|\bullet}$.

$$\text{Var}^{*}_{\bullet+h|\bullet} = \sum_{s=1}^{h} \rho^{2(s-1)} \sigma^2_{\omega} \tag{4}$$

The time-series behavior of h-period-ahead uncertainty, i.e. $\text{Var}_{t+h|t}$, $\text{Var}_{t+h+1|t+1}$, etc., depends on the true process of y. Specifically, it depends on whether σ^2_{ω} is time-varying. If time-invariant, h-period-ahead uncertainty is simply a constant. In the baseline case, I make such an assumption, in general, it may not be true. In the extension, I make alternative assumptions of the inflation process allowing for stochastic volatility.[13]

Another testable implication of rationality lies in the revision of uncertainty. Hereafter, we refer to revision (instead of change) as the difference of moments across vintages of the forecast with the fixed terminate date of realization. For instance, the uncertainty revision for from $t-1$ to t the h-period-ahead forecast is the difference between the uncertainty about y_{t+h} at time t and the uncertainty about y_{t+h} at time $t-1$.

Moving from t to $t+1$, for instance, the revision in uncertainty is simply a negative constant independent of the time. There is an unambiguous reduction in uncertainty (or information gain in the forecasting literature) as more and more shocks have realized. In the most intuitive case, from the one-step-ahead forecast at $t-1$ to that of t, i.e. $h=1$, the variance drops exactly by the resolution of uncertainty of ω_t, which is σ^2, to zero uncertainty.

$$\text{Var}^{*}_{t+h|t+1} - \text{Var}^{*}_{t+h|t} = -\rho^{2(h-1)} \sigma^2_{\omega} \tag{5}$$

Lastly, FIRE has predictions about the disagreement. As agents perfect update the same information, there is no disagreement at any point of the time.

$$Disg^{*}_{t+h|t} = 0 \quad \forall t \tag{6}$$

[12]This is the same to Jurado et al. (2015)'s terminology.

[13]For example, Justiniano and Primiceri (2008), Vavra (2013) on time-varying volatility of inflation.

Table 2 summarizes all other expectation moments of 1-period-ahead inflation ($h = 1$) as predicted by FIRE and a process of AR1. Both variances of forecast errors ($FEVar$) and average uncertainty Var are equal to the size of the shock to inflation σ^2, while in contrast, the disagreement is always zero, hence has zero correlation with $FEVar$ and Var. In Section 2.4, I discuss in greater detail how the data counterparts of the forecast moments are inconsistent with these predictions.

Table 2: Moments of Inflation and Expectations

	SPF	SCE	FIRE+AR	FIRE+SV
InfAV	0	0	0	0
InfVar	1.145	3.959	$\sigma^2/(1-\rho^2)$	N/A
InfATV	0.918	3.763	$\rho\sigma^2/(1-\rho^2)$	N/A
FE	-0.15	1.062	0	0
FEVar	1.032	3.794	σ^2	$\bar{\sigma}^2_\eta + \bar{\sigma}^2_\epsilon$
FEATV	0.006	0.631	0	0
Disg	0.2	2.695	0	0
DisgVar	0.042	0.146	0	0
DisgATV	0.01	-0.015	0	0
Var	0.246	1.738	σ^2	$\bar{\sigma}^2_\eta + \bar{\sigma}^2_\epsilon$
VarVar	0.002	0.04	0	>0
VarATV	0.001	-0.005	0	>0

This table reports the moments of demeaned inflation and inflation expectations of both SPF and SCE used in the model estimation. Core CPI inflation is used for SPF forecasts, and the headline inflation is used for SCE moments. The data sample is 2007M1-2022M5 for SPF and 2013M1-2021M6 for SCE, for which the expectation data is available for each survey, respectively. For SCE moments, both disagreement (Disg) and uncertainty (Var) are computed using the regression residuals of individual mean forecast and uncertainty after controlling for individual fixed effects.

2.3 Data

This paper utilizes one special feature of professional and household inflation expectation survey to test various theories of expectation formation, which is that both surveys contain density forecasts by asking individual respondents to fill a number of externally specified bins representing different ranges of values.

Survey of Professional Forecasters(SPF) has reported the individual-level density forecasts of core CPI and core PCE inflation since 2007. In addition, for these two inflation measures, both forecasts for current-year inflation, basically nowcast, and one-year-ahead forecasts are elicited via density surveys. This makes it possible to directly test the implications of the revisions in uncertainty across different vintages of forecasts.

The New York Fed Survey of Consumer Expectation(SCE), which started in 2013 also asked households to provide their perceived probabilities about 1-year-ahead and

3-year-ahead inflation for various ranges of values each month[14]. This allows for comparing 3-year-ahead forecast at time $t-3$ with 1-year-ahead forecast at $t-1$. Since the maximum duration for households to stay in the panel is 12 months (for about one-third of the households), forecast revision can be only examined at the population level. The advantage of SCE compared to SPF is its monthly frequency. This provides an invaluable chance to explore the dynamics of uncertainty. A summary of the two surveys is in Table 3.

Converting expressed probability forecasts based on externally divided bins into an underlying subjective distribution requires a density distribution. I closely follow Engelberg et al. (2009)'s method with a small modification to estimate the density distribution of each individual respondents in SPF. Answers with at least 3 bins with positive probabilities or 2 bins but open-ended from either left or right are fit with a generalized beta distribution. Depending on if there is open-ended bin on either side with positive probability, 2-parameter or 4-parameter beta distribution are estimated, respectively. Those with only two bins with positive probabilities and adjacent are fit with a triangular distribution. Answers with only one bin of positive probability are fit by a uniform distribution. [15] This is the same approach adopted by the New York Fed researchers Armantier et al. (2017).

To avoid the biases introduced by extreme answers or data errors, I undertake some winsorization for both data sets. For SPF, I drop the outliers of mean forecast and uncertainty estimates at both the top and bottom one percentile, as these are typically abnormal that are due to measurement errors or other reasons. For SCE, I drop the top and bottom 5 percent of mean forecasts and uncertainty, as households' mean forecasts are inclined to give extreme values. All the results in this paper are robust to different thresholds, such as 10 and 1 percentile. [16]

Table 3: Information of Data

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Var in Density	1-yr inflation	1-yr-ahead Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

Throughout the paper, I use three measures of inflation: headline CPI, core CPI, and core PCE. Depending on the specific variable of forecast in the survey series, the realization of the corresponding inflation is used to compute moments such as forecast

[14]Most importantly, the survey respondents are guaranteed to assign probabilities to all bins that sum up to one, as a feature of the online survey design.

[15]See the Python program with detailed steps of estimation.

[16]For mean forecasts and uncertainty, respectively, this means dropping 6528 and 5096 observations, out of 68887 observations in total.

errors. Specifically, SPF has density forecasts for both core CPI and core PCE [17]. For SCE, as the household responders are asked about the overall inflation, it is the most reasonable to interpret it as headline CPI inflation. To simplify the expression, from now on, core CPI and core PCE are simply referred to as CPI and PCE, respectively.

2.4 Stylized facts

2.4.1 Relationship between the size of forecast errors, disagreement and uncertainty

Despite the significant differences in magnitudes between professionals' forecasting moments and households, both types of agents share common patterns in terms of the relationship across various moments. Figure 1a, 1b, and 1c plot the population uncertainty against expected inflation, forecast errors, and disagreements in the first, second, and third rows, respectively.

It is both documented in the literature [18] and cited in popular narratives that high inflation (or expected inflation) is typically associated with high inflation uncertainty. The observable correlation between the realized inflation and the directly measured forecasting uncertainty by both professionals and households is indeed consistent with such a pattern, with such a positive correlation primarily driven by co-movement of the two since the Covid pandemic. Over the entire period between 2007-2021, the correlation coefficients are 0.3, 0.35, and 0.67 for SPF's CPI forecast, SPF's PCE forecast, and SCE's CPI expectation, respectively.[19] Although there is weak evidence of comovement of inflation rates and uncertainty during the pre-Pandemic period of persistently low and stable inflation, the fast rise in inflation since middle 2021 was accompanied by a notable rise in uncertainty in the same period.

Figure 1b looks into the relationship between the size of the forecast error and uncertainty. According to the benchmark prediction under FIRE, the variance of forecast errors and forecast uncertainty should be equal to each other on average, as seen in Table 2. In the data, the correlation coefficients of the two are 0.26, 0.24, and 0.39 for SPF CPI forecasts, SPF PCE forecasts, and SCE's forecasts, respectively. Such a positive correlation is consistent with the FIRE prediction, but the coefficients are much smaller than 1, as predicted in the benchmark model. In particular, the variance of forecast errors is bigger than the average of uncertainty for both professionals and households.

Figure 1c examines the relationship between disagreement and uncertainty. At the same time, a large body of empirical literature in macroeconomics use disagreement, which is often more available than the uncertainty, as a proxy of latter[20]. But under benchmark FIRE, regardless of inflation process, disagreement should be always zero,

[17]SPF also has density forecasts for GDP deflator (for GNP prior to 1992: Q1) going back to 1968. Since the ranges of the values and the definition are not consistent over time, I do not use them in this paper.

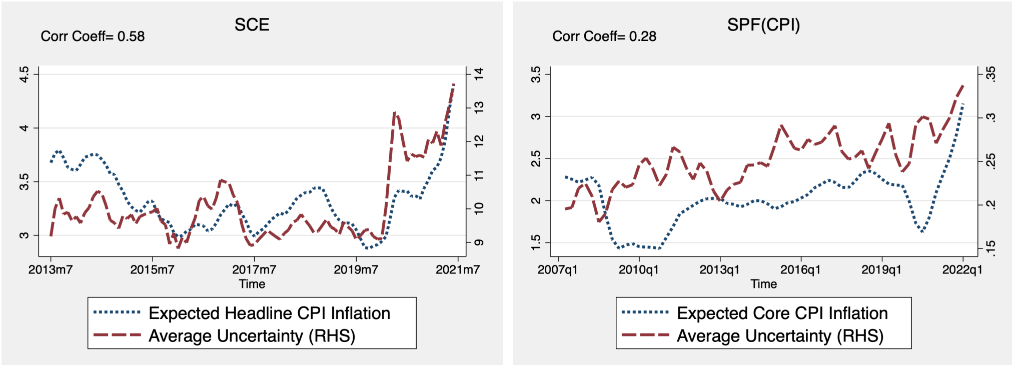
[18]For example Ball et al. (1990); Ball (1992).

[19]The same coefficients before 2020 were 0.14, 0.12 and -0.24

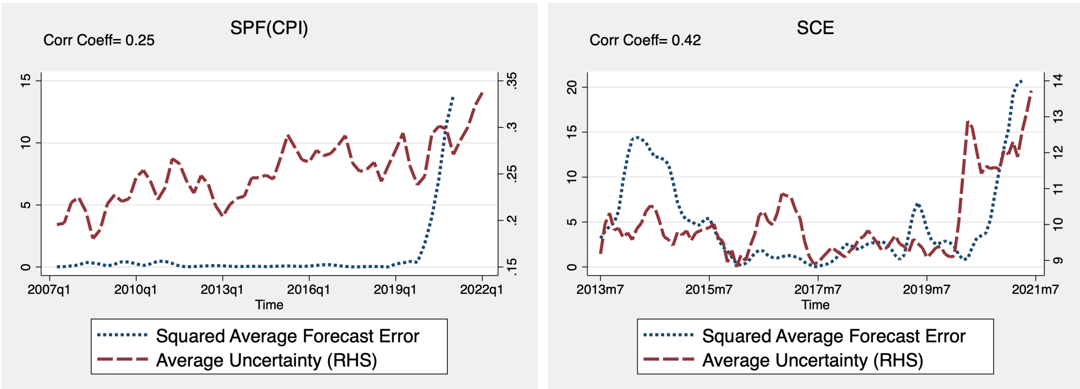
[20]For instance, Bachmann et al. (2013) used ex-ante disagreement and ex-post forecast errors as two measures of uncertainty and find that both uncertainty indicators lead to a reduction in real economic activity.

Figure 1: Uncertainty and Other Moments

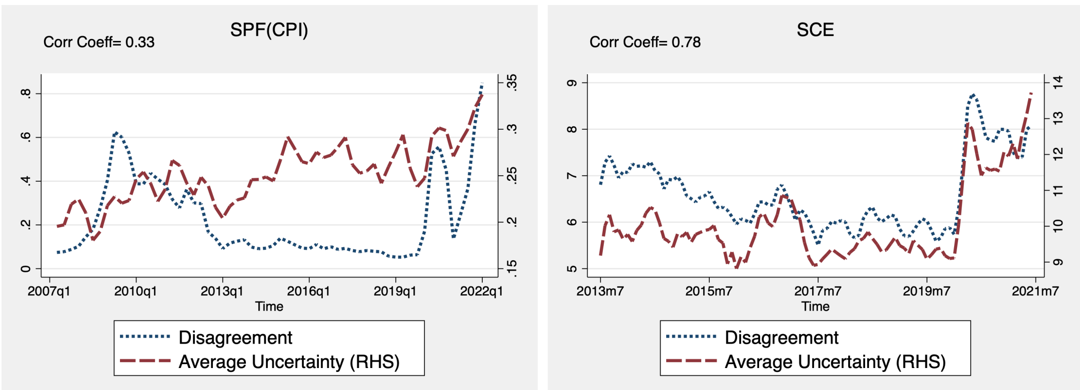
(a) Expected Inflation and Uncertainty



(b) Squared Forecast Errors and Uncertainty



(c) Disagreement and Uncertainty



Note: From the left to right: SPF's forecasts of core CPI and SCE's household forecast of headline CPI. From the top to the bottom: uncertainty (dash line) versus expected inflation (dot) with correlation coefficient of 0.58, and 0.28, respectively; uncertainty (dash) versus square of the realized forecast errors (dot) with correlation coefficient of 0.25, 0.42, respectively; uncertainty (dash) versus disagreements (dot) with correlation coefficient of 0.33, and 0.78, respectively.

and it is therefore not correlated with the average uncertainty.[21] Despite such a clear distinction in theory, the empirical correlation between disagreement and uncertainty is indeed sufficiently positive, which are 0.33 and 0.23 for SPF forecasts, and 0.78 for households. The positive correlation between the two was particularly salient since early 2020, a period of rapidly rising inflation.

The most striking fact from the discussion above is the co-movement of ex-ante uncertainty, disagreement, and the size of ex-post forecast errors over time, a pattern that is seen in both professionals and households. Our assumed benchmark data generating process of inflation and the few theories on expectation formation under consideration are not able to account for such a pattern, as discussed in Section 3.

2.4.2 Dispersion in forecast uncertainty

A persistent disagreement in expectations has been used as important stylized evidence inconsistent with the assumption of identical expectation embedded in FIRE (Mankiw et al. (2003)). A similar argument can be made with the dispersion in forecasting uncertainty. FIRE predicts individuals share an equal degree of uncertainty. In contrast, SE predicts that the uncertainty of individuals differs in that agents are not equally updated at a point of the time (Equation 8). NI generates a homogeneous degree of uncertainty only under the stringent conditions of an equal precision of signals and the same prior for uncertainty (Equation 25). DE predicts an equal degree of uncertainty across agents (Equation 30). Therefore, the presence of dispersion of uncertainty across agents is not consistent with predictions from FIRE and the canonical version of DE.

Figure 2 plots the median inflation expectation along with its 25/75 percentiles in the left and its counterpart in uncertainty in the right column. Not only there is long-lasting dispersion in individual forecasts, i.e. disagreement, but also notable heterogeneity in uncertainty across agents. And not surprisingly, the dispersion of both forecasts and the uncertainty of households are both of a much greater magnitude than that of the professionals. The 25/75 interquartile range (IQR) of households point forecasts is 4-5 percentage points, compared to 1 percentage point of professionals. And the IQR of the uncertainty of households is around 150-200 times(12-14 times in standard deviation terms) of that of professional forecasters.

Besides, in terms of the distribution of uncertainty, there are differences and similarities between households and professionals. Households' uncertainty is more skewed toward the right (higher uncertainty), meaning there is a wide dispersion in the high values of uncertainty. This can be also seen in Figure 3, where I plot the kernel density estimated distribution of uncertainty by year. [22].

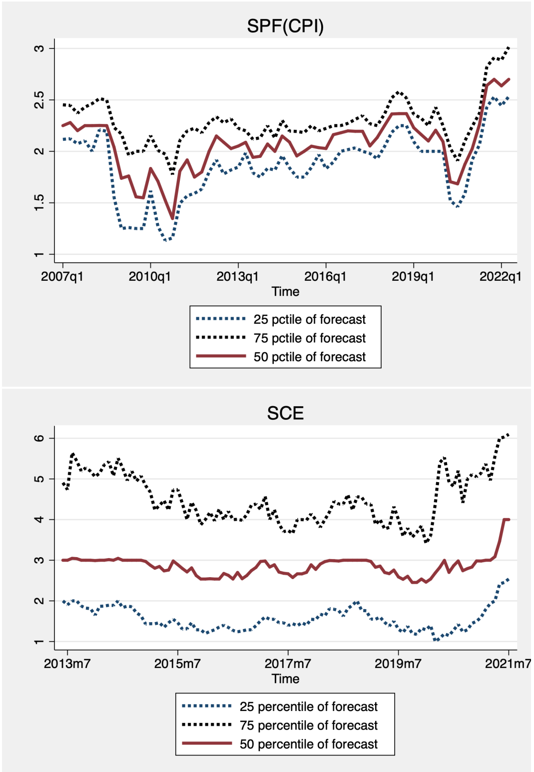
Another pattern worth discussing in Figure 2 is that there is a notable rise in the dispersion of professional forecasts in the recent 2-3 years, primarily driven by an increase of upper side of the forecast (i.e. 75 percentile forecast increases from 1% to

[21] This point was made very clearly by Zarnowitz and Lambros (1987). Manski (2018) also points out that much empirical work has confused the dispersion with uncertainty.

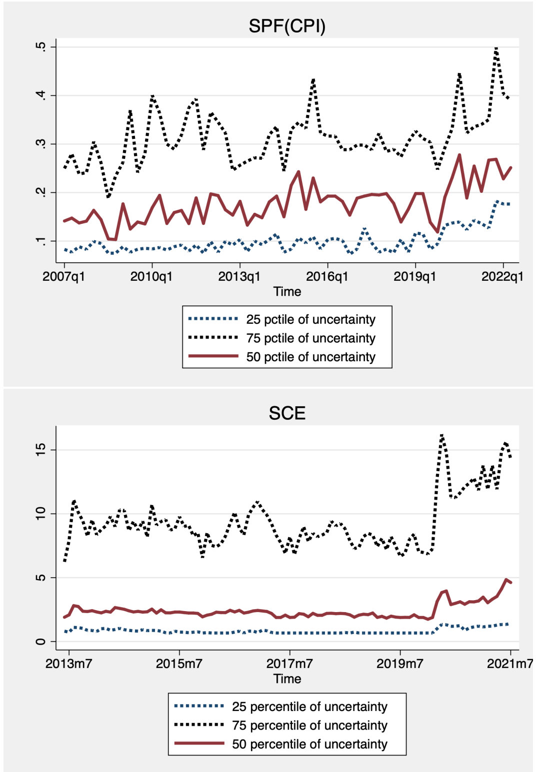
[22] Kumar et al. (2015) also presents the dispersion in uncertainty using a shorter-period of sample for SCE.

Figure 2: Dispersion of Mean Forecasts and Uncertainty

(a) Mean Forecasts



(b) Uncertainty



2%). [23] This is consistent with the observation in the top left two graphs of Figure 3 that the distribution of inflation forecasts in recent years have become flattened.

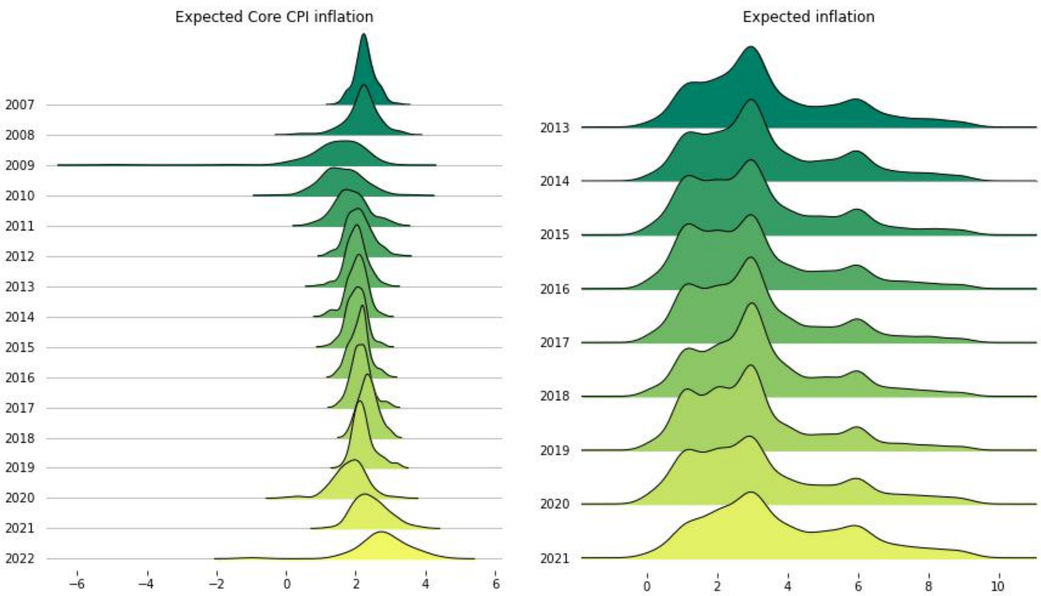
2.4.3 Revision in uncertainty

FIRE also predicts an unambiguous reduction in uncertainty as one approaches the date of realization, where the drop is exactly equal to the volatility of the realized shocks. Although quantitatively it is hard to check this, one can look if the distribution of the uncertainty revision concentrates in the negative range. Figure 4 plots the average revision in mean forecasts and uncertainty from 1-year-ahead forecast in year $t-1$ to the current-year nowcast in year t. The more negative range in which the revision lies, the more "rational" of the forecast. Looking from the histograms, uncertainty revision shows left-skewness relative to zero. This implies, on average, forecasters feel more certain for her nowcasts relative to her forecast made one year before. Unfortunately, since SCE does not provide the data structure for this purpose, I cannot make a comparison between two types of agents. A formal test of revision equal to zero or being negative will be carried out in Section 4.

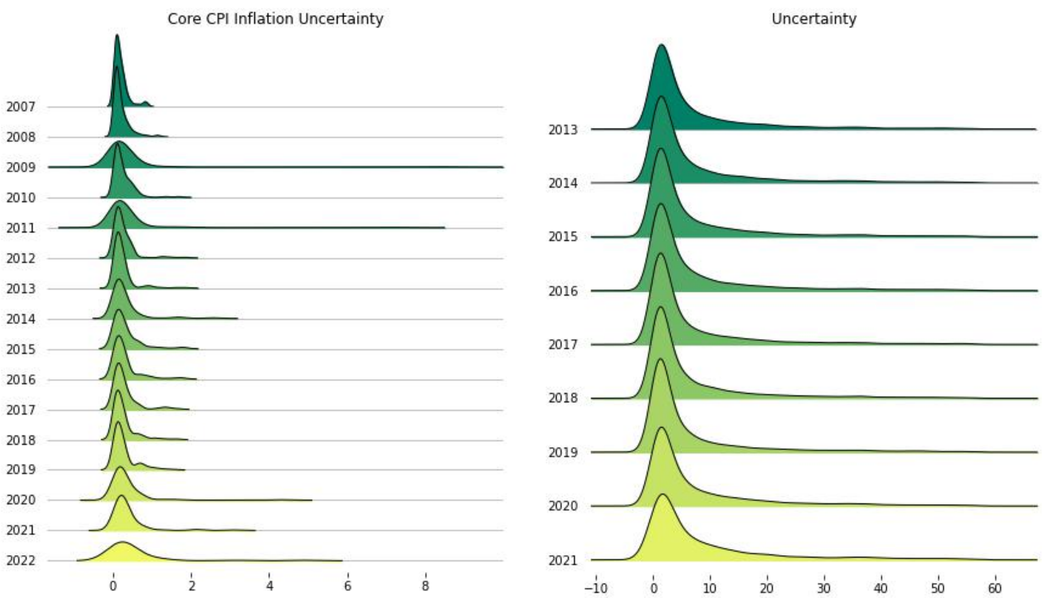
[23] This should be interpreted with caution, since the disagreements of SPF forecasts shown in Figure 1c actually exhibits a gradual decline.

Figure 3: Distribution of Mean Forecast and Uncertainty

(a) Mean Forecasts



(b) Uncertainty



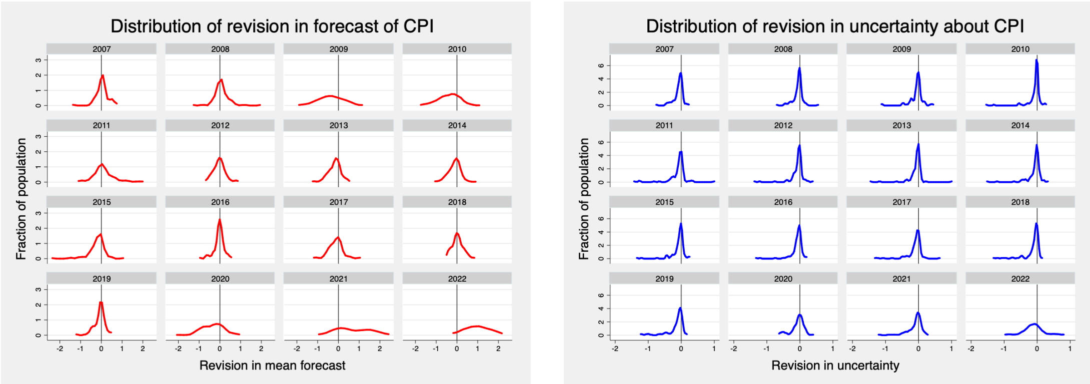
3 Theories of Expectation Formation

3.1 Sticky expectation (SE)

The theory of sticky expectation (Mankiw and Reis (2002), Carroll (2003) etc.), regardless of various micro-foundations, builds upon the assumption that agents do not update information instantaneously as they do in FIRE. One tractable assumption is that, by a homogenous and time-independent probability, λ, agents update their

Figure 4: Distribution of Revision in Forecasts and Uncertainty

(a) Revision in Mean Forecasts (b) Revision in Uncertainty



information up-to-date. Specifically, at any point of time t, each agent learns about the up-to-date realization of y_t with the probability of λ; otherwise, they form the expectation based on the most recent up-to-date realization of $y_{t-\tau}$, where τ is the time experienced since the last update.

Denote the mean forecast of a non-updater since $t-\tau$ as $y_{i,t+h|t-\tau}$ since her forecast conditions upon the information up till $t-\tau$.

$$y^{se}_{i,t+h|t-\tau} = \rho^{h+\tau} y_{t-\tau} \tag{7}$$

$$\text{Var}^{se}_{i,t+h|t-\tau} = \text{Var}^{*}_{t+h|t-\tau} = \sum_{s=1}^{h+\tau} \rho^{2(s-1)} \sigma^2_{\omega} \tag{8}$$

For non-updaters, the uncertainty is essentially dependent upon the time between the most recent update $t-\tau$ and the forecast date $t-h$, which is $\tau+h$. FIRE basically assumes $\tau = 0$ for all the agents and all the time, namely all agents' last update takes place in the previous period. So setting $\tau = 0$ in the above equation gives the uncertainty in FIRE common to everyone.

At the individual level, the key difference between FIRE and SE is that the latter does not reduce uncertainty as efficiently as in the former, primarily because of the rigidity of incorporating new information. But note that the rigidity in updating according to SE cannot be systematically observable at the individual level, both in terms of forecasts errors and uncertainty. This is because the behaviors of each individual forecast specifically depend on if she updates or not in that period.

Relying upon the law of large numbers, one can derive testable predictions about population moments that allow us to conduct tests of sticky expectation and recover rigidity parameter λ.

With the SE mechanism, the average forecast is a weighted average of update-to-date rational expectation and lagged average expectation, as reproduced below.[24] It

[24]See Coibion and Gorodnichenko (2012) for detailed steps.

can also be expressed as a weighted average of all the past realizations of y. Setting $\lambda = 1$, then the SE collapses to FIRE, and the average forecast is equal to y's long-run mean of zero.

$$\bar{y}^{se}_{t+h|t} = \lambda \underbrace{y^*_{t+h|t}}_{\text{rational expectation at t}} + (1-\lambda) \underbrace{\bar{y}^{se}_{t+h|t-1}}_{\text{average forecast at } t-1} \tag{9}$$

It follows that the average forecast errors are serially correlated, as described in Equation 10.

$$\overline{FE}^{se}_{t+h|t} = (1-\lambda)\rho\overline{FE}^{se}_{t+h-1|t-1} + \lambda FE^{*2}_{t+h|t} \tag{10}$$

The unconditional variance of the h-period-ahead forecast error is proportional to that of the FIRE model.

$$\overline{FE}^{se2}_{\bullet+h|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2} FE^{*2}_{\bullet+h|\bullet} \tag{11}$$

Regarding the uncertainty, average uncertainty at any point of time is now a weighted average of uncertainty to agents whose last updates have taken place in different periods of the past. Since at any point of the time, there are agents who have not updated the recent realization of the shocks, thus with higher uncertainty, the population uncertainty is unambiguously higher than the case of FIRE. (See Appendix 6 for detailed derivations.)

$$\begin{aligned} \overline{\mathrm{Var}}^{se}_{t+h|t} &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^\tau}_{\text{fraction of non-updater until } t-\tau} \underbrace{\mathrm{Var}^*_{t+h|t-\tau}}_{\text{uncertainty based on updating by } t-\tau} \\ &= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^\tau \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma^2_\omega \\ &= (\frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2} - 1)\frac{1}{\rho^2-1}\sigma^2_\omega \\ &\geq \overline{\mathrm{Var}}^*_{t+h|t} \end{aligned} \tag{12}$$

With respect to revision, the inefficiency of reducing uncertainty in SE takes the following form at the aggregate level. Since not all agents incorporate the recently realized shocks, the revision in average uncertainty exhibits a serial correlation, as described in Equation 13. It is a weighted average of the resolution of uncertainty from the most recent shocks and its lagged counterpart.

$$\overline{\mathrm{Var}}^{se}_{t+h|t+1} - \overline{\mathrm{Var}}^{se}_{t+h|t} = (1-\lambda)(\overline{\mathrm{Var}}^{se}_{t+h|t} - \overline{\mathrm{Var}}^{se}_{t+h|t-1}) - \lambda\rho^{2(h-1)}\sigma^2_\omega \tag{13}$$

In particular, the second component is the information gain from the most recent realization of the shock, underweighted by $\lambda < 1$. The first component is the inefficiency sourced from the stickiness of updating. The higher rigidity (lower λ), the smaller the efficiency gain or uncertainty reduction compared to in FIRE.

Lastly, SE also predicts non-zero disagreements and sluggish adjustment compared to FIRE. This is because of different lags in updating across populations.

$$\overline{Disg}^{se}_{t+h|t} = \lambda \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} (y_{t+h|t-\tau} - \bar{y}_{t+h|t})^2 \tag{14}$$

From time t to $t+1$, the change in disagreement comes from two sources. One is newly realized shock at time $t+1$. The other component is from people who did not update at time t and update at time $t+1$. [25]

3.1.1 Summary of predictions of SE

- Average forecast errors are zero across time, but have a positive serial correlation and have a higher variance across time than in FIRE.
- Population disagreement is positive on average across time.
- Population average uncertainty is higher than FIRE, and its revision under-reacts to the realized volatility of the shocks.

3.2 Noisy information (NI)

A class of models (Lucas (1972), Sims (2003), Woodford (2001), and Maćkowiak and Wiederholt (2009), etc), noisy information (NI hereafter), describes the expectation formation as a process of extracting or filtering a true variable y_t from a sequence of realized signals. The starting assumption is that the agent cannot observe the true variable perfectly. But unlike SE, NI assumes agents keep track of the realizations of signals instantaneously all the time.

I adopt the identical framework to Coibion and Gorodnichenko (2015) by assuming that an agent i observes two signals, s^{pb} being a public signal common to all agents, and s^{pr}_i, being a private signal specific to the agent i. The generating process of two signals is assumed to be the following.

$$\begin{aligned} s^{pb}_t &= y_t + \epsilon_t, \quad \epsilon_t \sim N(0,\sigma^2_\epsilon) \\ s^{pr}_{i,t} &= y_t + \xi_{i,t} \quad \xi_{i,t} \sim N(0,\sigma^2_\xi) \end{aligned} \tag{15}$$

Stacking the two signals into one vector $s_{i,t} = [s^{pb}_t, s^{pr}_{i,t}]'$ and $v_{i,t} = [\epsilon_t, \xi_{i,t}]'$, the equations above can be rewritten as

$$\begin{aligned} s_{i,t} &= Hy_t + v_{i,t} \\ &\text{where } H = [1,1]' \end{aligned} \tag{16}$$

[25] Coibion and Gorodnichenko (2012) derive the impulse response of disagreement at time $t+k$ to a shock that realized at t to be $\rho^{2(h+k)}(1-(1-\lambda)^{k+1})(1-\lambda)^{k+1}\omega^2_t$. Disagreement increases following the shock and gradually returns to its steady-state level.

Now any agent trying to forecast the future y has to form her expectation of the contemporaneous y. Denote it as $y^{ni}_{i,t|t}$, which needs to be inferred from the signals particular to the agent i. The agent's best h-period ahead forecast is simply iterated h periods forward based on the AR(1) process, and it is equal to $\rho^h y_{i,t|t}$. This is the same as FIRE.

What is different from FIRE is that the agent makes her best guess of y_t using Kalman filtering at the time t. Specifically, the mean forecast of an individual i is the posterior mean based on her prior and realized signals $s_{i,t}$.

$$\begin{aligned} y^{ni}_{i,t|t} &= \underbrace{y^{ni}_{i,t|t-1}}_{\text{prior}} + P \underbrace{(s_{i,t|t} - s_{i,t|t-1})}_{\text{innovations to signals}} \\ &= (1-PH)y^{ni}_{t|t-1} + PHy_t + Pv_{i,t} \end{aligned} \tag{17}$$

where the Kalman gain P is a vector of size of two that determines the degrees of reaction to signals.

$$P = [P_\epsilon, P_\xi] = \text{Var}^{ni}_{i,t|t-1} H (H' \text{Var}^{ni}_{i,t|t-1} H + \Sigma^v)^{-1} \tag{18}$$

$\text{Var}^{ni}_{i,t|t-1}$ is the forecast uncertainty of y_t based on prior beliefs up to $t-1$. And Σ^v is a 2-by-2 matrix indicating the noisiness of the two signals.

$$\Sigma^v = \begin{bmatrix} \sigma^2_\epsilon & 0 \\ 0 & \sigma^2_\xi \end{bmatrix} \tag{19}$$

Individual forecast partially responds to new signals as $PH < 1$. $PH = 1$ is a special case when both signals are perfect, thus $\Sigma^v = 0$, then the formula collapses to FIRE.

A comparable parameter with $1-\lambda$ in SE that governs rigidity in NI is $1-PH$. It is a function of previous period uncertainty, $\text{Var}_{t-1|t-1}$, and noisiness of the signals determined by Σ^v. Note that beyond steady state, P is time-varying as the variance is updated by the agent each period (as in Equation 23). With a constant volatility in AR(1), we can focus on the Kalman gain in steady state, corresponding to a constant variance. Therefore, I drop time t from P.

What differentiates average forecast from individual's is the role played by private signals. On average, private signals cancel out across agents, therefore, only public signals enter the average forecast, hence, average forecast errors (Equation 21).

$$\begin{aligned} \bar{y}^{ni}_{t+h|t} &= \rho^h[(1-PH) \underbrace{\bar{y}^{ni}_{t+h|t-1}}_{\text{Average prior}} + P \underbrace{\bar{s}_t}_{\text{Average signals}}] \\ &= (1-PH)\bar{y}^{ni}_{t+h|t-1} + P_\epsilon \epsilon_t \end{aligned} \tag{20}$$

The population average forecast error under NI takes a similarly recursive form as in SE.

$$\overline{FE}^{ni}_{t+h|t} = (1-PH)\rho\overline{FE}^{ni}_{t+h-1|t-1} + \rho^h P_\epsilon \epsilon_t + FE^*_{t+h|t} \tag{21}$$

The unconditional variance of the forecast errors is unambiguously greater than $FE^{*2}_{t+h|t}$ in FIRE, as shown in Equation 22.

$$\overline{FE}^{ni2}_{\bullet+1|\bullet} = \frac{\rho^{2h}P^2_\epsilon\sigma^2_\epsilon + FE^{*2}_{\bullet+h|\bullet}}{(PH)^2} \tag{22}$$

Kalman filtering also updates the variance recursively according to the rule of normal updating. The posterior variance at time t is a linear function of uncertainty in the previous period and variance of signals.

$$\text{Var}^{ni}_{i,t|t} = \text{Var}^{ni}_{i,t|t-1} - \text{Var}^{ni}_{i,t|t-1}H'(H\text{Var}^{ni}_{i,t|t-1}H' + \Sigma^v)^{-1}H\text{Var}^{ni}_{i,t|t-1} \tag{23}$$

The unconditional nowcasting variance can be solved as the steady-state value of the Equation 23. In the steady-state, there is no heterogeneity across agents in forecasting uncertainty and the nowcasting uncertainty becomes a constant. Thus, we can drop the subscript i. Note that the average uncertainty non-linearly depend on the noisiness of the two signals σ^2_ϵ and σ^2_ξ, as well as the volatility of inflation itself. The relationship is not monotonic, either.

Beyond steady-state, the Equation 23 also directly gives the revision in uncertainty from time $t-1$ to t. The newly arrived information, albeit noisy, still brings about information gains, thus leading to an unambiguous drop in uncertainty. But due to the signal is not perfect, i.e. $\Sigma^v \neq 0$, there is inefficiency in reducing uncertainty compared to in FIRE.

$$\text{Var}^{ni}_{i,t|t} - \text{Var}^{ni}_{i,t|t-1} = -\text{Var}^{ni}_{i,t|t-1}H'(H\text{Var}^{ni}_{i,t|t-1}H' + \Sigma^v)^{-1}H\text{Var}_{i,t|t-1} < 0 \tag{24}$$

The h-period-ahead forecasting uncertainty comes from both nowcasting uncertainty and volatility of unrealized shocks in the future. (Equation 25)

$$\text{Var}^{ni}_{i,t+h|t} = \rho^{2h}\text{Var}^{ni}_{i,t|t} + \sum_{s=1}^{h}\rho^{2(s-1)}\sigma^2_\omega \geq \text{Var}^*_{t+h|t} \tag{25}$$

As a result of NI mechanism, the revision in h-period-ahead uncertainty from $t-1$ to t only partially reacts to the resolution of uncertainty from newly realized shock ω_t in the past period.

NI also predicts non-zero disagreement in the presence of private signals. The size of the disagreement increases with the noisiness of the private signals.

$$\overline{Disg}^{ni}_{t+h|t} > 0 \tag{26}$$

3.2.1 Summary of predictions of NI

- Individual and population forecast errors are zero on average across time, but are positively serially correlated and have a higher variance than that in FIRE.
- The steady-state/unconditional uncertainty of both individual and population are higher than FIRE, and it depends on, but does not monotonically increase with the noisiness of private and public signals. Uncertainty revision is smaller than that in FIRE due to the underreaction to noisy signals.
- Population disagreement is positive on average across time, and it is a non-linear and non-monotonic function of the volatility of private signals σ^2_ξ.

3.3 Diagnostic expectations (DE)

Different from the previous two theories featuring informational rigidity, diagnostic expectation (Bordalo et al. (2018)) introduces an extrapolation mechanism in expectation formation that results in overreactions to the news (Bordalo et al. (2020)). Both SE and NI deviate from FIRE in terms of the information set available to the agents, while DE deviates from FIRE in terms of the processing of an otherwise fully updated information set.

Skipping over its micro foundation, the following equation captures the essence of DE's mechanism. Each individual i's h-period-ahead forecast consists of two components. The first component can be considered as a rational forecast based on the fully updated y_t. The second component corresponds to the potential overreaction to the unexpected surprises from $t-1$ to t. The degree of overreaction is governed by the parameter θ. The premise of DE models is that $\theta \geq 0$, capturing the fact that the agent overly responded to the realized forecast errors. The forecast collapses to the FIRE when $\theta = 0$.

$$\bar{y}^{de}_{i,t+h|t} = \rho^h y_t + \theta_i(\rho^h y_t - \bar{y}^{de}_{i,t+h|t-1}) \tag{27}$$

There is no room for disagreement with a homogenous degree of overreaction. To account for the possibility of a positive disagreement, I assume θ to be different across different agents, therefore, I add the subscript i to the parameter. Since agents are equally informed about the realizations of the variable, the only room for disagreement to be positive is heterogeneous degrees of overreaction. To capture this, I assume θ_i to follow a normal distribution across the population, $N(\hat{\theta}, \sigma^2_\theta)$. So the DE model has two parameters. Disagreement increases with the dispersion of overreaction, σ_θ.

The average forecast takes exactly the same form, with the individual-specific θ_i replaced by the population average $\hat{\theta}$. Therefore, we focus on average forecast errors directly, as written below. (See Appendix 6 for derivations)

$$\overline{FE}^{de}_{t+h|t} = FE^*_{t+h|t} + \hat{\theta}(FE^*_{t+h-1|t} - \rho\overline{FE}^{de}_{t+h-1|t-1}) \tag{28}$$

The formula contains the intuition behind the extrapolation mechanism in DE: moving from $t-1$ to t, the h-period-ahead forecast error exceeds that of FIRE forecast error by exactly the surprise to the expectation formed at $t-1$ with a degree of overreaction $\hat{\theta}$.

The unconditional variance of h-period-ahead forecast errors is equal to the following.

$$\overline{FE}^{de2}_{\bullet+h|\bullet} = \frac{(1+\hat{\theta})^2}{1+\hat{\theta}^2\rho^2} FE^{*2}_{\bullet+h-1|\bullet} + \frac{\sigma^2_\omega}{1+\hat{\theta}^2\rho^2} \tag{29}$$

Finally, as to the uncertainty, since the mechanism of extrapolation in DE does not change the agent's perceived distribution of the future shocks, benchmark DE theory predicts the forecast uncertainty to remain the same as in FIRE.

$$\overline{Var}^{de}_{t+h|t} = \overline{Var}^{*}_{t+h|t} \tag{30}$$

3.3.1 Summary of predictions of DE

- Average forecast errors are zero on average across time, but have **negative** serial correlation and higher variance than FIRE.
- The average uncertainty is equal to that in FIRE.
- Population disagreements are positive on average across time if there is heterogeneity in the degree of overreaction.

3.4 Diagnostic Expectation (DE) augmented with heterogeneous information (DENI)

Bordalo et al. (2020) embeds heterogeneous information in a standard DE model. Their motivation is primarily to generate cross-sectional disagreement in forecasts because the baseline version predicts zero-dispersion unless heterogeneity in the degree of overreaction is introduced in the first place, as we did in the previous section. The framework is essentially a hybrid of the NI and DE. It maintains the assumption regarding how agents overreact to new information at individual levels, but the information is no longer the real-time realization of the variable but a noisy signal of it. Since the mechanism is explicitly discussed in previous sections, I omit the detailed derivations here.

3.4.1 Summary of predictions of DENI

- Population forecast errors are zero on average across time. The sign of the auto-correlation depends on the relative size of the positive serial correlation induced by rigidity and the negative one induced by overreaction.
- Average uncertainty is now the same as NI due to the noisiness of the signal, and it increases with the noisiness of signals and volatility of inflation per se.
- Disagreement is positive on average due to the presence of private information. It also has positive auto-correlation and higher variance across time than FIRE.

3.5 An inflation process with stochastic volatility (SV)

This section considers an alternative data generating process in which the volatility of shocks to the inflation is stochastic, following the unobservable components/stochastic volatility (UCSV) model by Stock and Watson (2007). One of the widely acknowledged findings from the paper is that univariate inflation process is better described by a two-component model with time-varying volatility.

For the point of this paper, the UCSV assumption has two additional purposes. First, the time-varying pattern of observed forecast uncertainty as well as its comovement with other moments such as disagreement and the size of forecast errors, as seen in Figure 2.4, is hardly consistent with a simple inflation process with constant volatility. Second, from the point of view of understanding expectation formation, extending the inflation process to allow for stochastic volatility also serves as a sensitivity test of various theories of expectation formation with respect to the underlying process of the forecast variable.

In particular, UCSV assumes that inflation has an unobserved permanent and transitory component.

$$\begin{aligned} y_t &= \zeta_t + \eta_t, \quad \text{where } \eta_t = \sigma_{\eta,t}\xi_{\eta,t} \\ \zeta_t &= \zeta_{t-1} + \epsilon_t, \quad \text{where } \epsilon_t = \sigma_{\epsilon,t}\xi_{\epsilon,t} \\ \log \sigma^2_{\eta,t} &= \log \sigma^2_{\eta,t-1} + \mu_{\eta,t} \\ \log \sigma^2_{\epsilon,t} &= \log \sigma^2_{\epsilon,t-1} + \mu_{\epsilon,t} \end{aligned} \tag{31}$$

The shocks to levels of the two components and their volatility are drawn from the following distributions, respectively. The only parameter of the model is γ, which determines the smoothness of the time-varying volatility.

$$\begin{aligned} \xi_t &= [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I) \\ \mu_t &= [\mu_{\eta,t}, \mu_{\epsilon,t}]' \sim N(0, \gamma I) \end{aligned} \tag{32}$$

The information set necessary for forecasting is different in SV from in AR(1) process. Consider first the benchmark case of FIRE. At time t, the FIRE agent sees the most recent and past realization of all stochastic variables as of t, including y_t, ζ_t, η_t, $\sigma_{\eta,t}$, $\sigma_{\epsilon,t}$. Using $*sv$ stands for FIRE benchmark under the stochastic volatility assumption.

$$\overline{y}^{*sv}_{t+h|t} \equiv y^{*sv}_{t+h|i,t} = \zeta_t \tag{33}$$

Under FIRE, forecast error is simply the cumulative sum of unrealized permanent and transitory shocks from t to $t+h$, which is equal to

$$\overline{FE}^{*sv}_{t+h|t} = \sum_{s=1}^{h} (\eta_{t+s} + \epsilon_{t+s}) \tag{34}$$

Disagreement is zero across agents in FIRE. h-step-ahead conditional variance or the forecast uncertainty is time-varying as the volatility is stochastic now.

$$\begin{aligned} \overline{Var}^{*sv}_{t+h|t} \equiv Var^{*sv}_{t+h|i,t} &= \sum_{k=1}^{h} E_{i,t}(\sigma^2_{\eta,t+k}) + E_{i,t}(\sigma^2_{\epsilon,t+k}) \\ &= \sigma^2_{\eta,t} \sum_{k=1}^{h} exp^{-0.5k\gamma} + \sigma^2_{\epsilon,t} exp^{-0.5h\gamma} \end{aligned} \tag{35}$$

Under the sticky expectation (SE), an agent whose most recent up-to-date update happened in $t-\tau$ only have seen the realizations of y, ζ, η, σ_η, σ_ϵ till $t-\tau$. Her forecast is hence the permanent component at $t-\tau$.

$$y^{sesv}_{t+h|i,t-\tau} = \zeta_{t-\tau} \tag{36}$$

The lag in updating is also reflected in a higher forecast uncertainty than in FIRE.

$$Var^{sesv}_{t+h|i,t-\tau} = \sigma^2_{\eta,t-\tau} \sum_{k=1}^{h+s} exp^{-0.5k\gamma} + \sigma^2_{\epsilon,t-\tau} exp^{-0.5(h+\tau)\gamma} \tag{37}$$

The population average of the two are, respectively, a weighted average of people whose the most update was in $t, t-1...t-s, t-\infty$, respectively. The key difference in SV from AR(1) is that the average uncertainty exhibits positive serial correlation under SV. Expectations being sticky further increases the positive serial correlation compared to that in FIRE due to the lag in updating of the shocks to the volatility. The predictions regarding both forecast errors and disagreements under SV are the same.

Under noisy information (NI), the agent at time t needs to forms her best real-time nowcast for the permanent component ζ_t using noisy signals, $\bar{\zeta}_{t|t}$, via Kalman filtering. We assume again that the noisy signals of ζ_t consists of a public signal s^{pb}_t and a private signal $s^{pr}_{i,t}$ around the true realization of ζ_t.

$$y^{nisv}_{t+h|t} \equiv y^{nisv}_{t+h|i,t} = \bar{\zeta}_{t|t} \tag{38}$$

The prediction regarding forecast errors and disagreements remain the same as in AR(1). What is different under time-varying volatility is that there is no steady-state Kalman gain and uncertainty independent of time because the underlying volatility of the variable is time-varying. This also implies that the rigidity induced by the noisiness of information is state-dependent. At each period, the agents in the economy will update their forecast based on the realized volatility. In periods with high (low) fundamental volatility, the Kalman gain from noisy signals is larger (smaller) thus the agents will be more (less) responsive to the new information. There is no such state-dependence of rigidity in SE.

The mechanisms of DE and DENI exactly mimic that under AR(1) except that the average volatility is time varying now. Therefore, I leave the derivations in the online appendix.

3.6 Comparing theories

We summarize the distinctions across different theories in their moment conditions. The three theories all predict average population FEs across time to be zero and have higher variances than in FIRE. The distinction lies in that two rigidity models SE and NI imply a positive serial correlation while DE implies a negative serial correlation. DENI allows the two forces to counteract each other. Therefore, the sign of the serial correlation is ambiguous.

In terms of disagreement across agents, both SE and NI predict non-zero disagreement, positive serial correlation, and higher volatility. Non-zero disagreement can only arise in DE only if heterogeneity in overreaction is introduced. But this still leads to a zero serial correlation. The prediction of DENI is the same as that of NI.

Forecast uncertainty also contains useful distinctions in moment conditions across theories. In particular, although SE and NI both predict higher uncertainty than FIRE, the magnitudes of that depend on the size of structural parameters in the respective model. In contrast, baseline DE predicts equal uncertainty to FIRE. DENI has similar predictions as NI.

The general takeaway is that not only the first moment such as forecast error but also higher moments, disagreement and uncertainty contains restrictions to identify the model parameters within each theory. We will utilize these moment conditions to estimate each theory in Section 5.

4 Reduced-form Tests of FIRE

As a credible starting benchmark, I first reproduce a number of reduced-form statistical tests of FIRE only using information from forecast errors primarily following Mankiw et al. (2003), and the results are reported in Table 11 in Appendix 6. Consistent with the existing findings, the results reject the null hypothesis of unbiasedness in forecasts, non-serial correlation of non-overlapping forecast errors, and efficient use of information in forecating.

This section presents a number of new tests relying on uncertainty in Table 4 and 5, in the spirit of forecasting efficiency by Nordhaus (1987). It is an extension of revision tests on mean forecasts by Fuhrer (2018) to the forecast uncertainty.

Table 4 focuses on estimating forecasting efficiency using revisions of mean forecasts and uncertainty, hereafter referred to as revision-based tests. In plain words, the revision from 1-year-ahead forecast to nowcast of current-year inflation is efficient if the following two criteria are satisfied: (1) forecast revision does not depend on past information, including the past revisions; (2) the drop in uncertainty is sufficiently rapid to reflect the uncertainty of all realized shocks.

The mean revision test by Fuhrer (2018) takes the following form (using 1 period as an example):

$$y_{i,t+1|t+1} - y_{i,t+1|t} = \alpha + \beta(y_{i,t+1|t} - y_{i,t+1|t-1}) + \epsilon_{i,t+1} \tag{39}$$

In the above equation $\beta = 0$ according to FIRE, because rational forecast revision only responds to newly realized shocks, thus it is not predictable by past revisions.[26]. Since we have four vintages of the forecasts from SPF, the above specification can include lagged revisions up to 4 quarters.

The test with uncertainty simply replaces the revision of forecast with revision in uncertainty, as shown below.

$$\mathrm{var}_{i,t+1|t+1} - \mathrm{var}_{i,t+1|t} = \alpha^{\mathrm{var}} + \beta^{\mathrm{var}}(\mathrm{var}_{i,t+1|t} - \mathrm{var}_{i,t+1|t-1}) + \zeta_{i,t+1} \quad (40)$$

This regression follows from Equation 13 for SE and Equation 24 for NI. The autocorrelation coefficient β^{var} speaks to the speed of the drop in uncertainty, which takes zero value of zero under FIRE and one with perfect rigidity. Depending on the model, one can interpret it as the particular structural parameter of rigidity.

The top panel in Table 4 presents the results for the mean forecast. Following Fuhrer (2018), I include the median forecast available at time t and $t-1$ as an indicator of past information for the revision regression. In the first column of each panel, I report the regression on a constant.

The mean revision in forecast is mildly negative and significant for CPI forecast. The second to fourth columns of each panel in Table 4 checks autocorrelation of revisions, including different lags. Revisions of forecasts are serially correlated over 4 quarters, and the coefficients are all positive and significant. Also, the median forecasts as the past information always predict a negative revision with significant coefficients. This is evidence against the null hypothesis of FIRE and my estimates are comparable with those by Fuhrer (2018).

The bottom panel reports autoregression results for revision in uncertainty. Again, the first column first test the mean revision against the null being zero. For professional forecasters, the mean revisions in uncertainty are negative (0.25-0.3 percentage points equivalence in standard deviation of uncertainty) and statistically significant, confirming our observation from Figure 4 that forecasters are more certain about current inflation compared to her previous year forecast. However, for households at the population level, moving from 3-year-ahead inflation to 1-year-ahead inflation 2-years later, the drop in uncertainty is not significantly different from zero, suggesting inefficient forecasting and inconsistent with the FIRE.

The second to fourth column shows a positive serial correlation of revision in uncertainty for both CPI and PCE forecasts. The revision to CPI seems more efficient as serial correlation is with only one-quarter lag. For PCE, the revisions in uncertainty are serially correlated with all past three quarters.

Table 5 presents the results with the revision replaced with change in mean forecasts and uncertainty, i.e. from $y_{t|t-1}$ to $y_{t+1|t}$. As we have discussed in Section 3, the auto-correlation of change in mean forecast and uncertainty do not bear testable predictions from FIRE. But if the forecasts and uncertainty are persistent in its first difference, it may imply that the agent does not react to the news and newly realized shocks sufficiently. In addition, the auto-correlation regressions of this kind is a useful characterization of the time series dynamics of forecasts. With the variable being the

[26]Adding $y_{t+1|t}$ to both sides of Equation 39 gives an equivalent null hypothesis used by Fuhrer (2018): coefficient of regression of $y_{t+1|t+1}$ on $y_{t+1|t}$ is $1-\beta = 1$.

Table 4: Tests of Revision Efficiency Using Mean Revision and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 1. Revision efficiency of mean forecast													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_rv		0.482***	0.406***	0.400***		0.569***	0.473***	0.456***	L.InfExp_Mean_rv		0.986***	0.844***	0.861***
		(0.037)	(0.040)	(0.044)		(0.050)	(0.069)	(0.080)			(0.048)	(0.111)	(0.114)
L2.InfExp_Mean_rv			0.153***	0.121**			0.164**	0.133*	L2.InfExp_Mean_rv			0.130	0.119
			(0.036)	(0.039)			(0.051)	(0.058)				(0.115)	(0.125)
L3.InfExp_Mean_rv				0.054				0.080	L3.InfExp_Mean_rv			0.047	0.250
				(0.045)				(0.040)				(0.116)	(0.157)
SPFCPI_ct50		0.393**	0.370*	0.354*					L4.InfExp_Mean_rv				-0.195
		(0.120)	(0.142)	(0.149)									(0.156)
SPFPCE_ct50						0.380**	0.341*	0.340*	L5.InfExp_Mean_rv				0.091
						(0.120)	(0.134)	(0.143)					(0.141)
									L6.InfExp_Mean_rv				-0.155
													(0.114)
Const	-0.079*	-0.858**	-0.809**	-0.773*	-0.056	-0.732**	-0.654**	-0.644*	Const	-0.032	0.015	0.025	0.014
	(0.036)	(0.249)	(0.290)	(0.307)	(0.033)	(0.222)	(0.244)	(0.263)		(0.045)	(0.018)	(0.017)	(0.018)
N	1765	1501	1295	1136	1513	1275	1086	945	N	86	85	83	80
R2	0.000	0.281	0.302	0.296	0.000	0.373	0.381	0.375	R2	0.000	0.858	0.876	0.884
Test 2. Revision efficiency of uncertainty													
	Mean revision	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_rv		0.339***	0.166	0.185*		0.395***	0.352***	0.285***	L.InfExp_Var_rv		0.749***	0.770***	0.844***
		(0.074)	(0.093)	(0.089)		(0.057)	(0.066)	(0.070)			(0.074)	(0.151)	(0.167)
L2.InfExp_Var_rv			0.162*	0.217**			0.206***	0.114	L2.InfExp_Var_rv			0.014	-0.097
			(0.078)	(0.073)			(0.058)	(0.058)				(0.174)	(0.203)
L3.InfExp_Var_rv				0.231**				0.271***	L3.InfExp_Var_rv			-0.005	-0.218
				(0.079)				(0.069)				(0.123)	(0.175)
Constant	-0.087***	-0.053***	-0.053***	-0.031**	-0.076***	-0.048***	-0.035***	-0.026***	L4.InfExp_Var_rv				0.378*
	(0.008)	(0.007)	(0.007)	(0.009)	(0.006)	(0.006)	(0.006)	(0.005)					(0.153)
									L5.InfExp_Var_rv				-0.134
													(0.231)
									L6.InfExp_Var_rv				0.066
													(0.144)
									Const	0.020	-0.006	0.037	0.037
										(0.157)	(0.103)	(0.103)	(0.106)
N	1696	1435	1237	1082	1622	1370	1171	1013	N	86	85	83	80
R2	0.000	0.132	0.092	0.210	0.000	0.146	0.211	0.263	R2	0.000	0.567	0.602	0.639

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

first difference, the panel structure of SCE and SPF allows for calculating changes in individual levels for greater sample size, especially for households. Besides autoregression, I also report the constant estimate of the changes in the first column of each sub-panel.

The most noticeable pattern for both professionals and households, and for both mean forecast and uncertainty, is that past change predict future changes with universally negative coefficients across different horizons. Most of the negative coefficients are statistically significant in the 1% level. For instance, one percentage point of increase in SPF's CPI forecast in the previous quarter predicts around 0.29 percentage points decrease in the next quarter. This negative correlation is smaller in size but remain significant into further past, i.e. -0.24 for two quarters lag and -0.1 for three quarters lag. Data on SCE is monthly, so lags are included up to six months. The negative correlation between past and current changes are all negative and significant for households. The sizes of the correlation coefficients are comparable with professional forecasts when the monthly coefficients are converted to their quarterly average, i.e. -0.3 to -0.4.

Such an auto-correlation of change in mean forecast is very much reflected in the same regression for uncertainty, as reported in the bottom panel of Table 5. For SPF of CPI and PCE, respectively, one unit increase in uncertainty about 1-year-ahead inflation in the previous quarter predicts around a 0.39 and 0.44 unit of the drop in the next quarter. The effect holds up to two quarters for professionals and 5 months for households.

These evidence suggest that both the mean forecast and uncertainty of individuals are mean-reverting. An essentially equivalent explanation is that both series are realizations of noisy signals around their respective long-run mean. This will lead to the exact negative correlation of the first differences we have seen. The mean-reverting patterns might also explain what is observed from Figure 3, according to which, there are no significant changes in the distribution across different years.

The second noticeable result lies in the constant regressions reported in the first column of each sub-panel in Table 5. It implies that households constantly lower their mean forecasts as well as uncertainty from month to month, while professional forecasts do not behave in such a pattern. In particular, the constant regression of the change in the mean forecast for SCE gives an estimated coefficient of -0.05 which is significant in the 5% level. Individual households' 1-year-ahead inflation expectations keep being downward adjusted each month compared to their previous answer. What is more interesting is that their uncertainty about 1-year-ahead inflation also decreases each month. The size of the downward adjustment is -1.39 unit and statistically significant in the level of 0.1%. This negative significant and constant coefficient remains throughout all auto-regressions, implying it is not driven by time-varying changes.

The most natural explanation for this, that repeatedly surveyed households have become more informative about inflation over time. Given the unconditional forecast errors of inflation by households are positive, a downward adjustment of inflation stands for a less-biased forecast. [27]

[27]The possibility that the surveys' information set is influenced by the survey itself, or "learning through surveys"(Kim and Binder, 2020), is a double-edged sword. On one hand, this poses a

Table 5: Weak Tests of Revision Efficiency Using Change in Forecasts and Uncertainty

	SPF CPI				SPF PCE					SCE			
Test 3. Weak. Efficiency of change in forecast													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean revision	t-1	t-1- t-2	t-1-t-3		Mean revision	t-1	t-1- t-2	t-1-t-3
L.InfExp_Mean_ch		-0.249***	-0.308***	-0.311***		-0.289***	-0.324***	-0.324***	L.InfExp_Var_ch		-0.408***	-0.569***	-0.612***
		(0.062)	(0.054)	(0.060)		(0.060)	(0.080)	(0.089)			(0.005)	(0.008)	(0.012)
L2.InfExp_Mean_ch			-0.194**	-0.177**			-0.100	-0.100	L2.InfExp_Var_ch			-0.305***	-0.385***
			(0.064)	(0.060)			(0.081)	(0.096)				(0.007)	(0.012)
L3.InfExp_Mean_ch				-0.075*				0.014	L3.InfExp_Var_ch			-0.126***	-0.241***
				(0.036)				(0.064)				(0.007)	(0.012)
									L4.InfExp_Var_ch				-0.142***
													(0.012)
									L5.InfExp_Var_ch				-0.078***
													(0.011)
									L6.InfExp_Var_ch				-0.038***
													(0.008)
Constant	-0.019	-0.015	-0.019	-0.021	0.017	0.026	0.019	0.017	Constant	-0.034**	-0.018	0.002	0.020
	(0.019)	(0.019)	(0.019)	(0.020)	(0.021)	(0.024)	(0.026)	(0.028)		(0.013)	(0.014)	(0.017)	(0.019)
N	4286	3519	2971	2581	1791	1538	1338	1189	N	85166	67555	43489	20894
R2	0.000	0.072	0.103	0.089	0.000	0.077	0.088	0.088	R2	0.000	0.187	0.259	0.276
Test 4. Weak. Efficiency of change in uncertainty													
	Mean change	t-1	t-1- t-2	t-1-t-3	Mean change	t-1	t-1- t-2	t-1-t-3		Mean change	t-1	t-1- t-2	t-1-t-3
L.InfExp_Var_ch		-0.381***	-0.542***	-0.601***		-0.352***	-0.428***	-0.513***	L.InfExp_Var_ch		-0.396***	-0.572***	-0.653***
		(0.071)	(0.075)	(0.069)		(0.043)	(0.053)	(0.054)			(0.008)	(0.010)	(0.016)
L2.InfExp_Var_ch			-0.280***	-0.428***			-0.237***	-0.430***	L2.InfExp_Var_ch			-0.307***	-0.430***
			(0.059)	(0.061)			(0.044)	(0.044)				(0.011)	(0.018)
L3.InfExp_Var_ch				-0.293***				-0.382***	L3.InfExp_Var_ch			-0.124***	-0.269***
				(0.078)				(0.059)				(0.006)	(0.016)
									L4.InfExp_Var_ch				-0.155***
													(0.015)
									L5.InfExp_Var_ch				-0.079***
													(0.012)
									L6.InfExp_Var_ch				-0.029**
													(0.009)
Constant	-0.002	-0.001	0.005	0.006	0.003	0.004	0.006	0.008		-0.710***	-0.717***	-0.593***	-0.448***
	(0.005)	(0.006)	(0.005)	(0.005)	(0.004)	(0.005)	(0.005)	(0.004)		(0.072)	(0.064)	(0.072)	(0.076)
N	1685	1439	1251	1104	1629	1406	1225	1079		88052	69979	45003	21476
R2	0.000	0.130	0.240	0.286	0.000	0.125	0.161	0.286		0.000	0.187	0.271	0.311

Standard errors are clustered by date. *** p<0.001, ** p<0.01 and * p<0.05.

In summary, the major additional insights that arise from the empirical tests of this section is that rigidity of incorporating new information in forming expectations imply noticeable inefficiency of revisions in forecasts and drop in uncertainty.

5 Model Estimation and Sensitivity Analysis

5.1 SMM Estimation

The reduced-form tests in Section 4 are sufficient in rejecting the null hypothesis of FIRE. But there are two limitations with these tests in terms of identifying differences among non-FIRE theories. First, the coefficient estimates from the reduced-form regression cannot always be mapped into a structural parameter of the particular model. Second, even if it does so, the tests fall short of simultaneously utilizing all the restrictions across moments implied by a particular non-FIRE theory, as discussed in great detail in Section 3. In this section, I undertake a structural estimation that jointly accounts for cross-moment restrictions.

Since many of the moment conditions cannot be easily derived as a closed-form function of parameters, I adopt the simulated method of moment (SMM). In a nutshell, the estimation chooses the best set of model parameters by minimizing the weighted distances between the data moments and the model-simulated moments. For a given process of inflation, and a particular theory of expectation formation, the vector of the parameters estimates is defined as the minimizer of the following objective function.

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

where Ω^o stands for the parameters of the particular pair of theory of expectation and inflation process, i.e. $o \in \{se, ni, de, deni\} \times \{ar, sv\}$. Γ^o represents the corresponding parameter space respecting the model-specific restrictions. M_{data} is a vector of the unconditional moments that is computed from data on expectations and inflation. F^o is the simulated model moments under the theory pair o. W is the weighting matrix used for the SMM estimation. I report estimation results using the 2-step feasible SMM approach, in which the inverse of the variance-covariance matrix from the 1st-step estimation using identity matrix is used as the W in the second step, which has been shown to give asymptotically efficient estimates of the model parameters.

Crucially, notice that the model-implied moments F^o are not just a function of model parameters Ω^o, but also a function of the corresponding information set available to the forecasters. I use H to represent the historical realizations of the variables in the agents' information set that are used as the inputs for forecasts. Although the real-time history is the same across models, the mapping between the history to data moments depends on model specifics. For instance, although the real-time inflation

methodological challenge to the survey designers of expectations as to if surveys can objectively elicit the "true" expectations held by the respondents. On the other hand, researchers can use the survey as a meaningful intervention tool to identify the effect of factors such as information provision and attention. Recent examples of this type of research includes: Coibion et al. (2018) for firms and Coibion et al. (2019) for households.

is the only variable in the information set for AR(1) process, the information set in SV contains both the permanent component of the inflation and the realized levels of volatility. Since the different components are not directly observed from historical data, I estimate it using the Markov Chain Monte Carlo (MCMC) procedure developed by Stock and Watson (2007) in this context.

It is also important to mimic the information set that were truly available to the agents at each point of the time in history[28]. Therefore, I use the real-time data on historical inflation that was publicly available at each point of the time instead of the historical data released later, since it is well known among macroeconomists that the latter typically incorporates many rounds of revisions over time. I obtain the data from the Real-Time Data Research Center hosted by Philadelphia Fed[29].

The estimation is also specific to the choices of moments included in computing the distances. I focus on the unconditional moments of (independent of the time) at the population level, defined in Table 1. In particular, they include the mean (FE), variance($FEVar$) and auto-covariance($FEATV$) of population forecast error, the mean($Disg$), variance($DisgVar$) and auto-covariance($DisgATV$) of disagreement, and the mean (Var), variance($VarVar$) and auto-covariance($VarATV$) of uncertainty. When the joint estimation is done, two unconditional moments of the inflation are used, the variance($InfVar$) and auto-covariance($InfVar$). Table 2 reports the size of these moments computed for both SPF and SCE.

The model-implied moment conditions also implicitly depend on the parameters of the inflation process for a given model. This point is illustrated well in Bordalo et al. (2020). For instance, the observed overreaction in DE is lower for an AR(1) process with higher persistence. In recovering the model parameters associated with the expectation formation, it is important to take into account the information contained in expectation data regarding the process of inflation per se. To handle this, I undertake both 2-step and joint estimation. The former refers to the first externally estimating the inflation process and then estimate expectation formation separately treating the inflation parameter as given. The latter refers to jointly estimating parameters of inflation and expectation.

These alternative specifications of the estimation also serve as a model sensitivity analysis with respect to following criteria: (1) different choices of moments; (2) AR(1) and SV for the process of inflation. (3) two-step and jointly. (4) for both professionals and households. A reasonable theory of expectation formation ought to be relatively robust to these criteria.

5.2 Moments matching and parameter estimates

I first focus on the professionals' expectations as a benchmark, since professionals are supposed to be more rational. A first look at the data moments of SPF, as reported in Table 2, already provide clues as to the degree of the deviations from FIRE benchmark predictions laid out in Section 3. When separately estimated, the core CPI inflation during 1995-2022 followed an AR(1) process with the persistence

[28] For the importance of using real-time data to study survey forecasts, see Faust and Wright (2008), Faust and Wright (2009) and so on.

[29] https://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/.

parameter $\hat{\rho} = 0.97$ and volatility $\hat{\sigma} = 0.40$. The FIRE benchmark predicts zero forecast errors, zero disagreement, an uncertainty level identical to the volatility of inflation shocks, and zero time-variation and zero auto-correlation of these population forecasting moments. But in the data, quarterly professional forecasts of the core CPI inflation has a mildly positive average forecast errors 0.15 percentage points, a mild degree of disagreement of 0.2, and an uncertainty of 0.246, larger than the inflation volatility $0.16 \approx 0.4^2$ estimated from the same period. These are indeed deviations from FIRE predictions. But at the same time, it is worth noting that the variance of all forecast moments and their auto-covariance of between two consecutive years (non-overlapping horizons) are all close to zero, implying that the deviation is not severe. These, together, suggest that the most important source of identification for alternative theories to FIRE benchmark about expectation formation has to come from the persistent disagreement and additional inflation uncertainty.

Table 6 presents the SMM estimates of different model parameters for professionals. For each theory, I estimate the theory both in two steps or jointly using expectations and inflation data moments. Different rows within each panel reports the estimates depending on various choices of moments used for estimation: forecast errors only (FE), forecast error and disagreement (FE+Disg), and the two plus uncertainty (FE+Disg+Var).

5.2.1 Cross-moment consistency within a theory of expectation

As discussed in Section 3, each theory of expectation formation has distinctive predictions about the unconditional moments of forecast error, disagreement, and uncertainty. Cross-moment restrictions help identify the parameter values for the particular theory. Comparing the sensitivity of parameter estimation from utilizing information from different moments serves as an over-identification test.

All theories under consideration in this paper have parameter estimates that are broadly consistent within the theory no matter what moments are used, although additional moments do cause variation in the magnitudes of the model parameters. This pattern is the most salient in the estimation for professional forecasts in Table 6.

For SE, as the top panel of the table shows, the estimated annual updating rate is 0.34-0.35 across different combinations of moments, suggesting an average updating frequency of every four months. Such a medium degree of information rigidity seems to reflect the counteracting evidence both for and against information rigidity. On one hand, average forecast errors of a much higher variance (FEVar = 1.032) than the inflation volatility $0.4^2 \approx 0.16$, and a positive disagreement of 0.20 and higher uncertainty of 0.246 than the inflation volatility are all in line with the patterns of the information rigidity. On the other hand, a very limited degree of serial correlation in average forecast errors $(0, 006)$ and in disagreements$(0, 01)$ is inconsistent with the predictions of SE.

For NI, various cross-moment restrictions produce an estimated noisiness of the private signals σ_{pr} between $0.84 - 0.87$, while the estimated noisiness of public signals seems to be extremely large and directly hits the externally imposed upper bound of 3.0. These are highly noisy signals compared to the conditional standard deviation of inflation shocks 0.4, suggesting that incompleteness of information does play a role.

Table 6: SMM Estimates of Different Models: Professionals

SE									
Moments Used	2-Step Estimate			Joint Estimate					
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ			
FE	0.33	1	0.9	0.19	1	0.02			
FE+Disg	0.34	1	0.9	0.2	0.98	0.1			
FE+Disg+Var	0.34	1	0.9	0.2	0.98	0.1			
NI									
Moments Used	2-Step Estimate				Joint Estimate				
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	
FE	N/A	N/A	1	0.9	N/A	N/A	1	0	
FE+Disg	3	1.02	1	0.9	3	3	1	0.07	
FE+Disg+Var	2.38	3	1	0.9	2.99	2.99	1	0	
DE									
Moments Used	2-Step Estimate				Joint Estimate				
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ	
FE	0.9	N/A	1	0.9	1.62	N/A	0.9	0.38	
FE+Disg	1.6	0.81	1	0.9	1.6	0.81	0.9	0.38	
FE+Disg+Var	1.6	0.81	1	0.9	1.6	0.81	0.9	0.41	
DENI									
Moments Used	2-Step Estimate					Joint Estimate			
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	N/A	N/A	N/A	1	0.9	N/A	N/A	N/A	1	0.06
FE+Disg	1.44	3	2.19	1	0.9	1.22	3	2.8	1	0.08
FE+Disg+Var	1.04	3	3	1	0.9	1.24	3	3	0.99	0.1

Since forecast errors at the population level are due to only public noises and the private noises average out across agents, forecast-error moments only are not sufficient to identify both model parameters. Combining information from disagreement, one could separately identify the σ_{pb} and σ_{pr}. The non-zero disagreement helps pin down the sensible estimates σ_{pr} of 0.87. In addition, uncertainty also contains information about the noisiness of signals, although the relationship is non-linear. In particular, the steady-state uncertainty does not equally depend on the noisiness of both signals, but whichever is more precise, as NI agents endogenously select two signals, as shown in Equation 23. This is also the reason why observed uncertainty does not particularly help shrink σ_{pb} to more sensible estimates.

DE estimates demonstrate good within-model consistency across specifications. With only information from forecast error, the estimated overreaction parameter of DE θ is around 1.6 on average for all forecasters. At the same time, the dispersion cannot be separately identified only using information from forecast errors. Disagreement identifies the population dispersion in the degree of overreaction, since the heterogeneity in responsiveness to the news is the only source of disagreement in DE. This results in an estimated heterogeneity in the overreaction parameter σ_θ of 0.81 for a similar mean estimate of 1.6.[30] With additional moments from uncertainty, the estimates stay similar. This is not surprising because DE does not have identification information for the two model parameters.

Compared to the previous theories, DENI sees less cross-moment consistency for professional forecasts. With the non-identification with only forecast error moments, additional information from disagreement helps narrow parameters to a sensible range of values of DENI. As dispersion of private signals serves as the source of the positive disagreement, the overreaction parameter could "focus on" matching the forecast errors and their variation. This reveals a positive overreaction parameter θ around 0.05 while a noisiness of both public and private nosiness of signals of 1.5 percentage points. This is in line with the original premise of the DENI model as in Bordalo et al. (2020), which reconciles both overreactions to news and a positive disagreement. However, incorporating information from uncertainty again leads to a negative overreaction parameter estimate $\theta = -1.87$. Uncertainty revealed from professional forecasts leads to a lower estimate of the noisiness of both signals to 0.19 and 0.73 respectively, which further implies higher responsiveness to the news. Therefore, the behaviors of forecast errors need to be again matched by some level of underreaction to the news instead of overreaction.

5.2.2 Interactions between expectation formation and inflation dynamics

The specific mechanisms of expectation formation could also interact with the underlying process of inflation. With the benchmark AR(1) process, both the persistence of the shock to inflation ρ and the overall volatility of the inflation shock σ determine what the FIRE forecasts moments should be and model-specific forecasting moments are not only a function of the model parameters but also the inflation process parameters itself. The differences between 2-step estimation and joint estimation reveal such inter-dependence. At the same time, in order to avoid inflation parameters to be overfitting expectation moments, for the joint estimation, I impose a loose lower bound for the persistence parameter ρ of 0.9.

For SE, letting professional forecasts reveal information about the inflation process leads to a less persistent and less volatile AR(1) process. The joint estimate of the persistence parameter ρ hit the externally set lower bound 0.9 and lower inflation volatility of $0.13 - 0.16$, compared to 0.99 and 0.23 when estimated separately. The joint estimation with disagreement or uncertainty under such a less volatile process generates an update rate estimate of $0.62 - 0.64$, implying a degree of the rigidity half of that from a separate estimation. This suggests that the underlying information rigidity could have been lower if the inflation process is less persistent and less volatile. This is consistent with the underlying mechanisms of SE. Think about an extreme case where shocks to inflation are purely transitory, i.e. $\rho = 0$, the infrequent updating of the realized shocks will not lead to any differences in forecasts and the uncertainty will be exactly equal to the inflation uncertainty. It means that less rigidity

[30] Estimates using only inflation expectation sample before 2020, however, yields a negative estimates of overreaction, which is consistent with the model assumption, as seen in Table 12 of Appendix. These findings are consistent with the rigidity/underreaction seen in the population level (Coibion et al. (2018)) in contrast to the overreaction seen at the individual level (Bordalo et al. (2018)).

is required in the expectation formation to justify the observed level of disagreement and uncertainty. Another way of interpreting such changes is via the question of how expectation moments shed light upon the inflation process itself. Since both disagreement and uncertainty increase with the inflation volatility σ, such change in estimates seem to suggest that the low disagreement and uncertainty as shown in professionals' forecasts only need to be justified by lower inflation volatility.

NI shows more sensitivity toward the estimation procedure. Although across different moments, the joint estimation reveals exactly the same estimate of the persistence parameter $\rho = 0.99$, the revealed inflation volatility and noisiness of signals all see sizable changes. For instance, the inflation volatility σ is estimated to be zero in joint estimation and the nosiness of signals becomes smaller with FE and Disg moments used and bigger when information from uncertainty is also used. This reflects some horserace between the noisiness of the signals and the volatility of inflation itself. This is not surprising because, in NI models, both shocks to inflation itself or to the signals contribute to forecast errors and forecast uncertainty. Although it is ex-ante clearly distinguishable from the point of view of the modeler, the distinction between a shock to inflation itself and simply to signals may be indistinguishable for the agents who try to form the forecasts. These results suggest that the data-implied information rigidity according to NI could be potentially sensitive to the assumed volatility to inflation itself.

In terms of the sensitivity against the estimation procedure, DE estimates perform rather well, although the results reveal underreaction instead of overreaction as shown in the separate estimation. Letting expectation data under DE speak to the inflation process leads to a lower persistence $(0.9 - 0.92)$ and lower volatility $(0.12 - 0.15)$ of inflation, but the average overreaction parameter θ remain in the range of -0.36 to -0.48 and its cross-sectional dispersion σ_θ stays in the range of $0.39 - 0.56$. Across moments conditions, most of the forecasters underreact to the news.

DENI performs the worst in terms of its sensitivity against the estimation procedure, implying it is very dependent on the persistence and volatility of the inflation process. Taking the most likely credible estimate that utilizes information from all moments, a mild degree of underreaction $\theta = -0.11$, very precise public signals, modestly noisy private signals, a less persistent inflation process $\rho = 0.9$ and equal degree of volatility $\sigma = 0.23$ fit the joint dynamics of inflation and professional forecasts the best.

5.2.3 Professionals versus households

In contrast with professionals, raw household forecasts see a more substantial deviation from FIRE in every dimension. During the sample period of 2013-2020, the monthly headline CPI inflation is estimated to have a persistence parameter of 0.98 and volatility of 0.41[31]. The household's forecasts had an average forecast error of 1.75, a disagreement of 6.15, and an average uncertainty of 9.70. It is obvious that these deviations are difficult to be directly accounted for by the theories under consideration because of their sheer sizes. In order for the theory to fit better with data, I

[31]The higher volatility compared to that of core CPI is due to a higher frequency and inclusion of more volatile items in the CPI basket.

first control for the individual fixed effects on individual forecasts, forecast errors, and uncertainty before computing the population average of these moments. This follows from the emerging evidence[32] that individual-specific effects such as demographics and experience play important roles in driving systematic differences in expectations. After controlling for this ex-ante heterogeneity in beliefs, households' expectations exhibit similar yet still more distorted expectations than that of professionals. It has an average forecast error close to zero, but a substantially larger disagreement of 2.48, and a slightly higher uncertainty of 1.72 than professionals.

Table 7: SMM Estimates of Different Models: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.83	1.85	0.21	0.9	0.7				
FE+Disg	0.35	0.83	1.85	0.2	0.98	0.1				
FE+Disg+Var	0.35	0.83	1.85	0.39	0.9	0.77				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	N/A	N/A	0.83	1.85	N/A	N/A	0.9	0.7		
FE+Disg	0	2.16	0.83	1.85	0	1.64	0.9	0.7		
FE+Disg+Var	3	3	0.83	1.85	3	3	0.9	0.77		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_{θ}	ρ	σ	$\hat{\theta}$	σ_{θ}	ρ	σ		
FE	1.1	N/A	0.83	1.85	1.05	N/A	0.9	0.7		
FE+Disg	1.04	1.01	0.83	1.85	1.04	1.01	0.9	0.7		
FE+Disg+Var	1.04	1.01	0.83	1.85	0.94	0.96	0.9	0.7		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	N/A	N/A	N/A	0.83	1.85	N/A	N/A	N/A	0.9	0.66
FE+Disg	N/A	N/A	N/A	0.83	1.85	0.61	0.42	0.16	0.93	0.56
FE+Disg+Var	N/A	N/A	N/A	0.83	1.85	0.58	0.56	0.45	0.91	0.11

Table 7 presents the estimates for households after excluding the fixed effects on forecasts, forecast errors, and uncertainty. Overall, the estimates are surprisingly similar to that for professionals and within each model, in some sense, exhibits even more model consistency across moment conditions and estimation procedures. Most noticeably, the updating rate in SE falls into a very small range of values of $0.31 - 0.35$, equivalent to an updating every four months. This is extremely close to our estimates for professionals. It is well documented in the literature that households are more irrational than professionals. But the results of our estimates show that the major

[32] Malmendier and Nagel (2015), Das et al. (2017), D'Acunto et al. (2019) and so forth.

differences are not least mostly due to the differences in updating rate in information. The second intuitive pattern revealed from the households estimates is that, unlike professionals, the NI estimates reveal a much larger noisiness of public signals (hitting the upper bound 3) than that of private signals ($0.45 - 0.65$) across all moments, and according to both 2-step and join estimations. This is very consistent with the fact that households have been found to pay very little attention to commonly accessible macroeconomic news than to the individual-specific information that is immediate to them.

Similar to that for professionals, the canonical DE model-based estimates suggest an over-reaction parameter values between -2(lower bound) to -0.99 on average and a larger degree of heterogeneity across populations with a standard deviation between 3.88 to 5.

As to DENI, it shows more model consistency for households. Across specifications, the estimated overreaction parameter is negative between -0.93 to -0.3. At the same time, the nosiness of both noisy signals is very similar to that of NI model estimates. It all suggests that households not only underreact to news in general, but also face highly noisy public signals and more precise private signals.

5.2.4 Alternative inflation process with stochastic volatility

Table 8 and 9 show the estimation of the model-specific parameters allowing the alternative inflation process featuring stochastic volatility in two separate unobserved components of different persistence. Compared to the benchmark AR(1) process, there are two crucial implications for expectation formation. The first is that now the SV model admits time-varying volatility, which has more potential to be consistent with the time-varying pattern of the forecast uncertainty, primarily in the expectations of the household. The second is that given the permanent component is a random walk, the shock to the permanent component is permanent instead of persistent. We examine if the alternative inflation process accommodates similar estimates of the model-specific parameters.

Among all theories, SE gives the closest parameter estimates to that of the benchmark. Surprisingly enough, admitting stochastic volatility reveals an almost identical information rigidity in SE, i.e. an annual updating rate of $0.34 - 0.35$ for both households and professionals. This is not a mechanical coincidence, since the SVSE model assumes agents do not only infrequently update realized shocks but also the shocks to the volatility, as explicitly discussed in the section 3.5. Therefore, the dynamics of uncertainty seen from data do provide useful additional information now to identify information rigidity than the benchmark model. This suggests that SE has a very good consistency against the assumed inflation process. Such information rigidity is also indirectly confirmed by the revealed underreaction according to DESV estimates. On average, both households and professionals underreact instead of overreacting to inflation news, even under the stochastic volatility model of inflation. More specifically, professionals show slightly more underreaction with SV than the benchmark estimates, and households show less.

The estimates for NI and DENI augmented with *SV* are a lot different from that of benchmark estimates. Since SV effectively allows the inflation to be more volatile and the shock to be more persistent, the estimated nosiness of both public and private signals for professionals increase in the sizes compared to the $AR(1)$ so much that they both hit the externally imposed upper bound of 3. But for households, the sizes of the public signals and privates signals flipped the relationship, i.e. the nosiness of public signals becomes 0 while privates signals are much noisier. This suggests that both NI and DENI have poor sensitivity toward the alternative inflation processes.

Table 8: SMM Estimates of Different Models under Stochastic Volatility: Professionals

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.34		
FE+Disg+Var	0.33		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	N/A	N/A	
FE+Disg	0.48	3	
FE+Disg+Var	0.48	3	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-1.97	N/A	
FE+Disg	-1.72	1.38	
FE+Disg+Var	-1.72	1.38	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	N/A	N/A	N/A
FE+Disg	2.78	0.26	1.32
FE+Disg+Var	2.13	3	3

5.3 The scoring card of different theories

Table 10 reports my evaluation of the four theories under consideration based on four sensitivity criteria laid out in the previous section. According to this evaluation, the SE seems to capture the average behavior of expectations better than the other two theories. DE shows a similar degree of performance according to these criteria, but the average estimation does not show overreaction in line with the premise of the theory, instead, underreaction, consistent with the existence of the information rigidity.

In comparison, NI and DENI perform less well in terms of these four criteria, except for the sensitivity against the moment conditions used for estimation. It is worth giving some diagnoses of the crucial challenges of the two theories. The central challenge

Table 9: SMM Estimates of Different Models under Stochastic Volatility: Households

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	N/A	N/A	
FE+Disg	0.1	2.99	
FE+Disg+Var	0.1	2.99	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_θ	
FE	0.85	N/A	
FE+Disg	0.85	1.04	
FE+Disg+Var	0.85	1.04	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	N/A	N/A	N/A
FE+Disg	2.35	0.17	3
FE+Disg+Var	0.77	0.15	0.41

that makes NI unable to explain the data well is that the level of information rigidity consistent with NI formulation requires too big sizes of the nosiness of the signals. And it is also very sensitive to the assumed level of inflation volatility and the degree of its persistence. Such a problem also exists in the DENI model, which in theory could reconcile the observed coexistence of information rigidity and overreaction better, as argued in Bordalo et al. (2020).

Table 10: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation ?	No	No	No	No
Sensitive to assumed inflation process?	No	Yes	Yes	Yes
Sensitive to two-step or joint estimate?	No	Yes	No	Yes
Sensitive to the type of agents?	No	Yes	No	Yes

6 Conclusion

Most of the studies on expectation formation documenting how it deviates from the FIRE benchmark have focused on the first moment, i.e. the mean forecasts and the cross-sectional dispersion of the forecasts. This paper shows that the surveyed forecasting uncertainty by professionals and households provides useful information to understanding the exact mechanisms of expectation formation. It not only provides additional reduced-form testing results of rejecting FIRE, such as persistent disagreements in forecasting uncertainties and its inefficient revisions, but also provides additional moment restrictions to any particular model of expectation formation, which helps identify the differences across theories.

At least two lines of questions remain unresolved in this paper and require future research. First, throughout the analysis, we maintain the normality assumptions of the shocks and ignore the beliefs in tail events or even higher moments. It is a natural extension of this paper to explore how different theories of expectation formation may contain different predictions on tail beliefs. Second, although this paper focuses only on macroeconomic expectations regarding inflation, it is worth asking if the belief formation regarding individual variables such as income bear similar mechanisms.

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Appendix

Detailed derivation

SE

$$
\begin{aligned}
\bar{V}ar_t(y_{t+h}) &= \sum_{\tau=0}^{+\infty} \underbrace{\lambda(1-\lambda)^{\tau}}_{\text{fraction who does not update until } t-\tau} \underbrace{Var_{t|t-\tau}(y_{t+h})}_{\text{Variance of most recent update at } t-\tau} \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \sum_{s=1}^{h+\tau} \rho^{2(s-1)}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} (1+\rho^2+...+\rho^{2(h+\tau-1)})\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{\rho^{2(h+\tau)}-1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{\rho^{2(h+\tau)}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \rho^{2\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \sum_{\tau=0}^{+\infty} \lambda((1-\lambda)\rho^2)^{\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 - \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \frac{\lambda}{(1-\rho^2+\lambda\rho^2)} \sum_{\tau=0}^{+\infty} (1-\rho^2+\lambda\rho^2)(1-(1-\rho^2+\lambda\rho^2))^{\tau} \frac{\rho^{2h}}{\rho^2-1}\sigma_{\omega}^2 \\
&- \sum_{\tau=0}^{+\infty} \lambda(1-\lambda)^{\tau} \frac{1}{\rho^2-1}\sigma_{\omega}^2 \\
&= \left(\frac{\lambda\rho^{2h}}{(1-\rho^2+\lambda\rho^2)(\rho^2-1)} - \frac{1}{\rho^2-1}\right)\sigma_{\omega}^2 \\
&= \left(\frac{\lambda\rho^{2h}}{1-\rho^2+\lambda\rho^2} - 1\right)\frac{\sigma_{\omega}^2}{\rho^2-1} \\
&= \left(\frac{\lambda\rho^{2h}-1+\rho^2-\lambda\rho^2}{1-\rho^2+\lambda\rho^2}\right)\frac{\sigma_{\omega}^2}{\rho^2-1}
\end{aligned} \tag{41}
$$

NI

The steady-state nowcasting uncertainty Var_{ss}^{ni} is solved using the updating equation (Equation 23).

$$
\begin{aligned}
&\mathrm{Var}^{ni}_{t|t} = \mathrm{Var}^{ni}_{t|t-1} - \mathrm{Var}^{ni}_{t|t-1} H'(H\mathrm{Var}^{ni}_{t|t-1}H' + \Sigma^v)^{-1} H\mathrm{Var}^{ni}_{t|t-1} \\
&\rightarrow \mathrm{Var}^{ni}_{t|t} = \rho^2(\mathrm{Var}^{ni}_{t-1|t-1} + \sigma^2) \\
&- \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2)H'(H\rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2)H' + \Sigma^v)^{-1} H\mathrm{Var}^{ni}_{ss} \\
&\rightarrow \mathrm{Var}^{ni}_{ss} = \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_\omega) \\
&- \rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_\omega)H'(H\rho^2(\mathrm{Var}^{ni}_{ss} + \sigma^2_\omega)H' + \Sigma^v)^{-1} H\mathrm{Var}^{ni}_{ss}
\end{aligned}
\tag{42}
$$

DE

$$
\begin{aligned}
FE^{de}_{i,t+h|t} &= y^{de}_{i,t+h|t} - y_{t+h} \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y^{de}_{i,t+h|t-1}) \\
&= \rho^h y_t - y_{t+h} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= FE^*_{t+h|t} + \theta_i(\rho^h y_t - y_{t+h} - FE^{de}_{i,t+h|t-1}) \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i FE^{de}_{i,t+h|t-1} \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i(\rho FE^{de}_{i,t+h-1|t-1} - \omega_{t+h}) \\
&= (1+\theta_i)FE^*_{t+h|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} + \theta_i\omega_{t+h} \\
&= (1+\theta_i)FE^*_{t+h|t} + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^*_{t+h-1|t} + (1+\theta_i)(-\omega_{t+h}) + \theta_i(\omega_{t+h} - \rho FE^{de}_{i,t+h-1|t-1}) \\
&= (1+\theta_i)FE^*_{t+h-1|t} - \omega_{t+h} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^*_{t+h|t} + \theta_i FE^*_{t+h-1|t} - \theta_i\rho FE^{de}_{i,t+h-1|t-1} \\
&= FE^*_{t+h|t} + \theta_i(FE^*_{t+h-1|t} - \rho FE^{de}_{i,t+h-1|t-1})
\end{aligned}
\tag{43}
$$

Reduced-form tests with forecast errors

The FE-based null-hypothesis of FIRE utilize the moment restrictions on forecast errors. In plain words, the null hypotheses of the three tests are the following. First, since the forecasts are on average unbiased according to FIRE, forecast errors across agents should converge to zero in a large sample. Second, forecast errors of non-overlapping forecasting horizon are not serially correlated. Third, forecast errors cannot be predicted by any information available at the time of the forecast, including the mean forecast itself and other variables that are in the agent's information set. This follows from Equation 2. In addition, I include what is called a weak version of the FE-based test which explores the serial correlation of forecast errors in overlapping periods, i.e. 1-year-ahead forecasts within one year. The forecast errors are correlated to the extent of the realized shocks in the overlapping periods. So the positive serial correlation does not directly violate FIRE. But the correlation of overlapping forecast errors still contains useful information about the size of the realized shocks.

Individual-level data are used whenever possible, utilizing the panel structure of both surveys. Since test 2 and 3 requires individual forecasts in vintages that are more than one year apart while SCE only surveys each household for 12 months, the two tests are done with the population average expectations for SCE. Also, the regressions are adjusted accordingly depending on the quarterly and monthly frequency of SPF and SCE. Since these regressions are based on 1-year inflation in overlapping periods, Newy-West standard error is computed for hypothesis testing.

First, all three forecast series easily reject the null hypothesis of unbiasedness at the significance level of 0.1%. There are upward biases in both professional forecasts of core PCE inflation and households' forecast of headline inflation [33], while at the same time professionals underpredicted core CPI inflation over the entire sample period. This was primarily driven by the under-prediction of the inflation over the recent two years since the Pandemic.

Second, the average point forecast one year ago predicts the forecast errors of both groups at the significance level of 0.1%. For headline CPI inflation, for instance, one percentage point inflation forecast corresponds to 0.35 percentage points of the forecast errors one year later. Thus, test 2 in Table 11 easily rejects the second hypothesis test of FIRE that past information does not predict future forecast errors. This suggests that both types of agents inefficiently utilize all information when making the forecasts.

Third, forecast errors are positively correlated with the forecast errors one year ago, with a significant coefficient ranging from 0.35 to 0.572. A higher positive auto-correlation coefficient of forecast errors by households is consistent with the common finding that households are subject to more information rigidity than attentive professionals.

Lastly, test 4 in Table 11 presents a higher serial correlation of forecast errors produced within a year. For SPF forecasts, the serial correlation does not exist beyond 2 quarters, implying the relative efficiency of professional forecasts. For the households, the forecast errors are more persistent over the entire year, in that current forecast errors are correlated with all past forecast errors over the past three quarters. Although the persistence of 1-year forecast errors within one year does not directly violate FIRE, the fact that households' forecast errors are more persistent than professionals' indicates that the former group is subject to a higher degree of rigidity than the latter one.

Estimates of stochastic volatility model of inflation

Figure 5 plots the estimated stochastic volatility of permanent and transitory component of inflation, respectively, as specified in Equation 31, using the same estimation method of Stock and Watson (2007).

Structural estimation using pre-Pandemic sample

Table 13, 12 report the estimates for households and professionals, respectively, using only sample before June 2020 for AR(1) inflation process. Table 15, 14 report the estimates based on the alternative UCSV model of inflation process.

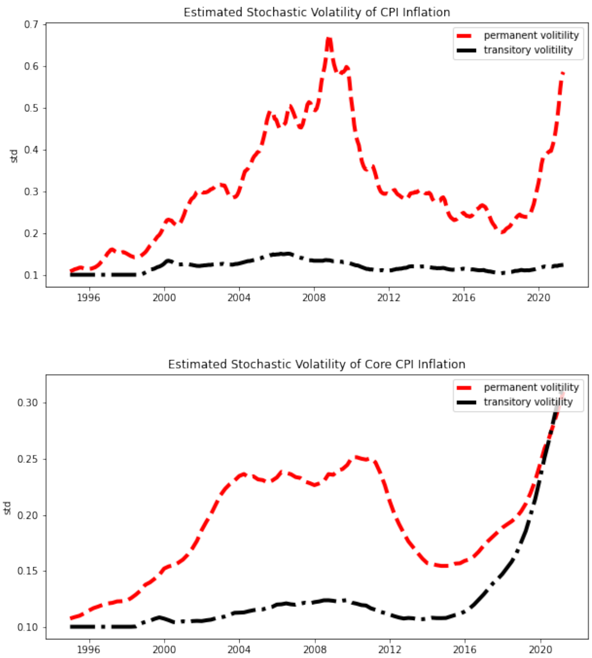
[33] Coibion et al. (2018) finds the same upward bias for firms' managers.

Table 11: Tests of Rationality and Efficiency Using Forecast Errors

	SPF CPI	SPF PCE	SCE
Test 1: Bias			
Constant	-3.021***	0.460***	1.673***
	(0.242)	(0.047)	(0.008)
N	5510	1610	112668
Test2: FE Depends on past information			
Forecast 1-yr before	0.350***	0.460***	4.190***
	(0.035)	(0.047)	(0.659)
Constant	-3.452***	-2.333***	-12.92***
	(0.386)	(0.192)	(2.213)
N	3945	1610	84
Rˆ2	0.828	0.826	0.311
Test3: FE of non-overllaping forecast horizons are serially correlated			
Forecast Error 1-year before	0.350***	0.460***	0.572**
	(0.035)	(0.047)	(0.195)
Constant	0.314	-1.351***	-0.149
	(0.231)	(0.156)	(0.445)
N	3945	1610	84
Rˆ2	0.828	0.826	0.0957
Time FE	Yes	Yes	No
Test4: Overlapping FE are serially correlated			
Forecast Error 1-q before	0.502***	0.551***	0.327***
	(0.060)	(0.075)	(0.010)
Forecast Error 2-q before	0.0901	0.231***	0.341***
	(0.064)	(0.060)	(0.024)
Forecast Error 3-q before	0.146*	0.0693	0.333***
	(0.065)	(0.052)	(0.023)
Constant	1.147***	-0.356***	0.509***
	(0.224)	(0.058)	(0.035)
N	2971	1338	4432
Rˆ2	0.890	0.903	0.243

Note: white standard errors reported in the parentheses of estimations. *** p<0.001, ** p<0.01 and * p<0.05.

Figure 5: Stochastic Volatility of Inflation



Note: this figure plots the estimated stochastic volatility of permanent and transitory components of monthly headline CPI (top) and quarterly core CPI inflation (bottom) using the same approach as in Stock and Watson (2007).

Table 12: SMM Estimates of Different Models: Professionals

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.36	0.99	0.23	0.37	0.9	0.13				
FE+Disg	0.35	0.99	0.23	0.64	0.9	0.13				
FE+Disg+Var	0.34	0.99	0.23	0.62	0.9	0.16				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	NI: ρ	NI: σ		
FE	3	2.14	0.99	0.23	1.52	1.59	0.99	0		
FE+Disg	2.18	2.56	0.99	0.23	0.83	2.09	0.99	0		
FE+Disg+Var	0.77	0.94	0.99	0.23	0.85	1.79	0.99	0		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ		
FE	-0.36	0.68	0.99	0.23	-0.48	0.56	0.9	0.13		
FE+Disg	-0.48	0.39	0.99	0.23	-0.5	0.38	0.92	0.12		
FE+Disg+Var	-0.44	0.38	0.99	0.23	-0.48	0.39	0.9	0.15		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	-0.81	2.91	3	0.99	0.23	-0.18	1.27	1.99	0.99	0
FE+Disg	0.05	1.5	1.54	0.99	0.23	0.98	1.93	1.12	0.99	0
FE+Disg+Var	-1.87	0.19	0.73	0.99	0.23	-0.11	0	0.32	0.9	0.23

Table 13: SMM Estimates of Different Models: Households

SE										
Moments Used	2-Step Estimate			Joint Estimate						
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ				
FE	0.35	0.99	0.41	0.33	0.93	0.27				
FE+Disg	0.33	0.99	0.41	0.31	0.93	0.26				
FE+Disg+Var	0.34	0.99	0.41	0.33	0.93	0.24				
NI										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ		
FE	3	0.65	0.99	0.41	3	0.45	0.9	0.31		
FE+Disg	3	0.45	0.99	0.41	3	0.51	0.9	0.36		
FE+Disg+Var	3	0.5	0.99	0.41	3	0.48	0.9	0.34		
DE										
Moments Used	2-Step Estimate				Joint Estimate					
	$\hat{\theta}$	σ_{θ}	ρ	σ	$\hat{\theta}$	σ_{θ}	ρ	σ		
FE	-2	5	0.99	0.41	-1.02	0.51	0.9	0.32		
FE+Disg	-0.99	3.88	0.99	0.41	-0.99	3.88	0.9	0.32		
FE+Disg+Var	-0.99	3.88	0.99	0.41	-1.14	4.01	0.91	0.3		
DENI										
Moments Used	2-Step Estimate					Joint Estimate				
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	-0.93	3	0.08	0.99	0.41	-0.51	3	0.07	0.9	0.31
FE+Disg	-0.45	3	0.15	0.99	0.41	-0.79	3	0.11	0.96	0.21
FE+Disg+Var	-0.3	2.47	0.53	0.99	0.41	NA	NA	NA	NA	NA

Table 14: SMM Estimates of Different Models under Stochastic Volatility: Professionals

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.34		
FE+Disg+Var	0.34		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	3	3	
FE+Disg	3	3	
FE+Disg+Var	3	2.59	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.63	0.79	
FE+Disg	-0.64	0.21	
FE+Disg+Var	-0.64	0.21	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	-0.62	2.17	3
FE+Disg	-0.55	3	3
FE+Disg+Var	-0.56	2.78	3

Table 15: SMM Estimates of Different Models under Stochastic Volatility: Households

SE			
Moments Used	2-Step Estimate		
	$\hat{\lambda}$		
FE	0.35		
FE+Disg	0.35		
FE+Disg+Var	0.35		
NI			
Moments Used	2-Step Estimate		
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	
FE	0	0.71	
FE+Disg	0	0.71	
FE+Disg+Var	0	0.71	
DE			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	σ_{θ}	
FE	-0.7	0.83	
FE+Disg	-0.7	0.58	
FE+Disg+Var	-0.7	0.58	
DENI			
Moments Used	2-Step Estimate		
	$\hat{\theta}$	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.8	0	0.32
FE+Disg	0.79	0	1.36
FE+Disg+Var	0.8	0	0.82