How Do Agents Form Inflation Expectations?

Evidence from the Forecast Uncertainty

Tao Wang

Johns Hopkins University

IAAE 2023

June 29, 2023

Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...

Inflation expectation formation

- Many competing models deviating from FIRE
  - Sticky expectations (SE)
  - Noisy information (NI)
  - Diagnostic expectations (DE)
  - ...
- Testing these models using survey expectations e.g. (Coibion and Gorodnichenko, 2012)
  - Forecast errors (FE)
  - Disagreement (Disg)
  - This paper: +Uncertainty (Var)

Why inflation uncertainty?

Uncertainty (or higher moments) matters for both

- individual economic decisions
  - through precautionary saving motives
  - through portfolio investment
- and aggregate outcomes
  - inflation dynamics
  - asset prices

Preview of the findings

- Competing theories have distinctive predictions about uncertainty

Preview of the findings

- Competing theories have distinctive predictions about uncertainty
- Sticky expectation (SE) best jointly explains FE, Disg and Var,
- ... and more robust to
    1. type of agents: households or professionals
    2. moments used: FE, Disg, Var, etc.
    3. separate or joint estimation of inflation + expectation formation
    4. various inflation processes: AR or stochastic volatility (SV)

Preview of the findings

- Competing theories have distinctive predictions about uncertainty
- Sticky expectation (SE) best jointly explains FE, Disg and Var,
- ... and more robust to
    1. type of agents: households or professionals
    2. moments used: FE, Disg, Var, etc.
    3. separate or joint estimation of inflation + expectation formation
    4. various inflation processes: AR or stochastic volatility (SV)
- Additional evidence rejecting FIRE
    1. Uncertainty is widely dispersed
    2. Revision is inefficient

Data

Density forecast of inflation

	SCE	SPF
Time period	2013-2021M7	2007-2022Q2
Frequency	Monthly	Quarterly
Sample Size	1,300	30-50
Density Variables	1-yr-ahead inflation	1-yr-ahead Core CPI and Core PCE
Panel Structure	stay up to 12 months	average stay for 5 years
Individual Info	Education, Income, Age, Location	Industry

Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

FIRE predictions

Inflation process (AR1)

$$y_t = \rho y_{t-1} + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

FIRE

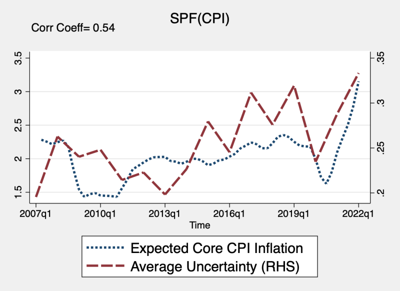
$$\overline{FE}^*_{t+1|t} = -\omega_{t+1} \rightarrow \quad \begin{aligned} \overline{FE}^{*2}_{\bullet+1|\bullet} &= \sigma_\omega^2 \\ \overline{\text{Var}}^*_{\bullet+1|\bullet} &= \sigma_\omega^2 \\ \overline{Disg}^*_{\bullet+1|\bullet} &= 0 \end{aligned}$$

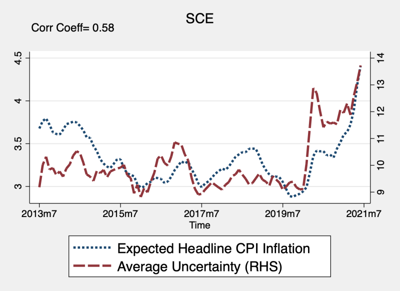
FIRE predictions v.s. data

	SPF	SCE	FIRE+AR	FIRE+SV
InfAV	0	0	0	0
InfVar	0.159	0.653	$\sigma_\omega^2/(1-\rho^2)$	N/A
InfATV	0.125	0.621	$\rho\sigma_\omega^2/(1-\rho^2)$	N/A
FE	0.136	1.772	0	0
FEVar	0.133	0.923	σ_ω^2	$\bar{\sigma}_\eta^2 + \bar{\sigma}_\epsilon^2$
FEATV	0.097	0.89	0	0
Disg	0.183	2.585	0	0
DisgVar	0.028	0.057	0	0
DisgATV	0.021	0.025	0	0
Var	0.242	1.75	σ_ω^2	$\bar{\sigma}_\eta^2 + \bar{\sigma}_\epsilon^2$
VarVar	0.001	0.023	0	$>$0
VarATV	0.001	0.004	0	$>$0

FIRE predictions v.s. data

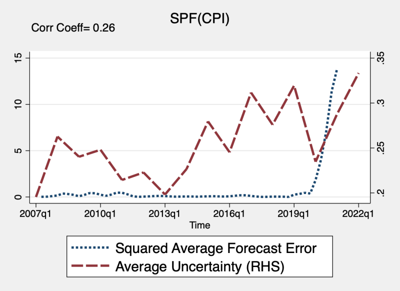
Expected inflation and uncertainty

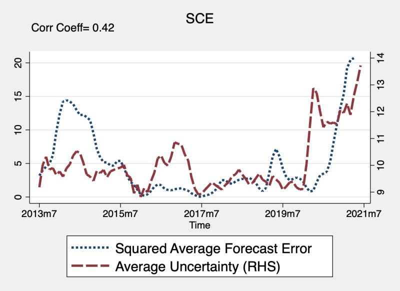




FIRE predictions v.s. data, continued

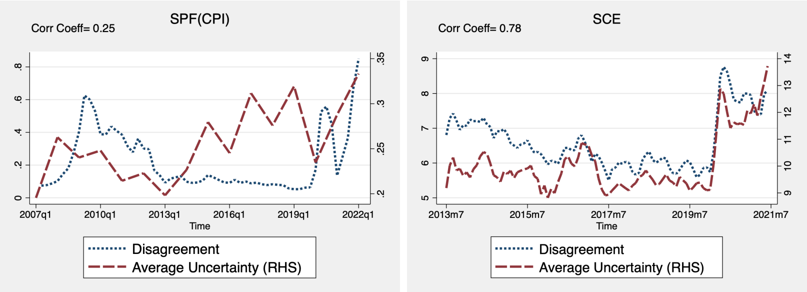
Forecast error and uncertainty





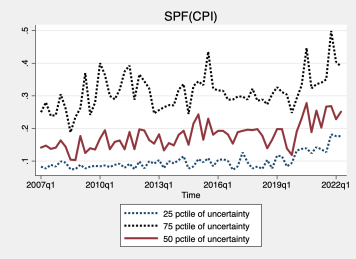
FIRE predictions v.s. data, continued

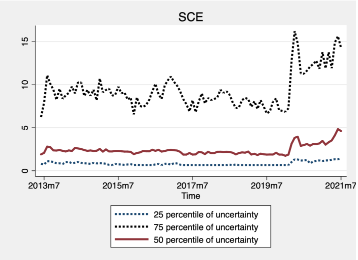
Disagreement and uncertainty



Other evidence rejecting FIRE

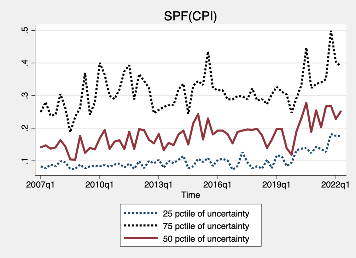
Dispersion in uncertainty

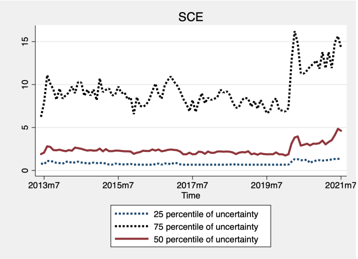




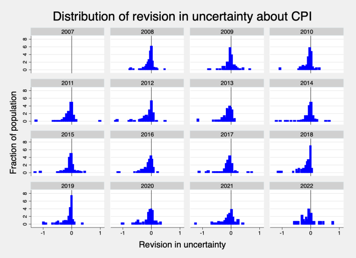
Other evidence rejecting FIRE

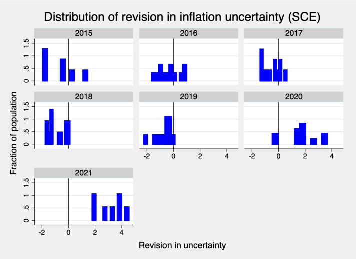
Dispersion in uncertainty





Inefficient revisions in uncertainty





Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - $\text{FE}^2 \leq$ Var

Taking stock

- Evidence rejecting FIRE
  - Heterogeneous Var
  - Inefficient revisions in Var
  - Disg > 0
  - FE^2 $\leq$ Var
- Also, these patterns help identify competing theories
  - SE$>$ NI, DE, DENI:
    - Disg and dispersion in Var because of different updating
    - Sticky updating implies inefficient revisions

Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Sticky expectations (SE)

[Mankiw and Reis, 2002, Carroll, 2003, etc]

With an updating rate of λ (FIRE when $\lambda = 1$)

$$\overline{FE}^{se}_{t+1|t} = (1-\lambda)\rho\overline{FE}^{se}_{t|t-1} - \lambda\omega_{t+1}$$

$$\rightarrow \overline{FE}^{se2}_{\bullet+1|\bullet} = \frac{\lambda^2}{1-(1-\lambda)^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+t|\bullet} = \sigma^2_\omega$$

$$\overline{\text{Var}}^{se}_{\bullet+1|\bullet} = \sum_{\tau=0}^{+\infty}\lambda(1-\lambda)^\tau\overline{\text{Var}}^{*}_{t+1|t-\tau} = \frac{1}{1-(1-\lambda)\rho^2}\sigma^2_\omega \geq \overline{\text{Var}}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{se}_{\bullet+1|\bullet} \geq 0$$

Noisy information (NI)

[Lucas, 1972, Woodford, 2001, Sims, 2003 and Maćkowiak and Wiederholt, 2009, etc]

With noisiness of public and private signals σ_{pb}^2 and σ_{pr}^2

$$\overline{FE}_{t+1|t}^{ni} = (1 - PH)\rho\overline{FE}_{t|t-1}^{ni} + \rho P_\epsilon \epsilon_t + \overline{FE}_{t+1|t}^{*}$$

$$\rightarrow \overline{FE}_{\bullet+1|\bullet}^{ni2} = \frac{\rho^2 P_\epsilon^2 \sigma_{pb}^2 + \sigma_\omega^2}{(PH)^2} \geq \overline{FE}_{\bullet+1|\bullet}^{*2} = \sigma_\omega^2$$

$$\text{Var}_{\bullet+1|\bullet}^{ni} = \rho^2 \text{Var}_{\bullet|\bullet}^{ni} + \sigma_\omega^2 \geq \text{Var}_{\bullet+1|\bullet}^{*} = \sigma_\omega^2$$

$$\overline{Disg}_{\bullet+1|\bullet}^{ni} = \frac{\rho^2 P_\xi^2}{1 - (1 - PH)^2 \rho^2} \sigma_{pr}^2 \geq 0$$

Kalman gain: $P = [P_\epsilon, P_\xi] = \overline{\text{Var}}_{\bullet|\bullet-1}^{ni} H (H' \overline{\text{Var}}_{\bullet|\bullet-1}^{ni} H + \Sigma^v)^{-1}$

Diagnostic expectations (DE)

[Bordalo, Gennaioli, and Shleifer, 2018, Bordalo, Gennaioli, Ma, et al., 2020, etc]

With overreaction parameter $\hat{\theta}(> 0)$ and dispersion σ_θ^2

$$\overline{FE}^{de}_{t+1|t} = \overline{FE}^{*}_{t+1|t} - \hat{\theta}\rho \mathsf{FE}^{de}_{t|t-1}$$

$$\rightarrow \overline{FE}^{de2}_{\bullet+1|\bullet} = \frac{1}{1+\hat{\theta}^2\rho^2}\sigma^2_\omega \leq \overline{FE}^{*2}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{\mathsf{Var}}^{de}_{\bullet+1|\bullet} = \overline{Var}^{*}_{\bullet+1|\bullet} = \sigma^2_\omega$$

$$\overline{Disg}^{de}_{\bullet+1|\bullet} \geq 0$$

Structural Estimation: SMM

$$\widehat{\Omega}^o = \underset{\{\Omega^o \in \Gamma^o\}}{argmin} (M_{\text{data}} - F^o(\Omega^o, H)) W (M_{\text{data}} - F^o(\Omega^o, H))'$$

- $o \in \{se, ni, de, deni\} \times \{ar, sv\}$
- Γ^o: parameter space
- H: real-time historical realizations
- W: weighting matrix

Structural Estimation: Professionals

SE								
Moments Used	2-Step Estimate			Joint Estimate				
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ		
FE	0.35	0.99	0.23	0.18	1	0.01		
FE+Disg	0.3	0.99	0.23	0.18	1	0		
FE+Disg+Var	0.32	0.99	0.23	0.21	1	0.02		
NI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	2.73	3	0.99	0.23	3	3	1	0.02
FE+Disg	3	2.95	0.99	0.23	3	3	1	0.02
FE+Disg+Var	3	3	0.99	0.23	1.97	1.17	1	0
DE								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ
FE	0.25	1.57	0.99	0.23	1.39	2.83	0.9	0.17
FE+Disg	1.12	1.62	0.99	0.23	0.05	1.73	1	0.01
FE+Disg+Var	0.29	1.81	0.99	0.23	1.19	1.68	0.91	0.16

Structural Estimation: Households

SE								
Moments Used	2-Step Estimate			Joint Estimate				
	$\hat{\lambda}$	ρ	σ	$\hat{\lambda}$	ρ	σ		
FE	0.35	0.98	0.41	0.2	0.98	0.16		
FE+Disg	0.35	0.98	0.41	0.2	0.98	0.1		
FE+Disg+Var	0.35	0.98	0.41	0.2	0.98	0.1		
NI								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$	ρ	σ
FE	3	0.45	0.98	0.41	2.98	0.28	0.95	0.24
FE+Disg	1.55	0.36	0.98	0.41	3	0.28	0.95	0.24
FE+Disg+Var	2.58	0.97	0.98	0.41	2.9	1.02	0.96	0.24
DE								
Moments Used	2-Step Estimate				Joint Estimate			
	$\hat{\theta}$	σ_θ	ρ	σ	$\hat{\theta}$	σ_θ	ρ	σ
FE	-0.47	0.81	0.98	0.41	-0.61	4.15	0.95	0.25
FE+Disg	-0.3	2.08	0.98	0.41	-0.08	2.13	0.95	0.25
FE+Disg+Var	-0.35	2.08	0.98	0.41	0.36	2.07	0.9	0.36

Roadmap

Motivations

FIRE benchmark v.s. data

Differentiating non-FIRE models

The role of stochastic volatility

Stochastic volatility (SV)

[Stock and Watson, 2007]

Process of inflation

$$y_t = \overbrace{\zeta_t}^{\text{Permanent component}} + \overbrace{\eta_t}^{\text{Transitory}}$$

$$\zeta_t = \zeta_{t-1} + z_t$$

$$z_t = \sigma_{z,t}\xi_{z,t}, \quad \eta_t = \sigma_{\eta,t}\xi_{\eta,t}, \quad \xi_t = [\xi_{\eta,t}, \xi_{\epsilon,t}] \sim N(0, I)$$

$$\log \sigma^2_{\eta,t} = \log \sigma^2_{\eta,t-1} + \mu_{\eta,t}, \qquad \log \sigma^2_{z,t} = \log \sigma^2_{z,t-1} + \mu_{z,t}$$

$$\mu_t = [\mu_{\eta,t}, \mu_{z,t}]' \sim N(0, \gamma I)$$

Structural Estimation with SV: Professionals

SE		
Moments Used	2-Step Estimate	
	$\hat{\lambda}$	
FE	0.35	
FE+Disg	0.34	
FE+Disg+Var	0.34	
NI		
Moments Used	2-Step Estimate	
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.27	0.04
FE+Disg	0.27	0.04
FE+Disg+Var	0.27	0.05
DE		
Moments Used	2-Step Estimate	
	$\hat{\theta}$	σ_{θ}
FE	-0.39	0.69
FE+Disg	-0.39	0.7
FE+Disg+Var	-0.39	0.7

Structural Estimation with SV: Households

SE		
Moments Used	2-Step Estimate	
	$\hat{\lambda}$	
FE	0.35	
FE+Disg	0.35	
FE+Disg+Var	0.35	
NI		
Moments Used	2-Step Estimate	
	$\hat{\sigma}_{pb}$	$\hat{\sigma}_{pr}$
FE	0.44	2.37
FE+Disg	0.44	2.61
FE+Disg+Var	0.44	2.9
DE		
Moments Used	2-Step Estimate	
	$\hat{\theta}$	σ_{θ}
FE	-0.05	0.58
FE+Disg	-0.05	0.56
FE+Disg+Var	-0.05	0.56

Scoring card

Table: Scoring card of different theories

Criteria	SE	NI	DE	DENI
Sensitive to moments used for estimation?	No	No	Yes	Yes
Sensitive to the assumed inflation process?	No	Yes	Yes	Yes
Sensitive to two-step or joint estimate?	Yes	Yes	Yes	Yes
Sensitive to the type of agents?	Yes	Yes	Yes	Yes

References I

Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, et al. (2020). “Overreaction in Macroeconomic Expectations”. *American Economic Review*.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2018). “Diagnostic expectations and credit cycles”. *The Journal of Finance* 73.1, pp. 199–227.

Carroll, Christopher D (2003). “Macroeconomic expectations of households and professional forecasters”. *the Quarterly Journal of economics* 118.1, pp. 269–298.

Coibion, Olivier and Yuriy Gorodnichenko (2012). “What can survey forecasts tell us about information rigidities?” *Journal of Political Economy* 120.1, pp. 116–159.

Lucas, Robert E (1972). “Expectations and the Neutrality of Money”. *Journal of economic theory* 4.2, pp. 103–124.

References II

Maćkowiak, Bartosz and Mirko Wiederholt (2009). "Optimal sticky prices under rational inattention". *American Economic Review* 99.3, pp. 769–803.

Mankiw, N Gregory and Ricardo Reis (2002). "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve". *The Quarterly Journal of Economics* 117.4, pp. 1295–1328.

Sims, Christopher A (2003). "Implications of rational inattention". *Journal of monetary Economics* 50.3, pp. 665–690.

Stock, James H and Mark W Watson (2007). "Why has US inflation become harder to forecast?" *Journal of Money, Credit and banking* 39, pp. 3–33.

Woodford, Michael (2001). *Imperfect common knowledge and the effects of monetary policy*. Tech. rep. National Bureau of Economic Research.