Forward Kinematics of the 6-6 Stewart Platform with Planar Base and Platform Using Algebraic Elimination

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Forward Kinematics of the 6-6 Stewart Platform with Planar Base and Platform Using Algebraic Elimination*

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***Abstract* - This paper presents a new algorithm to solve the closed-form forward kinematics of the general 6-6 Stewart platform with planar base and moving platform. Based on the presented algebraic method, we derive the 20th degree univariate polynomial from the determinant of the 15×15 Sylvester's matrix, which is relatively small in size, without factoring out or deriving the greatest common divisor. We also find that the highest degree of the univariate polynomial is in accordance with the determinant of the univariate coefficient matrix, however, many prevenient methods are not so. The algorithm is comparatively concise and requires fairly less computation time enough to be used for real-time applications. The result is verified by a numerical example.**

Index Terms – Direct kinematics, Parallel mechanism, Closed-form solutions.

I. Introduction

In the past decades, Stewart platforms have received a great deal of attention from many researchers due to their inherent advantages over the conventional serial mechanism, such as, simpler structure, higher stiffness, better accuracy, heavier loading ability. Stewart platforms have now been found a wide variety of applications, such as, flight simulators, parallel robot manipulators, machine tools, 6 degrees of freedom coordinate measuring devices, entertainment and health equipment.

The inverse kinematics problem, which is to determine the lengths of the six legs given the positions and orientations of the moving platform, is nearly straightforward. However, the direct kinematics problem, which is to determine the positions and orientations of the moving platform given the lengths of the six legs, is more difficult. This is in contrast to serial chain manipulators where the opposite is true. The motion planning and control of a Stewart platforms calls for the solution of the direct kinematics, which is a basic and challenging problem as well.

The direct kinematics problems of the Stewart platforms lead naturally to system of nonlinear algebraic and/or transcendental equations. Devising an algorithm to solve, in polynomial form, the direct kinematics problem of the 6-6 Stewart platform has proved to be a challenging undertaking. Although a numerical iterative procedure can find forward kinematics solutions, it is not suitable to a Stewart platform because it leads to a heavy computational burden and requires a good initial value. Besides, no correct solution is guaranteed from this approach, so it is difficult for real-time applications. A closed-form forward kinematics solution will provide more information about the geometry and kinematic behaviour of a parallel mechanism. What's more, the input-output closed-form univariate polynomial equations derived by using the closed-form methods have highly theoretical values based on which many kinematic problems will be solved easily.

There have been many literatures on the closed-form solutions for the forward kinematics of different types and geometry. Griffis and Duff [1] and Innocenti and Parenti-Cartelli [2] derived a 16th degree univariate polynomial on the general 3-6 Stewart platform. Innocenti [3] found that the forward kinematics of the type 4-6 admits of 32 closure configurations in the complex field, and Nielson and Roth [4] reduced a system of equations for the type 5-6 to a 40th degree univariate after factoring out trivial roots. Lin et al. [5] [6] deal with the type 4-4 and 4-5 and Chen and Song [7] with the type 4-6. Wen and Liang [8] and Lee [9] gave the closed-form solutions for the Stewart platform with planar base and platforms. However, we find that the highest degree of the univariate polynomial is not in accordance with the determinant of the univariate coefficient matrix in [8] and [9]. Dhingra et al. [10] used Gröbner-Sylvester hybrid method to obtain a 20th degree input-output polynomial directly from the 15×15 Sylvester's matrix formed by calculating Gröbner basis for the Stewart platform with planar base and platforms.

The approach mentioned above provide algorithms to obtain all solutions for various types of Stewart platforms, but were not intended for real-time applications. In fact, the mathematical complexity of the forward kinematics of Stewart platforms has become a serious deficiency that prevents Stewart platform systems from being used in many high-speed, real-time and on-line engineering situations.

This paper presents a novel and relatively concise algorithm, by borrowing techniques from Wu and Huang [11], to provide all the solutions of the forward kinematic analysis of the 6-6 Stewart platform with planar base and moving platform. Based on the presented algebraic method, we derive the 20th degree univariate polynomial from the determinant of the 15×15 Sylvester's matrix, which is relatively small in size, without factoring out or deriving the greatest common divisor. We also find that the highest degree of the univariate

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polynomial is in accordance with the determinant of the univariate coefficient matrix, however, many prevenient methods are not so. The algorithm is comparatively concise and requires fairly less computation time enough to be used for real-time applications. A numerical example is given to verify the algorithm and its results without extraneous roots agree with the original equations.

The rest of the paper is organized as follows. In Section II, we give the kinematic constraint equations. In Section Ⅲ, we present the elimination process for solving the kinematic constraint equations. In Section Ⅳ, we give a numerical example verifies the algorithm presented in this paper. In Section Ⅴ, conclusions are given.

II. The Kinematic Constraint Equations

Fig. 1 shows the geometric model of the 6-6 Stewart platform with planar base and moving platform. All the joints of its base and moving platform are located in respective planes. The six inputs necessary to describe the location and orientation of the upper platform are the leg lengths controlled by each prismatic joint. For a general case, the absolute local frame system $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$ and the relative moving frame system $\boldsymbol{O}_2$-$\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$ are fixed to the arbitrary points $\boldsymbol{O}_1$ on the base platform and $\boldsymbol{O}_2$ on the moving platform, respectively. The direct kinematics problem is to find the position and orientation of the moving platform supposing that the pose of the base platform is known and values for the six constraints connecting to the base and the platform are given.

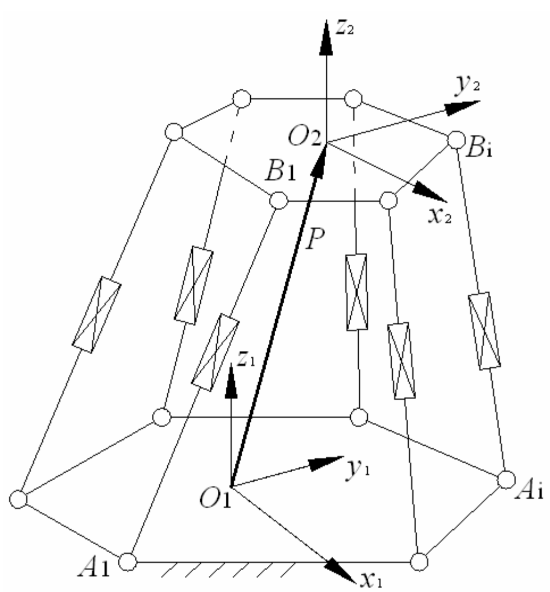

Fig.1 The geometric model of the 6-6 Stewart platform

Let the coordinates of point A_i is (x_i, y_i, 0) (i = 1, 2, 3, 4, 5, 6) in the absolute frame $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$, the coordinates of point B_i is (p_i, q_i, 0) (i = 1, 2, 3, 4, 5, 6) in the moving frame $\boldsymbol{O}_2$-$\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$, and the coordinates of origin point $\boldsymbol{O}_2$ in the absolute frame $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$ is (x_i, y_i, 0). The lengths of $\boldsymbol{A}_i\boldsymbol{B}_i$ are denoted as L_i. Given the position vector $\boldsymbol{P}$ between the two origin points $\boldsymbol{O}_1$ and $\boldsymbol{O}_2$, and the transformation matrix R between the two coordinate systems, the leg vectors can be easily represented as

$$\boldsymbol{L}_i = \boldsymbol{R}\boldsymbol{B}_i + \boldsymbol{P} - \boldsymbol{A}_i,\ i = 1, 2, 3, 4, 5, 6 \quad (1)$$

Where

$$\boldsymbol{R} = \begin{bmatrix} r_1 & r_4 & r_7 \\ r_2 & r_5 & r_8 \\ r_3 & r_6 & r_9 \end{bmatrix} \quad (2)$$

With given leg lengths, the kinematic constraint equations corresponding to the conditions of constraint length of each leg are as follows

$$(\boldsymbol{P} + \boldsymbol{R}\boldsymbol{B}_i - \boldsymbol{A}_i)^T(\boldsymbol{P} + \boldsymbol{R}\boldsymbol{B}_i - \boldsymbol{A}_i) = L_i^2 \quad i=1, 2, 3, 4, 5, 6 \quad (3)$$

Substituting all coordinates above and (2) into (3), we get

$$(p_i r_1 + q_i r_4 + x - x_i)^2 + (p_i r_2 + q_i r_5 + y - y_i)^2 + (p_i r_3 + q_i r_6 + z)^2 - L_i^2 = 0 \quad i=1,2,3,4,5,6 \quad (4)$$

Since $\boldsymbol{R}$ is orthogonal, r_i (i = 1, …, 9) satisfy the following relations

$$r_1^2 + r_2^2 + r_3^2 - 1 = 0 \quad (5)$$

$$r_4^2 + r_5^2 + r_6^2 - 1 = 0 \quad (6)$$

$$r_1 r_4 + r_2 r_5 + r_3 r_6 = 0 \quad (7)$$

$$r_4 r_8 - r_5 r_7 - r_3 = 0 \quad (8)$$

$$r_2 r_7 - r_1 r_8 - r_6 = 0 \quad (9)$$

$$r_1 r_5 - r_2 r_4 - r_9 = 0 \quad (10)$$

Equations (4), (5), (6), (7) are devoid of unknown variables r_7, r_8, and r_9. If it is necessary, the unknown variables r_7, r_8, and r_9 will be obtained by (8), (9), (10) when other variables are known. So (4), (5), (6), (7), which represent 9 equations in 9 unknowns r_1, r_2, r_3, r_4, r_5, r_6, x, y, z, are the kinematic constraint equations describing the direct kinematics of the 6-6 Stewart platform.

Ⅲ. The Elimination Process

A. *Intermediate Polynomials in Three Variables*

Equation (4) can be reduced to

$$p_i x_i r_1 + p_i y_i r_2 - p_i u + q_i x_i r_4 + q_i y_i r_5 - q_i v + x_i x + y_i y - w/2 + m_i = 0 \quad i = 1, 2, 3, 4, 5, 6 \quad (11)$$

where

$$m_i = (L_i^2 - x_i^2 - y_i^2 - p_i^2 - q_i^2)/2$$

$$u = r_1 x + r_2 y + r_3 z \quad (12)$$

$$v = r_4 x + r_5 y + r_6 z \quad (13)$$

$$w = x^2 + y^2 + z^2 \quad (14)$$

Equations (11) are linear with respect to r_1, r_2, r_4, r_5, u, v, x, y, and w, and can be arranged as follows

$$\boldsymbol{M}_{6\times10}\boldsymbol{t} = 0$$

where the i row of the matrix $\boldsymbol{M}_{6\times10}$ is

$$(p_i x_i, p_i y_i, -p_i, q_i x_i, q_i y_i, -q_i, x_i, y_i, -1/2, m_i)$$

and

$$\boldsymbol{t} = (r_1, r_2, u, r_4, r_5, v, x, y, w, 1)^{\mathrm{T}}$$

Solving the system (11) symbolically regarding r_1, r_2, r_4, r_5, u, v as linear unknowns, we obtain the linear expressions of those variables with respect of x, y, w by Cramer algorithm

$$a_0 r_1 + a_{11}x + a_{12}y + a_{13}w + a_{14} = 0 \quad (15)$$

$$a_0 r_2 + a_{21}x + a_{22}y + a_{23}w + a_{24} = 0 \quad (16)$$

$$a_0 u + a_{31}x + a_{32}y + a_{33}w + a_{34} = 0 \quad (17)$$

$$a_0 r_4 + a_{41}x + a_{42}y + a_{43}w + a_{44} = 0 \quad (18)$$

$$a_0 r_5 + a_{51}x + a_{52}y + a_{53}w + a_{54} = 0 \quad (19)$$

$$a_0 v + a_{61}x + a_{62}y + a_{63}w + a_{64} = 0 \quad (20)$$

where

$$a_0 = \det(c_1, c_2, c_3, c_4, c_5, c_6)$$

$$a_{ij} = \det(c_1, \cdots, c_{i-1}, c_{j+6}, c_{i+1}, \cdots, c_6)$$

and c_j denotes the jth column of $\boldsymbol{M}_{6\times 10}$.

Once the values of x, y, and w are obtained, other unknown variables are easily computed by the six equations above.

B. Deriving a Univariate Polynomial in x

By (5), (6), (7), (12), (13), (14), we obtain

$$f_1 = r_3^2 = 1 - r_1^2 - r_2^2 = 1 - a_0^{-2}(A^2 + B^2) \quad (21)$$

$$f_2 = r_6^2 = 1 - r_4^2 - r_5^2 = 1 - a_0^{-2}(D^2 + F^2) \quad (22)$$

$$f_3 = r_3 r_6 = -r_1 r_4 - r_2 r_5 = 1 - a_0^{-2}(AD + BF) \quad (23)$$

$$f_4 = r_3 z = u - r_1 x - r_2 y = a_0^{-1}(-C + Ax + By) \quad (24)$$

$$f_5 = r_6 z = v - r_4 x - r_5 y = a_0^{-1}(-G + Dx + Fy) \quad (25)$$

$$f_6 = z^2 = w - x^2 - y^2 \quad (26)$$

where

$$A = a_{11}x + a_{12}y + a_{13}w + a_{14}$$

$$B = a_{21}x + a_{22}y + a_{23}w + a_{24}$$

$$C = a_{31}x + a_{32}y + a_{33}w + a_{34}$$

$$D = a_{41}x + a_{42}y + a_{43}w + a_{44}$$

$$F = a_{51}x + a_{52}y + a_{53}w + a_{54}$$

$$G = a_{61}x + a_{62}y + a_{63}w + a_{64}$$

From the 6 equations, we obtain obviously

$$p_1 = f_1 f_6 - f_4^2 = 0 \quad (27)$$

$$p_2 = f_2 f_6 - f_5^2 = 0 \quad (28)$$

$$p_3 = f_3 f_6 - f_4 f_5 = 0 \quad (29)$$

$$p_4 = f_1 f_5 - f_3 f_4 = 0 \quad (30)$$

$$p_5 = f_2 f_4 - f_3 f_5 = 0 \quad (31)$$

$$p_6 = f_1 f_2 - f_3^2 = 0 \quad (32)$$

Equations p_i (i = 1, 2, 3, 4, 5, 6) are in terms of x, y, w and the total degree of every equation with respect to x, y, and w is 4.

From p_i (i = 1, 2, 3, 4, 5, 6), we construct the following nine polynomials which are simpler than that of Wu and Huang [11].

$$p_7 = -Dp_1 + Ap_3 - xp_4 \quad (33)$$

$$p_8 = Fp_1 - Bp_3 + yp_4 \quad (34)$$

$$p_9 = Dp_3 - Ap_2 - xp_5 \quad (35)$$

$$p_{10} = -Fp_1 + Bp_3 + yp_5 \quad (36)$$

$$p_{11} = Ap_5 + Dp_4 + xp_6 \quad (37)$$

$$p_{12} = Bp_5 + Fp_4 + yp_6 \quad (38)$$

$$p_{13} = -Fp_7 - Bp_9 + yp_{11} \quad (39)$$

$$p_{14} = -Cp_5 + Dp_7 + Bp_{10} - a_0^2 p_2 - xp_{11} \quad (40)$$

$$p_{15} = p_{14} - Dp_7 - Fp_8 - Ap_9 - Bp_{10} + xp_{11} + yp_{12} \quad (41)$$

It is easy to check that p_i (i = 7, …, 15) are in terms of x, y, w and the total degree of every equation with respect to x, y, and w is still 4.

Suppressing the unknowns x, the 15 equations can be viewed as a linear system in y and w. They can be written in the form of matrix as follows.

$$\boldsymbol{M}_{15\times 15}\boldsymbol{T} = 0 \quad (42)$$

where

$$\boldsymbol{T} = [w^4, w^3 y, w^2 y^2, wy^3, y^4, w^3, w^2 y, wy^2, y^3, w^2, wy, y^2, w, y, 1]^{\mathrm{T}}$$

The system (42) has a solution, if and only if

$$\det(\boldsymbol{M}_{15\times 15}) = 0 \quad (43)$$

It is easy to check that the highest degrees with respect to x in every column of $\boldsymbol{M}_{15\times 15}$ are 4, 3, 2, 1, 0, 3, 2, 1, 0, 2, 1, 0, 1, 0, 0. Obviously, the degree of the univariate polynomial derived from $\det(\boldsymbol{M}_{15\times 15})$ in x is at most 20.

The condition (43) can yield the following 20th degree univariate polynomial without factoring out or deriving the greatest common divisor.

$$\sum_{i=0}^{20} s_i x^i = 0 \quad (44)$$

where s_i (i = 0, …, 20) are real constants depending on input data only. Equation (44) is a 20th degree polynomial in x. Solving (44), we obtain 20 roots x_i (i = 1, …, 20) for x in the complex domain.

C. Back Substitution for Other Unknowns

Solving the linear system, which is obtained by removing any one row from the matrix $\boldsymbol{M}_{15\times 15}$ in (42) with x replaced by x_i (i = 1, …, 20), solutions of y and w can be easily computed in the complex domain. For one solution of x there will be one solution of y and w.

Substituting x, y, w into (15), …, (20), solutions of r_1, r_2, r_4, r_5, u, v can be gained easily. For one solution of x, y, w there will be one solution of r_1, r_2, r_4, r_5, u, *and* v.

By (12), (13), and (14), we can get the solutions of z, r_3, and r_6. For one solution of x, y, w, there will be two groups opposite sign solutions of each z, r_3, and r_6.

Ⅳ. NUMERICAL EXAMPLE

A set of input data for the direct kinematics of the 6-6 Stewart platform with planar base and moving platform is considered. With reference to Fig. 1, the coordinates of the base attachment points A_i (i =1, 2, 3, 4, 5, 6) with respect to the absolute local frame system $\boldsymbol{O}_1$-$\boldsymbol{X}_1\boldsymbol{Y}_1\boldsymbol{Z}_1$, the coordinates of the moving platform attachment points B_i (i =1, 2, 3, 4, 5, 6) with respect to the relative moving frame system $\boldsymbol{O}_2$-$\boldsymbol{X}_2\boldsymbol{Y}_2\boldsymbol{Z}_2$ and the actuator lengths L_i (i =1, 2, 3, 4, 5, 6) are given in Table I.

The 40 sets of solutions in the complex domain shown in Table Ⅱ are obtained. All solutions have been verified by the inverse positions analysis.

TABLE I
INPUT DATA FOR THE NUMERICAL EXAMPLE

i	x_i	y_i	p_i	q_i	L_i
1	9	3	3	1	$\sqrt{36205}/13$
2	6	8	2	3	$2\sqrt{188630}/65$
3	0	14	1	5	$3\sqrt{101465}/65$
4	-8	13	-3	4	$\sqrt{237}$
5	-7	-6	-2	2	$\sqrt{462}$
6	-3	-5	-1	-4	$6\sqrt{46670}/65$

TABLE Ⅱ
SOLUTIONS FOR THE NUMERICAL EXAMPLE

No.	x	y	z	r_1	r_2	r_3	r_4	r_5	r_6
1,2	8	9	±10	0.6	0.3077	±0.7385	-0.8	0.2308	±0.5538
3,4	-2.1867	10.7203	∓9.2147	0.0434	-0.8201	±0.5705	-0.0336	-0.5720	∓0.8196
5,6	9.9574+ 6.8118*i	2.9276+ 3.0687*i	∓11.5817± 10.3310*i	-3.6512- 2.0511*i	-3.1638+ 0.3122*i	±1.4488∓ 4.4873*i	-0.9394- 1.6711*i	-0.1816- 0.7516*i	±2.0358∓ 0.8381*i
7,8	9.9574- 6.8118*i	2.9276- 3.0687*i	∓11.5817∓ 10.3310*i	-3.6512+ 2.0511*i	-3.1638- 0.3122*i	±1.4488± 4.4873*i	-0.9394+ 1.6711*i	-0.1816+ 0.7516*i	±2.0358± 0.8381*i
9,10	15.9630	-3.6668	±10.3431*i	-2.2469	-3.1072	∓3.7017*i	-0.2951	1.3506	±0.9546*i
11,12	-24.7650	31.4610	±37.6345*i	1.3569	1.7816	±2.004*i	-0.4691	-3.8023	∓3.6984*i
13,14	-3.9530	4.6726	∓5.8285*i	-4.9267	-5.1238	∓7.0375*i	0.09634	-1.3546	∓0.9188*i
15,16	-8.0731	32.9140	±30.7157*i	1.3669	4.0516	±4.1574*i	-2.5801	-3.3213	∓4.0851*i
17,18	-4.3926- 0.0869*i	10.7561- 0.5922*i	±9.8320∓ 0.3967*i	0.7319- 0.4911*i	-0.6842- 0.4139*i	∓0.6500∓ 0.1174*i	0.3510+ 0.0021*i	-0.4564- 0.07454*i	±0.8221∓ 0.0423*i
19,20	-4.3926+ 0.0869*i	10.7561+ 0.5922*i	±9.8320± 0.3967*i	0.7319+ 0.4911*i	-0.6842+ 0.4139*i	∓0.6500± 0.1174*i	0.3510- 0.0021*i	-0.4564+ 0.07454*i	±0.8221± 0.0423*i
21,22	20.6782	13.6647	±26.2915*i	-7.0775	-1.5011	∓7.1654*i	-4.5370	-2.2003	∓4.9422*I
23,24	10.8047+ 0.9346*i	7.0721- 1.6629*i	∓10.3329± 0.4648*i	0.9961- 0.4159*i	0.4386- 0.4849*i	∓0.8652∓ 0.7246*i	-0.7331+ 0.0702*i	0.7164+ 0.1298*i	∓0.1720± 0.2416*i
25,26	10.8047- 0.9346*i	7.0721+ 1.6629*i	∓10.3329∓ 0.4648*i	0.9961+ 0.4159*i	0.4386+ 0.4849*i	∓0.8652± 0.7246*i	-0.7331- 0.0702*i	0.7164- 0.1298*i	∓0.1720∓ 0.2416*i
27,28	2.2139- 3.9851*i	5.4240- 0.4501*i	∓15.9057± 0.1681*i	1.2014- 0.5583*i	-0.8072- 0.8758*i	∓0.1704± 0.2123*i	0.5378+ 0.4063*i	0.6508- 0.2697*i	±0.7270∓ 0.0591*i
29,30	2.2139+ 3.9851*i	5.4240+ 0.4501*i	∓15.9057∓ 0.1681*i	1.2014+ 0.5583*i	-0.8072+ 0.8758*i	∓0.1704∓ 0.2123*i	0.5378- 0.4063*i	0.6508+ 0.2697*i	±0.7270± 0.0591*i
31,32	17.5675	-4.8684	±5.5032*i	0.9414	-1.3942	∓1.3529*i	0.3656	2.5388	±2.362*i
33,34	-16.1829	28.9639	±0.5362*i	22.3821	14.0219	∓26.3927*i	3.2620	3.2406	∓4.4880*i
35,36	-23.1686+ 56.8836*i	15.3952+ 19.0136*i	±65.5915± 14.4444*i	25.8663- 4.3498*i	12.2132+ 8.3318*i	∓0.3983∓ 27.0008*i	6.8940- 10.2988*i	5.7983- 1.5358*i	∓9.7613∓ 8.1859*i
37,38	-23.1686- 56.8836*i	15.3952- 19.0136*i	±65.5915∓ 14.4444*i	25.8663+ 4.3498*i	12.2132- 8.3318*i	∓0.3983± 27.0008*i	6.8940+ 10.2988*i	5.7983+ 1.5358*i	∓9.7613± 8.1859*i
39,40	-58.8080	16.1191	±49.0381*i	27.9725	9.3892	∓29.4893*i	11.2369	4.9135	∓12.2233*i

Ⅴ. CONCLUSIONS

This paper presents a novel and relatively concise algorithm method for solving the forward kinematic problem of the 6-6 Stewart platform with planar base and moving platform. Base on the presented method, we derive the 20th degree univariate polynomial from the determinant of the 15×15 Sylvester's matrix without factoring out or deriving the greatest common divisor. We also obtain all the closed-form solutions for the forward kinematics of the platform. The algorithm is comparatively concise and requires fairly less computation time enough to be used for real-time applications. The result is verified by a numerical example.

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