Project 9B - Find Energy Levels

Use a well width of 1E-10 m.

1. The quantum well (V=0, $0 < x < a$, else $V = \infty$) is a boundary value problem because one knows the values of $\psi(x)$ at the edges of the well. Suppose the well were divided into 4 points, $x_0 = 0$, $x_1 = \frac{a}{3}$, $x_2 = \frac{2a}{3}$, $x_3 = a$. The value of $\psi(x)$ is known for two of these points from the boundary conditions. It is also known that $\psi(x)$ must satisfy $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V\psi = E\psi$, where V=0 for these values of x. Using S.I. units, create an object of type matrix that represents the operation, $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$. Your matrix should only be a 2x2 because two of the values of $\psi(x)$ are known.

2. Computationally determine the eigenvalues of this matrix. Check your work using pencil and paper. What do the eigenvalues represent? When your program prints them, label them as what they are along with appropriate units.

3. Scale up your matrix to be able to handle more points. This should make your estimates for the allowed energy levels more accurate. Divide the region into 10 points instead of 4. Determine the first three energy levels and print them. Print the closed-form values alongside of your calculated values for comparison.

4. @ Try using more points, 10, 15, 20. Graph the time it takes your program to run as a function of the number of points. Do a polynomial fit and explain.

Due Monday, April 22th.