Imran Yafai
Computational Physics
Project 6B

Closed form solution for circuit:
Capacitor Charged to Initial Voltage of 5V with Capacitance 3 F connected to an 8Ω Resistor

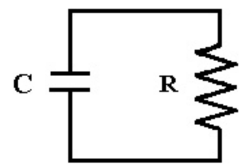

Figure 1: High-Quality Circuit Diagram

Voltage Drop over Resistor:

$$V_R = I_R R$$

Charge on Capacitor:

$$Q_C = V_C C$$

$$V_R = \frac{dQ}{dt} R$$

This current entering the resistor, is the negative of current leaving the capacitor

$$V_R = -\frac{dQ_C}{dt} R$$

Subsituting for V_R and rewritting Q_C as Q

$$\frac{Q}{RC} = -\frac{dQ}{dt}$$

This can be rewritten as

$$-\frac{dQ}{Q} = \frac{dt}{RC}$$

And then we integrate the from time 0 to .5 s

$$\int_{Q_0}^{Q_f} -\frac{dQ}{Q} = \int_0^{t_f} \frac{dt}{RC}$$

This gives us:

$$-\ln(Q)\Big|_{Q_0}^{Q_f} = \frac{1}{RC} t\Big|_0^{t_f}$$

$$\ln Q_0 - \ln Q_f = \ln \frac{Q_0}{Q_f} = \frac{t_f}{RC}$$

This equation can be rewritten as

$$\frac{Q_0}{Q_f} = e^{t_f/RC}$$

and so the charge on the capacitor at .5 seconds is

$$Q_f = Q_0 e^{-t_f/RC} = (3 * 5) e^{-.5/8*3}\,\mathrm{C} = 14.69073272\,\mathrm{C}$$