Dynamic trajectory planning for automated lane-changing

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ABSTRACT

LC (Lane-changing) trajectory planning is one of the key algorithms in supporting autonomous vehicles to complete the LC maneuver safely. This paper proposes a novel DLTP (dynamic LC trajectory planning) algorithm for automated lane-changing. Through introducing the core parameter of planning step size, this algorithm is capable of adjusting its LC trajectory dynamically according to the states of the surrounding vehicles. For each planning step, the time-based quintic polynomial function is introduced to model the LC trajectory equation, which is the most commonly-used curve equation. The problem of solving the corresponding parameters is then transformed into an optimization problem, which takes driver's safety, comfort and efficiency into account. Hereafter, the trajectory information for each planning step can be acquired through introducing the SQP (Sequential Quadratic Programming) algorithm to solve this optimization problem. Finally, a systematic numerical simulation has been conducted to verify the effectiveness of the proposed algorithm. We believe this paper may provide certain valuable insights for developing more reliable LC algorithms for autonomous vehicles.

Keywords: Autonomous vehicle, Lane-changing maneuver, Trajectory Planning Algorithm, Quintic Polynomial Function, Sequential Quadratic Programming.

INTRODUCTION

Recent years, the research on autonomous vehicle technology has received extensive attentions(*1-4*). And numerous researches indicate that the advent of autonomous vehicles may effectively relieve traffic congestion and improve traffic safety(*1; 5-7*). One of the indispensable components of autonomous vehicle technology is to model the automated lane-changing (LC) maneuver, which describes the longitudinal and lateral movement of the autonomous vehicle from the current lane to the target lane. Researching on LC can not only help us reveal and understand the essence and characteristics of traffic phenomena, but also help us develop more reliable and safe automated LC algorithms for autonomous vehicles.

Generally speaking, the research on automated LC can be roughly divided into: modeling the process of making LC decision(*8; 9*) (i.e. whether to change lane or not), modeling the impact of LC on surrounding vehicles(*10*), modeling the duration of LC(*11-13*) and planning the LC trajectory for the autonomous vehicle(*1; 2; 5-7; 14; 15*). Among these directions, LTP (LC trajectory planning) algorithm is one of the most important algorithms in supporting the autonomous vehicle to complete the LC maneuver safely, which has received considerable attentions from scholars over the past decades(*1; 2; 4-6; 14-16*).

When the autonomous vehicle has decided and is about to execute LC maneuver, the vehicle needs to calculate a specified LC trajectory in advance, and it will drive along this trajectory until it arrives the center-line in the target lane. As the schematic diagram of LC shown in **Figure 1**, starting from the center-line of the current lane, the autonomous vehicle gradually moves to the center-line of the target lane, and the center position of the autonomous vehicle at each time step forms the entire shape of the LC trajectory. The LTP algorithm is mainly to settle several questions related to this reference trajectory as shown in **Figure 1**: (1) What's the mathematical equation form of this curve? (2) How to obtain the corresponding parameters values? (3) How to ensure the safe completion of the LC maneuver of the autonomous vehicle? Therefore, this research will be unfolded around these three questions closely.

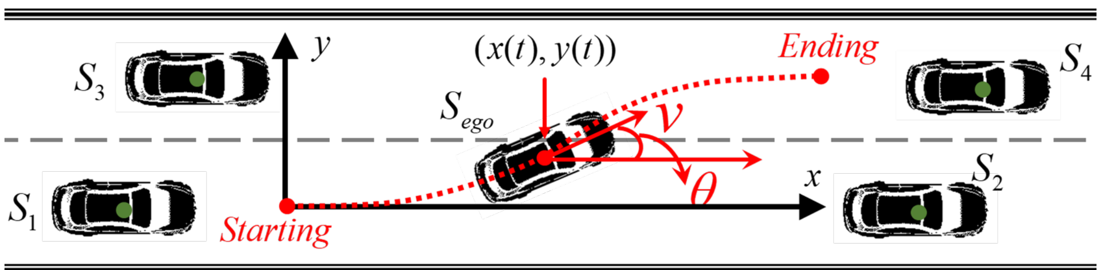


Figure 1 The schematic diagram of LC of the autonomous vehicle (the red dot line represents the LC trajectory of the autonomous vehicle)

This paper takes a further step to develop an LTP algorithm for the autonomous vehicle. In this algoritm, we introduce the core parameter of planning step size to achieve the effect of dynamically LC trajectory planning. Since the autonomous vehicle is equipped with sensors, the autonomous vehicle is capable of receiving the real-time states of surroundings, and thus adjusting its LC trajectory according to these changes. For each planning step, the polynomial function is introduced to model the LC trajectory, and the corresponding parameters of current planning step is obtained through transforming this problem into an optimization problem, which takes driver's safety, comfort and efficiency into account. Finally, a comprehensive numerical simulation has been carried out to verify the effectiveness of the proposed algorithm.

The subsequent sections of this paper are organized as follows: a detailed literature review is presented in "LITERATURE REVIEW". The section "MODEL FRAMEWORK" presents the detailed modeling process of the proposed LTP algorithm. The section "NUMERICAL SIMULATION" presents the complete numerical simulations on the proposed algorithm. The section "DISCUSSION" presents the discussion on the proposed LTP algorithm. The summary of this paper is presented in "CONCLUSION".

LITERATURE REVIEW

Over the past decades, a considerable amount of works has been made to research the LTP algorithms(*1; 2; 4-6; 14-17*). The methods in developing LTP algorithms can be mainly divided into: analytical method and data-driven method. Analytical method usually refers to the method of deriving mathematical formulas to obtain the optimal LC trajectory. Data-driven method usually refers to the method of using the machine learning or deep learning algorithm, which aims to extract LC dynamics from massive data instead of describing the nature of things (*16; 17*). Although adopting the data-driven method could achieve better modeling performance(*16; 17*), the modeling process of LC trajectory is actually like a black box, and it is impossible for us to know what really happened in that black box. At the same time, using this kind of methods may be limited by the dataset, which requires acquisition and training of large amounts of LC trajectory data. While if we use the analytical method, each parameter in this method has a clear physical meaning. Meanwhile, the analytical method is capable of modeling the LC trajectory in various situations without being limited by the dataset.

Therefore, in this paper, we will use the analytical method to model the LC trajectory, and we will mainly focus on reviewing the literatures which are closely related to our research theme. If we adopt the analytical method, the approximate curve form of LC trajectory has to be determined in the first place. So far, the most commonly-used mathematics equations are quintic polynomial function(*2; 6; 14; 15*), cubic polynomial function(*1; 5*), sine(cosine) curve, trapezoidal curve, etc. After determining the form of curve equation, the problem of obtaining the values of corresponding parameters are generally transformed into an optimization problem, which usually takes driving comfort, driving efficiency and driving safety into account. Afterwards, the optimization algorithm (i.e. Sequential Quadratic Programming(*4; 15*), Interior-Point Algorithm(*14*), etc.) is introduced to solve this optimization problem. Afterwards, we could acquire the mathematical equation of the LC trajectory of the autonomous vehicle, and the autonomous vehicle will then drive along with this trajectory until it arrives the center-line of the target lane.

Up until now, the following studies have produced some remarkable results in this regard. Based on the vehicle-to-vehicle communication, the dynamic automated lane change maneuver algorithm is proposed(*14*), which is composed of trajectory planning algorithm and trajectory tracking algorithm. The time-based quintic polynomial function is introduced to model the reference trajectory, which satisfies the safety, comfort and efficiency of the automated vehicle. Finally, simulation and experiments results demonstrate the effectiveness of the proposed algorithm. Using the same mathematics equation, Bai et al. (*2)* introduces the quintic polynomial function to model the accelerated lane-changing characteristics, which considers the collaboration with the following vehicle in the target lane. Furthermore, Bai et al. (*2)* establishes the rectangular collision boundary of the subject vehicle so as to analyze the possible collision points. Finally, this algorithm is verified under different accelerated lane-changing scenarios. Yue et al. (*6)* introduces the time-based quintic polynomial function for the implementation of trajectory planning, which takes safety, comfort and traffic efficiency.

In order to satisfy different drivers' personalized LC needs, Huang et al. (*15)* incorporates personalized driving style into the automated lane-changing trajectory planning algorithm. The time-based quintic polynomial function is also introduced to model the LC trajectory. Simulation results demonstrate that the driving style adaptive LC trajectory model can meet the driver's personalized LC needs very well. On the other hand, in order to develop cooperative LTP algorithm, a multi-vehicle cooperative automated LC trajectory planning algorithm is proposed in Luo et al. (*4)*. Due to multiple cars are involved at the same time, the cooperative safety spacing model is proposed to guarantee and improve the safety of the vehicles. The idea of solving the optimal trajectory equation is similar to that used in the above research. Different from previous researches, the prediction of the states of surrounding vehicles is also integrated into the LTP algorithm so as to reduce the risk of possible collisions. In this regard, the prediction of the leading vehicle (using the hidden Markov model) is integrated in the LTP algorithm to avoid collisions(*7*). The collision area of the preceding vehicle is defined to tolerate the disturbances and uncertainties. Then, the generated trajectory is not allowed to cross the collision area, and thus obtaining the final optimal trajectory.

Although the above-mentioned studies have provided valuable insights into developing LTP algorithm, these existing studies all share a common flaw. The algorithms adopted in these studies can all be viewed as SLTP (static LTP) algorithm(*1*), which fails to react to the changes of the states of surroundings. Specifically, these algorithms simply assume that the surrounding vehicles maintain a constant speed during the LC of the subject vehicle(*1*). At the same time, the autonomous vehicle plans the LC trajectory only once, which is based on the state of surroundings before the execution of LC. Nevertheless, in the actual traffic flow environment, the states of the surrounding vehicles may change randomly (i.e. suddenly accelerates or decelerates), if the autonomous vehicle drives along with the original LC trajectory, there may be a traffic accident, or even death or injury(*1*).

In order to bridge this research gap, the DLTP (dynamic LTP) algorithm is proposed byYang et al. (*1)*, which is capable of re-planning the LC trajectory at every certain time (planning step size). A comprehensive LC trajectory model is presented including trajectory decision module, trajectory generation module and starting-point determination module. As for the trajectory planning part, Yang et al. (*1)* assumes that the lateral position of the vehicle is a cubic polynomial function of the longitudinal position of the vehicle(*1*), which is quite different from the methods used in the above studies. This algorithm will make adaptive adjustments to its LC trajectory according to the changes of the state of the surroundings. Finally, this algorithm is verified using the field LC data and Carsim simulation. And the results demonstrate that the proposed DLTP algorithm can provide more safer trajectory for autonomous vehicle than traditional SLTP algorithm. Although this research has indeed achieved certain research results in developing DLTP algorithm, this research can be further improved from the following points. The first improvement can be the LC trajectory curve equation form selected in this article. There may not exist a cubic polynomial mapping relationship between the lateral movement distance and the longitudinal movement distance in real traffic flow environment. Consequently, simply adopting this curve equation form may fail to accurately model the LC trajectory in real traffic flow environment. Meanwhile, as one of the most important parameters in DLTP algorithm, the parameter of planning step size is not well introduced and analyzed. Moreover, the trajectory information at each planning step is also not well presented in detail.

Therefore, the objective of this paper is to address the above research gaps, and present our research progress in modeling DLTP algorithm. The paper's main contributions can be stated in two headlines: first, we introduce the time-based quintic polynomial function to develop the DLTP algorithm, which is the most commonly used equation form in the existing literatures. Meanwhile, to the best of our knowledge, this is the first time we adopt this underlying curve equation form to develop the DLTP algorithm. Second, a comprehensive numerical simulation, including the detailed analysis of the parameter of planning step size, has been conducted to verify the effectiveness of the proposed algorithm.

MODEL FRAMEWORK

A typical LC schematic involves five vehicles, including the ego vehicle and the surrounding four vehicles. For the convenience of subsequent research, we denote these five vehicles as $S_{ego}, S_1, S_2, S_3, S_4$. The schematic diagram of DLTP of the autonomous vehicle is given in **Figure 2.** The typical LC process can be divided into two regions: (1) Preparation region: when the autonomous vehicle has decided to perform LC (the decision point shown in **Figure 2**), it will still drive forward until there is a safe gap distance appeared in the target lane. This relates to the research of the process of LC decision, which is beyond the research scope of this study. (2) Execution region: when the autonomous vehicle is about to execute LC, it will gradually move from the current lane to the target lane (from the starting point to the ending point as shown in **Figure 2**). This is the main research object of this article. It is worth noting that the trajectory tracking algorithm is beyond the scope of this research, and this part will be completed in the future research. The proposed DLTP algorithm is given in **Table 1**. The input of this algorithm is the perception of the state of surrounding vehicles, and the output is the real-time state of the ego vehicle.

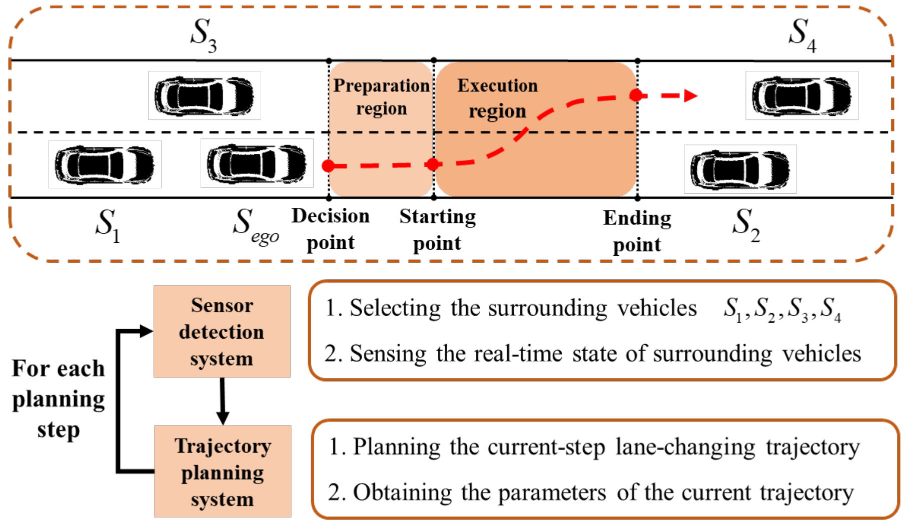


Figure 2 The schematic diagram of DLTP of the autonomous vehicle

Table 1 The proposed DLTP algorithm

Algorithm 1 Dynamic LC trajectory planning algorithm

Input: Real-time perception of the state of the surrounding vehicles.
Output: Real-time longitudinal and lateral state of the ego vehicle.
Determining the t_{pss}, D_0 in the proposed algorithm.

If $\sum_{1}^{k}\sum_{0}^{t_{pss}} y_k(t) \le D_0$ (k represents the k^{th} planning step) **then**

  For k in $1, 2, \ldots, m$, **do** (m represents the final planning step)

    Planning the trajectory for the k^{th} planning step

$$\begin{cases} x_k(t) = a_{0,k} + a_{1,k}t + a_{2,k}t^2 + a_{3,k}t^3 + a_{4,k}t^4 + a_{5,k}t^5 \\ y_k(t) = b_{0,k} + b_{1,k}t + b_{2,k}t^2 + b_{3,k}t^3 + b_{4,k}t^4 + \mathrm{b}_{5,k}t^5 \end{cases}$$

    Determining the initial and final state of the ego vehicle at current planning step.
    Determining the objective function and corresponding constraints at current planning step.
    Solving function $J_k(t_{fin,k}, x_{fin,k}) \rightarrow$ current curve coefficients, $t_{fin,k}$ and $x_{fin,k}$.

    Generating k^{th} trajectory $\rightarrow$ calculating remaining lateral distance $D_0 - \sum_{1}^{k}\sum_{0}^{t_{pss}} y_k(t)$

    If remaining LC duration nearly equals to planning step size **then**
      break
  break
  End
End

The core parameter of this algorithm is the planning step size (t_{pss}). t_{pss} is introduced for re-planning the LC trajectory every t_{pss} second interval, thus enabling the autonomous vehicle to adjust its LC trajectory in real-time according to the surrounding driving environment proactively. t_{pss} is one of the most important parameters in the proposed DLTP algorithm. It is worth noting that an excessively large planning step size may make it difficult for autonomous vehicles to cope with the dynamic changes of the states of the surrounding vehicles in real time, thereby causing a risk of traffic accidents. On the other hand, an excessively small planning step size may lead to the increase of the calculation and processing costs of autonomous vehicles, however this could reduce the risk of traffic accidents. Meanwhile, the planning step size can be adjusted according to the real-time state of the driving surroundings. If the LC conditions are comfortable (i.e. no surrounding vehicles or too far apart), the autonomous vehicle could choose a large planning step size. On the other hand, the autonomous vehicle could choose a small planning step size when the LC conditions is more complicated. The lateral movement distance D_0 represents the lateral distance between the current lane and the target lane, which could be obtained before executing the LC maneuver.

After determining the above two parameters, for each planning step, the time-based quintic polynomial function is introduced to model the LC trajectory, and the corresponding parameter values are obtained through solving an optimization problem. Finally, the trajectory information for each planning step can be calculated. In the next few subsections, we will present detailed explanation of these components in the proposed algorithm.

Modeling the LC trajectory

For the k^{th} planning step, the quintic polynomial function is introduced to model the LC trajectory, which exhibits better performance in fitting the LC trajectory and has been widely used in the existing studies (*14; 18*). The longitudinal and lateral trajectory with respect to time is defined as below (**Equation 1**):

$$\begin{cases} x_k(t) = a_{0,k} + a_{1,k}t + a_{2,k}t^2 + a_{3,k}t^3 + a_{4,k}t^4 + a_{5,k}t^5 \\ y_k(t) = b_{0,k} + b_{1,k}t + b_{2,k}t^2 + b_{3,k}t^3 + b_{4,k}t^4 + \mathrm{b}_{5,k}t^5 \\ \theta_k(t) = ar\tan\left(\dot{y}_k(t) / \dot{x}_k(t)\right) \end{cases} \tag{1}$$

Where $x_k(t), y_k(t), \theta_k(t)$ denote the longitudinal position, lateral position and course angle of the ego vehicle. $a_{i,k}, i = 0,1,...5$ and $b_{j,k}, j = 0,1,...5$ are the corresponding coefficients of the polynomial curve.

At the first planning step, it is reasonable for us to assume that the velocity and the acceleration of the ego vehicle are desired to be zero at the start and end state of LC. Therefore, we could simplify the above formula to get the LC trajectory at the first planning step (**Equation 2**).

$$\begin{cases} x_1(t) = \dot{x}(t_{ini})t - \left[\dot{x}(t_{ini})t_{fin,1} - x_{fin,1}\right]\left[6(\dfrac{t}{t_{fin,1}})^5 - 15(\dfrac{t}{t_{fin,1}})^4 + 10(\dfrac{t}{t_{fin,1}})^3\right] \\ y_1(t) = 6D_0(\dfrac{t}{t_{fin,1}})^5 - 15D_0(\dfrac{t}{t_{fin,1}})^4 + 10D_0(\dfrac{t}{t_{fin,1}})^3 \end{cases} \tag{2}$$

Where t_{ini} denotes the initial time of LC, $\dot{x}(t_{ini})$ denotes the initial speed of LC and D_0 denotes the lateral distance. It can be found that the simplified formula contains only two unknown parameters: $t_{fin,1}$ and $x_{fin,1}$. As for the other planning steps, it is reasonable to assume that the position and speed of the ego vehicle should be continuous at the intersection of the adjacent LC trajectory curves. Actually, the trajectory formula at any planning step can be reduced to contain only two parameters ($t_{fin,k}$ and $x_{fin,k}$) if we want to, since we are able to obtain the state of the autonomous vehicle at the start and end points of current planning step.

Optimization objective function

During the process of LC, the autonomous vehicle hopes that the driver could have a high level of comfort, so we hope the acceleration or the deceleration of the autonomous vehicle changes smoothly rather than significantly. Hence, we introduce the variable of jerk serving as a measure of driver's comfort, which is defined as the rate of change of acceleration or deceleration. As mentioned in the previous subsection, for the k^{th} planning step, the trajectory formula at any planning step can be reduced to contain only two unknown parameters $t_{fin,k}$ and $x_{fin,k}$.

Therefore, we define the formula of comfort cost function $J_k^{comfort}(t_{fin,k}, x_{fin,k})$ (**Equation 3**), which is composed of longitudinal and lateral comfort parts. The numerator is the sum of the discomfort felt by the driver within the remaining lane-changing duration, and the denominator is the product of the maximum acceleration and jerk of the autonomous vehicle.

$$J_k^{comfort}(t_{fin,k}, x_{fin,k}) = \frac{\int_{t_{ini}+(k-1)t_{pss}}^{t_{fin,k}} j_{x,k}(t)^2 \, dt}{j_{x,\max} * a_{x,\max}} + \frac{\int_{t_{ini}+(k-1)t_{pss}}^{t_{fin,k}} j_{y,k}(t)^2 \, dt}{j_{y,\max} * a_{y,\max}} \quad (3)$$

Where $k = 1, 2, 3, ..., m$ denotes the current planning step. m denotes the total planning steps. $j_{x,k}$, $j_{y,k}$ denote the longitudinal and lateral jerk of the autonomous vehicle. $j_{x,\max}$, $j_{y,\max}$ denote the maximum and minimum jerk. $a_{x,\max}$, $a_{y,\max}$ denote the maximum and minimum acceleration.

On the other hand, the autonomous vehicle hopes to complete the LC maneuver as soon as possible to minimize the impact of LC on the surrounding vehicles or avoid excessive occupation of the surrounding road resources. At the initial point of trajectory planning, the lateral movement distance is known, while the longitudinal movement distance will vary with the parameters of the trajectory curve. Hence, we introduce the ratio of the longitudinal movement distance and lateral movement distance as a measure of LC efficiency of the autonomous vehicle. The higher the ratio, the lower the efficiency of the autonomous vehicle to complete the LC.

The formula of efficiency cost function $J_k^{efficiency}(t_{fin,k}, x_{fin,k})$ is given in **Equation 4**, which is composed of the remaining longitudinal and lateral distance of the autonomous vehicle.

$$J_k^{efficiency}(t_{fin,k}, x_{fin,k}) = \frac{x_{fin,k} - \sum_{1}^{k\text{-}1} \sum_{0}^{t_{pss}} x_k(t)}{D_0 \text{-} \sum_{1}^{k\text{-}1} \sum_{0}^{t_{pss}} y_k(t)} \quad (4)$$

Where D_0 denotes the lane width. $\sum_{1}^{k\text{-}1} \sum_{0}^{t_{pss}} x_k(t)$ denotes the longitudinal moving distance with respect to the start point. $\sum_{1}^{k\text{-}1} \sum_{0}^{t_{pss}} y_k(t)$ denotes the lateral moving distance with respect to the start point.

Therefore, we define $J_k^{total}(t_{fin,k}, x_{fin,k})$ as the total cost function of the autonomous vehicle at the k^{th} planning step (**Equation 5**).

$$J_k^{total}(t_{fin,k}, x_{fin,k}) = \omega_1 * J_k^{comfort}(t_{fin,k}, x_{fin,k}) + \omega_2 * J_k^{efficiency}(t_{fin,k}, x_{fin,k}) \quad (5)$$

Where ω_1 and ω_2 are the corresponding weight coefficients of these two parts, which reflect the trade-off between the comfort part and efficiency part. It is worth noting that these two values may vary differently among different drivers and different driving environments. Different settings for these two values may lead to different optimal objective function values.

For the k^{th} planning step, there are twelve coefficients needed to be determined in the above quintic polynomial function. Thus, we transform the problem of solving these coefficients into an optimization problem. And it is reasonable for us to assume that the autonomous vehicle attempts to minimize the discomfort and inefficiency for the current trajectory planning step.

$$\min J_k^{total}(t_{fin,k}^*, x_{fin,k}^*) = \omega_1 * J_k^{comfort}(t_{fin,k}^*, x_{fin,k}^*) + \omega_2 * J_k^{efficiency}(t_{fin,k}^*, x_{fin,k}^*) \quad (6)$$

Where $t_{fin,k}^*$ and $x_{fin,k}^*$ are optimal corresponding parameters when the total cost function is the lowest. It is worth noting that when solving this optimization problem, the corresponding constraint must be considered (It will be elaborated in next subsection).

Constrains for the vehicle

When solving this optimization problem, it is inevitable for us to consider the corresponding constraints: Speed constraint, Stability and comfort constraints; Safety constraint.

(1) Speed constraint: the speed of the autonomous vehicle should not exceed the maximum speed limit and should exceed the minimum speed limit. The formula of the speed constraint is given below in **Equation 7**.

$$\begin{cases} v_{\min} \le \dot{x}(t) \le v_{\max} \\ v_{\min} \le \dot{y}(t) \le v_{\max} \\ v_{\min} \le \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2} \le v_{\max} \end{cases} \quad (7)$$

Where $v_{\min}$ represents the minimum speed limit and $v_{\max}$ represents the maximum speed limit.

(2) Stability and comfort constraints: the acceleration and the jerk of the autonomous vehicle should within the reasonable range. The corresponding constraint formula is given below in **Equation 8.**

$$\begin{cases} a_{\min} \le \ddot{x}(t) \le a_{\max} \\ a_{\min} \le \ddot{y}(t) \le a_{\max} \\ j_{\min} \le \dddot{x}(t) \le j_{\max} \\ j_{\min} \le \dddot{y}(t) \le j_{\max} \end{cases} \quad (8)$$

Where $a_{\min}$, $a_{\max}$, $j_{\min}$, $j_{\max}$ denote the maximum acceleration, minimum acceleration, minimum jerk and maximum jerk respectively.

(3) Safety constraint: we must ensure that autonomous vehicle do not collide with the surrounding vehicles at any time. The definition of the collision boundary area is shown in **Figure 2**. The l_a, l_b, C_a, C_b are defined as the vehicle length, vehicle width, ellipse long radius and ellipse short radius respectively.

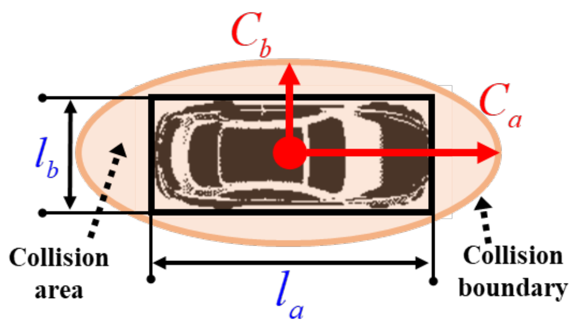


Figure 3 The boundary of the collision area of the ego vehicle

Taking the starting point of LC as the origin of coordinates, suppose at time t, let $P_j(t) = \left(x_j(t), y_j(t)\right), j = S_{ego}, S_1, S_2, S_3, S_4$ denotes the center position of the vehicle j. The real-time boundary of collision area of each vehicle is defined as $G_j(x, y)$.

$$\begin{cases} M^2 / {C_a}^2 + N^2 / {C_b}^2 = 1 \\ M = \left(x - x_j(t)\right) * \cos\theta_j - \left(y - y_j(t)\right) * \sin\theta_j \\ N = \left(x - x_j(t)\right) * \sin\theta_j + \left(y - y_j(t)\right) * \cos\theta_j \end{cases} \tag{9}$$

It is worth noting that the four corners of the smallest circumscribed rectangle of the vehicle outline should fall on the ellipse or with the ellipse (**Equation 10**). If these four points fall exactly on the ellipse boundary, we can consider the rectangle to be the largest inscribed rectangle of this ellipse.

$$ {l_a}^2 / {C_a}^2 + {l_b}^2 / {C_b}^2 \geq 4 \tag{10}$$

The real-time minimum distance between two collision boundary ellipses should be greater than the corresponding threshold MSS_0 (Minimum Safe Space). The reason for setting this threshold is to avoid collision if emergency situation occurs. If the value of MSS_0 is too small, it is difficult for the ego vehicle to cope with unexpected situations, otherwise, the efficiency of LC may be affected.

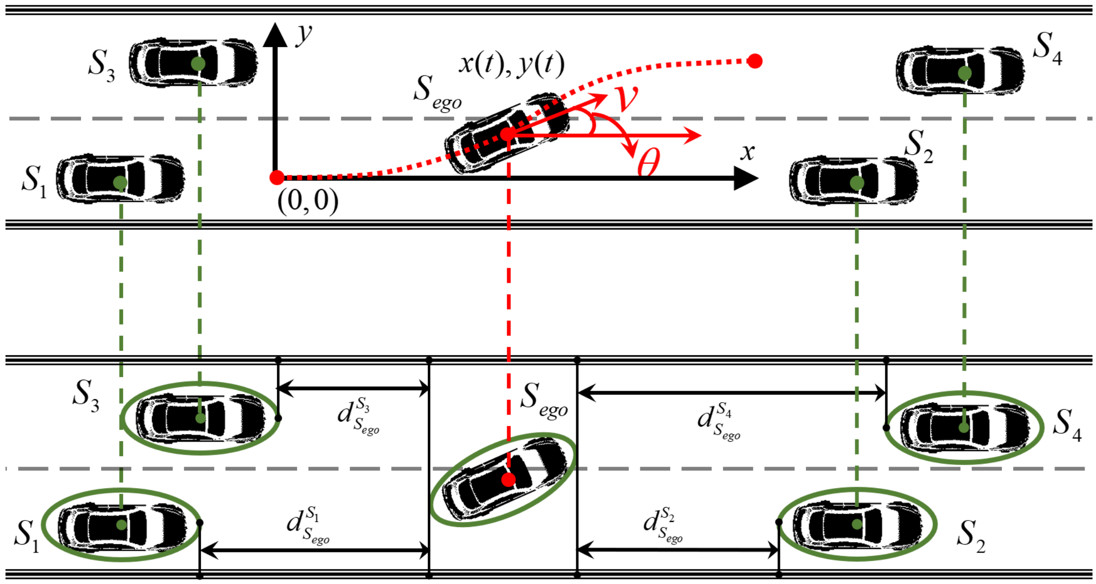


Figure 4 The real-time minimum distance between the current-lane rear vehicle and the autonomous vehicle

SQP algorithm

To get the solution of the above optimization objective function, we introduce the SQP algorithm. The idea of the SQP algorithm is to transform the nonlinear optimization problem with equality and inequality constraints into a quadratic programming problem. The optimization problem of general nonlinear constraints can be expressed as:

$$\min_{x \in R} f(x) \\ s.t. \begin{cases} c_i(x) = 0 \\ c_j(x) \geq 0 \end{cases} \tag{11}$$

Where $f(x)$ refers to the **Equation 5**. x denotes the corresponding variables. $c_i(x)$ denote the equality constraints and $c_j(x)$ denotes the inequality constraints.

After the initial point is given, the Lagrange equation of the objective optimization function is approximated quadratically, and then the subproblem of quadratic programming is obtained as below.

$$\min_{x \in R} \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d \\ s.t. \begin{cases} c_i(x_k) + \nabla c_i(x_k)^T d = 0 \\ c_j(x) + \nabla c_j(x_k)^T d \geq 0 \end{cases} \tag{12}$$

Where H_k denotes the positive definite approximation of $\nabla_{xx}^2 L(x_k, \lambda_k)$. k denotes the current number of iterations. The quasi-Newton method can be used to approximate the solution of the quadratic programming subproblem as the search direction of the next iteration, thus converging to the final solution set.

NUMERICAL SIMULATION

In this section, we will conduct numerical simulation to verify the effectiveness of the proposed

DLTP algorithm. The simulated parameters used in this section is given in **Table 2**. This section mainly contains three parts of numerical simulation.

(1) Prelimiary analysis of the proposed DLTP algorithm.

(2) Sensitivity analysis of the parameters in the proposed DLTP algorithm.

(3) Comparison between the proposed DLTP algorithm and the traditional SLTP algorithm.

Table 2 The numerical parameters used in this section

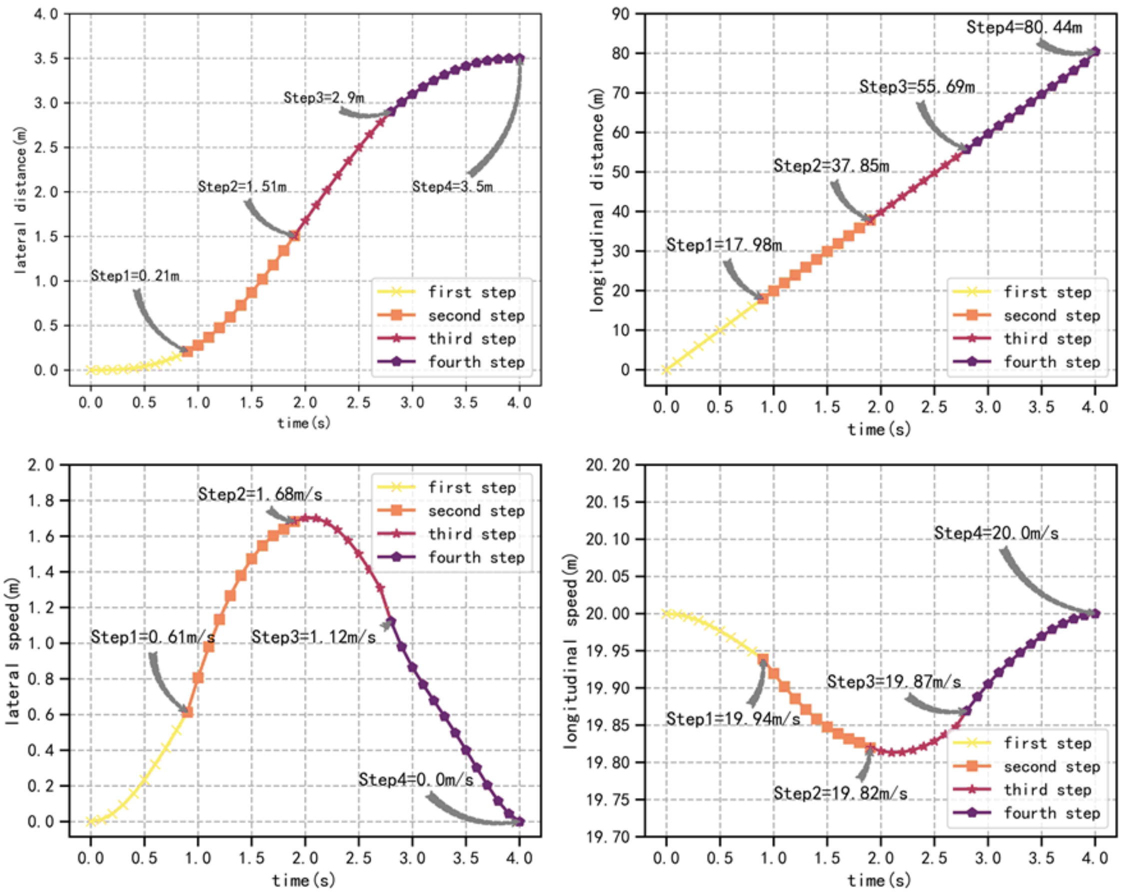
Variable	Denotation	Value	Variable	Denotation	Value
Vehicle length	l_a	$5m$	Maximum acceleration	$a_{\max}$	$8m/s^2$
Vehicle width	l_b	$2m$	Minimum acceleration	$a_{\min}$	$-8m/s^2$
Ellipse long radius	C_a	$2.5m$	Maximum jerk	$j_{\max}$	$8m/s^3$
Ellipse short radius	C_b	$1m$	Minimum jerk	$j_{\min}$	$-8m/s^3$
Maximum speed	$v_{\max}$	$30m/s$	Lateral moving distance	D_0	$3.5m$
Minimum speed	$v_{\min}$	$5m/s$	Minimum safe space	MSS_0	$5m$

Preliminary analysis

In this scenario, we assume that the initial speed of the $S_{ego}, S_1, S_2, S_3, S_4$ vehicles are all $20m/s$,the surrounding vehicles main the constant speed during the process of LC, the planning step size t_{pss} is set as $1s$ and the value of comfort weight coefficient is set as 0.5. The initial longitudinal relative position between the surrounding vehicles and the ego vehicle is all $50m$ .The preliminary numerical result is given in **Figure 4** with detailed trajectory planning information.

It can be found that there are total four steps of lane-changing trajectory planning and the total lane-changing duration is about $4s$. At the end of the first-step trajectory planning, the autonomous vehicle arrives at $(17.98m, 0.21m)$. Then, the autonomous vehicle re-plans its second-step trajectory planning and arrives at $(37.85m, 1.51m)$. After the third-step and fourth-step trajectory planning, the autonomous vehicle finally reaches the center line of the target lane. Meanwhile, the lateral speed of the vehicle gradually increases to $1.7m/s$ and then gradually decreases to $0m/s$. The longitudinal speed drops slightly, but it is always above $20m/s$. The cost value $\left(J^{total}, J^{comfort}, J^{efficiency}\right)_k$ at each planning step is $(15.22, 5.09, 25.34)_{k=1}$, $(13.43, 7.18, 19.69)_{k=2}$, $(13.2, 5.84, 20.56)_{k=3}$, $(22.58, 8.05, 37.1)_{k=4}$ respectively.

Overall, the above preliminary numerical simulation verifies the effectiveness of the proposed DLTP algorithm. This algorithm indeed achieves the effect of dynamically planning the LC trajectory. There are total four steps of trajectory planning in this simulation scenario.



	first step	second step	third step	fourth step
Total cost value	**15.22**	**13.43**	**13.2**	**22.58**
Comfort cost value	**5.09**	**7.18**	**5.84**	**8.05**
Efficiency cost value	**25.34**	**19.69**	**20.56**	**37.1**

Figure 5 Preliminary numerical result of the proposed algorithm (four planning steps in this scenario)

Sensitivity analysis

In this subsection, we conduct sensitivity analysis on the parameters in the proposed algorithm. The parameters we selected mainly include: planning step size (t_{pss}), the weight coefficient (ω_1), initial speed of the ego vehicle and different initial relative position with surrounding vehicles.

(1) Planning step size: **Table 3** and **Figure 6** presents the sensitivity analysis results of the planning step size. It can be found that with the decrease of the planning step size (from 0.5s to 2s), there are more steps that the autonomous vehicle needed to plan. When the t_{pss} equals to 0.5s, there are nine steps of LC trajectory planning. When the t_{pss} equals to 2s, there are only two steps of trajectory planning. It is interesting to find that with the increase of the planning step size, the final longitudinal position of the autonomous vehicle also increases. The corresponding total cost value, comfort cost value and efficiency cost value at each planning step is also detailed presented in **Table 3. Figure 6** presents the average total cost, average comfort cost and average efficiency cost of the autonomous vehicle. The baseline histogram represents the results of the traditional SLTP algorithm, and the other histograms represent the results of the DLTP algorithm under different planning step size. It can be found that the difference between different planning step size is not so significant in the above three costs. The average comfort cost remains at about

6, the average efficiency cost remains at about 25 and the average total cost remains at about 25. When t_{pss} equals 0.5s, the autonomous vehicle has the highest costs (total, comfort, efficiency), and when t_{pss} equals 2s, the autonomous vehicle has the lowest costs.

(2) Weight coefficient: **Figure 7** presents the sensitivity analysis results of the weight coefficient. It can be found that: when ω_1 equals to 0.1, it means that the driver is most concerned about LC efficiency compared to LC comfort. At that time, the corresponding average comfort cost and average efficiency cost is 36.96 and 17.52 separately. The comfort cost is twice that of efficiency cost. Thereafter, with the gradually increase of ω_1 from 0.1 to 0.9, the driver becomes more concerned about LC comfort. What follows is the increase of the LC duration and the final longitudinal distance. The LC duration gradually increase from 2.7s to 5.4s and the final longitudinal distance gradually increase from 57.32m to 111.46m. Meanwhile, the corresponding average comfort cost gradually decreases from 36.96 to 1.35, and average efficiency cost gradually increases from 17.52 to 46.44.

(3) Initial speed of the ego vehicle: **Figure 8** presents the sensitivity analysis results of the initial speed of the ego vehicle. it can be found that: with $\dot{x}(t_{ini})$ increase from 15m/s to 30m/s, the LC duration gradually decreases from 4s to 3.6s. Meanwhile, the final longitudinal position also increases from 69.69m to 120.68m, the corresponding average comfort cost gradually increases from 4.00 to 8.33, and the corresponding average efficiency cost gradually increases from 19.91 to 34.48.

(4) Initial relative position with surrounding vehicles: **Figure 9** presents the sensitivity analysis results of the initial relative position with surrounding vehicles. It can be found that: with the distance the between surrounding vehicles (we choose the target front vehicle and target rear vehicle as the subject vehicles.) and the autonomous vehicle becomes smaller (from 11m to 5m), the autonomous vehicle needs more time to complete its LC maneuver and the farther the vehicle moves in the longitudinal direction. It is also interesting to find that the comfort cost and efficiency cost also increase with the decrease of the distance between the autonomous vehicle and the surrounding vehicles.

Overall, in this subsection, we present comprehensive sensitivity analysis of the parameters in the proposed algorithm.

Table 3 Sensitivity analysis result of planning step size (trajectory information for each planning step)

t_{pss}	Step	Final longitudinal position(m)	Remaining lcd(s)	Total cost value	Comfort cost value	Efficiency cost value
0.5s	1	88.71	4.45	15.22	5.09	25.34
	2	83.91	3.81	13.04	4.24	21.83
	3	77.33	3.09	14.98	11.38	18.58
	4	72.35	2.44	19.25	21.50	17.00
	5	70.60	1.95	15.89	13.45	18.34
	6	71.89	1.62	14.44	3.95	24.94
	7	73.84	1.31	28.65	16.55	40.75
	8	73.76	0.91	39.49	13.49	65.48
0.8s	1	88.71	4.45	15.22	5.09	25.34
	2	82.87	3.46	13.16	6.02	20.29
	3	78.54	2.55	14.58	10.51	18.64
	4	78.14	1.83	13.50	3.07	23.93
	5	78.48	1.14	28.57	11.83	45.32
1s	1	88.71	4.45	15.22	5.09	25.34

	2	82.80	3.26	13.43	7.18	19.69
	3	80.23	2.23	13.20	5.84	20.56
	4	80.44	1.34	22.58	8.05	37.10
1.5s	1	88.71	4.45	15.22	5.09	25.34
	2	83.72	2.80	13.68	7.85	19.50
	3	83.24	1.38	22.24	7.53	36.95
2s	1	88.71	4.45	15.22	5.09	25.34
	2	85.19	2.38	13.35	5.42	21.28

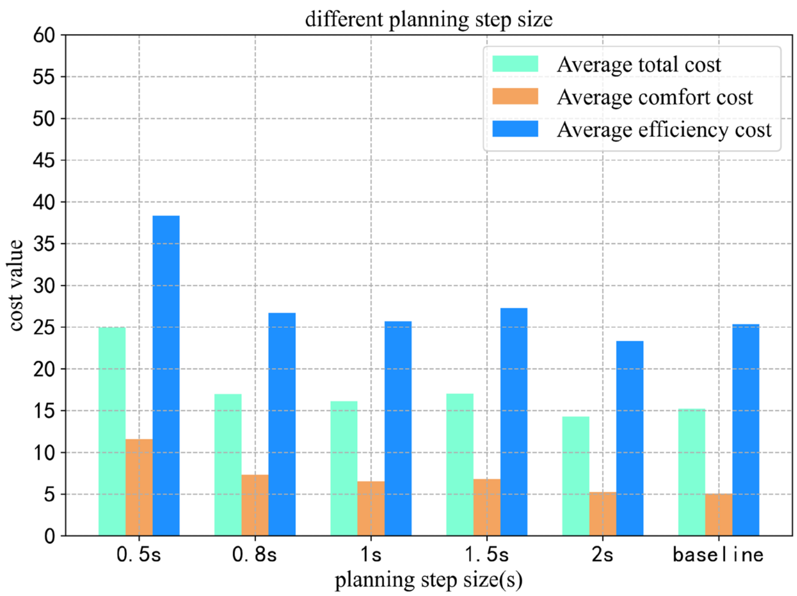


Figure 6 Sensitivity analysis result of the planning step size (average cost for the autonomous vehicle)

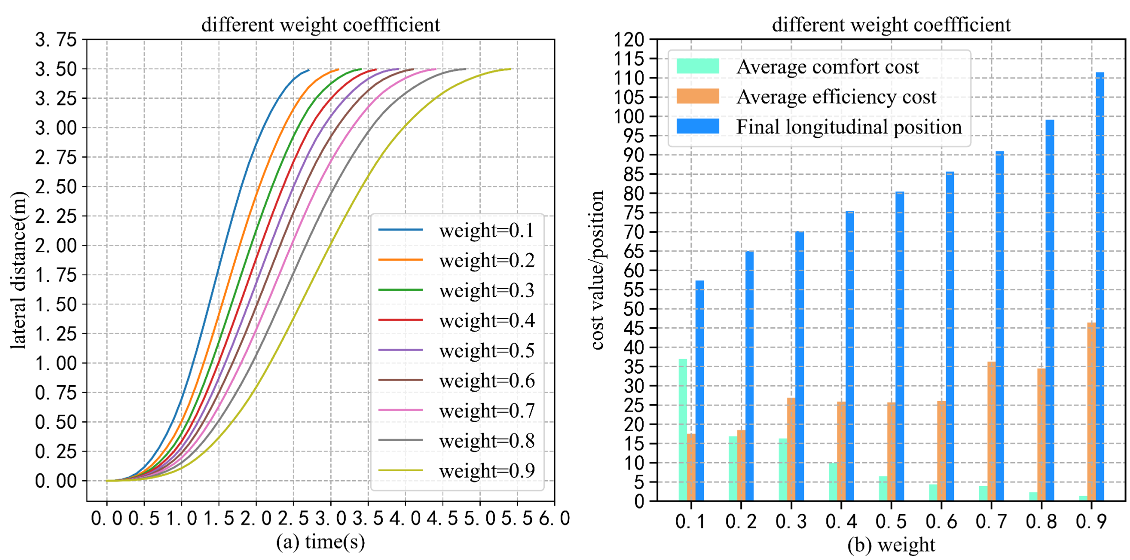


Figure 7 Sensitivity analysis of the weight coefficient of the objective function

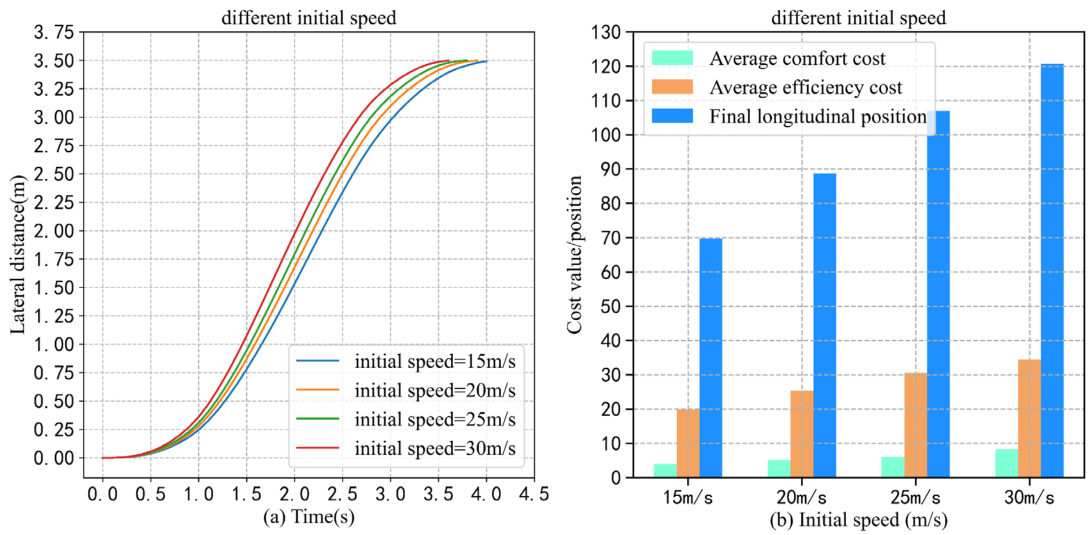


Figure 8 Sensitivity analysis of the initial speed of the autonomous vehicle

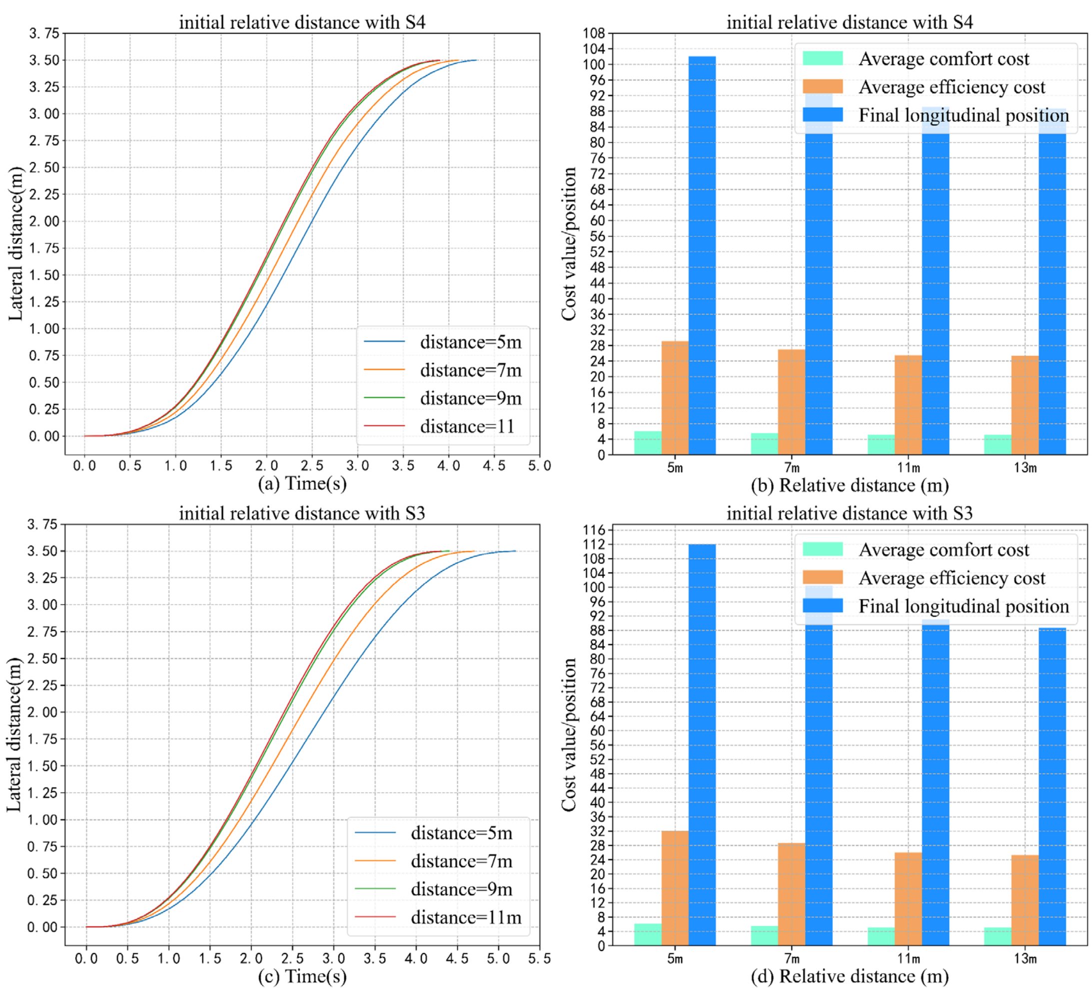


Figure 9 Sensitivity analysis of the initial relative position with surrounding vehicles

Comparison with SLTP

In this subsection, we present the comparison between the traditional SLTP algorithm and the proposed DLTP algorithm.

First, we assume that the planning step size is set as $1.5s$, the initial relative distance is all $20m$. The target front vehicle suddenly decelerates at $3,4,5m/s^2$, and the target rear vehicle suddenly accelerates at $3,4,5m/s^2$ separately. **Figure 10** presents the comparison between the traditional SLTP algorithm and the proposed DLTP algorithm. The blue curve represents the result generated by the traditional SLTP algorithm, and the other three curves represent the results of the proposed DLTP algorithm. When the target rear vehicle suddenly accelerates at $5m/s^2$, the autonomous vehicle increases its longitudinal speed from $19.89m/s$ to $20.9m/s$. The corresponding LC duration decreases from $4.1\mathrm{s}$ to $3.5\mathrm{s}$. When the target front vehicle suddenly decelerates at $5m/s^2$, the autonomous vehicle gradually decreases its longitudinal speed from $19.89m/s$ to $19.2m/s$. Meanwhile, it is interesting to find that in order to ensure the safe completion of LC, a certain degree of comfort has been sacrificed.

Second, to further evaluate the effectiveness of the proposed algorithm, we assume that the initial relative distance with the target front vehicle is $10m$. The target front vehicle suddenly decelerates at $5m/s^2$. The TTC (Time to collision) indicator is introduced to evaluate the real-time risk between the front vehicle and the autonomous vehicle in the longitudinal direction. Simulation results given in **Figure 11** demonstrate that if we adopt the SLTP algorithm to control the autonomous vehicle, the autonomous vehicle will crash with the target front vehicle. Correspondingly, the TTC value gradually decreases from 2s to 0s. While if we adopt the proposed DLTP algorithm, the autonomous vehicle is capable of adjusting its longitudinal speed (decreasing from $20m/s$ to about $15m/s$), thus keeping a safe distance with the target front vehicle. Correspondingly, the TTC value always maintains above $1.58s$.

Overall, the numerical results in this subsection demonstrate the proposed DLTP algorithm can provide safer LC trajectory for the autonomous vehicle that the traditional SLTP algorithm.

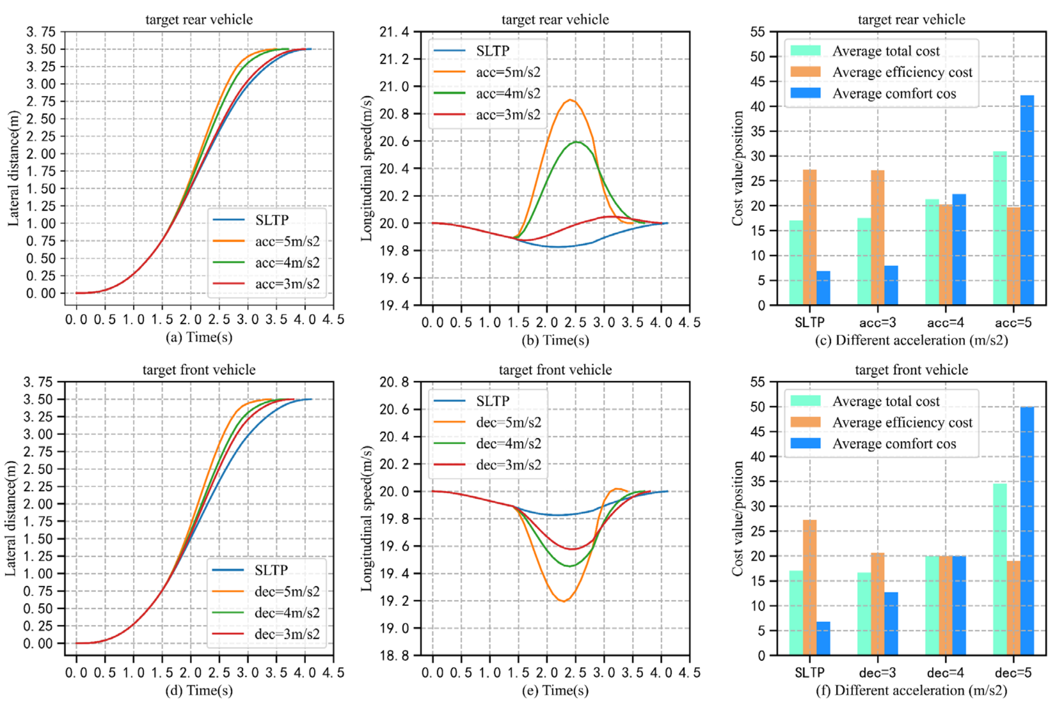


Figure 10 Comparison between the traditional SLTP algorithm and the proposed DLTP algorithm 1

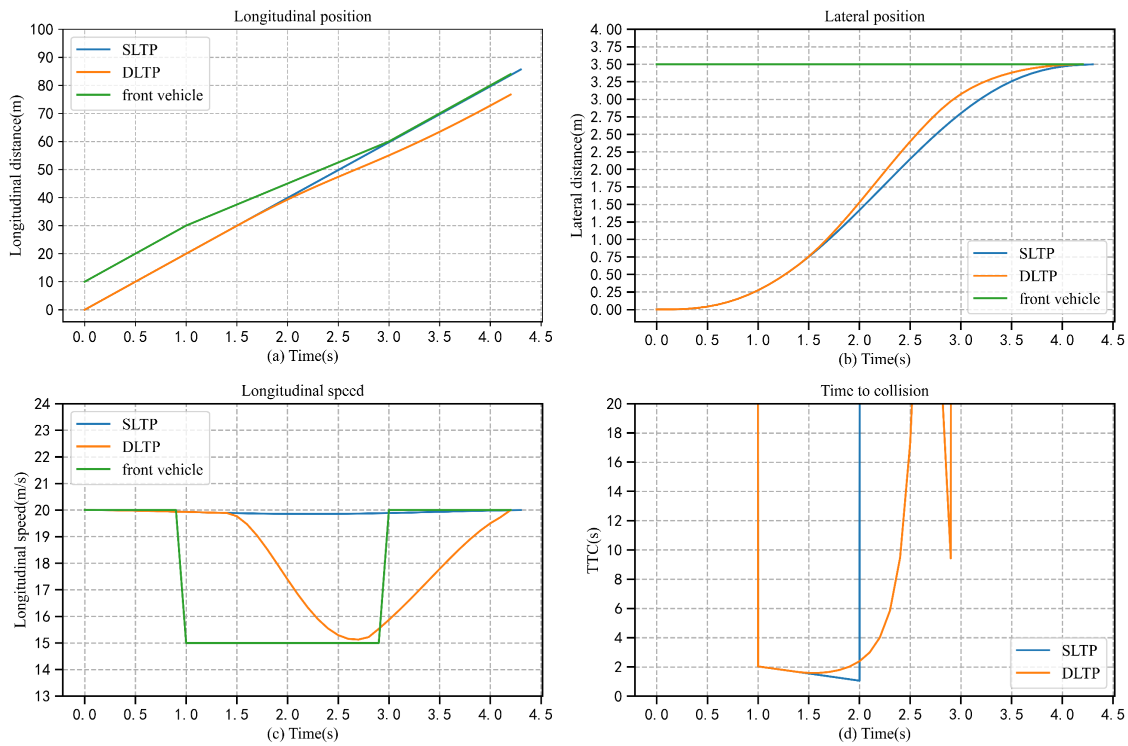


Figure 11 Comparison between the traditional SLTP algorithm and the proposed DLTP algorithm 2

In this section, we have conducted complete numerical simulation on the proposed DLTP algorithm. In summary: (1) We have verified the effectiveness of the proposed DLTP algorithm. The algorithm we proposed can indeed achieve the effect of planning the LC trajectory dynamic. (2) We have carried out comprehensive sensitivity analysis of the parameters in the proposed algorithm. The numerical results are discussed in detail. (3) Compared with the traditional SLTP algorithm, the proposed DLTP algorithm can provide safer LC trajectory for the autonomous vehicle.

DISCUSSION

The LTP algorithms in the existing studies can be mainly considered static, which fail to dynamically adjust its real-time LC trajectory(*1*) according to the states of the surrounding vehicles. Although the DLTP (dynamic LTP) algorithm proposed in Yang et al. (*1*) has shed light on this research regard, there are still some limitations needed improving as mentioned in the “LITERATURE REVIEW”. Therefore, this paper aims to overcome the limitations in the existing literatures and present our current research progress in developing LTP algorithm.

In this study, we have developed a novel DLTP algorithm for the autonomous vehicle. Through introducing the parameter of planning step size to the time-based quintic polynomial function, the proposed algorithm is capable of adjusting its real-time LC trajectory according to the changes of the states of the surrounding vehicles, thus providing safer LC trajectory for the autonomous vehicle. This was, to the best of our knowledge, the first study that develops the DLTP algorithm based on the time-based quintic polynomial function. The main reason why we select this underlying curve equation form is due to its generalization in modeling LC trajectory and its popularization in the existing literatures(*6; 7; 14*). Afterwards, compared with the existing literatures (*1; 2; 6; 14*), we have conducted more comprehensive numerical simulation on the proposed DLTP algorithm.

Although this research has achieved certain results in developing LTP algorithm, many aspects of this novel DLTP algorithm need further research. Due to the limitation of the article length, we only conducted numerical simulation on the proposed algorithm in this study. Ongoing research is extracting the field LC trajectory data to verify the proposed DLTP algorithm, like the NGISM dataset (*19*) and the HighD dataset(*20*). Another subject that remains to be explored is to develop the corresponding LTT (lane-changing trajectory tracking) algorithm, which is also an indispensable part in supporting automated lane-changing for the autonomous vehicle(*6; 7*). Since the main focus of this study is to further bridge the research gaps in developing DLTP algorithm, the corresponding LTT algorithm is not covered in this paper, and it will be completed in the future research of this study. At the same time, the idea in Chen, Hu and Wang (*7)* of incorporating prediction algorithm into the LTP algorithm can also be applied to the DLTP algorithm we proposed in this paper. The optimal LC trajectory, for each planning step, may be different if we consider the future states of the surrounding vehicles into the optimization problem. This may further reduce the risk of collisions with the surrounding vehicles.

CONCLUSIONS

In this study, we have proposed a novel DLTP algorithm for the autonomous vehicle. Specifically, for each planning step, we introduce the time-based quintic polynomial function to model the LC trajectory dynamically. The problem of obtaining the corresponding parameter is then transformed into an optimization problem, which takes driver's safety, comfort and efficiency into account. Thereafter, the effectiveness of the proposed DLTP algorithm has been verified in the numerical simulation. Numerical results indicate that the proposed DLTP algorithm can dynamically adjust its LC trajectory according to the states of the surrounding vehicles, thus providing safer LC trajectory than the traditional SLTP algorithm. The insights gained from this study may be of assistance to develop safer LTP algorithm for the autonomous vehicle.

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AUTHOR CONTRIBUTIONS

The authors confirm contribution to the paper as follows: study conception and design: Linbo Li. Author, Yang Li. Author, Daiheng Ni. Author; analysis and interpretation of results: Daiheng Ni. Author, Yue Zhang. Author; draft manuscript preparation: Yang Li. Author, Linbo Li. Author, Yue Zhang. Author. All authors reviewed the results and approved the final version of the manuscript.

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