# Tutorial No-4

**Ques-1.** $T(n) = 3T(n/2) + n^2$

**Sol** $T(n) = aT(n/b) + f(n^2)$

$a \geq 1, b \geq 1$

On comparing

$a = 3, b = 2, f(n) = n^2$

Now, $c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.534} > n^2$.

$\therefore f(n) > n^2$

$\therefore T(n) = O(n^2)$

**Ques-2.** $T(n) = 4T(n/2) + n^2$.

$a \geq 1, b > 1$

$a = 4, b = 2, f(n) = n^2$

$c = \log_2 4 = 2$

$n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$

**Ques-3** $T(n) = T(n/2) + 2^n$

$a = 1$

$b = 2$

$f(n) = 2^n$.

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$f(n) > n^2$

$\rightarrow T(n) = \theta(2^n)$

**Ques 4.** $T(n) = 2^n T(n/2) + n^2$

$a = 2^n$.

$b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 2^n$.

$= n$.

$n^c = n^n$

$f(n) = n^c$

$f(n) = \Theta(n^2 \log_2 n)$

**Ques 5** $T(n) = 16T(n/4) + n$.

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 (4)^2 = 2\log_4 4$

$= 2$

$n^c = n^2$.

$f(n) < n^c$

$\therefore T(n) = \Theta(n^2)$

**Question 6.** $T(n) = 2T(n/2) + n\log n$.

$a = 2$, $b = 2$

$f(n) = n\log n$.

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n\log n > n$

$f(n) > n^c$

$T(n) = \Theta(n\log n)$

**Ques 7.** $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$.

$\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

**Ques 8.** $T(n) = 2T(n/4) + n^{0.5}$.

$a = 2, b = 4, f(n) = n^{0.5}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.5}$

$f(n) > n^c$

$T(n) = \theta(n^{0.5})$

**Ques 9.** $T(n) = 0.5T(n/2) + 1/n$.

$a = 0.5, b = 2$

$a \geq 1$ but here a is 0.5

So we can't apply master's theorm.

**Ques 10** $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

**Ques 7.** $T(n) = 2T(n/2) + n/\log n$.

$a = 2, b = 2, f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$.

$\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = \theta(n)$

**Ques 8.** $T(n) = 2T(n/4) + n^{0.51}$.

$a = 2, b = 4, f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

$n^{0.5} < n^{0.51}$

$f(n) > n^c$

$T(n) = \theta(n^{0.51})$

**Ques 9.** $T(n) = 0.5T(n/2) + 1/n$.

$a = 0.5, b = 2$

$a \geq 1$ but here $a$ is $0.5$

So we can't apply master's theorem.

**Ques 10** $T(n) = 16T(n/4) + n!$

$a = 16, b = 4, f(n) = n!$

$\therefore c = \log_b a = \log_4 16 = 2$

$n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \theta(n!)$

**Ques 11** $4T(n/2) + \log n$.

$a = 4, b = 2, f(n) = \log n$.

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$.

$\therefore \log(n) < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^c)$

**Ques 12** $T(n) = sqrt(n) T(n/2) + \log n$

$a = \sqrt{n}, b = 2$

$c = \log_b a = \log_2 \sqrt{n} = 1/2 \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log n$

$\therefore f(n) > n^c$

$T(n) = \theta(n^{1.5849})$

**Ques 13** $T(n) = 3T(n/2) + n$.

$a = 3; b = 2 \quad f(n) = n$.

$c = \log_b a = \log_2 3 = 1.5849$.

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$T(n) = \theta(n^{1.5849})$

**Ques 11** $4T(n/2) + \log n$.

$a = 4, b = 2, f(n) = \log n$.

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$f(n) = \log n$.

$\therefore \log(n) < n^2$

$f(n) < n^c$

$T(n) = \theta(n^c)$

$= \theta(n^c)$

**Ques 12** $T(n) = sqrt(n) T(n/2) + \log n$

$a = \sqrt{n}, b = 2$

$c = \log_b a = \log_2 \sqrt{n} = 1/2 \log_2 n$

$\therefore \frac{1}{2} \log_2 n < \log n$

$\therefore f(n) > n^c$

$T(n) = \theta(n^{1.5849})$

**Ques 13** $T(n) = 3T(n/2) + n$.

$a = 3; b = 2 \quad f(n) = n$.

$c = \log_b a = \log_2 3 = 1.5849$.

$n^c = n^{1.5849}$

$n < n^{1.5849}$

$f(n) < n^c$

$T(n) = \theta(n^{1.5849})$

**Ques 14** $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, \; b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$.

$f(n) < n^c$

$T(n) = \theta(n)$

**Ques 15** $T(n) = 4T(n/2) + n$.

$a = 4 \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$.

$n < n^2$ (for any constant.

$f(n) < n^2$.

$f(n) = \theta(n^2$.

**Ques 16** $T(n) = 3T(n/4) + n \log n$.

$a = 3, \; b = 4, \; f(n) = n \log n$.

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n \log n$.

$T(n) = \theta(n \log n)$

**Ques 14** $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, \; b = 3$

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$.

$f(n) < n^c$

$T(n) = \theta(n)$

**Ques 15** $T(n) = 4T(n/2) + n$.

$a = 4 \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$.

$n < n^2$ (for any constant.

$f(n) < n^2$.

$f(n) = \theta(n^2$.

**Ques 16** $T(n) = 3T(n/4) + n \log n$.

$a = 3, \; b = 4, \; f(n) = n \log n$.

$c = \log_b a = \log_4 3 = 0.792$

$n^c = n^{0.792}$

$n^{0.792} < n \log n$.

$T(n) = \theta(n \log n)$

**Ques 17** $T(n) = 3T(n/3) + n/2$

$a = 3 ; b = 3$

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) < n^c$

$\therefore T(n) = \theta(n)$

**Ques 18** $T(n) = 6T(n/3) + n^2 \log n$

$a = 6 ; b = 3$

$c = \log_b a = \log_3 6 = 1.6309$

$n^c = n^{1.6309}$

As $n^{1.6309} < n^2 \log n$

$\therefore T(n) = \theta(n^2 \log n)$

**Ques 19** $T(n) = 4T(n/2) + n \log n$

$a = 4, b = 2, f(n) = n/\log n.$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\frac{n}{\log n} < n^2$

$T(n) = \theta(n^2)$

**Ques 20** $T(n) = 64\,T(n/8) - n^2 \log n$.

$a = 64 \quad b = 8$

$c = \log_b a = \log_8 64 \Rightarrow \log_8 (8)^2$

$c = 2$.

$n^c = n^2$.

$n^2 \log n > n^2$

$T(n) = \theta(n^2 \log n)$

**Ques 21** $T(n) = 7T(n/3) + n^2$

$a = 7 ; \; b = 3 ; \; f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$

$n^c = n^{1.7712}$

$n^{1.7712} < n^2$.

$T(n) = \theta(n^2)$

**Ques 22** $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1 , \; b = 2$

$c = \log_b a = \log_2 1 = 0$

$n^c = n^0 = 1$

$n(2 - \cos n) > n^c$

$T(n) = \theta(n(2 - \cos n))$