Tutorial -2

Ques-1 What is the time complexity of below code and how ?

```
void func (int n)
{
  int j=1, i=0
  while (i<n)
    {
      i+=j;
      j++;
    }
}
```

→ $j=1$ $i=1$

$j=2$ $i=1+2$

$j=3$ $i=1+2+3$

] m-level

for (i)

$\because 1+2+3+ \dots + < n$

$\because 1+2+3+m<n$

$\because \frac{m(m+1)}{2} < n$

$m \simeq \sqrt{n}$

By summation method

$$\sum_{i=1}^{n} 1 \Rightarrow 1+1+\ldots+\sqrt{n} \text{ times}$$

$$\boxed{T(n) = \sqrt{n}} \quad \text{— Ans}$$

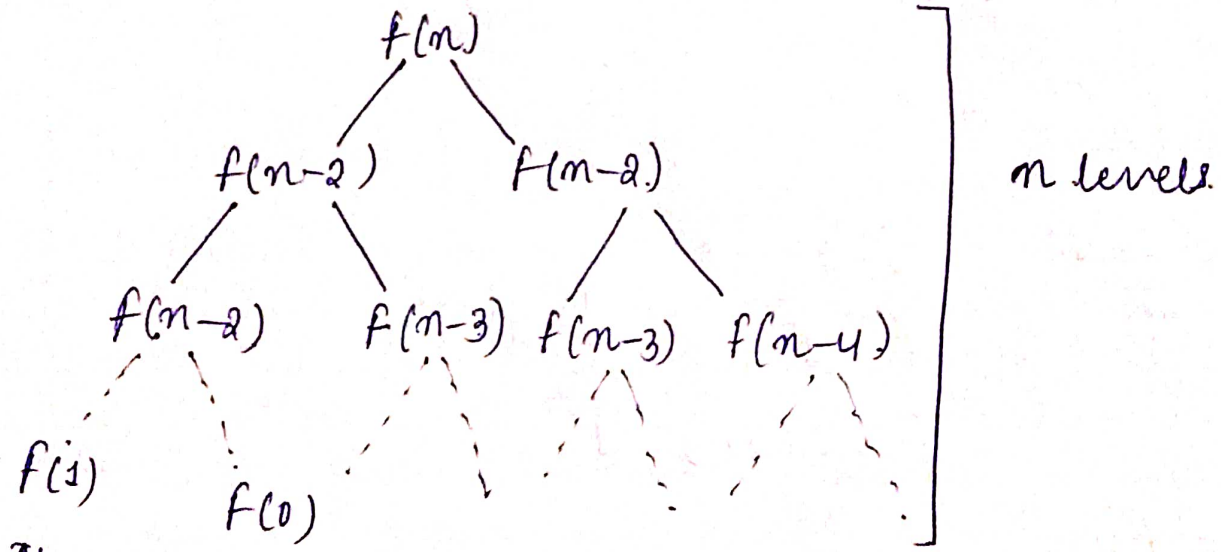
Ques2 Write recurrence relation for function that prints fibonacci series. Solve it to get the time complexity. What will be the space complexity and why?

For Fibonacci series

$$f(n) = f(n-2) + f(n-2)$$

$f(0) = 0$
$f(1) = 1$

By forming a tree



∵ At every function call we get 2 func calls
∴ for n levels

we have $= 2 \times 2 \ldots$ n times

$$\therefore \boxed{T(n) = 2^n}$$

Maximum Space → considering Recursive

Stack:

no. of calls maximum = n

for each call we have space complexity O(1)

$\therefore T(n) = O(n)$

without considering recursive stack;

each call we have time complexity O(1)

$\therefore T(n) = O(1)$

Ques 3 write programs which have complexity:

n(log n), n^3, log(log n)

1) n(log n) — Quick sort

```
Void quicksort (int arr[], int low, int high)
{
  if (low < high)
  {
    int pi = partition (arr, low, high);
    quicksort (arr, low, pi-1)
    quicksort (arr, pi+1, high);
  }
}
int partition (int arr[], int low, int high)
{
    int pivot = arr[high];
    Int i = (low-1);
    for (int j=low; j<= high-1; j++)
    {
      if (arr[i] < pivot)
      { i++;
```

```
        swap(&arr[i], &arr[j]);
      }
    }
    swap(&arr[i+1], &arr[high])
    return (i+1);
}
```

2) n^3 → multiplication of 2 square matrix

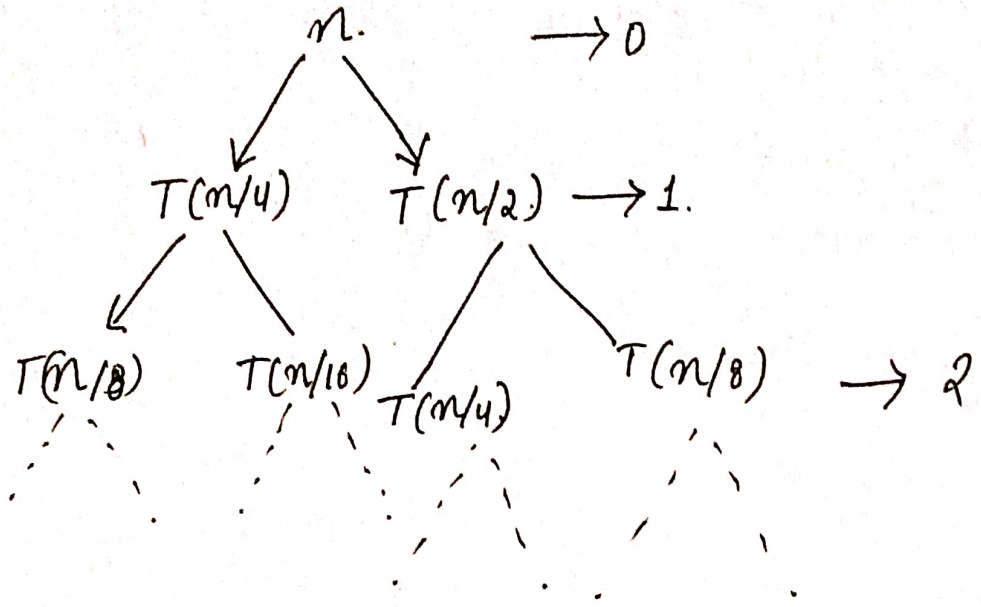
```
for(i=0; i<r1; i++)
  {
    for(j=0; j<c2; j++)
    for(k=0; k<c1; k++)
      {
        res[i][j] += a[i][k] * b[k][j];
      }
```

3) log(log n) →

```
for(i=2; i<n; i=i*i)
  {
    count++;
  }
```

Ques 4 Solve the following recurrence relation

$$T(n) = T(n/4) + T(n/2) + cn^2$$



At level

$0 \rightarrow cn^2$

$1 \rightarrow \frac{n^2}{4^2} + \frac{n^2}{2^2} \Rightarrow \frac{5cn^2}{16}$

$2 \rightarrow \frac{n^2}{8^2} + \frac{n^2}{16^2} + \frac{n^2}{4^2} + \frac{n^2}{8^2} = \left(\frac{5}{16}\right)^2 n^2 c$

$\vdots$

$$\text{max level} = \frac{n}{2^k} = 1$$

$$= k = \log_2 n.$$

$$T(n) = c\left(n^2 + (5/16) + (5/16)^2 n^2 + \dots + (5/16)^{\log n} n^2\right)$$

$$T(n) = cn^2\left[1 + \left(\frac{5}{16}\right) + \left(\frac{5}{16}\right)^2 + \dots + \left(\frac{5}{16}\right)^{\log n}\right]$$

$$T(n) = cn^2 \times 1 \times \left(\frac{1 - (5/16)^{\log n}}{1 - (5/16)}\right)$$

$$T(n) = cn^2 \times \frac{11}{5} \times \left(1 - \left(\frac{5}{16}\right)^{\log n}\right)$$

$T(n) = O(n^2 C)$

$O(Cn^2)$

Ques-3 What is the time complexity of following fun()?

```
int fun (int n){
    for (int i = 1; i<=n; i++){
        for (int j=1; j<n; j+=i){
            // some O(1) task
        }
    }
}
```

for	i	j
	1	1
	2	1 + 3 + 5
	3	1 + 4 + 7
	!	
	n	

$j = (n-1)/i$ times

$$\sum_{i=1}^{n} \frac{(n-1)}{i}$$

$$\therefore T(n) = \frac{(n-1)}{1} + \frac{(n-1)}{2} + \frac{(n-1)}{3} + \dots + \frac{(n-1)}{n}$$

$$T(n) = n\left(1 + 1/2 + 1/3 + \dots + 1/n\right) - 1 \times \left[1 + 1/2 + 1/3 + \dots + 1/n\right]$$

$$= n \log n - \log n$$

$$T(n) = O(n \log n) \quad - \text{Ans}$$

Ques: What should be time complexity of

```
for(int i=2; i<=n; i=pow(i,k))
{
    //some O(1)
}
```

where K is a constant.

For

i
2^1
2^k
2^{k^2}
2^{k^3}
.
.
.
2^{k^m}

where →

$$2^{k^m} < = n.$$

$$k^m = \log_2 n$$

$$m = \log_k \log_2 n$$

$$\sum_{i=1}^{m} * 1$$

$1+1+1+ \ldots$ m times

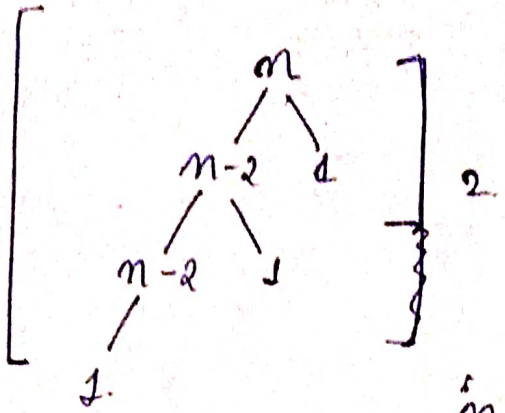
$$T(n) = O(\log_k \log n) \text{ — Ans}$$

Ques-2. Write a recurrence relation when quick sort repeatedly divides array into 2 parts of 99% and 1%. Derive time complexity in this case. Show the recurrence tree while deriving time complexity & find difference in heights of both extreme parts. What do you understand by this analysis?

Given algo divides arr in 99% and 1% part

$\therefore T(n) = T(n-1) + O(1)$

n levels



'n' work is done at each level

$$T(n) = (T(n-1) + T(n-2) + \ldots\ldots + T(1) + O(1)) \times n.$$

$$= n \times n.$$

$$\therefore \boxed{T(n) = O(n^2)}$$

Lowest height = 2

highest height = n

$$\therefore \boxed{\text{difference} = n-2} \quad n \geq 1$$

The given algo produces linear result.

Ques 8 Arrange the following in increasing order of rate growth.

a) $n, n!, \log n, \log\log n, \text{rev}(n), \log(n!), n\log n, \log^2(n), 2^n, 2^{2n}, 4^n, n^2, 100$.

→ $100 < \log\log n < \log n < (\log n)^2 < \sqrt{n} < n < n\log n < \log(n!) < n^2 < 2^n < 4^n < 2^n$

b) $2(2^n), 4n, 2n, 1, \log(n), \log(\log(n)), \sqrt{\log(n)}, \log 2n, 2\log(n), n\log(n!), n!, n2, n\log(n)$

→ $1 < \log\log n < \sqrt{\log n} < \log n < \log 2n < 2\log n < n < n\log n < 2n < 4n < \log(n!) < n^2 < n! < 2^{2n}$

c) $8^{2n}, \log_2(n), n\log_6(n), n\log_2(n), \log(n!), n!, \log_8(n), 96, 8n2, 7n^3, 5n$.

→ $96 < \log_8(n) < \log 2n < 5n < n\log_6(n) < n\log_2 n < \log(n!) < 8n^2 < 7n^3 < n! < 8^{2n}$.