Tutorial No-6

Q1 What do you mean by Minimum spanning Tree? What are the application of MST?

Ans A minimum spanning Tree is a subset of edges of a connected weighted undirected graph that connects all the vertices together with the minimum possi-ble total edge weight.

Applications -

(i) Consider n stations are to be linked using a communication network and lying of communication ~~geo~~ link between any two station involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning Tree.

(ii) Designing LAN.

(iii) Suppose you want to construct highways or railroads spanning several cities. Then we can use the concept of MST.

(iv) laying pipelines connecting offshore drilling sites, refineries of consumer market.

Ques 2. Analyze Time and space complexity of Prim, Kruskal's, Dijkstra and Bellman Ford Algorithm.

Ans Time complexity of Prim's Algorithm : $O(|E| \log |V|)$

~~Space~~ Space complexity of Prim's Algo : $O|V|$

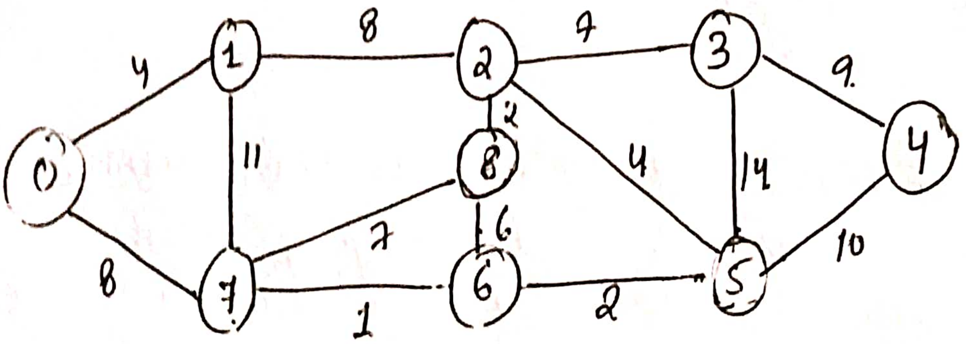
Time complexity of Kruskal's Algorithm : $O|E| \log |E|$

Space complexity of Kruskal's Algorithm : $O|V|$

Time complexity of Dijkstra's Algorithm $= O(V^2)$
Space complexity of Dijkstra's Algorithm $= O(V^2)$
Time complexity of Ford's Algorithm $= O(VE)$
Space complexity of Bellman Ford's Algo $= O(E)$

Question 3.

Apply Kruskal's and Prim's Algorithm on given graph to compute MST and it's weight.

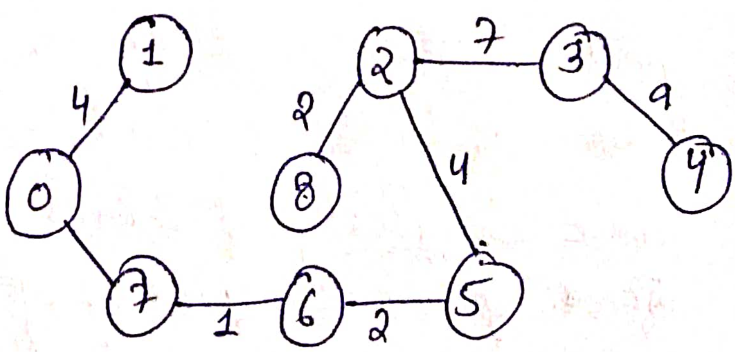


Kruskal's Algorithm

0	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	X
2	3	7	✓
7	8	7	X
0	7	8	✓
1	2	8	X
4	3	9	✓
4	5	10	X
1	7	11	X
3	5	14	X

Prim's Algorithm

weight =
$= 4+8+2+4+2+7+9+3$
$= 37$

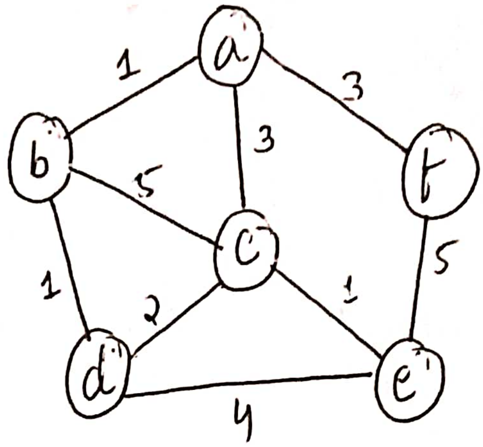


weight $= 1+2+2+4+4+7+8+9$
$= 37$

Ques4. Given a directed weighted graph. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following cases.

(i) If weight of every edge is increased by 10 units.

(ii) If weight of every edge is multiplied by 10 units.



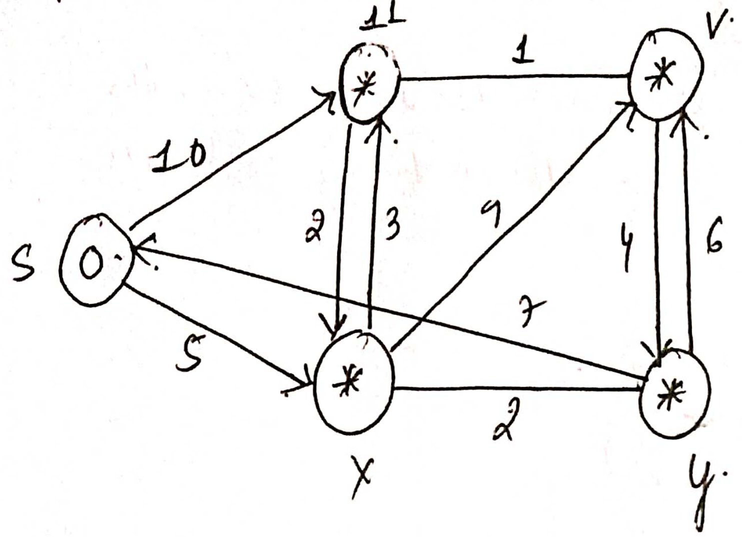
Ans

The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'.

For ex: Let the shortest path of weight 15 and edges 5. Let there be another path with 2 edges and total weight 25. The weight of shortest path is increased by 5*10 and becomes 15+50 weight of other path is increased by 2*10 that becomes 25+50. So the shortest path changes to other path with weight as 45.

(ii) If we multiply all edges weight by 10, the shortest path doesn't change. The reason is that weights of all path from 's' to 't' gets multiplied by same unit. The number of edges or path doesn't matter.

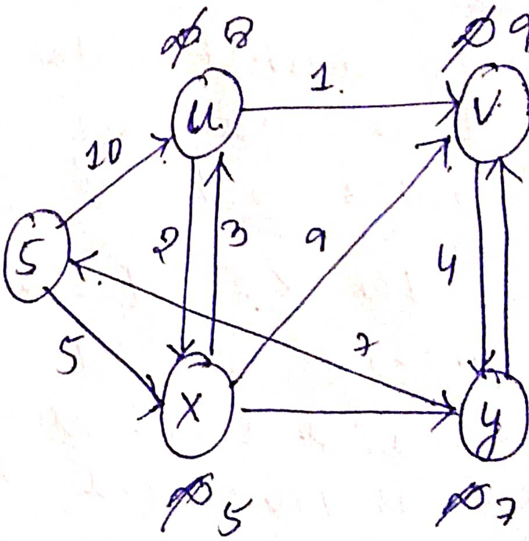
Ques 5. Apply Dijkstra and Bellman Ford's Algorithm on graph given right side to compute shortest path to all nodes from node s.



Ans

Dijkstra's Algorithm :

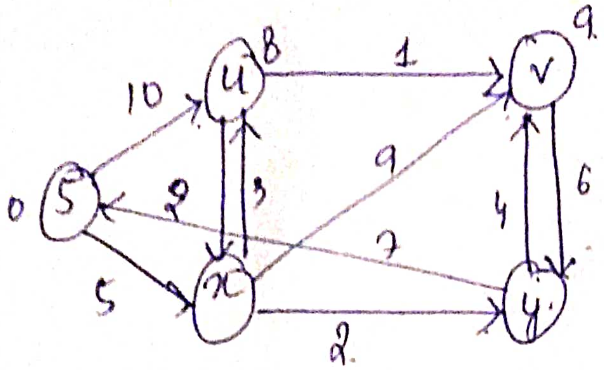
Node	Shortest Distance from source node
u	8
v	9
x	5
y.	7.



Bellman's Ford Algorithm :

	S	u	v	x	y
1st →	0	~~10~~∞.	∞	~~∞~~ 5	∞
2nd →	0	10	~~∞~~ 11	5	∞
3rd →	0	~~10~~ 8	~~11~~ 9	5	~~∞~~ 7
4th →	0	8	9	5	7.

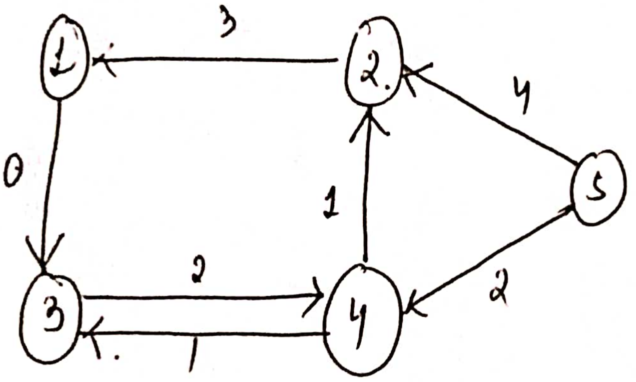
* graph doesn't have negative cycle.



→ final graph.

Question 6.

Apply all pair shortest path algorithm. Floyd Warshall on below mentioned graph. Also analyze space and time complexity.



$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & \infty & \infty & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & 0
\end{array}
\longrightarrow
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & \infty & 1 & 1 & 0 & \infty \\
5 & \infty & 4 & \infty & 2 & 0
\end{array}
$$

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & 3 & 1 & 1 & 0 & \infty \\
5 & 6 & 4 & 12 & 2 & 0
\end{array}
\longrightarrow
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & \infty & 6 & 3 & \infty \\
2 & 2 & 0 & 8 & 5 & \infty \\
3 & \infty & \infty & 0 & 2 & \infty \\
4 & 3 & 1 & 1 & 0 & \infty \\
5 & 6 & 4 & 12 & 1 & 0
\end{array}
$$

Time complexity – $O(|V|^3)$; Space complexity = $O(|V|^2)$