OR Lecture 3

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Mathematical Programming Formulations

Real life problems (complex, non-linear relationships) $\xrightarrow{\text{Mathematical tools, Probability Theory, Measure Theory}}$ Mathematical programming formulations

Mathematical programming formulations $\xrightarrow{\text{Computational Analysis, Combinatorics, Convex analysis, Algebraic geometry}}$ Optimal solutions

Parameters of the problem: The data describing the physical system/process.

Decision Variables: Decisions that we need to take (optimization problem variables).

Objective function: (measurable real valued function) $f: \mathbb{R}^n \to \mathbb{R}$. (function of the decision variables)

Constraints: Restrictions on the functions of decision variables that help us represent a physical system/phenomenon/process. (algebraic/semi-algebraic).

Knapsack Problem

n items: $\{1, 2 \ldots n\}$. Each item i has utility/unit u_i and volume/unit c_i.

Items can be divided into three categories:

- I – divisible items (water, wood etc.)
- J – indivisible items (bags of chips, cookie boxes etc.)
- K – unique items (tent, stove etc.)

The volume of the knapsack: B

Parameters of the problem: Utility/unit – u_i

Volume/unit – c_i

Volume of knapsack – B

Decision Variables:

→ For each item $i \in I$, amount of i to be added to the knapsack,

$x_i \in \mathbb{R}_+$ continuous variables

→ For each item $j \in J$, the number of units of j to be added to the knapsack, $x_j \in \mathbb{Z}_+$ discrete variables

→ For each item $k \in K$, whether k is added to the knapsack or not.

$x_k \in \{0, 1\}$. binary variables

Objective function:

Total utility of the knapsack: $\sum_{i \in I} u_i x_i + \sum_{j \in J} u_j x_j + \sum_{k \in K} u_k x_k$

objective sense: maximization

Constraints:

Volume of the knapsack:

$$\sum_{i \in I} u_i x_i + \sum_{j \in J} u_j x_j + \sum_{k \in K} u_k x_k \leq B$$

Variable constraints:

$$x_i \geq 0 \quad \forall i \in I$$

$$x_j \geq 0,\ x_j \in \mathbb{Z}_+ \quad \forall j \in J$$

$$x_k \in \{0, 1\} \quad \forall k \in K$$