OR Lecture 1 Introduction to Optimization Defining a mathematical optimization problem: max/min f(x) = objective function s.t. $g(x) \le \alpha + i \in I$ constraints $\alpha \in \mathbb{R}^{t}$ or $\alpha \in S \subseteq \mathbb{R}^{n}$ decision variables 9:10 i & parameters of the problem let': F:= {x \in R: gi(x) \le \alpha_i, \text{ \text{ \in Called the feasible} set/region of the methematical programming problem (MP) ZEF- feasible solution, ZFF- infeasible solution Def: If $f=\emptyset$, then we say that the MP problem is infeasible $F\neq\emptyset$, then the MP problem is feasible. Def' : If IXEF s.t. f(x) = > HAER, then the MP problem is unbounded. (WLG assume miximization) Eg max x over x < R is unbounded. Def : (Bounded set) If $\exists r > 0$, s.t. $F \subseteq B(0,r)$ then F is said to be bounded. Eopen ball Remark: A MP problem cannot be unbounded unless its feasible region is inbounded. F bounded = 7 MP problem bounded What about the converse? Here's an example that the converse needn't be true Consider the problem $\max(-x^2)$ $2 \le 0$ $\text{over } x \in \mathbb{R}$ $\text{over } x \in \mathbb{R}$ f(x) is bounded above on R, hence the MP problem is bounded. Note that, as we are dealing with a maximization problem, the upper bound is the only relevant bound (we don't want the objective to how up). However, the feasible region for the constraint x <0 is the Mustrated half plane, which is clearly unbounded. Hence the converse is not true! (Max) $z \in F$ is called an optimal solution if $f(z) \ge f(z) \forall z \in F$ (min) Def : (Local optimal solution) $\overline{\chi} \in F$ is called a local optimal solution if $\overline{J} \in F \geqslant 0$ such that $f(\overline{\chi}) \geq f(\chi) \neq \chi \in \mathbb{N}_{\Xi}(\chi) = ||\overline{\chi} - \chi|| < \varepsilon \int (\max) (f(\overline{\chi}) \leq f(\chi))$ Does every nathematical programming problem have an OPTIMAL solution? (1) F = \$ (infeasible MP problem) (fg max 26 over Ph) 2) F is unbounded (recessary condition) (3) Consider the following case 7 does not F = (a, b)have an optimal solution optimization over open set (4) What if f(z) is discontinuous over F? min f(x) does not have an $x \in [a, b]$ optimed solutions Def: Infimum of fover f, denoted as $\alpha = \inf \{ f(x) : x \in F \}$ Is the greatest lower bound on f over Fi.e. & Lof(x) & X EF and AXXX and XX+(x) +xEF. eg inf {x:x>0} = 0 5 initiarly, we define the supremum (sup $2f(x): x \in F_3$) as the least upper bound on f over F. Weier strass theorem (Fundamental theorem of optimization) Let f: R - 1 R be a continuous function and F < R be non-empty, bounded and closed. Then min $\{f(z): z \in F\}$ has an optimal solution.

(sufficient) Consider K= inf (f(x): x = F] Let FK = {x < F: x < f(x) < x + EK}, 0 < E<1, k=1,2... Consider a convergent subsequence $\{\chi_{k}\}_{k \in K}$, $\chi \in F^{k}$. lim {xx} \rightarrow \overline{\pi} \in \forall \text{ (as F is closed)} As f is continuous, $\lim_{k\to\infty} f(x^k) = f(\bar{x}) = \alpha$ Thus is optimal.