

$X$  是  $n \times h$  维  $X_i$  是  $1 \times h$   $u, w$  是  $h \times h$ .  $V$  是  $n \times h$   $V_i$  是  $1 \times h$ .  
 $h$  是  $1 \times h$ .  ~~$X_u$~~

$$L = - \sum_u \sum_{i,j} \ln(\sigma(\hat{X}_{u,i,j})) + \frac{\lambda}{2} \|\theta\|^2$$

$$\hat{X}_{u,i,j}(\theta) = h(t) V_{i(t)}^T - h(t) \cdot V_{j(t)}^T \quad \text{数字.}$$

$$h(t) = \sigma(b(t)) \quad 1 \times h.$$

$$b(t) = X_{i(t)} \cdot u(t) + h(t-1) w(t) \quad 1 \times h.$$

求导:  $\frac{\partial L}{\partial X_{i(t)}} = \frac{\partial L}{\partial b(t)} \cdot (u(t))^T = \frac{\partial L}{\partial h} \times \frac{\partial h}{\partial b} \cdot (u(t))^T =$

$$= \frac{\partial L}{\partial X_{u,i,j}} \cdot V_{i(t)} \times \frac{\partial L}{\partial X_{u,i,j}} \times \frac{\partial X_{u,i,j}}{\partial h(t)} \times \frac{\partial h(t)}{\partial b(t)} \cdot \frac{\partial b(t)}{\partial X_i}$$

$$= [-(1 - \sigma(X_{u,i,j}))] \times (V_{i(t)} - V_{j(t)}) \times (\sigma(b(t)) \times (1 - \sigma(b(t)))) \cdot (u(t))^T + \lambda |X_{i(t)}|$$

$$\frac{\partial L}{\partial V_i} = \frac{\partial L}{\partial X_{u,i,j}} \left( (h(t))^T \cdot \frac{\partial L}{\partial X_{u,i,j}} \right)^T + \lambda |V_{i(t)}| = \frac{\partial L}{\partial X_{u,i,j}} \times h(t) + \lambda |V_{i(t)}|.$$

$$\frac{\partial L}{\partial V_j} = - \frac{\partial L}{\partial X_{u,i,j}} \times h(t) + \lambda |V_{j(t)}|.$$

$$\frac{\partial L}{\partial u(t)} = X_{i(t)}^T \cdot [-(1 - \sigma(X_{u,i,j}))] \times (V_{i(t)} - V_{j(t)}) \times (\sigma(b(t)) \times (1 - \sigma(b(t))))$$

$$= X_{i(t)}^T \cdot \frac{\partial L}{\partial X_{u,i,j}} \times \frac{\partial X_{u,i,j}}{\partial h} \times \frac{\partial h}{\partial b} + \lambda |u(t)|$$

$$\frac{\partial L}{\partial w} = h(t-1)^T \cdot \frac{\partial L}{\partial X_{u,i,j}} \times \frac{\partial X_{u,i,j}}{\partial h} \times \frac{\partial h}{\partial b} + \lambda |w(t)|.$$