Department of Mathematics

National University of Singapore

(2022/23) Semester I MA1521 Calculus for Computing Tutorial 10

(1) Find the volume of the solid whose base is the region in the xy-plane that is bounded by the parabola $y = 4 - x^2$ and the line $y = 3x$, while the top of the solid is bounded by the plane $x - z + 4 = 0$.

Ans. 625/12.

(2) Find the area of the surface consisting of the part of the sphere $x^2 + y^2 + z^2 = 2^2$ that lies above the horizontal plane $z = 1$.

Ans. 4π.

(3) Find the exact value of the surface area of the portion of the upper cone

$$z = \sqrt{x^2 + y^2}$$

above the region $D = \{(x, y) \in \mathbb{R}^2 : \quad x^2 \le y \le x + 2, \quad -1 \le x \le 2\}$.

Ans. $\frac{9\sqrt{2}}{2}$.

(4) Find the area cut from the saddle surface $az = x^2 - y^2$ by the cylinder $x^2 + y^2 = a^2$. Here a is a positive constant.

Ans. $\frac{\pi a^2}{6}\left(5^{\frac{3}{2}} - 1\right)$.

(5) For each of the following, find a function $f(x, y)$ with continuous second order partial derivatives (if one exists) such that

(a) $\nabla f = (4x^3 y - \frac{1}{1+x^2} + e^y)\mathbf{i} + (x^4 + xe^y + x)\mathbf{j}$.

(b) $\nabla f = (4x^3 y + y + e^y)\mathbf{i} + (x^4 + xe^y + x + y)\mathbf{j}$.

Ans. (a) does not exist (b) $x^4 y + xy + xe^y + \frac{1}{2}y^2 + C$.

(6) Solve the following differential equations:

(a) $x(x + 1)y' = 1$

(b) $y' = e^{(x-3y)}$

(c) $(1 + y)y' + (1 - 2x)y^2 = 0$

Ans. (a) $y = \ln\left|\frac{x}{x+1}\right| + C$, (b) $e^{3y} = 3e^x + C$, (c) $\ln|y| - \frac{1}{y} = x^2 - x + C$.

Further Exercises (not to be discussed)

1. Using polar coordinates, evaluate $\displaystyle\iint\limits_D \frac{1}{(1+x^2+y^2)^{3/2}}\,dA$, where D is the region in the first quadrant enclosed by the circle $x^2+y^2=16$.

   Ans. $\frac{\pi}{2}(1-\frac{1}{\sqrt{17}})$.

2. Find the volume of the solid bounded by the cylinders $x^2+y^2=r^2$ and $y^2+z^2=r^2$.

   Ans. $\frac{16}{3}r^3$.

3. Solve the differential equation $(x+2y-1)+3(x+2y)y'=0$ by letting $z=x+2y$.

   Ans. $x+3y+c=3\ln|x+2y+2|$, $x+2y+2=0$.