Recitation - 01

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Problem

A building has 100 floors. One of the floors is the highest floor from which an egg can be dropped without breaking. The egg will break from all the floors above, and from all the floors below, the egg will not break. You have been given with two eggs. How many drops are needed to find the highest floor from where the egg drop does not break the egg?

Solution 1

Divide a Little, Conquer a Bit. The following solution takes a little bit of the linear approach and mixes in splitting from our binary approach. We can start off by dropping an egg at floor 10, increasing the drop floor by 10 at a time, then going back to drop one floor at a time until we find that n. If our egg breaks at floor 10, we know n is one of the 9 floors below us.

Worst case, the egg drops and doesn't break until floor 100 (10 drops) and we drop the second egg but don't break it for floors $91 - 99$.

It brings our worst-case drop count to 19 drops.

It seems relatively straightforwards, but we can still improve our number of drops.

Solution 2

This approach can reduce our worst-case scenario by balancing the linear drops and our 10-floor drop increment. If getting to the higher floors means more drops overall, we need to decrease the drops we need to perform linearly. We're essentially trying to make all possible scenarios take the same number of drops to solve.

If we drop our first egg from floor x (10 in our 10 floor strategy), the linear portion of our strategy is $x - 1$ (9 in the above strategy). Our drop count is:

$$x + (x - 1) \tag{1}$$

If the egg does not break on the first drop, our drop count increases by one, so we will need to remove a drop from our floor by floor drop count. The next drop should be from $x - 1$ floors up. Every additional floor jump will need to have one less floor so that when we get to the linear portion of the solution, we will have one less floor to check. We continue removing one floor until we only have 1 floor to check:

$$\begin{aligned} x + (x-1) + (x-2) + (x-3) + ... + 1 \\ = x(x+1)/2 \end{aligned} \tag{2}$$

Because there are 100 floors in our problem, we solve for x when the entire summation is equal to 100:

$$\begin{aligned} \frac{x(x+1)}{2} &= 100 \\ \rightarrow x &\approx 13.651 \end{aligned} \tag{3}$$

This means we want to start dropping from floor 14, jump up 13 floors to drop from floor 27, jump up 12 floors to drop from floor 39, and so on. Our worst-case scenario is then a drop count total of 14.