1\. a) $\iint_R (x^2+y^2)\,dA = \int_0^b \int_0^a (x^2+y^2)\,dx\,dy$

$= \int_0^b \left[\frac{1}{3}x^3 + y^2x \right]_0^a dy$

$= \int_0^b \frac{1}{3}a^3 + ay^2 \, dy$

$= \left[\frac{1}{3}a^3y + \frac{a}{3}y^3 \right]_0^b$

$= \frac{1}{3}a^3b + \frac{1}{3}ab^3$

$= \frac{1}{3}ab(a^2+b^2)$

b) $\iint_R \frac{xy}{\sqrt{4-x^2}}\,dA \quad x=0,\ x=1,\ y=1,\ y=2$

$= \int_1^2 \int_0^1 \frac{xy}{\sqrt{4-x^2}}\,dx\,dy$

$= \int_1^2 y\left[-\sqrt{4-x^2} \right]_0^1 dy$

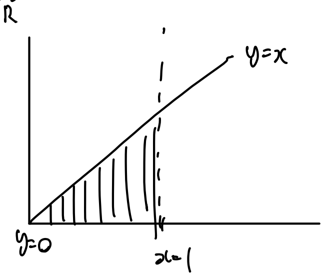
$= \int_1^2 y(-\sqrt{3}+2)\,dy$

$= \left[-\frac{\sqrt{3}}{2}y^2 + y^2 \right]_1^2$

$= -2\sqrt{3}+4+\frac{\sqrt{3}}{2}-1$

$= 3 - \frac{3\sqrt{3}}{2}$

c) $\iint_R e^{x^2}\,dA \quad y=0,\ y=x,\ x=1$



$\int_0^1 \int_0^x e^{x^2}\,dy\,dx$

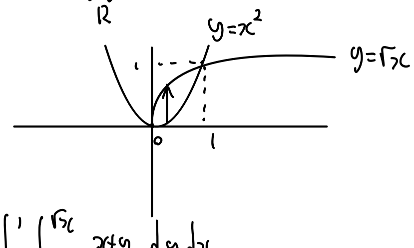
$= \int_0^1 \left[e^{x^2}y \right]_0^x dx$

$= \int_0^1 xe^{x^2}\,dx$

$= \left[\frac{1}{2}e^{x^2} \right]_0^1$

$= \frac{1}{2}e - \frac{1}{2} = \frac{1}{2}(e-1)$

d) $\iint_R (x+y)\,dA \quad y=\sqrt{x},\ y=x^2$



$\int_0^1 \int_{x^2}^{\sqrt{x}} x+y\,dy\,dx$

$= \int_0^1 \left[xy + \frac{1}{2}y^2 \right]_{x^2}^{\sqrt{x}} dx$

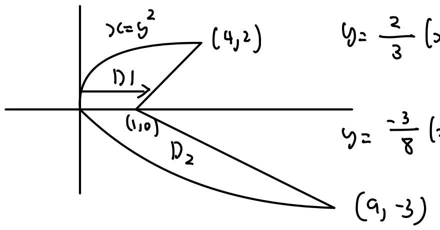
$= \int_0^1 x^{\frac{3}{2}} + \frac{1}{2}x - x^3 - \frac{1}{2}x^4\,dx$

$= \left[\frac{2}{5}x^{\frac{5}{2}} + \frac{1}{4}x^2 - \frac{1}{4}x^4 - \frac{1}{10}x^5 \right]_0^1$

$= \frac{2}{5} + \frac{1}{4} - \frac{1}{4} - \frac{1}{10} = \frac{4}{10} - \frac{1}{10}$

$= \frac{3}{10}$

2\. $\iint_D x\,dA$



$y = \frac{2}{3}(x-1) = \frac{2}{3}x - \frac{2}{3} \qquad y+\frac{2}{3} = \frac{2}{3}x$

$x = \frac{3}{2}y+1$

$y = -\frac{3}{8}(x-1) = -\frac{3}{8}x + \frac{3}{8} \qquad y - \frac{3}{8} = -\frac{3}{8}x$

$x = 1 - \frac{8}{3}y$

$\iint_{D_1} x\,dA = \int_0^2 \int_{y^2}^{\frac{3}{2}y+1} x\,dx\,dy$

$= \int_0^2 \left[\frac{1}{2}x^2 \right]_{y^2}^{\frac{3}{2}y+1} dy$

$\frac{1}{2}\left(\frac{3}{2}y+1\right)^2$

$= \frac{1}{2}\left(\frac{9}{4}y^2+3y+1\right)$

$= \frac{9}{8}y^2 + \frac{3}{2}y + \frac{1}{2}$

$= \int_0^2 \frac{9}{8}y^2 + \frac{3}{2}y + \frac{1}{2} - \frac{1}{2}y^4\,dy$

$= \left[\frac{9}{24}y^3 + \frac{3}{4}y^2 + \frac{1}{2}y - \frac{1}{10}y^5 \right]_0^2$

$= 3+3+1-\frac{16}{5} = 7 - \frac{16}{5} = \frac{19}{5}$

$\iint_{D_2} x\,dA = \int_{-3}^0 \int_{y^2}^{1-\frac{8}{3}y} x\,dx\,dy$

$= \int_{-3}^0 \left[\frac{1}{2}x^2 \right]_{y^2}^{1-\frac{8}{3}y} dy$

$= \int_{-3}^0 \frac{32}{9}y^2 - \frac{8}{3}y + \frac{1}{2} - \frac{y^4}{2}\,dy$

$= \left[\frac{32}{27}y^3 - \frac{8}{6}y^2 + \frac{1}{2}y - \frac{1}{10}y^5 \right]_{-3}^0$

$= \frac{106}{5}$

$\iint_{D_1} x\,dA + \iint_{D_2} x\,dA = \frac{106}{5} + \frac{19}{5} = 25$

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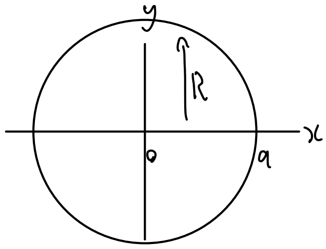
3. $\displaystyle\int_0^a \int_0^{\sqrt{a^2-x^2}} \frac{1}{1+x^2+y^2}\, dy\, dx$ ← Type 1 region

a) $R = \{ (r,\theta) : 0 \le r \le a ,\ 0 \le \theta \le \frac{\pi}{2} \}$



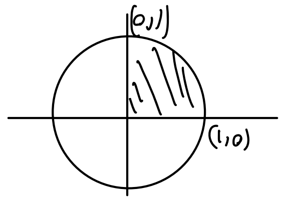
$y^2 = a^2 - x^2$
$y^2 + x^2 = a^2$

$$\int_0^{\frac{\pi}{2}} \int_0^a \frac{1}{1+r^2\cos\theta + r^2\sin\theta}\, r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^a \frac{r}{1+r^2}\, dr\, d\theta$$

$$= \frac{\pi}{4} \ln(a^2+a)$$

b) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$



$$\int_0^{\frac{\pi}{2}} \int_0^1 e^{r^2\cos^2\theta + r^2\sin^2\theta}\, r\, dr\, d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 r e^{r^2}\, dr\, d\theta$$

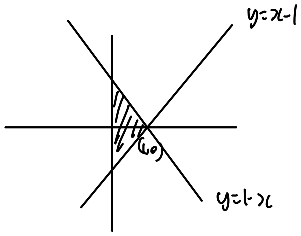
$$= \int_0^{\frac{\pi}{2}} \left[\frac{1}{2} e^{r^2} \right]_0^1 d\theta$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}e - \frac{1}{2}\, d\theta$$

$$= \left[\frac{1}{2}e\,\theta - \frac{1}{2}\theta \right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{4}\theta\,(e-1)$$

4. a) $\displaystyle\int_0^1 \int_{x-1}^{1-x} x\, dy\, dx$



$$2\int_{-1}^{0} \int_0^{y+1} x\, dx\, dy$$

$$= 2\int_{-1}^{0} \left[\frac{1}{2}x^2 \right]_0^{y+1} dy$$

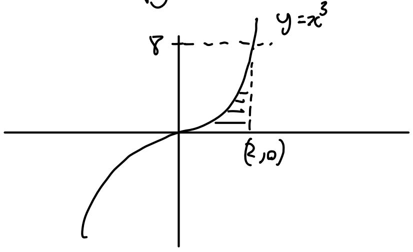
$$= 2\int_{-1}^{0} \frac{1}{2}(y^2+2y+1)\, dy = 2\int_{-1}^{0} \frac{1}{2}y^2 + y + \frac{1}{2}\, dy$$

$$= 2\left[\frac{1}{6}y^3 + \frac{1}{2}y^2 + \frac{1}{2}y \right]_{-1}^{0}$$

$$= 2 \cdot -\left(-\frac{1}{6} + \frac{1}{2} - \frac{1}{2}\right)$$

$$= 2 \times \frac{1}{6} = \frac{1}{3}$$

b) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^{2} e^{x^4}\, dx\, dy$



$$\int_0^2 \int_0^{x^3} e^{x^4}\, dy\, dx$$

$$= \int_0^2 \left[y e^{x^4} \right]_0^{x^3} dx$$

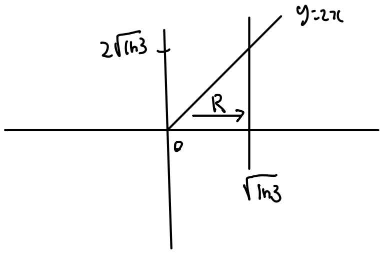
$$= \int_0^2 x^3 e^{x^4}\, dx$$

$$= \left[\frac{1}{4} e^{x^4} \right]_0^2$$

$$= \frac{1}{4}e^{16} - \frac{1}{4}$$

$$= \frac{1}{4}(e^{16} - 1)$$

c) $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\,dx\,dy$ Type II



$$\int_0^{\sqrt{\ln 3}} \int_0^{2x} e^{x^2}\,dy\,dx$$

$$= \int_0^{\sqrt{\ln 3}} \left[ye^{x^2} \right]_0^{2x} dx$$

$$= \int_0^{\sqrt{\ln 3}} 2xe^{x^2}\,dx$$

$$= \left[e^{x^2} \right]_0^{\sqrt{\ln 3}} = e^{\ln 3} - 1 = 2$$