Department of Mathematics

National University of Singapore

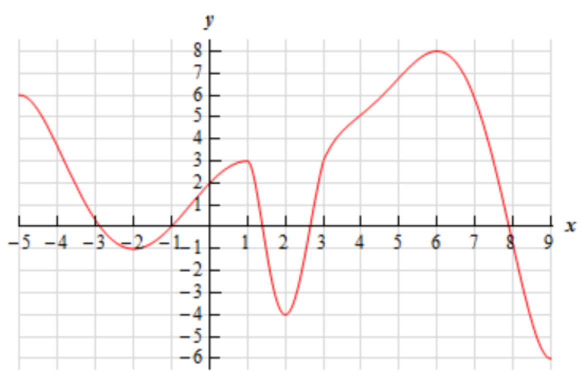
(2022/23) Semester I MA1521 Calculus for Computing Tutorial 3

(1) Sketch the graph of a function which satisfies the conditions $f(0) = 2$, $f(2) = 4$, $f'(x) < 0$ for $x < 0$, and f has a local maximum between $x = 0$ and $x = 2$, and finally f has a local minimum at some point c with $c > 2$.

(2) Let $f(x) = \sec x + \tan x$. Find all values of x for which f is defined and $f'(x) > 0$ in the interval $(0, 2\pi)$.

Ans $(0, 2\pi) - \{\pi/2, 3\pi/2\}$.

(3) Find x at which the local maximun, local minimun and absolute extrema is attained if the graph of f is given as follows:



(4) For each of the following functions:

(a) $y = \dfrac{x+1}{x^2+1}$, $x \in [-3, 3]$, (b) $y = (x-1)\sqrt[3]{x^2}$, $x \in (-\infty, \infty)$,

determine

(i) the critical points,

(ii) the intervals where it is increasing and decreasing,

(iii) the local and absolute extreme values.

Ans. (a) (iii) local minimum value $-\frac{1}{2(\sqrt{2}+1)}$ at $x = -1 - \sqrt{2}$ which is also the absolute minimum value; local maximum value $\frac{1}{2(\sqrt{2}-1)}$ at $x = -1 + \sqrt{2}$ which is also the absolute maximum value. (b) (iii) local minimum value $-\frac{3}{5}\left(\frac{2}{5}\right)^{2/3}$ at

$x = \frac{2}{5}$, local maximum value 0 at $x = 0$. No absolute minimum and no absolute maximum.

(5) Ornithologists have determined that some species of birds tend to avoid flights over large bodies of water during daylight hours. It is believed that more energy is required to fly over water than land because air generally rises over land and falls over water during the day. A bird with these tendencies is released from an island that is 5 km from the nearest point B on a straight shoreline, flies to a point C on the shoreline, and then flies along the shoreline to its nesting area D. Assume that the bird instinctively chooses a path that will minimize its energy expenditure. Points B and D are 13 km apart. If it takes 1.4 times as much energy to fly over water as land, find the distance between B and C.

Ans. 5.1 km.

(6) Use L'Hopital's rule to find the following limits.

(a) $\displaystyle\lim_{x\to\pi/2} \frac{1-\sin x}{1+\cos 2x}$, (b) $\displaystyle\lim_{x\to 0} \frac{\ln(\cos ax)}{\ln(\cos bx)}$, $a,\ b > 0$,

(c) $\displaystyle\lim_{x\to\infty} x\tan\frac{1}{x}$, (d) $\displaystyle\lim_{x\to 0+} x^a \ln x$, $a > 0$,

(e) $\displaystyle\lim_{x\to 1} x^{\frac{1}{1-x}}$, (f) $\displaystyle\lim_{x\to 0^+} x^{\sin x}$.

Ans. (a) $\frac{1}{4}$, (b) $\frac{a^2}{b^2}$, (c) 1, (d) 0, (e) e^{-1}, (f) 1.

Further Exercises (Not to be be discussed during tutorial)

(1) The curve $2(x^2+y^2)^2 = 25(x^2-y^2)$ is called a *lemniscate*. Using implicit differentiation, find $\dfrac{dy}{dx}$. Find also the equation of the tangent line to the curve at $(3, 1)$.

Ans. $9x + 13y = 40$.

(2) Evaluate

(a) $\displaystyle\lim_{x\to\infty} \frac{x^3}{3^x}$.

(b) $\displaystyle\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.

Ans. (a) 0, (b) $e^{-1/6}$.

(3) Express the limit $\displaystyle\lim_{x\to 1} \frac{\sqrt{x}-1}{g(x)-g(1)}$ in terms of $g'(1)$, where $g'(1) \neq 0$.