<u>GET1024 / GEC1036 Lecture 3</u>

Atomic Nucleus & Radioactivity

Properties of Nucleus

Size, charge, density, etc.

Constituents of Nucleus

Nuclear Binding Energy

Decay of Unstable Nucleus

Mainly α, β and γ decays

Laws of Radioactivity

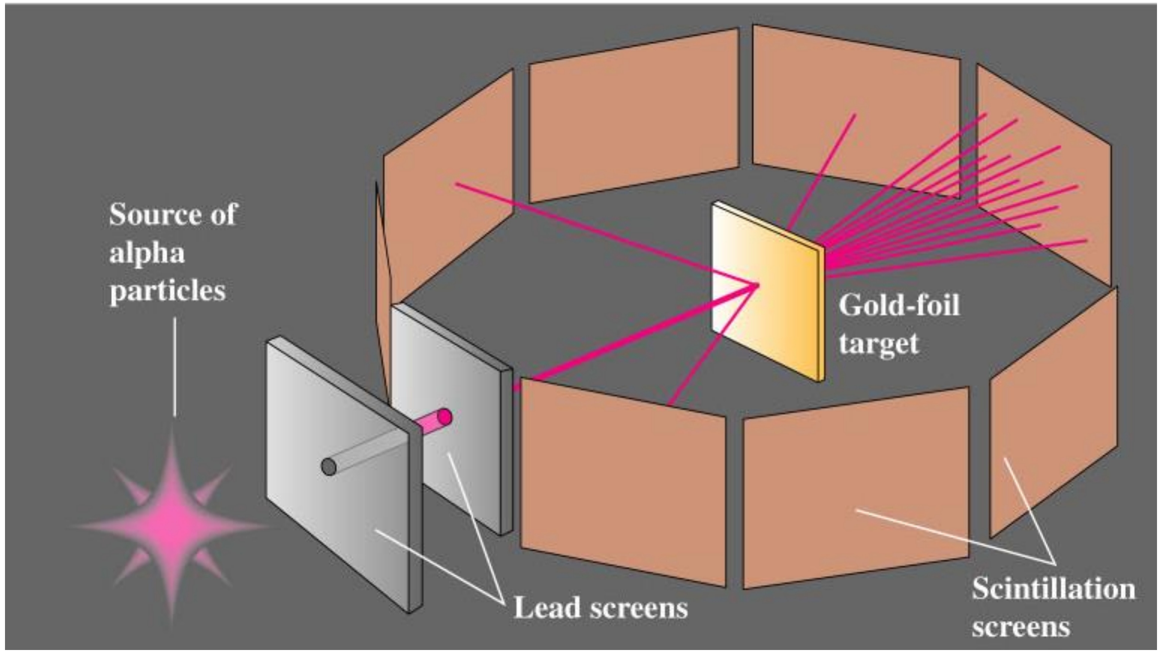
Introductory Comments

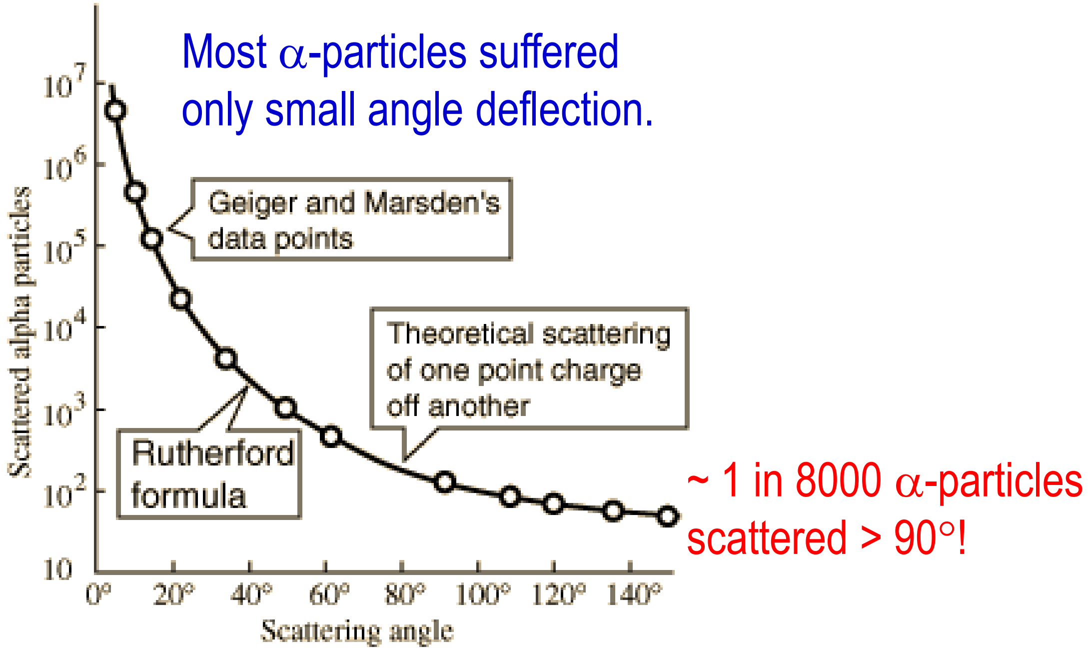
- The nucleus contains almost all the mass of the atom. In most of the chemical reactions that we are used to, the nucleus plays an inactive role.
- While orders of magnitude smaller in size than the atom, the energy involved is approximately millions of times higher. This potential has already been realized especially in power production and will likely be the ultimate source of energy for mankind!
- In this lecture, we will study its structure and properties and the activities originated from an unstable nucleus. The nuclear reactions leading to nuclear plants and nuclear bombs will be covered in a later lecture.
- *Important concepts to take away from this lesson: Constituents of nucleus (protons and neutrons), size of nucleus,* ***binding energy****, nomenclature used, unit of energy in MeV (conversion between J and MeV or eV), half-lives, α-, β- and γ-decays.*

Discovery of the Atomic Nucleus

Recall

- Experiment by Rutherford, Geiger and Marsden in 1909 - first experiment to give the hint of the size of the nucleus.



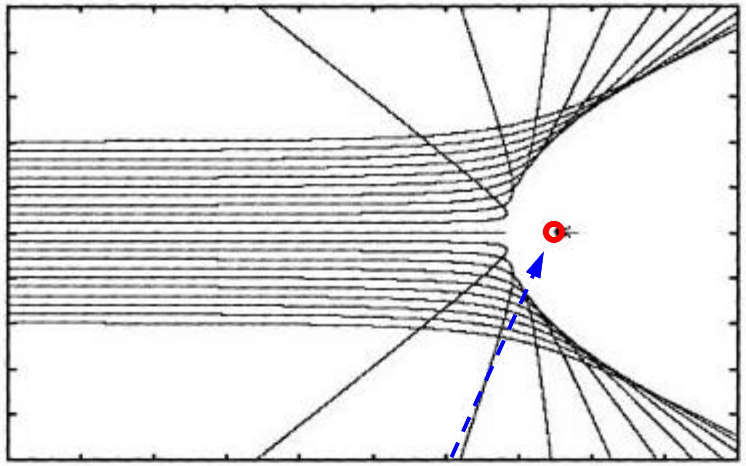


It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you. *– Ernest Rutherford, 1908 Nobel Prize (Chemistry) co-winner*

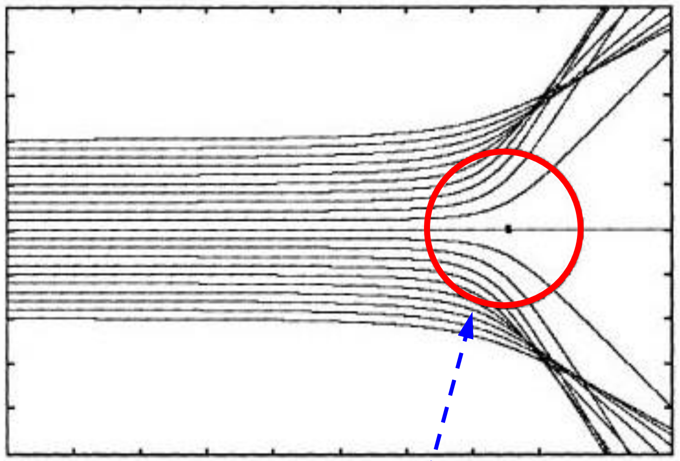
Size of the Nucleus

- From the large angle scattering, we can also estimate the size of the nucleus!

Computer simulation of the scattering of α-particle is size of nucleus is varied



Actual size of gold nucleus, $R_{\mathrm{Au}} = 7.0 \times 10^{-15}$ m

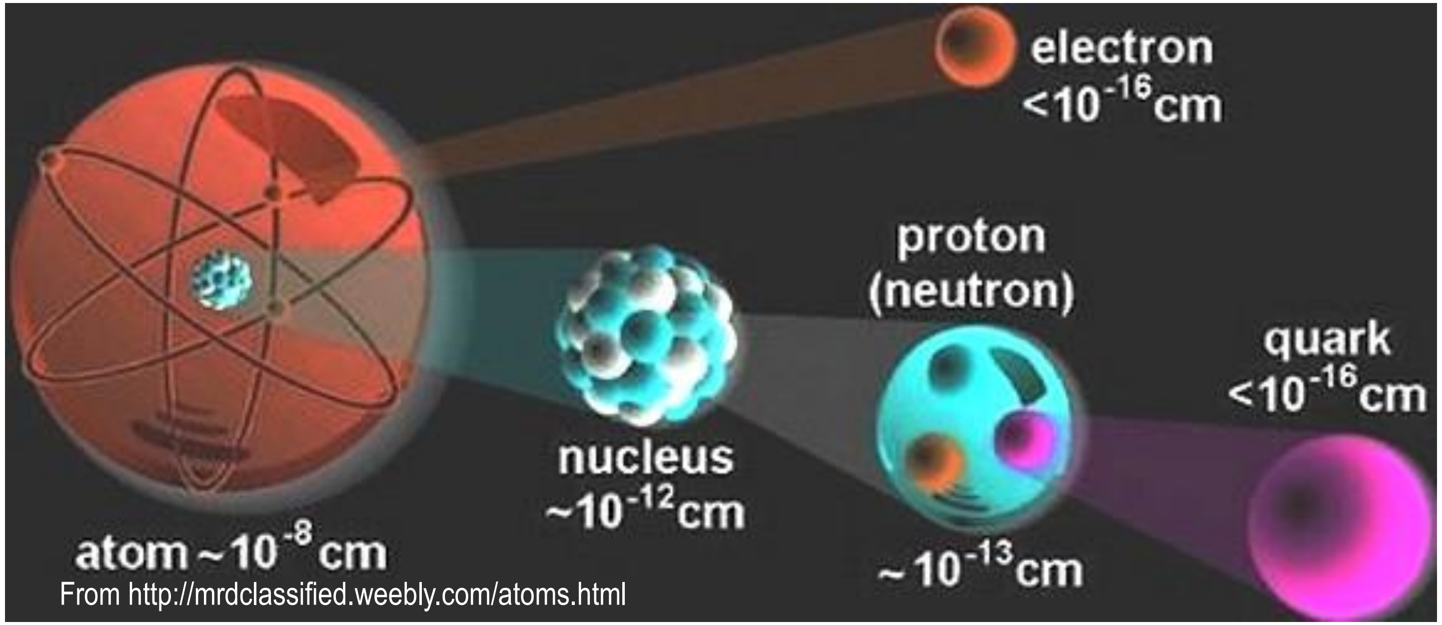


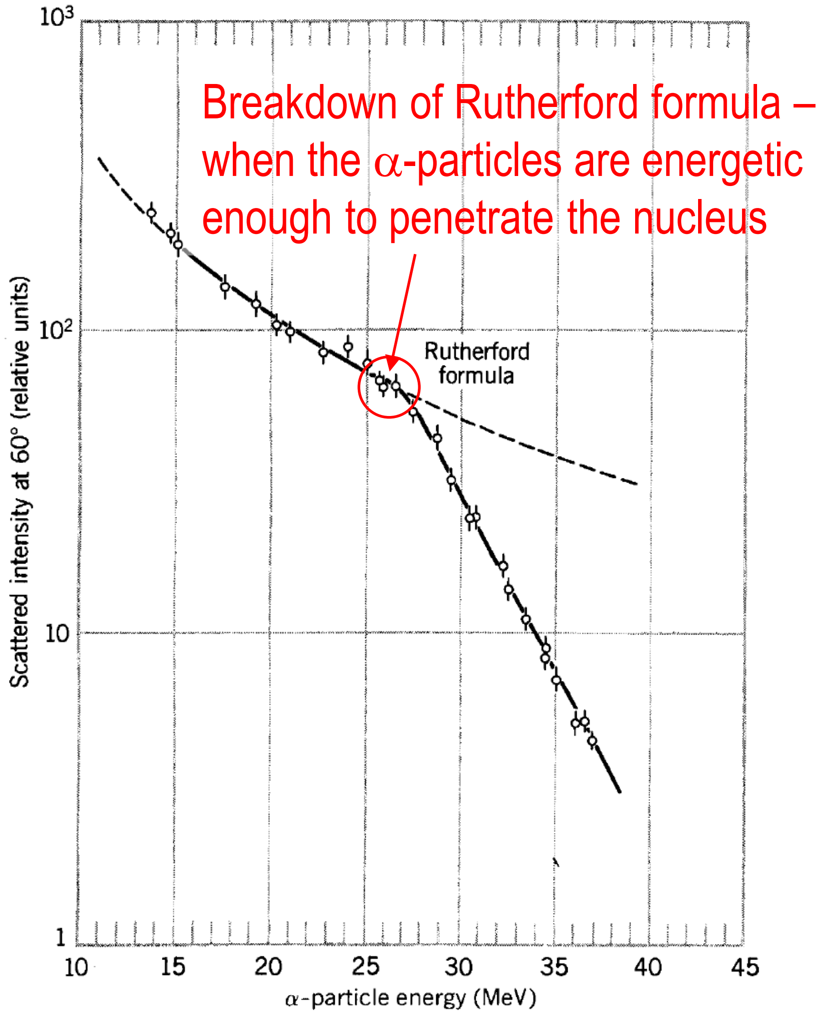
If $R_{\mathrm{Au}} = 7.0 \times 10^{-14}$ m

- Notice that if the nucleus were ten times bigger, large angle scattering is no longer observed. The experimental result can only be explained if the mass is concentrated within a radius < 10^{-14} m.

Size of Atomic Nucleus

- Nuclei are extremely small with almost all the mass of the atoms but not point-like (infinitesimally small) as demonstrated in Rutherford scattering experiments.
- Analogy: marble for nucleus ⇒ Football stadium with height of 50-storey building for atom.





Protons in Atomic Nucleus

- Rutherford scattering experiment (1909 – 11) firmly established a nucleus at the center of the atom – **positively charged** and very small.
- Nucleus comprises even smaller particles – protons (+ ?).
- Rutherford can also be credited for the discovery of protons – in the first induced nuclear reaction that was recorded: $\alpha + N \rightarrow O + H$. [The nucleus of hydrogen is just the proton itself.]
- Mass of proton is very large compared to electrons: $m_p \approx 1836\ m_e$. But magnitude of charge is exactly the same as the charge of the electrons though with opposite sign.
- Size ~ 10^{-15} m. Very small but we have shown that it is definitely not point-like.
- For a while, most scientists thought that the proton was an elementary particle. However, deep inelastic electron-proton scattering seemed to suggest otherwise, i.e., it is made up of some other even more elementary particles.

Atomic Masses

- Below is a table of the atomic masses of elements sorted according to the number of protons in nucleus (atomic number)

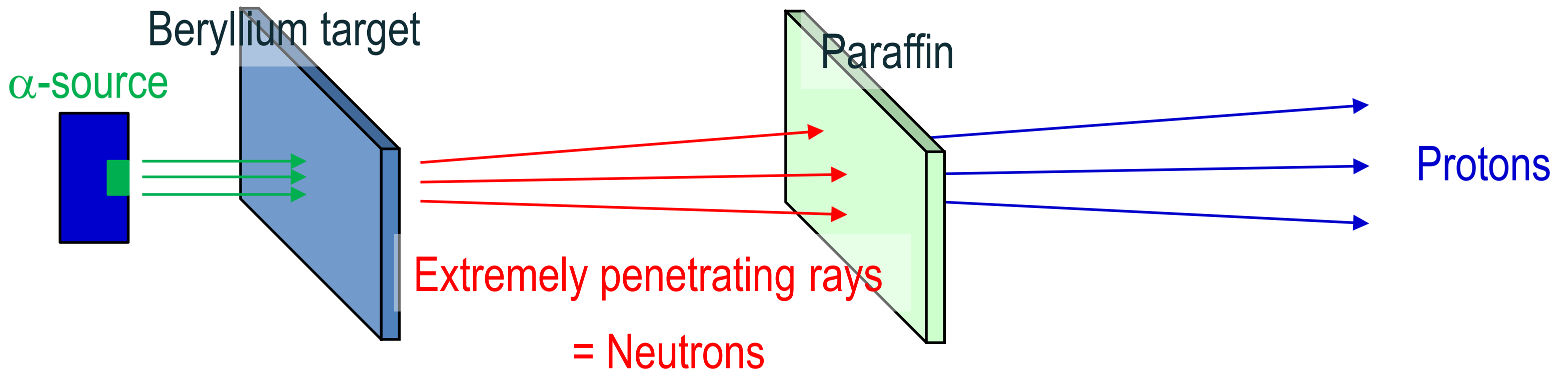
Element	Atomic Number	Atomic Mass
Hydrogen	1	1.0079
Helium	2	4.0026
Lithium	3	6.941
Beryllium	4	9.0122
Boron	5	10.811
Carbon	6	12.0107
Nitrogen	7	14.0067
Oxygen	8	15.9994
Fluorine	9	18.9984
Neon	10	20.1797

Element	Atomic Number	Atomic Mass
Sodium	11	22.9897
Magnesium	12	24.305
Aluminium	13	26.9815
Silicon	14	28.0855
Phosphorus	15	30.9738
Sulphur	16	32.065
Chorine	17	35.453
Argon	18	39.948
Potassium	19	39.0983
Calcium	20	40.078

- Note the different mass to charge ratio of various elements.
- The atomic mass of many elements are closed to whole numbers but there some that are not.
- For some elements, the atomic mass is larger than the next higher Z element!

Another New Particle

- 1930 – Walther Bothe & Herbert Becker – observed that when beryllium was bombarded by α-particle, a very penetrating radiation is produced – originally thought to be γ-rays (but actually more penetrating than any γ-rays known).



- 1932 – Irene & Frederic Joliot-Curie found that this radiation can eject protons from paraffin – still tried to interpret it as very high energy γ-rays.
- 1932 – Chadwick repeated the experiments and did more of his own – and concluded that it must be the **neutron** he and Rutherford were already looking for (a **neutral** particle with mass similar to that of proton)! The correct equation* for the reaction should be:

$$ {}^{4}_{2}\mathrm{He} + {}^{9}_{4}\mathrm{Be} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} $$

** More on how to read this equation and symbols later*

Neutrons

- The discovery of the neutrons complete the picture of the nucleus – initially scientists were puzzled by the different charge-to-mass ratio of the nuclides.
- Note that while neutrons in the nucleus can be stable and does not decay, a free neutron is not stable and will beta-decay with a half life of 10.3 minutes changing into a proton and emitting an electron* in the process.

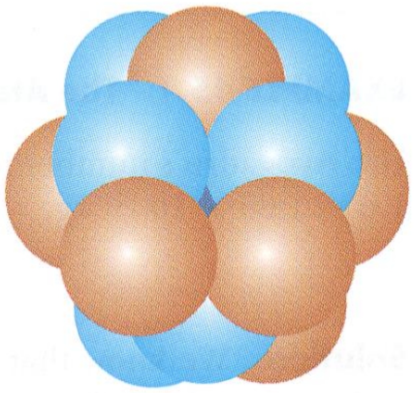
With neutrons, we appeared to have completed our search on the building blocks of matter – protons + neutrons at the nucleus and electrons around the nucleus.

* *This description is incomplete – more explanation later.*

Nomenclature

- **Atomic Number** Z**:** Number of protons in nucleus

  Charge of nucleus is Ze where e = electronic charge



- **Neutron Number** N**:** Number of neutrons in nucleus

- **Mass Number** A**:** Number of nucleons (protons or neutrons) in nucleus

$$\boxed{A = Z + N}$$

- **Nuclide**: Nuclear species have specific values of Z and N.

- **Isotope**: Nuclides having same Z but different N. Same chemical properties but different nuclear properties.

- **Notation for Nuclide**: $^A_Z\mathrm{El}$ ← **Chemical symbol of element**

e.g., $^1_1\mathrm{H}$ $^2_1\mathrm{H}$ $^3_1\mathrm{H}$ $^3_2\mathrm{He}$ $^4_2\mathrm{He}$ $^{235}_{92}\mathrm{U}$ $^{238}_{92}\mathrm{U}$

(Note that Z is often omitted from the above notation since it is redundant.)

Isotopes

- With neutrons, we can now explain the different mass to charge ratios of various elements as having different abundance of stable (or very long-lived) isotopes.

Element	Z	**Atomic Mass**	**Natural Abundance of Isotopes**
Hydrogen	1	1.0079	$^{1}_{1}\mathrm{H}$ (99.985%), $^{2}_{1}\mathrm{H}$ (0.015%)
Helium	2	4.0026	$^{3}_{2}\mathrm{He}$ (1.4×10^{-4}%), $^{4}_{2}\mathrm{He}$ (99.99986%)
Lithium	3	6.941	$^{6}_{3}\mathrm{Li}$ (7.5%), $^{7}_{3}\mathrm{Li}$ (92.5%)
Beryllium	4	9.0122	$^{9}_{4}\mathrm{Be}$ (100%)
Boron	5	10.811	$^{10}_{5}\mathrm{B}$ (19.8%), $^{11}_{5}\mathrm{B}$ (80.2%)
Carbon	6	12.0107	$^{12}_{6}\mathrm{C}$ (98.89%), $^{13}_{6}\mathrm{C}$ (1.11%)
Nitrogen	7	14.0067	$^{14}_{7}\mathrm{N}$ (99.634%), $^{15}_{7}\mathrm{N}$ (0.366%)
Oxygen	8	15.9994	$^{16}_{8}\mathrm{O}$ (99.76%), $^{17}_{8}\mathrm{O}$ (0.038%), $^{18}_{8}\mathrm{O}$ (0.204%)
Fluorine	9	18.9984	$^{19}_{9}\mathrm{F}$ (100%)
Neon	10	20.1797	$^{20}_{10}\mathrm{Ne}$ (90.51%), $^{21}_{10}\mathrm{Ne}$ (0.27%), $^{22}_{10}\mathrm{Ne}$ (9.22%)

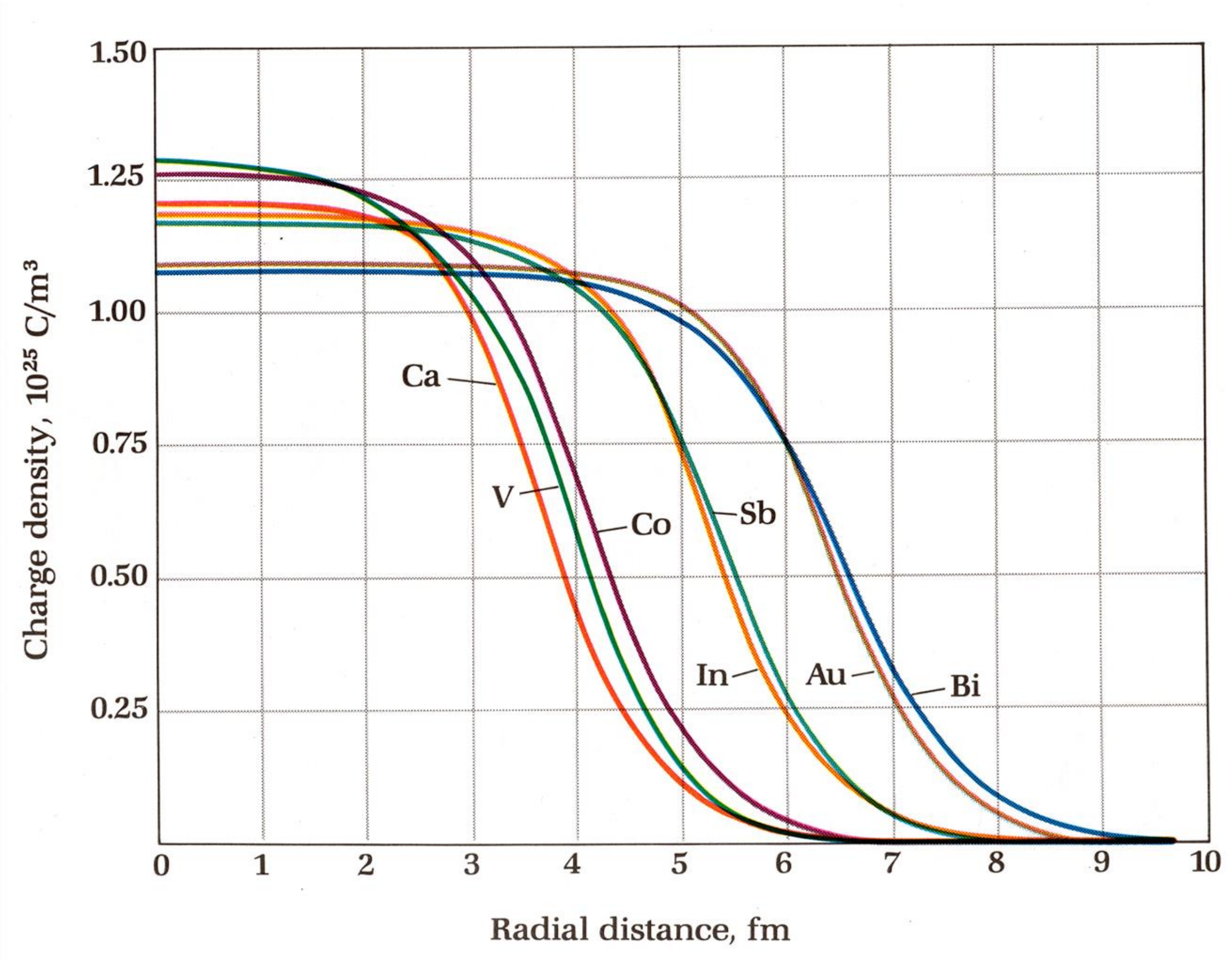
Isotopes (continued)

Element	Z	Atomic Mass	Natural Abundance of Isotopes
Sodium	11	22.9897	$^{23}_{11}Na$ (100%)
Magnesium	12	24.305	$^{24}_{12}Mg$ (78.99%), $^{25}_{12}Mg$ (10.00%), $^{26}_{12}Mg$ (11.01%)
Aluminium	13	26.9815	$^{27}_{13}Al$ (100%)
Silicon	14	28.0855	$^{28}_{14}Si$ (92.23%), $^{29}_{14}Si$ (4.67%), $^{30}_{14}Si$ (3.10%)
Phosphorus	15	30.9738	$^{31}_{15}P$ (100%)
Sulphur	16	32.065	$^{32}_{16}S$ (95.02%), $^{33}_{16}S$ (0.75%), $^{34}_{16}S$ (4.21%)
Chorine	17	35.453	$^{35}_{17}Cl$ (75.77%), $^{37}_{17}Cl$ (24.23%)
Argon	18	39.948	$^{36}_{18}Ar$ (0.337%), $^{38}_{18}Ar$ (0.063%), $^{40}_{18}Ar$ (99.60%)
Potassium	19	39.0983	$^{39}_{19}K$ (93.26%), $^{40}_{19}K$ (0.0117%), $^{41}_{19}K$ (6.73%)
Calcium	20	40.078	$^{40}_{20}Ca$ (96.94%), $^{42}_{20}Ca$ (0.647%), $^{43}_{20}Ca$ (0.135%), $^{44}_{20}Ca$ (2.09%), $^{46}_{20}Ca$ (0.0035%), $^{48}_{20}Ca$ (0.187%)
Thorium	90	232.038	$^{232}_{90}Th$ (100%)
Uranium	92	238.029	$^{234}_{92}U$ (0.005%), $^{235}_{92}U$ (0.720%), $^{238}_{92}U$ (99.275%)

Note: Isotopes in red are radioactive. They have very long half-lives or products of ongoing radioactive decay.

Mass and Charge Distribution in Nucleus

- Can also use very energetic electrons as probes into the nucleus. (Can you think of reasons why electrons rather than proton or α-particles are better probes?)
- Researchers found the following charge distribution:



- Charge (or mass) density is very similar for most nuclei.
- The radius of the nucleus is approximately given by

Mass number

$$\boxed{R = R_0 A^{1/3}}$$

$$R_0 = 1.2 \times 10^{-15}\ \text{m}$$

Nuclear Density & Packing

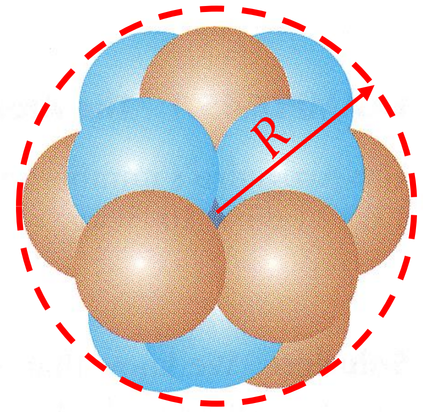
- Assuming a spherical nucleus, then the nuclear density is:

$$\rho_{\text{nucl}} = \frac{\text{mass}}{\text{volume}} \approx \frac{Au}{\frac{4}{3}\pi R^3}$$

Mass number

Unified atomic mass unit ≈ $m_p \approx m_n$

$$\rho_{\text{nucl}} \approx \frac{3Au}{4\pi R_0^3 A} = \frac{3 \times 1.66 \times 10^{-27}}{4\pi(1.2 \times 10^{-15})^3} = \boxed{2.3 \times 10^{17}\ \text{kg/m}^3}$$



- **<u>All nuclei have approximately the same density</u>** (independent of *A*) and is extremely high! 1 cm^3 of nuclear material has a mass of 230 million tons.

- We can imagine that the nucleons are packed into the nucleus like tightly packed spheres, and that all nuclei have the same density, similar to a drop of liquid.

Are there any physical macroscopic objects that have the density of the nucleus as calculated above?

Unified Atomic Mass Unit

- Mass is usually given in terms of the **unified atomic mass unit** u.

$$\boxed{1\text{u} \equiv \frac{1}{12} \times \text{mass of } {}^{12}_{6}\text{C atom} \ = 1.660538782(83) \times 10^{-27} \text{ kg}}$$

- In terms of u, the mass of proton, neutron and electron are:

$$\begin{aligned} m_p &= 1.00727646677(10) \text{ u} &=& \ 1.672621637(83) \times 10^{-27} \text{ kg} \\ m_n &= 1.00866491591(43) \text{ u} &=& \ 1.674927211(84) \times 10^{-27} \text{ kg} \\ m_e &= 0.00054857990943(23) \text{ u} &=& \ 9.10938215(45) \times 10^{-31} \text{ kg} \end{aligned}$$

- The mass given in most tables (in references and textbooks) are the mass of a neutral atom, i.e., nucleus + electrons, such as the one on the next slide from CODATA (2006), compiled by NIST (National Institute of Standards & Technology). If you need the mass of nucleus alone, you need to minus the mass of electrons (neglecting binding energy of electrons).

Measured Masses of Some Atoms

Measured mass of some atoms of interest. Notice the high level of precision.

You can check the latest values of some of these values at the NIST website or in the CODATA paper published in 2016 listed below:

https://ws680.nist.gov/publication/get_pdf.cfm?pub_id=920687

TABLE II Values of the relative atomic masses of the neutron and various atoms as given in the 2003 atomic mass evaluation together with the defined value for ^{12}C.

Atom	Relative atomic mass $A_r(X)$	Relative standard uncertainty u_r
n	1.008 664 915 74(56)	5.6×10^{-10}
^{1}H	1.007 825 032 07(10)	1.0×10^{-10}
^{2}H	2.014 101 777 85(36)	1.8×10^{-10}
^{3}H	3.016 049 2777(25)	8.2×10^{-10}
^{3}He	3.016 029 3191(26)	8.6×10^{-10}
^{4}He	4.002 603 254 153(63)	1.6×10^{-11}
^{12}C	12	(exact)
^{16}O	15.994 914 619 56(16)	1.0×10^{-11}
^{28}Si	27.976 926 5325(19)	6.9×10^{-11}
^{29}Si	28.976 494 700(22)	7.6×10^{-10}
^{30}Si	29.973 770 171(32)	1.1×10^{-9}
^{36}Ar	35.967 545 105(28)	7.8×10^{-10}
^{38}Ar	37.962 732 39(36)	9.5×10^{-9}
^{40}Ar	39.962 383 1225(29)	7.2×10^{-11}
^{87}Rb	86.909 180 526(12)	1.4×10^{-10}
^{107}Ag	106.905 0968(46)	4.3×10^{-8}

Mass in MeV/c^2

- Because of the **equivalence of mass and energy** ($E = mc^2$), often the mass is given in energy units, e.g., energy equivalent of 1 u is:

$$E(1\ \mathrm{u}) = 1.660538782 \times 10^{-27} \times (2.99792485 \times 10^8)^2 = 1.492418099 \times 10^{-10}\ \mathrm{J}$$

$$= \frac{1.492418099 \times 10^{-10}\ \mathrm{J}}{1.602176487 \times 10^{-19}\ \mathrm{J/eV}}$$

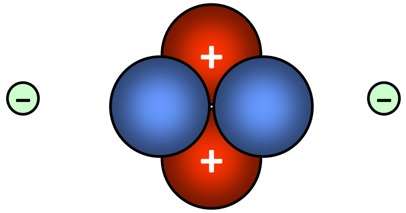
$$= 9.31494196 \times 10^8\ \mathrm{eV} \quad (= 931.494196\ \mathrm{MeV})$$

$$\boxed{1\ \mathrm{u} \approx 931.494\ \mathrm{MeV}/c^2}$$

eV (electron-volt) is a unit for energy used mainly for atomic processes. I eV is the (kinetic) energy gain by an electron or a proton when accelerated through a potential difference of 1 V. ***1 eV*** $= \mathbf{1.602 \times 10^{-19}}$ ***J.***

Nuclear Binding Energy

- If we take any atomic mass given and compare it to the sum of the masses of its constituents, you will find that the atomic mass is always less than the sum.
- Take helium-4, for example. The neutral He atom contains 2 protons, 2 neutrons and 2 electrons.



$$m\left({}^4_2\mathrm{He}\right) = 4.002603\ \mathrm{u}$$ (measured and given in tables)

$$2m_p + 2m_n + 2m_e = 2(1.007276 + 1.008665 + 0.0005486)\ \mathrm{u} = 4.032979\ \mathrm{u}$$

$$\Delta m = 4.032979 - 4.002603 = 0.030376\ \mathrm{u} = 28.295\ \mathrm{MeV}/c^2$$

Mass defect

- We need ~ 28 MeV to separate the helium atom into its individual constituents – a factor for its stability. Alternatively, we can say that if we assemble the helium atom from its constituents, 28 MeV will be given off as energy. This is known as its **binding energy**.

Nuclear Binding Energy

- In general, we can use the following formula to calculate the binding energy of nuclide:

Mass of neutral atom

$$\boxed{E_B = (ZM_H + Nm_n - {}^A_Z M)c^2}$$

Mass of hydrogen atom **Mass of neutron**

- We have lumped the mass of electron and mass of proton together as the mass of hydrogen, and it is easy to see that this will take care of all the electrons – since we are usually given the mass of neutral atoms.
- Some of you may see that there is a slight difference between $(m_p + m_e)$ and m_H at the precision given in the tables earlier. Calculate this difference and see if you agree on the origin of this difference.

Nuclear Binding Energy

- Using the formula and the values of mass of neutral atoms:

$$\boxed{E_B = (ZM_H + Nm_n - {}^A_Z M)c^2}$$

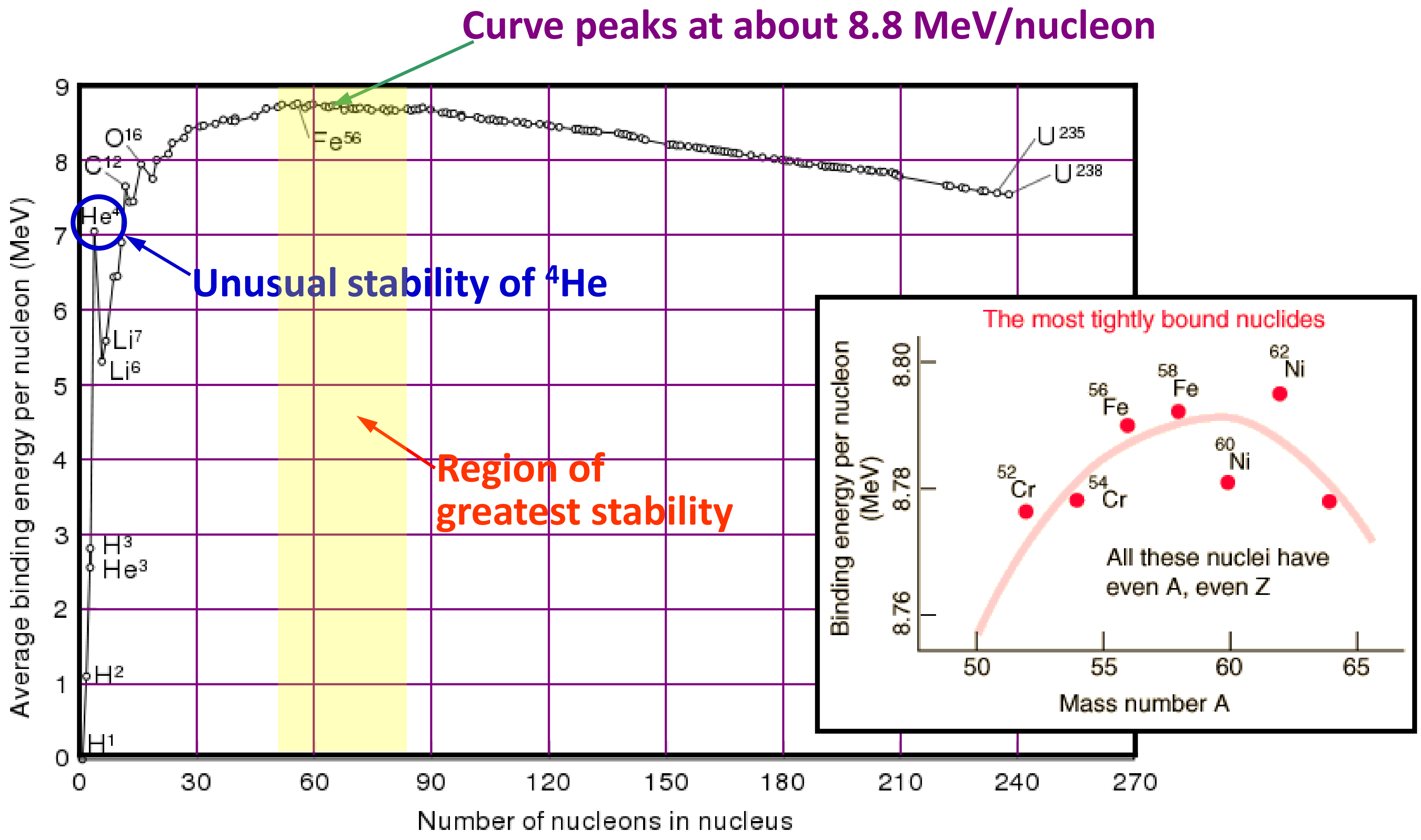
- We found that the binding energy per nucleon for each element is different – also different for different isotopes within the same element. E.g.:

$$
\begin{array}{lll}
{}^2_1\mathrm{H} & : & E_B/A = 1.112\ \mathrm{MeV/nucleon} \\
{}^3_2\mathrm{He} & : & E_B/A = 2.573\ \mathrm{MeV/nucleon} \\
{}^4_2\mathrm{He} & : & E_B/A = 7.074\ \mathrm{MeV/nucleon} \\
{}^{12}_{6}\mathrm{C} & : & E_B/A = 7.684\ \mathrm{MeV/nucleon} \\
{}^{62}_{28}\mathrm{Ni} & : & E_B/A = 8.795\ \mathrm{MeV/nucleon}
\end{array}
$$

(Most tightly bound nuclide)

Nuclear Binding Energy

- Plot of binding energy per nucleon against A:



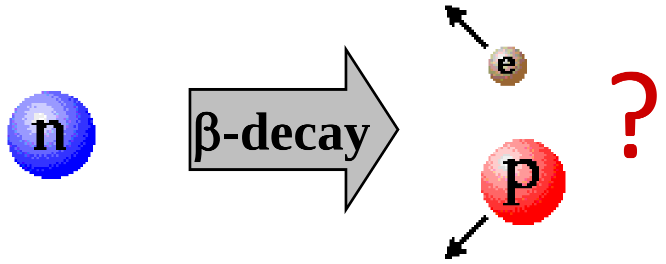
Nuclear Forces

- The most stable nuclides, Fe and Ni, have 26 and 28 protons respectively, packed into a very small volume. There must exist a very strong force that binds the nucleons together despite the repulsive Coulomb interaction.
- Though we cannot write down the force in a closed form like the electrostatic or gravitational forces, we have deduced that this **strong nuclear force** has the following properties:
  - **Independent of charge**: acts on protons and neutrons equally.
  - **Short-range**: only 10^{-15} m. Acts only on nearest neighbours – different from Coulomb interaction. Size of nucleus limited – Coulomb force overwhelms nuclear force for large nucleus.
  - **Favors pairing:** pair of protons, pair of neutrons, and pair of pairs – as seen in ^{4}He.

Yet Another New Particle: Neutrinos

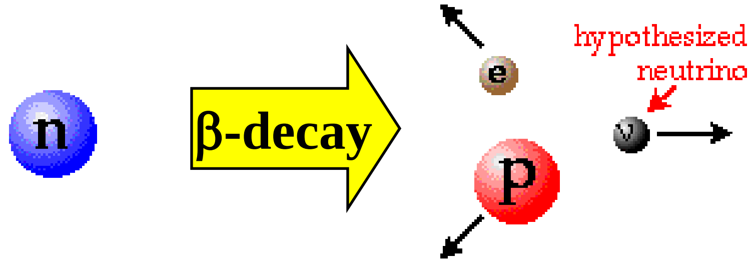
- Some nuclei that are "rich" in neutrons are unstable and decay by "converting" a neutron to a proton (which will still reside within the nucleus) and ejecting an electron (known as beta-particle) out of the nucleus. The expected equation is:

$$\boxed{{}^{1}_{0}\mathrm{n} \longrightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\beta^{-}}$$



- Note that while the charge is conserved above, measurement of the energy, momentum and angular momentum showed that these were **<u>not</u>** conserved.
- Pauli proposed a third undetected neutral particle (neutrino) which carries away the remaining energy, etc. So the equation should in fact be:

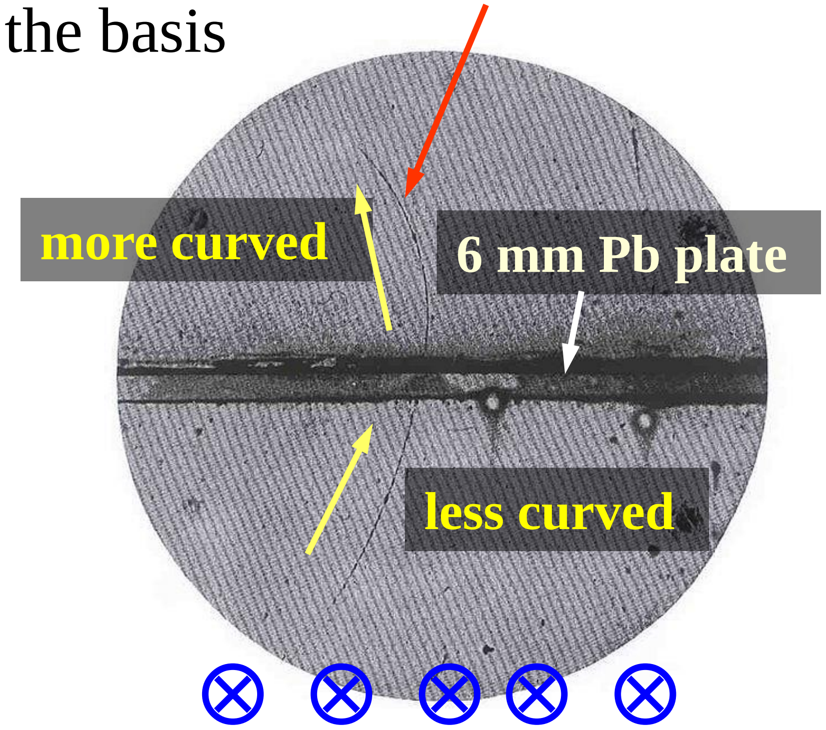
$$\boxed{{}^{1}_{0}\mathrm{n} \longrightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\beta^{-} + \bar{\nu}}$$



Discovery of Positron – Carl Anderson, 1932

"On August 2, 1932, during the course of photographing cosmic-ray tracks... the tracks shown in Fig. 1 were obtained, which seemed to be interpretable only on the basis of the existence in this case of a particle carrying a positive charge but having a mass of the same order of magnitude as that normally possessed by a free negative electron... It is concluded, therefore, that the magnitude of the charge of the positive electron which we shall henceforth contract to positron is very probably equal to that of a free negative electron which from symmetry considerations would naturally be called a negatron."

track of positron in cloud chamber



B (into slide)

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

Anti-Matter

- The observational confirmation of the anti-particle of electron opened up the possibility that there could be anti-particles for all the particles that we have so far mentioned.
- Anti-proton was discovered in 1955. Stable but will annihilate with proton.
- Anti-neutron was discovered in 1956. Neutral but different from neutron – its magnetic moment and its composition in terms of quarks are different. Will annihilate with neutrons – and results in γ-radiation or formation of new particles.
- Anti-hydrogen (positron and anti-proton) atom first created in CERN in 1994. Anti-helium has also been created.
- Storage – not in normal container – will annihilate. Only in magnetic traps – similar to those used for cold atoms.

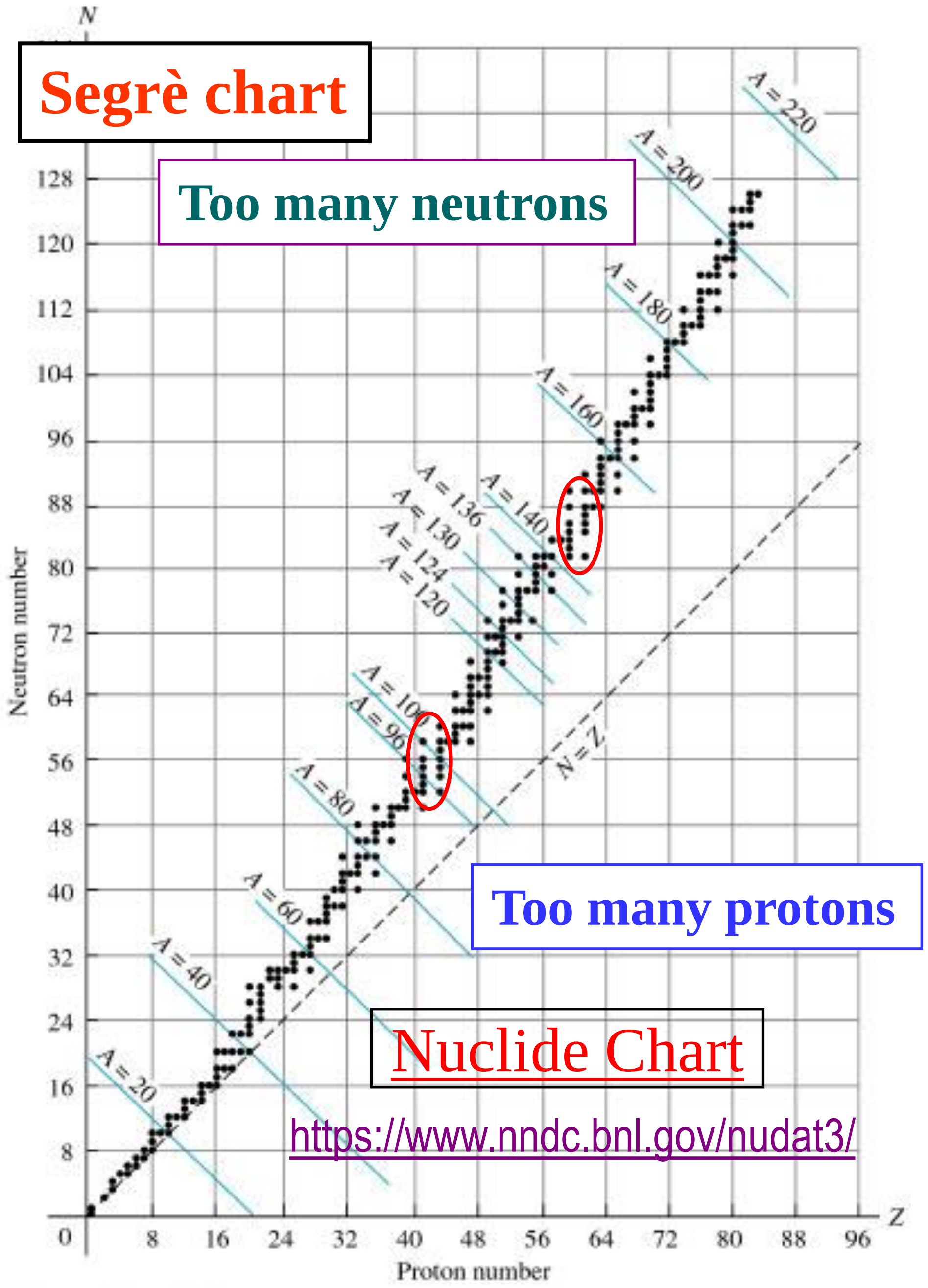
Anti-matter is real – not science fiction. However, for some reasons, our universe is filled almost entirely by normal matter though anti-matter are continuously being created and annihilated. Some nuclides decay by emitted anti-matter (positrons)!

Nuclear Stability

- Segrè Chart on right shows all the 254 stable nuclides plotted on N vs Z graph.
- For low A, $N \approx Z$. N/Z increases to 1.6 for large A.
- For every value of A (blue diagonal lines), there are usually 1 or 2 stable nuclides.
  - No stable nuclide for $A = 5$ and $A = 8$.
  - 3 stable nuclides for $A = 96, 124, 130$ and 136.
- Also no stable nuclide for $Z = 43$ (technetium) or $Z = 61$ (promethium).
- Only 4 stable nuclides with odd Z and N

$$^{2}_{1}\mathrm{H} \quad ^{6}_{3}\mathrm{Li} \quad ^{10}_{5}\mathrm{B} \quad ^{14}_{7}\mathrm{N}$$

- No stable nuclide with $A > 209$ or $Z > 83$.



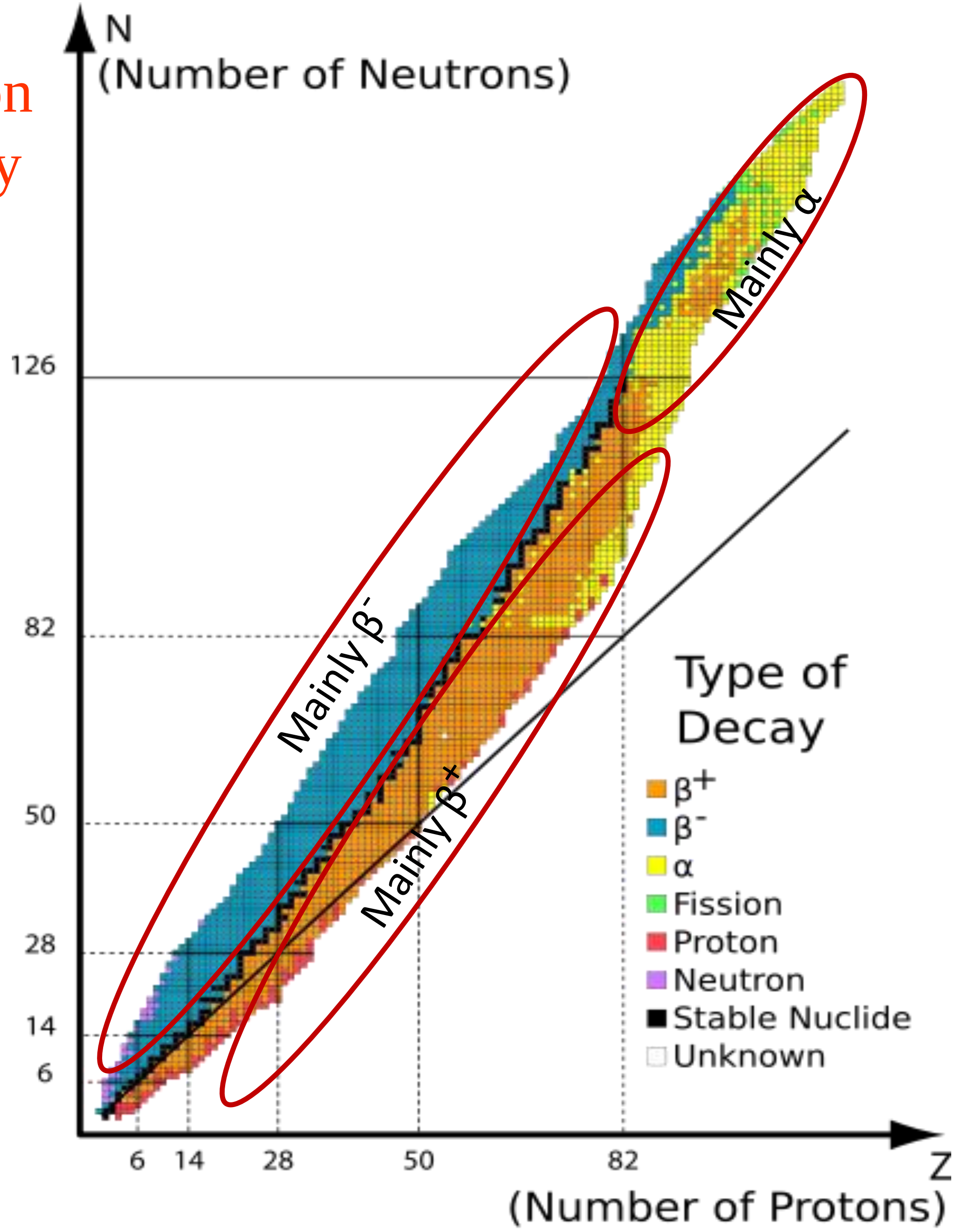
Ways that Unstable Nuclides Decay

- Alpha Decay – Helium nucleus
- Beta Decay – Usually electrons
- Gamma Decay – EM radiation

Most common and named by Rutherford

<u>Other Less Common Ways of decay</u>

- Other β-decays –
  - Positron emission
  - electron capture
- Proton emission
- Neutron emission
- Clusters decay – small nucleus
- Spontaneous fission



Alpha Decay

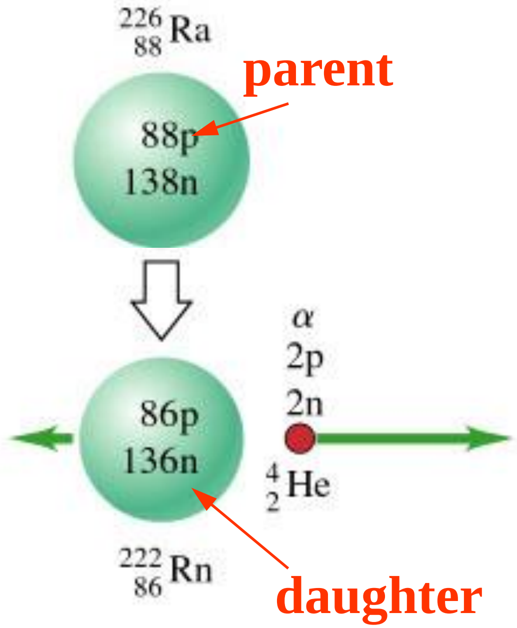
- Mainly for heavier atoms – most common way to get rid of excess nucleons when nuclide mass is too large.

$$\boxed{{}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He}}$$

- Spontaneous when:

$$m_{\mathrm{X}} > m_{\mathrm{Y}} + m_{\mathrm{He}}$$

- **Disintegration energy *Q*** = difference in mass is released as kinetic energy (KE) of new nuclide and helium nuclide. Process must obey conservation of KE and linear momentum.



- Alpha decays move the nuclide towards the more stable part of the Segre chart (may be followed by β-decay or more α-decays).

Examples of α-Decay Rate

- Table shows some examples of the decay time for α-decay and the energies of the α-particle for these decays.
- Notice the vast difference among the decay time – tend to be short for those with higher energy.

Parent nucleus	**α-particle energy (MeV)**	**Mean time to decay**
Po-212	8.8	4.4×10^{-7} s
Rn-220	6.3	79 s
Ra-224	5.7	5.3 days
Ra-226	4.8	2300 years
U-238	4.3	6.5×10^{9} years

Beta Decays

- There are actually three different types of β-decay, The most common is that of **an emission of an electron**, known as **beta-minus decay**. It happens when N/Z in the nucleus is too high. Symbolically given by:

$$\boxed{{}_{Z}^{A}\mathrm{X} \rightarrow {}_{Z+1}^{\;\;A}\mathrm{Y} + {}_{-1}^{\;\;0}\beta^{-} + \bar{\nu}}$$

anti-neutrino

- On the other hand, if the Z/N in the nucleus is too large, the nucleus may decay by emitting a positron (known as **beta-plus decay**). During this process, a neutrino is emitted too. The reaction is

$$\boxed{{}_{Z}^{A}\mathrm{X} \rightarrow {}_{Z-1}^{\;\;A}\mathrm{Y} + {}_{+1}^{\;\;0}\beta^{+} + \nu}$$

neutrino

positron

Beta Decay (Electron Capture)

- Another process that may happen when *Z*/*N* is too large is that the nucleus may seize an orbiting electron and effectively convert a proton into a neutron in the nucleus, meanwhile emitting a neutrino.

$$\boxed{{}^{A}_{Z}\mathrm{X} + {}^{0}_{-1}\beta^{-} \rightarrow {}_{Z-1}^{\ \ A}\mathrm{Y} + \nu}$$

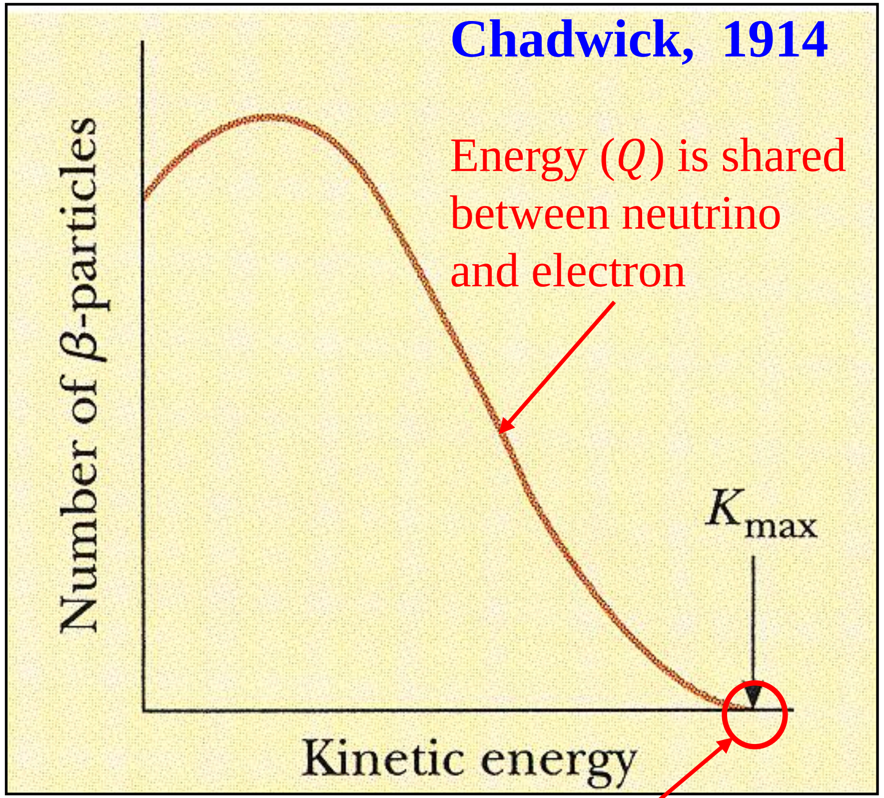
neutrino

electron

Beta Decay – KE of electrons

- Note that when the kinetic energy of the electrons is measured, it was found to follow a distribution similar to the graph on the right. We cannot easily use the energy of the emitted electron to identify nuclide that decays.

- Some energy could be carried away by the neutrino and thus unlike α and γ decays, the kinetic energy of the electrons show a continuous spectrum with a maximum value $K_{max} \approx Q$ calculated from the mass difference.



All energy (Q) carried by the electron

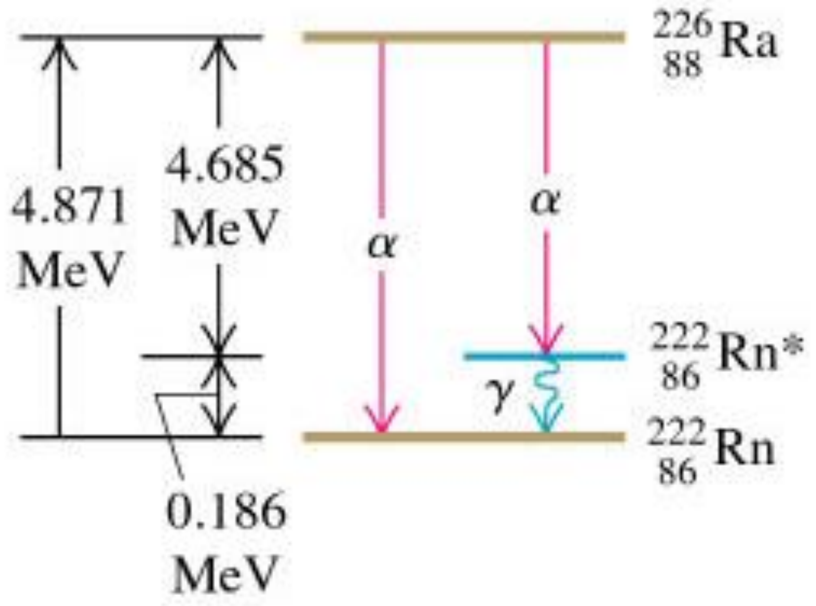
Gamma Decay

- An atom in an excited state may lower its energy by emitting a photon. Similar, a nucleus in an excited state may also do so, but the energy of the photon is usually higher resulting in gamma radiation.

- Unlike α- and β-decays, γ-decay does not change A, N nor Z.

$$\boxed{{}^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X} + \gamma}$$

excited nucleus

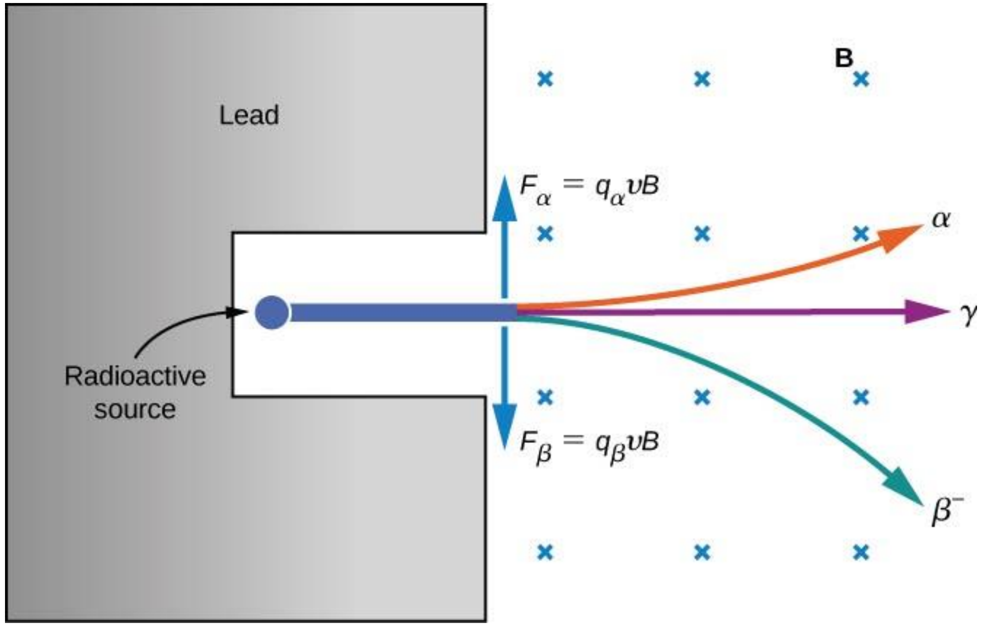


- γ-decay often accompanies α- or β-decay, as the daughter nucleus is left in excited states, e.g., in the α-decay of radium discussed, a different pathway would be an α–decay with E = 4.685 MeV + a photon of 0.186 MeV.

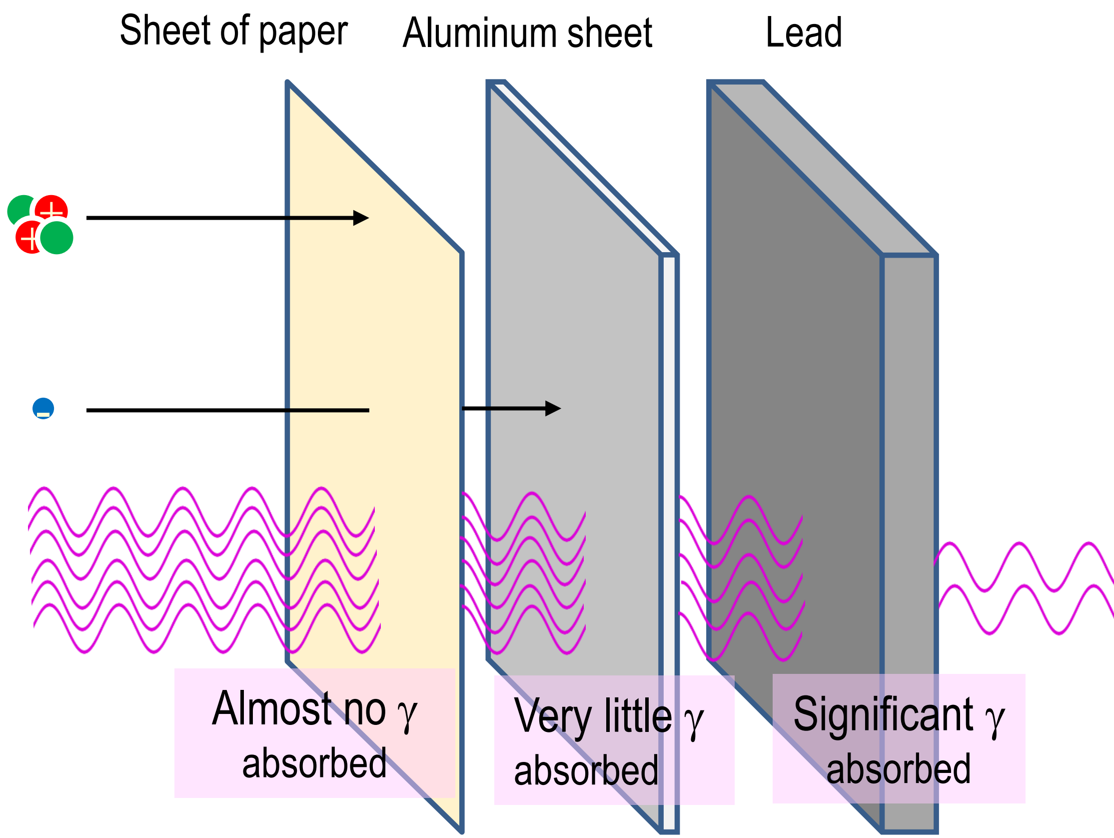
<u>Observed</u>
α particles: 4.685 MeV & 4.871 MeV
γ-rays: 0.186 MeV

Some Properties of α- and β-particles and γ-rays

Radiation	Identity	Penetration depth in air	Charge	Speed
α particle	Helium nucleus	A few cm	+ve	Fast
β particle	Fast electron	About 1 m	-ve	Very fast
γ ray	Energetic EM radiation	Very long	0	Speed of light



α and β particles and γ-rays in magnetic field



Penetration of α and β particles and γ-rays in paper, aluminum sheet and lead block

Radioactive Decay Law

- Radioactivity is a random process – while we can determine the probability of a nucleus decaying within a certain period of time, we cannot know whether a particular nucleus will or will not decay. However, if we have a large sample of radioactive atoms, we can predict quite accurate how many will decay within this period.

- Let $N(t)$ be the number of radioactive nuclei in a sample at time t, and $dN(t)$ be the (negative) change in N during a short time interval dt. The rate of change of $N(t)$ is called the **decay rate** or **activity** $(= -dN(t)/dt)$ of the specimen. The larger $N(t)$ is, the larger the activity A will be, i.e.,

$$\boxed{-\frac{dN(t)}{dt} = \lambda N(t)}$$

Negative sign to make sure that both sides of equation are positive

decay constant

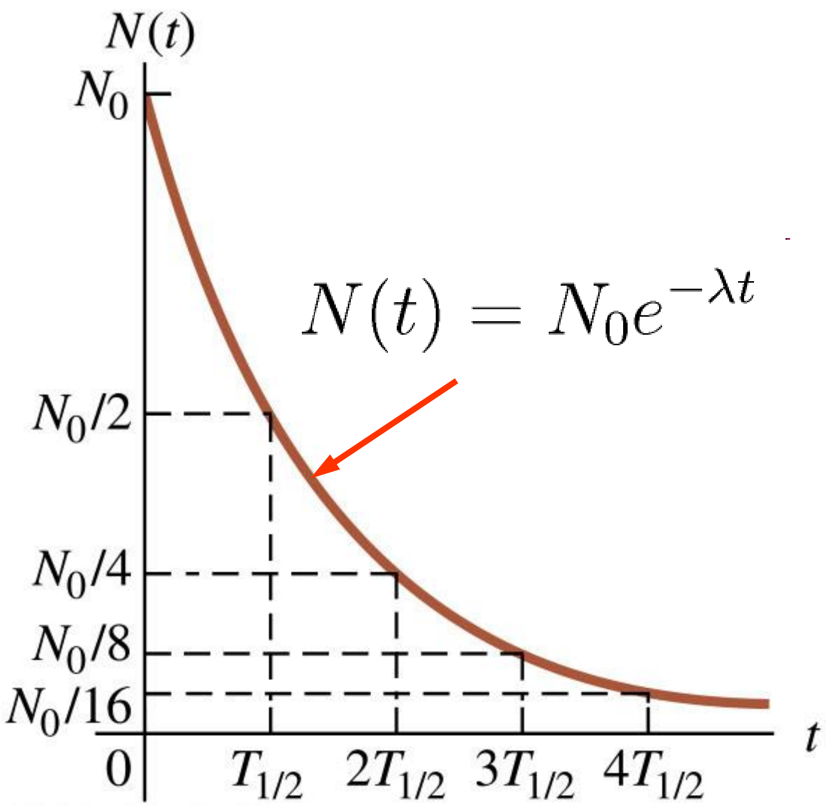
Decay Constant

- The **decay constant λ** is different for different nuclides. Larger value means more decay per unit time. l is also the ratio of number of decay per unit time to the number of remaining radioactive nuclei. We can interpret it as the probability per unit time that any particular nucleus will decay.

$$\boxed{-\frac{dN(t)}{dt} = \lambda N(t)} \xrightarrow{\textbf{Solving}} \boxed{N(t) = N_0 e^{-\lambda t}}$$

where N_0 is the number of nucleus at time $t = 0$.

Half-life



- Another useful term is **half-life $T_{1/2}$** which is the time required for the number of radioactive nucleus to decrease to half its original value.
- Thus after say *n* half-lives, only $(1/2)^n$ of the original nuclei are still radioactive.
- To relate $T_{1/2}$ to λ:

$$\frac{N_0}{2} = N_0 e^{\lambda T_{1/2}} \Rightarrow \frac{1}{2} = e^{-\lambda T_{1/2}}$$

$$\ln 2 = \lambda T_{1/2}$$

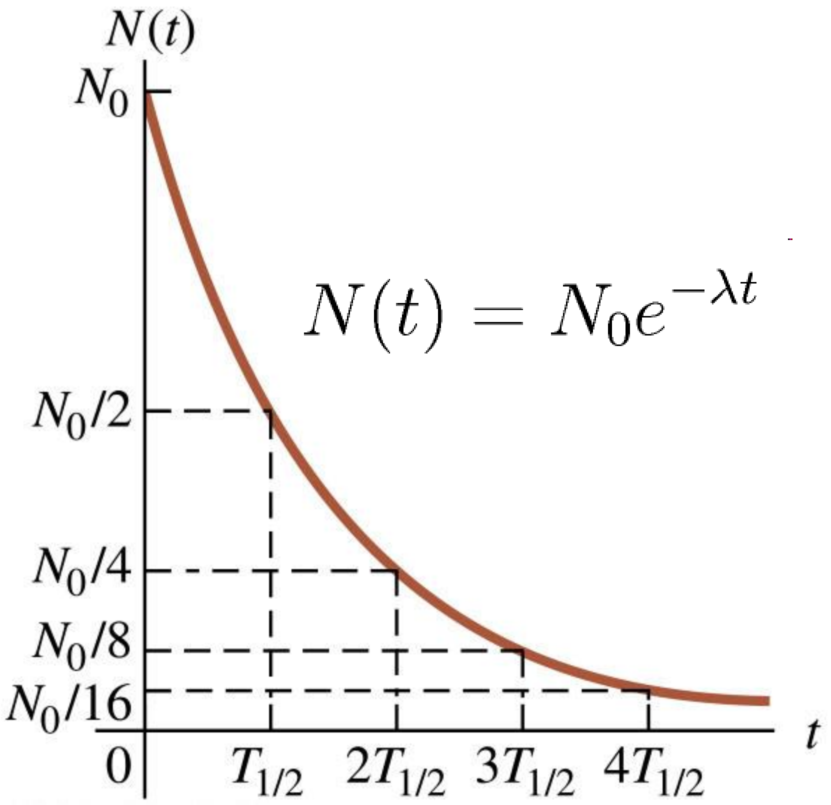
$$\boxed{T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}}$$

Mean Lifetime and Activity

- We may also define the **mean lifetime T_{mean}** as the time for the $N(t)$ to decrease to $1/e$ of its original value (similar to the time constant τ in the RC circuit). Then

($\ln e = 1 \Rightarrow e = 2.71\,828\,18$)

$$\boxed{T_{\text{mean}} = \frac{1}{\lambda} = \frac{T_{1/2}}{\ln 2} = \frac{T_{1/2}}{0.693}}$$



- Note that since the **activity** is proportional to $N(t)$, it has the same relation wrt time as $N(t)$, i.e., an exponential decay type of dependence.
- After n half-lives, the activities will also drop to $(½)^n$ of its original activity level.

Units for Activity

- The modern unit for activity is **bacquerel** (**Bq**, named after Henri Becquerel who discovered radioactivity accidentally in 1896) which is one decay per second.
- Another common and easier to work with unit is **curie** (**Ci**, named after Pierre Curie) which is the level of activity of 1 gram of Ra-226. The conversion between Ci and Bq is:

$$\boxed{1\ \mathrm{Ci} = 3.70 \times 10^{10}\ \mathrm{Bq}}$$

- 1 curie is a very large unit and usually mCi ($= 10^{-3}\ \mathrm{Ci} = 37\ \mathrm{MBq}$) or μCi ($= 10^{-6}\ \mathrm{Ci} = 37\ \mathrm{kBq}$) are used.

Example: ^{60}Co

An unstable isotope of cobalt, ^{60}Co, is a β^- emitter with a half-life of 5.27 years. A certain radiation source in a hospital contains 0.0360 mg of ^{60}Co.

a) What is the decay constant of this isotope?
b) How many atoms are in the source?
c) How many decays occur per second?
d) What is the activity of the source in curies? How does this compare with the activity of an equal mass of radium-226 with a half-life of 1600 year?

Given:

$$T_{1/2} = 5.27 \text{ yrs}, \qquad m = 3.6 \times 10^{-8} \text{ kg}, \qquad A = 60$$

Example: ^{60}Co

$$\text{Given:} \quad T_{1/2} = 5.27 \text{ yrs}, \qquad m = 3.6 \times 10^{-8} \text{ kg}, \qquad A = 60$$

a) Decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{5.27 \times 365 \times 24 \times 3600} = \boxed{4.19 \times 10^{-9} \text{ s}^{-1}}$$

b) Number of atoms in source *N* is

$$N = \frac{m}{m_{\text{atom}}} = \frac{3.6 \times 10^{-8}}{60 \times 1.66 \times 10^{-27}} = \boxed{3.61 \times 10^{17}}$$

c) Number of decays per second is

$$-\frac{dN(t)}{dt} = \lambda N = 4.19 \times 10^{-9} \times 3.61 \times 10^{17} = \boxed{1.51 \times 10^{9} \text{ Bq}}$$

Example: ^{60}Co

d) Activity in curie is

$$-\frac{dN(t)}{dt} = \frac{1.51 \times 10^9 \text{ Bq}}{3.70 \times 10^{10} \text{Bq/Ci}} = \boxed{0.0408 \text{ Ci}}$$

By definition, 1 Ci is the activity of 1 gram of ^{226}Ra. So, 0.036 mg of ^{226}Ra will have activity 3.6×10^{-5} Ci. Ratio of activity of Co over that of Ra is therefore:

$$\frac{R_{\text{Co}}}{R_{\text{Ra}}} = \frac{0.0408}{3.6 \times 10^{-5}} = \boxed{1130}$$

Alternatively, we can calculate directly too:

$$\frac{R_{\text{Co}}}{R_{\text{Ra}}} = \frac{\lambda_{\text{Co}} N_{\text{Co}}}{\lambda_{\text{Ra}} N_{\text{Ra}}} = \frac{T_{1/2,\text{Ra}} A_{\text{Ra}}}{T_{1/2,\text{Co}} A_{\text{Co}}} = \frac{1600 \times 226}{5.27 \times 60} = \boxed{1140}$$