

GET1024 / GEC1036 Lecture 2

Atoms, Electrons & EM Radiation

What is an Atom?

Experimental Evidence

Models of an Atom

Wave and Particle Nature

Schrödinger Wave Equation

Periodic Table of Element

Introductory Comments

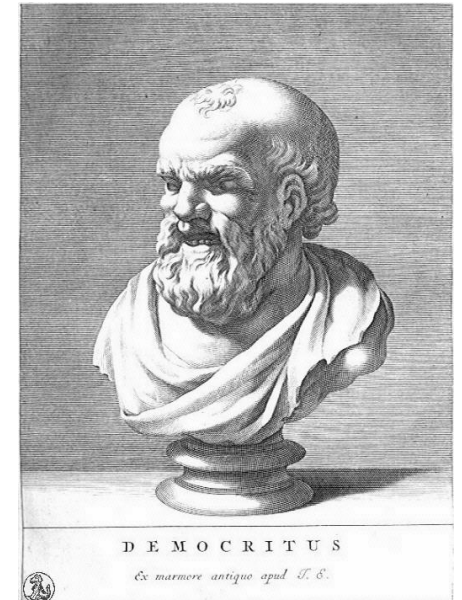
- This lecture is probably the most “technical” lecture in the whole module. It appears to be a pure Physics lecture and put together many ideas in Modern Physics to bring you to our understanding (or interpretation) of the world at the atomic scale.
- It aims to give a more complete picture of what we know about an atom from its speculative beginnings through the observations of scientists over the ages to various models built to explain these observations.
- Some of the contents or even concepts, e.g., the Schrödinger Equation, may not be directly relevant to later lectures in this module. However, these are some of the most important breakthroughs in science that led to our final understanding of nature including something that almost of us learnt when we started our journey in Science – the Periodic Table of Elements.
- There are also important concepts such as the idea of photon being the “quantum” emitted when a bound electron changes from a higher energy state to a lower energy state in an atom, which will be important throughout this module.
- As a General Education Module, we hope to broaden students’ perspectives and go beyond boundaries. While most calculations and proofs have to be left out and many ideas are rather hand-waving, I hope that by including them, you get a better understanding of how physicists view the world.
- Finally, don’t be discouraged by the contents. I will indicate during the lecture what you need to know.

Ancient Greek Philosophical Speculations

- Greek philosopher Empedocles (492 – 432 BCE) proposed that all matters were composed of four **elements**: fire, air, water, and earth.
- Democritus (460 – 370 BCE) observed that if you cut a stone in half, each half had the same properties as the original stone. He reasoned that if you continued to cut the stone into smaller and smaller pieces, at some point you would reach a basic unit of stone (presumably of very tiny size) that could no longer be divided. Democritus called these basic unit of matter **atomos**, meaning “**indivisible**”. He visualized that atomos were eternal and could not be destroyed.
- Democritus further theorized that atomos were specific to the material that they made up, meaning that:
 - The atomos of a particular material, e.g., stone, were all the same.
 - Different materials are made up of different atomos.



Empedocle's.

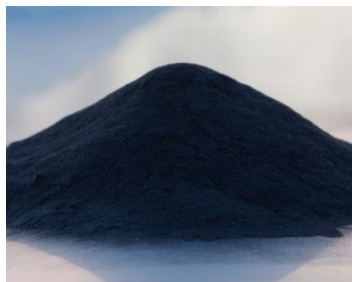


DEMOCRITUS

Ex marmore antiquo apud H. G.

Evidence Based Theory of Atoms

- The philosophical speculations were the products of creative imagination and logical reasoning. But they are not science – not based on evidence – does not lead to deeper understanding nor find ways to get things done better.
- In early 1800s, John Dalton used the concept of atoms to explain why elements always react in ratios of small whole numbers (the law of multiple proportions).
- Example: there are two types of tin oxides. Mass ratios of tin (Sn) and oxygen (O) are :



88.1% (Sn), 11.9% (O)
100 g of Sn combines with 13.5 g of O



78.7% (Sn), 21.3% (O)
100 g of Sn combines with 27.0 g of O





















13.5 and 27.0 form a ratio of 1:2, a ratio of small whole numbers.

- This common pattern in chemistry suggested to Dalton that elements react in multiples of discrete units — in other words, atoms. In the case of tin oxides, one tin atom will combine with either one or two oxygen atoms.



Dalton's List of Elements (1808)

- Identification of the elements was a tedious project.
- Substances were removed one by one from the list when they were found to decompose into other substances.
- The search for elements was a continuation of the ancient quest: to identify the basic, pure components that make up all matters in the universe.
- It was soon clear that there were many elements, not just four, as the Greek thought.

ELEMENTS				
	Hydrogen	1		Strontian (Strontium) 46
	Azote (Nitrogen)	5		Barytes (Barium) 68
	Carbon	5		Iron 50
	Oxygen	7		Zinc 56
	Phosphorus	9		Copper 56
	Sulphur	13		Lead 90
	Magnesia (Magnesium)	20		Silver 190
	Lime (Calcium)	24		Gold 190
	Soda (Sodium)	28		Platina (Platinum) 190
	Potash (Potassium)	42		Mercury 167

List of elements proposed by Dalton in 1808

Dmitri Mendeleev's Periodic Table

- By 1863, 56 elements had been identified and their atomic weight determined.
- Dmitri Mendeleev published in 1869 his first periodic table. He discovered that by arranging the elements in order of atomic weight, elements with similar properties tended to occur in certain periods.
- Such periodic tables had in fact been proposed by earlier chemists, but Mendeleev was the first to follow up on the periodic structure seriously and noted that some slots in the table should be left empty for some as yet undiscovered elements.
- He also made predictions on the properties of the missing elements. The predictions mostly proved correct when the elements were later discovered.
- Such periodic structure strongly hinted at some underlying principle in the making of the atoms of the elements.

ОПЫТЪ СИСТЕМЫ ЭЛЕМЕНТОВЪ.
ОСНОВАННОЙ НА ИХЪ АТОМНОМЪ ВѢСѢ И ХИМИЧЕСКОМЪ СХОДСТВѢ.

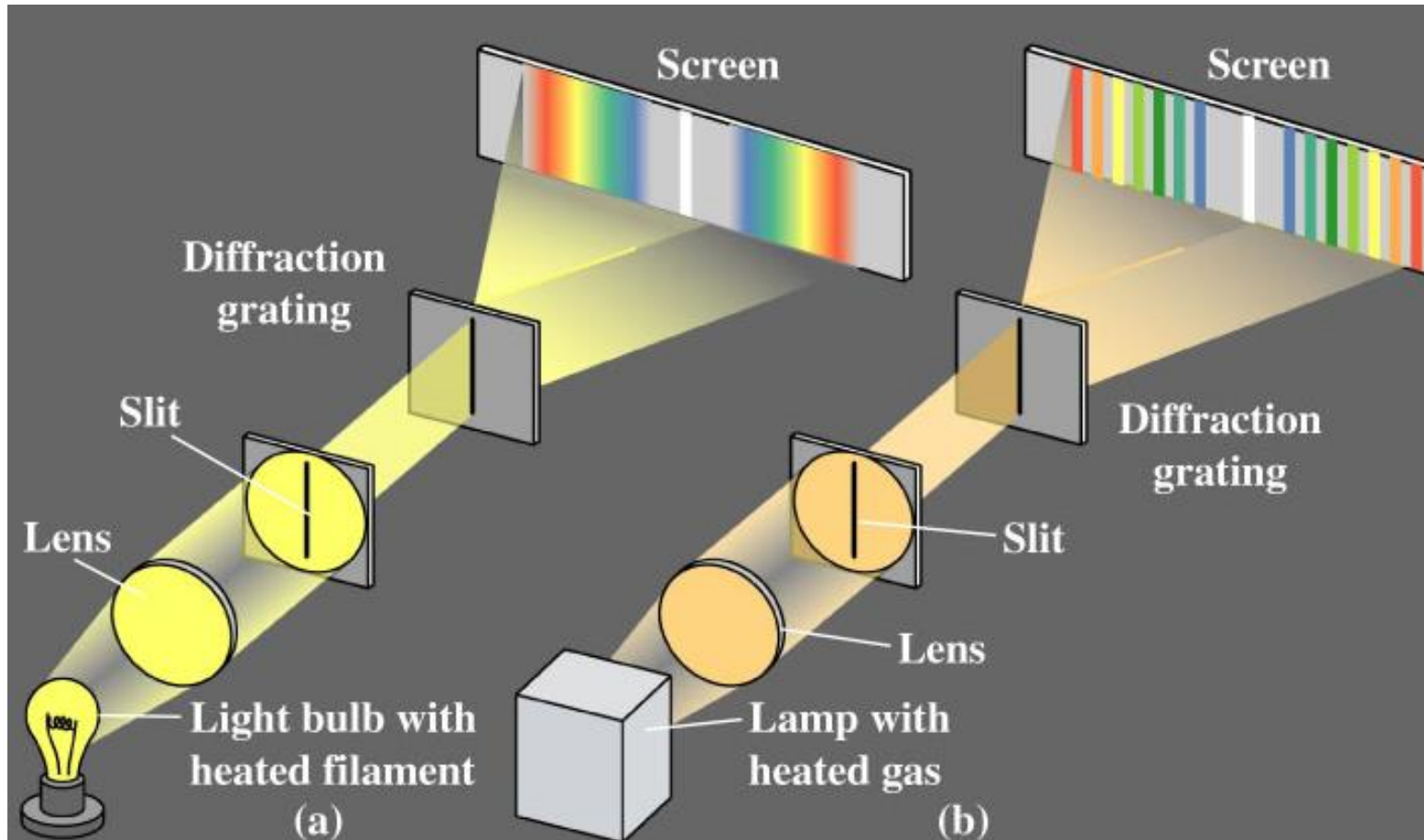
			Ti = 50	Zr = 90	? = 180.
			V = 51	Nb = 94	Ta = 182.
			Cr = 52	Mo = 96	W = 186.
			Mn = 55	Rh = 104,4	Pt = 197,1.
			Fe = 56	Ru = 104,4	Ir = 198.
			Ni = 59	Pd = 106,8	Os = 199.
			Cu = 63,4	Ag = 108	Hg = 200.
H = 1	Be = 9,4	Mg = 24	Zn = 65,2	Cd = 112	
	B = 11	Al = 27,1	? = 68	Ur = 116	Au = 197?
	C = 12	Si = 28	? = 70	Sn = 118	
	N = 14	P = 31	As = 75	Sb = 122	Bi = 210?
	O = 16	S = 32	Se = 79,4	Te = 128?	
	F = 19	Cl = 35,5	Br = 80	I = 127	
Li = 7	Na = 23	K = 39	Rb = 85,4	Cs = 133	Tl = 204.
		Ca = 40	Sr = 87,6	Ba = 137	Pb = 207.
		? = 45	Ce = 92		
		?Er = 56	La = 94		
		?Yt = 60	Di = 95		
		?In = 75,5	Th = 118?		



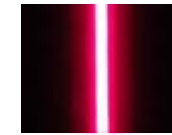
Д. Менделѣевъ

What Really is an Atom?

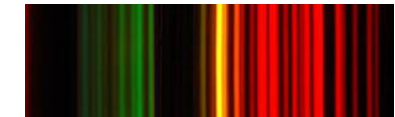
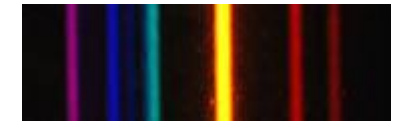
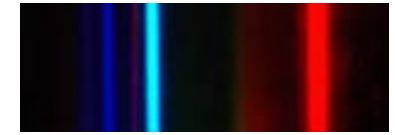
- From Chemistry alone, there is no clue on what really is an atom or is it “indivisible” as described by Democritus. Are there any structures in the atoms?
- Surprisingly, the breakthrough came from the observation of light emitted from different sources.



Color of lamp



Spectrum



Hydrogen

Helium

Neon

Hydrogen Spectrum

- **Hydrogen** is the lightest and simplest – consisting of one proton and one electron. It is therefore not surprising that the first effort was to use the spectrum of hydrogen to understand the physics behind the spectrum, and as a result how the atom is “structured”.
- In the visible range, there are only **four intense lines**. The wavelengths of these lines had been measured accurately by A J Angstrom in 1862. The four visible lines occur at wavelengths 656.3 nm* (red), 486.1 nm (blue), 434.1 (violet), and 410.2 nm (violet).
- In 1885, a school teacher, Johann Jacob Balmer, through weeks of fitting equations, he found that the wavelengths of these lines can be described by the empirical equation



$$\lambda = C \frac{n^2}{n^2 - 4}$$

for $n = 3, 4, 5$ and 6 where the best fitted value of $C = 364.6 \text{ nm}$



* $1 \text{ nm} = 10^{-9} \text{ m} = 0.000\,000\,001 \text{ m}$

Rydberg Formula

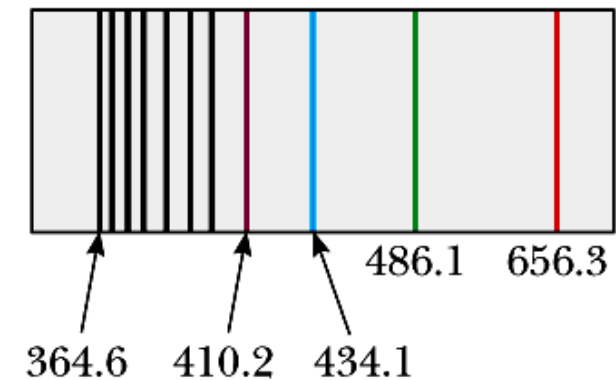
- In 1889, Rydberg rewrote the Balmer's formula in term of wavenumber ($\bar{\lambda} = 1/\lambda = \text{number of waves occupying a unit length}$) to the form that are more useful:

$$\lambda = C \frac{n^2}{n^2 - 4} \quad \xrightarrow{\text{Taking reciprocal \& simplifying with } R = 4/C} \quad \bar{\lambda} = \frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$$

where the Rydberg constant $R = 1.097 \times 10^7 \text{ m}^{-1}$, and $n = 3, 4, 5, 6, \dots$



- Balmer predicted the existence of other lines in the hydrogen spectrum for $n = 7, 8, 9$, etc. These weaker lines (in intensity) in the violet and near ultra-violet region were eventually discovered and found to obey the above formula.



Other Series of Hydrogen Spectrum

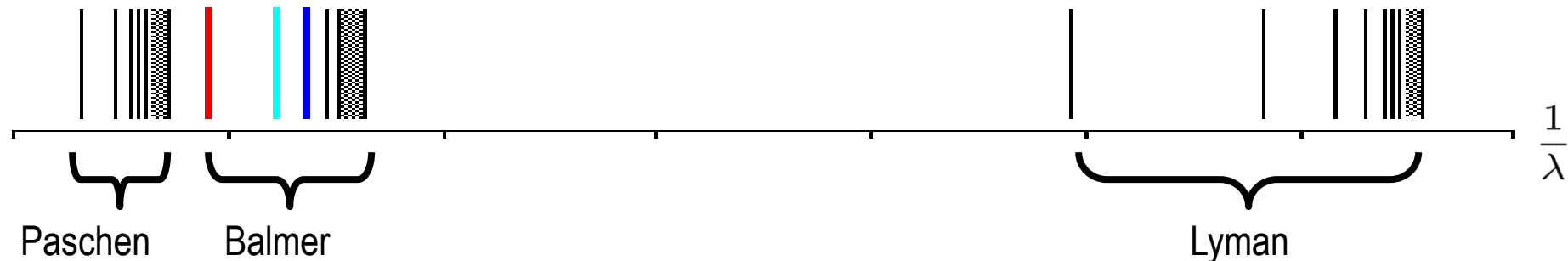
- Balmer also predicted that there may be more series with the 2^2 substituted by $1^2, 3^2, 4^2$ etc., so that in general the formula (using Rydberg form) should yield many more lines with wavelengths given by

$$\bar{\lambda} = \frac{1}{\lambda} = R \left(\frac{1}{m^2} - \frac{1}{n^2} \right)$$

Lyman	Ultraviolet	$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$	$n = 2, 3, 4, \dots$
Balmer	Mainly Visible	$\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{n^2} \right)$	$n = 3, 4, 5, \dots$
Paschen	Infrared	$\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{n^2} \right)$	$n = 4, 5, 6, \dots$
Brackett	Infrared	$\frac{1}{\lambda} = R \left(\frac{1}{4^2} - \frac{1}{n^2} \right)$	$n = 5, 6, 7, \dots$
Pfund	Infrared	$\frac{1}{\lambda} = R \left(\frac{1}{5^2} - \frac{1}{n^2} \right)$	$n = 6, 7, 8, \dots$

where $m = 1, 2, 3$, etc., and n can take any integers greater than m .

- Other than the Balmer's series, the others are not in the visible range, and required further development in UV and IR spectroscopy before they were subsequently measured and verified to obey the Rydberg formula.



Discovery of the Electron

- In 1897, Joseph John Thomson confirmed that the cathode rays observed in discharge tube consisted of beams of negatively charged particles, now known as the electrons.
- He noted that the particles were the same no matter what cathodes, anodes or the gases were used in the experiment, hence establishing the **electron as a universal component of all matter**.
- He used a combination of electrostatic field and magnetic field to deflect the cathode rays, and determined the ratio of the electron charge to the electron mass.

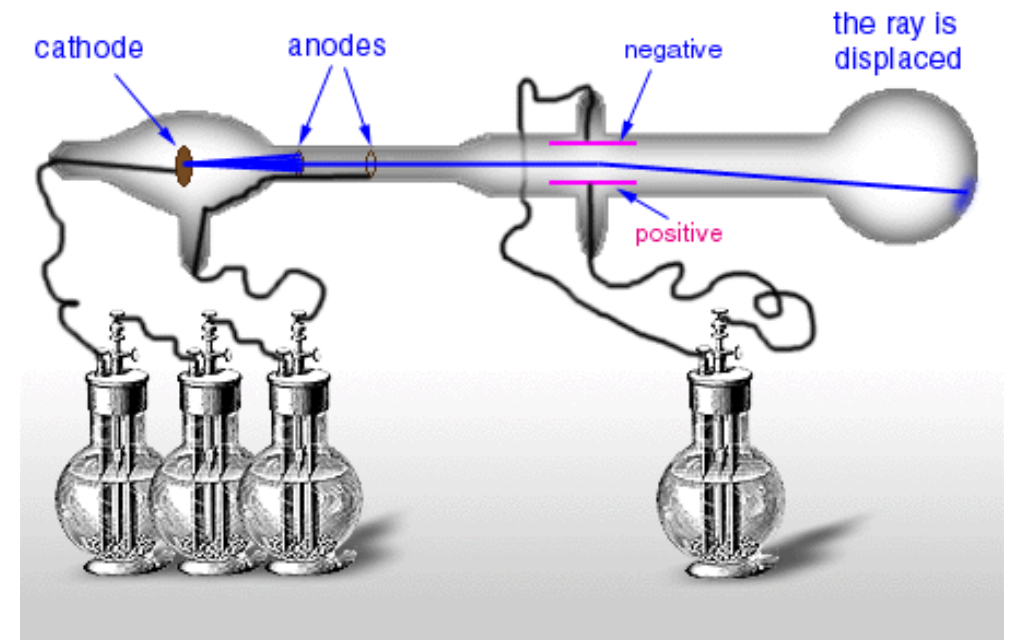
- Robert Andrews Millikan determined the electron charge with high precision in 1909 with the oil drop experiment.
- The electron charge, usually denoted e , is the fundamental unit of charge. Electric charge on any body consists of exact multiples of e . The current value of

$$e = 1.602\,176\,6208 \times 10^{-19} \text{ C.}$$

- The current value of electron mass is

$$m_e = 9.109\,383\,56 \times 10^{-31} \text{ kg.}$$

It is about 1,836 times lighter than a hydrogen atom.



One of the discharge tubes of J.J. Thomson. Source: http://www.brooklyn.cuny.edu/bc/ahp/LAD/C3/C3_Electrons.html

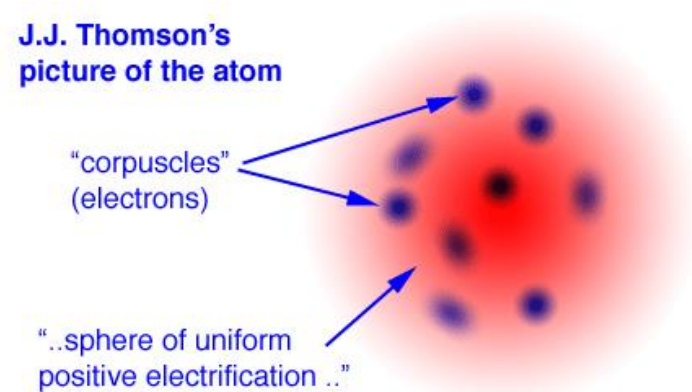
Thomson's Atomic Model

- Realizing that the electron must be a universal component of the atoms of all materials, Thomson boldly put forward in 1904 a model for the atom:



"... suppose that the atom consists of a number of corpuscles moving about in a sphere of uniform positive electrification ..."

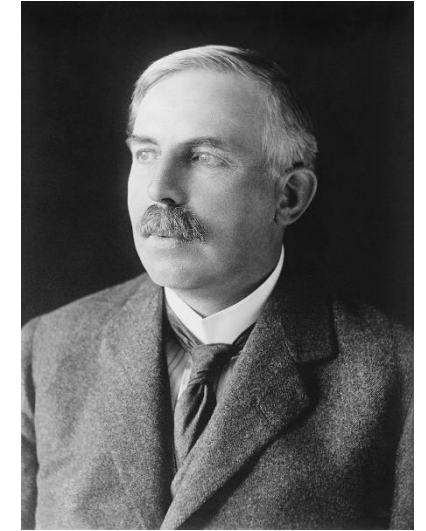
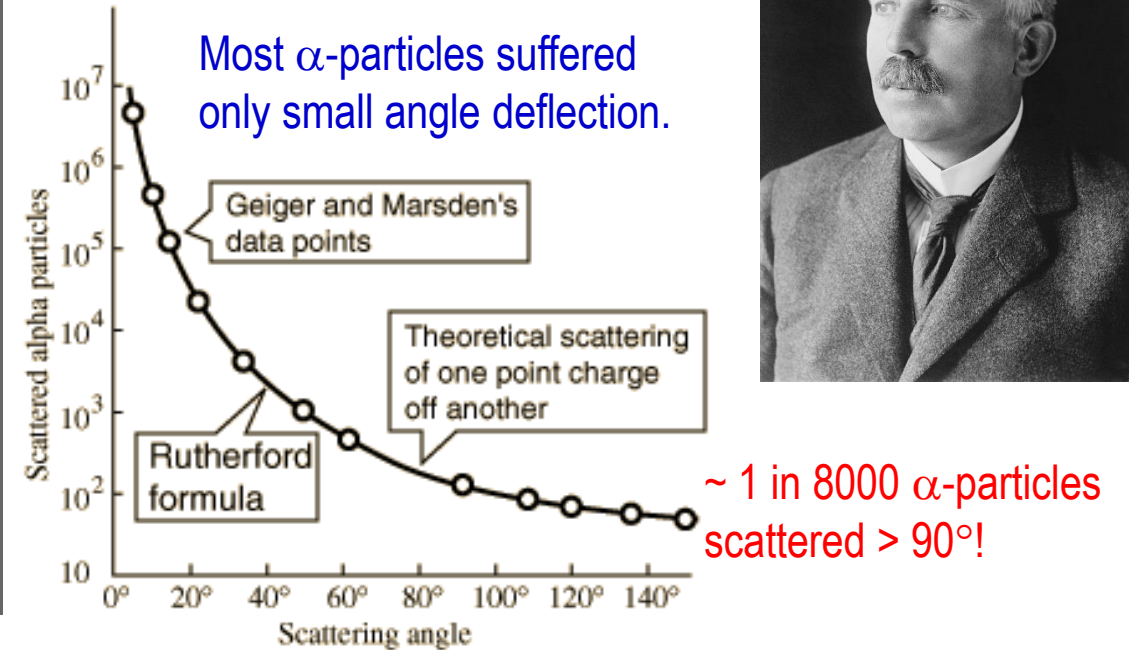
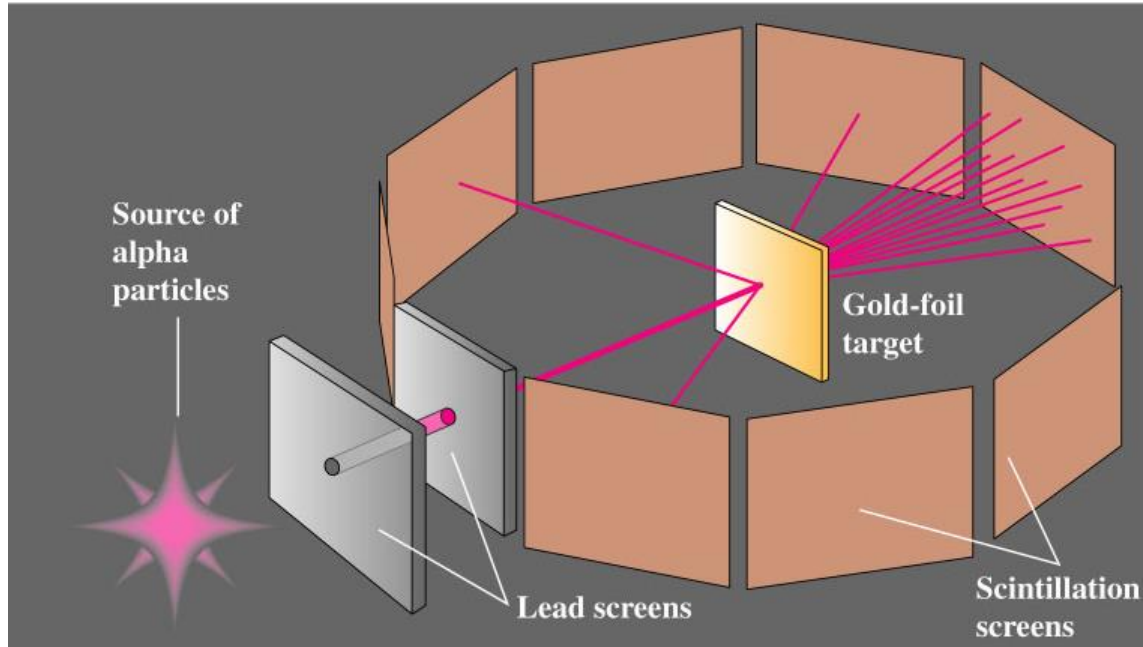
J.J. Thomson's picture of the atom



- This was known as the "plum pudding model".
- Atoms had been considered the ultimate, indivisible unit of matter. Thomson's model was the first to suggest that atom had an internal structure.
- Thomson knew that as the atom is neutral in charge, there must be positive charges of exactly the same amount to balance the electron charges. However, he did not have any information about the part of the atom that held the positive charges, and made a guess.
- Some physicists pointed out that the plum pudding atom could not be valid, as a collection of electric charges cannot be in stable equilibrium, according to a theorem in electrostatics.

Discovery of the Atomic Nucleus

- Experiment by Rutherford, Geiger and Marsden in 1909 - first experiment to give the hint of the size of the nucleus.

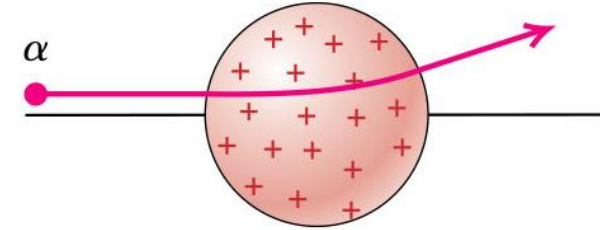


It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.

— Ernest Rutherford, 1908 Nobel Prize (Chemistry) co-winner

Thomson vs Rutherford Models

- **Thomson Model**: Plum pudding model – almost all the mass & all +ve charges of atom spread out over all space. Electrons are like plums in pudding.

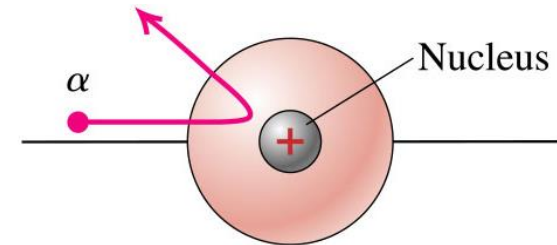


$$m_{\alpha} \approx 7300 m_e$$



Only small angle deflections through collisions with electrons

- **Rutherford Model**: Very small nucleus containing most of mass & all +ve charges. Electrons going around the nucleus (similar to solar system).



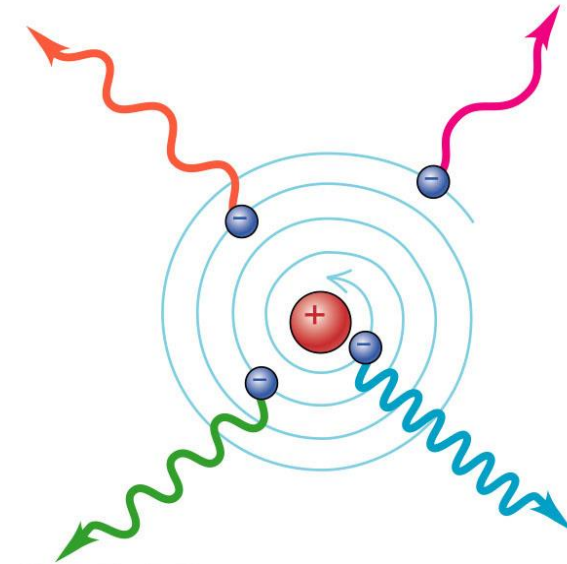
$$m_{\alpha} < m_{\text{nucl}}$$



Some α -particles come very close to nucleus and deflected at large angle via electrostatic interaction.

Rutherford Atom

- Following the Newtonian planetary model, it was initially thought that the atom may follow a similar scheme.
- However, it is also known from studies of E&M that an accelerating charge radiates and loses energy. Thus a planetary model atom is unstable. The electrons will spiral into the nucleus within 10^{-10} s, radiating its energy away during the process.
- The spectrum of such EM Radiation would also be a continuous spectrum instead of the line spectrum observed.



An electron that moving in circular orbit is accelerating (because direction of motion is changing) will radiate!

Bohr's Model for Hydrogen Atom (1913)

- Bohr, who had worked with Rutherford for a while, was trying to resolve the stability of the Rutherford atom. His approach was semi-classical, as it retains most of the features of the planetary model. However, he made two crucial (and radical) postulates that did not explain away the main problem of Rutherford atom but tried to accommodate it.



1. An electron in an atom moves in a circular orbit about the nucleus under the influence of Coulomb attraction between the electron and nucleus and obeying the laws of classical mechanics.
2. But instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum $L (= mvr)$ is an integral multiple of Planck's constant h divided by 2π .
3. Despite the fact that it is constantly accelerating, an electron moving in such an allowed orbit does not radiate electromagnetic energy. Thus, its total energy E remains constant.
4. Electromagnetic energy is emitted if an electron, initially moving in an orbit of total energy E_m , discontinuously changes its motion so that it moves in an orbit of total energy E_n . The frequency of the emitted radiation f is equal to the quantity $(E_m - E_n)$ divided by Planck's constant h .

Bohr's Model for Hydrogen Atom (1913)

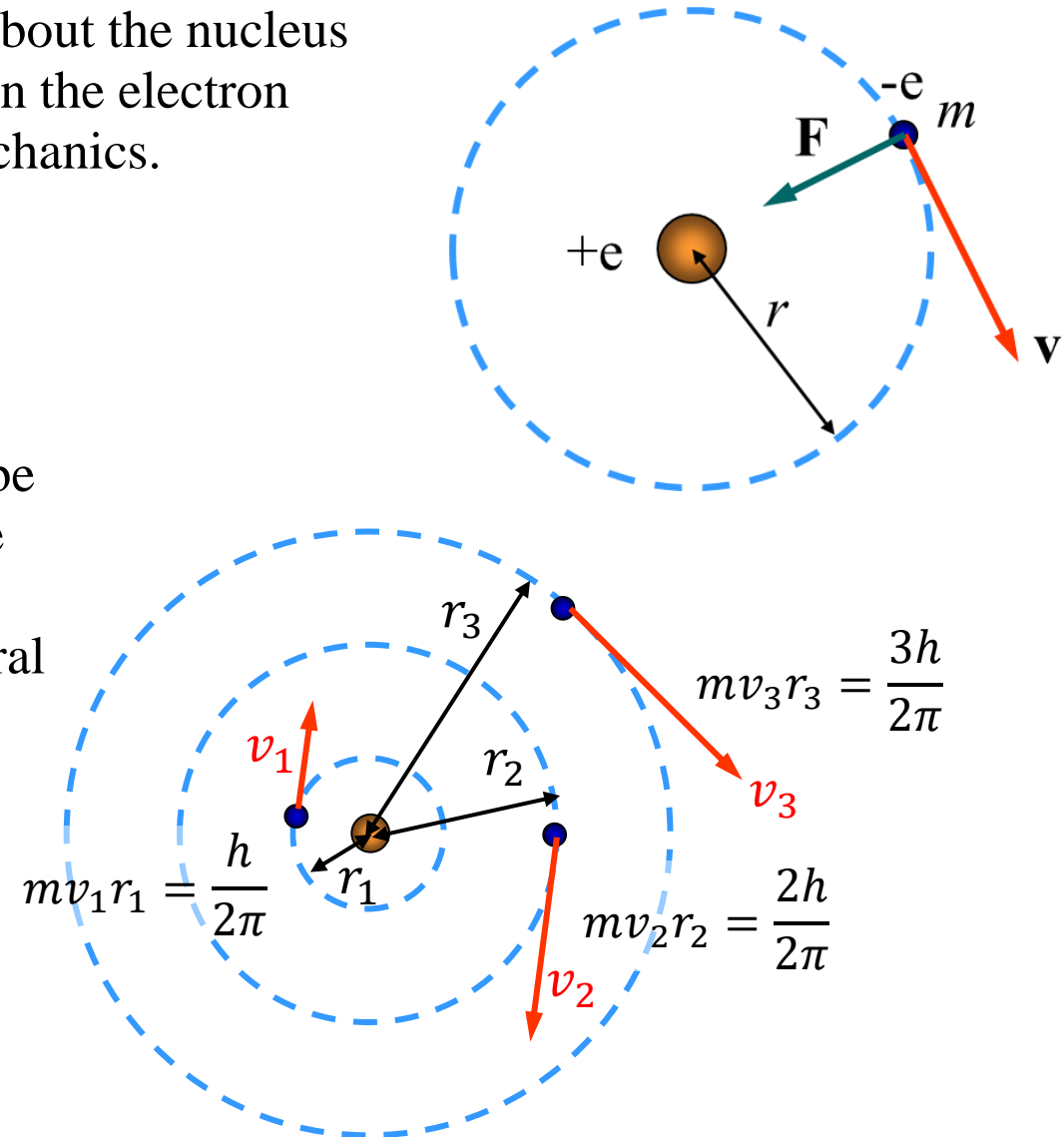
1. An electron in an atom moves in a circular orbit about the nucleus under the influence of Coulomb attraction between the electron and nucleus and obeying the laws of classical mechanics.

$$F = ma = \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

2. But instead of the infinity of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum L ($= mvr$) is an integral multiple of Planck's constant h divided by 2π .

$$L_n = mv_n r_n = \frac{nh}{2\pi}$$

Can solve for r and v for each value of n .



Bohr's Model for Hydrogen Atom (1913)

3. Despite the fact that it is constantly accelerating, an electron moving in such an allowed orbit does **not** radiate electromagnetic energy. Thus, its total energy E remains constant.

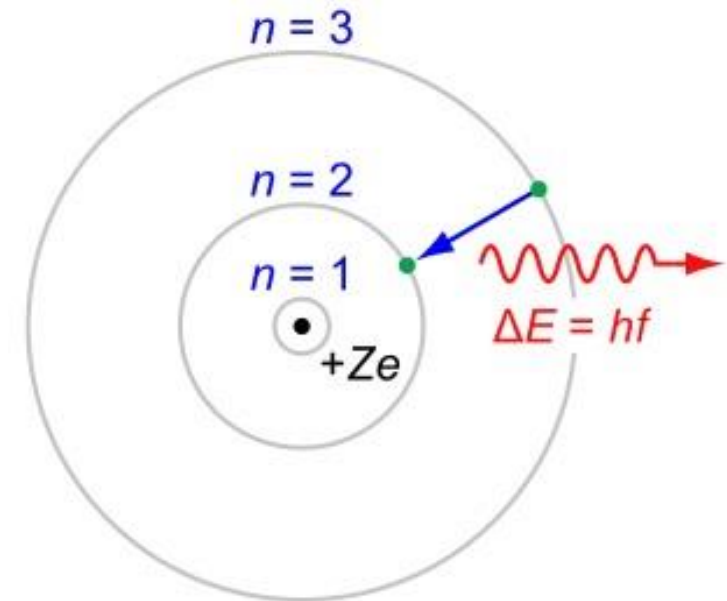
$$E = \text{KE} + \text{PE} = \frac{1}{2}mv^2 - \frac{e^2}{4\pi\epsilon_0 r} = \text{constant}$$

4. Electromagnetic energy is emitted if an electron, initially moving in an orbit of total energy E_m , discontinuously changes its motion so that it moves in an orbit of total energy E_n . The frequency of the emitted radiation f is equal to the quantity $(E_m - E_n)$ divided by Planck's constant h .

$$f = \frac{E_m - E_n}{h} \quad \text{or} \quad \Delta E = E_m - E_n = hf$$

From expressions of r , v and E , it can be shown that an electron at the n^{th} stationary orbit has an energy given by

$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$



Success of Bohr's Model for Hydrogen

- Hence the change in energy when an electron jumps from the m th orbit to the n th orbit will be

$$\Delta E = E_m - E_n = hf = \frac{hc}{\lambda} = \frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} - \frac{1}{m^2} \right)$$

in agreement with the empirical expression derived from experimental observations.

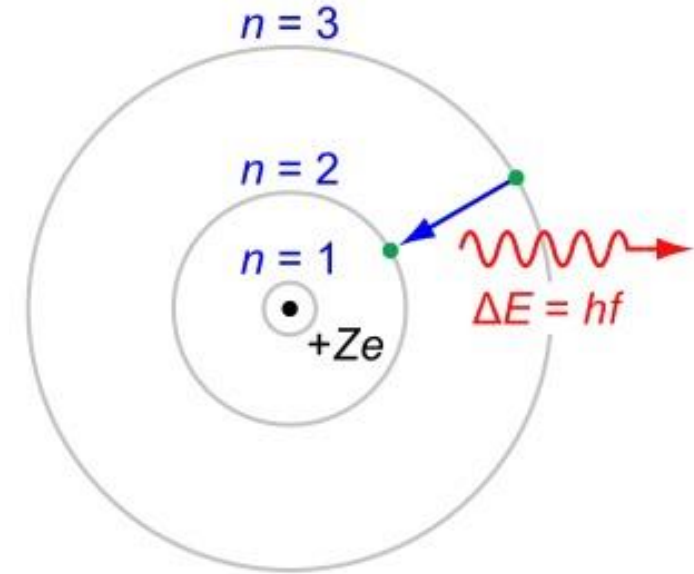
- The theory predicted the Rydberg constant:

$$R = \frac{me^4}{8\epsilon_0^2 ch^3}$$

which when the values of constants are substituted gives excellent agreement with the value determined in the study of atomic spectra.

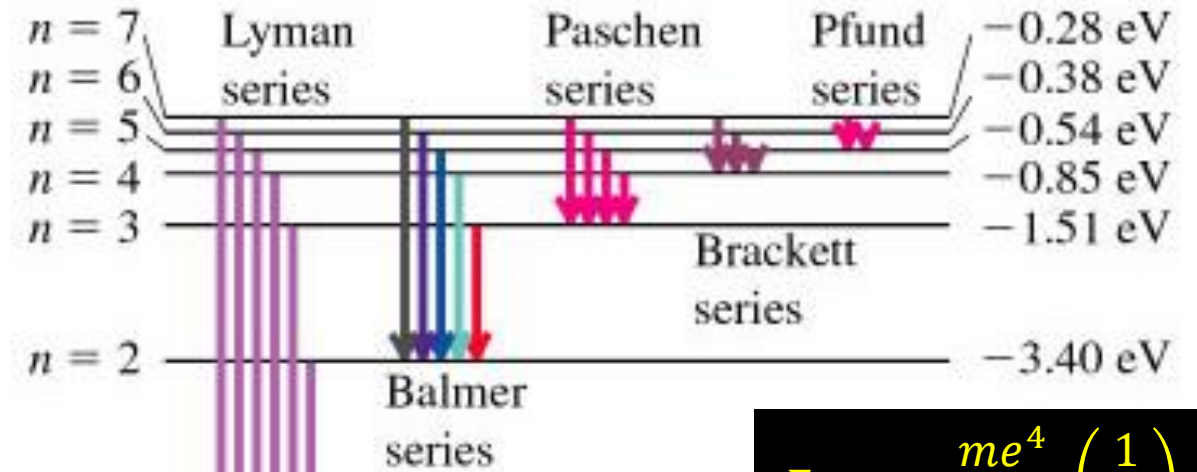
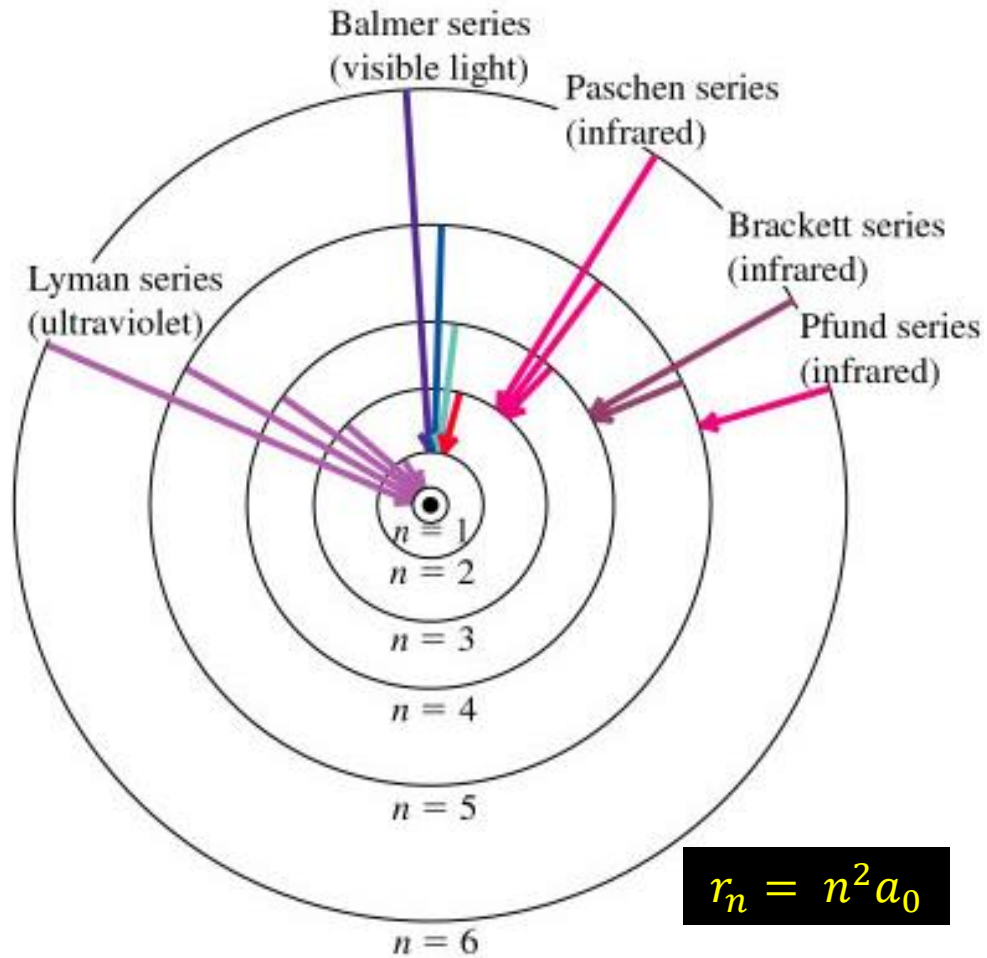
- The calculations also gives the radius of the lowest energy orbit (known as the **Bohr radius**) to be

$$a_0 = \frac{\epsilon_0 h^2}{me^2 \pi} = 0.0529 \text{ nm}$$



Energy-Level Diagram

This diagram shows the permitted orbits in the hydrogen atom according to Bohr's model.



$$E_n = -\frac{me^4}{8\epsilon_0^2 h^2} \left(\frac{1}{n^2} \right)$$

Useful to show the various possible energy values as horizontal lines on an **energy-level diagram**. The lowest energy level, the ground state is drawn at the bottom. There are no lower energy state beyond this. There are infinite levels as n can take any values from 1 to ∞ . The gap between different levels get smaller and smaller as n increases.



Limitations / Failure of Bohr's Model

- Despite its great success in interpreting the hydrogen spectrum, limitation in Bohr's theory soon became evident:
 - The theory could not calculate the spectrum of atoms other than hydrogen accurately.
 - The theory could not explain why some spectral lines are brighter than the others.
 - The theory could not explain the presence of fine details in atomic spectra revealed with high-resolution spectroscopy or in the presence of magnetic field.
- Right from the beginning, it was felt to be an incomplete theory – as it was a mix between classical theories and some postulates made to accommodate the “unexplainable” experimental observations. It is just intellectually unsatisfying!
- Nonetheless, Bohr model gives a very useful mental picture that many of us will resort to when we think about quantum processes – the new “correct” quantum mechanics are very difficult to visualize.

Wave & Particle Nature of Light

- Explanations of black body radiation and the photoelectric effect require a non-classical approach, and require that *light waves* behave as **particles** (photons) whose properties are:

$$\text{Energy } E = hf$$

and as a massless particle, its momentum p is given by

$$\text{Momentum } p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

Momentum p for a usual physical object (with mass) is mass $m \times$ velocity v , i.e.,
 $p = mv$

So is light a wave or a particle?

The principle of complementarity: The wave and particle descriptions of light are complementary: We need both descriptions to explain nature, but we never need to use at the same time, i.e., one or the other descriptions will suffice.

– Niels Bohr, 1928

De Broglie's Hypothesis

- In 1923, in his PhD thesis, Louis Victor de Broglie proposed that because photons have both wave and particle characteristics, perhaps all forms of matter have wave as well as particle properties.
- If a photon, which has a rest mass of zero, has a momentum $p = h/\lambda$, then de Broglie suggested that a particle with momentum $p (= mv)$ also has a wavelength associated with it given by:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

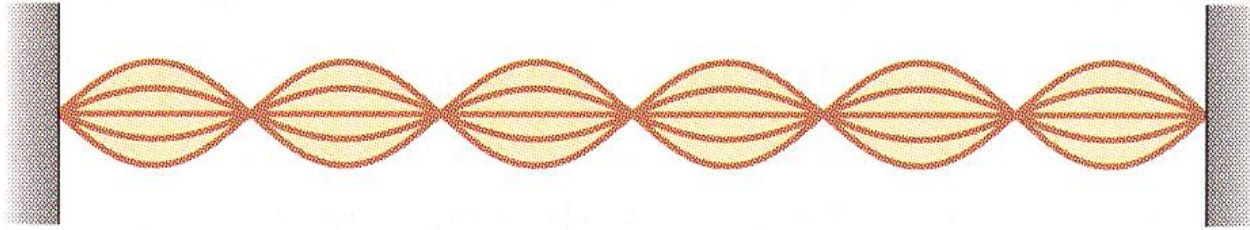
de Broglie wavelength
of a particle



Louis de Broglie (1892-1987)
– Nobel prize 1929

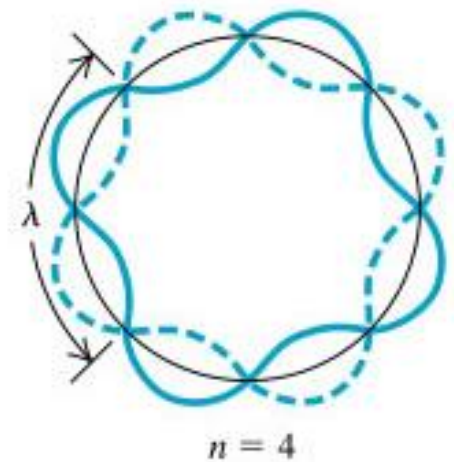
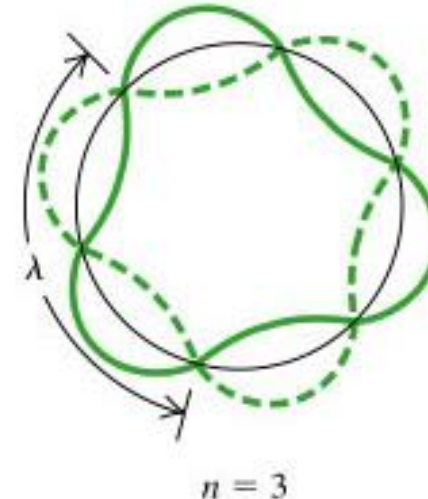
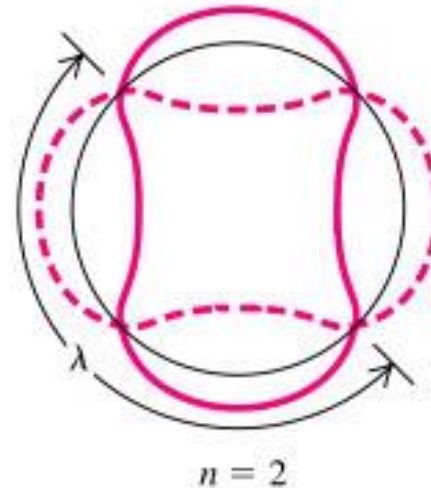
The Atom According to de Broglie

- The adoption of thinking that matter had a wave like nature removed some of the shortcomings of the Bohr model of the atom: It was a way of explaining the question '**why do electrons only have specific orbits**'?
- Consider a plucked guitar string: While initially subjected to a wide range of frequencies, the string will end up supporting only standing waves that have nodes at each end.



Standing wave patterns showing three full wavelength on a stretched string.

- The electrons can then be seen as 'standing waves' in their atomic orbit: Only those orbits which can support a standing wave can exist.
- This places restrictions on the radii of allowed orbits, as well as the energy of the electrons in the orbits.

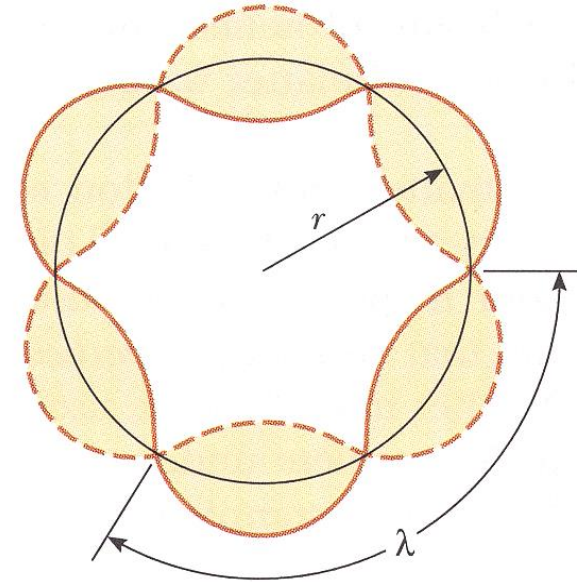


The Atom According to de Broglie

- Electron matter waves are bent in a circle around a nucleus! Electrons only have orbits which correspond to an integral number of wavelengths around the nucleus.

$$n\lambda = 2\pi r \quad \Rightarrow \quad n \frac{h}{mv} = 2\pi r$$

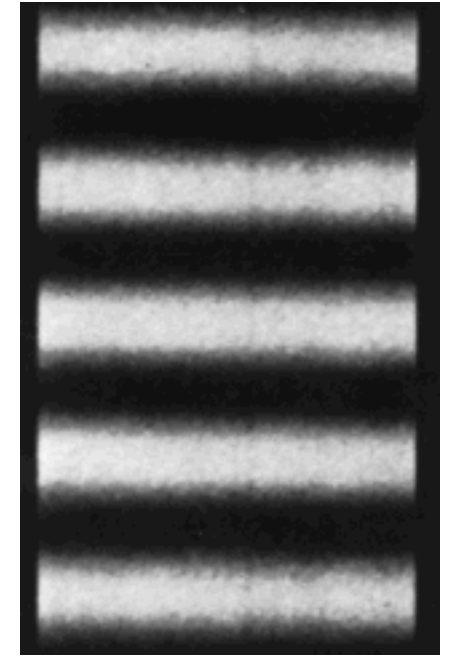
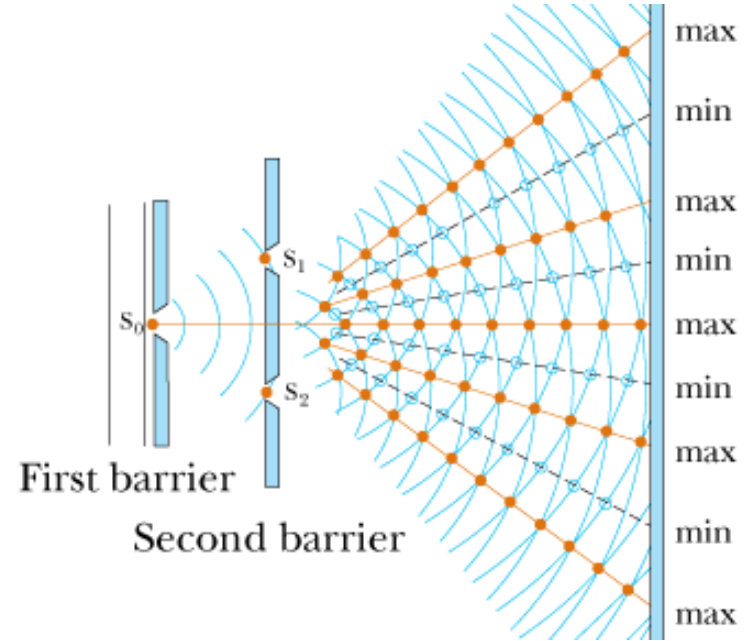
$$L = mvr = n \frac{h}{2\pi} \quad \Leftarrow$$



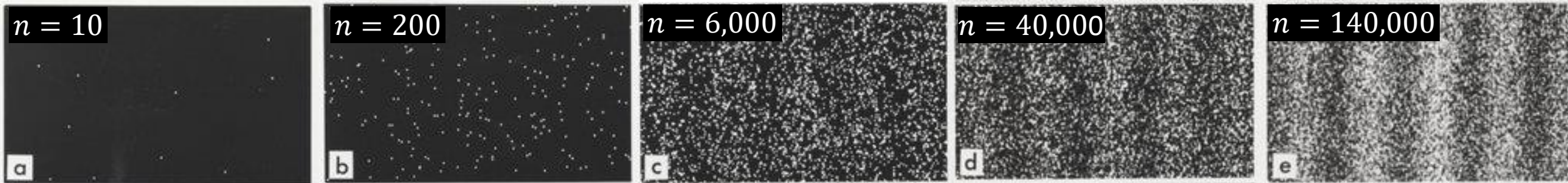
- Last equation is the same as Bohr's postulate on the quantization of angular momentum that gave the energy levels and numerical values of the lines in the Balmer's and other series in hydrogen.

Wave & Particle Nature of Matter

- The most representative experiment that demonstrates wave nature of light is the double-slit experiment, first performed by English scientist Thomas Young in 1801 to resolve the question of whether light was composed of particles (Newton's "corpuscular" theory), or rather consisted of waves.



- What if we performed a Young's double-slit experiments, as first performed in 1961 by Claus Jönsson? Below are what appeared on the screen after n electrons hit the screen.



- How can we describe the wave nature of matter, especially bound electrons in atoms?

Schrödinger Wave Equation

- Many attempts were made to describe the wave nature of matter.
- In 1926, Erwin Schrödinger proposed a theory which describes a dynamic system by a complex-valued wave function. His theory is known as wave mechanics.
- The simplest form of Schrodinger Equation (1-dimensional, time-independent) is:

Wave function of particle – it is used to determine the probability of finding the particle in a certain interval around position x .

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x)$$

$\hbar/2\pi$

Mass of particle

Potential energy function that the particle is in – depends on configuration and x

Allowed energies of the particle (pure numbers)

The materials from this slide onwards are meant for completeness and will not be tested. These give us a consistent picture of how the atoms of different elements may be described and allow us to finally complete the periodic table that Mendeleev started.

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- Basically, the problem reduces to solving this second order differential equation to find the wave function $\psi(x)$ and the energy E that the particle can take when given the potential energy function and that $\psi(x)$ must satisfy conditions such as being single value, differentiable, finite, etc.

Schrödinger Wave Equation for Hydrogen Atom

- For the hydrogen atom, we can put in the potential energy function for electrons due to electrostatic attraction and try to solve it.

$$\sin^2 \theta \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r} \right) + \sin \theta \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{\partial^2 \psi}{\partial \phi^2} + \frac{2mr^2 \sin^2 \theta}{\hbar^2} \left(\frac{e^2}{4\pi\epsilon_0 r} + E \right) \psi = 0$$

- As it is 3-dimensional, it is a lot more complicated and many tricks are needed to solve it. Can be done but is usually attempted by Physics majors in their Year 2 or 3. So we will not do it here but just state some of the important results. Notice that we have changed the 3 usual dimensions (x, y, z) to the spherical coordinates (r, θ, ϕ) to take advantage of the spherical symmetry of the problem.
- The most important result coming out of this exercise is the presence of **3 quantum numbers** (n, l, m_l) in every solution of $\psi(x)$ [n here is similar to n in Bohr model] and these numbers can take on only certain values.

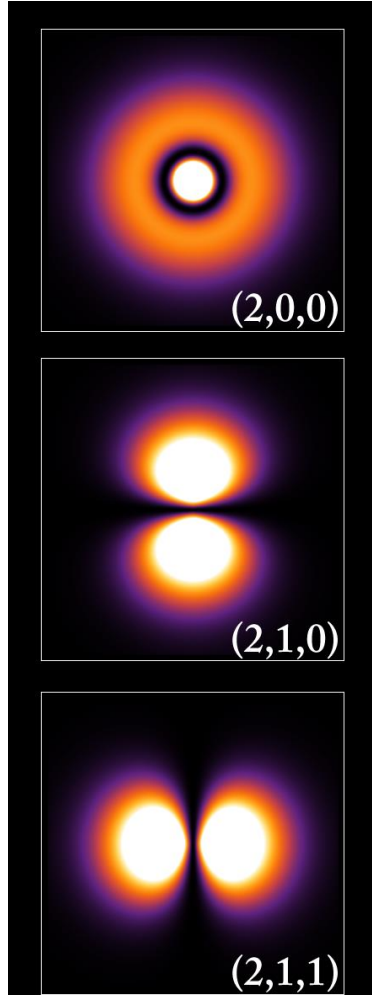
Principal quantum number: $n = 1, 2, 3, \dots$

Orbital quantum number: $l = 0, 1, 2, \dots, (n - 1)$ for a particular n

Magnetic quantum number: $m_l = 0, \pm 1, \pm 2, \dots, \pm l$ for a particular l

Schrödinger Wave Equation for Hydrogen Atom

- Here are some of the wave functions and how they can be visualized (just for your interest) .



(Shown is ψ_{nlm_l} at xz plane)

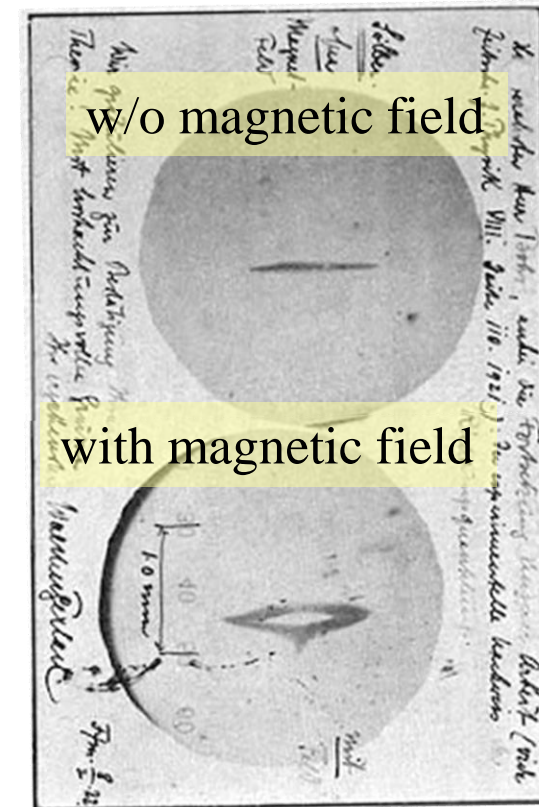
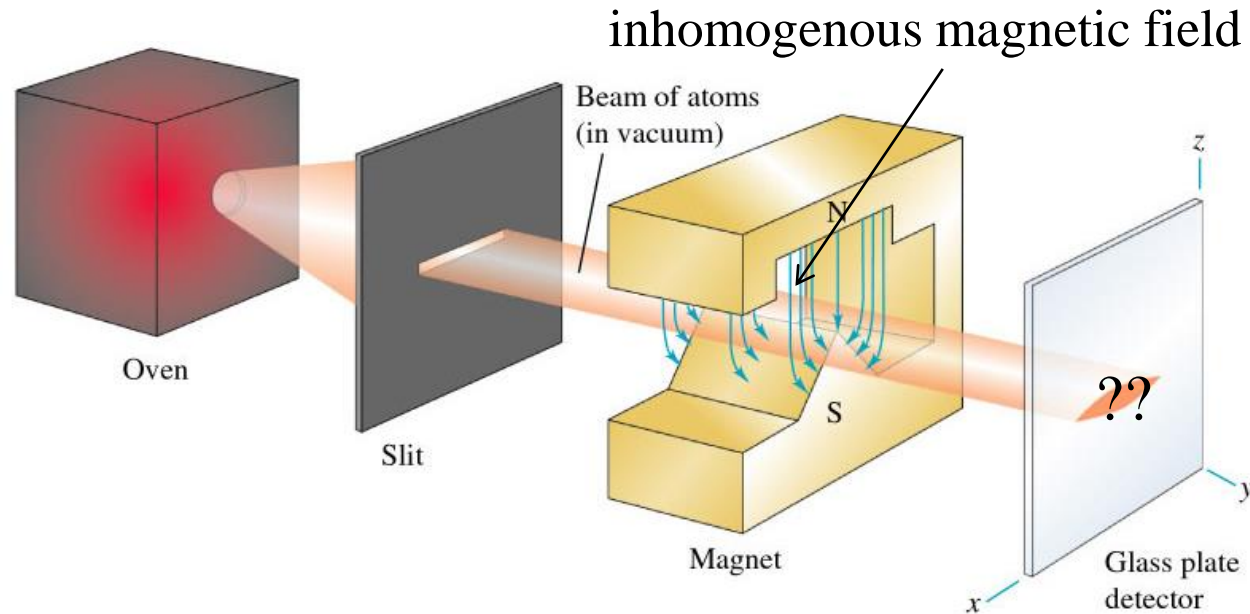
$$\psi_{200}(r, \theta, \phi) = \frac{1}{2\sqrt{2\pi}a_0^{3/2}} e^{-r/2a_0} \left(1 - \frac{r}{2a_0}\right)$$

$$\psi_{210}(r, \theta, \phi) = \frac{1}{2\sqrt{2\pi}a_0^{3/2}} e^{-r/2a_0} \left(\frac{r}{2a_0}\right) \cos \theta$$

$$\psi_{21\pm 1}(r, \theta, \phi) = \mp \frac{1}{4\sqrt{\pi}a_0^{3/2}} e^{-r/2a_0} \left(\frac{r}{2a_0}\right) \sin \theta e^{\pm i\phi}$$

Electron Spin (Final Concept to Specify State of Electron)

- Through experiments (e.g., Stern-Gerlach experiment), scientists discovered that electron and other particles has an intrinsic angular momentum which is called spin.



- Spin is an intrinsic property, and electrons are not spinning in the usual way we know it. The familiar building blocks of matter (electron, proton, neutron) each has spin equal to $\hbar/2$.
- More importantly, this intrinsic spin provide another important quantum number for electrons

$$m_s = \pm \frac{1}{2}$$

Quantum Numbers (Summary)

- We need four different quantum numbers to describe a state of hydrogen fully (n, l, m_l, m_s) .

- n is an integer 1, 2, 3, ...

- For each value of n there are n possible values of l

$$l = 0, 1, 2, \dots, (n - 1)$$


- For each value of l , there are $(2l + 1)$ possible values of m_l .

$$m_l = 0, \pm 1, \pm 2, \dots, \pm l$$

- There are always only two value of m_s :

$$m_s = \pm \frac{1}{2}$$

- The number of states for each value of n is given by:


$$\text{No of states} = 2 \sum_{l=0}^{n-1} 2l + 1 = 2 [1 + 3 + \dots + (2n - 1)]$$

$$\text{No of states} = 2 \left[\frac{n}{2} (1 + 2n - 1) \right] = 2n^2$$

Pauli Exclusion Principle

- Electrons obey the Pauli exclusion principle – **No two electrons can occupy the same quantum mechanical state** – or in other words – **no two electrons in an atom can have the same values of all four quantum numbers.**

n	l	m_l	m_s	Spectroscopic Notation	Number of States	Shell
1	0	0	$\pm 1/2$	1s	2	K
2	0	0	$\pm 1/2$	2s	2	L
	1	-1, 0, 1	$\pm 1/2$	2p	6	
3	0	0	$\pm 1/2$	3s	2	M
	1	-1, 0, 1	$\pm 1/2$	3p	6	
	2	-2, -1, 0, 1, 2	$\pm 1/2$	3d	10	
4	0	0	$\pm 1/2$	4s	2	N
	1	-1, 0, 1	$\pm 1/2$	4p	6	
	2	-2, -1, 0, 1, 2	$\pm 1/2$	4d	10	
	3	-3, -2, -1, 0, 1, 2, 3	$\pm 1/2$	4f	14	

Periodic Table of Elements

<p>Frequently used fundamental physical constants</p> <p>For the most accurate values of these and other constants, visit physics.nist.gov/constants</p> <p>1 second = 9 192 631 770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of ^{133}Cs</p> <p>speed of light in vacuum c 299 792 458 m s⁻¹ (exact)</p> <p>Planck constant h 6.6261 x 10⁻³⁴ J s ($\hbar = h/2\pi$)</p> <p>elementary charge e 1.6022 x 10⁻¹⁹ C</p> <p>electron mass m_e 9.1094 x 10⁻³¹ kg</p> <p>proton mass m_p 1.6726 x 10⁻²⁷ kg</p> <p>fine-structure constant α 1/137.036</p> <p>Rydberg constant R_∞ 10 973 732 m⁻¹</p> <p>Boltzmann constant k 1.3807 x 10⁻²³ J K⁻¹</p>																	
<p>Physics Laboratory physics.nist.gov</p>																	
<p>Standard Reference Data www.nist.gov/srd</p>																	
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19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104
87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104

This is the familiar periodic table of elements that every student who studied chemistry is familiar with.

Arranged accordingly to the number of electrons in neutral atom (or protons in nucleus).

Completes Mendeleev's scheme of arranging the elements according to the number of electrons in neutral atom (or protons in nucleus) and grouped by their properties.

Note that some elements, e.g., Tc, Pm, Pu, etc. do not exist naturally.

¹Based upon ¹²C. () indicates the mass number of the longest-lived isotope.

For a description of the data, visit physics.nist.gov/data

NIST SP 966 (September 2010)

Periodic Table of Elements

We can use the Pauli Exclusion Principle to obtain the ground state of atoms by progressively filling each of the lower energy states starting with $1s$, $2s$, $2p$, $3s$, $3p$, $4s$, $3d$, The number of electrons that can go into each shell (or subshells for each l) was determined earlier. The pattern of filling the shells provides an explanation for the periodic table proposed by Mendeleev in 1869.

Element	Sym.	Atomic No. (Z)	Electronic Configuration
Hydrogen	H	1	$1s$
Helium	He	2	$1s^2$
Lithium	Li	3	$1s^2 2s$
Beryllium	Be	4	$1s^2 2s^2$
Boron	B	5	$1s^2 2s^2 2p$
Carbon	C	6	$1s^2 2s^2 2p^2$
Nitrogen	N	7	$1s^2 2s^2 2p^3$
Oxygen	O	8	$1s^2 2s^2 2p^4$
Fluorine	F	9	$1s^2 2s^2 2p^5$
Neon	Ne	10	$1s^2 2s^2 2p^6$

Element	Sym.	Atomic No. (Z)	Electronic Configuration
Sodium	Na	11	$1s^2 2s^2 2p^6 3s$
Magnesium	Mg	12	$1s^2 2s^2 2p^6 3s^2$
Aluminium	Al	13	$1s^2 2s^2 2p^6 3s^2 3p$
Silicon	Si	14	$1s^2 2s^2 2p^6 3s^2 3p^2$
Phosphorus	P	15	$1s^2 2s^2 2p^6 3s^2 3p^3$
Sulphur	S	16	$1s^2 2s^2 2p^6 3s^2 3p^4$
Chlorine	Cl	17	$1s^2 2s^2 2p^6 3s^2 3p^5$
Argon	Ar	18	$1s^2 2s^2 2p^6 3s^2 3p^6$
Potassium	K	19	$1s^2 2s^2 2p^6 3s^2 3p^6 4s$
Calcium	Ca	20	$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s	16s	17s	18s	19s	20s	21s	22s	23s	24s	25s	26s	27s	28s	29s	30s
1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s	16s	17s	18s	19s	20s	21s	22s	23s	24s	25s	26s	27s	28s	29s	30s
1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s	16s	17s	18s	19s	20s	21s	22s	23s	24s	25s	26s	27s	28s	29s	30s
1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s	16s	17s	18s	19s	20s	21s	22s	23s	24s	25s	26s	27s	28s	29s	30s
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1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s	16s	17s	18s	19s	20s	21s	22s	23s	24s	25s	26s	27s	28s	29s	30s
1s	2s	3s	4s	5s	6s	7s	8s	9s	10s	11s	12s	13s	14s	15s</															

Note that the magnitudes of the energy of levels do not follow strictly according to n . It is also affected by values of l . The order goes roughly as:

$$\begin{array}{c} 1s, \\ 2s, 2p, \\ 3s, 3p, \\ 4s, 3d, 4p, \\ 5s, 4d, 5p, \\ 6s, 4f, 5d, 6p, \\ 7s, 5f, 6d, 7p \end{array}$$

[†]Based upon ¹²C. () indicates the mass number of the longest-lived isotope.