# CS2040S – Data Structures and Algorithms

## Lecture 10 – UFDS

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## Motivation for UFDS

- We have seen that we can use Hashtable to implement simple Set ADT with the following operations
  - Find
  - Insert
  - remove

- But what if we need to represent multiple disjoint sets and also to union them?

take all the keys from one hashtable and rehash them into another table -> take O(n) time but we can do better

A simple yet effective data structure to model disjoint sets…

https://visualgo.net/**en/ufds**

CP4 Book 1, Section 2.4.2

## UNION-FIND DISJOINT SETS DATA STRUCTURE

# Union-Find Disjoint Sets (UFDS)

UFDS is a collection of disjoint sets

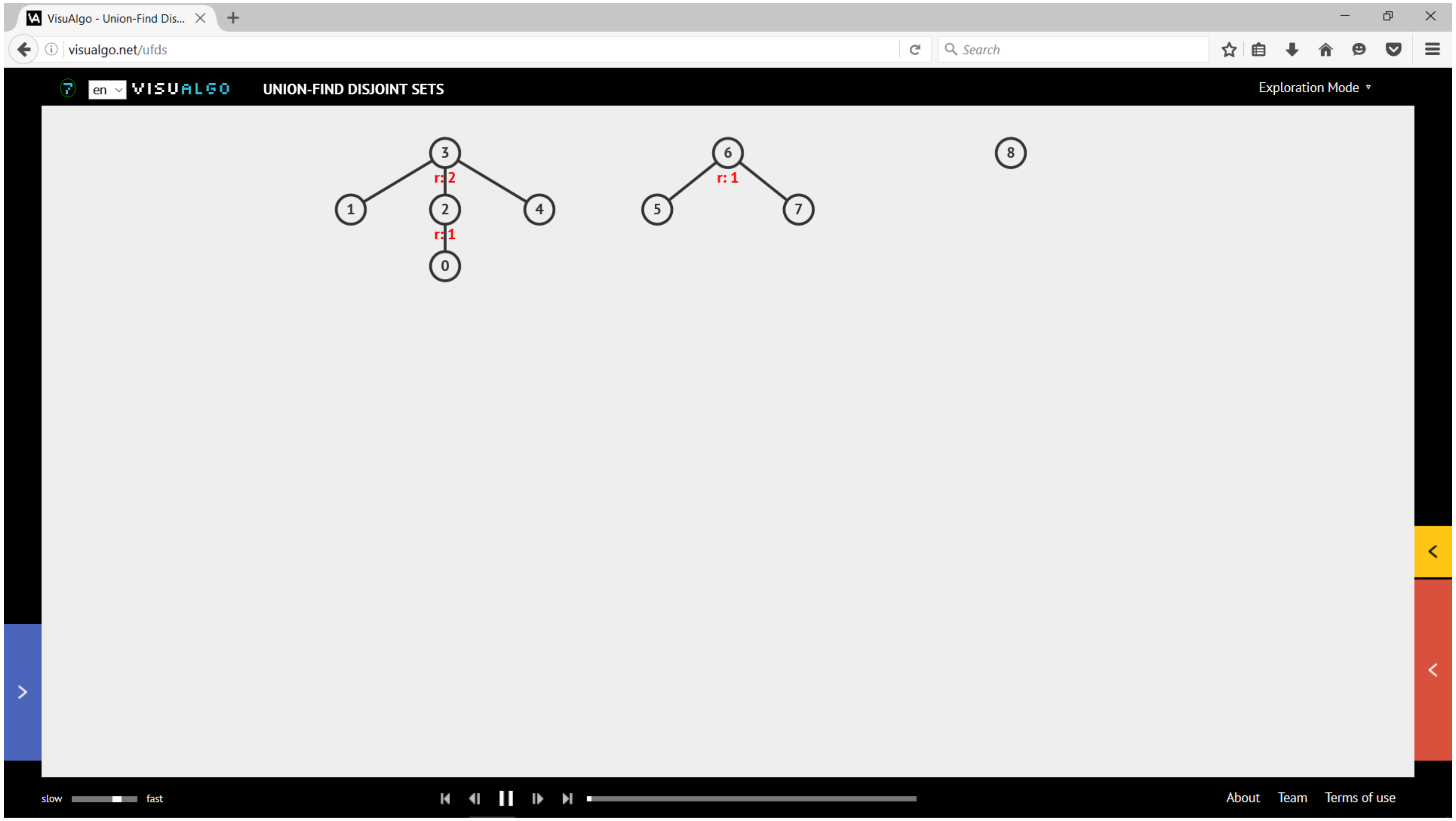
Given several disjoint sets in the UFDS the operations we have are

- Union two disjoint sets when needed
- Find which set an item belongs to
- Check if two items belong to the same set

Key ideas:

- Each set is modeled **as a tree**
  - Thus a collection of disjoint sets form **a forest of trees**
- Each set is represented by a representative item
  - Which is the root of the corresponding tree of that set

# Example with 3 Disjoint Sets



# Data Structure to store UFDS

We can record this forest of trees with an array **p** of size n

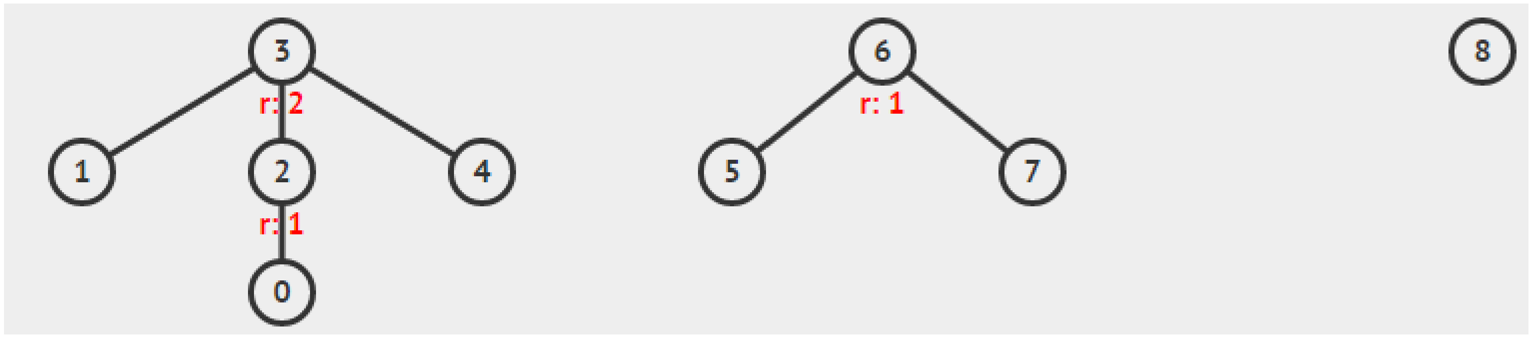
- **p[i]** records the parent of item **i**
- if **p[i] = i**, then **i** is a root
  - And also the representative item of the set that contains **i**

only need one p array to store all the disjoint sets

For the example below, we have **p = {2,3,3,3,3,6,6,6,8}**

index: 0,1,2,3,4,5,6,7,8

if we want to store items of other types such as strings we have to map it to integers 0 to n-1

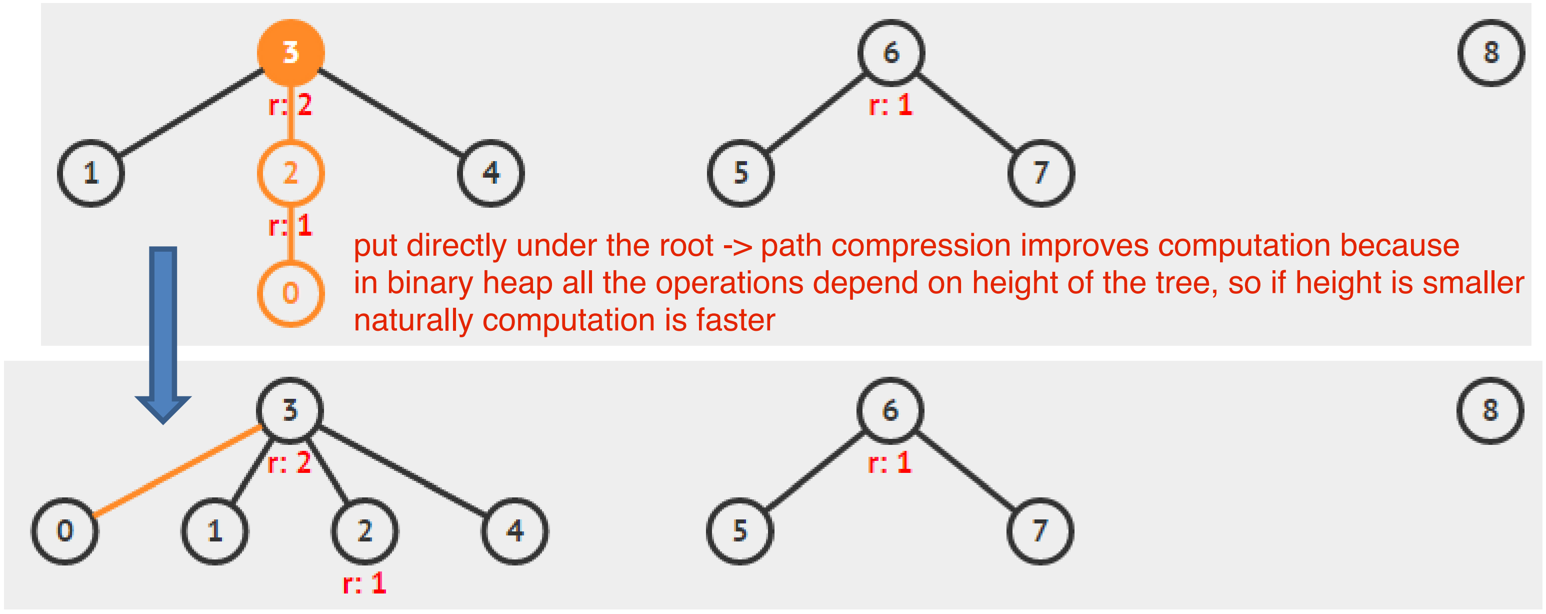


# UFDS – findSet(i) Operation

the representative item is the the root of the tree

For each item **i**, we can **find** the representative item of the set that contains item **i** by recursively visiting **p[i]** until **p[i] = i**; Then, we *compress the path* to make future find operations (very) fast, i.e. O(1)

- Example of findSet(0), *ignore attribute 'r' for now*



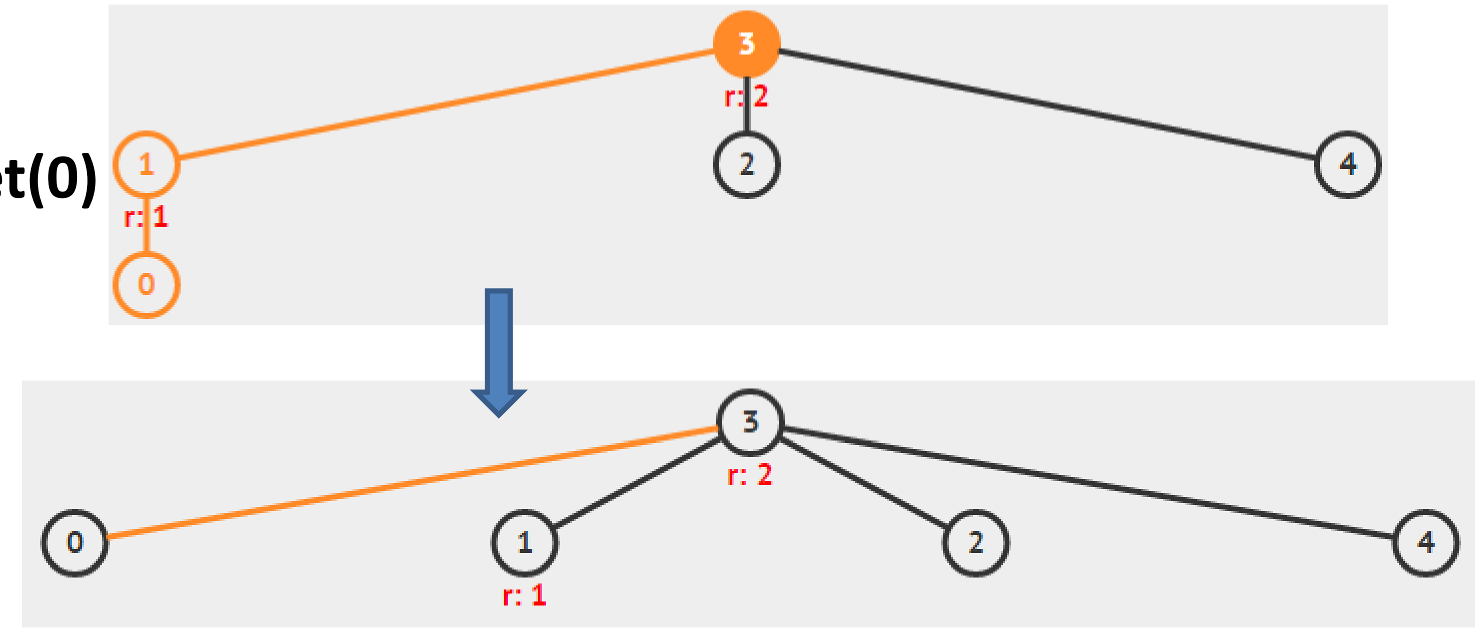
put directly under the root -> path compression improves computation because in binary heap all the operations depend on height of the tree, so if height is smaller naturally computation is faster

# findSet code

```
public int findSet(int i) {
  if (p[i] == i)
    return i;
  else {
    p[i] = findSet(p[i]);
    return p[i];
  }
}
```

ufds doesnt care about the parent, it cares about the set

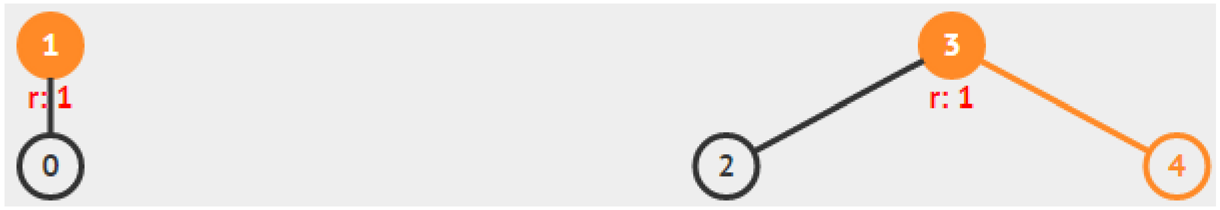
**findSet(0)**



# UFDS – isSameSet(i,j) Operation

For item **i** and **j** we can check whether they are in the same set in O(1) by finding the representative item for i and j and checking if they are the same or not

- Example: isSameSet(0,4) will return false



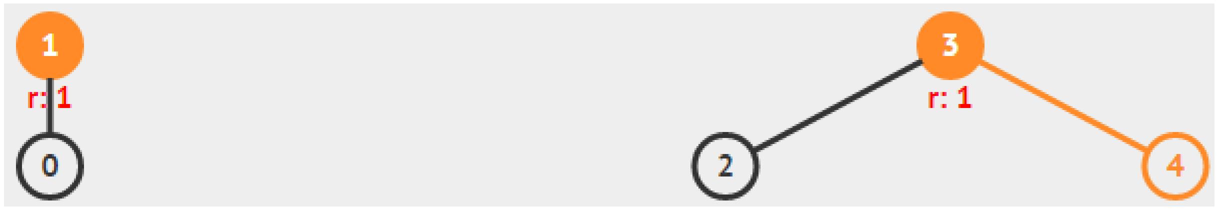
# isSameSet code

```
public Boolean isSameSet(int i, int j) {
  return findSet(i) == findSet(j);
}
```

path compression is indirectly used here as we call findSet

As the representative items of the sets that contains item 0 and 4 are different, we say that 0 and 4 are **not** in the same set!

**isSameSet(0,4)**

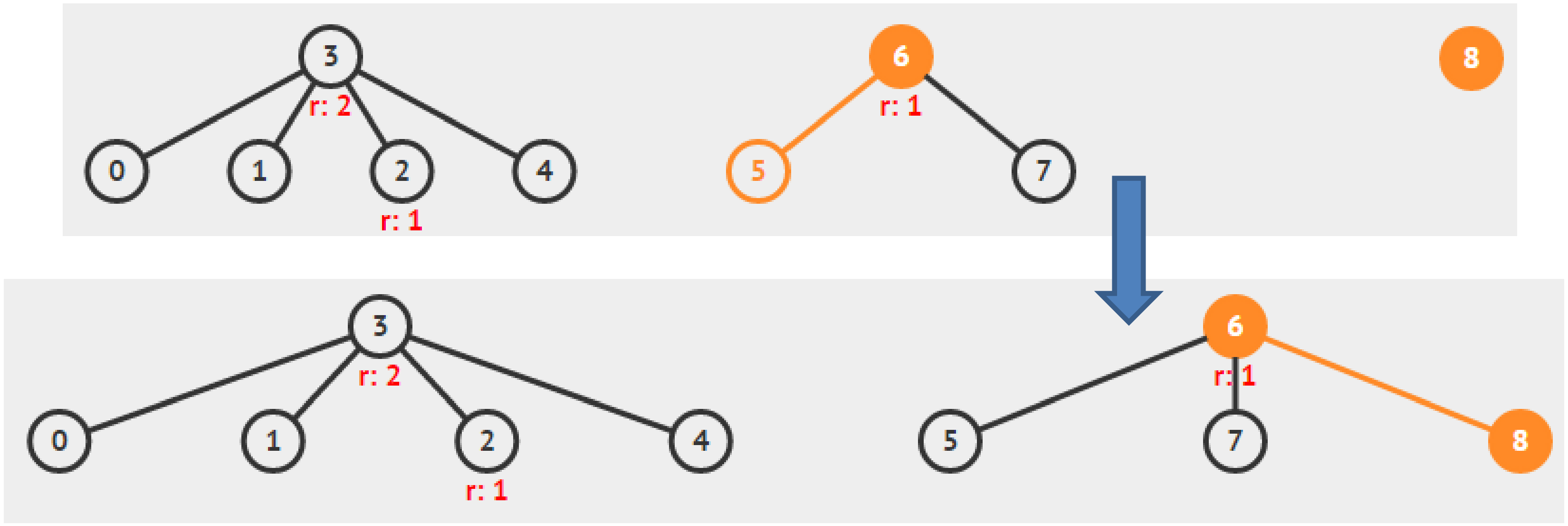


# UFDS – unionSet(i,j) Operation (1)

If two items **i** and **j** currently belong to different disjoint sets, we can **union** them by setting the representative item of *the one with taller* tree* to be the new representative item of the combined set

place the shorter tree directly under the root of the taller tree -> resultant tree will not change in height

- Example of unionSet(5, 8), *see attribute 'r' (elaborated soon)*



## UFDS – unionSet(i,j) Operation (2)

if equally tall, tree of i is placed under tree of j

This is called the *"Union-by-Rank"* ***heuristic***

- This helps to make the resulting combined tree shorter
  - Convince yourself that doing the opposite action will make the resulting tree taller (we do not want this)

If both trees are equally tall, this heuristic is not used

of size N

We use another integer array **rank,** where **rank[i]** stores the upper bound of the height of (sub)tree rooted at **i**

- This is just an upper bound as path compressions can make (sub)trees shorter than its upper bound and we do not want to waste effort maintaining the correctness of **rank[i]** can only union with another tree if it has a rank bigger or equal to each other

# unionSet code

finds representative item, does path compression so if we call anything other than the root, the path compression may change the structure of the tree

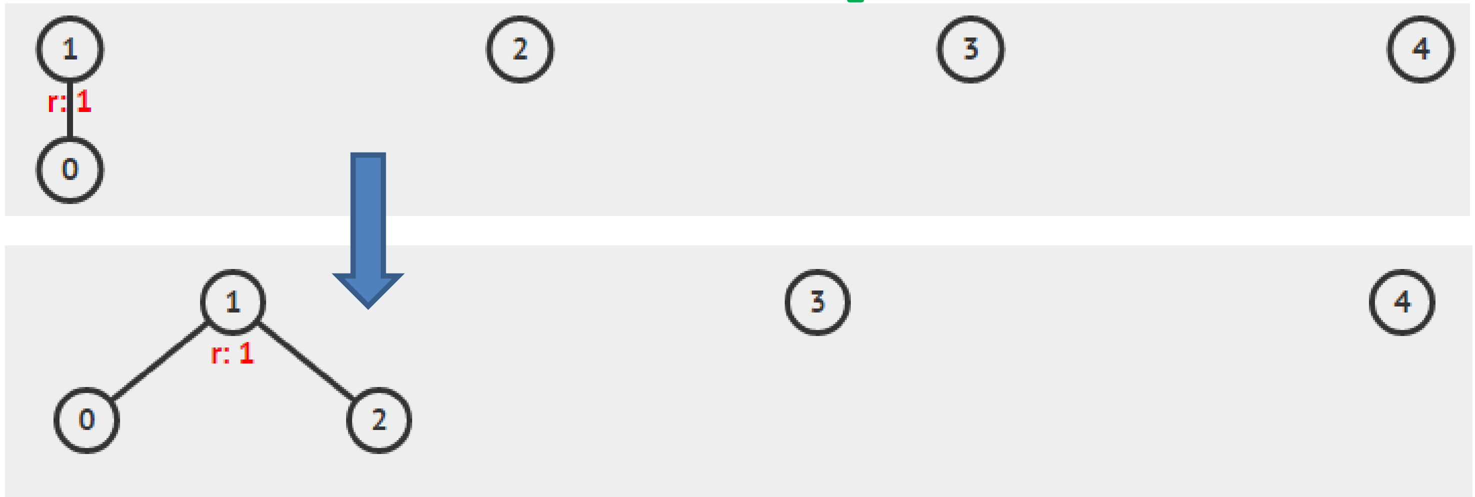
```
public void unionSet(int i, int j) {
  if (!isSameSet(i, j)) {
    int x = findSet(i), y = findSet(j);
    // rank is used to keep the tree short
    if (rank[x] > rank[y])
      p[y] = x;
    else {
      p[x] = y;
      if (rank[x] == rank[y])// rank increases
        rank[y] = rank[y]+1; // only if both trees
    }                        // initially have the same rank
  }
}
```

is rank x > rank y, place y directly underneath x

else place x directly under y

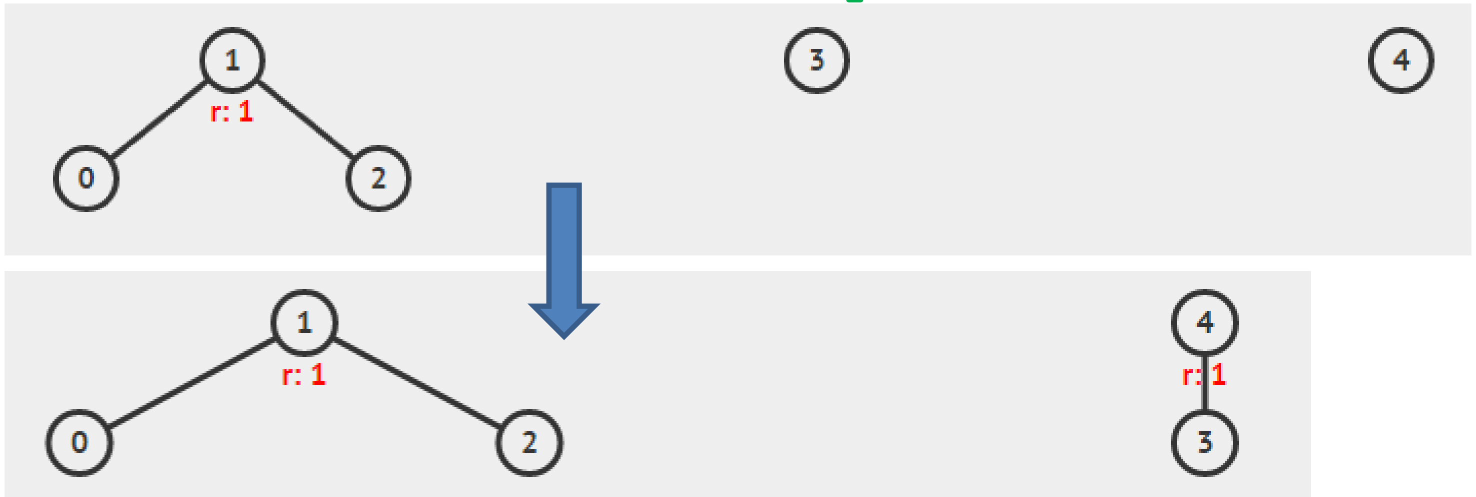
if they have the same rank, the height increases -> rank + 1 because x is placed UNDER y

**unionSet(0,2)**



# unionSet code

```
public void unionSet(int i, int j) {
  if (!isSameSet(i, j)) {
    int x = findSet(i), y = findSet(j);
    // rank is used to keep the tree short
    if (rank[x] > rank[y])
      p[y] = x;
    else {                              Text
      p[x] = y;
      if (rank[x] == rank[y])// rank increases
        rank[y] = rank[y]+1; // only if both trees
    }                        // initially have the same rank
  }
}
```



**unionSet(3,4)**

# Constructor, UnionFind(N)

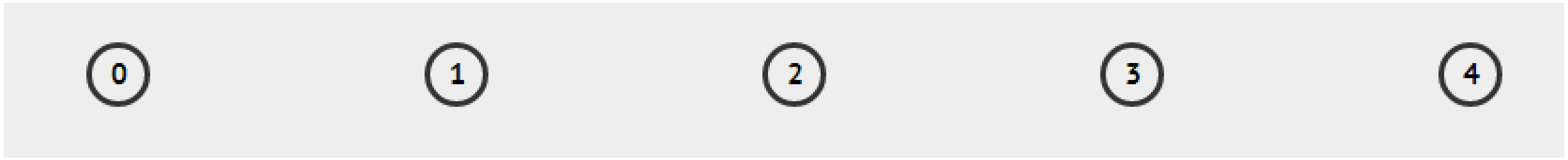
```
class UnionFind {
  public int[] p;
  public int[] rank;

  public UnionFind(int N) {
    p = new int[N];
    rank = new int[N];
    for (int i = 0; i < N; i++) {
      p[i] = i;
      rank[i] = 0;
    }
  }

  // ... other methods in the previous slides
}
```

at the start determine total num of items as there is no operations to add items dynamically

**UnionFind(5)**



# UFDS – Summary

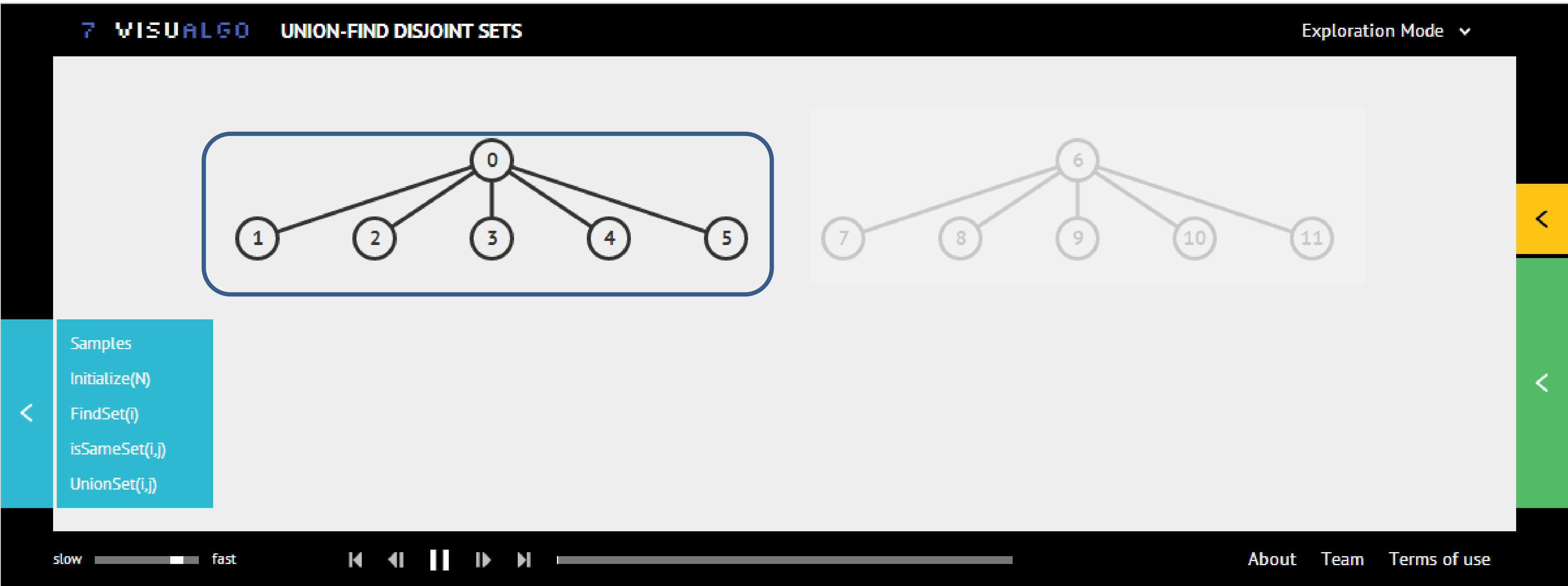
That's the basics… we will not go into further details

- UFDS operations runs in just O($\alpha$(N)) if UFDS is implemented with both "union-by-rank" and "path-compression" heuristics
  - $\alpha$(N) is called the **inverse Ackermann** function
    - This function grows very slowly
    - You can assume it is "constant", i.e. O(1) for practical values of N (<= 1M)
- Review UFDS at https://visualgo.net/en/ufds and again train lots on Visualgo

# VisuAlgo UFDS Exercise (1)

First, click “**Initialize(N)**”, enter **6**, then click “**Go**”

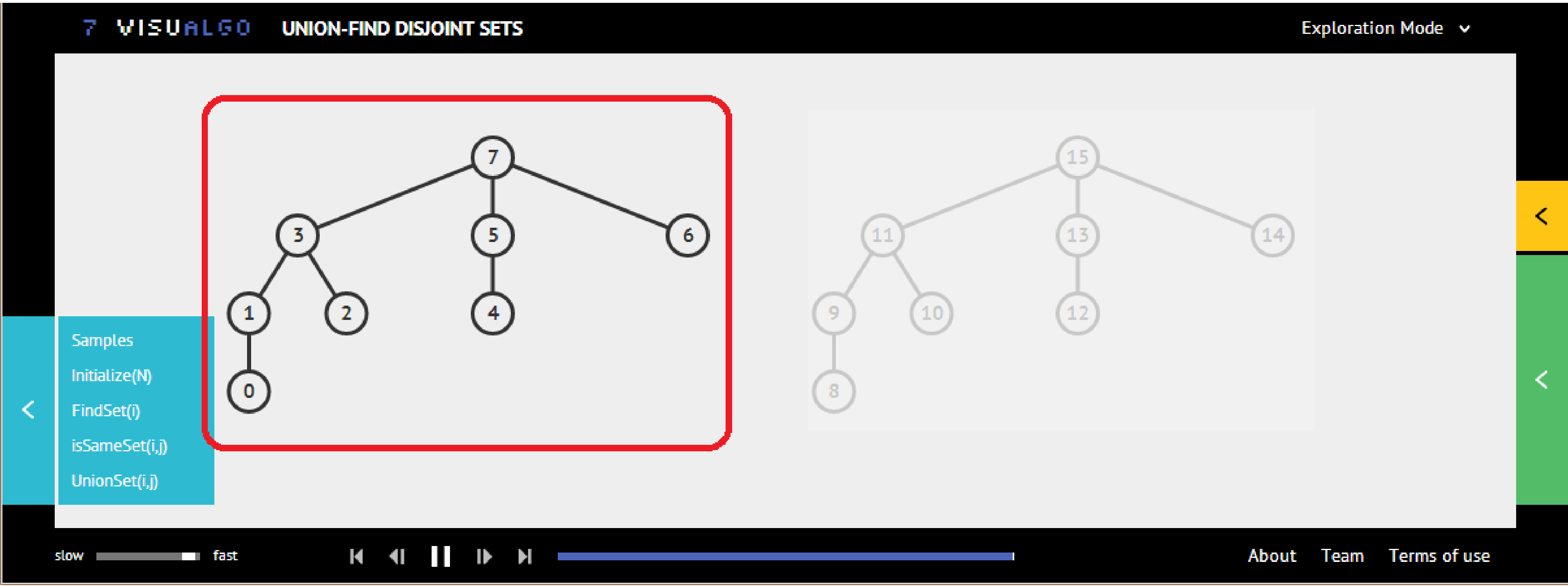
Do a sequence of union and/or find operations to get the left subtree of (**Samples: 2 Trees of Rank 1**)



# VisuAlgo UFDS Exercise (2)

First, click "**Initialize(N)**", enter **8**, then click "**Go**"

Do a sequence of union and/or find operations to get the left subtree of (**Samples: 2 Trees of Rank 3**)


VISUALGO
UNION-FIND DISJOINT SETS
Exploration Mode
7
3
5
6
1
2
4
0
15
11
13
14
9
10
12
8
Samples
Initialize(N)
FindSet(i)
isSameSet(i,j)
UnionSet(i,j)
slow
fast
About
Team
Terms of use