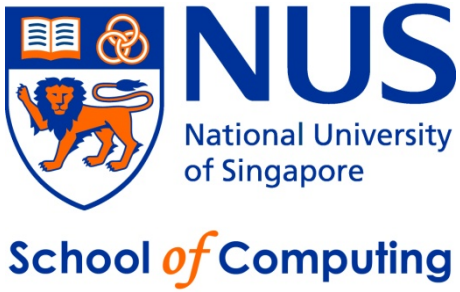
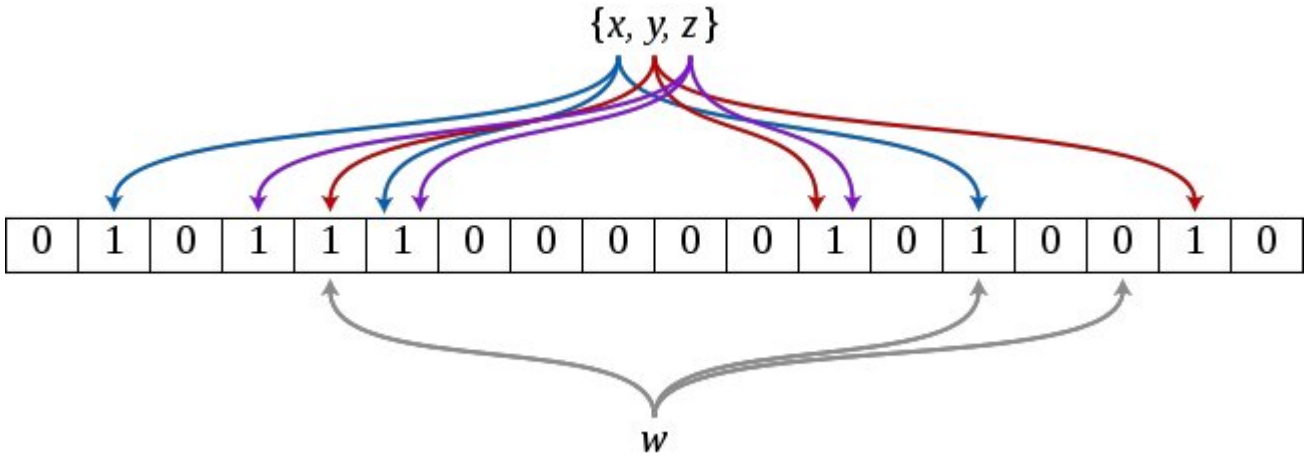
CS2040S – Data Structures and Algorithms

Lecture 8* – Bloom Filter

chongket@comp.nus.edu.sg



Bloom Filter – A Probabilistic Map



Motivation for using Bloom Filter

- When a hash table (specifically a **hash set**) is too large to fit into RAM, data will have to be read/written to and from hard disk which is much slower than RAM

- A possible solution is to use Bloom Filter instead of a classical hash table, as it is able to “compress” the amount of memory required to store the data

Introduction to Bloom Filter (1)

- Most important characteristic of a Bloom Filter is that retrieval of a key in the Bloom Filter is probabilistic
  - "No" means the key is definitely not in the filter
  - "Yes" has a certain probability that the key is actually not in the Bloom Filter (a false positive)

Introduction to Bloom Filter (2)

- Data Structure used – A bit array (in Java there is no bit primitive data type so have to use the **BitSet** class) which is initialized to 0

- Basic Operations
  - Insertion
  - Retrieval

- Can’t do removal in a Bloom Filter !

Bloom Filter - Insert operation (1)

- Inserting an integer key r into the bloom filter requires setting up to K bits of the bit array to 1. To do this,
  - use K independent and uniformly distributed hash functions $H_1(r)$ to $H_K(r)$
  - Each hash function returns an (hopefully different) index of the bit array which is then set to 1

- K is usually bounded by a small value so can be considered a constant

Bloom Filter - Insert operation (2)

3 hash functions:

$H_1(key) = (key\%17)\%10$

$H_2(key) = (key\%29)\%10$

$H_3(key) = (key\%37)\%10$

just choose k random prime numbers for the hash fcn

Bit array of size 10

{123}

Inserting 123:

$H_1(123) = 4$

$H_2(123) = 7$

$H_3(123) = 2$

Bit array of size 10

{123,306}

Inserting 306:

$H_1(306) = 0$

$H_2(306) = 6$

$H_3(306) = 0$

Bit array of size 10

Bloom Filter - Insert operation (3)

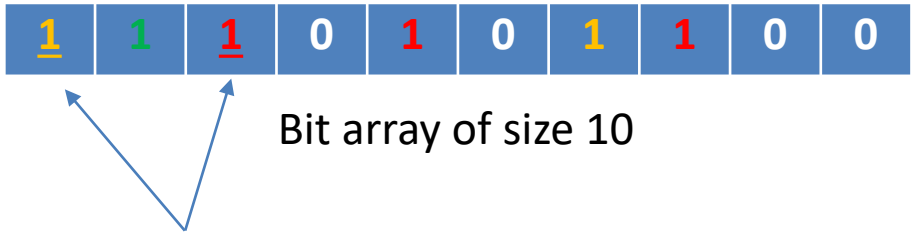
Inserting 7125:

$H_1(7125) = 2$

$H_2(7125) = 0$

$H_3(7125) = 1$

{123,306,7125}



Overlap with bit representation for 123, 306 with 7125

Bloom Filter - Retrieval operation (1)

- To retrieve a key, again use the K hash functions to get K indices in the filter
  - If all K indices are set to 1 then the key is possibly in the filter
  - If at least one index is 0 then the key is definitely not in the filter

Bloom Filter - Retrieval operation (2)

{123,306,7125}

Given

Bit array of size 10

Retrieving 7125:

$H_1(7125) = 2$

$H_2(7125) = 0$

$H_3(7125) = 1$

{123,306,7125}

Bit array of size 10

All 1's so return 7125 is in the Bloom Filter

Bloom Filter - Retrieval operation (3)

Retrieving 513:

$H_1(513) = 3$

$H_2(513) = 0$

$H_3(513) = 2$

{123,306,7125}

Bit array of size 10

Retrieving 74:

$H_1(74) = 6$

$H_2(74) = 6$

$H_3(74) = 0$

{123,306,7125}

even though 0 and 6 are 1, 74 is not inside because these indexes were set by 306 -> false positive

Bit array of size 10

Considerations when designing a good Bloom Filter (1)

- In general, the bigger the bloom filter and the more hash functions used, the less likely the bits set for any key will coincide with any other key
- However this defeats the purpose of the bloom filter to “compress” the memory requirements
- Need to find a good compromise

Considerations when designing a good Bloom Filter (2)

- 4 variables for a bloom filter
  - N: the number of keys to be inserted
  - S: the size of the bit array
  - K: the number of hash functions used
  - P: the probability of a false positive

if we dk the no. of keys, we should find an upper bound

we dont want too many hash functions so that the insertion/retrieval can be fast

- If we fix a value for N and P, then S and K can be computed as follows

Considerations when designing a good Bloom Filter (3)

$$S = Ceil\left(-\frac{N * lnP}{(ln2)^2}\right)$$

$$K = round(\frac{S}{N} * ln2)$$

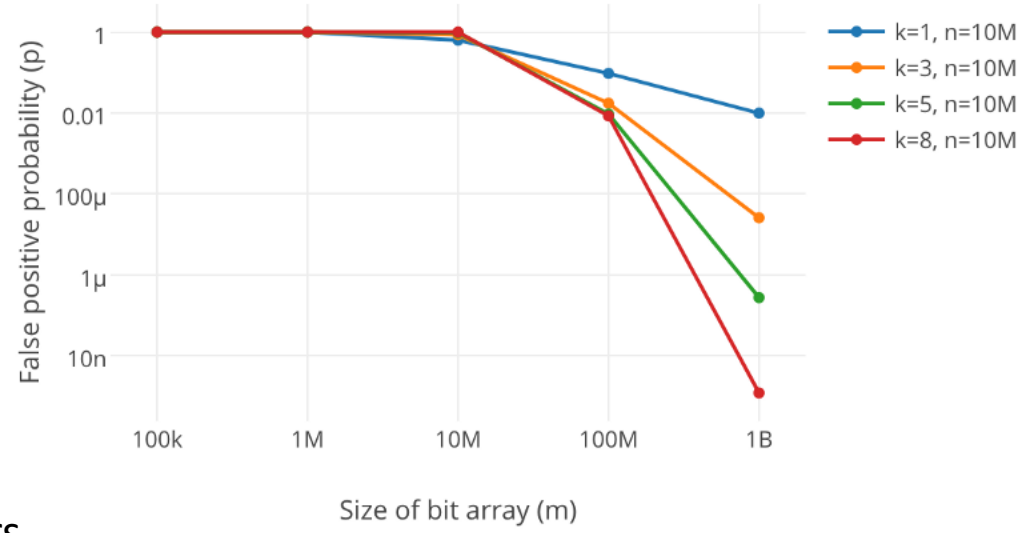


Image Credit: Abishek Bhat's article about bloom filter

Compression Ratio :

Assuming keys = 32 bits or 4 byte integers
At N = 10M and P = 0.01
Bit array size = 100Mbit = 12.5 Mbytes

If use hashtable need at least 40Mbytes.

Compression ratio = 40/12.5 = **3.2 times**

the larger the num of key, the higher the compression ratio because we dont store the key, we store its bit representation

those with more hash functions will have lower false positive rate compared to those with less fcns for the same no. of insertions

Considerations when designing a good Bloom Filter (4)

- In general the number of bits required to store each key in a bloom filter with false positive rate P is

$$Average\ \#\ of\ bits\ per\ key = \frac{S}{N} = 2.1 \ln \frac{1}{P}$$

- The number of bits required per key is independent of the size of the key itself, thus the greater the size of a key the greater the compression ratio

Bloom Filter Conclusion

- Pros
  - Useful when number of keys too large for classical hash table to fit into RAM
  - Useful when we can accept a small amount of errors (false positives) during retrieval
  - Use K bits to represent a key regardless of the size of the key (especially useful when key is a long string)
- Cons
  - Larger overhead to insert/retrieve a key since need to compute M hash functions instead of 1 hash function
  - Not useful for problems where no error is tolerated
  - Not cache-friendly since the K bits for a key may not be close together thus cannot fit into cache

bloom filter is replacement for hash set when it is too big