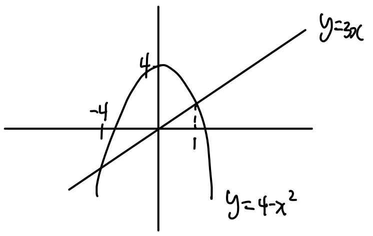
1.



$z = x+4$

$3x = 4 - x^2$
$x^2+3x-4=0$
$(x+4)(x-1)=0$
$x=-4, 1$

$$\int_{-4}^{1}\int_{3x}^{4-x^2} x+4 \; dy\,dx$$
$$= \int_{-4}^{1} \Big[xy+4y \Big]_{3x}^{4-x^2} dx$$
$$= \int_{-4}^{1} 4x - x^3 + 16 - 4x^2 - 3x^2 - 12x \; dx$$
$$= \int_{-4}^{1} -x^3 - 7x^2 - 8x + 16 \; dx$$
$$= \frac{625}{12}$$

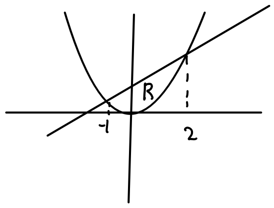
2. $\iint_D ds = \iint_D \sqrt{f_x^2 + f_y^2 + 1}\; dA$

$x^2+y^2+z^2 = 2^2$ above $z=1$
$x^2+y^2+z^2 = 3$

$\frac{d}{dx}(x^2+y^2+z^2) = \frac{d}{dx}4$
$2x + 2z f_x = 0$ because $\frac{d}{dx}(z^2) = 2z\frac{dz}{dx} = 2z f_x$
$f_x = -\frac{x}{z}$, $f_y = -\frac{y}{z}$

$$\int_0^{2\pi}\int_0^{\sqrt3} \sqrt{\frac{x^2}{z^2}+\frac{y^2}{z^2}}\; r\,dr\,d\theta$$

3.

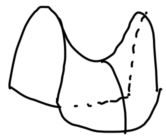


$x^2 = x+2$
$x^2-x-2=0$
$(x+1)(x-2)=0$

$z = \sqrt{x^2+y^2}$ $\quad z_x = \frac12 \frac{2x}{\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}}$ $\quad z_y = \frac{y}{\sqrt{x^2+y^2}}$

$$\iint_D ds = \iint_D \sqrt{4x^2+4y^2+1}\; dA = \iint_D \sqrt{1+\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}}\; dA$$
$$= \iint_D \sqrt2 \; dA$$
$$= \int_{-1}^{2}\int_{x^2}^{x+2} \sqrt2 \; dy\,dx$$
$$= \sqrt2 \int_{-1}^{2} (x+2-x^2)\,dx$$
$$= \frac92\sqrt2$$

4. $az = x^2-y^2$ $\quad x^2+y^2=a^2$



$z = \frac1a (x^2-y^2)$
$z_x = \frac{2x}{a}$ $\quad z_y = -\frac{2y}{a}$
$\sqrt{1+\frac{4x^2}{a^2}+\frac{4y^2}{a^2}} = \frac1a\sqrt{a^2+4x^2+4y^2}$

$$S = \iint_D \frac1a\sqrt{a^2+4x^2+4y^2}\; dA = \int_0^{2\pi}\int_0^{a} \frac1a\sqrt{a^2+4r^2}\; dr\,d\theta$$
$$= \int_0^{2\pi} \Big[(a^2+4r^2)^{\frac32}\Big]_0^a$$
$$= \frac{\pi a^2}{6}\left(5^{\frac32}-1\right)$$

5. a) $f(x,y)$
$\nabla f = \left(4x^3y - \frac{1}{1+x^2} + e^y\right)i + (x^4 + xe^y + x)j$

$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial y \partial x}$ by Clairaut's theorem

If such $f(x,y)$ exists, then
$\nabla f = \langle f_x, f_y \rangle$
$\therefore f_x = 4x^3y - \frac{1}{1+x^2} + e^y$, $f_y = x^4 + xe^y + x$
$f_{xy} = f_{yx}$ by Clairaut's theorem
$4x^3 + e^y \neq 4x^3 + e^y + 1$

b) $\nabla f = (4x^3y + y + e^y)i + (x^4 + xe^y + x + y)j$

$f_{xy} = 4x^3 + 1 + e^y = f_{yx} = 4x^3 + 1 + e^y$

$f_x = 4x^3y + y + e^y$, $f_y = x^4 + xe^y + x + y$

$f = \int 4x^3y + y + e^y \, dx = x^4y + xy + e^y x + g(y)$ could be a function of y as the derivative will go to zero
$f_y = x^4 + x + xe^y + g'(y)$
$g'(y) = y$
$\Rightarrow g(y) = \frac{y^2}{2} + C$
$f(x,y) = x^4y + xy + e^y x + \frac{y^2}{2} + C$

6. a) $x(x+1)y' = 1$

$y' = \frac{1}{x(x+1)}$

$\int 1 \, dy = \int \frac{1}{x(x+1)} dx$

$y = \int \frac{1}{x} - \frac{1}{x+1} \, dx$

$y = \ln\left|\frac{x}{x+1}\right| + C$

b) $y' = e^{(x-3y)}$

$e^{3y} y' = e^{x}$

$\int e^{3y} dy = \int e^{x} dx$

$= \frac{1}{3} e^{3y} = e^{x} + C$

c) $(1+y)y' + (1-2x)y^2 = 0$

$(1+y)\frac{dy}{dx} = (2x-1)y^2$

$\frac{1+y}{y^2} dy = (2x-1)dx$

$\int \frac{1+y}{y^2} dy = \int (2x-1) dx$, $\underline{\underline{y \neq 0}}$

need to take this condition

$= \ln|y| - \frac{1}{y} = x^2 - x + C$

$y = 0$ is also solution

So extra solution exist