

**Department of Mathematics**  
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(2022/23) Semester I      MA1521 Calculus for Computing      Tutorial 1

- (1) For each of the following functions, find all values of  $x$  for which it is defined, i.e. the maximal domain of each function.

(a)  $f(x) = \frac{4-x^2}{(4+x^2)(8-x^3)(1-x^4)}$

(b)  $g(x) = \sqrt{2 - \ln(x-3)}$

(c)  $h(x) = \frac{\ln(\sqrt{16-2x}+1)}{\sqrt{\ln x}-1}$

**Ans.** (a)  $x \in \mathbb{R} \setminus \{-1, 1, 2\}$ , (b)  $3 < x \leq 3 + e^2$ , (c)  $1 \leq x \leq 8$  and  $x \neq e$ .

(2) Let  $f(x)$  be defined on  $(-\infty, \infty)$  such that  $f(x) = \begin{cases} 2 & \text{if } x \leq -5 \\ x^2 - 1 & \text{if } -5 < x \leq -1 \\ 0 & \text{if } -1 < x \leq 1 \\ \frac{1}{x-1} & \text{if } x > 1 \end{cases}$ .

Find all  $x$  such that  $f$  is not continuous at  $x$ .

**Ans.**  $x = -5, 1$ .

- (3) The function  $f$  is defined on  $[0, 8]$  by

$$f(x) = \begin{cases} p\sqrt{x} & 0 \leq x < 4 \\ 7 & x = 4 \\ q(x-2)^2 + 5 & 4 < x \leq 6 \\ \frac{r}{x-5} & 6 < x \leq 8 \end{cases}.$$

It is given that  $f$  is continuous at  $x = 4$  and  $\lim_{x \rightarrow 6} f(x)$  exists. Find the values of  $p$ ,  $q$  and  $r$ .

**Ans.**  $p = \frac{7}{2}, q = \frac{1}{2}, r = 13$ .

- (4) Evaluate each of the following limits if it exists

- (a)  $\lim_{x \rightarrow 1} \frac{4+x}{2-x}$
- (b)  $\lim_{x \rightarrow 2} \frac{4-x^2}{x^2-3x+2}$
- (c)  $\lim_{x \rightarrow -2} \frac{4-x^2}{\sqrt{x^2-x-2}-\sqrt{2-x}}$
- (d)  $\lim_{x \rightarrow 1} \frac{3-\sqrt{x+8}}{\sqrt{x+3}-\sqrt{5-x}}$
- (e)  $\lim_{x \rightarrow 1} \frac{(x^2-1)}{(x-1)^2}$

**Ans.** (a) 5, (b)  $-4$ , (c)  $-4$ , (d)  $-\frac{1}{3}$  (e) undefined

(5) Evaluate the following limits

- (a)  $\lim_{x \rightarrow \infty} \sqrt{\frac{9x^{10}+3x-1}{(x^2+3x+5)^3(2x-5)^4}}$
- (b)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^{10}+3x-1}}{(1+2x)^5}$
- (c)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{9x^{10}+3x-1}}{(1+2x)^2(x^2+x-1)}$

**Ans.** (a)  $\frac{3}{4}$ , (b)  $-\frac{3}{32}$ , (c)  $\infty$ .

(6) Let  $f$  and  $g$  be continuous functions on  $\mathbb{R}$ . Given that  $f(3) = 2$  and  $\lim_{x \rightarrow 3} [2f(x) - g(x)] = 4$ . Determine  $g(3)$ .

**Ans** 0.

### Further Exercises (not to be discussed)

- (1) Suppose  $|x+3| < \frac{1}{2}$ . Show that  $|4x+13| < 3$ .
- (2) Let  $f(x) = \frac{x+1}{x-2}$ .
- (a) Find the domain  $D$  of the function  $f$ .
- (b) Is there an  $x \in D$  such that  $f(x) = 1$ ?

- (c) If  $c \in \mathbb{R} \setminus \{1\}$ , find an  $x \in D$  such that  $f(x) = c$ .
- (d) Find the range of  $f$ .