**Department of Mathematics**

**National University of Singapore**

**(2022/23) Semester I MA1521 Calculus for Computing Tutorial 7**

1. Write the vector $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ as a sum of two vectors $\mathbf{u} + \mathbf{v}$ such that $\mathbf{u}$ is parallel to $\mathbf{w} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{v}$ is perpendicular to $\mathbf{w}$. (Hint: Use projection)

   **Ans**. $\frac{1}{2}(\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}) + \frac{1}{2}(5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

2. Find the equation of a line through $P(1, -2, 3)$ and perpendicular to two lines $\ell_1$ and $\ell_2$ given by:

$$\ell_1 : x = 3t + 2, y = -4t - 1, z = 4t - 9.$$

$$\ell_2 : x = 3t - 4,\ y = -t + 6,\ z = 5t.$$

   **Ans**. $x = 1 - 16t, y = -2 - 3t, z = 3 + 9t$.

3. Consider the two lines:

$$\ell_1 : x = 2t - 1,\ y = -3t + 2,\ z = 4t - 3.$$

$$\ell_2 : x = 4t + 7, y = 2t - 2, z = -3t + 2.$$

   Show that $\ell_1$ and $\ell_2$ intersect. Find the point of intersection and an equation of the plane containing $\ell_1$ and $\ell_2$.

   **Ans**. $(3, -4, 5), x + 22y + 16z = -5$.

4. (a) Find an equation of the plane $\Pi$ passing through the points $A(3, 3, 0)$, $B(3, 0, 1)$ and $C(0, 2, 1)$.

   (b) Find the shortest distance from $O(0, 0, 0)$ to $\Pi$.

   (c) Let $D = (4, 2, 1)$. Find the coordinates of the point of intersection of the plane $\Pi$ in part (a) and the line segment $OD$.

   **Ans**. (a) $2x + 3y + 9z = 15$, (b) $15/\sqrt{94}$, (c) $\frac{15}{23}(4, 2, 1)$.

5. Find the shortest distance between the two planes:

$$\Pi_1 : \quad 2x + 2y - z = 1 \qquad \text{and} \qquad \Pi_2 : \quad 4x + 4y - 2z = 5.$$

   **Ans**. $1/2$.

6. Two particles travel along the space curves which, at time $t$, are given by:

$$\mathbf{r}_1(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}, \quad \mathbf{r}_2(t) = (1+2t)\mathbf{i} + (1+6t)\mathbf{j} + (1+14t)\mathbf{k}.$$

   Do the particles collide? Do their paths intersect?

7. Let $\mathbf{A}$ and $\mathbf{B}$ be two non-zero constant vectors such that $\|\mathbf{B}\| = 1$. If the angle between them is equal to $\frac{\pi}{4}$, find the value of $\lim\limits_{x\to 0} \frac{\|\mathbf{A}+x\mathbf{B}\|-\|\mathbf{A}\|}{x}$.

   **Ans**. $\frac{\sqrt{2}}{2}$.

**Further Exercises (Not to be discussed during tutorial)**

1. Find parametric equations of the line through the point $(0, 1, 2)$ that is parallel to the plane $x + y + z = 2$ and perpendicular to the line $x = 1 + t$, $y = 1 - t$, $z = 2t$.

   **Ans**. $x = 3t$, $y = 1 - t$, $z = 2 - 2t$.

2. Show that the lines $x = t, y = t, z = t$ and $x = s - 1, y = 2s, z = 3s$ are skew. Find the distance between these two lines.

   **Ans**. $\frac{1}{\sqrt{6}}$.

3. Find the equation of the surface obtained by rotating the line $x = 3y$ about the $x$-axis.

   **Ans**. $x^2 - 9y^2 - 9z^2 = 0$.