Department of Mathematics

National University of Singapore

(2022/23) Semester I MA1521 Calculus for Computing Tutorial 9

1. Evaluate the following double integrals:

   (a) $\displaystyle\iint_R (x^2+y^2)\,dA$ where R is bounded by the lines, $x=0, x=a,\ y=0$ and $y=b$.

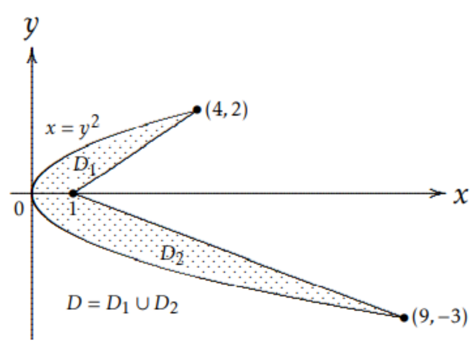
   (b) $\displaystyle\iint_R \frac{xy}{\sqrt{4-x^2}}\,dA$ where R is bounded by the lines $x=0, x=1, y=1$ and $y=2$.

   (c) $\displaystyle\iint_R e^{x^2}\,dA$ where R is the region bounded by $y=0,\ y=x,\ x=1$.

   (d) $\displaystyle\iint_R (x+y)\,dA$, where R is the region bounded by the two curves $y=\sqrt{x},\ y=x^2$.

   Ans. (a) $\frac{1}{3}ab(a^2+b^2)$, (b) $3-3\sqrt{3}/2$, (c) $\frac{1}{2}(e-1)$, (d) $\frac{3}{10}$.

2. Evaluate the double integral $\iint_D x\ dA$ where D is the region as shown below.



   Ans. 25.

3. Evaluate the following integrals by converting it to polar coordinates:

   (a) $\displaystyle\int_0^a\int_0^{\sqrt{a^2-x^2}} \frac{1}{1+x^2+y^2}\,dydx$, (b) $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\,dydx$

   Ans. (a) $\frac{\pi}{4}\ln(a^2+a)$, (b) $\frac{1}{4}\pi(e-1)$.

4. Evaluate the following integrals by reversing the order of integration.

(a) $\displaystyle\int_0^1 \int_{x-1}^{1-x} x\, dydx$ (b) $\displaystyle\int_0^8 \int_{\sqrt[3]{y}}^{2} e^{x^4}\, dxdy$ (c) $\displaystyle\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dxdy.$

Ans. (a) $\frac{1}{3}$, (b) $\frac{1}{4}(e^{16}-1)$, (c) 2.

Further Exercises

1. Evaluate the following double integrals

(a) $\displaystyle\iint_D \frac{2y}{x^2+1}\, dA, \quad D = \{(x,y) \mid 0 \le x \le 1, 0 \le y \le \sqrt{x}\}$

(b) $\displaystyle\iint_D x\cos y\, dA,$ D is the region bounded by $y=0, y=x^2, x=1$.

(c) $\displaystyle\iint_D (2x-y)\, dA,$ D is the region bounded by the circle $x^2+y^2=4$.

Ans. (a) $\frac{1}{2}\ln 2$, (b) $\frac{1}{2}(1-\cos(1))$, (c) 0.

2. Find the volume of the solid bounded by the elliptic paraboloid $z = 1+(x-1)^2+4y^2$, the planes $x=3$ and $y=2$, and the coordinate planes.

Ans. 44.

3. Let R be the smaller segment of the circular region $x^2+y^2 \le a^2$ cut off by the line $x+y=a$, $(a>0)$. Sketch R and evaluate $\displaystyle\iint_R xy^2\, dA$.

Ans. $\frac{a^5}{20}$.