CS2040S – Data Structures and Algorithms

Lecture 13 – *Splay Tree

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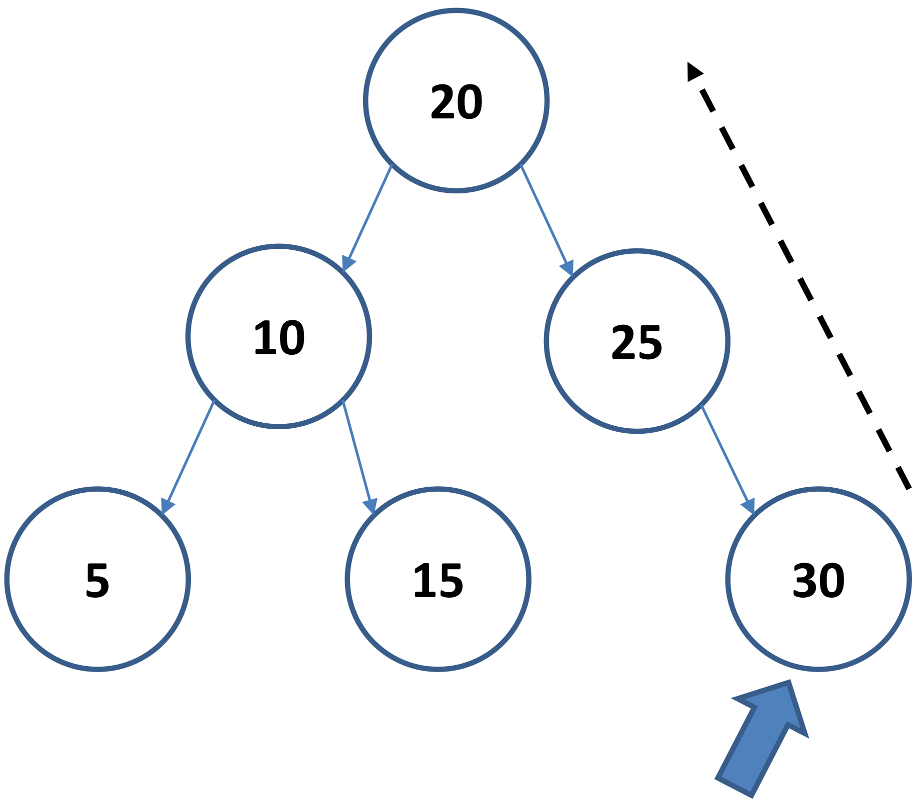
Splay Tree – Another self-balancing BST

- Balancing is based on the following heuristic:
  - The most frequently accessed key will most likely be accessed again and so should be placed at the top of the tree, making future accesses O(1) time
  - No need to keep height information

- Search is modified so that whenever a search key X is found, the node containing X is shifted to the root using a series of rotation operations (Splay Steps) ← Splaying

- Insertion/deletion is their standard BST counterpart with additional Splaying after the insertion/deletion

Example of balancing in Splay Tree

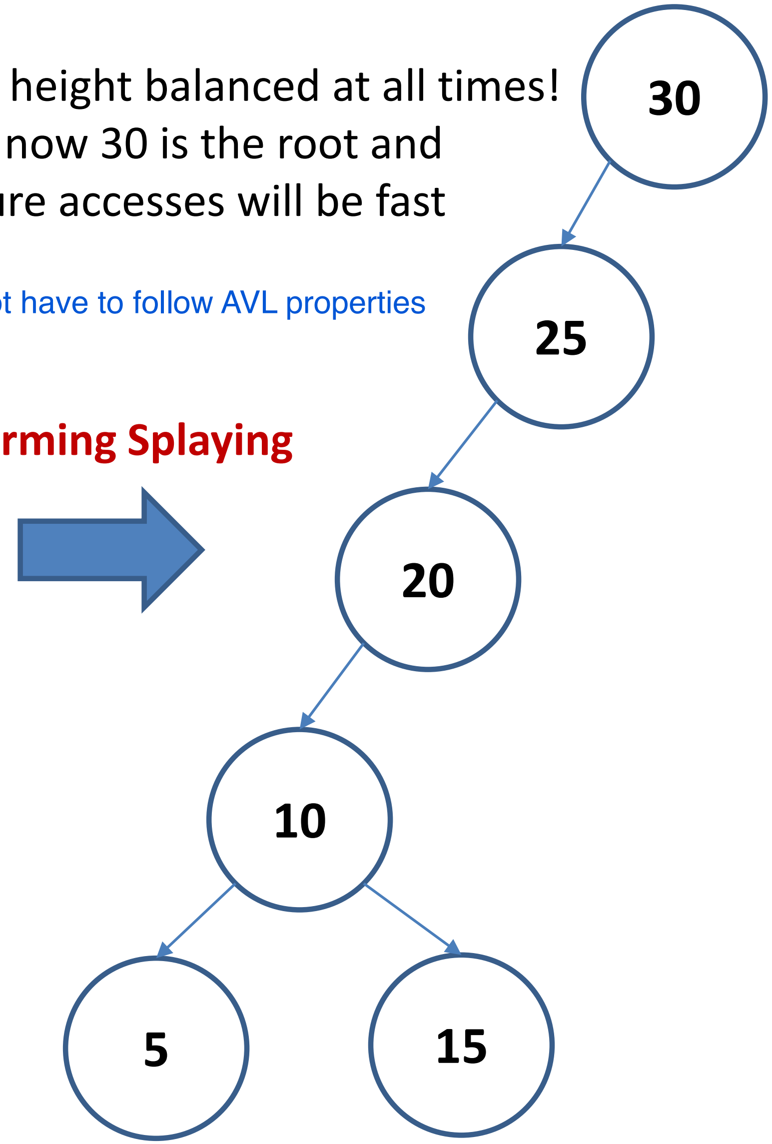


If 30 was searched, thus increasing frequency of its access, 30 will be shifted towards the root of the tree

Not height balanced at all times! But now 30 is the root and future accesses will be fast

does not have to follow AVL properties

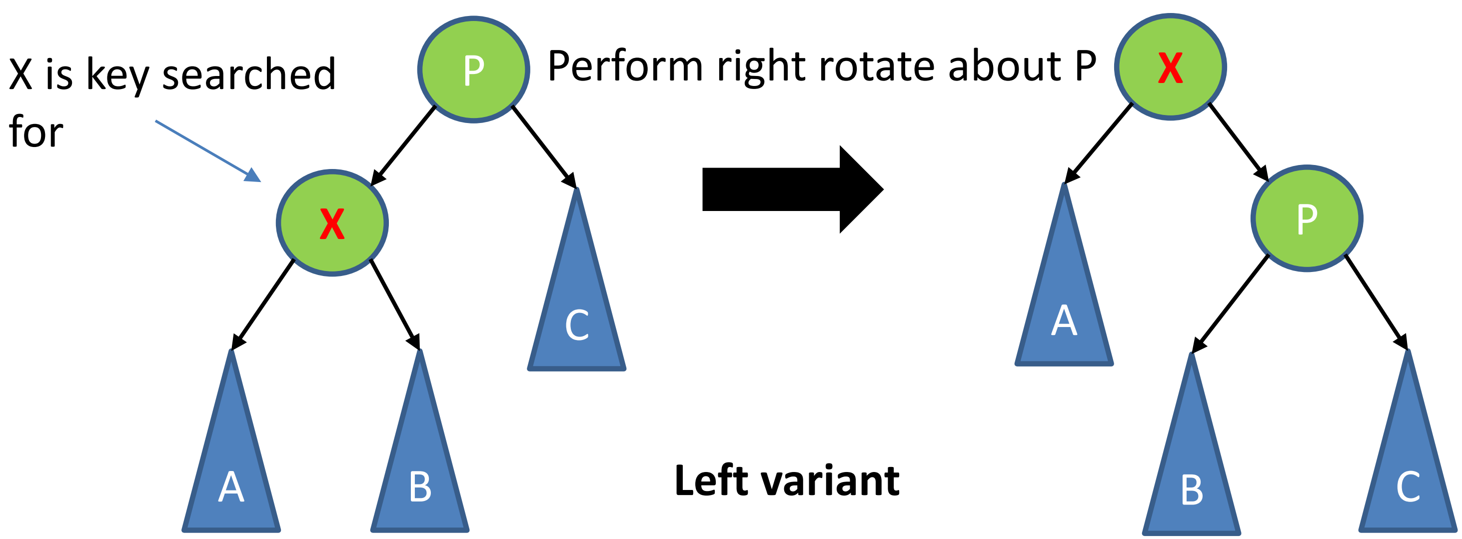
Performing Splaying



Splay Step – 6 cases to consider (1)

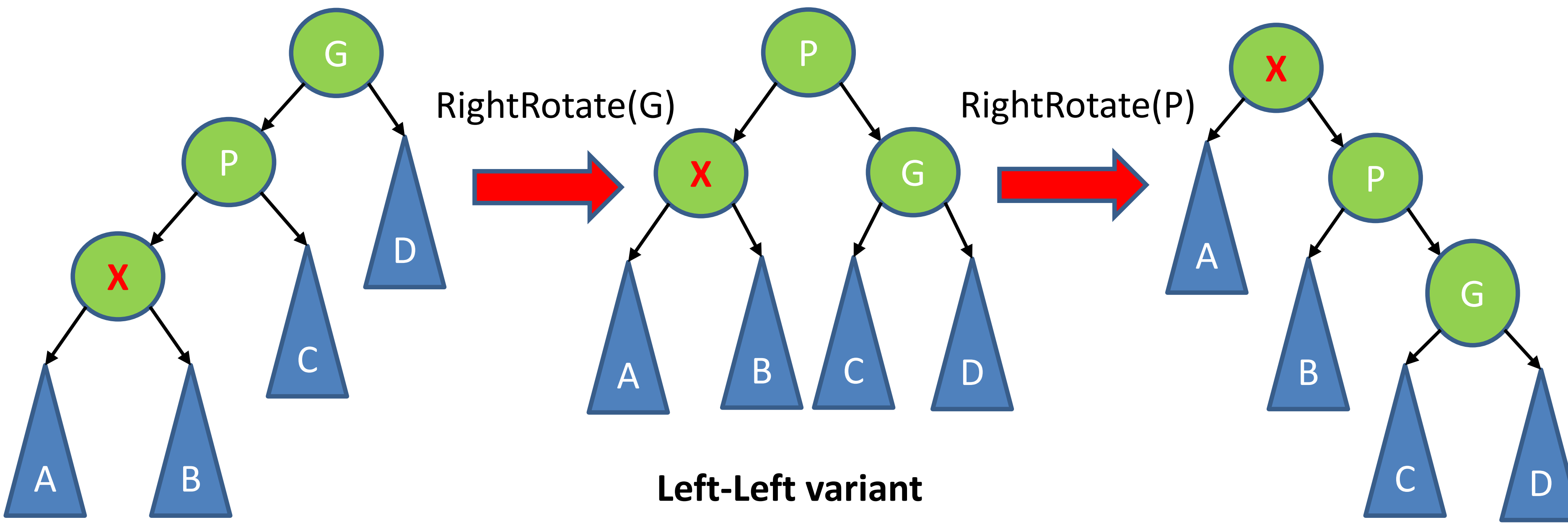
- 3 cases (each case has two variant so 6 cases in all)
- Case 1: Zig Step (last step in a series of splay steps)
  - If X = P.left and P = root (left variant) → RightRotate(P)
  - If X = P.right and P = root (right variant) → LeftRotate(P)

X: curr node
P: parent



Splay Step – 6 cases to consider (2)

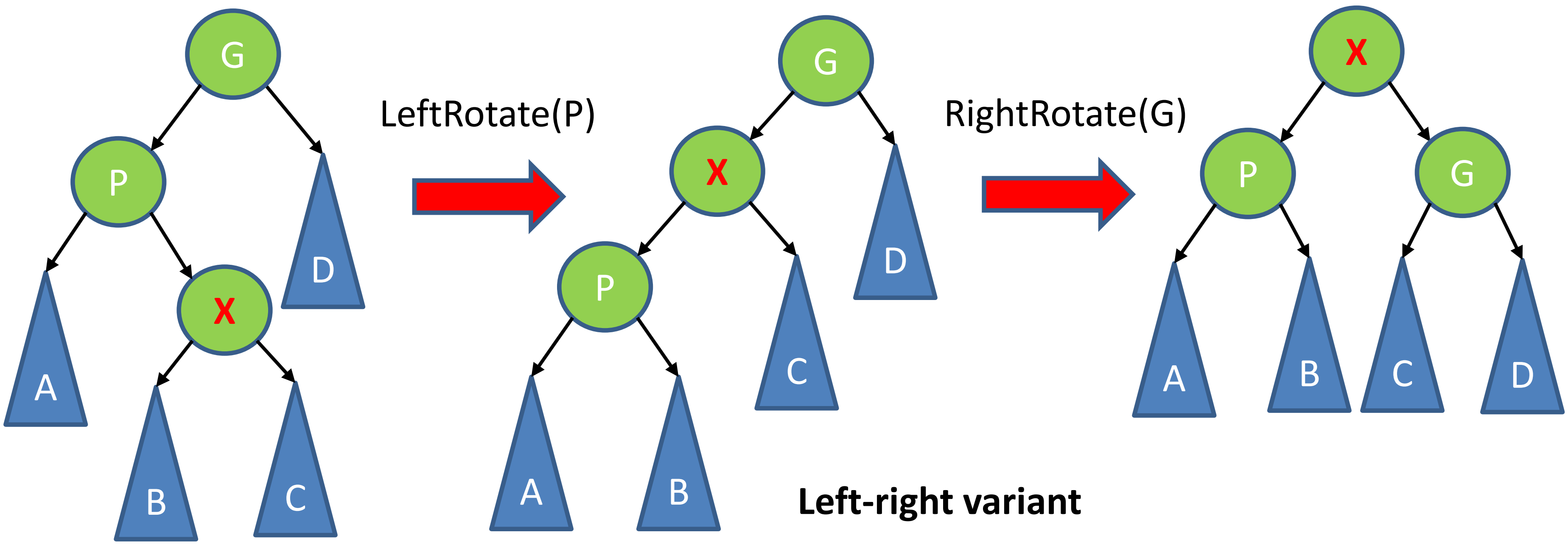
- Case 2: Zig-Zig Step
  - If X = P.left and P = G.left (left-left variant)
    → RightRotate(G) then RightRotate(P)
  - If X = P.right and P = G.right (right-right variant)
    → LeftRotate(G) then LeftRotate(P)



Left-Left variant

Splay Step – 6 cases to consider (3)

- Case 3: Zig-Zag Step
  - If P = G.left and X = P.right (left-right variant)
    → LeftRotate(P) then RightRotate(G)
  - If P = G.right and X = P.left (right-left variant)
    → RightRotate(P) then LeftRotate(G)



Left-right variant

Splay Tree Operations

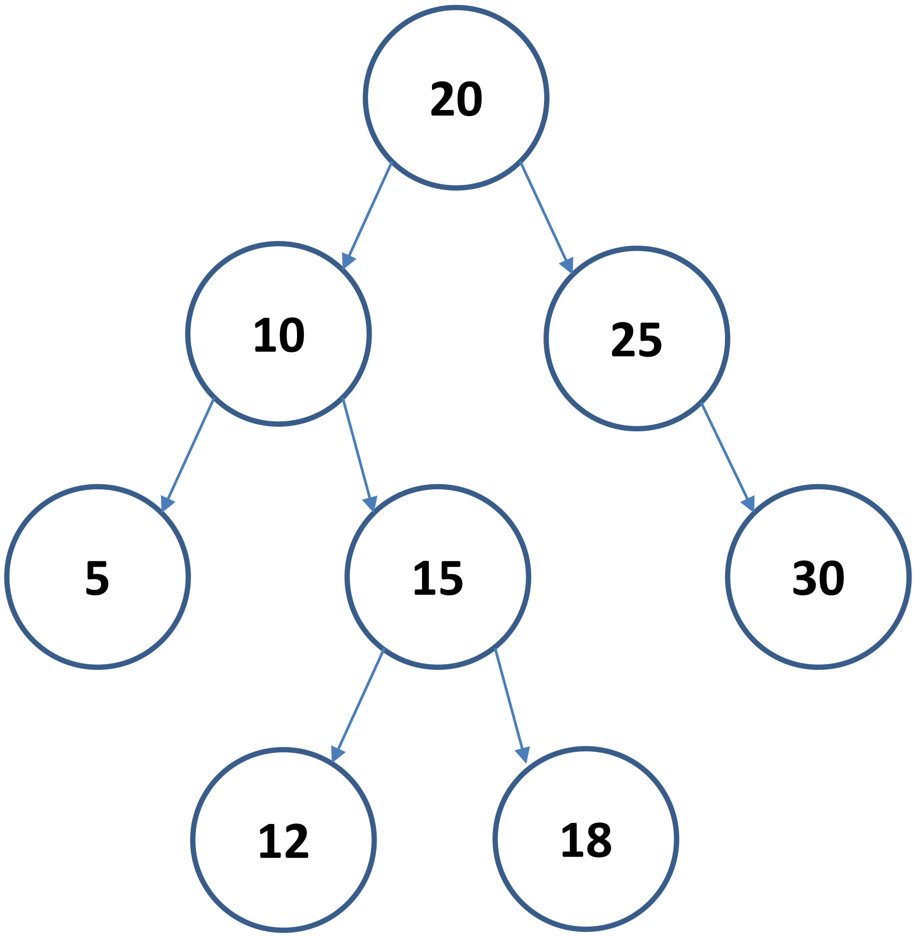
- Search
  - If successful, for the searched node x, repeated perform splay steps on x until it is the root
  - If unsuccessful, repeated splay the last node before null was reached
- Insert
  - After the new node x is inserted, repeated perform splay steps on x until it is the root

Splay Tree Operations

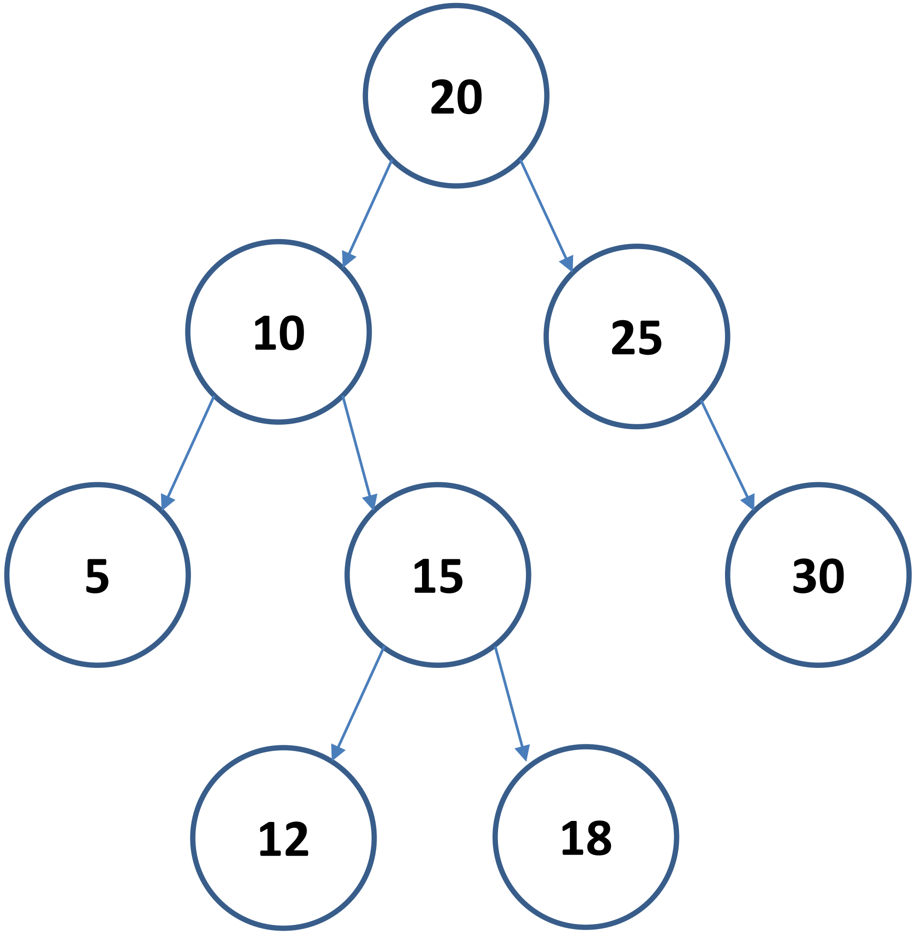
- Delete
  - If node x to be deleted is the only node in the tree, do nothing after x is deleted
  - If node x to be deleted has 0 or 1 child, after x is deleted (standard BST deletion), splay x's parent to the root
  - If node x to be deleted has 2 children, after x's successor is deleted (again standard BST deletion), splay x's successor's parent to the root

Exercises

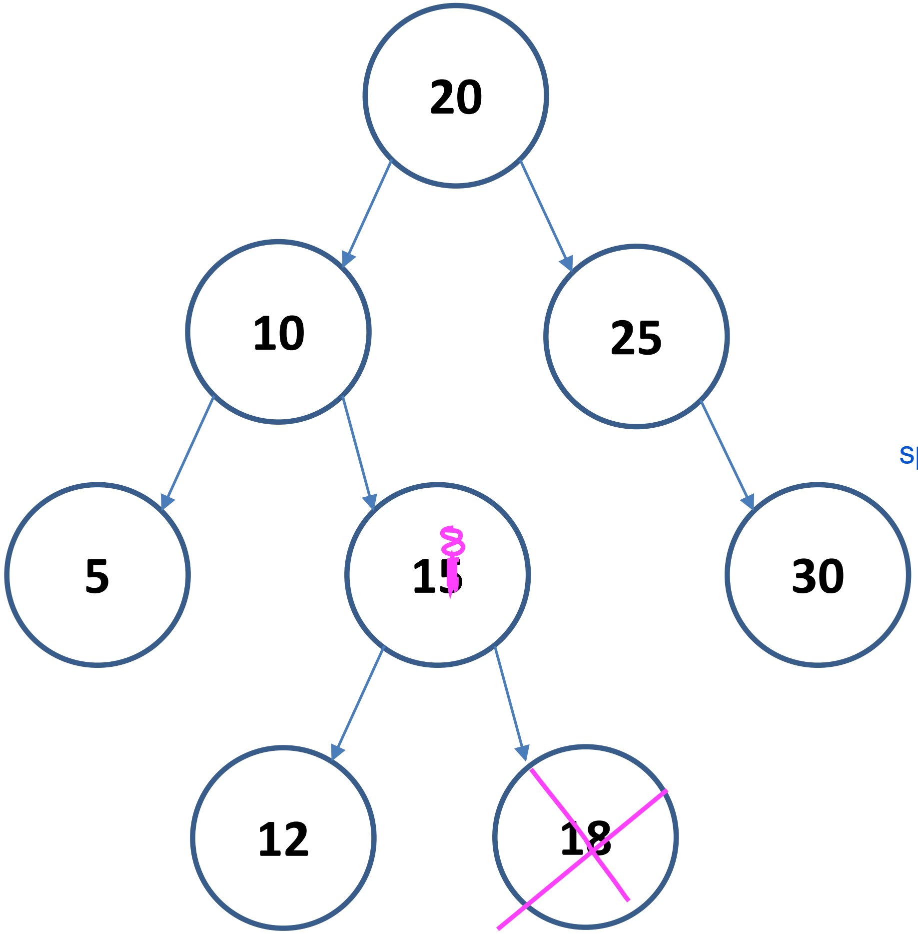
- Search for 12



- Insert for 35



- Remove 15



splay the parent of the successor

Summary Splay Tree

- Heuristic approach to balancing a BST which can achieve good results with real life applications
- No need to height balance the tree!
- Amortized cost for search/insert/delete will still run in O(logN) time