Q1. a.

Prefix	Symbol	Power
atto	a	10^{-18}
femto	f	10^{-15}
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
		10^{0}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

i. 400 – 700 nm = 400 – 700 × 10^{-9} m = 4 – 7 × 10^{-7} m

ii. 4 Tbytes = 4 × 10^{12} bytes

iii. 400 fs = 400 × 10^{-15} s = 4 × 10^{-13} s

iv. 0.10 pF = 0.10 × 10^{-12} F = 1.0 × 10^{-13} F; 5 μF = 5 × 10^{-6} F

v. 13.8 Gyrs = 13.8 × 10^{9} yrs = 1.38 × 10^{10} yrs

b. Emphasis on ASSUMPTIONS

i. Mass of the universe, $m_{\text{universe}} \approx 3 \times 10^{52}$ kg

Information needed is the mass of one atom to get the total no. of atoms.

1 atomic mass unit (or 1 amu, or 1 u) = 1.66 × 10^{-27} kg (roughly the mass of one proton or one neutron)

A rough estimate using *order of magnitude* would be ~$10^{52}/10^{-27} = 10^{79}$ atoms.

To get a better estimate, you may refer to the "abundance of the chemical elements" (table from the Wikipedia page shown below) and derive the average mass of atoms in the universe using the 10 most common elements in the Milky Way.

Average mass of 1 atom, $m_{\text{atom}} \approx$ (0.739 × 1 u) + (0.240 × 2 u) + (0.0104 × 8 u) + (0.0046 × 6 u) + … = 1.401 u = 2.326 × 10^{-27} kg

Better estimate = (3 × 10^{52} kg)/(2.326 × 10^{-27} kg/atom) = 1.29 × 10^{79} atoms.

Proton number, Z	Element	Mass fraction [parts per million, ppm]
1	Hydrogen	739,000
2	Helium	240,000
8	Oxygen	10,400
6	Carbon	4,600
10	Neon	1,340
26	Iron	1,090
7	Nitrogen	960
14	Silicon	650
12	Magnesium	580
16	Sulphur	440

b. ii. Conversion of 4.5 billion years to seconds:

$$\left(4.5\times10^{9}\text{years}\right)\times\frac{365\text{ days}}{\text{year}}\times\frac{24\text{ hours}}{\text{day}}\times\frac{60\text{ mins}}{\text{hour}}\times\frac{60\text{ s}}{\text{min}}=1.42\times10^{17}\text{ s}$$

b. iii. Volume of disc = $\pi r^2 h = (\pi d^2/4)h = \pi(8\ \mu\text{m})^2/4 \times (2\ \mu\text{m}) = 1.005 \times 10^{-16}\ \text{m}^3$

Density of water (at room temperature) = 997 kg m^{-3}

Mass of cell = Volume × Density = 1.002×10^{-13} kg

This approach assumes that red blood cells can be represented by discs. If we want to have a more accurate answer, we will need to study the exact shape of RBCs. Improvement in the description of the shape: e.g. two curved segments at the top and bottom removed. We also know that there are medical conditions where people have RBCs that are sickle-shaped, a geometry that is very different from a disc.

c. i. 1 eV = 1.6×10^{-19} J

1 kWh = (1000 J s^{-1}) × (60 min/hr × 60 s/min) = 3.6×10^{6} J

1 kton TNT = 4.184×10^{12} J [Look up 'TNT equivalent']

1 Å = 10^{-10} m = 0.1 nm (Angstroms used because atomic distances are on this order of magnitude.)

Q2. a. $E = mc^2 = (10^{-3}\text{ kg}) \times (3 \times 10^8\text{ m s}^{-1})^2 = 9 \times 10^{13}\text{ J}$

For a 60-kg man to get this amount of kinetic energy ($E_{kin} = ½mv^2$), he needs to have a velocity of $v = (2 \times E_{kin}/m)^{1/2} = [2 \times (9 \times 10^{13}\text{ J})/(60\text{ kg})]^{1/2} = 1.73 \times 10^6\text{ m s}^{-1}$.

b. Emphasis on ANTIMATTER

Mass of electron, m_e = Mass of positron = 9.1×10^{-31} kg

When they annihilate, the total mass of the electron plus positron will be converted into energy in the form two gamma photons. The mass-equivalent energy of the electron is $m_ec^2 = (9.1 \times 10^{-31}\text{ kg}) \times (3 \times 10^8\text{ m s}^{-1})^2 = 8.19 \times 10^{-14}\text{ J} = 511\text{ keV} = 0.511\text{ MeV}$, which is the same for that of the positron. Each photon will carry this amount of energy, and the total energy produced from this process is 2×0.511 MeV = 1.022 MeV.

For 0.25 g of antimatter, another 0.25 g of matter will be involved in the annihilation process, so the total mass we need to consider is 0.5 g = 5×10^{-4} kg. Total energy explosion = $(5 \times 10^{-4}\text{ kg}) \times (3 \times 10^8\text{ m s}^{-1})^2 = 4.5 \times 10^{13}\text{ J}$ = 10.76 kton TNT. According to a Washington state Department of Health report, for a 10 kton explosion:

Air blast	~ 600 m radius
Heat	~1.8 km radius
Initial radiation with 50% mortality (1st minute within explosion)	~1.2 km radius
Secondary radiation with 50% mortality (1st hour within explosion)	~10 km radius

For context, the Hiroshima bomb was ~15 kton TNT, while the Nagasaki bomb was ~20 kton TNT. Almost everything within ~1.6 km radius of the Hiroshima bomb was completely destroyed by the blast; lethal radiation within ~1.3 km radius.

Dimensions of the Vatican City ~1.05 km × 0.85 km, therefore this explosion would more or less destroy the whole of the Vatican City.

c. $m = E/c^2 = (200\text{ MeV}) / (3 \times 10^8\text{ m s}^{-1})^2$

$= (200 \times 10^6 \times 1.6 \times 10^{-19}\text{ J}) / (3 \times 10^8\text{ m s}^{-1})^2$

$= 3.56 \times 10^{-28}\text{ kg}$

No. of atoms in 1 g of U-235 = $(0.001\text{ kg})/(235 \times 1.66 \times 10^{-27}\text{ kg per atom})$

$= 2.56 \times 10^{21}$ U-235 atoms

Fission energy = $(2.56 \times 10^{21}) \times 200$ MeV $= 5.12 \times 10^{23}$ MeV

$= 8.20 \times 10^{10}$ J

$= 2.28 \times 10^{4}$ kWh

Family can rely on this energy for $\frac{2.28 \times 10^{4}\ \text{kWh}}{20\ \text{kWh/day}}$ = 1140 days = 3 yrs 45 days

Q3. Nuclear reaction.

a.

$$He_2^4 + N_7^{14} \rightarrow O_8^{17} + H_1^1$$

$(m_{He} + m_N) - (m_O + m_H) = -0.00128\ u = -2.12 \times 10^{-30}$ kg

$Q = (-2.12 \times 10^{-30}\ \text{kg})(3 \times 10^{8}\ \text{ms}^{-1})^2 = -1.91 \times 10^{-13}\ \text{J} = -1.2$ MeV

b. Why this reaction is not spontaneous:

From the negative Q value calculated, more energy is required to form the RHS products than the LHS reactants. Lower mass favoured (i.e. larger binding energy),

Q > 0 (left heavier): forward reaction favoured

Q < 0 (right heavier): backward reaction favoured

c. Minimum velocity of alpha particles:

$0.5(m_{He})v^2 = 0.5(4.002603 \times 1.66 \times 10^{-27}\ \text{kg})v^2 = 1.91 \times 10^{-13}$ J

$v = 7.58 \times 10^{6}\ \text{m.s}^{-1}$

d. Why this energy is insufficient:

1. Conservation of momentum: Threshold energy is the energy below which the reaction does not occur. Energy available from the reaction is maximum when two equal-mass particles collide head-on with the same energy. Conversely, this is minimum when the target is at rest. The actual value varies depending on the angle of the products etc. For this reaction, $E_{threshold}$ ~1.5 MeV.

2. Coulomb barrier: This is very complex as it depends on the nuclear radius, and using the well-known equation for Coulomb potential (or use hyperphysics calculator) this value is supposedly ~10.2 MeV.

Q4. Beta decay of K-40:

$$K_{19}^{40} \rightarrow Ca_{20}^{40} + \beta_{-1}^{0} + \bar{\nu}_e + 1.311\ \text{MeV}$$

where β^{0}_{-1} is a beta-particle (which is also an electron when negatively charged, or positron when positively charged) and $\bar{\nu}_e$ is an *electron anti-neutrino*.

A 150-g banana contains 537 mg of potassium.

Mass of K-40 = 0.012% × 537 mg = 6.44 × 10⁻⁸ kg.

No of K-40 atoms, $N_{K\text{-}40} = \dfrac{6.44 \times 10^{-8}\ \text{kg}}{40 \times 1.66 \times 10^{-27}\ \text{kg/atom}} = 9.71 \times 10^{17}$ atoms

$$\frac{(6.44 \times 10^{-5}\ \text{g})\ (6.022140857 \times 10^{23}\text{atoms.mol}^{-1})}{39.96399848\ \text{g.mol}^{-1}} = 9.70 \times 10^{17}\ \text{atoms}$$

Activity: $-\frac{dN}{dt} = \lambda N$ with the solution $N(t) = N_0\, e^{-\lambda t}$

After one half-life, $N(T_{1/2}) = N_0\, e^{-\lambda T_{1/2}} = \frac{N_0}{2}$

Solve $\lambda = \dfrac{\ln 2}{(1.23 \times 10^{9}\ \text{yrs}) \times (365 \times 24 \times 60 \times 60\ \text{secs/yr})} = 1.787 \times 10^{-17}\ \text{s}^{-1}$

Activity = $\lambda N_{K\text{-}40} = (1.787 \times 10^{-17}\ \text{s}^{-1}) \times (9.71 \times 10^{17}) = 17.4$ Bq

Q6. Postulate 1 equates the circular motion of the electron's orbit around the proton $\frac{m_e v^2}{r}$ to the electrostatic attraction $\frac{e^2}{4\pi\varepsilon_0 r^2}$

Multiply by $\frac{r}{m_e v}$ to get $v = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{m_e v r} = \frac{e^2}{4\pi\varepsilon_0}\frac{1}{nh/2\pi}$ where the last term is from Postulate 2.

From $m_e v r = n\frac{h}{2\pi}$ rearrange to get $\frac{1}{r} = \frac{m_e v}{nh/2\pi}$

Substitute into v equation: $\frac{1}{r} = \frac{m_e}{nh/2\pi}\frac{e^2}{4\pi\varepsilon_0}\frac{1}{nh/2\pi} = \frac{e^2}{4\pi\varepsilon_0}\frac{m_e}{n^2\left(\frac{h}{2\pi}\right)^2}$

Total Energy = $E_{\text{kinetic}} + E_{\text{potential}} = \frac{1}{2}m_e v^2 - \frac{e^2}{4\pi\varepsilon_0 r}$

Again from $\frac{m_e v^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$ we multiply by $\frac{r}{2}$ to get $E_{\text{kinetic}} = \frac{m_e v^2}{2} = \frac{e^2}{8\pi\varepsilon_0 r}$

so Total Energy = $\frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 r}$

Substitute $\frac{1}{r}$ equation to get Total Energy = $-\frac{e^2}{8\pi\varepsilon_0}\frac{e^2}{4\pi\varepsilon_0}\frac{m_e}{n^2\left(\frac{h}{2\pi}\right)^2} = -\frac{e^4 m_e}{8\varepsilon_0^2 h^2}\frac{1}{n^2} = (-13.6\ \text{eV})\frac{1}{n^2}$

When $n = 1$, Total Energy = −13.6 eV = −2.17 × 10⁻¹⁸ J