Recitation - 03

CS2040S Recitation Team

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Problem[1]

A bunch of people are living on an island, when a visitor comes with a strange order: all blue-eyed people must leave the island as soon as possible. There will be a flight out 8.00 PM every evening. Each person can see everyone else's eye color, but they do not know their own(nor is anyone allowed to tell them). Additionally, they do not know how many people have blue eyes, although they do know that at least one person does. How many days will it take the blue-eyed people to leave?

Solution

Let's apply the Base Case and build approach. Assume that there are n people on the island and c of them have blue eyes. We are explicitly told that $c > 0$.

Case $c = 1$: Exactly one person has blue eyes

Assuming all people are intelligent, the blue eyed- person should look around and realize that no one else has blue eyes. Since he knows that at least one person has blue eyes, he must conclude that it is he who has blue eyes. Therefore, he would take the flight that evening.

Case $c = 2$: Exactly two people have blue eyes

The two blue-eyed people see each other, but are unsure whether c is 1 or 2. They know, from previous case, that if $c = 1$, the blue-eyed person would leave on the first night. Therefore, if other blue-eyed person is still there, he must deduce $c = 2$, which means he himself has blue eyes. Both men would then leave on the second night.

[1] Puzzle Credits - Book:Cracking the Coding Interview

Case $c > 2$: The General Case

As we increase c, we can see that this logic continues to apply. If $c = 3$, then those three people will immediately know that there are either 2 or 3 people with blue eyes. If there were two people, then those two people would have left on the second night. So, when the others are still around after that night, each person would conclude that $c = 3$ and that they, therefore, have blue eyes too. They would leave that night. This same pattern extends up through any value of c. Therefore, if c mean have blue eyes, it will take c nights for the blue-eyed men to leave. All will leave on the same night.