Department of Mathematics

National University of Singapore

(2022/23) Semester I   MA1521 Calculus for Computing   Tutorial 4

(1) Evaluate the following definite integrals.

(a) $\displaystyle\int_1^{\sqrt{2}} \frac{s^2+\sqrt{s}}{s^2}\, ds\,,$

(b) $\displaystyle\int_{-4}^{4} |x|\, dx\,,$

(c) $\displaystyle\int_0^{\pi} \frac{1}{2}(\cos x + |\cos x|)\, dx\,,$

(d) $\displaystyle\int_0^{\pi} \sin^2\left(1+\frac{\theta}{2}\right) d\theta\,.$

Ans. (a) $1+\sqrt{2}-2^{3/4}$,   (b) 16,   (c) 1,   (d) $\frac{1}{2}\pi + \sin 2$.

(2) Using the fundamental theorem of Calculus, find the derivative dy/dx for the following functions.

(a) $\displaystyle y = \int_0^{\sqrt{x}} \cos t\, dt\,,$

(b) $\displaystyle y = \int_0^{x^2} \cos\sqrt{t}\, dt\,,$

(c) $\displaystyle y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}, \qquad |x| < \frac{\pi}{2}\,.$

Ans. (a) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$,   (b) $2x\cos x$,   (c) 1.

(3) Using the *substitution* method, or otherwise, find the following integrals.

(a) $\displaystyle\int x^{1/2}\sin(x^{3/2}+1)\, dx\,,$

(b) $\displaystyle\int \csc^2 2t\, \cot 2t\, dt\,,$

(c) $\displaystyle\int \frac{1}{\theta^2}\sin\frac{1}{\theta}\cos\frac{1}{\theta}\, d\theta\,,$

(d) $\displaystyle\int \frac{18\tan^2 x\sec^2 x}{(2+\tan^3 x)}\, dx\,,$

(e) $\displaystyle\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta}\cos^3\sqrt{\theta}}\,d\theta\,.$

Ans. (a) $-\frac{2}{3}\cos(x^{3/2}+1)+C$, (b) $-\frac{1}{4}\cot^2 2t+C$, (c) $-\frac{1}{2}\sin^2\frac{1}{\theta}+C$, (d) $6\,\ln|\tan^3 x+2|+C$, (e) $\sec^2\sqrt{\theta}+C$.

(4) Applying the method of *integration by parts*, or otherwise, find the following integrals.

(a) $\displaystyle\int x\,\sin\left(\frac{x}{2}\right)\,dx\,,$

(b) $\displaystyle\int t^2\,e^{4t}\,dt\,,$

(c) $\displaystyle\int e^{-y}\,\cos y\,dy\,,$

(d) $\displaystyle\int \theta^2\,\sin(2\theta)\,d\theta\,,$

(e) $\displaystyle\int z\,(\ln z)^2\,dz\,.$

Ans. (a) $-2\,\left[x\cos\left(\frac{x}{2}\right)-2\sin\left(\frac{x}{2}\right)\right]+C$, (b) $(\frac{t^2}{4}-\frac{t}{8}+\frac{1}{32})e^{4t}+C$, (c) $\frac{e^{-y}}{2}(\sin y-\cos y)+C$, (d) $-\frac{1}{2}\,\left[\theta^2\cos(2\theta)-\theta\sin(2\theta)-\frac{1}{2}\cos(2\theta)\right]+C$, (e) $\frac{1}{2}\left[z^2(\ln z)^2-z^2(\ln z)+\frac{z^2}{2}\right]+C$.

(5) Evaluate the following improper integrals.

(a) $\displaystyle\int_0^1 \frac{1}{(x-1)^{\frac{4}{5}}}\,dx,$

(b) $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx.$

Ans. (a) 5, (b) $\frac{1}{4}$.

Further Exercises (Not to be discussed during tutorial

(1) Find the exact value of $\displaystyle\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}}$, where a is a positive constant.

Ans. $\frac{\pi}{4}$.

(2) Use the result

$$\int_a^b f(x)\,dx = bf(b) - af(a) - \int_{f(a)}^{f(b)} f^{-1}(x)\,dx$$

to evaluate the integral

$$\int_1^2 \sec^{-1} x\,dx.$$

Ans. $\frac{2\pi}{3} - \ln(2+\sqrt{3})$.

(3) (a) Find the value of $\displaystyle\int_0^\pi \frac{\sin x}{\sqrt{9-\cos^2 x}}\,dx$.

(b) Use the result $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ and (a) to show that

$$\int_0^\pi \frac{x\sin x}{\sqrt{9-\cos^2 x}}\,dx = \pi \sin^{-1}\frac{1}{3}.$$