GET1024/GEC1036: Radiation – Scientific Understanding and Public Perception

Tutorial 1 – Introduction, Atoms and Nucleus

<u>Note from the lecturer.</u> As a General Education Module, we do not intend the module to be too technical but the concepts must be understood. It is unavoidable that some jargons or conventions will be used during the lectures. In fact, as you will find when you go over articles later, it is necessary for you to be familiar with terms such as 10^9, MeV, µm, U-235 or ^{235}U, etc. which appear even in popular writing. As this is the first out of five tutorial sessions that you will be having for this module, we will start by introducing some of the conventions and symbols. We will also go through some simple calculations which illustrate the immense power with nuclear reactions that may be possibly used by mankind.

1. **Conventions and notations**.

   a. **Units and prefix**. You are familiar with units like m, **m**m (= 0.001 m), **c**m (= 0.01 m) and **k**m (1000 m) or even mile (~ 1609 m), etc. when one measures the distance or length of some objects. The letter "m", "c" and "k" are prefixes that give a fraction or a multiple of the base unit metre. Write out the values of the following quantities (you may check out https://en.wikipedia.org/wiki/Unit_prefix).
      i. The wavelength of visible light ranges approximately from **400** to **700 nm**.
      ii. A portable **4 Tbyte** hard disk costs about S$130.
      iii. The pulse of some laser lasted for only **400 fs**.
      iv. The capacitance of these capacitors ranges from **0.10 pF** to **5 µF**.
      v. The age of the universe is about **13.8 Gyrs**.

   b. **Scientific Notation**. When we talk about very big or very small numbers, it may be inconvenient to use our usual notation, e.g., there are about 14 trillion but rather say **1.4 x 10^{13}** human cells in our body. The following are some instances when the scientific notations are useful.
      i. The mass the universe is estimated to be **3 x 10^{52} kg**. Roughly how many atoms are there?
      ii. The Earth is formed about **4.5 billions** years ago. What is this age in terms of seconds?
      iii. The human red blood cell is roughly a disc with a diameter of **8 µm** and thickness of **2 µm**, what is the volume of the cell? Estimate its mass assuming that its density is similar to that of water.

   c. **Other units** that may be used in this module
      i. Find out what these units mean and provide the conversion factor to the more traditional units such as metres or joules (units for energy).
         1. MeV
         2. kWh
         3. kton TNT
         4. angstrom (Å)

2. **Calculations with *E* = *mc*²**.

   a. What is the equivalent energy of 1 g of mass? You may recall that the kinetic energy of a moving object is $\frac{1}{2}mv^2$. How fast must a typical human (say m = 60 kg) run in order to have this energy?

   b. There exists in this universe antimatter which will annihilate the normal matter that exists in our normal environment.

      i. For example, positron is the anti-particle of electron. When they come together, all their energies are converted to radiation (in fact two photons of equal energy). What are the energy (in J and keV), frequency and wavelength of the radiation due to this annihilation process? In what range of EM radiation do these photons fall into?

      ii. In the novel "Angels and Demons" by Dan Brown, the hero has with him a canister containing about ¼ gram of antimatter. He exploded it in the atmosphere and saved the Vatican City. How much energy is released? Give your answer in joules and also convert it to kton TNT. Do you think the antimatter could have destroyed the Vatican City as originally planned by the villains? Provide some justifications to your answers. (You may want to compare with the atomic bombs that were dropped in Hiroshima and Nagasaki during World War II.)

   c. In the splitting of nucleus (known as nuclear fission), the mass of the products of the process is less than that of the original nucleus (plus that of the neutron causes the fission). The energy released in each fission process is about 200 MeV.

      i. How much mass is converted to energy in each reaction?
      ii. Estimate the number of atoms in one gram of U-235. How much energy would be released if all these atoms are fissioned? A typical Singaporean family uses about 20 kWh of energy a day. For how long would one g of U-235 supply the energy needs of this family? (Mass of one U-235 atom is 3.9×10^{-25} kg)

3. **Nuclear Reaction**. Consider the world's first recorded induced nuclear reaction, performed by Rutherford and colleagues in 1917 – 1919, encountered in Lecture 3.

$$ {}^{4}_{2}\mathrm{He} + {}^{14}_{7}\mathrm{N} \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H} $$

   The atomic masses of the above nuclides are $M_N = 14.003074$ u, $M_{He} = 4.002603$ u, $M_O = 16.999132$ u and $M_H = 1.007825$ u where u is the unified atomic mass unit.

   a. What is the reaction energy Q of this reaction?
   b. Explain why this reaction cannot be spontaneous, i.e., putting helium and nitrogen gas together would not produce oxygen and hydrogen gas.

c. The experiment was done by bombarding the nitrogen gas with "energetic" alpha particles. Based on the value of Q that you have calculated earlier, what is the minimum speed that the alpha particles must have for this reaction to occur if all the kinetic energy of the alpha particles is converted to mass.

d. In fact, this energy is not sufficient. Can you guess why?

4. **Radioactivity**. It is known that in 100 g of banana, there is 358 mg of potassium. The radioactive isotope K-40 of potassium makes up 0.012% of the total amount of potassium found in nature. K-40 has a half-life of 1.23×10^9 years. Calculate the activity of a typical 150 g banana in Bq.

   Data you may need: Isotope mass of K-40 = 39.96399848(21) u, Avogadro's number = $6.022140857 \times 10^{23}$;

   Hints: The data on bananas in the question were obtained from:

   http://www.myfit.ca/nutrition/banana_facts_recipes_origin.asp &
   https://en.wikipedia.org/wiki/Potassium-40

5. **Reading Assignment**. Read the article (https://en.wikipedia.org/wiki/Electromagnetic_spectrum) on **EM radiation**. You will see how some of the notations in question 1 are used in the article. More information on various types of EM radiation can be found. You should note that the range for each type of radiation are only approximate and whether a photon with energy of 200 keV or 1.5 MeV is designated as an x-ray photon or gamma-ray photon is often not based on its frequency but rather the source that produce these photons. Note also that speed of light in vacuum is always the same irrespective of the types of radiation and thus there is a simple relationship occurs between the frequency and wavelength of the radiation. In this module, we will be dealing mainly with x-rays and gamma-rays which fall into the short wavelength end of the spectrum.

   Two of the more important applications of EM radiation are:

   a. **Microwave oven** which makes use of microwave to heat up our food. Perform an internet search to find out more on how the microwave oven works. You should be able to conclude for yourself, based on basic scientific principles, whether consuming food that have been microwaved present any (additional) risk to our health.

   b. **Food irradiation** which makes use of ionising radiation, most commonly gamma radiation, to kill germs that can cause food poisoning. Again, perform an internet search to find out how it works, and conclude, based on science, whether consuming food that have been irradiated with gamma-rays present any additional risk to our health.

6. (Optional) **Bohr's Model of the Hydrogen Atom**. The first two postulates that Bohr made allow us to solve for the radii and energies of the various electronic states of the hydrogen atom.

   Postulate 1: An electron in an atom moves in a circular orbit about the nucleus under the influence of Coulomb attraction between the electron and nucleus and obeying the laws of classical mechanics, that is:

$$F = ma = \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

   Postulate 2: But instead of the infinite number of orbits which would be possible in classical mechanics, it is only possible for an electron to move in an orbit for which its orbital angular momentum *L* is an integral multiple of Planck's constant divided by 2π, that is:

$$L = mvr = n\frac{h}{2\pi} \quad \text{where } n \text{ is a positive integer}$$

   a. Solve the above simultaneous equations to express *r* and *v* in terms of the other constants (*n*, *m*, *h*, ϵ_0, etc.).

   b. The energy of the electron consists of two parts:
      i. Kinetic Energy $= \frac{1}{2}mv^2$, and
      ii. Electrostatic Potential Energy $= -\frac{e^2}{4\pi\epsilon_0 r}$

      Substitute the expressions you have obtained for *r* and *v* in part a. to obtain the expression for the total energy of atoms as given in the lecture.

   c. The smallest radius of the electronic orbits is obtained when *n* = 1. What is the energy of this state, in unit of joule and eV?