Department of Mathematics

National University of Singapore

(2022/23) Semester I MA1521 Calculus for Computing Tutorial 5

(1) Find the area of the following region.

(a) The region bounded between $y = \dfrac{1}{2}\sec^2 x$, $y = -4\sin^2 x$, $x = -\dfrac{\pi}{3}$ and $x = \dfrac{\pi}{3}$.

(b) The region in the first quadrant bounded by $y = x$, $y = \dfrac{1}{4}x^2$ and below $y = 1$.

[Hint: Easier to integrate with respect to y.]

(c) The region between the graphs of $y = 4 - x^2$ and $y = 2 - x$ from $x = -2$ to $x = 2$.

Ans. (a) $\dfrac{4}{3}\pi$, (b) $\dfrac{5}{6}$, (c) $\dfrac{19}{3}$.

(2) Calculate the length of the following curves.

(a) $y = \ln(\cos x)$, $0 \le x \le \frac{\pi}{3}$.

(b) $y = \frac{x^5}{15} + \frac{1}{4x^3}$, $1 \le x \le 2$.

Ans. (a) $\ln(2 + \sqrt{3})$, (b) $\frac{1097}{480}$.

(3) Suppose $f(x)$ is defined as follows:

$$f(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ 2x - 1 & \text{if } 1 \le x \le 2 \end{cases} .$$

Let R be the region bounded by the graph of $f(x)$ and the lines $x = 2$ and $y = 0$. Find the volume of the solid generated if R is revolved about the x-axis.

Ans. $\frac{14\pi}{3}$.

(4) (a) Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

(b) The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

Ans. (a) $\dfrac{64}{15}\sqrt{2}\pi$, (b) $\dfrac{8}{3}\pi$.

(5) Let a be a positive constant. Find the area of the finite region bounded by the curves $y^2 = x + 4a^2$ and $x - ay + 2a^2 = 0$.

Ans. $\frac{9}{2}a^3$.

(6) A finite region R is bounded by the curve $y = \sqrt{\tan x}$, and the lines $x = \frac{\pi}{4}$ and $y = 0$. Find the volume of the solid formed by revolving R one complete round about the x-axis.

Ans. $\frac{\pi}{2} \ln 2$.

Further Exercises (not to be discussed)

1. Evaluate $\displaystyle\int_1^2 \frac{1}{x^7 + x}\, dx$.

   Ans. $\frac{1}{6} \ln(\frac{128}{65})$.

2. Use Riemann sum to show that

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\sqrt{3kn + n^2}} = \frac{2}{3}.$$

3. Find the length of the curve $8x = 4e^{2y} + e^{-2y}$, $\ln 2 \le y \le \ln 3$.

   Ans. $\frac{725}{288}$.

The volume V is given by
$V = 2\pi \int_0^1 x^2\,dx + 2\pi \int_1^2 x(2x-1)\,dx = 2\pi \left[\frac{x^3}{3}\right]_0^1 + 2\pi \left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_1^2$
$= 2\pi[\frac{1}{3} + (\frac{16}{3} - 2) - (\frac{2}{3} - \frac{1}{2})] = 7\pi.$

$\int_0^1 \pi x^2 dx + \int_1^2 \pi(2x-1)^2 dx = [\pi x^3/3]_0^1 + [\pi(2x-1)^3/6]_1^2 = \pi/3 + [27\pi/6 - \pi/6] = 14\pi/3.$