CS2100 COMPUTER ORGANISATION

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Lecture #19

Sequential Logic

NUS National University of Singapore | School of Computing

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6. Synchronous Sequential Circuits

- Building blocks: logic gates and flip-flops.
- Flip-flops make up the memory while the gates form one or more combinational sub-circuits.
- We have discussed *S-R* flip-flop, *J-K* flip-flop, *D* flip-flop and *T* flip-flop.

6.1 Flip-flop Characteristic Tables

- Each type of flip-flop has its own behaviour, shown by its characteristic table.

J	*K*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	*Q(t)'*	Toggle

S	*R*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	?	Unpredictable

D	*Q(t+1)*	
0	0	Reset
1	1	Set

T	*Q(t+1)*	
0	*Q(t)*	No change
1	*Q(t)'*	Toggle

6.2 Sequential Circuits: Analysis (1/7)

- Given a sequential circuit diagram, we can analyze its behaviour by deriving its *state table* and hence its *state diagram*.
- Requires *state equations* to be derived for the flip-flop inputs, as well as *output functions* for the circuit outputs other than the flip-flops (if any).
- We use ***A(t)*** and ***A(t+1)*** (or simply ***A*** and ***A***$^{+}$) to represent the present state and next state, respectively, of a flip-flop represented by *A*.

6.2 Sequential Circuits: Analysis (2/7)

- Example using *D* flip-flops

State equations:

$A^+ = A{\cdot}x + B{\cdot}x$

$B^+ = A'{\cdot}x$

Output function:

$y = (A + B){\cdot}x'$

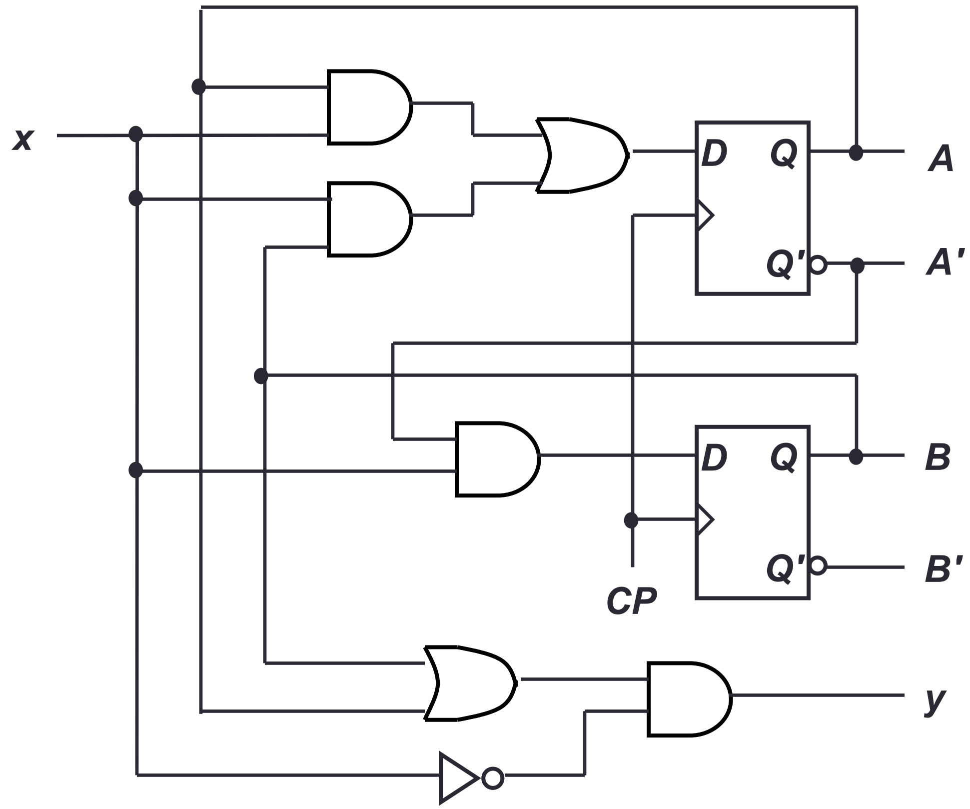


Figure 1

6.2 Sequential Circuits: Analysis (3/7)

- From the *state equations* and *output function*, we derive the ***state table***, consisting of all possible binary combinations of present states and inputs.
- State table
  - Similar to truth table.
  - Inputs and present state on the left side.
  - Outputs and next state on the right side.
- m flip-flops and n inputs $\rightarrow 2^{m+n}$ rows.

6.2 Sequential Circuits: Analysis (4/7)

- ***State table*** for circuit of Figure 1:

State equations:

$A^+ = A \cdot x + B \cdot x$

$B^+ = A' \cdot x$

Output function:

$y = (A + B) \cdot x'$

Present State		Input	Next State		Output
A	***B***	***x***	A^+	B^+	***y***
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

6.2 Sequential Circuits: Analysis (5/7)

- Alternative form of state table:

Full table

Present State		Input	Next State		Output
A	B	x	A^+	B^+	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

Compact table

Present State	Next State		Output	
	x=0	x=1	x=0	x=1
AB	A^+B^+	A^+B^+	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

6.2 Sequential Circuits: Analysis (6/7)

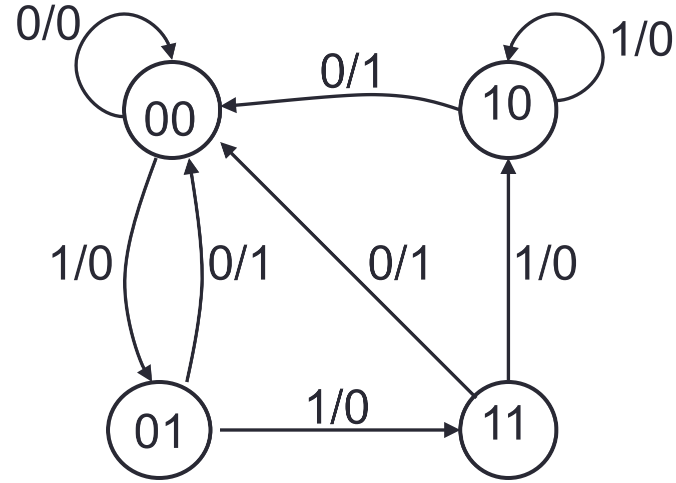
- From the *state table*, we can draw the ***state diagram***.
- State diagram
  - Each state is denoted by a circle.
  - Each arrow (between two circles) denotes a transition of the sequential circuit (a row in state table).
  - A label of the form *a*/*b* is attached to each arrow where *a* (if there is one) denotes the inputs while *b* (if there is one) denotes the outputs of the circuit in that transition.
- Each combination of the flip-flop values represents a state. Hence, m flip-flops $\rightarrow$ up to 2^m states.

6.2 Sequential Circuits: Analysis (7/7)

- **State diagram** of the circuit of Figure 1:

Present State	Next State		Output	
	x=0	*x*=1	*x*=0	*x*=1
AB	A^+B^+	A^+B^+	y	y
00	00	01	0	0
01	00	11	1	0
10	00	10	1	0
11	00	10	1	0

DONE!



6.2 Flip-flop Input Functions (1/3)

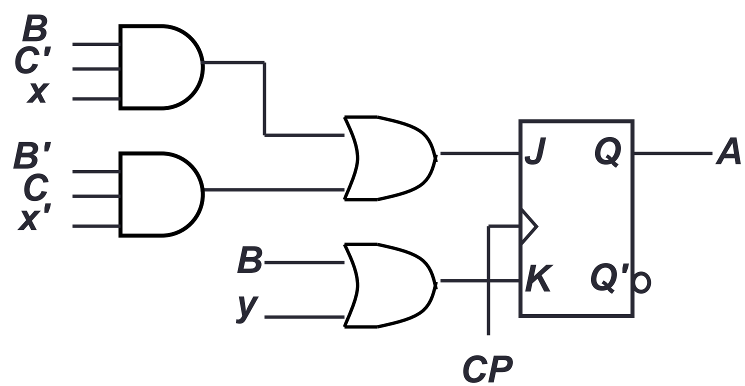
- The outputs of a sequential circuit are functions of the present states of the flip-flops and the inputs. These are described algebraically by the *circuit output functions*.
  - In Figure 1: $y = (A + B) \cdot x'$
- The part of the circuit that generates inputs to the flip-flops are described algebraically by the *flip-flop input functions* (or *flip-flop input equations*).
- The flip-flop input functions determine the next state generation.
- From the flip-flop input functions and the characteristic tables of the flip-flops, we obtain the next states of the flip-flops.

6.2 Flip-flop Input Functions (2/3)

- Example: circuit with a *JK* flip-flop.
- We use 2 letters to denote each flip-flop input: the first letter denotes the input of the flip-flop (*J* or *K* for *J-K* flip-flop, *S* or *R* for *S-R* flip-flop, *D* for *D* flip-flop, *T* for *T* flip-flop) and the second letter denotes the name of the flip-flop.

$JA = B \cdot C' \cdot x + B' \cdot C \cdot x'$

$KA = B + y$



6.2 Flip-flop Input Functions (3/3)

- In Figure 1, we obtain the following state equations by observing that $Q^+ = DQ$ for a D flip-flop:

$\boldsymbol{A^+ = A{\cdot}x + B{\cdot}x}$ (since $DA = A{\cdot}x + B{\cdot}x$)

$\boldsymbol{B^+ = A'{\cdot}x}$ (since $DB = A'{\cdot}x$)

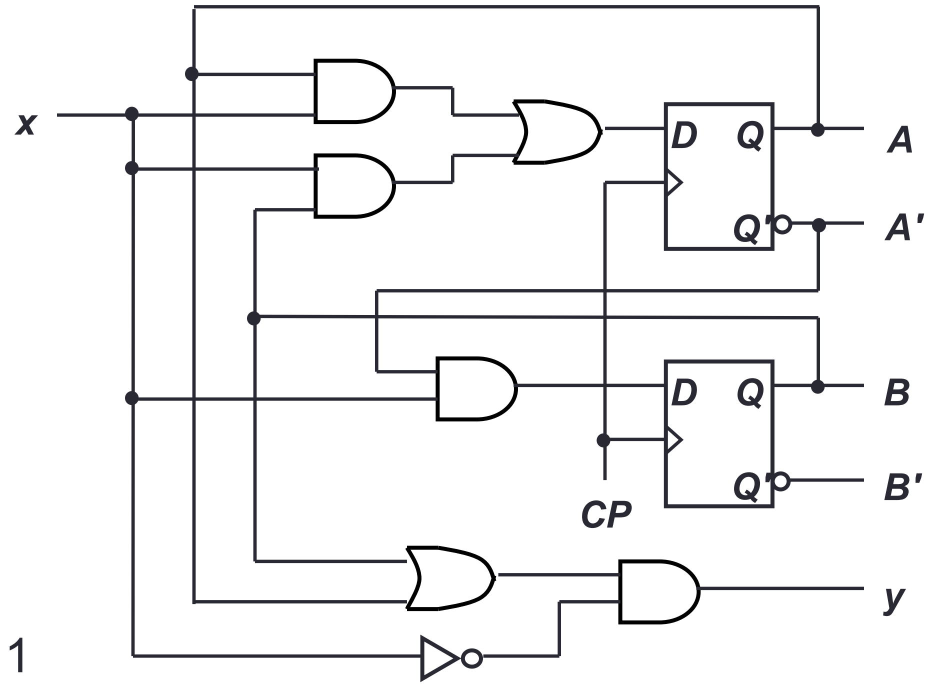


Figure 1

6.2 Analysis: Example #2 (1/3)

- Given Figure 2, a sequential circuit with two *J-K* flip-flops *A* and *B*, and one input *x*.

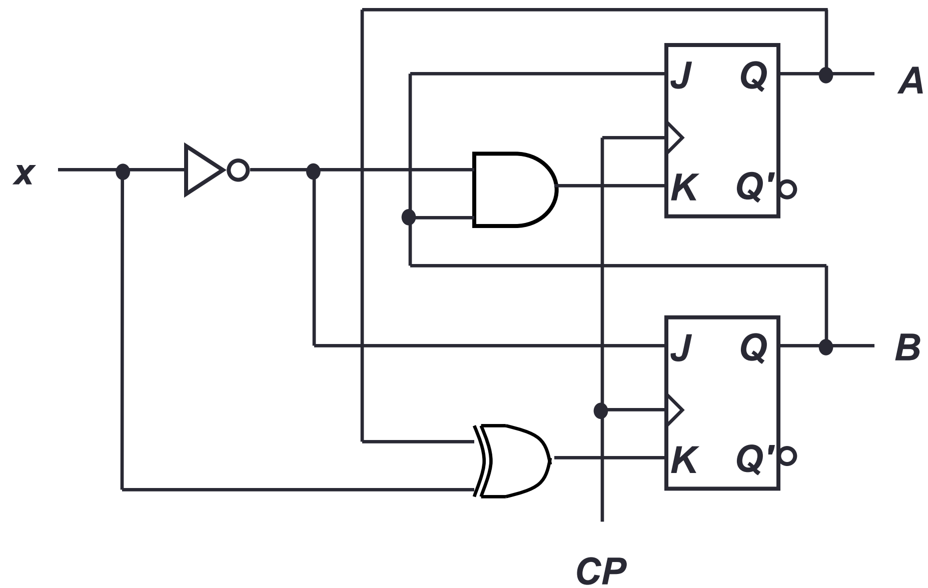


Figure 2

- Obtain the flip-flop input functions from the circuit:

$$JA = B \qquad JB = x'$$

$$KA = B \cdot x' \qquad KB = A' \cdot x + A \cdot x' = A \oplus x$$

6.2 Analysis: Example #2 (2/3)

$JA = B \qquad JB = x'$

$KA = B \cdot x' \qquad KB = A' \cdot x + A \cdot x' = A \oplus x$

- Fill the state table using the above functions, knowing the characteristics of the flip-flops used.

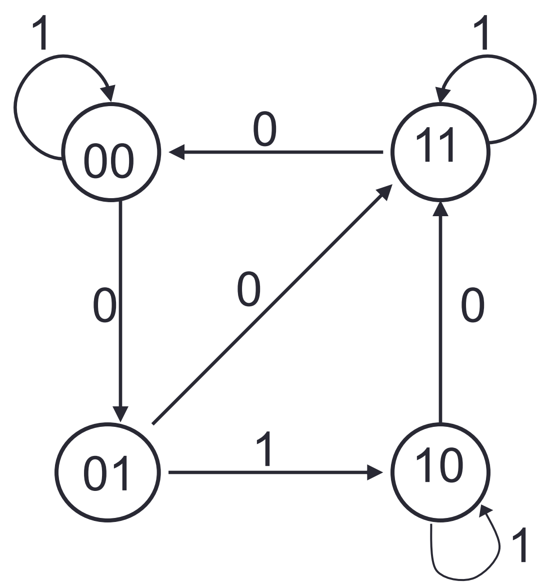
J	*K*	*Q(t+1)*	Comments
0	0	*Q(t)*	No change
0	1	0	Reset
1	0	1	Set
1	1	*Q(t)'*	Toggle

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0

6.2 Analysis: Example #2 (3/3)

- Draw the state diagram from the state table.

Present state		Input	Next state		Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*JA*	*KA*	*JB*	*KB*
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	1
1	0	1	1	0	0	0	0	0
1	1	0	0	0	1	1	1	1
1	1	1	1	1	1	0	0	0



6.2 Analysis: Example #3 (1/3)

- Derive the state table and state diagram of this circuit.

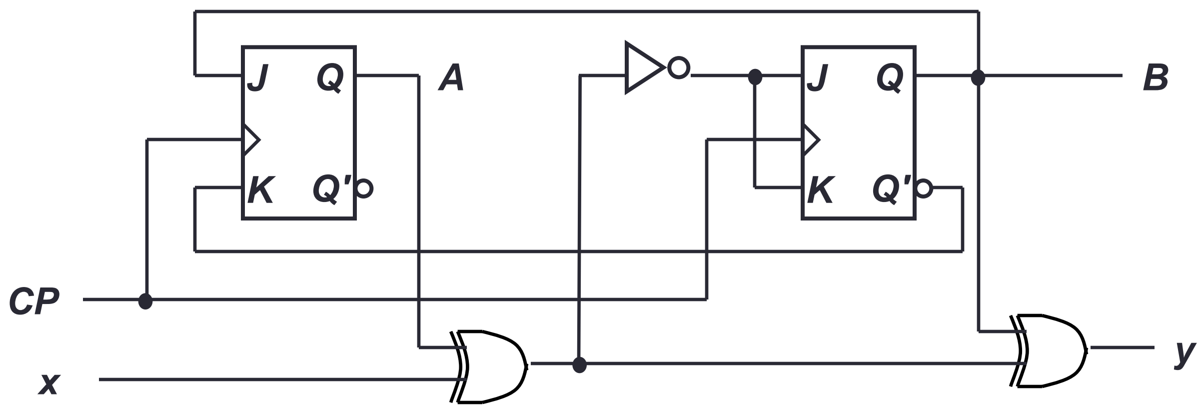


Figure 3

- Flip-flop input functions:

$JA = B$ $JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$

$KA = B'$

6.2 Analysis: Example #3 (2/3)

- Flip-flop input functions:

  $JA = B$ $\quad$ $JB = KB = (A \oplus x)' = A \cdot x + A' \cdot x'$

  $KA = B'$

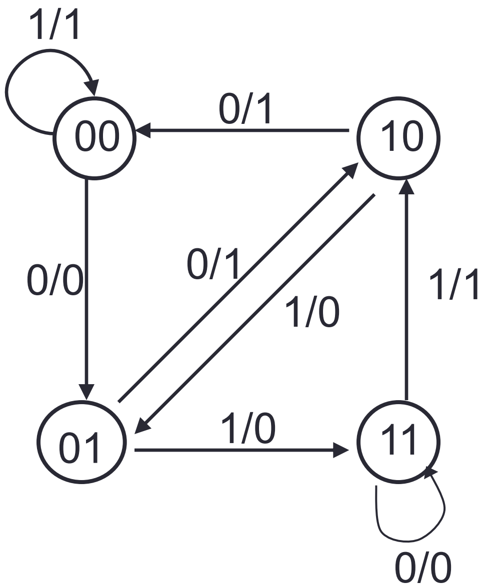
- State table:

Present state		Input	Next state		Output	Flip-flop inputs			
A	B	x	A^+	B^+	y	JA	KA	JB	KB
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1

6.2 Analysis: Example #3 (3/3)

- State diagram:

Present state		Input	Next state		Output	Flip-flop inputs			
A	*B*	*x*	A^+	B^+	*y*	*JA*	*KA*	*JB*	*KB*
0	0	0	0	1	0	0	1	1	1
0	0	1	0	0	1	0	1	0	0
0	1	0	1	0	1	1	0	1	1
0	1	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0	0
1	0	1	0	1	0	0	1	1	1
1	1	0	1	1	0	1	0	0	0
1	1	1	1	0	1	1	0	1	1



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