CS1231S Assignment #1

AY2022/23 Semester 1

Deadline: Monday, 12 September 2022, 1:00pm

IMPORTANT: Please read the instructions below

This is a graded assignment worth 10% of your final grade. There are **seven questions** with a total score of 40 marks. Please work on it by yourself, not in a group or in collaboration with anybody. Anyone found committing plagiarism (submitting other's work as your own), or sending your answers to others, or other forms of academic dishonesty will be penalised with a straight zero for the assignment, and possibly an F grade for the module.

You are to submit your assignment to **Canvas > Assignments** before the deadline.

Your answers may be typed or handwritten. Make sure that it is legible (for example, don't use very light pencil or ink if it is handwritten, or font size smaller than 11 if it is typed) or marks may be deducted.

You are to submit **a SINGLE pdf file**, where each page is A4 size. Do not submit files in other formats.

You may test out your submission folder before the deadline, but make sure you remove any test files you have submitted earlier.

Late submission will NOT be accepted. We will set the closing time of the submission folders to slightly later than 1pm to give you a few minutes of grace, but in your mind, you should treat **1pm** as the deadline. If you think you might be too busy on the day of the deadline, please submit earlier. Also, avoid submitting in the last minute; the system may get sluggish due to overload and you will miss the deadline.

Note the following as well:

- Name your pdf file with your **Student Number**. Your student number begins with 'A' (eg: A0234567X). (Do not mix up your student number with your NUSNET-id which begins with 'e'.)
- At the top of the first page of your submission, write your **Name** and **Tutorial Group**..
- To keep the submitted file short, please submit your answers without including the questions.
- As this is an assignment given well ahead of time, we expect you to work on it early. You should submit **polished work**, not answers that are untidy or appear to have been done in a hurry, for example, with scribbling and cancellation all over the places. Marks may be deducted for untidy work.

To combine all pages into a single pdf document for submission, you may find the following scanning apps helpful if you intend to scan your handwritten answers:

* for Android: https://fossbytes.com/best-android-scanner-apps/

* for iphone:
https://www.switchingtomac.com/tutorials/ios-tutorials/the-best-ios-scanner-apps-to-scan-documents-images/

If you need any clarification about this assignment, please do NOT email us or post on telegram, but post on the **Canvas > Discussions> Assignments** forum so that all queries are at one place and everybody can read the answers to the queries.

Note: Do not use any methods that have not been covered in class. When using a theorem or result that has appeared in class (lectures or tutorials), please quote the theorem number/name, the lecture and slide number, or the tutorial number and question number in that tutorial, failing which marks may be deducted. Remember to use numbering and give justification for important steps in your proof, or marks may be deducted.

Question 0. (2 marks)
Check that …

- you have submitted a pdf file with your Student Number as the filename. [1 mark]
- you have written both your name and tutorial group number (eg: T08A) at the top of the first page of your file. [1 mark]

Question 1. (6 marks)

(a) Prove the following statements, using the truth table method for (i) and Theorem 2.1.1 for (ii).

(i) $p \wedge (\sim p \vee q) \equiv p \wedge q$ [1 mark]

(ii) $p \vee (\sim p \wedge q) \equiv p \vee q$ [1 mark]

The above are variants of the absorption laws in Theorem 2.1.1. To distinguish them from the absorption laws, we shall call them the "variant absorption laws" and you may cite them in your work from now on.

(b) In tutorial #1, we assume that the following statement is a tautology:

$$\big((p \to q) \wedge (q \to r)\big) \to (p \to r)$$

Now, prove it using Theorem 2.1.1, and if necessary, the implication law and the variant absorption laws in part (a). If you are using the same law x times on x consecutive steps, you may write "by ⋯ law x times" and this is counted as one step. You should write your answer <u>within 15 steps</u> for full credit. The first 3 steps are done for you and are not counted in your steps. [4 marks]

$\big((p \to q) \wedge (q \to r)\big) \to (p \to r)$

$\equiv \sim\big((\sim p \vee q) \wedge (\sim q \vee r)\big) \vee (\sim p \vee r)$ (Implication law four times)

$\equiv \big((\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim r)\big) \vee (\sim p \vee r)$ (De Morgan's law three times)

$\equiv \big((p \wedge \sim q) \vee (q \wedge \sim r)\big) \vee (\sim p \vee r)$ (Double negative law twice)

Question 2. (8 marks)

Mastermind© is a code-breaking game for two players. The codemaker selects four pegs from pegs of six colours and places them on the board, hidden from the codebreaker's view. The codebreaker then attempts to guess the pattern by placing her pegs on a row on the board. Once placed, the codemaker provides feedback by placing from zero to four hint pegs. A black hint peg is placed for each code peg from the guess that is correct in both colour and position; this is called a Sink. A white hint peg indicates the existence of a correct colour peg but placed in the wrong position; this is called a Hit.

In this question, we will adopt the version where the codemaker does not put pegs of duplicate colours. We will use A, B, C, D, E and F to represent the six colours, and S for a sink and H for a hit.

(a) Consider this statement: It is possible for the codemaker to place three Sink hint pegs and one Hit hint peg for a codebreaker's guess.

Is the statement true or false? If true, prove it; if false, disprove it. [2 marks]

(b) Suppose you are the codebreaker and you make the following sequence of guesses and receive the codemaker's hints at each guess:

A B C D	1S 2H
A B E F	0S 3H
A C B F	0S 2H
B E C F	2S 0H

(i) After the third guess, you are able to tell the colours of the 4 pegs in the codemaker's code. What are the colours? Explain how you deduce it. [3 marks]

(ii) After the fourth guess, you are able to break the code. What is the code? Explain how you deduce it. [3 marks]

Question 3. (4 marks)

One definition of prime is given in Lecture #4:

An integer n is prime iff

$$(n > 1) \wedge \forall r, s \in \mathbb{Z}^+ \big(n = rs \rightarrow (r = 1 \wedge s = n) \vee (r = n \wedge s = 1)\big).$$

(a) To prove that 3 is prime, Aiken has decided to use the above definition and proof by division into cases and his partial proof is given below. Complete his proof without changing those parts that are given. You may add more steps if necessary. [2 marks]

1. As 3 > 1, it suffices to show that $n = 3$ is prime by showing

$$\forall r, s \in \mathbb{Z}^+ \big(3 = rs \rightarrow (r = 1 \wedge s = 3) \vee (r = 3 \wedge s = 1)\big).$$

2. Suppose $3 = rs$.

 2.1 Case 1: $r = 1$
 2.1.1 Then …

 2.2 Case 2: $r = 2$
 2.2.1 Then …

 2.3 Case 3: $r = 3$
 2.3.1 Then …

 2.4 Case 4: $r > 3$
 2.4.1 Then …

3. In all cases, $\forall r, s \in \mathbb{Z}^+ \big(3 = rs \rightarrow (r = 1 \wedge s = 3) \vee (r = 3 \wedge s = 1)\big)$ is true.

4. Therefore, 3 is a prime.

(b) Prove this statement: (p is prime and $3|p$) iff $p = 3$. [2 marks]

Question 4. (5 marks)

For each $k \in \mathbb{Z}^+$, let $A_k = \{n \in \mathbb{Z}_{\geqslant 2} : k = mn \text{ for some } m \in \mathbb{Z}_{\geqslant 2}\}$. Also, let $B = \{0,1,4,9\}$, $C = \{1,2,3,4,5\}$ and $E = \{\, n \in \mathbb{Z} : n > 2 \wedge n = 2k \text{ for some integer } k\}$.

(a) What is the following? [1 mark]

$$\bigcap_{x \in E} A_x$$

(b) Is the following true or false? If it is false, give a counter-example. [2 marks]

$$\forall x \in B\ \exists y \in C\ (x = y^2)$$

(c) Is the following true or false? If it is false, give a counter-example. [2 marks]

$$\forall x \in A_{21}\ \exists y \in B\ \exists z \in C\ (y < x \wedge x < z)$$

Question 5. (6 marks)

Refer to the Tarski's world introduced in lecture, with the possibility of many other colours besides blue, gray and black. Note the following:

- You may use <u>only</u> the predicates given in Lecture #3 Slide 64 and additional predicates for the other colours (eg: $White(x)$, $Yellow(x)$, etc.) but not any other predicates of your own.
- You may omit the domain of discourse in your answer, assuming that the domain is the set of all objects in the Tarski's world.
- Use x, y, z for your variable names.
- Use the single-line conditional symbol $\rightarrow$ instead of the double-line symbol $\Rightarrow$.

Write a quantified statement for each of the following.

(a) There is a circle that is either red or white. For this question, you may use the inclusive-or instead of exclusive-or. [1 mark]

(b) All squares are blue or green. For this question, you may use the inclusive-or instead of exclusive-or. [1 mark]

(c) For every square, there is a triangle with matching colour. [2 marks]

(d) All blue squares are above all triangles. For this question, do not use the conjunction or disjunction connectives. [2 marks]

Question 6. (9 marks)

(a) Let $\mathcal{P}(S)$ denote the power set of set S. Prove: $A \subseteq B \Leftrightarrow \mathcal{P}(A) \subseteq \mathcal{P}(B)$. [4 marks]

(b) Is the following statement true or false? If true, prove it; if false, disprove it.

$$(A \cup B = A \cup C) \Rightarrow B = C. \quad [1 \text{ mark}]$$

(c) Is the following statement true or false? If true, prove it; if false, disprove it.

$$(A \cap B = A \cap C) \Rightarrow B = C. \quad [1 \text{ mark}]$$

(d) Is the following statement true or false? If true, prove it; if false, disprove it.

$$(A \cup B = A \cup C) \wedge (A \cap B = A \cap C) \Rightarrow B = C. \quad [3 \text{ marks}]$$