CS2040S – Data Structures and Algorithms

Lecture 11 – Census Problem

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Outline

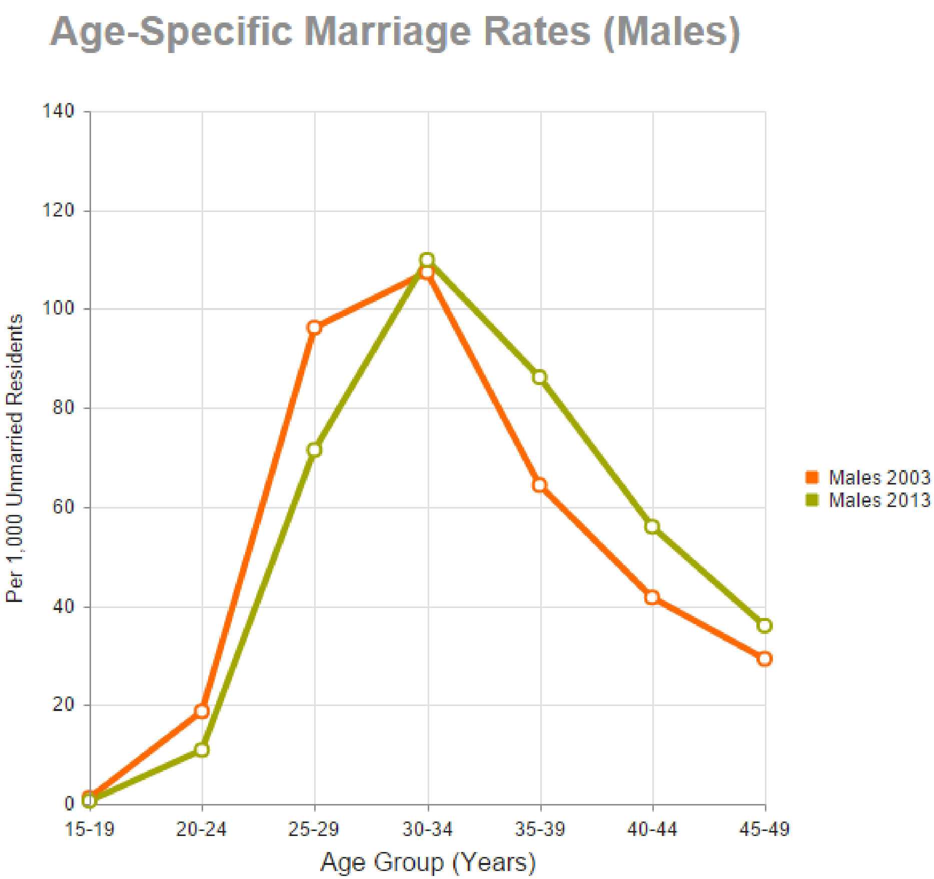
Motivation: Census Problem

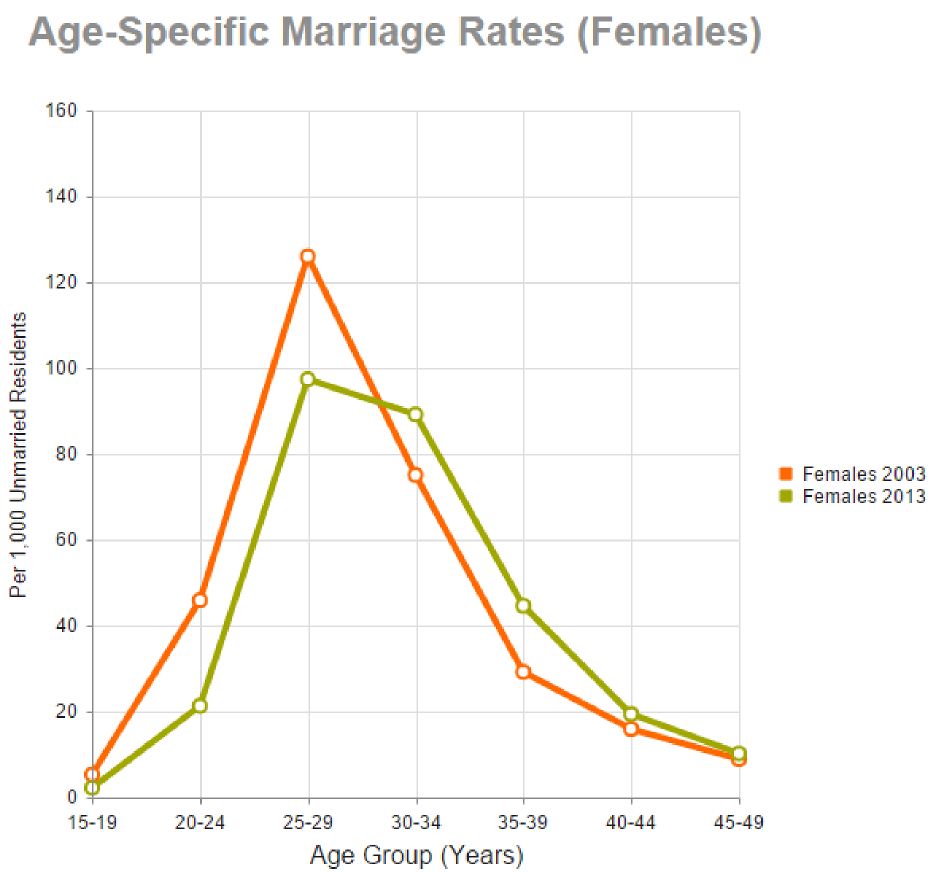
- Abstract Data Type (ADT) Ordered Map
- Solving Census Problem with CS2040S 1st Half Knowledge
- The “performance issue”

Binary Search Tree (BST)

- Heavy usage of VisuAlgo Binary Search Tree Visualization
- Simple analysis of BST operations
- Java Implementation

Census is Important!





Source: http://www.singstat.gov.sg

Sun Tzu's Art of War

Chapter 1 "The Calculations"

知彼知己百战不殆

zhī bǐ zhī jǐ bǎi zhàn bù dài

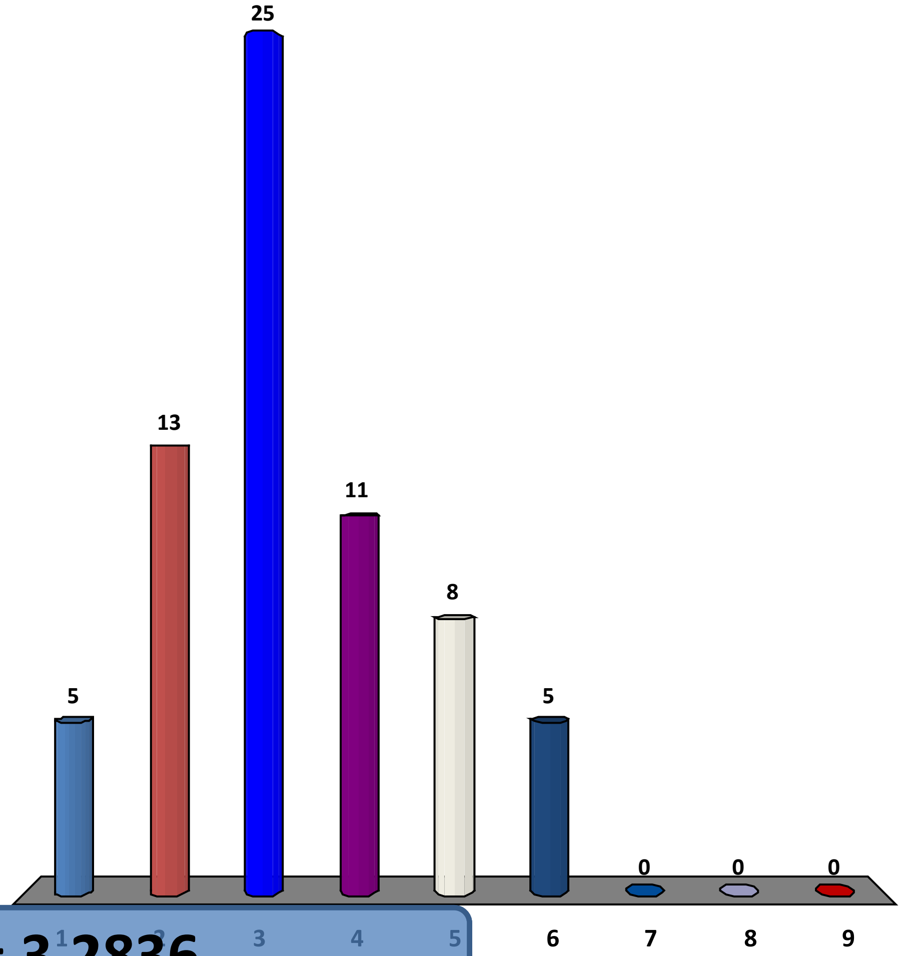
(If you know your enemies and know yourself, you will not be imperiled in a hundred battles)

‘[‘ (or ‘]’) means that endpoint is included (closed)

Your Age (2016 data)

1. [24 … ∞)
2. [23 … 24)
3. [22 … 23)
4. [21 … 22)
5. [20 … 21)
6. [19 … 20)
7. [18 … 19)
8. [17 … 18)
9. [0 … 17)

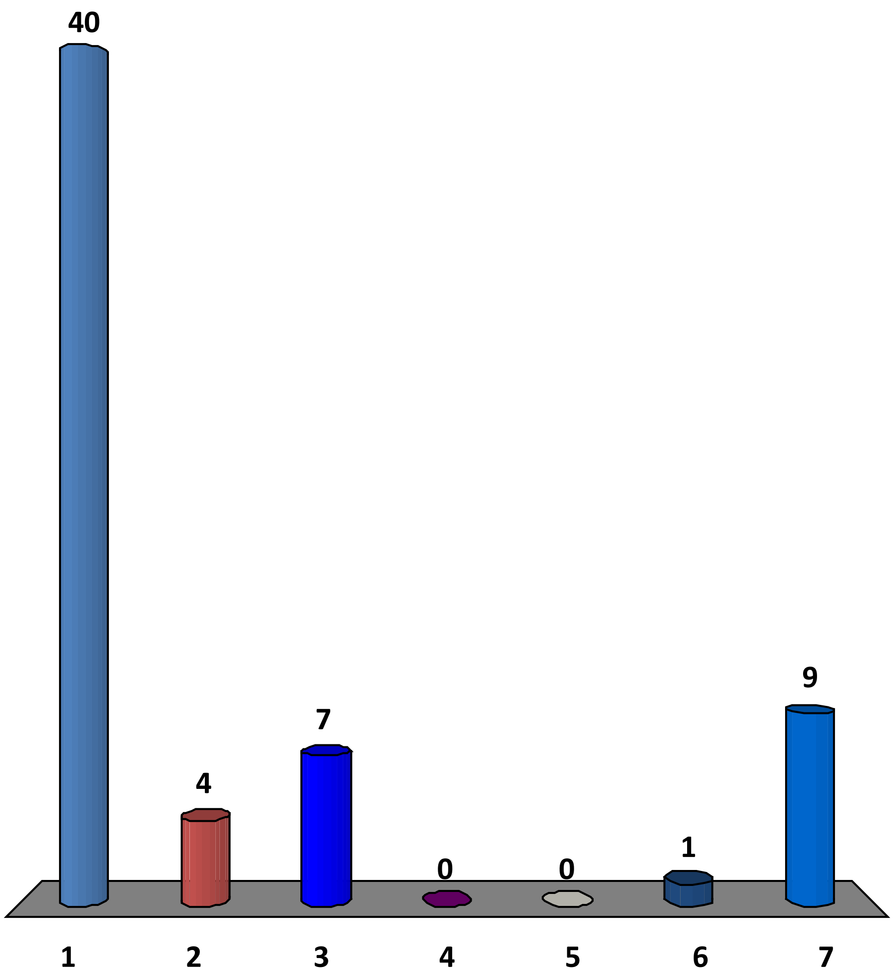
‘(‘ (or ‘)‘) means that endpoint is **not** included (open)



Mean = 3.2836

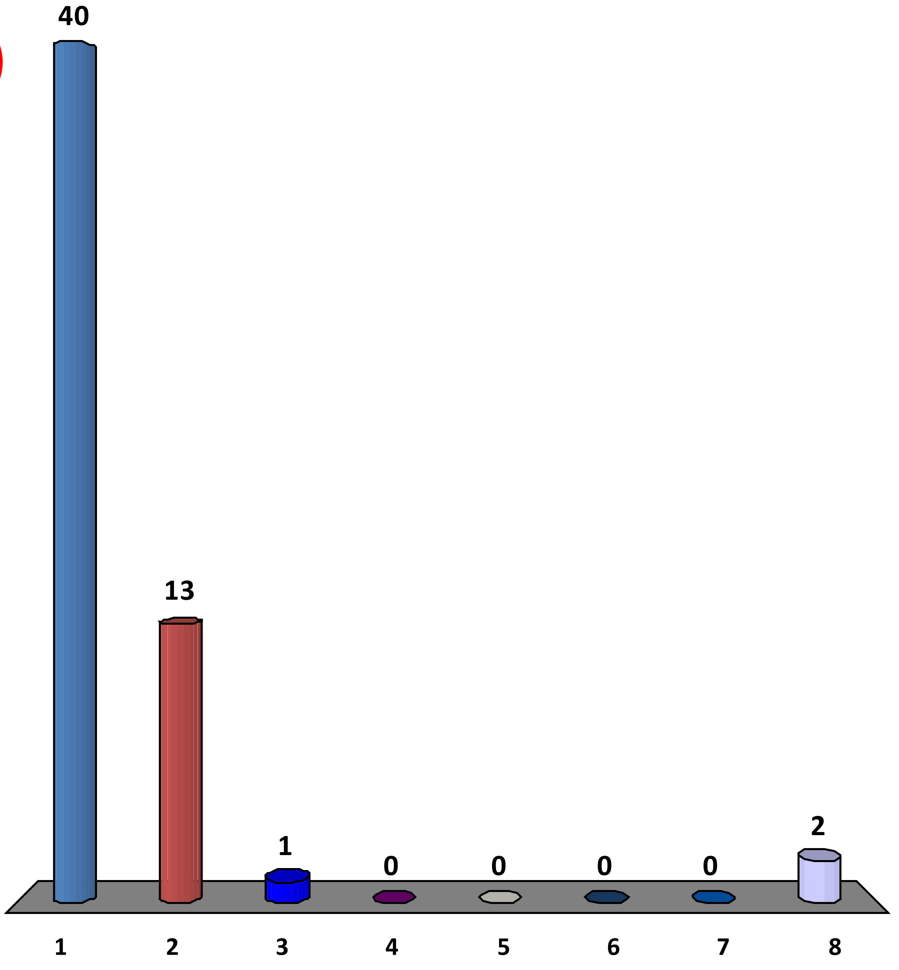
Your Major (2016 data)

1. Computer Science (CS)
2. Business Analytics (BZA)
3. Computer Engineering (CEG/CEC)
4. Comp. Biology (CB)
5. Information System (IS)
6. Science Maths (SCI)
7. None of the above :O



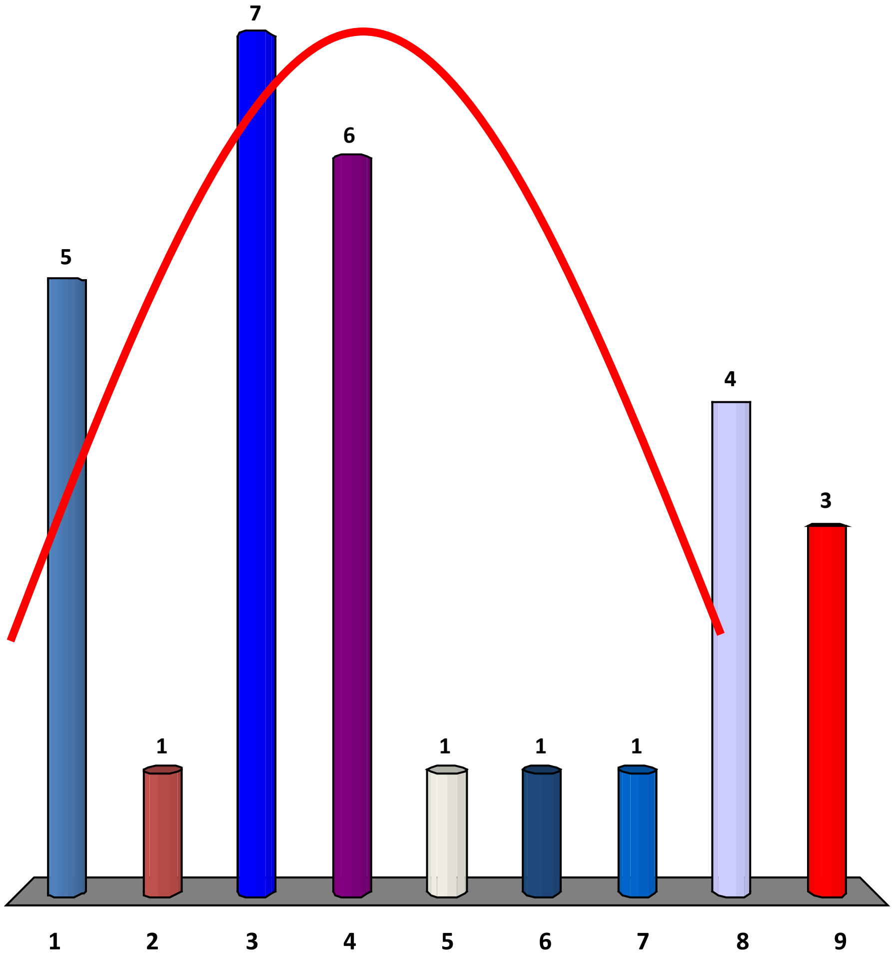
Your Nationality (2016 data)

1. Singapore (should be ≥ 70% according to MOE rules)
2. Chinese
3. Indian
4. Indonesian
5. Vietnamese
6. Malaysian
7. European
8. None of the above



Your CAP (2013 data) <- very old data

1. [4.5 … 5.0]
2. [4.25 … 4.5)
3. [4.0 … 4.25)
4. [3.75 … 4.0)
5. [3.5 … 3.75)
6. [3.25 … 3.5)
7. [3.0 … 3.25)
8. [0.0 … 3.00)
9. I do not want to tell



What Happen After Census?

Data Mining

Statistical Analysis

Some statistical analysis required

Let’s deal with one aspect of our census : **Age**

To simplify this lecture, we assume that students’ age ranges from [0 … 100), all integers, and distinct

Some required operations:

1. Search whether there is a student with a certain age?
2. Insert a new student (insert using his/her age)
3. Determine the youngest and oldest student
4. List down the ages of students in sorted order
5. Find a student slightly older than a certain age!
6. Delete existing student (remove using his/her age)
7. Determine the median age of students
8. How many students are younger than a certain age?

Ordered Map ADT

hashtable

- If we use a Map ADT to store the student data, there are some operations which are not well supported
  - Find a student slightly older than a certain age
  - List down student in sorted order of age
  - …
- This is because there is no notion of ordering in a Map
- Instead we required a more advanced Map called an Ordered Map
  - Items in the Ordered Map are still accessed and manipulated using the key (age attribute in our example)
  - In addition the items are also given an ordering

Ordered Map Implementation – Unsorted Array

Index	0	1	2	3	4	5	6	7	
A	5	7	71	50	23	4	6	15	

No	Operation	Time Complexity
1	Search(age)	O(**N**)
2	Insert(age)	O(**1**)
3	FindOldest()	O(**N**)
4	ListSortedAges() radix sort O(n) because age is bounded (at most 100+)	O(**N** log **N**)
5	NextOlder(age) find and store the smallest age that is larger than this age	O(**N**)
6	Remove(age)	O(**N**)
7	select the item in the middle GetMedian()	O(**N** log **N**)/O(**N**)
8	NumYounger(age)	O(**N** log **N**)/O(**N**)

Use QuickSelect to get median
GetMedian() = QuickSelect(**N**/2)
Expected O(**N**)

sort it then linear search to find age just below age

Ordered Map Implementation – Sorted Array

Index	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	
A	4	5	6	7	15	23	50	71	

No	Operation	Time Complexity
1	Search(age) binary search	O(log **N**)
2	Insert(age) cannot insert to back, must create gap	O(N)
3	FindOldest()	O(**1**)
4	ListSortedAges()	O(**N**)
5	NextOlder(age) binary search and return index + 1 of age	O(log **N**)
6	Remove(age) must left shift to close gap	O(**N**)
7	GetMedian() return middle value	O(**1**)
8	NumYounger(age) binary search and return index - 1 of age	O(log **N**)

With Just 1st Half Knowledge

No	Operation	Unsorted Array	Sorted Array
1	Search(age)	O(**N**)	O(log **N**)
2	Insert(age)	O(**1**)	O(**N**)
3	FindOldest()	O(**N**)	O(**1**)
4	ListSortedAges()	O(**N** log **N**)	O(**N**)
5	NextOlder(age)	O(**N**)	O(log **N**)
6	Remove(age)	O(**N**)	O(**N**)
7	GetMedian()	O(**N** log **N**)/O(**N**)	O(**1**)
8	NumYounger(age)	O(**N** log **N**)/O(**N**)	O(log **N**)

Dynamic data structure operations

If N is large, our queries are slow…

O(N) versus O(log N): A Perspective

N = 8

$\log_2$ **N** = 3

N = 16

$\log_2$ **N** = 4

N = 32

$\log_2$ **N** = 5

Try larger **N**, e.g. **N** = 1 000 000…

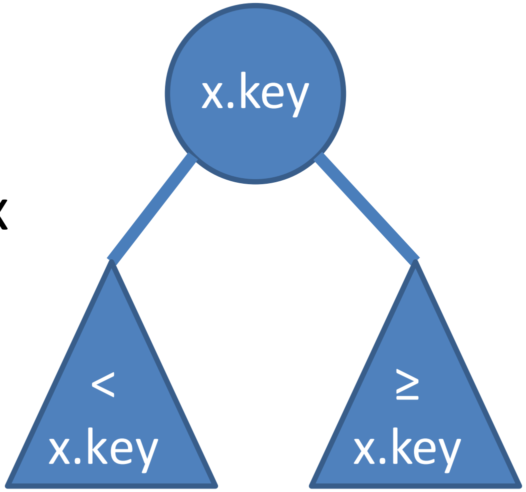
A Versatile, Non-Linear Data Structure

BINARY SEARCH TREE (BST)

Binary Search Tree (BST) Vertex

For every vertex x, we define:

- x.left = the left child of x
- x.right = the right child of x
- x.parent = the parent of x
- x.key (or x.value, x.data) = the value stored at x



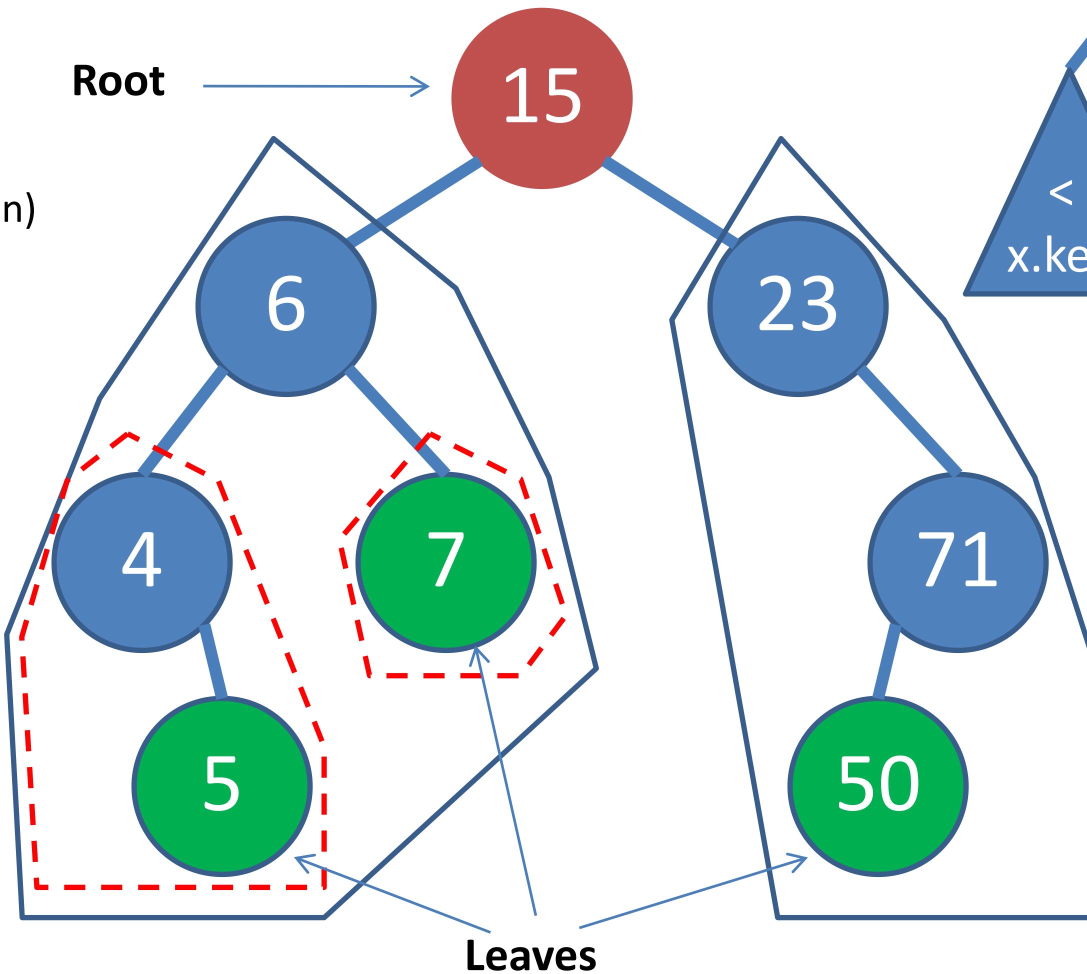
BST Property:

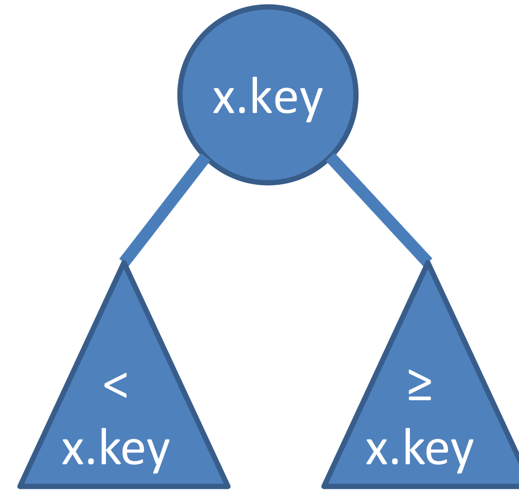
- For every vertex **x** and **y**
  y.key < x.key if **y** is in left subtree of **x** everything in left subtree of x must be smaller than x
  y.key ≥ x.key if **y** is in right subtree of **x** everything in right subtree of x must be bigger than x
- For simplicity, we assume that the keys are unique so that we can change ≥ to >

BST: An Example, Keys = Ages

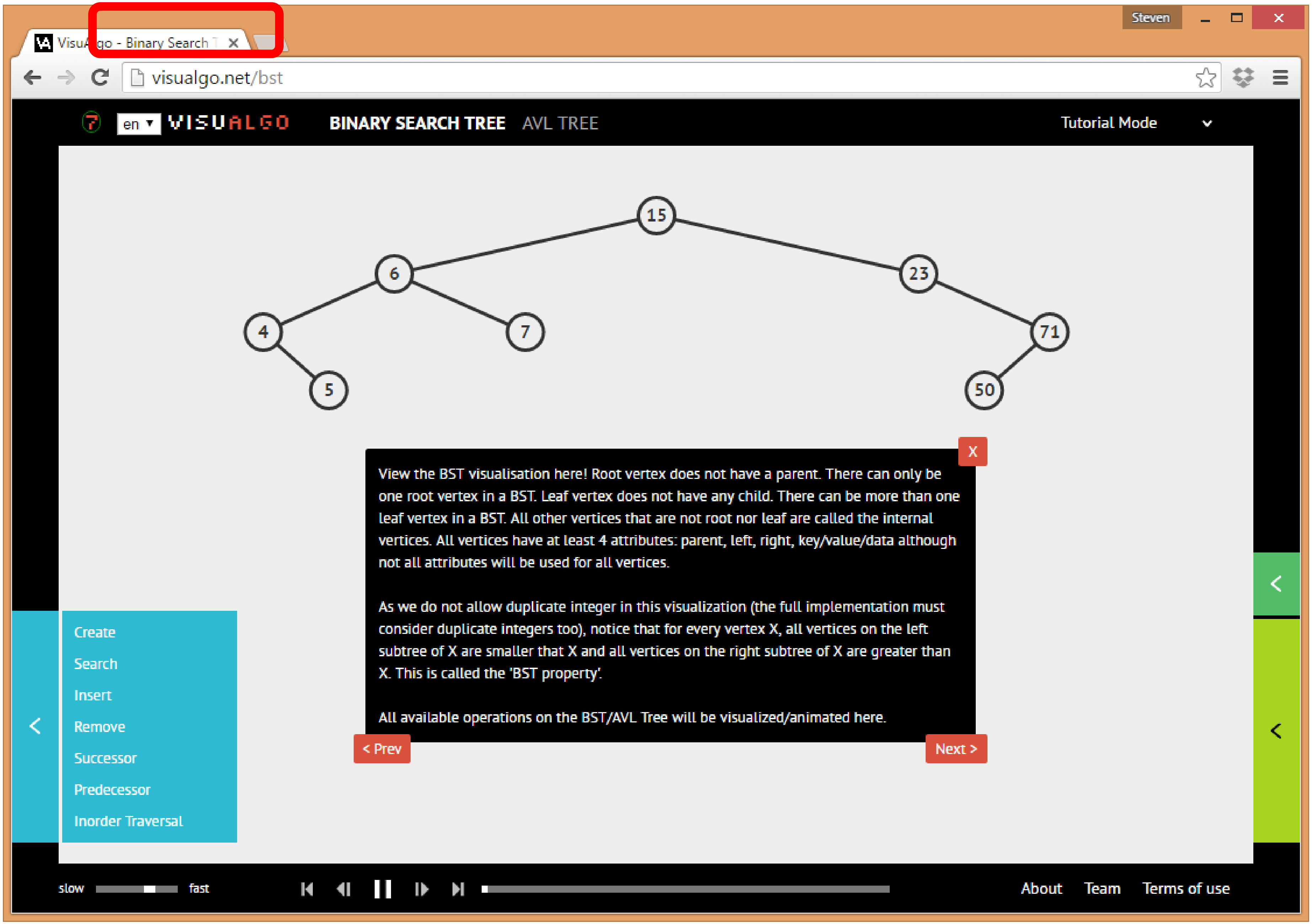
Recursive definition

Root is not an internal node
(VisuAlgo definition)





All other vertices other than 15, 5, 7, and 50 are **Internal vertices**

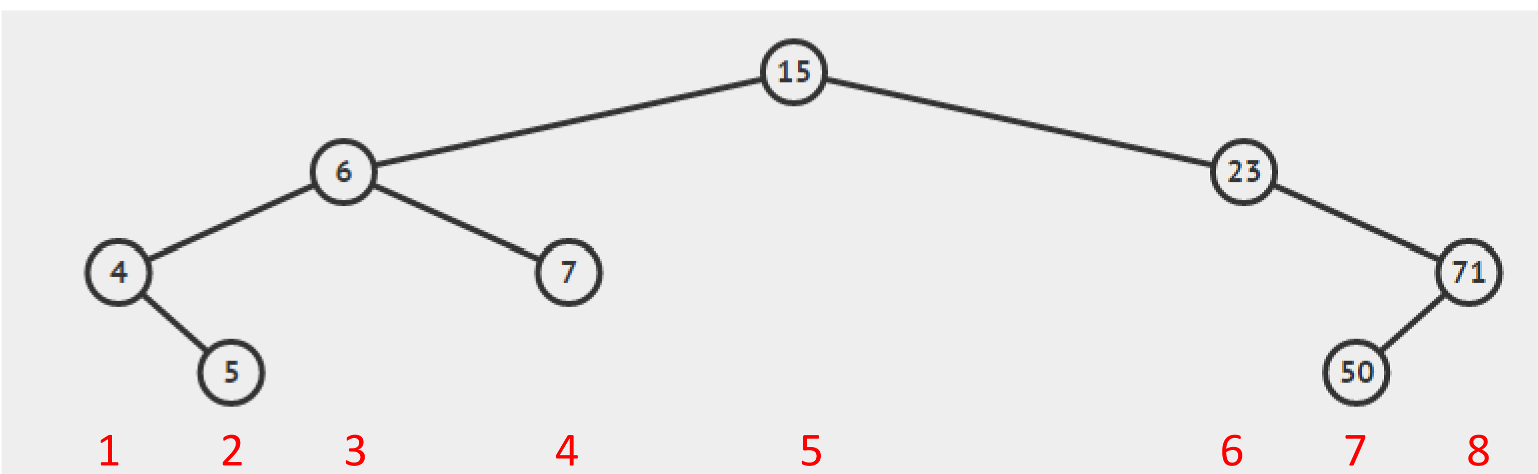


VisuAlgo - Binary Search T
Steven
visualgo.net/bst
en
VISUALGO
BINARY SEARCH TREE
AVL TREE
Tutorial Mode
15
6
23
4
7
71
5
50
X
View the BST visualisation here! Root vertex does not have a parent. There can only be one root vertex in a BST. Leaf vertex does not have any child. There can be more than one leaf vertex in a BST. All other vertices that are not root nor leaf are called the internal vertices. All vertices have at least 4 attributes: parent, left, right, key/value/data although not all attributes will be used for all vertices.
As we do not allow duplicate integer in this visualization (the full implementation must consider duplicate integers too), notice that for every vertex X, all vertices on the left subtree of X are smaller that X and all vertices on the right subtree of X are greater than X. This is called the 'BST property'.
All available operations on the BST/AVL Tree will be visualized/animated here.
< Prev
Next >
Create
Search
Insert
Remove
Successor
Predecessor
Inorder Traversal
slow
fast
About
Team
Terms of use

BST: NEW Select/Rank Operations

These 2 operations are not yet in VisuAlgo BST visualization; for now, here are the concepts:

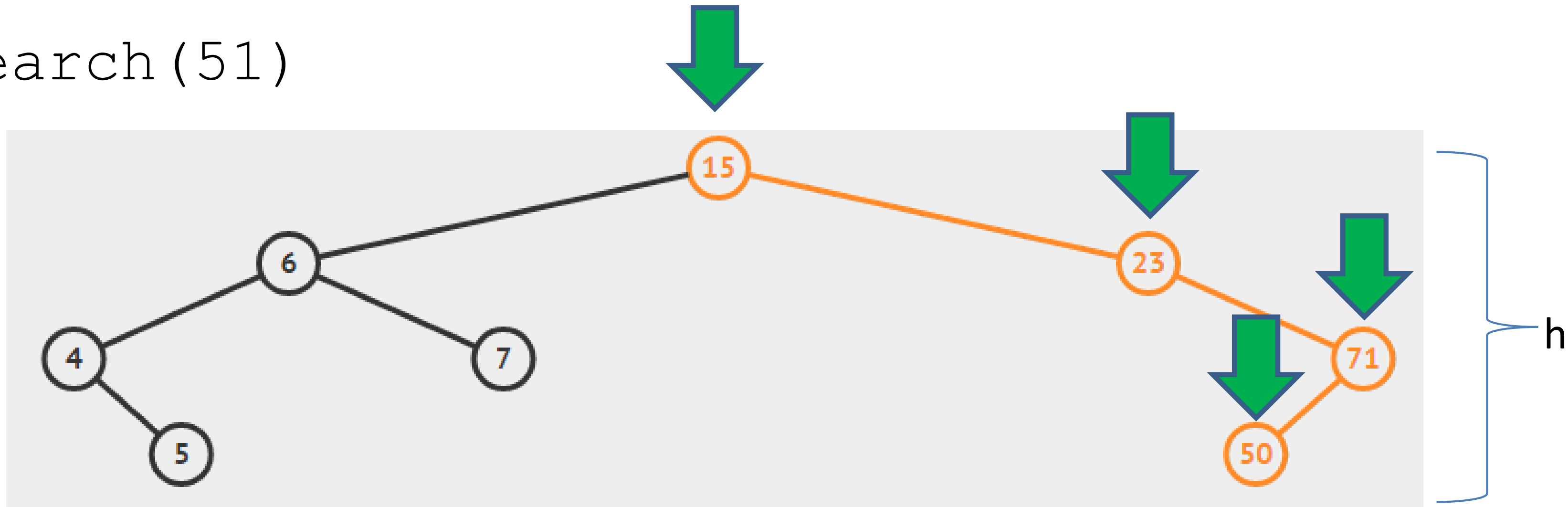
- Select(k) – Return the value v of k-th smallest* element
  - Examples: Select(1) = 4, Select(3) = 6, Select(8) = 71, etc (1-based index)
- Rank(v) – Return the rank* k of element with value v
  - Examples: Rank(4) = 1, Rank(6) = 3, Rank(71) = 8, etc
- Details will be discussed in topic for AVL trees



ANALYSIS OF BST OPERATIONS

BST: Search Analysis

`search(51)`



51 is not found ☹

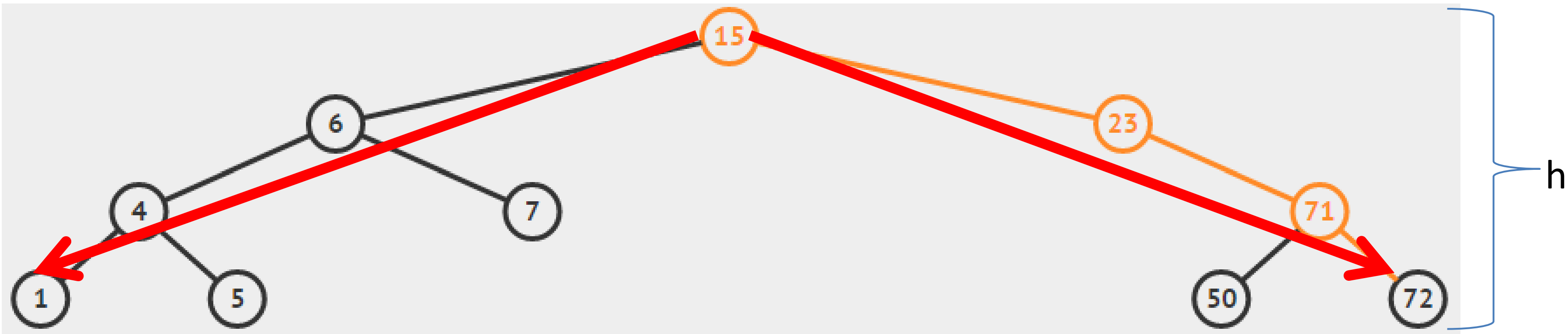
Quick analysis:

`search` runs in **O(h)**

binary search if value is not found (is null) return false

BST: FindMin/FindMax Analysis

`findMin()/findMax()`

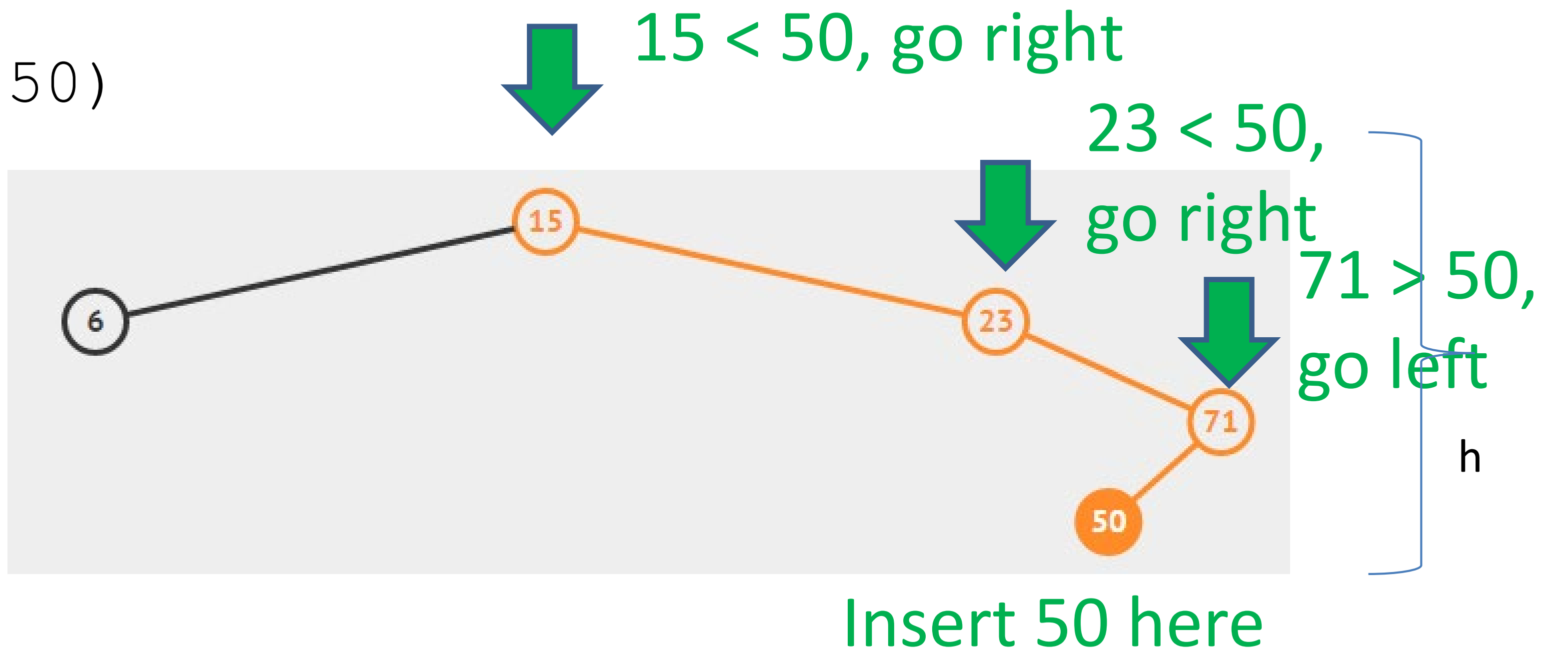


Quick analysis:

`findMin()/findMax` also runs in **O(h)**

BST: Insertion Analysis

`insert(50)`



Quick analysis:

`insert` also runs in **O(h)**

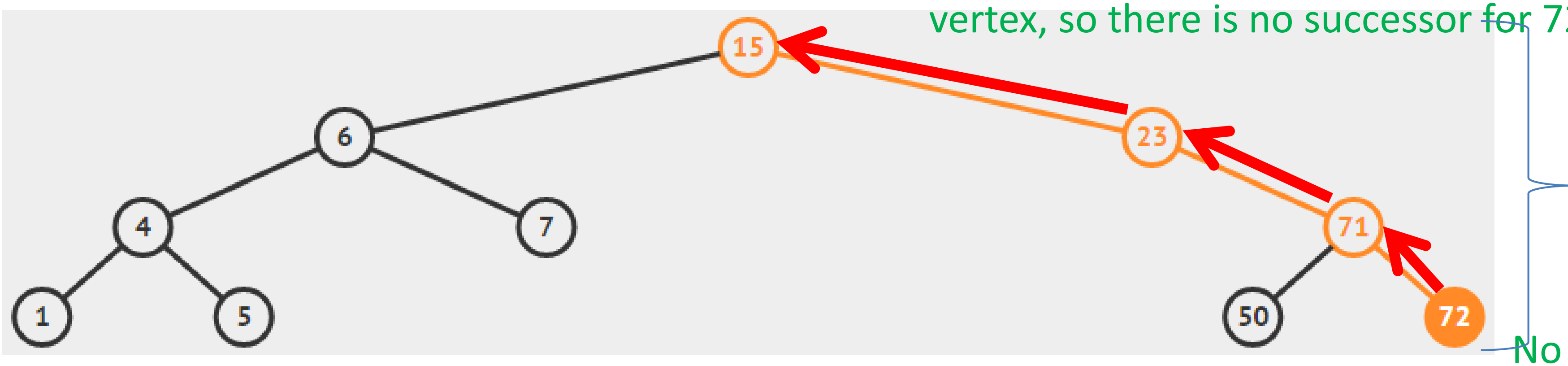
binary search if value is null we insert it there

BST: Successor/Predecessor Analysis

Assumption, we already done an O(h) search(72) before

`successor(72)` assumption that the key exists in the tree

Keep going up until we make a 'right turn', but here we do not find such vertex, so there is no successor for 72



No right child

Quick analysis:

O(h) again, similarly for predecessor

predecessor
1. search for the value
2. the predecessor must be the largest value in the left subtree
-> find max in left sub tree

successor
1. search for the value
2. the successor must be the smallest value in the right subtree
-> find min in right sub tree

BST: Inorder Traversal Analysis

pre order :
process node,
go left
go right

post order :
go left
go right
process node

Using a *new* analysis technique

recursively go left, process the node(ie print the value) then go right
inorder traverses the tree in ascending order

Ask this question:

- How many times is a vertex *visited* during inorder traversal from the start until the end? 3 times if it has left and right children

Answer:

- Three times: from parent and from left + right children (even if one or both of them is/are empty/NULL)
- O(3***N**) = O(**N**)

inorder traversal: O(N)
same for preorder and postorder

Why is successor of x used for deletion of a BST vertex x with 2 children?

Claim: Successor of **x** has at most 1 child!

- Easier to delete and will not violate BST property

Proof:

- Vertex **x** has two children
- Therefore, vertex **x** must have **a right child**
- Successor of **x** must then be the minimum of the right subtree
- A minimum element of a BST has no left child!!
- *So, successor of **x** has at most 1 child!* ☺

BST: Deletion Analysis

Delete a BST vertex **v**, find **v** in O(**h**), then three cases:

- Vertex **v** has no children: if a leaf just delete it by pointing parents left/right ref to null
  - Just remove the corresponding BST vertex **v** → O(**1**)
- Vertex **v** has 1 child (either left or right):
  - Connect **v**.left (or **v**.right) to **v**.parent and vice versa → O(**1**)
  - Then remove v → O(**1**) make the child of the node to be deleted the child of its parents node the delete it -> doesnt violate bst property because it is till smaller than parent node
- Vertex **v** has 2 children: find the node's successor and replace w successor then delete the original pos of successor
  - Find **x** = successor(**v**) → O(**h**)
  - Replace **v**.key with **x**.key → O(**1**) **successor cannot have 2 children as it is the smallest node in the right subtree of the root, at most it can have a right child which is the same as vertex with 1 child case**
  - Then delete **x** in **v**.right (otherwise we have duplicate) → O(**h**)

Running time: O(**h**)

BST: Select/Rank Analysis

We have not explored the operations in detail yet

This will be discussed in more details in the next lecture

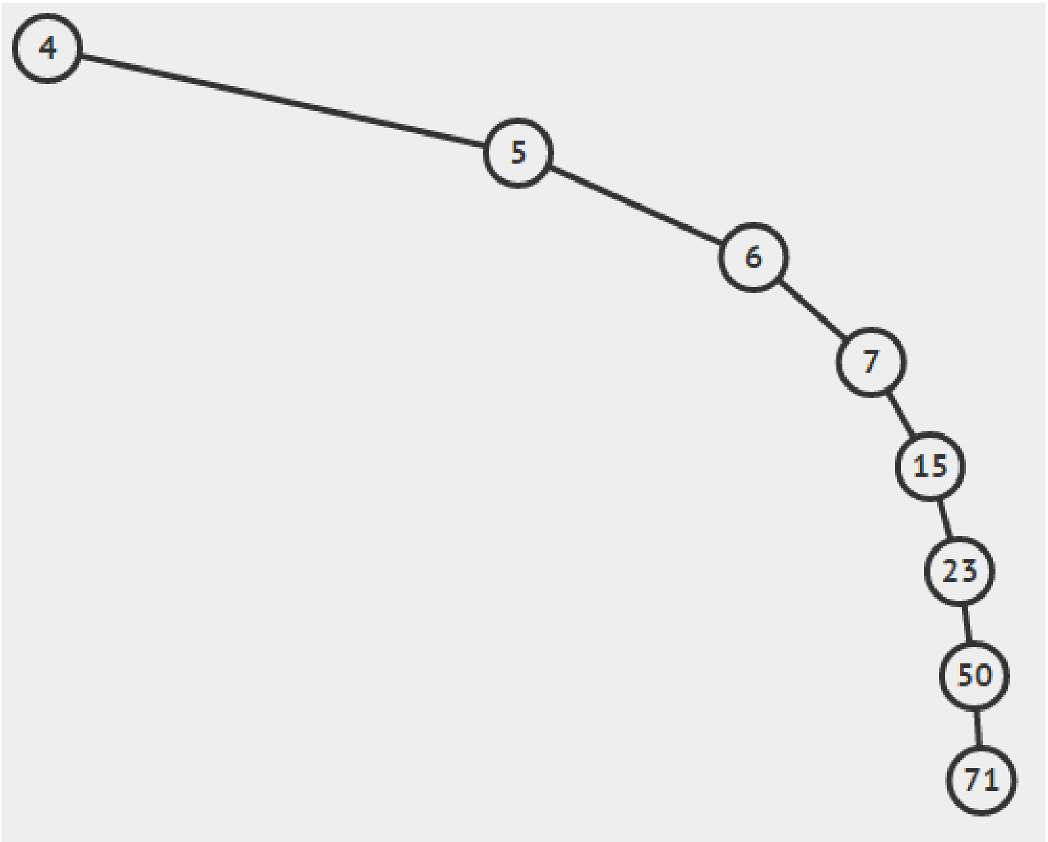
Now, after we learn BST…

No	Ordered Map Operations	Unsorted Array	Sorted Array	BST
1	Search(age)	O(**N**)	O(log **N**)	O(h)
2	Insert(age)	O(**1**)	O(**N**)	O(h)
3	FindOldest()	O(**N**)	O(**1**)	O(h)
4	ListSortedAges()	O(**N** log **N**)	O(**N**)	in order traversal O(**N**)
5	NextOlder(age)	O(**N**)	O(log **N**)	successor O(h)
6	Remove(age)	O(**N**)	O(**N**)	O(h)
7	GetMedian()	O(**N** log **N**)/O(**N**)	O(**1**)	select ?
8	NumYounger(age)	O(**N** log **N**)/O(**N**)	O(log **N**)	rank ?

It is all now depends on **'h'**… → next topic ☺

Worst case height of a BST

h = O(**N**)... ☹



Can you spot one more worst case scenario using the same set of numbers?

Can we do better?
YES, h = O(log **N**) → next topic ☺