

# Recitation - 08

CS2040S Recitation Team

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## Problem<sup>1</sup>

You have 1000 bottles of soda, and exactly one is poisoned. You have 10 test strips which can be used to detect poison. A single drop of poison will turn the test strip positive permanently. You can put any number of drops on a test strip at once and you can reuse a test strip as many times as you'd like (as long as the results are negative). However, you can only run tests once per day and it takes seven days to return a result. How would you figure out the poisoned bottle in as few days as possible?

## Solution

### Naive Approach - 28 Days

A simple approach is to divide the bottles across the 10 test strips, first in groups of 100. Then, we wait seven days. When the results come back, we look for a positive result across the test strips. We select the bottles associated with the positive strip, toss / ignore all the other bottles, and repeat the process. We perform this operation until there is only one bottle left in the test set.

### Optimization I - 10 Days

If we divide the bottles up into 10 groups (with bottles 0-99 going to strip 0, bottles 100-199 going to strip 1, bottles 200-299 going to strip 2, and so on), then day 7 will reveal the first digit of the bottle number. A positive result on strip  $i$  at day 7 shows that the first digit (100's digit) of the bottle number is  $i$ . Dividing the bottles in a different way can reveal the second or third digit. We just need to run these tests on different days so that we don't confuse the results.

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<sup>1</sup>Puzzle Credits - Book:Cracking the Coding Interview

	Day 0 $\rightarrow$ 7	Day 1 $\rightarrow$ 8	Day 2 $\rightarrow$ 9
Strip 0	0XX	X0X	XX0
Strip 1	1XX	X1X	XX1
Strip 2	2XX	X2X	XX2
Strip 3	3XX	X3X	XX3
Strip 4	4XX	X4X	XX4
Strip 5	5XX	X5X	XX5
Strip 6	6XX	X6X	XX6
Strip 7	7XX	X7X	XX7
Strip 8	8XX	X8X	XX8
Strip 9	9XX	X9X	XX9

This idea works for most of the bottle numbers. It fails for the following edge case. For instance; in day 7 and in day 8, strip 2 and strip 3 becomes positive respectively. But in day 9, there is no new positive results on the strips. It means that the final digit is either 2 or 3. For these cases, we have to run one more additional test case to find the digit. (Challenge for you - how can we do this additional test?)

## Optimization II - 7 Days

This can be done in 7 days. Think about the idea. (Hint - binary representation of a bottle number can be expressed in 10 bits. ( $2^{10} > 1000$ ))