

1. Which of the following are pure functions?

(a) Problem #A

```
1  int f(int i) {
2      if (i < 0) {
3          throw new IllegalArgumentException();
4      } else {
5          return i + 1;
6      }
7  }
```

(b) Problem #B

```
1  int g(int i) {
2      System.out.println(i);
3      return i + 1;
4  }
```

(c) Problem #C

```
1  int h(int i) {
2      return new Random().nextInt() + i;
3  }
```

(d) Problem #A

```
1  int k(int i) {
2      return Math.abs(i);
3  }
```

2. Consider the following lambda expression:

$x \rightarrow y \rightarrow z \rightarrow f(x, y, z)$

where x , y , and z are of some type T and f returns a value of type R .

(a) What kind of lambda expression is this?

(b) Suppose that:

- T and R are of type `Integer`
- $f(x, y, z)$ is given by $x + y + z$
- The above lambda expression implements the `Immutator` functional interface

Initialize the appropriate lambda expression and assign it to a variable `trisum`. Given three inputs x , y , and z , show how you can evaluate the lambda expression with x , y , and z to obtain $f(x, y, z)$.

3. The following depicts a classic tail-recursive implementation for finding the sum of values of n (given by $\sum_{i=0}^n i$) for $n \geq 0$.

```
1  static long sum(long n, long result) {
2      if (n == 0) {
3          return result;
4      } else {
5          return sum(n - 1, n + result);
6      }
7  }
```

In particular, the implementation above is considered **tail-recursive** because the recursive function is at the tail end of the method (*i.e., no computation is done after the recursive call returns*). As an example, `sum(100, 0)` gives 5050.

Although the tail-recursive implementation can be simply rewritten in an iterative form using loops, we desire to capture the original intent of the tail-recursive implementation using delayed evaluation via the `Producer` functional interface.

We present each recursive computation as a `Compute<T>` object. A `Compute<T>` object can be either:

- **Recursive Case:** Represented by a `Recursive<T>` object, that can be re-cursed, or
- **Base Case:** Represented by a `Base<T>` object, that can be evaluated to a value of type `T`.

As such, we can rewrite the `sum` method as:

```

1  static Compute<Long> sum(long n, long s) {
2      if (n == 0) {
3          return new Base<>(() -> s);
4      } else {
5          return new Recursive<>(() -> sum(n - 1, n + s));
6      }
7  }
```

Then we can evaluate the sum of n terms via the `summer` method below:

```

1  static long summer(long n) {
2      Compute<Long> result = sum(n, 0);
3
4      while (result.isRecursive()) {
5          result = result.recurse();
6      }
7
8      return result.evaluate();
9  }
```

- Complete the program by writing the `Compute`, `Base`, and `Recursive` classes.
- By making use of a suitable client class `Main`, show how the "tail-recursive" implementation is invoked.
- Redefine the `Main` class so that it now computes the factorial of n recursively.