# Graph

- Introduction
- Graph API
- Elementary Graph Operations
  - **DFS: Depth first search**
  - BFS: Breadth first search
  - CC: Connected Components

Major references:

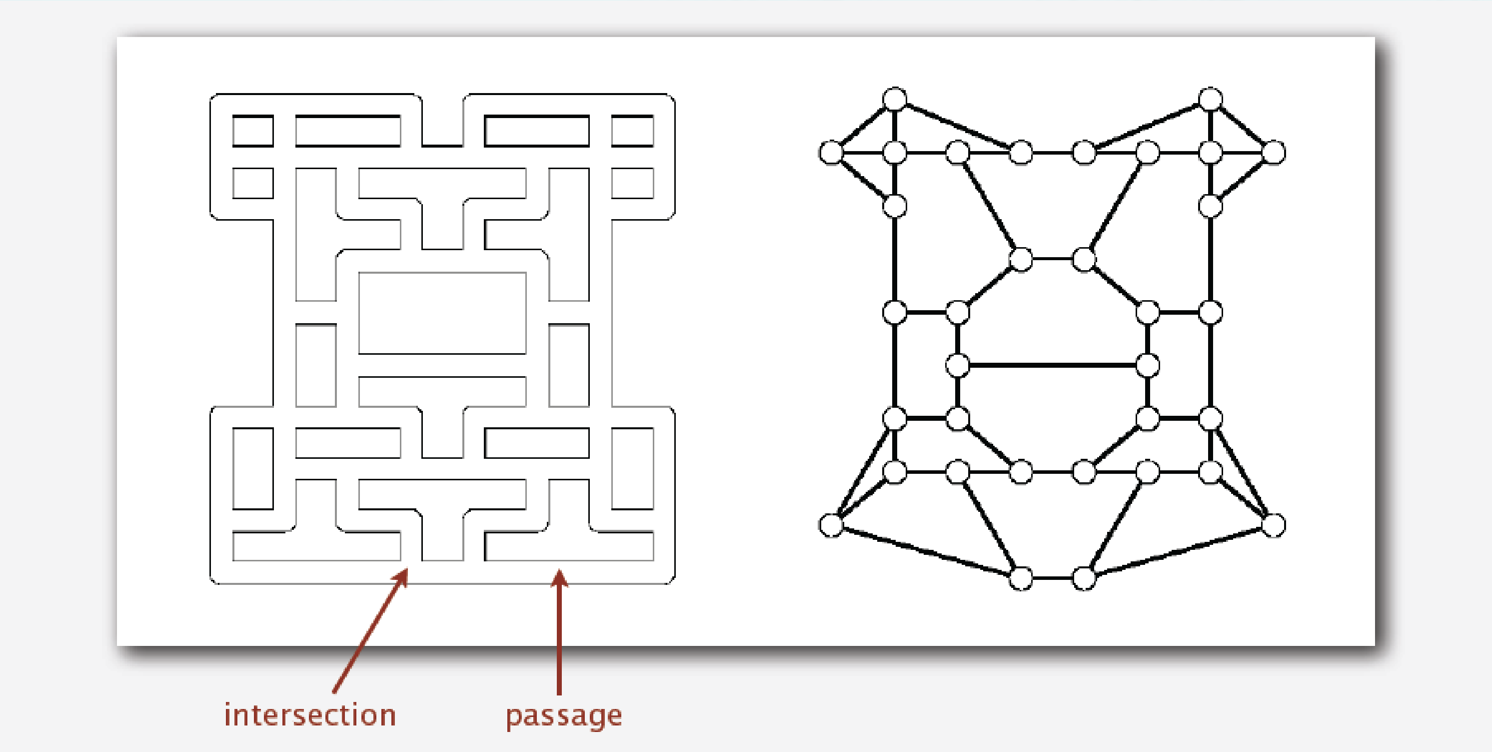
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

## DFS: Depth-First Search

**Algorithm:**

- Vertex = intersection
- Edge = passage

pacman

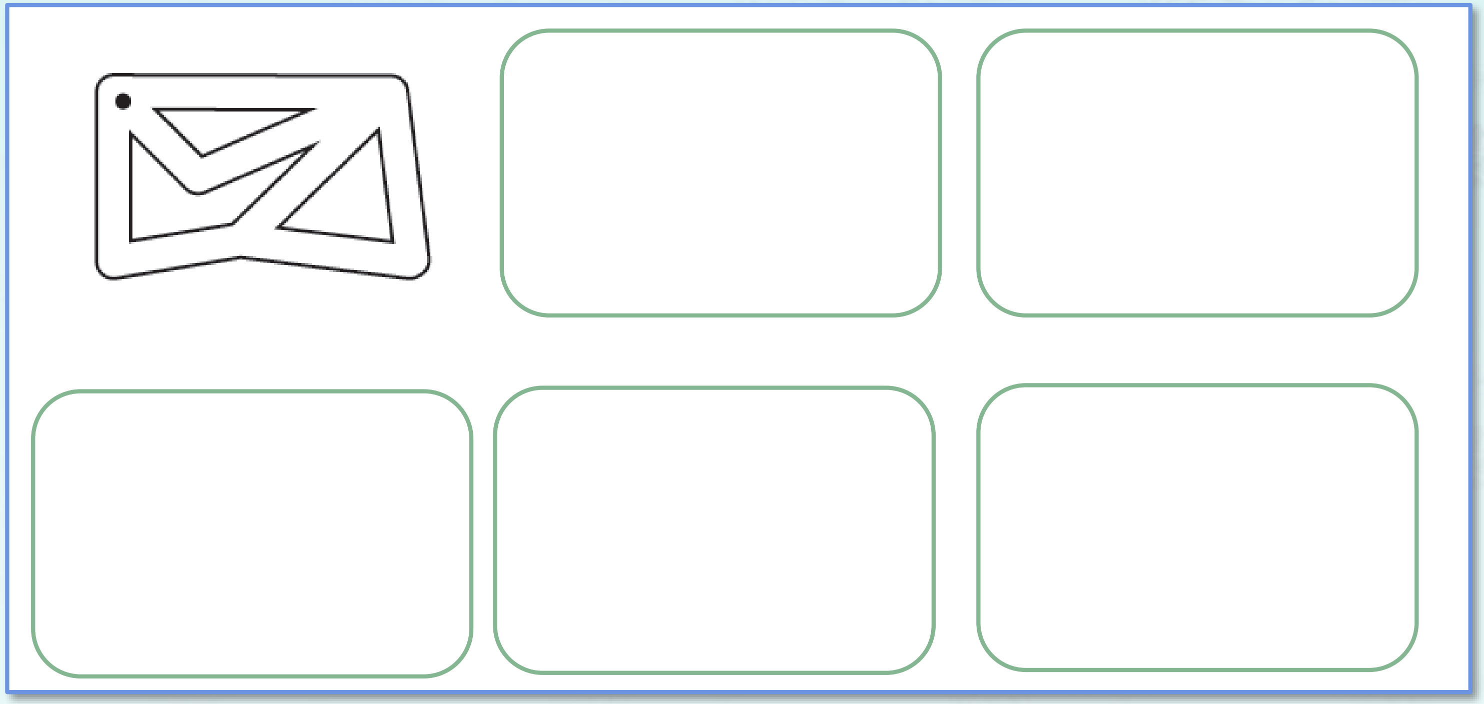



**Maze Goal:** Explore every intersection in the maze.

# DFS: Depth-First Search

**Maze graph:**

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options



**Maze Goal:** Explore every intersection in the maze.

**Good Visualization:** https://www.cs.usfca.edu/~galles/visualization/DFS.html

# DFS: Depth-First Search

**Maze graph:**

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- Mark each visited intersection and each visited passage.
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Theseus, a hero of Greek mythology, is best known for slaying a monster called the Minotaur. When Theseus entered the Labyrinth where the Minotaur lived, he took a ball of yarn to unwind and mark his route. Once he found the Minotaur and killed it, Theseus used the string to find his way out of the maze.

Read more: http://www.mythencyclopedia.com/Sp-Tl/Theseus.html#ixzz30wFO3ofe

**Maze Goal:** Explore every intersection in the maze.

## DFS: Depth-First Search

**Maze graph:**

- Unroll a ball of string behind you.
- Mark each visited intersection and each visited passage.
- Retrace steps when no unvisited options

Shannon and his famous electromechanical mouse *Theseus* (named after Theseus from Greek mythology) which he tried to have solve the maze in one of the first experiments in artificial intelligence.

**The Las Vegas connection:** Shannon and his wife Betty also used to go on weekends to Las Vegas with MIT mathematician Ed Thorp, and made very successful forays in blackjack using game theory.

**Maze Goal:** Explore every intersection in the maze.

# DFS: Depth-First Search

**Design pattern:** Decouple graph data type
**Idea:** Mimic maze exploration

**DFS (to visit a vertex v)**

- **Mark v as visited.**
- **Recursively visit all unmarked vertices w adjacent to v.**

**Typical applications:**
- Find all vertices connected to a given source vertex.
- Find a path between two vertices.

**Challenge:**
- How to implement?

## DFS: Depth-First Search

**Goal:** Systematically search through a graph from graph processing

- **Create a graph object**
- **Pass the graph to a graph processing routine**
- **Query the graph-processing routine**

## DFS: Depth-First Search

**Goal:** Systematically search through a graph from graph processing

- **Create a graph object**
- **Pass the graph to a graph processing routine**
- **Query the graph-processing routine**

| | | |
|---|---|---|
| | `public class Paths` | |
| | `Paths(Graph G, int s)` | *find paths in G from source s* |
| `boolean` | `hasPathTo(int v)` | *is there a path from s to v?* |
| `Iterable<Integer>` | `pathTo(int v)` | *path from s to v; null if no such path* |

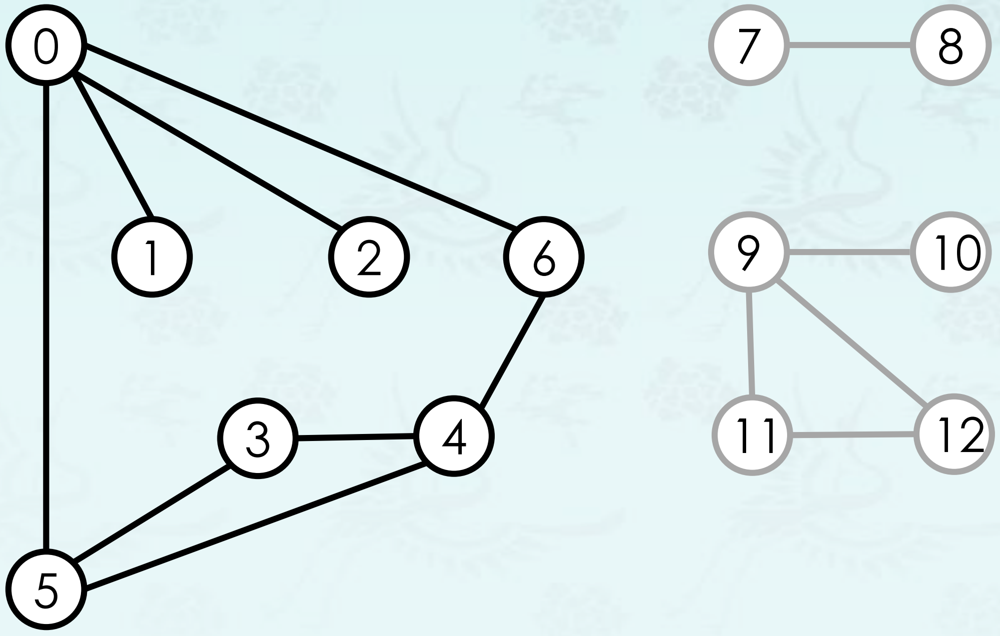
```
Paths paths = new Paths(G, s);
for (int v = 0; v < G.V(); v++)
    if (paths.hasPathTo(v))
        StdOut.println(v);
```

print all vertices connected to s

## Graph – Coding

For each edge(v, w) in the list

- Insert front each vertex both (adj[v] ,  w) and (adj[w], v)
  addEdgeFromTo(g, v, w);        // add an edge from v to w.




Graph g:

V-E lists →

```
graph3.txt
13   ← V
13   ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

**Challenge:** build adjacency lists?

# Graph Coding – graph.cpp

```
struct Graph {
  int V;                 // number of vertices in the graph
  int E;                 // number of edges in the graph
  gnode adj;             // an array of adjacency lists (or gnode pointers)

  Graph(int v = 0) { // constructs a graph with v vertices
    V = v;
    E = 0;

    // initialize each adjacency list as an empty list;
    for (int  i = 0; i < V; i++) {
        g->adj[i].next = nullptr;
        g->adj[i].item = i;
    }
  }
  ~Graph() {}
};
using graph = Graph *;
```

set each adj list nullptr

unused; but may store the size of degree.

## Graph Coding

```
// add an edge to an undirected graph

void addEdgeFromTo(graph g, int v, int w) {
  // add an edge from v to w.
  // A new vertex is added to the adjacency list of v.
  // The vertex is added at the beginning

  gnode node = new Gnode(w);
  g->adj[v].next = node;
  g->E++;
}

// add an edge to an undirected graph
void addEdge(graph g, int v, int w) {
  addEdgeFromTo(g, v, w); // add an edge from v to w.
  addEdgeFromTo(g, w, v); // if graph is undirected, add both
}
```

**With a bug**

instantiate a node w insert it at **the front of** adjacency list[v]

add an edge for undirected graph

## Graph – Build Adjacency list

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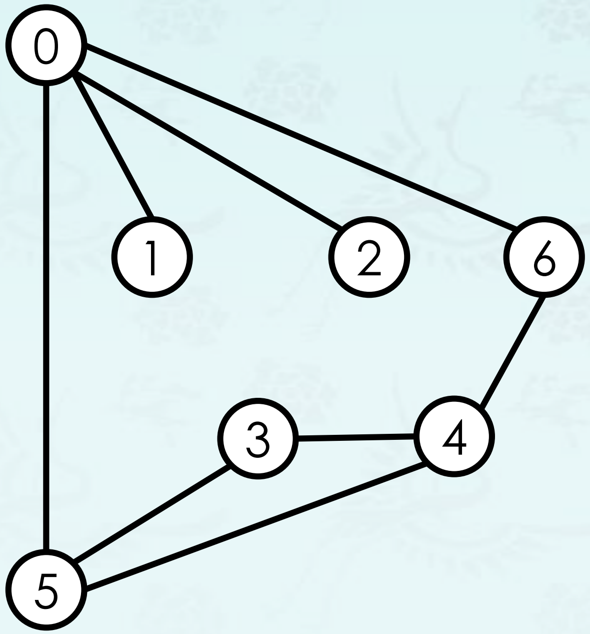
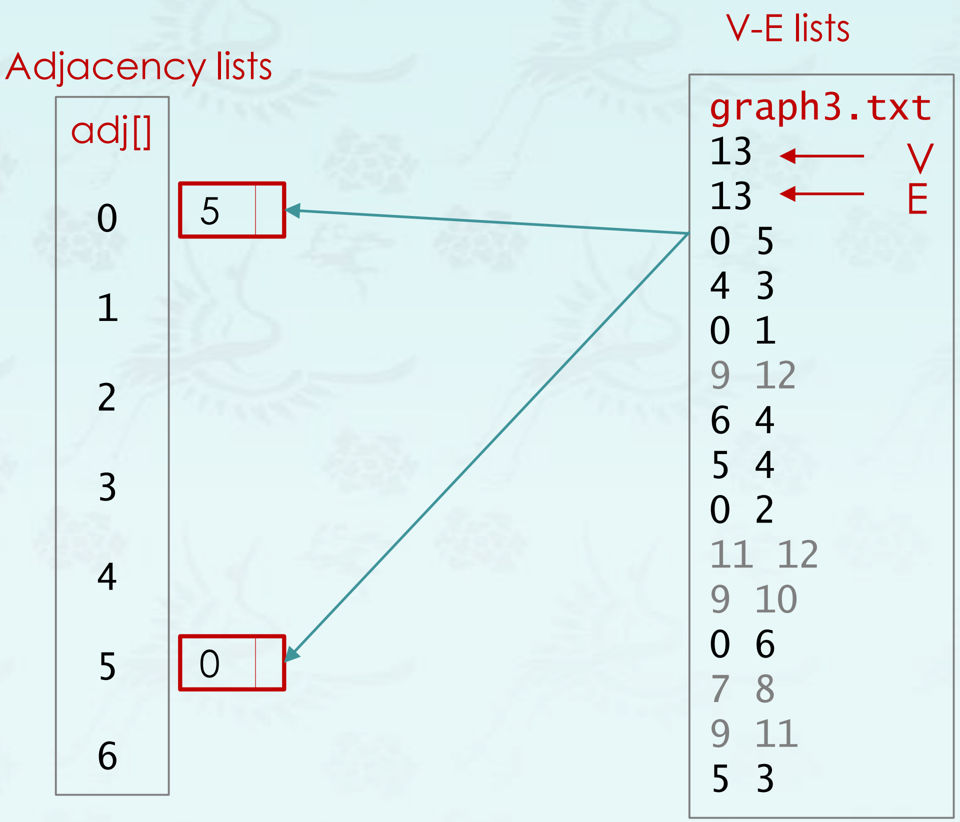



Graph g



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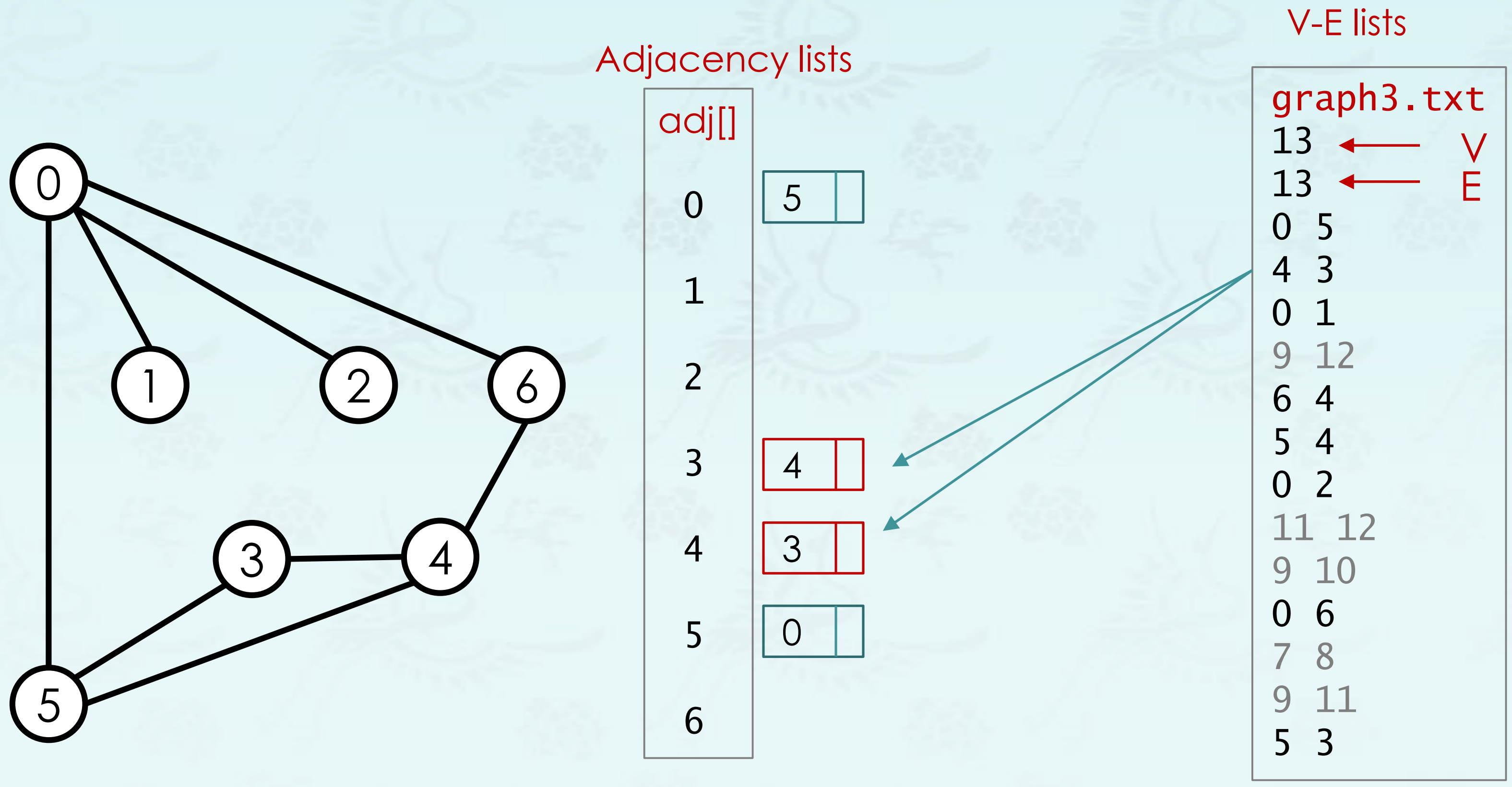


Graph g

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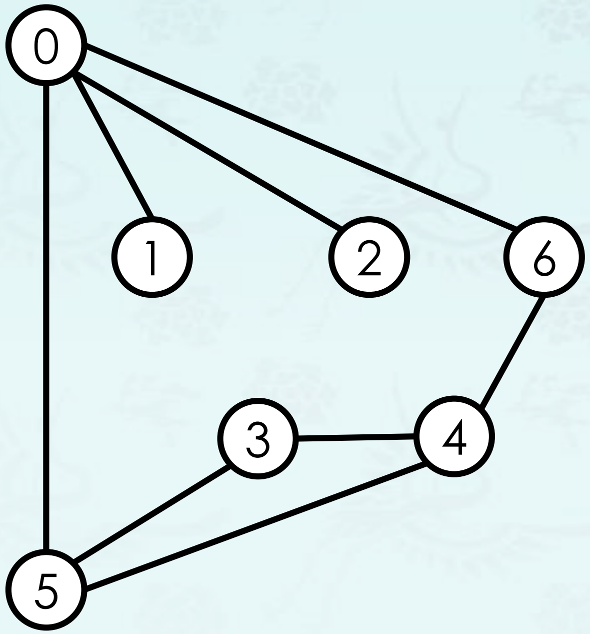
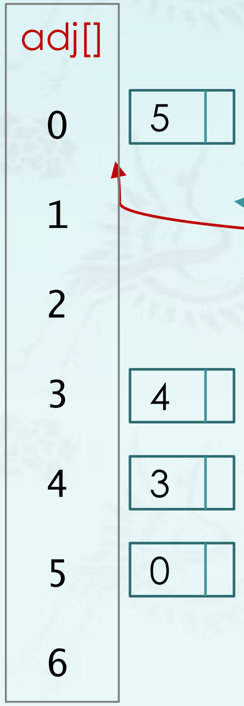



Graph g

Adjacency lists




V-E lists

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13   ← V
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0 5
4 3
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6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
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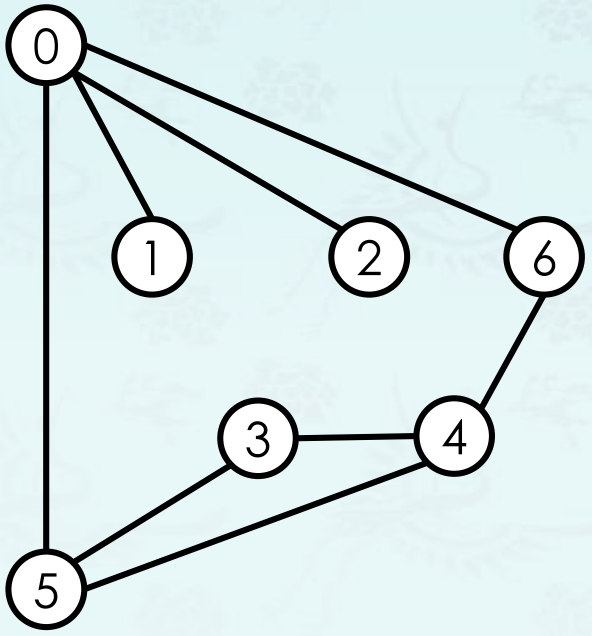
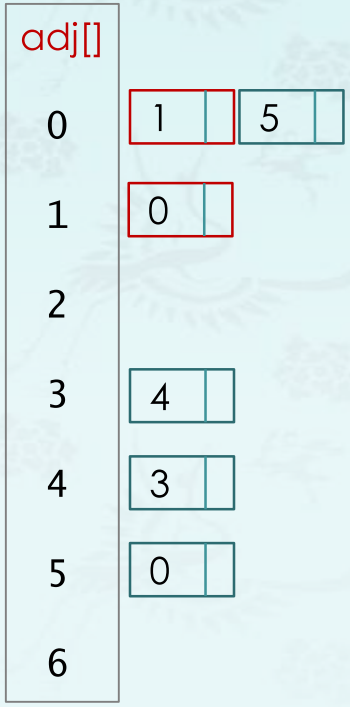



Graph g

Adjacency lists

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13   V
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9 10
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7 8
9 11
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```

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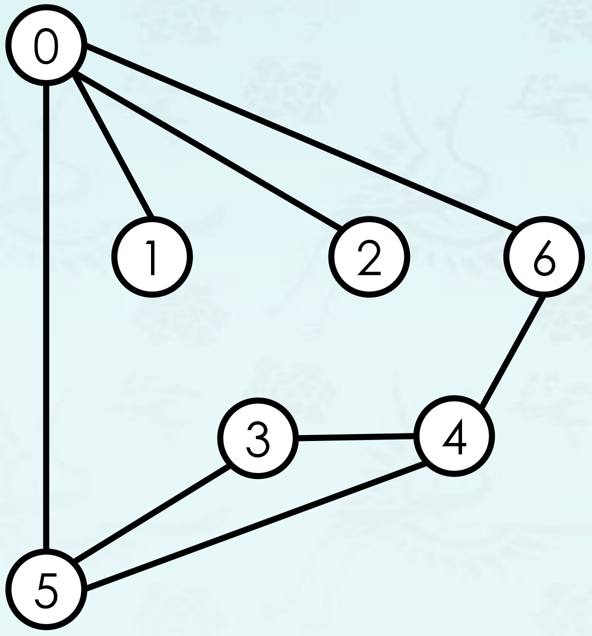


Graph g

Adjacency lists

| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

V-E lists

graph3.txt

```
13  ← V
13  ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

## DFS: Depth-First Search Demo

**To visit a vertex v:**

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.

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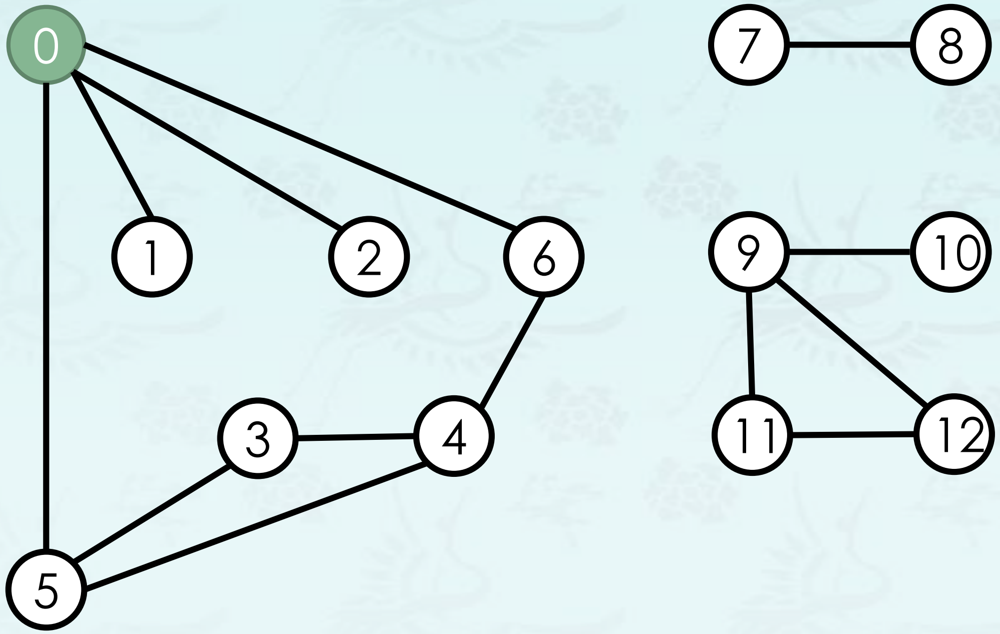
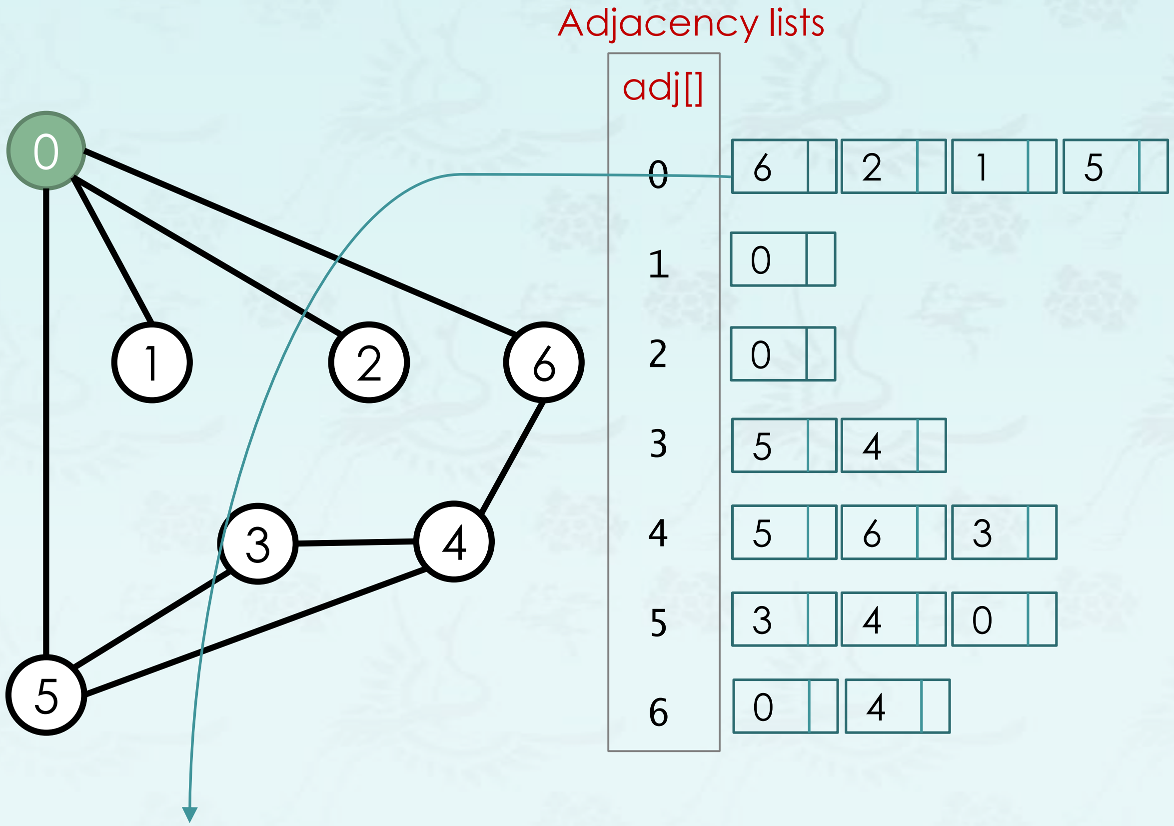



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | F | – |
| 5 | F | – |
| 6 | F | – |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

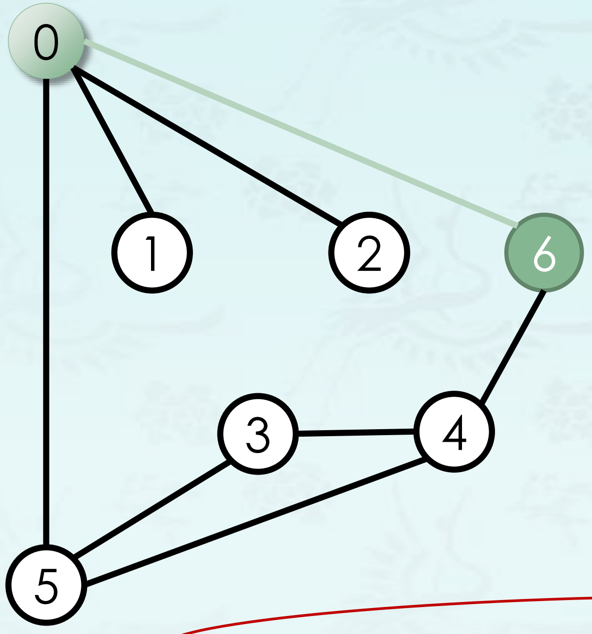
visit 0: **Which one first?**




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | F | – |
| 5 | F | – |
| 6 | F | – |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 0: **check 6**, **check 2, check 1, and check 5**

DFS 0



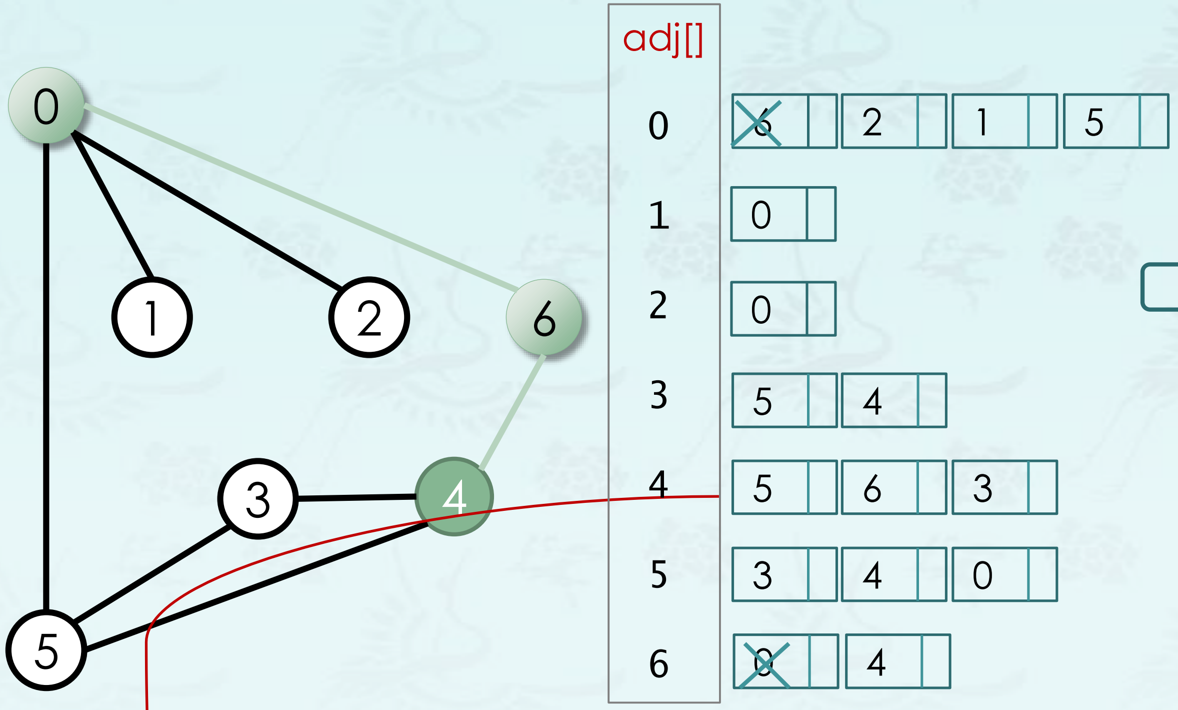

Adjacency lists

| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | F | – |
| 5 | F | – |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 6: **~~check 0~~**, **check 4**

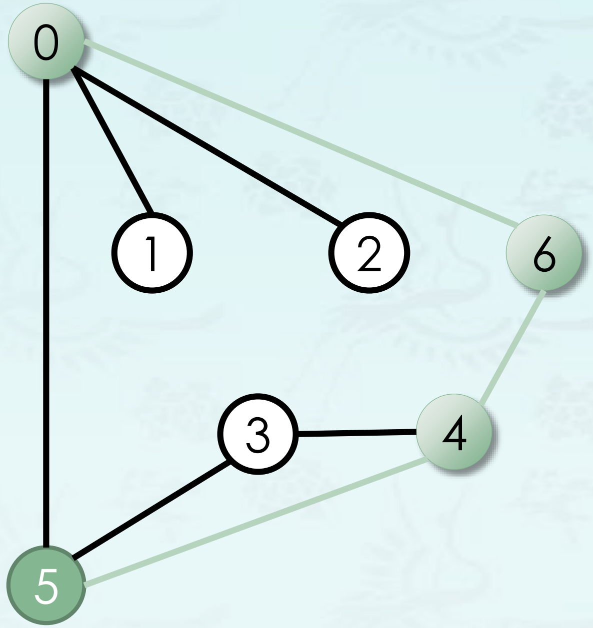
DFS 0 6

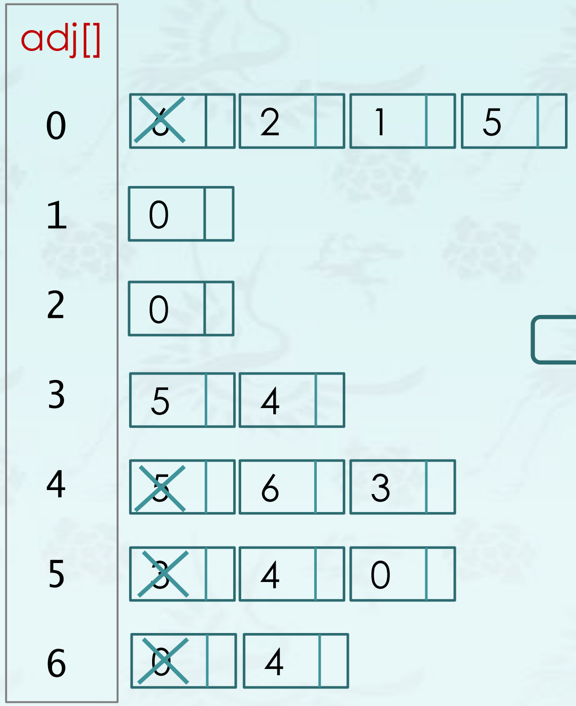



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | T | 6 |
| 5 | F | – |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 4: **check 5**, **check 6**, **check 3**

DFS 0 6 4

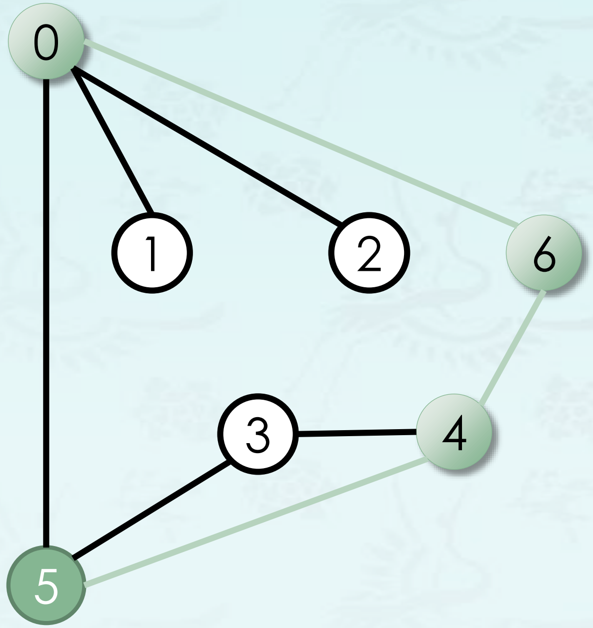



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 5: **check 3**, **check 4**, **check 0**

DFS 0 6 4 5

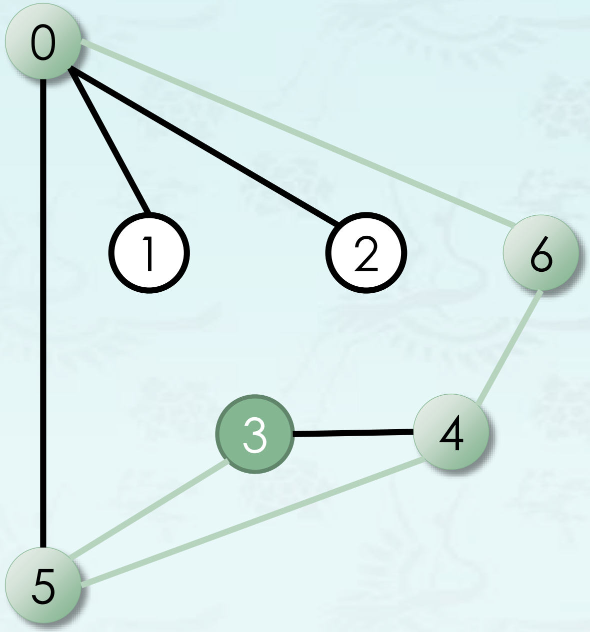



adj[]

| | |
|---|---|
| 0 | 6 → 2 → 1 → 5 |
| 1 | 0 |
| 2 | 0 |
| 3 | 5 → 4 |
| 4 | 5 → 6 → 3 |
| 5 | 3 → 4 → 0 |
| 6 | 0 → 4 |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | F | – |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

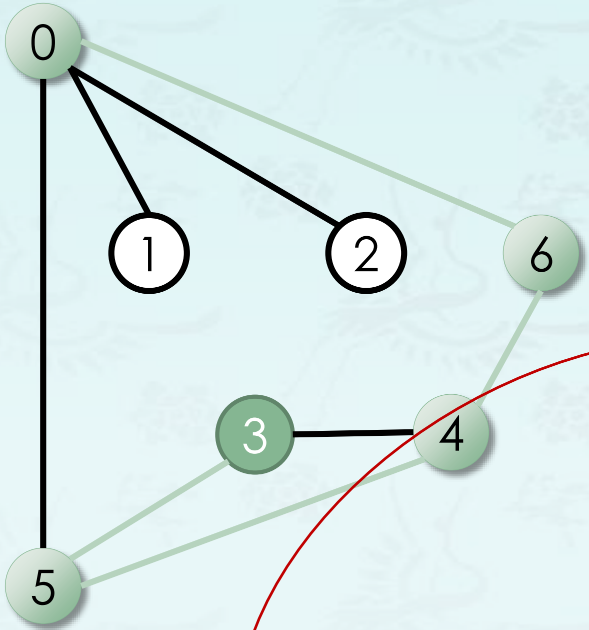
visit 5: **check 3**, **check 4**, **check 0**




| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 5: **check 3**, **check 4**, **check 0**



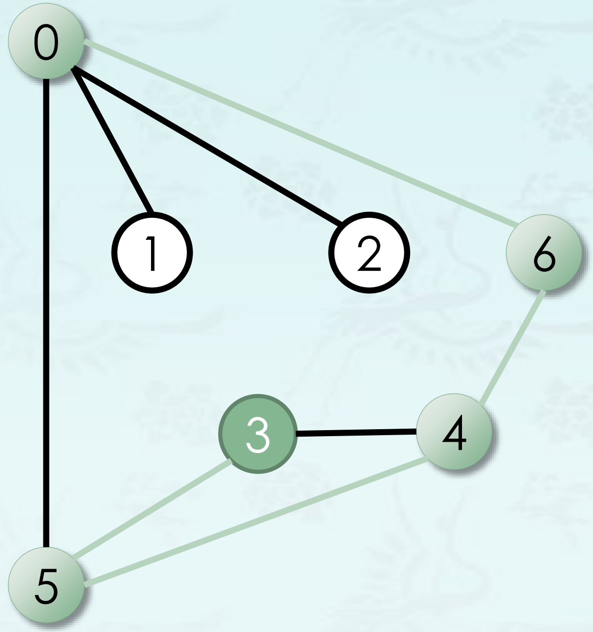

Adjacency lists

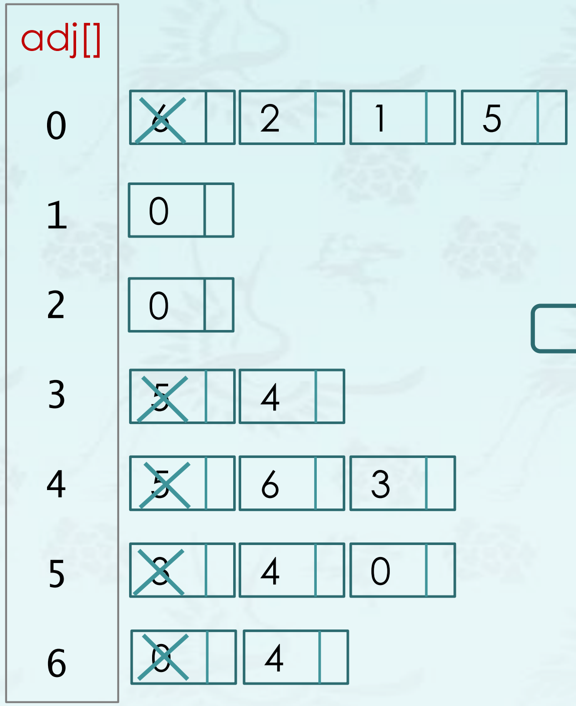
| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 3: **check 5**, **check 4**

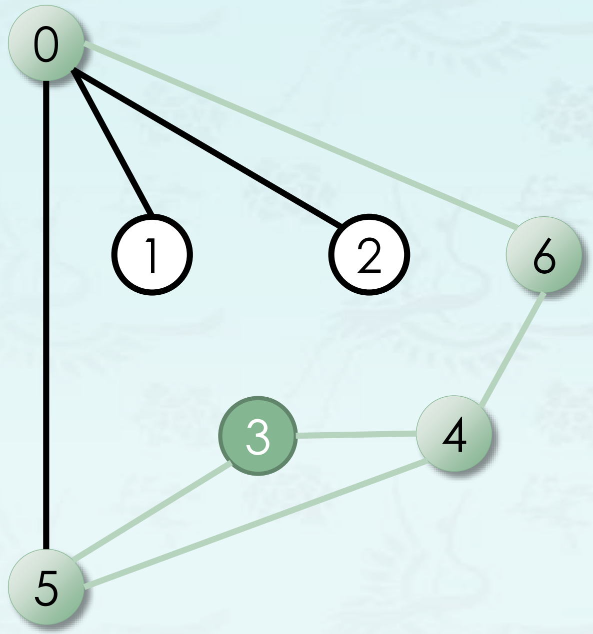
DFS 0 6 4 5 3




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 3: ~~**check 5**~~, **check 4**

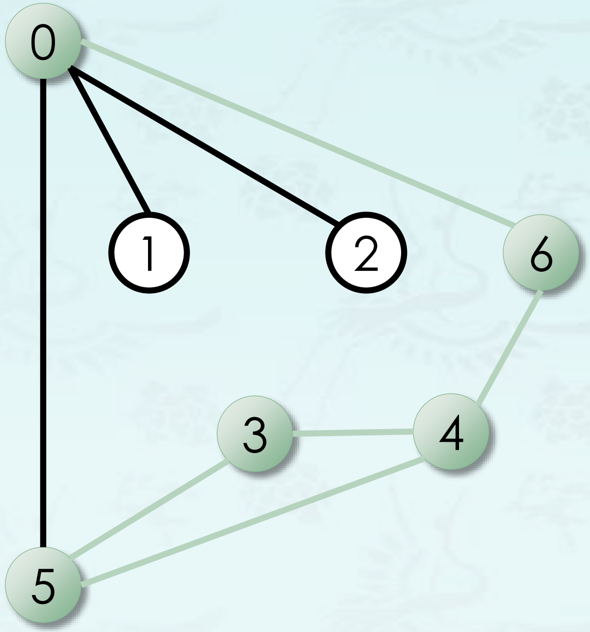



adj[]

- 0: ~~6~~ → 2 → 1 → 5
- 1: 0
- 2: 0
- 3: ~~5~~ → 4
- 4: ~~5~~ → 6 → 3
- 5: ~~3~~ → 4 → 0
- 6: ~~0~~ → 4

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

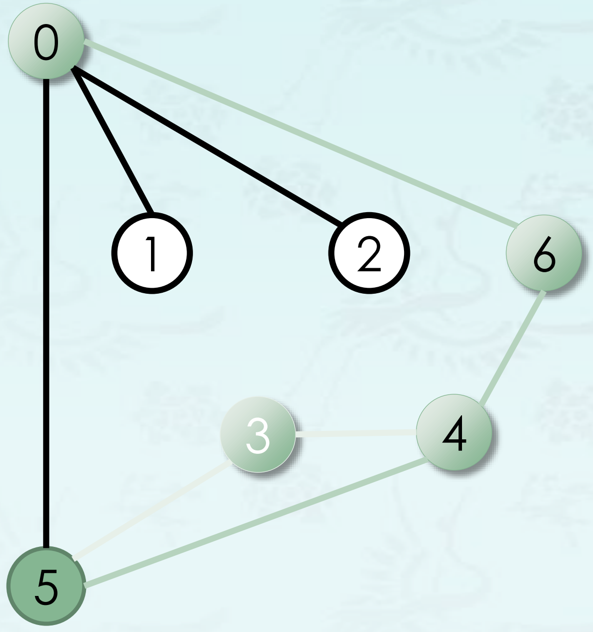
visit 3: check 5, check 4

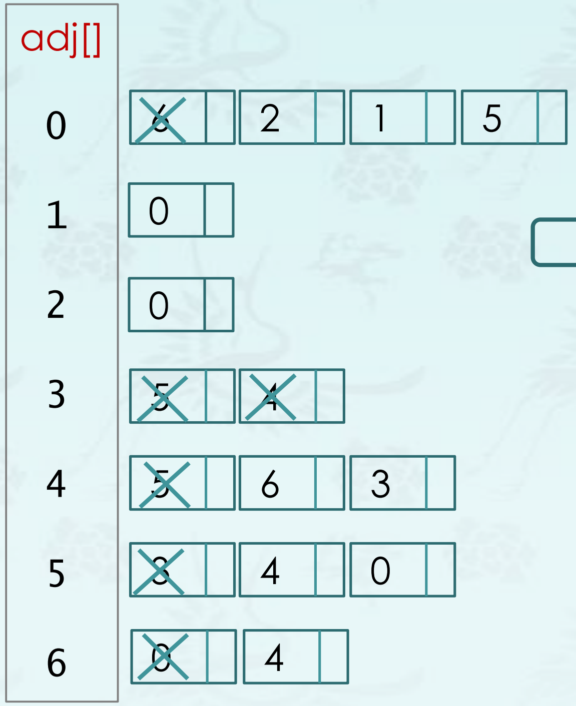



| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | 4 | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

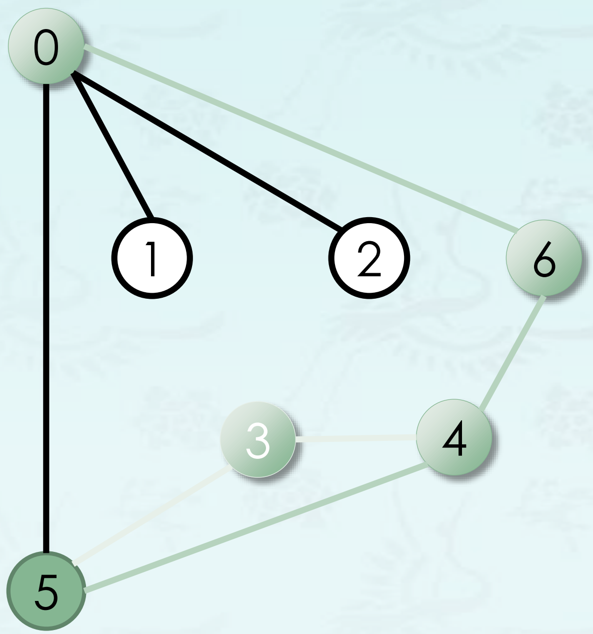
visit 3: check 5, ~~check 4~~




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

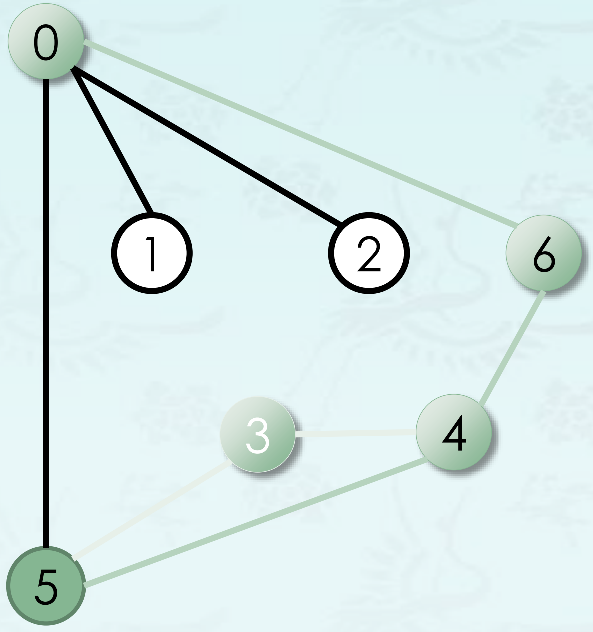
3 done: What's the next?




| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | 4 | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

3 done: What's the next? **Backtrack!**

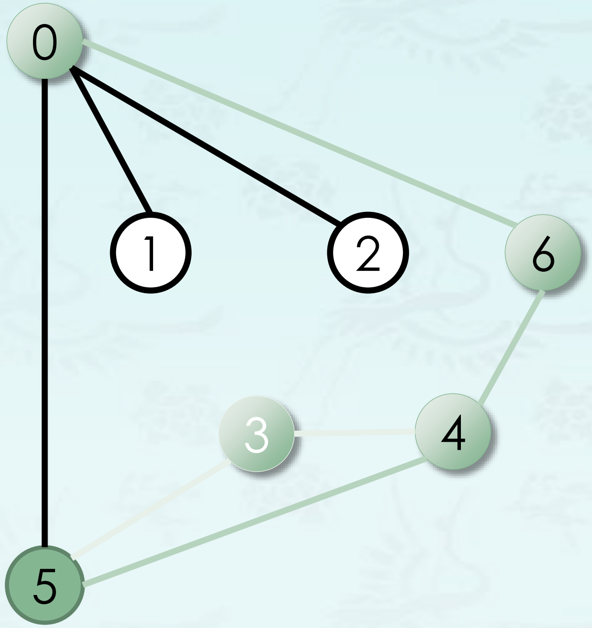



adj[]

- 0: ~~6~~ → 2 → 1 → 5
- 1: 0
- 2: 0
- 3: ~~5~~ → ~~4~~
- 4: ~~5~~ → 6 → 3
- 5: ~~3~~ → 4 → 0
- 6: ~~0~~ → 4

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

3 done: What's the next? **Backtrack!**
How to?

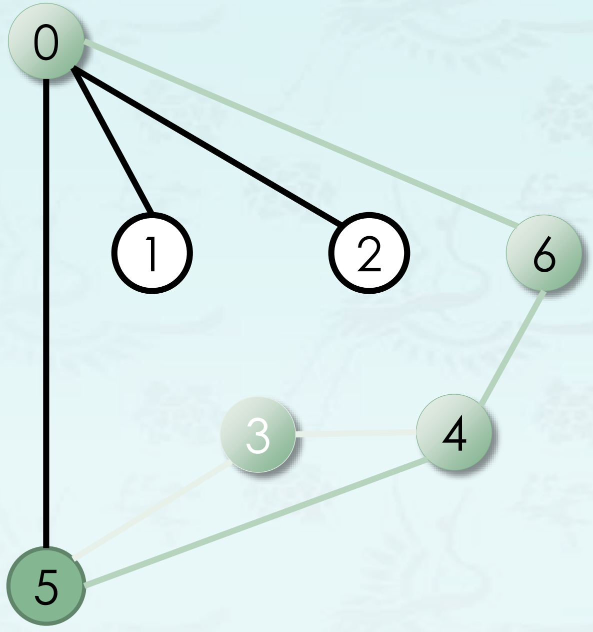



| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | 4 | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

3 done: What's the next? **Backtrack!**

How to? **Use parent[v]** parent[3] = 5

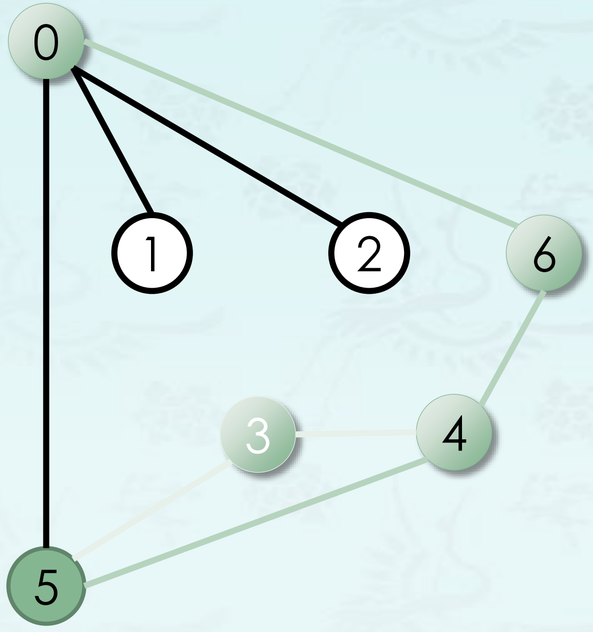


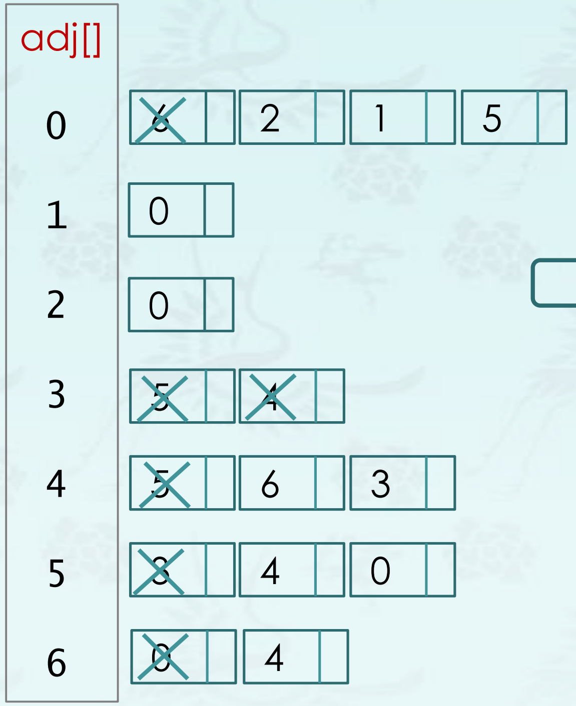

| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | 4 | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

done

visit 5: check 3, **check 4**, **check 0**

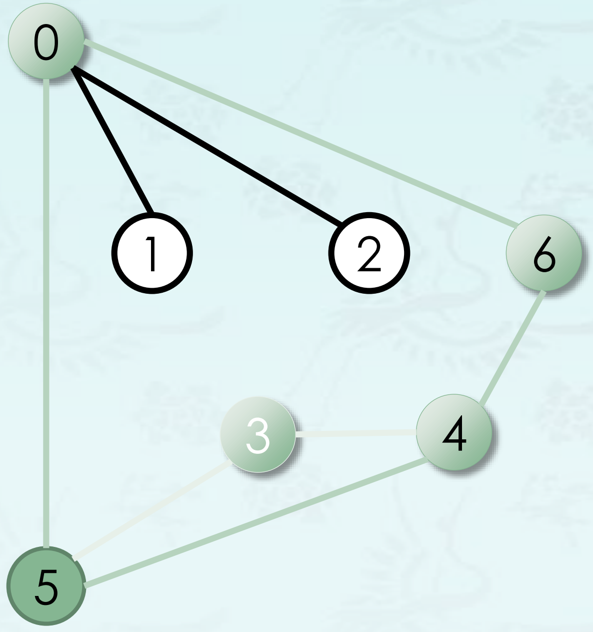



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | - |
| 1 | F | - |
| 2 | F | - |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | - |
| 8 | F | - |
| 9 | F | - |
| 10 | F | - |
| 11 | F | - |
| 12 | F | - |

done

visit 5: check 3, **check 4**, **check 0**

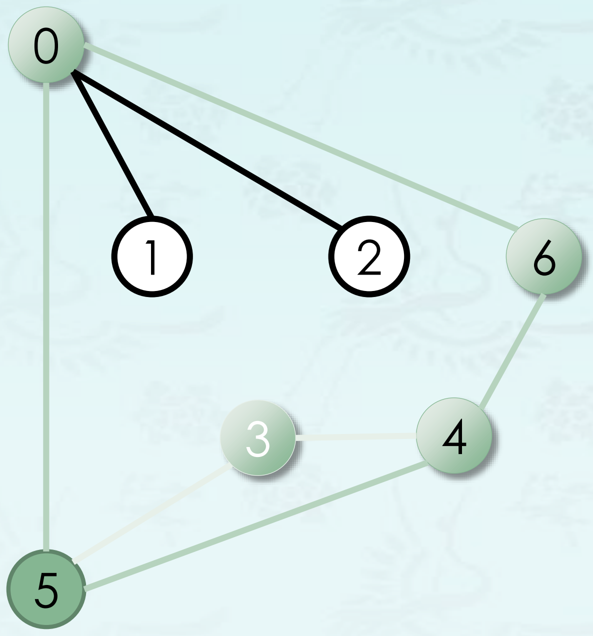



| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | ~~4~~ | 0 | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

done

visit 5: ~~check 3~~, ~~**check 4**~~, **check 0**




| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

done

visit 5: check 3, check 4, **check 0**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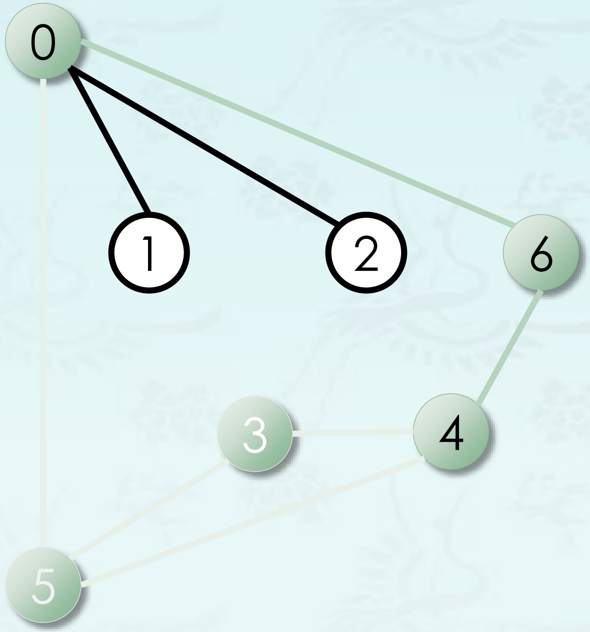
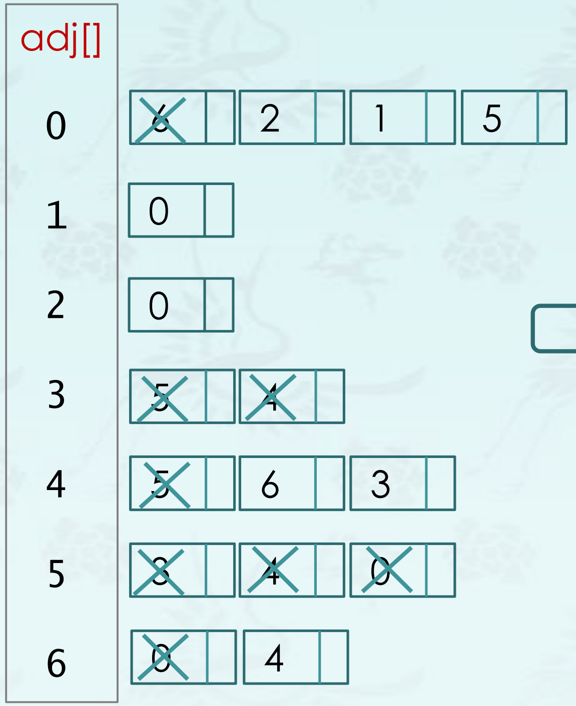
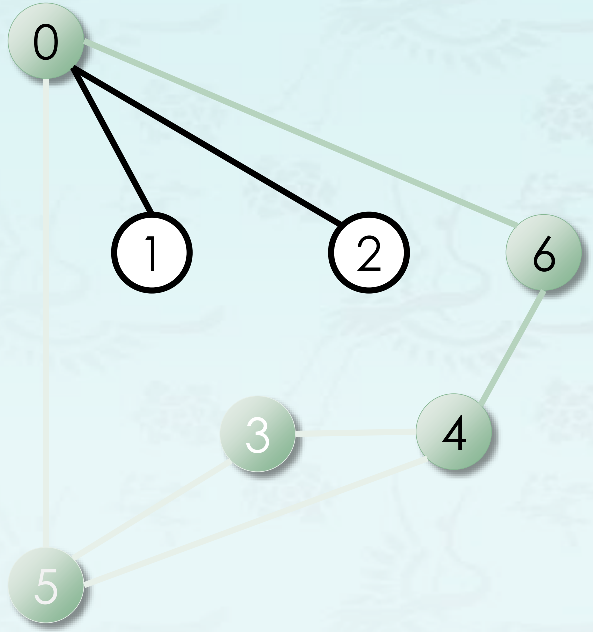



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

5 done

What's the next? **Backtrack!**
How to?




| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | 6 | 3 | |
| 5 | ~~3~~ | ~~4~~ | ~~0~~ | |
| 6 | ~~0~~ | 4 | | |

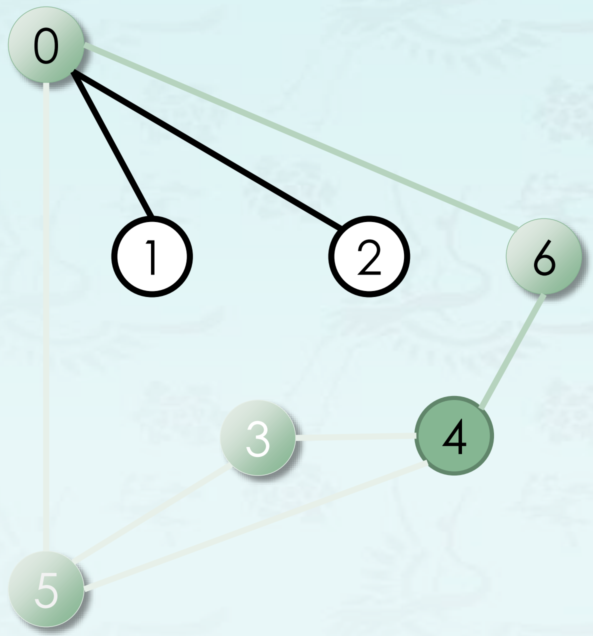
| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

5 done

What's the next? **Backtrack!**

How to? **Use parent[v]**

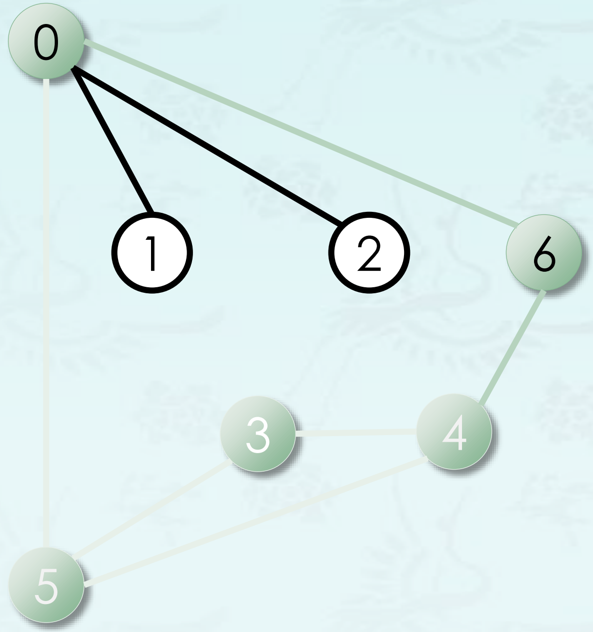
parent[5] = 4




| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

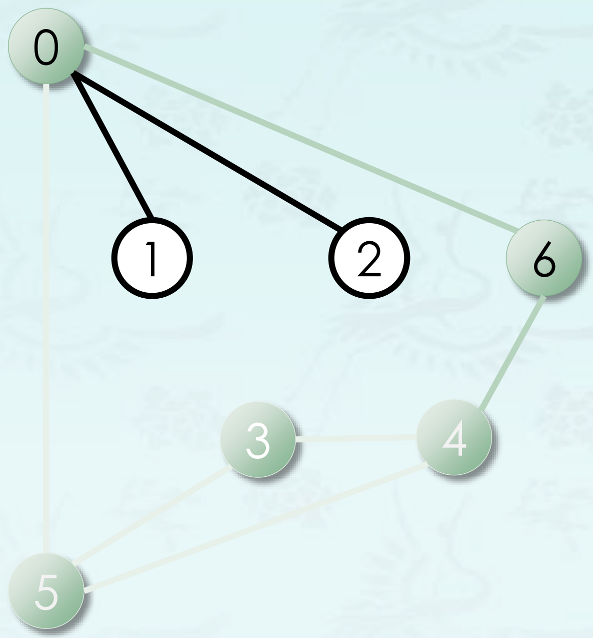
visit 4: check 5, **check 6**, **check 3**




| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | ~~6~~ | ~~3~~ | |
| 5 | ~~3~~ | ~~4~~ | ~~0~~ | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 4: **check 5**, **check 6**, **check 3**     4 done




| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | ~~6~~ | ~~3~~ | |
| 5 | ~~3~~ | ~~4~~ | ~~0~~ | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 4: check 5, check 6, **check 3** 4 done **Backtrack!** **parent[4] = 6**



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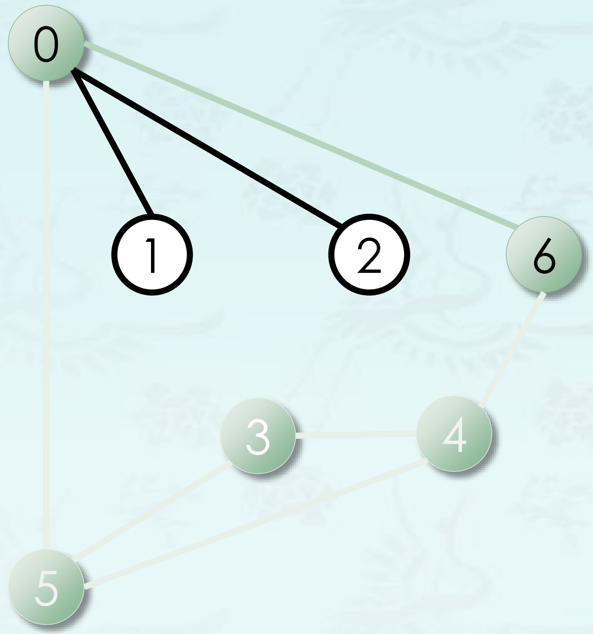
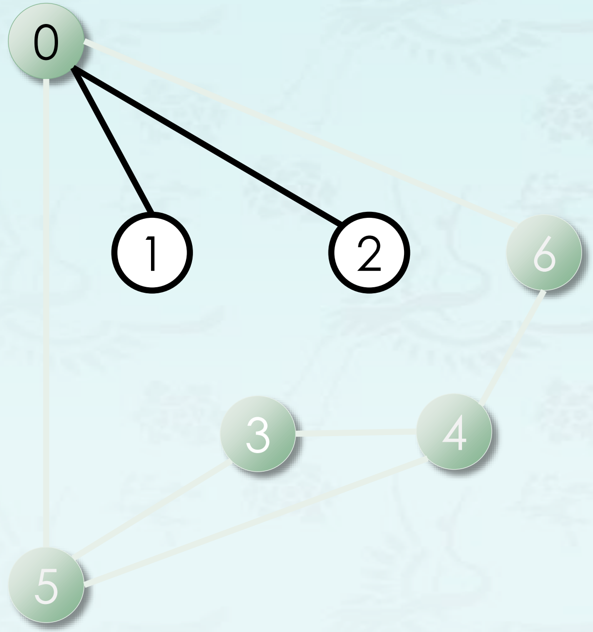
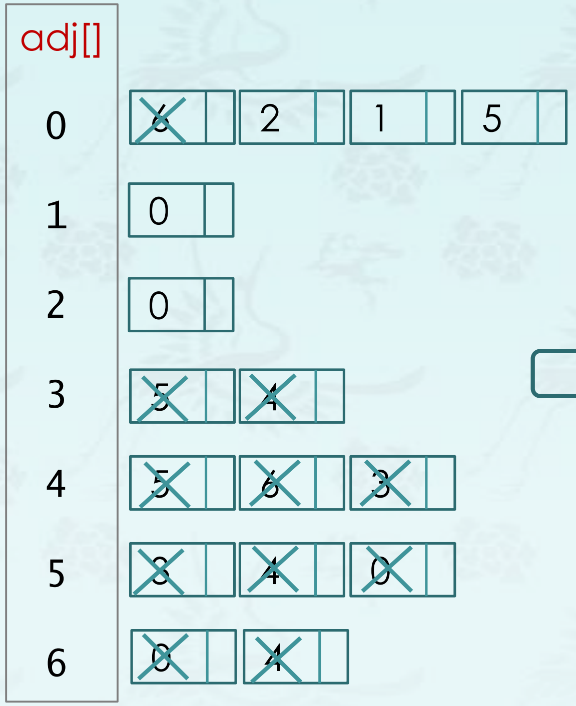



| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | ~~6~~ | ~~3~~ | |
| 5 | ~~3~~ | ~~4~~ | ~~0~~ | |
| 6 | ~~0~~ | 4 | | |

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

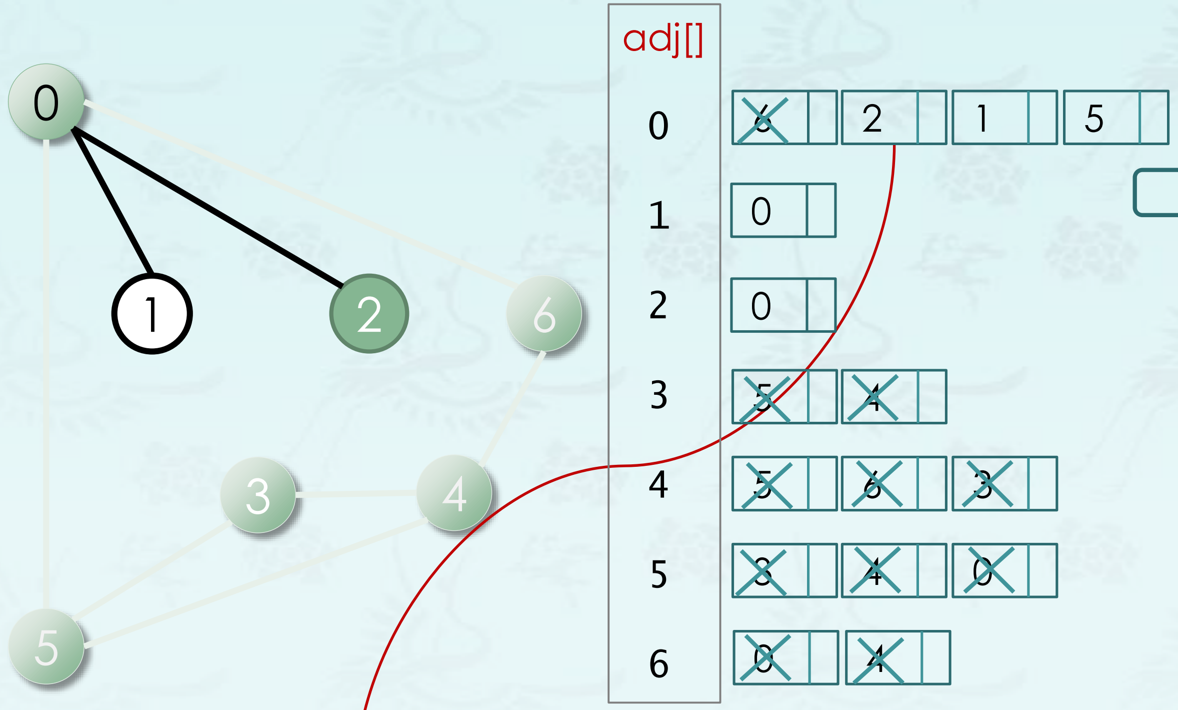
visit 6: **check 0**, **check 4**



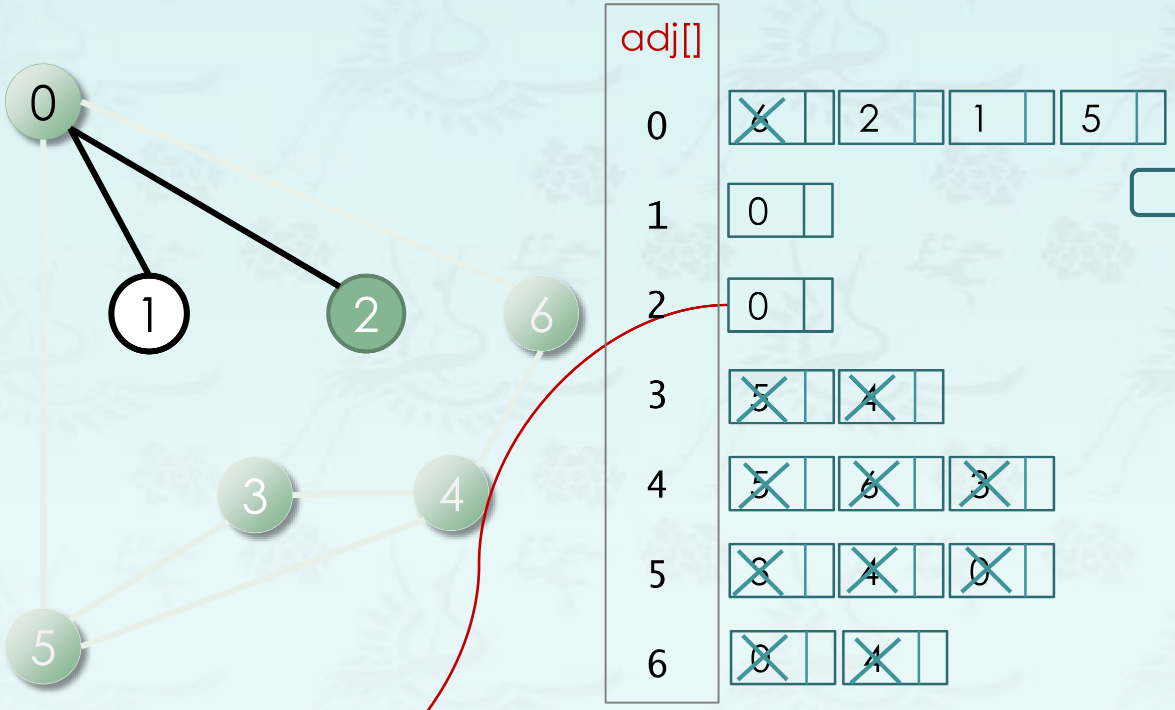

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | F | – |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 6: check 0, check 4 **done 6** **Backtrack!** **parent[6] = 0**




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

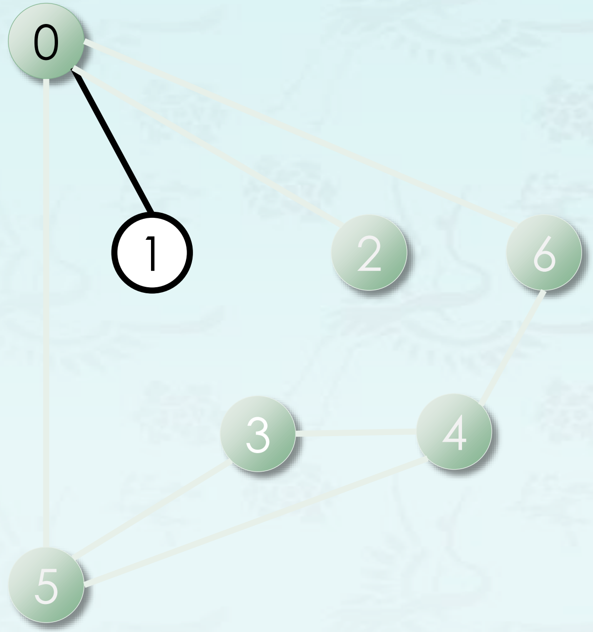
visit 0: **check 6**, **check 2**, **check 1, and check 5**




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 2: check 0

DFS: 0 6 4 5 3 2



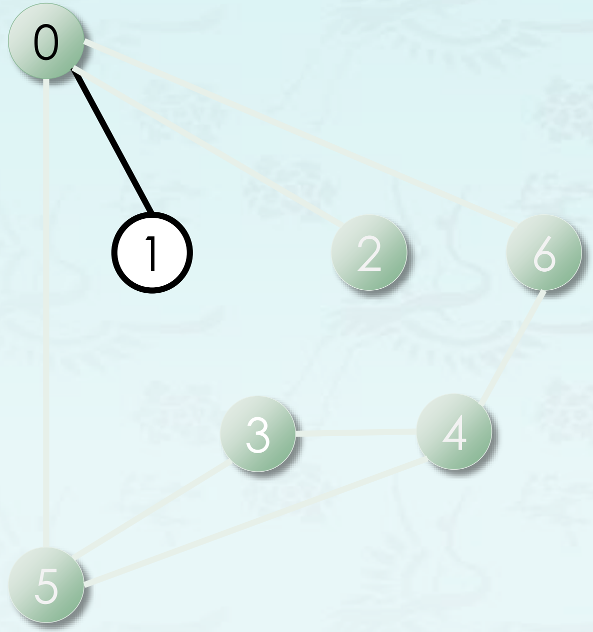

| adj[] | | | | |
|---|---|---|---|---|
| 0 | ~~6~~ | ~~2~~ | 1 | 5 |
| 1 | 0 | | | |
| 2 | ~~0~~ | | | |
| 3 | ~~5~~ | ~~4~~ | | |
| 4 | ~~5~~ | ~~6~~ | ~~3~~ | |
| 5 | ~~3~~ | ~~4~~ | ~~0~~ | |
| 6 | ~~0~~ | ~~4~~ | | |

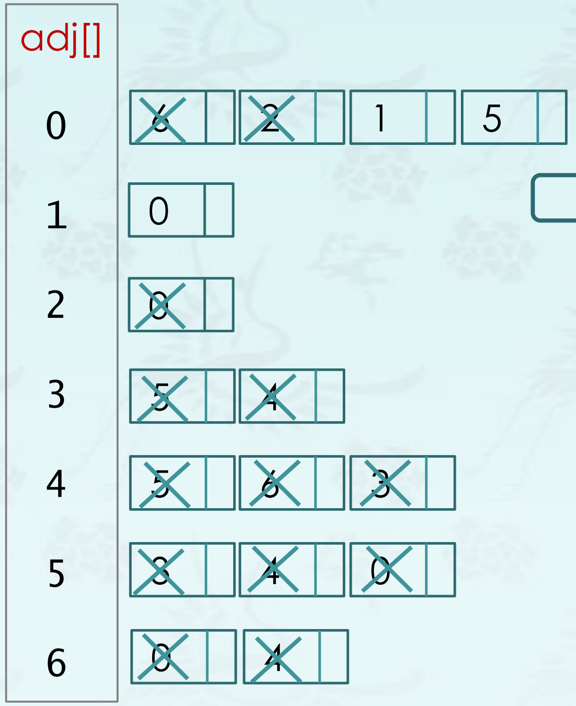
| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 2: check 0

**2 done**

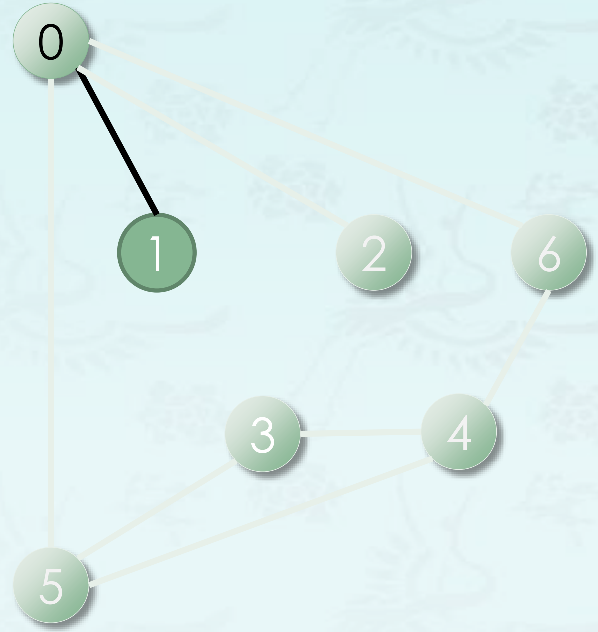
**Backtrack!** **parent[2] = 0**

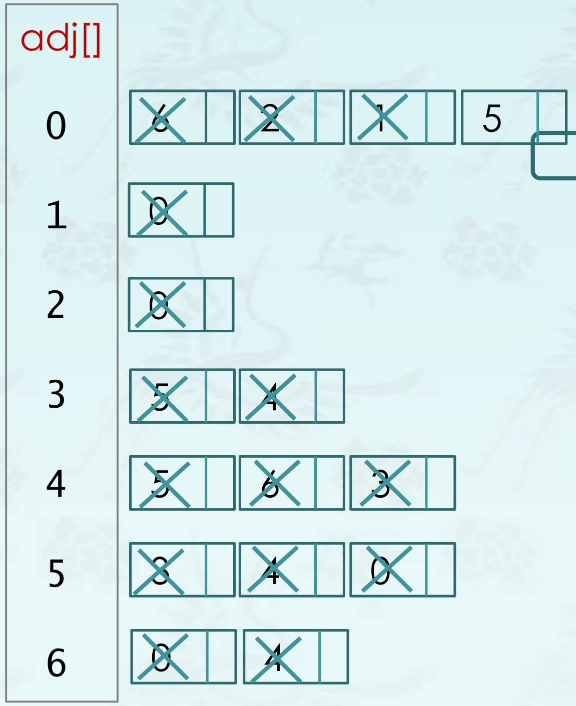



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | F | – |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 0: **check 6**, **check 2**, **check 1**, **and check 5**

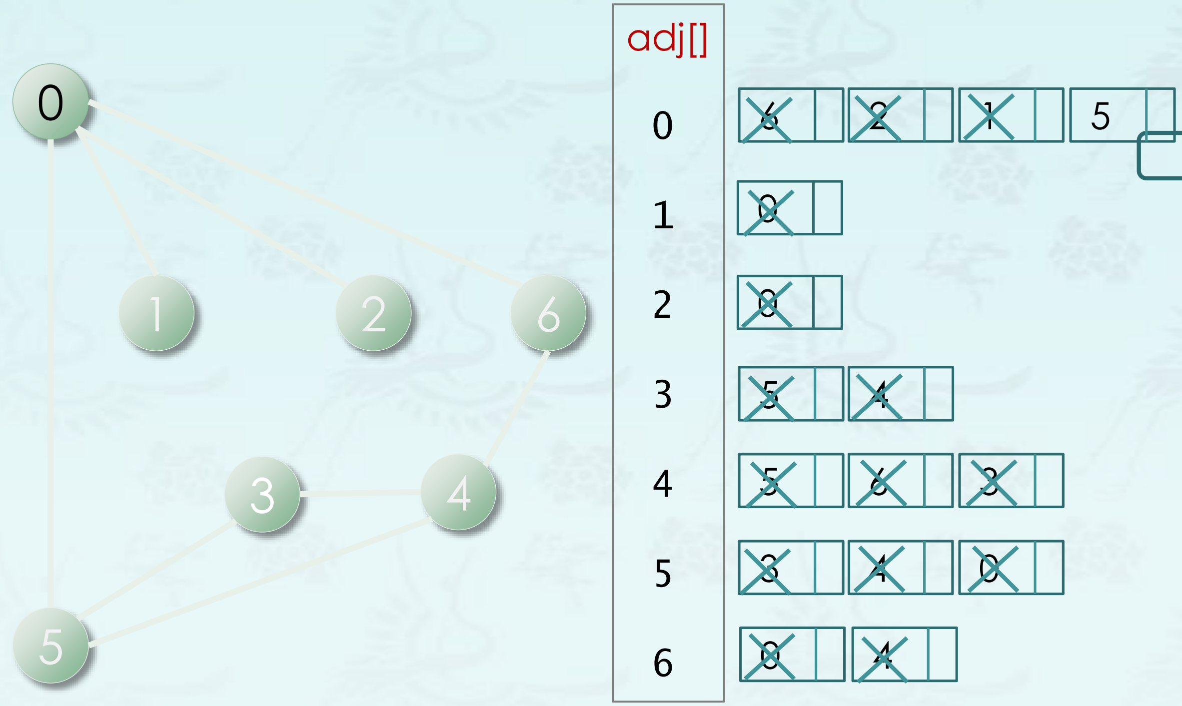



| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

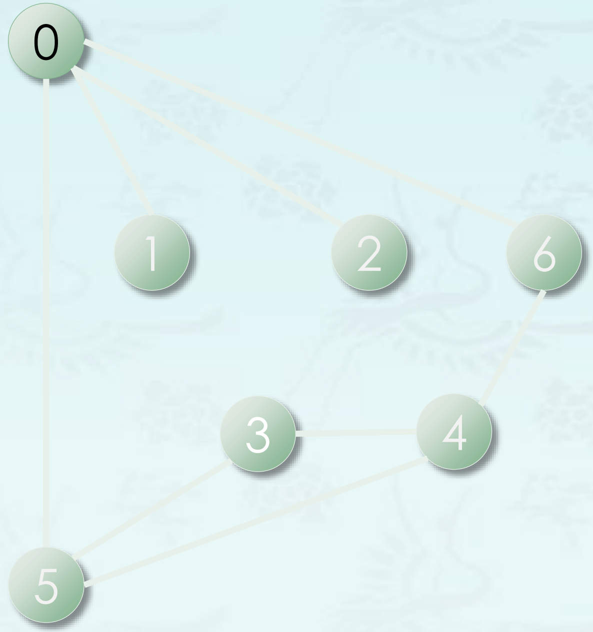
visit 1: check 0 **1 done** **Backtrack!** **parent[1] = 0**

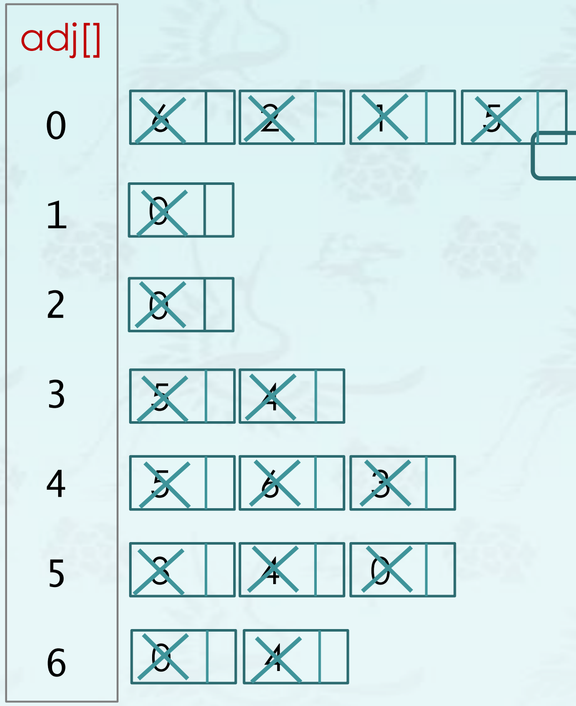
DFS: 0 6 4 5 3 2 1




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 0: **check 6, check 2, check 1, and check 5**



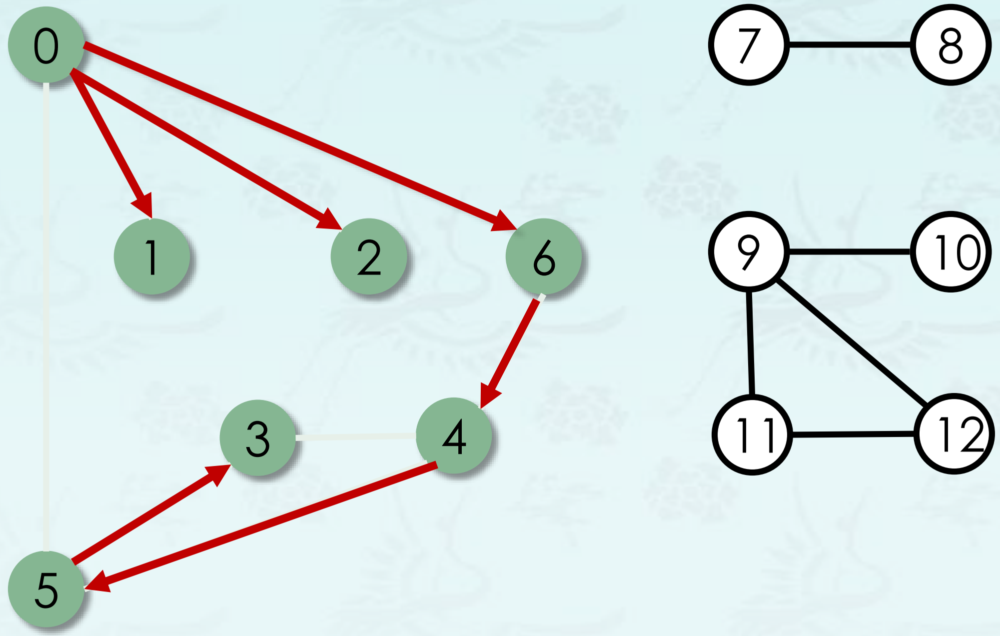

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

visit 0: **check 6, check 2, check 1, and check 5**
**0 done**

# DFS: Depth-First Search Demo

**To visit a vertex v:**

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.




| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | – |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | – |
| 8 | F | – |
| 9 | F | – |
| 10 | F | – |
| 11 | F | – |
| 12 | F | – |

**DFS Output:** DFS: 0 6 4 5 3 2 1

- found vertices reachable from 0
- build a data structure **parent[v]**

## DFS: Depth-First Search Demo

**Goal:** Find all vertices connected to s (and a corresponding path).
**Idea:** Mimic maze exploration

**Algorithm:**

- Use recursion (ball of string).
- Mark each visited vertex (and keep track of edge taken to visit it).
- Return (retrace steps) when no unvisited options.

**Data Structures:**

- `Boolean[] marked` to mark visited vetices.
- `int[] parent` to keep tree of paths.
  (`parent[w] == v`) means that edge v-w taken to visit w for first time

# DFS: Depth-First Search Coding

```
// DFS - find vertices connected to v
void DFS(graph g, int v, queue<int>& que) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++)
    g->marked[i] = false;
  _DFS(g, v, que);    // recursive _DFS at v
  g->DFSv = que;      // save result at DFSv at v
}
```

## DFS: Depth-First Search Coding

```
// DFS - find vertices connected to v
void DFS(graph g, int v, queue<int>& que) {
  if (empty(g)) return;
  for (int i = 0; i < V(g); i++)
    g->marked[i] = false;
  _DFS(g, v, que);    // recursive _DFS at
  g->DFSv = que;     // save result at DF
}
```

```
// Recursive _DFS does the work
void _DFS(graph g, int v, queue<int>& que) {
  g->marked[v] = true;
  que.push(v);                // save the path

  for (gnode w = g->adj[v].next; w; w = w->next) {
    if (!g->marked[w->item]) {
      _DFS(g, w->item, que);
      g->parentDFS[w->item] = v;
    }
  }
}
```

'v' current visiting vertex

keep where it reached from

## DFS: Depth-First Search Properties

**Proposition:** After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.

**Proof:** parent[] is parent-link representation of a tree rooted at s.

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | - |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | - |
| 8 | F | - |
| 9 | F | - |
| 10 | F | - |
| 11 | F | - |
| 12 | F | - |

```
// returns a path from v to w using the result of DFS's parent[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;

  // DFS at v, starting vertex
  queue<int> q;
  DFS(g, v, q);

  path = {};
  for (int x = w; x != v; x = g->parentDFS[x])
    path.push(x);

  path.push(v);
}
```

## DFS: Depth-First Search Coding

**Proposition:** After DFS, can find vertices connected to s in constant time and can find a path to s (if one exists) in time proportional to its length.

**Proof:** parent[] is parent-link representation of a tree rooted at s.

**What is the path from vertex 0 to vertex 3?**
**In this case, what is in the stack when parnet() returns?**

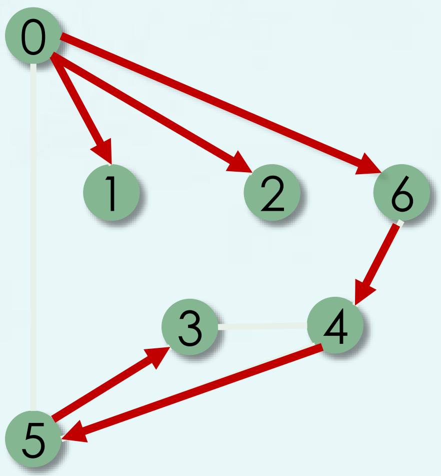
```
// returns a path from v to w using the result of DFS's parent[].
// It has to use a stack to retrace the path back to the source.
// Once the client(caller) gets a stack returned,
void DFSpath(graph g, int v, int w, stack<int>& path) {
  if (empty(g)) return;

  // DFS at v, starting vertex
  queue<int> q;
  DFS(g, v, q);

  path = {};
  for (int x = w; x != v; x = g->parentDFS[x])
    path.push(x);

  path.push(v);
}
```

| v | marked[] | parent[v] |
|---|---|---|
| 0 | T | - |
| 1 | T | 0 |
| 2 | T | 0 |
| 3 | T | 5 |
| 4 | T | 6 |
| 5 | T | 4 |
| 6 | T | 0 |
| 7 | F | - |
| 8 | F | - |
| 9 | F | - |
| 10 | F | - |
| 11 | F | - |
| 12 | F | - |



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  queue<int> q;
  DFS(g, v, q);

  path = {};
  for (int x = w; x != v; x = g->parentDFS[x])
    path.push(x);

  path.push(v);
}
```

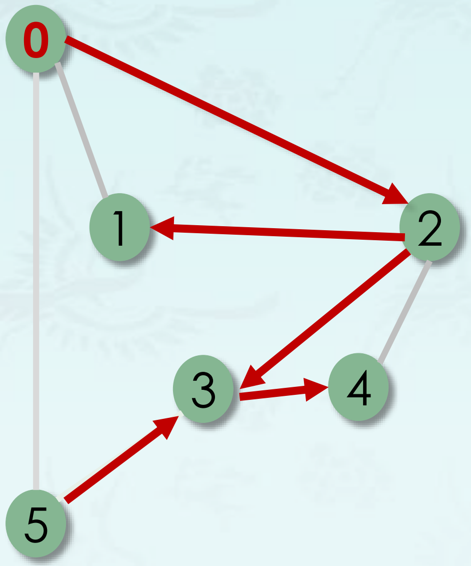


parent[]

| | |
|---|---|
| 0 | |
| 1 | 2 |
| 2 | 0 |
| 3 | 2 |
| 4 | 3 |
| 5 | 3 |

a wrong edge between ?

# Graph

- Introduction
- Graph API
- Elementary Graph Operations
    - **DFS: Depth first search**
    - BFS: Breadth first search
    - CC: Connected Components

Major references:
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
3. Wikipedia and many resources available from internet

Prof. Youngsup Kim, idebtor@handong.edu, 2014 Data Structures, CSEE Dept., Handong Global University

# Graph

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## Connectivity Queries

**Def.:** Vertices v and w are connected if there is a path between them.

**Goal:** Preprocess graph to answer queries of the form "*is v connected to w?*" in constant time.

| public class CC | | |
|---|---|---|
| | `CC(Graph G)` | *find connected components in G* |
| `boolean` | `connected(int v, int w)` | *are v and w connected?* |
| `int` | `count()` | *number of connected components* |
| `int` | `id(int v)` | *component identifier for v* |

**Depth-first search?** Yes …

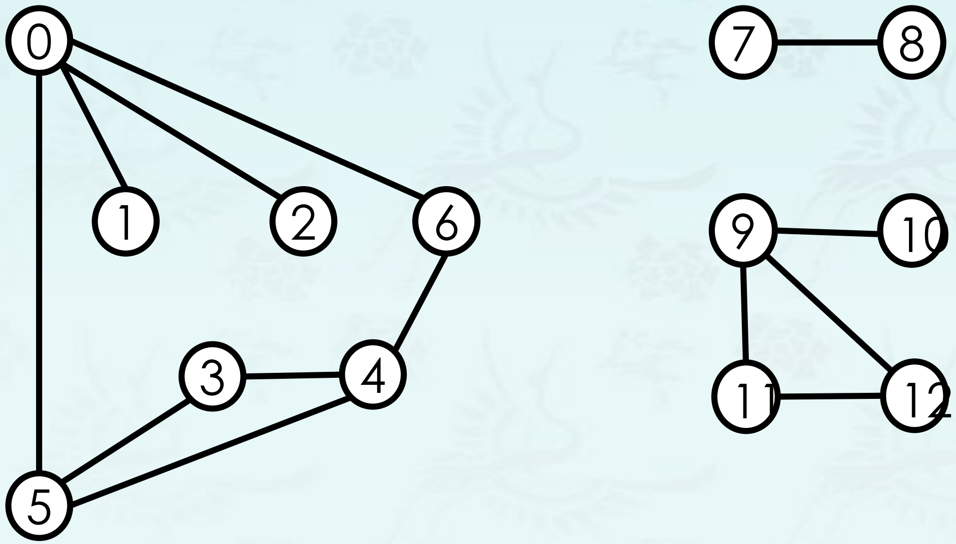
## Connected Components

The relation "is connected to" is equivalence relation:

**Reflexive:** v is connected to v.

**Symmetric:** if v is connected to w, then w is connected v.

**Transitive:** if v connected to w and w connected to x, then v connected to x



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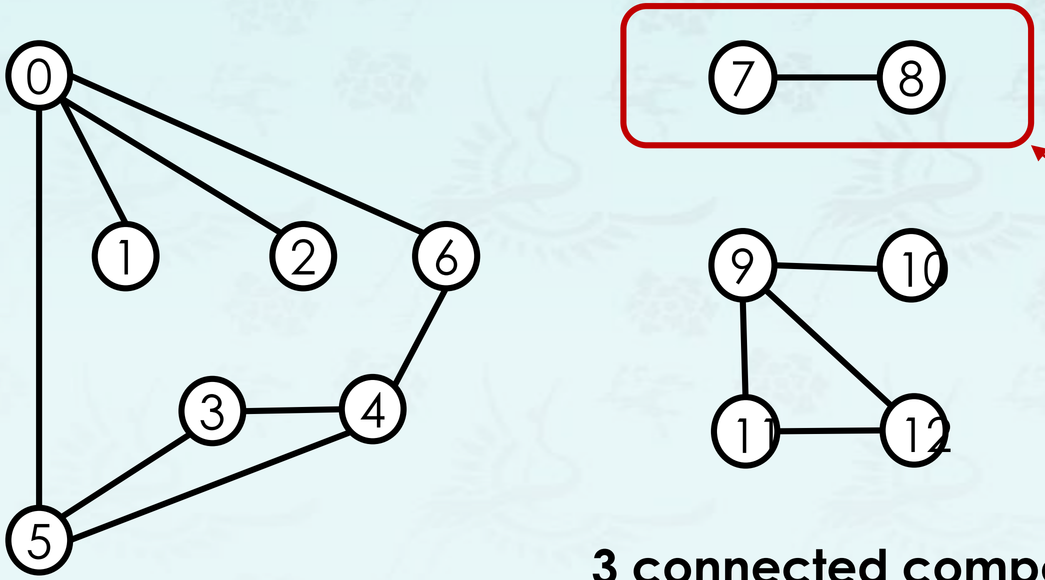
**Def.:** A connected component is a maximal set of connected vertices.




**3 connected components**

| v | id[v] |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 1 |
| 5 | 1 |
| 6 | 1 |
| 7 | 2 |
| 8 | 2 |
| 9 | 3 |
| 10 | 3 |
| 11 | 3 |
| 12 | 3 |

## Connected Components

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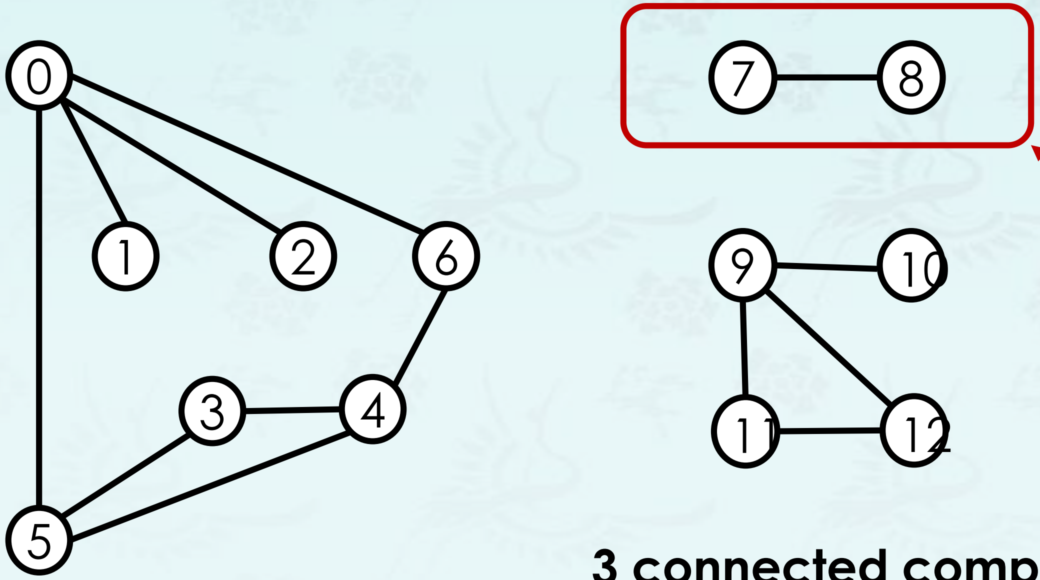



**3 connected components**

| v | id[v] |
|---|---|
| 0 | 1 |
| 1 | 1 |
| 2 | 1 |
| 3 | 1 |
| 4 | 1 |
| 5 | 1 |
| 6 | 1 |
| 7 | 2 |
| 8 | 2 |
| 9 | 3 |
| 10 | 3 |
| 11 | 3 |
| 12 | 3 |

**Remark:** Given connected components, can answer queries in constant time.

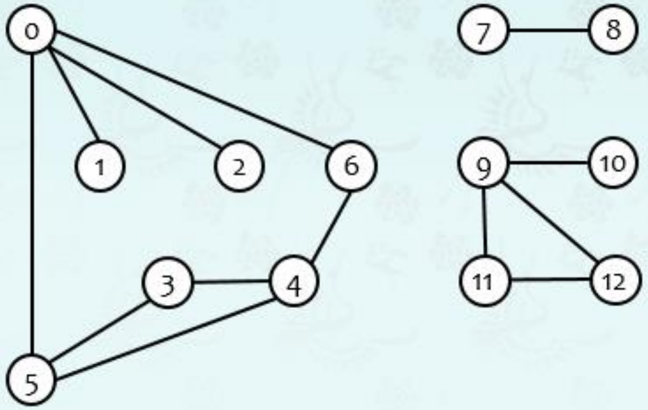
# Connected Components

Goal: Partition vertices into connected components.

**Connected components**

**Initialize all vertices v as unmarked.**

**For each unmarked vertex v, run DFS to identify all vertices discovered as part of the same component.**

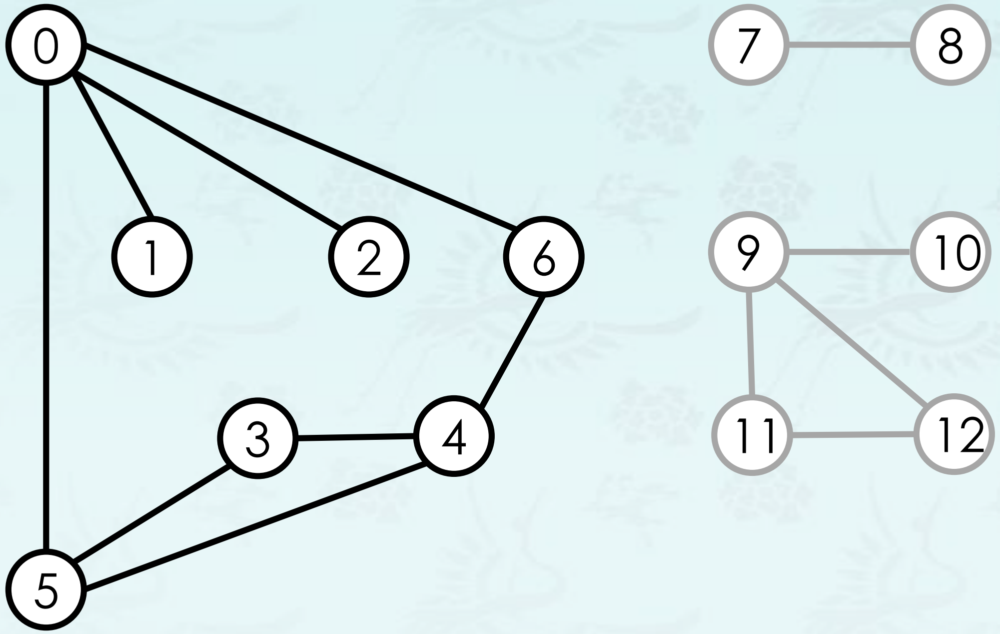



```
graph3.txt
13
13
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

## Connected Components

**To visit a vertex v:**

- Mark vertex v as visited.
- Recursively visit all unmarked vertices adjacent to v.



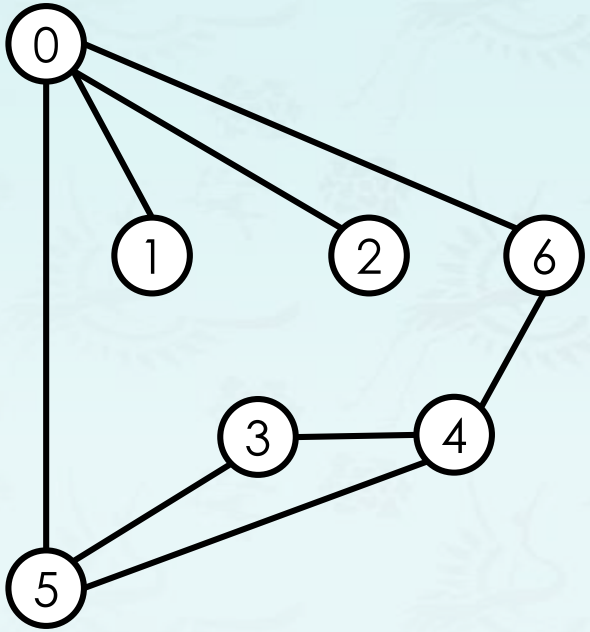

Graph g:

V-E lists →

```
graph3.txt
13   ← V
13   ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

**Challenge:** build adjacency lists?

# Connected Components



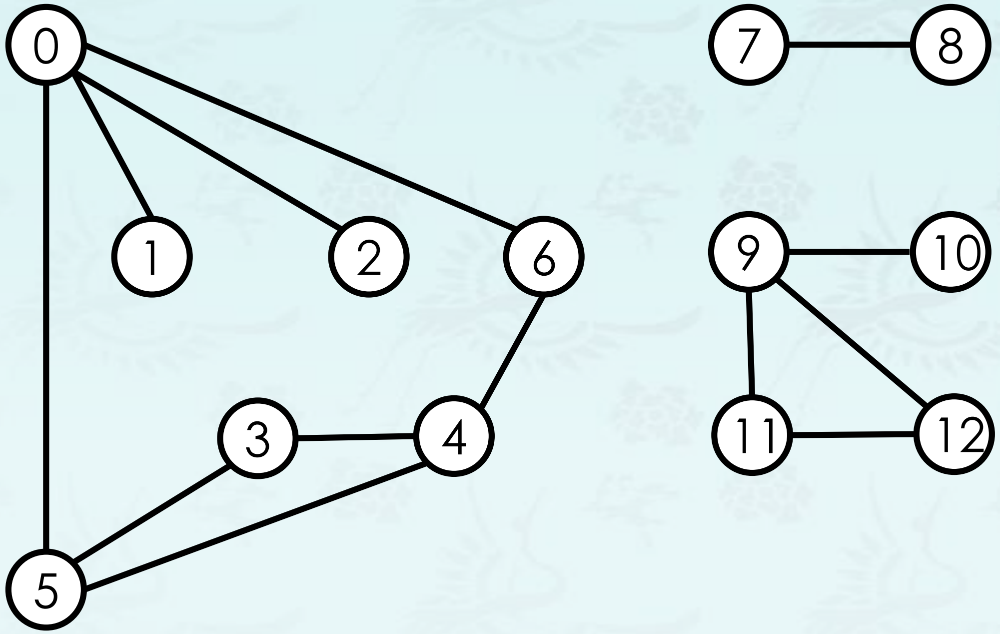

Graph g

Adjacency lists

| adj[] | | | | |
|---|---|---|---|---|
| 0 | 6 | 2 | 1 | 5 |
| 1 | 0 | | | |
| 2 | 0 | | | |
| 3 | 5 | 4 | | |
| 4 | 5 | 6 | 3 | |
| 5 | 3 | 4 | 0 | |
| 6 | 0 | 4 | | |

V-E lists

```
graph3.txt
13   ← V
13   ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

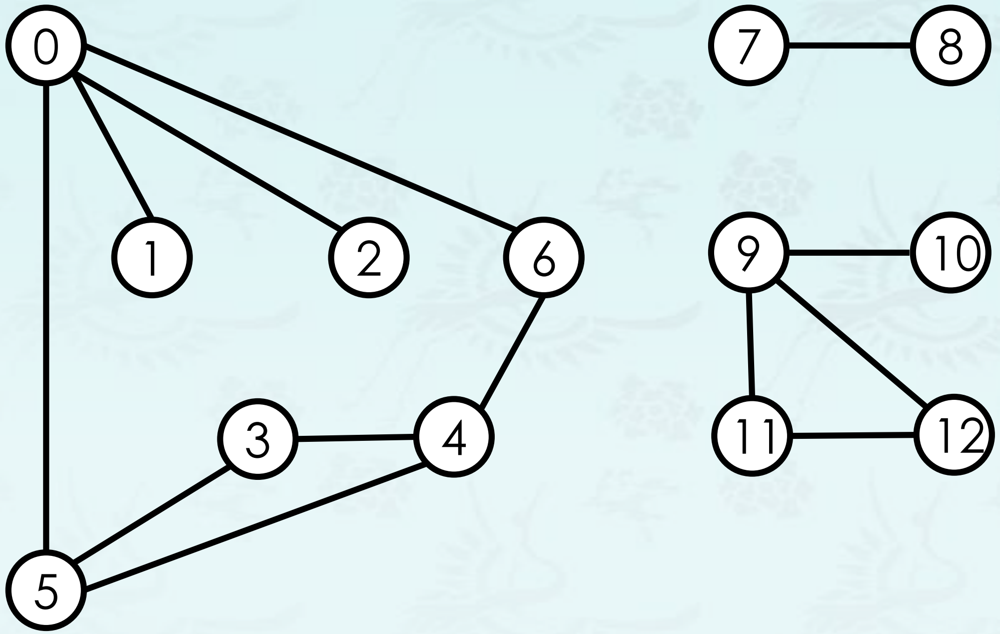



V-E lists →

```
graph3.txt
13  ← V
13  ← E
0 5
4 3
0 1
9 12
6 4
5 4
0 2
11 12
9 10
0 6
7 8
9 11
5 3
```

Graph g:

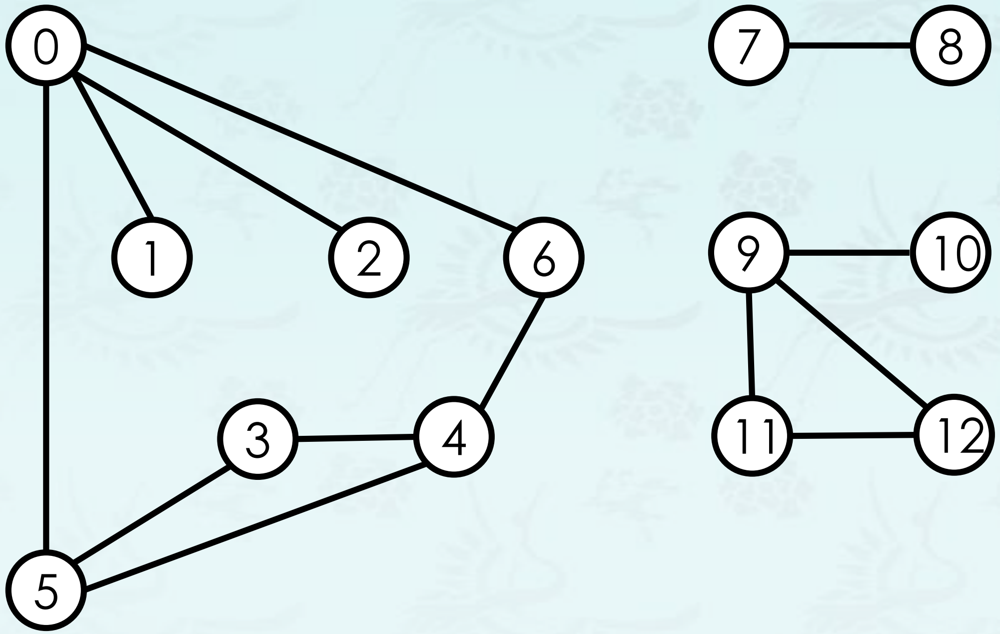
## Connected Components




| v | marked | ccid[v] |
|---|---|---|
| 0 | F | 0 |
| 1 | F | 0 |
| 2 | F | 0 |
| 3 | F | 0 |
| 4 | F | 0 |
| 5 | F | 0 |
| 6 | F | 0 |
| 7 | F | 0 |
| 8 | F | 0 |
| 9 | F | 0 |
| 10 | F | 0 |
| 11 | F | 0 |
| 12 | F | 0 |

Graph g:

# Connected Components




| v | marked | ccid[v] |
|---|---|---|
| 0 | F | 1 |
| 1 | F | 1 |
| 2 | F | 1 |
| 3 | F | 1 |
| 4 | F | 1 |
| 5 | F | 1 |
| 6 | F | 1 |
| 7 | F | 2 |
| 8 | F | 2 |
| 9 | F | 3 |
| 10 | F | 3 |
| 11 | F | 3 |
| 12 | F | 3 |

Done:

# Connected Components – Coding

```
// returns true if v and w are connected.
bool connected(graph g, int v, int w) {
  if (empty(g)) return true;

  queue<int> q;
  DFS(g, v, q);

  return g->CCID[v] == g->CCID[w];
}
```

```
// returns number of connected components.
int nCCs(graph g) {
  int id = g->CCID[0];
  int count = 1;
  for (int i = 0; i < V(g); i++)
    if (id != g->CCID[i]) {
      id = g->CCID[i];
      count++;
    }
  return id == 0 ? 0 : count;
}
```

# Graph

- Introduction
- Graph API
- Elementary Graph Operations
  - DFS: Depth first search
  - BFS: Breadth first search
  - **CC: Connected Components**
- pset – graph.cpp
  - **implement DFS, BFS, CC and others**

Major references:
1. Fundamentals of Data Structures by Horowitz, Sahni, Anderson-Freed,
2. Algorithms 4th edition - Part 1 & Part 2 by Robert Sedgewick and Kevin Wayne
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# pset graph – 5 points by June 6.

for g++, Mac

for Visual Studio

- Files provided
  - graph.h, graph.o, graph.obj
  - graphDriver.cpp – your code goes here
  - antenna.txt – your answer goes here
  - `g++ -std=c++11 graphDriver.cpp graph.o –I../include –L../lib –llib –o graph`

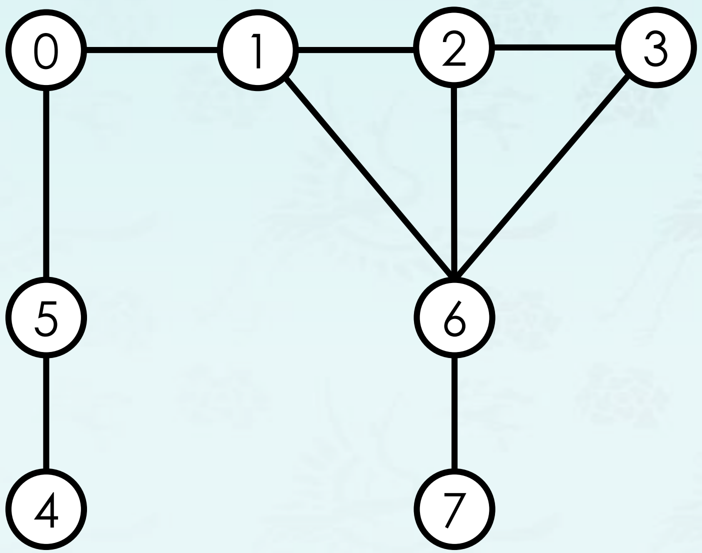
- Step 1: Code cyclic_check() in graphDriver.cpp
  - Don't use graph_by_file(), but use Graph() and addEdge().
  - Run DFS and BFS at v = 0, then print results saved in the graph structure. Don't use print_DFS() and print_BFS().
- Step 2: Using a graph called "antenna" provided in this ppt:
  - Find the edge list from an adjacency list of a graph.
  - Complete antenna.txt provided including graph figure and results.
- Step 3: Code **"case p"** in the main() of graphDriver.cpp.
  - It should function as shown graphx.exe provided.
- Step 4: Code bigraph_check() in graphDriver.cpp

- Files to submit by June 6, 11:55 PM: graphDriver.cpp, antenna.txt
  Points: one point per step except step 4 two points

## DFS/BFS – Exercise "antenna.txt"

**Problem set step 2:**

- Complete **antenna.txt file** which contains a following figure as well as an edge list such that it produces the **exact adj-list below**.
- Test this graph file (antenna.txt) and adj-list using graphx.exe provided
- Run DFS and BFS and compare the output with your results.




| edge list |
|---|
| V 8 |
| E 9 |
| 0 5 |
| 1 0 |
| 2 1 |
| … |
| … |

| **adjacent list** |
|---|
| **0: 1 5** |
| **1: 6 2 0** |
| **2: 3 6 1** |
| **3: 2 6** |
| **4: 5** |
| **5: 4 0** |
| **6: 7 1 2 3** |
| **7: 6** |

Graph g:

| |
|---|
| DFS: 0 1 6 …?… 5 4 |
| BFS: 0 1 5 …?… 7 3 |

```
dfs(0)
  dfs(1)
    dfs(6)
      dfs(7)
        check 6
      7 done
      check 1
      dfs(2)
        dfs(3)
          check 2
          check 6
        3 done
        check 6
        check 1
      2 done
      check 3
    6 done
    check 2
    check 0
  1 done
  dfs(5)
    dfs(4)
      check 5
    4 done
    check 0
  5 done
0 done
check 1
check 2
check 3
check 4
check 5
check 6
check 7
```