

# Problem Sheet #4

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## **Problem 4.1:** *Prefix order relations*

Let  $\Sigma$  be a finite set (called an alphabet) and let  $\Sigma^*$  be the set of all words that can be created out of the symbols in the alphabet  $\Sigma$ . ( $\Sigma^*$  is the Kleene closure of  $\Sigma$ , which includes the empty word  $\epsilon$ ). A word  $p \in \Sigma^*$  is called a prefix of a word  $w \in \Sigma^*$  if there is a word  $q \in \Sigma^*$  such that  $w = pq$ . A prefix  $p$  is called a proper prefix if  $p \neq w$ .

a) Let  $\preceq \subseteq \Sigma^* \times \Sigma^*$  be a relation such that  $p \preceq w$  for  $p, w \in \Sigma^*$  if  $p$  is a prefix of  $w$ . Show that  $\preceq$  is a partial order.

Let's assume that the relation is a partial order relation. We will see if it holds or not.

In order for a relation to be a partial order, it must possess three properties:

- a) The relation must be **reflexive**.
- b) The relation must be **antisymmetric**.
- c) The relation must be **transitive**.

Reflexive relation is defined by:

A relation is reflexive if  $\forall p \in A, (p, p) \in R$ , where  $R$  is the universal set.

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