Problem 2.1: proof by contrapositive

Let n be a natural number. If n is not divided by 3, then n is also not divisible by 15.

Mathematical notation,

$n \in \mathbb{N}$ let $A := \{n : 3|n\}$ and $B := \{n : 15|n\}$

According to question,

$$\neg A \rightarrow \neg B$$

Let's assume the opposite that if n is divisible by 3, then n is divisible by 15.

$$B \rightarrow A$$

which is true because 15 itself is a multiple of 3 (3×5)

So, any number that is divisible by ~~3~~ 15 is automatically divisible by 3.

Therefore,

since,

$B \rightarrow A$ holds

And,

$\neg A \rightarrow \neg B \Leftrightarrow B \rightarrow A$ through contrapositive

$\neg A \rightarrow \neg B$ also holds True.

Aayush Sharma Acharya

Problem 2.2

To prove,

$$1^2+3^2+5^2+\dots(2n-1)^2 = \sum_{k=1}^{n}(2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}.$$

so,

~~For~~ Let's check if the above statement holds true for $n=0$,

Base step,

$$(2\times1-1)^2 = \frac{\cancel{2}\times1\,(2\times1-1)\,(2\times1+1)}{\cancel{6}\;3}$$

$$1 = \frac{1\times\cancel{3}}{\cancel{3}}$$

$1=1$ so the statement holds.

Induction step.

Let assume the statement holds for $n=k$ and find out if it actually holds for $n=k+1$.

$$\sum_{k=1}^{k+1}(2k-1)^2 = \sum_{k=1}^{k}(2k-1)^2 + \left(2(k+1)-1\right)^2$$

$$= \frac{2k(2k-1)(2k+1)}{6} + (2k+2-1)^2$$

$$= \frac{2k(4k^2-1)}{6} + (4k^2+4k+1)$$

$$= \frac{8k^3+24k^2+22k+6}{6}$$

Aayusen Shankar Acharya

$$= \frac{8k^3 + 12k^2 + 12k^2 + 18k + 4k + 6}{6}$$

Aayush Sharma
Acharya.

$$= \frac{(4k^2 + 6k + 2)(2k + 3)}{6}$$

$$= \frac{(2k+2)(2k+1)(2k+3)}{6}$$

$$= \frac{2(k+1)(2k+2-1)(2k+2+1)}{6}$$

$$= \frac{2(k+1)[2(k+1)-1][2(k+1)+1]}{6}$$ //

This proves that ~~if~~ equations holds for n=k and n=k+1. //

Therefore,

The equation holds true for all n.

2.3.

Source codes are attached inside ~~with~~ zip files

(a) code:-

```
rotate :: Int -> [a] -> [a]
rotate n [] = []
rotate 0 (x:xs) = (x:xs)
rotate n (x:xs) = rotate (n-1) (xs ++ [x])
```

(b)

```
circle :: [a] -> [[a]]
circle s = circlehelper (length s) s

circlehelper 0 s = []
```

~~circlehelper n s = [rotate n s] ++ circlehelper (n-1) s~~

```
circlehelper n s = [rotate n s] ++ circlehelper (n-1) s.
```

A.S.A