Eigenvalues

Peer Member 1: λ_1 (Cedric Izabayo) => 2.6746

$$A = \begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ 1 & -13 & -14 & -13-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$$

Using Newton Raphson method

$$\lambda_{n+1} = \lambda_n - \frac{f(\lambda_n)}{f'(\lambda_n)}$$

$$\rightarrow f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$$

$$f'(\lambda) = 4\lambda^3 + 39\lambda^2 - 438\lambda - 835$$

$$\frac{\lambda(4\lambda^3 + 39\lambda^2 - 438\lambda - 835) - (\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

$$\rightarrow \frac{4\lambda^4 + \cancel{39}\lambda^3 - \cancel{438}\lambda^2 - \cancel{835}\lambda - \cancel{\lambda^4} - \cancel{13}\lambda^3 + \cancel{219}\lambda^2 + \cancel{835}\lambda - 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

$$\rightarrow \frac{3\lambda^4 + 26\lambda^3 - 219\lambda^2 - 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835} \rightarrow \lambda_{n+1}$$

$\rightarrow \lambda_0 = 2$, (Divisible by 3500)

$$\rightarrow \frac{3(2)^4 + 26(2^3) - 219(2)^2 - 3500}{4(2)^3 + 39(2)^2 - 438(2) - 835}$$

$\rightarrow 2.6746 \rightarrow \lambda_1$

Peer Member 2: λ_2 (Eliel Ntwali) => 11.0540

$$\boxed{\lambda = 2.6746}$$

$$\Rightarrow \frac{3\lambda^4 + 26\lambda^3 - 219\lambda^2 - 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

$$\Rightarrow \frac{3(2.6746)^4 + 26(2.6746)^3 - 219(2.6746)^2 - 3500}{4(2.6746)^3 + 39(2.6746)^2 - 436(2.6746) - 835}$$

$\Rightarrow 11.0540 \Rightarrow \lambda_2$

Peer Member 3: λ_3 (Edith Githinji) => -5.6040.

Edith

Eingen value 3

$$\frac{3\lambda^4 + 26\lambda^3 - 219\lambda^2 - 3500}{4\lambda^3 + 39\lambda^2 - 438\lambda - 835}$$

$\lambda = 11.0540$

$$= \frac{3(11.0540)^4 + 26(11.0540)^3 - 219(11.0540)^2 - 3500}{4(11.0540)^3 + 39(11.0540)^2 - 438(11.0540) - 835}$$

$= -5.6040 = \lambda_3.$

Eigenvectors

Peer Member 1: (Cedric Izabayo)

$\lambda_1 \approx 2.675$

$$A - \lambda_3 \underset{..}{I} \approx \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

$Av = \lambda v$

$(A - \lambda \underset{..}{I}) \cdot v = 0$

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (0.754)$$

$$R_1 / (1.325) \to R_1 \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (2)$$

$$R_2 - (-2) \cdot R_1 \to R_2 \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (1) \quad \cancel{R_4 + (-1) \cdot R_1 \to}$$

$$R_4 - (-1) \cdot R_1 \to R_4 \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times (2.518)$$

$$R_2 / (0.397) \to R_2 \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times (-10)$$

$\lambda_1 \approx 2.675$

$$A - \lambda_3 \underset{\sim}{I} \approx \begin{bmatrix} 1.325 & 8 & -1 & -2 \\ -2 & -11.675 & -2 & -4 \\ 0 & 10 & 2.325 & -10 \\ -1 & -13 & -14 & -15.675 \end{bmatrix}$$

$Av = \lambda v$

$(A - \lambda \underset{\sim}{I}) \cdot v = 0$

$$\left[\begin{array}{cccc|c} 1.325 & 8 & -1 & -2 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (0.754)$$

$$R_1 / (1.325) \to R_1 \quad \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ -2 & -11.675 & -2 & -4 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (2)$$

$$R_2 - (-2) \cdot R_1 \to R_2 \quad \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ -1 & -13 & -14 & -15.675 & 0 \end{array}\right] \times (1) \quad \cancel{R_4 - (-1) \cdot R_1 \to}$$

$$R_4 - (-1) \cdot R_1 \to R_4 \quad \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 0.397 & -3.509 & -7.018 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times (2.518)$$

$$R_2 / (0.397) \to R_2 \quad \left[\begin{array}{cccc|c} 1 & 6.036 & -0.754 & -1.509 & 0 \\ 0 & 1 & -8.834 & -17.669 & 0 \\ 0 & 10 & 2.325 & -10 & 0 \\ 0 & -6.964 & -14.754 & -17.184 & 0 \end{array}\right] \times (-10)$$

$x_3 = -1.838\, x_4 \; (1)$

$x_2 = 1.428\, x_4 \; (1) \rightarrow$

$x_1 = -8.494\, x_4 \; (1)$

$x_1 = -8.494\, x_4$

$x_2 = 1.428\, x_4$

$x_3 = -1.838\, x_4$

$x_4 = x_4$

$$X = \begin{bmatrix} -8.494\, x_4 \\ 1.428\, x_4 \\ -1.838\, x_4 \\ x_4 \end{bmatrix}$$

$$x_4 \cdot \begin{bmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{bmatrix}$$

Let $x_4 = 1$

$$\text{vector} \approx \begin{bmatrix} -8.494 \\ 1.428 \\ -1.838 \\ 1 \end{bmatrix}$$

Peer Member 2: (Eliel Ntwali)

4) $\lambda_4 \approx 11.054$

$$A - \lambda_4 I \approx \begin{pmatrix} -7.054 & 8 & -1 & -2 \\ -2 & -20.054 & -2 & -4 \\ 0 & 10 & -6.054 & -10 \\ -1 & -13 & -14 & -24.054 \end{pmatrix}$$

$Av = \lambda v$

$(A - \lambda I)^* v = 0$

So we have a homogeneous system of linear equations, We solve it By Gaussian Elimination

$$\left(\begin{array}{cccc|c} -7.054 & 8 & -1 & -2 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times (-0.142)$$

$$R_1/(-7.054) \to R_1 \left(\begin{array}{cccc|c} 1 & -1.136 & 0.142 & 0.284 & 0 \\ -2 & -20.054 & -2 & -4 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ -1 & -13 & -13 & -24.054 & 0 \end{array}\right) \times 2$$

$R_2-(-2)\cdot R_1 \to R_2$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 1 & -13 & -14 & -24.054 & 0 \end{array}\right) \times 1$

$R_4-(-1)\cdot R_1 \to R_4$

$\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & -22.322 & -1.716 & -3.433 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times(-0.045)$

$R_2/(-22.322) \to R_2$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 10 & -6.054 & -10 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \times(-10)$

$R_3-10R_2 \to R_3$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.823 & -11.538 & 0 \\ 0 & -14.134 & -13.858 & -23.771 & 0 \end{array}\right) \cdot 14.134$

$R_4-(-14.134)R_3 \to R_4$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & -6.828 & -11.538 & 0 \\ 0 & 0 & -12.771 & -21.597 & 0 \end{array}\right) -0.14$

$R_3(-6.823) \rightarrow R_3$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.154 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 1 & 0 & -12.771 & -21.597 & 0 \end{array}\right)$ 12.771

$R_4 - -12.771 R_3 \rightarrow R_4$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0.077 & 0.159 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array}\right)$ -0.077

$R_2 - 0.077 R_3 \rightarrow R_2$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0.142 & 0.284 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ -0.142

$R_1 - 0.142 R_3 \rightarrow R_1$ $\left(\begin{array}{cccc|c} 1 & -1.134 & 0 & 0.044 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$ $+1.134$

$R_1 - -1.134 R_2 \rightarrow R_1$ $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0.071 & 0 \\ 0 & 1 & 0 & 0.024 & 0 \\ 0 & 0 & 1 & 1.691 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$$\begin{cases} x_1 \qquad + 0.071x_4 = 0 \\ x_2 \quad + 0.024x_4 = 0 \\ x_3 + 1.691x_4 = 0 \end{cases} \quad \begin{aligned} x_1 &= -0.071x_4 \\ x_2 &= -0.024x_4 \\ x_3 &= -1.691x_4 \\ x_4 &= x_4 \end{aligned}$$

$$X = \begin{pmatrix} -0.071x_4 \\ -0.024x_4 \\ -1.691x_4 \\ x \end{pmatrix} \Rightarrow x_4 \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix}$$

$\boxed{\text{Let } x_4 = 1}$

$$\boxed{V_4 \approx \begin{pmatrix} -0.071 \\ -0.024 \\ -1.691 \\ 1 \end{pmatrix}}$$

Peer Member 3: (Edith Githinji)

Date: / /

Eingen vectors

$\lambda_3 = -5.604$

$$A-\lambda_3 I = \begin{bmatrix} 9.604 & 8 & -1 & -2 \\ -2 & -3.396 & -2 & -4 \\ 0 & 10 & 10.604 & -10 \\ -1 & -13 & -14 & -7.396 \end{bmatrix}$$

$AV = \lambda V$

$(A-\lambda I)\cdot V = 0$

Using Gaussian Eliminition

$$\left[\begin{array}{cccc|c} 9.604 & 8 & -1 & -2 & 0 \\ -2 & -3.396 & -2 & -4 & 0 \\ 0 & 10 & 10.604 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right] \times (0.1)$$

$R_1 / (9.604) \to R_1$

$$\left[\begin{array}{cccc|c} 1 & 0.833 & -0.104 & -0.208 & 0 \\ 0 & -1.730 & -2.208 & 7.416 & 0 \\ 0 & 10 & 10.608 & -10 & 0 \\ -1 & -13 & -14 & -7.396 & 0 \end{array}\right] \times 1$$

$R_2 / (-2) \cdot R_1 \to R_2$.

$$\left[\begin{array}{cccc|c} 1 & 0.318 & -0.040 & \cancel{0.080} & 0 \\ 0 & 12.761 & -2.080 & -6.159 & 0 \\ 0 & 10 & 26.125 & -10 & 0 \\ 0 & -12.682 & -14.040 & 8.045 & 0 \end{array}\right] \times 0.07$$

Date: / /

$R_4 / (27.754) \rightarrow R_3$

$$\begin{bmatrix} 1 & 0.318 & -0.040 & -0.08 & | & 0 \\ 0 & 1 & -0.163 & -0.326 & | & 0 \\ 0 & 0 & 1 & -0.243 & | & 0 \\ 0 & 0 & -16.106 & 3.912 & | & 0 \end{bmatrix} \times (16.106)$$

$R_4 - -16.106 R_3 \rightarrow R_4$

$$\begin{bmatrix} 1 & 0.318 & -0.04 & -0.08 & | & 0 \\ 0 & 1 & -0.163 & -0.326 & | & 0 \\ 0 & 0 & 1 & -0.243 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$R_1 - -0.04 \cdot R_3 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0.318 & 0 & -0.089 & | & 0 \\ 0 & 1 & 0 & -0.365 & | & 0 \\ 0 & 0 & 1 & -0.243 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix} \times (0.318)$$

$R_1 - 0.318 \cdot R_2 \rightarrow R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0.027 & | & 0 \\ 0 & 1 & 0 & -0.365 & | & 0 \\ 0 & 0 & 1 & 0 & | & 0 \\ 0 & 0 & 0 & 0 & | & 0 \end{bmatrix}$$

$x_1 + 0.027 = 0$

$x_2 + 0.365 = 0$

$x_3 + 0.243 = 0$

Variable for system (1)

$x_1 = -0.027$

$x_2 = -0.365$

$x_3 = -0.243$

$x_4 = x_4$

General solution

$$\begin{bmatrix} -0.027 x_4 \\ -0.365 x_4 \\ -0.243 x_4 \\ x_4 \end{bmatrix}$$

Solution set $= x_4 \cdot \begin{bmatrix} 0.027 \\ 0.365 \\ -0.243 \\ 1 \end{bmatrix}$