Escola de Economia de São Paulo - Fundação Getulio Vargas

Course: Econometria 2

Instructor: Vitor Possebom

Problem Set 1 - Deadline: May 16 at 9:00 am - Total = 390 points

Question 1 (Forecasting GDP Growth - 150 points)

Our goal is to forecast Brazilian Annual GDP Growth between 2021 and 2030. You will use the data in file `data_gdp_brazil.csv` and the following models:

1. *AR(1)*
2. *AR(2),*
3. *MA(1),*
4. *MA(2),*
5. *ARMA(1,1),*
6. *ARMA(2,1),*
7. *ARMA(1,2),*
8. *ARMA(2,2).*

For each model, you will report the following objects:

a. *All estimated coefficients (including the intercept), all coefficients' standard errors and all coefficients' p-values (testing whether each coefficient is zero or not) (5 points for each model - 40 points in total)*

b. *Estimated BIC and AIC (5 points for each model - 40 points in total)*

c. *Plot with a 10-year-ahead forecast and its 95%-confidence interval (5 points for each model - 40 points in total)*

Moreover, answer the following question (30 points): "Among these eight models, which one would you choose as the best model? Explain your choice."

Question 2 (ARMA(p,q) MLE Estimator's Asymptotic Behavior - 200 points) *In class, we used a Monte Carlo simulation to have a better understanding of the a MA(1) MLE estimator's asymptotic behavior. In particular, we focused on the estimator for the first moving average coefficient associated with the following MA(1) process:*

$$Y_t = c + \theta \cdot \epsilon_{t-1} + \epsilon_t,$$

where ϵ_t *was i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$.

In this question, you will use sample sizes of $30, 31, \ldots, 100, 110, 120, \ldots, 200, 250, 300, \ldots, 500$ *and follow the same steps to analyze the following topics:*

a. *convergence in probability*

b. *convergence in distribution*

c. *test size control*

of the following estimators:

1. *the intercept estimator of* $Y_t = c + \theta \cdot \epsilon_{t-1} + \epsilon_t$*, where* $c = 0$*,* $\theta = 0.5$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

2. *the intercept estimator of* $Y_t = c + \phi \cdot Y_{t-1} + \epsilon_t$*, where* $c = 0$*,* $\phi = 0.3$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

3. *the estimator for the first autoregressive coefficient of* $Y_t = c + \phi \cdot Y_{t-1} + \epsilon_t$*, where* $c = 0$*,* $\phi = 0.3$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

4. *the intercept estimator of* $Y_t = c + \phi \cdot Y_{t-1} + \theta \cdot \epsilon_{t-1} + \epsilon_t$, *where* $c = 0$, $\phi = 0.3$, $\theta = 0.5$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

5. *the estimator for the first autoregressive coefficient of* $Y_t = c + \phi \cdot Y_{t-1} + \theta \cdot \epsilon_{t-1} + \epsilon_t$, *where* $c = 0$, $\phi = 0.3$, $\theta = 0.5$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

6. *the estimator for the first moving average coefficient of* $Y_t = c + \phi \cdot Y_{t-1} + \theta \cdot \epsilon_{t-1} + \epsilon_t$, *where* $c = 0$, $\phi = 0.3$, $\theta = 0.5$ *and* ϵ_t *is i.i.d* $N(0,1)$ *or i.i.d* $\exp(1)$. *(5 points for each analyzed topic of each stochastic process - 30 points in total)*

Moreover, answer the following question (20 points): Based on the simulations above, do you feel comfortable with a sample size of 500 periods for any stochastic process? Explain your answer.

Question 3 (Examples: Stochastic Processes - 20 points)

1. *Give one example of a weakly stationary stochastic process. (10 points)*

2. *Give one example of a stochastic process that is not stationary. (10 points)*

Question 4 (Properties of a Weakly Stationary Process - 10 points)

Let Y_t *denote a weakly stationary process and* γ_j *denote its autocovariance. Show that* $\gamma_j = \gamma_{-j}$.