

Escola de Economia de São Paulo - Fundação Getulio Vargas

Course: Econometria 2

Instructor: Vitor Possebom

Problem Set 3 - Deadline: June 6 at 9:00 am - Total = 430 points

Question 1 (General Time Series Regression - 90 points)

*In this question, you will use the dataset `data_brazil.csv`. It contains four variables:*

- *`date`: Year (1901-2021)*
- *`real_gdp_growth_pct`: Annual Real GDP Growth measured in %.*
- *`exchange_rate_real_dolar_annual_average`: Annual Average Exchange Rate between Real and Dollar measured in R\$/US\$.*
- *`ipc_fipe_pct`: Annual Inflation measured by IPC-FIPE in %.*

*Your goal is to forecast GDP growth in 2020. To do so, we will use data from 1942 to 2019.*

1. *Model 1: Run an ADL(2,1) using GDP growth as your dependent variable and Exchange Rate as your predictor.*
  - (a) Report the estimated coefficient and their standard errors. (10 points)*
  - (b) Predict GDP growth in 2020 using model 1. (10 points)*
2. *Model 2: Run an ADL(2,2) using GDP growth as your dependent variable and inflation as your predictor.*
  - (a) Report the estimated coefficient and their standard errors. (10 points)*
  - (b) Predict GDP growth in 2020 using model 2. (10 points)*
3. *Model 3: Run an general time series regression model using GDP growth as your dependent variable and two lags of GDP growth, Exchange Rate and Inflation as your predictors.*

(a) Report the estimated coefficient and their standard errors. (10 points)

(b) Predict GDP growth in 2020 using model 3. (10 points)

4. Model 4: Run an ARMA(2,0) using GDP growth as your dependent variable.

(a) Report the estimated coefficient and their standard errors. (10 points)

(b) Predict GDP growth in 2020 using model 4. (10 points)

5. Which model generate the prediction that is closest to the realized value? (10 points)

**Question 2 (Expected Value of a VAR(p) Process - 20 points points)**

Assume that the vector process  $\{Y_t\}$  is covariance-stationary and follows a VAR(p) model.

Find its mean  $\mu =: \mathbb{E}[Y_t]$ .

**Question 3 (The j-th Autocovariance Matrix - 20 points)**

Let  $\{Y_t\}$  be a covariance-stationary  $n$ -dimensional vector process with mean  $\mu$ . Its  $j$ -th autocovariance is defined to be the following  $(n \times n)$  matrix:

$$\Gamma_j = \mathbb{E}[(Y_t - \mu)(Y_{t-j} - \mu)'].$$

Prove that

$$\Gamma'_j = \Gamma_{-j}.$$

**Question 4 (Vector MA(q) Process - 40 points)**

A vector moving average process of order  $q$  (VAR(p)) takes the form

$$Y_t = \mu + \epsilon_t + \Theta_1 \epsilon_{t-1} + \Theta_2 \epsilon_{t-2} + \dots + \Theta_q \epsilon_{t-q},$$

where  $\{\epsilon_t\}$  is a vector white noise process and  $\Theta_j$  denotes a  $(n \times n)$  matrix of MA coefficients for  $j \in \{1, 2, \dots, q\}$ .

Prove that any vector MA(q) process is covariance-stationary.

**Question 5 (A Type of LLN - 40 points)**

Prove Proposition 1 in the Lecture Notes. (I wrote this proposition below for your convenience.)

**Proposition 1** Let  $\{Y_t\}$  be a covariance-stationary process with moments given by

$$\mathbb{E}[Y_t] = \mu,$$

$$\mathbb{E}[(Y_t - \mu)(Y_{t-j} - \mu)'] = \Gamma_j$$

and with absolutely summable autocovariances (i.e.,  $(\sum_{v=-\infty}^{+\infty} \Gamma_v) \in \mathbb{R}$ ). Assume we have a sample of size  $T$  drawn from  $\{Y_t\}$ . Then, the sample mean  $\bar{Y}_T := \frac{\sum_{t=1}^T Y_t}{T}$  satisfies

1.  $\bar{Y}_T \xrightarrow{P} \mu$
2.  $\lim_{T \rightarrow +\infty} \left\{ T \cdot \mathbb{E}[(\bar{Y}_T - \mu)(\bar{Y}_T - \mu)'] \right\} = \sum_{v=-\infty}^{+\infty} \Gamma_v.$

**Question 6 (VAR(p) Model - 220 points)**

In this question, you will, once more, use the dataset `data.brazil.csv`. It contains four variables:

- `date`: Year (1901-2021)
- `real_gdp_growth_pct`: Annual Real GDP Growth measured in %.
- `exchange_rate_real_dollar_annual_average`: Annual Average Exchange Rate between Real and Dollar measured in  $\text{R\$}/\text{US\$}$ .
- `ipc_fipe_pct`: Annual Inflation measured by IPC-FIPE in %.

Your goal is to forecast GDP growth in 2020. To do so, we will use data from 1942 to 2019.

1. First, we will use a VAR(p) to forecast GDP growth in 2020.

(a) Model A: Set  $p = 1$  and estimate a reduced-form VAR(1) model.

- i. Report the estimated coefficient and their standard errors. (10 points)

- ii. Predict GDP growth in 2020 using model A. (10 points)
  - (b) Model B: Set  $p = 2$  and estimate a reduced-form VAR(2) model.
    - i. Report the estimated coefficient and their standard errors. (10 points)
    - ii. Predict GDP growth in 2020 using model B. (10 points)
  - (c) Model C: Set  $p = 3$  and estimate a reduced-form VAR(3) model.
    - i. Report the estimated coefficient and their standard errors. (10 points)
    - ii. Predict GDP growth in 2020 using model C. (10 points)
  - (d) Which model generate the prediction that is closest to the realized value? (10 points)
2. Now, we will focus on a VAR(2) model and focus on structural IRFs.
- (a) Choose the order of your variables and justify your exclusion restrictions. (40 points)
  - (b) Estimate and plot all nine structural impulse response functions and their 90%-confidence intervals based on 1,000 bootstrap repetitions. (90 points)
  - (c) Do you believe your results are credible? Justify your answer. (20 points)