

# INDEX

1. If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

(a) 4-cell reuse.                      (b) 7-cell reuse.                      (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

2. If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a)  $n = 4$ .                                      (b)  $n = 3$ .

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

3. How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

(a) 1,                      (b) 5,                      (c) 10,                      (d) 20,                      (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

4. An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

5. A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

- a) The number of cells in the service area,
- b) The number of channels per cell,
- c) Traffic intensity of each cell,
- d) The maximum carried traffic,
- e) The total number of users that can be served for 2% GOS,
- f) The number of mobiles per channel, and
- g) The theoretical maximum number of users that could be served at one time by the system.

6. If a transmitter produces 50 watts of power, express the transmit power in units of
  - a) dBm,
  - and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

  - a) Find the received power in dBm at a free space distance of 100 m from the antenna,
  - b) What is P (10 km)?

Assume unity gain for the receiver antenna.
7. Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4Km.
8. Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.
9. A mobile is located 5 km away from a base station and uses a vertical  $\lambda/4$  monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.
  - a) Find the length and the gain of the receiving antenna.
  - b) Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.
10. A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and  $\lambda =$  call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-
  - a) How many users per square kilometer will this system support?
  - b) What is the probability that a delayed call will have to wait for more than 10s?
  - c) What is the probability that a call will be delayed for more than 10 seconds?

**Problem-1:** If a total of 33 MHz of bandwidth is allocated to a particular FDD cellular telephone system which uses two 25 kHz simplex channels to provide full duplex voice and control channels, compute the number of channels available per cell if a system uses-

- (a) 4-cell reuse.                      (b) 7-cell reuse.                      (c) 12-cell reuse.

If 1 MHz of the allocated spectrum is dedicated to control channels, determine an equitable distribution of control channels and voice channels in each cell for each of the three systems.

**Solution:**

Have given,

$$\begin{aligned}
 \text{Total bandwidth} &= 33 \text{ MHz,} \\
 &= 33,000 \text{ kHz} \\
 \text{Channel bandwidth} &= 25 \text{ kHz} \times 2 \text{ simplex channels} \\
 &= 50 \text{ kHz / duplex channel} \\
 \text{Total available channels} &= \text{Total Bandwidth / Channel Bandwidth} \\
 &= 33,000 / 50 \\
 &= 660 \text{ channels}
 \end{aligned}$$

*If 1 MHz of the allocated spectrum is dedicated to control channels,*

$$\text{i.e. Control channel bandwidth} = 1000 \text{ kHz}$$

$$\begin{aligned}
 \text{The number of available control channel} &= \text{Control channel bandwidth / Channel bandwidth} \\
 &= 1000 / 50 \\
 &= 20 \text{ channels}
 \end{aligned}$$

**(a)**

Have given,

$$\text{Cluster size, } N = 4$$

$$\begin{aligned}
 \text{Total number of channels available per cell} &= \text{Total available channels} / N \\
 &= 660 / 4 \\
 &\approx 165 \text{ channels.}
 \end{aligned}$$

**∴ Equitable distribution of,**

$$\begin{aligned}
 \text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\
 &= (660 - 20) / 4 \\
 &\approx 160 \text{ channels}
 \end{aligned}$$

$$\begin{aligned}
 \text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\
 &= 165 - 160 \\
 &= 5 \text{ channels}
 \end{aligned}$$

**(b)**

Have given,

Cluster size,  $N = 7$

$$\begin{aligned}\text{Total number of channels available per cell} &= \text{Total available channels} / N \\ &= 660/7 \\ &\approx 94 \text{ channels.}\end{aligned}$$

**∴ Equitable distribution of,**

$$\begin{aligned}\text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\ &= (660 - 20) / 7 \\ &\approx 91 \text{ channels}\end{aligned}$$

$$\begin{aligned}\text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\ &= 94 - 91 \\ &= 3 \text{ channels}\end{aligned}$$

**(c)**

Have given,

Cluster size,  $N = 12$

$$\begin{aligned}\text{Total number of channels available per cell} &= \text{Total available channels} / N \\ &= 660/12 \\ &\approx 55 \text{ channels.}\end{aligned}$$

**∴ Equitable distribution of,**

$$\begin{aligned}\text{Voice Channel} &= (\text{Total available channels} - \text{The number of available control channel}) / N \\ &= (660 - 20) / 12 \\ &\approx 53 \text{ channels}\end{aligned}$$

$$\begin{aligned}\text{Control Channel} &= \text{Total number of channels available per cell} - \text{Voice Channel} \\ &= 55 - 53 \\ &= 2 \text{ channels}\end{aligned}$$

**Source Code (MATLAB):**

```
clc;
clear all;
close all;
cluster_sizes = input('Enter Cluster Sizes with [ ] around Them :'); % Such as [4 7 12]
```

```

bw = 33000;           % Total Bandwidth in kHz
sim_ch_bw = 25;       % Simplex channel bandwidth in kHz
dup_ch_bw = 2*sim_ch_bw; % Duplex channel bandwidth in kHz

t_ch = (bw/dup_ch_bw); % Total available channels

cc_bw = 1000;         % Control channel bandwidth
t_cc = cc_bw/dup_ch_bw; % The number of available control

for N = cluster_sizes

    % Calculate Desired Result For each system uses
    ch_per_cell = round(t_ch/N); % Channels available per cell
    vc = round((t_ch-t_cc)/N); % Voice channel
    cc = ch_per_cell-vc; % Control channel

    % Result Print
    fprintf('For Cluster size N = %d\n',N);
    disp('-----');
    fprintf('Total number of channels available per cell : %d channels\n',ch_per_cell);
    fprintf('Voice Channel : %d channels\n',vc);
    fprintf('Control Channel : %d channels\n',cc);
    fprintf('\n\n');
end

```

### **Input:**

Enter Cluster Sizes with [ ] around them: [4 7 12].

### **Output:**

#### **For Cluster size N = 4**

Total number of channels available per cell	: 165 channels
Voice Channel	: 160 channels
Control Channel	: 5 channels

#### **For Cluster size N = 7**

Total number of channels available per cell	: 94 channels
Voice Channel	: 91 channels
Control Channel	: 3 channels

#### **For Cluster size N = 12**

Total number of channels available per cell	: 55 channels
Voice Channel	: 53 channels
Control Channel	: 2 channels

**Problem-2:** If a signal to interference ratio of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss exponent is-

(a)  $n = 4$ .

(b)  $n = 3$ .

Assume that there are 6 co-channels cells in the first tier and all of them are at the same distance from the mobile. Use suitable approximations.

**Solution:**

Have given,

Minimum Required Signal-to-Noise interference ratio,  $S/I = 15$  dB,

The number of Co-channel interfering cells,  $i_0 = 6$

We Know,

Number of cell reuse,  $N = i^2 + i*j + j^2$  ..... (i)

First, let us consider a 7-cell reuse pattern,  $N = 7$  For  $i=1, j=2$

Also,

The Frequency Reuse Factor,  $Q = D/R$

$= \sqrt{(3N)}$  ..... (ii)  
 $= 4.583$ .

Where,

$D$  = Distance between centers of the nearest Co-channel cells.

$R$  = Radius of the cell.

**(a)**

Have given,

Path Loss exponent,  $n = 4$

**Frequency Reuse Factor,  $Q = 4.583$ .**

We know,

Signal-to-Noise interference ratio,  $S/R = 10 \log (Q^n / i_0)$   
 $= 10 \log ((4.583)^4 / 6)$   
 $= 18.66$  dB.

Since this is greater than the minimum required  $S/I$  ( $18.66 > 15$ ),  $N = 7$  can be used.

**(b)**

Have given,

Path Loss exponent,  $n = 3$

We know,

Signal-to-Noise interference ratio,  $S/R = 10 \log (Q^n / i_0)$  ..... (iii)  
 $= 10 \log ((4.583)^3 / 6)$   
 $= 12.05$  dB.

*Since this is less than the minimum required S/I ( $12.05 < 15$ ), we need to use a larger N.*

Using equation (i), the next possible value of  $N = 12$ ; For  $i = j = 2$ .

The corresponding co-channel ratio is given by equation (ii) as-

**Frequency Reuse Factor,  $Q = 6$ .**

Using equation (iii) the signal-to-interference ratio,  $S/I = 15.56$  dB.

**Since, this is greater than the minimum required S/I ( $15.56 > 15$ ),  $N = 12$  can be used.**

### **Source Code (MATLAB):**

```
clc;
clear all;
close all;

pl_exponent = input('Enter Path Loss exponent with [ ] around Them :'); % Such as [4 3]
r_si = 15; % Minimum Required S/I in dB
i0 = 6; % The number of Co-channel interfering cells
for n = pl_exponent
    N = 7; % Cluster size

    % Calculate Result
    Q = sqrt(3*N); % Frequency reuse factor
    si = 10*(log10((Q^n)/i0)); % Signal to interference ratio in dB

    % If First Condition is Not Satisfied
    if (si < r_si)
        i = 2; j = 2;
        N = (i*i)+(i*j)+(j*j);
        Q = sqrt(3*N);
        si = 10*(log10((Q^n)/i0));
    end

    % Result Print
    fprintf('For Path Loss Exponent, n = %d\n',n);
    disp('-----');
    fprintf('Signal-to-Noise interference Ratio S/I: %7.3f dB > %d dB\n',si,r_si);
    fprintf('Hence, Cluster size N: %d\n',N);
    fprintf('Frequency Reuse Factor Q: %7.3f\n',Q);
    fprintf('\n\n');
end
```

**Input:**

Enter Path Loss exponent with [ ] around them: [4 3].

**Output:****For Path Loss Exponent, n = 4**

Signal-to-Noise interference Ratio, S/ I: 18.663 dB > 15 dB

Hence, Cluster size, N : 7

Frequency Reuse Factor, Q : 4.583

**For Path Loss Exponent, n = 3**

Signal-to-Noise interference Ratio, S/I : 15.563 dB > 15 dB

Hence, Cluster size, N : 12

Frequency Reuse Factor, Q : 6.000



**Problem-3:** How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

- (b) 1, (b) 5, (c) 10, (d) 20, (e) 100.

Assume each user generates 0.1 Erlangs of traffic.

**Solution:**

Have given,

Blocking Probability,  $P_B = 0.5\%$ ,

Traffic Intensity,  $A_u = 0.1$  Erlangs

We Know,

For Erlangs B, Grade of Service,  $GOS = P_B$

$$= 0.005$$

And, Total number of user,  $U = A / A_u \dots\dots\dots (i)$

Where,

$A$  = Offered Traffic Intensity.

Also,

**Table 3.1:** Capacity of an Erlang B System.

Number of Channels $C$	Capacity (Erlangs) for GOS			
	= 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

From Table 3.1, we can find the total capacity in Erlangs for the 0.5% GOS for different numbers of channels.

**(a)**

Have given,

Trunked channels,  $C = 1$

From table 3.1, For  $C = 1$  we obtain,  $A = 0.005$

From equation (i), we have-

$$\begin{aligned} \text{Total number of user, } U &= A / A_u \\ &= 0.05 \text{ users.} \end{aligned}$$

**But, actually one user could be supported on one channel. So,  $U = 1$ .**

**(b)**

Have given,

Trunked channels,  $C = 5$

From table 3.1, For  $C = 5$  we obtain,  $A = 1.13$  Erlang.

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &\approx 11 \text{ users.}\end{aligned}$$

**(c)**

Have given,

Trunked channels,  $C = 10$

From table 3.1, For  $C = 10$  we obtain,  $A = 3.96$  Erlang

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &\approx 39 \text{ users.}\end{aligned}$$

**(d)**

Have given,

Trunked channels,  $C = 20$

From table 3.1, For  $C = 20$  we obtain,  $A = 11.10$  Erlang

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &\approx 110 \text{ users.}\end{aligned}$$

**(e)**

Have given,

Trunked channels,  $C = 100$

From table 3.1, For  $C = 100$  we obtain,  $A = 80.9$  Erlang.

From equation (i), we have-

$$\begin{aligned}\text{Total number of user, } U &= A / A_u \\ &\approx 809 \text{ users.}\end{aligned}$$

### **Source Code (MATLAB):**

```
clc;
clear all;
close all;
GOS = 0.5/100;    % Blocking probability (0.5%)
Au = 0.1;        % Traffic intensity per user

% From Erlang B chart Offered Traffic Intensity, A
A = [0.005 1.13 3.96 11.1 80.9];
% Trunked Channels
C = [1 5 10 20 100];

U = round(A/Au); % Total number of user

% Result Print
fprintf('Grade of Service, GOS = %7.3f\n',GOS);
disp('Trunked Channels, C :');
disp(C);
disp('From table 3.1, we obtain Offered Traffic Intensity, A For all Channels, C :');
disp(A);
disp('Total number of user, U');
disp('-----');
disp(U);
```

### **Input:**

Trunked Channels, C = [1 5 10 20 100];

### **Output:**

Grade of Service, GOS = 0.005

Trunked Channels,	C:	1	5	10	20	100
Offered Traffic Intensity,	A:	0.0050	1.1300	3.9600	11.1000	80.9000

<b><u>Total number of user,</u></b>	<b>U:</b>	0	11	40	111	809
-------------------------------------	-----------	---	----	----	-----	-----

**Problem-4:** An urban area has a population of 2 million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages 2 calls per hour at an average call duration of 3 minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

**Solution:**

Have given,

Blocking Probability,  $P_B$  = 2%,  
 The average number of call requests per unit time  $\lambda$  = 2.  
 The average duration of a call,  $H$  = 3/60 seconds  
 There are 2 million residents in the given urban area = 2000000

We Know,

For Erlangs B, Grade of Service,  $GOS = P_B$   
 = 0.02  
 And, Traffic Intensity,  $A_u$  =  $\lambda H$   
 = 0.1 Erlangs

Also, Total number of user,  $U$  =  $A / A_u$  ..... (i)

Where,

$A$  = Offered Traffic Intensity.

Also,

**Table 4.1:** Capacity of an Erlang B System.

Erlang B Traffic Table												
Maximum Offered Load Versus B and N												
N/B	B is in %											
	0.01	0.05	0.1	0.5	1.0	2	5	10	15	20	30	40
1	.0001	.0005	.0010	.0050	.0101	.0204	.0526	.1111	.1765	.2500	.4286	.6667
2	.0142	.0321	.0458	.1054	.1526	.2235	.3813	.5954	.7962	1.000	1.449	2.000
3	.0868	.1517	.1938	.3490	.4555	.6022	.8994	1.271	1.603	1.930	2.633	3.480
4	.2347	.3624	.4393	.7012	.8694	1.092	1.525	2.045	2.501	2.945	3.891	5.021
5	.4520	.6486	.7621	1.132	1.361	1.657	2.219	2.881	3.454	4.010	5.189	6.596
9	1.826	2.302	2.558	3.333	3.783	4.345	5.370	6.546	7.551	8.522	10.58	13.05
10	2.260	2.803	3.092	3.961	4.461	5.084	6.216	7.511	8.616	9.685	11.95	14.68
16	5.339	6.250	6.722	8.100	8.875	9.828	11.54	13.50	15.18	16.81	20.30	24.54
17	5.911	6.878	7.378	8.834	9.652	10.66	12.46	14.52	16.29	18.01	21.70	26.19
18	6.496	7.519	8.046	9.578	10.44	11.49	13.39	15.55	17.41	19.22	23.10	27.84
19	7.093	8.170	8.724	10.33	11.23	12.33	14.32	16.58	18.53	20.42	24.51	29.50
20	7.701	8.831	9.412	11.09	12.03	13.18	15.25	17.61	19.65	21.64	25.92	31.15
56	33.49	36.13	37.46	41.23	43.32	45.88	50.54	56.06	60.98	65.94	77.00	90.97
57	34.27	36.95	38.29	42.11	44.22	46.82	51.55	57.14	62.14	67.18	78.43	92.64
94	64.25	68.07	69.98	75.41	78.43	82.17	89.10	97.53	105.3	113.2	131.2	154.3
95	65.08	68.93	70.85	76.33	79.37	83.13	90.12	98.63	106.4	114.4	132.6	155.9
100	69.27	7~.25	75.24	80.91	84.06	87.97	95.24	104.1	112.3	120.6	139.7	164.3

N is the number of servers. The numerical column headings indicate blocking probability B in %. Table generated by Dan Dexter

From Table 4.1, we can find the total capacity in Erlangs for the 2% GOS for different numbers of channels.

### **For System-A**

Have given,

Number of channels per cell used in the system,  $C = 19$

From table 4.1, For  $C = 19$  and  $GOS = 0.02$  we obtain,  $A = 12$  Erlangs

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $= 120$  users.

*Since there are 394 cells, the total number of subscribers that can be supported by **System A** is equal to  $120 \times 394 = 47280$ .*

**Since, the percentage market penetration**  $= 47280/2000000 = 2.36\%$

### **For System-B**

Have given,

Number of channels per cell used in the system,  $C = 57$

From table 4.1, For  $C = 57$  and  $GOS = 0.02$  we obtain,  $A = 45$  Erlangs

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $= 450$  users.

*Since there are 98 cells, the total number of subscribers that can be supported by **System B** is equal to  $450 \times 98 = 44,100$ .*

**Since, the percentage market penetration**  $= 44100/2000000 = 2.205\%$

### **For System-C**

Have given,

Number of channels per cell used in the system,  $C = 100$

From table 4.1, For  $C = 100$  and  $GOS = 0.02$  we obtain,  $A = 88$  Erlangs

From equation (i), we have-

Total number of user,  $U = A / A_u$   
 $= 880$  users.

*Since there are 49 cells, the total number of subscribers that can be supported by **System C** is equal to  $880 \times 49 = 43,120$ .*

**Since, the percentage market penetration**  $= 43,120/2000000 = 2.156\%$

Therefore, total number of cellular subscribers that can be supported by these three systems are  $(47280 + 44100 + 43120) = 134500$  users.

***The market penetration of the three systems combined is equal to  $134500/2000000 = 6.725$***

## Source Code (MATLAB):

```
clc;
clear all;
close all;
blocking_probability = 2/100;      % GOS
population = 2000000;
Au = (2/60)*3;                    % Traffic intensity per user
disp('For system A:');
disp('-----');
C1 = 19;                          % Number of channels per cell
A1 = 12;                          % Total traffic intensity from
erlang B chart, GOS=0.02, C=19
U1 = A1/Au;                       % Total number of user
Aa = U1*394;                      % Total Number of Subscriber
percentage_A = (Aa/population)*100;
fprintf('Total number of users for system A: %d\n',Aa);
fprintf('Percentage market penetration for System A: %7.3f%%\n',percentage_A);

fprintf('\n\nFor system B:\n');
disp('-----');
C2 = 57;                          % Number of channels per cell
A2 = 45;                          % Total traffic intensity from
erlang B chart, GOS=0.02, C=57
U2 = A2/Au;                       % Total number of user
Bb = U2*98;                       % Total Number of Subscriber
percentage_B = (Bb/population)*100;

fprintf('Total number of users for system B: %d\n',Bb);
fprintf('Percentage market penetration for System B: %7.3f%%\n',percentage_B);

fprintf('\n\nFor system C:\n');
disp('-----');
C3 = 100;                         % Number of channels per cell
A3 = 88;                         % Total traffic intensity from
erlang B chart, GOS=0.02, C=100
U3 = A3/Au;                       % Total number of user
Cc = U3*49;                       % Total Number of Subscriber
percentage_C = (Cc/2000000)*100;

fprintf('Total number of users for system C: %d\n',Cc);
fprintf('Percentage market penetration for System C: %7.3f%%\n',percentage_C);

fprintf('\n\nFor all three systems:\n');
disp('-----');

T = Aa+Bb+Cc; % Total Subscribers
percentage_T = (T/2000000)*100;
fprintf('Total number of users of all three system: %d\n',T);
fprintf('Percentage market penetration for all three System: %7.3f%%\n',percentage_T);
```

**Input:**

Trunked Channels,  $C = [19\ 57\ 100]$ ;

**Output:****For system A:**

Total number of users for system A: 47280

Percentage market penetration for System A: 2.364%

**For system B:**

Total number of users for system B: 44100

Percentage market penetration for System B: 2.205%

**For system C:**

Total number of users for system C: 43120

Percentage market penetration for System C: 2.156%

**For all three systems:**

Total number of users of all three system: 134500

Percentage market penetration for all three System: 6.725%

**Problem-5:** A certain city has an area of 1,300 square miles and is covered by a cellular system using a 7-cell reuse pattern. Each cell has a radius of 4 miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute-

- The number of cells in the service area,
- The number of channels per cell,
- Traffic intensity of each cell,
- The maximum carried traffic,
- The total number of users that can be served for 2% GOS,
- The number of mobiles per channel, and
- The theoretical maximum number of users that could be served at one time by the system.

**Solution:**

**(a)**

Have given,

Total coverage area = 1300 miles

Cell radius = 4 miles

We know,

The area of a cell (hexagon) can be shown to be  $2.5981R^2$

Thus each cell covers  $2.5981 \times (4)^2 = 41.57$  sq km.

**Hence, the total number of cells,  $N_c = 1300/41.57 = 31$  cells**

**(b)**

Have given,

Allocated spectrum = 40, 000,000 Hz

Channel width = 60,000 Hz

Frequency reuse factor, N = 7 cells

We know,

The total number of channels per cell,  $C = \text{Allocated spectrum} / (\text{Channel width} \times N)$

$= 40, 000,000 / (60,000 \times 7)$

$= 95$  channels/cell

**(c)**

Have given,

From (b) No, C = 95

And, GOS = 0.02

From the table 4.1 (Erlang B chart) For C = 95 and GOS = 0.02, we have-

Traffic intensity per cell, A = 84 Erlangs/cell



**(d)**

Have given,

From (a), Number of cells = 31 cells

From (c), Traffic intensity per cell = 84 Erlangs/cell

We Know,

$$\begin{aligned}\text{Maximum carried traffic} &= \text{Number of cells} \times \text{Traffic intensity per cell} \\ &= 31 \times 84 \\ &= 2604 \text{ Erlangs.}\end{aligned}$$

**(e)**

Have given,

Traffic per user,  $A_u$  = 0.03 Erlangs

From (d), Total traffic,  $A$  = 2604 Erlangs.

We Know,

$$\begin{aligned}\text{Total number of users, } U &= A / A_u \\ &= 2604 / 0.03 \\ &= 86,800 \text{ users.}\end{aligned}$$

**(f)**

Have given,

Allocated spectrum = 40,000,000 Hz

Channel width = 60,000 Hz

From (e), Number of users,  $U$  = 86,800 users.

We Know,

$$\begin{aligned}\text{Number of channels} &= \text{Allocated Spectrum} / \text{Channel Width} \\ &= 40,000,000 / 60,000 \\ &\approx 666\end{aligned}$$

$$\begin{aligned}\text{Number of mobiles per channel} &= \text{Number of users} / \text{Number of channels} \\ &= 86,800 / 666 \\ &\approx 130 \text{ mobiles/channel}\end{aligned}$$

**(g)**

Have given,

From (b) No,  $C$  = 95 channels/cell

From (a), the total number of cells,  $N_c$  = 31 cells.

From (e) Total number of users,  $U$  = 86,800 users.

We Know,

The theoretical maximum number of served mobiles is the number of available channels in the system (all channels occupied)

$$\begin{aligned}&= C \times N_c \\ &= 95 \times 31 = 2945 \text{ users,}\end{aligned}$$

Which is  $(2945/86,800) \times 100 = 3.4\%$  of the customer base.

### **Source Code (MATLAB):**

```
clc;
clear all;
close all;
% Question (a)
total_city_coverage_area = 1300;
radius = 4;
a = (2.591*radius^2); % Each cell covers
Nc = round(total_city_coverage_area/a); % Total number of cells, Nc
fprintf('(a) Total number of cells, Nc : %d cells\n\n',Nc);

% Question (b)
allocated_spectrum = 40000; % Allocated spectrum=40MHz,
channel_width = 60; % Full duplex channel BW = 60KHz
N = 7;
C = round(allocated_spectrum/(channel_width*N));
fprintf('(b) The total number of channels per cell, C : %d channels/cell\n\n',C);

% Question (c)
A = 84; % C=95, GOS=0.02 from erlang B chart
fprintf('(c) Traffic intensity per cell, A : %d Erlangs/cell\n\n',A);

% Question (d)
max_c_t = floor(Nc*A); % Number of Cell, Nc & Traffic Intensity per cell, A
fprintf('(d) Maximum carried traffic : %d Erlangs\n\n',max_c_t);

% Question (e)
U = round(max_c_t/0.03); % Traffic per user, Au = 0.03
fprintf('(e) Total number of users, U : %d users\n\n',U);

% Question (f)
no_of_channel = floor(allocated_spectrum/channel_width);
no_of_m_p_c = floor(U/no_of_channel); % Number of mobiles per channel
fprintf('(f) Number of mobiles per channel : %d mobiles/channel\n\n',no_of_m_p_c);

% Question (g)
g = C*Nc;
fprintf('(g) Theoretical maximum no of user that could be served : %d users\n\n',g);
```

**Output:**

- (a) Total number of cells,  $N_c$  : 31 cells
- (b) The total number of channels per cell,  $C$  : 95 channels/cell
- (c) Traffic intensity per cell,  $A$  : 84 Erlangs/cell
- (d) Maximum carried traffic : 2604 Erlangs
- (e) Total number of users,  $U$  : 86800 users
- (f) Number of mobiles per channel : 130 mobiles/channel
- (g) Theoretical maximum no of user that could be served: 2945 users

**Problem-6:** If a transmitter produces 50 watts of power, express the transmit power in units of

a) dBm, and b) dBW.

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

c) Find the received power in dBm at a free space distance of 100 m from the antenna,

d) What is P (10 km)?

Assume unity gain for the receiver antenna.

### **Solution:**

Have given,

Transmitter power,  $P_t = 50 \text{ W}$

Carrier frequency,  $f_c = 900 \text{ MHz}$

**(a)**

We know,

$$\begin{aligned} \text{Transmitter power, } P_t(\text{dBm}) &= 10 \log[P_t(\text{mW})/(1\text{mW})] \\ &= 10 \log [50 \times 10^3] \\ &= 47.0 \text{ dBm} \end{aligned}$$

**(b)**

We know,

$$\begin{aligned} \text{Transmitter power, } P_t(\text{dBW}) &= 10 \log[P_t(\text{W})/(1\text{W})] \\ &= 10 \log [50] \\ &= 17.0 \text{ dBW} \end{aligned}$$

**(c)**

If 50 watts is applied to a unity gain antenna with a 900 MHz carrier frequency,

Have given,

$$\begin{aligned} \text{Transmitter Gain, } G_t &= 1 \\ \text{Receiver Gain, } G_r &= 1 \\ \text{Wave length } \lambda &= c / f = 1 / 3 \text{ m} \\ \text{The T-R separation distance, } d &= 100\text{m} \\ \text{The system loss factor, } L &= 1 \end{aligned}$$

We know,

$$\begin{aligned} \text{The received power, } P_r &= (P_t \times G_t \times G_r \times \lambda^2) / (4\pi^2 \times d^2 \times L) \\ &= (50 \times 1 \times 1 \times (1/3)^2) / ((4\pi)^2 \times 100^2 \times 1) \\ &= 3.5 \times 10^{-3} \text{ mW} \\ \text{Received power, } P_r(\text{dBm}) &= 10 \log[P_r(\text{mW})] \\ &= 10 \log[P_r(3.5 \times 10^{-3})] \\ &= -24.5 \text{ dBm} \end{aligned}$$

(d)

Have given,

$$d_o = 10 \text{ km} = 10000 \text{ m}$$

We Know,

The received power at 10 km can be expressed in terms of dBm, we have

$$\begin{aligned}\therefore P_r(10 \text{ km}) &= P_r(100) + 20 \log[d / d_o] \\ &= P_r(100) + 20 \log[100 / 10000] \\ &= -24.5 - 40 \\ &= -64.5 \text{ dBm}\end{aligned}$$

### Source Code (MATLAB):

```
clc;
clear all;
close all;
pt = 50;      % Transmitted power
fc = 900;     % Carrier frequency
gt = 1;       % Transmitter antenna gain
gr = 1;       % Receiver antenna gain
d = 100;      % Free space distance
L = 1;
lambda = (3*10e8)/(fc*10e6); % lambda = c/f = 1/3

% Quesstion (a)
tr_dBm = ceil(10*log10(pt*1000));
fprintf('(a) Transmitter power,Pt in dBm: %d dBm\n\n',tr_dBm);

% Quesstion (b)
tr_dBW = ceil(10*log10(pt*1));
fprintf('(b) Transmitter power,Pt in dBW: %d dBW\n\n',tr_dBW);

% Quesstion (c)
c = ((pt*gt*gr*(lambda)^2)/((4*3.1416)^2*d^2*L))*1000;
Pr = 10*log10(c);
fprintf('(c) Received power,Pr in dBm: %7.2f dBm\n\n',Pr);

% Quesstion (d)
d = Pr + (20*log10(d/10000));
fprintf('(d) Received power,Pr at 10km in dBm: %7.2f dBm\n\n',d);
```

### Output:

(a) Transmitter power, $P_t$ in dBm	: 47 dBm
(b) Transmitter power, $P_t$ in dBW	: 17 dBW
(c) Received power, $P_r$ in dBm	: -24.54 dBm
(d) Received power, $P_r$ at 10km in dBm	: -64.54 dBm

**Problem-7:** Determine the path loss of a 900MHz cellular system in a large city from a base station with the height of 100m and mobile station installed in a vehicle with antenna height of 2m. The distance between mobile and base station is 4 km.

**Solution:**

Have given,

The frequency, $f_c$	= 900 MHz (150 MHz to 1500MHz)
The effective transmitter (base station) antenna height, $h_{te}$	= 100m
The effective transmitter (mobile) antenna height, $h_{re}$	= 2m
T-R separation distance, $d$	= 4 km

Now, The correction factor for effective mobile antenna height,

$$a(h_{re}) = 3.2 (\log 11.75 h_{re})^2 - 4.97 \text{ dB for } f_c \geq 300 \text{ MHz}$$

From Okumura-Hata Model we know,

The path loss in urban areas is given by

$$\begin{aligned} L_{50}(\text{urban})(\text{dB}) &= 69.55 + 26.16 \log f_c - 13.82 \log h_{te} - a(h_{re}) \\ &\quad + (44.9 - 6.55 \log h_{te}) \log d \\ &= 69.55 + 26.16 \times 2.954 - 13.82 \times 2 - 1.045 + (44.9 - 13.1) \times 0.6 \\ &= 137.3 \text{ dB} \end{aligned}$$

**Source Code (MATLAB):**

```
clc;
clear all;
close all;
hte = 100;    % Effective transmitter (base station) antenna height in meter
hre = 2;      % Effective receiver (mobile) antenna height in meter
fc = 900;     % Frequency in MHz
d = 4;        % T-R separation distance in kilometer

a_hre = (3.2*(log10(11.75*hre))^2)-4.97;    % The correction factor(using okumura-hata model)

% Path Loss in urban areas
Lp = 69.55 + 26.16*log10(fc)-13.82*log10(hte)-a_hre+((44.9-6.55*log10(hte))*log10(d));
fprintf('The path loss in urban areas, Lp = %7.2f dB\n',Lp);
```

**Input:**

```
hte = 100;    % Effective transmitter (base station) antenna height in meter
hre = 2;      % Effective receiver (mobile) antenna height in meter
fc = 900;     % Frequency in MHz
d = 4;        % T-R separation distance in kilometer
```

**Output:**

The path loss in urban areas,  $L_p = 137.29 \text{ dB}$ .

**Problem-8:** Determine the path loss between base station (BS) and mobile station (MS) of a 1.8GHz PCS system operating in a high-rise urban area. The MS is located in a perpendicular street to the location of the BS. The distances of the BS and MS to the corner of the street are 20 and 30 meters, respectively. The base station height is 20m.

### Solution:

Have given,

$$\begin{aligned} \text{The frequency, } f_c &= 1.8 \text{ GHz (0.9 to 2 GHz)} \\ \text{The effective transmitter (base station) antenna height, } h_b &= 20\text{m} \\ \text{T-R separation distance, } d &= \sqrt{(20^2 + 30^2)} = 0.036 \text{ km} \end{aligned}$$

From Okumura-Hata Model we know,

The path loss in a high-rise urban areas with Perpendicular Street to the location of the Base Station is given by-

$$\begin{aligned} L_p &= 135.41 + 12.49 \log f_c - 4.99 \log h_b + [46.84 - 2.34 \log h_b] \log d \\ &= 135.41 + 12.49 \times \log (1.8) - 4.99 \times \log 20 + [46.84 - 2.34 \log 20] \times \log 0.036 \\ &= 68.91 \text{ dB} \end{aligned}$$

### Source Code (MATLAB):

```
clc;
clear all;
close all;
fc = 1.8;      % Frequency in GHz
hb = 20;      % Effective transmitter (base station) antenna height in meter

d = (sqrt(20^2+30^2))/1000;    % T-R separation distance in kilometer

% Path Loss in a high-rise urban areas
Lp = 135.41+(12.49*log10(fc))-(4.99*log10(hb))+((46.84-2.34*log10(hb))*log10(d));
fprintf('The path loss in a high-rise urban areas, Lp = %7.2f dB\n',Lp);
```

### Input:

```
hb = 20;      % Effective transmitter (base station) antenna height in meter
fc = 1.8;      % Frequency in GHz
```

### Output:

The path loss in a high-rise urban areas,  $L_p = 68.91 \text{ dB}$

**Problem-9:** A mobile is located 5 km away from a base station and uses a vertical  $\lambda/4$  monopole antenna with a gain of 2.55 dB to receive cellular 3 radio signals. The E-field at 1 km from the transmitter is measured to be V/m. The carrier frequency used for this system is 900 MHz.

- Find the length and the gain of the receiving antenna.
- Find the received power at the mobile using the 2-ray ground reflection model assuming the height of the transmitting antenna is 50 m and the receiving antenna is 1.5m above ground.

**Solution:**

Have given,

Frequency of operation, f	= 900 MHz
Gain of antenna, G	= 1.8 = 2.55 dB

**(a)**

We Know,

Wave length,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{900 \times 10^6} = 0.333 \text{ m.}$$

$$\text{Length of the antenna, } L = \lambda/4 = 0.333/4 = 0.0833 \text{ m} = 8.33 \text{ cm.}$$

And, Gain of antenna, G = 2.55 dB.

**(b)**

Have given,

T-R separation distance, d	= 5 km
E-field at a distance of 1 km, E <sub>o</sub>	= 10 <sup>-3</sup> V/m
Transmitter distance d <sub>o</sub>	= 1km
Transmitting antenna height, h <sub>t</sub>	= 50m
Receiving antenna height, h <sub>r</sub>	= 1.5m
Wave length, $\lambda$	= 0.333

We Know,

Since  $d \gg \sqrt{h_t h_r}$ , the electric field is given by

$$\begin{aligned} E_R(d) &\approx \frac{2E_0 d_0}{d} \frac{2\pi h_t h_r}{\lambda d} \approx \frac{k}{d^2} \text{ V/m} \\ &= \frac{2 \times 10^{-3} \times 1 \times 10^3}{5 \times 10^3} \left[ \frac{2\pi (50) (1.5)}{0.333 (5 \times 10^3)} \right] \\ &= 113.1 \times 10^{-6} \text{ V/m.} \end{aligned}$$



Here, Effective Aperture

$$A_e = \frac{\lambda^2}{4\pi} G$$

$$= 0.016 \text{ m}^2$$

Now, the received power at a distance d can be obtained using

$$P_r(d) = P_d A_e = \frac{|E|^2}{120\pi} A_e$$

$$= ((113.1 \times 10^{-6})^2 \times 0.016) / 337$$

$$= 5.4 \times 10^{-13} \text{ W}$$

$$= -122.68 \text{ dBW}$$

$$= -92.68 \text{ dBm}$$

### Source Code (MATLAB):

```
clc;
clear all;
close all;
f = 900;      % Frequency in MHz
g = 2.55;     % Gain of antenna in dB

% Question (a)

gain = (10^(g/10));
lemda = (3*10^8)/(f*10^6);
L = lemda/4;      % Antenna Length
disp('For (a)');
disp('-----');
fprintf('Length of the antenna : %7.3f m\n',L);
fprintf('Gain of the antenna : %7.1f = %7.2fdB\n\n',gain,g);

disp('For (b)');
disp('-----');
d = 5000;      % T-R seperation distance
E0 = 10^-3;    % Electric-field
d0 = 1000;     % Transmitter distance
ht = 50;       % Transmitting antenna height, ht (m)
hr = 1.5;      % Receiving antenna height, hr (m)

Er_d = (2*E0*d0*2*3.1416*ht*hr)/(lemda*d^2); % Electric Field
Ae = (gain*lemda^2)/(4*3.1416); % Effective Aperture
Pr_d = (Er_d^2/(120*3.1416))*Ae; % The received power at a distance d
Pr_dB = 10*log10(Pr_d);
fprintf('Electric Field, Er(d) : %7.9f v/m\n',Er_d);
fprintf('Effective Aperture, Ae : %7.3f m^2\n',Ae);
fprintf('Receiveed power at 5km distance Er(5 km) : %7.3f dbW\n',Pr_dB);
```

**Input:**

f = 900;	% Frequency in MHz
g = 2.55;	% Gain of antenna in dB
d = 5000;	% T-R separation distance
E0 = 10^-3;	% Electric-field
d0 = 1000;	% Transmitter distance
ht = 50;	% Transmitting antenna height, ht (m)
hr = 1.5;	% Receiving antenna height, hr (m)

**Output:****For (a)**

Length of the antenna, L	: 0.083 m
Gain of the antenna, G	: 1.8 = 2.55dB

**For (b)**

Electric Field, $E_r(d)$	: 0.000113098 v/m
Effective Aperture, $A_e$	: 0.016 m <sup>2</sup>
Received power at 5 km distance $E_r(5 \text{ km})$	: -122.679 dbW

**Problem-10:** A hexagonal cell within a 4-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and  $\lambda$  = call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call-

- How many users per square kilometer will this system support?
- What is the probability that a delayed call will have to wait for more than 10s?
- What is the probability that a call will be delayed for more than 10 seconds?

**Solution:**

Have given,

Cell radius, R	= 1.387 km
Area covered per cell is $2.598 \times (1.387)^2$	$\approx 5$ sq km
Number of cells per cluster, n	= 4
Total number of channels, N	= 60
Therefore, number of channels per cell = $60 / 4$	= 15 channels.
From Erlang C chart, for 5% probability of delay with C = 15,	
Traffic intensity, A	= 9.0 Erlangs.

**(a)**

Have given,

Traffic per user,  $A_u = 0.029$  Erlangs.

We know,

$$\begin{aligned} \text{The number of users, } U &= A / A_u \\ &= 9.0 / 0.029 \\ &= 310 \text{ users} \end{aligned}$$

$$\begin{aligned} \text{The number of users per square km} &= 310 \text{ users} / 5 \text{ sq km} \\ &= 62 \text{ users /sq km} \end{aligned}$$

**(b)**

Have given,

$$\begin{aligned} \text{Wave length, } \lambda &= 1 \text{ call/hour} \\ \text{Holding time, } H &= A_u / \lambda \\ &= 0.029 \text{ hour} \\ &= 104.4 \text{ seconds.} \\ \text{Time, } t &= 10s \end{aligned}$$

We know,

$$\begin{aligned}\text{The conditional probability that a delayed call will have to wait for more than } t \\ \text{seconds is } P_r[\text{delay} > t \mid \text{delay}] &= \exp(-(C-A)t/H) \\ &= \exp(-(15-9)10/104.4) \\ &= 56.29 \%\end{aligned}$$

(c)

Have given,

$$\text{The probability of delayed call, } P_r[\text{delay} > 0] = 5 \% = 0.05$$

We know,

Probability that a call is delayed more than 10 seconds,

$$\begin{aligned}P_r[\text{delay} > 10] &= P_r[\text{delay} > 0] \times P_r[\text{delay} > t \mid \text{delay}] \\ &= 0.05 \times 0.5629 \\ &= 2.81 \%\end{aligned}$$

### Source Code (MATLAB):

```
clc;
clear all;
close all;
R = 1.387;      % Cell Radius
n = 4;          % No of cell
N = 60;         % Total no of channel
area = round(2.5981*R^2); % Area covers per cell
C = N/4;        % No of channel per cell
A = 9;          % Traffic intensity at c=15, GOS=0.05, Au=0.029 from erlang C chart

% Question (a)
Au = 0.029;     % Traffic per user
U = floor(A/Au); % Number of user
U_per = round(U/area); % Number of users per square km
fprintf('(a) Number of users per square km: %d users/sq km\n',U_per);

% Question (b)
lemda = 1;      % lamda = 1 hour
H = (Au/lemda)*3600; % Holding Time hour to second
Prb = (exp(-(C-A)*10/H)); % t=10s, C=15, A=9, H=104.4
fprintf('(b) The probability that a delayed call will have to wait: %7.2f%%\n',Prb*100);

% Question (c)
Prc = 0.05*Prb*100; % 5% probability of delayed call
fprintf('(c) The probability that a call will be delayed : %7.2f%%\n',Prc);
```

### Output:

(a) Number of users per square km	: 62 users/sq km
(b) The probability that a delayed call will have to wait	: 56.29%
(c) The probability that a call will be delayed	: 2.81%