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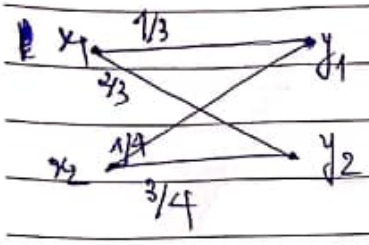
Bài 1:

$p(x_1) = 1/4 \Rightarrow p(x_2) = 1 - 1/4 = 3/4$



a) $I(x_2) = ?$

$I(x_i) = -\log(p(x_i)) \Rightarrow I(x_2) = -\log(p(x_2))$

$= -\log(3/4) = 0,41504$ (bit)

$I(x_2|y_1) = -\log(p(x_2|y_1))$

$p(x_k, y_l) = p(x_k) \cdot p(y_l|x_k) = p(y_l) \cdot p(x_k|y_l) \Rightarrow p(x_k|y_l) = \frac{p(x_k) \cdot p(y_l|x_k)}{p(y_l)}$

$p(y_l) = \sum_k p(x_k) \cdot p(y_l|x_k)$

$\Rightarrow p(y_1) = p(x_1) p(y_1|x_1) + p(x_2) p(y_1|x_2)$

$= 1/4 \cdot 1/3 + 3/4 \cdot 1/4 = 13/48$

$p(x_2|y_1) = \frac{p(x_2) \cdot p(y_1|x_2)}{p(y_1)} = \frac{3/4 \cdot 1/4}{13/48} = 9/13$

$\Rightarrow I(x_2|y_1) = -\log(9/13) = 0,53051$ (bit)

$I(x_1; y_2) = I(x_1) - I(x_1|y_2) = \log\left(\frac{p(x_1|y_2)}{p(x_1)}\right)$

$p(x_1|y_2) = \frac{p(x_1) p(y_2|x_1)}{p(y_2)} = \frac{p(x_1) p(y_2|x_1)}{p(x_1) \cdot p(y_2|x_1) + p(x_2) \cdot p(y_2|x_2)}$

$= \frac{1/4 \cdot 2/3}{1/4 \cdot 2/3 + 3/4 \cdot 3/4} = \frac{1/6}{35/48} = 8/35$

$\Rightarrow I(x_1; y_2) = \log\left(\frac{8/35}{1/4}\right) = -0,12928$ (bit)

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b) $H(X) = -\sum_{k=1}^{2} p(x_k) \log(p(x_k)) = -[p(x_1)\log(p(x_1)) + p(x_2)\log(p(x_2))]$

$= -[1/4 \cdot \log(1/4) + 3/4 \cdot \log(3/4)] = 0,81128 \text{ (bit)}$

$H(Y) = -\sum_{l=1}^{2} p(y_l) \cdot \log(p(y_l)) = -[p(y_1)\log(p(y_1)) + p(y_2)\cdot\log(p(y_2))]$

$= -\left[\frac{13}{48} \cdot \log\left(\frac{13}{48}\right) + \frac{35}{48} \cdot \log\left(\frac{35}{48}\right)\right] = 0,84266 \text{ (bit)}$

$H(X|y_1) = \sum_{k=1}^{2} p(x_k|y_1) \log(p(x_k|y_1))$

$p(x_k|y_l) = \frac{p(x_k) \cdot p(y_l|x_k)}{p(y_l)}$

$p(x_1|y_1) = 4/13, \quad p(x_2|y_1) = 9/13$

$p(x_1|y_2) = \frac{8}{35}, \quad p(x_2|y_2) = 27/35$

$H(X|y_1) = -[p(x_1|y_1)\log(p(x_1|y_1)) + p(x_2|y_1)\log(p(x_2|y_1))]$

$= -\left[4/13 \cdot \log\left(\frac{4}{13}\right) + \frac{9}{13} \cdot \log\left(\frac{9}{13}\right)\right]$

$= 0,89050 \text{ (bit)}$

$H(Y|x_2) = -\sum_{l=1}^{2} p(y_l|x_2) \cdot \log(p(y_l|x_2))$

$= -[p(y_1|x_2)\log(p(y_1|x_2)) + p(y_2|x_2)\log(p(y_2|x_2))]$

$= -[1/4 \cdot \log(1/4) + 3/4 \cdot \log(3/4)] = 0,81128 \text{ (bit)}$

~~$H(X|Y) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l) \cdot \log(p(x_k|y_l))$~~

$= -\left[1/3 \cdot \log\left(\frac{4}{13}\right) + \frac{2}{3} \cdot \log\left(\frac{8}{35}\right) + \frac{1}{4} \cdot \log\left(\frac{9}{13}\right) + \frac{3}{4}\log\left(\frac{27}{35}\right)\right]$

$= 2,39976 \text{ (bit)}$

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$$H(Y|X) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l)\cdot \log\left(p(y_l|x_k)\right)$$

$$p(x_k, y_l) = p(x_k)\cdot p(y_l|x_k)$$

$p(x_1, y_1) = 1/12$, $p(x_1, y_2) = 1/6$;

$p(x_2, y_1) = 3/16$, $p(x_2, y_2) = 9/16$.

$$H(Y|X) = -\left[\frac{1}{12}\cdot\log\frac{1}{3} + \frac{1}{6}\cdot\log\frac{2}{3} + \frac{3}{16}\cdot\log\frac{1}{4} + \frac{9}{16}\cdot\log\frac{3}{4}\right]$$

$$= 0,83803 \text{ bit}$$

$$H(X|Y) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l)\cdot\log\left(p(x_k|y_l)\right)$$

$$= -\left[\frac{1}{12}\cdot\log\frac{1}{13} + \frac{1}{6}\cdot\log\frac{8}{35} + \frac{3}{16}\cdot\log\frac{9}{13} + \frac{9}{16}\cdot\log\frac{27}{35}\right]$$

$$= 0,80665 \text{ bit}$$

$$I(X;Y) = \sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l)\cdot\log\left(\frac{p(x_k, y_l)}{p(x_k)\cdot p(y_l)}\right)$$

$$= \left[\frac{1}{12}\cdot\log\left(\frac{\frac{1}{12}}{\frac{1}{4}\cdot\frac{13}{48}}\right) + \frac{1}{6}\cdot\log\left(\frac{\frac{1}{6}}{\frac{1}{4}\cdot\frac{35}{48}}\right) + \frac{3}{16}\cdot\log\left(\frac{3/16}{3/4\cdot 13/48}\right)\right.$$

$$\left. + \frac{9}{16}\cdot\log\left(\frac{9/16}{3/4\cdot 35/48}\right)\right]$$

$$= 4,62533 \cdot 10^{-3} \text{ (bit)}$$

$$H(X,Y) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k, y_l)\cdot\log\left(p(x_k, y_l)\right)$$

$$= -\left[\frac{1}{12}\log\frac{1}{12} + \frac{1}{6}\cdot\log\frac{1}{6} + \frac{3}{16}\cdot\log\frac{3}{16} + \frac{9}{16}\cdot\log\frac{9}{16}\right]$$

$$= 1,64931 \text{ (bit)}$$

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Bài 2:

$P(x_k, y_k)$	x_1	x_2
y_1	1/3	1/3
y_2	1/3	0

$\sum_k P(y_e|x_k) = 1 \Rightarrow P(y_1|x_1) + P(y_2|x_1) = 1$ (1)

$P(x_1, y_1) = P(x_1) \cdot P(y_1|x_1) = 1/3$

$P(x_1, y_2) = P(x_1) \cdot P(y_2|x_1) = 1/3$

$\Rightarrow P(x_1)\left[P(y_1|x_1) + P(y_2|x_1)\right] = 1/3 + 1/3 = 2/3$

$\Rightarrow P(x_1) \cdot 1 = 2/3 \Rightarrow P(x_1) = 2/3$

$\Rightarrow P(x_2) = 1 - P(x_1) = 1 - 2/3 = 1/3$

$P(y_e) = \sum_k P(x_k) \cdot P(y_e|x_k) = \sum_k P(x_k, y_e)$

$\Rightarrow P(y_1) = P(x_1, y_1) + P(x_2, y_1) = 1/3 + 1/3 = 2/3$

Tương tự: $P(y_2) = P(x_1, y_2) + P(x_2, y_2) = 1/3 + 0 = 1/3$

a) $I(x_2) = -\log(P(x_2)) = -\log\left(\frac{1}{3}\right) = 1{,}58496$ (bit)

$I(x_2|y_1) = -\log\left(P(x_2|y_1)\right) = -\log\left(\frac{P(x_2, y_1)}{P(y_1)}\right)$

$= -\log\left(\frac{1/3}{2/3}\right) = -\log\left(\frac{1}{2}\right) = 1$ (bit)

$I(x_1, y_2) = \log\left(\frac{P(x_1|y_2)}{P(x_1)}\right) = \log\left(P(x_1|y_2)\right) - \log\left(P(x_1)\right)$

$P(x_1|y_2) = \frac{P(x_1, y_2)}{P(y_2)} = \frac{1/3}{1/3} = 1$

$\Rightarrow I(x_1, y_2) = \log\frac{1}{2/3} = \log\frac{3}{2} = 0{,}58496$ (bit)

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b) $H(X) = -\sum_{k=1}^{2} p(x_k) \cdot \log(p(x_k))$

$= -\left(\frac{2}{3} \log\frac{2}{3} + \frac{1}{3} \cdot \log\frac{1}{3}\right) = 0,91830 \text{ (bit)}$

$H(Y) = -\sum_{k=1}^{2} p(y_k) \log(p(y_k))$

$= -\left(\frac{2}{3} \cdot \log\frac{2}{3} + \frac{1}{3} \cdot \log\frac{1}{3}\right) = 0,91830 \text{ (bit)}$

$H(X|Y) = -\sum_{k=1}^{2} \sum_{l=1}^{2} p(x_k, y_l) \log(p(x_k|y_l))$

$p(x_k|y_l) = \frac{p(x_k, y_l)}{p(y_l)}$

$\Rightarrow p(x_1|y_1) = \frac{p(x_1, y_1)}{p(y_1)} = \frac{1/3}{2/3} = 1/2$

$p(x_1|y_2) = 1$

$p(x_2|y_1) = 1/2$

$p(x_2|y_2) = 0$

$H(X|Y) = -\left[\frac{1}{3} \cdot \log\frac{1}{2} + \frac{1}{3} \log 1 + \frac{1}{3} \cdot \log\frac{1}{2} + 0\right] = \frac{2}{3} \text{ (bit)}$

$H(Y|X) = -\sum_{k=1}^{2} \sum_{l=1}^{2} p(x_k, y_l) \log(p(y_l|x_k))$

$p(y_l|x_k) = \frac{p(x_k, y_l)}{p(x_k)}$

$p(y_1|x_1) = \frac{1}{2}$, $p(y_2|x_1) = 1$

$p(y_1|x_2) = \frac{1}{2}$, $p(y_2|x_2) = 0$

$H(Y|X) = -\left[\frac{1}{3} \log\frac{1}{2} + \frac{1}{3} \log 1 + \frac{1}{3} \log\frac{1}{2} + 0\right] = \frac{2}{3} \text{ (bit)}$

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$$H(X|y_1) = -\sum_{k=1}^{2} p(x_k|y_1)\log\left(p(x_k|y_1)\right)$$

$$= -\left[\frac{1}{2}\cdot\log\frac{1}{2} + \frac{1}{2}\log\frac{1}{2}\right] = 1\ (\text{bit})$$

$$H(y|x_2) = -\sum_{l=1}^{2} p(y_l|x_2)\cdot\log\left(p(y_l|x_2)\right)$$

$$= -\left(\frac{1}{2}\log\frac{1}{2} + 0\right) = \frac{1}{2}\ (\text{bit})$$

$$H(X,Y) = -\sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k,y_l)\cdot\log\left(p(x_k,y_l)\right)$$

$$= -\left(\frac{1}{3}\cdot\log\frac{1}{3} + \frac{1}{3}\log\frac{1}{3} + \frac{1}{3}\log\frac{1}{3} + 0\right)$$

$$= 1{,}58496\ (\text{bit})$$

$$I(X;Y) = \sum_{k=1}^{2}\sum_{l=1}^{2} p(x_k,y_l)\cdot\log\left(\frac{p(x_k,y_l)}{p(x_k)\cdot p(y_l)}\right)$$

$$= \frac{1}{3}\cdot\log\left(\frac{\frac{1}{3}}{\frac{2}{3}\cdot\frac{2}{3}}\right) + \frac{1}{3}\cdot\log\frac{\frac{1}{3}}{\frac{2}{3}\cdot\frac{1}{3}} + \frac{1}{3}\cdot\log\frac{\frac{1}{3}}{\frac{1}{3}\cdot\frac{2}{3}} + 0$$

$$= 0{,}251163\ (\text{bit})$$