Date: . . No:

* Entropy

$$H(X) = E|I(x_k)| = \sum_{k=1}^{N} p(x_k) I(x_k)$$

$$= -\sum_{k=1}^{N} p(x_k) \log(p(x_k)) = E[-\log(p(x_k))] \quad (\text{bit, nat, hartley})$$

* Entropy đồng thời

$$H(X,Y) = \cancel{E[H(} -\sum_{x_1 \in X} \sum_{y_1 \in Y} p(x_k, y_1) \log(p(x_k, y_1))$$

$$= E[-\log(p(x_k, y_1))]_{(x_1, y_1) \in (X,Y)}$$

* Entropy có điều kiện từng phần

$$H(X | Y = y_1) = E[I(x_k | Y = y_1)]_{x_k \in X | Y = y_1}$$

$$= \sum_{x_k \in X} p(x_k | Y = y_1) \cdot I(x_k | Y = y_1)$$

$$= -\sum_{x_k \in X} p(x_k | Y = y_1) \log(x_k | Y = y_1)$$

$$= -\sum_{x_k \in X} p(x_k | y_1) \log(x_k | y_1)$$

$$H(Y | X = x_k) = E[I(y_1 | X = x_k)]_{y_1 \in Y | X = x_k}$$

$$= \sum_{y_1 \in Y} p(y_1 | X = x_k) I(y_1 | X = x_k)$$

$$= -\sum_{y_1 \in Y} p(y_1 | X = x_k) \log(y_1 | X = x_k)$$

$$= -\sum_{y_1 \in Y} p(y_1 | x_k) \log(y_1 | x_k)$$

* Entropy có điều kiện

$$H(X|Y) = E\left[H(X|Y=y_1)\right]_{y_1 \in Y} = \sum_{y_1 \in Y} p(y_1)\, H(X|Y=y_1)$$

$$= -\sum_{y_1 \in Y} p(y_1) \sum_{x_k \in X} p(x_k|y_1) \log\left(p(x_k|y_1)\right)$$

$$= -\sum_{y_1 \in Y} \sum_{x_k \in X} p(x_k, y_1) \log\left(p(x_k|y_1)\right)$$

$$= E\left[\log\left(p(X|Y)\right)\right]_{p(x_k, y_1)}$$

* ~~Entropy tương đối~~ Lượng thông tin tương hỗ

$$I(X;Y) = E\left[I(x_k; y_1)\right] = \sum_{x_k \in X} \sum_{y_1 \in Y} p(x_k, y_1) \log\left(\frac{p(x_k, y_1)}{p(x_k)\, p(y_1)}\right)$$

$$= D\left(p(x_k; y_1) \,||\, p(x_k) p(y_1)\right)$$

* ~~Lượng thông tin tương hỗ~~

* Entropy tương đối

$$D(p||q) = \sum_{x_k \in X} p(x_k) \log\left(\frac{p(x_k)}{q(x_k)}\right)$$