

DFT $N = 4$ điểm $\tilde{x}(n) = \{1, 5, 0, 8\}$ chu kỳ 4
FFT phân theo thời gian

Đặt $f_1(n) = x(2n) = \{1, 0\}$

$$f_2(n) = x(2n+1) = \{5, 8\}$$

$$\Rightarrow F_1(k) = \underset{N/2}{\text{DFT}} [f_1(n)] \quad \left| \quad F_2(k) = \underset{N/2}{\text{DFT}} [f_2(n)]\right.$$

~~$F_1(k)$~~

$$F_1(0) = f_1(0)W_2^0 + f_1(1)W_2^0 = f_1(0) + f_1(1) = 1 + 0 = 1$$

$$\begin{aligned} F_1(1) &= f_1(0)W_2^0 + f_1(1)W_2^1 = f_1(0)W_2^0 - f_1(1)W_2^0 \\ &= [f_1(0) - f_1(1)]W_2^0 = 1 - 0 = 1 \end{aligned}$$

$$F_2(0) = f_2(0)W_2^0 + f_2(1)W_2^0 = f_2(0) + f_2(1) = 5 + 8 = 13$$

$$\begin{aligned} F_2(1) &= f_2(0)W_2^0 + f_2(1)W_2^1 = f_2(0)W_2^0 - f_2(1)W_2^0 \\ &= \cancel{f_2(0)W_2^0} [f_2(0) - f_2(1)]W_2^0 = 5 - 8 = -3 \end{aligned}$$

$$X(k) = F_1(k) + F_2(k) \cdot W_N^k$$

$$\Rightarrow X(0) = F_1(0) + F_2(0)W_4^0 = 1 + 13 = 14$$

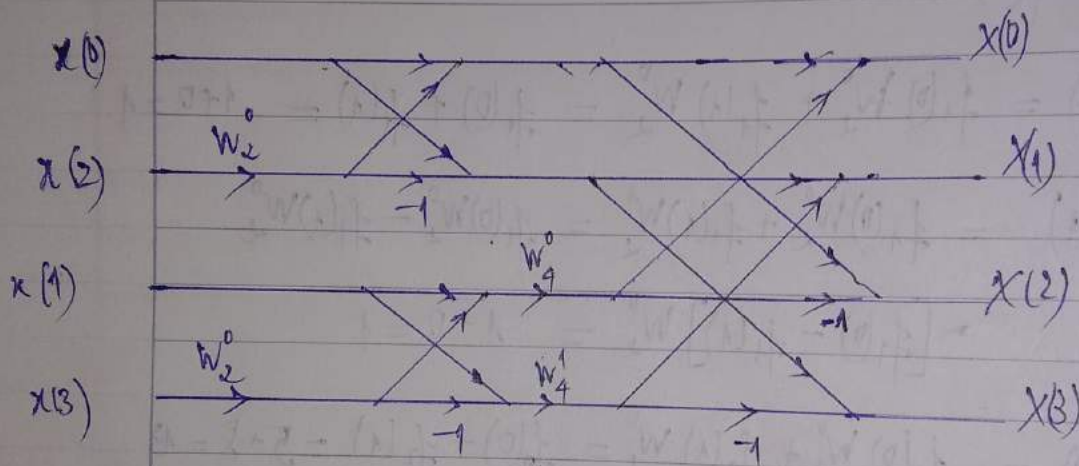
$$X(1) = F_1(1) + F_2(1)W_4^1 = 1 + (-3) \cdot (-j) = 1 + 3j$$

$$X(k + N/2) = F_1(k) - F_2(k)W_N^k$$

$$\Rightarrow X(2) = F_1(0) - F_2(0)W_4^0 = 1 - 13 = -12$$

$$X(3) = F_1(1) - F_2(1)W_4^1 = 1 - (-3)(-j) = 1 - 3j$$

$$\Rightarrow X(k) = \{1, 1+3j, -12, 1-3j\}$$



DFT $N=4$ điểm $\tilde{x}(n) = \{1, 8, 0, 9\}$ chu kỳ 4
FFT phân theo tần số

$$\text{Đặt } g_1(n) = x(n) + x(n+N/2)$$

$$n=0, 1, \dots, N/2$$

$$g_2(n) = [x(n) - x(n+N/2)] W_N^n$$

$$\Rightarrow \begin{cases} g_1(0) = x(0) + x(2) = 1 + 0 = 1 \\ g_1(1) = \cancel{x(0)} - \cancel{x(2)} \quad x(1) + x(3) = 8 + 9 = 17 \end{cases}$$

$$\Rightarrow \begin{cases} g_2(0) = [x(0) - x(2)] W_4^0 = (1 - 0) \cdot 1 = 1 \\ g_2(1) = [x(1) - x(3)] W_4^1 = (8 - 9)(-j) = j \end{cases}$$

$$X(2k) = \text{DFT}[g_1(n)]$$

$$X(2k) = \sum_{n=0}^{N/2-1} g_1(n) W_{N/2}^{kn}$$

$$X(2k+1) = \text{DFT}[g_2(n)]$$

$$X(2k+1) = \sum_{n=0}^{N/2-1} g_2(n) W_{N/2}^{kn}$$

$$X(0) = g_1(0) W_2^0 + g_1(1) W_2^0 = g_1(0) + g_1(1) = 1 + 17 = 18$$

$$X(2) = g_1(0) W_2^0 + g_1(1) W_2^1 = g_1(0) W_2^0 - g_1(1) W_2^0$$

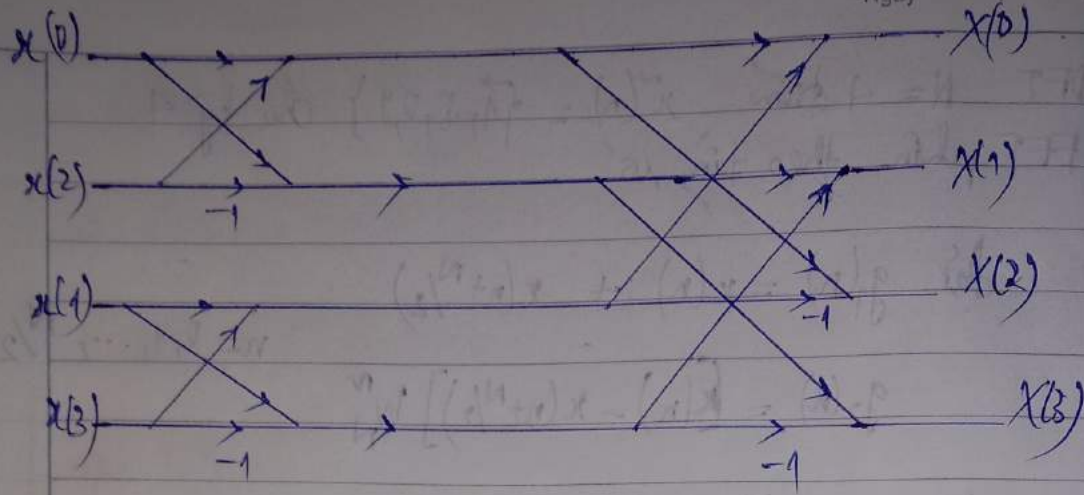
$$= [g_1(0) - g_1(1)] W_2^0 = g_1(0) - g_1(1) = 1 - 17 = -16$$

$$X(4) = g_2(0) W_2^0 + g_2(1) W_2^0 = g_2(0) + g_2(1) = 1 + j$$

$$X(3) = g_2(0) W_2^0 + g_2(1) W_2^1 = g_2(0) W_2^0 - g_2(1) W_2^0$$

$$= [g_2(0) - g_2(1)] W_2^0 = g_2(0) - g_2(1) = 1 - j$$

$$\Rightarrow X(k) = \{18, 1+j, -16, 1-j\}$$



$$b = a + b = (1)x + (0)x \rightarrow (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow$$

$$F_1 = 0 + 3 = (0)x + (3)x \rightarrow 3x = (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$b = 1 \cdot (0 - 1) = {}^0W[(1)x - (0)x] = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow$$

$$c = (-1)(0 - 3) = {}^0W[(3)x - (0)x] = (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix} \end{bmatrix} = (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{ad. } \frac{{}^0W}{\sigma = 0} = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{ad. } \frac{{}^0W}{\sigma = 0} = (3) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$3b = 3 \cdot 1 = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (1)x$$

$${}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (1)x$$

$$3b = 3 \cdot 1 = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W[(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix}] =$$

$$3b = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (1)x$$

$${}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + {}^0W(0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = (1)x$$

$$3b = (1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix} = {}^0W[(1) \begin{bmatrix} 1 \\ 0 \end{bmatrix} + (0) \begin{bmatrix} 0 \\ 1 \end{bmatrix}] =$$

$$3b = 3 \cdot 1 = 3 \cdot 1 = 3 \cdot 1 = (1)x$$