Práctica 2 – Parte 1

Percepción y Control para Sistemas Empotrados

Curso 2020-21

Izar Castorina – Grupo 1

Tarea 1: Ecuación del sistema

$$M = 1000 \quad b = 80 \quad K = 20 \quad T = 0.1$$

La expresión del sistema completo es dada por su función de transferencia, es decir, la salida partida por la entrada. En este caso:

$$\frac{Y(z)}{X(z)} = \frac{C(z) * G'(z)}{1 + C(z) * G'(z)}$$

con $C(z) = K\frac{z-0.9}{z-1} = 20\frac{z-0.9}{z-1}$ y $G'(z) = \frac{40}{b}\frac{(1-e^{-\left(\frac{b}{M}\right)T})}{z-e^{-\left(\frac{b}{M}\right)T}} = \frac{40}{80}\frac{(1-e^{-\left(\frac{80}{1000}\right)0.1})}{z-e^{-\left(\frac{80}{1000}\right)0.1}} = \frac{1}{2}\frac{1-e^{-0.008}}{z-e^{-0.008}}$

$$\frac{Y(z)}{X(z)} = \frac{20\frac{z-0.9}{z-1} * \frac{1}{2}\frac{1-e^{-0.008}}{z-e^{-0.008}}}{1 + 20\frac{z-0.9}{z-1} * \frac{1}{2}\frac{1-e^{-0.008}}{z-e^{-0.008}}} \rightarrow$$

$$Substituimos\ e^{-0.008}\ por\ \alpha$$

$$\rightarrow \frac{10\frac{z-0.9}{z-1} * \frac{1-\alpha}{z-\alpha}}{1 + 10\frac{z-0.9}{z-1} * \frac{1-\alpha}{z-\alpha}} = \frac{10\frac{z-\alpha z-0.9+0{,}9\alpha}{z^2-\alpha z-z+a}}{1 + 10\frac{z-\alpha z-0.9+0{,}9\alpha}{z^2-\alpha z-z+a}}$$

$$= \frac{\frac{10z-10\alpha z-9+9\alpha}{z^2-\alpha z-z+a}}{1 + \frac{10z-10\alpha z-9+9\alpha}{z^2-\alpha z-z+a}}$$

Tratamos numerador y denominador por separado, y deshacemos la substitución

Numerador:

$$\frac{10z-10e^{-0.008}z-9+9e^{-0.008}}{z^2-e^{-0.008}z-z+e^{-0.008}} = \frac{10z-10*0{,}992z-9+9*0{,}992}{z^2-0{,}992z-z+0{,}992}$$

$$= \frac{10z-9{,}9203z-9+8{,}9283}{z^2-0{,}992z-z+0{,}992} = \frac{0{,}0797z-0{,}0717}{z^2-1{,}992z+0{,}992}$$

Denominador:

$$1 + NUM. = 1 + \frac{0{,}0797z - 0{,}0717}{z^2 - 1{,}992z + 0{,}992} = \frac{z^2 - 1{,}992z + 0{,}992 + 0{,}0797z - 0{,}0717}{z^2 - 1{,}992z + 0{,}992}$$
$$= \frac{z^2 - 1{,}9123z + 0{,}9203}{z^2 - 1{,}992z + 0{,}992}$$

Resultado:

$$\frac{Y(z)}{X(z)} = \frac{\frac{0{,}0797z - 0{,}0717}{z^2 - 1{,}992z + 0{,}992}}{\frac{z^2 + 0{,}0717z + 0{,}9203}{z^2 - 1{,}992z + 0{,}992}} = \frac{0{,}0797z - 0{,}0717}{z^2 - 1{,}992z + 0{,}992} * \frac{z^2 - 1{,}992z + 0{,}992}{z^2 + 0{,}0717z + 0{,}9203}$$
$$= \frac{\boldsymbol{0{,}0797z - 0{,}0717}}{\boldsymbol{z^2 + 0{,}0717z + 0{,}9203}}$$

Tarea 2: Forma Directa I

$$\frac{Y(z)}{X(z)} = \frac{0{,}0797z - 0{,}0717}{z^2 - 1{,}9123z + 0{,}9203} * \frac{z^{-2}}{z^{-2}} = \frac{0{,}0797z^{-1} - 0{,}0717z^{-2}}{1 - 1{,}9123z^{-1} + 0{,}9203z^{-2}} \rightarrow$$
$$\rightarrow Y(z) * (1 - 1{,}9123z^{-1} + 0{,}9203z^{-2}) = X(z) * (0{,}0797z^{-1} - 0{,}0717z^{-2}) \rightarrow$$
$$\rightarrow Y(z) = \frac{1{,}9123}{1} Y(z)z^{-1} - \frac{0{,}9203}{1} Y(z)z^{-2} + \frac{0{,}0797}{1} X(z)z^{1-2} - \frac{0{,}0717}{1} X(z)z^{1-2-1} \rightarrow$$
$$\rightarrow y[k] = 1{,}9123y[k-1] - 0{,}9203y[k-2] + 0{,}0797x[k-1] - 0{,}0717x[k-2]$$

Tarea 3: Forma Directa II

$$\frac{Y(z)}{X(z)} = \frac{0{,}0797z - 0{,}0717}{z^2 - 1{,}9123z + 0{,}9203} * \frac{z^{-2}}{z^{-2}} = \frac{0{,}0797z^{-1} - 0{,}0717z^{-2}}{1 - 1{,}9123z^{-1} + 0{,}9203z^{-2}} = \frac{Y(z)}{F(z)} \frac{F(z)}{X(z)}$$

$$X(z) = (1 - 1{,}9123z^{-1} + 0{,}9203z^{-2}) F(z)$$

$$Y(z) = (0{,}0797z^{-1} - 0{,}0717z^{-2}) F(z)$$

$$x[k] = f[k] - 1{,}9123f[k-1] + 0{,}9203f[k-2] \rightarrow f[k] = x[k] + 1{,}9123f[k-1] - 0{,}9203f[k-2]$$

$$y[k] = 0{,}0797f[k-1] - 0{,}0717f[k-2]$$