$$A = 1 + 2 + 3 + \cdots + 100$$

$$+ \quad A = 100 + 99 + 98 + \cdots + 1$$

$$2A = 101 + 101 + 101 + \cdots + 101 = 100 \cdot 101$$

$$A = \frac{1}{2} \, 100 \cdot 101$$

$$S - \frac{1}{4}S \quad \rightarrow \quad \frac{3}{4}S$$

$$= \left(1 - \frac{1}{4}\right) S$$

$$2S - \frac{1}{3}S = \left(2 - \frac{1}{3}\right) S$$

$$S = 1 + x + x^2 + \cdots + x^n$$

$$xS = \quad\quad x + x^2 + \cdots + x^n + x^{n+1}$$

$$S - xS = 1 - x^{n+1}$$

$$(1-x) S = 1 - x^{n+1}$$

$$\therefore S = \frac{1 - x^{n+1}}{1 - x} = \frac{x^{n+1} - 1}{x - 1}$$

$$1 + x + x^2 + \cdots + x^n = \frac{1 - x^{n+1}}{1 - x}$$