University of Glasgow

UESTC4004
Digital Communications

Channel Coding

CONVOLUTIONAL ENCODING

- A convolutional code is described by three integers, n, k, and K where the ratio k/n is called the rate of the code
- The integer K is constraint length; it represents number of k-tuple stages in the encoding shift register.
- Encoder has memory—the n-tuple emitted by the convolutional encoding procedure is not only a function of an input k-tuple, but is also a function of the previous K-1 input k-tuples

Connection Representation

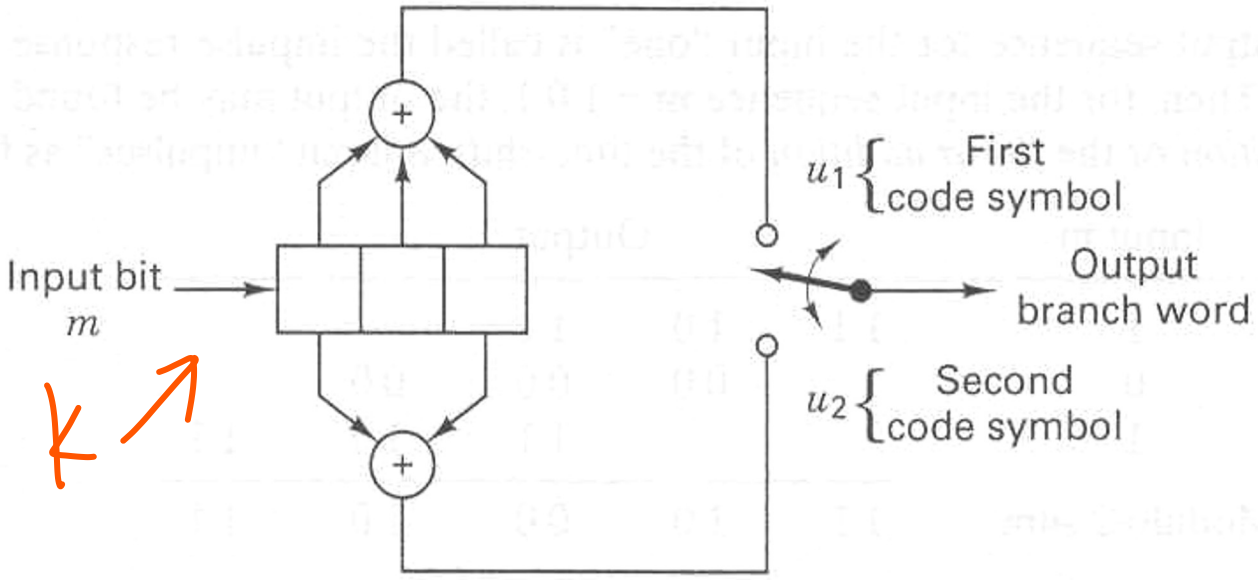
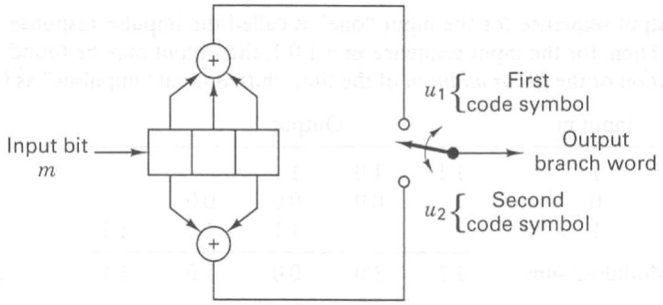


Figure: Convolutional Encoder (rate ½, K=3)

State Representation and State Diagram



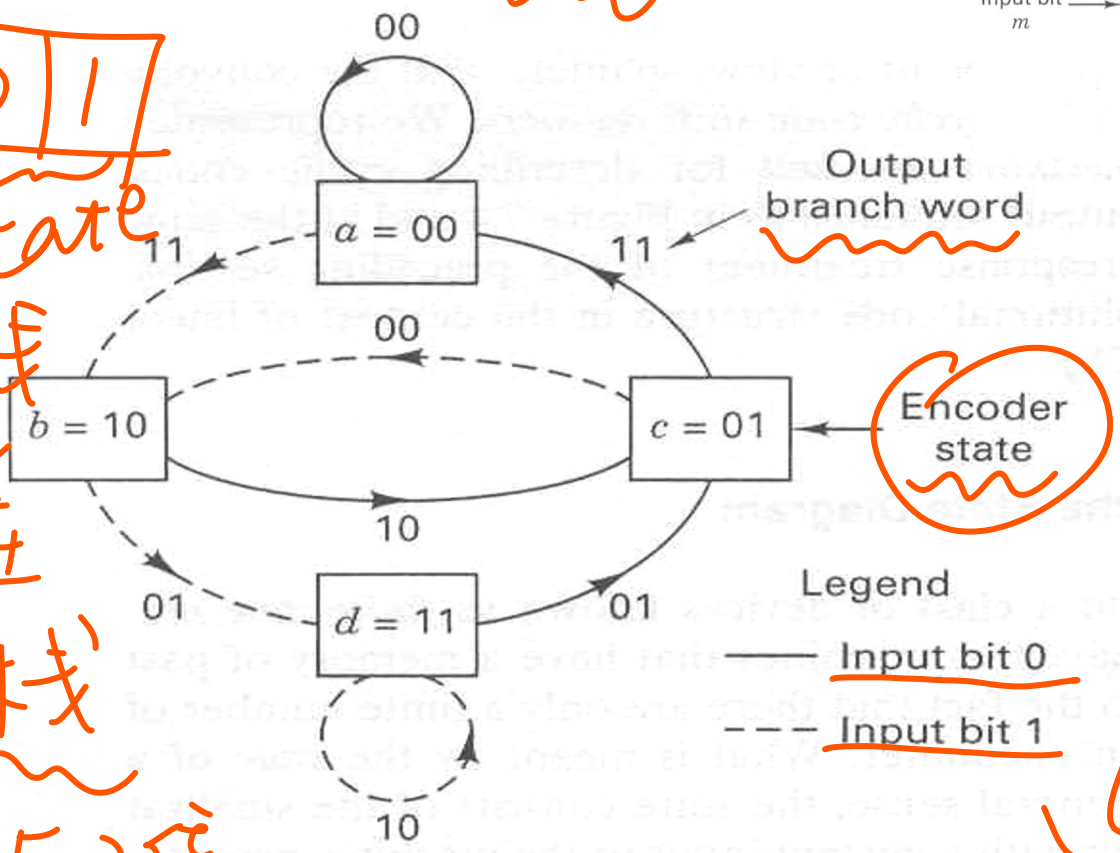
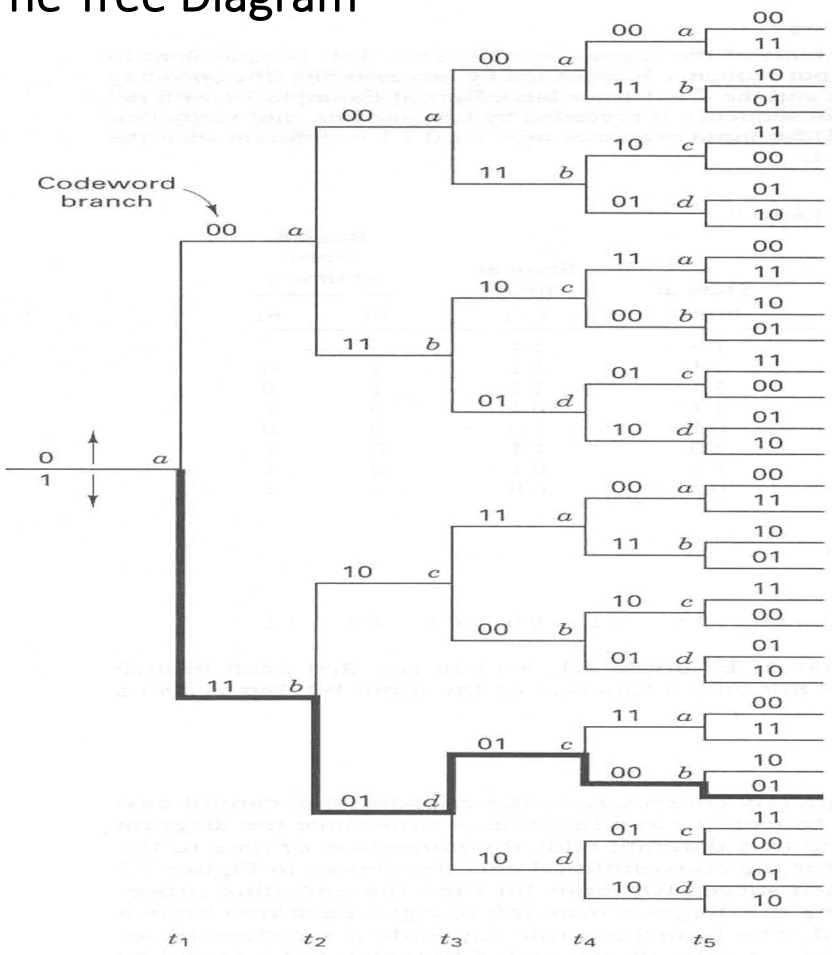


Figure: Encoder state (the contents of the K-1 leftmost registers) diagram (rate ½ ,k=3)

The Tree Diagram

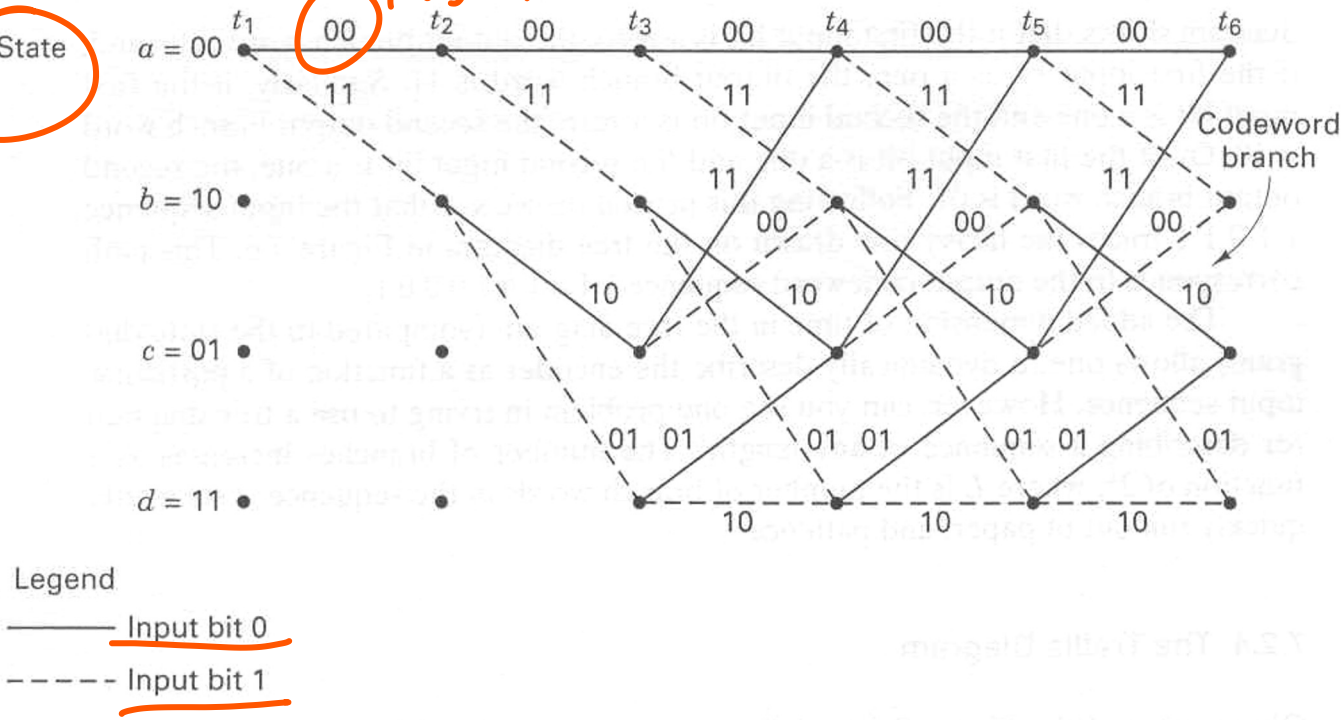


■Tree diagram adds the dimension of time to the state diagram

t

Figure: Tree representation of encoder (rate ½, k=3)

The Trellis Diagram



■The trellis diagram, by exploiting the repetitive structure, provides a more manageable encoder description

Figure: Encoder trellis diagram (rate ½, K=3)

The Viterbi convolutional decoding Algorithm

Input data sequence	m:	1	1	0	1	1 ...
Transmitted codeword	U:	11	01	01	00	01 ...
Received sequence	Z:	11	01	01	10	01 ...

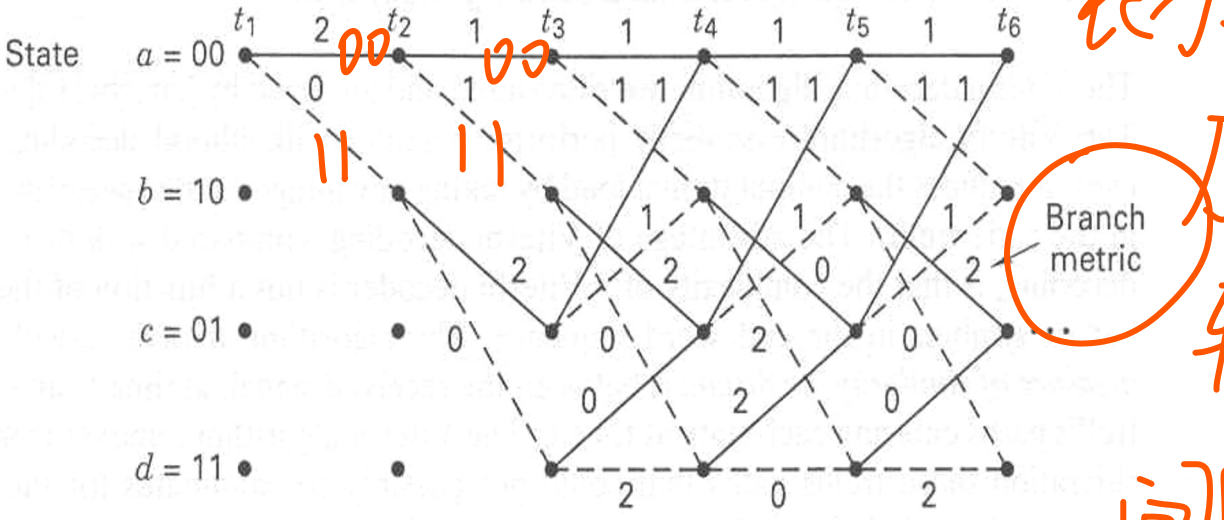


Figure: Decoder trellis diagram (rate ½ , K=3)

Steps in Decoding

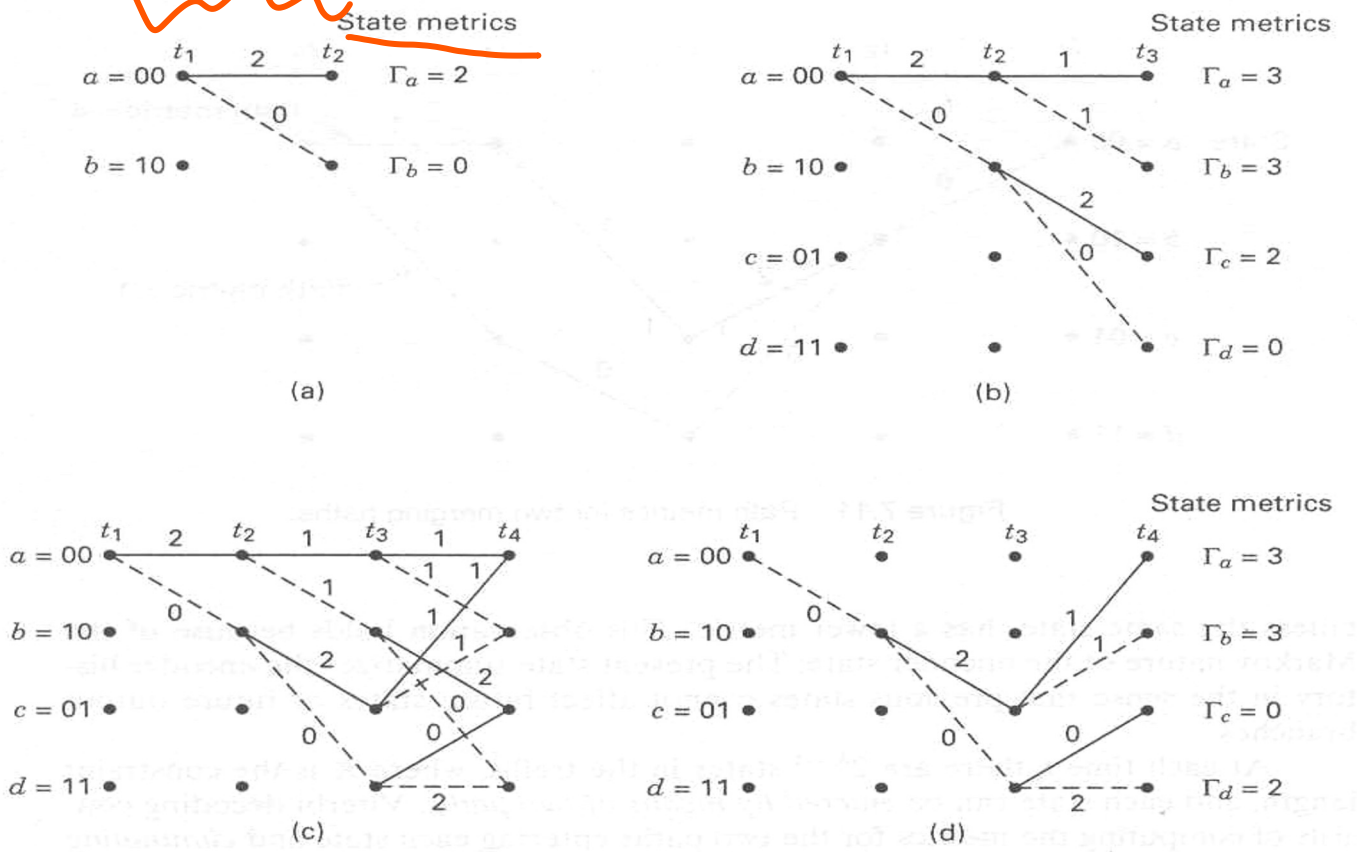
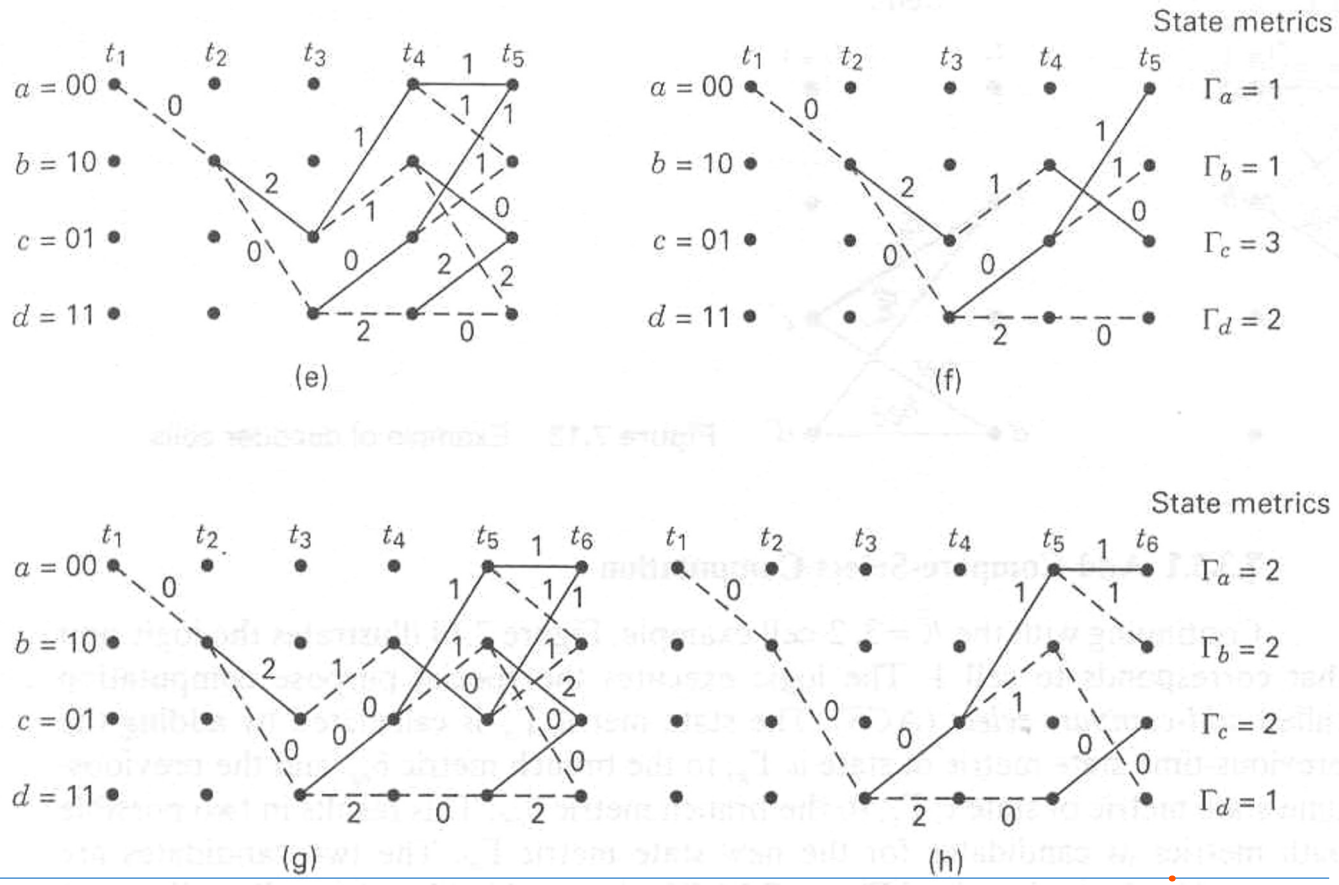


Figure: selection of survivors (a) survivors at t2 (b) survivors at t3 (c) metric comparison at t4 (d) survivors at t4



保留该状态下到达的最短路径

Figure: Selection of survivors (e) metric comparison at t5 (f) survivors at t5 (g) metric comparison at t6 (h) survivors at t6

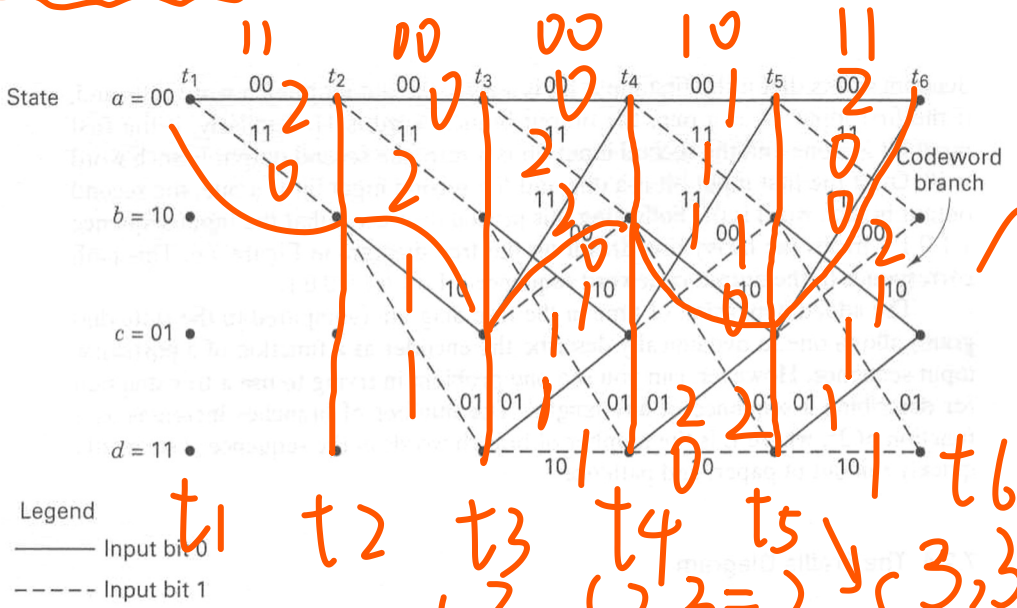
Best Known Convolution Codes

Rate	Constraint Length	Free Distance	Code Vector
$\frac{1}{2}$	3	5	111 101
$\frac{1}{2}$	4	6	1111 1011
$\frac{1}{2}$	5	7	10111 11001
$\frac{1}{2}$	6	8	101111 110101
$\frac{1}{2}$	7	10	1001111 1101101
$\frac{1}{2}$	8	10	10011111 11100101
$\frac{1}{2}$	9	12	110101111 100011101
$\frac{1}{3}$	3	8	111 111 101
$\frac{1}{3}$	4	10	1111 1011 1101
$\frac{1}{3}$	5	12	11111 11011 10101
$\frac{1}{3}$	6	13	10111 110101 111001
$\frac{1}{3}$	7	15	1001111 1010111 1101101
$\frac{1}{3}$	8	16	11101111 10011011 10101001

Table: Optimum Short Constraint Length Convolutional Codes

Practice Question 译码

- For the trellis diagram, given in figure below, of a K=3, rate=1/2 convolutional encoder and the received sequence **Z**= (1 1 0 0 0 0 1 0 1 1 rest all '0') (starts with MSB).
- Determine the first 5 decoded information bits. If a tie occurs between any two merged paths, choose the upper branch entering the particular state.



$\hat{m} = 10100$

A brief introduction to Turbo Codes

- Turbo codes are forward error correction codes based on Convolution codes
- Turbo codes have performance close to Shannon theoretical limit. In fact, when they were presented in a paper, the idea was rejected as it was not considered possible to have such large coding gain
- Turbo coders are formed by the parallel concatenation of two convolutional codes separated by an interleaver
- Turbo codes are widely used in communication e.g., WiMAX, UMTS, CDMA 2000, LTE etc.

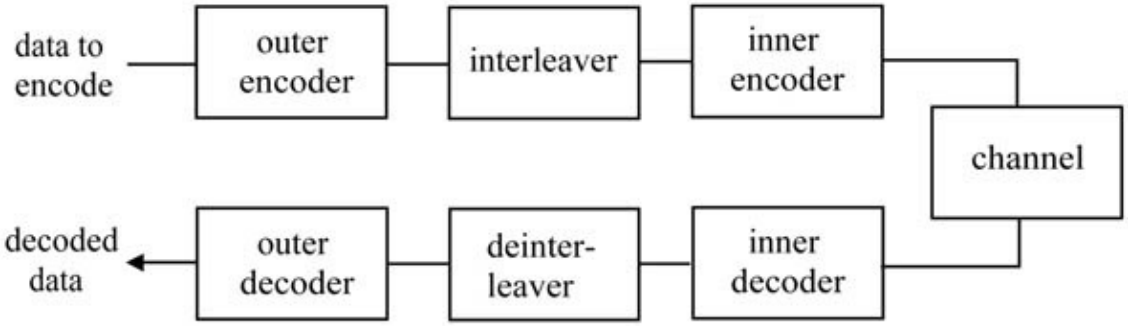
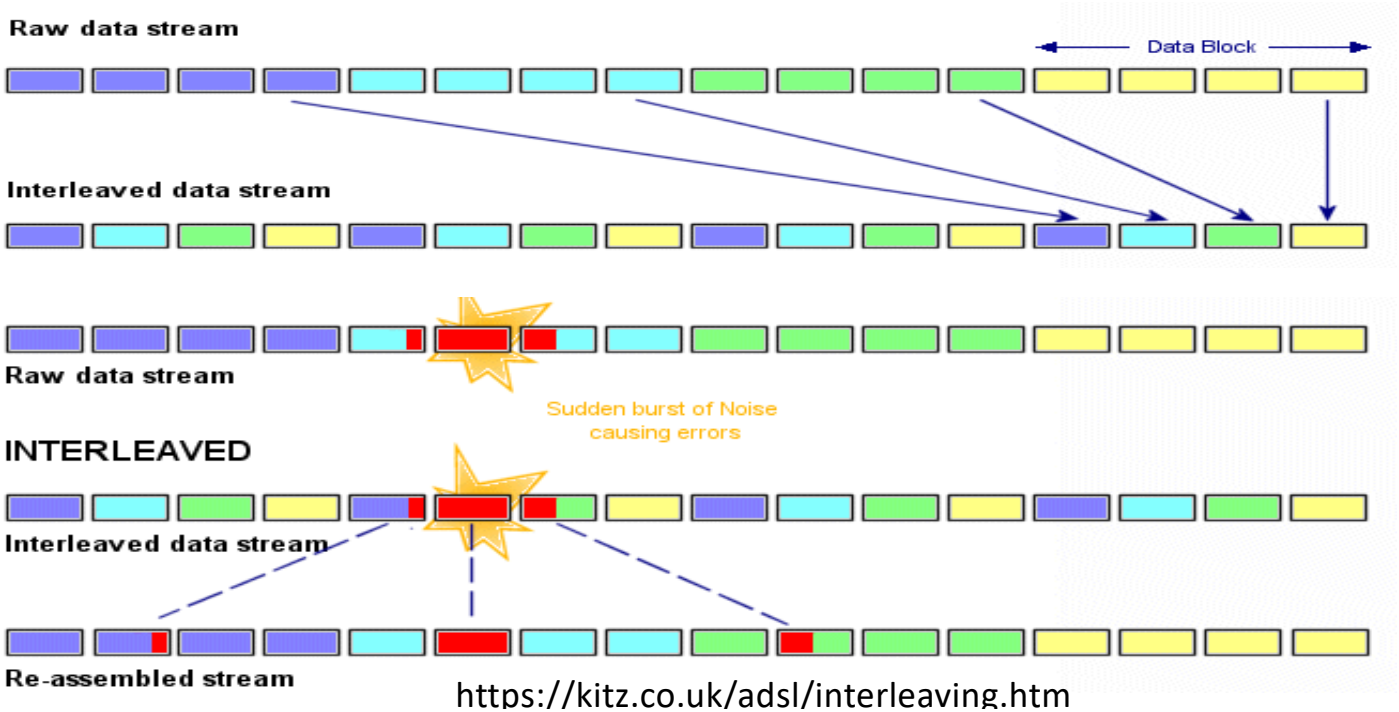


Figure: Concatenated encoder and decoder [1]

[1] http://www.scholarpedia.org/article/Turbo_code

Interleaver

- The performance of error correction coding is improved by introducing an interleaver
- An interleaver is used to improve the error correction in the presence of burst errors



https://kitz.co.uk/adsl/interleaving.htm