Circuit Analysis and Design

Academic year 2019/2020 – Semester 1 – Presentation 7

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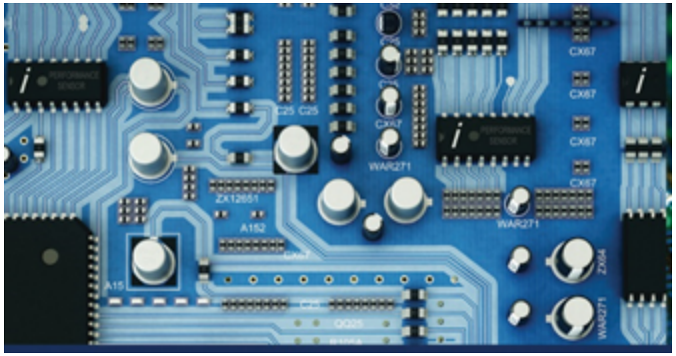
"A good student never steal or cheat"

Agenda



- **Introduction**
- **Superposition principle**
- **Source transformations**
- **Summary**

Introduction

- The theorems are superposition principle, source transformation, Thévenin's theorem, Norton's theorem, and maximum power transfer.

- If a circuit contains more than one source, the circuit can be analyzed by summing the response from each source with all other sources deactivated. This is called the superposition principle.

- A voltage source with a series resistor is interchangeable with a current source in parallel to a resistor : **source transformation.**

- According to Thévenin's theorem, a given circuit is equivalent to a voltage source V_{th} and a series resistor R_{th} between terminals *a* and *b*.

- According to Norton's theorem, a given circuit is equivalent to a current source I_n and a parallel resistor R_n between terminals *a* and *b*.

- The load resistance R_L that maximizes the power delivered to the load is given by the Thévenin equivalent resistance R_{th}.

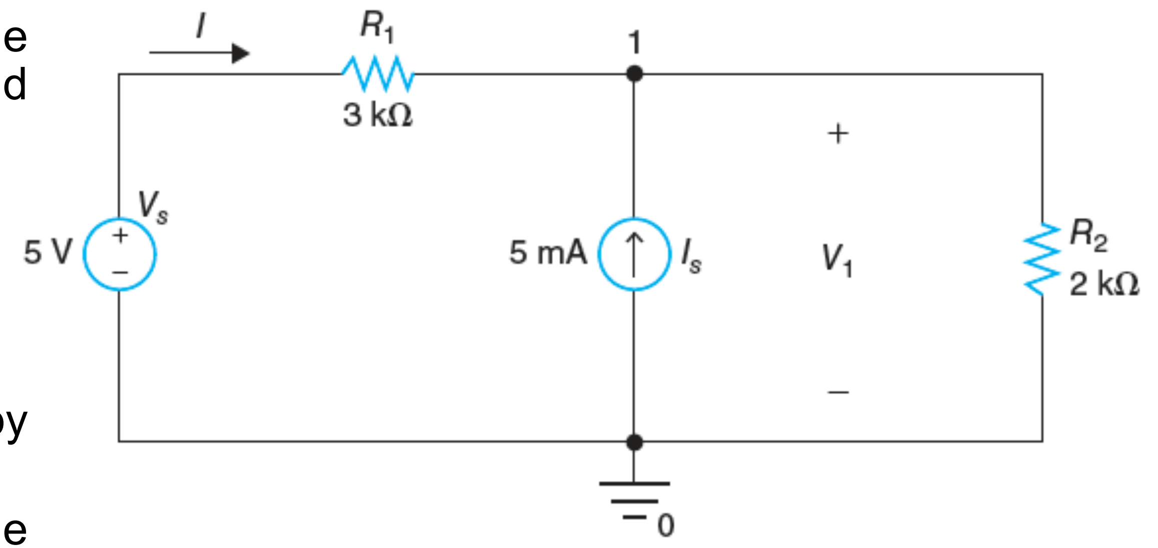
Superposition Principle

- Suppose that a circuit has N independent sources with $N \geq 2$.
- Create N circuits from the original circuit with only one independent source by deactivating the other N–1 independent sources.
- Deactivating a current source is to open-circuit it and deactivating a voltage source is to short-circuit it.
- The unknown voltages and currents of the original circuit can be found by adding the voltages and currents from the N circuits with one independent source: superposition principle.
- The superposition principle reveals the contribution of each source to the voltages and currents in the circuit.
- It makes it easier to interpret the response of the circuit because we can trace the sources of the response.

Superposition Principle

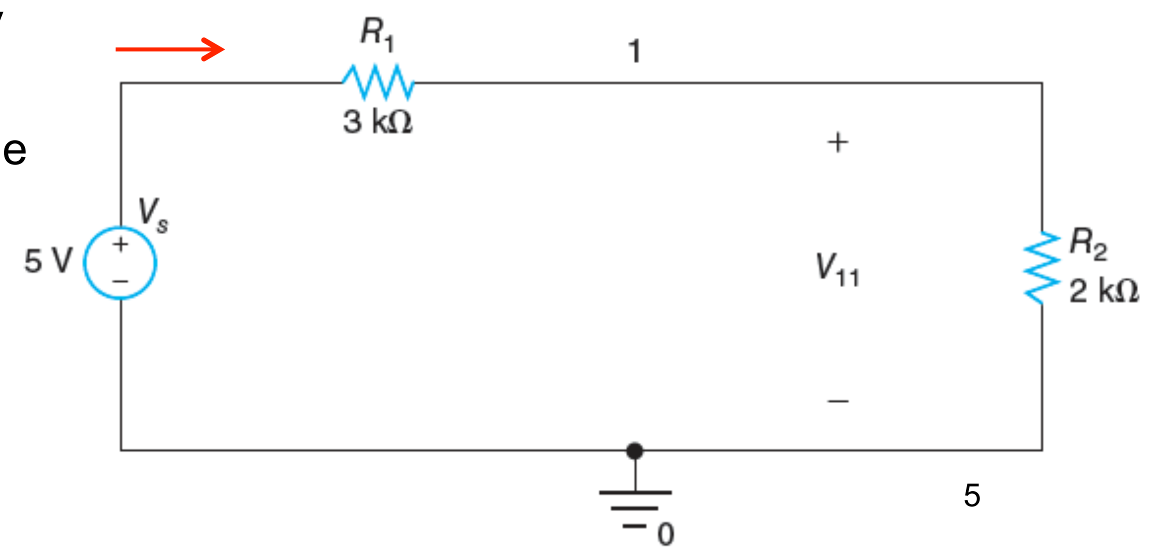
- Consider a circuit with a voltage source V_s, a current source I_s, and two resistors R_1 and R_2.

  Finding V_1 and I
  using superposition principle?



- Deactivate the current source by removing it from the circuit.
- The circuit contains only one independent source V_s.

- Applying the voltage divider rule,
  $V_{11}= V_s{\times}R_2/(R_1+R_2)=5 \times 2/5 = 2$ V



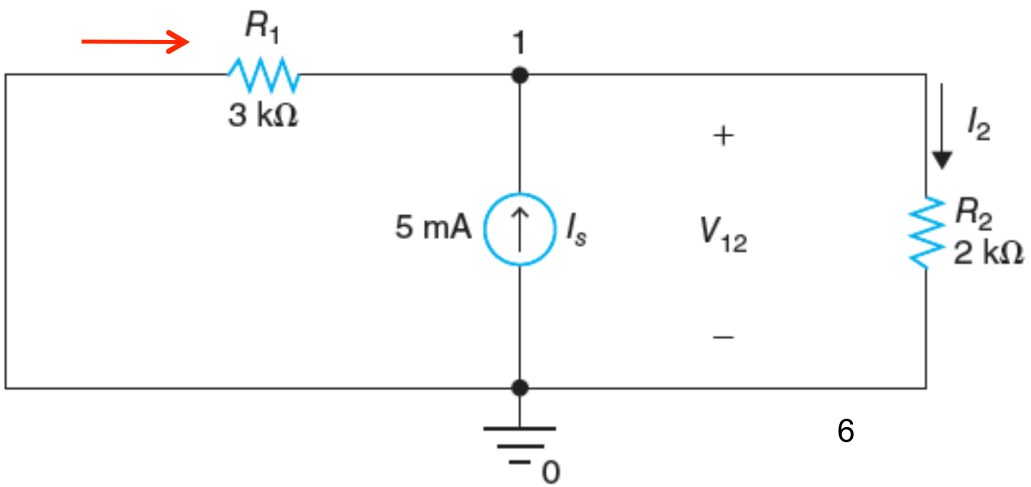
- The contribution of the voltage source to the voltage across R_2 is 2 V.

- Applying Ohm's law,
  $I_a = V_s/(R_1 + R_2) = 5\ /5k = 1$ mA

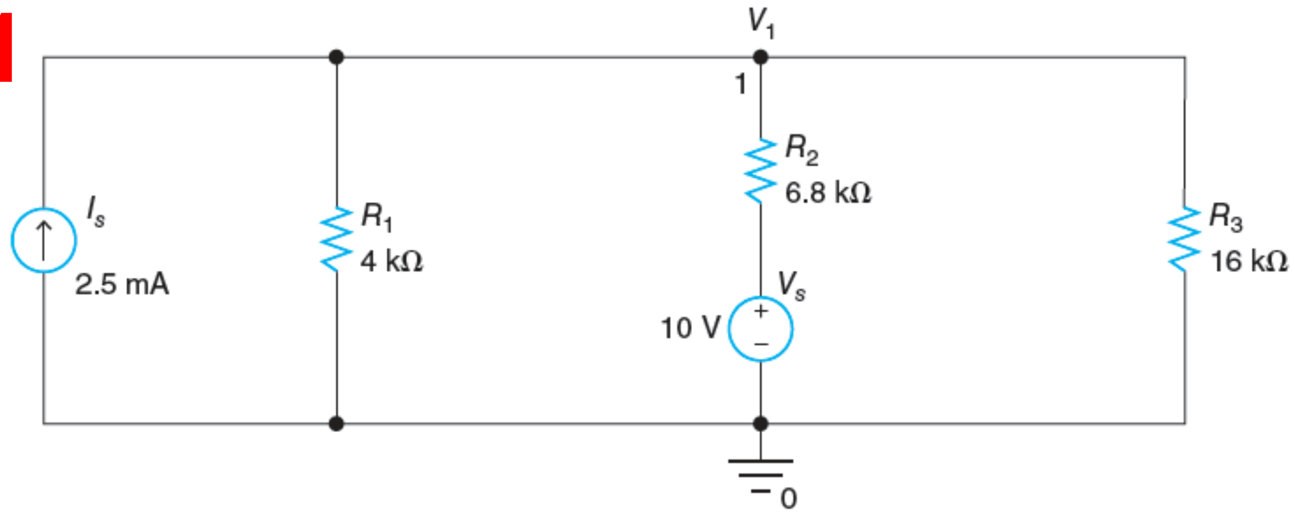
Superposition Principle

- Deactivate the voltage source by short-circuiting.
- The circuit contains only one independent source I_s.
- Applying the current divider rule, we obtain the current through R_2:
  $I_2 = I_s \times R_1/(R_1 + R_2) = 5m \times 3/5 = 3$ mA
- The voltage across R_2 is given by $V_{12} = R_2 I_2 = 2000 \times 0.003 = 6$ V
- The contribution of the current source to the voltage across R_2 is 6 V.
- Applying KCL ➔ $I_b = -(I_s - I_2) = -2$ mA
- The voltage across R_2 is given by the sum of V_{11} and V_{12} ➔
  $V = V_{11} + V_{12} = 2\text{ V} + 6\text{ V} = 8\text{ V}$
- The current I is the sum of I_a and I_b ➔

  $I = I_a + I_b = -1$ mA



EXAMPLE 4.1

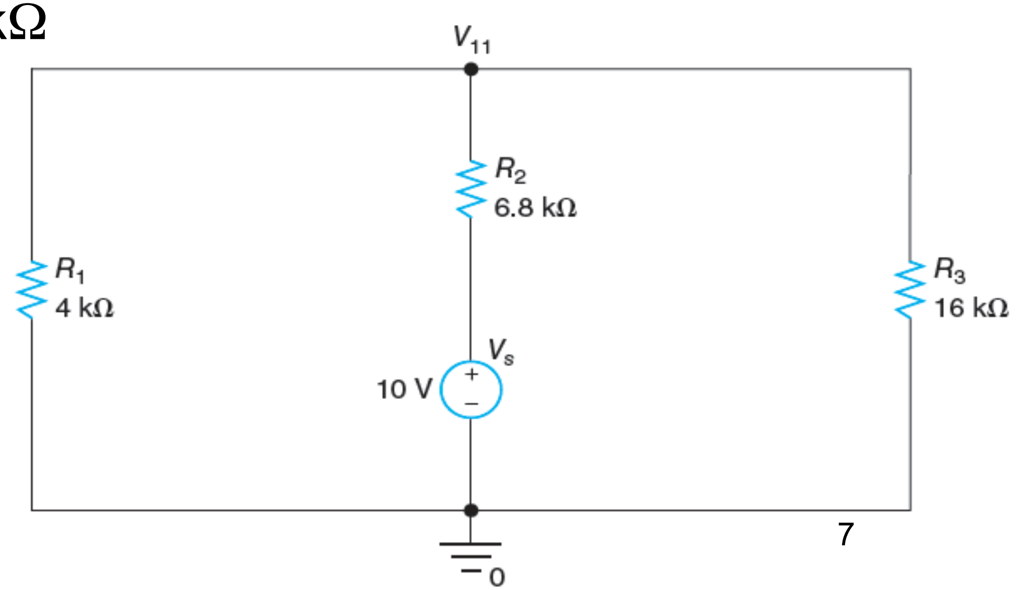


- Use superposition principle to find V_1 in the circuit.

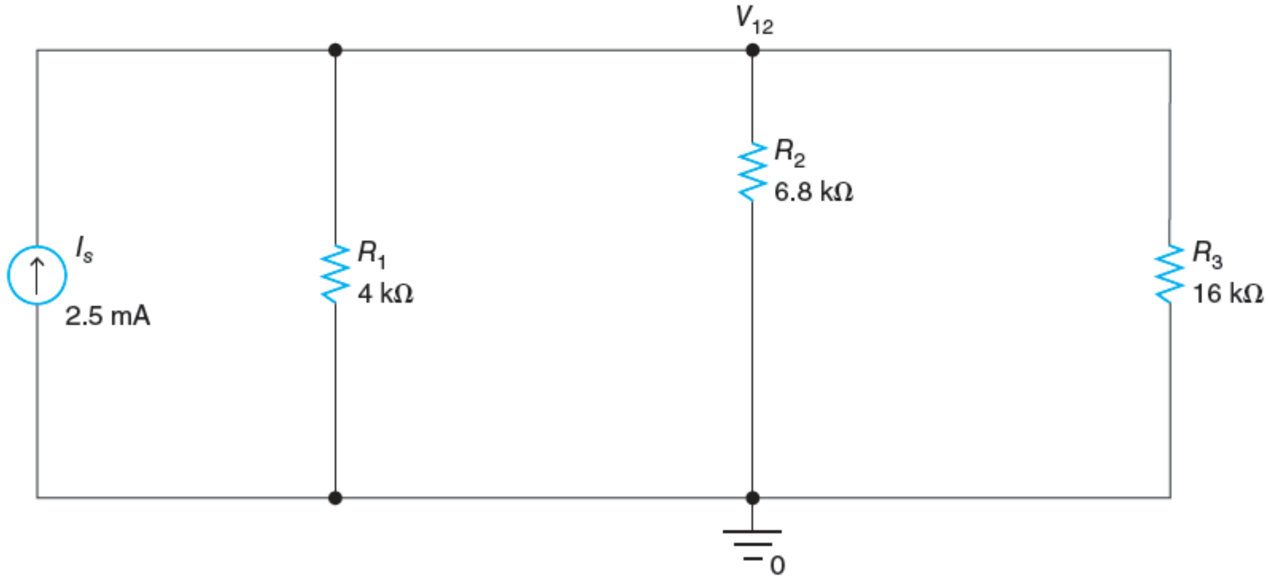
- When the current source is deactivated, the circuit reduces to .
- $R_a = R_1 \,||\, R_3 = 4{\times}16/20$ kΩ = 3.2 kΩ

- Voltage divider rule:

  $V_{11} = V_s \times R_a/(R_2 + R_a)$
  $V_{11} = 10 \times 3.2/10 = 3.2$ V



EXAMPLE 4.1



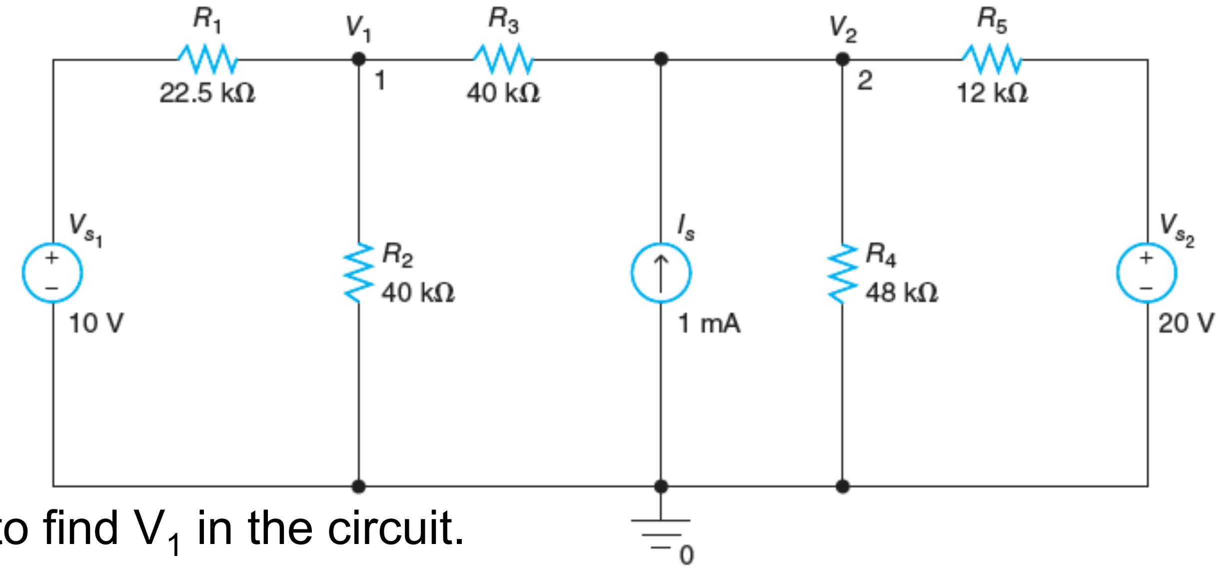
- When the voltage source is deactivated, the circuit reduces to ☝.

- $R_a = R_1 \parallel R_3 = 4{\times}16/20\ k\Omega = 3.2\ k\Omega$
- $R_b = R_a \parallel R_2 = 3.2{\times}6.8/10 = 2.176\ k\Omega$

- Ohm's law ➔ $V_{12} = R_b \times I_s = 2.176 \times 2.5 = 5.44$ V

- The voltage V_1 is the sum of V_{11} and V_{12}:
  $V_1 = V_{11} + V_{12} = 3.2\ V + 5.44\ V = 8.64\ V$

EXAMPLE 4.2

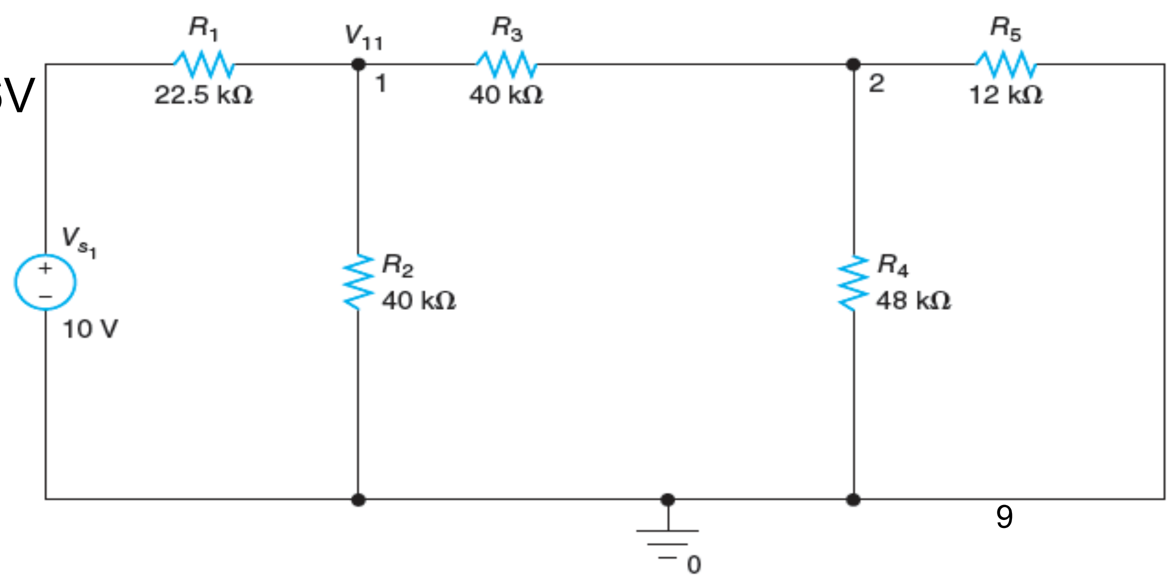


- Use superposition principle to find V_1 in the circuit.
- When I_s and V_{s2} are deactivated, the circuit reduces to ☟.
- $R_a = R_4 \,||\, R_5 = 48 \times 12/(48 + 12) = 9.6\ k\Omega$

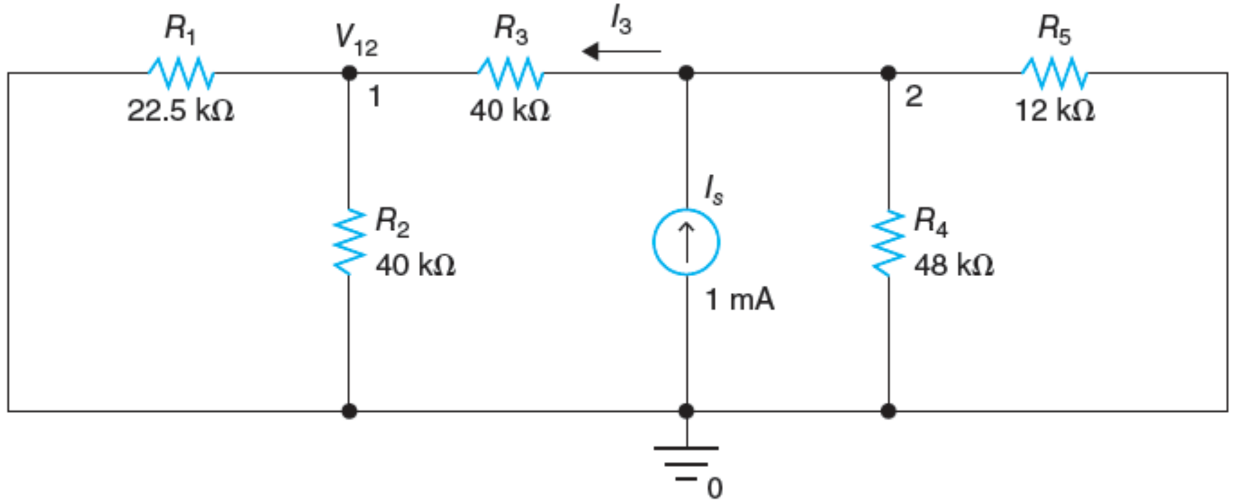
  $R_b = R_3 + R_a = 49.6\ k\Omega$

  $R_c = R_2 \,||\, R_b = 40 \times 49.6/(40 + 49.6) = 22.1429\ k\Omega$
- Voltage divider rule:

  $V_{11} = V_{s1} \times R_c/(R_1 + R_c) = 4.96V$

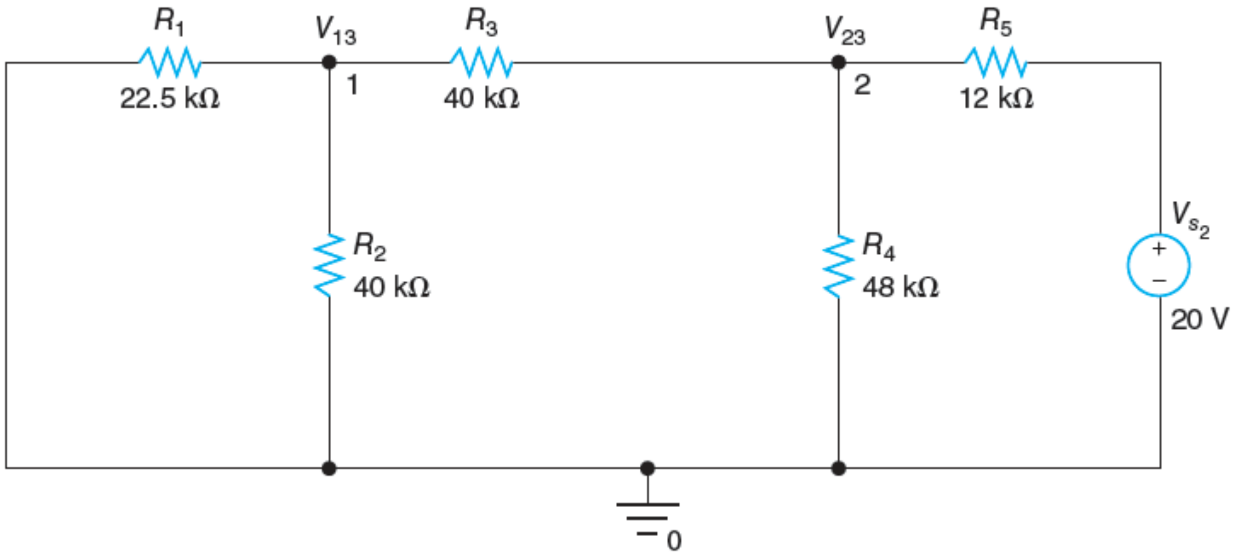


EXAMPLE 4.2



- When V_{s1} and V_{s2} are deactivated, the circuit reduces to ☝.
- $R_a = R_4 \,||\, R_5 = 48 \times 12/(48 + 12) = 9.6$ kΩ
  $R_d = R_1 \,||\, R_2 = 22.5 \times 40/(22.5 + 40) = 14.4$ kΩ
  $R_e = R_3 + R_d = 54.4$ kΩ
- Current divider rule:
  $I_3 = I_s \times R_a/(R_a + R_e) = 0.15$ mA
- Ohm's law:
  $V_{12} = R_d I_3 = 2.16$ V

EXAMPLE 4.2



- When V_{s1} and I_s are deactivated, the circuit reduces to ☝.
- $R_d = R_1 \,||\, R_2 = 22.5 \times 40/(22.5 + 40) = 14.4\ \text{k}\Omega$

  $R_e = R_3 + R_d = 54.4\ \text{k}\Omega$

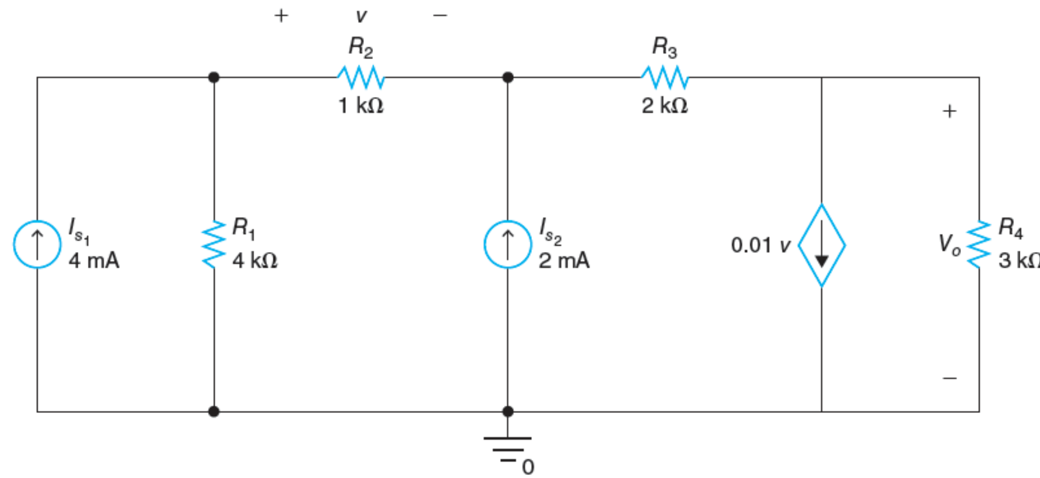
  $R_f = R_4 \,||\, R_e = 48 \times 54.4/(48 + 54.4) = 25.5\ \text{k}\Omega$
- Voltage divider rule:

  $V_{23} = V_{s2} \times R_f/(R_5 + R_f) = 13.6\ \text{V}$

  $V_{13} = V_{23} \times R_d/(R_3 + R_d) = 3.6\ \text{V}$
- The voltage V_1 is given by

  $V_1 = V_{11} + V_{12} + V_{13} = 4.96\ \text{V} + 2.16\ \text{V} + 3.6\ \text{V} = 10.72\ \text{V}$

EXAMPLE 4.3

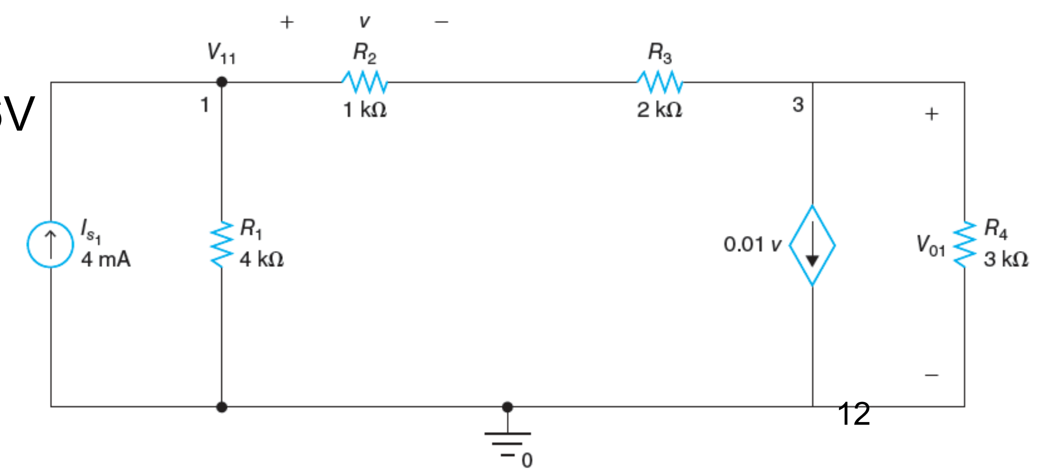


- Use superposition principle to find V_o in the circuit.
- When I_{s2} is deactivated, the circuit reduces to ☟.
- $v = (V_{11} - V_{01}) \times R_2/(R_2 + R_3) = (V_{11} - V_{01})/3$
- Node 1➔ $-0.004 + \dfrac{V_{11}}{4000} + \dfrac{V_{11} - V_{01}}{3000} = 0$ Multiply by 12000 ➔ $7V_{11} - 4V_{01} = 48$ (1)
- Node 2 ➔ $\dfrac{V_{01} - V_{11}}{3000} + \dfrac{V_{01}}{3000} + 0.01\dfrac{V_{11} - V_{01}}{3} = 0$ Multiply 3000 ➔ $-8V_{01} + 9V_{11} = 0 \Rightarrow$

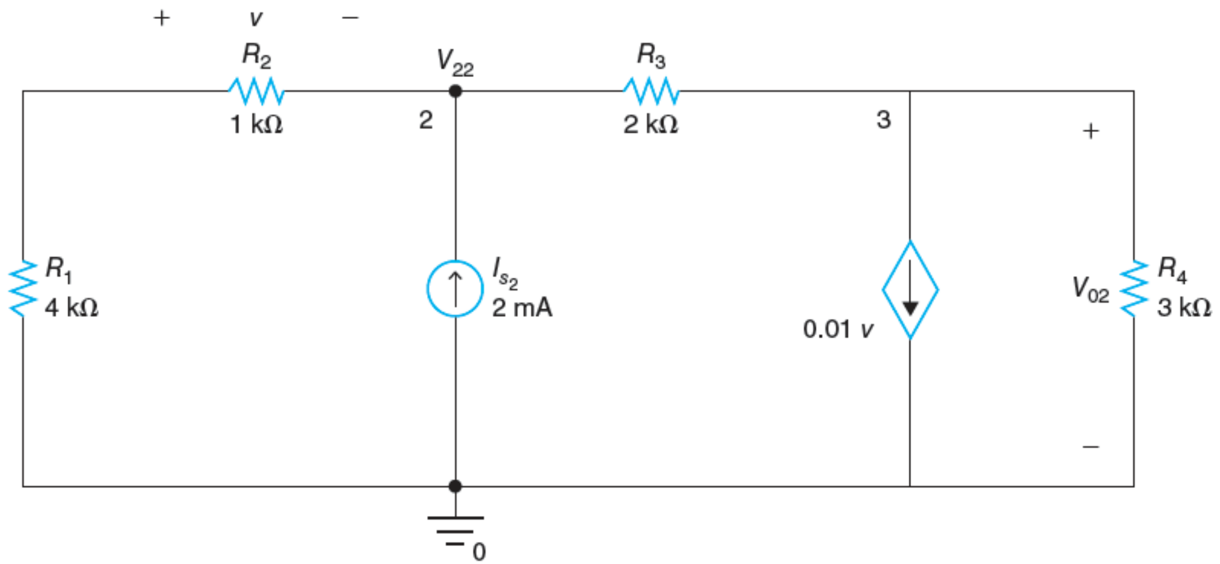
$V_{11} = (8/9)V_{o1}$ (2),

(2)→(1):$(7 \times 8/9)\, V_{01} - 4V_{01} = 48 \Rightarrow$

$(20/9)\, V_{o1} = 48 \Rightarrow V_{o1} = 48 \times 9/20 = 21.6\text{V}$



EXAMPLE 4.3



- When I_{s1} is deactivated, the circuit reduces to the one shown in Figure 4.15.
- $v = (-V_{22}) \times R_2/(R_1 + R_2) = (-V_{22})/5$
- Node 2: $\dfrac{V_{22}}{5000} - 0.002 + \dfrac{V_{22} - V_{02}}{2000} = 0$ Multiply by 10000: $7V_{22} - 5V_{02} = 20$ (3)
- Node 3: $\dfrac{V_{02} - V_{22}}{2000} + \dfrac{V_{02}}{3000} + 0.01\dfrac{-V_{22}}{5} = 0$ Multiply 6000: $3V_{02} - 3V_{22} + 2V_{02} - 12V_{22} = 0$,

$V_{22} = (1/3)V_{02}$ (4), Substitute Equation (4) into Equation (3)➔
$(7/3)V_{02} - 5V_{02} = 20 \Rightarrow (-8/3)\, V_{02} = 20 \Rightarrow V_{02} = -60/8 = -7.5$ V

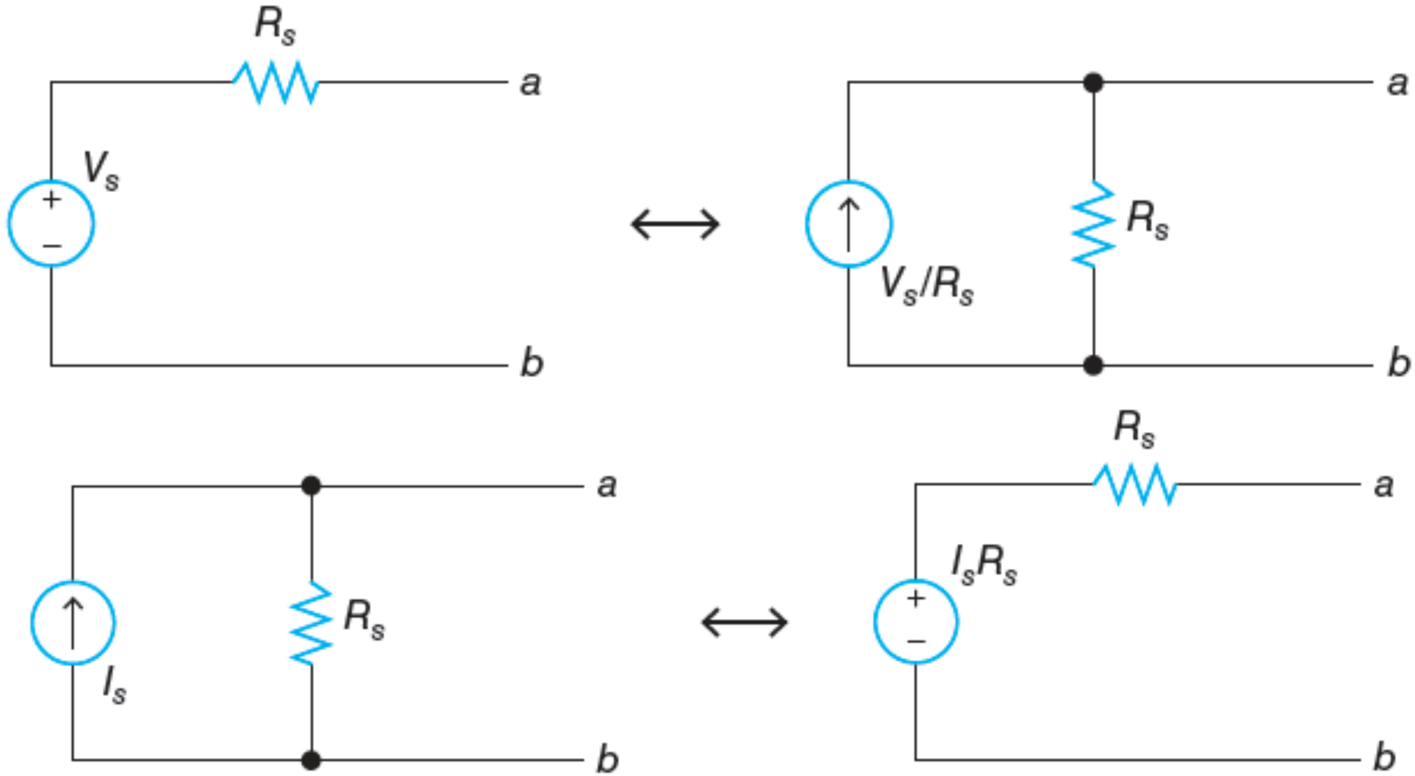
- $V_0 = V_{01} + V_{02} = 21.6$ V – 7.5 V = 14.1 V

Source Transformation

- A circuit consisting of a voltage source with voltage V_s and a series resistor with resistance R_s, is equivalent to a circuit consisting of a current source with current V_s/R_s and a parallel resistor with resistance R_s.

- Equivalence means that the circuits have the same open-circuit voltage across *a* and *b*, the same short-circuit current through *a* and *b*, and the same resistance looking into the circuit from *a* and *b* after deactivating the source.

- A circuit consisting of a current source with current I_s and a parallel resistor with resistance R_s is equivalent to a circuit consisting of a voltage source with voltage I_sR_s and a series resistor with resistance R_s.



Source Transformation

- The source transformations apply to dependent sources as well. Figures 4.19 and 4.20 show the equivalence of a voltage source and a series resistor, and a current source and a parallel resistor.

FIGURE 4.19

A dependent voltage source and a series resistor are equivalent to a dependent current source and a parallel resistor.

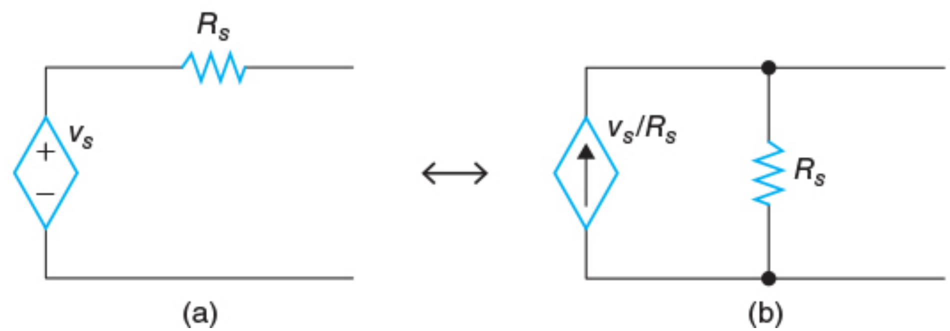
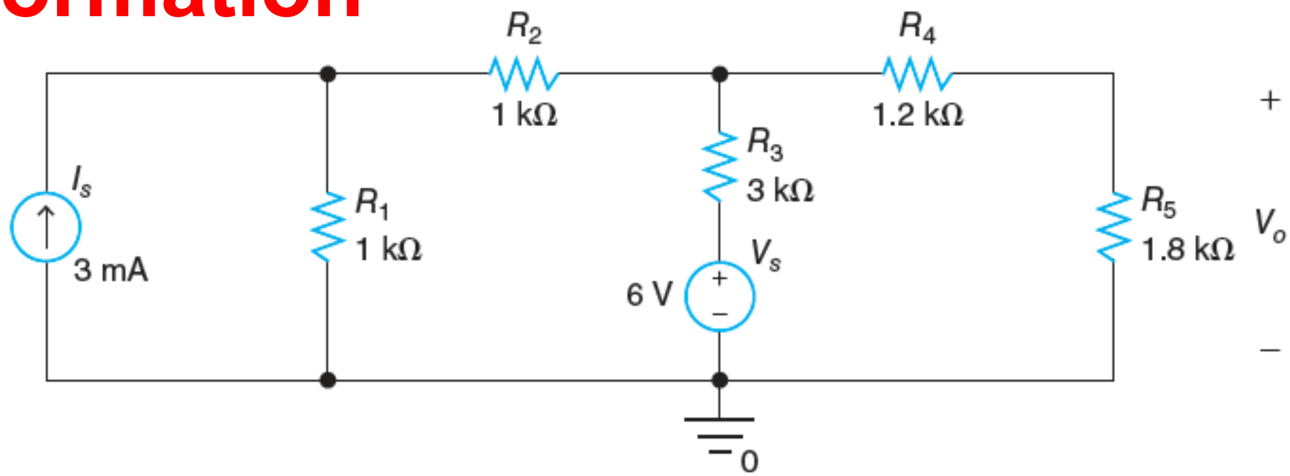


FIGURE 4.20

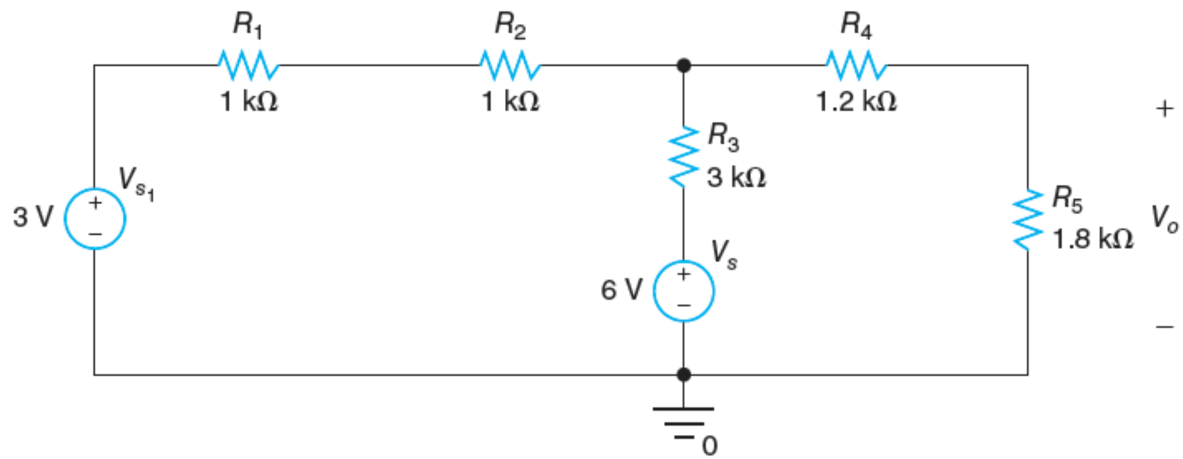
A dependent current source and a parallel resistor are equivalent to a dependent voltage source and a series resistor.

Source Transformation

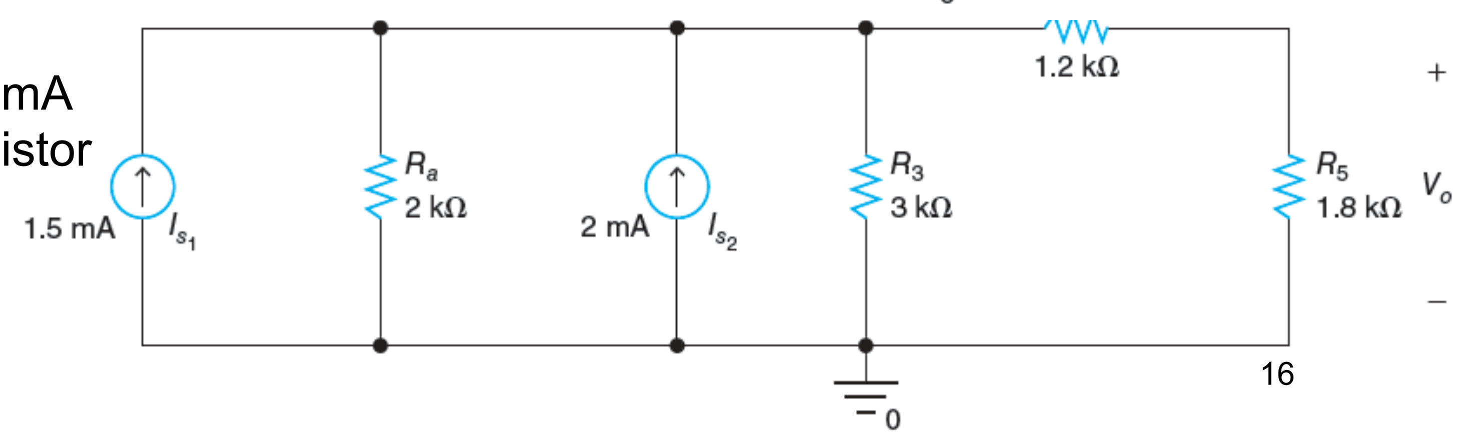
- We are interested in finding V_o using source transformation.



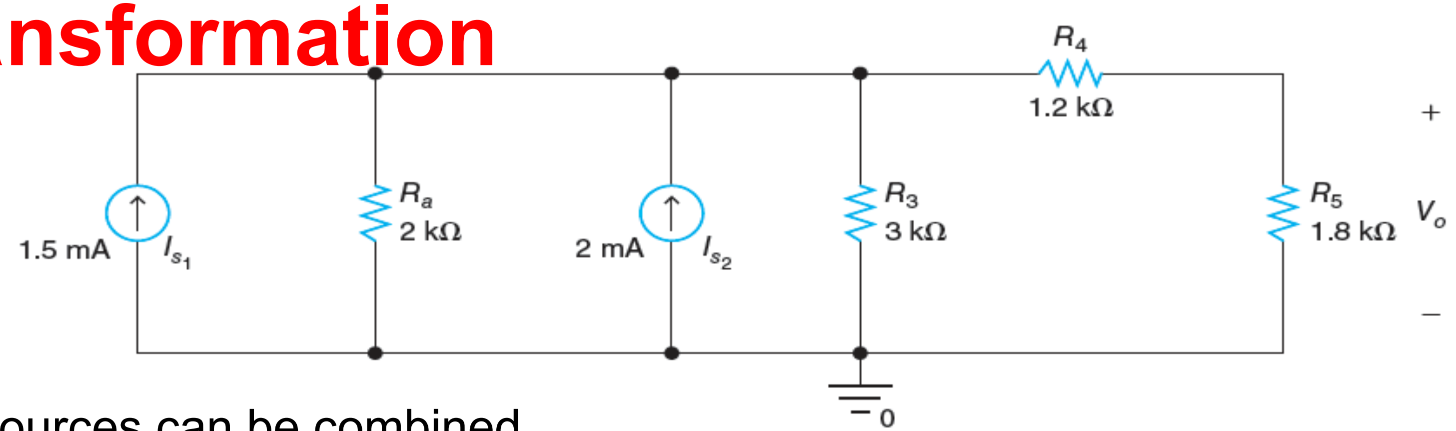
- I_s and R_1 can be transformed to a voltage source V_{s1} and a series resistor R_1. Let $R_a = R_1 + R_2 = 2\text{k}\Omega$.



- V_{s1} and R_a can be transformed to a current source $I_{s1} = V_{s1}/R_a = 1.5$ mA and a parallel resistor R_a.

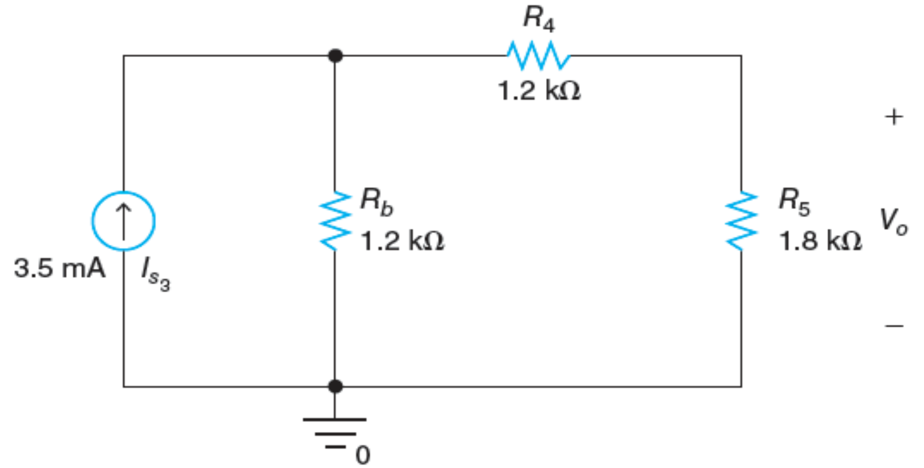


Source Transformation



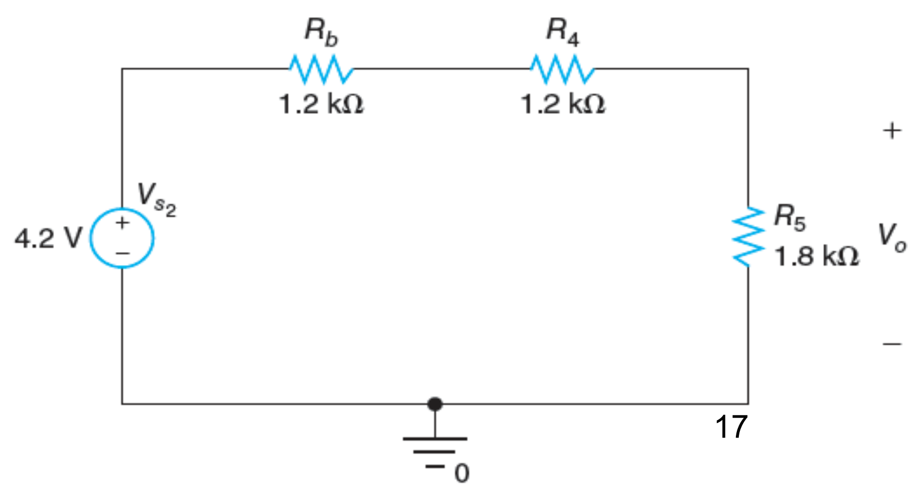
- The two current sources can be combined into one current source with current

  $I_{s3} = I_{s1} + I_{s2} = 3.5$ mA, and parallel resistors R_a and R_3 can be combined into an equivalent resistor with resistance $R_b = R_a \,||\, R_3 = 1.2$ kΩ.

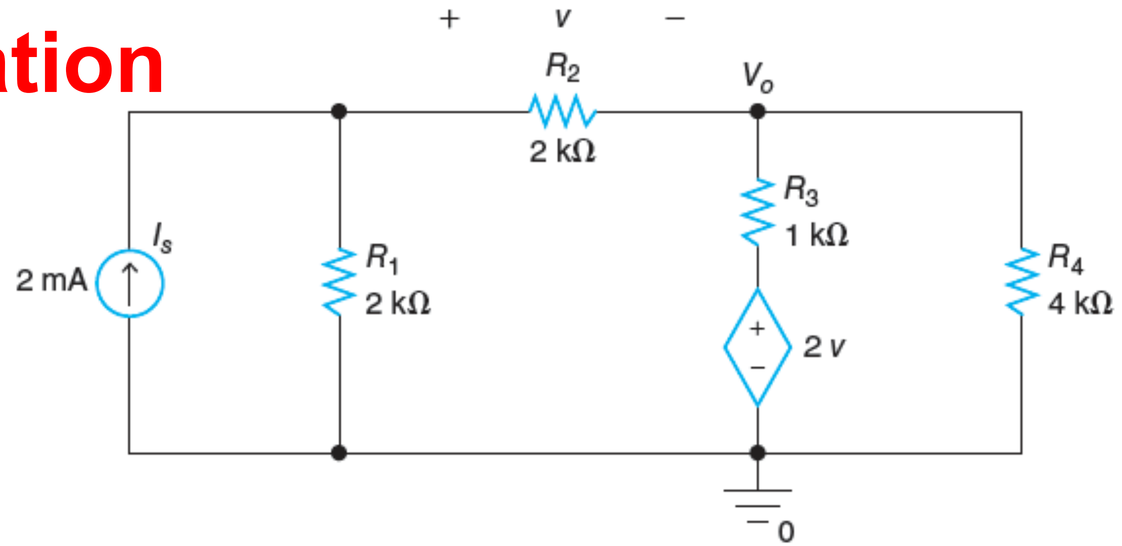


- I_{s3} and R_b can be transformed to a voltage source with voltage $V_{s2} = R_b I_{s3} = 4.2$ V and a series resistor R_b.
-
- According to the voltage divider rule, we obtain

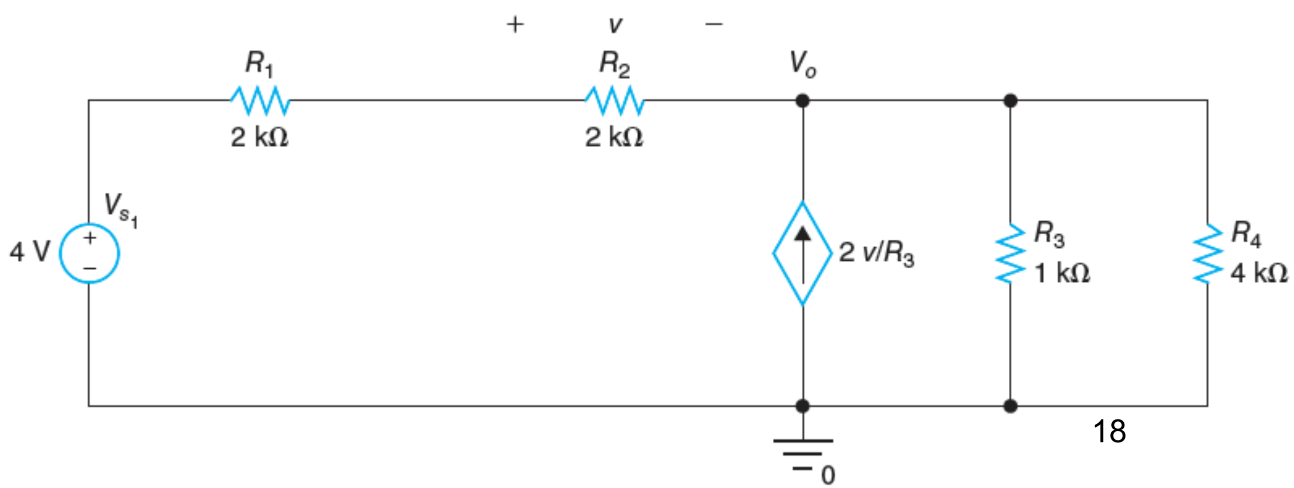
  $V_o = V_{s2} \times R_5/(R_b + R_4 + R_5)$
  $= 4.2 \text{ V} \times 1.8/(1.2 + 1.2 + 1.8) = 1.8$ V



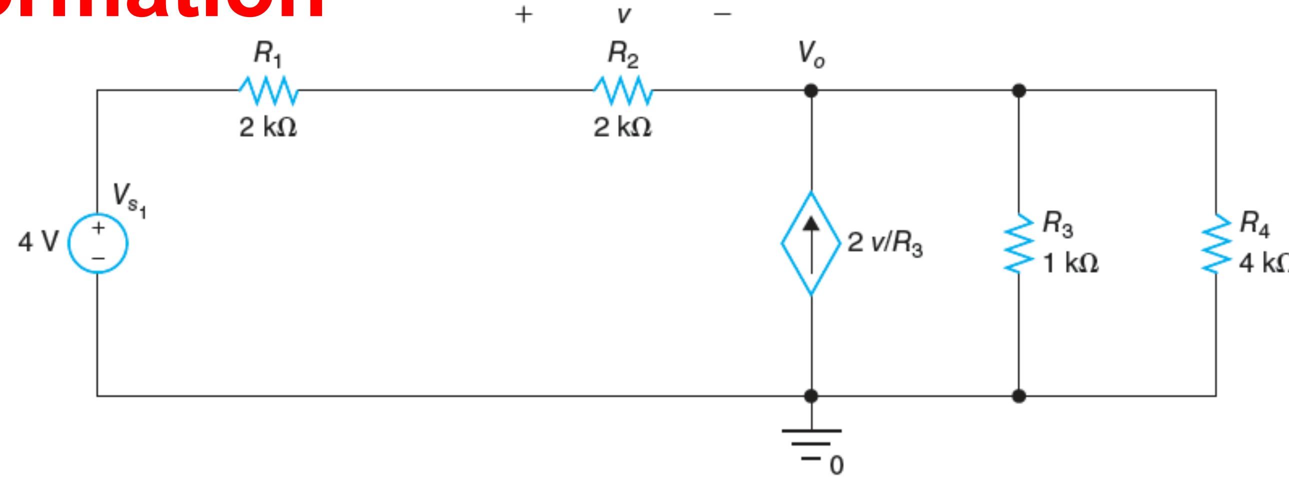
Source Transformation



- We are interested in finding V_o in the circuit.
- I_s and R_1 are transformed to V_{s1} with voltage 4 V and a series resistor R_1
- The voltage source 2v and series resistor R_3 can be transformed to a current source with current $2v/R_3$ and a parallel resistor R_3.
- Let R_a be the equivalent resistance of the parallel connection of R_3 and R_4. Then, we have $R_a = R_3 \mid\mid R_4 = 0.8$ kΩ.



Source Transformation



- The current source with current $2v/R_3$ and a parallel resistor R_a can be transformed to a voltage source with voltage $2vR_a/R_3$ and a series resistor R_a .

- Collecting the voltage drops around the mesh, we obtain

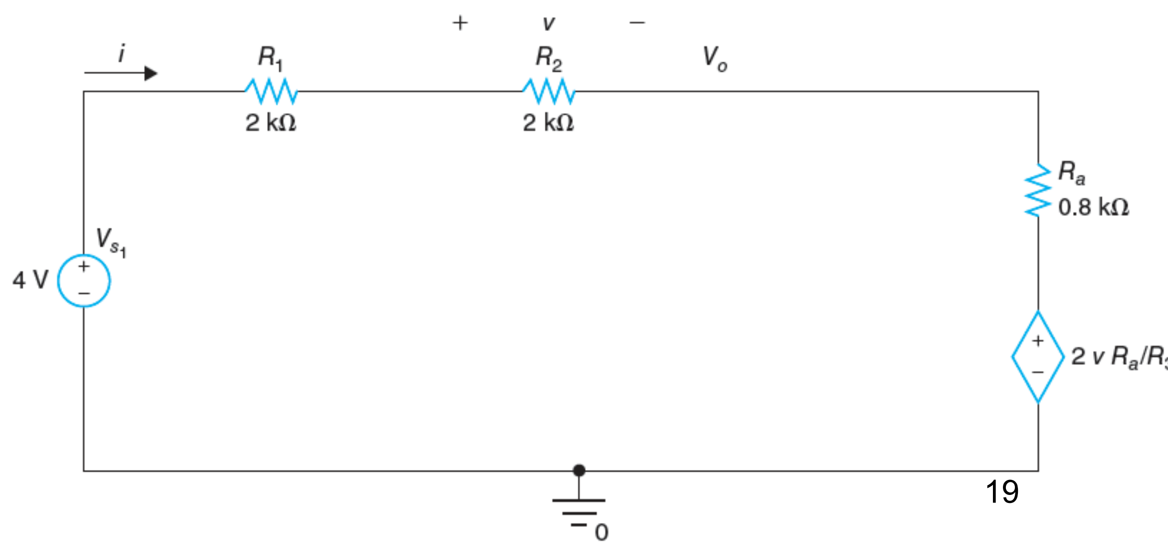
  $– 4 + 2000i + 2000i + 800i + 2(2000i)800/1000 = 0$

  $i = 4/8000 = 0.5$ mA

- The voltage V_o is given by

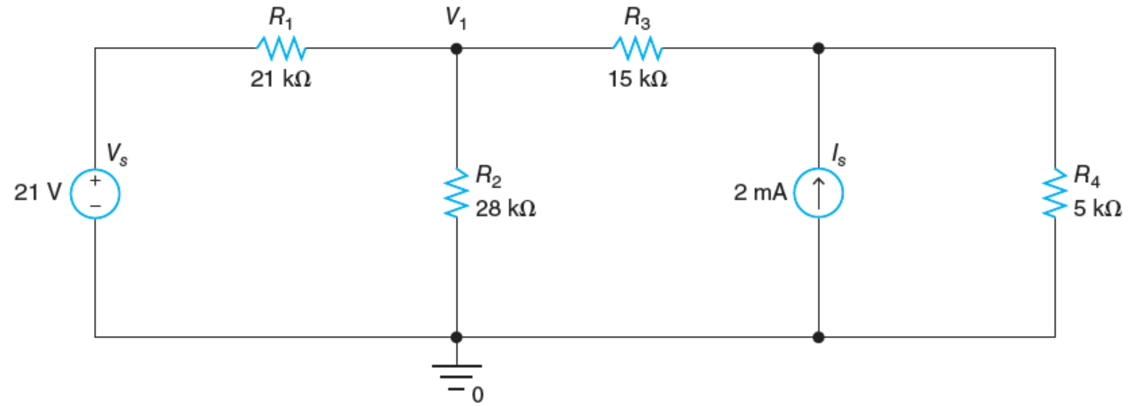
  $V_o = V_s – 2000i – 2000i$

  $V_o = 4$ V – 1 V – 1 V = 2 V

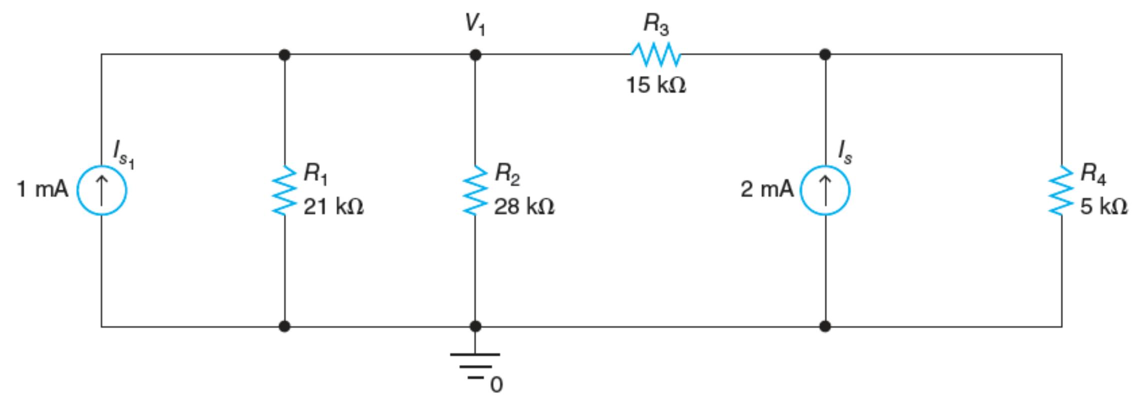


EXAMPLE 4.4

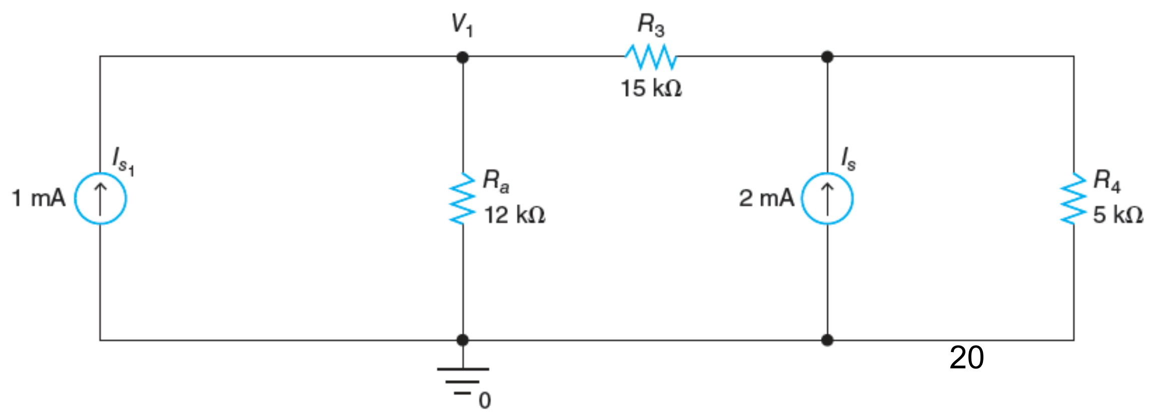
- Use source transformation to find V_1 in the circuit.



- V_s and R_1 are transformed to $I_{s1} = V_s/R_1 = 1$ mA and R_1.

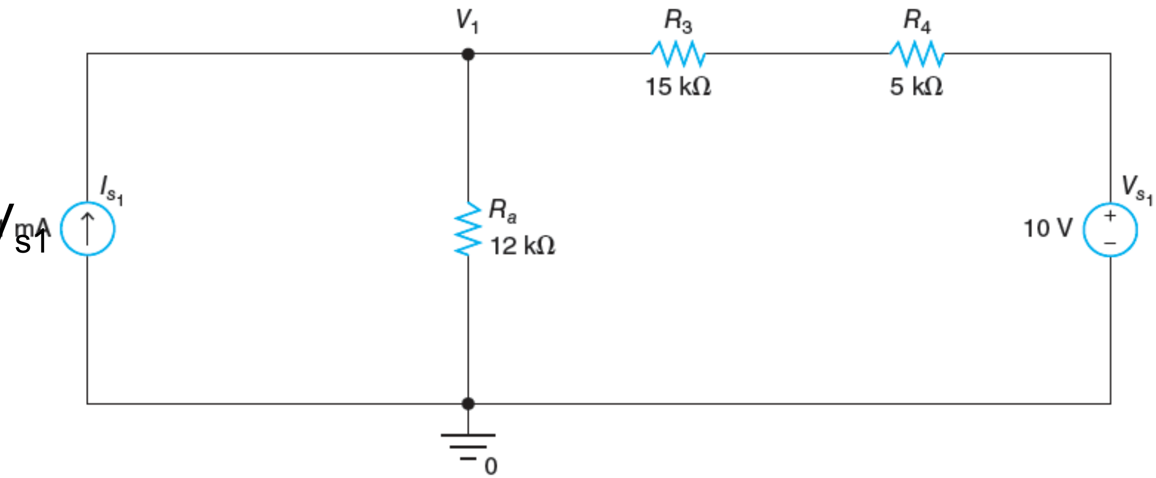


- Let $R_a = R_1 \parallel R_2 = 12\ \text{k}\Omega$. Then, the parallel connection of R_1 and R_2 can be replaced by R_a.

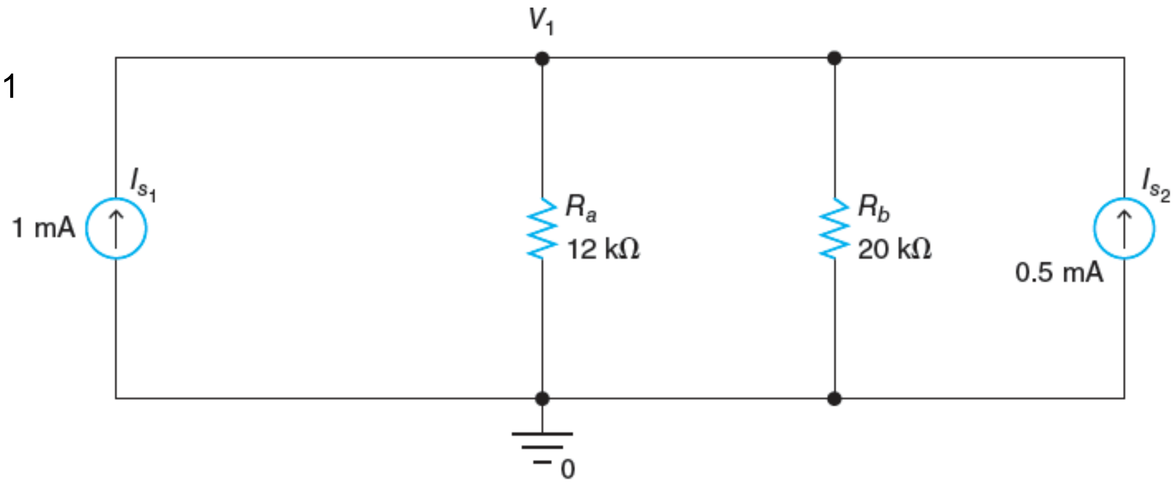


EXAMPLE 4.4

- I_s and R_4 are transformed to $V_{s1} = R_4 I_s = 10$ V and R_4.

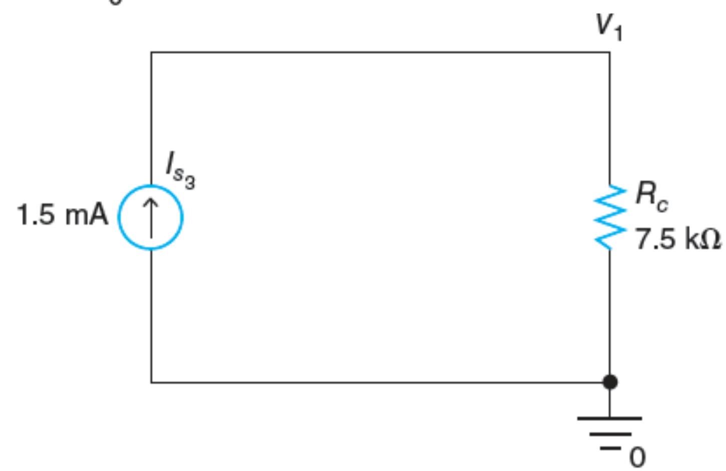


- Let $R_b = R_3 + R_4 = 20$ kΩ. V_{s1} and R_b are transformed to $I_{s2} = V_{s1}/R_b = 0.5$ mA and R_b in parallel.

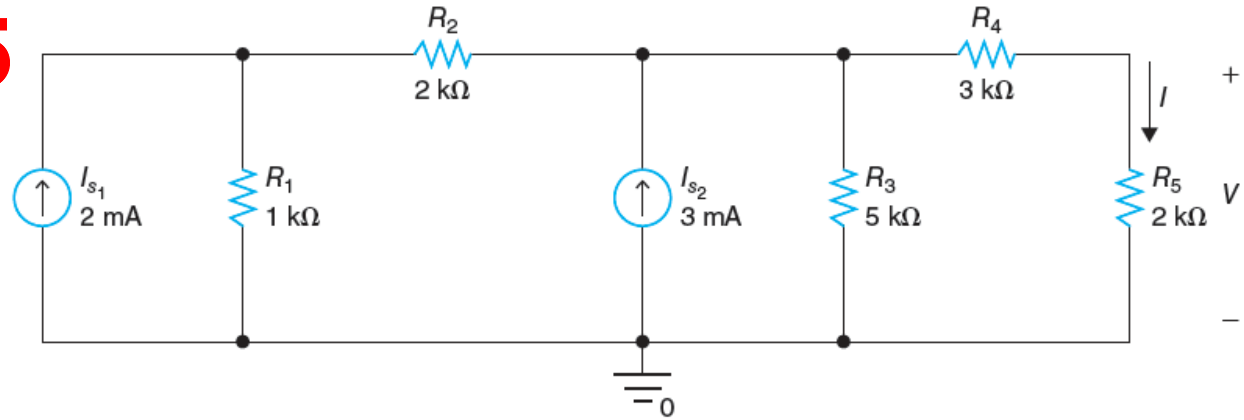


- $R_c = R_a \,||\, R_b = 7.5$ kΩ, $I_{s3} = I_{s1} + I_{s2} = 1.5$ mA

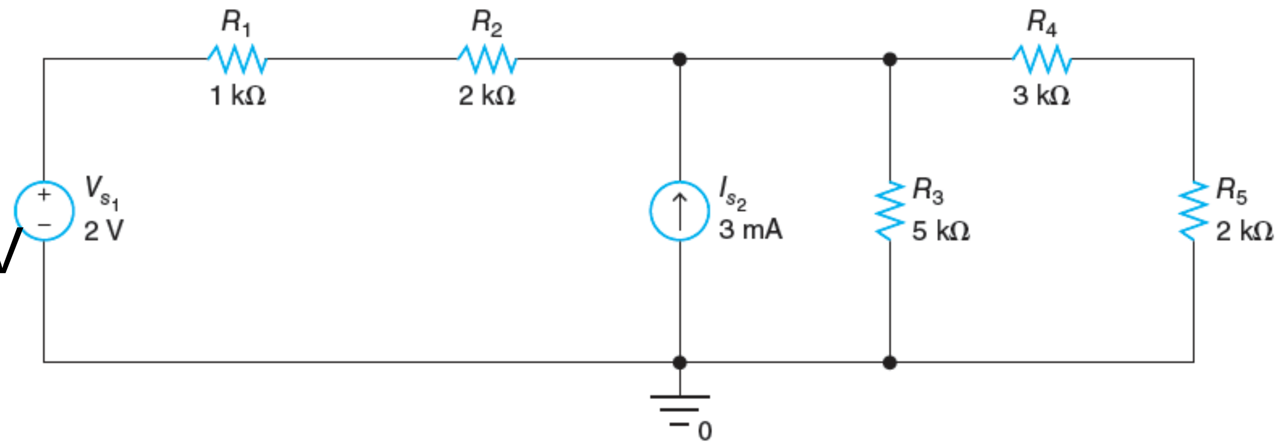
- $V_1 = R_c I_{s3} = 11.25$ V



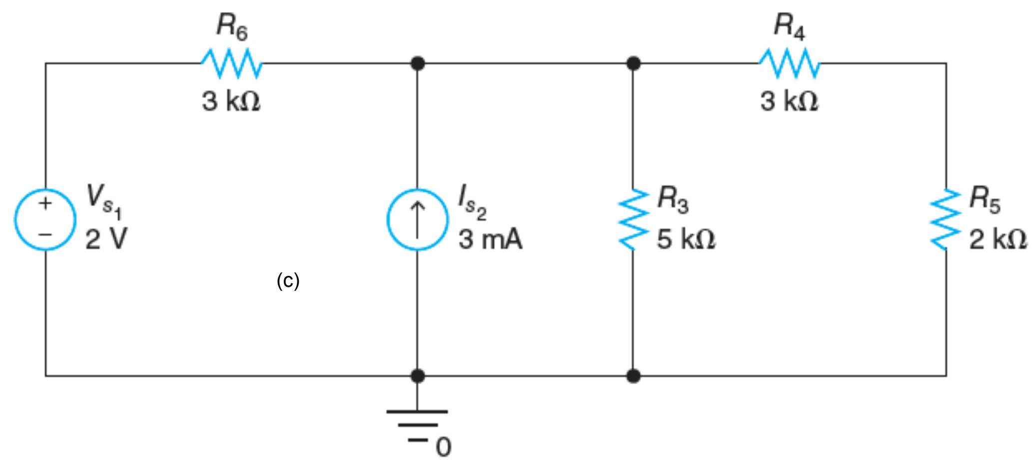
EXAMPLE 4.5



- Use source transformation to find V and I for the circuit.



- Transform I_{s1} and R_1 to $V_{s1}=R_1 I_{s1}= 2$ V and R_1.

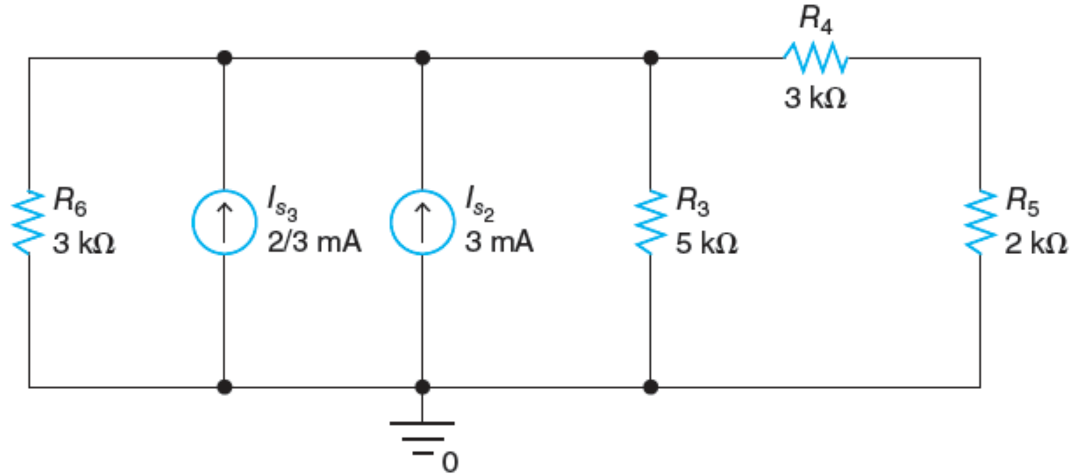


(c)

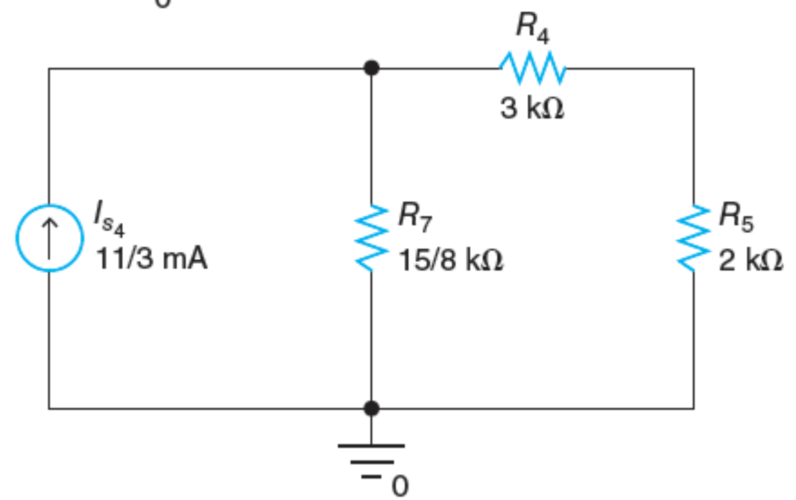
- $R_6 = R_1 + R_2 = 3$ kΩ.

EXAMPLE 4.5

- Transform V_{s1} and R_6 to $I_{s3} = V_{s1}/R_6 = 2/3$ mA and R_6.

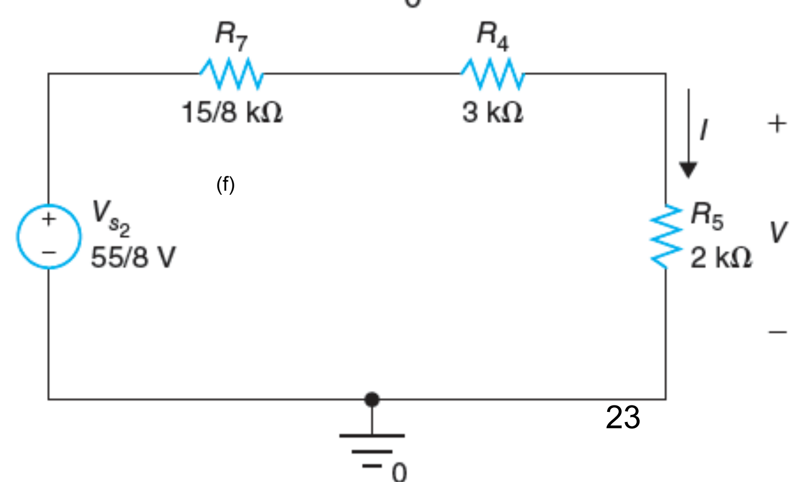


- $R_7 = R_3 \parallel R_6 = 15/8$ kΩ, $I_{s4} = I_{s2} + I_{s3} = 11/3$ mA.

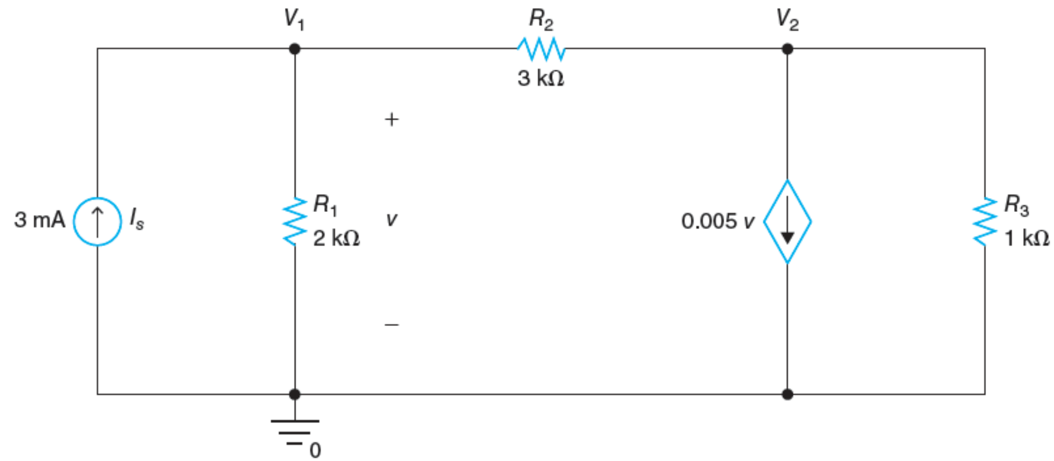


- Transform I_{s4} and R_7 to $V_{s2} = R_7 I_{s4} = 55/8$ V and R_7.

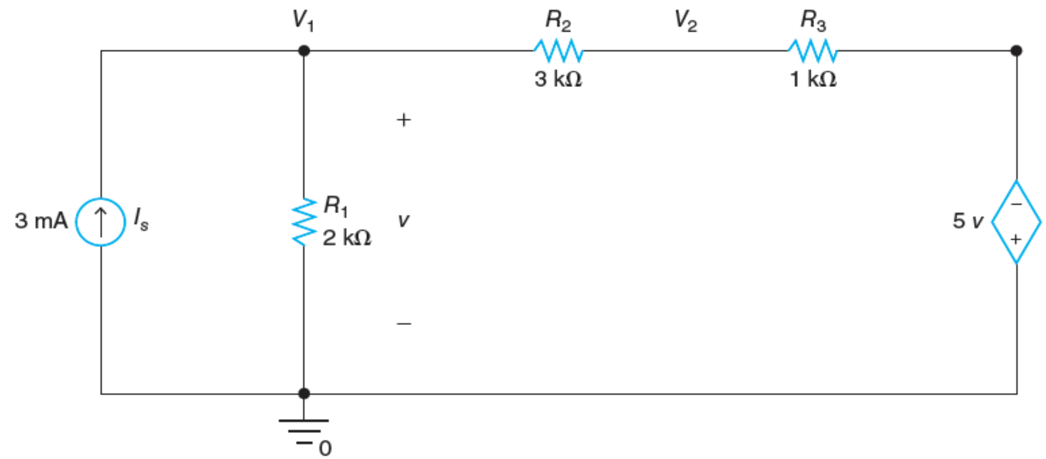
- $I = V_{s2}/(R_7 + R_4 + R_5) = 1$ mA
- $V = R_5 I = 2$ V



EXAMPLE 4.6

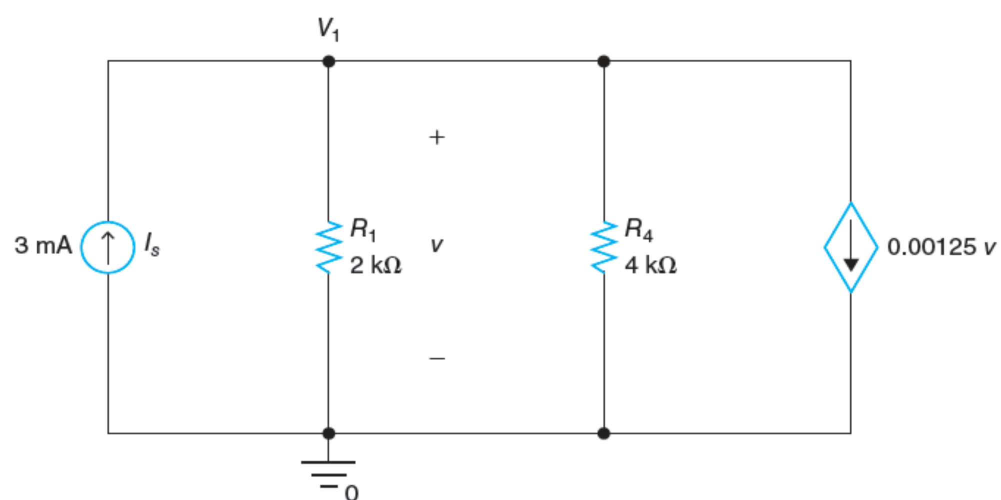


- Use source transformation to find V_1.



- Transform the current source and R_3 to a voltage source with voltage $0.005v \times 1000 = 5v$ and R_3.

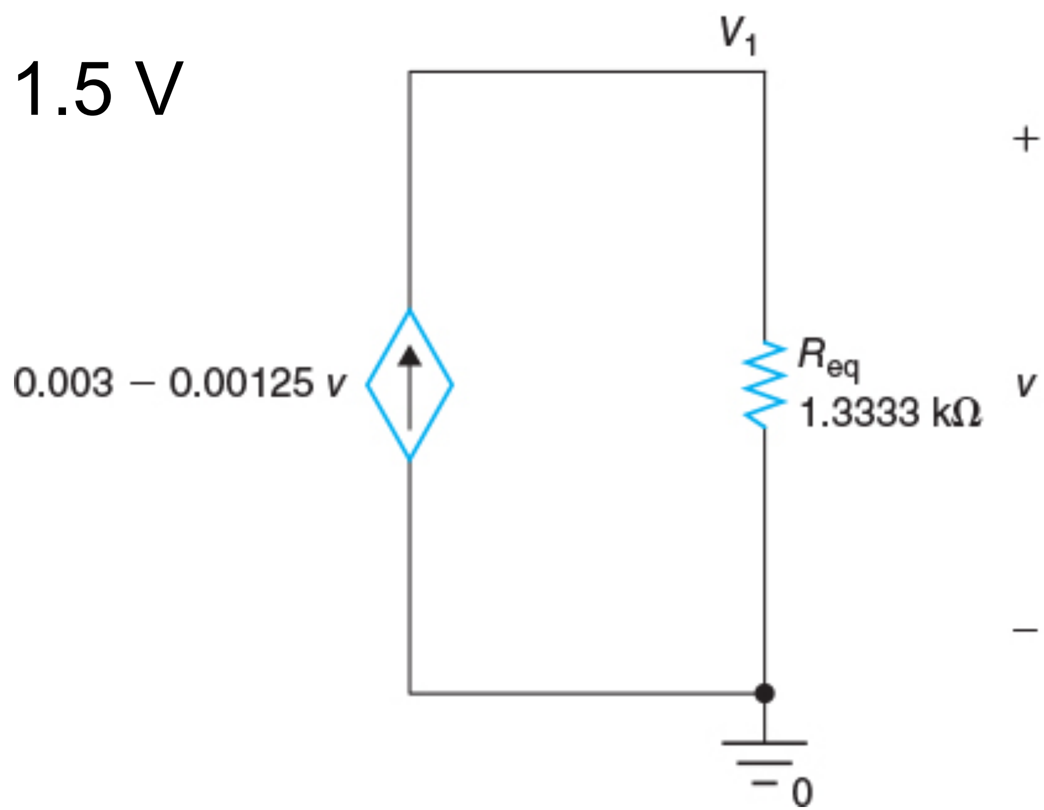
- $R_4 = R_2 + R_3 = 4$ kΩ



- Transform current source 5v and R_4 to voltage source with current $5v/R_4 = 0.00125v$ and R_4.

EXAMPLE 4.6

- $R_{eq} = R_1 \| R_4 = 1.3333\ k\Omega$
- Add the currents to get 0.003 – 0.00125v.
- $v = (0.003 - 0.00125v) \times 1333.3333 = 4 - 1.6667v$
- $v = V_1 = 4/2.66667\ V = 1.5\ V$



Summary

- Superposition principle
- Source transformations
- What will we study in next lecture.