Lecture 3

Electronic Devices

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Carrier Transport

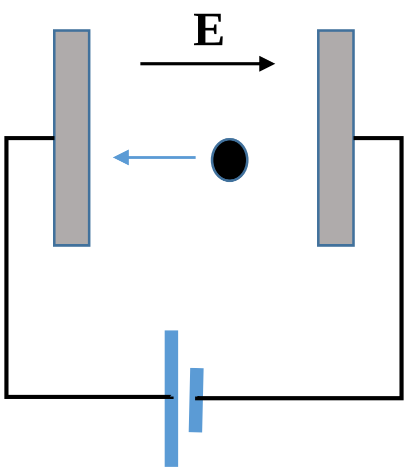
Three primary types of carrier action occur inside a semiconductor:

- **Drift:** charged particle motion under the influence of an electric field.
- **Diffusion:** particle motion due to concentration gradient or temperature gradient.
- **Recombination-generation** (R-G)

Today, we will focus on the highlighted areas.

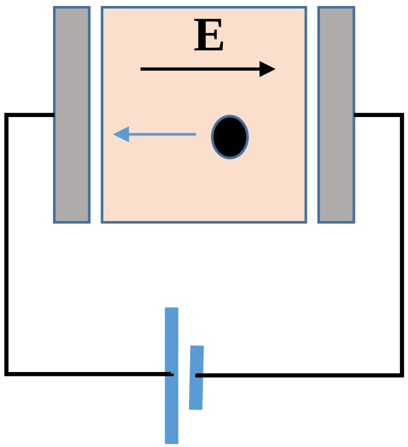
Carrier Transport - Drift

In vacuum



$$F = (-q)\mathbf{E} = m_o a$$

In semiconductor



$$F = (-q)\mathbf{E} = m_n{}^{*} a$$

where $m_n{}^{*}$ is the ***effective* mass**

Carrier Transport - Drift

Under the influence of an **electric field (E-field)**, an electron or a hole is **accelerated**:

$$a = \frac{-q\mathcal{E}}{m_n^*} \quad \text{electrons}$$

$$a = \frac{q\mathcal{E}}{m_p^*} \quad \text{holes}$$

Electron and hole effective masses

	Si	Ge	GaAs
m_n^*/m_o	**0.26**	0.12	0.068
m_p^*/m_o	**0.39**	0.30	0.50

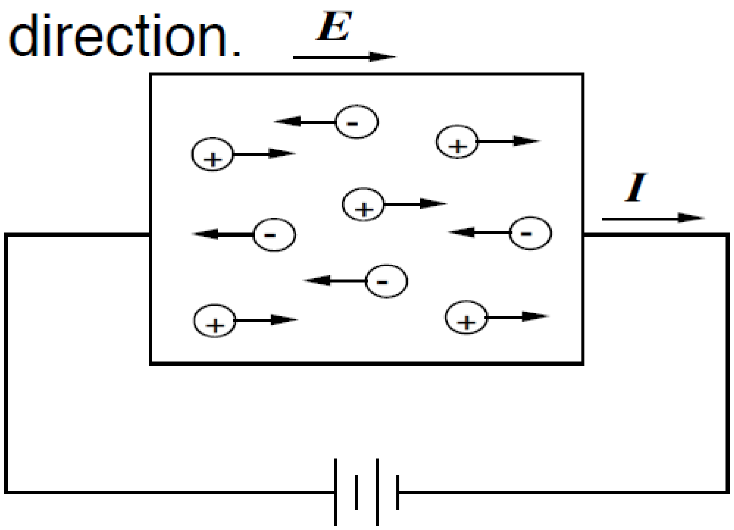
$m_o = 9.1\times10^{-31}$ kg

Carrier Transport - Drift

The current in semiconductors is carried by two types of carriers - electrons and holes.

Drift current

When an electric field is applied across a slab of semiconductor the carriers are accelerated - holes in the direction of the E-field and electrons in the opposite direction.



Due to scattering this results in an average *drift* velocity v_d parallel to the field and corresponding drift current. At low electric field

$$v_d = \mu E$$

The mobility for electrons μ_{n}, and for holes μ_p are different and depend on temperature, doping concentration and other factors.

Carrier Transport - Drift

The current density is given by the number of carries crossing a unit area per unit time multiplied by their charge. In the case of electrons and holes the drift currents are given by

$$J_{n,drift} = qnv_d = qn\mu_n E$$

$$J_{p,drift} = qpv_d = qp\mu_p E$$

The total drift current in a semiconductor is a sum of the electron and hole drift currents

$$J_{drift} = J_{n,drift} + J_{p,drift}$$

The conductivity σ is the coefficient of proportionality between the current density and the applied electric field $J = \sigma E$. In the case of semiconductors the conductivity has a component σ_n associated with the electrons and a component σ_p associated with holes.

$$\sigma_n = qn\mu_n \qquad \sigma_p = qp\mu_p \qquad \sigma = \sigma_n + \sigma_p$$

The resistivity of of the semiconductor is given by

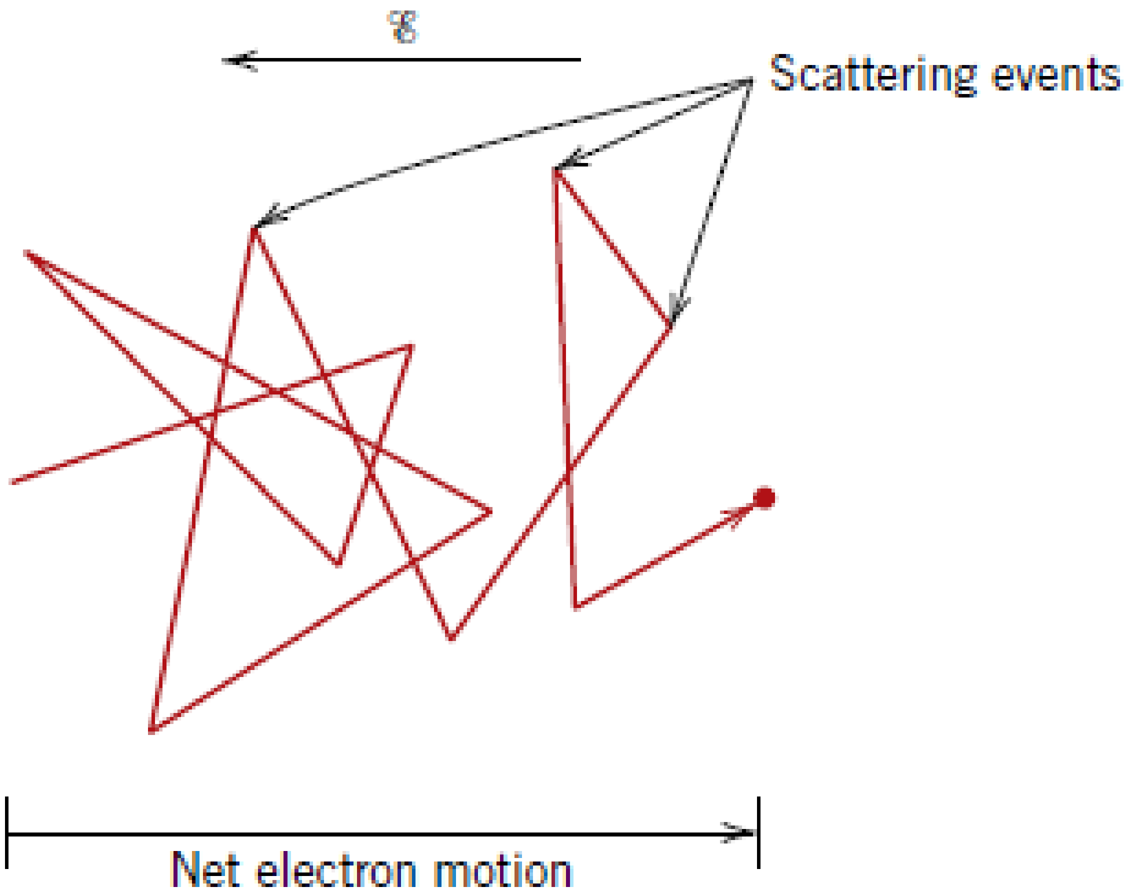
$$\rho = \frac{1}{\sigma} = \frac{1}{\sigma_n + \sigma_p} = \frac{1}{qn\mu_n + qn\mu_p}$$

Electron Mobility

- When **electric field** is applied, electrons experience an **acceleration** in a direction opposite to that of the field, by virtue of their **negative charge**. This acceleration eventually comes to halt.
- **Scattering** of electrons by imperfections in the crystal lattice, including impurity atoms, vacancies, dislocations, and even the thermal vibrations of the atoms themselves.
- Each scattering causes electron to **lose kinetic energy and to change its direction of motion**.
- There is, however, some **net** electron motion in the direction opposite to the field, and **this flow of charge is the electric current**.

Electron Mobility

Path of an electron that is deflected by **scattering** events.



Electron Mobility

- **The drift velocity *v_d*** represents the average electron velocity in the direction of the force imposed by the applied electric field.

$$\boxed{v_d = \mu_e E}$$

- The constant of proportionality *μ_e* is called the electron mobility

- Its units are (m^2/Vs)

Electron Mobility

- The conductivity σ of most materials may be expressed as:

$$\boxed{\sigma = n \times q \times \mu_e}$$

where <u>**n is the number of free electrons per unit volume**</u> (e.g., per cubic meter) and $\boldsymbol{q}$ is the absolute magnitude of the electrical charge on an electron (1.6×10^{-19}).

- Thus, the electrical **conductivity** is proportional to both the number of **free electrons** and the **electron mobility**.

Example (1)

(a) Calculate the **drift velocity** of electrons in germanium at room temperature and when the magnitude of the electric field is 1000 V/m. Assume that the electron mobility for Ge is 0.38 m^2/Vs.

(b) Under these circumstances, how long does it take an electron to traverse a 25mm (1-in.) length of crystal?

Intrinsic Semiconductivity

The expression for **<u>electrical conduction in a semiconductor</u>**

$$\sigma = nq\mu_e + pq\mu_h$$

Number of holes and electrons are equal in intrinsic semiconductors

$$n = p = n_i$$
$$\sigma = nq(\mu_e + \mu_h) = pq(\mu_e + \mu_h)$$
$$\sigma = n_i q(\mu_e + \mu_h)$$

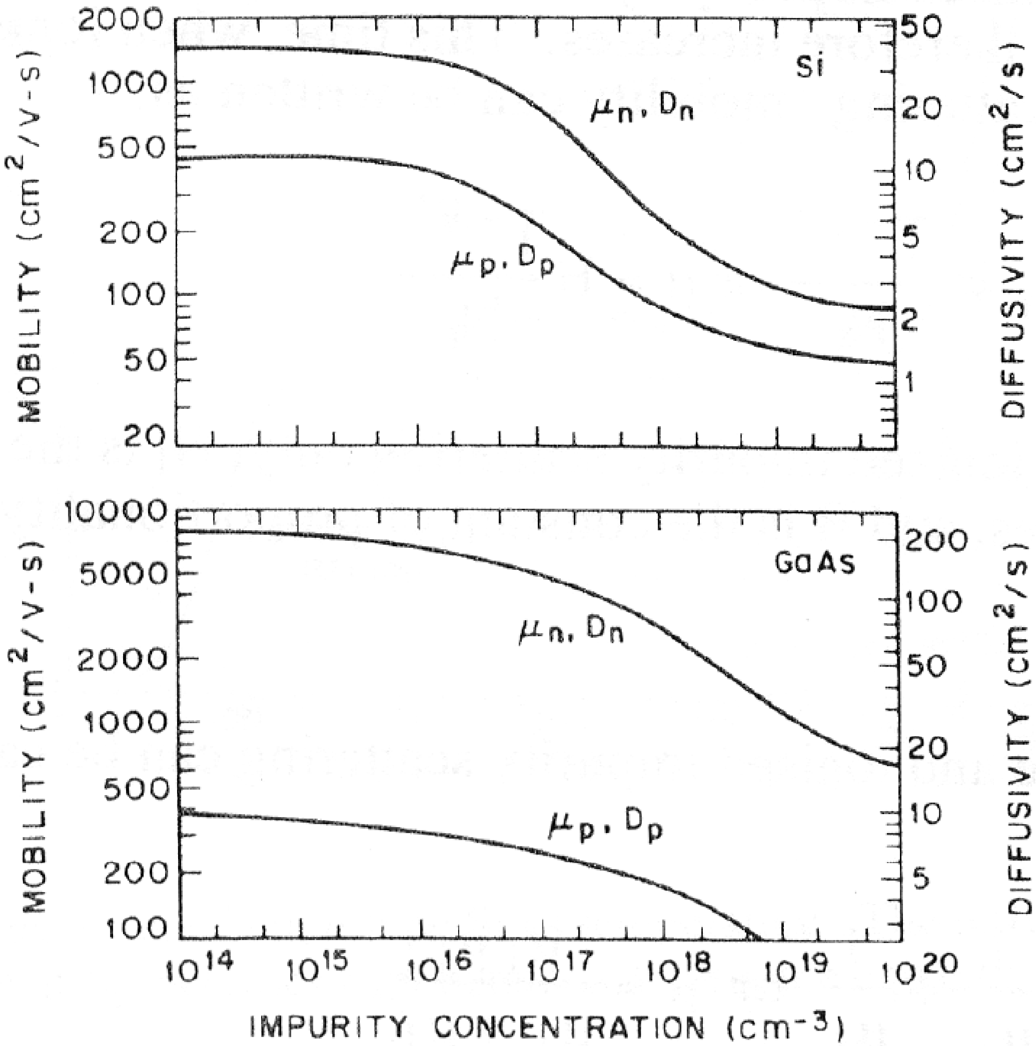
Example (2)

For intrinsic gallium arsenide, the room-temperature electrical conductivity is $3 \times 10^{-7}\ (\Omega m)^{-1}$. The electron and hole mobility are 0.80 and 0.04 $m^2/(Vs)$.

Compute the **intrinsic carrier concentration** $\boldsymbol{n_i}$ at room temperature.

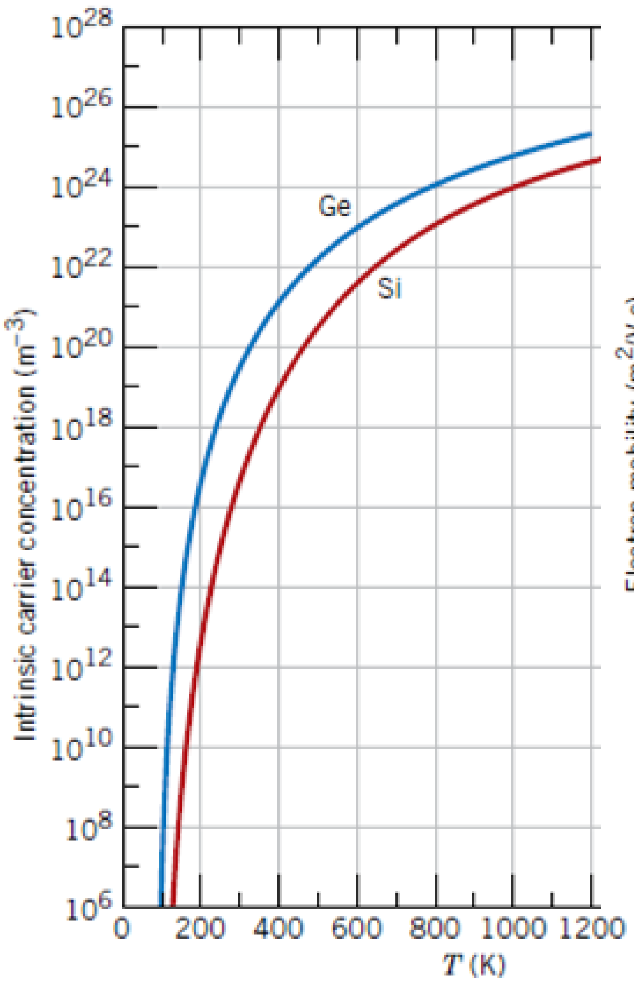
Carrier Transport - Drift

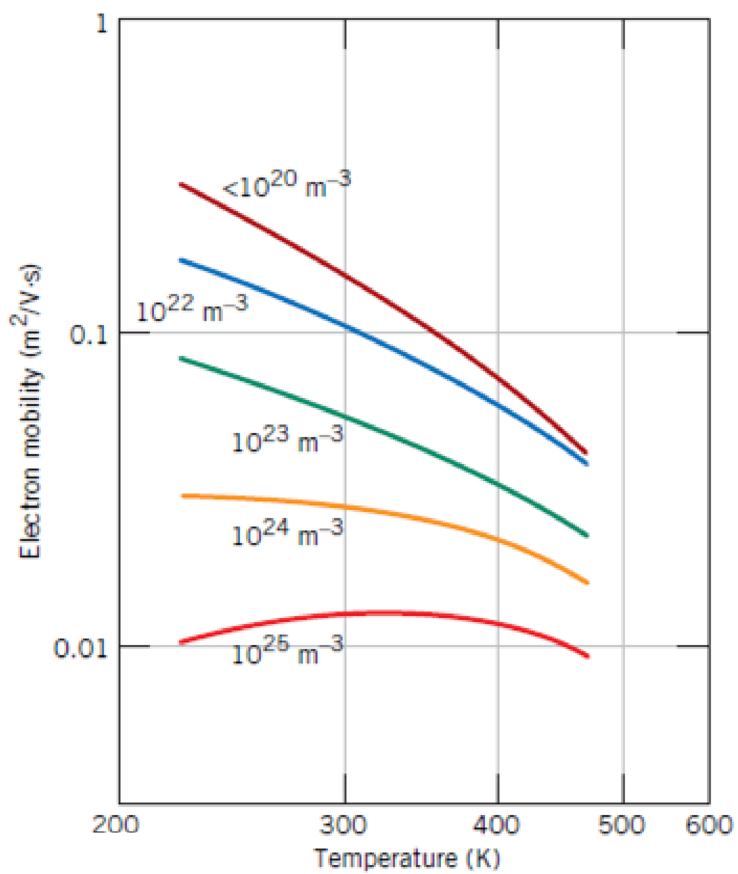
The figures below show the variation of both electron and hole mobilities in two semiconductors, Si and GaAs, as a function of doping concentration. The small effective masses for GaAs result in higher values of μ_n and μ_p for all impurity concentrations compared with Si.

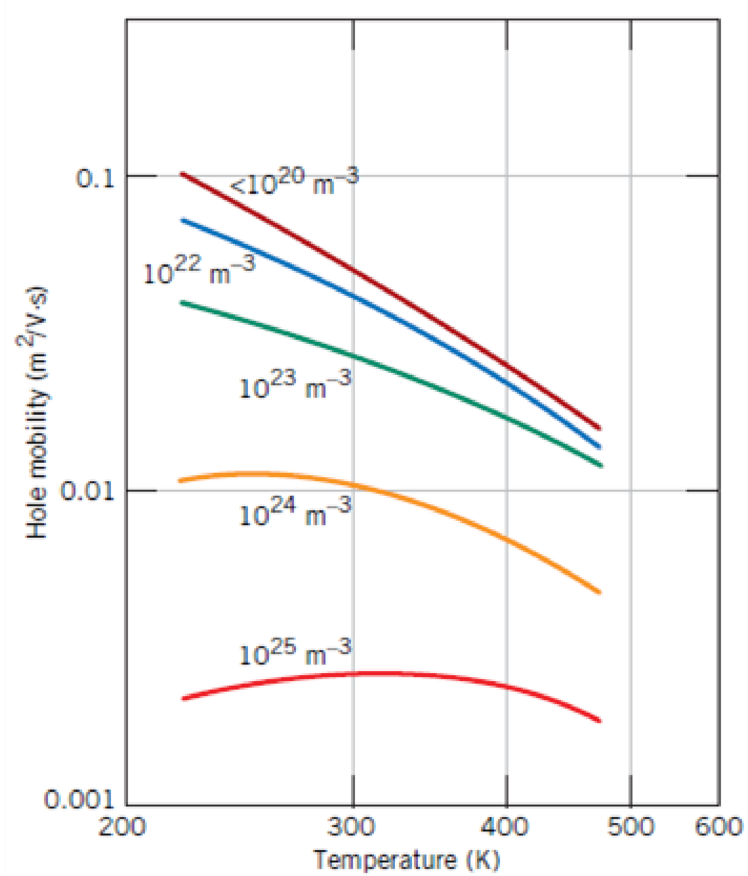


Example (3)

What is the electrical conductivity of intrinsic silicon at 300 K ?







Carrier Transport - Diffusion

Gas particles diffuse from **high** concentration regions to **low**-concentration regions. **Whenever there is a concentration gradient of particles**, there is a **net diffusional motion of particles** in the direction of decreasing concentration.

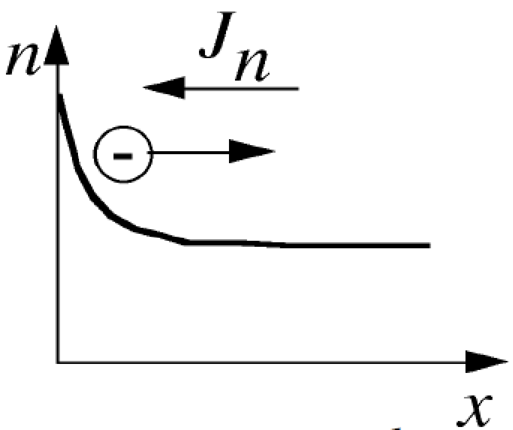
$$\Gamma_n = -D_n \frac{dn}{dx}$$

Thus, the **net electron flux Γ_n** at a position x is proportional to the **concentration gradient** and the **diffusion coefficient, D_n**.

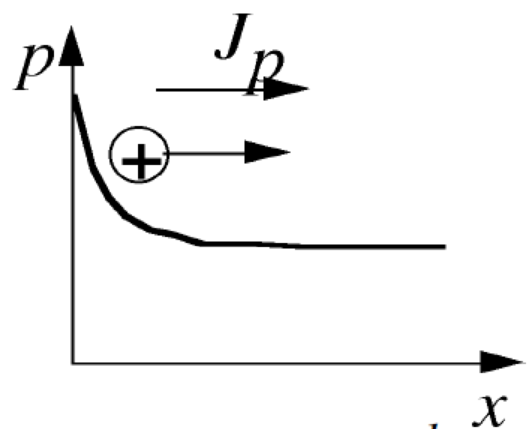
Diffusion

In semiconductors the carrier concentrations of electrons and holes can vary by orders of magnitude when moving from one region to another.

In the presence of a concentration gradient there will be diffusion of carriers from regions of high carrier concentration to regions of low carrier concentration. This is a directed motion of charged particles which constitutes diffusion current.



$$J_{n,diff} = qD_n \frac{dn}{dx}$$



$$J_{p,diff} = -qD_p \frac{dp}{dx}$$

The diffusion coefficients of electrons and holes D_n and D_p are related to the corresponding mobilities through the *Einstein relation*

$$\frac{D_p}{\mu_p} = \frac{D_n}{\mu_n} = \frac{k_B T}{q}$$

Diffusion

$\boldsymbol{D_n}$ has the dimensions of m^2s^{-1} and is a measure of how readily the particles (in this case, electrons) diffuse in the medium. Electron diffusion current density:

$$\boxed{J_{D,n} = q\Gamma_n = qD_n\frac{dn}{dx}}$$

In the case of a hole concentration gradient, the hole flux and hole current density is given by:

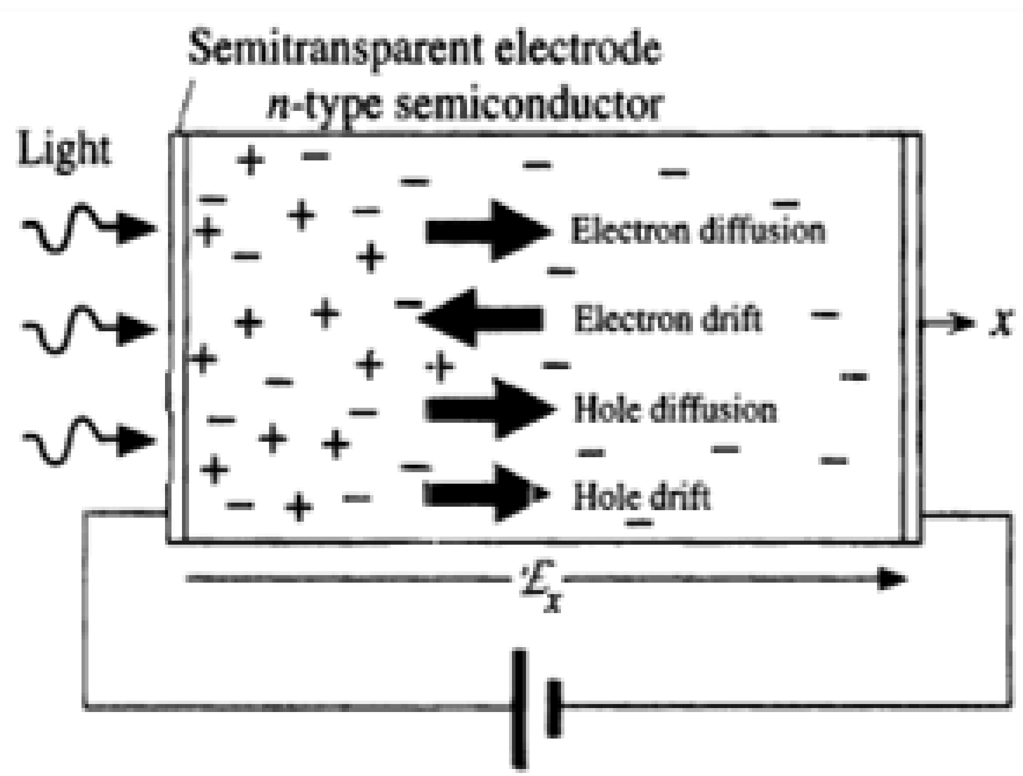
$$\boxed{\Gamma_p = -D_p\frac{dp}{dx}}$$

$$\boxed{J_{D,p} = q\Gamma_p = -qD_p\frac{dp}{dx}}$$

Diffusion & Conduction

Electrons	**Holes**
$\Gamma_n = qD_n \frac{dn}{dx}$	$\Gamma_p = -qD_p \frac{dp}{dx}$
$J_{D,n} = q\Gamma_n = qD_n \frac{dn}{dx}$	$J_{D,p} = q\Gamma_p = -qD_p \frac{dp}{dx}$
$J_n = qn\mu_n\ E_x + qD_n \frac{dn}{dx}$	$J_p = q\mu_h E_x - qD_p \frac{dp}{dx}$
$\frac{D_n}{\mu_n} = \frac{KT}{q}$	$\frac{D_p}{\mu_p} = \frac{KT}{q}$

Diffusion & Conduction



In the general case the total current in a semiconductor is carried by electrons and holes and has drift and diffusion components

$$J = J_n + J_p = J_{n,drift} + J_{n,diff} + J_{p,drift} + J_{p,diff}$$

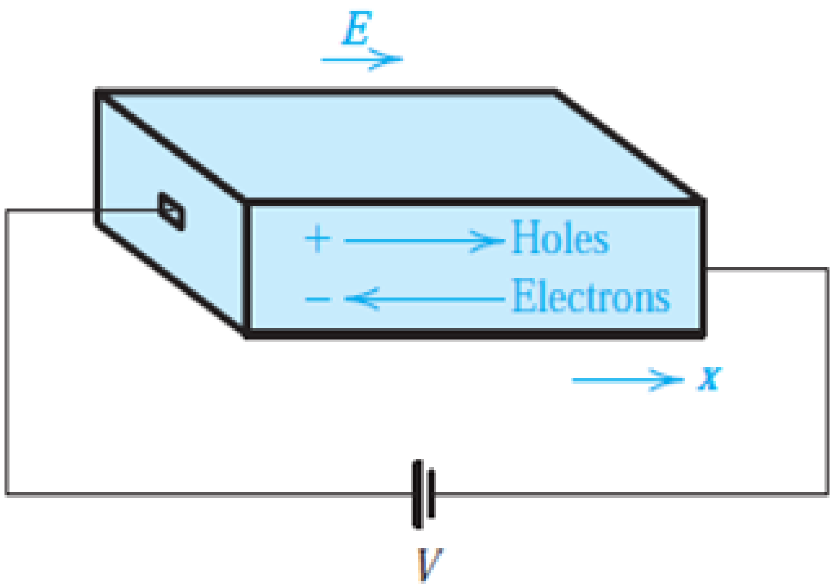
Example - 4

Calculate the diffusion co-efficient of electrons in n-type Si with electron drift mobility of $1300 cm^2V^{-1}s^{-1}$

Carrier Drift - Revision

Let the concentration of holes be ***p*** and that of free electrons ***n***.

Holes	Electrons
$v_{p\text{-drift}} = \mu_p E$	$v_{n\text{-drift}} = -\mu_n E$
$I_p = Aqpv_{p\text{-drift}}$	$I_n = -Aqnv_{n\text{-drift}}$
$I_p = Aqp\mu_p E$	...
$J_{p_{\text{drift}}} = \frac{I_p}{A} = qp\mu_p E$	$J_{n_{\text{drift}}} = qn\mu_n E$



TOTAL

$$J_{\text{drift}} = J_p + J_n = q(p\mu_p + n\mu_n)E$$

Carrier Drift - Revision

This relationship can be written as

$$J_{drift} = \sigma E$$

or

$$J_{drift} = E/\rho$$

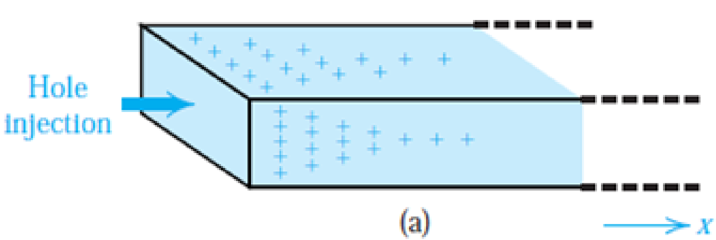
where the **conductivity** σ is given by

$$\sigma = q(p\mu_p + n\mu_n)$$

and the **resistivity** ρ is given by

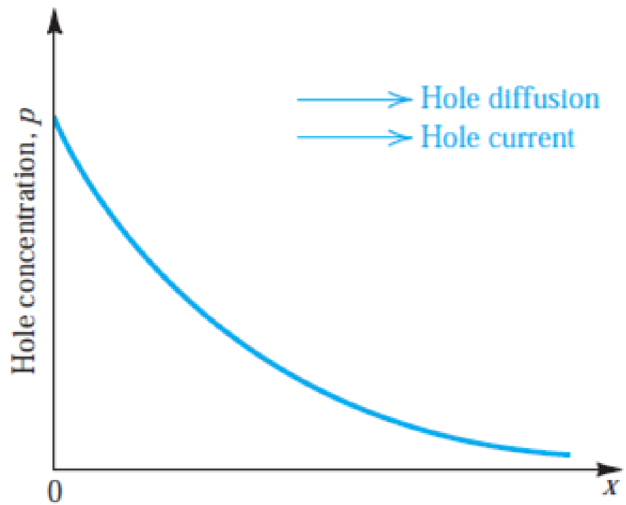
$$\rho \equiv \frac{1}{\sigma} = \frac{1}{q(p\mu_p + n\mu_n)}$$

Diffusion - Revision



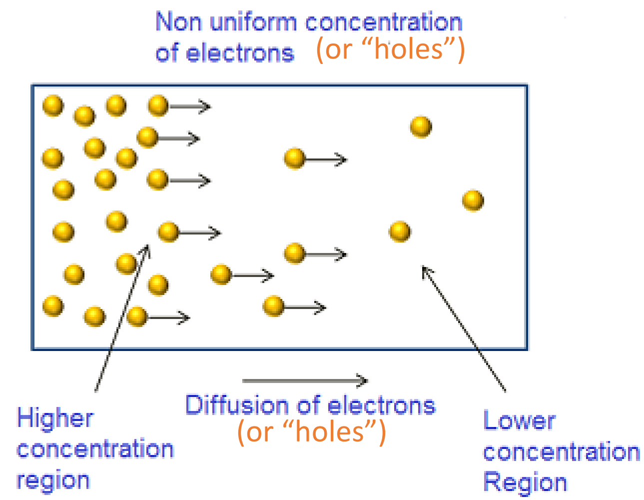
(a)

Bar of silicon with holes injected in one side, thus creating hole concentration profile along x-axis.



(b)

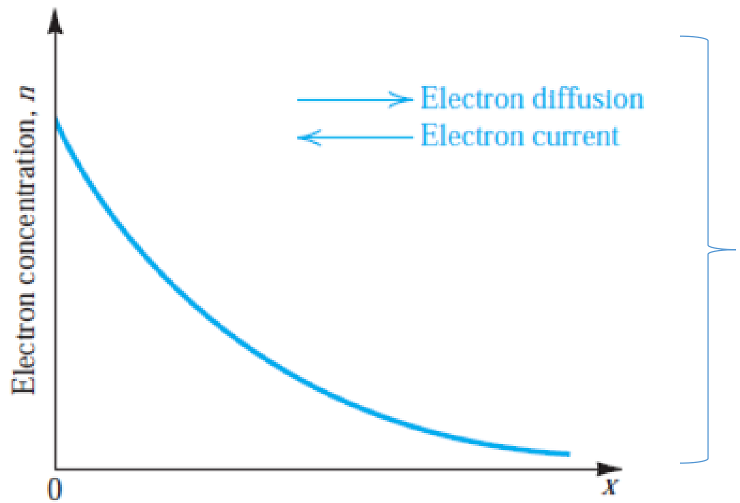
Holes diffuse in the positive direction of x and gives rise to a hole-diffusion current in the same direction.



$$\boxed{J_{p_{\text{diff}}} = -qD_p \frac{dp(x)}{dx}}$$

where $J_{p_{\text{diff}}}$ is the hole-current density (A/cm^2), q is the magnitude of electron charge, D_p is a constant called the **diffusion constant** or **diffusivity** of holes; and $p(x)$ is the hole concentration at point x. Note that the gradient (dp/dx) is negative, resulting in a positive current in the x direction, as should be expected.

Diffusion - Revision



Electrons diffuse in the direction of x and gives rise to an electron diffusion current in the **negative** *(-x)* direction.

$$\boxed{J_{n_{diff}} = qD_n \frac{dn(x)}{dx}}$$

where D_n is the diffusion constant or diffusivity of electrons. Observe that a negative (dn/dx) gives rise to a negative current, a result of the convention that the positive direction of current is taken to be that of the flow of positive charge (and opposite to that of the flow of negative charge). For holes and electrons diffusing in intrinsic silicon, typical values for the diffusion constants are $D_p = 12\ \mathrm{cm^2/s}$ and $D_n = 35\ \mathrm{cm^2/s}$.

Example 5

Consider a bar of silicon with hole concentration profile given by:

$$p(x) = p_0 e^{-x/L_p}$$

Find hole current density, I_p at x=0, given the following:

$$p_o = 10^{16}/cm^3$$
$$L_p = 1\mu m$$
$$A = 100\mu m^2$$
$$D_p = 12cm^2/s$$

Recombination

- During the diffusion process, an electron will experience **recombination:**
  → They will not travel in space indefinitely but will be stopped when it recombines
  → Encounters with a **hole**.

- It is possible to express the effects of recombination using a characteristic time:
  → Called the electron recombination lifetime: τ_n

- We can then define a distance called **diffusion length** for electrons and holes given by:

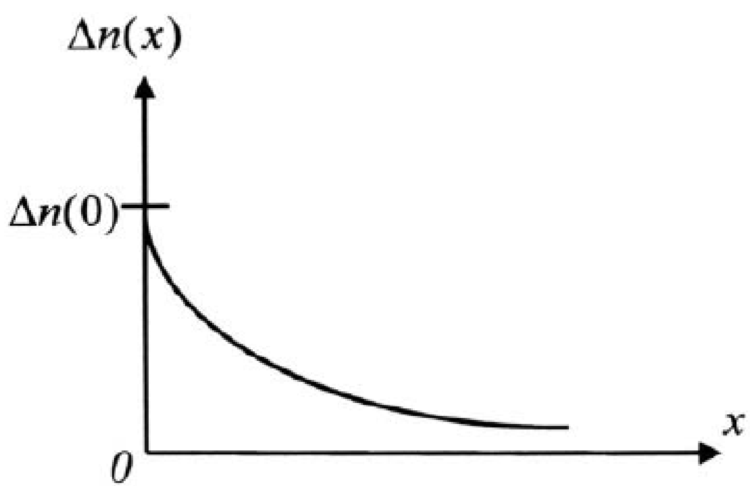


Fig. 9.7. Excess electron concentration in a one-dimensional model. The excess concentration decreases as it gets deeper into the material as a result of recombination. The decrease follows an exponential dependence.

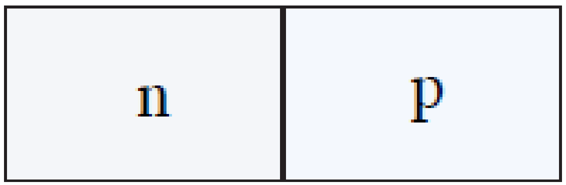
$$L_p = \sqrt{D_p \tau_p}$$

$$L_n = \sqrt{D_n \tau_n}$$

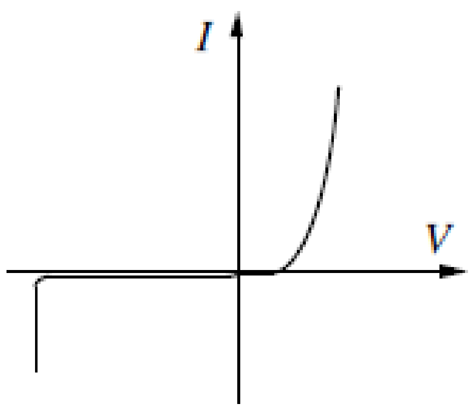
Example 6

Assume that in n-type silicon the characteristic (recombination) time for minority (hole) carriers is 0.2ns, estimate the diffusion length of these carriers at 300K, given the hole mobility is 450cm^2/V.s.

pn Junction

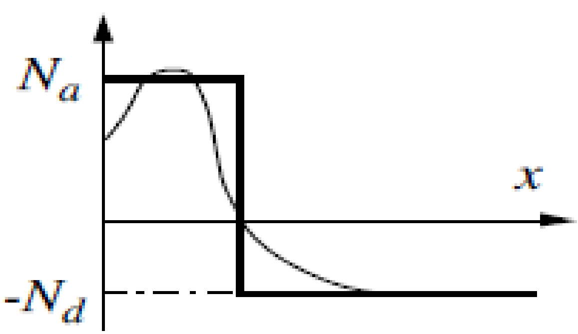


The metallurgical boundary between an *n*-type and *p*-type region in a single semiconductor crystal is called *p-n* junction.



The most remarkable property of the *p-n* junction is that it rectifies. It passes current only if a positive voltage is applied on the *p*-side of the junction.

A single *p-n* junction can be used as rectifier, light detector, solar cell, microwave generator, light emitter or laser. It is also a building block of almost all other semiconductor devices.



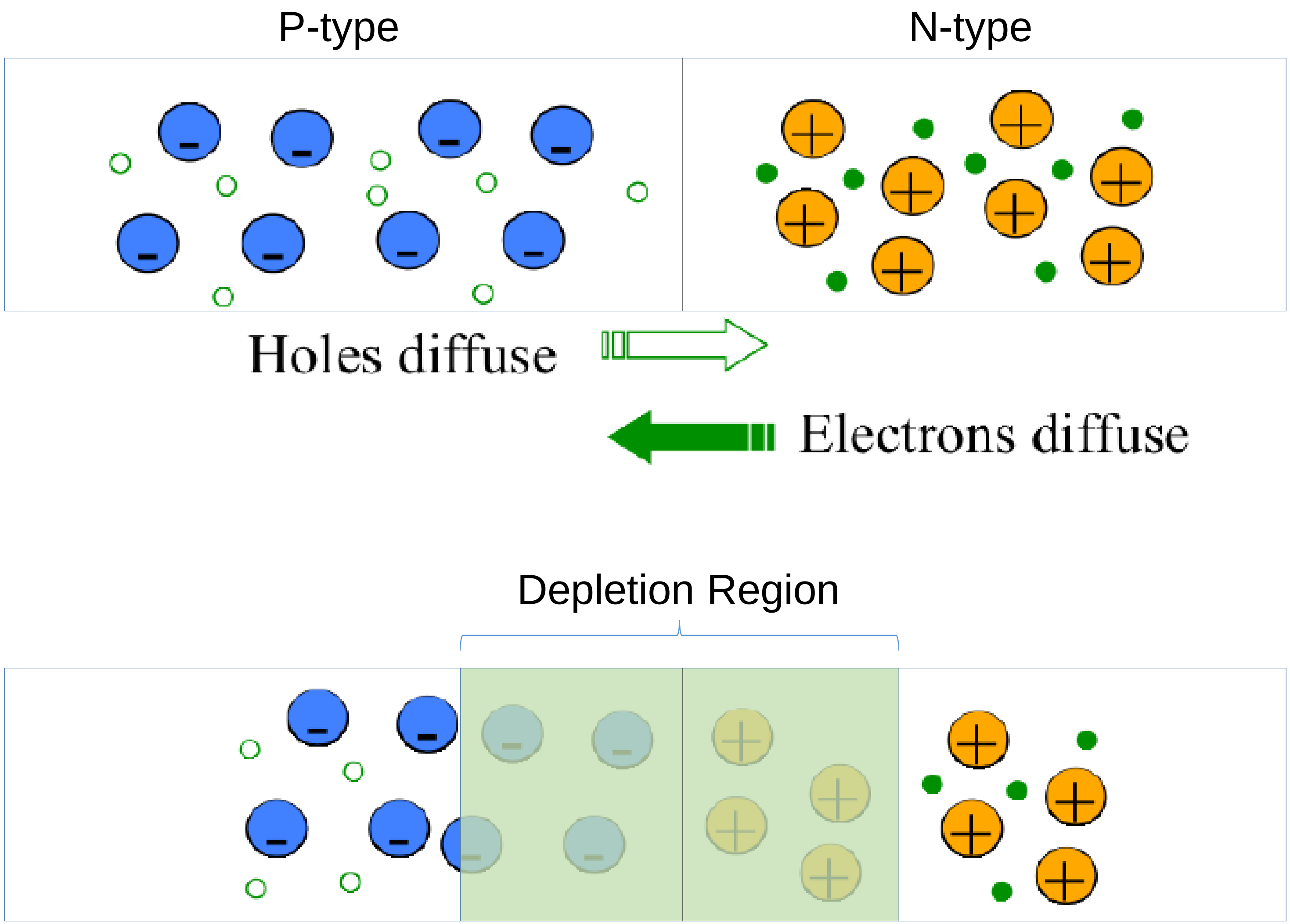
A *p-n* junction is formed by adding acceptors in an *n*-type wafer or by adding donors in a *p*-type wafer. The most common method for this is impurity diffusion or ion implantation.

pn Junction

Electrons will diffuse into p-type material where they will recombine with holes (fill in holes). And **holes** will diffuse into the n-type materials where they will recombine with electrons.

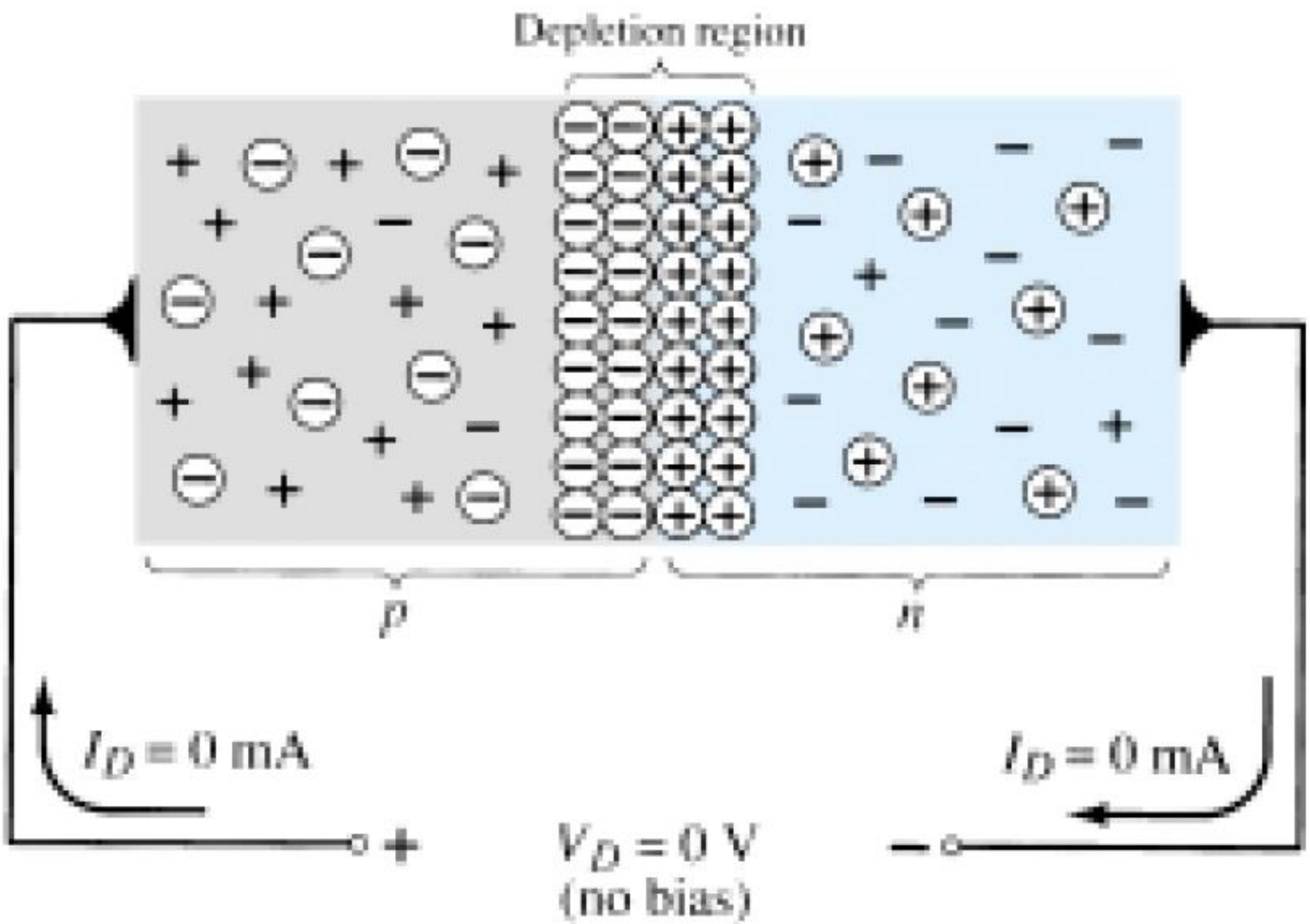
This means that eventually in vicinity of the junction all free carriers will be **depleted** leaving stripped ions behind, which would produce an **electric field** across the junction:

pn Junction


P-type
N-type
Holes diffuse
Electrons diffuse
Depletion Region

pn Junction

P-type N-type



Depletion Region

- Due to the movement of (majority) charge carriers there is a resulting **Diffusion Current** flow from the *p* to the *n*-side.
- As a result of diffusion, we have **fixed** (**immobile**) negative ions on the *p*-side and **fixed** **positive ions** on the *n*-side, near the junction.
- The magnitude of the ions on either side of the junction are equal.
- Thus, due to the space charge we have built-in E-field.

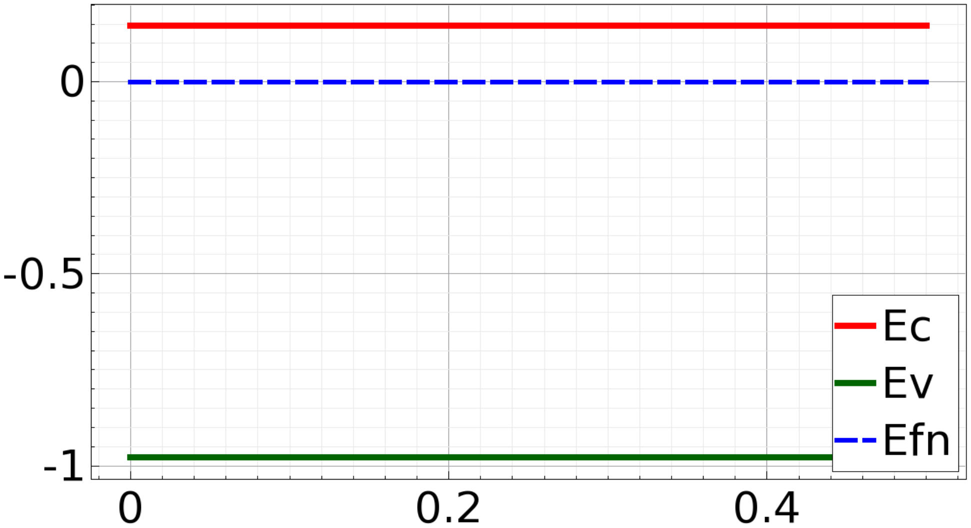
Depletion Region

- **This E-field opposes movement of majority carriers across junction**.
- However, it helps minority carriers to cross junction by attracting them.
- Hence, movement of e's from p-region to n-region and movement of holes from n-region to p-region.
- Hence, we have a **drift current** due to **minority** carriers.

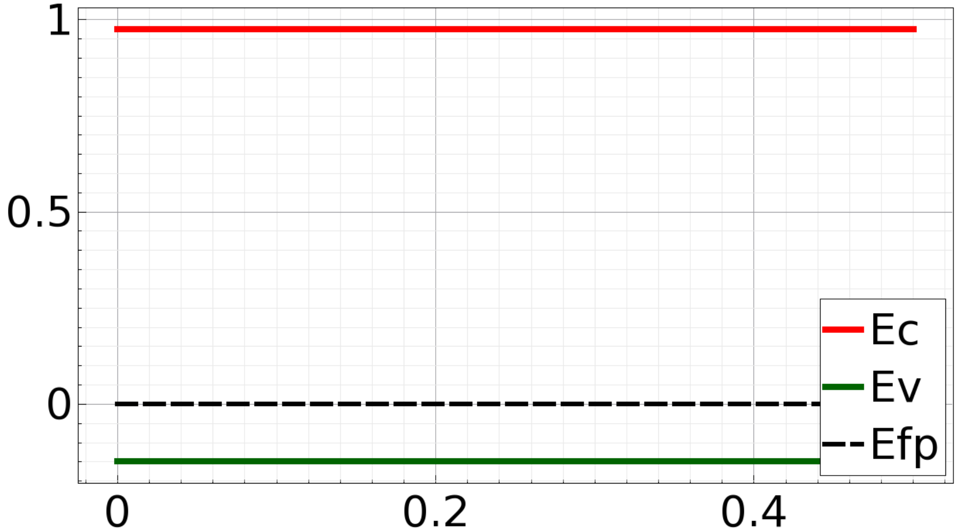
pn Junction

When materials are put in contact the carriers flow under driving force of diffusion until we reach an equilibrium.

n-type



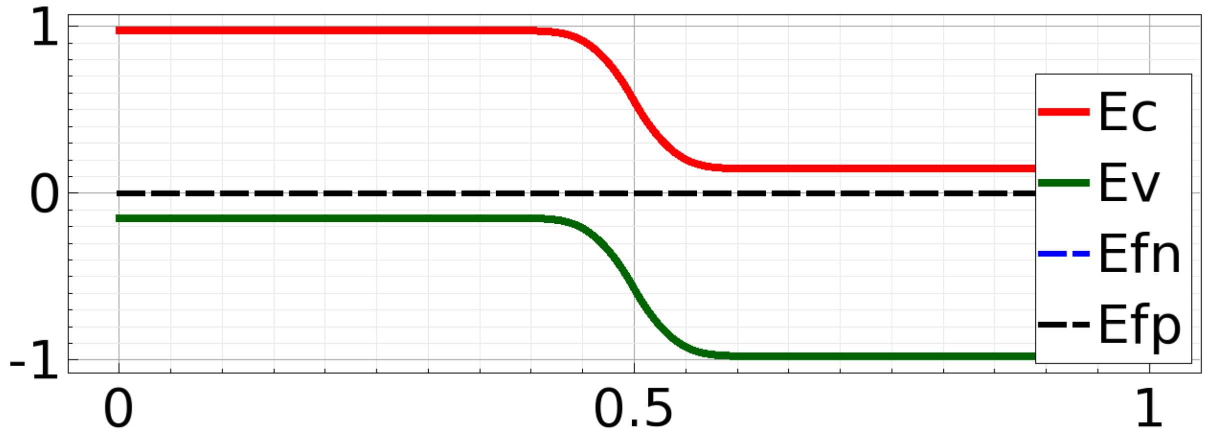
p-type



pn Junction

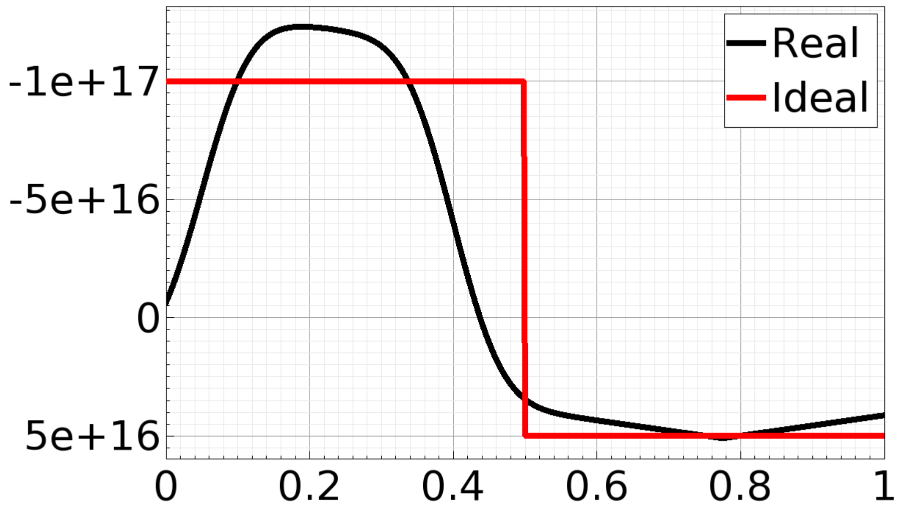
When materials are put in contact the carriers flow under driving force of diffusion until we reach an equilibrium.

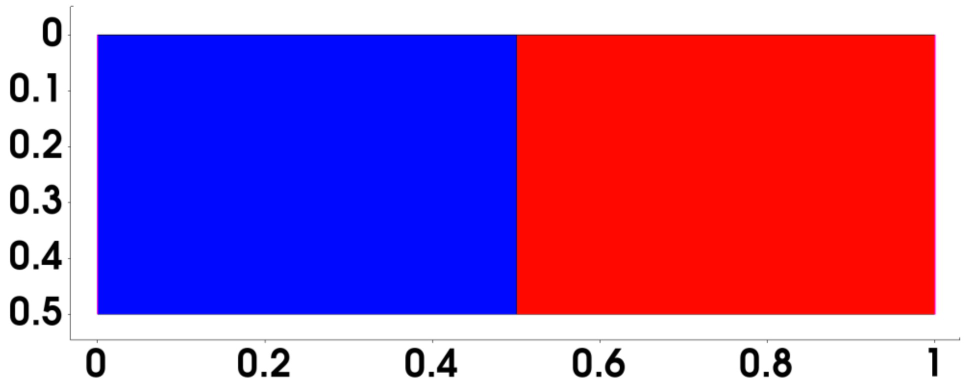
Position of the Fermi level must be the same in both p and n sides. This results in band bending:

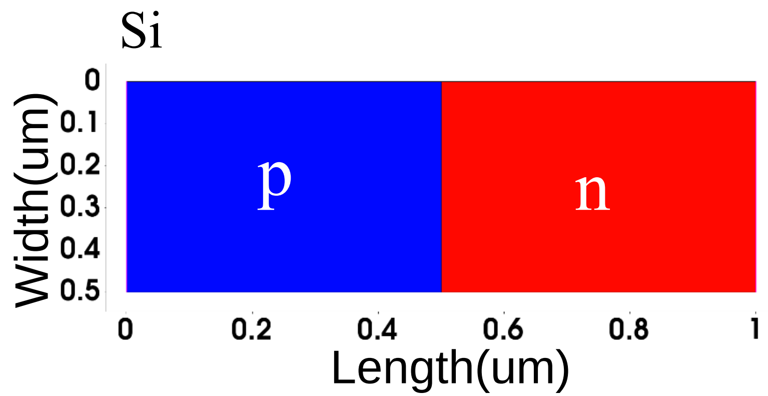


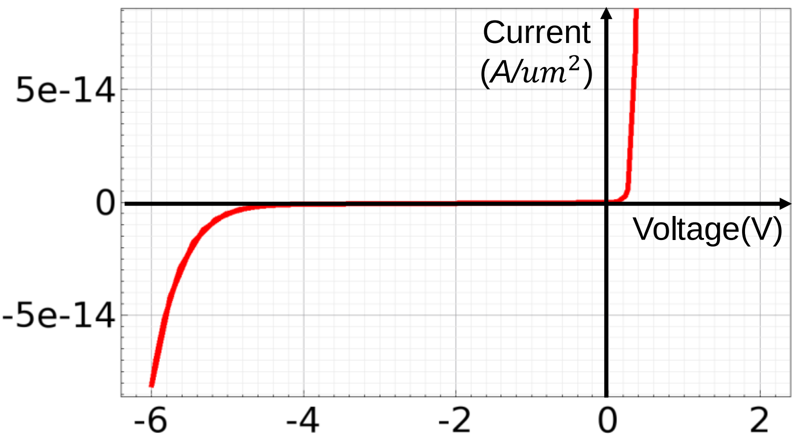
pn Junction

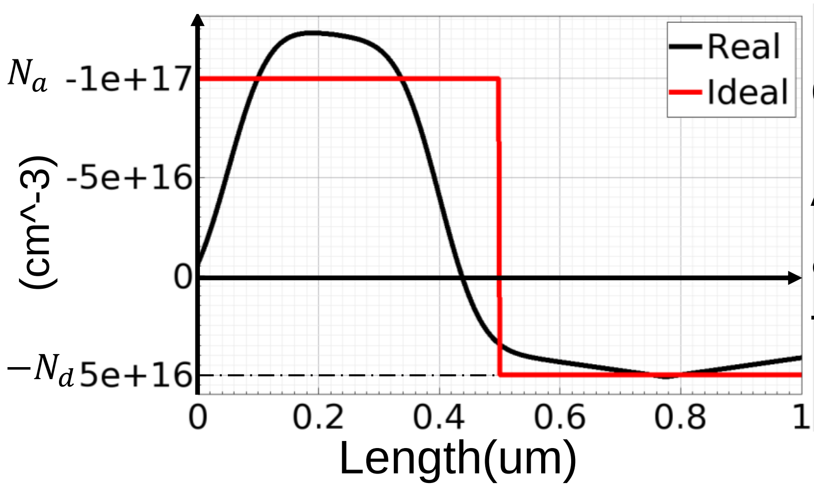
Comparison of ideal and realistic doping profiles for a PN junction. p-doping at 1e17, n-doping at 5e16











The metallurgical boundary between an *n*-type and *p*-type region in a single semiconductor crystal is called *p-n* junction.

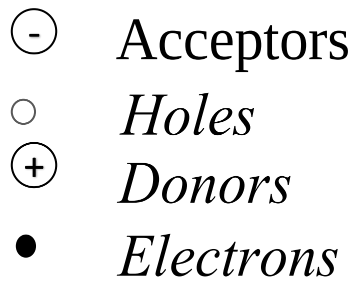
The most remarkable property of the *p-n* junction is that it rectifies. It passes current only if a positive voltage is applied on the *p*-side of the junction.

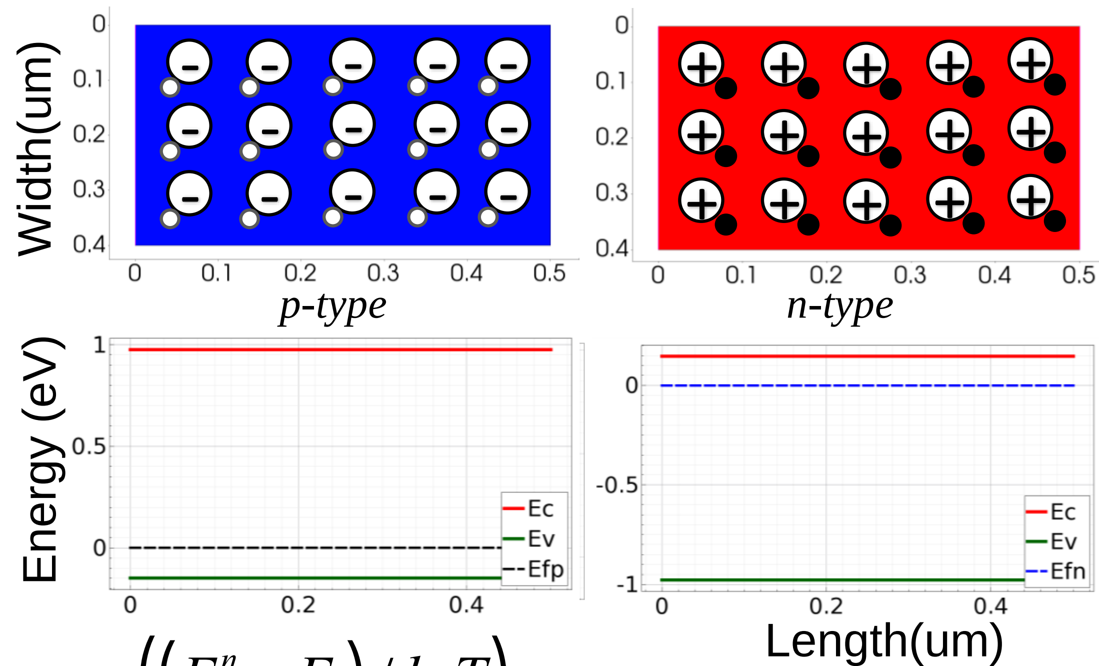
A single *p-n* junction can be used as rectifier, light detector, solar cell, microwave generator, light emitter or laser. It is also a building block of almost all other semiconductor devices.

A *p-n* junction is formed by adding acceptors in an *n*-type wafer or by adding donors in a *p*-type wafer. The most common method for this is impurity diffusion or ion implantation.

Let us consider initially that the two parts of the *p-n* junction are separated.

Both parts are electro-neutral. Electrons compensate the ionized donors and holes compensate the ionized acceptors.





Taking into account that

$$n \approx N_D = n_i \exp\left(\left(E_F^n - E_i\right)/k_B T\right)$$

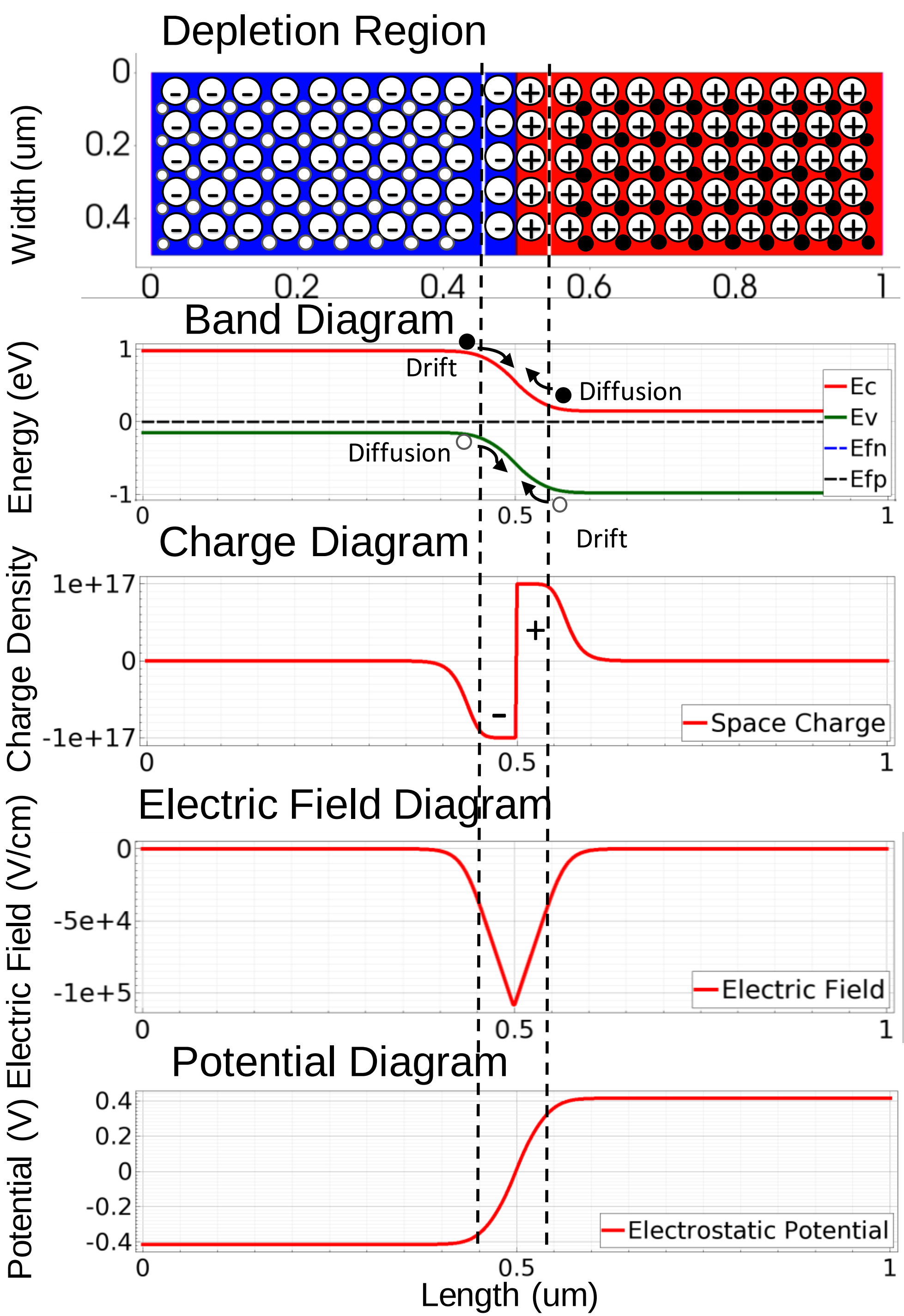
$$p \approx N_A = n_i \exp\left(\left(E_i - E_F^p\right)/k_B T\right)$$

We can calculate the distance between the Fermi levels on the both sides

$$E_F^n - E_F^p = \left(E_F^n - E_i\right) + \left(E_i - E_F^p\right)$$

$$= k_B T \ln\left(N_D / n_i\right) + k_B T \ln\left(N_A / n_i\right)$$

$$= k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$



Contact potential

When the two parts are in contact electrons diffuse to the p-side and holes to the n-side living donors and acceptors in the depletion layer uncompensated.

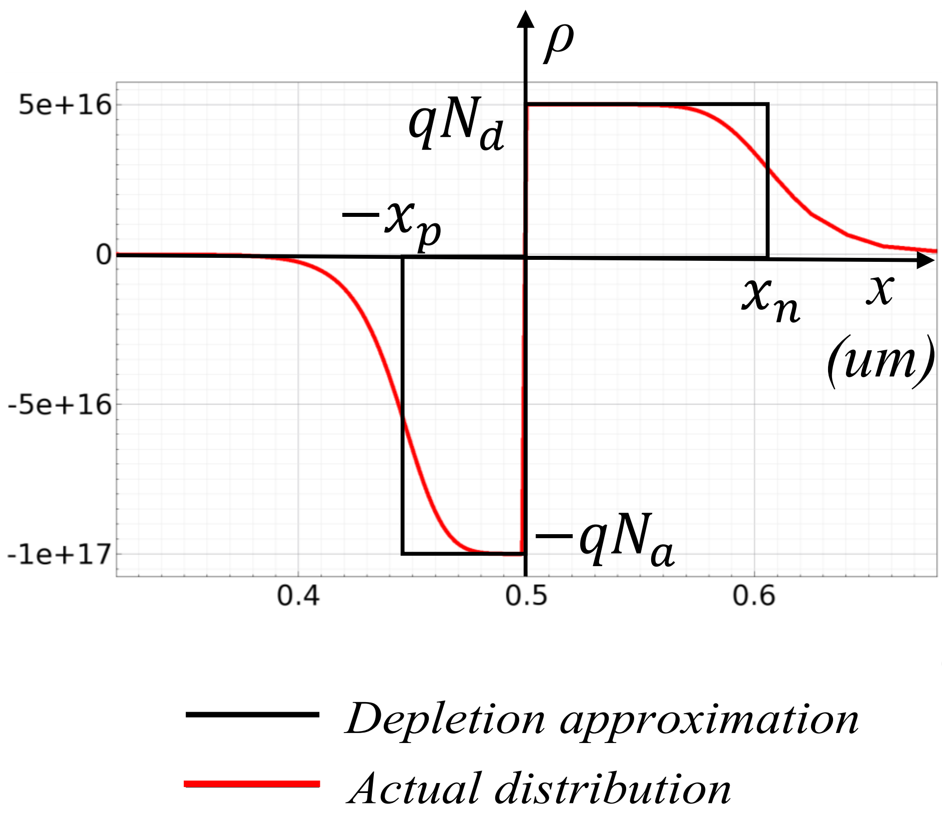
The resulting space charge gives rise to an electric field.

In equilibrium the drift current associated to this field compensated the diffusion current. The resulting current is zero.

The height of the potential barrier that supports the junction field is called contact potential

$$\psi_0 = \frac{E_F^n - E_F^p}{q} = \frac{k_B T}{q}\ln\left(\frac{N_D N_A}{n_i^2}\right)$$

Depletion approximation



The zone void of mobile charges is called depletion region. In depletion approximation the boundaries of the depletion region are abrupt and the charge density is equal to the charge density of the ionised acceptors or donors.

In order to obtain x_p and x_n and the depletion layer width $w = x_p + x_n$ the Poisson's equation must be solved. The solution gives

$$x_n = \left(\frac{2\varepsilon_0 \varepsilon_S (\psi_0 + V) N_A}{q N_D (N_A + N_D)} \right)^{1/2}, \quad x_p = \left(\frac{2\varepsilon_0 \varepsilon_S (\psi_0 + V) N_D}{q N_A (N_D + N_A)} \right)^{1/2}$$

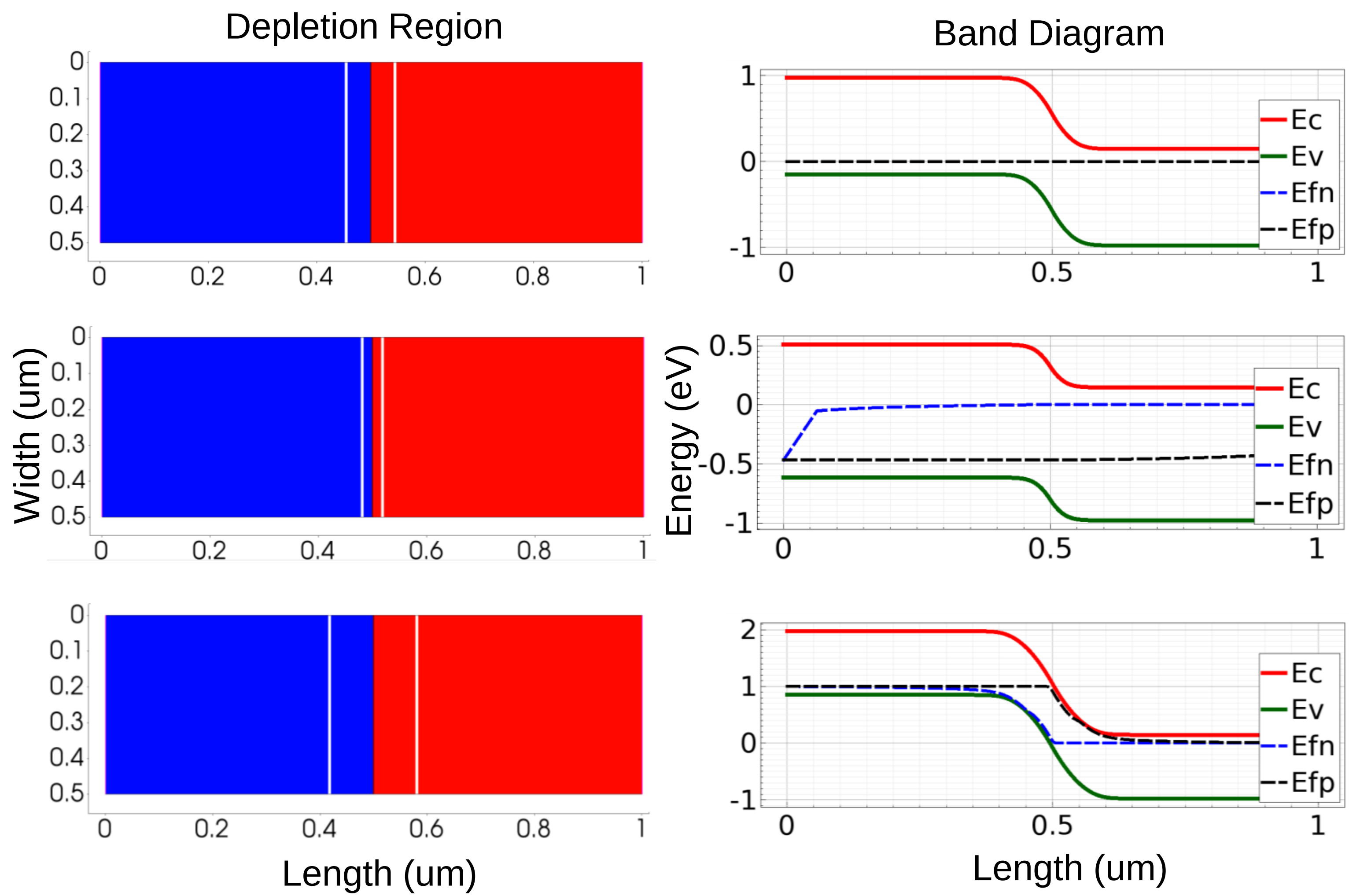
$$w = x_p + x_n = \left(\frac{2\varepsilon_0 \varepsilon_S (\psi_0 + V)(N_D + N_A)}{q N_A N_D} \right)^{1/2}$$

When an external voltage source is connected across the junction the equilibrium is disturbed. Since the depletion region has much higher resistance than the resistance in the rest part of the junction, *the applied voltage drop will almost entirely be in the depletion region*

Equilibrium: $V = 0$

Forward bias: $V = V_F$

Reverse bias: $V = V_R$



Forward Bias: Positive voltage applied on the *p*-side lowers the potential barrier to $q(\psi_0 - V_F)$. The depletion layer contracts in response to the barrier lowering. The reduction in the barrier favors the diffusion from the both sides and large current flows.

Reverse Bias: Negative voltage applied on the *p*-side increases the height of the potential barrier to $q(\psi_0 + V_R)$. The depletion layer expends to support the barrier growth. Only a small leakage current flows through the junction.

Depletion layer capacitance

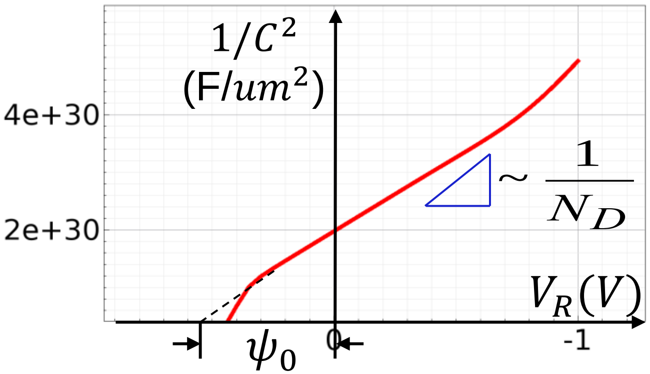
The small signal capacitance of the space charge layer is given by $C \equiv \dfrac{dQ}{dV_R}$ where Q is the unipolar charge in the depletion region

$$Q = qAN_D x_n = qAN_A x_p = A\left(\frac{2q\varepsilon_0\varepsilon_S(\psi_0 + V_R)N_A N_D}{(N_A + N_D)}\right)^{1/2}$$

Therefore

$$C = A\left(\frac{q\varepsilon_0\varepsilon_S N_D N_A}{2(N_A + N_D)(\psi_0 + V_R)}\right)^{1/2}$$

For asymmetrical p-n junction with $N_A >> N_D$

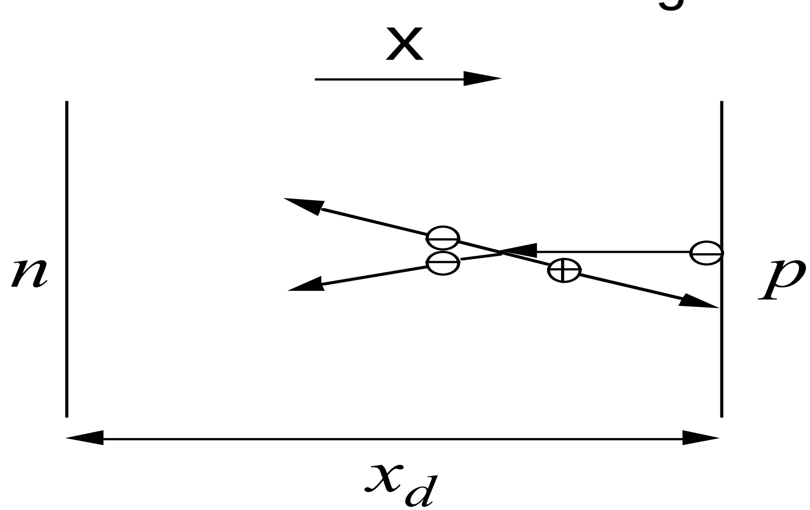


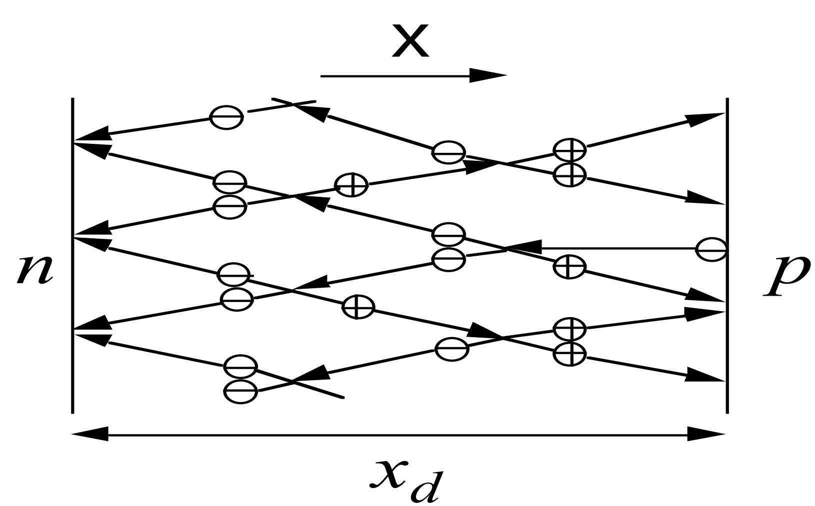
$$C = A\left(\frac{q\varepsilon_0\varepsilon_S N_D}{2(\psi_0 + V_R)}\right)^{1/2}$$

$$\frac{1}{C^2} = \frac{2}{A^2 q\varepsilon_0\varepsilon_s N_D}(V_R + \psi_0)$$

Avalanche breakdown

A free electron (hole) gaining sufficient energy from the electric field can break Si-Si bond and liberate new electron (hole). This is called *impact ionisation*. The new electron (hole) also can gain enough energy to break new bond. This process can cascade (avalanche) providing a large number of free carriers and large current - *avalanche breakdown*.

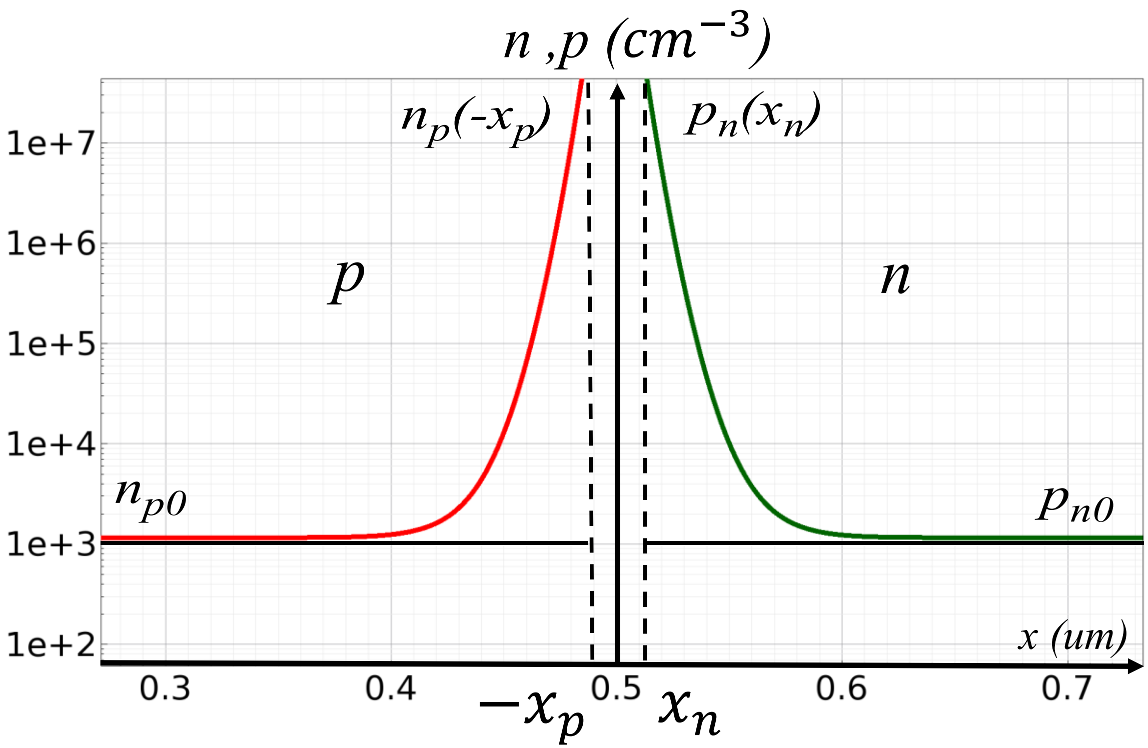




Current voltage characteristics

Under equilibrium the drift and diffusion components of the electron current are equal in magnitude but opposite in direction so that the net electron current is zero. The same is true for the net hole current.

Under forward bias the voltage reduces the potential barrier of the *p*-*n* junction. The electric field in the depletion region is reduced, and the drift components are decreased. In the same time the diffusion component remain virtually unchanged. This leads to *injection* of electrons into the *p*-side and *injection* of holes into the *n*-side.



The subscripts *n* and *p* denote the *n*- and *p*-side of the junctions and the subscript *0* denote equilibrium.

The injected minority carriers on the both side of the *p-n* junction diffuse away from the depletion layer edges and recombine with majority carriers. As a result, their concentration decay exponentially.

$$p_n(x) = \left(p_n(x_n) - p_{n0}\right)\exp\left(-\frac{(x - x_n)}{L_p}\right) + p_{n0}$$

$$n_p(x) = \left(n_p(-x_p) - n_{p0}\right)\exp\left(\frac{(x + x_p)}{L_n}\right) + n_{p0}$$

where L_p, L_n are the minority carriers diffusion lengths. The minority carrier diffusion currents are given by

$$I_p(x) = -qAD_p\frac{dp_n(x)}{dx} = qA\frac{D_p}{L_p}\left(p_n(x_n) - p_{n0}\right)\exp\left(-\frac{(x - x_n)}{L_p}\right)$$

$$I_n(x) = qA\frac{D_n}{L_n}\left(n_p(-x_p) - n_{p0}\right)\exp\left(\frac{(x + x_p)}{L_n}\right)$$

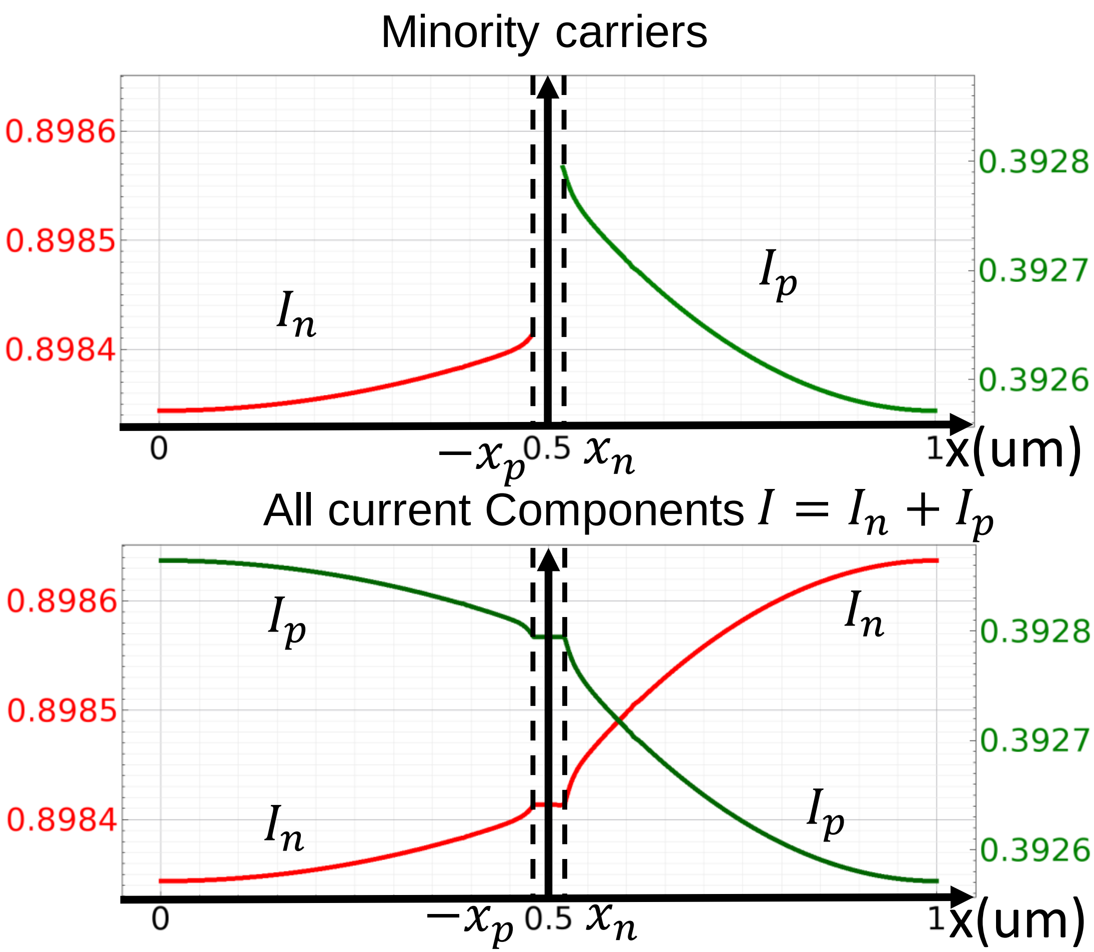
At *low level of injection* at the depletion layer edges, it can be shown that

$$n_p(-x_p) = n_{p0}\exp\left(\frac{V_F}{\phi_T}\right), \quad p_n(x_n) = p_{n0}\exp\left(\frac{V_F}{\phi_T}\right)$$

Therefore the minority carrier currents at the depletion layer edges are given by

$$I_p(x_n) = qA\frac{D_p}{L_p}p_{n0}\left[\exp\left(\frac{V_F}{\phi_T}\right)-1\right]\,,\quad I_n(-x_p) = qA\frac{D_n}{L_n}n_{p0}\left[\exp\left(\frac{V_F}{\phi_T}\right)-1\right]$$

The Shockley (ideal) diode equation



The current continuity requires the total current to be constant at each point along the device.

$$I(x) = I_n(x) + I_p(x) = \mathrm{C}$$

This means that away from the depletion layer edges the minority diffusion current is gradually transferred in majority drift current and this happened via recombination.

By assuming:

I. low level of injection and

II. no recombination and generation in the depletion layer

the total junction current is given by

$$I = I_p(x_n) + I_n(-x_p) = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right)\left(\exp\left(\frac{V_F}{\phi_T}\right) - 1\right)$$

Introducing the reverse bias saturation current I_0

$$I_0 = qA\left(\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right) = qAn_i^2\left(\frac{D_p}{L_p N_A} + \frac{D_n}{L_n N_D}\right)$$

We finally have the Shockley equation

$$I = I_0\left(\exp\left(\frac{V_F}{\phi_T}\right) - 1\right)$$

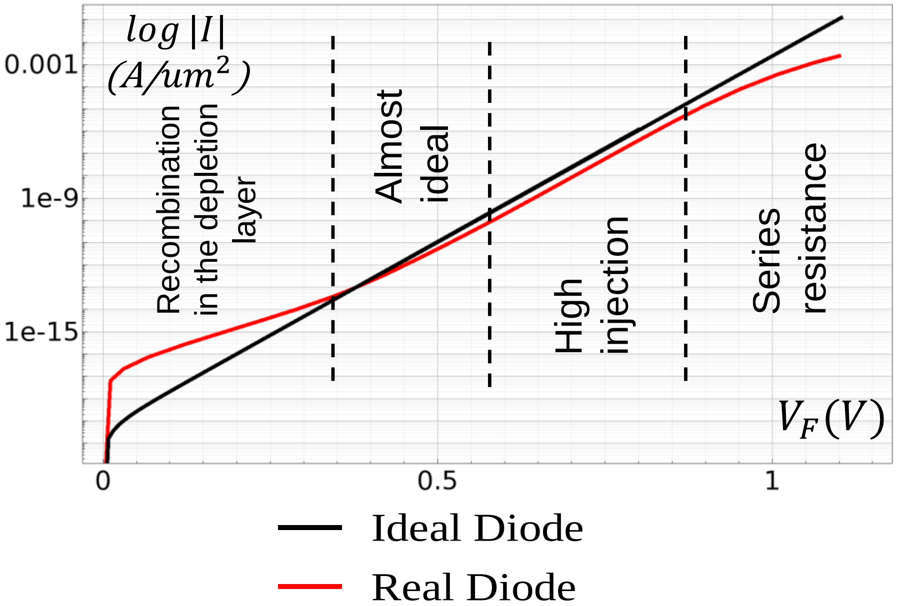
Normally $exp(V_F/\phi_T)$ >> 1 and therefore

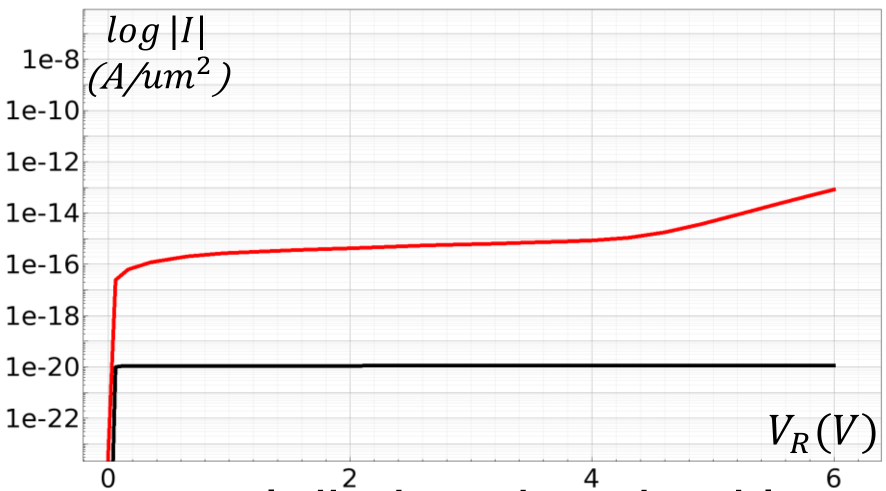
$$I = I_0 \exp\left(\frac{V_F}{\phi_T}\right)$$

In reverse bias conditions

$$I = I_0\left(\exp\left(-\frac{V_R}{\phi_T}\right) - 1\right) \approx -I_0$$

Real diodes





Forward bias

Recombination in the depletion layer increases the current at low V_F.

Space charge effects reduce the injection current at moderate V_F.

The series resistances of the un depleted *p-* and *n-* regions reduce the current at high V_F.

Forward bias

Generation in the depletion layer increase the reverse bias saturation current.

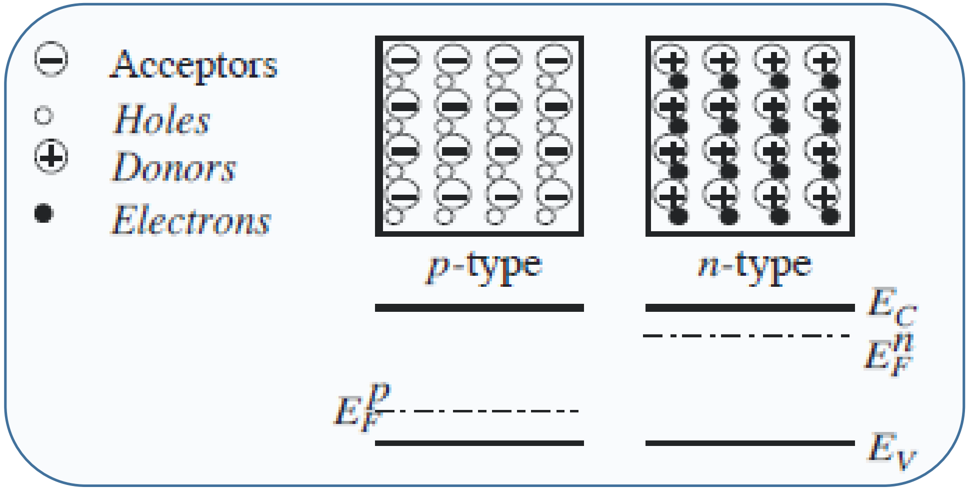
For real diodes, the Shockley equation is modified to

$$I = I_0 \exp\left(\frac{V_F}{n\varphi_T}\right)$$

Where n is called ideality factor which have values between 1 and 2.

pn Junction

Let us consider initially that the two parts of the *p-n* junction are separated.



Both parts are electro-neutral. Electrons compensate the ionised donors and holes compensate the ionised acceptors.

Taking into account that

$$\boxed{\begin{aligned} n &\approx N_D = n_i \exp\left(\left(E_F^n - E_i\right)/k_B T\right) \\ p &\approx N_A = n_i \exp\left(\left(E_i - E_F^p\right)/k_B T\right) \end{aligned}}$$

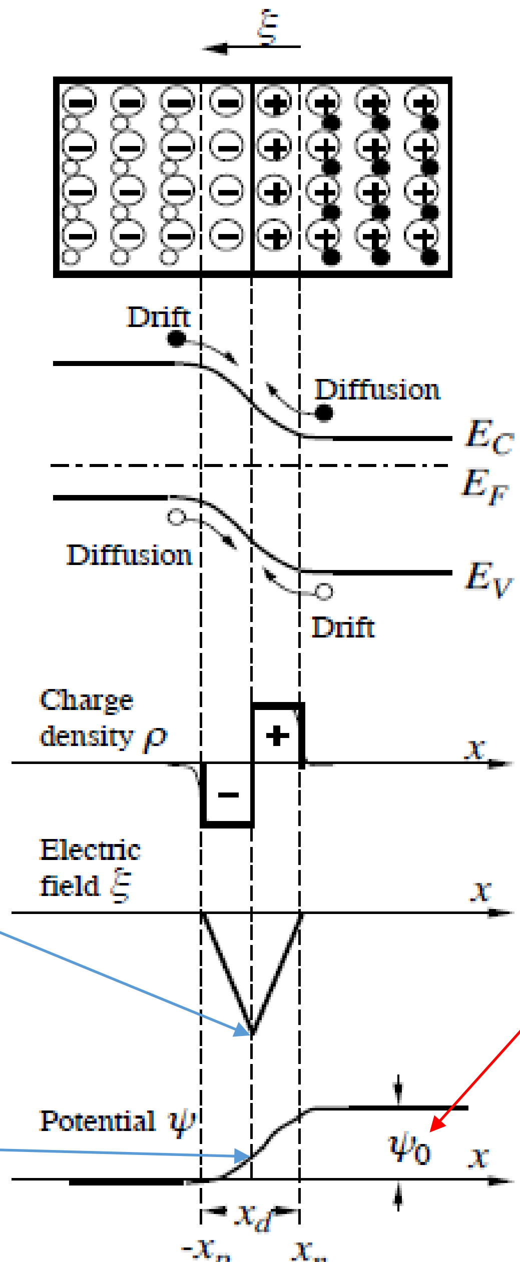
We can calculate the distance between the Fermi levels on the both sides

$$\begin{aligned} E_F^n - E_F^p &= \left(E_F^n - E_i\right) + \left(E_i - E_F^p\right) \\ &= k_B T \ln\left(N_D / n_i\right) + k_B T \ln\left(N_A / n_i\right) \\ &= k_B T \ln\left(\frac{N_D N_A}{n_i^2}\right) \end{aligned}$$

Depletion Region

The height of the potential barrier that supports the junction field is called the **contact potential**

At equilibrium, both drift and diffusion currents are equal and cancel each other.



$$qV_{bi} = E_{Fn} - E_{Fp}$$

$$E_{Fp} = E_{Fi} - k_B T \ln\left(\frac{N_A}{n_i}\right)$$

$$E_{Fn} = E_{Fi} + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

$$E(0) = \frac{-eN_d x_n}{\epsilon} = \frac{-eN_a x_p}{\epsilon}$$

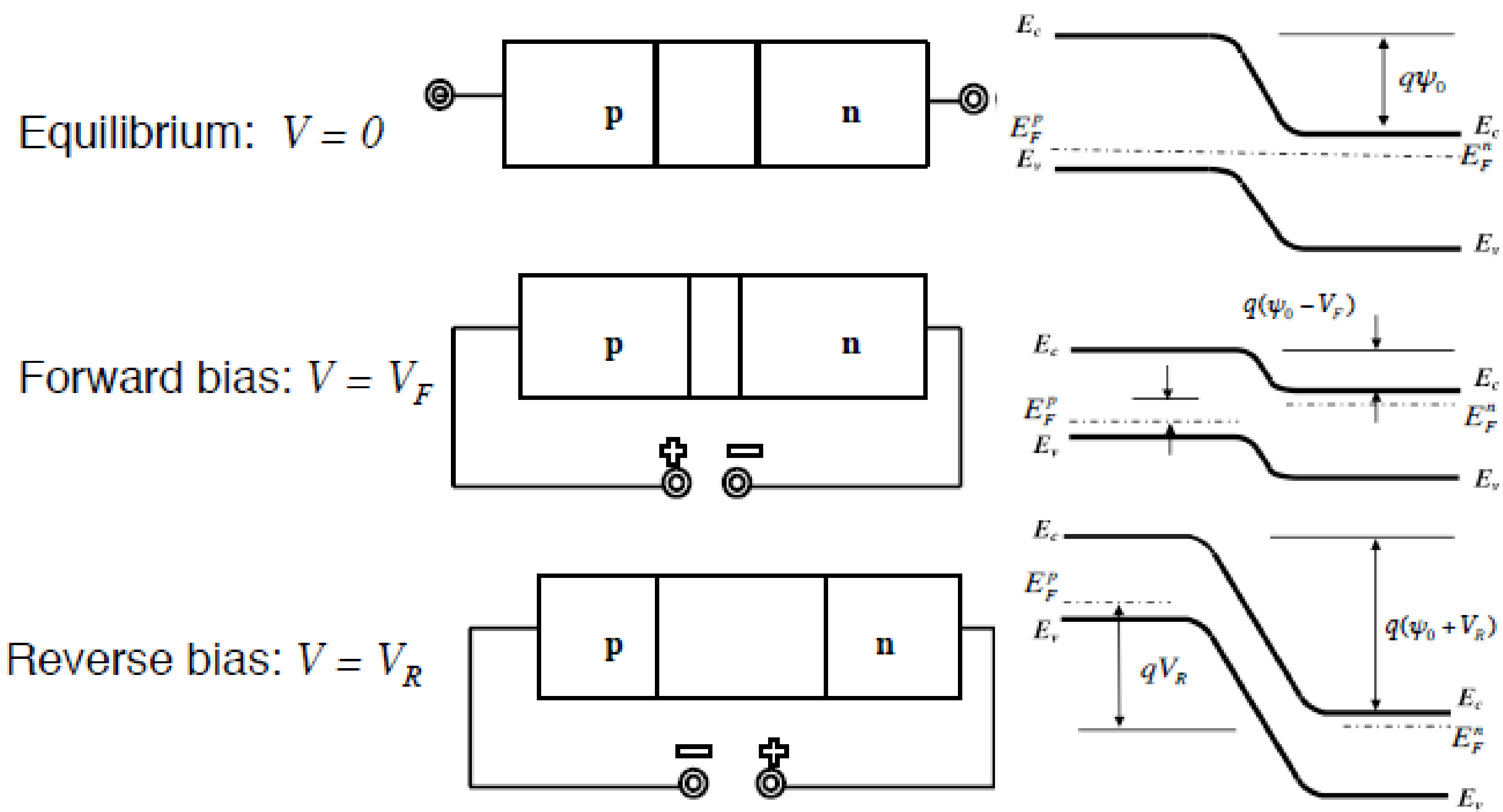
ψ_0

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

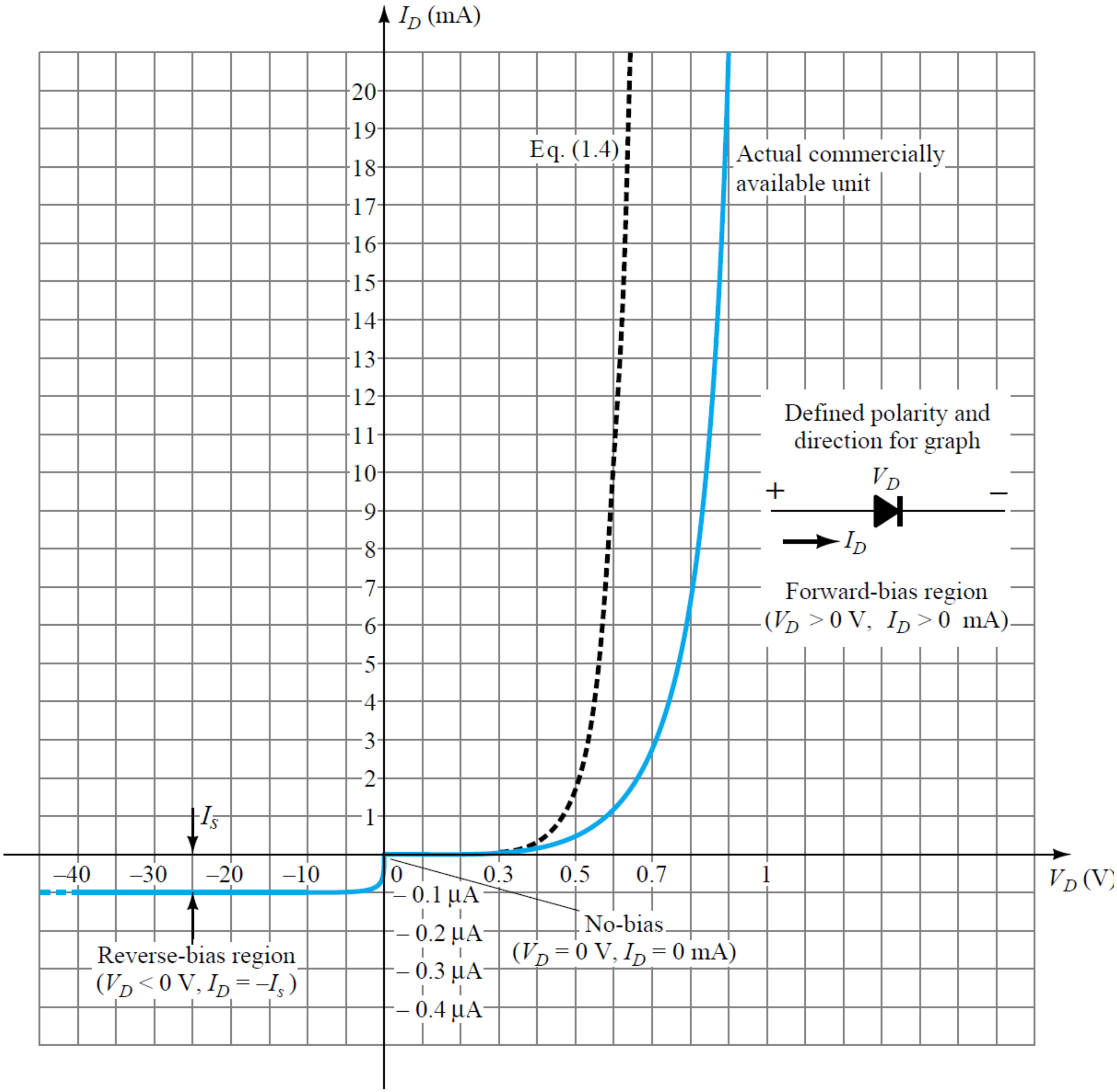
$$V(0) = \frac{eN_a . x_p^2}{2\epsilon}$$

Effect of Applied Bias

- Applying a potential to the ends of a diode does NOT increase current through drift rather it changes the potential barrier to diffusion.
  - Forward bias lowers potential barrier, decreases depletion region and also increases diffusion current exponentially.
  - Reverse bias increases barrier, increases depletion region and lowers current flow to only leakage current.



Effect of Applied Bias



Depletion Capacitance

The small signal capacitance of the space charge layer is given by $C \equiv \frac{dQ}{dV_R}$ where Q is the unipolar charge in the depletion region

$$Q = qAN_D x_n = qAN_A x_p = A\left(\frac{2q\varepsilon_0\varepsilon_S(\psi_0 + V_R)N_A N_D}{(N_A + N_D)}\right)^{1/2}$$

Therefore

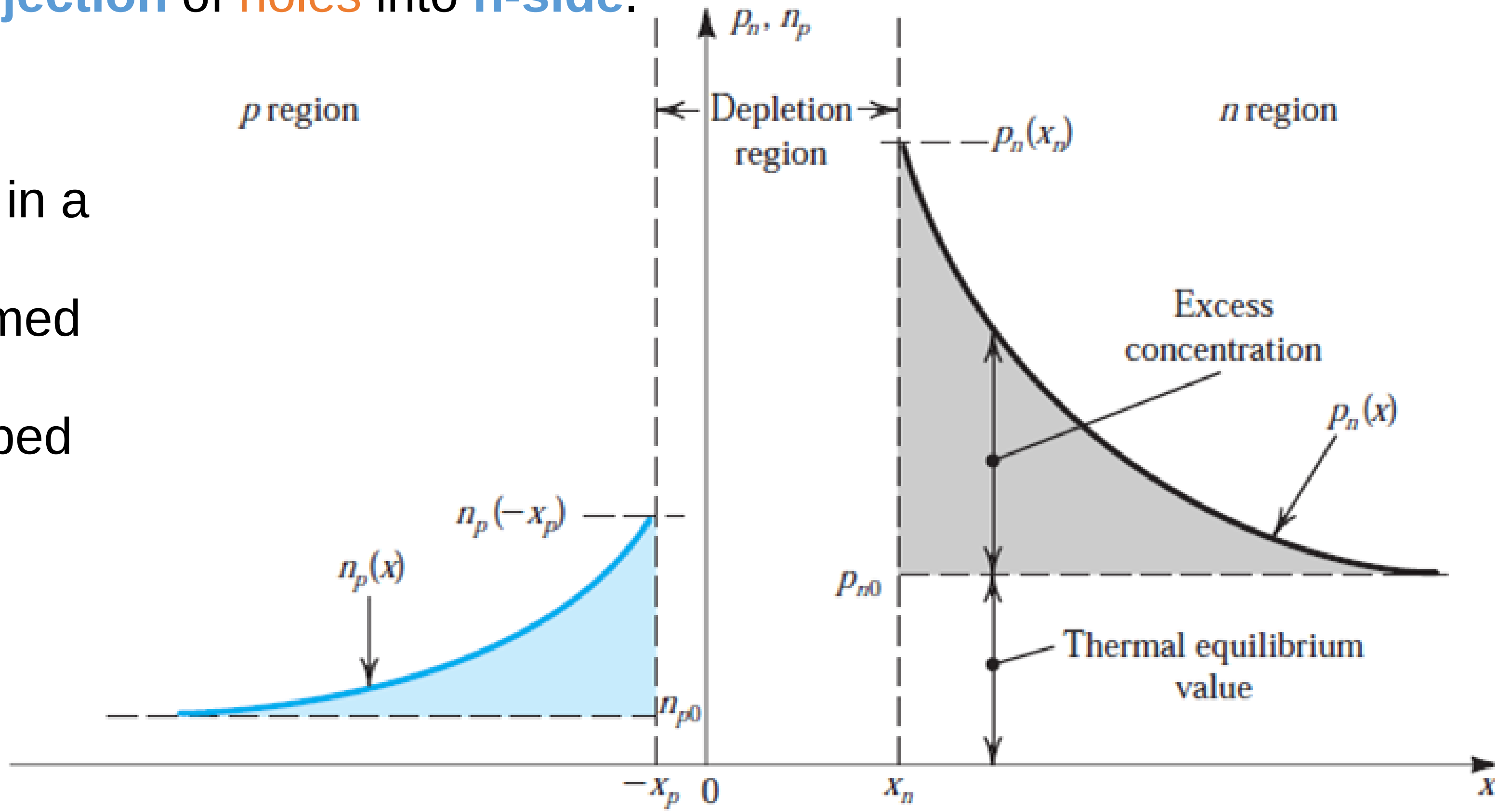
$$C = A\left(\frac{q\varepsilon_0\varepsilon_S N_D N_A}{2(N_A + N_D)(\psi_0 + V_R)}\right)^{1/2}$$

I-V Characteristics

Under equilibrium - **drift and diffusion** components of the electron current are equal but opposite in direction. Net electron current is zero. Same is true for net hole current.

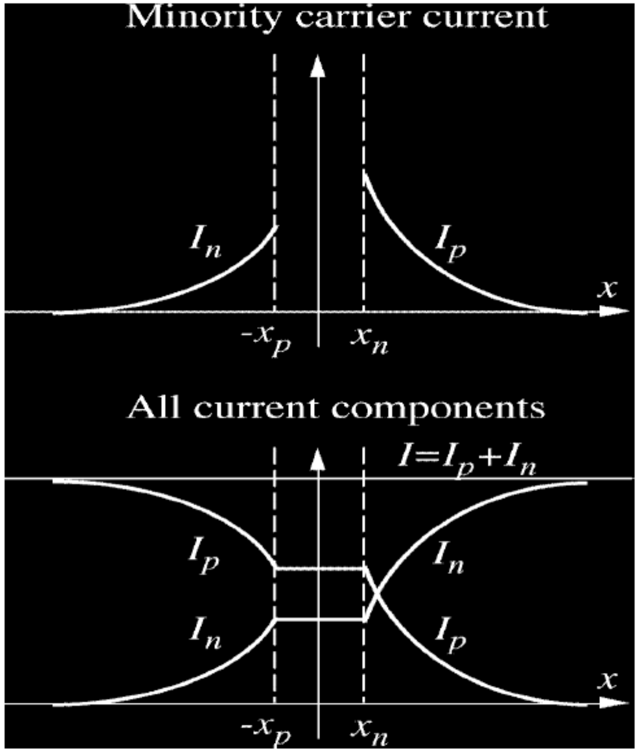
Under forward bias, depletion region is reduced, E-field in depletion region is reduced and drift current is reduced. However, there is an **injection** of electrons into **p-side** and **injection** of holes into **n-side**.

Carrier distribution in a forward biased ***pn*** junction. It is assumed that the ***p* region** is more heavily doped than the *n* region: NA >>ND.



I-V Characteristics

Shockley Relationship



The current continuity requires the total current to be constant at each point along the device.

$$\boxed{I(x) = I_n(x) + I_p(x) = \mathrm{C}}$$

This means that away from the depletion layer edges the minority diffusion current is gradually transferred in majority drift current and this happened via recombination.

I-V Characteristics

Hole diffusion into n-region:

$$p_n(x) = p_{n0} + p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

$$J_p(x) = -qD_p \frac{dp_n(x)}{dx}$$

Injected minority carriers diffuse away from the depletion region edges and recombine with majority carriers. Thus, their concentration decays exponentially.

$$J_p(x) = q\left(\frac{D_p}{L_p}\right)p_{n0}(e^{V/V_T} - 1)e^{-(x-x_n)/L_p}$$

$$J_p(x_n) = q\left(\frac{D_p}{L_p}\right)p_{n0}(e^{V/V_T} - 1) \xrightarrow{\text{NB for electron diffusion into p-region}} J_n(-x_p) = q\left(\frac{D_n}{L_n}\right)n_{p0}(e^{V/V_T} - 1)$$

$$I = A(J_p + J_n)$$

$$I = Aq\left(\frac{D_p}{L_p}\,p_{n0} + \frac{D_n}{L_n}\,n_{P0}\right)(e^{V/V_T} - 1)$$

Substituting for $p_{n0} = n_i^2/N_D$ and for $n_{p0} = n_i^2/N_A$ gives

$$I = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)(e^{V/V_T} - 1) \longrightarrow I_S = Aqn_i^2\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)$$

$$\boxed{I = I_S(e^{V/V_T} - 1)}$$

Famous **SHOCKLEY** equation

Where I_S is also know as the "**Reverse Bias Saturation**" Current

I-V Characteristics

$$p_n(x) = \left(p_n(x_n) - p_{n0}\right)\exp\left(-\frac{(x - x_n)}{L_p}\right) + p_{n0}$$

$$n_p(x) = \left(n_p(-x_p) - n_{p0}\right)\exp\left(\frac{(x + x_p)}{L_n}\right) + n_{p0}$$

where L_p, L_n are the minority carriers diffusion lengths. The minority carrier diffusion currents are given by

$$I_p(x) = -qAD_p\frac{dp_n(x)}{dx} = qA\frac{D_p}{L_p}\left(p_n(x_n) - p_{n0}\right)\exp\left(-\frac{(x - x_n)}{L_p}\right)$$

$$I_n(x) = qA\frac{D_n}{L_n}\left(n_p(-x_p) - n_{p0}\right)\exp\left(\frac{(x + x_p)}{L_n}\right)$$

At *low level of injection* at the depletion layer edges, it can be shown that

$$n_p(-x_p) = n_{p0}\exp\left(\frac{V_F}{V_T}\right) \quad , \quad p_n(x_n) = p_{n0}\exp\left(\frac{V_F}{V_T}\right) \longrightarrow V_T = \frac{K_B T}{q}$$

Therefore the minority carrier currents at the depletion layer edges are given by

$$I_p(x_n) = qA\frac{D_p}{L_p}p_{n0}\left(\exp\left(\frac{V_F}{V_T}\right) - 1\right) \quad , \quad I_n(-x_p) = qA\frac{D_n}{L_n}n_{p0}\left(\exp\left(\frac{V_F}{V_T}\right) - 1\right)$$

I-V Characteristics

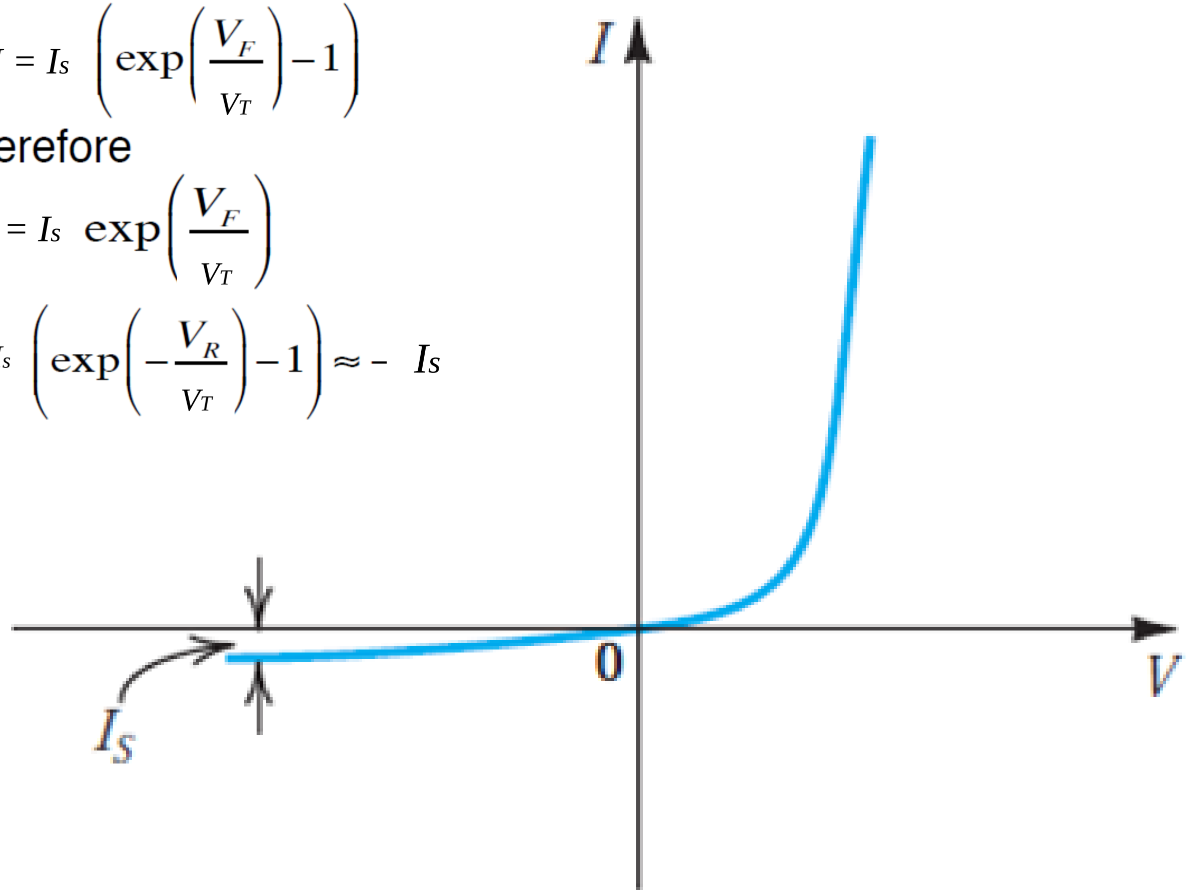
$$I = I_S \left(\exp\left(\frac{V_F}{V_T}\right) - 1 \right)$$

Normally $\exp(V_F/V_T) >> 1$ and therefore

$$I = I_S \ \exp\left(\frac{V_F}{V_T}\right)$$

In reverse bias conditions

$$I = I_S \left(\exp\left(-\frac{V_R}{V_T}\right) - 1 \right) \approx - \ I_S$$



Example

Assume we have a ***pn*** junction with $N_a = 10^{18}/cm^3$, $N_d = 10^{16}/cm^3$, $A = 10^{-4}cm^2$, $\mathrm{n_i} = 1.5\mathrm{X}10^{10}/cm^3$, L_p=5µm, L_n=10µm, D_p (in the *n*-region)=10cm^2/s and D_n (in the *p*-region) = 18cm^2/s. The ***pn*** junction is forward biased and conducting a current of I=0.1mA.

Calculate:

a) Reverse Bias Saturation Current.

b) Forward bias voltage.

c) Component of current due to hole injection and that due to electron injection across the junction.

Quantity	Relationship	Values of Constants and Parameters (for Intrinsic Si at $T = 300$ K)
Carrier concentration in intrinsic silicon (cm^{-3})	$n_i = BT^{3/2}e^{-E_g/2kT}$	$B = 7.3 \times 10^{15}\ \text{cm}^{-3}\text{K}^{-3/2}$ $E_g = 1.12$ eV $k = 8.62 \times 10^{-5}$ eV/K $n_i = 1.5 \times 10^{10}/\text{cm}^3$
Diffusion current density (A/cm^2)	$J_p = -qD_p\frac{dp}{dx}$ $J_n = qD_n\frac{dn}{dx}$	$q = 1.60 \times 10^{-19}$ coulomb $D_p = 12\ \text{cm}^2/\text{s}$ $D_n = 34\ \text{cm}^2/\text{s}$
Drift current density (A/cm^2)	$J_{drift} = q(p\mu_p + n\mu_n)E$	$\mu_p = 480\ \text{cm}^2/\text{V}\cdot\text{s}$ $\mu_n = 1350\ \text{cm}^2/\text{V}\cdot\text{s}$
Resistivity ($\Omega\cdot$cm)	$\rho = 1/[q(p\mu_p + n\mu_n)]$	μ_p and μ_n decrease with the increase in doping concentration
Relationship between mobility and diffusivity	$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = V_T$	$V_T = kT/q \simeq 25.8$ mV
Carrier concentration in n-type silicon (cm^{-3})	$n_{n0} \simeq N_D$ $p_{n0} = n_i^2/N_D$	
Carrier concentration in p-type silicon (cm^{-3})	$p_{p0} \simeq N_A$ $n_{p0} = n_i^2/N_A$	
Junction built-in voltage (V)	$V_0 = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$	
Width of depletion region (cm)	$\frac{x_n}{x_p} = \frac{N_A}{N_D}$ $W = x_n + x_p$ $= \sqrt{\frac{2\epsilon_s}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)(V_0 + V_R)}$	$\epsilon_s = 11.7\epsilon_0$ $\epsilon_0 = 8.854 \times 10^{-14}$ F/cm
Charge stored in depletion layer (coulomb)	$Q_J = q\frac{N_A N_D}{N_A + N_D}AW$	
Forward current (A)	$I = I_p + I_n$ $I_p = Aqn_i^2\frac{D_p}{L_p N_D}(e^{V/V_T} - 1)$ $I_n = Aqn_i^2\frac{D_n}{L_n N_A}(e^{V/V_T} - 1)$	
Saturation current (A)	$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)$	
I-V Relationship	$I = I_S(e^{V/V_T} - 1)$	
Minority-carrier lifetime (s)	$\tau_p = L_p^2/D_p \quad \tau_n = L_n^2/D_n$	$L_p, L_n = 1\ \mu\text{m}$ to $100\ \mu\text{m}$ $\tau_p, \tau_n = 1$ ns to 10^4 ns

Examples

Question 1

Assume we have a ***pn*** junction operated at room temperature with T=300 K, $N_a = 2 \times 10^{15}/cm^3$, $N_d = 5 \times 10^{18}/cm^3$ on the p- and n-side respectively.

a) Calculate the barrier potential, V_{bi}.

b) Calculate the doping concentration on the *p*-side required to achieve a barrier potential of $V_{bi} = 0.85$ V.

c) Calculate the depletion layer width at equilibrium, at reverse bias $V_R = 5$ V and at forward bias $V_F = 0.5$ V.

d) Calculate the doping concentration in the *p*-region to achieve a depletion layer width of $w = 1\ \mu m$.

Examples

Question 2

Sketch the band diagram of a silicon *p-n* junction clearly indicating the conduction and the valence bands, the position of the Fermi levels and the applied voltage in the following cases:

(a) Reverse bias condition.

(b) Forward bias condition.

Question 3

Calculate, at room temperature (T=300K), the reverse saturation current in a *p-n* junction ideal diode (**n=1**) with a cross-sectional area of $A = 5\times10^{-2}\,cm^{-2}$ if the forward bias current at $V_F = 0.4\text{V}$ is $I = 2\times10^{-2}\,\text{A}$.