UESTC3001 Dynamics & Control
Lecture 3

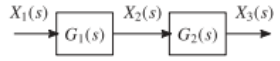
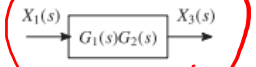
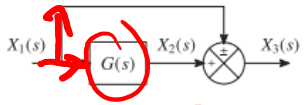
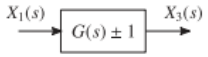
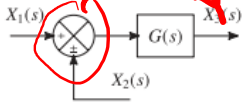
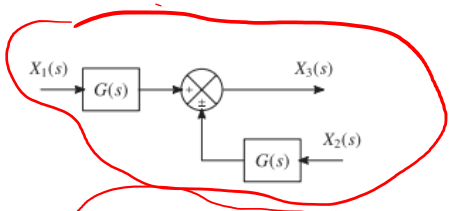
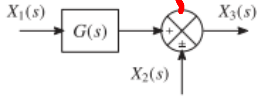
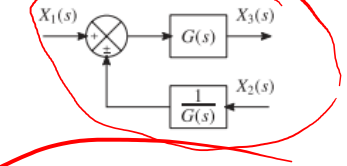
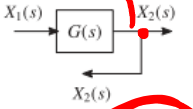
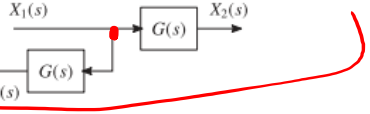
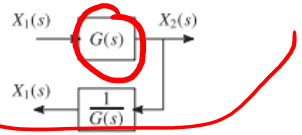
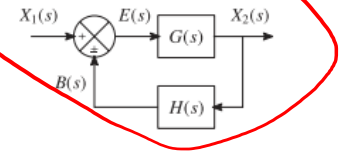
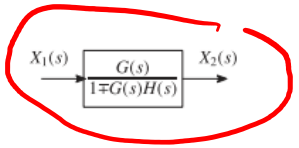
Block Diagram Reduction

Dr Kelum Gamage kelum.gamage@glasgow.ac.uk
Associate Professor (Senior Lecturer)
School of Engineering, University of Glasgow, UK

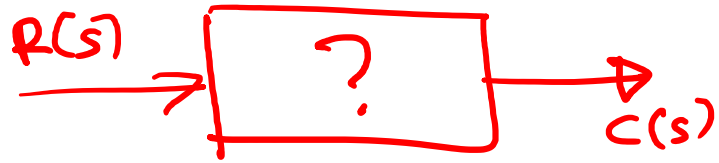
Outline

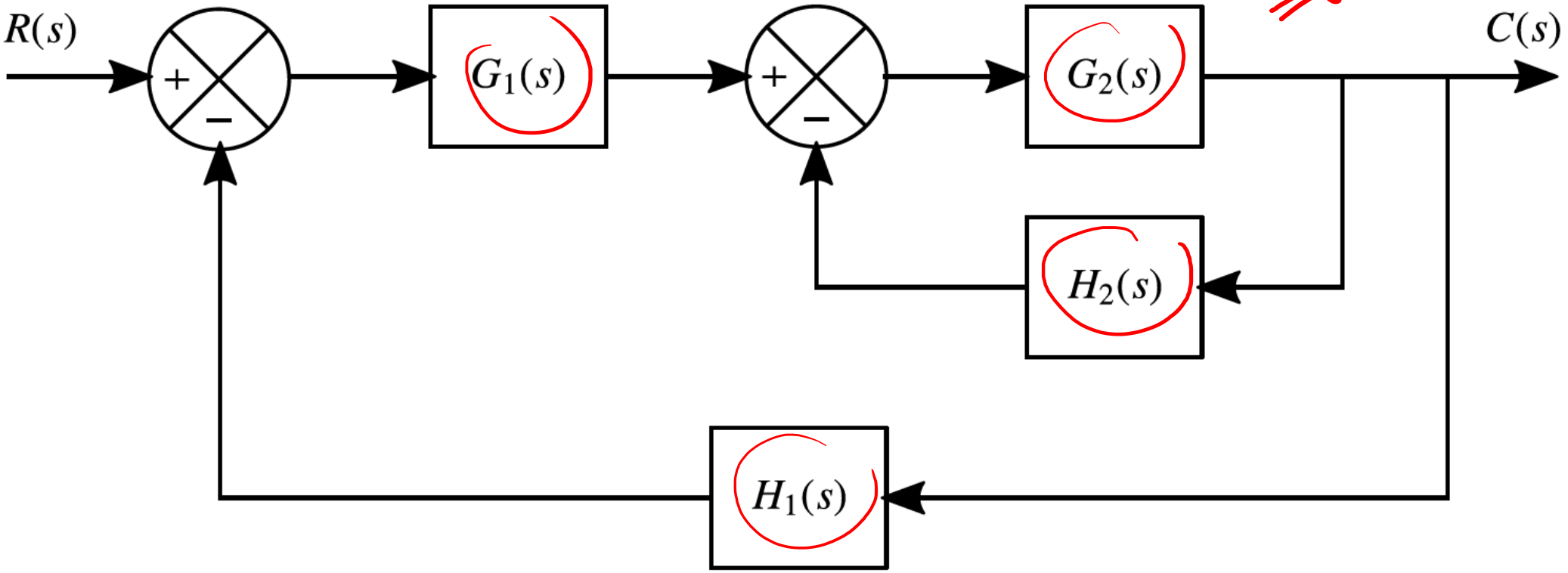
- Apply Block Diagram Reduction Rules
- Analyse Control System Subjected to a Disturbance

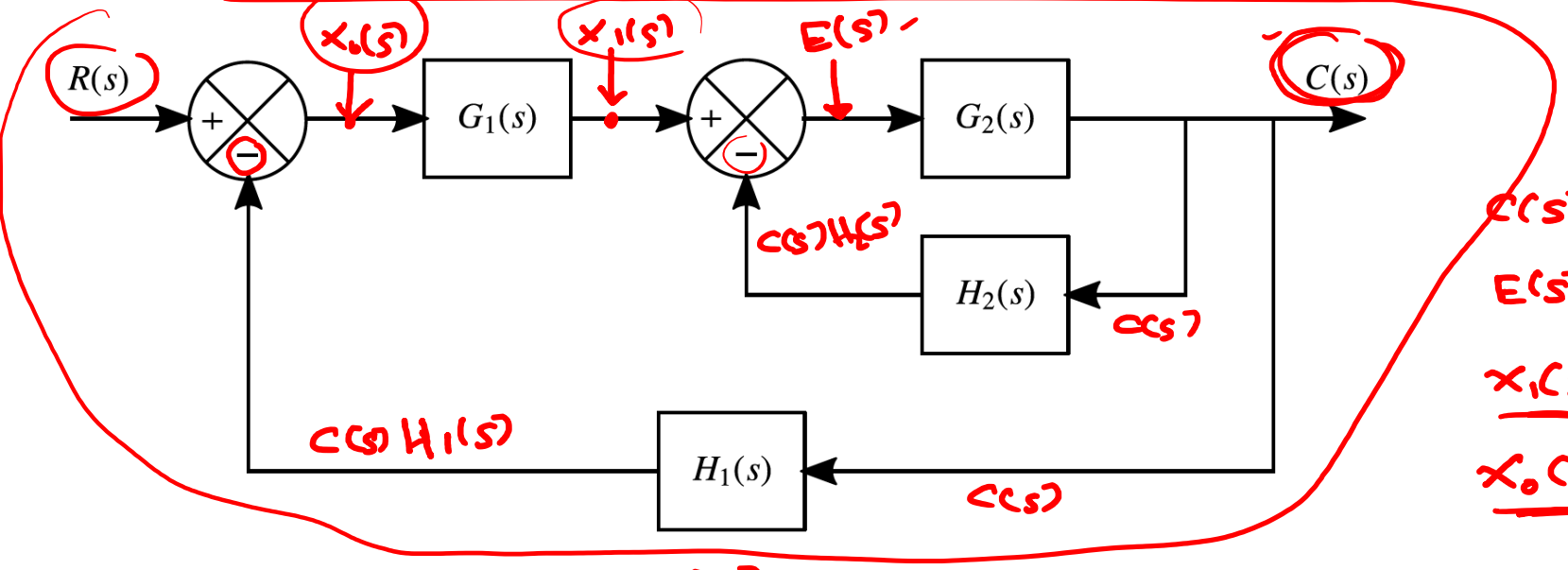
Rules for block diagram reduction - Summary

Rule	Original	Equivalent
1. Cascaded blocks		
2. Summing two signals		
3. Moving a summing point behind a block		
4. Moving a summing point ahead of a block		
5. Moving a branch point ahead of a block		
6. Moving a branch point behind a block		
7. Eliminating a feedback loop		

Example: Derive the overall transfer function







$$C(s) = E(s)\,G_2(s) \quad \text{——} (1)$$

$$E(s) = X_1(s) - C(s)\,H_2(s) \quad \text{——} (2)$$

$$X_1(s) = X_0(s)\,G_1(s) \quad \text{——} (3)$$

$$X_0(s) = R(s) - C(s)H_1(s) \quad \text{——} (4)$$

$$C(s) = E(s)\,G_2(s)$$

$$= (X_1 - CH_2)\,G_2$$

$$= (X_0 G_1 - CH_2)\,G_2$$

$$= [(R - CH_1)\,G_1 - CH_2]\,G_2$$

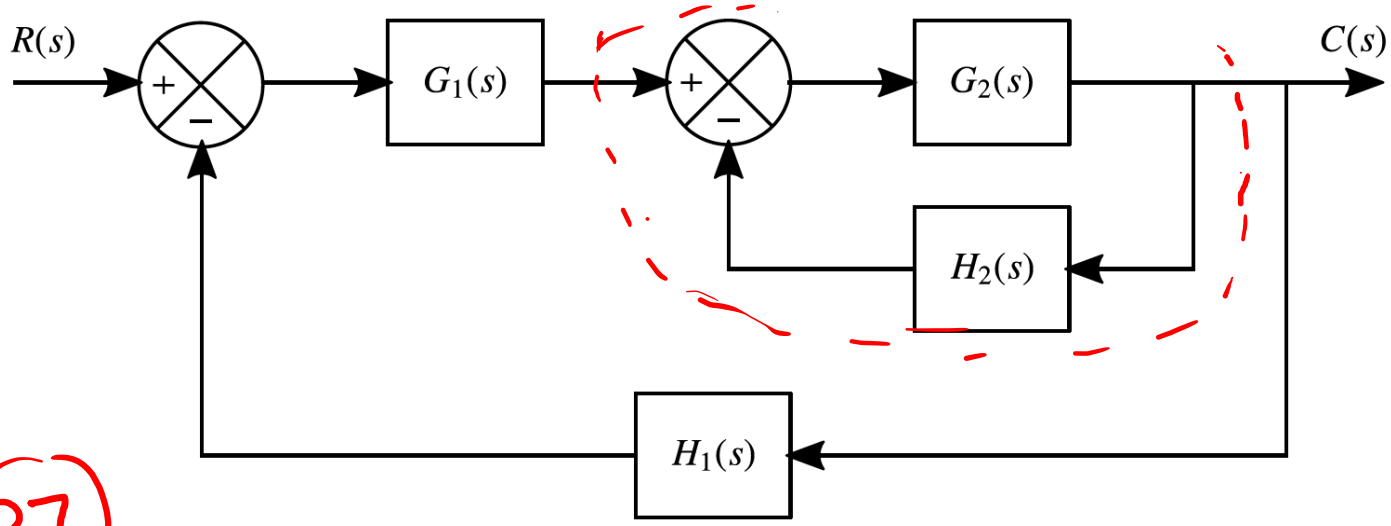
$$C(s) = RG_1G_2 - CH_1G_1G_2 - CH_2G_2$$

$$C(s) + CH_1G_1G_2 + CH_2G_2 = RG_1G_2$$

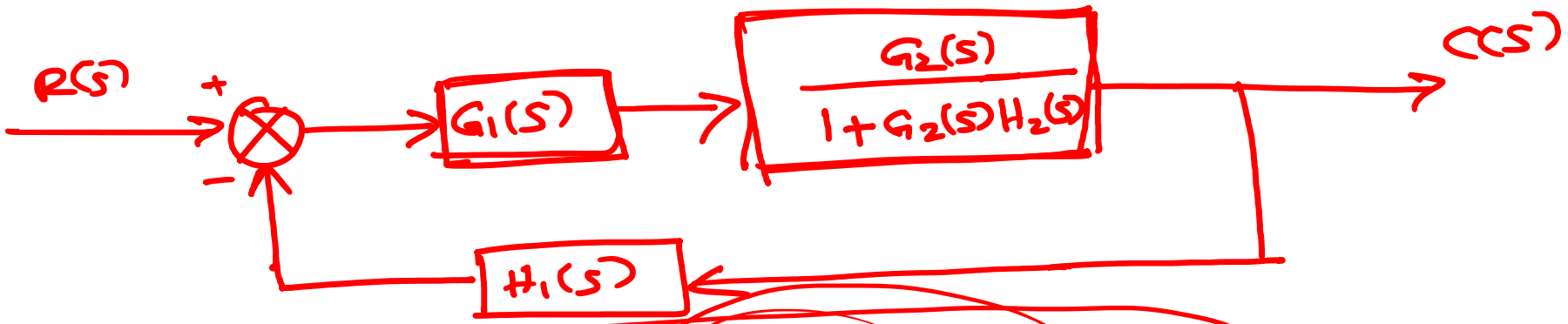
$$C(s)\left[1 + H_1(s)G_1(s)G_2(s) + H_2(s)G_2(s)\right] = R(s)\,G_1(s)\,G_2(s)$$

$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + H_1(s)G_1(s)G_2(s) + H_2(s)G_2(s)}$$

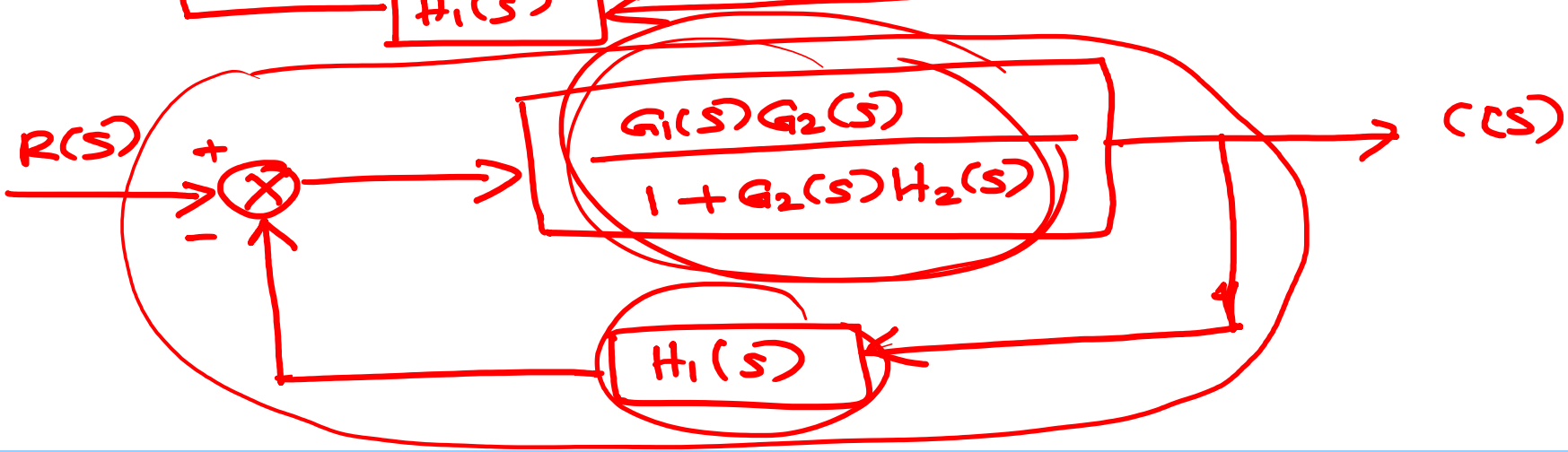
*

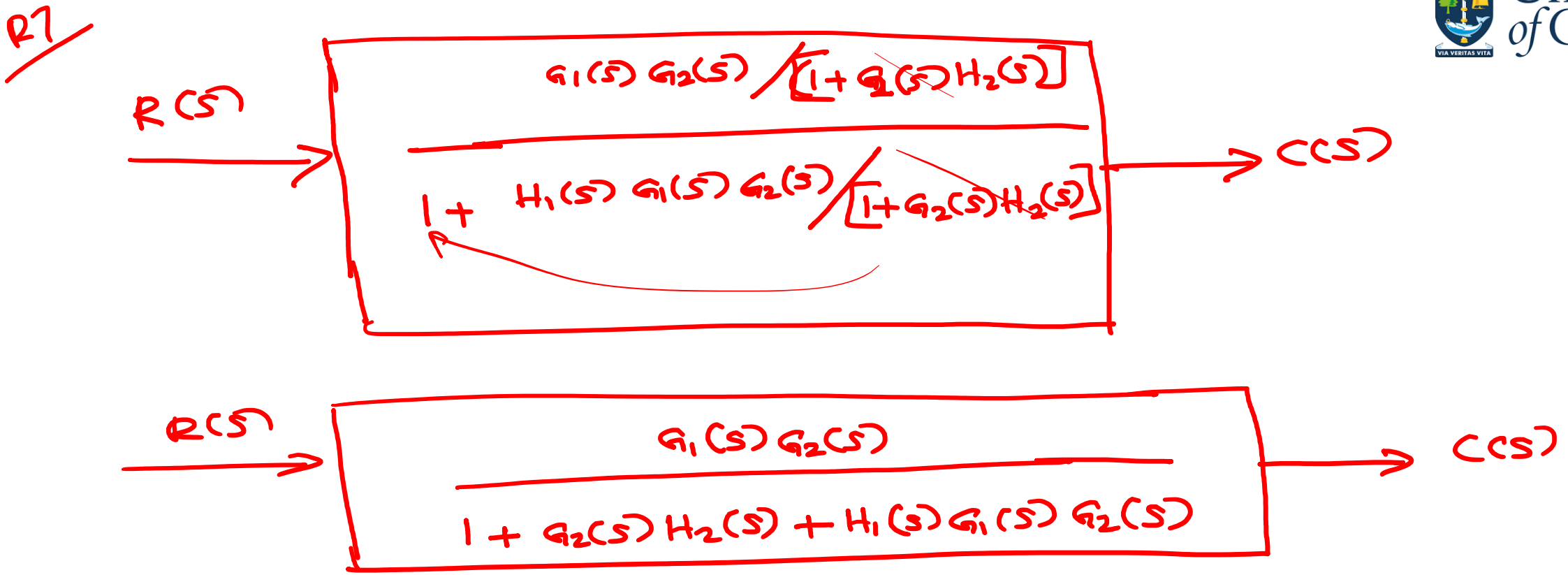

R(s)
+
−
G1(s)
+
−
G2(s)
C(s)
H2(s)
H1(s)

R7

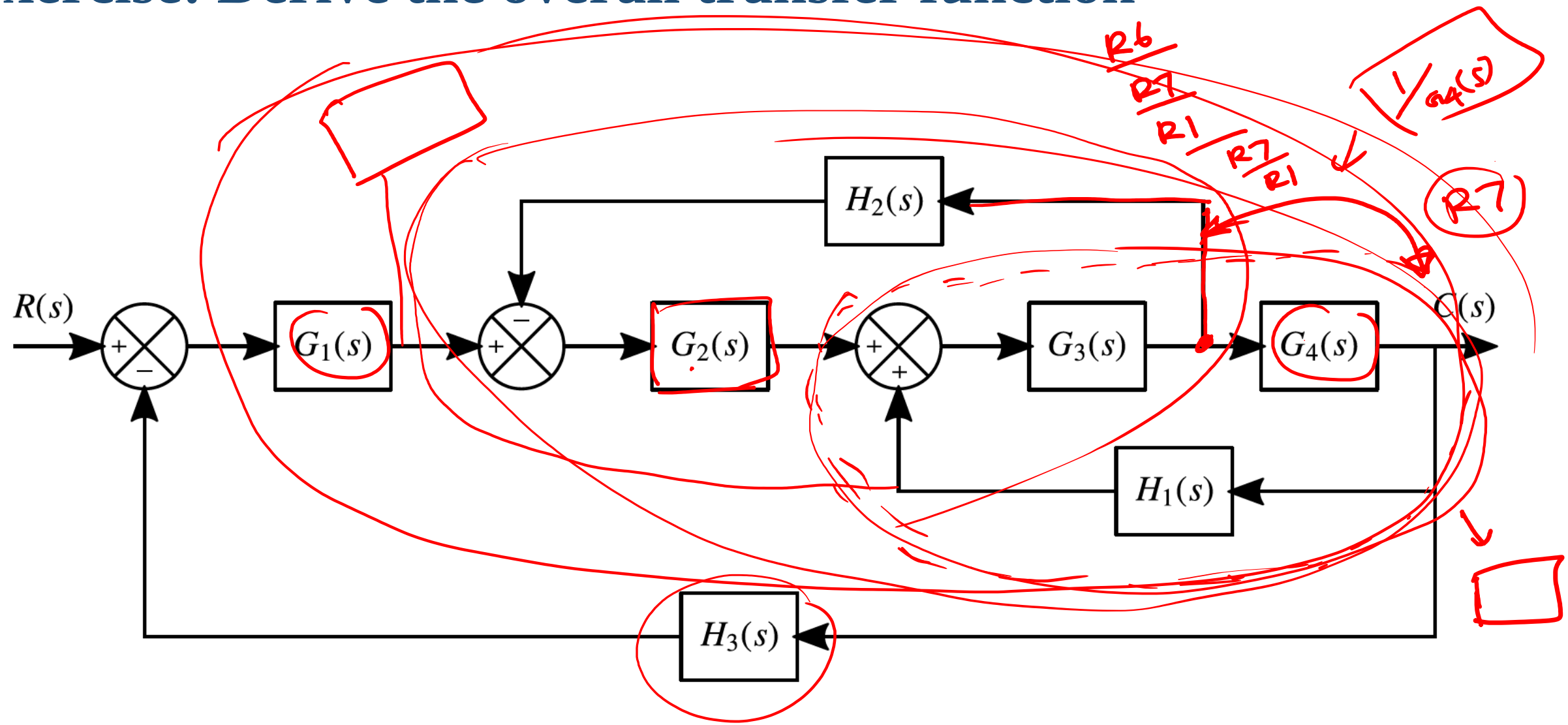

R(s)
+
−
G1(s)
G2(s)
1+G2(s)H2(s)
C(s)
H1(s)

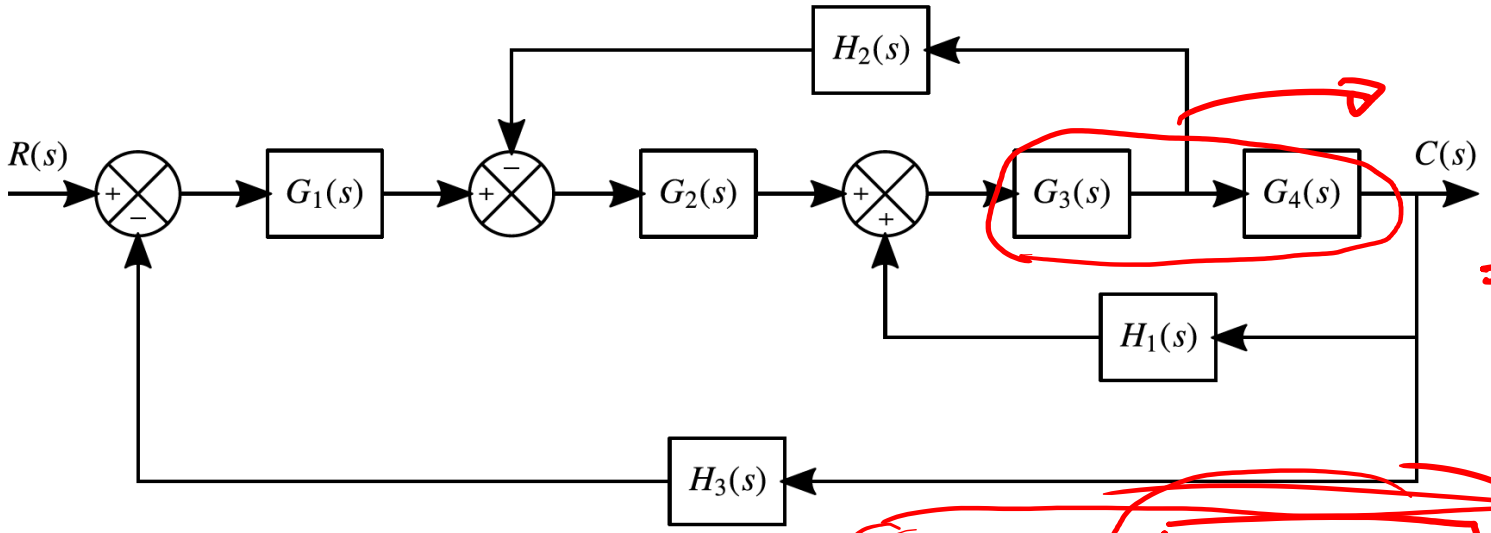
R1

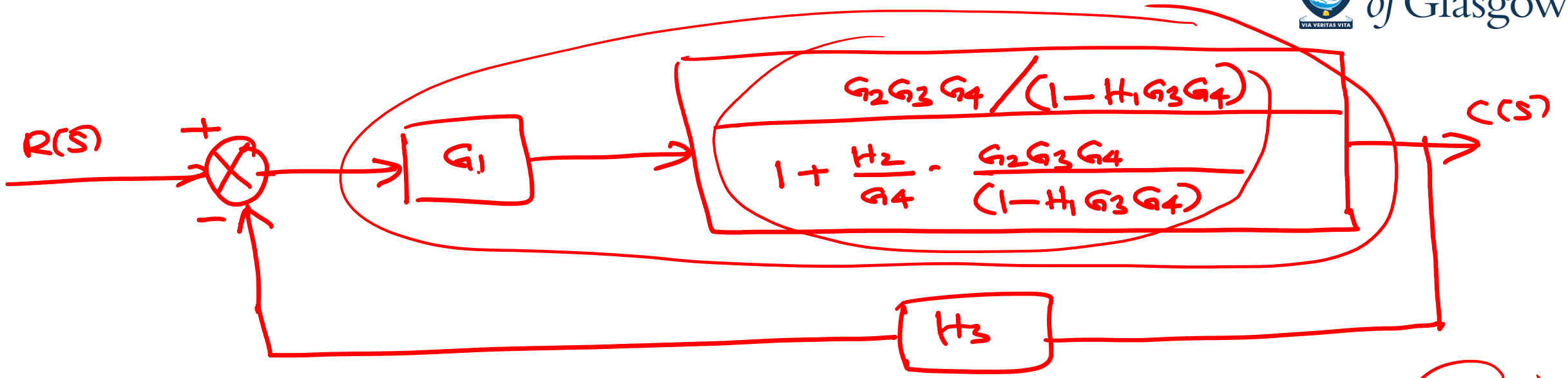

R(s)
+
−
G1(s)G2(s)
1+G2(s)H2(s)
C(s)
H1(s)

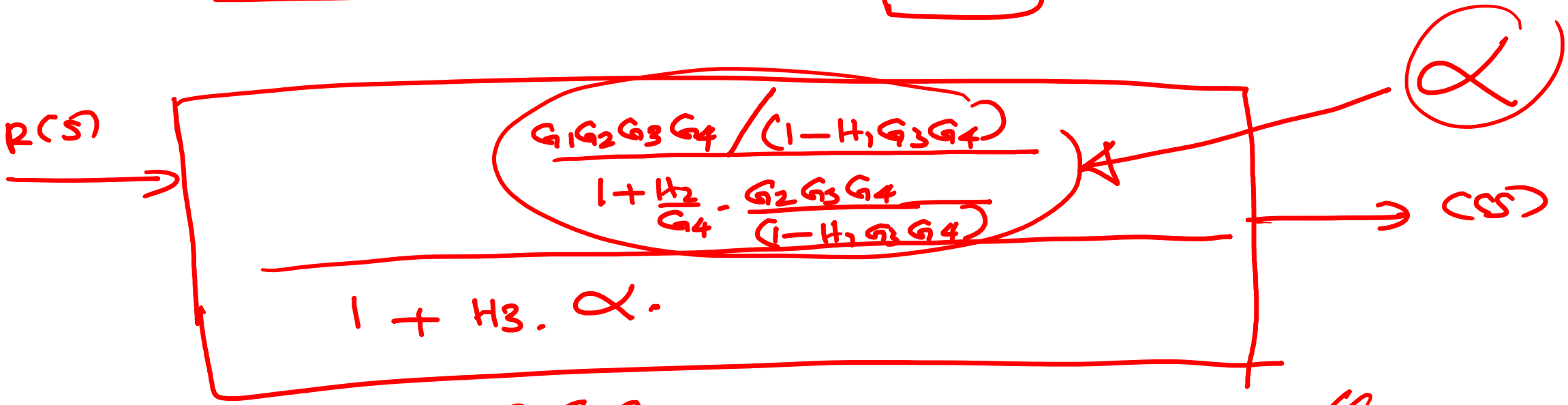

R7
R(s)
$G_1(s)\,G_2(s) \,/\, [1 + G_2(s)H_2(s)]$
$1 + H_1(s)\,G_1(s)\,G_2(s) \,/\, [1 + G_2(s)H_2(s)]$
C(s)
R(s)
$G_1(s)\,G_2(s)$
$1 + G_2(s)\,H_2(s) + H_1(s)\,G_1(s)\,G_2(s)$
C(s)

Exercise: Derive the overall transfer function




$H_2(s)$
$R(s)$
$G_1(s)$
$G_2(s)$
$G_3(s)$
$G_4(s)$
$C(s)$
$H_1(s)$
$H_3(s)$

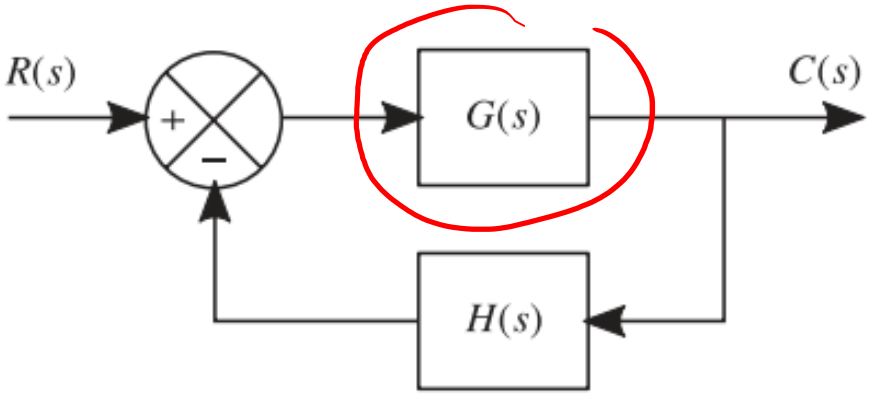


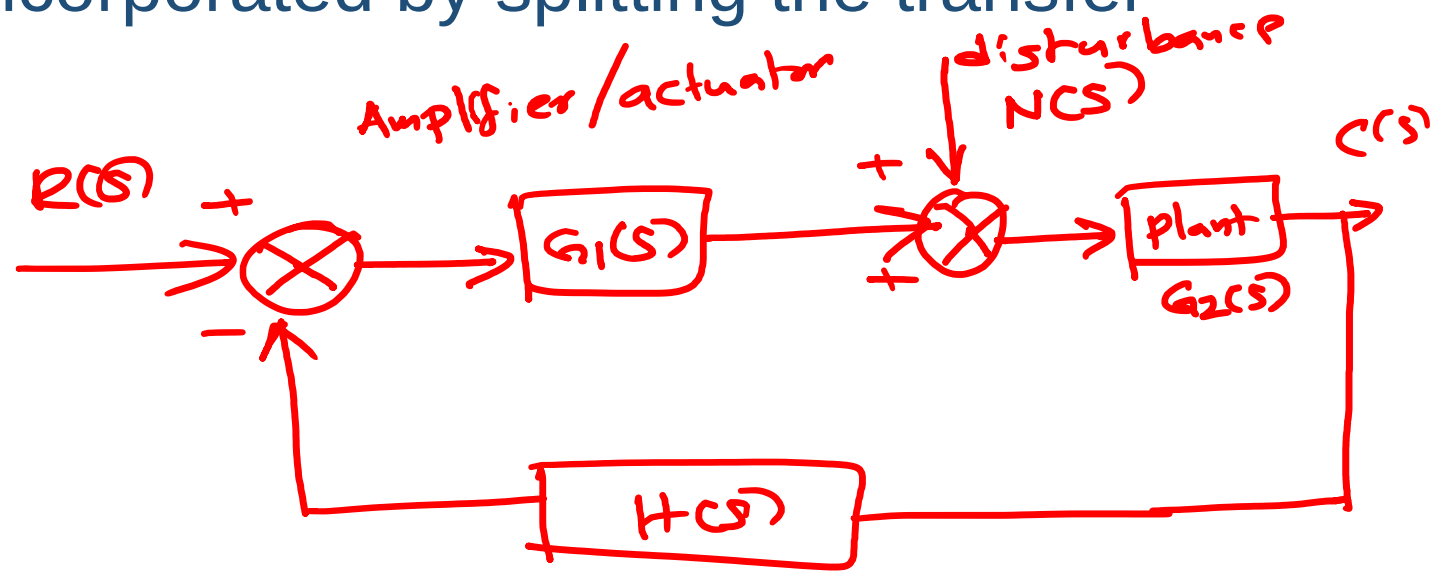


$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 - H_1 G_3 G_4 + H_2 G_2 G_3 + H_3 G_1 G_2 G_3 G_4}$$

Closed-Loop Control System Subjected to a Disturbance

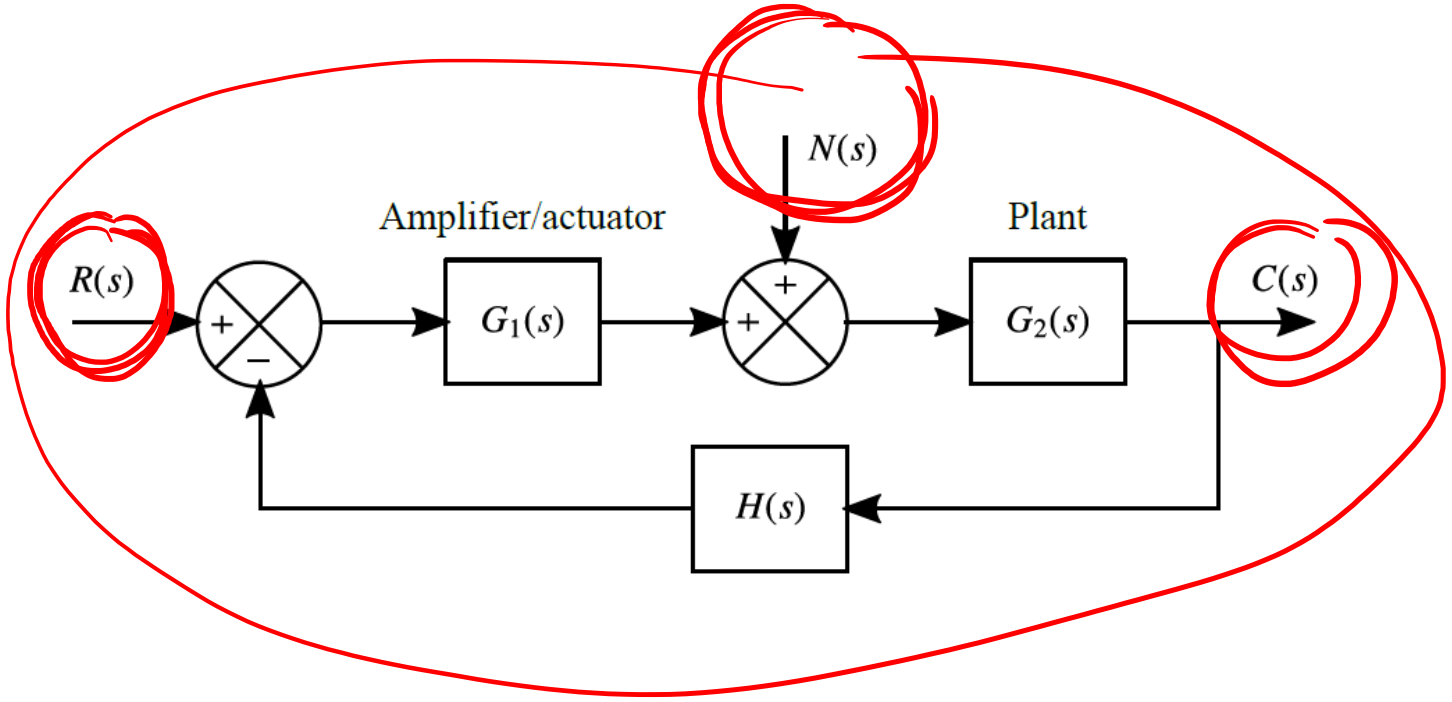
- Suppress disturbances in the steady state operation of the plant.
- Disturbance to the plant is incorporated by splitting the transfer function





Closed-Loop Control System Subjected to a Disturbance cont.

- Response to inputs and disturbances can be evaluated individually



$N(s)=0$ $\quad R(s) \rightarrow C_R(s)$

$R(s)=0$ $\quad N(s) \rightarrow C_N(s)$

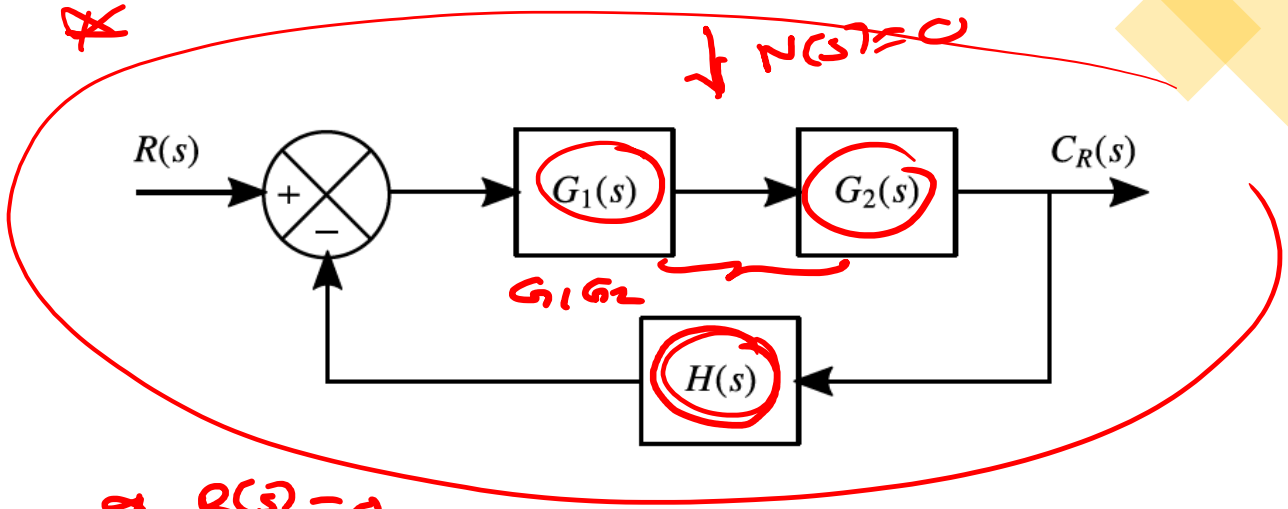
$$C(s) = C_R(s) + C_N(s)$$

C/L Control System Subjected to a Disturbance cont.

- No external disturbance

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

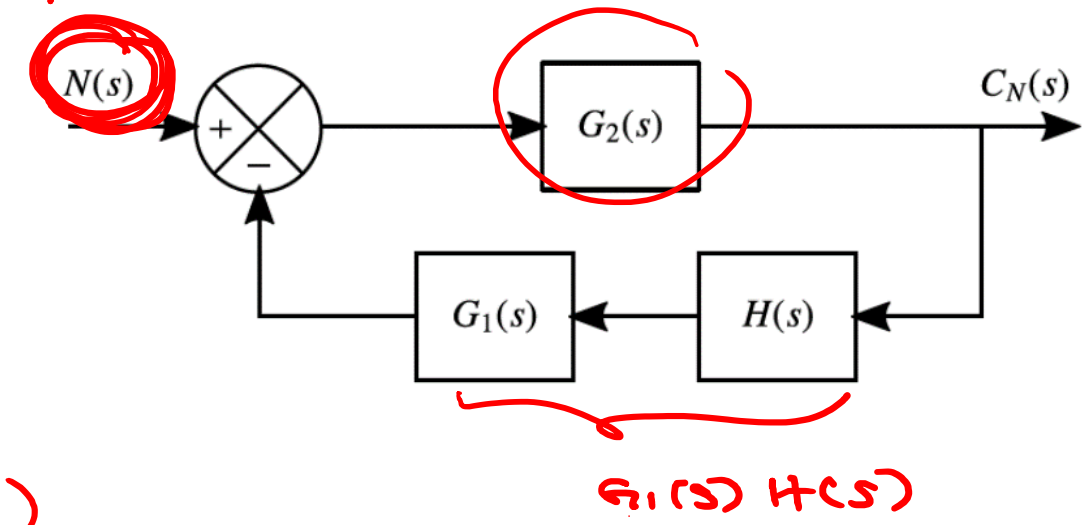
$$C_R(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} R(s)$$



- No reference input

$$\frac{C_N(s)}{N(s)} = \frac{G_2(s)}{1 + G_2(s)G_1(s)H(s)}$$

$$C_N(s) = \frac{G_2(s)}{1 + G_2(s)G_1(s)H(s)} \cdot N(s)$$

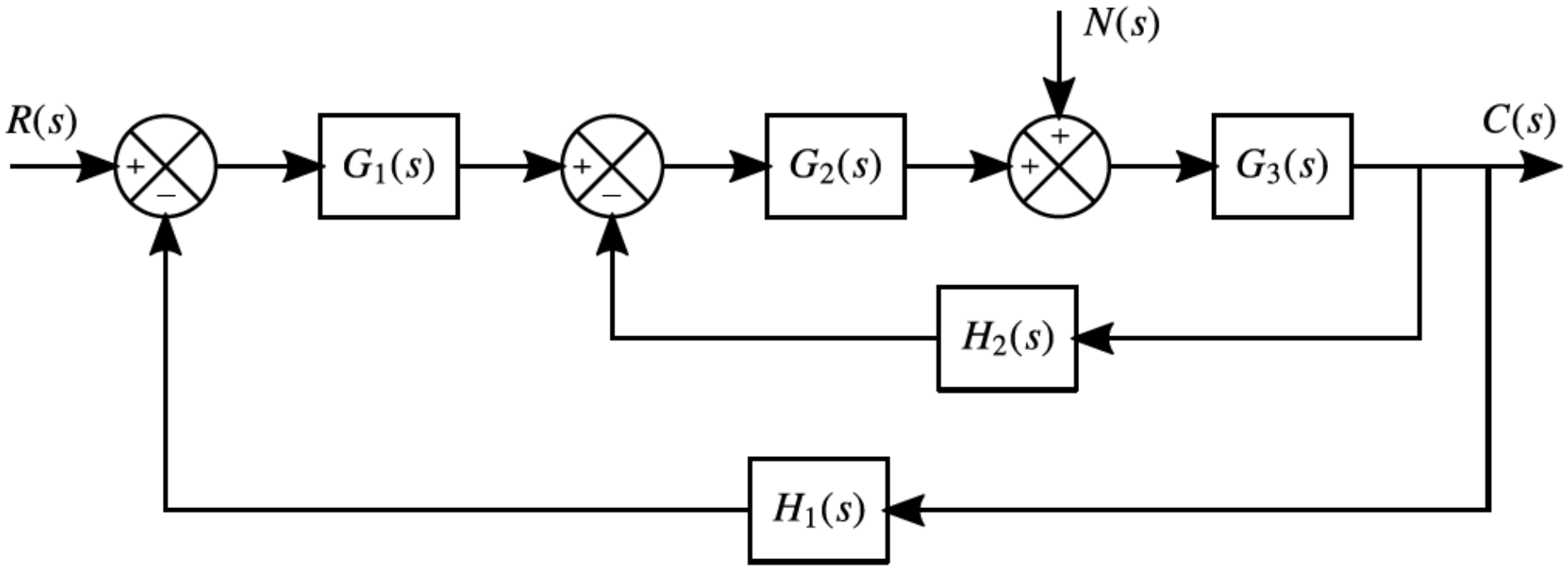


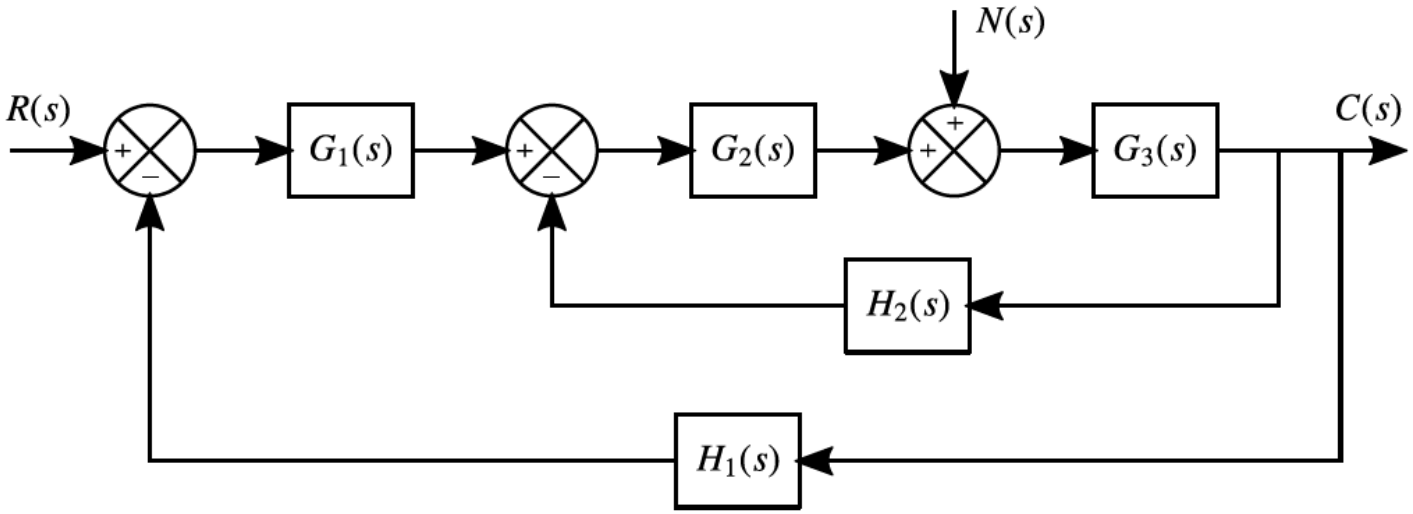
C/L System Subjected to a Disturbance cont.

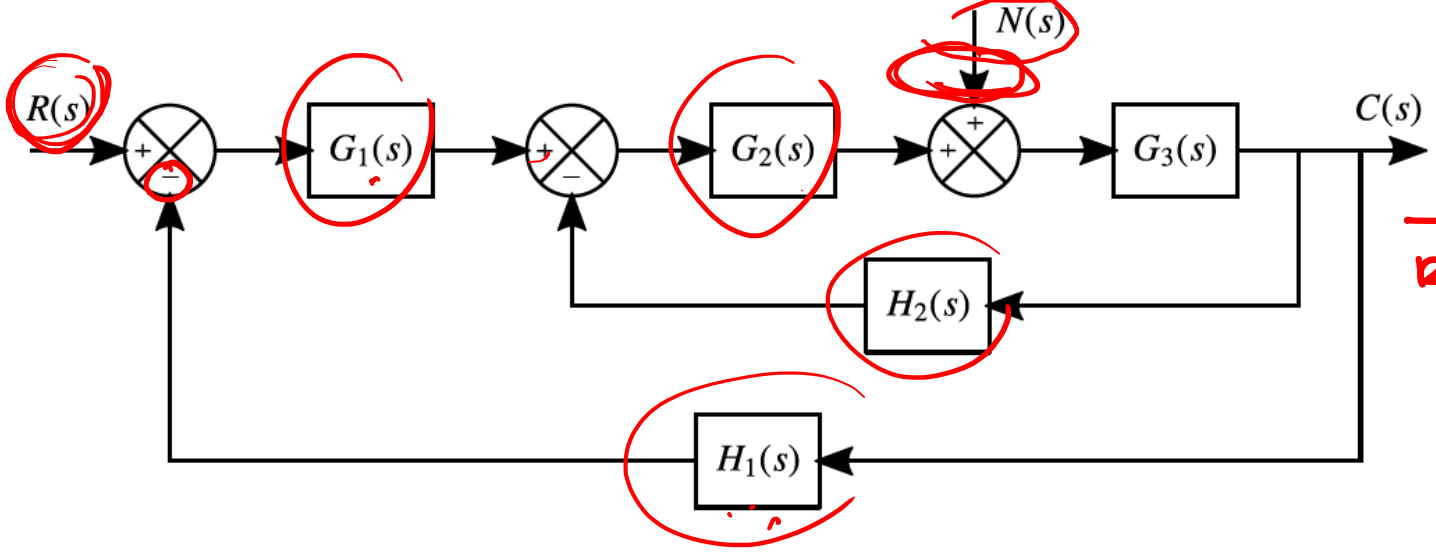
- Overall transfer function: $C(s) = C_R(s) + C_N(s)$

$$C(s) = \frac{R(s)[G_1(s)G_2(s) + G_2(s)N(s)}{(1 + G_1(s)G_2(s)H(s)}$$

Exercise:Derive the overall transfer function




N(s)
R(s)
G1(s)
G2(s)
G3(s)
C(s)
H2(s)
H1(s)



N(s) = 0

R(s) → G1 → G2G3 → C(s)

H2 (feedback around G2G3)

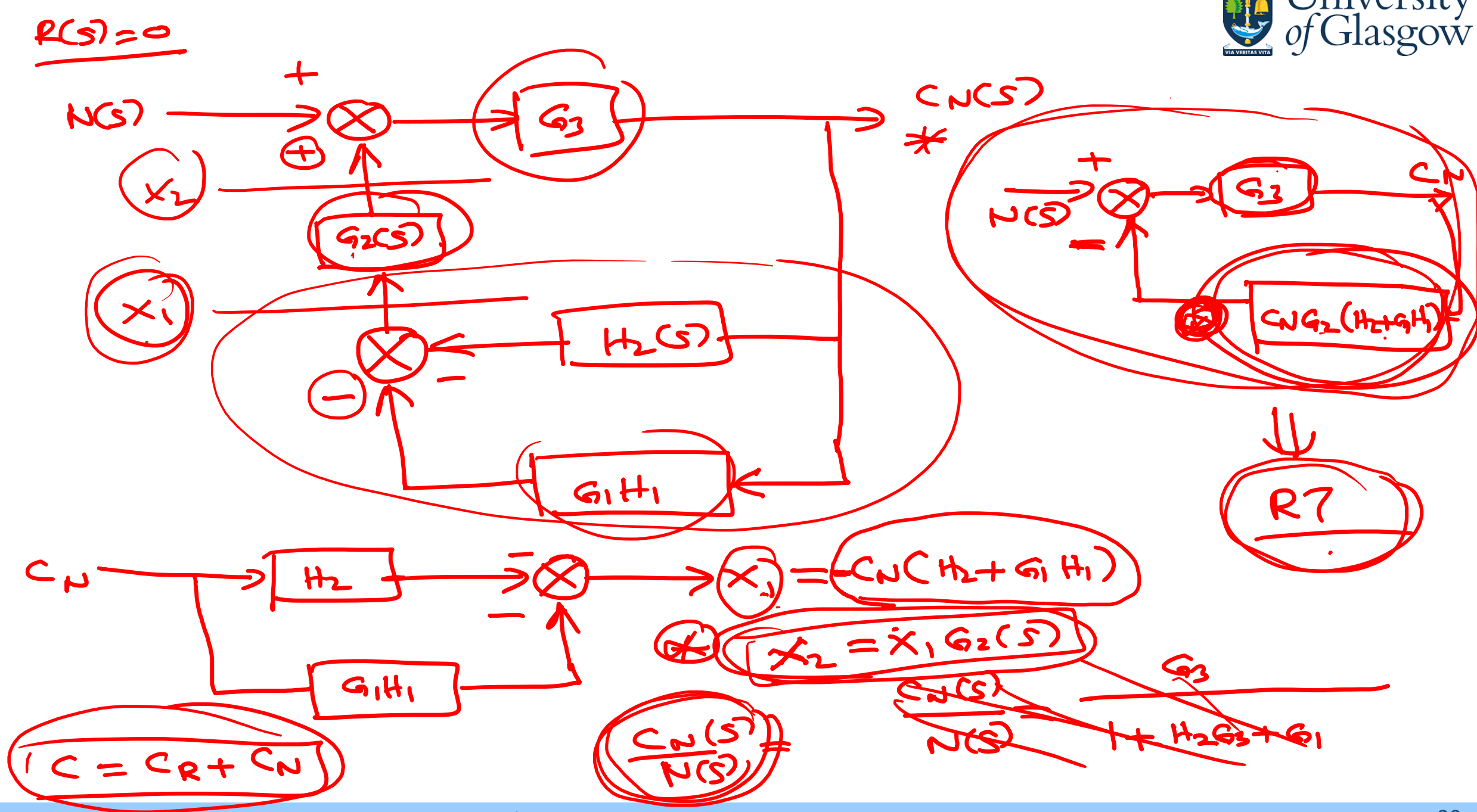
H1(s) (feedback around G1G2G3)

R(s) → G1G2G3 / (1 + H2G2G3) → C(s)

H1(s) (feedback)

G2G3G1 / (1 + H2G2G3)

$$\frac{C_{R(s)}}{R(s)} = \frac{G_1G_2G_3/(1+H_2G_2G_3)}{1 + H_1G_1G_2G_3/(1+H_2G_2G_3)} = \frac{G_1G_2G_3}{1 + H_1G_1G_2G_3 + H_2G_2G_3}$$


R(s)=0
N(s)
+
+
G3
CN(s)
X2
G2(s)
X1
H2(s)
−
−
G1H1
+
N(s)
−
G3
CN
CNG2(H2+G1H1)
R7
CN
H2
−
G1H1
X1 = CN(H2+G1H1)
X2 = X1G2(s)
G3
CN(s)
N(s)
1 + H2G3+G1
C = CR + CN
CN(s)/N(s)

Summary

- Apply Block Diagram Rules to Derive Overall Transfer Function
- Overall Transfer Function of Control Systems Subjected to Disturbances

Reference:

-Control Systems Engineering, 7th Edition, N.S. Nise
-UESTC3001 2019/20 Notes, J. Le Kernec