University of Glasgow

Lecture 2

Electronic Devices

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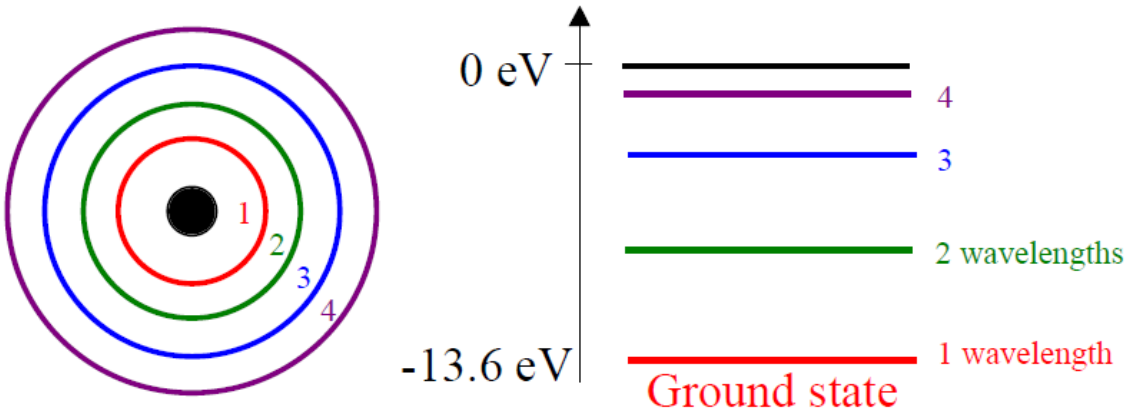
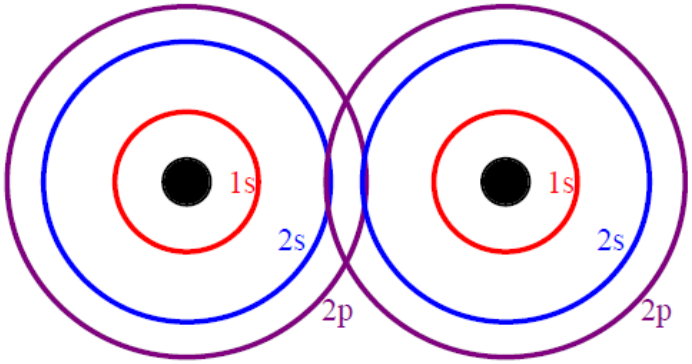
Energy Band Structures

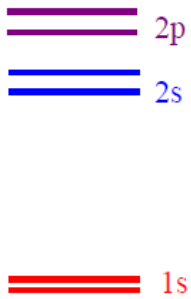


The electrons in an atom occupy discrete energy levels. For helium the the discrete levels are given by:

Atomic number

$$h\upsilon = \frac{Z^2 me^4}{8h^2\varepsilon_0^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = -13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]eV$$

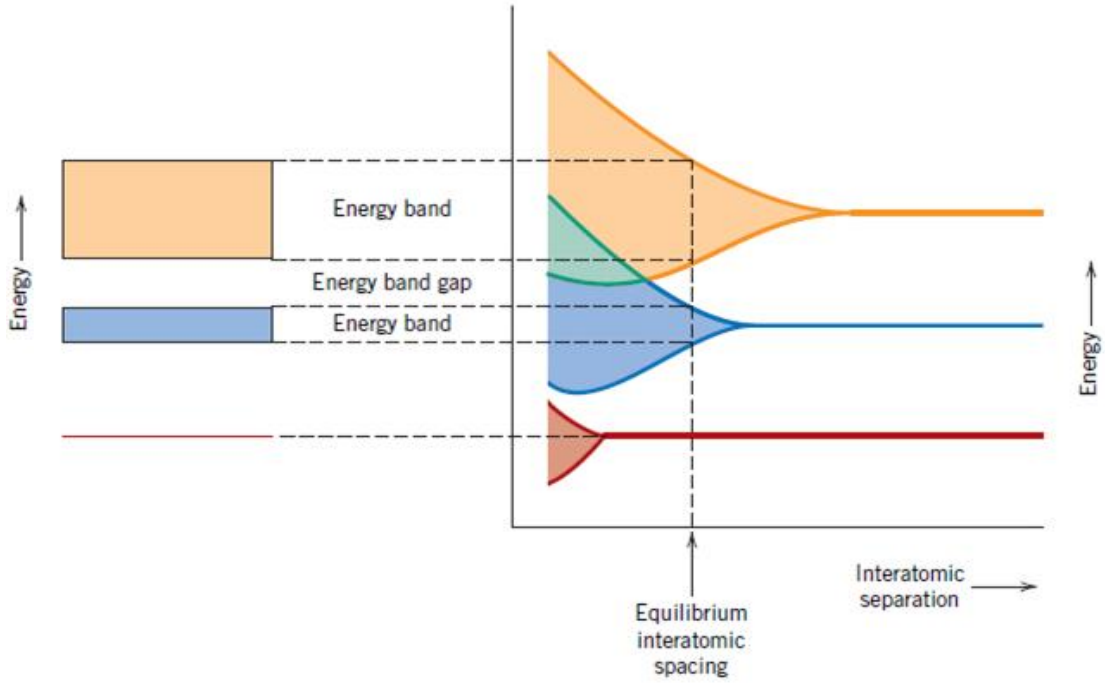




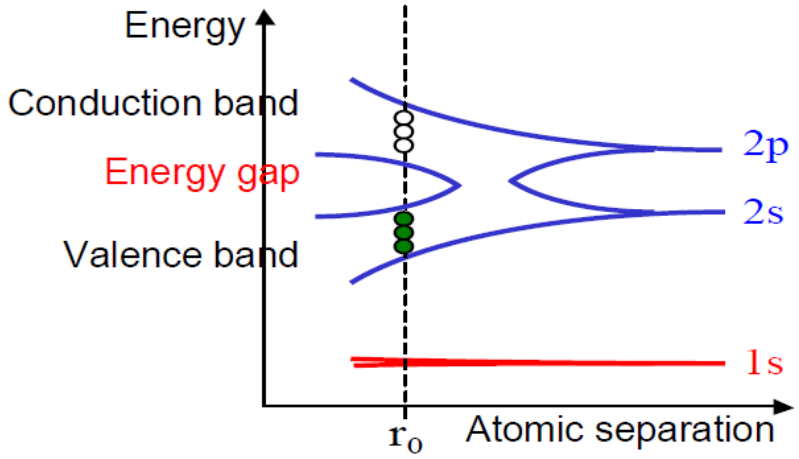
The interaction between two atoms when they are close together results in splitting of the energy levels in two.

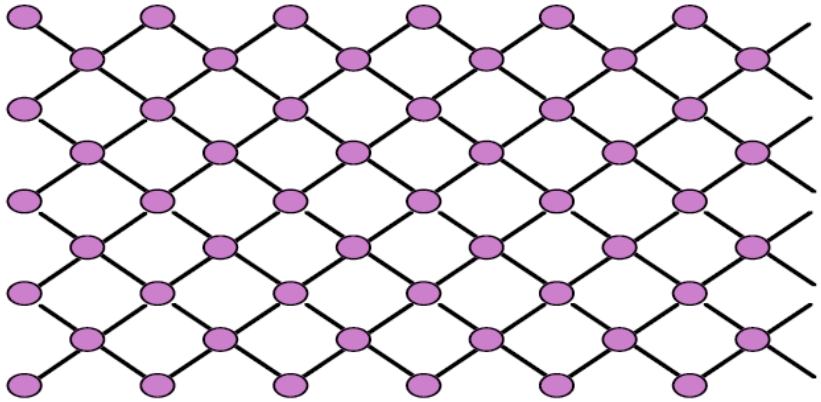
Energy Band Structures

When **atoms** combine to form **substances**, the outermost shells, subshells and orbitals merge, providing a greater number of available energy levels for electrons to assume. When large numbers of atoms are close to each other, these available energy levels form a nearly continuous band wherein electrons may move.



Energy Band Structures





Diamond crystal

When the atoms are packed together in a crystal each energy levels splits in a band with large number of very closely packed states. The bands are separated by regions of forbidden energies for the electrons called *forbidden gaps* or *band gaps* E_g.

The last band completely full of electrons is called *valence band* E_V. The next band which may be either empty of partially filled with electrons is called *conduction band* E_C.

The electrons in the valence band can not carry current because they can not accelerate (change energy when electric field is applied).

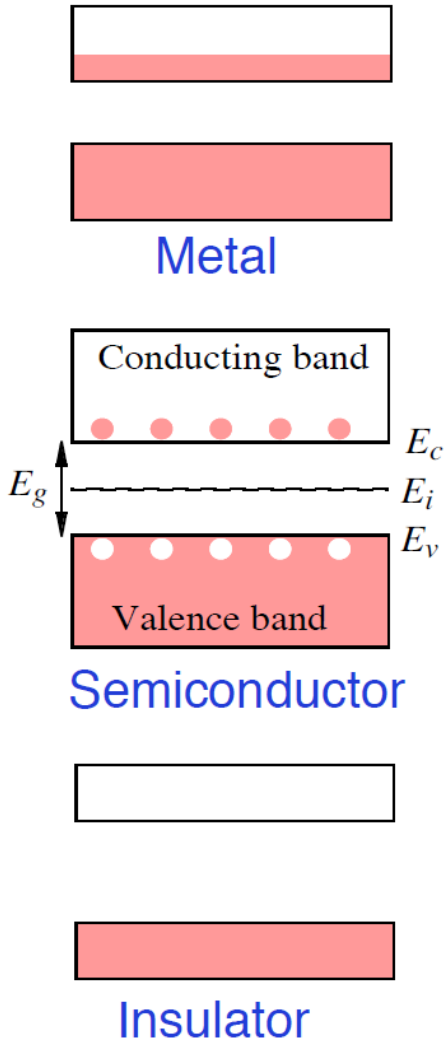
Energy Band Structures

Valence & Conduction Bands

- The valence band is completely filled with electrons and is separated from an empty conduction band.
- An energy band gap lies between them.
- The difference between the two band structures lies in the magnitude of the energy gap
- For insulators the band gap is wide.
- For semiconductors it is narrow.

Energy Band Structures

The occupancy of E_C and the magnitude of E_g determine the difference between *metals*, *semiconductors* and *insulators*.

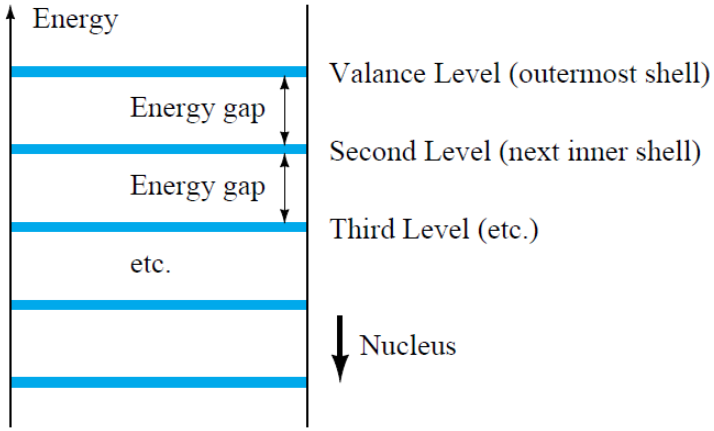


In *metals* the conduction band is partially filled with electrons. When electric field is applied they can accelerate (can change their energy) and therefore can carry current

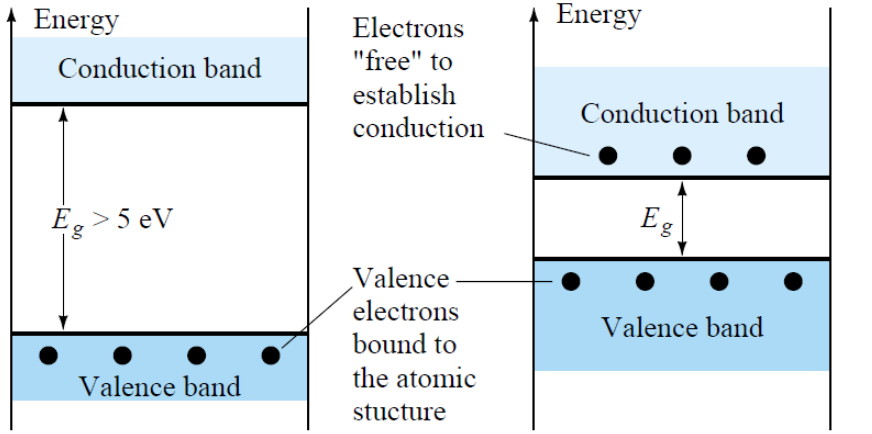
In *semiconductors* there are no electrons in the conduction band at $T = 0^0$ K. At room temperature however some electrons can acquire enough energy from vibrations of the lattice to jump into the conduction band and therefore carry current. However the number is much smaller compared to metals which results in much lower conductivity.

In *insulators* like in the semiconductors there are no electrons in the conduction band at T=0. The band gap however is much larger and electrons can not jump to the conduction band to carry current.

The more distant the electron from the nucleus, the higher the energy state, and any electron that has left its parent atom has a higher energy state than any electron in the atomic structure.



(a)

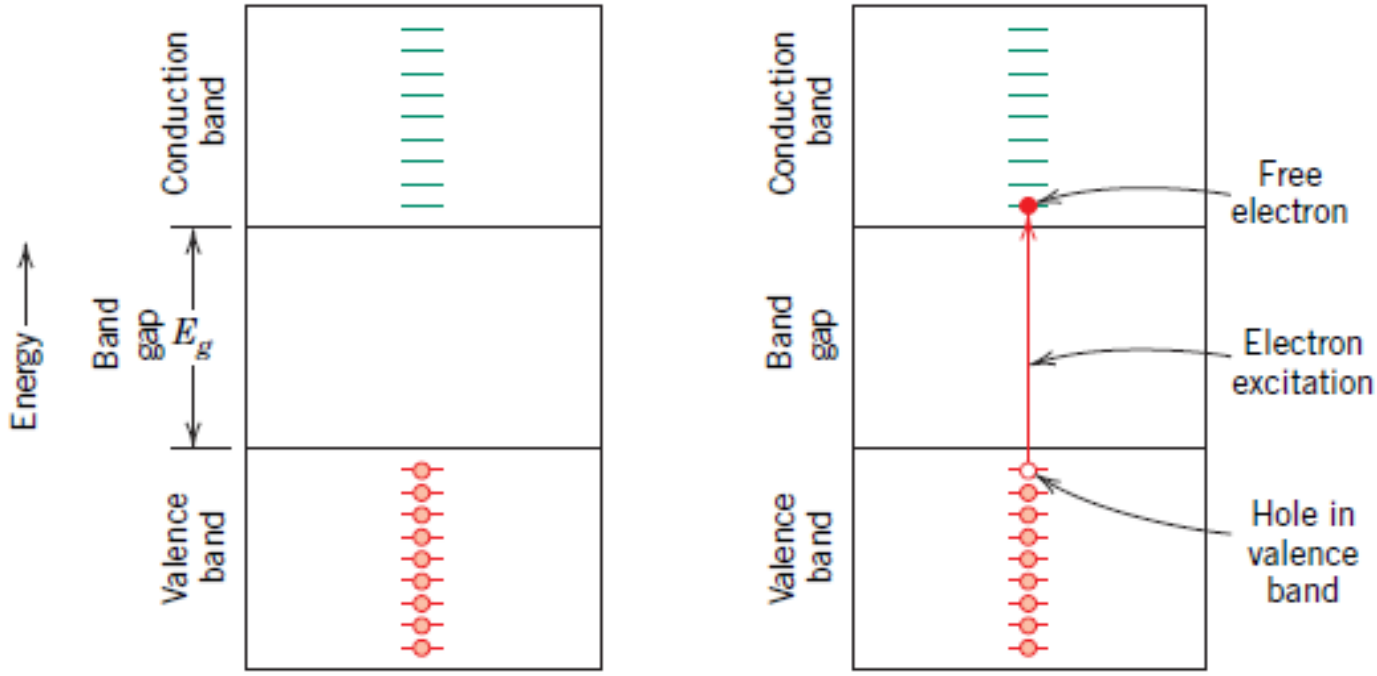


$E_g = 1.1$ eV (Si)
$E_g = 0.67$ eV (Ge)
$E_g = 1.41$ eV (GaAs)

Insulator | Semiconductor | Conductor

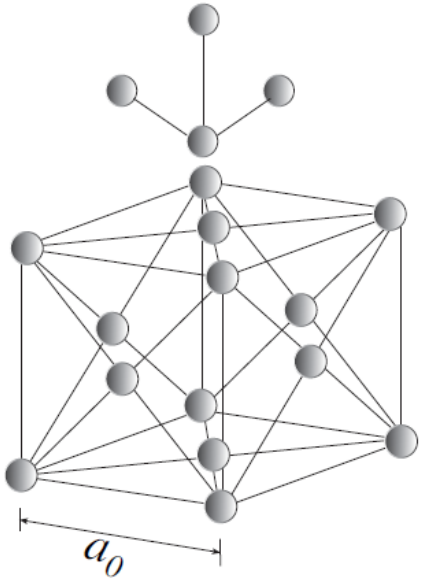
Conduction & Energy States

- Energy States & Excitation in Semiconductor:



Important Semiconductors

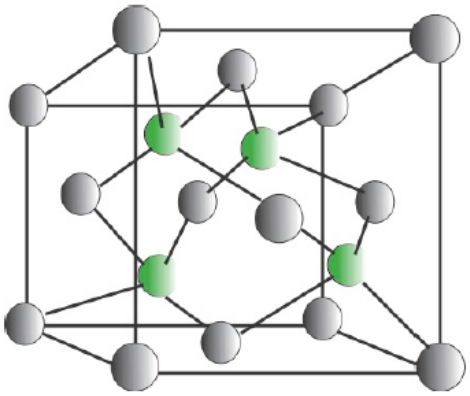
Group IV semiconductors: Si, Ge



Group IV atoms have 4 valence electrons and bond to four nearest neighbours forming a tetrahedron. The tetrahedrons form two interpenetrating face centered cubic sub-lattices displaced by 1/4 of the lattice constant a_0 which for Si is 0.543 nm.

Practically all Integrated Circuits are made of Si. The band gap of Si is $E_g \sim 1.1$ eV.

Compound semiconductors: GaAs



Formed with equal numbers of atoms from Group III and group V and are also called III-V semiconductors. Each group III atom is bounded to four group V atoms and vice versa.

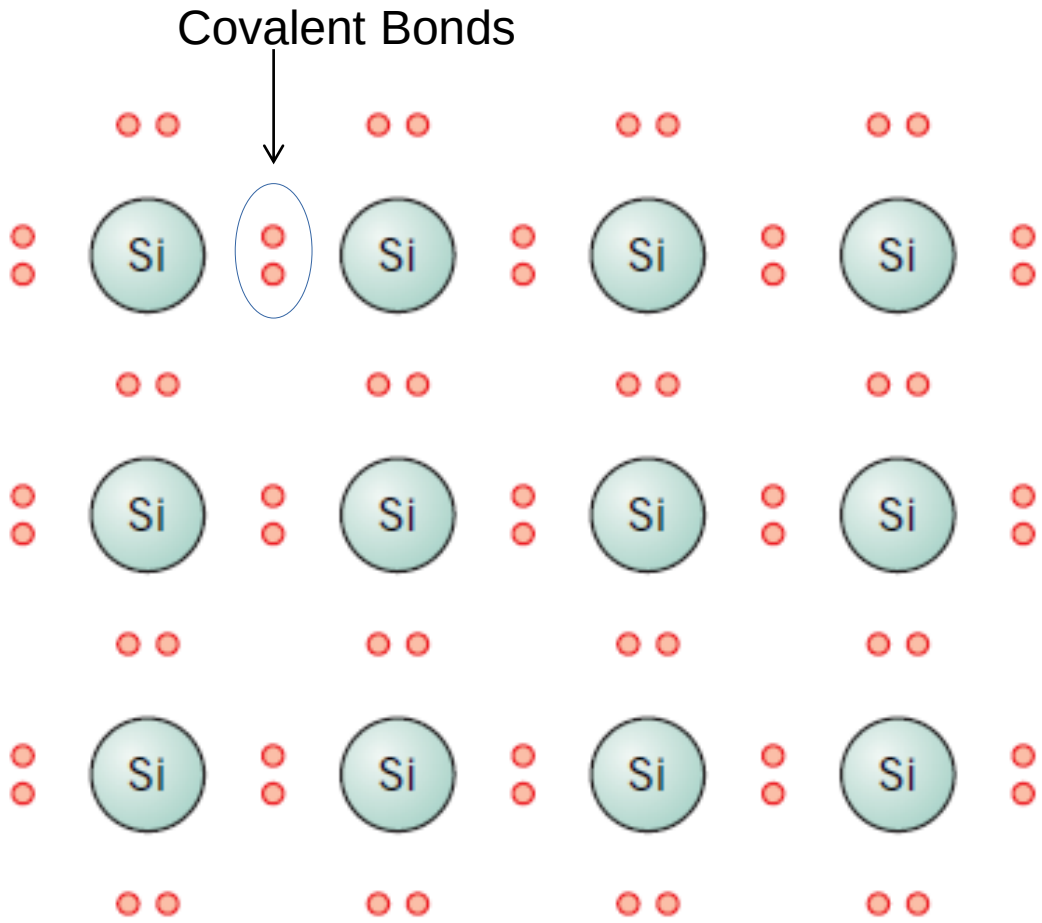
The compound semiconductors are the material of choice for optoelectronic applications because of their direct gap. The band gap of GaAs is $E_g \sim 1.4$ eV.

Semiconductivity

- In this lecture we develop a basic understanding of the properties of **intrinsic** and **extrinsic** semiconductors. Although most of our discussions and examples will be based on Si, the ideas are applicable to Ge and to the compound semiconductors such as GaAs, InP, and others.

- By **intrinsic** Si we mean **an ideal perfect crystal of Si that has no impurities or crystal defects such as dislocations and grain boundaries**. The crystal thus consists of Si atoms perfectly bonded (**covalent**) to each other in the **diamond** structure.

Semiconductivity

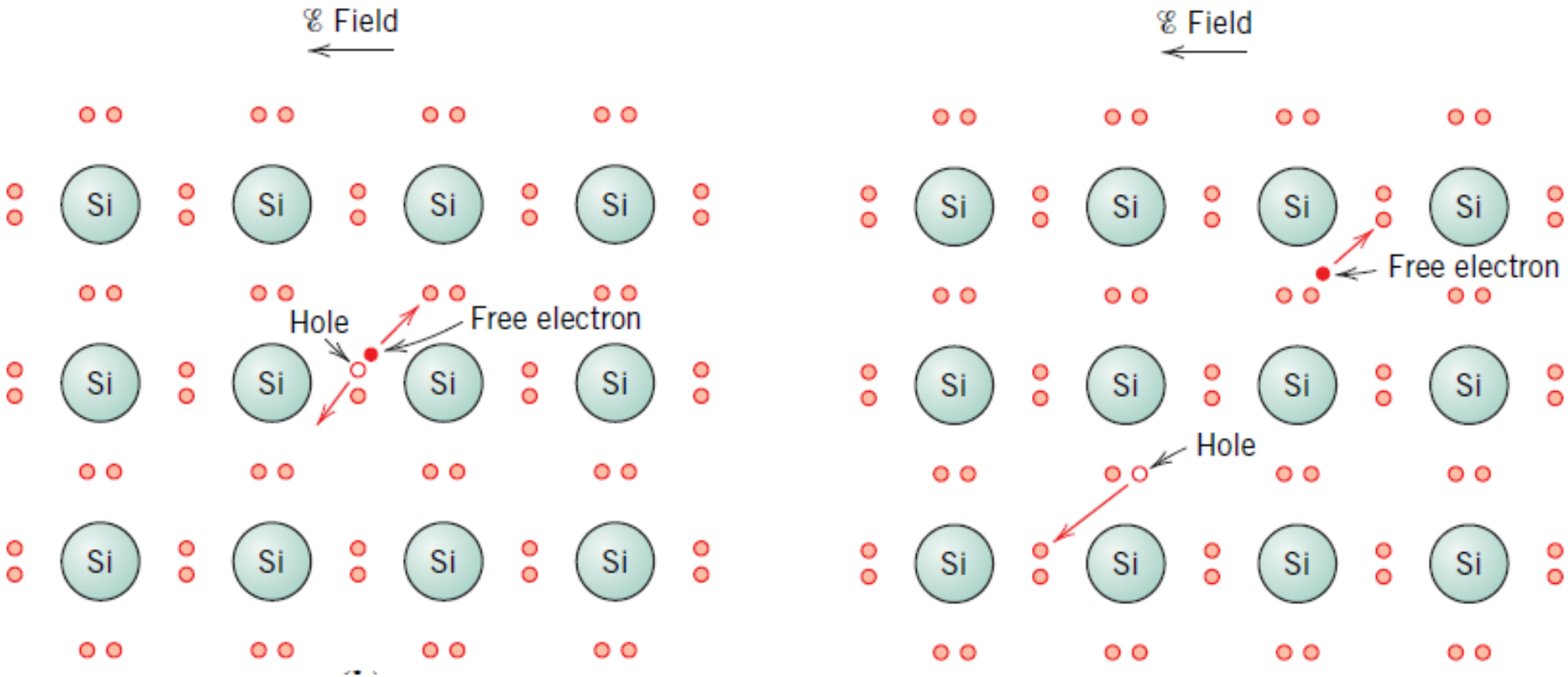


Semiconductivity

- When a Si-Si bond is broken, a "free" electron is created that can wander around the crystal and also contribute to electrical conduction in the presence of an applied field.
- The broken bond has a missing electron that causes this region to be positively charged.
- The vacancy left behind by the missing electron in the bonding orbital is called a **hole**.

Semiconductivity

Motion of electrons and holes



The concentration of intrinsic electrons n_0 and intrinsic holes p_0 are equal.

$$n_0 = p_0 = n_i$$

Semiconductivity

- In an intrinsic semiconductor, the number of electrons is equal to the number of holes (broken bonds). In an **extrinsic** semiconductor, impurities are added to the semiconductor that can contribute either excess electrons or excess holes.

- In intrinsic semiconductors, for every electron excited into the conduction band there is left behind a missing electron in one of the covalent bonds

- Under the influence of an electric field, the position of this missing electron within the crystalline lattice – **A hole**

- **A hole** is considered to have a charge +1.6exp-19 (opposite sign of electron).

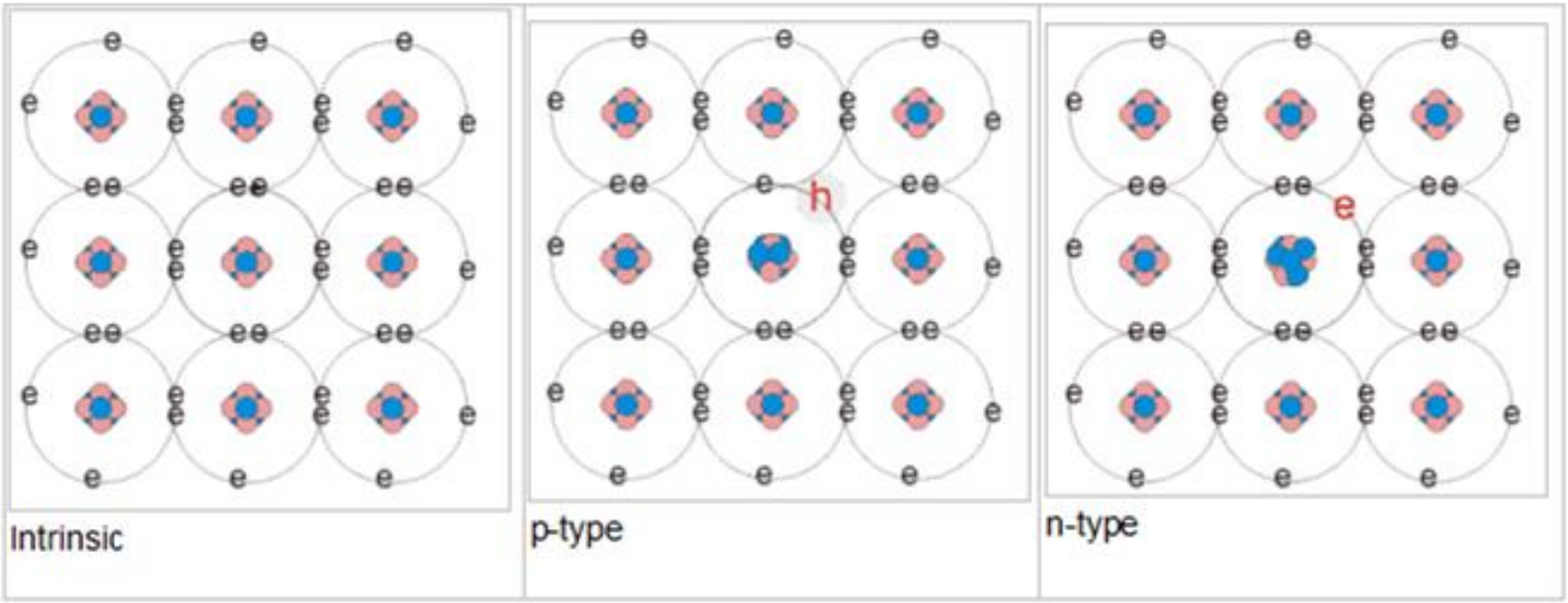
- **Electrons** and **holes** move in **opposite** directions.

Doped Semiconductors

An **extrinsic semiconductor** can be formed from an intrinsic semiconductor by adding impurity atoms to the crystal in a process known as **doping** .

For Si appropriate dopants are elements from group III and group V of the periodical table.

Extrinsic Semiconductors



FERMI LEVEL

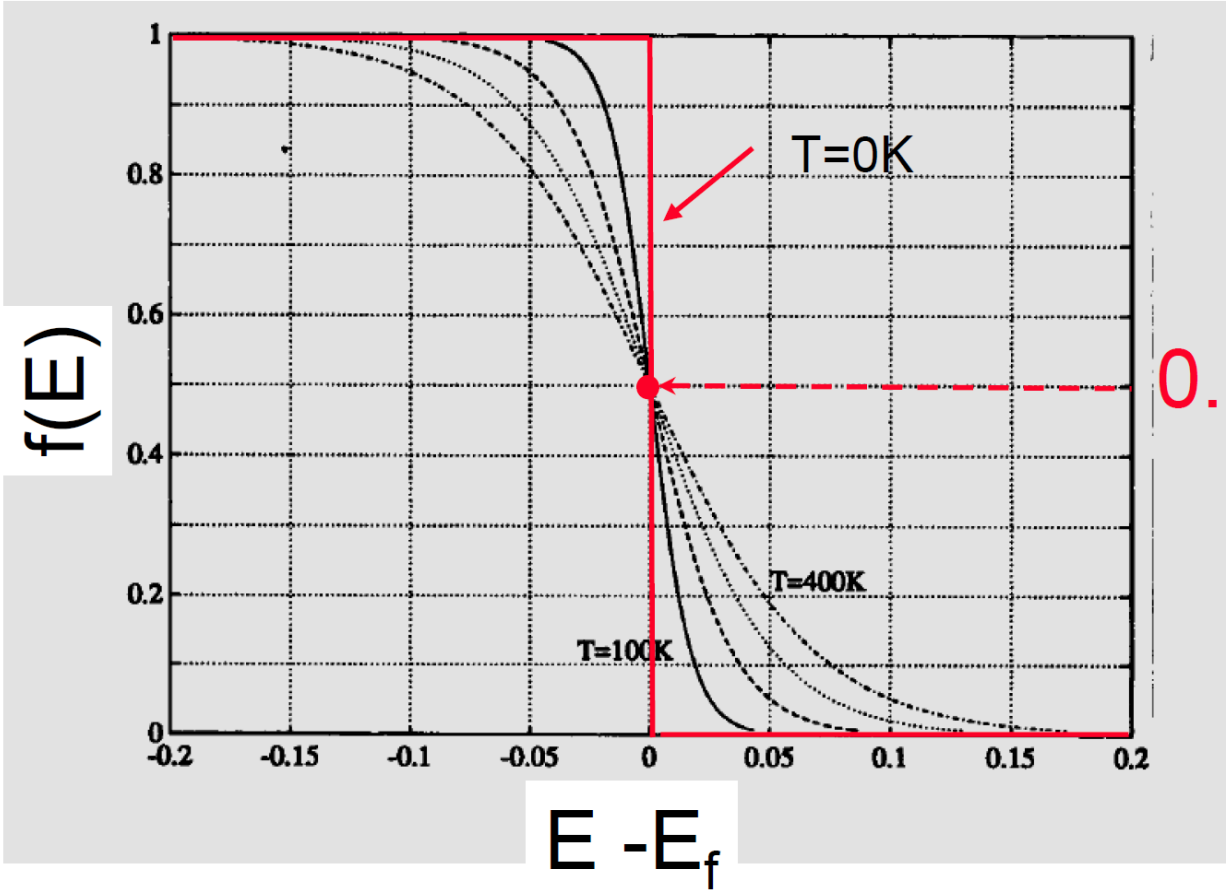
- "Fermi level" is the term used to describe the top of the collection of electron energy levels at absolute zero temperature.
- This concept comes from Fermi-Dirac statistics. Electrons are fermions .
- So at absolute zero they pack into the lowest available energy states and build up a "Fermi sea" of electron energy states.
- The Fermi level is the surface of that sea at absolute zero where no electrons will have enough energy to rise above the surface.

The Fermi-Dirac Distribution (Fermi Function)

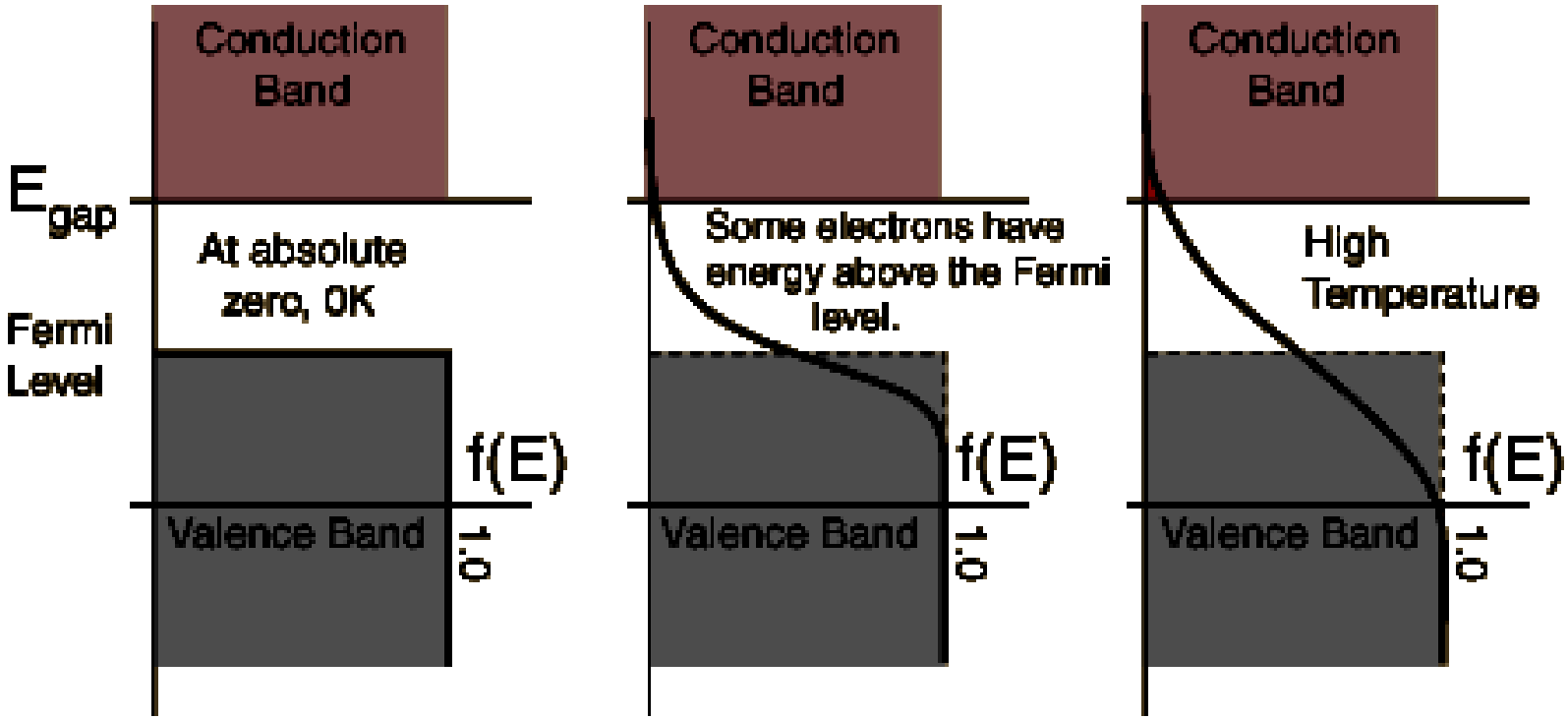
Probability of available states at energy E being occupied

$f(E) = 1/ [1+ \exp (E- E_f) / kT]$

where E_f is the Fermi energy and k = Boltzmann constant=$8.617 * 10^{-5}$ eV/K



Energy Band Semiconductors



No electrons can be above the valence band at 0K, since none have energy above the Fermi level and there are no available energy states in the band gap.

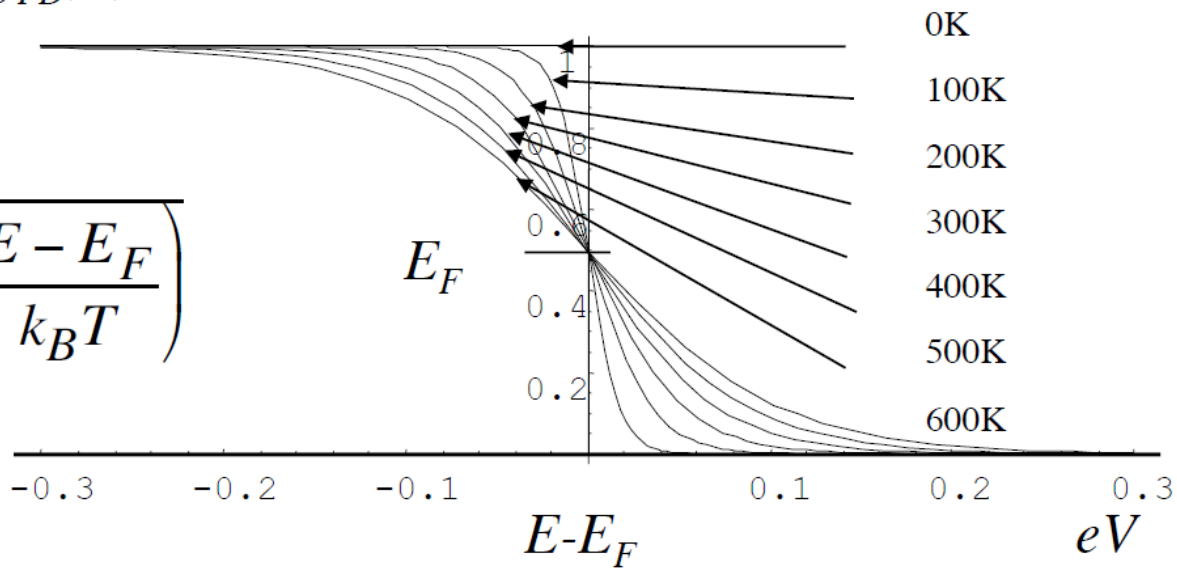
At high temperatures, some electrons can reach the conduction band and contribute to electric current.

Carrier Statistics

Fermi-Dirac distribution function

Electrons which have a spin 1/2 are fermions. The probability that electron (fermion) occupies an energy level E is given by the *Fermi-Dirac distribution function* $f_{FD}(E)$:

$$f_{FD}(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_B T}\right)}$$



Fermi-Dirac distribution function at different T

The Fermi energy E_F is the energy with a probability of occupation 1/2. At T=0, *the Fermi-Dirac distribution is a step function* and at finite temperatures the distribution is smeared out.

Energy Band Semiconductors

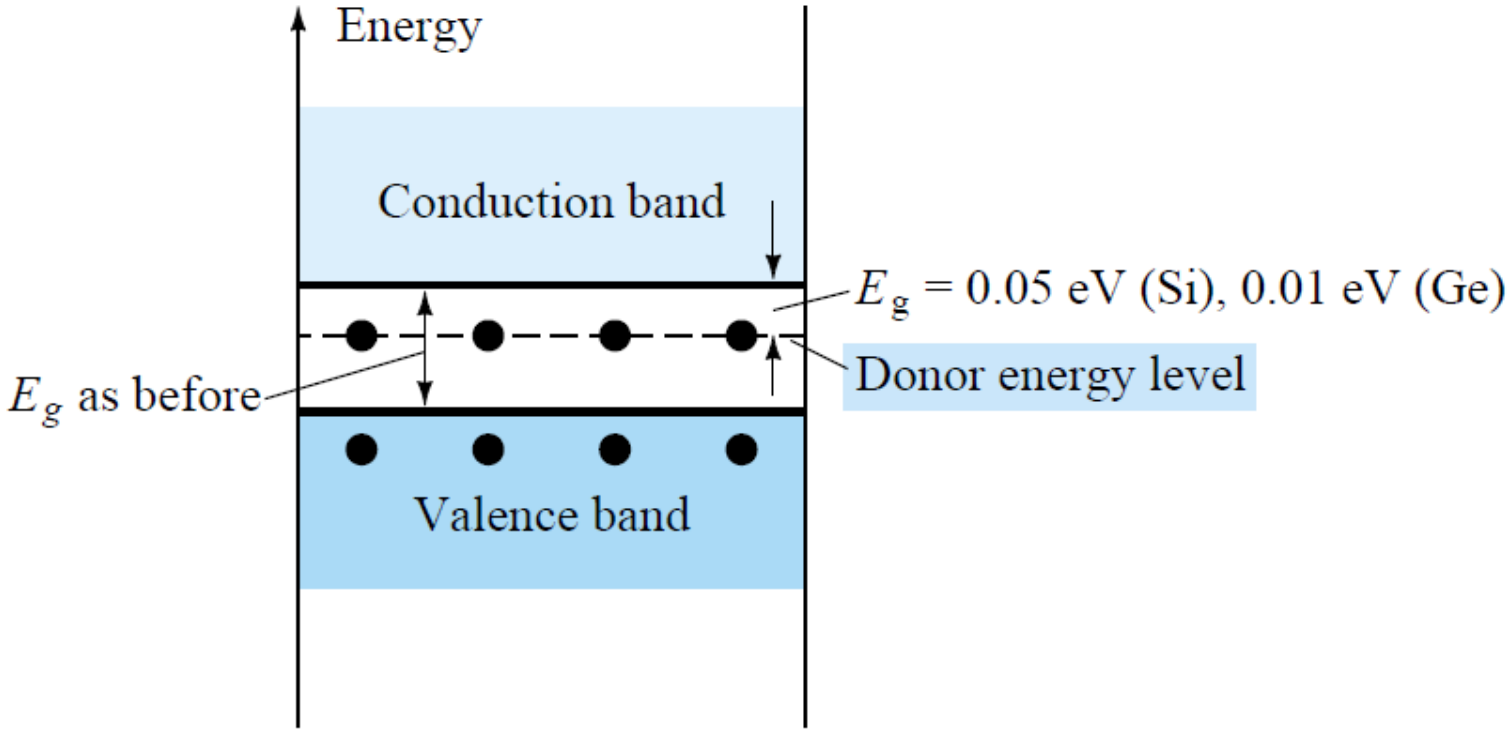
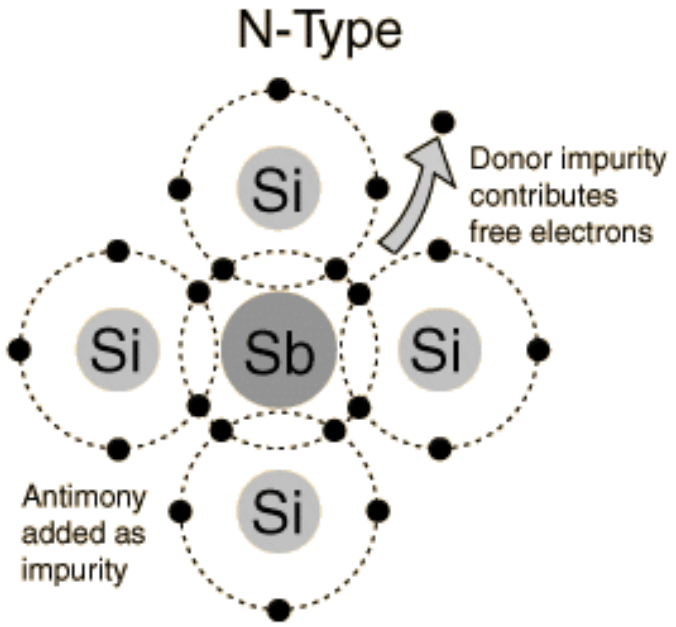
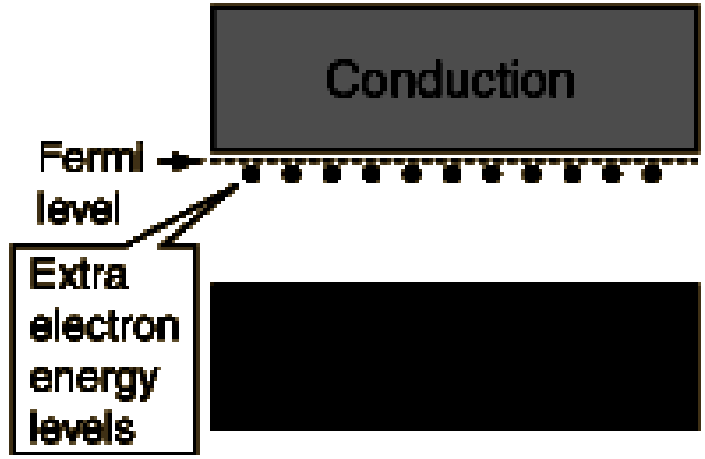


Figure 1.10 Effect of donor impurities on the energy band structure.

N-type Semiconductor



Antimony, arsenic or phosphorous contributes free electrons, greatly increasing the conductivity of the intrinsic semiconductorArsenic has five valence electrons, whereas Si has four. Thus when an As atom bonds with four Si atoms, it has one electron left unbonded.



The electrons are majority carriers.
Fermi level is shifted upward in the band gap

N-type Semiconductor

Elements that belong to group V of the periodic table such as As, P, Sb have an extra electron in the valence band. When added as a dopant to intrinsic Silicon, the dopant atom contributes an additional electron to the crystal. Dopants that add electrons to the crystal are known as **donors** and the semiconductor material is said to be n-type.

Usually the concentration of donors *ND* is much larger than the intrinsic carrier concentration. Therefore in n-type semiconductors:

$$N_D >> n_i$$

$$n \approx N_D$$

N-type Semiconductor

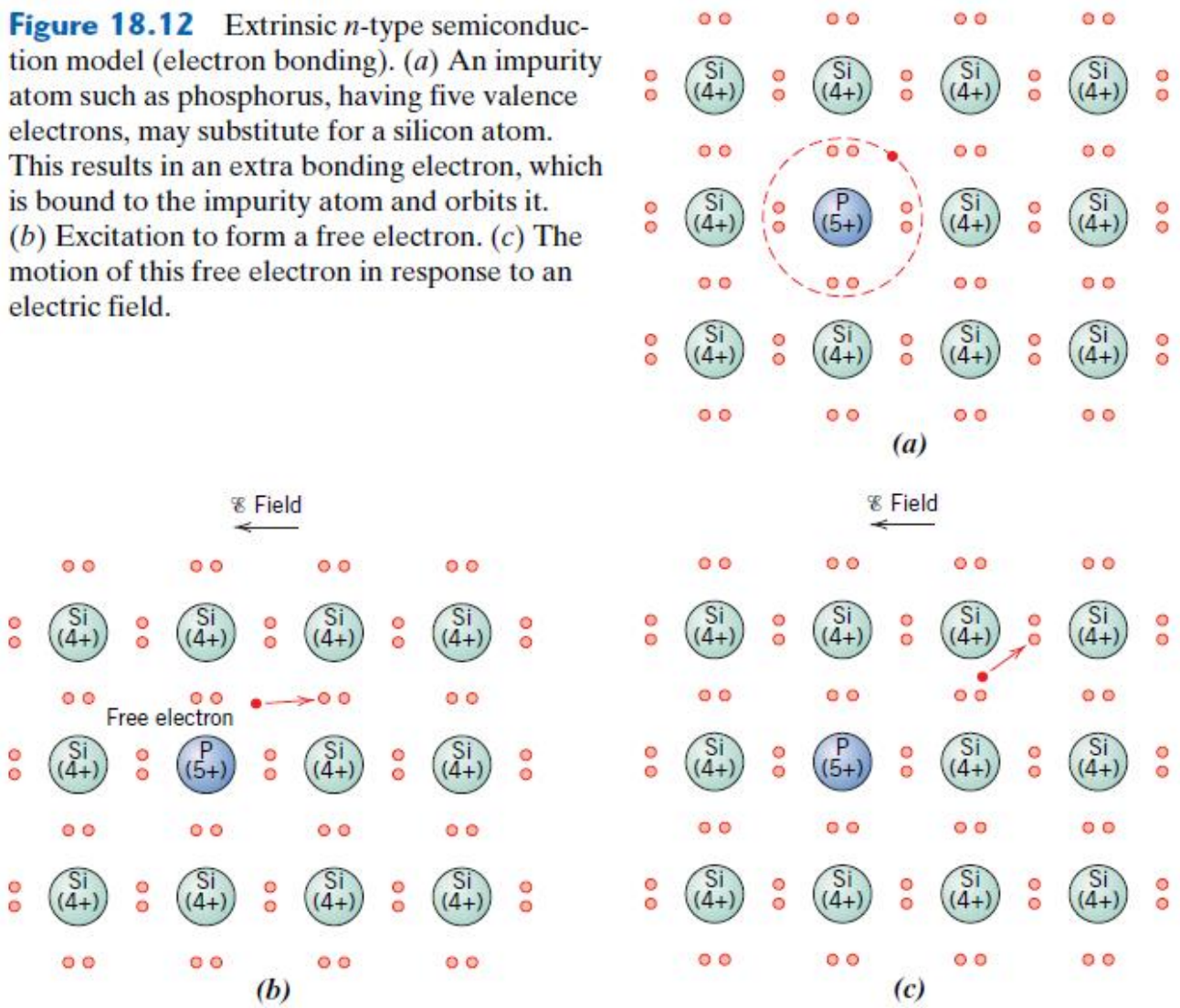
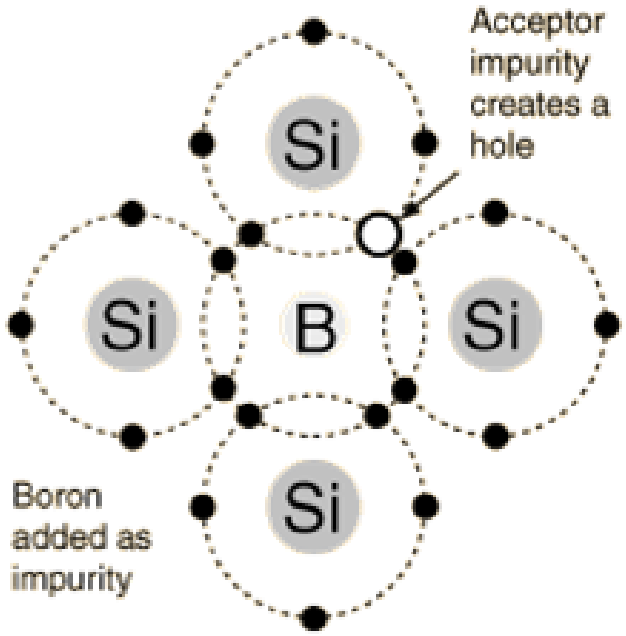
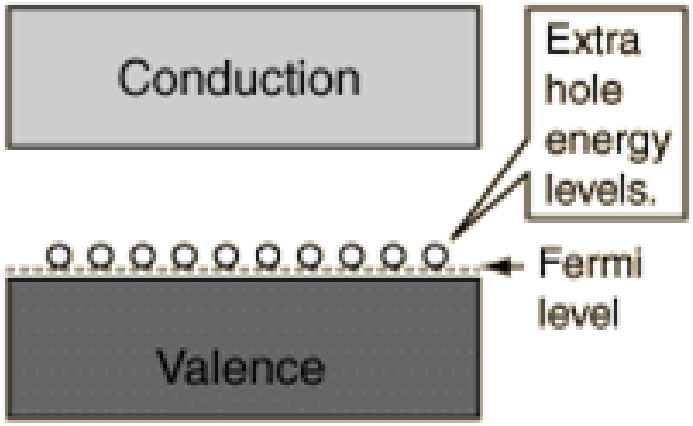


Figure 18.12 Extrinsic *n*-type semiconduction model (electron bonding). (*a*) An impurity atom such as phosphorus, having five valence electrons, may substitute for a silicon atom. This results in an extra bonding electron, which is bound to the impurity atom and orbits it. (*b*) Excitation to form a free electron. (*c*) The motion of this free electron in response to an electric field.

P-type Semiconductor





Dopants that create holes are known as **acceptors**. This type of extrinsic semiconductor is known as **p-type** as it creates positive charge carriers.

Usually the concentration of acceptors N_A is much larger than the intrinsic carrier concentration

Therefore in p-type semiconductors

$$N_A >> n_i$$

$$p \approx N_A$$

Summary

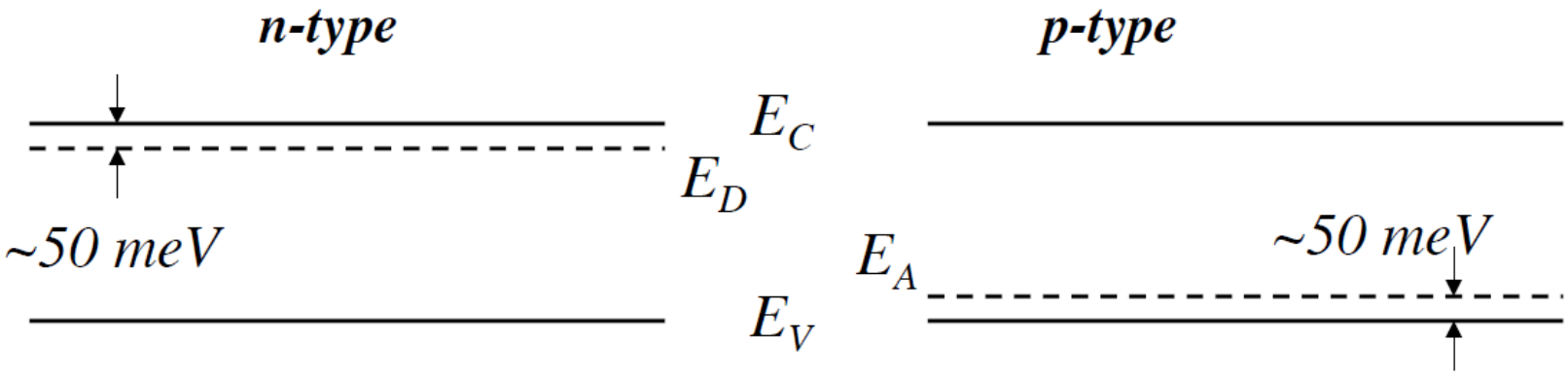
	Elem	Symb	Q	Conc	Type	Majority Carriers	Minority Carriers
Acceptor	B, Al	N_A	(-)	$p > n$	p-type	holes	electrons
Donor	P, As	N_D	(+)	$n > p$	n-type	electrons	holes

Carrier Statistics

In thermal equilibrium always $np = n_i^2$

Knowing the doping concentration we can calculate the *majority* and the *minority* carrier concentrations

	n-type	***p-type***
Doping Concentration	$N_D >> n_i$	$N_A >> n_i$
Majority carrier Concentration	$n = N_D$	$p = N_A$
Minority carrier Concentration	$p = \frac{n_i^2}{N_D}$	$n = \frac{n_i^2}{N_A}$



Carrier Statistics

For $E - E_F >> k_B T$ the Fermi-Dirac distribution reduces to Maxwell-Boltzmann distribution

$$f_{MB}(E) = \exp\left(-\frac{E - E_F}{k_B T}\right)$$

Carrier concentration

When the Fermi level is in the band gap, far away from the conduction and valence bands and from the acceptor and donor levels the concentration of electrons in the conduction and holes in the valence band are given by the expressions.

$$n = N_C \exp\left((E_F - E_C)/k_B T\right)$$

$$p = N_V \exp\left((E_V - E_F)/k_B T\right)$$

At finite temperatures the Fermi-Dirac distribution is smeared out. Where N_C and N_V are called effective density of states for electrons and holes respectively given by the expressions:

$$N_c = 2\left\{\frac{2\pi m_e^* k_b T}{h^2}\right\}^{3/2} \qquad N_V = 2\left\{\frac{2\pi m_h^* k_b T}{h^2}\right\}^{3/2}$$

Carrier Statistics

Bearing in mind that $np = n_i^2$ we can calculate the intrinsic carrier concentration:

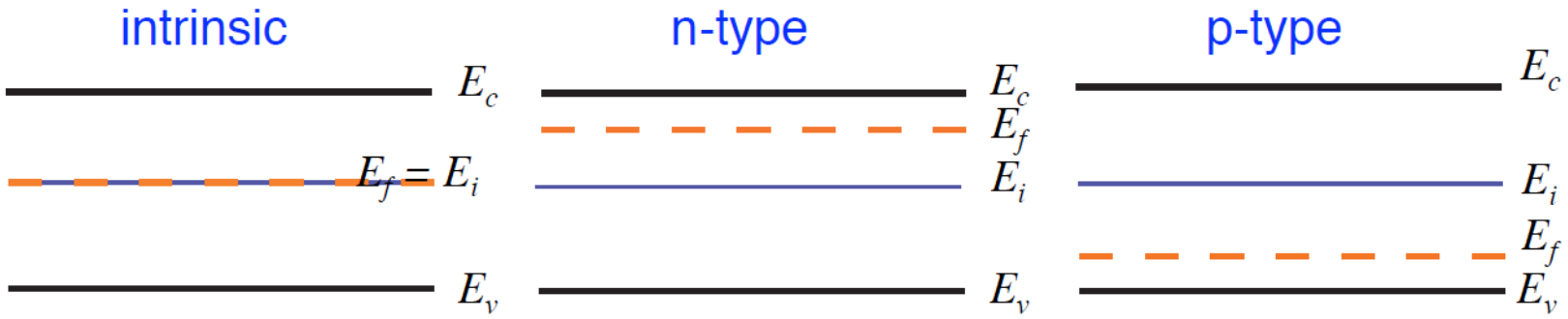
$$n_i = (N_c N_V)^{1/2} \exp\left(\frac{-E_g}{2k_b T}\right)$$

Also bearing in mind that in an intrinsic semiconductor $n = p$ we can calculate the position of the *intrinsic Fermi level*:

$$E_i = E_F = \frac{E_C + E_V}{2} - \frac{k_B T}{2} \ln\left(\frac{N_C}{N_V}\right)$$

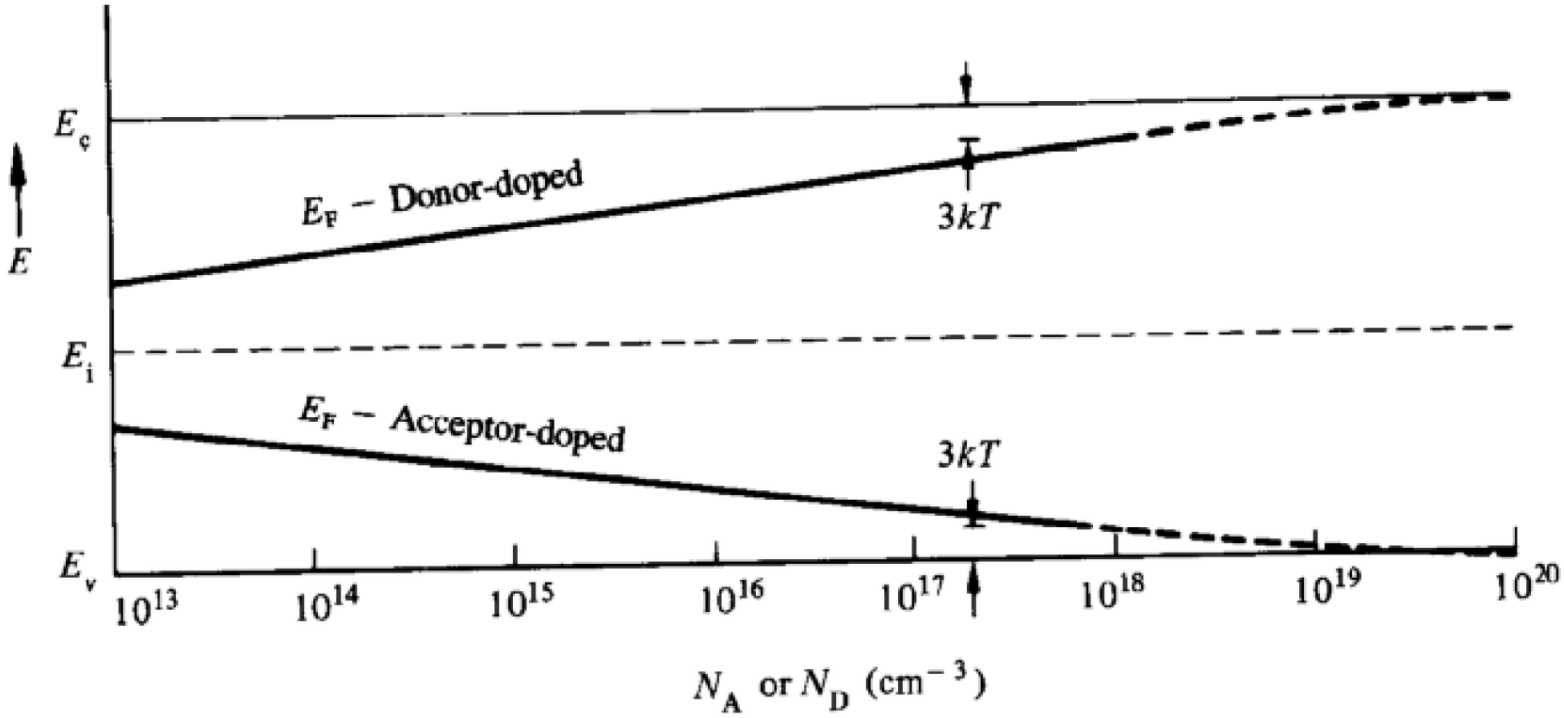
Also we can express n and p in terms of n_i.

$$\begin{aligned} n &= N_C \exp\left((E_F - E_i + E_i - E_C)/k_B T\right) \\ &= n_i \exp\left((E_F - E_i)/k_B T\right) \\ p &= N_V \exp\left((E_V - E_i + E_i - E_F)/k_B T\right) \\ &= n_i \exp\left((E_i - E_F)/k_B T\right) \end{aligned}$$



Dependence of Fermi Level with Doping Concentration

$E_i \equiv (E_C+E_V)/2$ Middle of energy gap



When Si is undoped, $E_f = E_i$; also $n = p = n_i$

Example

INTRINSIC CONCENTRATION AND CONDUCTIVITY OF Si Given that the density of states related effective masses of electrons and holes in Si are approximately $1.08m_e$ and $0.60m_e$, respectively, and the electron and hole drift mobilities at room temperature are 1350 and 450 $cm^2\ V^{-1}\ s^{-1}$, respectively, calculate the intrinsic concentration and intrinsic resistivity of Si.

Electron and hole conductivity effective masses

	Si	Ge	GaAs
m_n^*/m_o	**0.26**	0.12	0.068
m_p^*/m_o	**0.39**	0.30	0.50

$m_o = 9.1\times10^{-31}$ kg

Example

SOLUTION

We simply calculate the effective density of states N_c and N_v by

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \qquad \text{and} \qquad N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Thus

$$N_c = 2\left[\frac{2\pi(1.08 \times 9.1 \times 10^{-31}\ \text{kg})(1.38 \times 10^{-23}\ \text{J K}^{-1})\ (300\ \text{K})}{(6.63 \times 10^{-34}\ \text{J s})^2}\right]^{3/2}$$

$$= 2.81 \times 10^{25}\ \text{m}^{-3} \qquad \text{or} \qquad 2.81 \times 10^{19}\ \text{cm}^{-3}$$

and

$$N_v = 2\left[\frac{2\pi(0.60 \times 9.1 \times 10^{-31}\ \text{kg})(1.38 \times 10^{-23}\ \text{J K}^{-1})(300\ \text{K})}{(6.63 \times 10^{-34}\ \text{J s})^2}\right]^{3/2}$$

$$= 1.16 \times 10^{25}\ \text{m}^{-3} \qquad \text{or} \qquad 1.16 \times 10^{19}\ \text{cm}^{-3}$$

The intrinsic concentration is

$$n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2kT}\right)$$

Example

so that

$$n_i = [(2.81 \times 10^{19}\ \text{cm}^{-3})(1.16 \times 10^{19}\ \text{cm}^{-3})]^{1/2} \exp\left[-\frac{(1.10\ \text{eV})}{2(300\ \text{K})(8.62 \times 10^{-5}\ \text{eV K}^{-1})}\right]$$
$$= 1.0 \times 10^{10}\text{cm}^{-3}$$

The conductivity is

$$\sigma = en\mu_e + ep\mu_h = en_i(\mu_e + \mu_h)$$

that is,

$$\sigma = (1.6 \times 10^{-19}\ \text{C})(1.0 \times 10^{10}\ \text{cm}^{-3})(1350 + 450\ \text{cm}^2\ \text{V}^{-1}\ \text{s}^{-1})$$
$$= 2.9 \times 10^{-6}\ \Omega^{-1}\ \text{cm}^{-1}$$

The resistivity is

$$\rho = \frac{1}{\sigma} = 3.5 \times 10^5\ \Omega\ \text{cm}$$

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Supplementary

Electron & Hole Concentration

The general equation for the conductivity of a semiconductor, depends on **n**, the electron concentration, and **p**, the hole concentration.

How do we determine these quantities?

Procedure involves multiplying **density of states $g_{cb}(E)$** by the **probability of a state being occupied f(E)** and **integrating over the entire CB for n** and over the entire **VB for p**.

Electron Concentration

Integrating this from the **bottom (Ec)** to the **top (Ec + x)** of the **CB** gives the **electron concentration (n)**, which is the number of electrons per unit volume in the CB.

$$n = \int_{E_c}^{E_c+\chi} n_E(E)\,dE = \int_{E_c}^{E_c+\chi} g_{cb}(E) f(E)\,dE$$

$$n_E\,dE = g_{cb}(E) f(E)\,dE$$

Electron Concentration

Note that the Fermi function gives the probability of occupying an available energy state,

$$f(E) = \frac{1}{e^{(E - E_F)/kT} + 1}$$

Now that we have introduced he Fermi function, we should define the **Density of States,** which is the ***number of available energy states*** to determine how many electrons would reach the conduction band. The density of states is given by:

$$g_{cb}(E) = \frac{(\pi 8\sqrt{2})m_e^{*3/2}}{h^3}(E - E_c)^{1/2}$$

Electron Concentration

Effective mass of electron, $m_e^* = 9.1 \text{exp-}31$ kg

$$n \approx \frac{(\pi 8\sqrt{2}) m_e^{*3/2}}{h^3} \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left[-\frac{(E - E_F)}{kT}\right] dE$$

which leads to

$$n = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

where

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

Hole Concentration

We can carry out a similar analysis for the concentration of holes in the VB. Multiplying the density of states $g_{vb}(E)$ in the VB with the probability of occupancy by a hole $[1 – f(E)]$.

Remember that the probability that an electron is absent gives p, the hole concentration per unit energy. Integrating this over the VB gives the hole concentration.

Hole Concentration

The hole concentration can therefore be expressed as:

$$p = \int_0^{E_v} p_E \, dE = \int_0^{E_v} g_{vb}(E)[(1 - f(E)] \, dE$$

With the assumption that E_F is a few kT above E_v, the integration simplifies to

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$

where N_v is the effective density of states at the VB edge and is given by

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

Intrinsic Carrier Concentration

Using the expressions for hole and electron concentrations, we can therefore express the **intrinsic** carrier concentration as:

$$np = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right] N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right] = N_c N_v \exp\left[-\frac{(E_c - E_v)}{kT}\right]$$

or

$$np = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

Intrinsic Carrier Concentration

An **intrinsic semiconductor** is a pure semiconductor crystal in which the electron and hole concentrations are equal. By pure we mean virtually no impurities in the crystal. In an intrinsic semiconductor, the Fermi-level is in the middle of the bank gap, as previously shown.

$$np = n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$