UESTC3001 Dynamics & Control
Lecture 1

Overview of Dynamic Systems

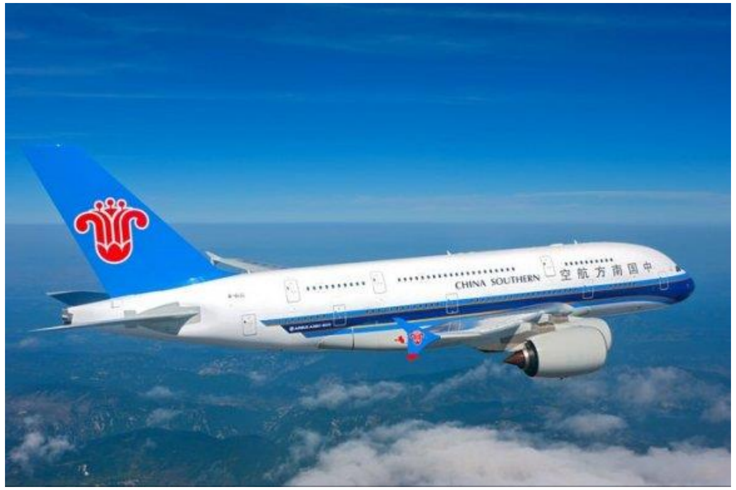
Dr Kelum Gamage kelum.gamage@glasgow.ac.uk
Associate Professor (Senior Lecturer)
School of Engineering, University of Glasgow, UK

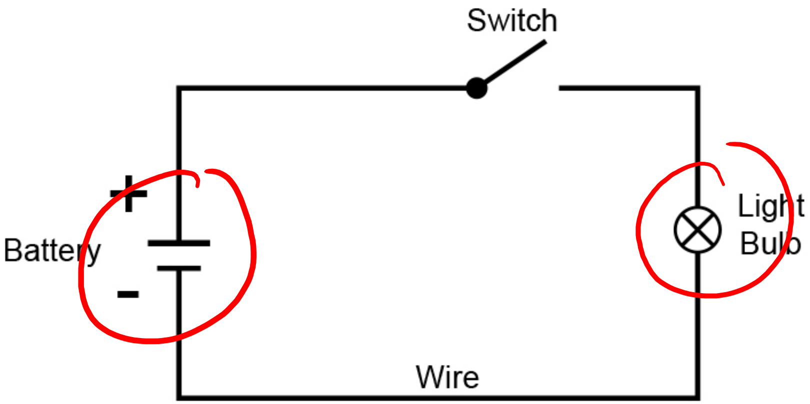
Systems

- Sets of connected objects or things
- They can be living things
- They can be mechanical things
- Behaviors of these systems are shaped by many factors
- All these systems exhibit some common behavior patterns

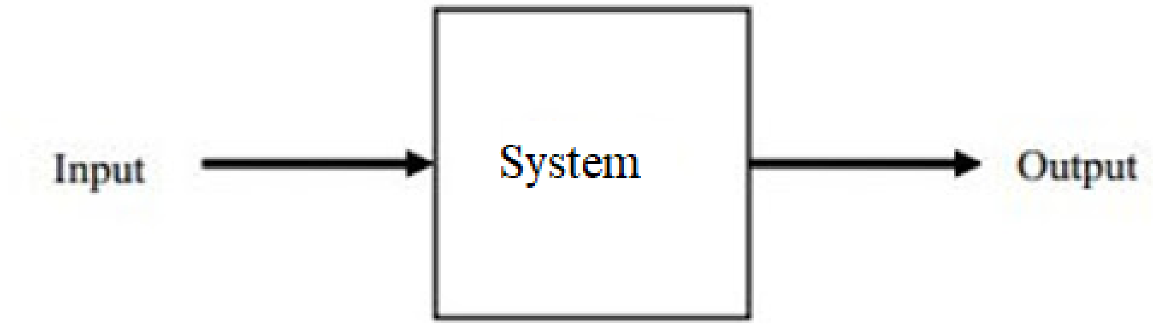
What is a Dynamic System?

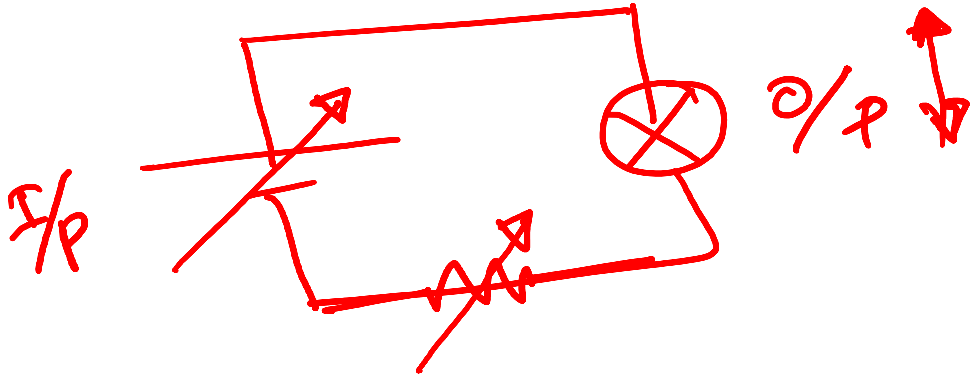
A dynamic system is one which is in motion.





The output of the system is dependent on the input and the behaviour of the system itself.



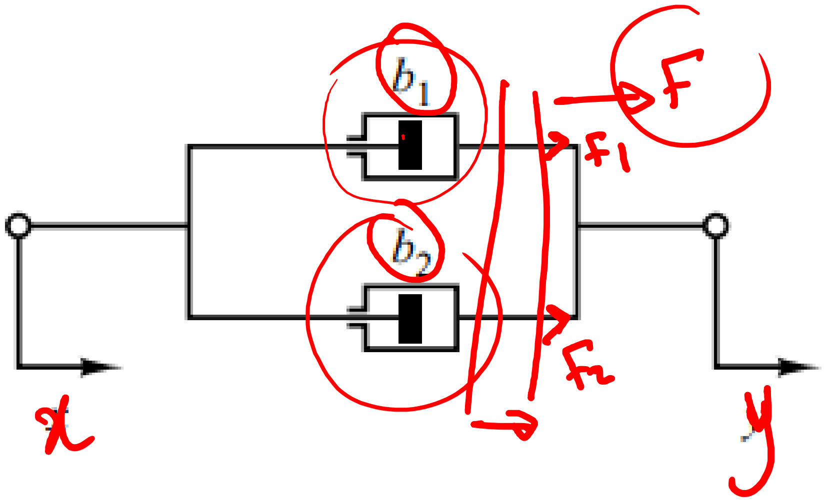


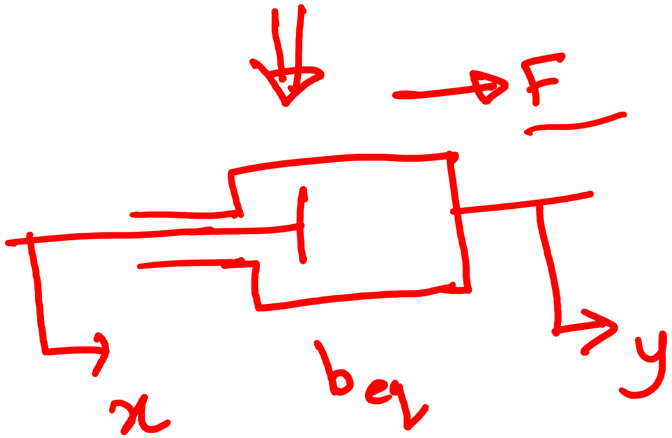
I/P and O/P relationship

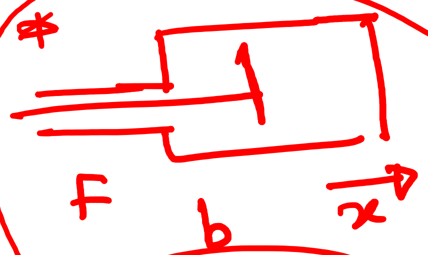
- Model dynamic systems in mathematical terms
- A set of equations that represents the dynamics of the system
- System may have many mathematical models
- The dynamics of systems may be described in terms of differential equations
- Use physical laws governing a particular system
- Testing a prototype of the device, or measuring its response to inputs
- Deriving reasonable mathematical models is the most important part

Mechanical System Modelling

System consisting of two dampers connected in parallel







$$F = b\dot{x}$$

$$F = F_1 + F_2$$

$$F = b_1(\dot{y} - \dot{x}) + b_2(\dot{y} - \dot{x})$$

$$F = (b_1 + b_2)(\dot{y} - \dot{x}) \quad \text{—— (1)}$$

$$F = b_{eq}(\dot{y} - \dot{x}) \quad \text{—— (2)}$$

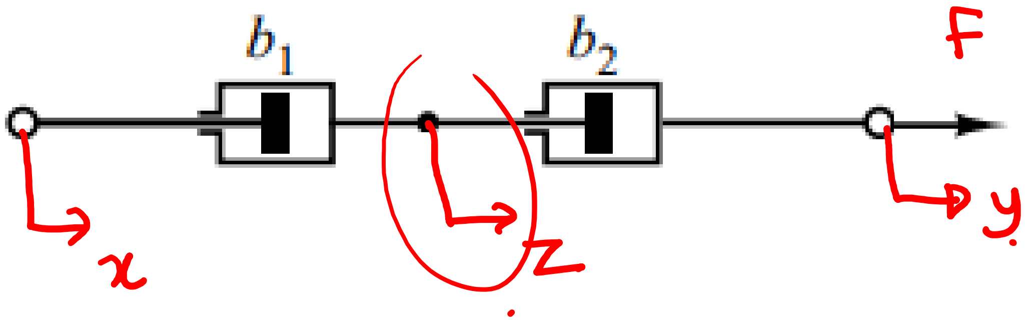
$$(1) \equiv (2)$$

$$\boxed{b_{eq} = b_1 + b_2}$$

Exercise: Mechanical System Modelling

System consisting of two dampers connected in series

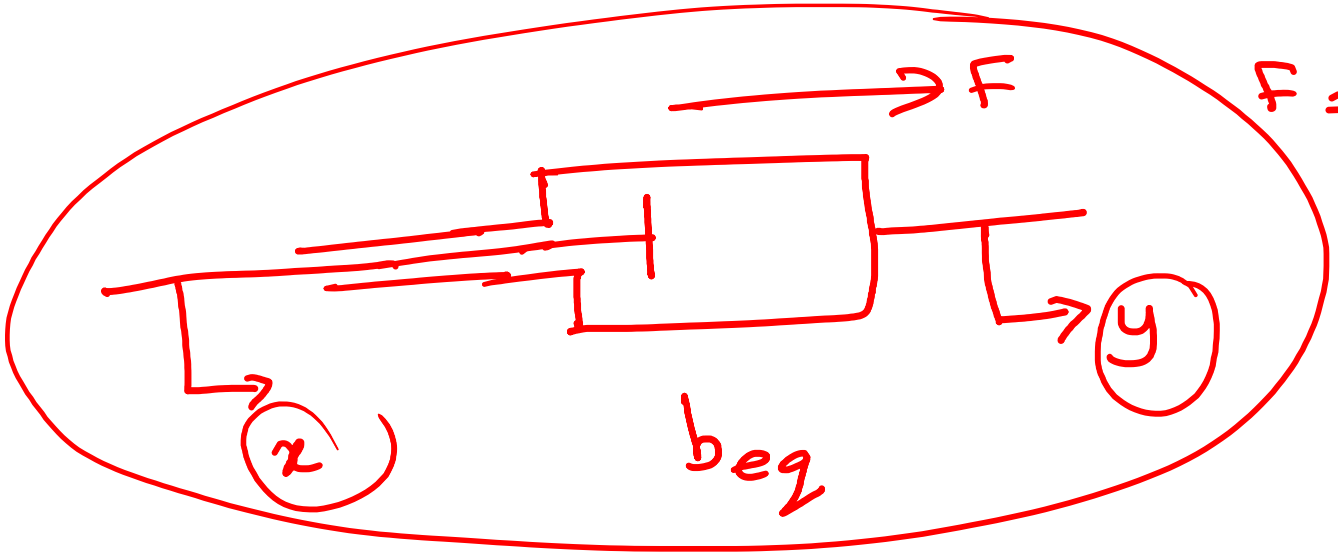
$$F = b\dot{x}$$



$$F = b_1(\dot{z} - \dot{x}) = b_2(\dot{y} - \dot{z})$$

$$b_1\dot{z} + b_2\dot{z} = b_1\dot{x} + b_2\dot{y}$$

$$\dot{z} = \frac{b_1\dot{x} + b_2\dot{y}}{b_1 + b_2}$$



$$F = b_2\left(\dot{y} - \frac{b_1\dot{x} + b_2\dot{y}}{b_1 + b_2}\right)$$

$$F = b_2\left(\dot{y} - \frac{b_2\dot{y} + b_1\dot{x}}{b_1+b_2}\right) = b_2\left[\frac{\dot{y}b_1 + \cancel{\dot{y}b_2} - \cancel{b_2\dot{y}} - b_1\dot{x}}{b_1+b_2}\right]$$

$$F = \frac{b_2 b_1(\dot{y}-\dot{x})}{b_1+b_2} \quad \text{——} \; 1$$

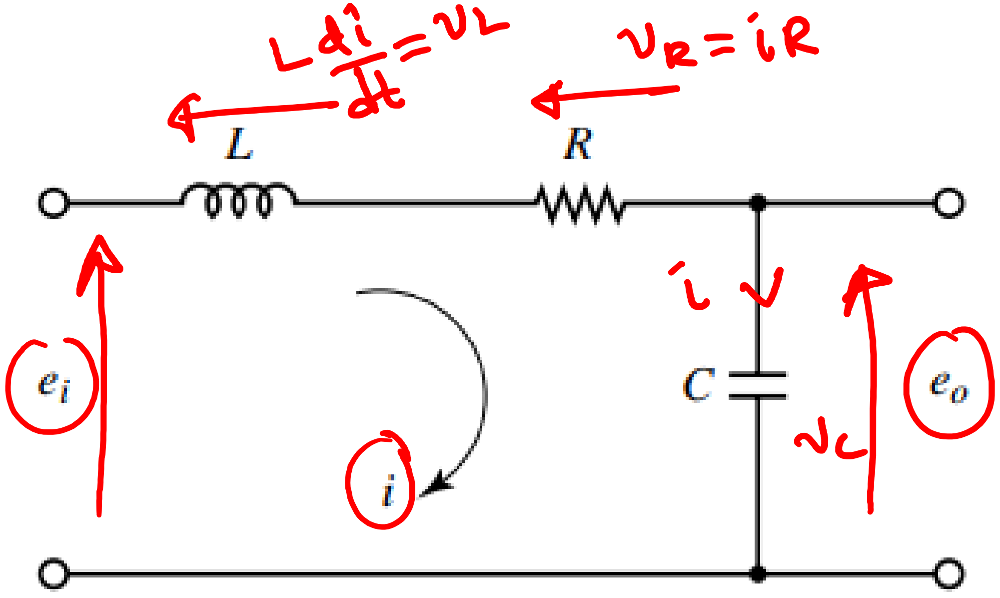
For the eq^n Sys $\Longrightarrow$ $\quad F = b_{eq}(\dot{y}-\dot{x}) \quad \text{——} \; 2$

$$1 = 2$$

$$\boxed{\frac{b_1 b_2}{b_1+b_2} = b_{eq}}$$

Electrical System Modelling

Consider the electrical circuit shown in the Figure



$i = C\frac{dv_c}{dt} \Rightarrow v_c = \frac{1}{C}\int i\,dt$

$e_i = v_L + v_R + v_c$

$e_i = L\frac{di}{dt} + iR + C\frac{d}{dt}v_c$

Describing a Dynamic System

- Equations describing the systems known as system’s equation of motion.
- Describes the O/P of the system
- Describes its derivatives as a function of the I/P
- Can have MIMO

First-Order System

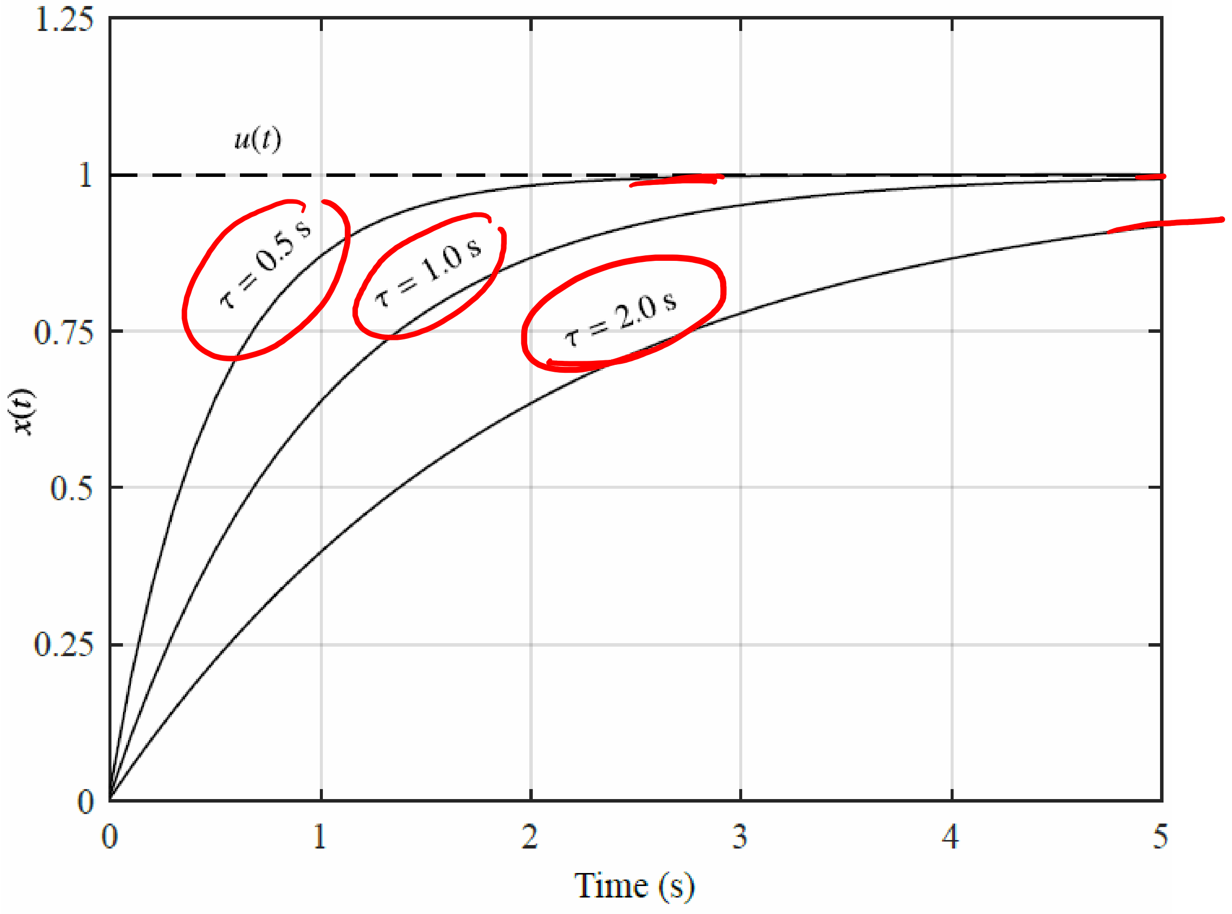
$$\boxed{\dot{x}(t) + \frac{1}{\tau}x(t) = Ku(t)}$$

gain

time constant

state

I/p



Second-Order System

natural frequency

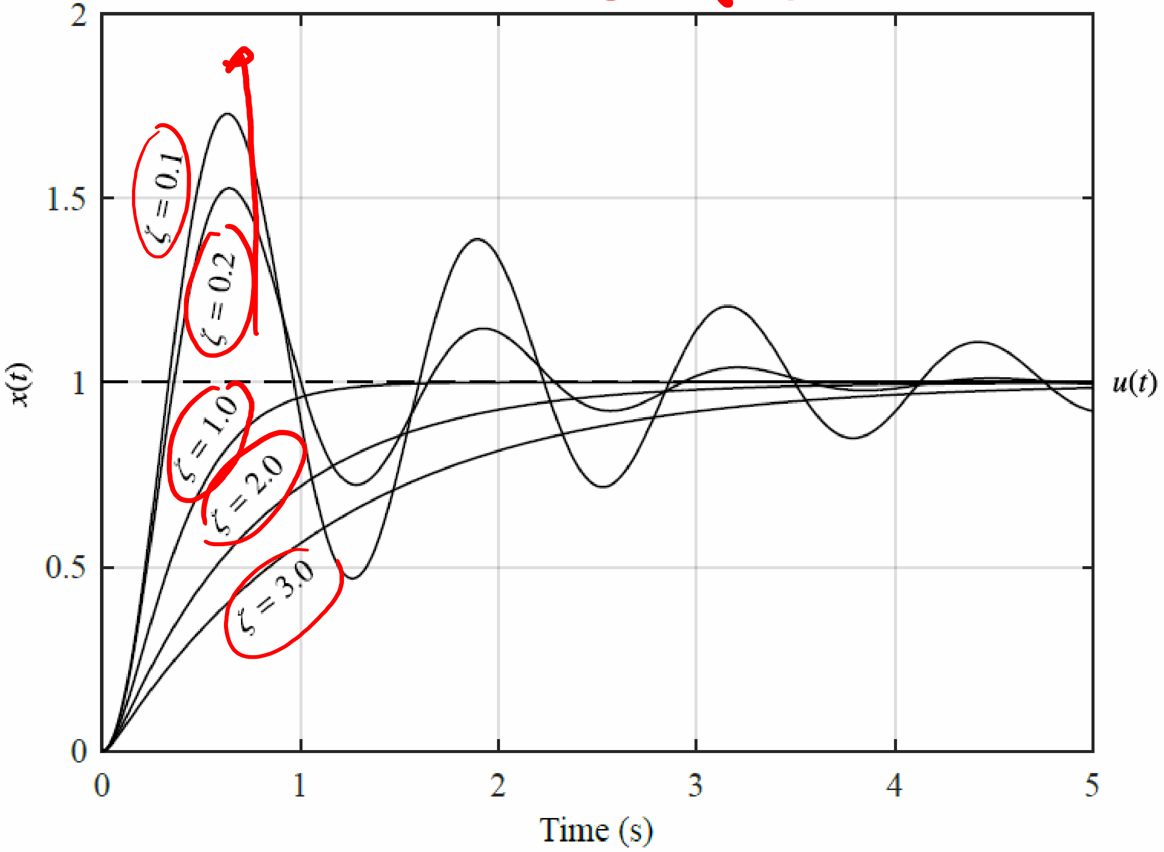
gain

$$\boxed{\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = K u(t)}$$

damping ratio

state/o/p

I/p



$\zeta < 1$ underdamped

$\zeta = 1$ critically damped

$\zeta > 1$ overdamped

Higher-Order Systems

An nth-order system has the general equation of motion

gain

$$* \quad \boxed{x^{(n)}(t) + a_1 x^{(n-1)}(t) + a_2 x^{(n-2)}(t) + \cdots + a_n x(t) = Ku(t)}$$

O/P / State

I/P

$$x^4(t) + a_1 x^3(t) + a_2 x^2(t) + a_1 x(t) = Ku(t)$$

The Laplace Transform

- Differential equation is difficult to model as a block diagram
- Represent the input, output, and system as separate entities
- Their interrelationship will be simply algebraic

$$\mathscr{L}[f(t)] = F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$$

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

$$\frac{d^4x}{dt^4} + 3\frac{d^3x}{dt^3} + 5\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 6x = 5u(t)$$

$$\mathcal{L} \Rightarrow \quad s^4X(s) + 3s^3X(s) + 5s^2X(s) + 2sX(s) + 6X(s) = 5U(s)$$

Transfer Functions

- Describe the relationship between O/P and I/P
- Transform it into the Laplace domain

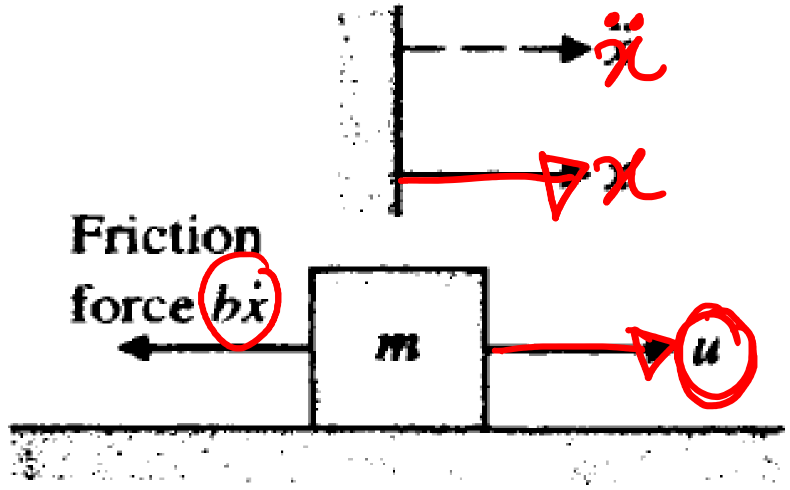
$$\boxed{\dot{x}(t) + \frac{1}{\tau}x(t) = Ku(t)}$$

$$sX(s) + \frac{1}{\tau}X(s) = KU(s)$$

$$X(s)\{s + 1/\tau\} = KU(s)$$

$$\frac{X(s)}{U(s)} = \frac{K}{s + 1/\tau}$$

Exercise: Transfer Functions



$$F = ma$$

$$u - b\dot{x} = m\ddot{x}$$

$$u = m\ddot{x} + b\dot{x}$$

$$\mathcal{L} \Rightarrow \quad U(s) = ms^2 X(s) + bsX(s)$$

$$U(s) = X(s)\{ms^2 + bs\}$$

$$\frac{X(s)}{U(s)} = \frac{1}{ms^2 + bs}$$

$$v = \dot{x}$$

$$V(s) = sX(s)$$

$$X(s) = \frac{V(s)}{s}$$

$$\frac{V(s)/\cancel{s}}{U(s)} = \frac{1}{\cancel{s}(ms+b)}$$

$$\boxed{\frac{V(s)}{U(s)} = \frac{1}{ms+b}}$$

Obtain transfer functions for the below systems

Second-order system: $\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = Ku(t)$ $\Rightarrow$ $\mathcal{L}$

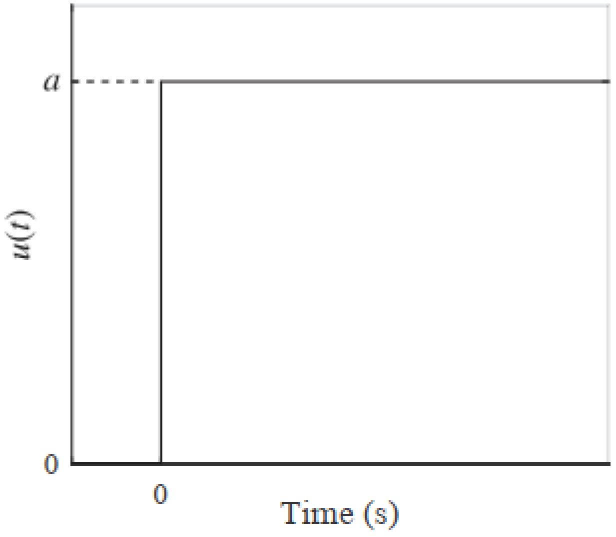
$$\frac{X(s)}{U(s)} =$$

nth-order system: $x^{(n)}(t) + a_1 x^{(n-1)}(t) + a_2 x^{(n-2)}(t) + \cdots + a_n x(t) = Ku(t)$

$$\frac{X(s)}{U(s)}$$

Types of Input

Step I/P



$t<0 \Rightarrow u(t)=0$

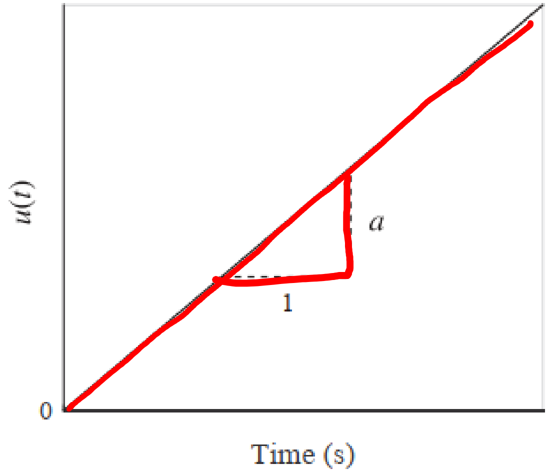
$t>0 \Rightarrow u(t)=a$

$a=1$

$\Downarrow$

unit step $f^{\underline{n}}$

Ramp I/P



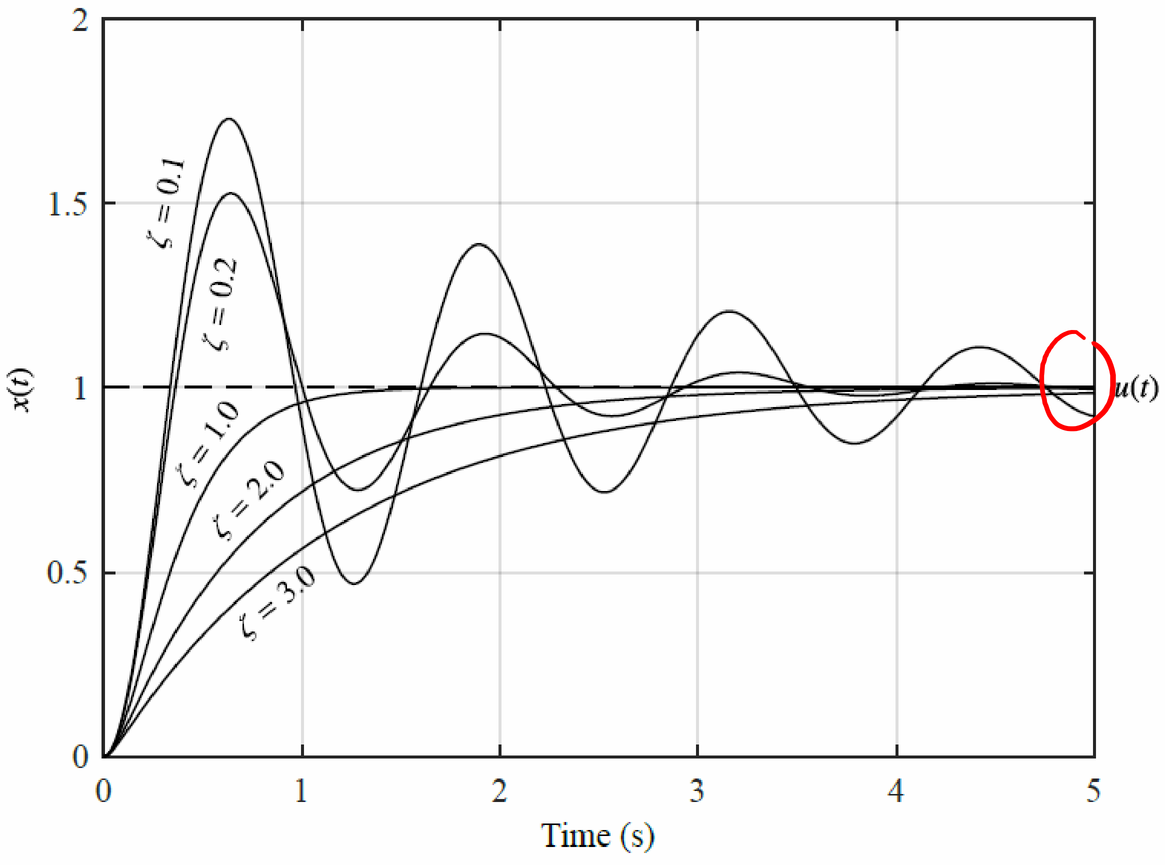
$t<0,\ u(t)=0$

$t>0,\ u(t)=at$

$u(s)=\dfrac{a}{s^2}$

Response Properties

- By applying a step input to a system which is at rest, we cause the system to be in forced vibration.

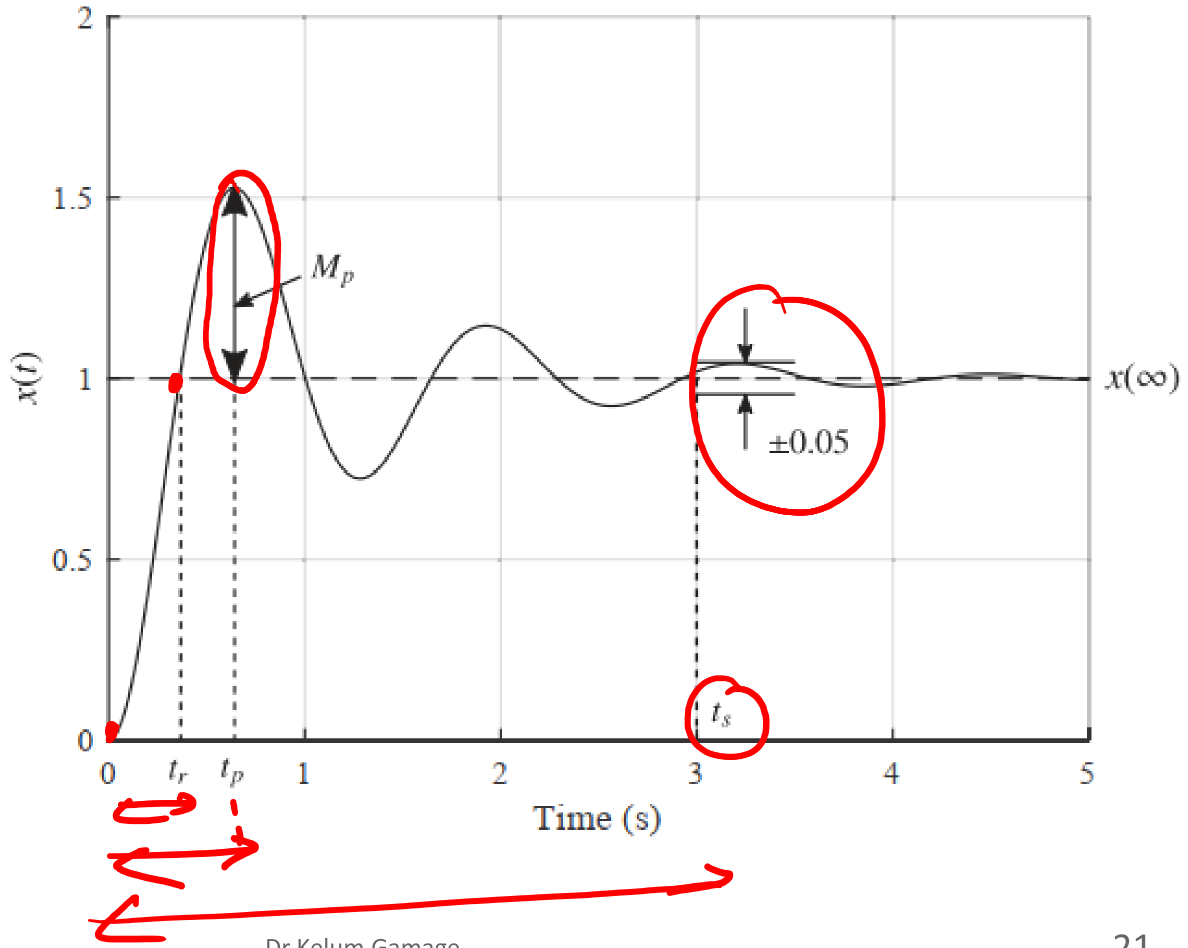


Final Value theorem

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$

Response Properties etc.

The response of a system can be described by several properties



Summary

- Basics of Dynamic Systems
- Mathematical Models
- Transfer Function
- Response Characteristics

Reference:

-Modern Control Engineering, 5th Edition, K. Ogata

-UESTC3001 2019/20 Notes, J. Le Kernec etc.