Circuit Analysis and Design

Academic year 2019/2020 – Semester 1 – Presentation 8

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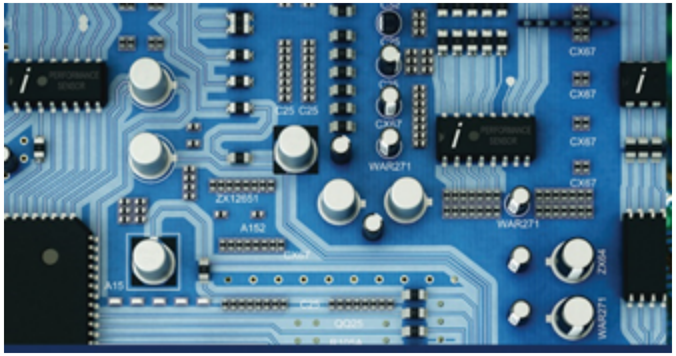
"A good student never steal or cheat"

Agenda

- Thévenin’s theorem
- Norton’s theorem
- Maximum power transfer
- Summary



Thévenin's Theorem

- A circuit consisting of a voltage source V_{th} and a series resistor R_{th}, representing the original circuit looking from a pair of terminals, is called a Thévenin equivalent circuit. The voltage V_{th} is called Thévenin equivalent voltage, and the resistance R_{th} is called Thévenin equivalent resistance, as shown in Figure 4.50.

- The Thévenin equivalent circuit can be used to simplify the circuit. When a load resistor is connected between terminals *a* and *b*, we can find the effects of the circuit on the load from the Thévenin equivalent circuit.

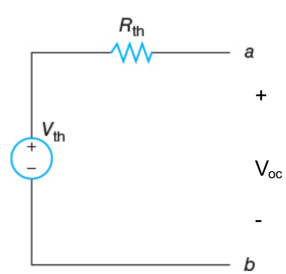
- We do not need all the details of the original circuit to find the voltage, current, and power on the load.

- Let the voltage across terminals *a* and *b* of the Thévenin equivalent circuit be V_{oc}. This voltage is called open-circuit voltage because terminals *a* and *b* are open (with an infinite resistance between *a* and *b*).

- No current flows through the Thévenin equivalent resistor R_{th}. Thus,
  $V_{oc} = V_{th}$

FIGURE 4.50

A Thévenin equivalent circuit.



Thévenin's Theorem (Continued)

- If the terminals *a* and *b* are short-circuited, as shown in Figure 4.51, the current through the short circuit is given by

$$I_{sc} = \frac{V_{th}}{R_{th}} = \frac{V_{oc}}{R_{th}}$$

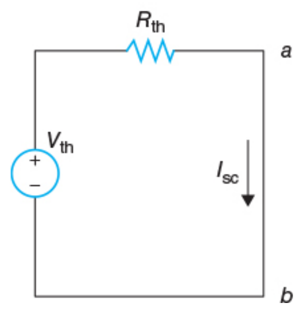
- If we solve this equation for R_{th}, we have

$$R_{th} = \frac{V_{oc}}{I_{sc}}$$

- This equation can be used to find the Thévenin equivalent resistance R_{th}.

FIGURE 4.51

Short-circuit current.



Finding the Thévenin Equivalent Resistance

Method 1:

- Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources.
- R_{th} is the equivalent resistance looking into the circuit from terminals *a* and *b.*
- This method cannot be used if the circuit contains dependent sources.

Method 2:

- Short-circuit terminals *a* and *b*. Find the short-circuit current I_{sc}.
- The Thévenin equivalent resistance is given by $R_{th} = V_{oc}/I_{sc} = V_{th}/I_{sc}$.

Method 3:

- Deactivate all the independent sources.
- Apply a test voltage of 1 V (or any other value) between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage.
- Measure the current flowing out of the positive terminal of the test voltage source.
- The Thévenin equivalent resistance R_{th} is given by the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source.
- Alternatively, apply a test current between terminals *a* and *b* after deactivating the independent sources, and measure the voltage across *a* and *b* of the test current source. The Thévenin equivalent resistance R_{th} is the ratio of the voltage across *a* and *b* to the test current.

Finding V_{th} and R_{th}

- Consider a circuit shown below. We are interested in finding V_{th} and R_{th} across terminals *a* and *b*.
- Sum the currents leaving node 1:

$$\frac{V_1 - 5}{5000} - 0.002 + \frac{V_1}{20000} + \frac{V_1 - V_2}{5000} = 0$$

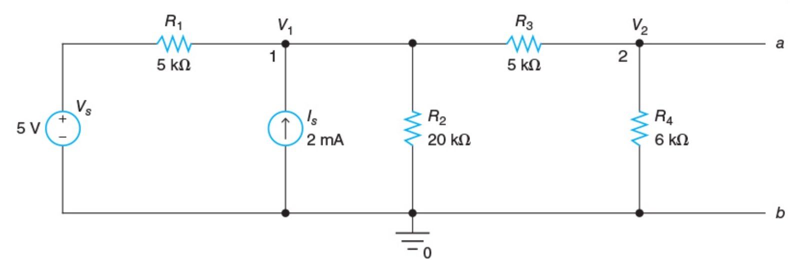
- Multiply by 20,000: $4V_1 – 20 – 40 + V_1 + 4V_1 – 4V_2 = 0 \Rightarrow 9V_1 – 4V_2 = 60$ (1)
- Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{5000} + \frac{V_2}{6000} = 0$$

- Multiplication by 30,000: $6V_2 – 6V_1 + 5V_2 = 0 \Rightarrow V_1 = 11/6V_2 = 60$
- Substituting in Equation 1: $V_2 = V_{th} = V_{oc} = 4.8$ V

FIGURE 4.52

A circuit with a pair of terminals.



Finding V_{th} and R_{th} (Continued)

- To find R_{th}, we deactivate V_s by short-circuiting it and I_s by open-circuiting it as shown in Figure 4.53, and find the equivalent resistance looking into the circuit from terminals *a* and *b* **(Method 1)**.

- $R_a = R_1 \parallel R_2 = 5 \times 20/(5 + 20)\ \text{k}\Omega = 100/25\ \text{k}\Omega = 4\ \text{k}\Omega$

- $R_b = R_3 + R_a = 9\ \text{k}\Omega$

- $R_{th} = R_4 \parallel R_b = 6 \times 9/(6 + 9)\ \text{k}\Omega = 54/15\ \text{k}\Omega = 3.6\ \text{k}\Omega$

- The Thévenin equivalent circuit is shown in Figure 4.57.

FIGURE 4.53

The circuit from Figure 4.52 with its sources deactivated.

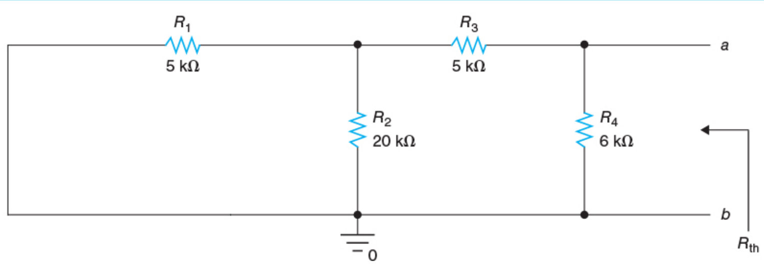
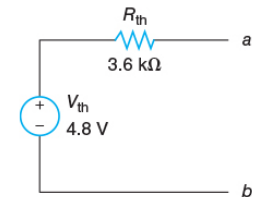


FIGURE 4.57

The Thévenin equivalent circuit.



Finding V_{th} and R_{th} (Continued)

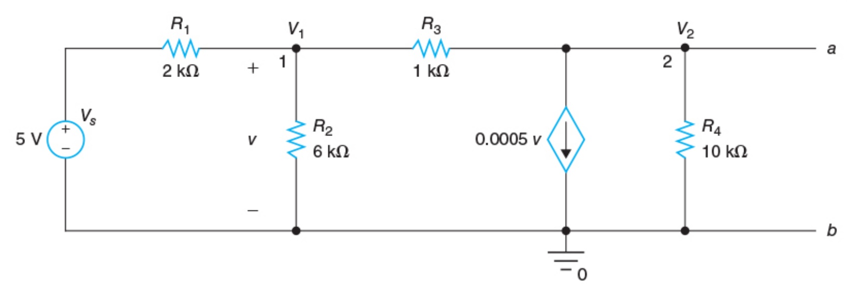
- Consider a circuit shown in Figure 4.58. We are interested in finding V_{th} and R_{th} across terminals *a* and *b*.
- Sum the currents leaving node 1: $\frac{V_1 - 5}{2000} + \frac{V_1}{6000} + \frac{V_1 - V_2}{1000} = 0$
- Multiply by 6000: $3V_1 - 15 + V_1 + 6V_1 - 6V_2 = 0 \Rightarrow 10V_1 = 6V_2 + 15 \Rightarrow V_1 = 0.6V_2 + 1.5$
- Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{1000} + 0.0005V_1 + \frac{V_2}{10000} = 0$$

- Multiply by 10000: $10V_2 - 10V_1 + 5V_1 + V_2 = 0 \Rightarrow 11V_2 - 5V_1 = 0 \Rightarrow 11V_2 - 5(0.6V_2 + 1.5) = 0$
- $8V_2 = 7.5$
- $V_{th} = V_{oc} = V_2 = 7.5/8 = 0.9375$ V

FIGURE 4.58

A circuit with VCCS.

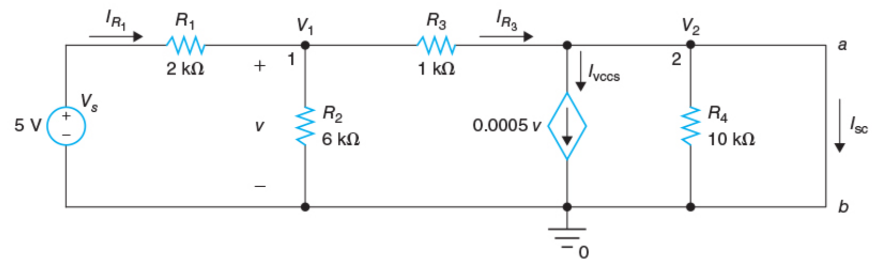


Finding V_{th} and R_{th} (Continued)

- Since there is a dependent source, Method 1 cannot be used to find R_{th}. Either Method 2 or Method 3 can be used.
- We will use **Method 2**. Terminals *a* and *b* are short-circuited as shown in Figure 4.59. $V_2 = 0$.
- Sum the currents leaving node 1: $\frac{V_1 - 5}{2000} + \frac{V_1}{6000} + \frac{V_1}{1000} = 0$
- Multiply by 6000: $3V_1 - 15 + V_1 + 6V_1 = 0 \Rightarrow 10V_1 = 15 \Rightarrow V_1 = 1.5$ V, $v = V_1 = 1.5$ V
- The current through $R_3(\rightarrow)$ is given by $I_{R3} = V_1/R_3 = 1.5\text{ V}/1\text{ k}\Omega = 1.5$ mA
- The current through VCCS($\downarrow$) is given by $I_{VCCS} = 0.0005V_1 = 0.0005 \times 1.5$ A = 0.75 mA
- $I_{sc} = I_{R3} - I_{VCCS} = 1.5\text{ mA} - 0.75\text{ mA} = 0.75$ mA
- $R_{th} = V_{th}/I_{sc} = 0.9675\text{ V}/0.75\text{ mA} = 1.25\text{ k}\Omega$

FIGURE 4.59

A circuit with a short circuit between *a* and *b*.

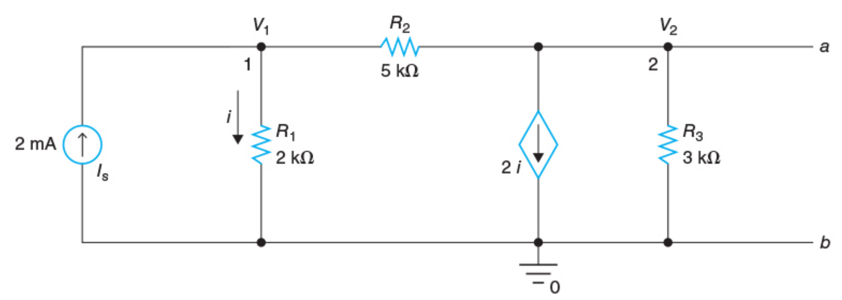


Finding V_{th} and R_{th} (Continued)

- Find V_{th} and R_{th} for the circuit shown in Figure 4.79.
- Sum the currents leaving node 1: $$-0.002+\frac{V_1}{2000}+\frac{V_1-V_2}{5000}=0$$
- Multiply by 10000: $-20 + 5V_1 + 2V_1 - 2V_2 = 0 \Rightarrow 7V_1 = 2V_2 + 20$ $\Rightarrow V_1 = (2/7)V_2 + 20/7$ (1)
- Sum the currents leaving node 2: $$\frac{V_2-V_1}{5000}+2\frac{V_1}{2000}+\frac{V_2}{3000}=0$$
- Multiply by 30000: $6V_2 - 6V_1 + 30V_1 + 10V_2 = 0$ (2), (1)$\rightarrow$(2): $24[(2/7)V_2 + 20/7] + 16V_2 = 0$
- $160V_2 = -480$
- $V_{th} = V_{oc} = V_2 = -3$ V

FIGURE 4.79

Circuit with a CCCS.

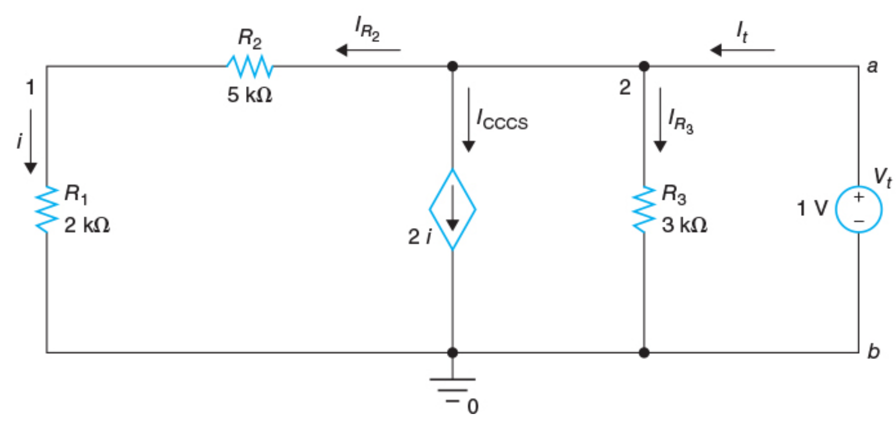


Finding V_{th} and R_{th} (Continued)

- To find R_{th}, I_s is open-circuited and a test voltage of 1 V is applied between *a* and *b* as shown in Figure 4.81 **(Method 3)**. V_t = 1 V. $i = I_{R2} = V_t/(R_1 + R_2)$ = (10/7) mA
- $I_{CCCS} = 2i$ = (20/7) mA, $I_{R3} = V_t/R_3$ = (1/3) mA
- The current flowing out of the positive terminal of the test voltage source is given by
- $I_t = I_{R2} + I_{CCCS} + I_{R3}$ = (3/21) mA + (6/21) mA + (7/21) mA = (16/21) mA
- The Thévenin equivalent resistance is the ratio of V_t to I_t: $R_{th} = V_t/I_t$ = 21/16 kΩ = 1.3125 kΩ

FIGURE 4.81

A circuit with a test voltage source.



EXAMPLE 4.7

- Find V_{th} and R_{th} for the circuit shown in Figure 4.63.
- Since the current through R_3 is zero, the voltage across R_3 is zero.
- The Thévenin equivalent voltage is the voltage across R_2.
- Applying the voltage divider rule, we obtain

  $V_{th} = V_{oc} = V_s \times R_2/(R_1 + R_2) = 20\text{ V} \times 56/80 = 14\text{ V}$
- When the voltage source is short-circuited, we obtain the circuit shown in Figure 4.64. The Thévenin equivalent resistance is the resistance looking into the circuit from *a* and *b*.
- $R_{th} = R_3 + (R_1 \,||\, R_2) = 3.2\text{ k}\Omega + 24 \times 56/(24 + 56)\text{ k}\Omega = 3.2\text{ k}\Omega + 16.8\text{ k}\Omega = 20\text{ k}\Omega$

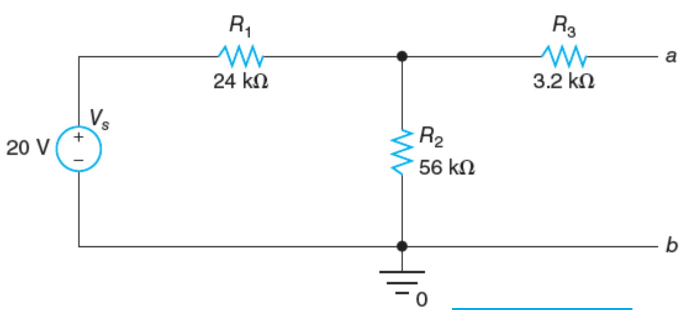


FIGURE 4.63

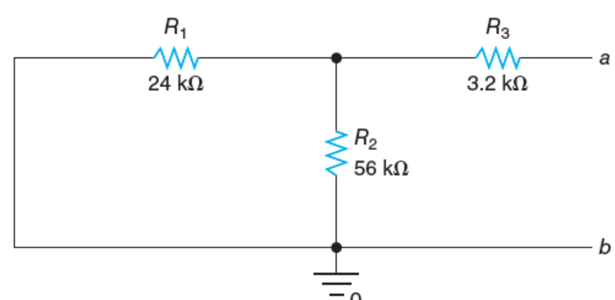


FIGURE 4.64

EXAMPLE 4.8

- Find V_{th} and R_{th} for the circuit shown in Figure 4.67.
- Sum the currents leaving node 1:

$$\frac{V_1 - 22.5}{15000} + \frac{V_1}{30000} + \frac{V_1 - V_2}{5000} = 0$$

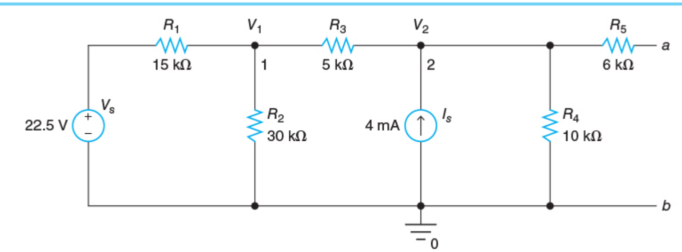
- Multiply by 30000: $2V_1 - 45 + V_1 + 6V_1 - 6V_2 = 0 \Rightarrow 9V_1 = 6V_2 + 45 \Rightarrow V_1 = (2/3)V_2 + 5$ (1)
- Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{5000} - 0.004 + \frac{V_2}{10000} = 0$$

- Multiply by 10000: $2V_2 - 2V_1 - 40 + V_2 = 0$ (2), (1)$\rightarrow$(2): $3V_2 - 2[(2/3)V_2 + 5] = 40 \Rightarrow$
- $(5/3)V_2 = 50$
- $V_{th} = V_{oc} = V_2 = 30$ V

FIGURE 4.67

Circuit for EXAMPLE 4.8.



EXAMPLE 4.8 (Continued)

- To find R_{th}, V_s is short-circuited and I_s is open-circuited as shown in Figure 4.70.
- $R_a = R_1 \parallel R_2 = 15 \times 30/(15 + 30)\ \text{k}\Omega = 450/45\ \text{k}\Omega = 10\ \text{k}\Omega$
- $R_b = R_3 + R_a = 5\ \text{k}\Omega + 10\ \text{k}\Omega = 15\ \text{k}\Omega$
- $R_c = R_4 \parallel R_b = 10 \times 15/(10 + 15)\ \text{k}\Omega = 150/25\ \text{k}\Omega = 6\ \text{k}\Omega$
- $R_{th} = R_5 + R_c = 6\ \text{k}\Omega + 6\ \text{k}\Omega = 12\ \text{k}\Omega$
- The Thévenin equivalent circuit is shown in Figure 4.71.

FIGURE 4.70

The circuit shown in Figure 4.67 after deactivating the sources.

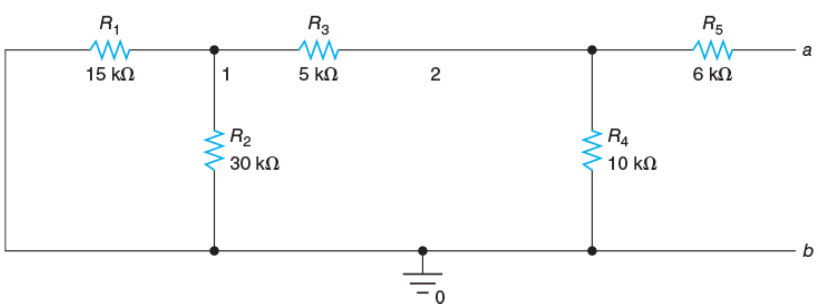
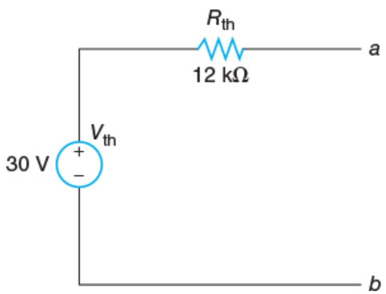


FIGURE 4.71

The Thévenin equivalent circuit.



EXAMPLE 4.9

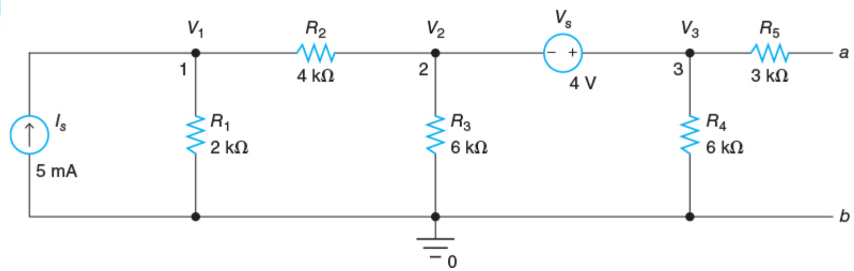
- Find V_{th} and R_{th} for the circuit shown in Figure 4.73.
- $V_3 = V_2 + 4$
- Sum the currents leaving node 1: $-0.005+\frac{V_1}{2000}+\frac{V_1-V_2}{4000}=0$
- Multiply by 4000: $-20 + 2V_1 + V_1 - V_2 = 0 \Rightarrow 3V_1 = V_2 + 20 \Rightarrow V_1 = (1/3)V_2 + (20/3)$ (1)
- Sum the currents leaving the supernode consisting of node 2 and node 3 (utilize (1) for V_1):

$$\frac{V_2-\left(V_2/3+20/3\right)}{4000}+\frac{V_2}{6000}+\frac{V_2+4}{6000}=0$$

- Multiply by 12000: $2V_2 - 20 + 2V_2 + 2V_2 + 8 = 0 \Rightarrow 6V_2 = 12 \Rightarrow V_2 = 2$ V
- $V_{th} = V_{oc} = V_3 = V_2 + 4 = 6$ V

FIGURE 4.73

Circuit for EXAMPLE 4.9.

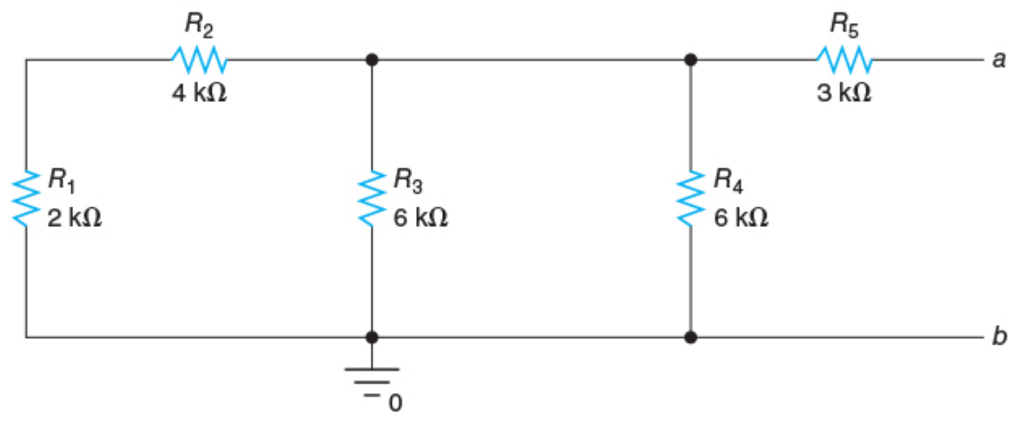


EXAMPLE 4.9 (Continued)

- To find R_{th}, V_s is short-circuited and I_s is open-circuited as shown in Figure 4.76.
- $R_a = (R_1 + R_2) \parallel R_3 = 6 \times 6/(6 + 6)$ kΩ = 36/12 kΩ = 3 kΩ
- $R_b = R_4 \parallel R_a = 6 \parallel 3$ kΩ = $6 \times 3/(6 + 3)$ kΩ = 18/9 kΩ = 2 kΩ
- $R_{th} = R_5 + R_b$ = 3 kΩ + 2 kΩ = 5 kΩ

FIGURE 4.76

Circuit shown in Figure 4.73 with the sources deactivated.



EXAMPLE 4.11

- Find V_{th} and R_{th} for the circuit shown in Figure 4.85.
- Since $i_2 = 0$, the voltage across CCVS is zero ($2i_2 = 0$). Thus,
- $i_1 = V_s/R_1 = 10\ \text{V}/5\ \Omega = 2\ \text{A}$
- $V_{th} = V_{oc} = 3i_1 = 6\ \text{V}$
- To find R_{th}, after deactivating V_s, a test voltage of 1 V is applied across *a* and *b* as shown in Figure 4.86 (Method 3).
- $i_1 = -2i_2/R_1 = -2i_2/5$
- $i_2 = (V_t - 3i_1)/4 = (1 + 6i_2/5)/4$
- $14i_2 = 5 \Rightarrow i_2 = 5/14\ \text{A}$
- $R_{th} = V_t/i_2 = 14/5\ \Omega = 2.8\ \Omega$

FIGURE 4.85

Circuit for EXAMPLE 4.11.

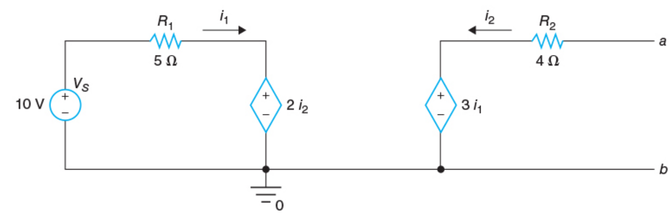
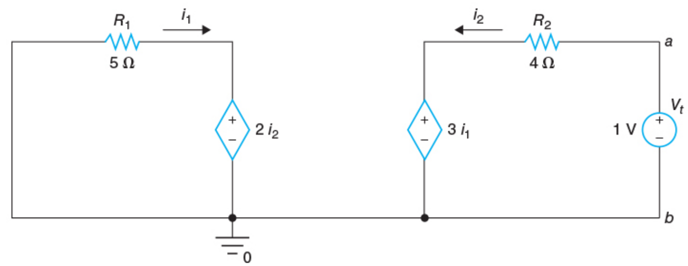


FIGURE 4.86

A circuit with a test voltage.



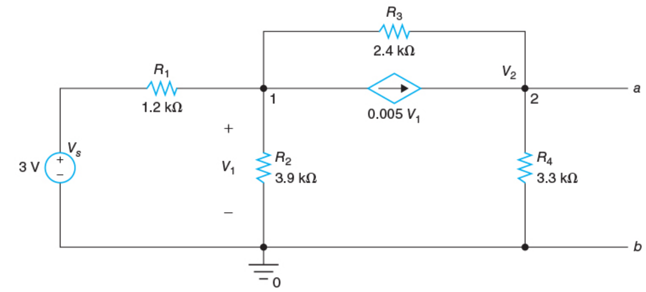
EXAMPLE 4.12

- Find V_{th} and R_{th} for the circuit shown in Figure 4.89.
- Sum the currents leaving node 1:

$$\frac{V_1 - 3}{1200} + \frac{V_1}{3900} + \frac{V_1 - V_2}{2400} + 0.005V_1 = 0$$

- Multiply by 31200: $26V_1 - 78 + 8V_1 + 13V_1 - 13V_2 + 156V_1 = 0 \Rightarrow 203V_1 = 13V_2 + 78 \Rightarrow$
  $V_1 = (13/203)V_2 + 78/203$ (1)
- Sum the currents leaving node 2: $$\frac{V_2 - V_1}{2400} - 0.005V_1 + \frac{V_2}{3300} = 0$$
- Multiply by 26400:
  $11V_2 - 11V_1 - 132V_1 + 8V_2 = 0 \Rightarrow$
  $19V_2 - 143V_1 = 0$ (2), (1)$\rightarrow$(2):
  $(19 - 1859/203)V_2 = 11154/203$
- $V_{th} = V_{oc} = V_2 = 5.5826$ V

FIGURE 4.89
Circuit for EXAMPLE 4.12.



EXAMPLE 4.12 (Continued)

- To find R_{th}, we short-circuit *a* and *b* as shown in Figure 4.90. $V_2 = 0$. Current through $R_4 = 0$.
- Sum the currents leaving node 1:

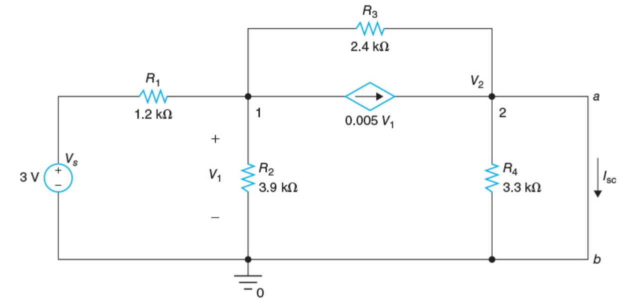
$$\frac{V_1 - 3}{1200} + \frac{V_1}{3900} \frac{V_1}{2400} + 0.005V_1 = 0$$

- Multiply by 31200: $26V_1 - 78 + 8V_1 + 13V_1 + 156V_1 = 0 \Rightarrow 203V_1 = 78$ $\Rightarrow$

  $V_1 = 78/203\text{ V} = 0.3842364532\text{ V}$
- $I_{R3}\ (\rightarrow) = V_1/2400 = 0.1601\text{ mA}$
- $I_{VCCS}\ (\rightarrow) = 0.005V_1 = 1.92118\text{ mA}$
- $I_{sc} = I_{R3} + I_{VCCS} = 2.08128\text{ mA}$
- $R_{th} = V_{th}/I_{sc} = 2.6823\text{ k}\Omega$

FIGURE 4.90

The circuit from Figure 4.89 with *a* and *b* short-circuited.



Norton's Theorem

- A circuit looking from terminals *a* and *b* can be replaced by a current source with current I_n and a parallel resistor with resistance R_n, as shown in Figure 4.93.
- This equivalent circuit consisting of a current source and a parallel resistor is called Norton equivalent circuit.
- The current I_n is called Norton equivalent current and the resistance R_n is called Norton equivalent resistance.
- When the terminals *a* and *b* are short-circuited in the Norton equivalent circuit, as shown in Figure 4.94, the short-circuit current I_{sc} is equal to I_n from the current divider rule.
- Thus, the Norton equivalent current can be obtained by finding the short-circuit current.

FIGURE 4.93

A Norton equivalent circuit.

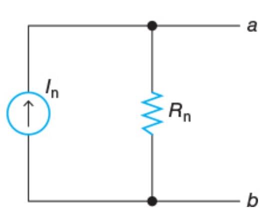
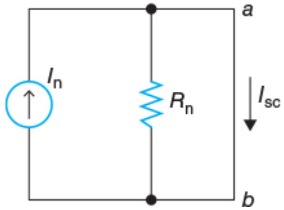


FIGURE 4.94

Short-circuit current.



Finding Norton Equivalent Resistance

Method 1:

- Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources.
- R_n is the equivalent resistance looking into the circuit from terminals *a* and *b.*
- This method can be used if the circuit does not contain dependent sources.

Method 2:

- Find the open-circuit voltage V_{oc} and the short-circuit current I_{sc}.
- The Norton equivalent resistance is given by $R_n = V_{oc}/I_{sc} = V_{oc}/I_n$.

Method 3:

- Deactivate all the independent sources by open-circuiting current sources and short-circuiting voltage sources.
- Apply a test voltage of 1 V (or any other value) between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage.
- Measure the current flowing out of the positive terminal of the test voltage source.
- The Norton equivalent resistance R_n is given by the ratio of the test voltage to the current flowing out of the positive terminal of the test voltage source.
- Alternatively, apply a test current between terminals *a* and *b* after deactivating the independent sources, and measure the voltage across *a* and *b* of the test current source. The Norton equivalent resistance R_n is the ratio of the voltage across *a* and *b* to the test current.

Thévenin Equivalent Circuit and Norton Equivalent Circuit

- Application of source transformation to the Norton equivalent circuit shown in Figure 4.95(a) yields the Thévenin equivalent circuit shown in Figure 4.95(b).
- Notice that the Thévenin equivalent voltage is $V_{th} = I_n R_n$ and the Thévenin equivalent resistance is $R_{th} = R_n$.
- The source transformation does not change the resistance value. Application of source transformation to the Thévenin equivalent circuit shown in Figure 4.96(a) yields the Norton equivalent circuit, as shown in Figure 4.96(b).
- Notice that the Norton equivalent current is $I_n = V_{th}/R_{th}$ and the Norton equivalent resistance is $R_n = R_{th}$.

FIGURE 4.95

Transformation from Norton equivalent circuit to Thévenin equivalent circuit.

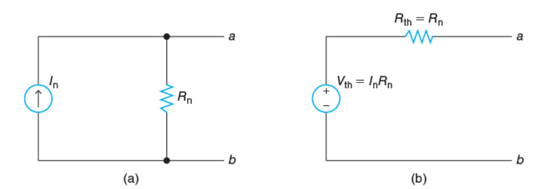
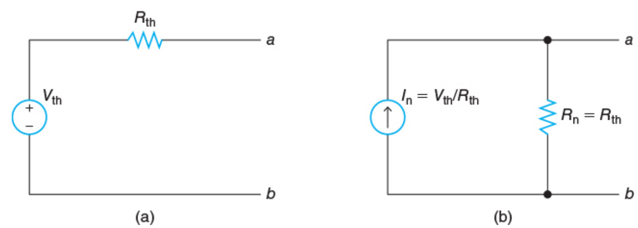


FIGURE 4.96

Transformation from Thévenin equivalent circuit to Norton equivalent circuit.



Finding I_n and R_n

- We are interested in finding I_n and R_n for the circuit shown in Figure 4.97.
- To find the short circuit current, we short-circuit *a* and *b* as shown in Figure 4.98. $V_3 = 0$. No current through R_5.
- Sum the currents leaving node 1:

$$-0.002 + \frac{V_1}{1500} + \frac{V_1 - V_2}{1500} = 0$$

- Multiply by 1500: $2V_1 - V_2 = 3$ (1)
- Sum the currents leaving node 2:

$$\frac{V_2 - V_1}{1500} + \frac{V_2 - 2.5}{1000} + \frac{V_2}{3000} = 0$$

- Multiply by 3000: $2V_2 - 2V_1 + 3V_2 - 7.5 + V_2 = 0 \Rightarrow$ $-2V_1 + 6V_2 = 7.5$ (2)
- (1) + (2): $5V_2 = 10.5 \Rightarrow V_2 = 2.1$ V, $V_1 = (V_2 + 3)/2 = 2.55$ V
- $I_n = V_1/R_4 + V_2/R_3 = 1.7$ mA + 0.7 mA = 2.4 mA

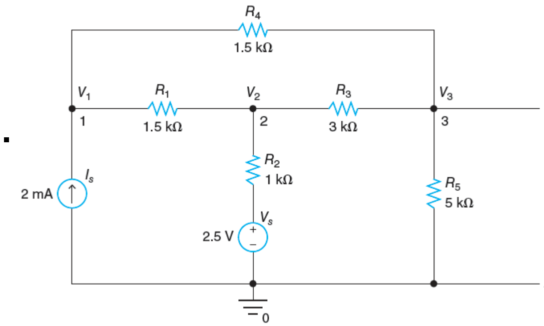


FIGURE 4.97

1

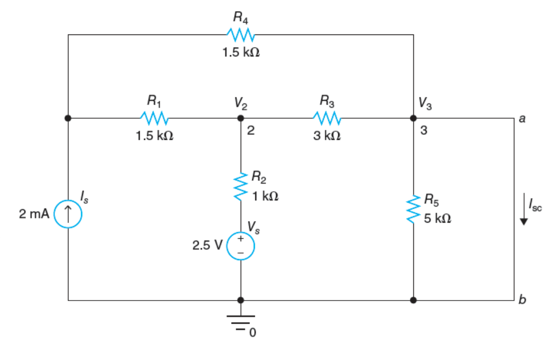


FIGURE 4.98

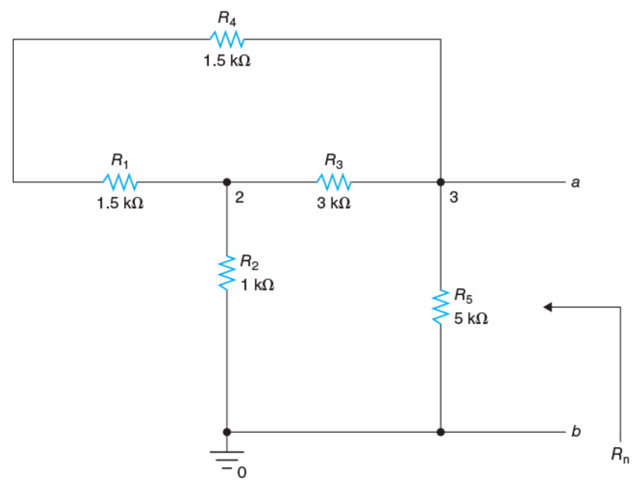
Finding I_n and R_n (Continued)

- To find R_n, V_s and I_s are deactivated as shown in Figure 4.101 **(Method 1)**.
- $R_a = R_1 + R_4 = 1.5\ k\Omega + 1.5\ k\Omega = 3\ k\Omega$
- $R_b = R_3 \parallel R_a = 3 \times 3/(3 + 3)\ k\Omega = 9/6\ k\Omega = 1.5\ k\Omega$
- $R_c = R_b + R_2 = 1.5\ k\Omega\ + 1\ k\Omega = 2.5\ k\Omega$
- $R_n = R_5 \parallel R_c = 5 \times 2.5/(5 + 2.5)\ k\Omega$

  $R_n = (12.5/7.5)\ k\Omega = 1.6667\ k\Omega$

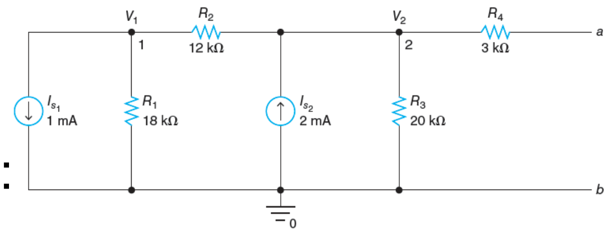
FIGURE 4.101

The circuit from Figure 4.97 with sources deactivated.



Finding I_n and R_n (Continued)

- Find I_n and R_n for the circuit shown in Figure 4.117.
- To find I_{sc}, we short circuited the terminals *a* and *b*.
- Sum the currents leaving node 1 (Figs 4.117, 4.118):



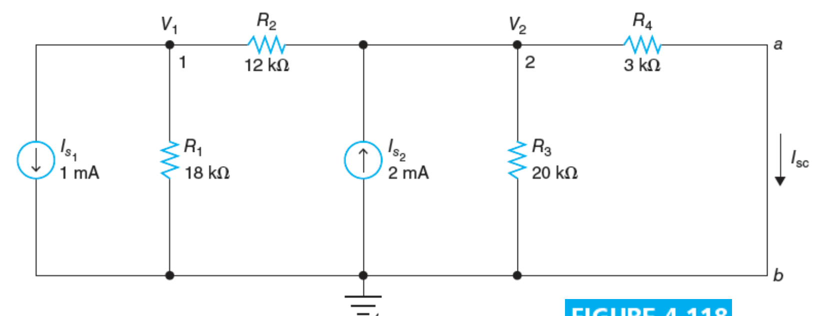
(a) FIGURE 4.117

$$0.001 + \frac{V_1}{18000} + \frac{V_1 - V_2}{12000} = 0$$

- Multiply by 36000: $36 + 2V_1 + 3V_1 - 3V_2 = 0 \Rightarrow$
- $5V_1 = 3V_2 - 36 \Rightarrow V_1 = 0.6V_2 - 7.2$ (1)
- Sum the currents leaving node 2 of Figure 4.118:

$$\frac{V_2 - V_1}{12000} - 0.002 + \frac{V_2}{20000} + \frac{V_2}{3000} = 0$$

- Multiply by 60000: $5V_2 - 5V_1 - 120 + 3V_2 + 20V_2 = 0$
- $28V_2 - 5V_1 = 120$ (2)
- Substitute Equation (1) into Equation (2):
- $28V_2 - 3V_2 = 84 \Rightarrow 25V_2 = 84$
- $V_2 = 84/25 = 3.36$ V
- $I_n = I_{sc} = V_2/R_4 = 1.12$ mA



(b) FIGURE 4.118

Finding I_n and R_n (Continued)

- We will apply **Method 2** to find the Norton Resistance finding V_{oc}.
- Sum the currents leaving node 1 (Figs 4.117, 4.118):

$$0.001 + \frac{V_1}{18000} + \frac{V_1 - V_2}{12000} = 0$$

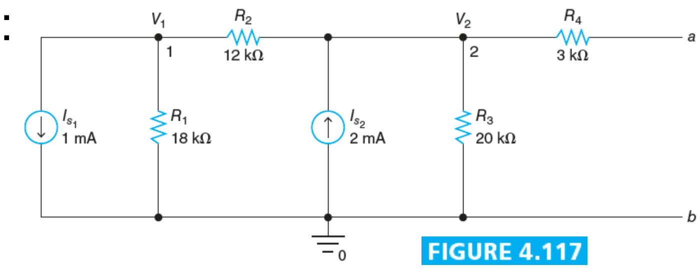


FIGURE 4.117

(a)

- Multiply by 36000: $36 + 2V_1 + 3V_1 - 3V_2 = 0 \Rightarrow$
- $5V_1 = 3V_2 - 36 \Rightarrow V_1 = 0.6V_2 - 7.2$ (1)
- Sum the currents leaving node 2 of Figure 4.117:

$$\frac{V_2 - V_1}{12000} - 0.002 + \frac{V_2}{20000} = 0$$

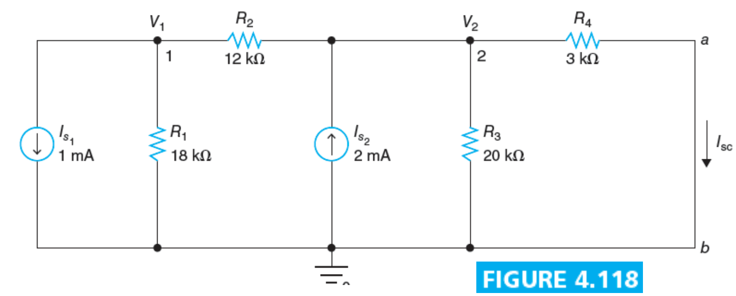


FIGURE 4.118

(b)

- Multiply by 60000: $5V_2 - 5V_1 - 120 + 3V_2 = 0$ (2)
- Substitute Equation (1) into Equation (2):
- $8V_2 - 3V_2 + 36 - 120 = 0 \Rightarrow 5V_2 = 84 \Rightarrow$
- $V_{oc} = V_2 = 16.8$ V
- $R_n = V_{oc}/I_{sc} = 16.8\text{ V}/1.12\text{ mA} = 15\text{ k}\Omega$
- The Norton equivalent circuit is shown in Figure 4.119.

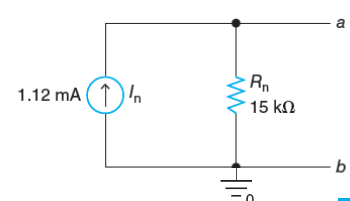


FIGURE 4.119

(c)

Finding I_n and R_n (Continued)

- We are interested in finding I_n and R_n for the circuit shown in Figure 4.103.
- To find the short circuit current, we short-circuit *a* and *b* as shown in Figure 4.104. $V_2 = 0$.
- Sum the currents leaving node 1: $-0.003+\frac{V_1}{3000}+\frac{V_1}{2000}=0$
- Multiply by 6000: $5V_1 = 18$
- $V_1 = 3.6$ V
- $i = V_1/3000 = 3.6\text{ V}/3000\ \Omega = 0.0012$ A
- $V_{CCVS} = 2000i = 2.4$ V
- $I_{R2} = V_1/R_2 = 1.8$ mA
- $I_{R3} = V_{CCVS}/R_3 = 2.4$ mA
- $I_n = I_{R2} + I_{R3} = 1.8\text{ mA} + 2.4\text{ mA} = 4.2$ mA

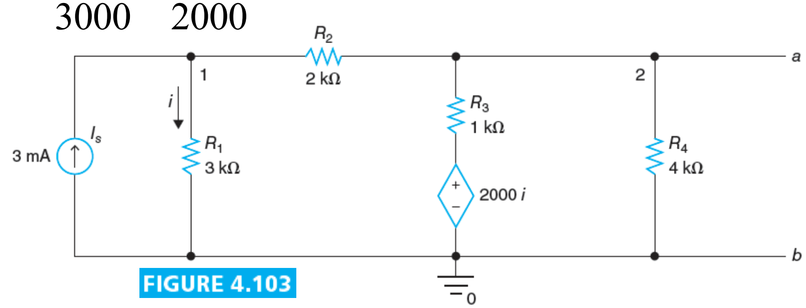


FIGURE 4.103

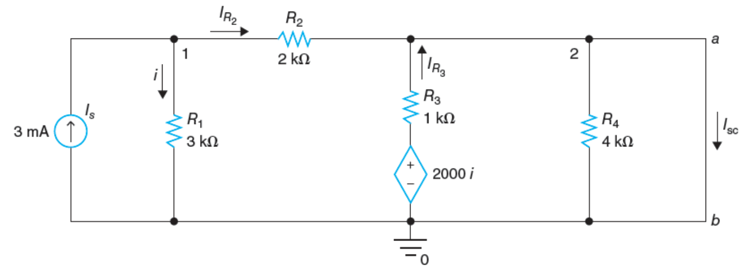


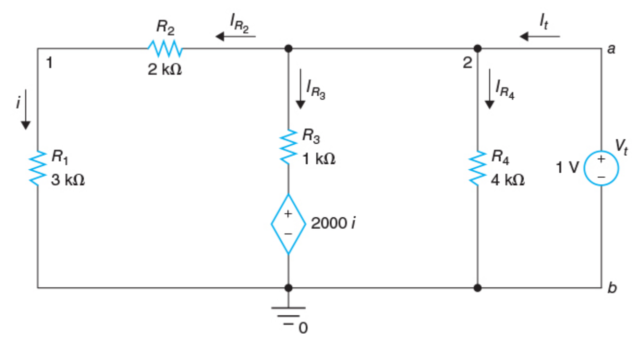
FIGURE 4.104

Finding I_n and R_n (Continued)

- To find R_n, after deactivating the current source, a test voltage of 1 V is applied across *a* and *b* **(Method 3)** as shown in Figure 4.105. V_t = 1 V.
- $I_{R2} = i = V_t/(R_1 + R_2) = 1\text{ V}/5\text{ k}\Omega = 0.2\text{ mA}$
- $V_{CCVS} = 2000i = 0.4\text{ V}$
- $I_{R3} = (V_t - V_{CCVS})/R_3 = 0.6\text{ mA}$
- $I_{R4} = V_t/R_4 = 0.25\text{ mA}$
- $I_t = I_{R2} + I_{R3} + I_{R4} = 0.2\text{ mA} + 0.6\text{ mA} + 0.25\text{ mA} = 1.05\text{ mA}$
- $R_n = V_t/I_t = 952.381\ \Omega$

FIGURE 4.105

A circuit with a test voltage source.



EXAMPLE 4.13

- Find I_n and R_n for the circuit shown in Figure 4.107.
- To find I_{sc}, terminals *a* and *b* are short-circuited in Figure 4.108.
- $R_a = R_2 \parallel R_3 = 3 \times 2/(3 + 2)\ \mathrm{k\Omega} = 1.2\ \mathrm{k\Omega}$
- $V_1 = V_s \times R_a/(R_1 + R_a) = 9\ \mathrm{V} \times 1.2/(0.6 + 1.2) = 6\ \mathrm{V}$
- $I_n = I_{sc} = V_1/R_3 = 3\ \mathrm{mA}$
- To find R_n, V_s is short-circuited as shown in Figure 4.109.
- $R_b = R_1 \parallel R_2 = 0.6 \times 3/(0.6 + 3)\ \mathrm{k\Omega} = 0.5\ \mathrm{k\Omega}$
- $R_n = R_3 + R_b = 2\ \mathrm{k\Omega} + 0.5\ \mathrm{k\Omega} = 2.5\ \mathrm{k\Omega}$
- The Norton equivalent circuit is shown in Figure 4.110.

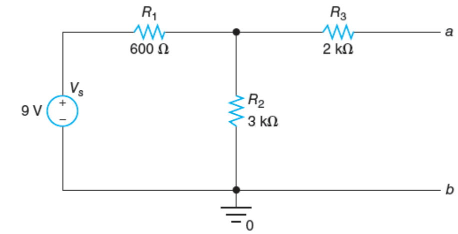


FIGURE 4.107

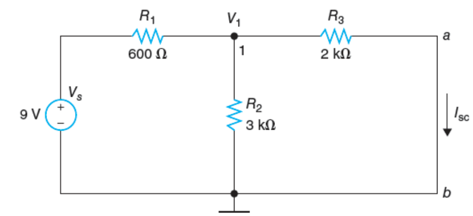


FIGURE 4.108

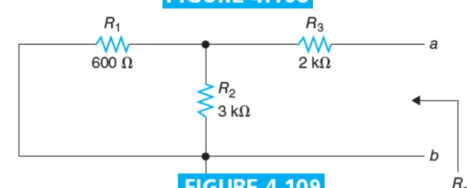


FIGURE 4.109

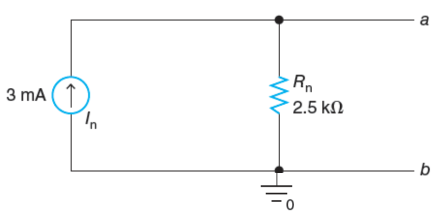


FIGURE 4.110

EXAMPLE 4.14

- Find I_n and R_n for the circuit shown in Figure 4.112.
- To find I_{sc}, terminals *a* and *b* are short-circuited in Figure 4.113.
- Sum the currents leaving node 1:

$$\frac{V_1-9}{33000}-0.002+\frac{V_1}{12000}+\frac{V_1}{3200}=0$$

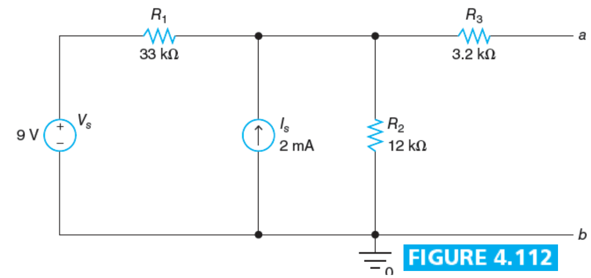


FIGURE 4.112

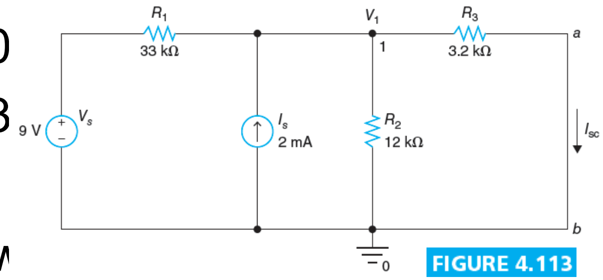


FIGURE 4.113

- Multiply by 33000: $V_1 - 9 - 66 + 2.75V_1 + 10$
- $14.0625V_1 = 75 \Rightarrow V_1 = 75/14.0625$ V = 5.33
- $I_n = I_{sc} = V_1/R_3$ = 1.6667 mA
- To find R_n, V_s and I_s are deactivated as show Figure 4.114.
- $R_a = R_1 \parallel R_2 = 33 \times 12/(33 + 12)$ kΩ = 8.8 kΩ
- $R_n = R_3 + R_a$ = 3.2 kΩ + 8.8 kΩ =12 kΩ

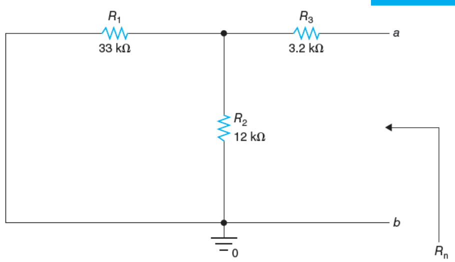


FIGURE 4.114

EXAMPLE 4.16

- Find I_n and R_n for the circuit shown in Figure 4.121. $g_m = 0.003$ S.
- To find I_{sc}, *a* and *b* are short-circuited as shown in Figure 4.122.
- $I = V_s/(R_1 + R_2) = 5\text{ V}/1500\ \Omega = (10/3)\text{ mA}$
- $V_a = R_2 I = 10/3\text{ V} = 3.3333\text{ V}$
- $I_n = I + g_m V_a = (10/3)\text{ mA} + 10\text{ mA} = (40/3)\text{ mA}$
  $= 13.3333\text{ mA}$
- To find R_n, a test voltage of 1 V is applied after short-circuiting V_s as shown in Figure 4.123.
- $V_a = -V_t \times R_2/(R_1 + R_2) = -(2/3)\text{ V}$
- $I_t = V_t/(R_1 + R_2) + V_t/R_3 - g_m V_a$
  $= (2/3)\text{ mA} + 4\text{ mA} + 2\text{ mA} = 20/3\text{ mA}$
- $R_n = V_t/I_t = 150\ \Omega$

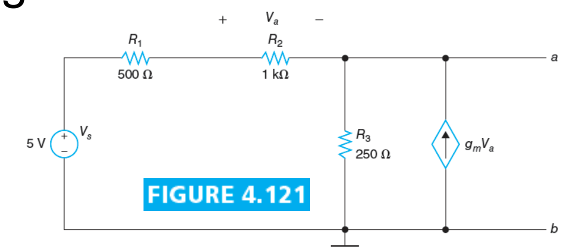


FIGURE 4.121

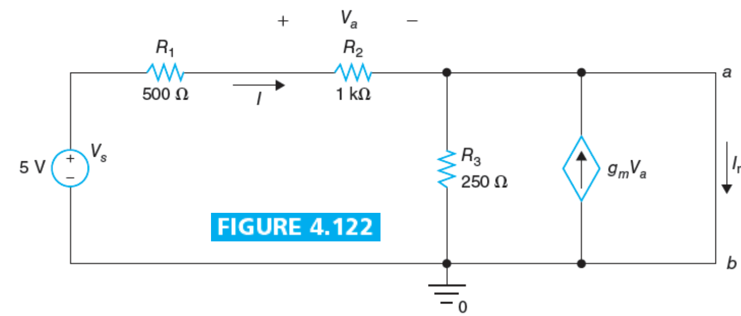


FIGURE 4.122

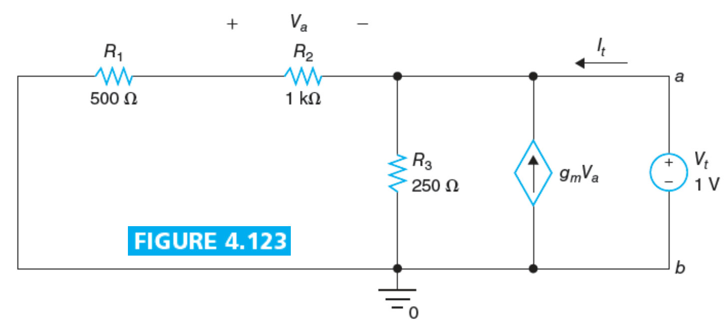


FIGURE 4.123

EXAMPLE 4.17

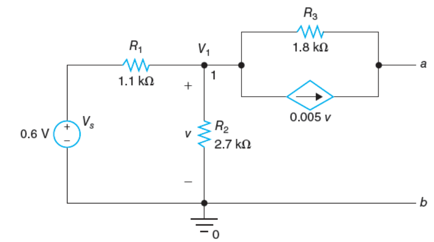


FIGURE 4.126

- Find I_n and R_n for the circuit shown in Figure 4.126.
- To find I_{sc}, *a* and *b* are short-circuited as shown in Figure 4.127.
- Sum the currents leaving node 1:

$$\frac{V_1 - 0.6}{1100} + \frac{V_1}{2700} + \frac{V_1}{1800} + 0.005V_1 = 0$$

- Multiply by 59400: $54V_1 - 32.4 + 22V_1 + 33V_1 + 297V_1 = 0 \Rightarrow$ $406V_1 = 32.4 \Rightarrow V_1 = 32.4/406 = 0.079803$ V
- $I_n = I_{sc} = V_1/R_3 + 0.005V_1 = 443.3498\ \mu A$
- To find R_n, a test voltage of 1 V is applied after short-circuiting V_s as shown in Figure 4.128.

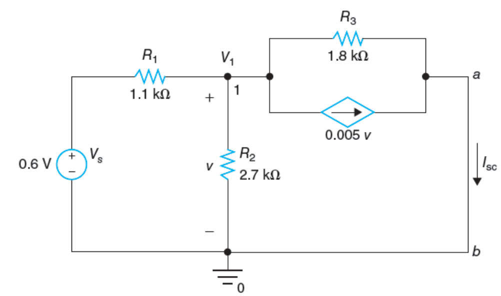


FIGURE 4.127

- Sum the currents leaving node 1: $$\frac{V_1}{1100} + \frac{V_1}{2700} + \frac{V_1 - 1}{1800} + 0.005V_1 = 0$$
- Multiply by 59400: $54V_1 + 22V_1 + 33V_1 + 297V_1 = 33 \Rightarrow V_1 = 33/406$ V $= 0.0812808$ V, $I_t = (1 - V_1)/R_3 - 0.005V_1 = 103.9956\ \mu A$
- $R_n = V_t/I_t = 9.6158\ k\Omega$

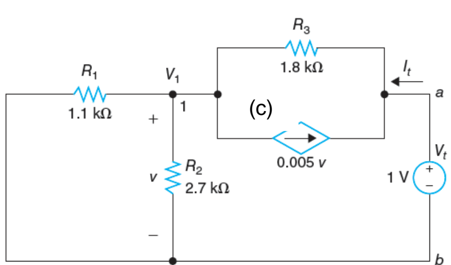


FIGURE 4.128

EXAMPLE 4.18

- Find I_n and R_n for the circuit shown in Figure 4.131.
- Sum the currents leaving node 1 of Figure 4.131: $$\frac{V_1-3}{1200}+\frac{V_1}{3500}+\frac{V_1+800\frac{V_1}{3500}}{5500}=0$$
- Multiply by 46200: $38.5V_1 - 115.5 + 13.2V_1 + 8.4V_1 + 1.92V_1 = 0 \Rightarrow$ $62.02V_1 = 115.5 \Rightarrow$

  $V_1 = 115.5/62.02 = 1.8623$ V, $V_{oc} = (V_1 + 800 \times V_1/R_2) \times R_4/(R_3 + R_4) =$ 1.3728 V
- To find I_{sc}, *a* and *b* are short-circuited as shown in Figure 4.132.

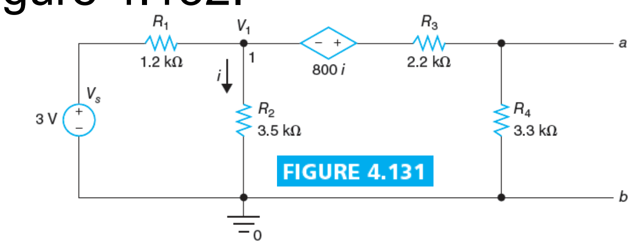


FIGURE 4.131

- Sum the currents leaving node 1 of Figure 4.132: $$\frac{V_1-3}{1200}+\frac{V_1}{3500}+\frac{V_1+800\frac{V_1}{3500}}{2200}=0$$
- Multiply by 46200: $38.5V_1 - 115.5 + 13.2V_1 + 21V_1 + 4.8V_1 = 0$
- $V_1 = 115.5/77.5 = 1.4903$ V
- $I_n = I_{sc} = (V_1 + 800 \times V_1/R_2)/R_3 = 832.2581\ \mu A$
- $R_n = V_{oc}/I_{sc} = 1.6495\ k\Omega$

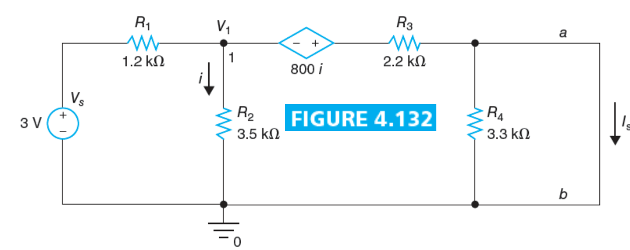


FIGURE 4.132

Maximum Power Transfer

- Suppose that a load with resistance R_L is connected to a circuit between terminals *a* and *b*.
- We are interested in finding the power p_L delivered to the load and finding the load resistance R_L that maximizes the power delivered to the load.
- We first find the Thévenin equivalent circuit with respect to the terminals *a* and *b.*
- Let V_{th} be the Thévenin equivalent voltage and R_{th} be the Thévenin equivalent resistance. With the original circuit replaced by the Thévenin equivalent circuit, we obtain the circuit shown in Figure 4.135.
- The current through the load resistor is given by

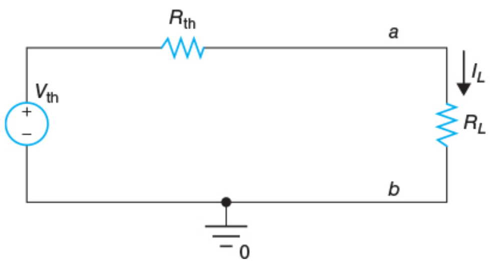
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

- The voltage across the load resistor is given by

$$V_L = R_L I_L = \frac{R_L V_{th}}{R_{th} + R_L}$$

FIGURE 4.135

A load connected to the Thévenin equivalent circuit.



Maximum Power Transfer (Continued)

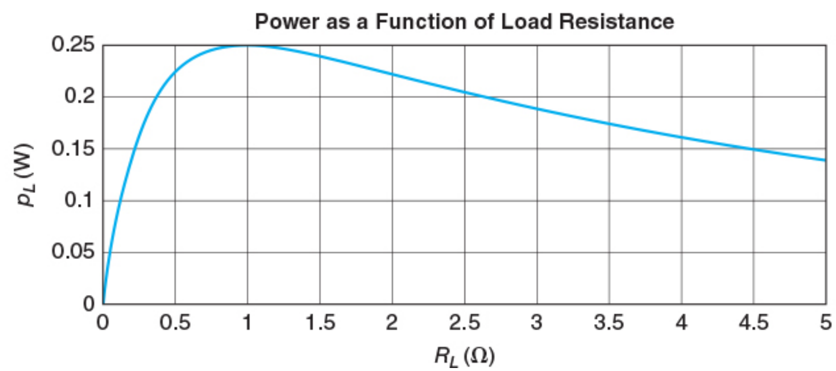
- The power delivered to the load is

$$p_L = I_L V_L = \frac{R_L V_{th}^2}{\left(R_{th} + R_L\right)^2} \quad (1)$$

- When $R_L = 0$, $p_L = 0$; and when $R_L = \infty$, $p_L = 0$.
- The power delivered to the load p_L must peak at a certain value.
- The plot shown in Figure 4.136 shows p_L as a function of R_L for $0 \le R_L \le 5R_{th}$ (V_{th} = 1 V, R_{th} = 1Ω).

FIGURE 4.136

Plot of the power on the load as a function of load resistance.



Maximum Power Transfer (Continued)

- The load resistance value for the maximum power transfer can be found by differentiating Equation (1) with respect to R_L and setting that equal to zero using :

$$\frac{d}{dt}\left(\frac{u(t)}{v(t)}\right) = \frac{v(t)\frac{du(t)}{dt} - u(t)\frac{dv(t)}{dt}}{v^2(t)}$$

$$\frac{dp_L}{dR_L} = \frac{d}{dR_L}\left(\frac{R_L V_{th}^2}{\left(R_{th}+R_L\right)^2}\right) = \frac{\left(R_{th}+R_L\right)^2 \frac{dR_L}{dR_L} - R_L \frac{d\left(R_{th}+R_L\right)^2}{dR_L}}{\left(R_{th}+R_L\right)^4} V_{th}^2 = \frac{\left(R_{th}+R_L\right)^2 \times 1 - R_L 2\left(R_{th}+R_L\right)}{\left(R_{th}+R_L\right)^4} V_{th}^2$$

$$\frac{(R_{th}+R_L)[(R_{th}+R_L) - 2R_L]}{(R_{th}+R_L)^4} {V_{th}}^2 = \frac{[(R_{th}+R_L) - 2R_L]}{(R_{th}+R_L)^3} {V_{th}}^2 = 0$$

- The answer is $R_L = R_{th}$. Thus, the load resistance that maximizes the power transfer to load is given by

  $R_L = R_{th}$ (2)

Maximum Power Transfer (Continued)

- The maximum power delivered to the load when the load resistance is $R_L = R_{th}$ is obtained by using Equation 2 in Equation 1:

$$p_{L,\max} = \frac{R_{th} V_{th}^2}{\left(R_{th} + R_{th}\right)^2} = \frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{4R_L} \quad (3)$$

- When a load resistor is connected to a Norton equivalent circuit as shown below, it can be shown that the load resistance value that provides maximum power to the load is given by

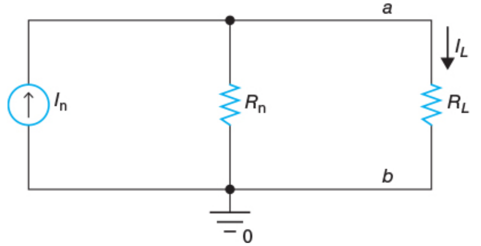
  $R_L = R_n$ (4)

- The maximum power delivered to the load when $R_L = R_n$ is given by

$$p_{L,\max} = \frac{I_n^2 R_n}{4} = \frac{I_n^2 R_L}{4} \quad (5)$$

FIGURE 4.137

A Norton equivalent circuit with load R_L.



EXAMPLE 4.19

- Find the load resistance value R_L that maximizes the power transfer to load for the circuit shown in Figure 4.138. Also find the maximum power delivered to load.

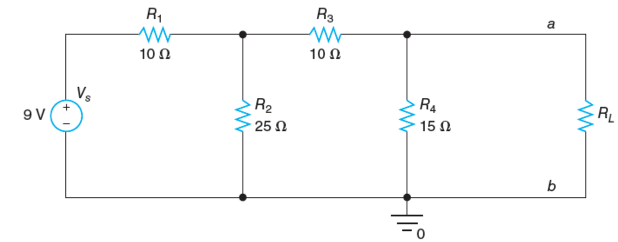


FIGURE 4.138

- Figure 4.139 shows circuit without R_L. Summing currents at node 1: $$\frac{V_1-9}{10}+\frac{V_1}{25}+\frac{V_1-V_2}{10}=0$$

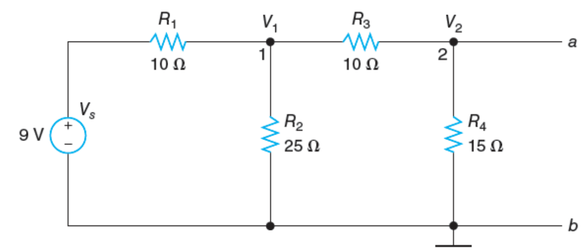


FIGURE 4.139

- Multiplying by 50: $5V_1 – 45 + 2V_1 + 5V_1 – 5V_2 = 0$
- $12V_1 – 5V_2 = 45$ (1)
- Summing currents at node 2:

$$\frac{V_2-V_1}{10}+\frac{V_2}{15}=0$$

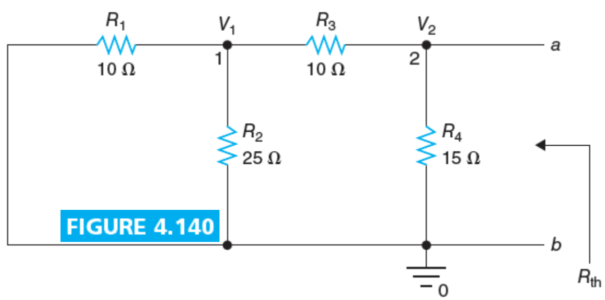


FIGURE 4.140

- Multiplying by 30: $5V_1 – 45 + 2V_1 + 5V_1 – 5V_2 = 0$
- $5V_2 – 3V_1 = 0 \Rightarrow V_2 = 3/5V_1$ (2)
- Substituting Equation 2 in 1: $9V_1 = 45 \Rightarrow V_1 = 5$ V
- $V_2 = V_{th} = 3/5 \times (5) = 3$ V

EXAMPLE 4.19 (Continued)

- We now find R_{th} using Method 1. Figure 4.140 shows the circuit.
- $R_a = R_1 \,||\, R_2 = 250/35\ \Omega = 50/7\ \Omega$
- $R_b = R_3 + R_a = 120/7\ \Omega$
- $R_{th} = R_4 \,||\, R_b = 1800/225\ \Omega = 8\ \Omega$
- $R_L = R_{th} = 8\ \Omega$
- The maximum power would be:
- $p_{L,max} = V_{th}^2/(4R_L) = 9/32\ \text{W} = 281.25\ \text{mW}$

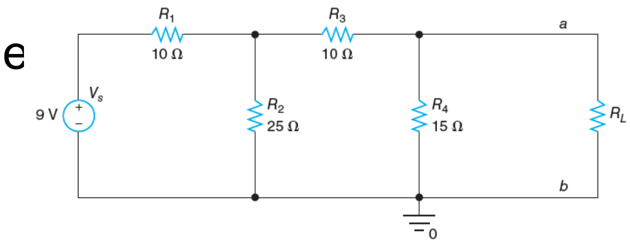


FIGURE 4.138

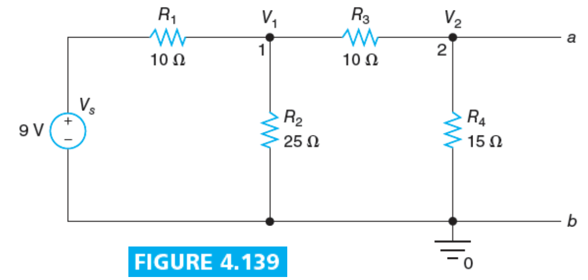


FIGURE 4.139

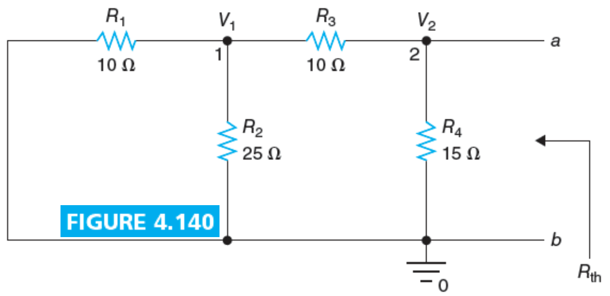


FIGURE 4.140

EXAMPLE 4.20

- Find the load resistance value R_L that maximizes the power transfer to load for the circuit shown in Figure 4.143. Also find the maximum power delivered to load. Figure 4.144 shows circuit without R_L.
- Sum the currents leaving node 1:

$$\frac{V_1 - 0.8}{1500} + \frac{V_1}{3500} + 0.006V_1 = 0$$

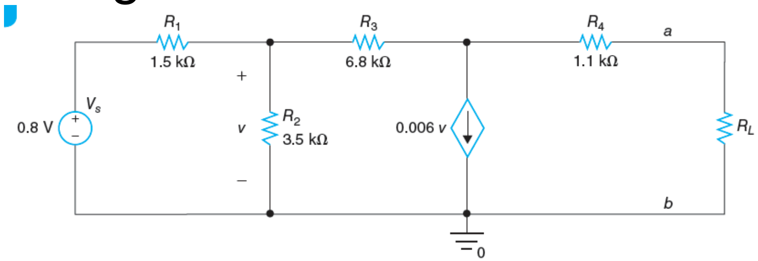


FIGURE 4.143

- Multiply by 10500: $7V_1 - 5.6 + 3V_1 + 63V_1 = 0$

$73V_1 = 5.6 \Rightarrow V_1 = 5.6/73$ V $= 0.07671233$ V

- $V_{th} = V_{oc} = V_1 - R_3 \times 0.006V_1 = -3.05315$ V

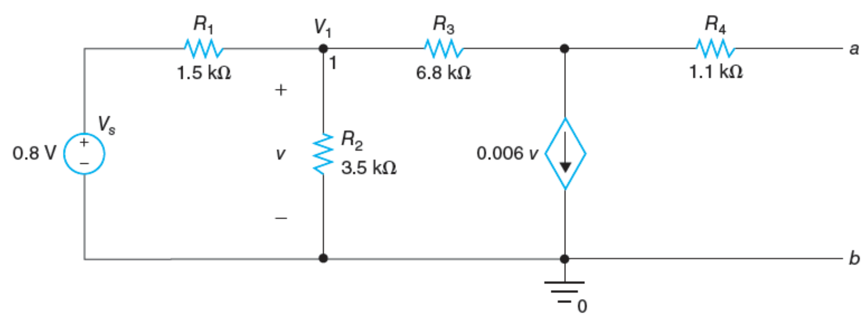


FIGURE 4.144

EXAMPLE 4.20 (Continued)

- To find the short-circuit current, *a* and *b* are short-circuited as shown in Figure 4.145.
- Sum the currents leaving node 1:

$$\frac{V_1 - 0.8}{1500} + \frac{V_1}{3500} + \frac{V_1 - V_2}{6800} = 0$$

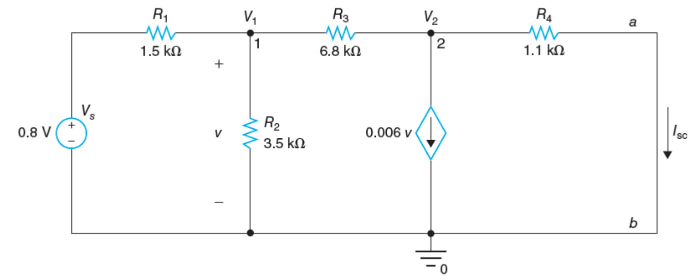


FIGURE 4.145

- Multiply by 71400:

$47.6V_1 - 38.08 + 20.4V_1 + 10.5V_1 - 10.5V_2 = 0$

$78.5V_1 = 10.5V_2 + 38.08 \Rightarrow V_1 = 0.1338V_2 + 0.4851$ (1)

- Sum the currents leaving node 2: $\frac{V_2 - V_1}{6800} + 0.006V_1 + \frac{V_2}{1100} = 0$

- Multiply by 7480: $1.1V_2 - 1.1V_1 + 44.88V_1 + 6.8V_2 = 0 \Rightarrow 7.9V_2 + 43.78V_1 = 0$ (2)
- (1)→(2) $7.9V_2 + 5.85592V_2 = -21.2375 \Rightarrow V_2 = -1.5488$ V, $I_{sc} = V_2/R_4 = -1.40353$ mA
- $R_{th} = V_{oc}/I_{sc} = 2.1753$ kΩ $= R_L$
- $p_{L,max} = V_{th}^2/(4R_L) = 1.0713$ mW

EXAMPLE 4.21

- Find the load resistance value R_L that maximizes the power transfer to load for the circuit shown in Figure 4.147. Also find the maximum power delivered to load. Figure 4.148 shows circuit without R_L. No current through R_4.
- Sum the currents leaving node 1: $-0.001 + V_1/4500 + V_1/3900 = 0$

  $V_1 = 0.001/(1/4500 + 1/3900) = 2.0893$ V

  $i = V_1/R_1 = 0.4642857$ mA
- $V_{th} = V_{oc} = V_1 \times R_3/(R_2 + R_3) + 2500i$

  $= 2.6071$ V

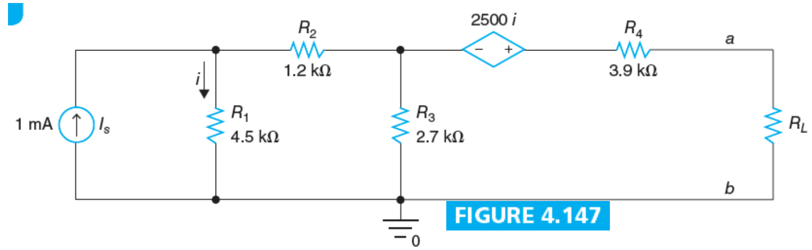


FIGURE 4.147

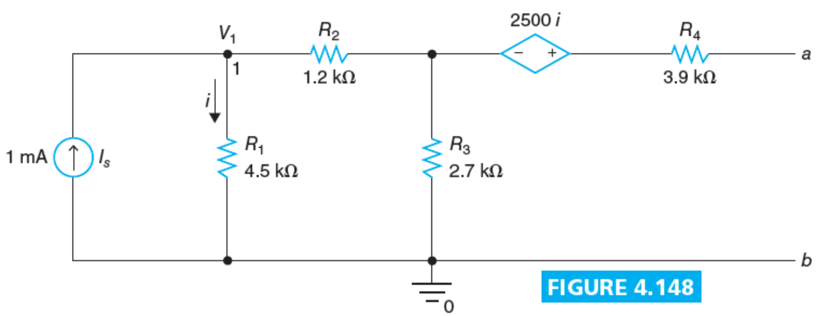


FIGURE 4.148

EXAMPLE 4.21 (Continued)

- To find the short-circuit current, *a* and *b* are short-circuited as shown in Figure 4.149.
- Sum the currents leaving node 1: $$-0.001+\frac{V_1}{4500}+\frac{V_1-V_2}{1200}=0$$
- Multiply by 18000: $4V_1 + 15V_1 - 15V_2 = 18 \Rightarrow 19V_1 - 15V_2 = 18$ (1)
- Sum the currents leaving node 2: $$\frac{V_2-V_1}{1200}+\frac{V_2}{2700}+\frac{V_2+2500\dfrac{V_1}{4500}}{3900}=0$$
- Multiply by 14040:

  $11.7V_2 - 11.7V_1 + 5.2V_2 + 3.6V_2 + 2V_1 = 0 \Rightarrow -9.7V_1 + 20.5V_2 = 0 \Rightarrow$
  $V_1 = (205/97)V_2$ (2)
- (2) $\rightarrow$ (1): $V_2 = 0.71557377$ V, $V_1 = 1.5123$ V
- $I_{sc} = (V_2 + 2500 \times V_1/R_1)/R_4 = 0.3989$ mA
- $R_{th} = V_{oc}/I_{sc} = 6.5357\ \text{k}\Omega = R_L$
- $p_{L,max} = V_{th}^2/(4R_L) = 0.26$ mW

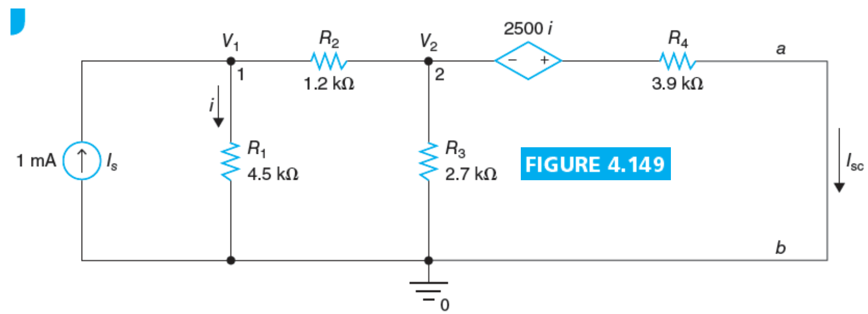


FIGURE 4.149

Summary

- Thévenin's Theorem
  A circuit consisting of a voltage source V_{th} and a series resistor R_{th}, representing the original circuit looking from a pair of terminals, is called a Thévenin equivalent circuit. The voltage V_{th} is called Thévenin equivalent voltage, and the resistance R_{th} is called Thévenin equivalent resistance. There are three methods to find Thévenin equivalent resistance.
- Method 1: Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources. Find the equivalent resistance looking into the circuit from terminals *a* and *b.*
- Method 2: Short-circuit terminals *a* and *b*. Find the short-circuit current I_{sc}. The Thévenin equivalent resistance is given by $R_{th} = V_{oc}/I_{sc} = V_{th}/I_{sc}$.
- Method 3: Deactivate all the independent sources. Apply a test voltage of 1 V between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage. Measure the current flowing out of the positive terminal of the test voltage source. The Thévenin equivalent resistance is the ratio of the voltage to current. Test current can be used also.

Summary (Continued)

- Norton's Theorem

  A circuit looking from terminals *a* and *b* can be replaced by a current source with current I_n and a parallel resistor with resistance R_n. This equivalent circuit consisting of a current source and a parallel resistor is called Norton equivalent circuit. The current I_n is called Norton equivalent current and the resistance R_n is called Norton equivalent resistance.
- Finding Norton equivalent resistance:
- Method 1: Deactivate all the independent sources by short-circuiting voltage sources and open-circuiting current sources. Find the equivalent resistance looking into the circuit from terminals *a* and *b.*
- Method 2: Short-circuit terminals *a* and *b*. Find the short-circuit current I_{sc}. The Norton equivalent resistance is given by $R_n = V_{oc}/I_{sc} = V_{oc}/I_n$.
- Method 3: Deactivate all the independent sources. Apply a test voltage of 1 V between terminals *a* and *b* with terminal *a* connected to the positive terminal of the test voltage. Measure the current flowing out of the positive terminal of the test voltage source. The Norton equivalent resistance is the ratio of the voltage to current. Test current can be used also.

Summary (Continued)

- Maximum Power Transfer

  Suppose that a load with resistance R_L is connected to a circuit between terminals *a* and *b*. The load resistance that maximizes the power transfer to the load is given by

  $R_L = R_{th}$

  where R_{th} is the Thévenin equivalent resistance when the circuit between terminals *a* and *b* looking from the load is replaced by Thévenin equivalent circuit.
- The maximum power delivered to the load when the load resistance is $R_L = R_{th}$ is given by

$$p_{L,\max} = \frac{V_{th}^2}{4R_{th}} = \frac{V_{th}^2}{4R_L}$$