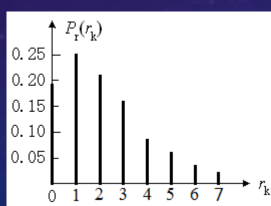
HISTOGRAM EQUALIZATION

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)P_r'(k)$$

Suppose a 64×64 image，with 8 gray levels([0,7])，has the intensity distribution shown in the table

r_k	0	1	2	3	4	5	6	7
n_k	790	1023	850	656	329	245	122	81
$P_k(r_k)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02



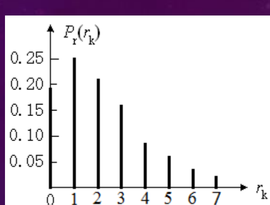
CDF $P_r'(0) = \sum_{j=0}^{0} p_r(r_j) = 0.19$

$$P_r'(1) = \sum_{j=0}^{1} p_r(r_j) = p_r(r_0) + p_r(r_1) = P_r'(0) + p_r(r_1) = 0.19 + 0.25 = 0.44$$

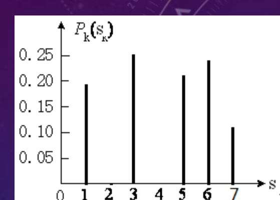
Similarly:

$P_r'(2) = 0.65$ $\quad P_r'(3) = 0.81$ $\quad P_r'(4) = 0.89$

$P_r'(5) = 0.95$ $\quad P_r'(6) = 0.98$ $\quad P_r'(7) = 1.00$

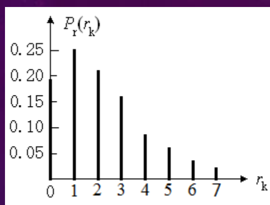


HISTOGRAM EQUALIZATION

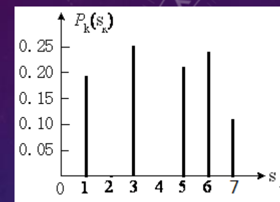


$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)P_r'(k)$$

Operations	Outputs							
Original gray level r_k	0	1	2	3	4	5	6	7
Pixel # of gray level in the ori hist	790	1023	850	656	329	245	122	81
Original hist $P(r_k)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
CDF $p(r'k)$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00
Rounding: $s_k = \mathrm{int}[(L-1)p(r'k)+0.5]$	1	3	5	6	6	7	7	7
Mapping relationship ($r_k \to s_k$)	0→1	1→3	2→5	3, 4→6		5, 6, 7→7		
Pixel # of gray level in the new hist		790		1023		850	985	448
New hist $P(s_k)$		0.19		0.25		0.21	0.24	0.11



HISTOGRAM EQUALIZATION



- Discrete histogram equalization has the general tendency to spread the histogram of the input image, resulting a contrast enhancement
- It is fully *automatic*

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

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AM EQUALIZATION

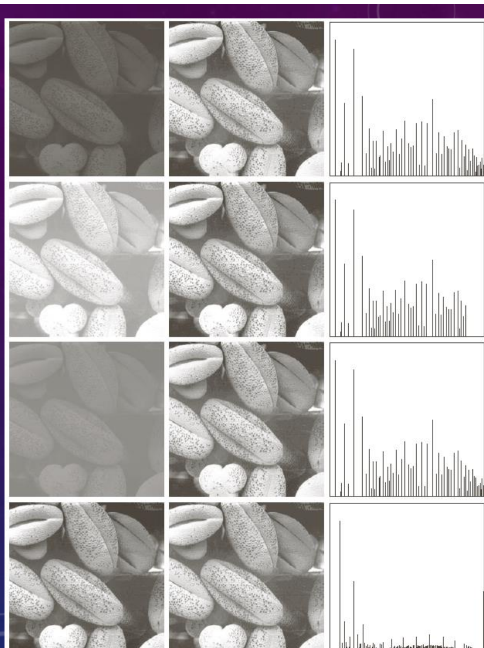


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

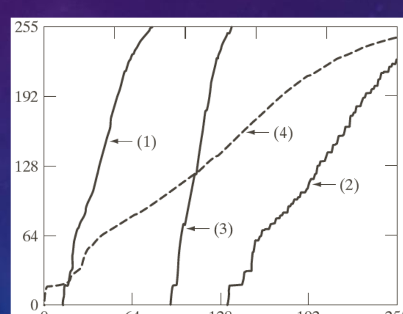
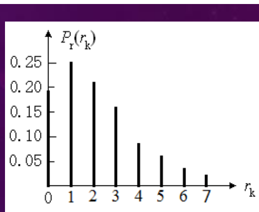


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).



HISTOGRAM EQUALIZATION

What should we do if we still want to produce uniform histogram in discrete histogram equalization?

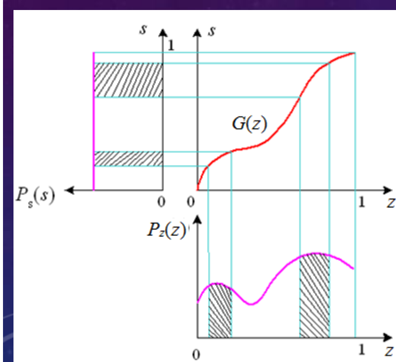
	s_0	s_1	s_2	s_3	s_4	s_5	s_6	s_7	b4
r_0	512	278							790
r_1		234	512	277					1023
r_2				235	512	103			850
r_3						409	247		656
r_4							265	64	329
r_5								245	245
r_6								122	122
r_7								81	81
after	512	512	512	512	512	512	512	512	4096

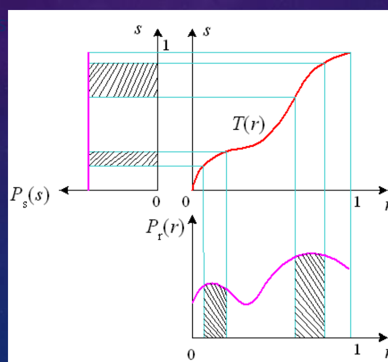
Pick up randomly v.s. Consider the gray level of neighbors

HISTOGRAM MODIFICATION

- Histogram Specification
  - There are applications in which a uniform histogram is not the best choice
  - The method used to generate a processed image that has a specified histogram is called *histogram specification/histogram matching*

HISTOGRAM SPECIFICATION





$$s = T(r) = \int_0^r p_r(\omega)d\omega$$

$$s = G(z) = \int_0^z p_r(t)dt$$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

HISTOGRAM SPECIFICATION

- Equalizing the input image
- Obtaining the equalized image in which the pixel values are the s values
- Performing the inverse mapping

HISTOGRAM SPECIFICATION

- Discrete histogram equalization transformation:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)P_r'(k) = \frac{L-1}{MN}\sum_{j=0}^{k} n_j$$

- The transformation function for the specified histogram:

$$s_k = G(z_q) = (L-1)\sum_{j=0}^{q} p_z(z_j) = (L-1)P_z'(q) = \frac{L-1}{MN}\sum_{j=0}^{q} n_j$$

Where $p_z(z_j)$ is the jth value of the specified histogram

$$z_q = G^{-1}(s_k)$$

HISTOGRAM SPECIFICATION

- Compute the histogram $p_r(r)$ of the given image, calculate s_k and round s_k to the integer range [0, L-1]

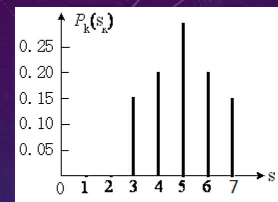
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

- Compute all values of the transformation function G, round the values of G to integers in the range [0, L-1], store the values in a table

$$G(z_q) = (L-1)\sum_{j=0}^{q} p_z(z_j)$$

- For every value of s_k, use the stored values of G to find the corresponding value of z_q so that $G(z_q)$ is closest to s_k and store these mappings from s to z(when the mapping is not unique, choose the smallest value)

$$z_q = G^{-1}(s_k)$$

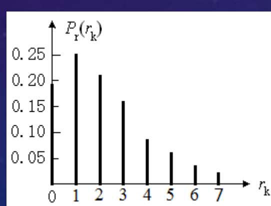


HISTOGRAM SPECIFICATION

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)P_r'(k)$$

Suppose a 64×64 image，with 8 gray levels([0,7])，has the intensity distribution shown in the table

r_k	0	1	2	3	4	5	6	7
n_k	790	1023	850	656	329	245	122	81
$P_k(r_k)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02



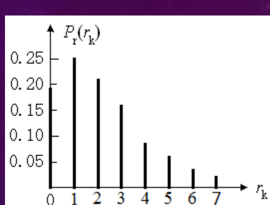
CDF $P_r'(0) = \sum_{j=0}^{0} p_r(r_j) = 0.19$

$$P_r'(1) = \sum_{j=0}^{1} p_r(r_j) = p_r(r_0) + p_r(r_1) = P_r'(0) + p_r(r_1) = 0.19 + 0.25 = 0.44$$

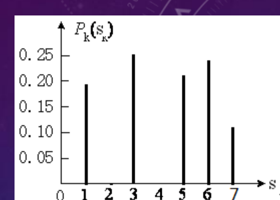
Similarly:

$$P_r'(2) = 0.65 \quad P_r'(3) = 0.81 \quad P_r'(4) = 0.89$$

$$P_r'(5) = 0.95 \quad P_r'(6) = 0.98 \quad P_r'(7) = 1.00$$



HISTOGRAM SPECIFICATION



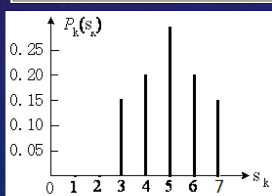
$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = (L-1)P_r'(k)$$

Operations	**Outputs**							
Original gray level r_k	0	1	2	3	4	5	6	7
Pixel # of gray level in the ori hist	790	1023	850	656	329	245	122	81
Original hist $P(r_k)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02
CDF $p(r'k)$	0.19	0.44	0.65	0.81	0.89	0.95	0.98	1.00
Rounding: s_k=int[(L-1)p(r'k)+0.5]	1	3	5	6	6	7	7	7
Mapping relationship ($r_k \to s_k$)	0→1	1→3	2→5	3, 4→6		5, 6, 7→7		
Pixel # of gray level in the new hist		790		1023		850	985	448
New hist $P(s_k)$		0.19		0.25		0.21	0.24	0.11

HISTOGRAM SPECIFICATION

the specified histogram shown in the table

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15



$$s_k = G(z_q) = (L-1)\sum_{j=0}^{q} p_z(z_j) = (L-1)P_z'(q)$$

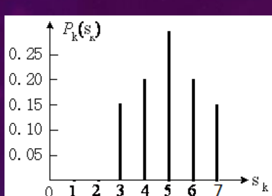
CDF $P_z'(0) = \sum_{j=0}^{0} p_z(z_j) = 0.00$

$$P_z'(1) = \sum_{j=0}^{1} p_z(z_j) = p_z(z_0) + p_z(z_1) = P_z'(0) + p_z(z_1) = 0.00 + 0.00 = 0.00$$

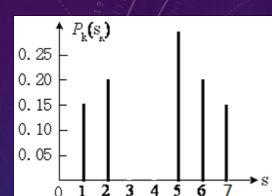
Similarly:

$P_z'(2) = 0.00$ $P_z'(3) = 0.15$ $P_z'(4) = 0.35$

$P_z'(5) = 0.65$ $P_z'(6) = 0.85$ $P_z'(7) = 1.00$



HISTOGRAM SPECIFICATION

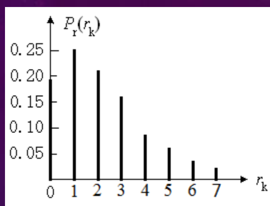


$$s_k = G(z_q) = (L-1)\sum_{j=0}^{q} p_z(z_j) = (L-1)P_z'(q)$$

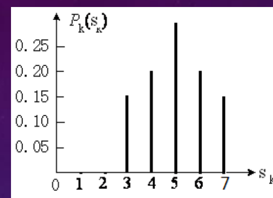
Operations	Outputs							
Specified gray level	0	1	2	3	4	5	6	7
Specified histogram p(zq)	0	0	0	0.15	0.2	0.3	0.2	0.15
CDF p(z'q)	0	0	0	0.15	0.35	0.65	0.85	1
Rounding:sk=INT[(L-1)p(z'q)+0.5]	0	0	0	1	2	5	6	7
Mapping relationship (zq->sk)	0,1,2->0			3->1	4->2	5->5	6->6	7->7
New hist P(sk)		0.15	0.2			0.3	0.2	0.15

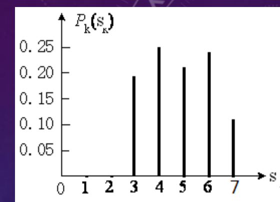
	Mapping							
s	0	1	2	3	4	5	6	7
z	0	3	4	4	5	5	6	7

z_q	$G(z_q)$
$z_0 = 0$	0
$z_1 = 1$	0
$z_2 = 2$	0
$z_3 = 3$	1
$z_4 = 4$	2
$z_5 = 5$	5
$z_6 = 6$	6
$z_7 = 7$	7



HISTOGRAM N





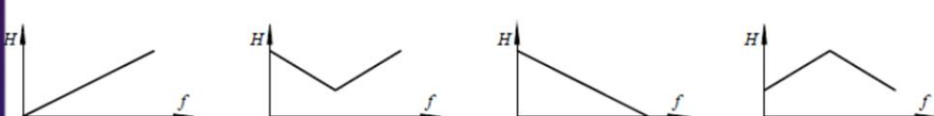
Operations	Outputs							
Original gray level r_k	0	1	2	3	4	5	6	7
Pixel								
Origi								
CDF								
Rounding: s_k=int[(L-1)p(r'k)+0.5]	~~1~~	~~3~~	~~5~~	~~6~~	~~6~~	~~7~~	~~7~~	~~7~~
Mapping relationship ($r_k \to s_k$)	0→1	1→3	2→5	3, 4→6		5, 6, 7→7		
Pixel # of gray level in the new hist		790		1023		850	985	448
New hist P(s_k)		0.19		0.25		0.21	0.24	0.11

Mapping								
s	0	1	2	3	4	5	6	7
z	0	3	4	4	5	5	6	7

r	s	z
0	1	3
1	3	4
2	5	5
3	6	6
4	6	6
5	7	7
6	7	7
7	7	7

HISTOGRAM SPECIFICATION

Which specified histogram gives better constrast of the input image ?

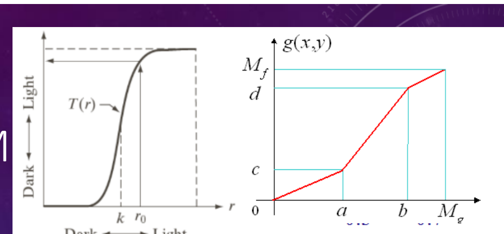


(A) (B) (C) (D)

IMAGE ENHANCEMENT

- Spatial domain processing $g(x, y) = T[f(x, y)]$
  - Spatial filtering
    - image sharpening(gradient/laplacian/unsharp masking & highboost filtering), smoothing(averaging/median filter)
  - Intensity transformation
    - Contrast manipulation, thresholding, histogram equalization, histogram specification
- Frequency domain processing $g(x, y) = F^{-1}\{T[F[f(x, y)]]\}$
  - Homomorphic filtering

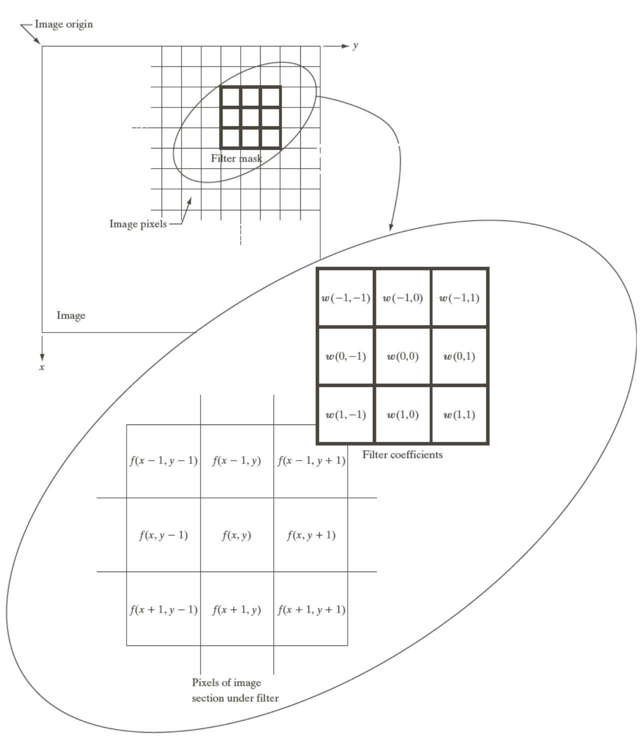
IMAGE ENHANCEM



- Contrast manipulation -> imadjust
- Thresholding -> im2bw
- histogram equalization -> histeq
  -> adapthisteq
- histogram specification -> histeq

SPATIAL DOMAIN FILTERING

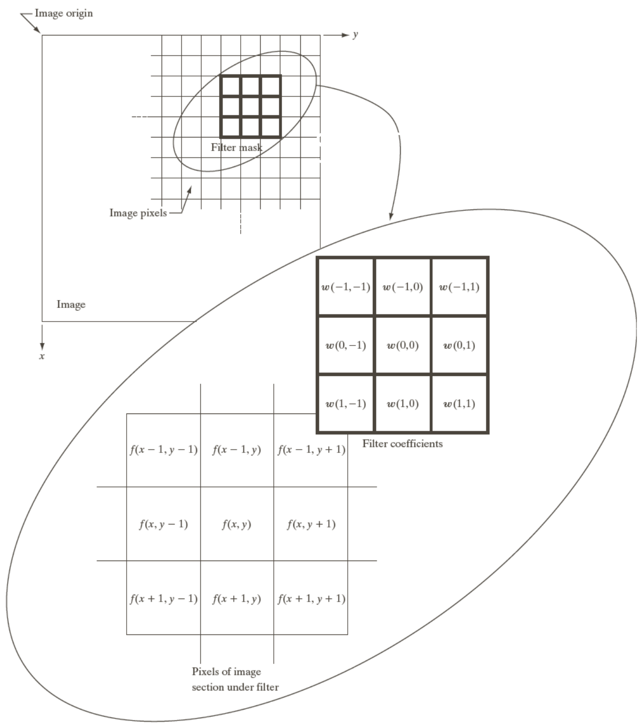
- Smoothing spatial filters
  - Smoothing linear filters
  - Order-statistic(nonlinear) filters
- Sharpening spatial filters
  - The laplacian
  - Unsharp masking and highboost filtering
  - The gradient



FILTERING

$$\begin{bmatrix} w_1 & w_2 & w_3 \\ w_4 & w_5 & w_6 \\ w_7 & w_8 & w_9 \end{bmatrix}$$

$$\begin{aligned} R &= w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9 \\ &= \sum_{k=1}^{8} w_k z_k \\ &= \mathbf{w}^T \mathbf{z} \end{aligned}$$



FILTERING

$$h(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

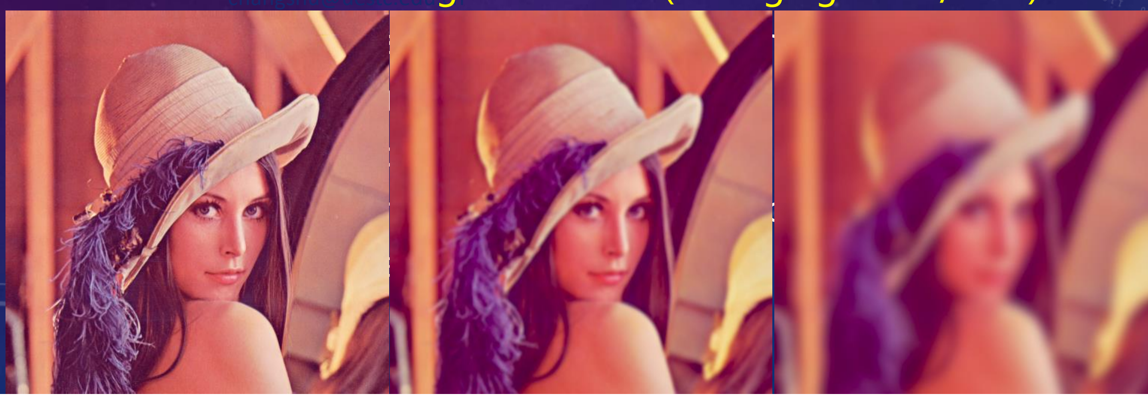
$$\begin{bmatrix} h(-1,-1) & h(-1,0) & h(-1,1) \\ h(0,-1) & h(0,0) & h(0,1) \\ h(1,-1) & h(1,0) & h(1,1) \end{bmatrix}$$

IMAGE SMOOTHING

- To smooth a data set is to create an approximating function that attempts to capture important patterns in the data, while leaving out noise or other fine-scale structures/rapid phenomena
- Image smoothing can be done in either spatial domain or frequency domain

SPATIAL DOMAIN FILTERING

- Smoothing spatial filters
  - Smoothing linear filters(averaging filters/LPFs)



SPATIAL DOMAIN FILTERING

$$h_1 = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad h_2 = \frac{1}{10}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad h_3 = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Box filter

An attempt to reduce blurring in the smoothing process

SPATIAL DOMAIN FILTERING

SPATIAL DOMAIN FILTERING

- Smoothing spatial filters
  - Smoothing linear filters
  - Order-statistic(nonlinear) filters --- median filter
- Sharpening spatial filters
  - The laplacian
  - Unsharp masking and highboost filtering
  - The gradient

SPATIAL DOMAIN FILTERING

- Median filter

For a 1D sequence $f_1, f_2, \ldots, f_n$

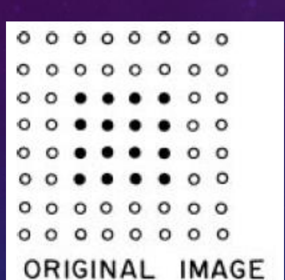
The median filter is of size m(m *is odd*);

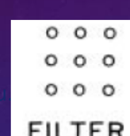
First sort the values of the pixel in the m neighborhood pixels, $f_{i-v}, \ldots, f_{i-1}, f_i, f_{i+1}, \ldots, f_{i+v}$, $v=(m-1)/2$, determine their median ξ, and assign that value to the corresponding pixel in the filtered image:

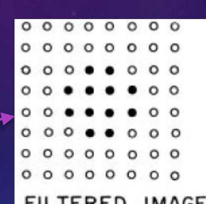
$$Y_i = Med\{f_{i-v}, \ldots, f_i, \ldots, f_{i+v}\} \qquad i \in Z, v = \frac{m-1}{2} \qquad \{0,3,4,0,7\}$$

$$Y_{ij} = \underset{A}{Med}\{X_{ij}\}$$ A is filter mask

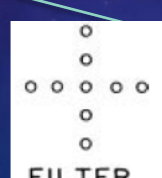
IMAGE SMOOTHING

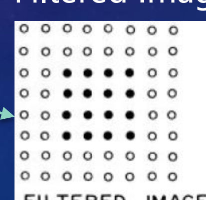






Filtered Image?





Filtered Image?