Signals and Systems

Chapter 6

Time and frequency characterization of S&S

6. Time and Frequency Characterization of Signals and Systems

6.1 The Magnitude-phase Representation of the Fourier Transform

For signal x(t) : $x(t) \xleftrightarrow{F} X(j\omega)$

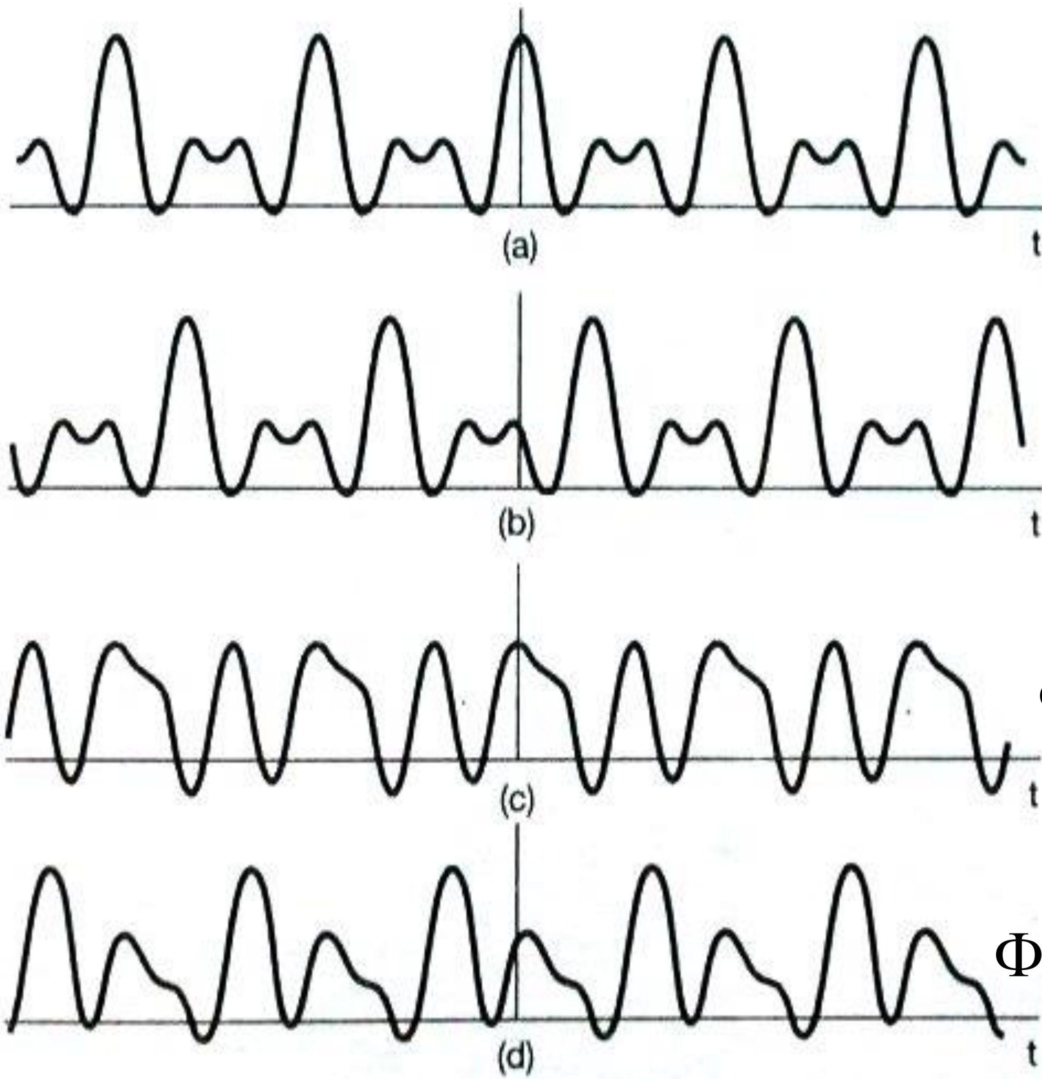
$$X(j\omega) = |X(j\omega)| e^{j\angle X(j\omega)}$$

$|X(j\omega)|$ --- *Magnitude Spectrum*

$\angle X(j\omega)$ --- *Phase Spectrum*

6 Time and frequency characterization of S&S

$$x(t)=1+\frac{1}{2}\cos(2\pi t+\Phi_1)+\cos(4\pi t+\Phi_2)+\frac{2}{3}\cos(6\pi t+\Phi_3)$$



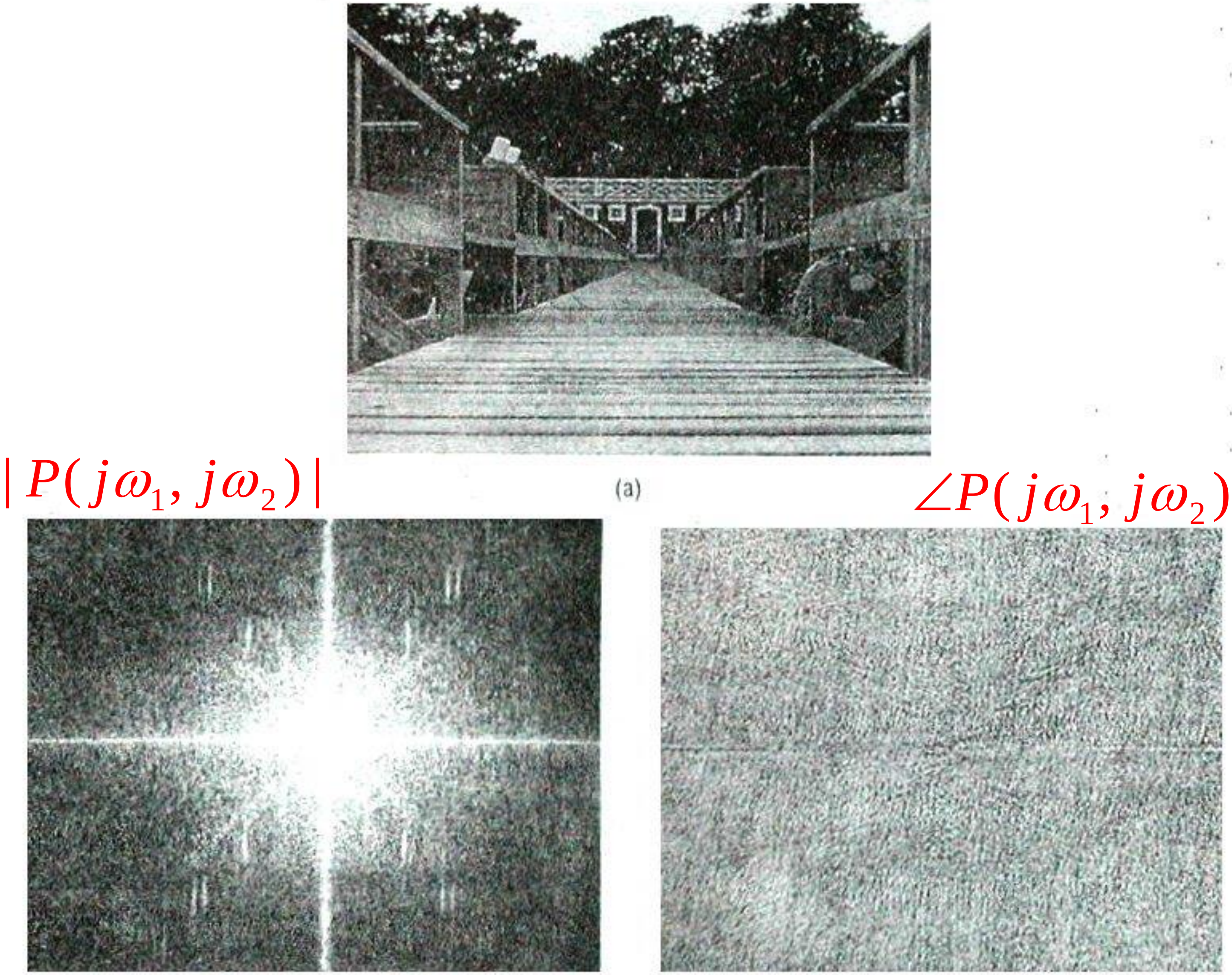
$$\Phi_1=\Phi_2=\Phi_3=0$$

$$\Phi_1=4,\Phi_2=8,\Phi_3=12$$

$$\Phi_1=6,\Phi_2=-2.7,\Phi_3=0.93$$

$$\Phi_1=1.2,\Phi_2=4.1,\Phi_3=-7.2$$

6 Time and frequency characterization of S&S

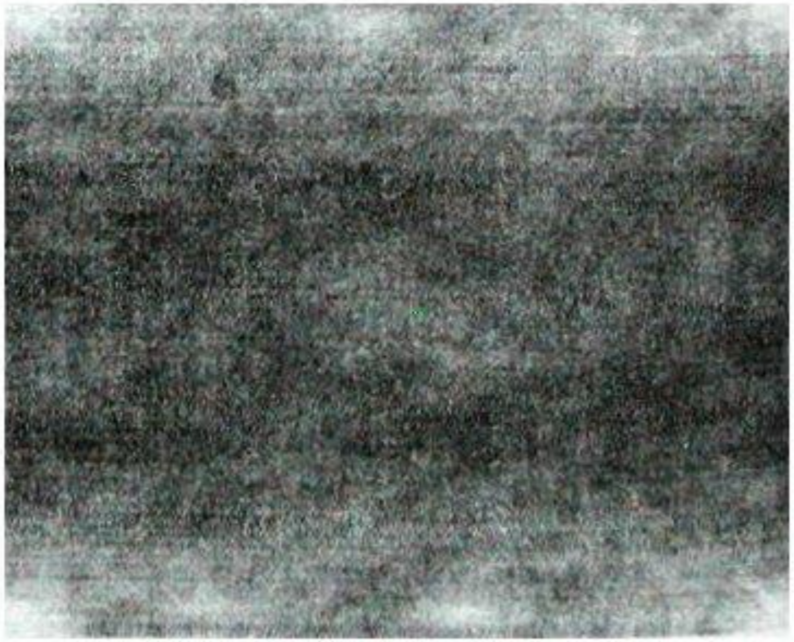


(a)

(b)

(c)

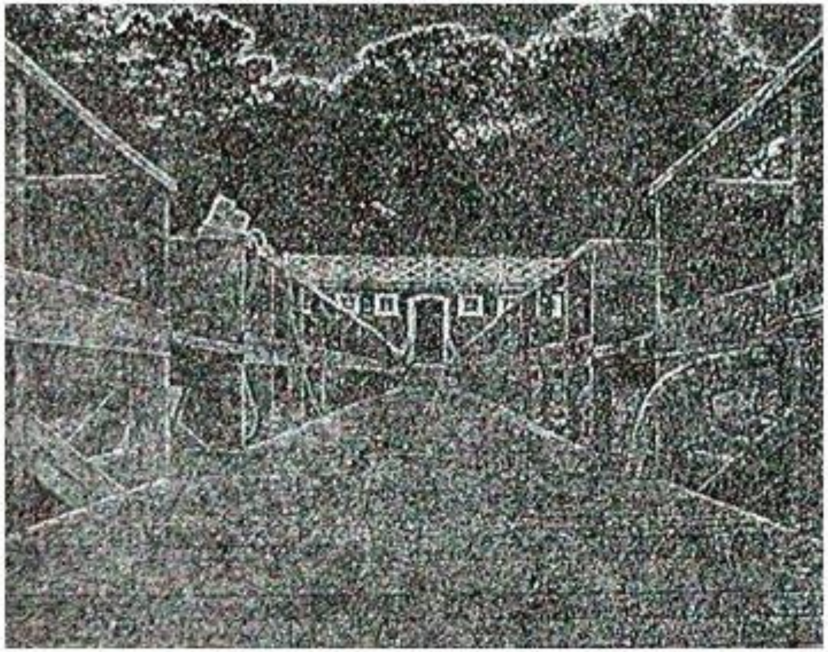
6 Time and frequency characterization of S&S



(d)

$$Magnitude: |P(j\omega_1, j\omega_2)|$$

$$Phase: 0$$



(e)

$$Magnitude: 1$$

$$Phase: \angle P(j\omega_1, j\omega_2)$$

6.2 The Magnitude-phase Representation of the Frequency Response of LTI System

System characterization:

Impulse response: $h(t) \xleftrightarrow{F} H(j\omega)$

Frequency response: $H(j\omega) = \dfrac{Y(j\omega)}{X(j\omega)}$

$$H(j\omega) = |H(j\omega)| e^{j\angle H(j\omega)}$$

$|H(j\omega)|$ --- *Magnitude Response*

$e^{j\angle H(j\omega)}$ --- *Phase Response*

6.2.1 Linear and Nonlinear Phase

Linear phase: $\angle H(j\omega) = k\omega$

Nonlinear phase: $\angle H(j\omega) = Nonlinear \ \ function$

Example: $y(t) = x(t - t_0)$

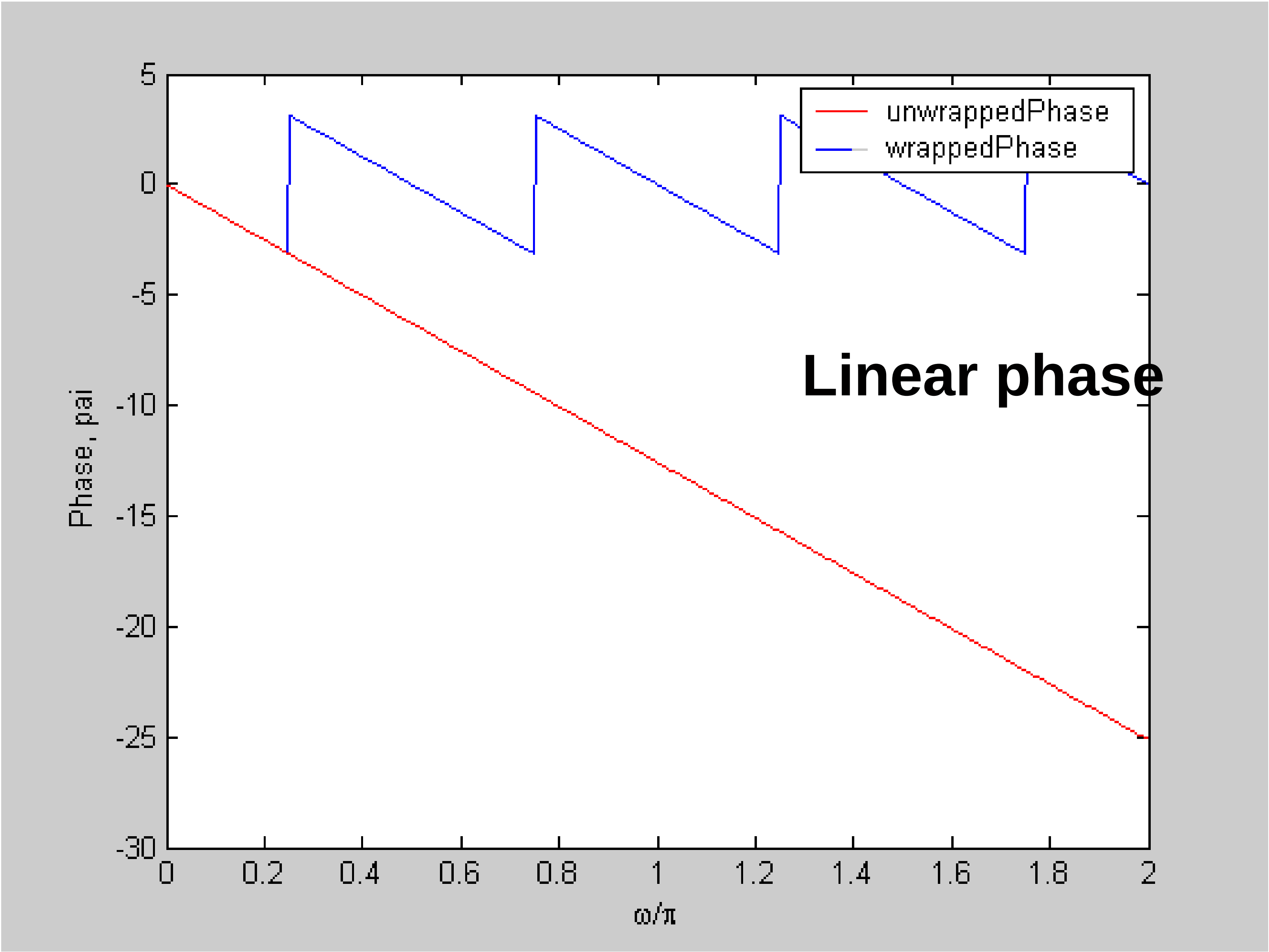
$$H(j\omega) = e^{-j\omega t_0}$$

$$\angle H(j\omega) = -\omega t_0 \ (Linear \ \ phase)$$

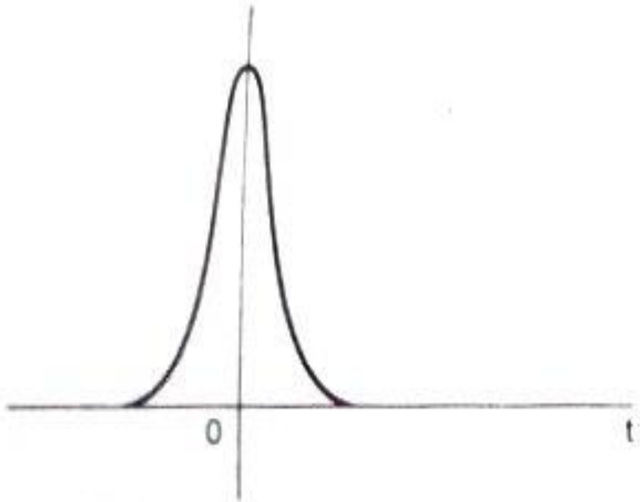
Effect: Linear phase means non-distortion of signal transmission.

($Magnitude \ \ Response$ **is constant**)

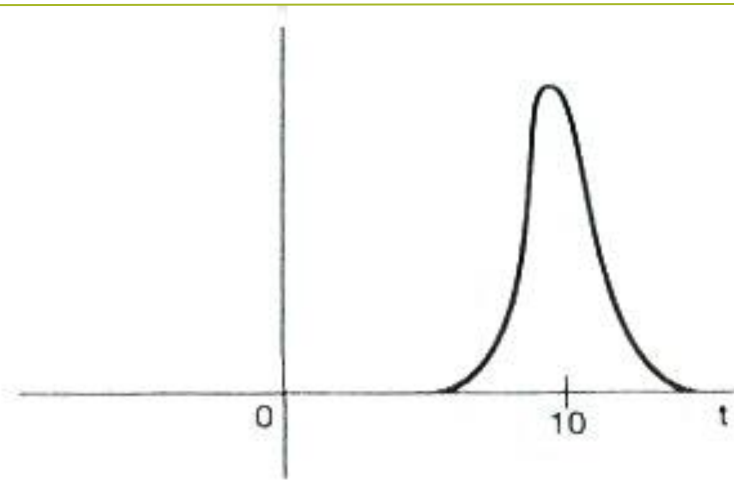
6 Time and frequency characterization of S&S



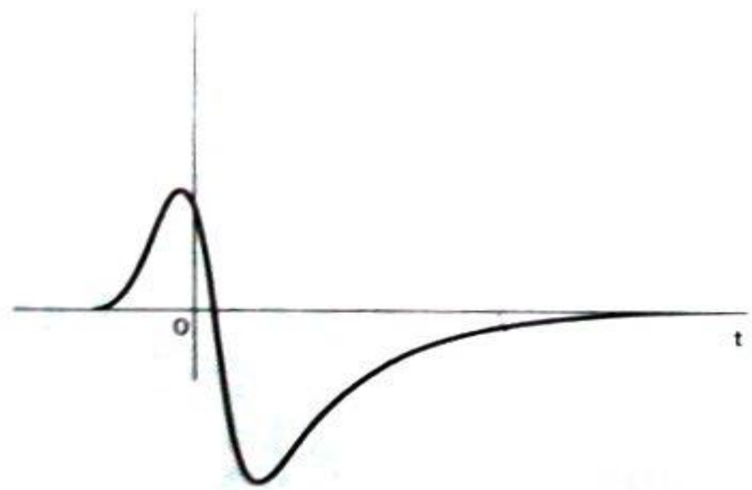
6 Time and frequency characterization of S&S



(Original signal)



(Linear phase)



(Nonlinear phase)

6.2.2 Group Delay

Definition: $$\tau(\omega) = -\frac{d}{d\omega}\angle H(j\omega)$$

Example: $$y(t) = x(t - t_0)$$

$$H(j\omega) = e^{-j\omega t_0}$$

$$\angle H(j\omega) = -\omega t_0$$

$$\tau(\omega) = t_0 \quad (signal\ delay)$$

Distortionless system : $\tau(\omega)$ **is flat.**

Phase Delay

Sinusoidal : $$\sin(\omega_0 t - \Phi) = \sin[\omega_0 (t - \frac{\Phi}{\omega_0})]$$

Phase Delay Definition: $$\frac{\Phi}{\omega_0}$$

6.2.3 Log-Magnitude and Bode Plots

Magnitude spectrum:

$$|H(j\omega)| \quad \sim \quad \omega$$

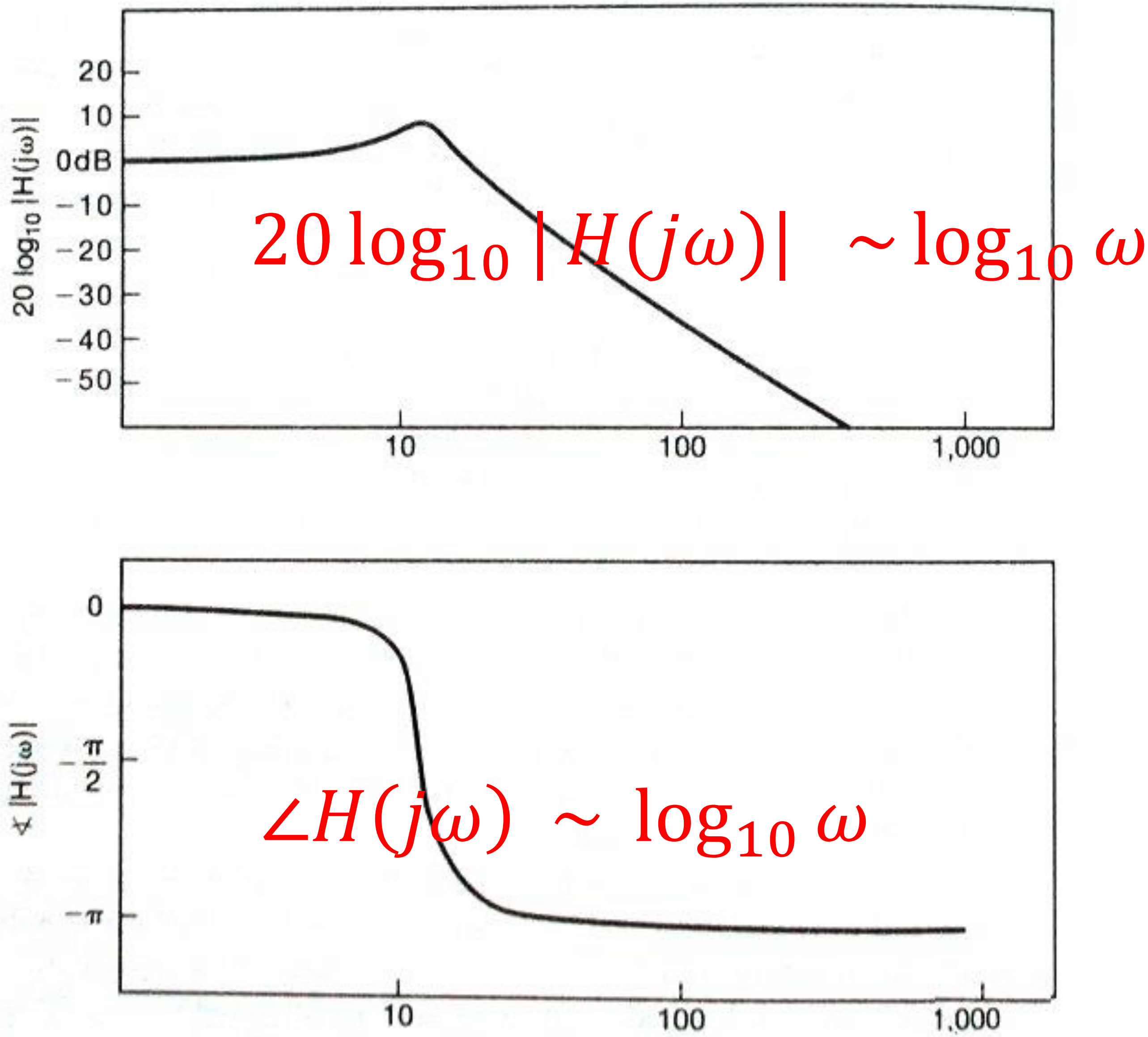
$$20\log_{10}|H(j\omega)| \;\sim \log_{10}\omega \;\; (Bode\ plots)$$

Phase spectrum:

$$\angle H(j\omega) \;\sim \quad \omega$$

$$\angle H(j\omega) \;\sim\; \log_{10}\omega \;\; (Bode\ plots)$$

6 Time and frequency characterization of S&S



6.3 Time-Domain Properties of Ideal Frequency-selective Filters

Lowpass filter:

(1) Continous time:

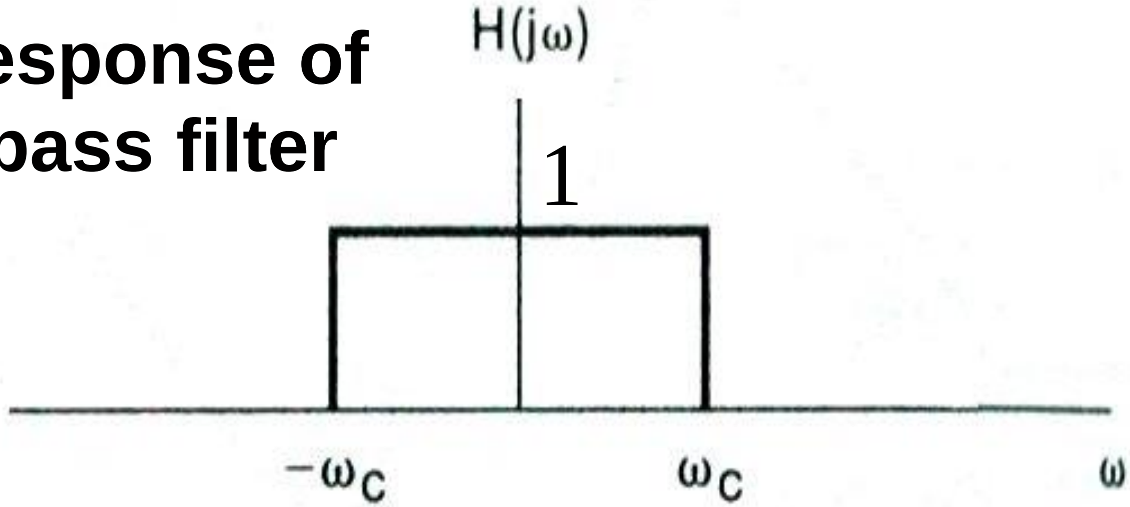
$$H(j\omega)=\begin{cases}1, & |\omega|\le\omega_c\\ 0, & |\omega|>\omega_c\end{cases}$$

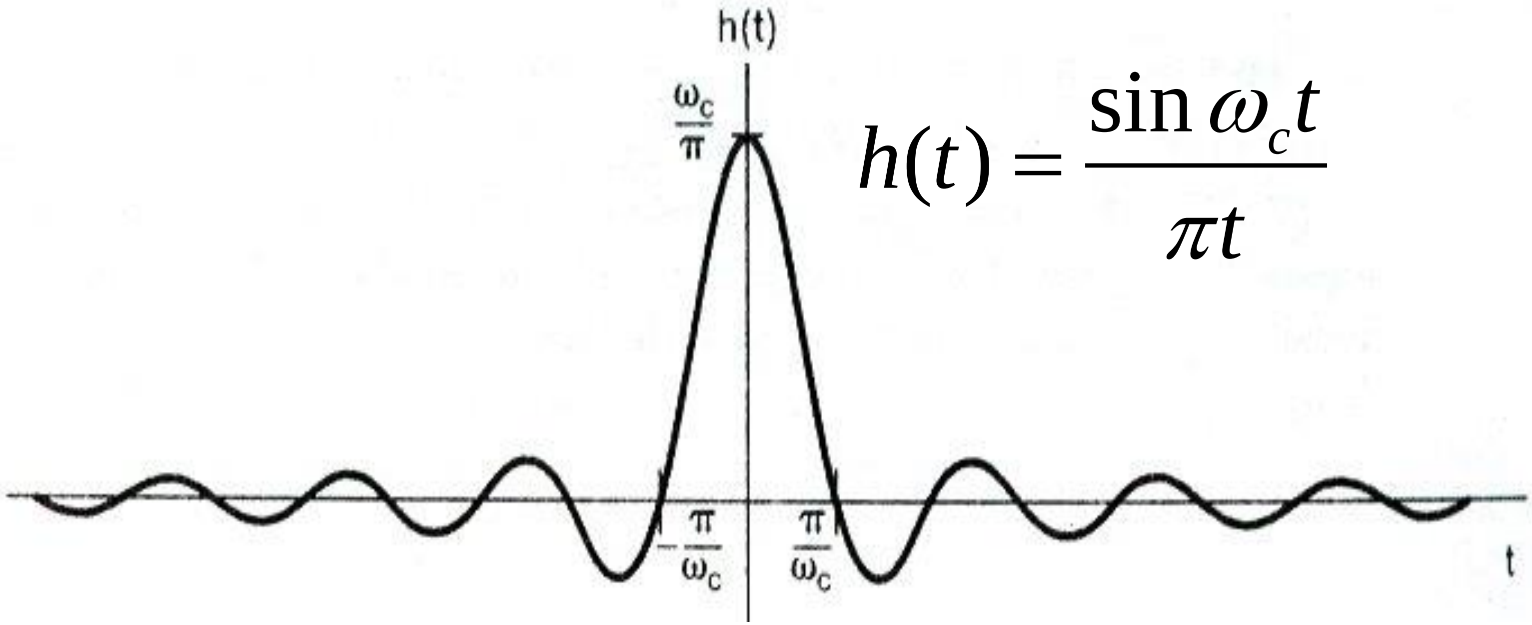
(2) Discrete time:

$$H(e^{j\omega})=\begin{cases}1, & |\omega|\le\omega_c\\ 0, & \omega_c<|\omega|\le\pi\end{cases}$$

6 Time and frequency characterization of S&S

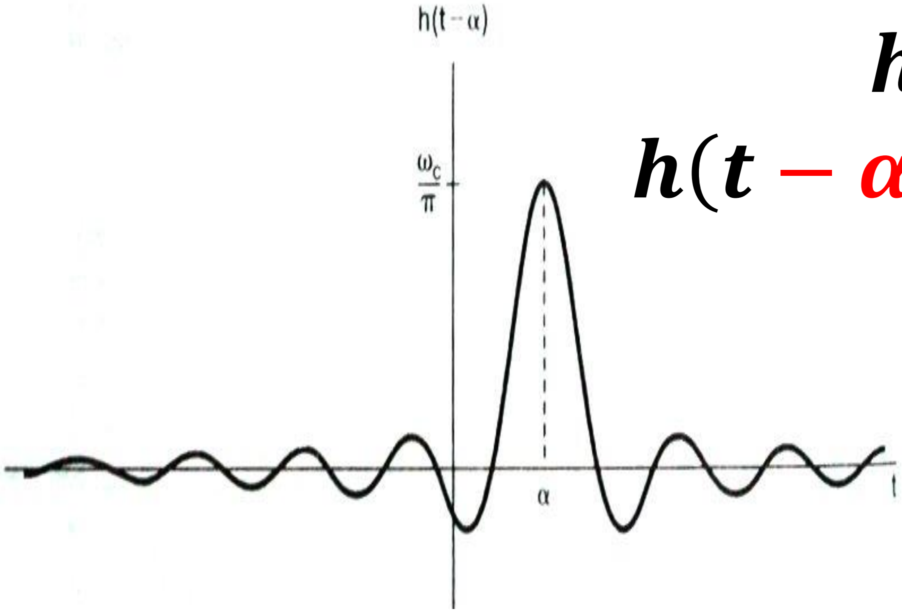
Impulse response of Ideal Lowpass filter





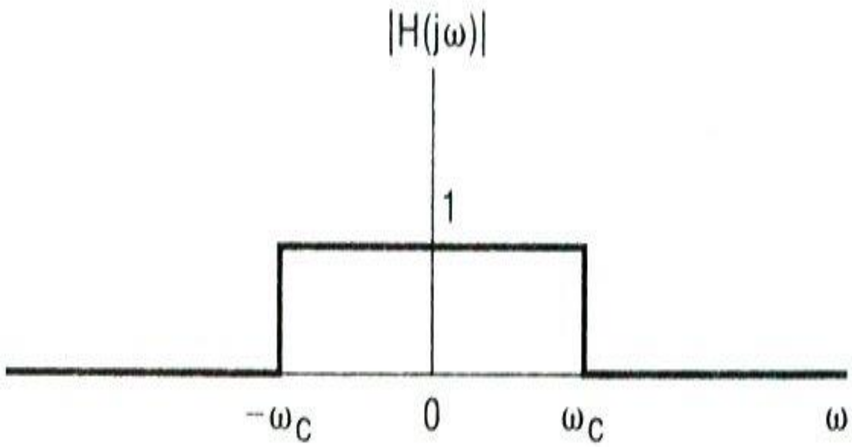
$$h(t) = \frac{\sin \omega_c t}{\pi t}$$

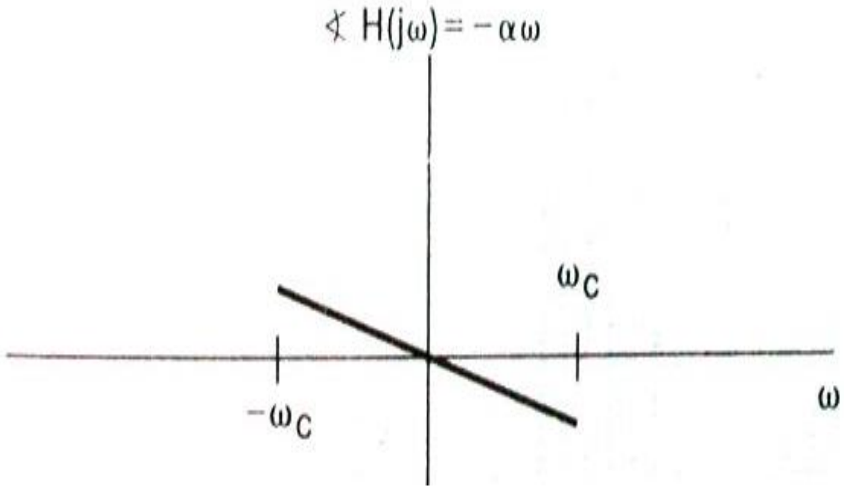
6 Time and frequency characterization of S&S



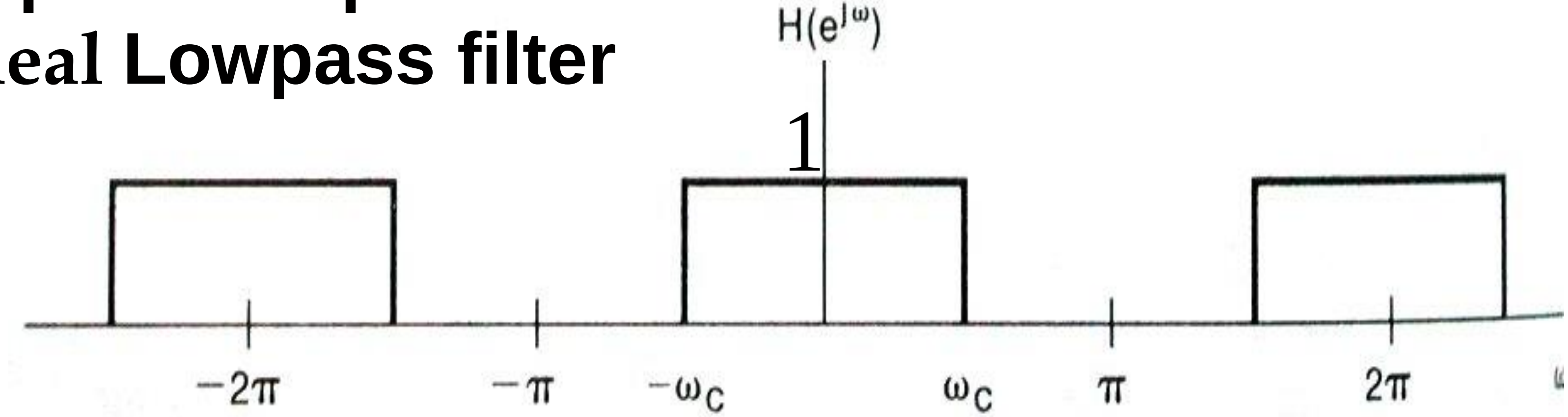
$$h(t) \quad \leftrightarrow \quad H(j\omega)$$

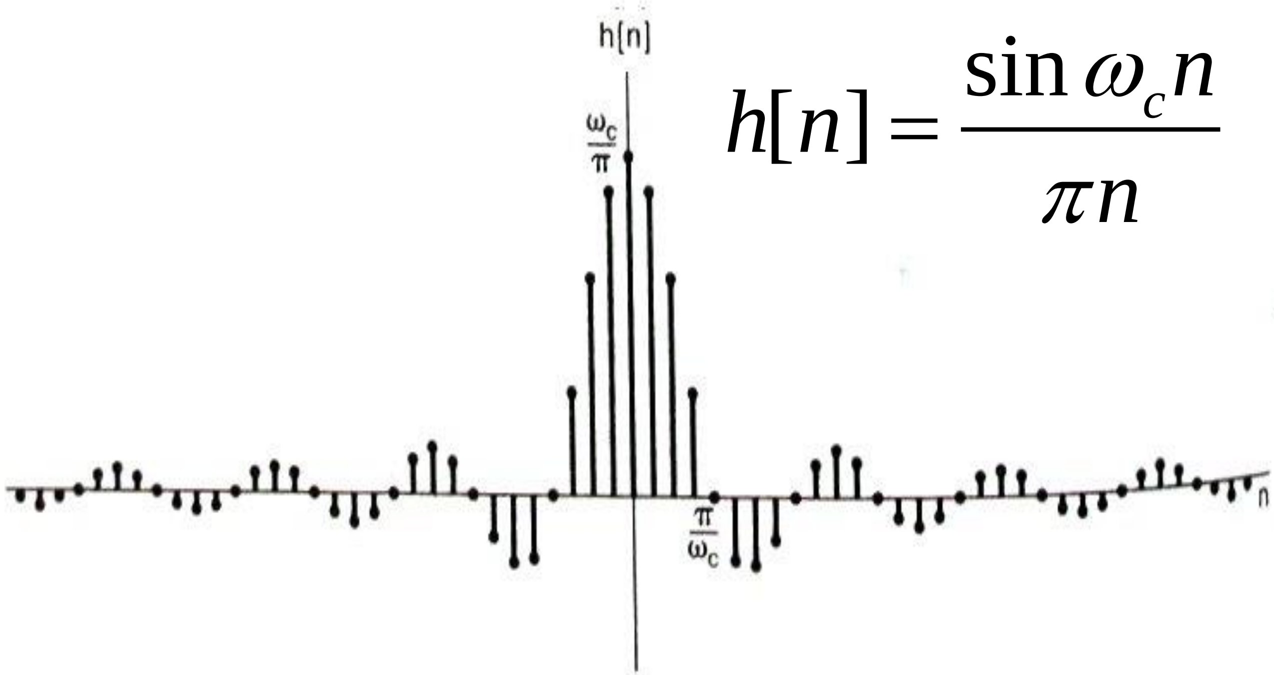
$$h(t-\alpha) \quad \leftrightarrow \quad e^{-j\omega\alpha}\, H(j\omega)$$





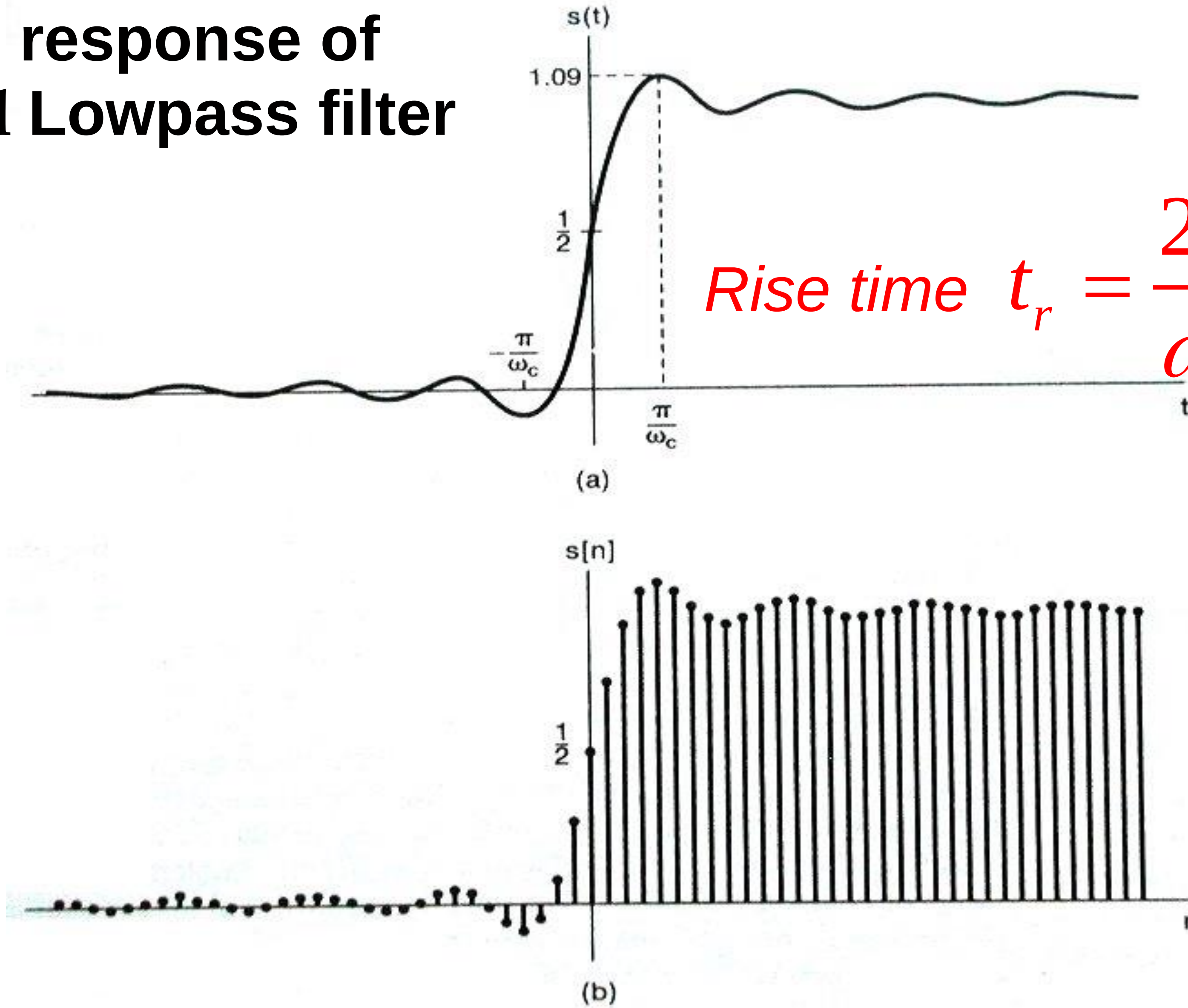
Impulse response of Ideal Lowpass filter





$$h[n] = \frac{\sin \omega_c n}{\pi n}$$

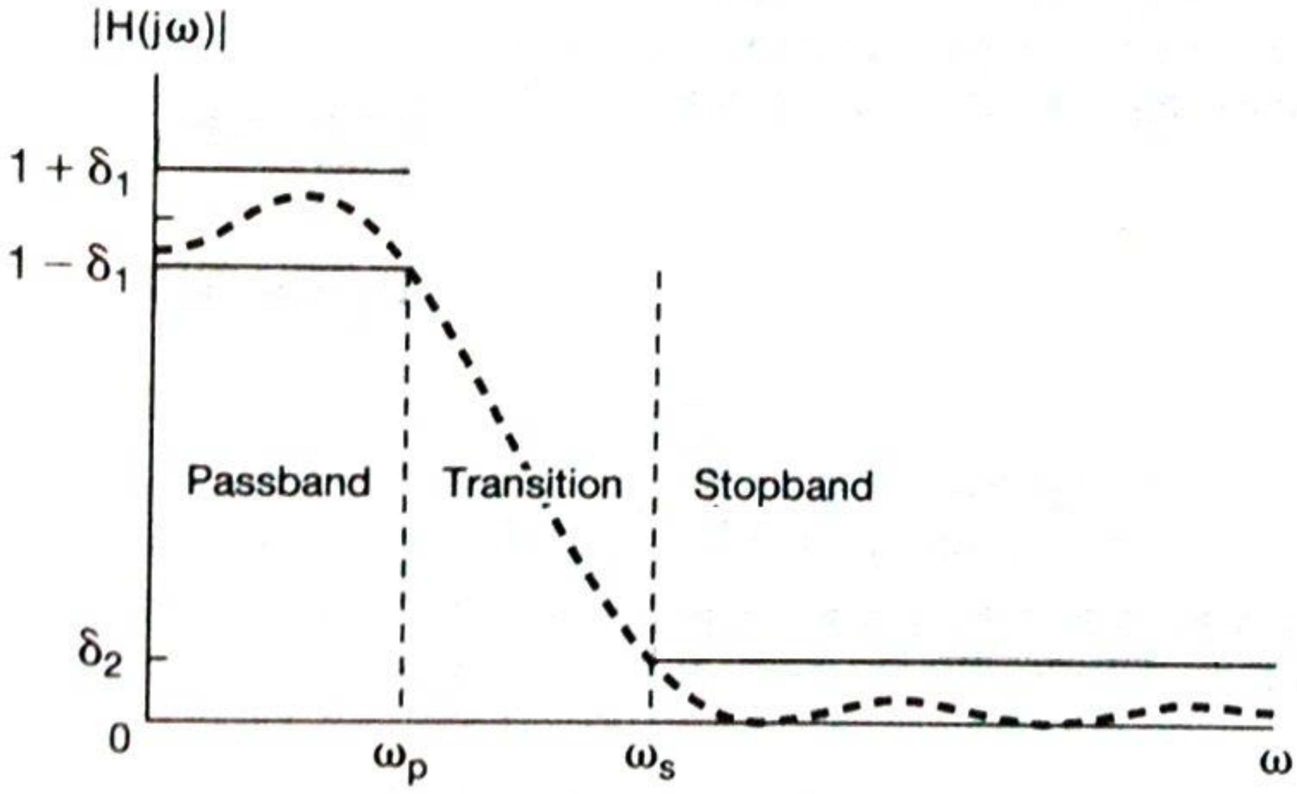
Step response of Ideal Lowpass filter



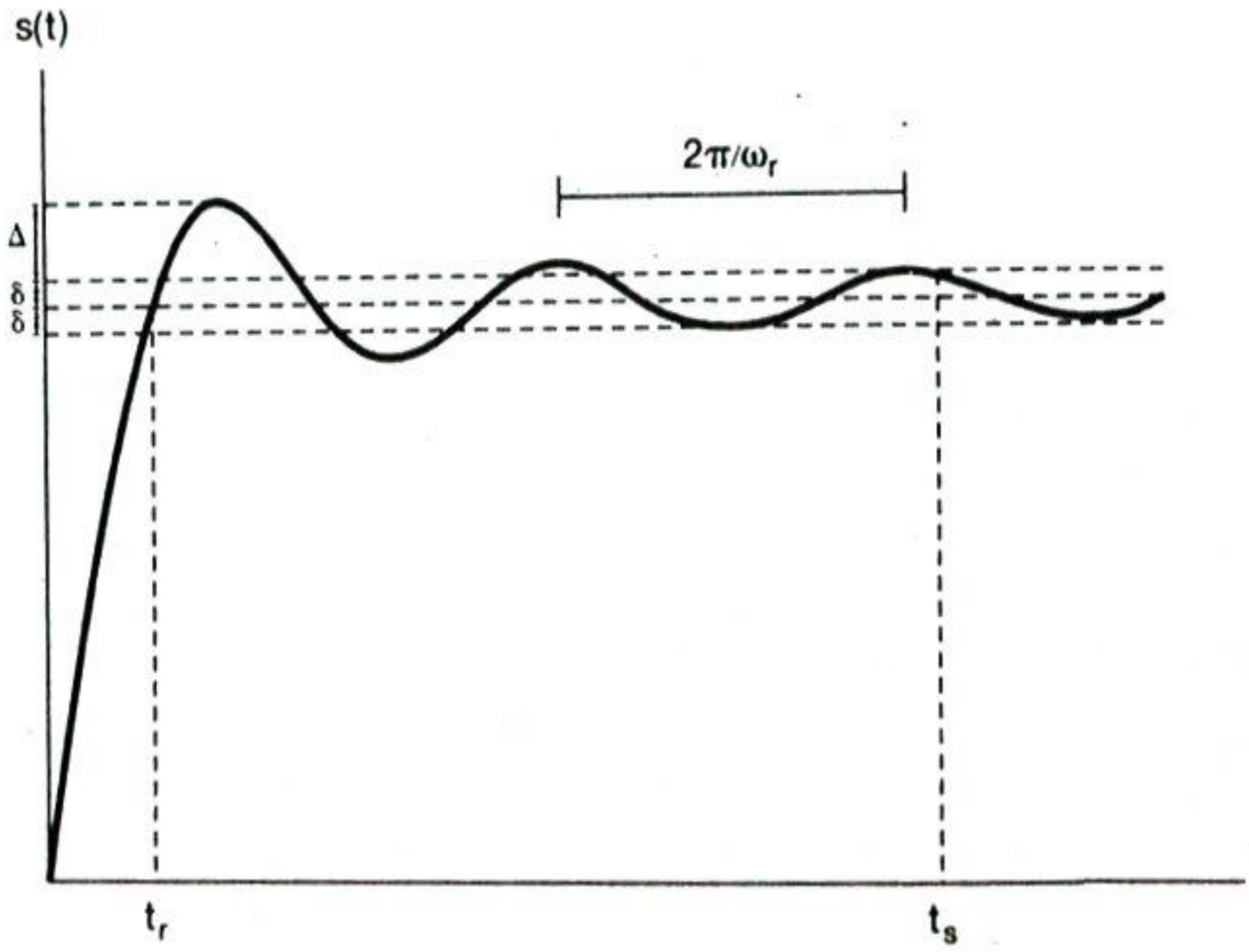
Rise time $t_r = \dfrac{2\pi}{\omega_c}$

6.4 Time-Domain and Frequency-domain Aspects of Non-ideal Filters

Basic parameter of lowpass filter:



6 Time and frequency characterization of S&S



Signals and Systems

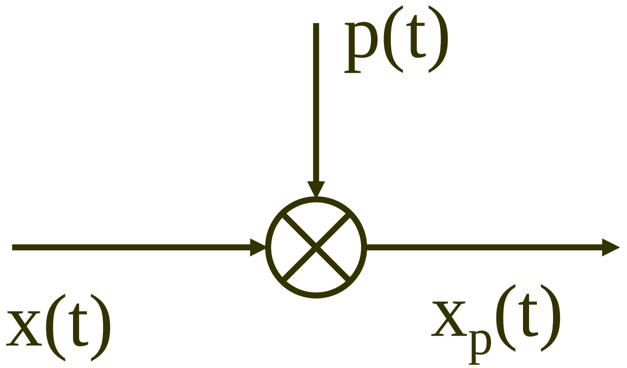
Chapter 7

Sampling

7.1 Representation of a Continuous-time Signal by its Samples: The Sampling Theorem

7.1.1 Impulse-train Sampling

(1) Sampling



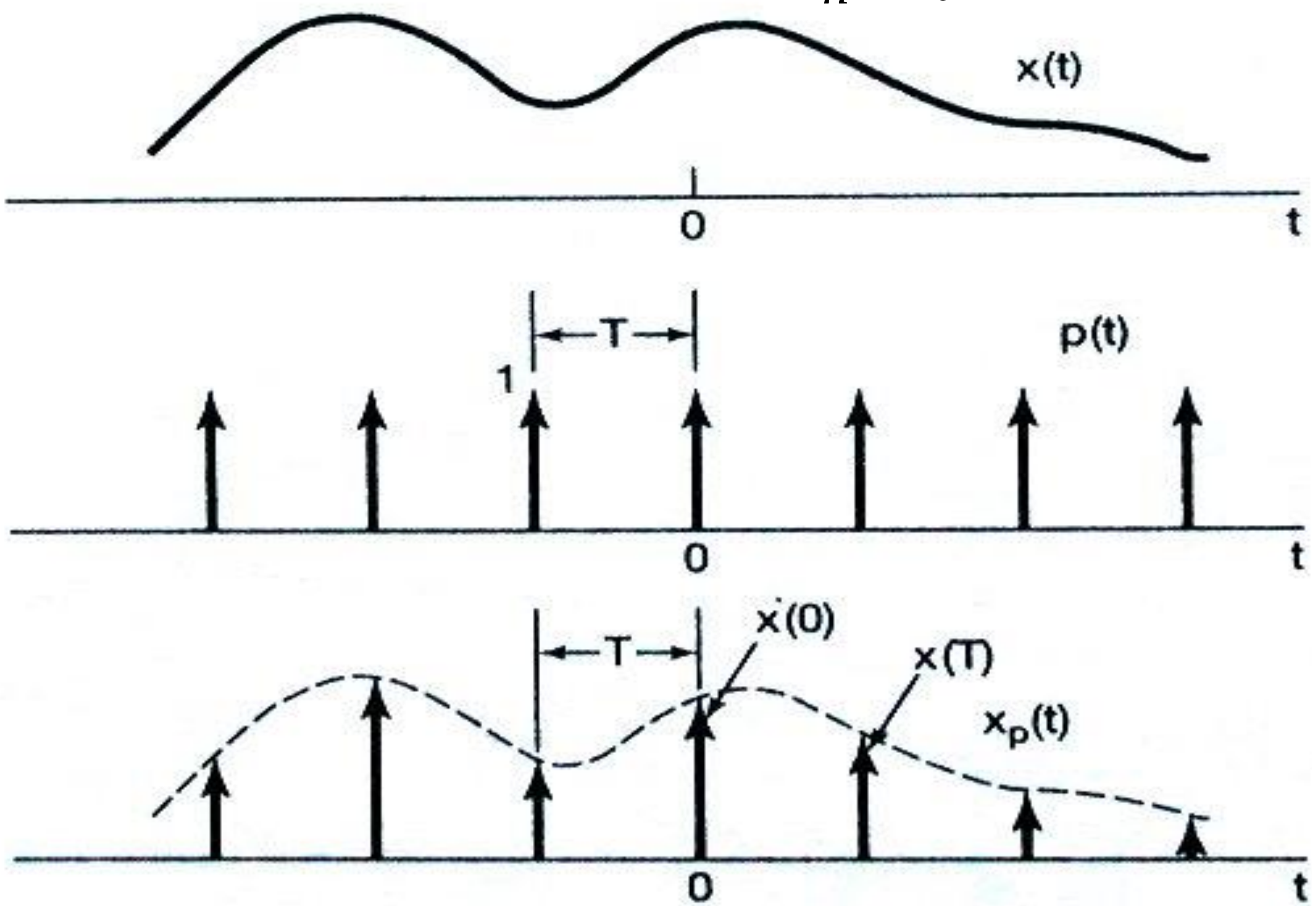
$$\begin{cases} x_p(t) = x(t)p(t) \\ X_p(j\omega) = \dfrac{1}{2\pi}[X(j\omega) * P(j\omega)] \end{cases}$$

$$\textit{where} \quad p(t) = \delta_T(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

7 Sampling

Time domain:

$$x_p(t) = x(t) \cdot \delta_T(t) = \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$



7 Sampling

Frequency domain: $$x(t) \xleftrightarrow{F} X(j\omega)$$

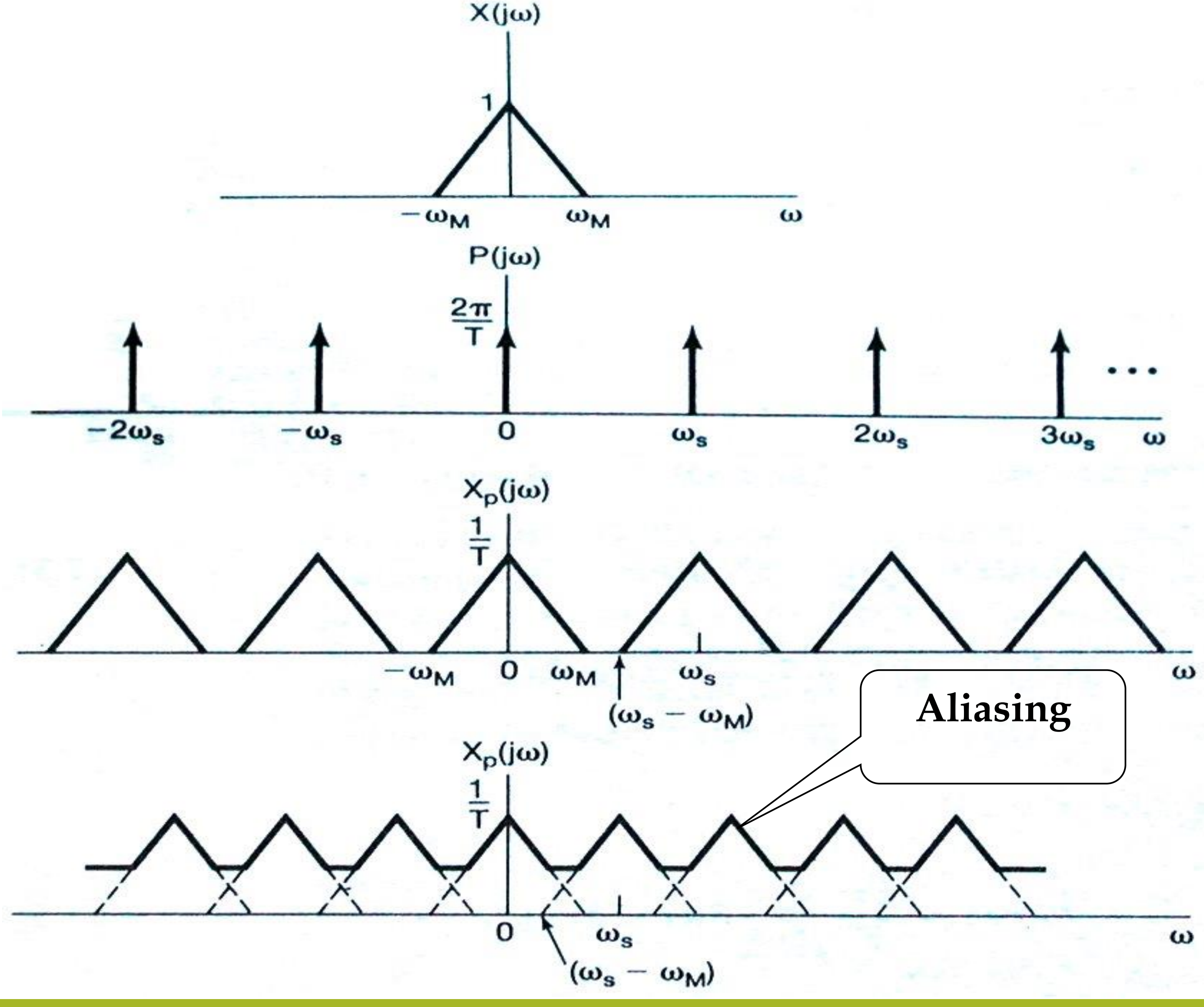
$$p(t) \xleftrightarrow{FT} P(j\omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta(\omega - k\frac{2\pi}{T})$$

$$= \sum_{k=-\infty}^{+\infty} \omega_s \delta(\omega - k\omega_s)$$

$$x_p(t) \xleftrightarrow{F} X_p(j\omega) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_s))$$

7 Sampling



X(jω)
1
−ω_M
ω_M
ω
P(jω)
2π/T
−2ω_s
−ω_s
0
ω_s
2ω_s
3ω_s
ω
X_p(jω)
1/T
−ω_M
0
ω_M
ω_s
(ω_s − ω_M)
ω
Aliasing
X_p(jω)
1/T
0
ω_s
(ω_s − ω_M)
ω

(2) Sampling theorem

Let $x(t)$ be a **band-limited** signal with $X(j\omega)=0$ for $|\omega|>\omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT), n=0,\pm1,\pm2,\ldots,$

if $\omega_s>2\omega_M$, where $\omega_s=2\pi/T$.

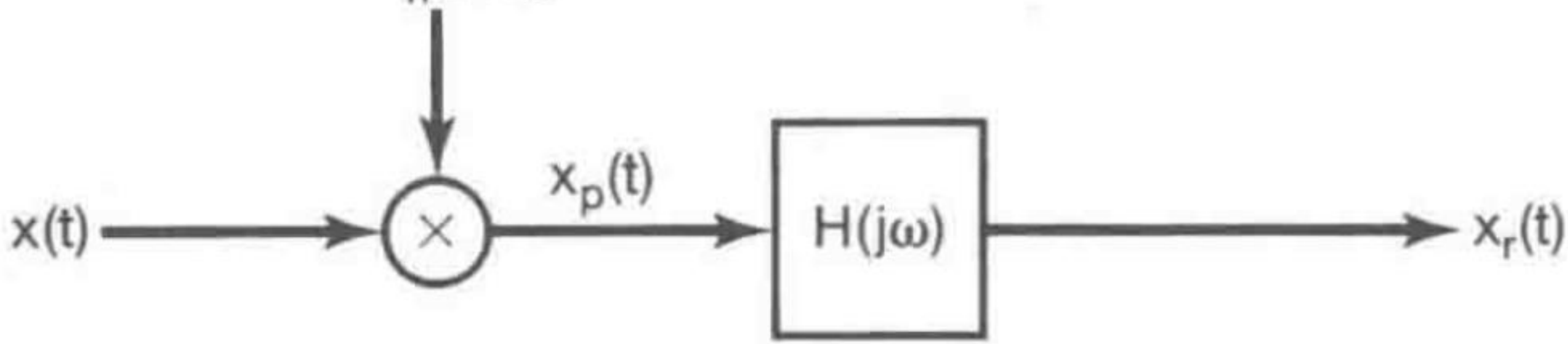
$2\omega_M$ is called **Nyquist Rate**.

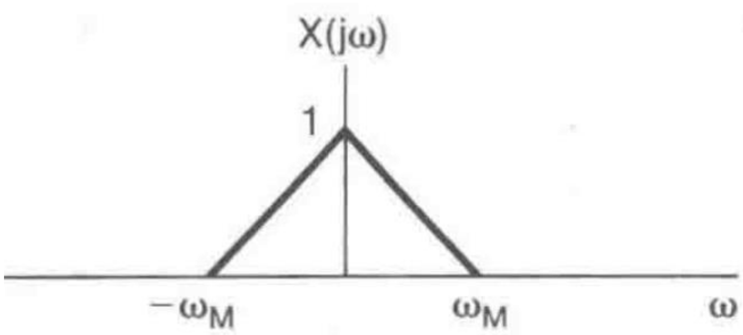
(Minimum distortionless sampling frequency)

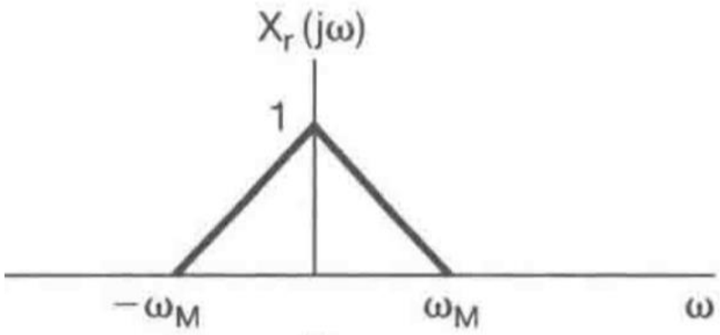
(3) Recovery

System for sampling and reconstruction:

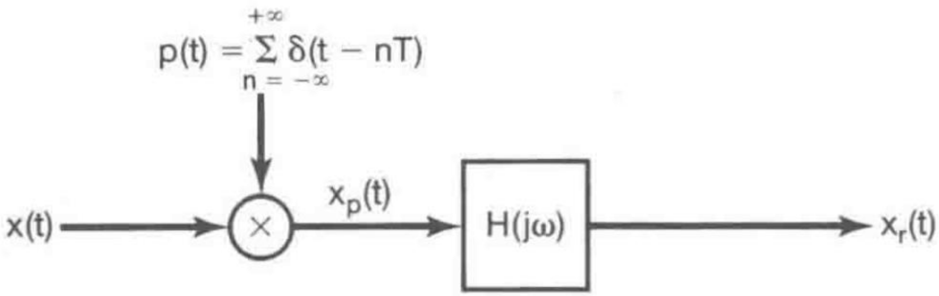
$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

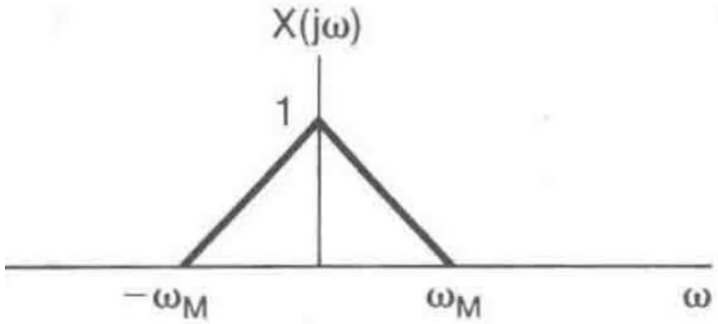






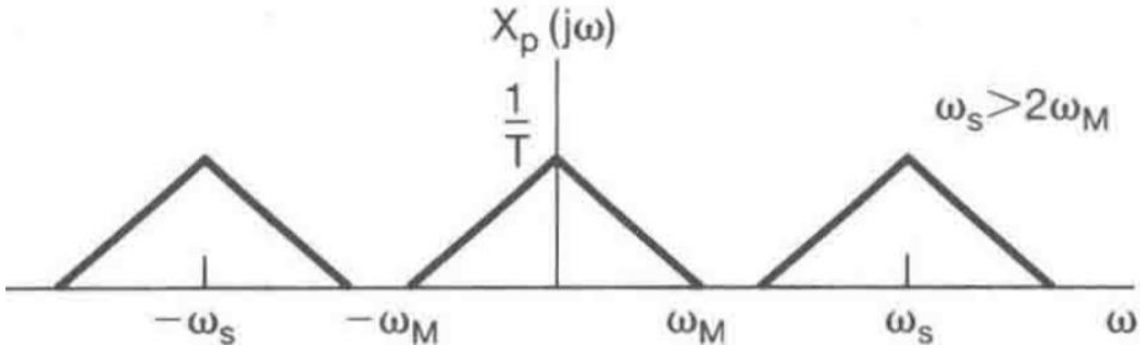
7 Sampling

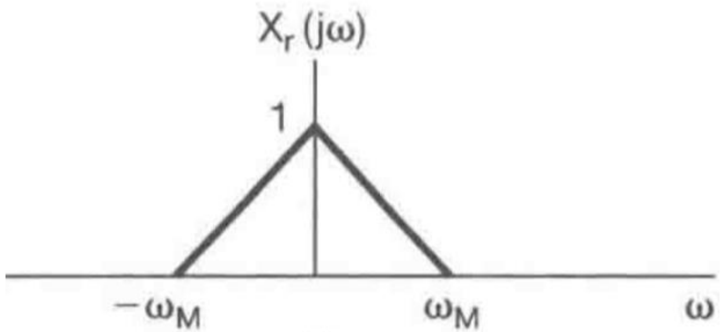


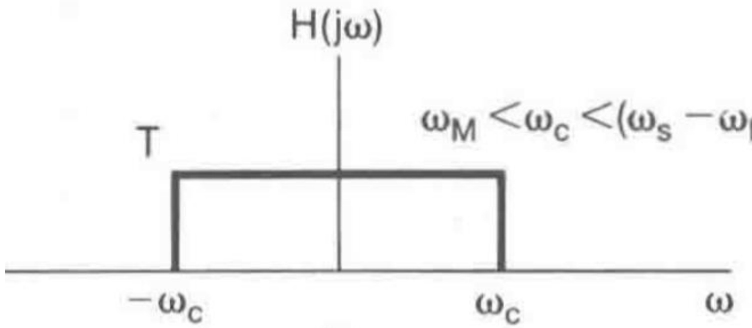


Normally,

$$\omega_c = \omega_s / 2$$







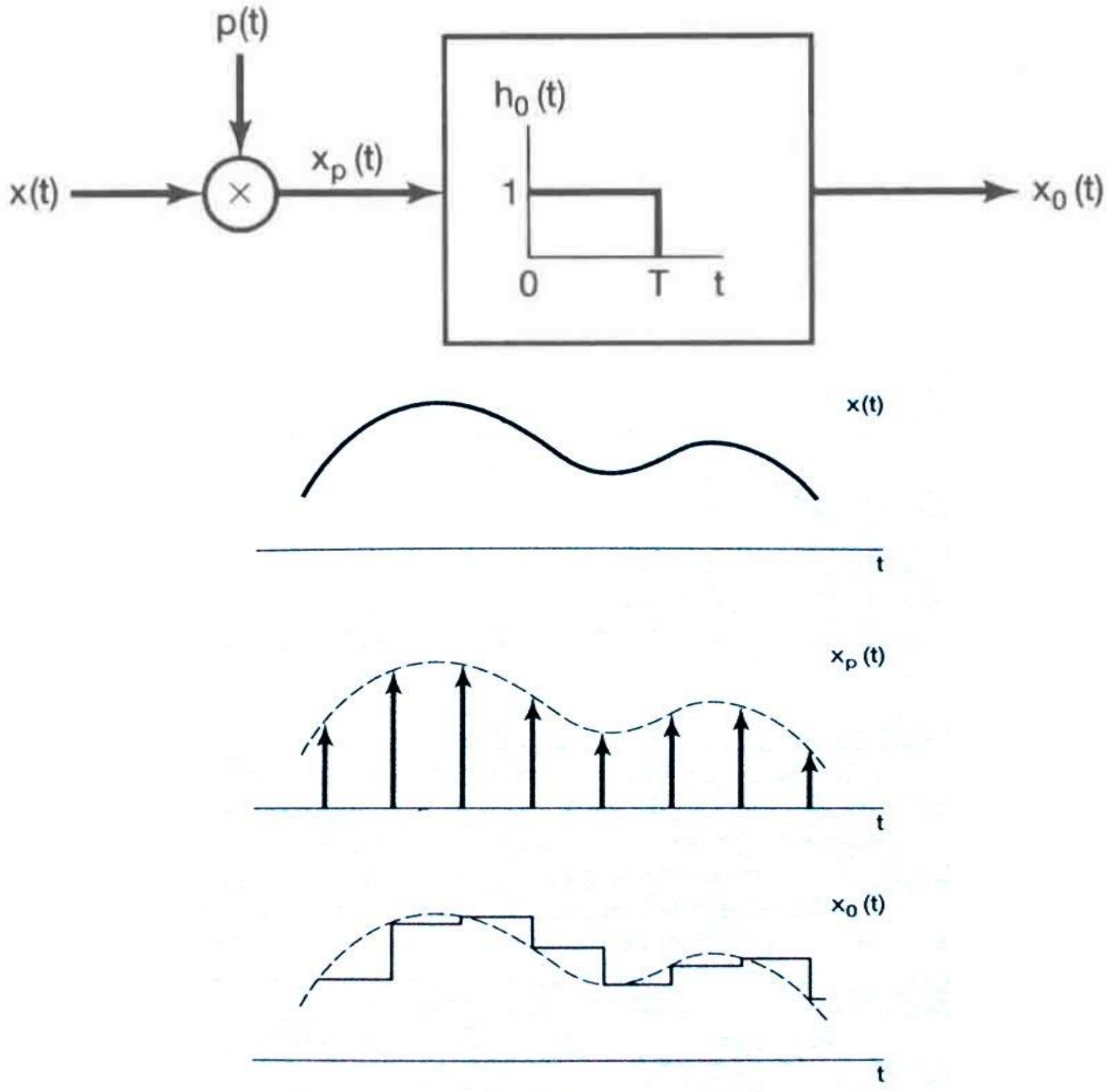
7.1.2 Sampling with a Zero-order Hold

Sampling system construction:

Key point:

at a given instant and holds the value until the next instant of sample is taken.

7 Sampling

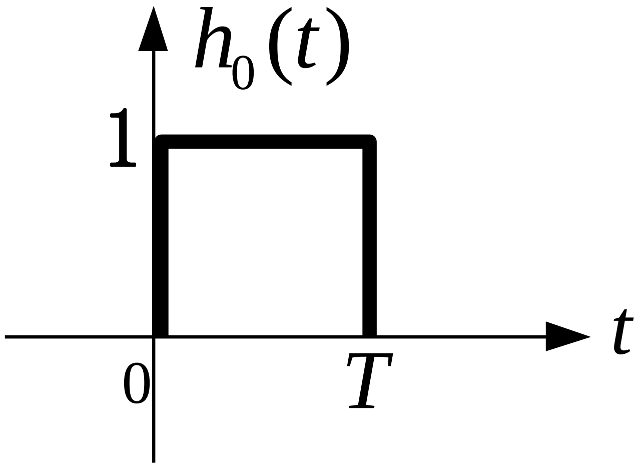


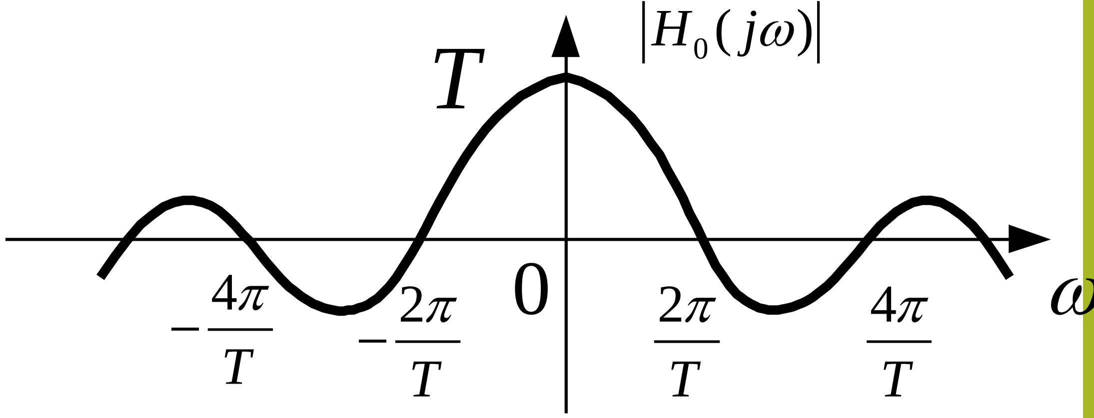
7 Sampling

$$H_0(j\omega)=\left(\frac{\sin(\omega T/2)}{\omega}\right)e^{-j\omega T/2}=TSa(\omega T/2)e^{-j\omega T/2}$$

$$x_0(t)\xleftrightarrow{F}X_0(j\omega)=H_0(j\omega)X_p(j\omega)$$

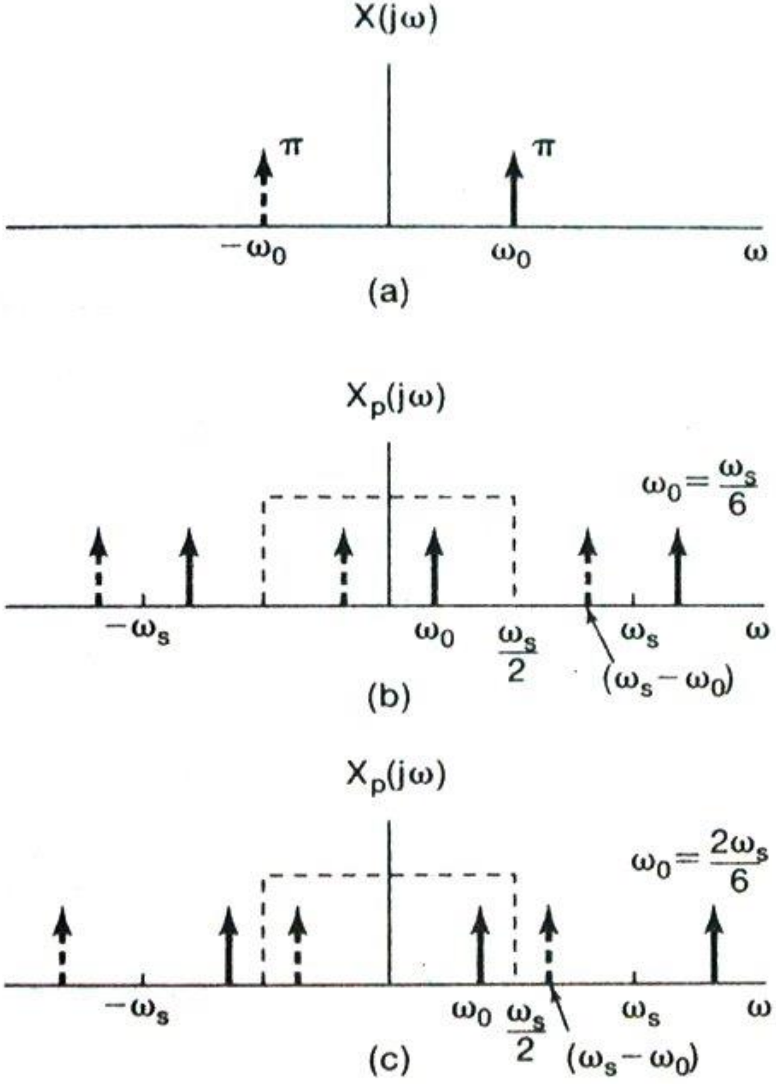
$$=\frac{\omega_s}{2\pi}H_0(j\omega)\{\sum_{k=-\infty}^{+\infty}X(\omega-k\omega_s)\}$$

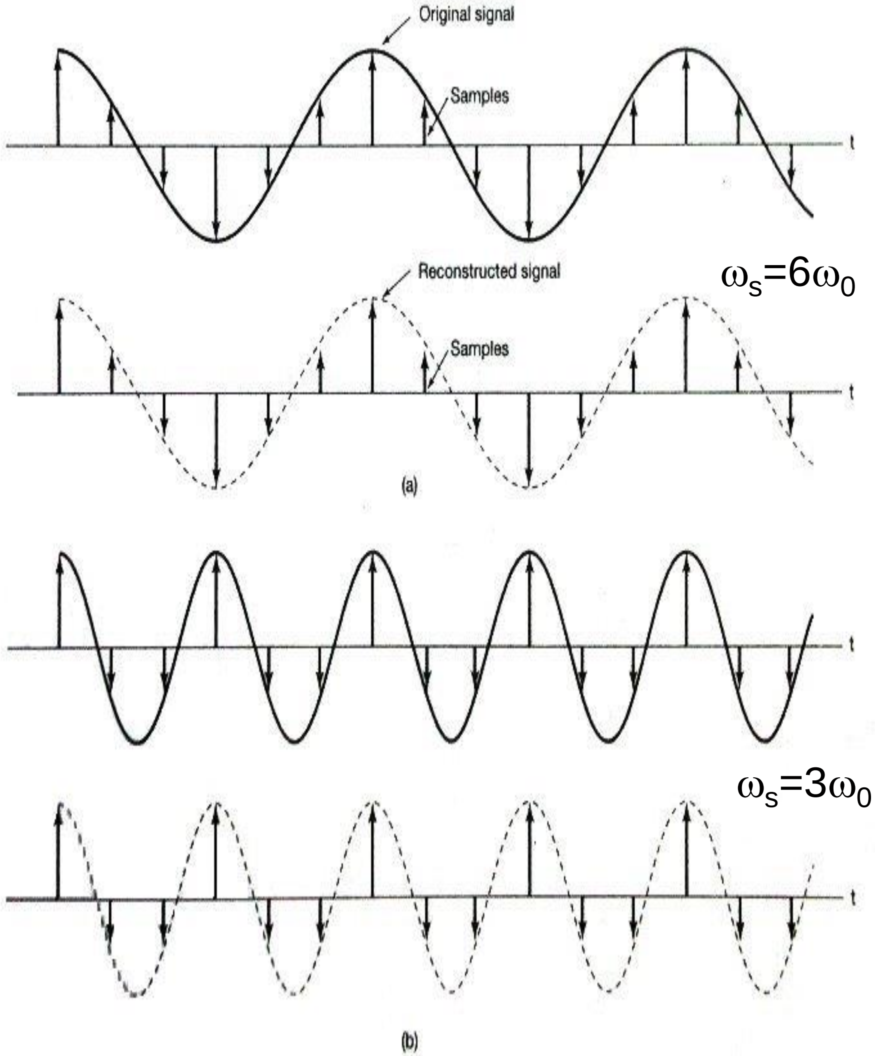




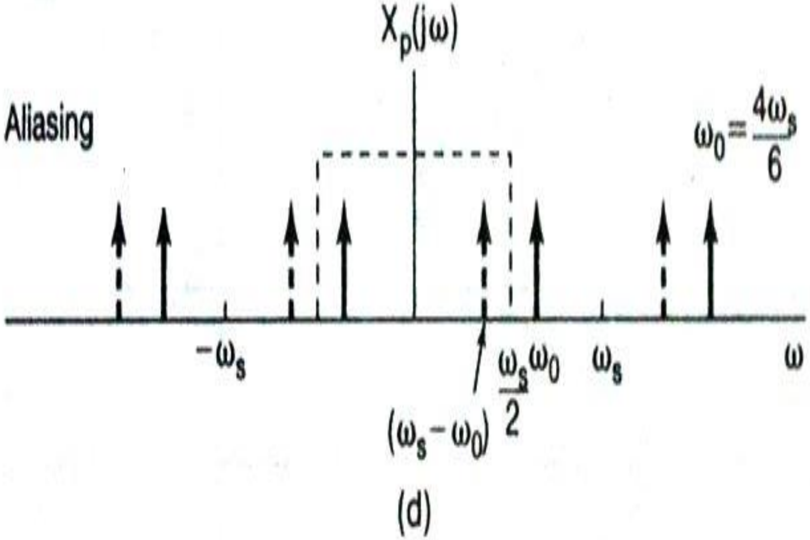
7 Sampling

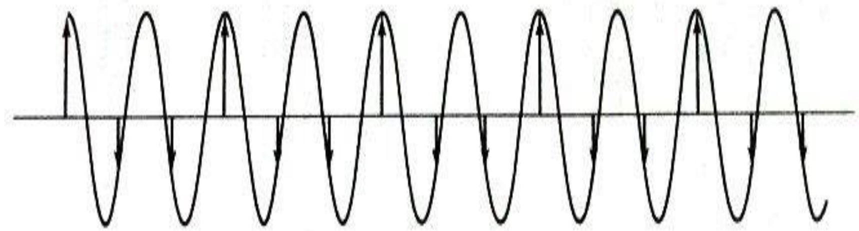
7.3 The Effect of Undersampling: Aliasing



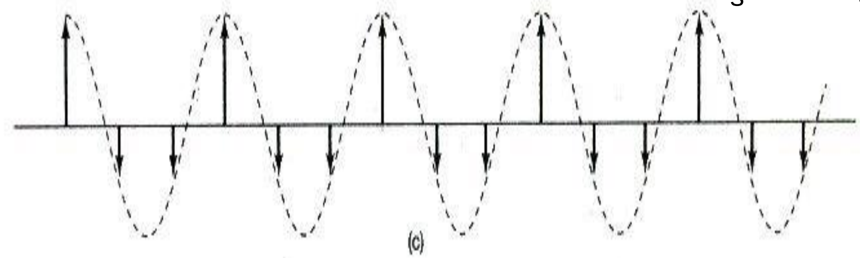


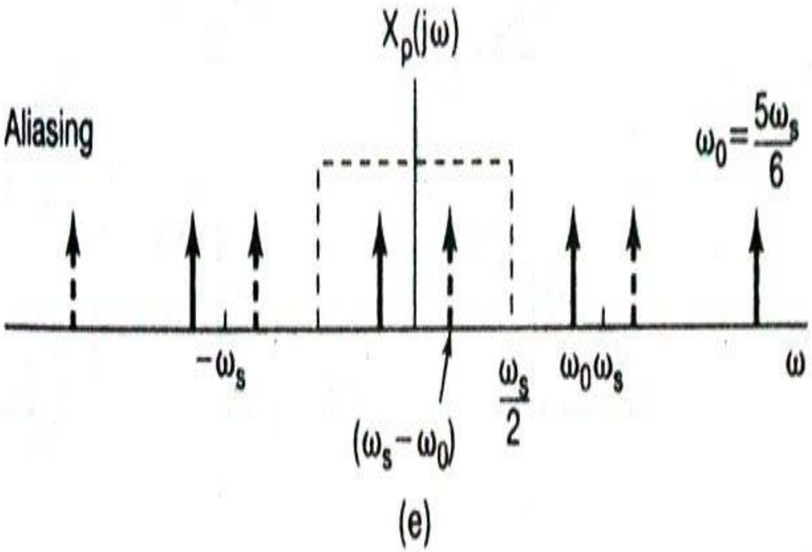
7 Sampling

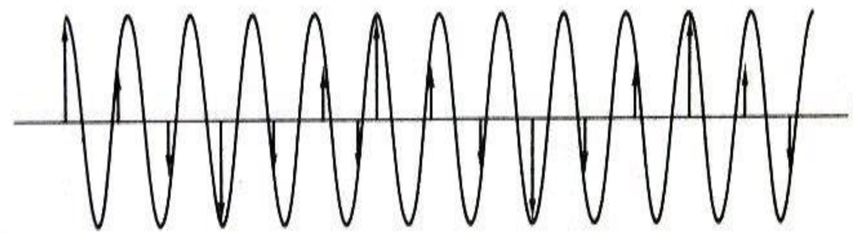




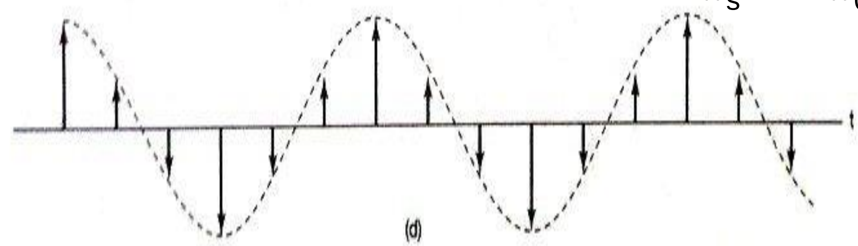
$\omega_s = 1.5\omega_0$



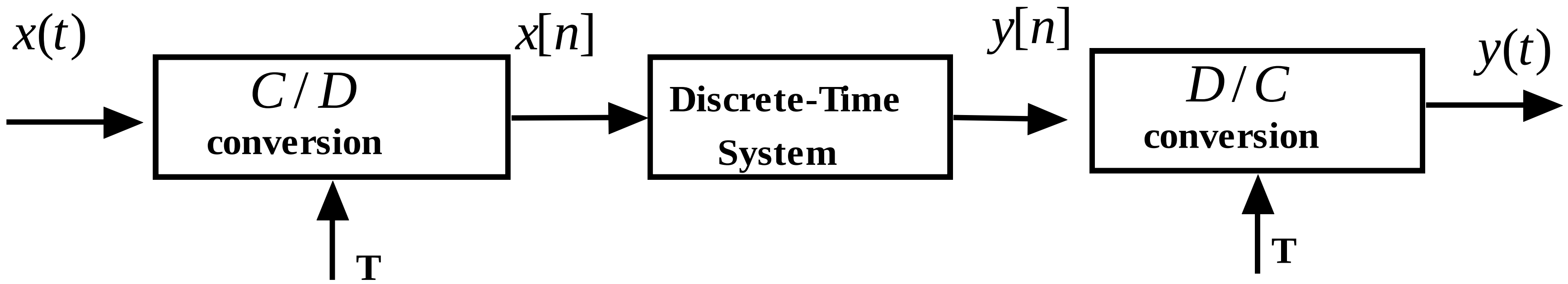


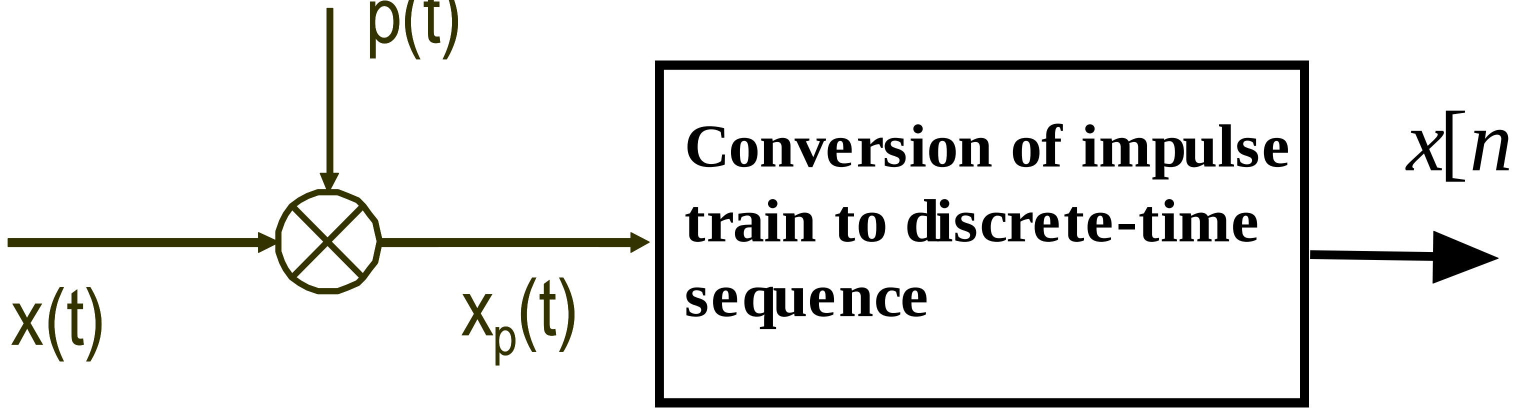


$\omega_s = 1.2\omega_0$



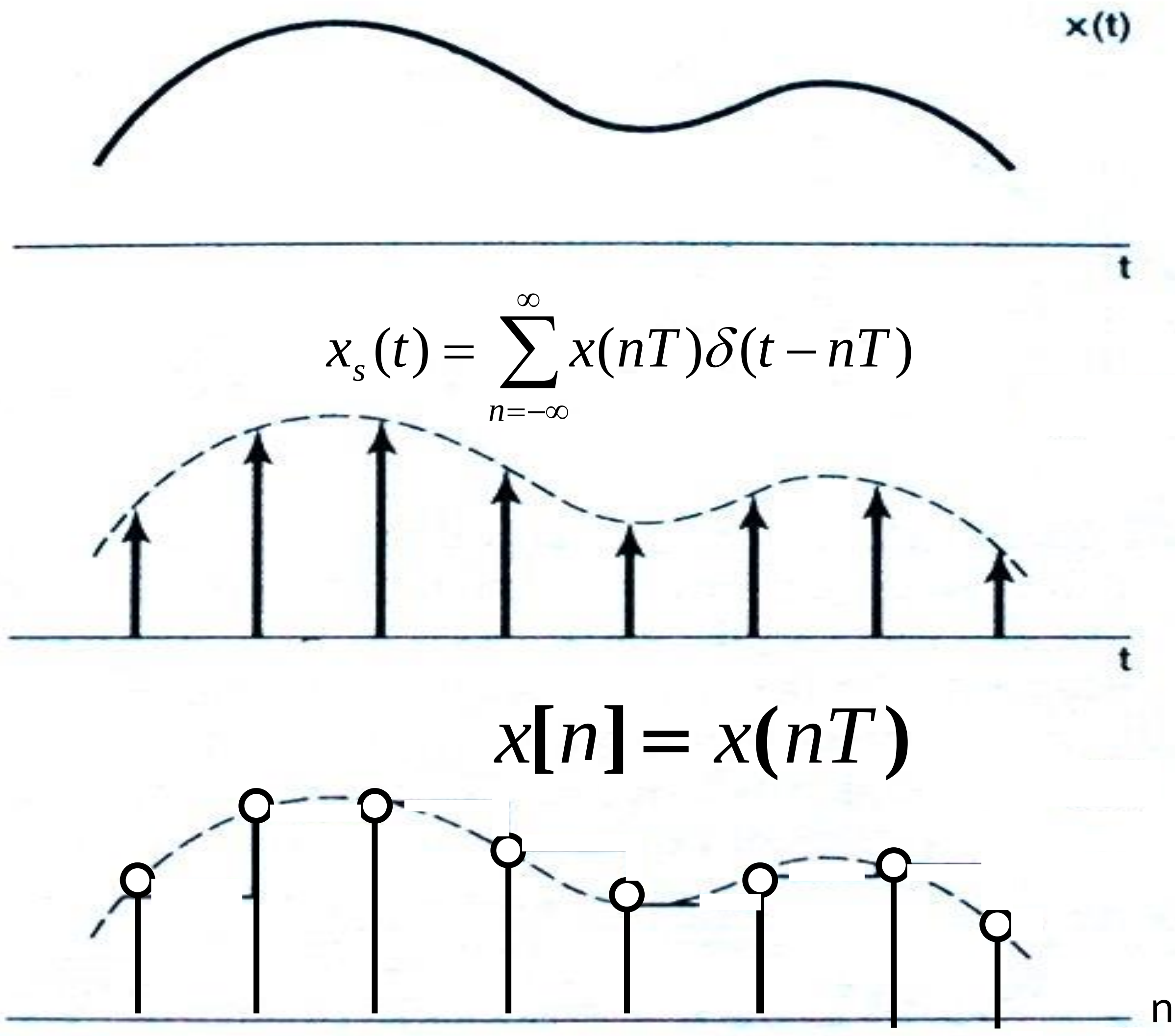
7.4 Discrete-Time Processing of Continuous-Time Signals (learn by yourself)





7 Sampling

Relationship between Continuous-time and Discrete-time



$$x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$

$$x[n] = x(nT)$$

Relationship between FT and DTFT

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)$$

$$x[n] = x(nT)$$

$$X_p(j\omega) = \sum_{n=-\infty}^{\infty} x(nT)e^{-j\omega Tn} \quad \text{------①}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n} \quad \text{------②}$$

let $$\Omega = \omega T$$

Then $$X(e^{j\Omega}) = X_p(j\omega)$$

Relationship between FT and DTFT

From 7.1, we know that $x(t) \xleftrightarrow{F} X(j\omega)$

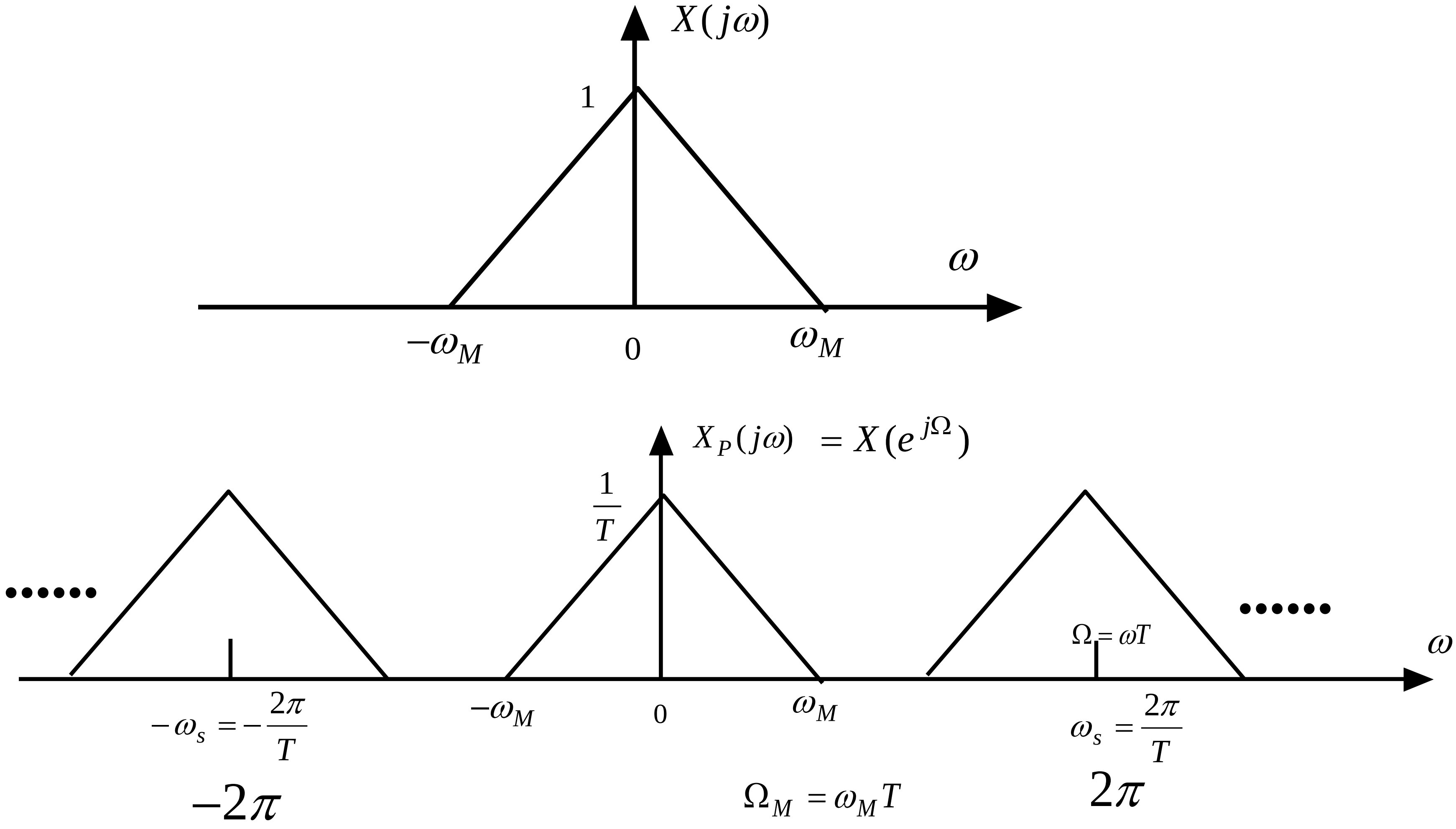
$$X_p(j\omega) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(j\omega - jm\frac{2\pi}{T}) \quad \text{------③}$$

$$\therefore X(e^{j\Omega}) = \frac{1}{T}\sum_{m=-\infty}^{\infty} X(j\omega - jm\frac{2\pi}{T})$$

where $\Omega = \omega T$

7 Sampling

Relationship between FT and DTFT



Poisson formula:

Compare ①and ③,we can get,

If $x(t) \xleftrightarrow{F} X(j\omega)$

Then

$$\frac{1}{T}\sum_{m=-\infty}^{\infty} X(j\omega - jm\frac{2\pi}{T}) = \sum_{n=-\infty}^{\infty} x(nT)e^{-j\omega Tn}$$

$$\sum_{n=-\infty}^{\infty} x(t-nT) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(jk\frac{2\pi}{T})e^{jk\frac{2\pi}{T}t}$$

Signals and Systems

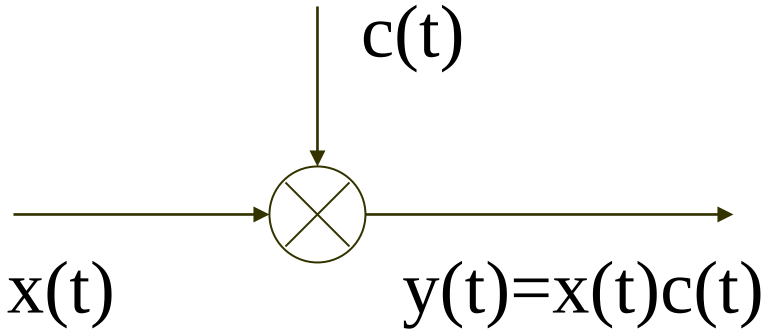
Chapter 8

Communication Systems

8. Communication Systems

8.1 Complex Exponential and Sinusoidal Amplitude Modulation

Modulating system model:



x(t) --- modulating signal
c(t) --- Carrier signal

8.1.1 Amplitude Modulation with Complex Exponential Carrier

(1) Modulation Theory

Exponential carrier: $c(t) = e^{j(\omega_c t + \theta_c)}$

For convenience, let θ_c=0, so $c(t) = e^{j\omega_c t}$

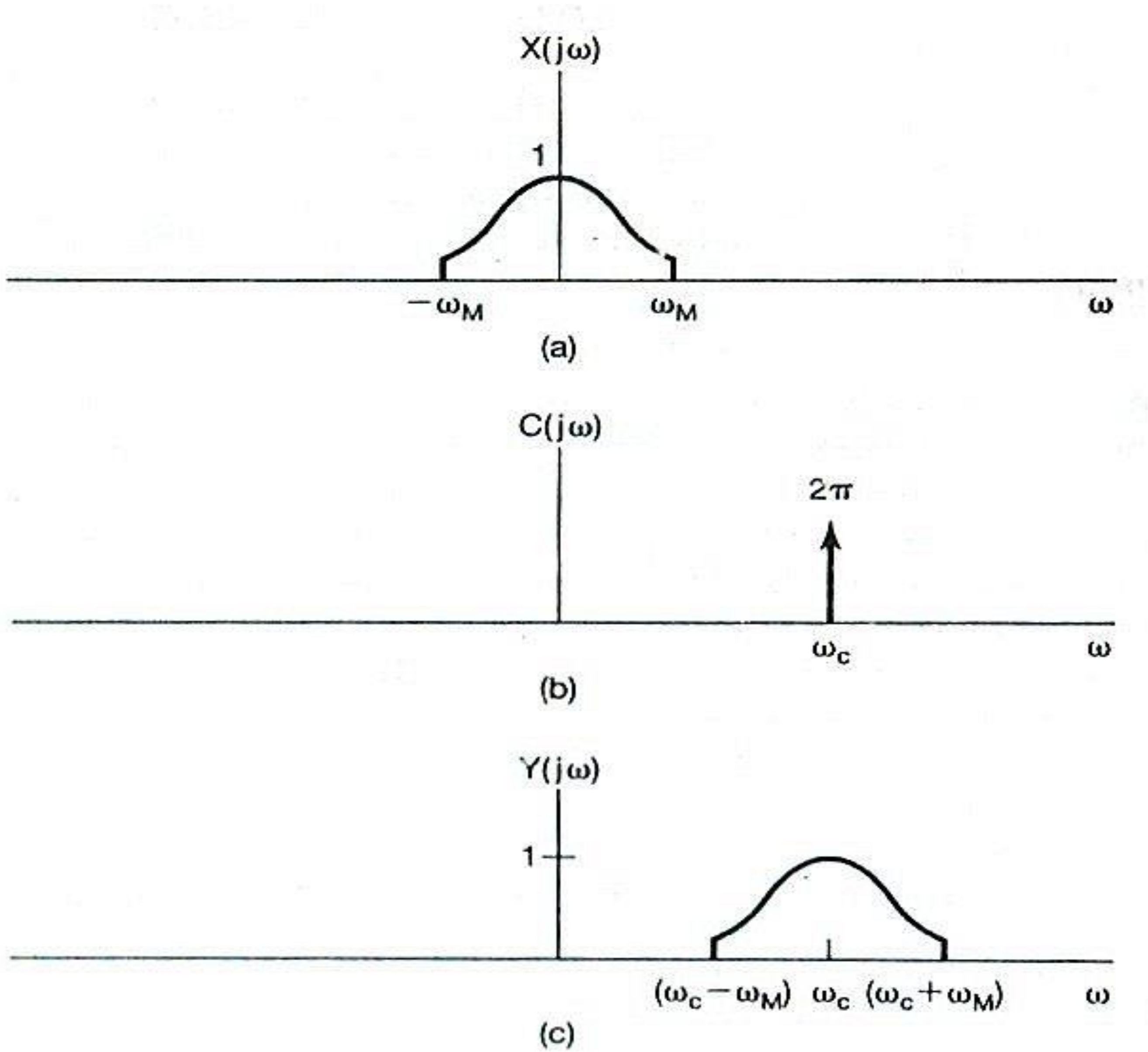
Output signal(modulated signal): $y(t) = x(t)e^{j\omega_c t}$

$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * C(j\omega)$$

$$C(j\omega) = 2\pi\delta(\omega - \omega_c)$$

$$Y(j\omega) = X(j\omega - j\omega_c)$$

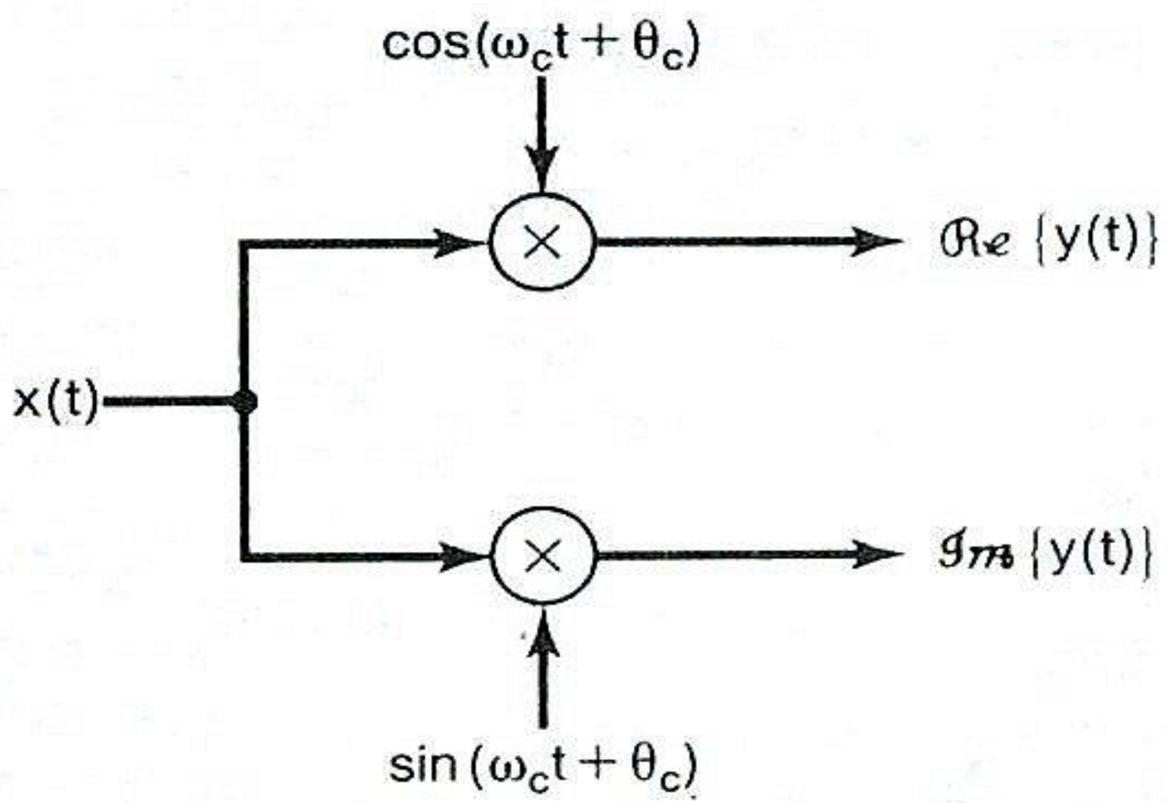
8 Communication systems



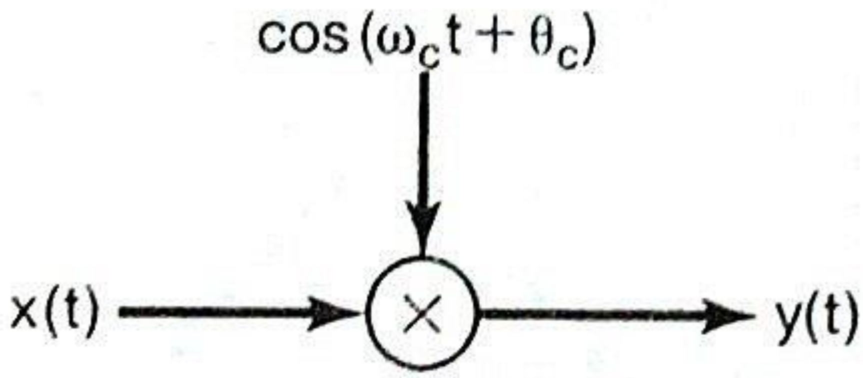
(1) Implementation

$$y(t) = x(t)e^{j\omega_c t}$$

$$= x(t)\cos\omega_c t + jx(t)\sin\omega_c t$$



8.1.2 Amplitude Modulation with Sinusoidal signal



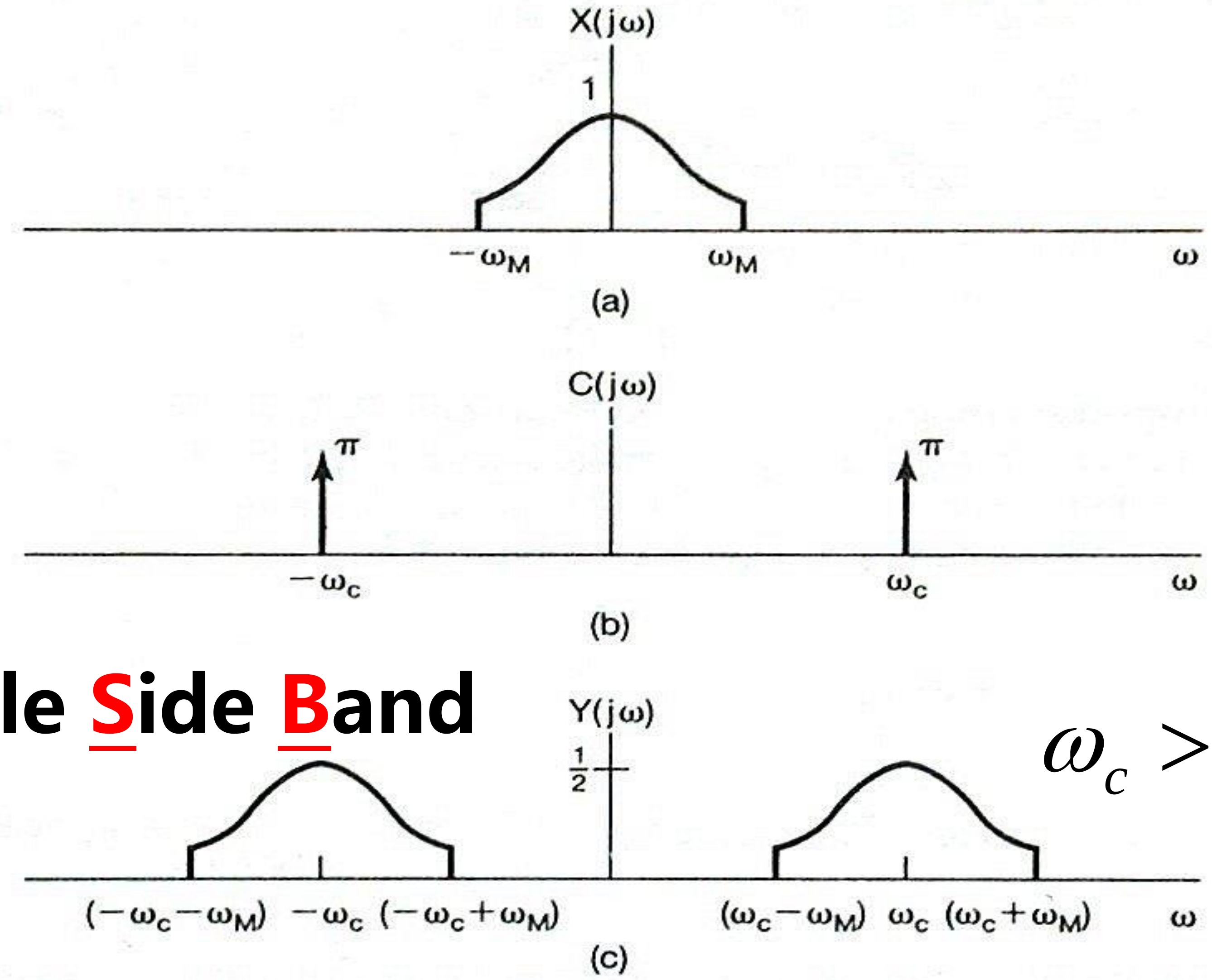
For convenience, choose θ_c=0, so

$$Y(j\omega)=\frac{1}{2\pi}X(j\omega)*C(j\omega)$$

$$C(j\omega)=\pi\delta(\omega-\omega_c)+\pi\delta(\omega+\omega_c)$$

$$Y(j\omega)=\frac{1}{2}X(j\omega-j\omega_c)+\frac{1}{2}X(j\omega+j\omega_c)$$

8 Communication systems



Double Side Band

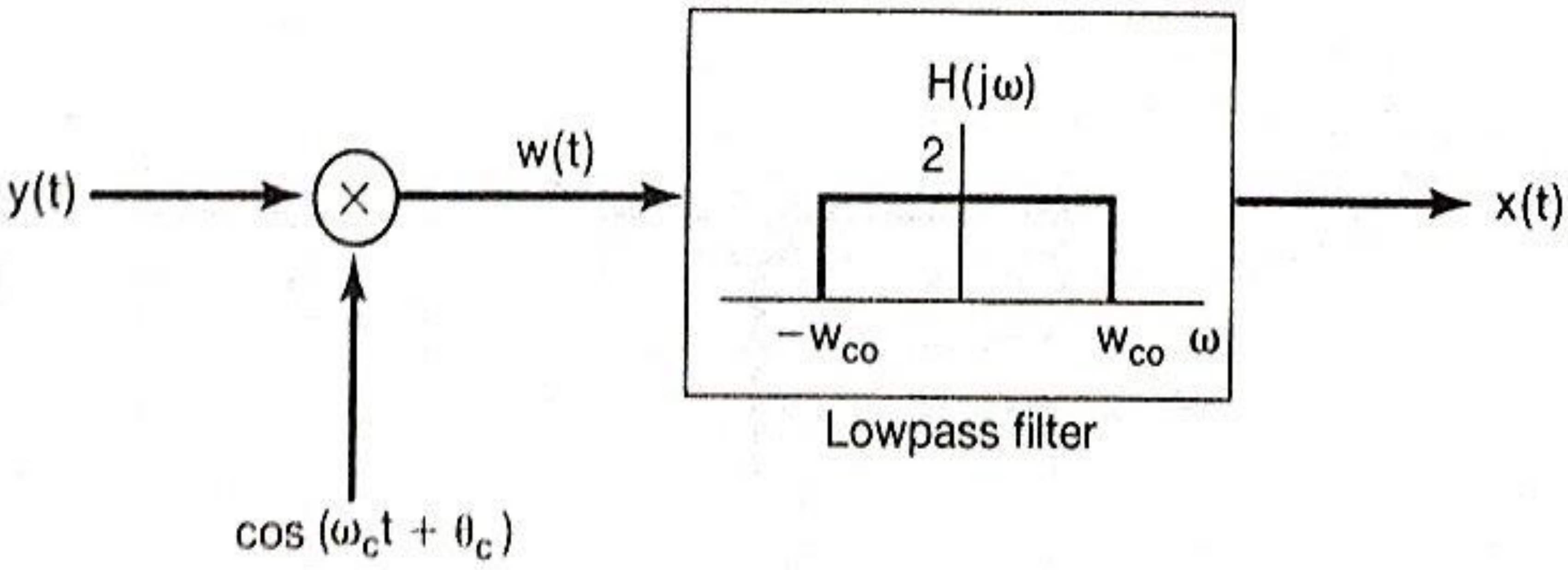
$$\omega_c > \omega_M$$

SingleSide Band in Problem 8.28.

8.2 Demodulation for Sinusoidal AM

8.2.1 Synchronous demodulation

(1) Demodulation process



In time domain: $$w(t) = y(t)c(t)$$
$$= x(t)\cos^2 \omega_c t$$
$$= \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos 2\omega_c t$$

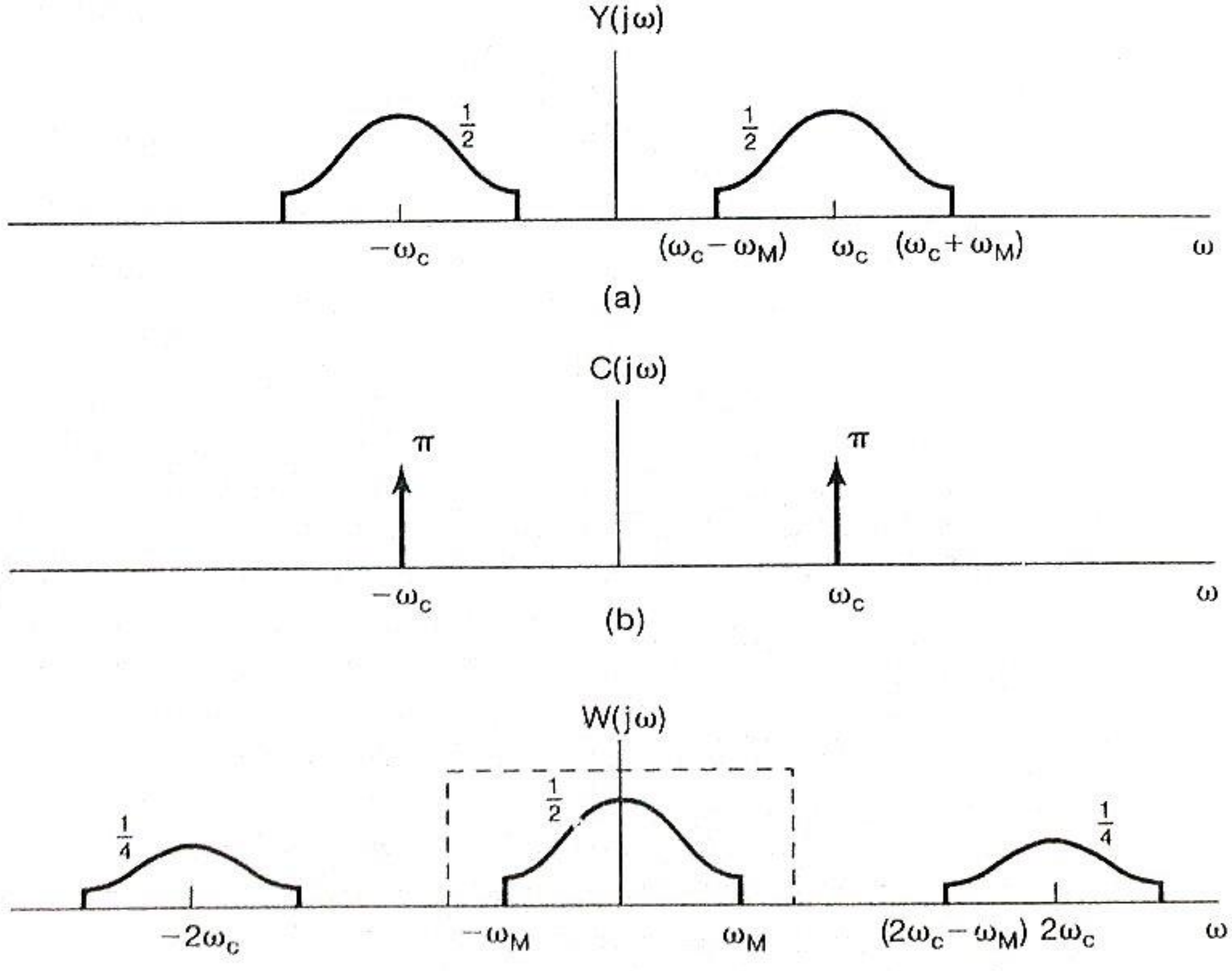
In frequency domain:

$$W(j\omega) = \frac{1}{2}X(j\omega) + \frac{1}{4}X(j\omega - j2\omega_c) + \frac{1}{4}X(j\omega + 2j\omega_c)$$

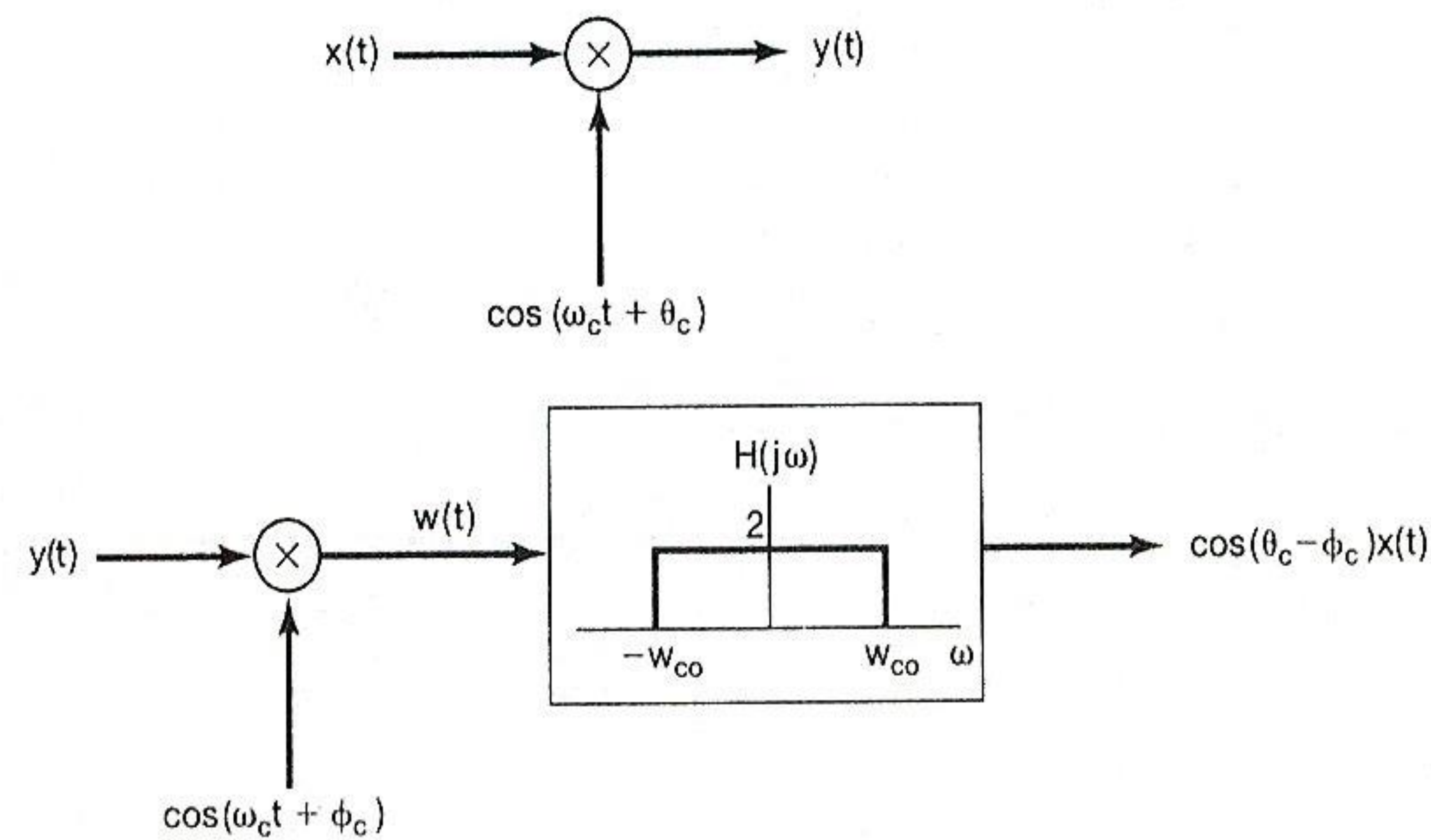
Expected signal:

$$x(t) \;\; or \;\; W(j\omega)$$

8 Communication systems



(2) Synchronous problem



Time domain:

$$
\begin{aligned}
w(t) &= y(t)\cos(\omega_c t + \phi_c) \\
&= x(t)\cos(\omega_c t + \theta_c)\cos(\omega_c t + \phi_c) \\
&= \frac{1}{2}x(t)\cos(\theta_c - \phi_c) + \frac{1}{2}x(t)\cos(2\omega_c t + \theta_c + \phi_c)
\end{aligned}
$$

The output of lowpass filter: $x(t)\cos(\theta_c - \phi_c)$

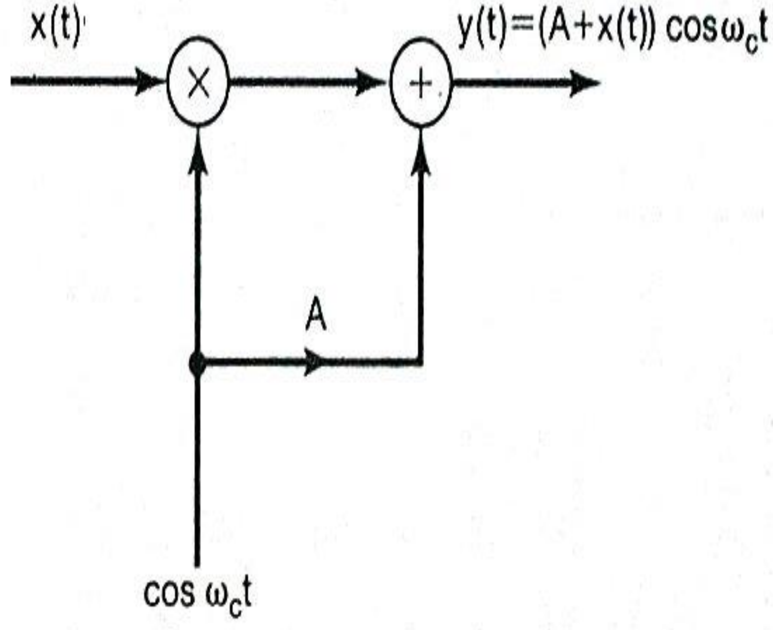
Ideal output: x(t) $(\theta_c = \phi_c \ is\ desired)$

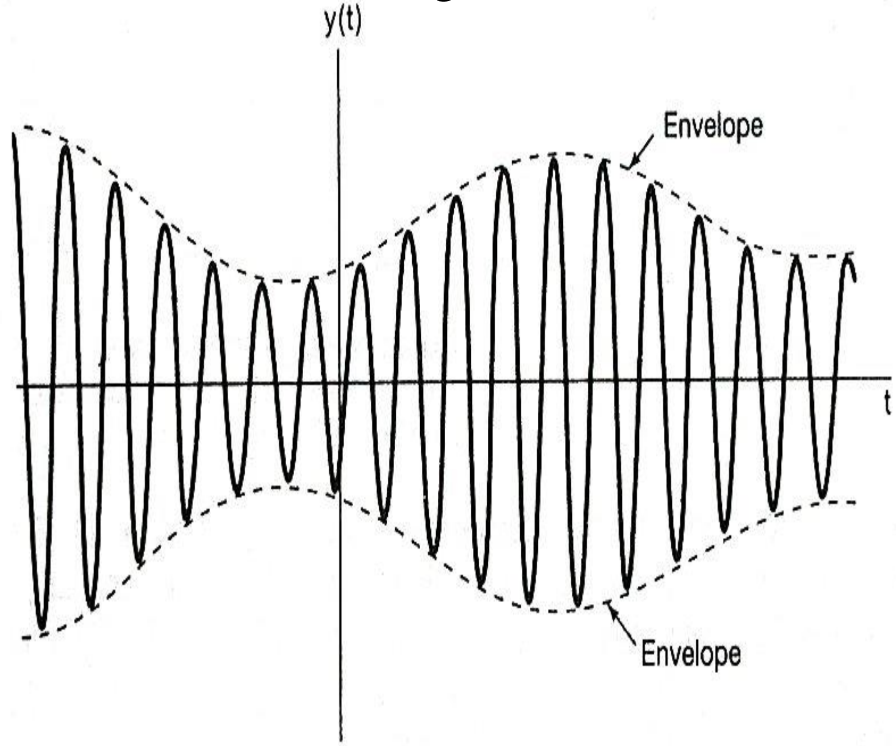
When $\theta_c = \phi_c$ **, it is referred to as synchronous demodulation.**

8.2.2 Asynchronous demodulation

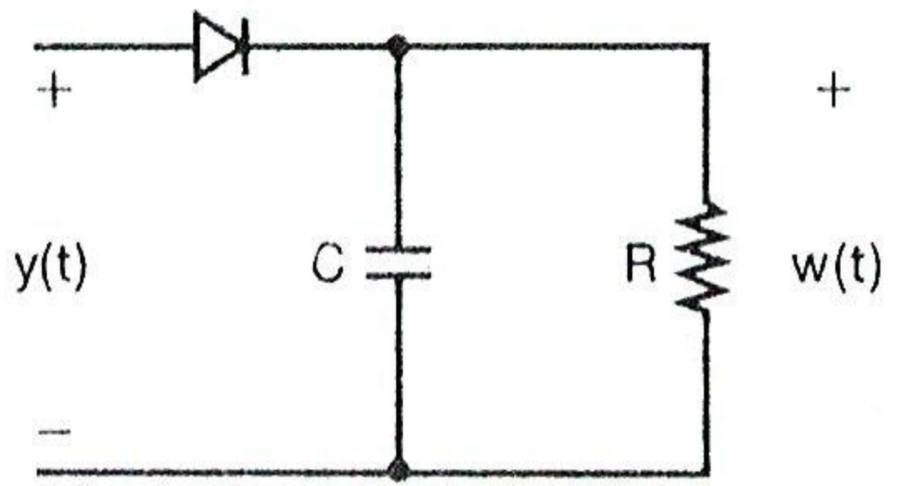
Amplitude-modulated signal:

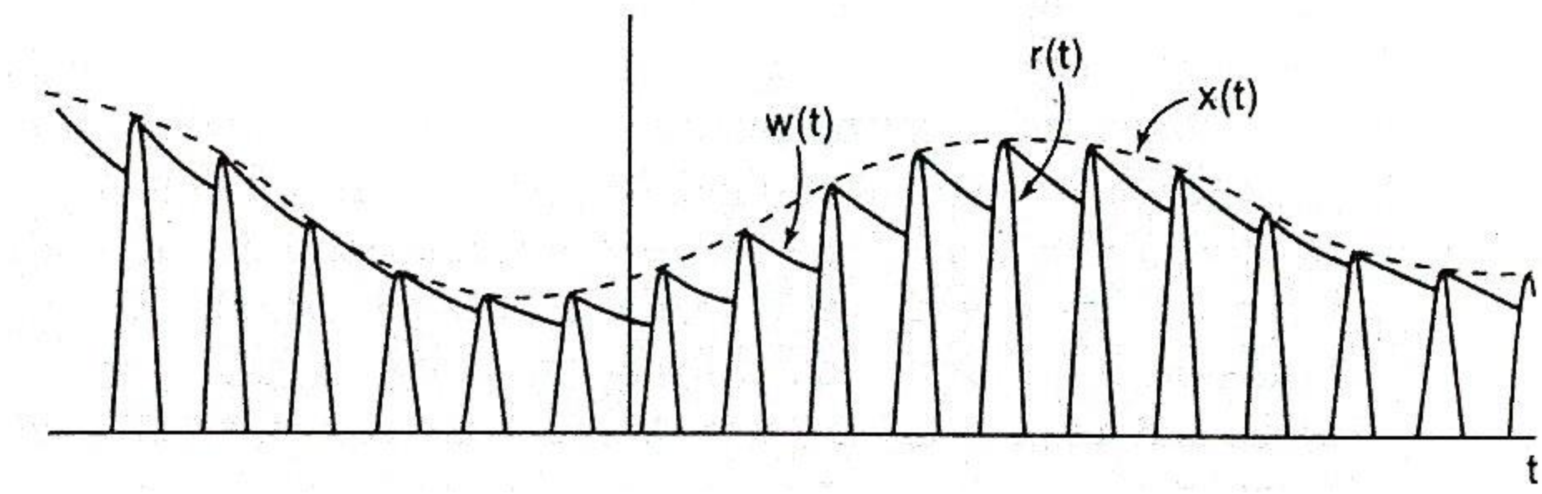
$$y(t) = [x(t) + A]\cos\omega_c t$$





Asynchronous demodulator





Homework for Chapters 6,7 & 8

Chapter 6

6.5, 6.23, **6.27**

Chapter 7

7.1, 7.2, **7.3,** 7.6, **7.9**

Chapter 8

8.1, 8.3, **8.22**, **8.28**