## SPATIAL DOMAIN FILTERING

- Smoothing spatial filters
  - Smoothing linear filters
  - Order-statistic(nonlinear) filters
- Sharpening spatial filters
  - The laplacian
  - Unsharp masking and highboost filtering
  - The gradient

isotropic 各向同性

## SPATIAL DOMAIN FILTERING

- The laplacian

$$\frac{\partial^2 f(y)}{\partial y^2} == f(j+1) + f(j-1) - 2f(j)$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\begin{aligned}\frac{\partial^2 f(x)}{\partial x^2} &= \nabla_x f(i+1) - \nabla_x f(i) \\ &= [f(i+1) - f(i)] - [f(i) - f(i-1)] \\ &= f(i+1) + f(i-1) - 2f(i)\end{aligned}$$

$f(i, j-1)$ $f(i, j)$ $f(i, j+1)$

# SPATIAL DOMAIN FILTERING

- The laplacian

$$H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

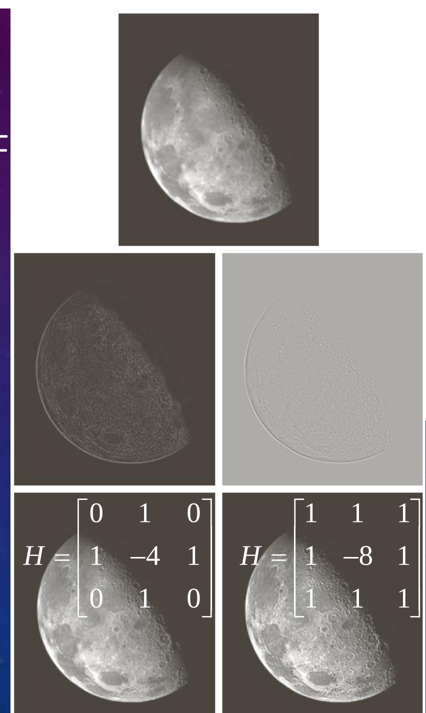
$$\begin{aligned}\nabla^2 f &= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \\ &= f(i+1,j) + f(i-1,j) - 2f(i,j) + f(i,j+1) + f(i,j-1) - 2f(i,j) \\ &= f(i+1,j) + f(i-1,j) + f(i,j+1) + f(i,j-1) - 4f(i,j)\end{aligned}$$

Chap3.6.2 p185

# SPATIAL DOMAIN F

- The laplacian

$$g = f - k\nabla^2 f$$




a
b c
d e

**FIGURE 3.38** (a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

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## SPATIAL DOMAIN FILTERING

- Unsharp masking and highboost filtering

$$g = f - \overline{f}$$

unsharp mask

$$f' = f + kg$$


a
b
c
d
e

**FIGURE 3.40** (a) Original image. (b) Result of blurring with a Gaussian filter. (c) Unsharp mask. (d) Result of using unsharp masking. (e) Result of using highboost filtering.

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## SPATIAL DOMAIN FILTERING

- The gradient

$$G[f(x,y)] = \begin{bmatrix} g_x \\ g_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla F = \frac{\partial F}{\partial x}\vec{i} + \frac{\partial F}{\partial y}\vec{j}$$

It points in the direction of the greatest rate of change of *f* at location (x,y)

magnitude

$$M(x,y) = Mag[f(x,y)] = \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

$$\frac{\partial f(x)}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \rightarrow \frac{\partial f(x,y)}{\partial x} \approx f(i+1,j) - f(i,j) \qquad \frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

# SPATIAL DOMAIN FILTERING

$$\frac{f(i+1,j) - f(i-1,j)}{2}$$

- For digital image

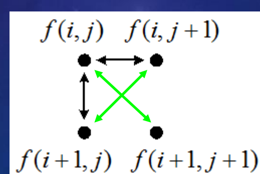
$$G[f(x,y)] = \{[f(i+1,j) - f(i,j)]^2 + [f(i,j+1) - f(i,j)]^2\}^{1/2}$$

or

$$G[f(x,y)] = | f(i+1,j) - f(i,j) | + | f(i,j+1) - f(i,j) |$$

| | | | |
|---|---|---|---|
| −1 | 0 | 0 | −1 |
| 0 | 1 | 1 | 0 |

Roberts cross gradient operators




| | | | | | |
|---|---|---|---|---|---|
| −1 | −2 | −1 | −1 | 0 | 1 |
| 0 | 0 | 0 | −2 | 0 | 2 |
| 1 | 2 | 1 | −1 | 0 | 1 |

Sobel operators

# SPATIAL DOMAIN FILTERING

- The gradient

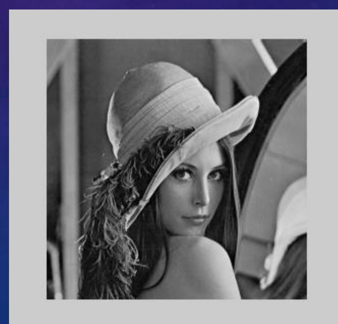
$$G[f(x,y)] = \left[ \left( \frac{\partial f}{\partial x} \right)^2 + \left( \frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

## SPATIAL DOMAIN FILTERING

- The gradient

$$g(x,y)=\begin{cases} G[f(x,y)] & G[f(x,y)] \geq T \\ f(x,y) & others \end{cases}$$

$$G[f(x,y)]=\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$

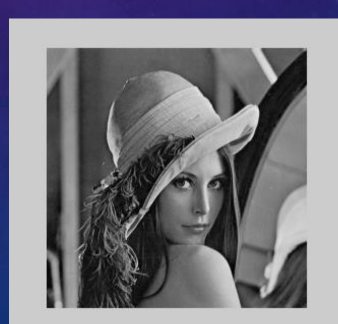


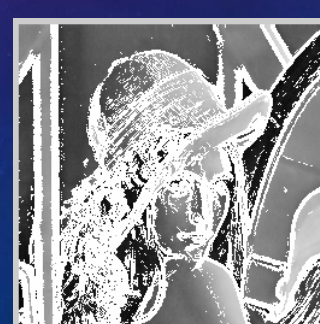
## SPATIAL DOMAIN FILTERING

- The gradient

$$g(x,y)=\begin{cases} L_G & G[f(x,y)] \geq T \\ f(x,y) & others \end{cases}$$

$$G[f(x,y)]=\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$



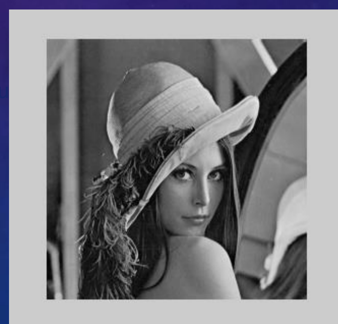


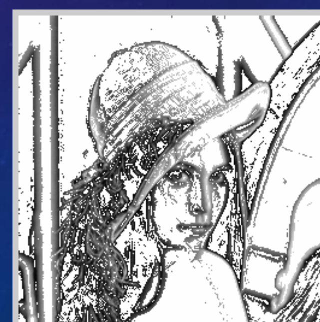
# SPATIAL DOMAIN FILTERING

- The gradient

$$g(x,y)=\begin{cases} G[f(x,y)] & G[f(x,y)]\geq T \\ L_G & others \end{cases}$$

$$G[f(x,y)]=\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$



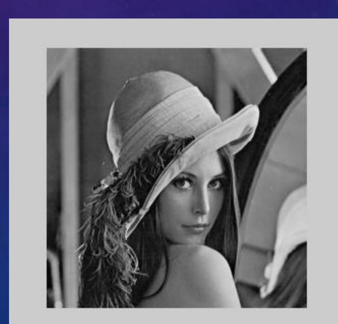


# SPATIAL DOMAIN FILTERING

- The gradient

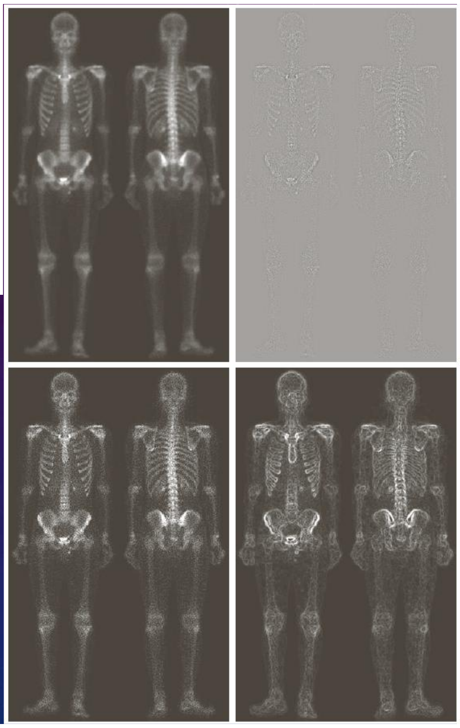
$$g(x,y)=\begin{cases} L_G & G[f(x,y)]\geq T \\ L_B & others \end{cases}$$

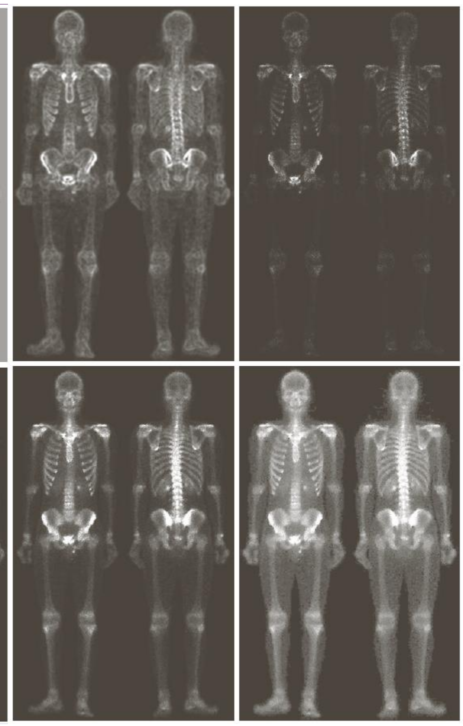
$$G[f(x,y)]=\left[\left(\frac{\partial f}{\partial x}\right)^2+\left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}$$



a b
c d

**FIGURE 3.43**
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).





e f
g h

**FIGURE 3.43**
*(Continued)*
(e) Sobel image smoothed with a $5 \times 5$ averaging filter. (f) Mask image formed by the product of (c) and (e).
(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)