UESTC4004
Digital Communications

Pulse shaping

University of Glasgow

Lecture Overview

星座图.

- Constellation diagram
- Pulse shaping 脉冲整形 → 为了消除ISI
- Eye diagram

眼图 → 描述ISI特性.

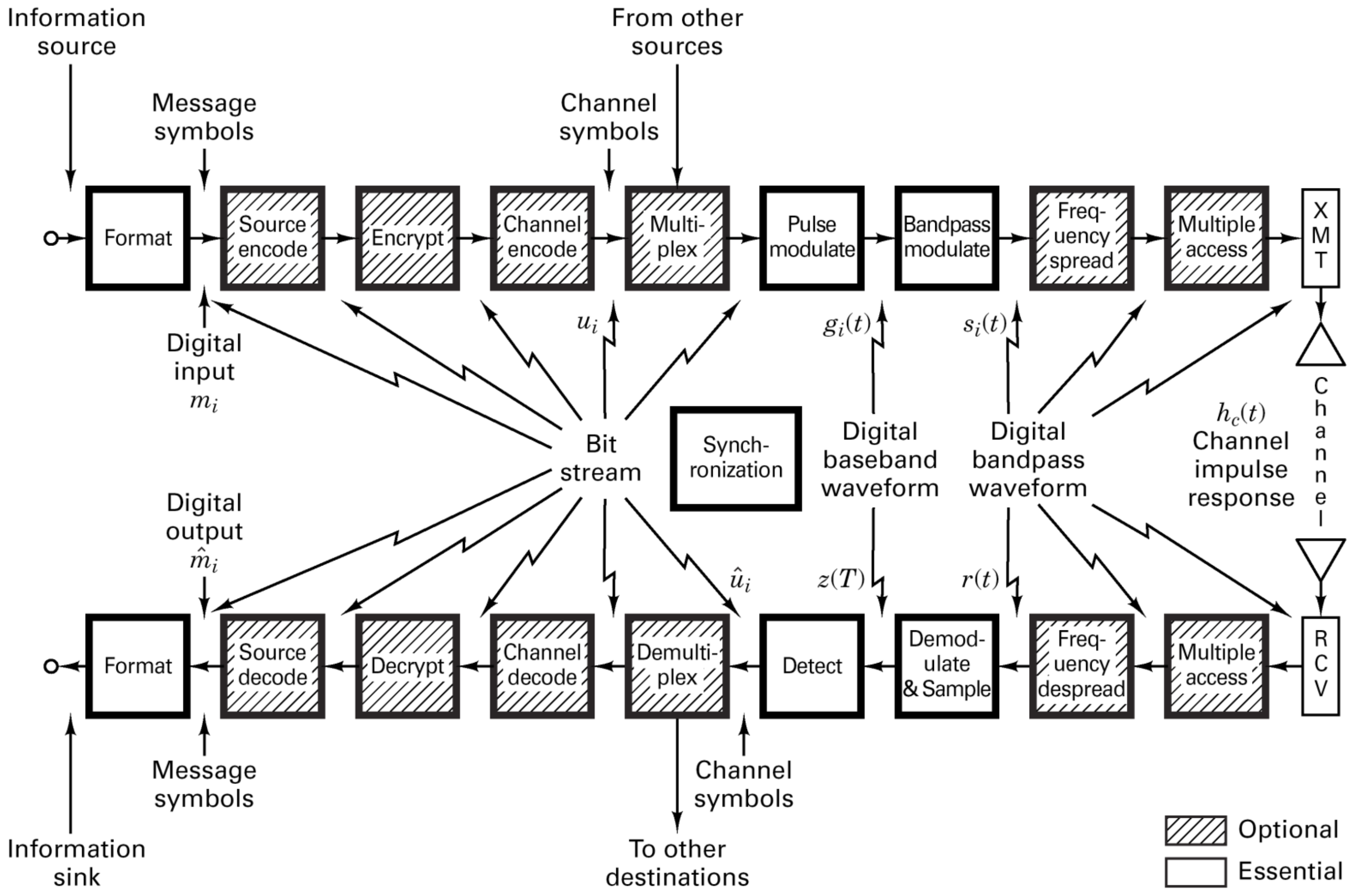
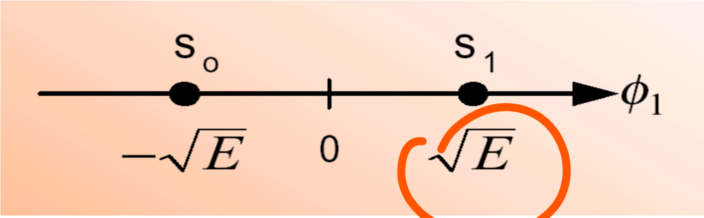


Figure 1.2 Block diagram of a typical digital communication system.

Constellation Diagram

- Is a method of representing the symbol states of modulated bandpass signals in terms of their amplitude and phase 幅相特性
- In other words, it is a geometric representation of signals
- There are three types of binary signals:
  - **Antipodal**
    - Two signals are said to be antipodal if one signal is the negative of the other $\Rightarrow s_1(t) = -s_0(t)$
    - The signal have equal energy with signal point on the real line



$$E_{avg} = \frac{E+E}{2} = E$$

  - **ON-OFF**
    - Are one dimensional signals either ON or OFF with signaling points falling on the real line

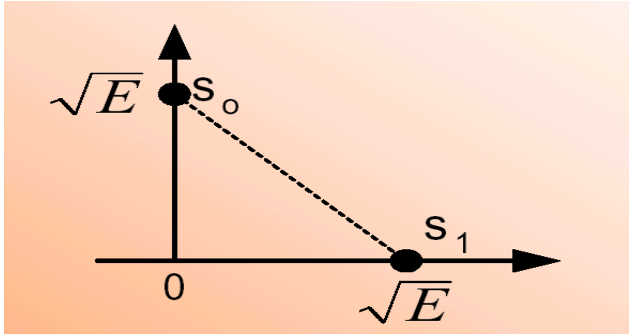
- With OOK, there are just 2 symbol states to map onto the constellation space
  - $a(t) = 0$ (no carrier amplitude, giving a point at the origin)
  - $a(t) = A \cos w_c t$ (giving a point on the positive horizontal axis at a distance A from the origin)

(BASK比如)

$$E_{avg} = \frac{0+E}{2} = \frac{E}{2}$$

- **Orthogonal**
  - Requires a two-dimensional geometric representation since there are two linearly independent functions $s1(t)$ and $s0(t)$

比如
BPSK



线性无关与复轴

$$E_{avg} = \frac{E+E}{2} = E$$

Typically, the horizontal axis is taken as a reference for symbols that are ***Inphase*** with the carrier $\cos w_c t$, and the vertical axis represents the ***Quadrature*** carrier component, $\sin w_c t$

Error Probability of Antipodal and Orthogonal Waveforms

$E_{avg} = \frac{d_1^2 + d_2^2}{2}$

- The error probability is given by

$$P_B = Q\left(\sqrt{\frac{E_d}{2N_0}}\right) \quad 3.63$$

怎么式在星图上

- E_d is the energy associated with the distance between the signals

距离能量

- N_0 is the noise spectral density

噪声功率强度

$-\sqrt{E}, \sqrt{E}$

Antipodal signals

Distance between signals: $2\sqrt{E}$

$$E_d = \left(2\sqrt{E}\right)^2 = 4E = 4E_{avg}$$

$$P_B = Q\left(\sqrt{\frac{2E_{avg}}{N_0}}\right)$$

Orthogonal signals

Distance between signals: $\sqrt{2E}$

$$E_d = \left(\sqrt{2E}\right)^2 = 2E = 2E_{avg}$$

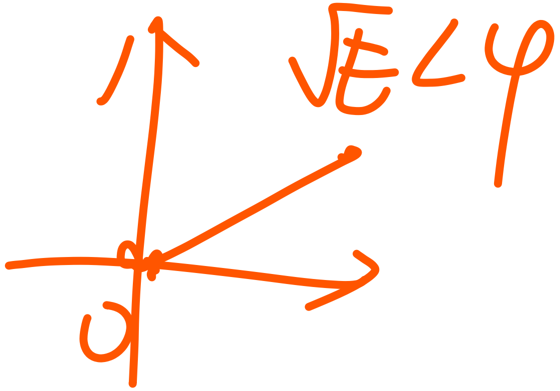
$$P_B = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right)$$

$\sqrt{E}\angle 90^\circ$

$\sqrt{E}\angle 0^\circ$

E_d = 距离的平方 $(d)^2$

Quiz:



Show that for ON-OFF or OOK (ON-OFF Keying)

$$\boxed{P_B = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right)}$$

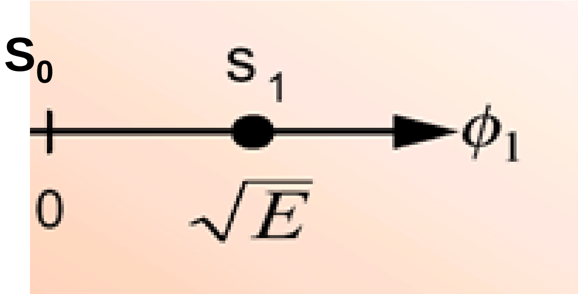
$$E_{avg} = \frac{(\sqrt{E})^2}{2} = \frac{E}{2}$$

$$E_d = (\sqrt{E})^2 = E = 2E_{avg} \Rightarrow P_B = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right)$$

Quiz:

Show that for ON-OFF

$$\boxed{P_B = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right)}$$



On-OFF

Distance between signals: $\sqrt{E}$

$$E_d = \left(\sqrt{E}\right)^2 = E = 2E_{avg}$$

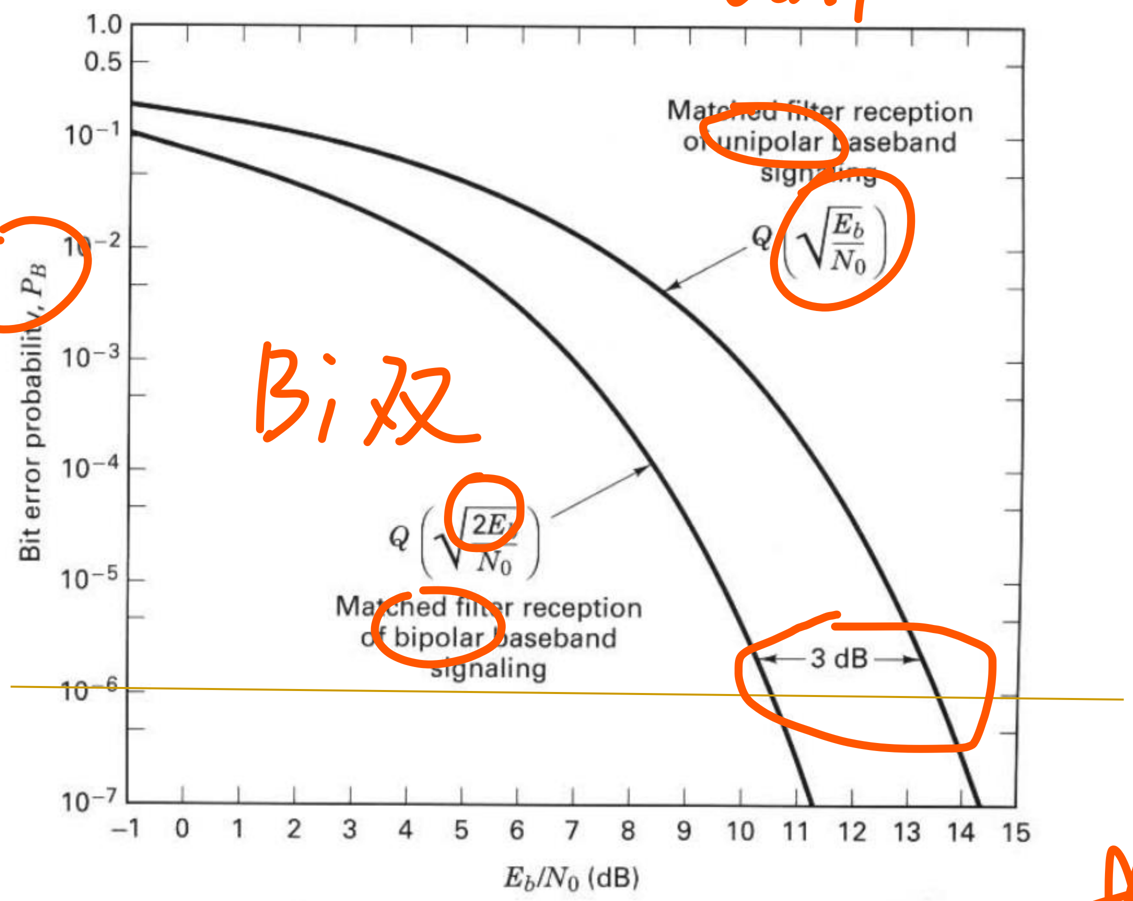
$$P_B = Q\left(\sqrt{\frac{E_{avg}}{N_0}}\right)$$

ON-OFF Keying (OOK) is also called Unipolar and its orthogonal

Error Probability Performance of Binary Signaling

- Bipolar signals require a factor of 2 increase in energy compared to Unipolar
- Since *$10\log_{10}2 = 3$ dB*, we say that bipolar signaling offers a *3 dB* better performance than Unipolar

uni —



Bi 双

Figure 3.14 Bit error performance of unipolar and bipolar signaling.

Note that average energy per bit $E_b \equiv E_{avg}$

Unipolar signalling is an example of **baseband Orthogonal signalling** while **Bipolar signalling** is a example of **baseband Antipodal signalling**

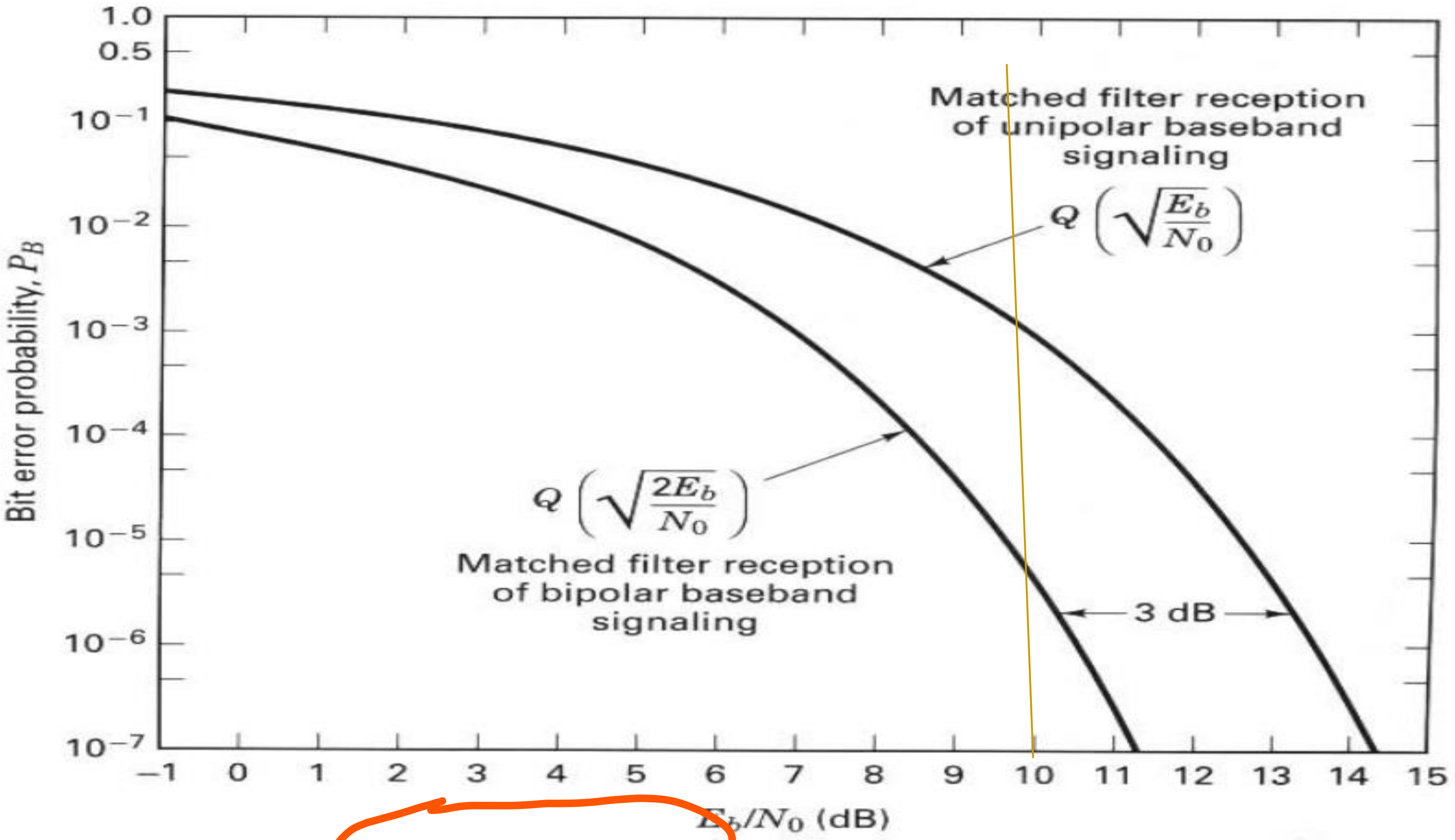
Q function Table

Table 1: Values of $Q(x)$ for $0 \le x \le 9$

x	$Q(x)$	x	$Q(x)$	x	$Q(x)$	x	$Q(x)$
0.00	0.5	2.30	0.010724	4.55	2.6823×10^{-6}	6.80	5.231×10^{-12}
0.05	0.48006	2.35	0.0093867	4.60	2.1125×10^{-6}	6.85	3.6925×10^{-12}
0.10	0.46017	2.40	0.0081975	4.65	1.6597×10^{-6}	6.90	2.6001×10^{-12}
0.15	0.44038	2.45	0.0071428	4.70	1.3008×10^{-6}	6.95	1.8264×10^{-12}
0.20	0.42074	2.50	0.0062097	4.75	1.0171×10^{-6}	7.00	1.2798×10^{-12}
0.25	0.40129	2.55	0.0053861	4.80	7.9333×10^{-7}	7.05	8.9459×10^{-13}
0.30	0.38209	2.60	0.0046612	4.85	6.1731×10^{-7}	7.10	6.2378×10^{-13}
0.35	0.36317	2.65	0.0040246	4.90	4.7918×10^{-7}	7.15	4.3389×10^{-13}
0.40	0.34458	2.70	0.003467	4.95	3.7107×10^{-7}	7.20	3.0106×10^{-13}
0.45	0.32636	2.75	0.0029798	5.00	2.8665×10^{-7}	7.25	2.0839×10^{-13}
0.50	0.30854	2.80	0.0025551	5.05	2.2091×10^{-7}	7.30	1.4388×10^{-13}
0.55	0.29116	2.85	0.002186	5.10	1.6983×10^{-7}	7.35	9.9103×10^{-14}
0.60	0.27425	2.90	0.0018658	5.15	1.3024×10^{-7}	7.40	6.8092×10^{-14}
0.65	0.25785	2.95	0.0015889	5.20	9.9644×10^{-8}	7.45	4.667×10^{-14}
0.70	0.24196	3.00	0.0013499	5.25	7.605×10^{-8}	7.50	3.1909×10^{-14}
0.75	0.22663	3.05	0.0011442	5.30	5.7901×10^{-8}	7.55	2.1763×10^{-14}
0.80	0.21186	3.10	0.0009676	5.35	4.3977×10^{-8}	7.60	1.4807×10^{-14}
0.85	0.19766	3.15	0.00081635	5.40	3.332×10^{-8}	7.65	1.0049×10^{-14}
0.90	0.18406	3.20	0.00068714	5.45	2.5185×10^{-8}	7.70	6.8033×10^{-15}
0.95	0.17106	3.25	0.00057703	5.50	1.899×10^{-8}	7.75	4.5946×10^{-15}
1.00	0.15866	3.30	0.00048342	5.55	1.4283×10^{-8}	7.80	3.0954×10^{-15}
1.05	0.14686	3.35	0.00040406	5.60	1.0718×10^{-8}	7.85	2.0802×10^{-15}
1.10	0.13567	3.40	0.00033693	5.65	8.0224×10^{-9}	7.90	1.3945×10^{-15}
1.15	0.12507	3.45	0.00028029	5.70	5.9904×10^{-9}	7.95	9.3256×10^{-16}
1.20	0.11507	3.50	0.00023263	5.75	4.4622×10^{-9}	8.00	6.221×10^{-16}
1.25	0.10565	3.55	0.00019262	5.80	3.3157×10^{-9}	8.05	4.1397×10^{-16}
1.30	0.0968	3.60	0.00015911	5.85	2.4579×10^{-9}	8.10	2.748×10^{-16}
1.35	0.088508	3.65	0.00013112	5.90	1.8175×10^{-9}	8.15	1.8196×10^{-16}
1.40	0.080757	3.70	0.0001078	5.95	1.3407×10^{-9}	8.20	1.2019×10^{-16}
1.45	0.073529	3.75	8.8417×10^{-5}	6.00	9.8659×10^{-10}	8.25	7.9197×10^{-17}
1.50	0.066807	3.80	7.2348×10^{-5}	6.05	7.2423×10^{-10}	8.30	5.2056×10^{-17}
1.55	0.060571	3.85	5.9059×10^{-5}	6.10	5.3034×10^{-10}	8.35	3.4131×10^{-17}
1.60	0.054799	3.90	4.8096×10^{-5}	6.15	3.8741×10^{-10}	8.40	2.2324×10^{-17}
1.65	0.049471	3.95	3.9076×10^{-5}	6.20	2.8232×10^{-10}	8.45	1.4565×10^{-17}
1.70	0.044565	4.00	3.1671×10^{-5}	6.25	2.0523×10^{-10}	8.50	9.4795×10^{-18}
1.75	0.040059	4.05	2.5609×10^{-5}	6.30	1.4882×10^{-10}	8.55	6.1544×10^{-18}
1.80	0.03593	4.10	2.0658×10^{-5}	6.35	1.0766×10^{-10}	8.60	3.9858×10^{-18}
1.85	0.032157	4.15	1.6624×10^{-5}	6.40	7.7688×10^{-11}	8.65	2.575×10^{-18}
1.90	0.028717	4.20	1.3346×10^{-5}	6.45	5.5925×10^{-11}	8.70	1.6594×10^{-18}
1.95	0.025588	4.25	1.0689×10^{-5}	6.50	4.016×10^{-11}	8.75	1.0668×10^{-18}
2.00	0.02275	4.30	8.5399×10^{-6}	6.55	2.8769×10^{-11}	8.80	6.8408×10^{-19}
2.05	0.020182	4.35	6.8069×10^{-6}	6.60	2.0558×10^{-11}	8.85	4.376×10^{-19}
2.10	0.017864	4.40	5.4125×10^{-6}	6.65	1.4655×10^{-11}	8.90	2.7923×10^{-19}
2.15	0.015778	4.45	4.2935×10^{-6}	6.70	1.0421×10^{-11}	8.95	1.7774×10^{-19}
2.20	0.013903	4.50	3.3977×10^{-6}	6.75	7.3923×10^{-12}	9.00	1.1286×10^{-19}
2.25	0.012224						

Comparing BER Performance

误码率



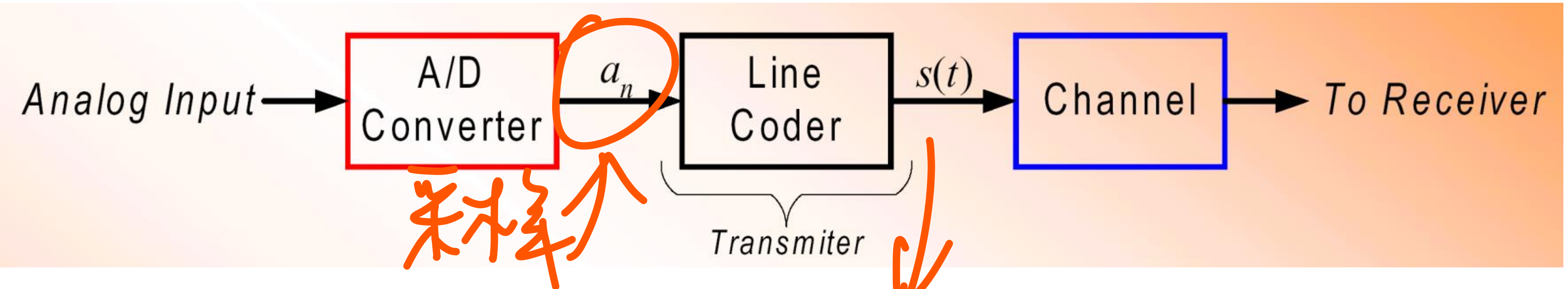
信噪比

Figure 3.14 Bit error performance of unipolar and bipolar signaling.

- For the same received E_b/N_0 ratio, bipolar signaling provides lower bit error rate than unipolar signaling

Baseband Communication System

- We have been considering the following baseband system



- The transmitted signal is created by the ***line coder*** according to

$$s(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT_b)$$

where a_n is the ***symbol mapping*** and *g(t)* is the ***pulse shape***

Problems with Line Codes

- The big problem with the line codes is that they are not ***bandlimited***
  - The absolute bandwidth is infinite
  - The power outside the 1st null bandwidth is not negligible
    - That is, the power in the sidelobes can be quite high

Inter Symbol Interference (ISI) 码间干扰

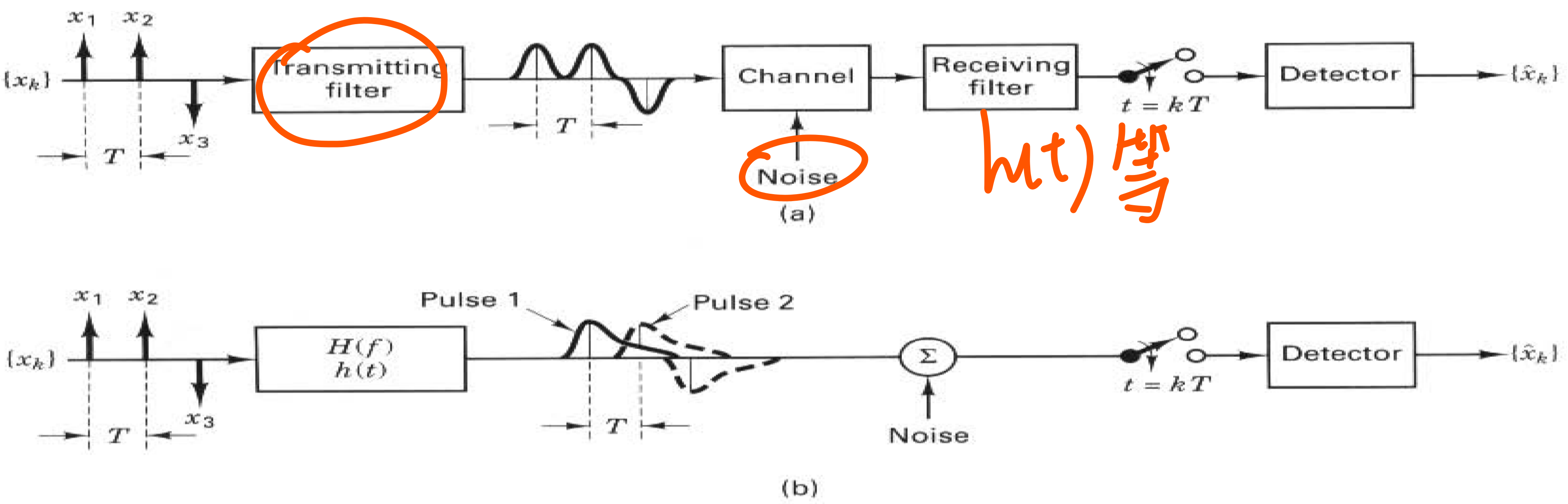


Figure 3.15 Intersymbol interference in the detection process. (a) Typical baseband digital system. (b) Equivalent model.

$$\boxed{H(f) = H_t(f)\, H_c(f)\, H_r(f)}$$

trans channel receive

Nyquist to help…

理论最小带宽

- Theoretical minimum system bandwidth needed to detect R_s symbols/sec without ISI, is $R_s/2$ Hz

奈奎斯特第一准则

- Sinc and Rect relationship $sinc(\frac{t}{T}) \leftrightarrow T\,rect(\frac{f}{1/T})$

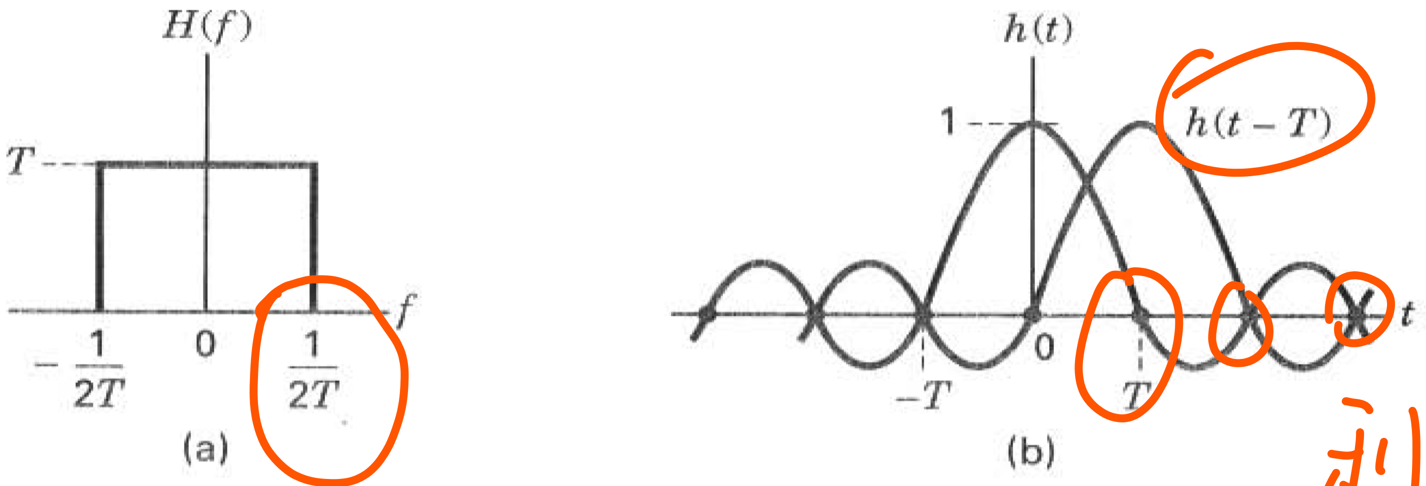


Figure 3.16 Nyquist channels for zero ISI. (a) Rectangular system transfer function $H(f)$. (b) Received pulse shape $h(t) = \text{sinc}(t/T)$.

Trade-off and Goals

Shrink the pulse to obtain better spectrum efficiency at the cost of increased ISI and vice versa.

利用t=kT时刻抽样

- **Problems with Sinc(.) function**
  - But there are problems with Sinc(.) shaped pulse
    - It is not possible to create Sinc shaped pulses due to
      - Infinite time duration
      - Sharp transition band in the frequency domain
  - Sinc(.) shaped pulse can cause ISI in the presence of timing errors
    - If sampling is not performed at exactly the correct sampling time, then ISI will occur

Raised Cosine Filter

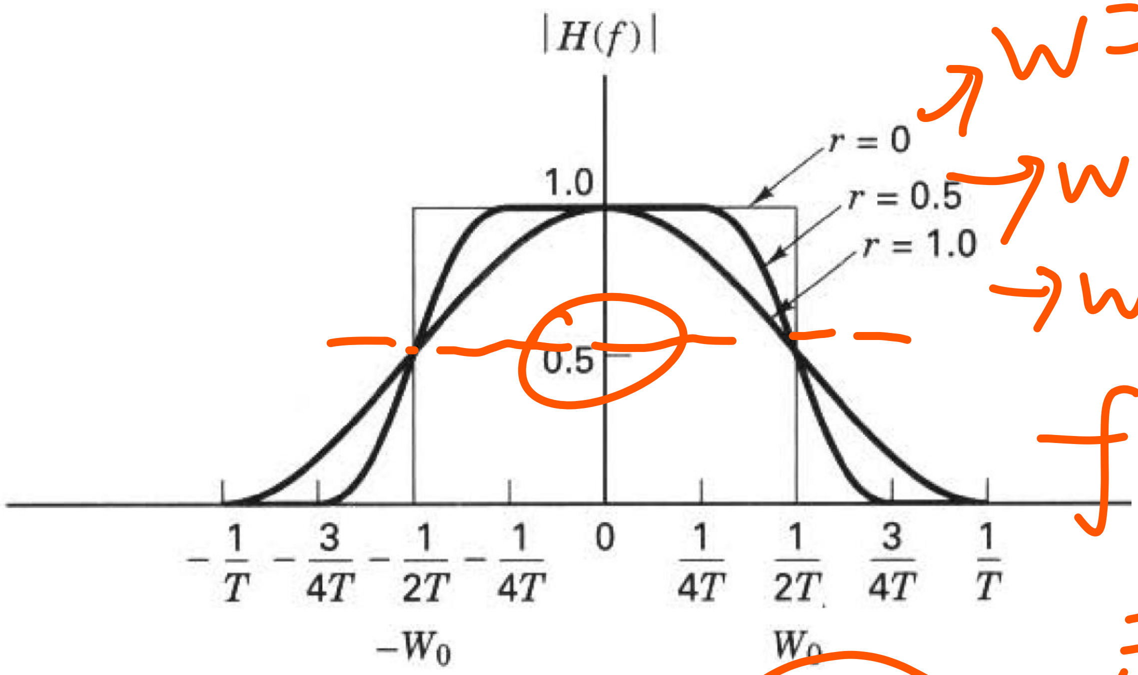
升余弦

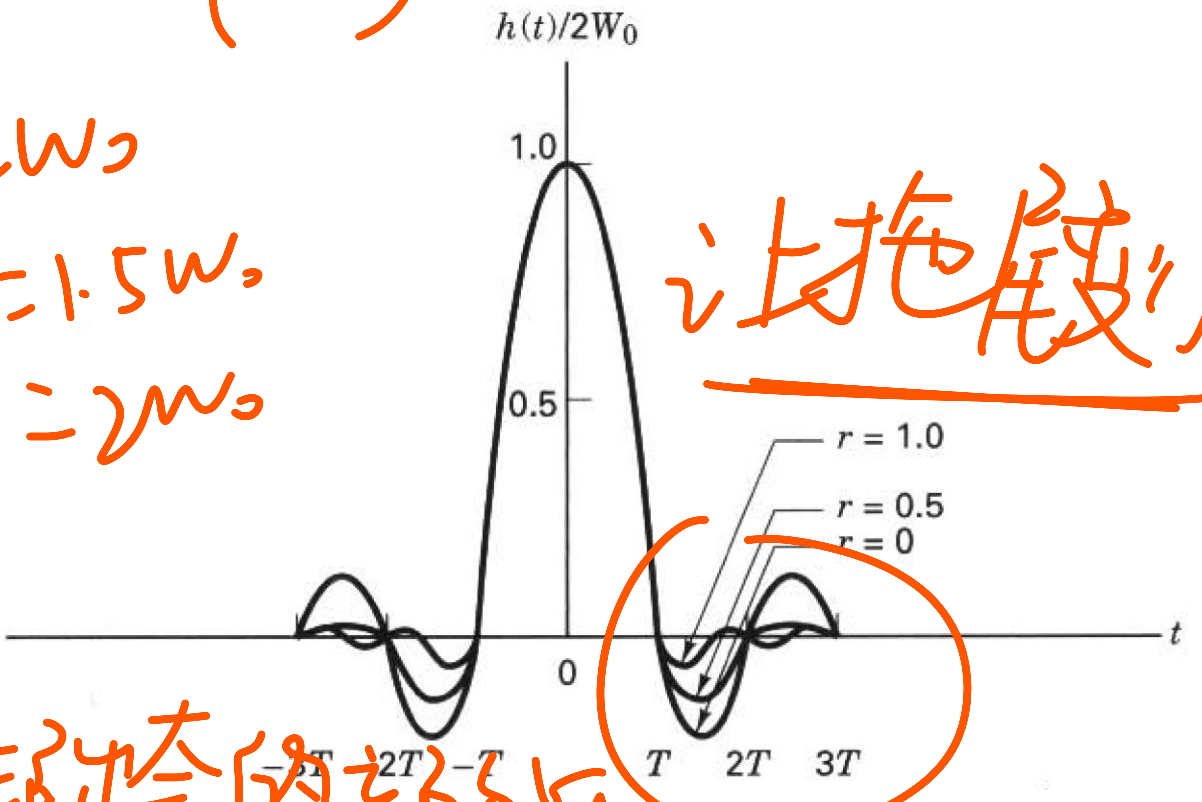
- Belongs to Nyquist class (zero ISI at sampling instants)

$$H(f) = \begin{cases} 1 & \text{for } |f| < 2W_0 - W \\ \cos^2\left(\frac{\pi}{4}\frac{|f| + W - 2W_0}{W - W_0}\right) & \text{for } 2W_0 - W < |f| < W \\ 0 & \text{for } |f| > W \end{cases}$$

$$h(t) = 2W_0(\text{sinc } 2W_0 t)\frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

$2W_0 - W = 2W_0 - (1+r)W_0 = (1-r)W_0$





$$W = rW_0 + W_0 = (1+r)W_0$$

$$r = \frac{W - W_0}{W_0} \quad \text{where} \quad 0 \le r \le 1$$

超出的部分的宽度

滚降系数

奈·带宽

Effect on bandwidth requirements

效率 $\boxed{E = \frac{R_s}{W}}$

$W = (1+r)W_0$

$$\boxed{W = \frac{1}{2}(1+r)R_s}$$

Find the minimum required bandwidth for the baseband transmission of a four level PAM pulse sequence having a data rate of $R = 2400$ bits/s if the system transfer characteristic consists of a raised-cosine spectrum with 100% excess bandwidth $(r = 1)$.

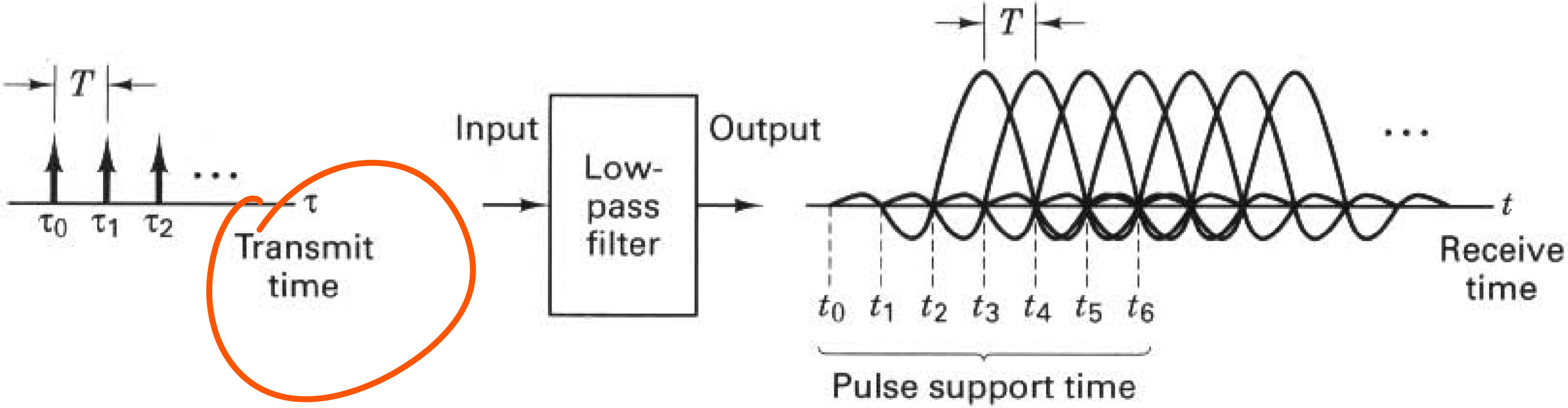
W, W_0, r, R_s 关系

$R_s \leq 2W_0$ (极限 $R_s = 2W_0$)

Practical Issues with Pulse Shaping

- Like the sinc(.) pulse, RC rolloff pulses extend infinitely in time
  - However, a very good approximation can be obtained by truncating the pulse
    - Can make *h(t)* extend from *-3T* to *+3T* 升余弦滚降
- RC rolloff pulses are *less sensitive to timing* errors than sinc(.) pulses
  - Larger values of r are more robust against timing errors
- US Digital Cellular (IS-54/136) uses root RC roll off pulse shaping with r=*0.35*
- IS-95 uses pulse shape that is slightly different from RC roll off shape
- European GSM uses Gaussian shaped pulses

Overall effect with pulse shape filtering



Root RC rolloff Pulse Shaping

- We saw earlier that the noise is minimized at the receiver by using a ***matched filter***
  - If the transmit filter is *H(f)*, then the receive filter should be *H*(f)*
- The combination of transmit and receive filters must satisfy Nyquist's first method for zero ISI

$$H_e(f) = H(f)H^*(f) \Rightarrow H(f) = \sqrt{H_e(f)}$$

根升余弦滚降

- Transmit filter with the above response is called the ***root raised cosine-rolloff filter***
- Root RC rolloff pulse shapes are used in many applications such as IS- 54 and IS-136

实际上
由2个环节共同实现 $H_e(f) \Rightarrow RCR$

$H(f) = \sqrt{H_e(f)}$

$(H_T(f)$和$H_R(f))$ RRCR

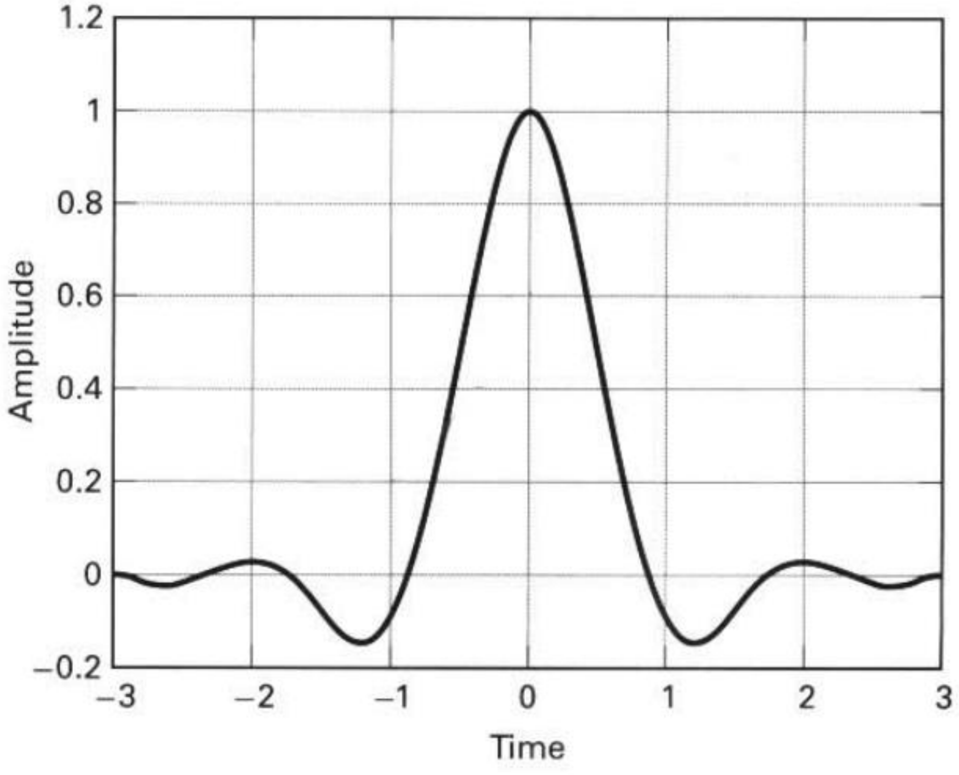


Figure 3.22a Square-root Nyquist pulse.

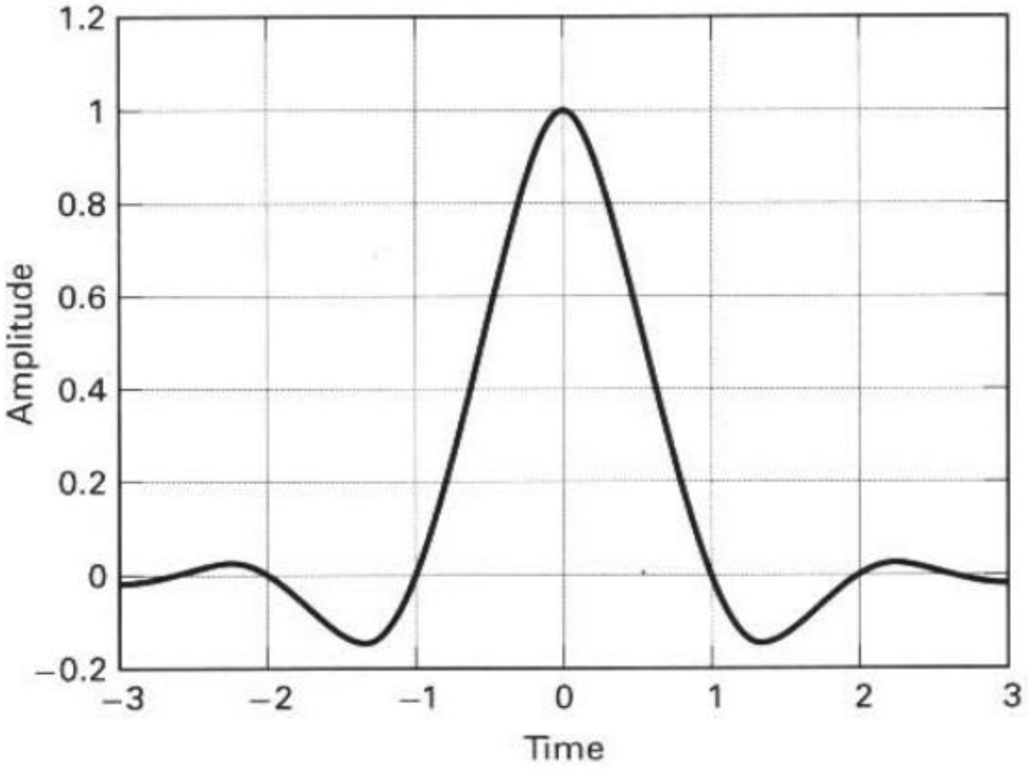


Figure 3.22b Nyquist pulse.

Notice the zero crossings!!!

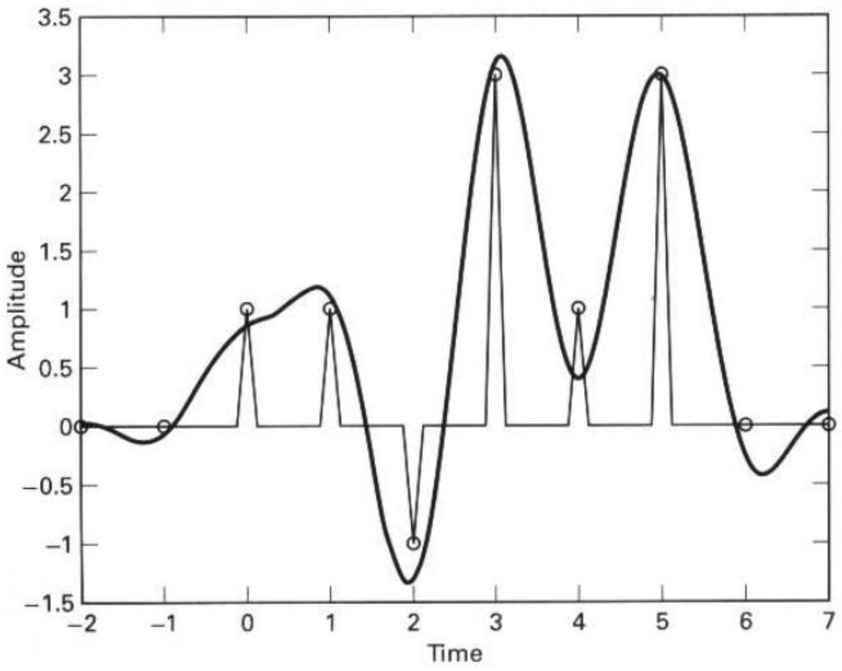


Figure 3.23a Square-root Nyquist-shaped *M*-ary waveform and delayed-input sample values.

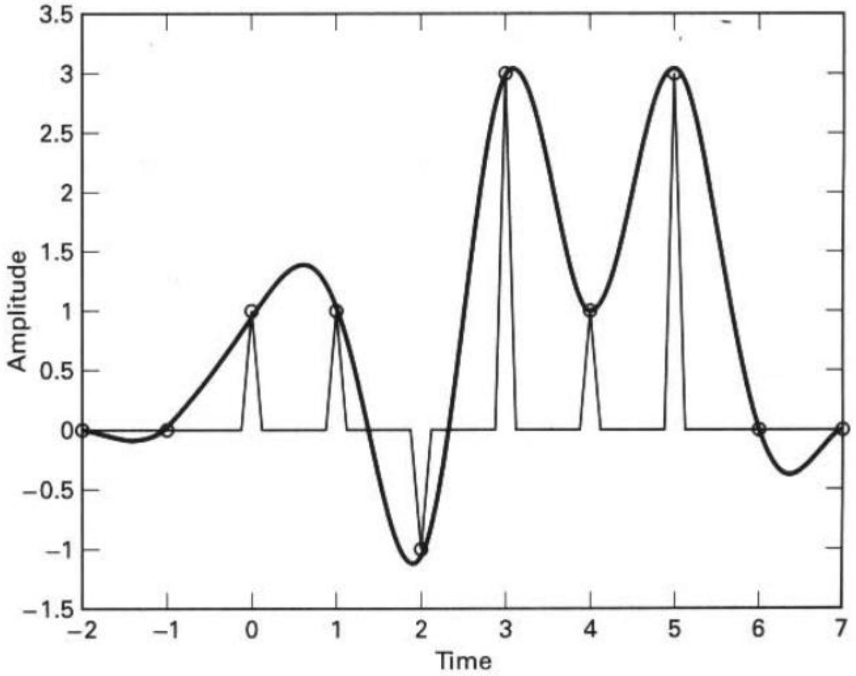
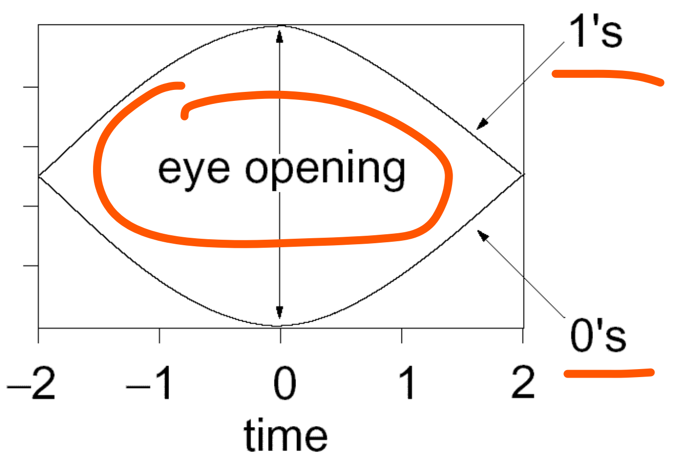


Figure 3.23b Output of raised-cosine matched filter and delayed-input sample values.

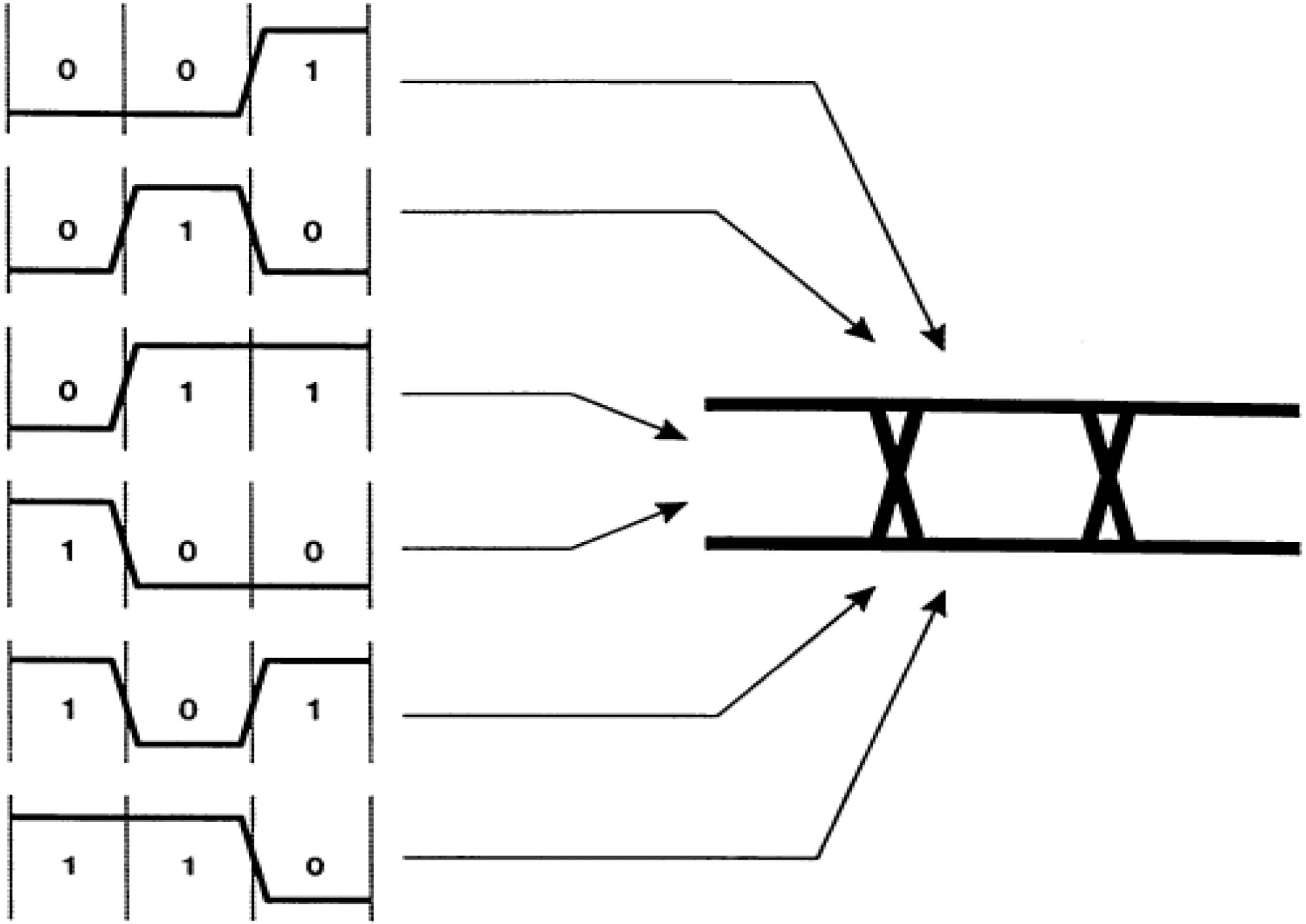
For data sequence of {1 1 -1 3 1 3}

Eye Patterns

- An **eye pattern** is obtained by superimposing the actual waveforms for large numbers of transmitted or received symbols
  - Perfect eye pattern for noise-free, bandwidth-limited transmission of an alphabet of two digital waveforms encoding a binary signal (1’s and 0’s)

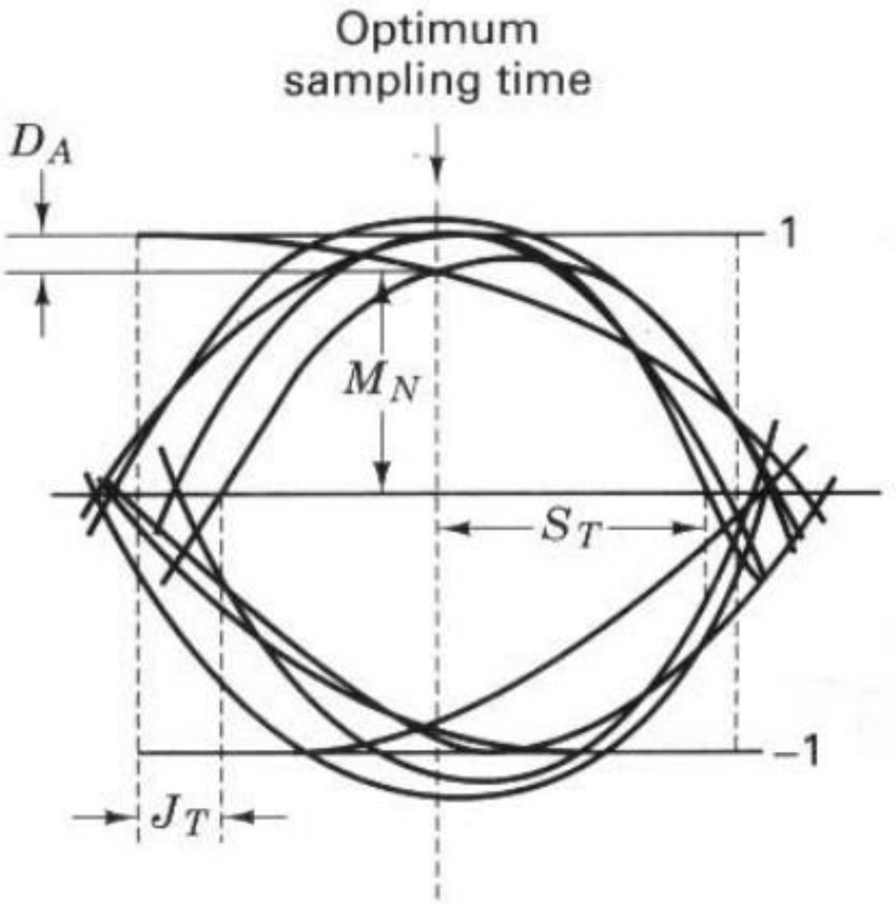


  - Actual eye patterns are used to estimate the bit error rate and the signal to- noise ratio

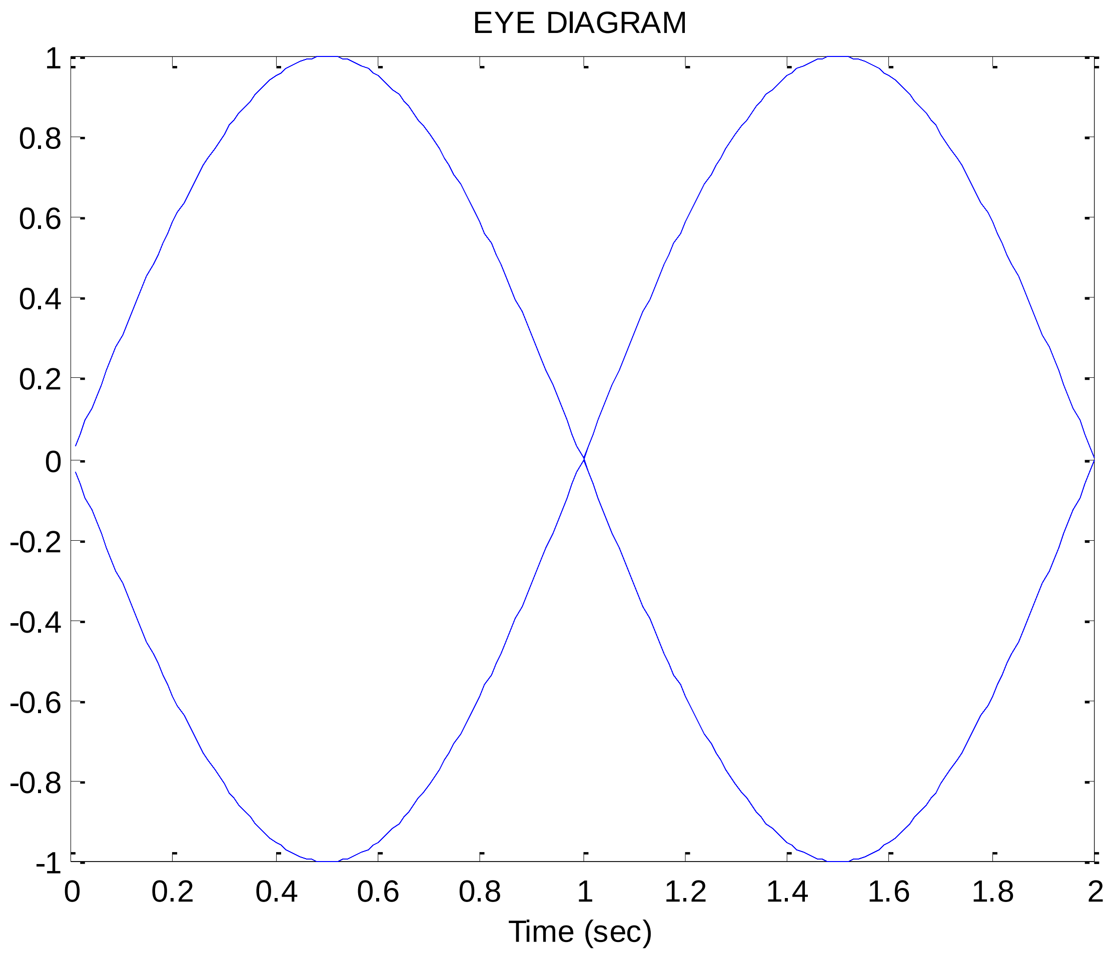


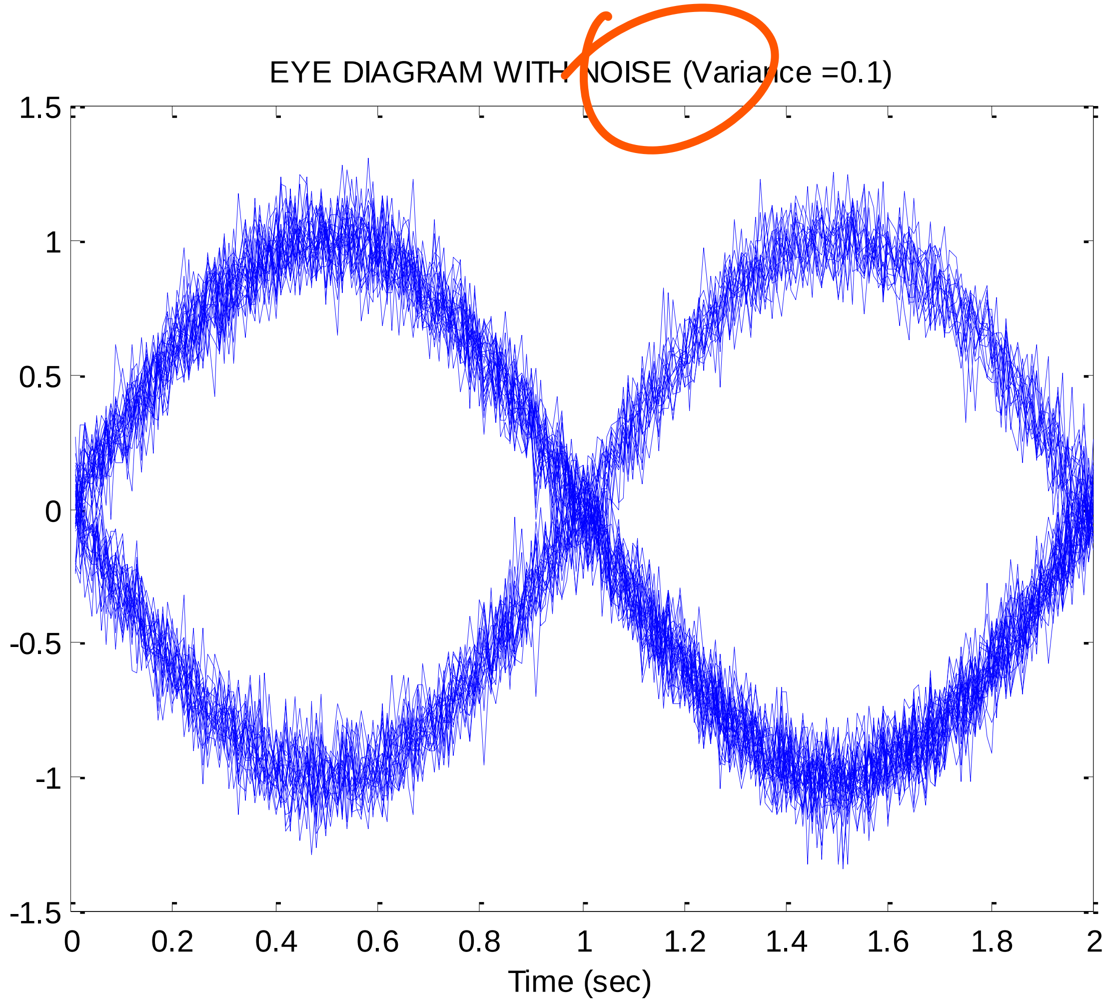
Concept of the **eye pattern**

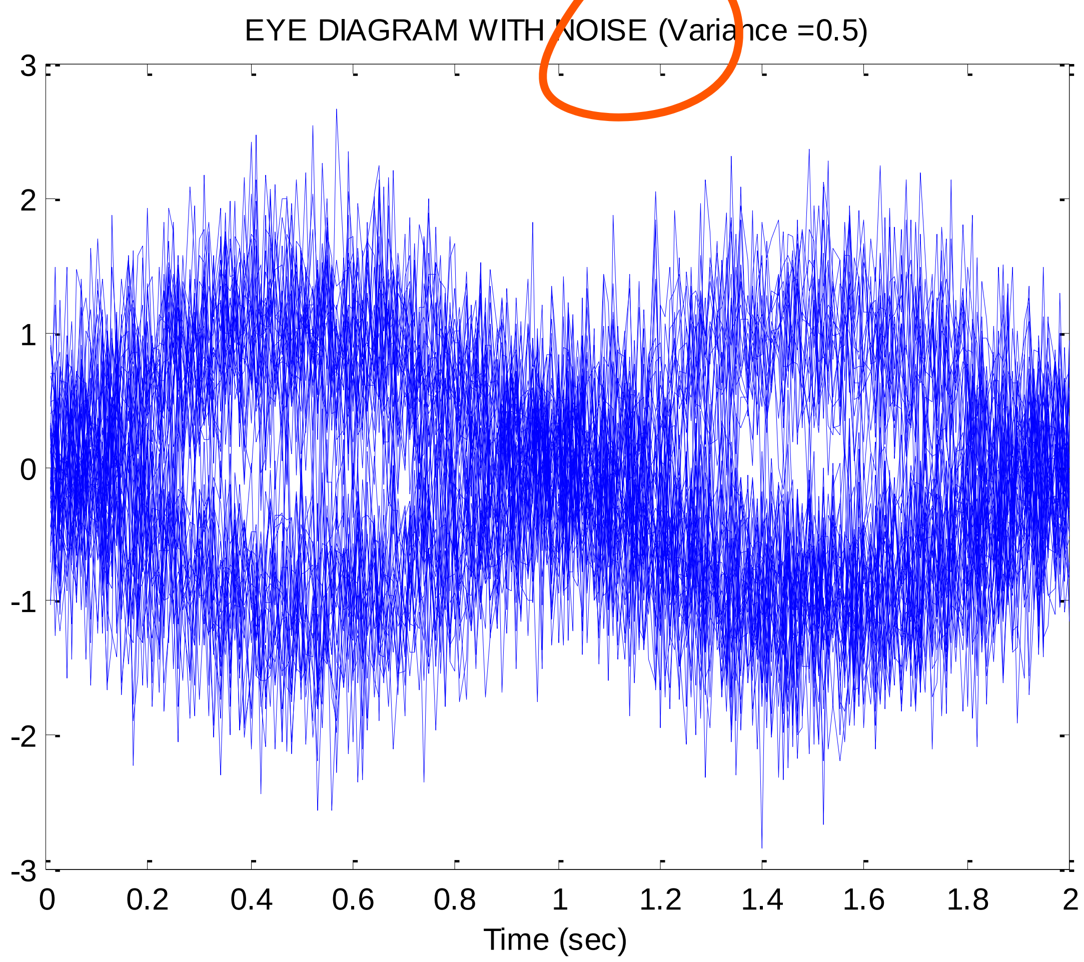
What does it tell?



D_A: Measure of distortion caused by ISI
J_T: Measure of Timing jitter
M_N: Measure of Noise margin
S_T: Measure of sensitivity to timing errors


EYE DIAGRAM
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time (sec)


EYE DIAGRAM WITH NOISE (Variance =0.1)
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time (sec)


EYE DIAGRAM WITH NOISE (Variance =0.5)
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time (sec)

Review Questions

8-ary ⇒ 2^3-ary ⇒ 1个sym 3bits

Binary data at 9.6 kbits/s are transmitted using 8-ary PAM modulation with a system using a raised cosine roll-off filter characteristic. The system has a frequency response out to 2.4 kHz. (a) What is the symbol rate? (b) What is the roll-off factor of the filter characteristic? (c) What is the theoretical minimum system bandwidth needed without ISI [a) 3200 symbols/s b) 0.5 c) 1600 Hz]

c).

a). $9.6k/3 = 3.2k$ sym/s $= R_s \Rightarrow f_m = \frac{R_s}{2} = 1.6$ kHz $= W_0$

b). $r = \frac{2.4-1.6}{1.6} = 0.5$

What is the theoretical minimum system bandwidth needed for a 10 Mbits/s signal using 16-level PAM without ISI? How large can the filter roll-off factor be if the allowable system bandwidth is 1.375 MHz? [1.25 MHz, 0.1]