UESTC4004
Digital Communications

Equalization

Equalization

考虑信道

- Nyquist filtering and pulse shaping schemes assumes that the channel is precisely known and its characteristics do not change with time

  $h(t)$ $H(f)$

- However, in practice we encounter channels whose frequency response are either unknown or change with time
  - For example, each time we dial a telephone number, the communication channel will be different because the communication route will be different
  - However, when we make a connection, the channel becomes time-invariant
  - The characteristics of such channels are not known a priori

- Examples of time-varying channels are radio channels
  - These channels are characterized by time-varying frequency response characteristics

Channel distortions

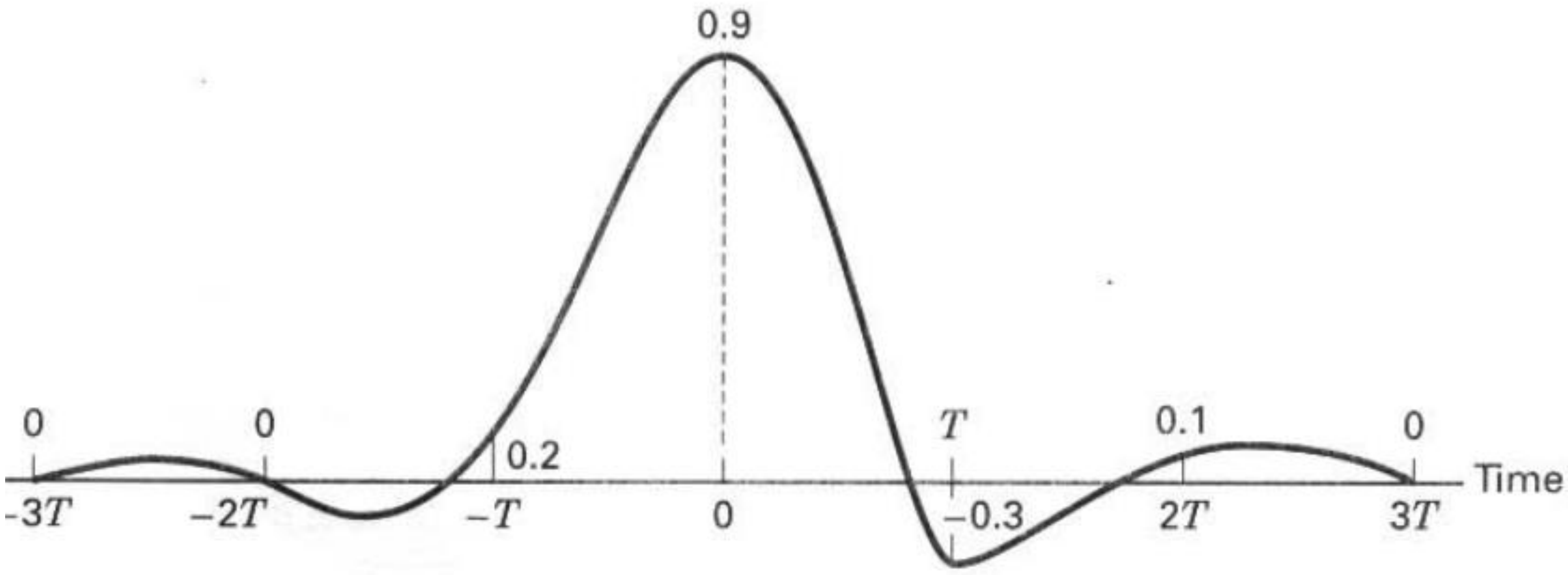
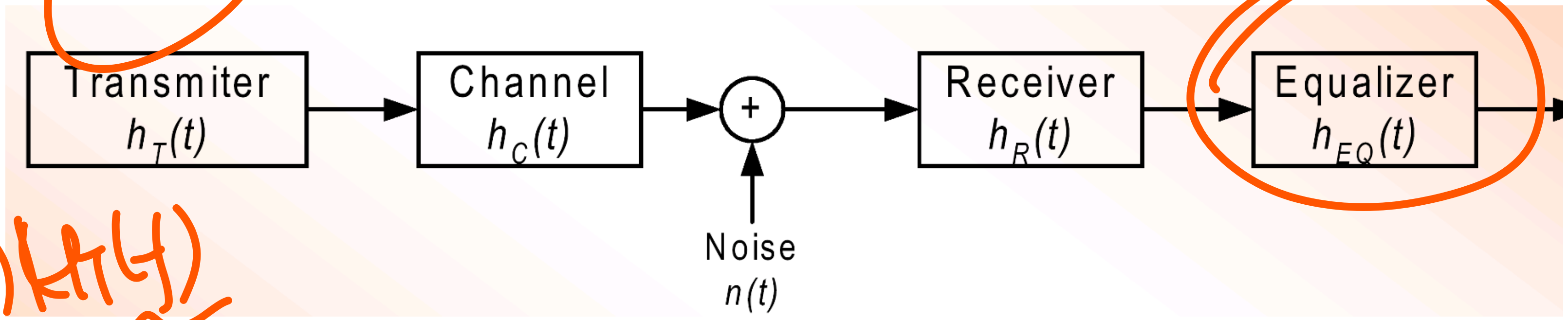


Figure 3.25 Received pulse exhibiting distortion.

- To compensate for channel induced ISI we use a process known as ***Equalization:*** a technique of correcting the frequency response of the channel
- The filter used to perform such a process is called an ***equalizer***



$He(f) = H_R(f)H_T(f)$

- Since $H_R(f)$ is matched to $H_T(f)$, we usually worry about $H_C(f)$
- The goal is to pick the frequency response $H_{EQ}(f)$ of the ***equalizer*** such that

总公式

$$H_c(f)H_{EQ}(f) = 1 \Rightarrow H_{EQ}(f) = \frac{1}{H_c(f)} = \frac{1}{|H_c(f)|}e^{-j\theta_c(f)}$$

where

$$|H_{EQ}(f)| = \frac{1}{|H_C(f)|}$$ and the phase characteristics $$\Theta_{EQ}(f) = -\Theta_C(f)$$

Note that $H_c(f) = |H_c(f)|e^{j\theta_c(f)}$

Problems with Equalization

- It can be difficult to determine the inverse of the channel response
  - If the channel response is zero at any frequency, then the inverse is not defined at that frequency
  - The receiver generally does not know what the channel response is. Channel changes in real time so equalization must be adaptive

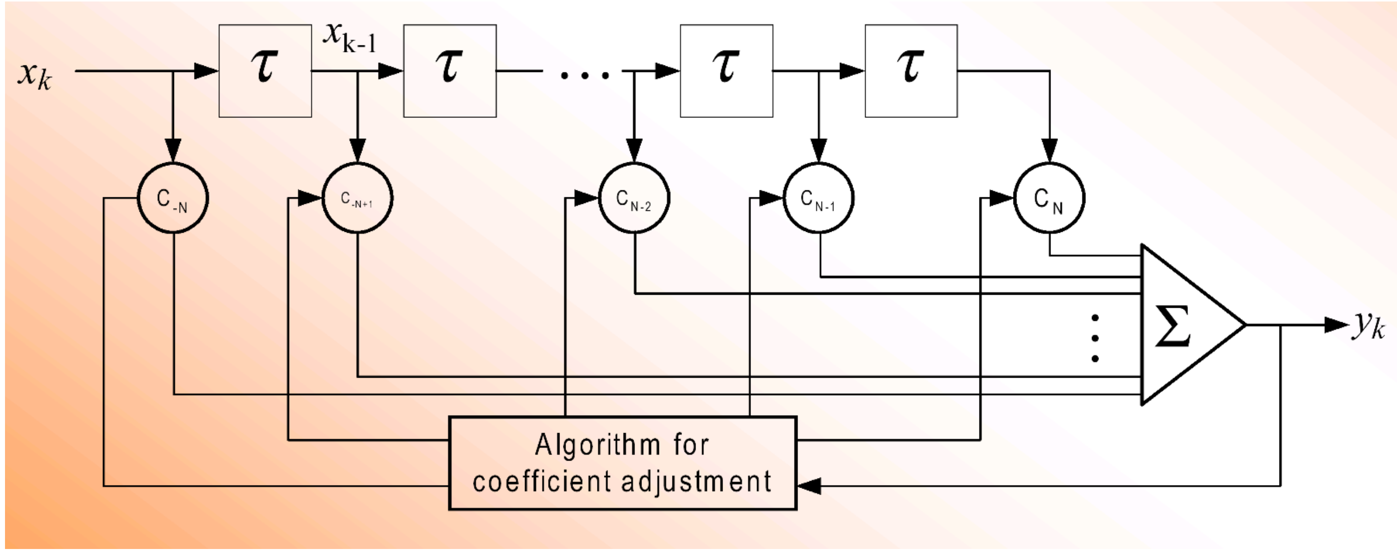
- The equalizer can have an infinite impulse response even if the channel has a finite impulse response
  - The impulse response of the equalizer must usually be truncated

Equalization Techniques or Structures

- Three Basic Equalization Structures
  - **Linear Transversal Filter**
    - Simple implementation using Tap Delay Line or FIR filters
    - FIR filter has guaranteed stability (although adaptive algorithm which determines coefficients may still be unstable)
  - **Decision Feedback Equalizer**
    - Extra step in subtracting estimated residual error from signal
  - **Maximal Likelihood Sequence Estimator (Viterbi)**
    - “Optimal” performance
    - High complexity and implementation problem (not heavily used)

Linear Transversal Equalizer

- This is simply a linear filter with adjustable parameters
- The parameters are adjusted on the basis of the measurement of the channel characteristics
- A common choice for implementation is the ***transversal filter*** (Tap Delay Line) or the FIR filter with adjustable tap coefficient



FIR

Fig. 3.26

- **Total number of taps = 2N+1**
- **Total delay = 2Nτ**

3taps = 2N+1 ⇒ N=1

- N is chosen sufficiently large so that equalizer spans length of the ISI.
- Normally the ISI is assumed to be limited to a finite number of samples
- The output y_k of the Tap Delay Line equalizer in response to the input sequence $\{x_k\}$ is

$$\boxed{y_k = \sum_{n=-N}^{N} x(k-n)c_n, \qquad k = -2N,\ldots,2N}$$

权重

where c_n is the weight of the n^{th} tap

- Ideally, we would like the equalizer to eliminate ISI resulting in

$$\boxed{y_k = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases}}$$

- But this cannot be achieved in practice.

$N, n, k \begin{cases} x(k-n) \\ y_k \\ c_n \end{cases}$

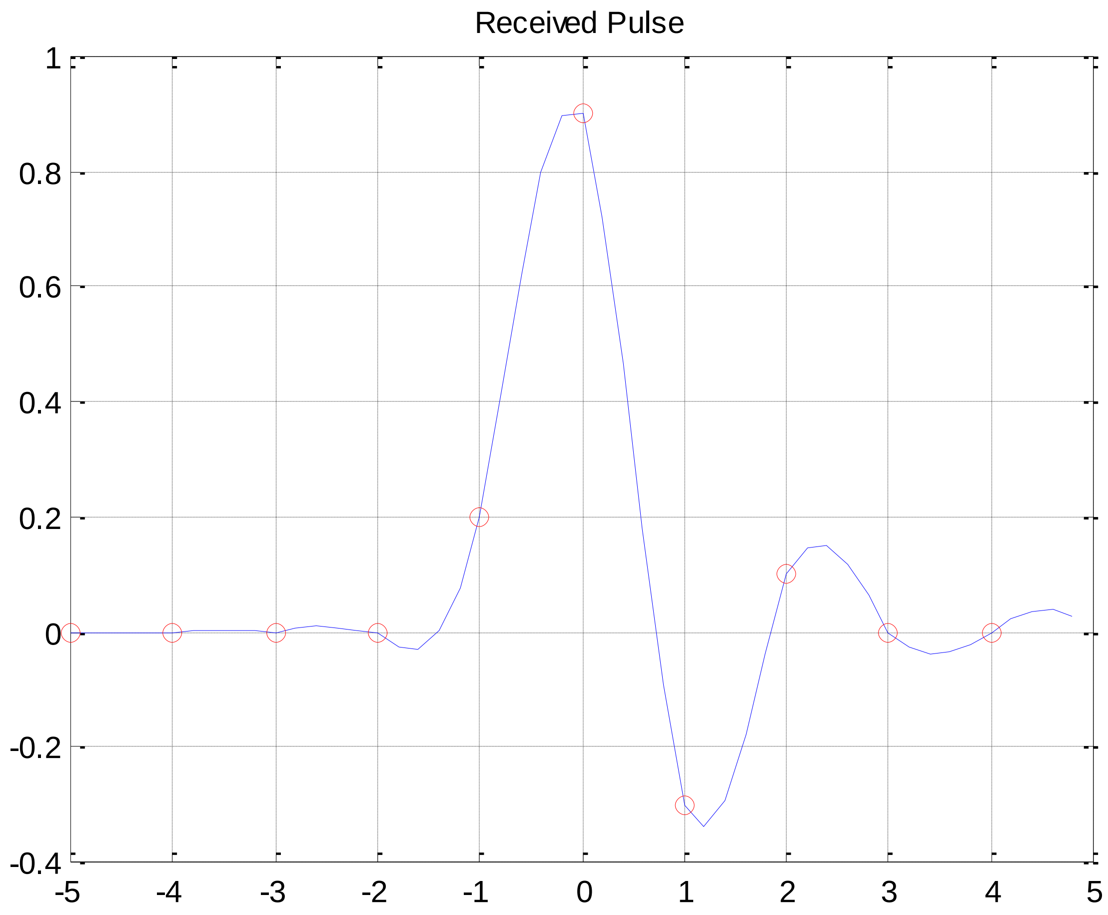
- Zero forcing Equalizer : However, the tap gains can be chosen such that

$$\boxed{y_k = \begin{cases} 1, & k = 0 \\ 0, & k = \pm 1, \pm 2, \ldots\ldots \pm N \end{cases}}$$

如何确定 y_k.

$$y = \begin{bmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

Example 3.5: (Page 155)



Zero-Forcing Solution

- For N=1

y_k

$\sum_n x(k-n)$

$$k=-1, \quad y(-1)=c_{-1}x(-1-(-1))+c_0x(-1-(0))+c_1x(-1-(1))$$

$$k=0, \quad y(0)=c_{-1}x(0-(-1))+c_0x(0-(0))+c_1x(0-(1))$$

$$k=1, \quad y(1)=c_{-1}x(1-(-1))+c_0x(1-(0))+c_1(1-(1))$$

$$\Rightarrow \begin{bmatrix} y(-1) \\ y(0) \\ y(1) \end{bmatrix} = \underbrace{\begin{bmatrix} x(0) & x(-1) & x(-2) \\ x(1) & x(0) & x(-1) \\ x(2) & x(1) & x(0) \end{bmatrix}}_{(2N+1)\times(2N+1)} \underbrace{\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}}_{(2N+1)\times 1}$$

- For N=2

$$\overset{y}{\begin{bmatrix} y(-2) \\ y(-1) \\ y(0) \\ y(1) \\ y(2) \end{bmatrix}} = \overset{X}{\begin{bmatrix} x(0) & x(-1) & x(-2) & x(-3) & x(-4) \\ x(1) & x(0) & x(-1) & x(-2) & x(-3) \\ x(2) & x(1) & x(0) & x(-1) & x(-2) \\ x(3) & x(2) & x(1) & x(0) & x(-1) \\ x(4) & x(3) & x(2) & x(1) & x(0) \end{bmatrix}} \overset{c}{\begin{bmatrix} c_{-2} \\ c_{-1} \\ c_0 \\ c_1 \\ c_2 \end{bmatrix}}$$

Generalizing results

$$\boxed{\mathbf{c} = \mathbf{X}^{-1} y}$$

$$where \quad \mathrm{y} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad for\ N = 1$$

Example

$N=1$

- Design a 3-taps zero forcing equalizer for input $x(n) = \{0, -0.1, 0.15, 0.87, 0.12, -0.2, 0\}$ in which $x(0) = 0.87$.

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3$

- Answer: c=[-0.236 1.220 -0.2226]'.

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 & x_{-1} & x_2 \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

Minimum MSE Solution

- The tap weights c_n are chosen to minimize the mean-square error (MSE) of all ISI terms plus noise power at the output of the equalizer
  - Set of overdetermined equations is used to obtain a minimum MSE solution by multiplying both sides of the equation by $\boldsymbol{X}^T$
  - $\boldsymbol{X}^T\boldsymbol{y} = \boldsymbol{X}^T\boldsymbol{X}\boldsymbol{c}$
    - $\boldsymbol{X}$ is a non-square matrix with dimension $4N+1$ by $2N+1$
    - $\boldsymbol{y}$ and $\boldsymbol{c}$ are vectors with $4N+1$ and $2N+1$, respectively
  - $\boldsymbol{R}_{Xy} = \boldsymbol{R}_{XX}\boldsymbol{c}$
    - $\boldsymbol{R}_{Xy} = \boldsymbol{X}^T\boldsymbol{y}$ is called the cross-correlation vector
    - $\boldsymbol{R}_{XX} = \boldsymbol{X}^T\boldsymbol{X}$ is called the auto-correlation vector
  - The tap weight can be obtained as
    - $\boxed{\boldsymbol{c} = \boldsymbol{R}_{XX}^{-1}\boldsymbol{R}_{Xy}}$
- Compared with zero-forcing, MSE solution is more robust in the presence of noise and large ISI

Review Questions

- Given a received distorted set of pulse samples {x(k)}, with voltage values 0.0, 0.2, 0.9, -0.3, 0.1 as shown in slide 10, design a 3 tap zero-forcing equalizer to reduce the ISI.

  [-0.2140 0.9631 0.3448]

- Given that the channel transfer function H(f)=1/(1+j2πf). Derive the expression for the corresponding equalizer transfer function and sketch both its amplitude and phase characteristics

$C_{-1} \quad C_0 \quad C_1$

$$N=1 \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix} \begin{bmatrix} C_{-1} \\ C_0 \\ C_1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\theta = \tan^{-1}\left(\frac{-w}{1}\right) = -\tan^{-1} w$$

$$H(f) = \frac{1}{1+j2\pi f} = \frac{1}{1+jw} = \frac{1-jw}{1-(jw)^2} = \frac{1-jw}{1+w^2} = \frac{1}{1+w^2} - j\frac{w}{1+w^2} = \frac{1}{1+w^2} e^{-j\tan^{-1} w}$$

$$H_{eq}(f) H_c(f) = 1 \Rightarrow H_{eq}(f) = (1+w^2) e^{j\tan^{-1} w}$$