Solutions to Buck tutorial questions

1.
$V_{In} = 100V$
$V_{Out} = \varphi.V_{In} = 65V$
$I_{Out\ (Ave)} = 5A$
$P_{Out} = 325W$

2.
$V_{In} = 100V$
$V_{Out} = \varphi.V_{In} = 72V$
$I_{Out\ (Ave)} = 7A$
$P_{Out} = 504W$

3.
$V_{In} = 100V$
$V_{Out} = \varphi.V_{In} = 20V$
$I_{In(Ave)}=2A$
$I_{Out\ (Ave)} = 10A$
$P_{Out} = 200W$

4.
$V_{In} = 100V$
$V_{Out} = \varphi.V_{In} = 40V$
$I_{In(Ave)}=16A \times 40\% = 6.4A$
$I_{Out\ (Ave)} = 16A$
$P_{Out} = 640W$

Typical DC to DC Converter applications.

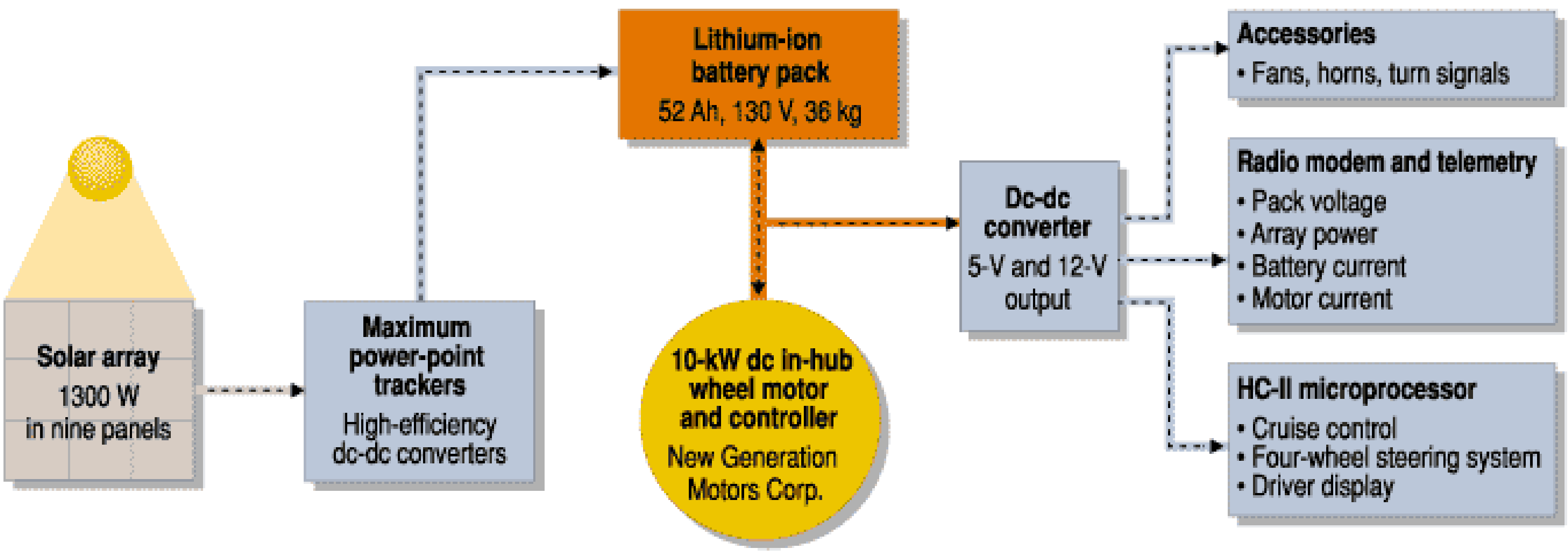
* DC Motor drives (e.g. Robotics)

* Power Factor Correction (PFC) and Active Filters.

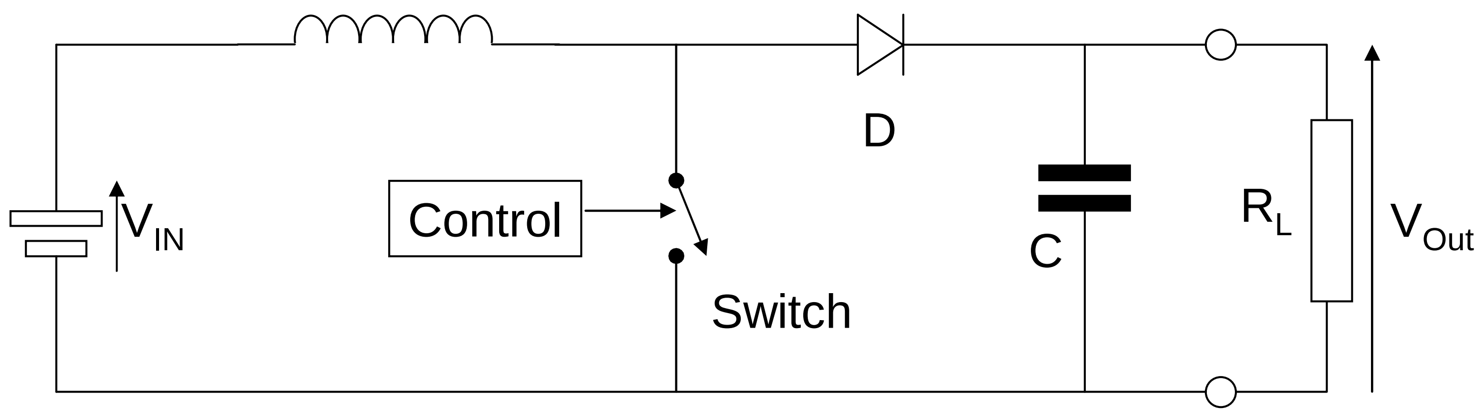
* Photovoltaic systems e.g. peak power tracking converters to transfer energy from the PV array to the load (satellites).

* Automotive applications – raising the battery voltage to another voltage e.g. High Intensity Discharge headlamps.

* Solar vehicles

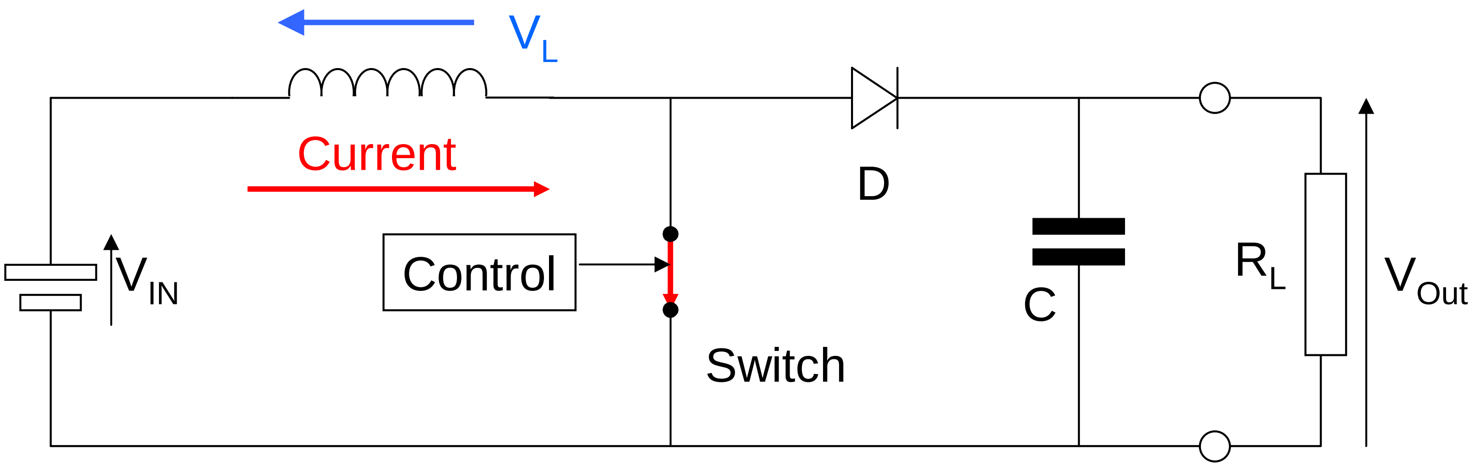


The boost (step-up) converter. ($V_{Out} > V_{In}$)



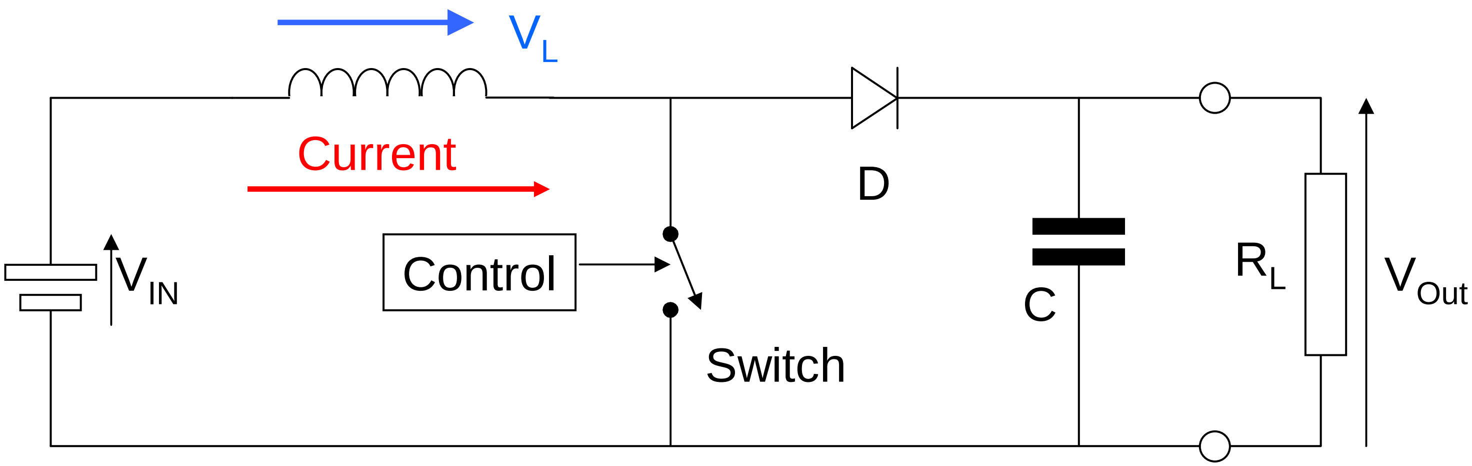
The boost converter is used to increase the output voltage (V_{Out}) above the input voltage (V_{In}). As in the case of the buck converter we will assume the circuit operates in continuous mode, i.e., current is always flowing in the inductor.

If we disregard the switch for a moment and examine the circuit, we see that in the steady state the output voltage is connected to the input voltage via an inductor (assume zero resistance) and a diode. Hence the output voltage will be one diode drop (1V) beneath the input voltage. As with the buck, ignore the voltage drop across the diode.



Now consider the circuit when the switch is closed. Any voltage on capacitor C will ensure the diode is reverse biased and therefore the output circuit is "disconnected" from the input. [Because the diode's anode is connected to ground via the switch.]

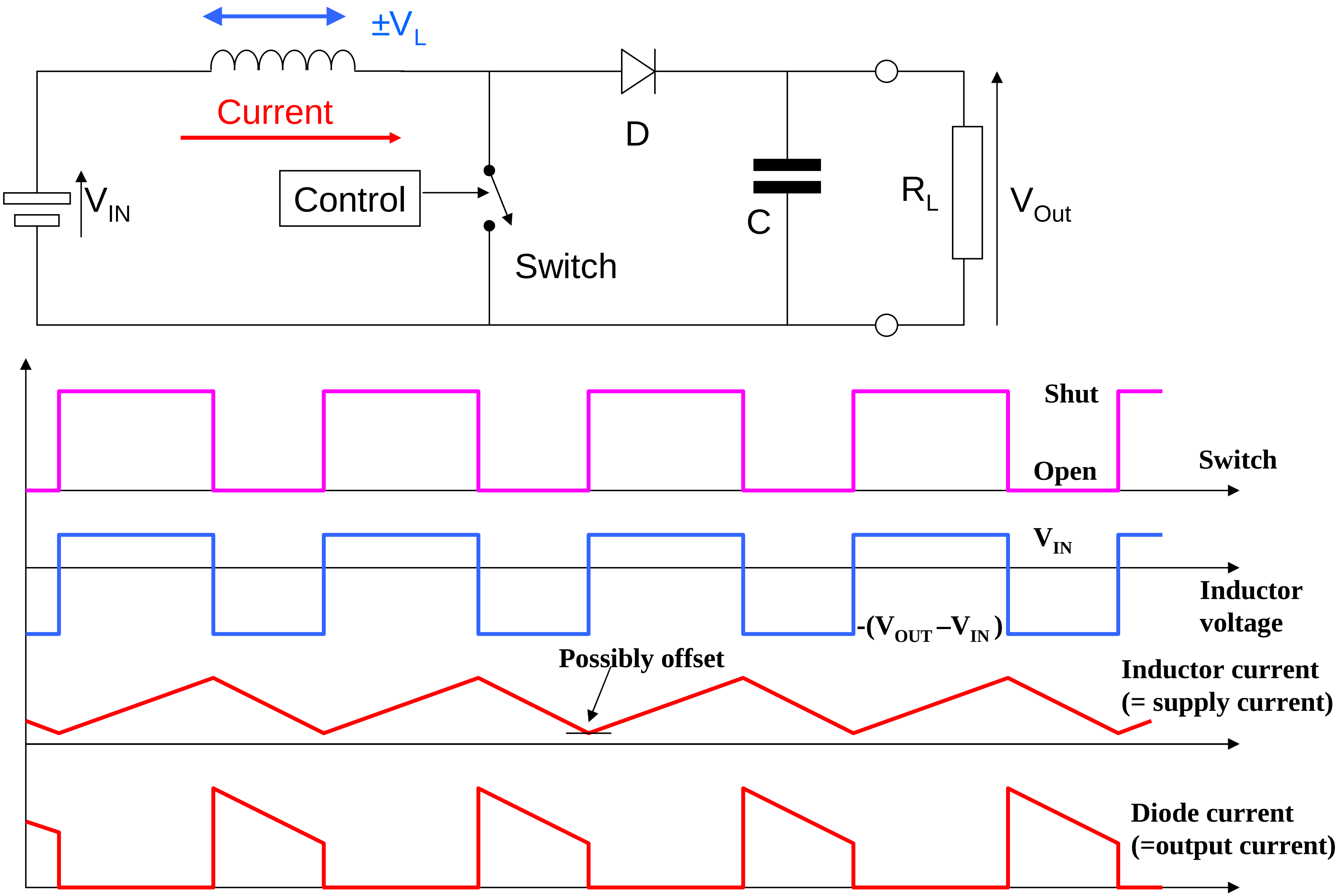
The inductor is connected across the supply (V_{In}) and hence current will start to build through it with the energy being stored in the magnetic field. During this phase, V_L = V_{In}.



When the switch is opened, the inductor wants to maintain the current flow and with the supply side of the inductor still held at V_{In}, the potential at the opposite side of the inductor rises until the diode starts to conduct. The inductor voltage during this phase is the difference between the input voltage (V_{In}) and the output voltage (V_{Out}). Note also that the sign of the voltage has changed when compared with the inductor charging period.

So $V_{Inductor} = -(V_{Out} - V_{In}) = V_{In} - V_{Out}$.

As energy is transferred from the inductor's magnetic field so the current flowing through the inductor and diode falls.



Note that current always flows in the inductor (continuous mode again)

Flux linkage analysis.

As with the buck converter, magnetic flux linkage Ψ is defined as the integral of the inductor voltage. During steady state load conditions the instantaneous value of the flux must be the same on consecutive PWM periods, otherwise there would be a net increase or decrease of flux.

From an energy point of view the magnetic field at any point on consecutive PWM periods is the same. Hence we can write:

$$\boxed{\Psi = \int_0^T V_L dt = 0}$$

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$$=> \int_0^{t_{ON}} V_{IN}\ dt + \int_{t_{ON}}^T -(V_{OUT} - V_{IN})dt = 0$$

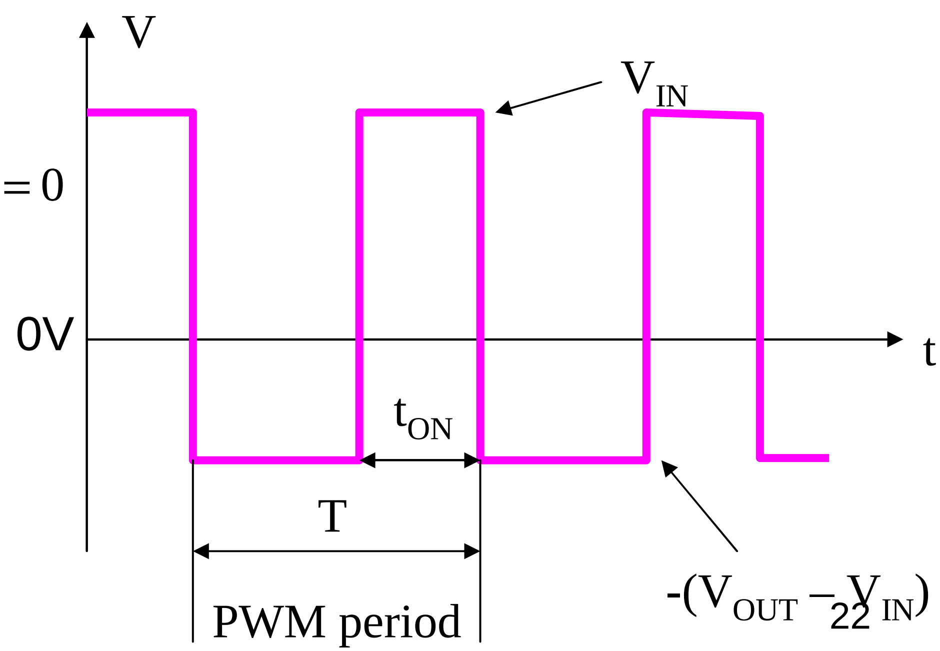
$$=> V_{IN}.t_{ON} - V_{OUT}.T + V_{IN}.T + V_{OUT}.t_{ON} - V_{IN}.t_{ON} = 0$$

$$=> T(V_{OUT} - V_{IN}) = V_{OUT}.t_{ON}$$

$$=> V_{OUT} - V_{IN} = \phi.V_{OUT}$$

$$=> V_{OUT}(1-\phi) = V_{IN}$$

$$=> V_{OUT} = \frac{V_{IN}}{1-\phi}$$

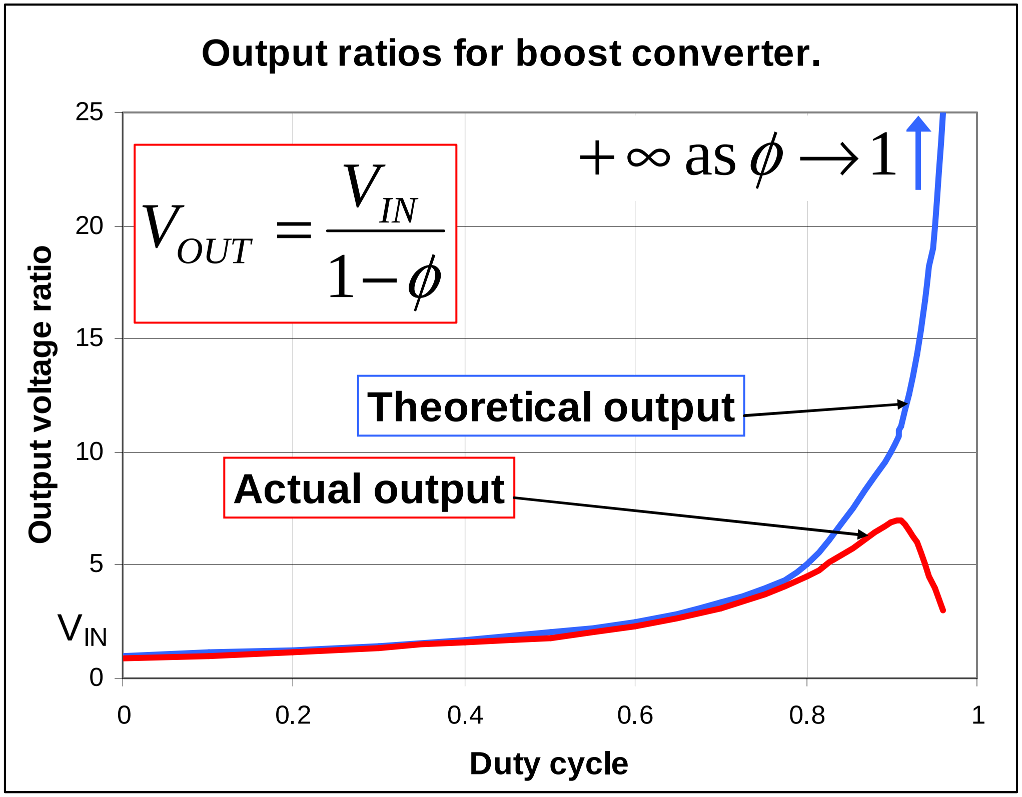


If we plot this relationship, we have:

Some important observations are:

1. The output voltage is always greater than the input voltage.

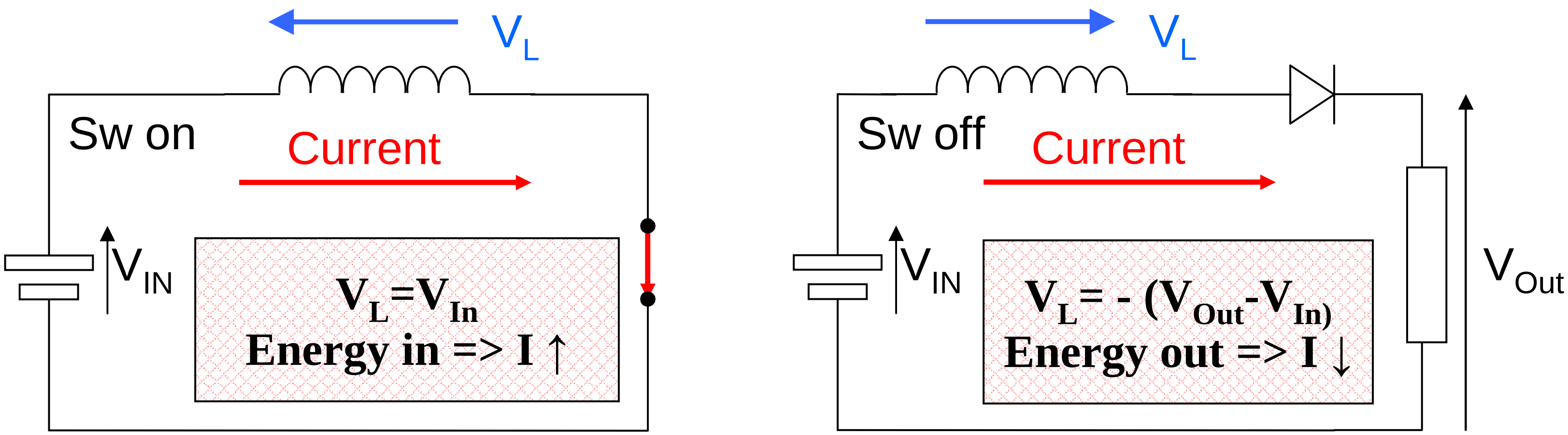
2. Assuming a lossless system, the output voltage is independent of the load on the output. The power delivered is proportional to the current flowing in the inductor.

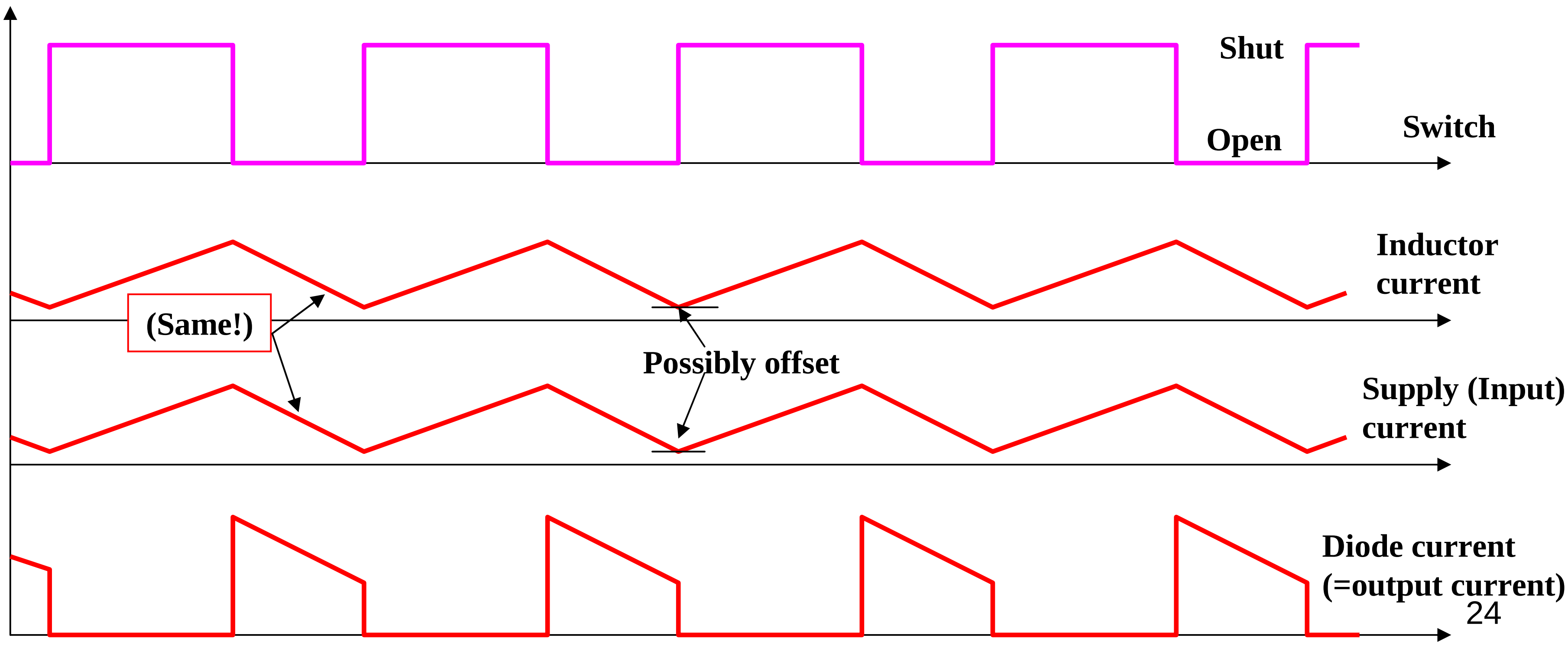


In reality, as φ approaches 1, the output voltage is limited due to loses in the system. If the switch were constantly closed the input would be short-circuited and no energy could be transferred to the output.

Note this relationship is true only if current is always flowing in the inductor, i.e. the converter is operating in continuous mode.

Reminding ourselves about the currents in the circuit:





Sticking with our assumption of a lossless system:

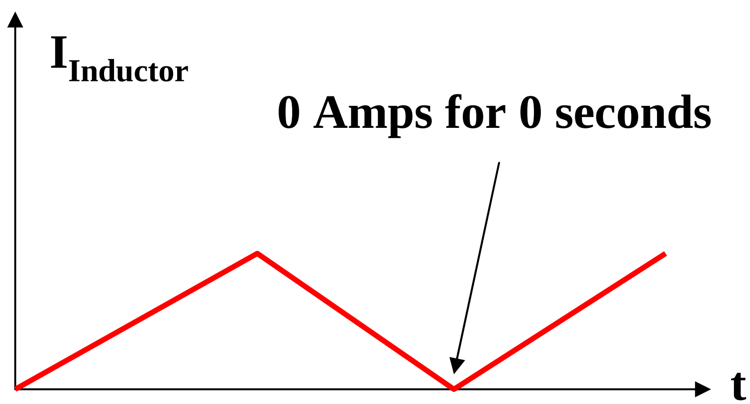
Output power = input power => $V_{OUT}I_{OUT(Mean)} = V_{IN}I_{IN(Mean)}$

Setting $V_{OUT} = V_{IN} / (1- \varphi)$, we have $I_{OUT(Mean)} = (1- \varphi).I_{IN(Mean)}$

(Remember for a DC voltage, average power = V_{DC} x I_{Mean})

Operation at the Boundary Condition.

If the inductor current falls to zero for a vanishingly short time at the end of the PWM period, this is called the *"boundary condition".* It is useful in that this permits the value of the inductance to be calculated for a given switching frequency and current.



Example.

A boost converter is required to provide a constant 24V / 100W output from an input voltage that varies between 9V and 15V. For a switching frequency of 20KHz and assuming 100% efficiency and operation in continuous mode, determine the following:

1. The required PWM duty cycle range to maintain a 24V DC output.

2. The average input current when the input voltage is 15V.

3. The value of the inductor required if the instantaneous current through the inductor at the end of the PWM period is zero. Calculate the value for both 9V and 15V input.

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Solution.

1. For continuous mode boost converter we have:

$$V_{OUT} = \frac{V_{IN}}{1-\phi} \Rightarrow \phi = 1 - \frac{V_{IN}}{V_{OUT}}$$

$$For\ V_{IN} = 9V, \phi = 1 - \frac{9}{24} = 62.5\%$$

$$For\ V_{IN} = 15V, \phi = 1 - \frac{15}{24} = 37.5\%$$

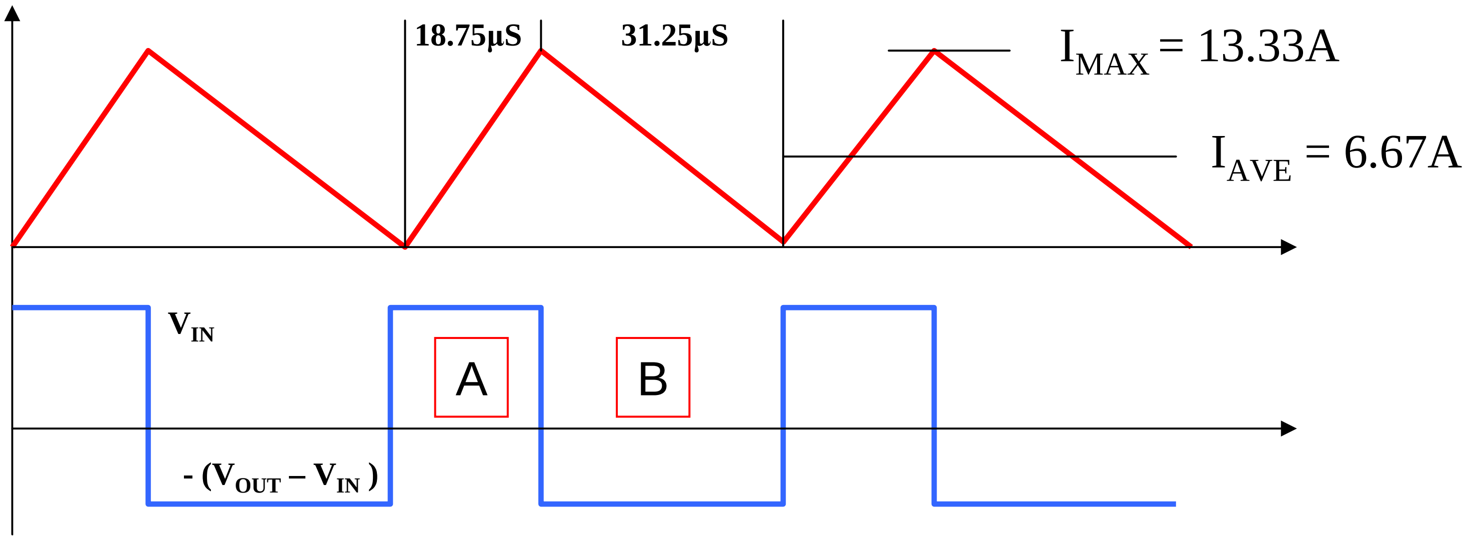
2. Assuming 100% efficiency:

$$V_{IN} I_{IN(MEAN)} = V_{OUT} I_{OUT(MEAN)} = 100W$$

$$For\ V_{IN} = 15V, I_{IN(MEAN)} = \frac{100W}{15V} = 6.67A$$

3. For a boost converter. the inductor current = input current. At 15V, φ=37.5% and $I_{IN(AVE)}$=6.67A. We know that at the end of the PWM period the inductor current is zero, hence we can construct the current waveform:

$I_{MAX} = 2 \text{ x } I_{AVE} = 13.33A$



F_{SWITCH} = 20KHz, φ = 37.5% => T_{ON} = 18.75µS

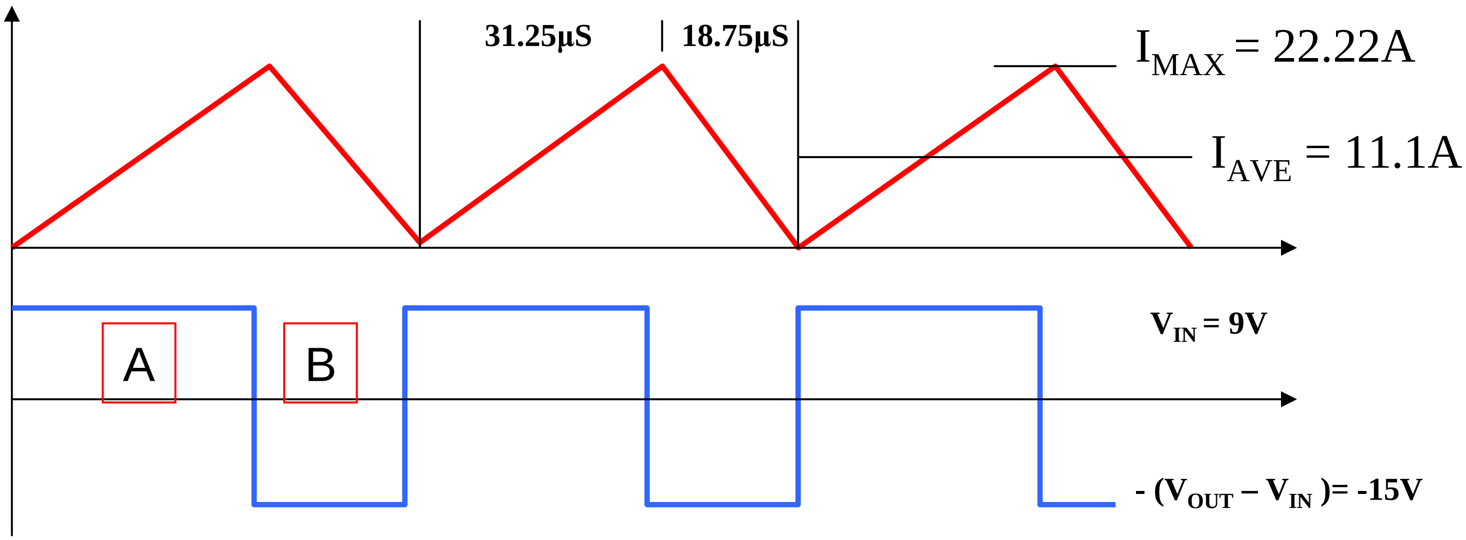
$$V = L\frac{di}{dt} \Rightarrow L = V\frac{\Delta t}{\Delta i}$$

$$For \;\; A: L = \frac{15V \times 18.75\mu S}{13.\dot{3}A} = \boxed{21\mu H}$$

$$For \;\; B: L = \frac{-(24V - 15V) \times 31.25\mu S}{-13.\dot{3}A} = \boxed{21\mu H}$$

Now we need to work out the average input current when the input voltage is 9V. At 9V, φ=62.5% and $I_{IN(AVE)}$=11.1A.

$I_{MAX} = 2 \text{ x } I_{AVE} = 22.22A$



F_{SWITCH} = 20KHz, φ = 62.5% => T_{ON} = 31.25µS

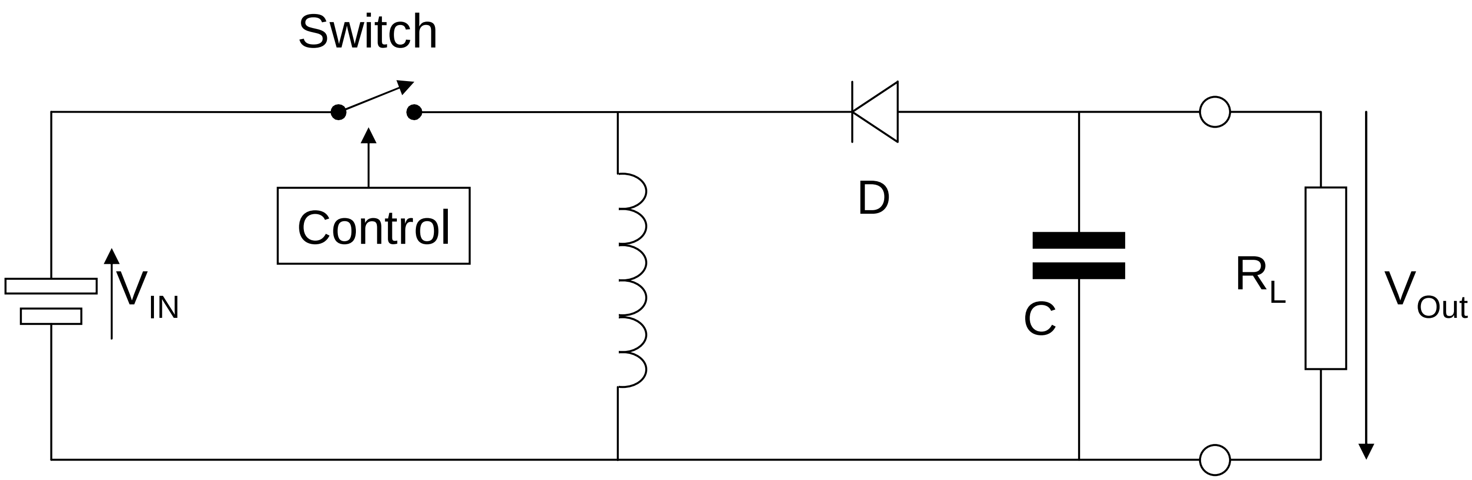
$$V = L\frac{di}{dt} \Rightarrow L = V\frac{\Delta t}{\Delta i}$$

$$For \quad A: L = \frac{9V \times 31.25\mu S}{22.\dot{2}A} \boxed{= 12.66\mu H}$$

$$For \quad B: L = \frac{-15V \times 18.75\mu S}{-22.\dot{2}A} \boxed{= 12.66\mu H}$$

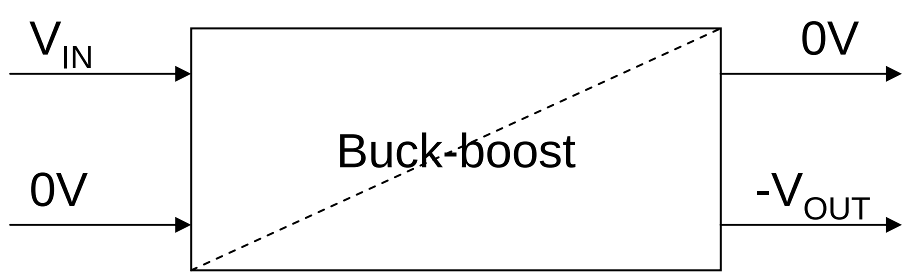
If we compare these values with the case for 15V we see the inductor value is smaller – this is expected since ΔI is greater because of the lower input voltage and the reduced input voltage able to establish the current in the inductor.

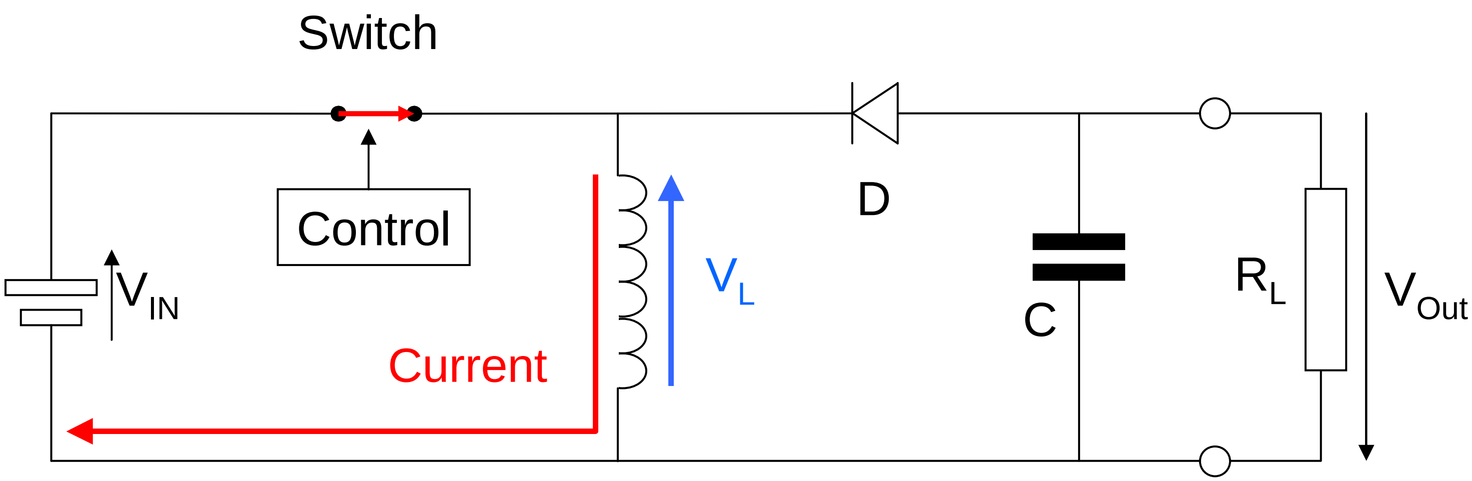
The buck-boost converter ($V_{OUT} <> V_{IN}$).



Note polarity of output voltage!

The Buck-Boost converter is able to produce output voltages that are higher or lower than the input voltage. Once again we will consider the continuous mode system. Note that the output is negative with respect to the 0V side of the input voltage.



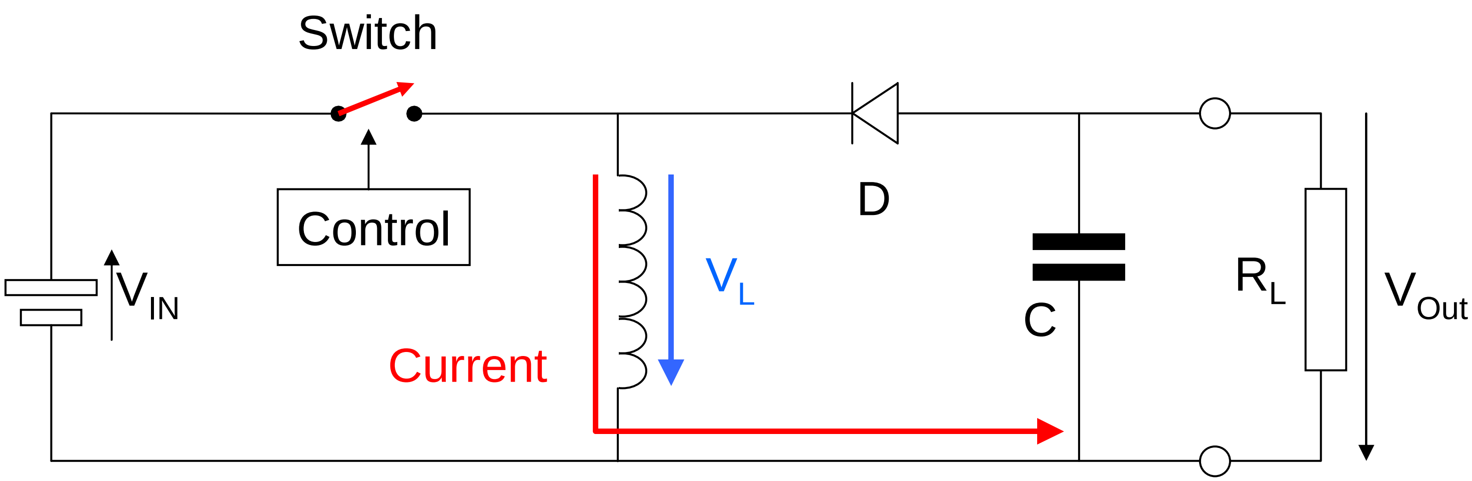


Before starting …
… Notice that when the switch is open there is no connection between the input to the supply and its output. Hence we can conclude that the supply is capable of delivering 0V output regardless of the input voltage for a 0% duty cycle on the switch.

With the switch closed, the input voltage is imposed across the inductor and the current through the inductor starts to rise. The diode is “off” since the cathode is more positive than the anode.

Hence for the “on” period, $V_L = V_{IN}$.

Note that with the diode reverse biased, the only route for the current to take that completes the circuit is back through the supply to the converter.



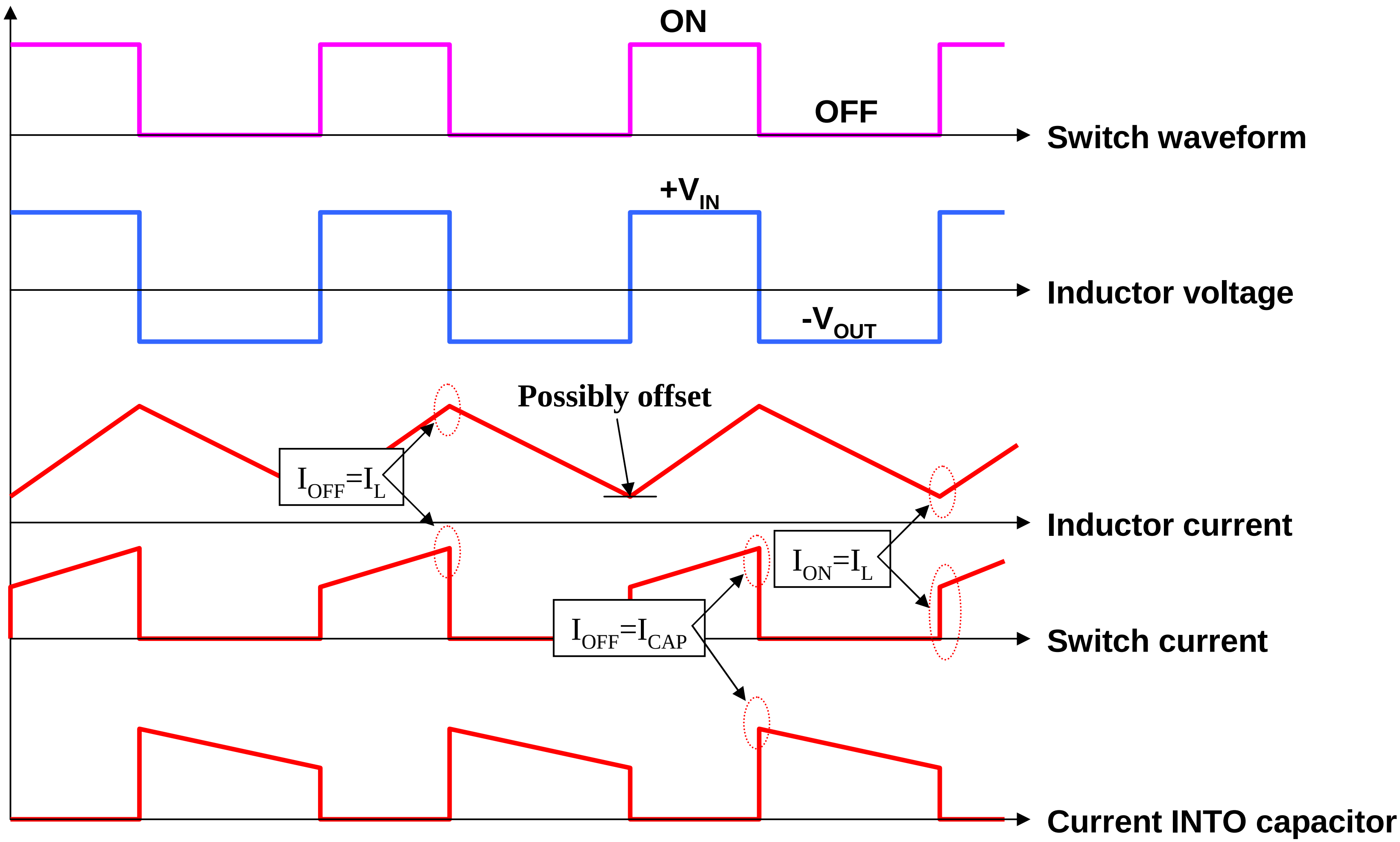
When the switch is opened the voltage on the inductor reverses polarity, the diode starts to conduct and hence the inductor current flows into the capacitor.

During this “off” period, $V_L = -V_{OUT}$ and this is maintained as the current in the inductor falls to its value at the end of the PWM period.

Note that with the switch opened there is no return path for current through the input to the converter and hence the only route the inductor current can follow is into the capacitor and load.

<u>*Also note that the capacitor will supply current to the load when there is no current contribution from the inductor.*</u>

Waveforms for the buck-boost converter.



Flux linkage analysis.

Once again from flux linkage considerations we have:

$$\boxed{\Psi = \int_0^T V_L \, dt = 0}$$

$$\Psi = \int_0^T V_L \, dt = 0$$

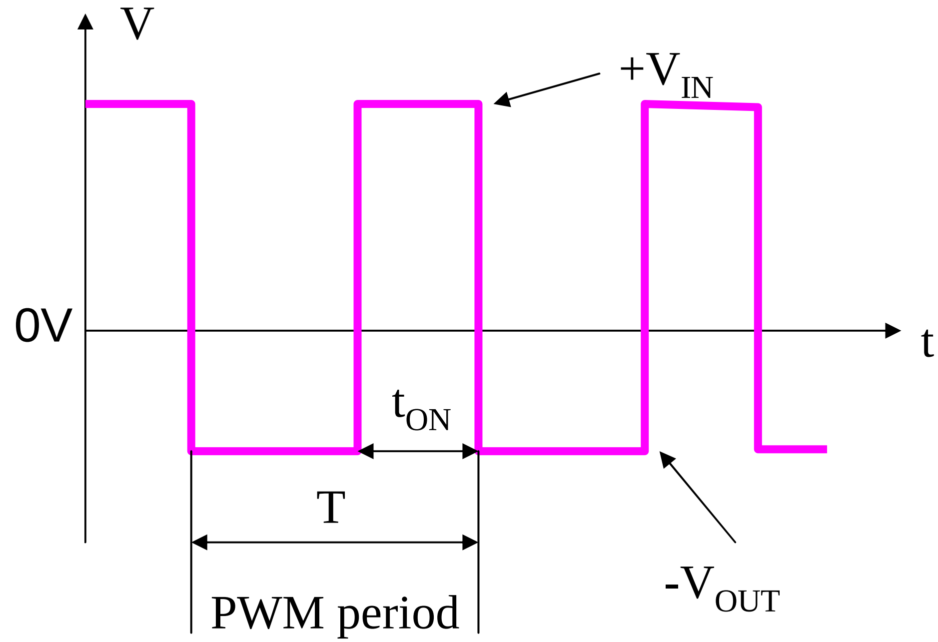
$$=> \Psi = \int_0^{t_{ON}} +V_{IN} \, dt + \int_{t_{ON}}^T -V_{OUT} \, dt = 0$$

$$=> \Psi = V_{IN}.t_{ON} - V_{OUT}(T - t_{ON}) = 0$$

$$=> V_{IN}.t_{ON} = V_{OUT}(T - t_{ON})$$

$$=> V_{IN}.\phi = V_{OUT}(1-\phi)$$

$$=> V_{OUT} = \frac{V_{IN}.\phi}{(1-\phi)}$$



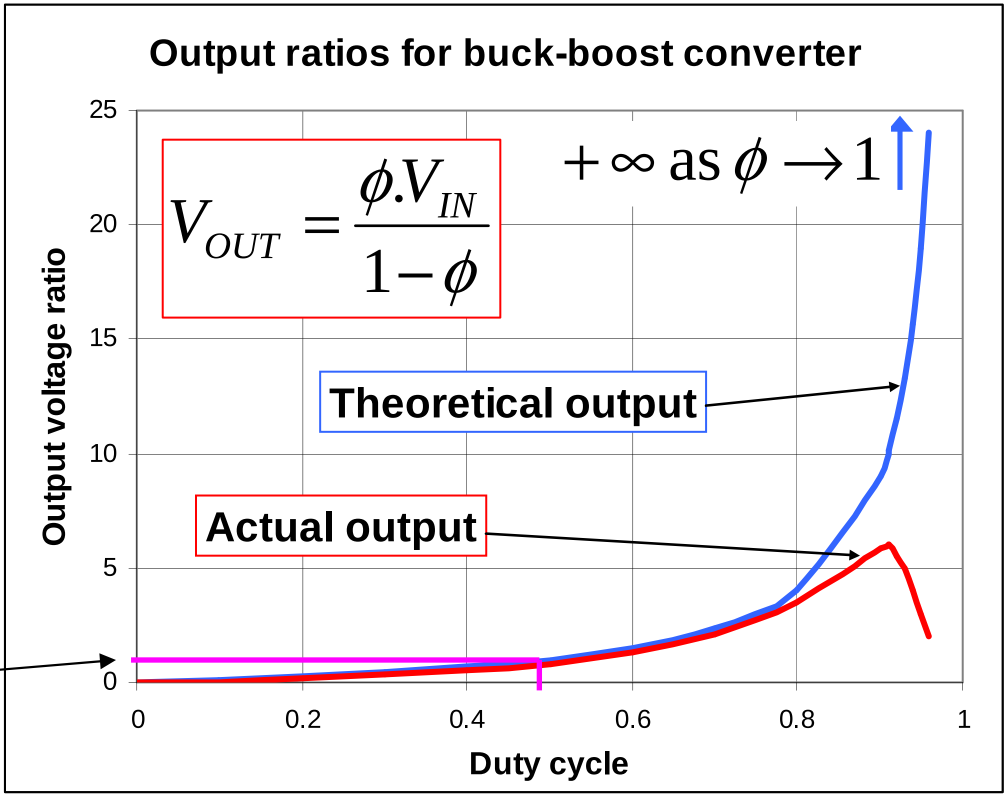
If we plot this relationship, we have:

Some important observations are:

1. The output voltage can be higher or lower than the input voltage.

2. Assuming a lossless system, the output voltage is independent of the load on the output. The power delivered is proportional to the current flowing in the inductor.

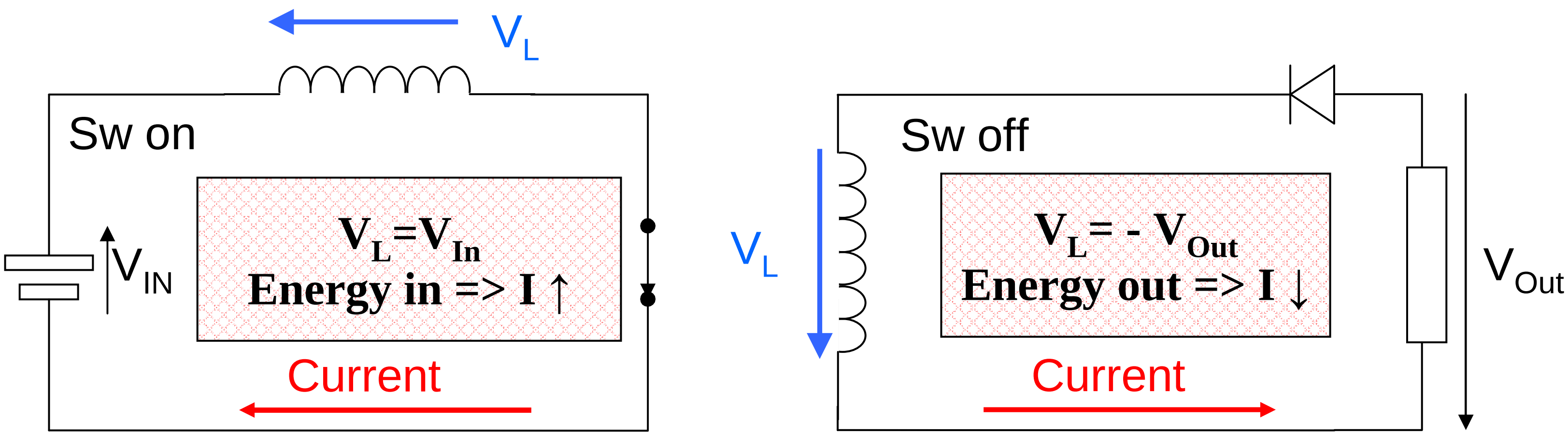
3. At φ=50%, $V_{OUT} = -V_{IN}$

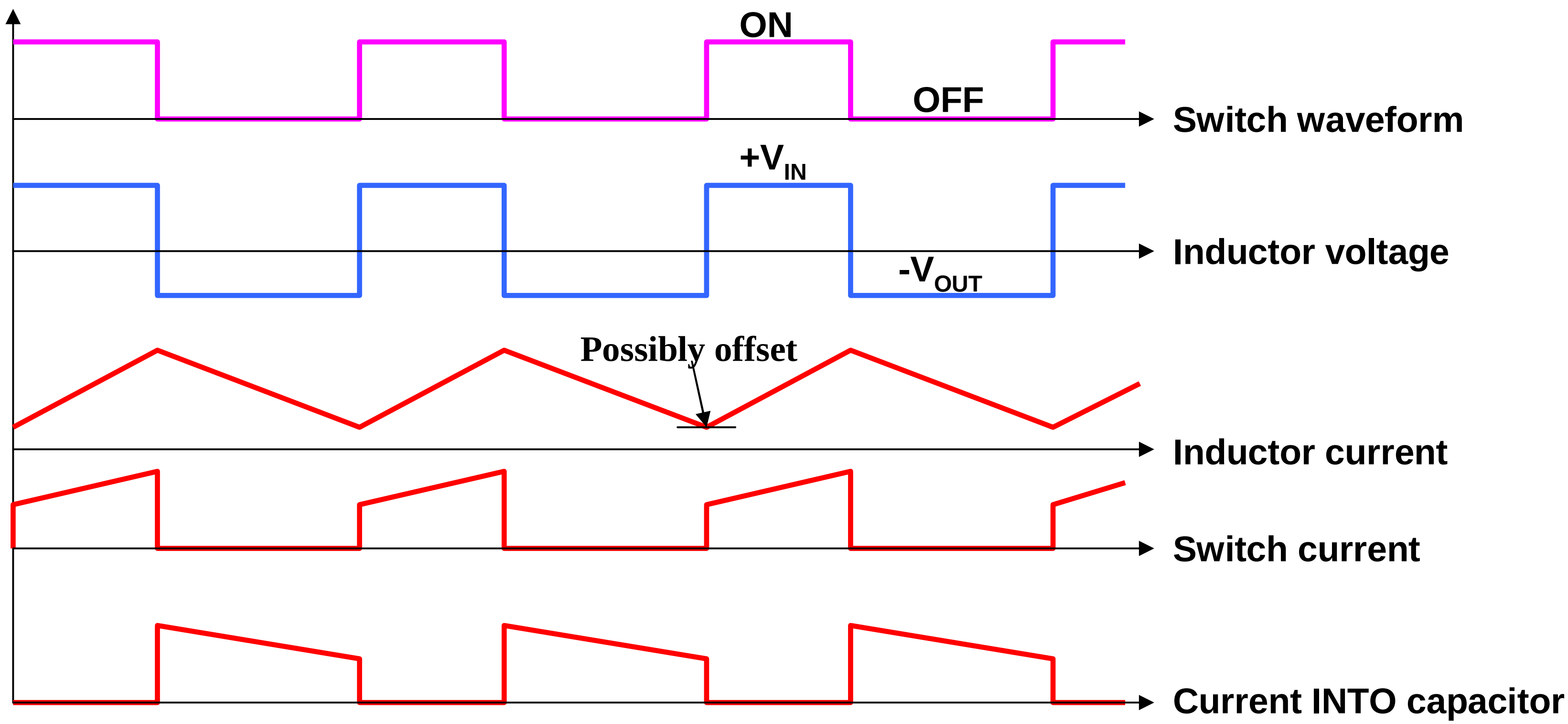


In reality, as φ approaches 1, the output voltage is limited due to loses in the system. If the switch were constantly closed the diode would be constantly reverse biased and no energy could be transferred to the output.

Note this relationship is true only if current is always flowing in the inductor, i.e. the converter is operating in continuous mode.

Reminding ourselves about the currents in the circuit:



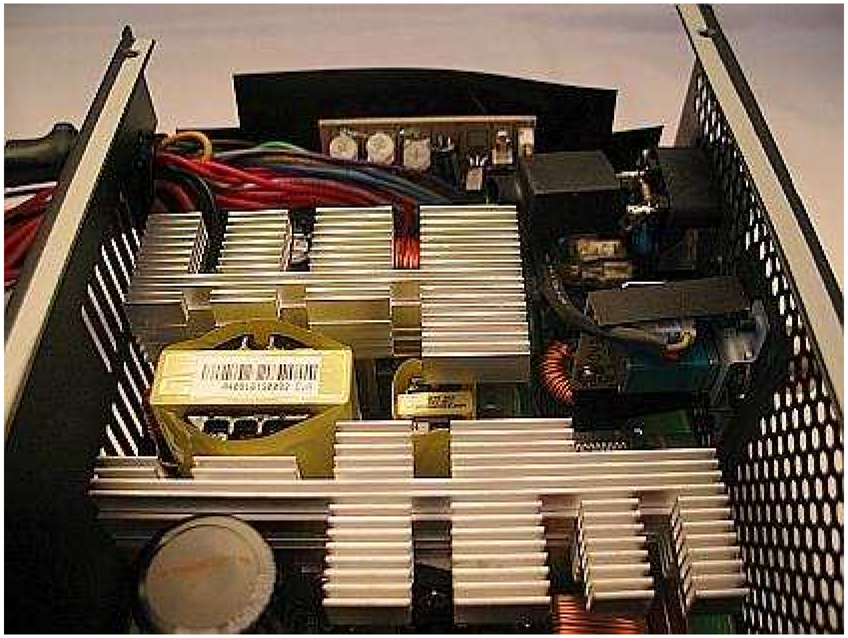


Sticking with our assumption of a lossless system:

Output power = input power => $V_{OUT}I_{OUT(Mean)} = V_{IN}I_{IN(Mean)}$

Setting $V_{OUT} = \varphi.V_{IN} / (1-\varphi)$, we have $I_{OUT(Mean)} = (1-\varphi).I_{IN(Mean)} / \varphi$

(Remember for a DC voltage, average power = $V_{DC} \times I_{Mean}$)

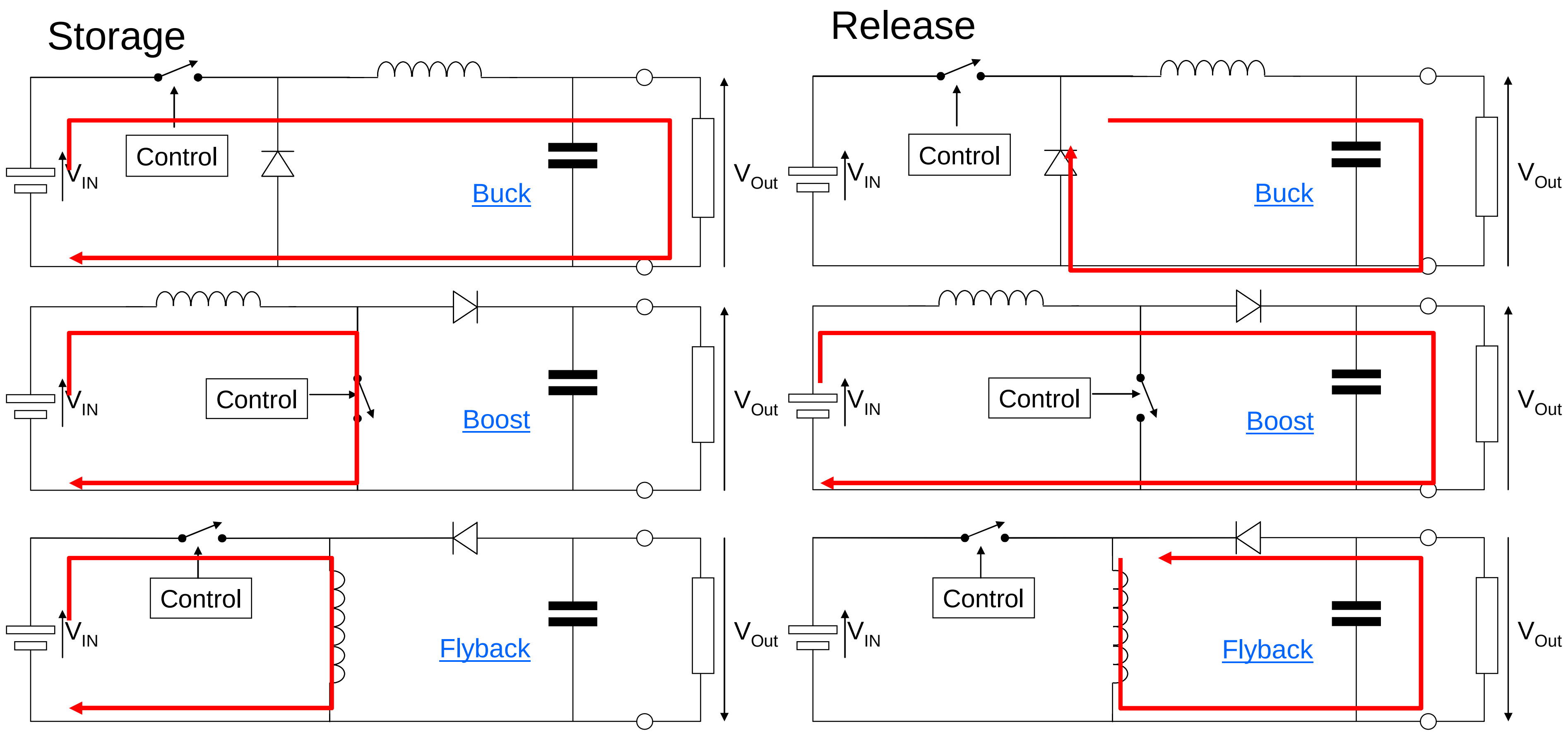


Converter summary

$$\boxed{\text{For all converters, assume } V_{IN} I_{IN(Ave)} = V_{OUT} I_{OUT(Ave)}}$$

Type	Voltage Equation	Current Equation
Buck ($V_{OUT} < V_{IN}$)	$V_{OUT} = \phi . V_{IN}$	$I_{OUT(Ave)} = \dfrac{I_{IN(Ave)}}{\phi}$
Boost ($V_{OUT} > V_{IN}$)	$V_{OUT} = \dfrac{V_{IN}}{1-\phi}$	$I_{OUT(Ave)} = (1-\phi).I_{IN(Ave)}$
Buck-boost (V_{OUT} <> V_{IN})	$V_{OUT} = \dfrac{\phi . V_{IN}}{1-\phi}$	$I_{OUT(Ave)} = \dfrac{(1-\phi).I_{IN(Ave)}}{\phi}$

The following diagrams summarise the energy flow in the circuit for each converter type during the storage and release phases of operation.



Question.

A small wind turbine generates 1.5V DC per MPH of wind speed.

For operation over the range 8 MPH to 32 MPH, determine the following:

1. The type of converter to be used;
2. The required range of duty cycle for these wind speeds;
3. The average input current to the converter at 32MPH;
4. The peak input current @ 32 MPH assuming operation at the boundary condition.