Circuit Analysis and Design

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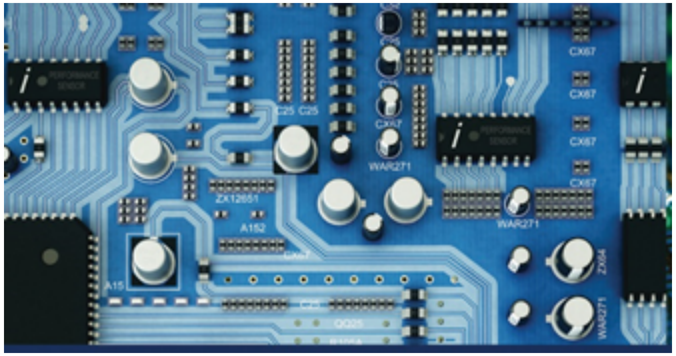
"A good student never steal or cheat"

Agenda



- Frequency response/Transfer function
- Filters

Introduction

- **The transfer function is the ratio of the output to input in the frequency domain. The absolute value of the transfer function is called the magnitude response, and the angle of the transfer function is called the phase response.**
- **Filters are circuits which are characterized according to their transfer function characteristics.**

Transfer Function

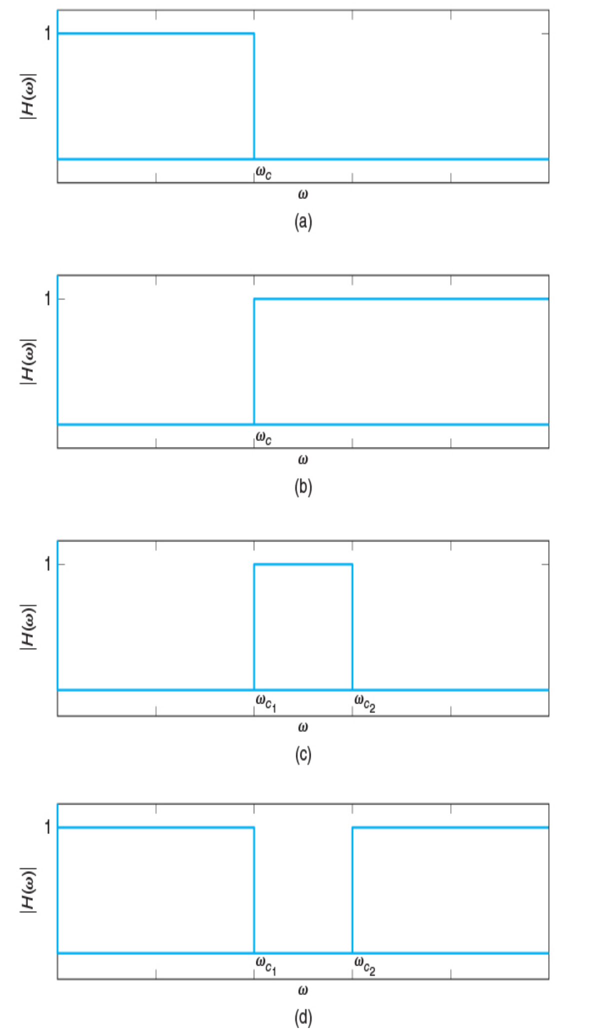
- **Transform a circuit to frequency domain by transforming capacitors by impedances,** $1/(j\omega C)$**, inductors by impedances,** $j\omega L$**, without specifying values of** ω**, and designating the input and output of the circuit as a function of** ω**. The transfer function** $H(\omega)$ **is defined as the ratio of the output to input.**
- **The transfer function is a function of radian frequency** ω **that characterizes the circuit or system in the frequency domain.**
- **By plotting the magnitude of the transfer function** $|H(\omega)|$**, called the magnitude response, and the phase of the transfer function** $\angle H(\omega)$**, called the phase response, we can gain more insight into the behavior of the circuit as the frequency is varied. The magnitude is called the gain.**

Ideal Filters

- A filter is a device that passes certain frequencies and blocks other frequencies.
- Common types of filters are lowpass filter (LPF), highpass filter (HPF), bandpass filter (BPF), and bandstop filter (BSF). The filters that cannot be physically realizable are called ideal filters. The magnitude responses of the ideal filters are shown in Figure 10.36.

FIGURE 10.36

Magnitude responses of ideal (a) LPF, (b) HPF, (c) BPF, and (d) BSF.

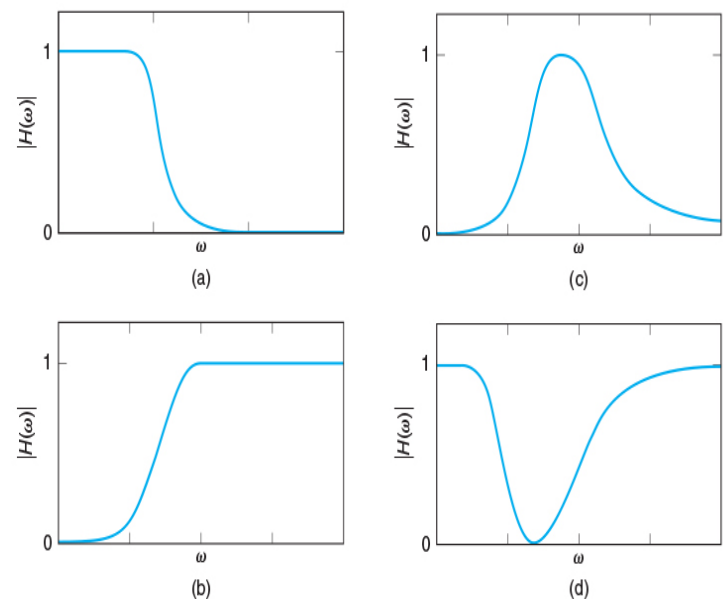


Practical Filters

- In practical filters, the gain in the passband cannot be one for all frequencies, and the gain in the stopband cannot be zero for all frequencies. Also, the transitions from passband to stopband and stopband to passband are gradual. Figure 10.37 shows the magnitude responses of practical LPF, HPF, BPF, and BSF. As can be seen, the magnitude response changes continuously as a function of radian frequency ω, and the transition from passband to stopband and stopband to passband happens over a finite width of ω.

FIGURE 10.37

Magnitude responses of practical (a) LPF, (b) HPF, (c) BPF, (d) BSF.



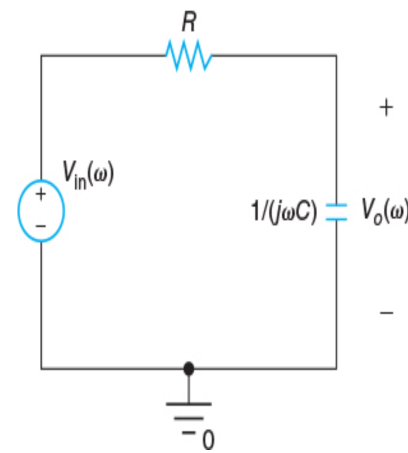
RC Circuit

- **Application of the voltage divider rule yields**

$$H(\omega)=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=\frac{\frac{1}{RC}}{j\omega+\frac{1}{RC}}=\frac{\frac{1}{RC}}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}}e^{-j\tan^{-1}(\omega RC)},\ |H(\omega)|=\frac{\frac{1}{RC}}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}},\ \angle H(\omega)=-\tan^{-1}(\omega RC)$$

FIGURE 10.39

RC circuit transformed into the ω domain.



- **At** $\omega = 1/(RC)$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20\log_{10}(1/\sqrt{2}) = -3.01$ dB

  $\angle H(\omega) = -\tan^{-1}(1) = -45^o$
- $\omega_0 = 1/(RC)$ **is called** *3 dB* **cutoff frequency.**
- **The gain decreased by** *3 dB*.
- **If** $R = 2\ k\Omega$, $C = 0.08\ \mu F$,

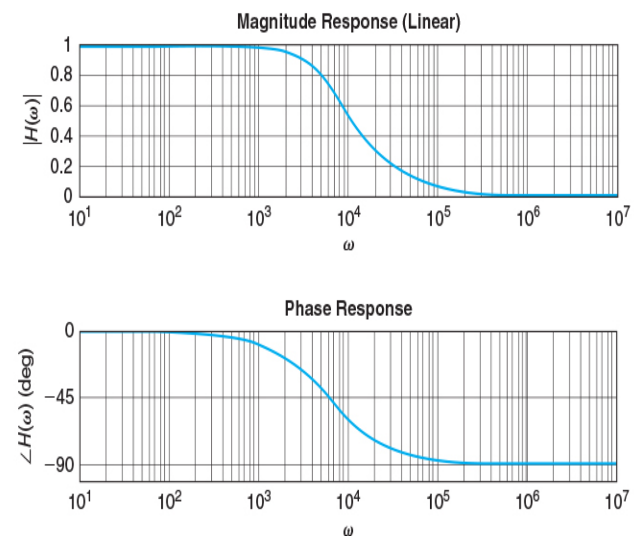
  $\omega_0 = 1/(RC) = 6250$ rad/s

  $f_0 = \omega_0/(2\pi) = 994.7184$ Hz

  Figure 10.40 shows $|H(\omega)|$ **and** $\angle H(\omega)$.
- **This is an LPF.**

FIGURE 10.40

Magnitude response and phase response for the circuit shown in Figure 10.39 with $R = 2\ k\Omega$ and $C = 0.08\ \mu F$.



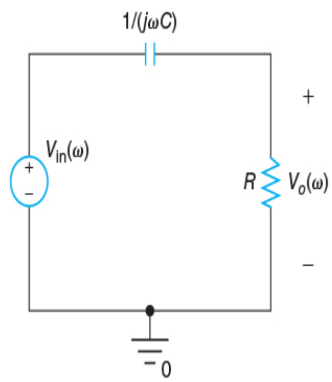
CR Circuit

- Application of the voltage divider rule yields

$$H(\omega)=\frac{R}{R+\frac{1}{j\omega C}}=\frac{j\omega}{j\omega+\frac{1}{RC}}=\frac{\omega}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}}e^{j\left[\frac{\pi}{2}-\tan^{-1}(\omega RC)\right]},\left|H(\omega)\right|=\frac{\omega}{\sqrt{\omega^2+\left(\frac{1}{RC}\right)^2}},\angle H(\omega)=\frac{\pi}{2}-\tan^{-1}\left(\omega RC\right)$$

FIGURE 10.41

CR circuit.



- **At** $\omega = 1/(RC)$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20\log_{10}(1/\sqrt{2}) = -3.01$ dB

$\angle H(\omega) = 90^o - \tan^{-1}(1) = 45^o$

- $\omega_0 = 1/(RC)$ **is called** *3 dB* **cutoff frequency.**
- **If** $R = 2\ k\Omega$, $C = 0.08\ \mu F$,

$\omega_0 = 1/(RC) = 6250$ rad/s

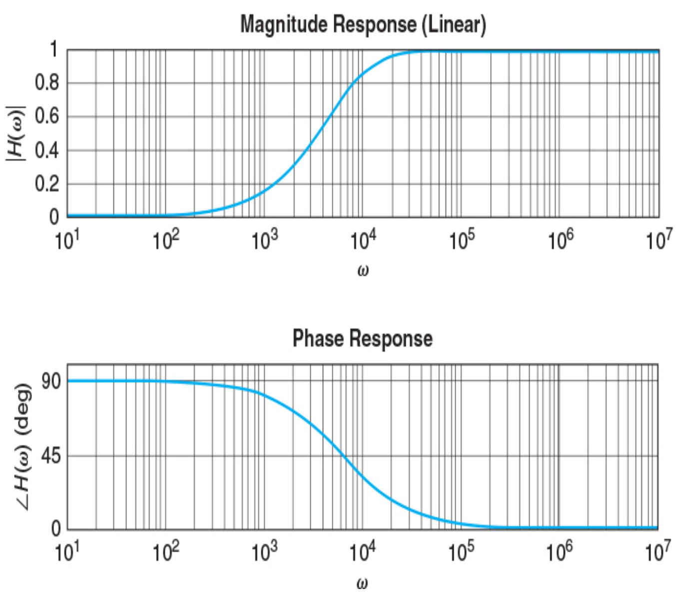
$f_0 = \omega_0/(2\pi) = 994.7184$ Hz

Figure 10.42 shows $|H(\omega)|$ **and** $\angle H(\omega)$**.**

- **This is an HPF.**

FIGURE 10.42

Magnitude response and phase response for the circuit shown in Figure 10.41 with $R = 2\ k\Omega$ and $C = 0.08\ \mu F$.



EXAMPLE 10.9

- **Find** $H(\omega) = V_o(\omega)/V_{in}(\omega)$ **and state filter type for the circuit shown in Figure 10.43.**

$$H(\omega)=\frac{R}{j\omega L+R}=\frac{\frac{R}{L}}{j\omega+\frac{R}{L}}=\frac{\frac{R}{L}}{\sqrt{\omega^2+\left(\frac{R}{L}\right)^2}}e^{-j\tan^{-1}\left(\frac{\omega L}{R}\right)},\ \left|H(\omega)\right|=\frac{\frac{R}{L}}{\sqrt{\omega^2+\left(\frac{R}{L}\right)^2}},\ \angle H(\omega)=-\tan^{-1}\left(\frac{\omega}{\frac{R}{L}}\right)$$

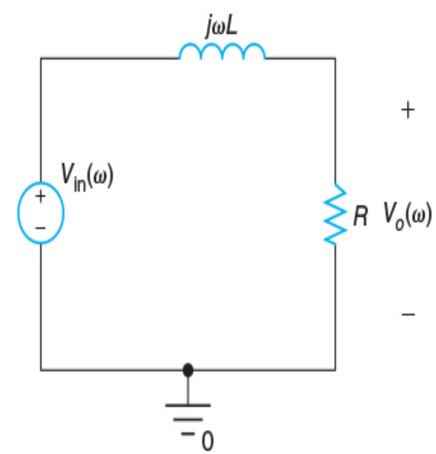
- **At** $\omega = R/L$, $|H(\omega)| = 1/\sqrt{2} = 0.7071$, $20 \log_{10}(1/\sqrt{2}) = -3.01$ dB

$\angle H(\omega) = -\tan^{-1}(1) = -45^{o}$

- $\omega_0 = R/L$ **is called** *3 dB* **cutoff frequency.**
- **This is an LPF.**

FIGURE 10.43

LR circuit.



EXAMPLE 10.10

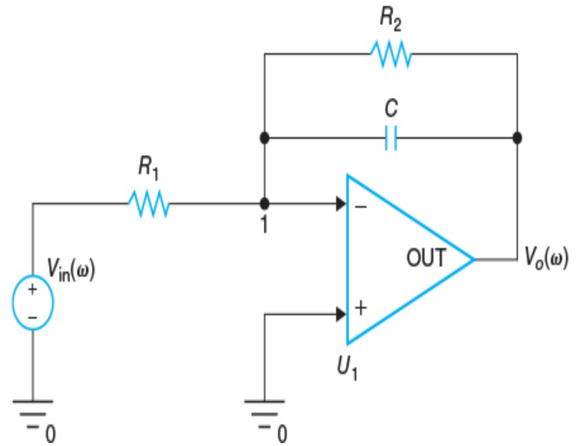
- Find $H(\omega) = V_o(\omega)/V_{in}(\omega)$ **and state filter type for the circuit shown in Figure 10.45.**
- **Sum the currents leaving node 1:**

$$\frac{0-V_{in}}{R_1}+\frac{0-V_o}{R_2}+\frac{0-V_o}{\frac{1}{j\omega C}}=0 \Rightarrow \left(j\omega C+\frac{1}{R_2}\right)V_o=\frac{-V_{in}}{R_1} \Rightarrow H(\omega)=\frac{V_o}{V_{in}}=-\frac{\frac{1}{R_1}}{j\omega C+\frac{1}{R_2}}=-\frac{\frac{1}{R_1C}}{j\omega+\frac{1}{R_2C}}=-\frac{R_2}{R_1}\frac{\frac{1}{R_2C}}{j\omega+\frac{1}{R_2C}}$$

- **This is an LPF.**

FIGURE 10.45

Circuit for EXAMPLE 10.10.



Series RLC LPF

- A series RLC circuit is shown in Figure 10.47.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{\frac{1}{j\omega C}}{j\omega L+R+\frac{1}{j\omega C}}=\frac{1}{(j\omega)^2 LC+RCj\omega+1}=\frac{\frac{1}{LC}}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\frac{1}{LC}}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=-\tan^{-1}\left(\frac{\frac{R}{L}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The corner frequency is $\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$
- This is a second order LPF.
- Figure 10.48 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.47

A series RLC circuit.

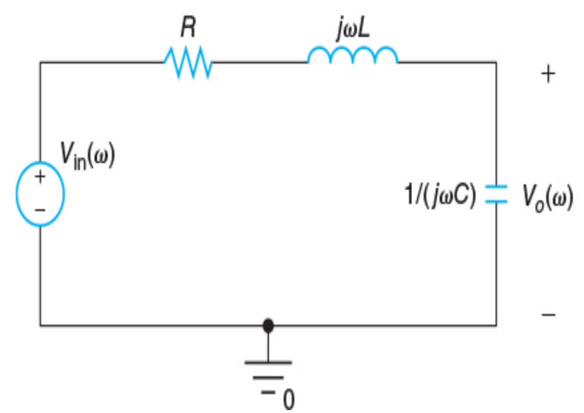
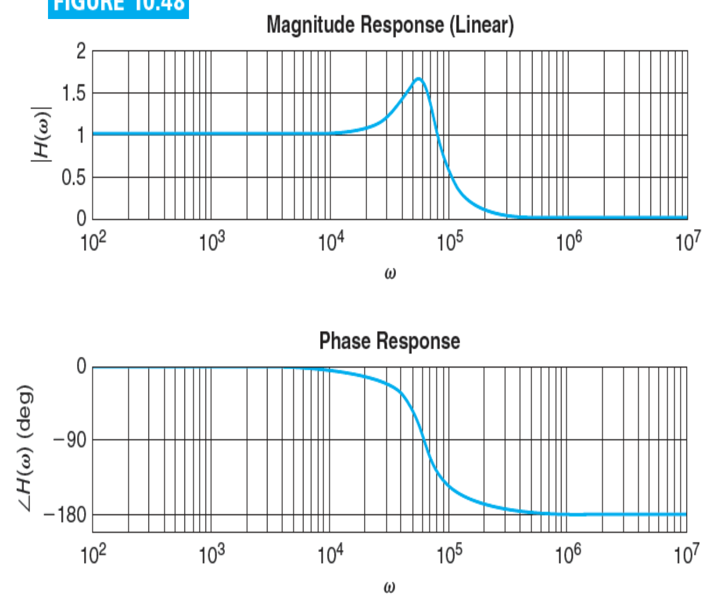


FIGURE 10.48



Series RLC HPF

- A series RCL circuit is shown in Figure 10.49.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{j\omega L}{j\omega L+R+\frac{1}{j\omega C}}=\frac{(j\omega)^2 LC}{(j\omega)^2 LC+RCj\omega+1}=\frac{-\omega^2}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\omega^2}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=\pi-\tan^{-1}\left(\frac{\frac{R}{L}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The corner frequency is $\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$
- This is a second order HPF.
- Figure 10.50 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.49

A series RCL circuit.

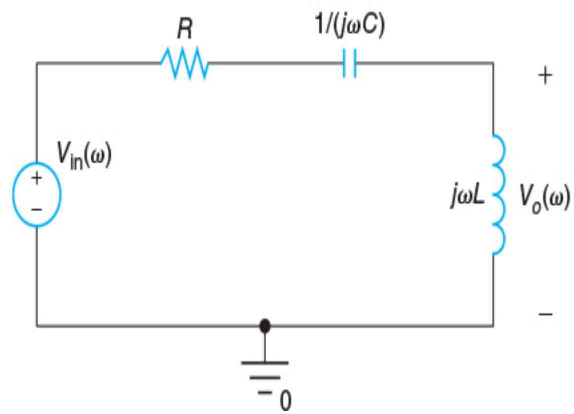
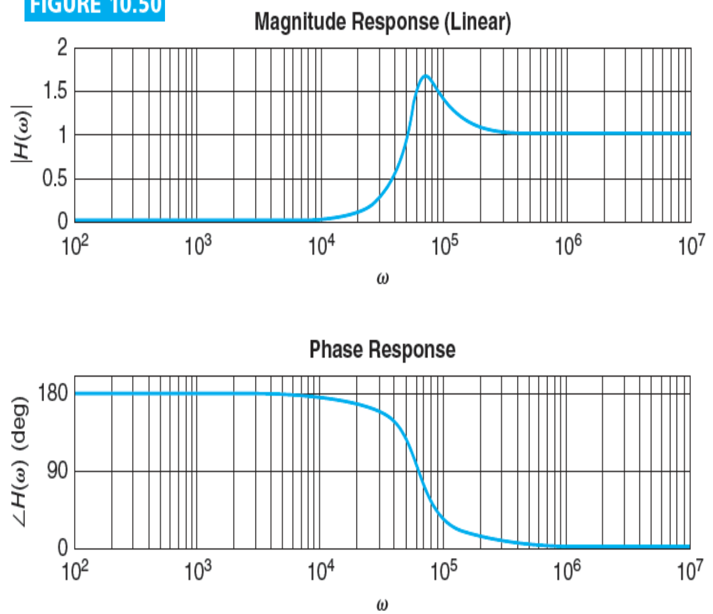


FIGURE 10.50



Series RLC BPF

- A series LCR circuit is shown in Figure 10.51.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{R}{j\omega L+R+\frac{1}{j\omega C}}=\frac{RCj\omega}{(j\omega)^2 LC+RCj\omega+1}=\frac{\frac{R}{L}j\omega}{-\omega^2+\frac{R}{L}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\frac{R}{L}\omega}{\sqrt{\left(\frac{1}{LC}-\omega^2\right)^2+\left(\frac{R}{L}\omega\right)^2}},\ \angle H(\omega)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\frac{R}{L}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The resonant frequency is $\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$
- This is a second order BPF.
- Figure 10.52 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.51

A series LCR circuit.

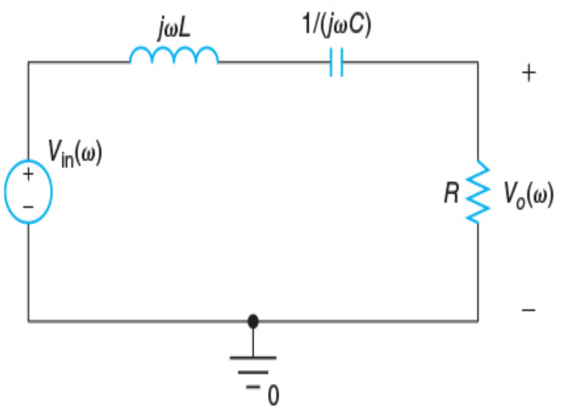
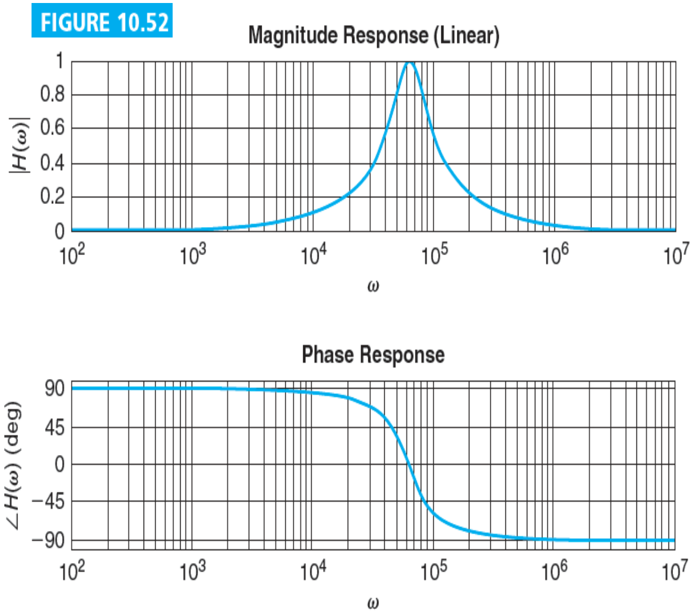


FIGURE 10.52



Series RLC BPF (Continued)

- **Lower** *3-dB* **cutoff frequency:**

$$\omega_1 = -\frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C} + 1}$$

- **Upper** *3-dB* **cutoff frequency:**

$$\omega_2 = \frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C} + 1}$$

- *3-dB* **bandwidth:** $\omega_{3dB} = \omega_2 - \omega_1 = \dfrac{R}{L}$

- **For** *$R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$,*

  $\omega_1 = 46{,}332.4958$ rad/s, $\omega_2 = 86{,}332.4958$ rad/s, $\omega_{3dB} = \omega_2 - \omega_1 = 40{,}000$ rad/s

Series RLC BSF

- A series RCL circuit is shown in Figure 10.53.
- Application of the voltage divider rule yields

$$H(\omega)=\frac{j\omega L+\dfrac{1}{j\omega C}}{j\omega L+R+\dfrac{1}{j\omega C}}=\frac{(j\omega)^2 LC+1}{(j\omega)^2 LC+RCj\omega+1}=\frac{-\omega^2+\dfrac{1}{LC}}{-\omega^2+\dfrac{R}{L}j\omega+\dfrac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\left|-\omega^2+\dfrac{1}{LC}\right|}{\sqrt{\left(\dfrac{1}{LC}-\omega^2\right)^2+\left(\dfrac{R}{L}\omega\right)^2}},\quad \angle H(\omega)=-\tan^{-1}\left(\frac{\dfrac{R}{L}\omega}{\dfrac{1}{LC}-\omega^2}\right)$$

- The null frequency is $\omega_0=\dfrac{1}{\sqrt{LC}}\ rad/s$
- This is a second order BSF.
- Figure 10.54 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.53

RCL circuit.

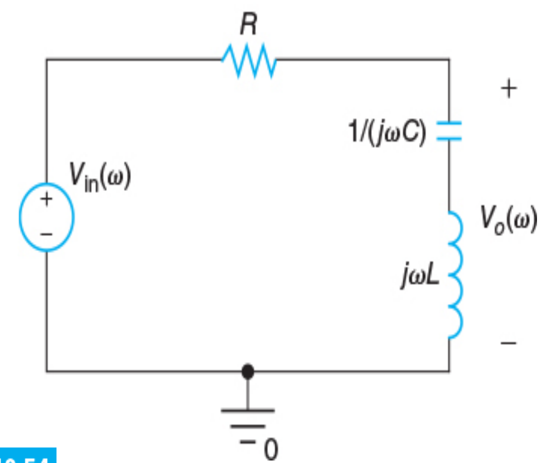
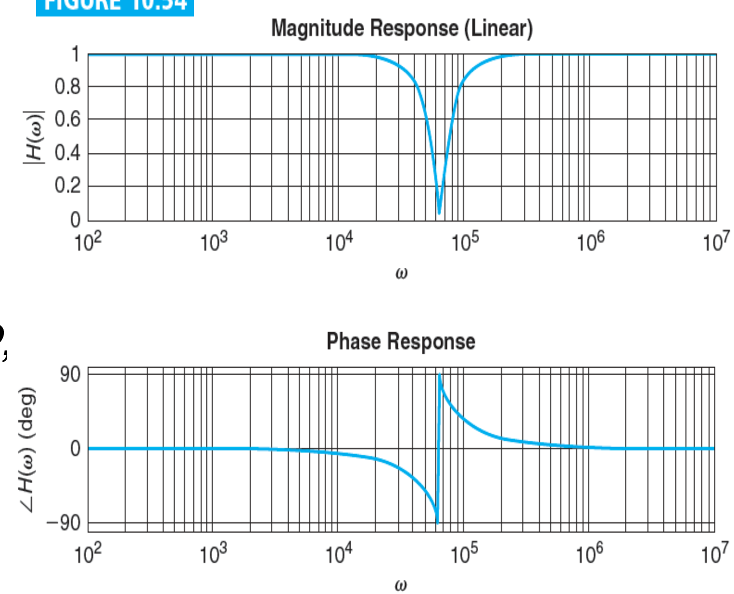


FIGURE 10.54



Series RLC BSF (Continued)

- **Lower** *3-dB* **cutoff frequency:**

$$\omega_1 = -\frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

- **Upper** *3-dB* **cutoff frequency:**

$$\omega_2 = \frac{R}{2L} + \frac{R}{2L}\sqrt{\frac{4L}{R^2C}+1}$$

- *3-dB* **bandwidth of null:** $\omega_{3dB} = \omega_2 - \omega_1 = \dfrac{R}{L}$

- **For** *R = 2 kΩ, L = 50 mH, C = 5 nF,*

  $\omega_1 = 46{,}332.4958$ *rad/s,* $\omega_2 = 86{,}332.4958$ *rad/s,* $\omega_{3dB} = \omega_2 - \omega_1 = 40{,}000$ *rad/s*

Parallel RLC LPF

- A parallel LRC circuit is shown in Figure 10.55.
- Nodal analysis yields

$$H(\omega)=\frac{\dfrac{1}{j\omega L}}{j\omega C+\dfrac{1}{R}+\dfrac{1}{j\omega L}}=\frac{1}{(j\omega)^2 LC+\dfrac{L}{R}j\omega+1}=\frac{\dfrac{1}{LC}}{-\omega^2+\dfrac{1}{RC}j\omega+\dfrac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\dfrac{1}{LC}}{\sqrt{\left(\dfrac{1}{LC}-\omega^2\right)^2+\left(\dfrac{1}{RC}\omega\right)^2}},\quad \angle H(\omega)=-\tan^{-1}\left(\frac{\dfrac{1}{RC}\omega}{\dfrac{1}{LC}-\omega^2}\right)$$

- The corner frequency is $\omega_0=\dfrac{1}{\sqrt{LC}}\ rad/s$
- This is a second order LPF.
- Figure 10.56 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.55

Parallel LRC circuit.

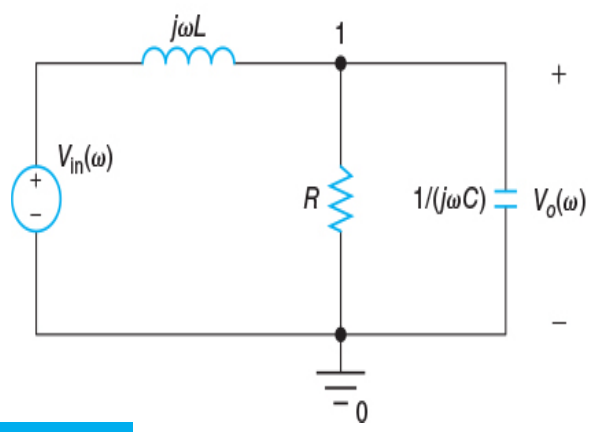
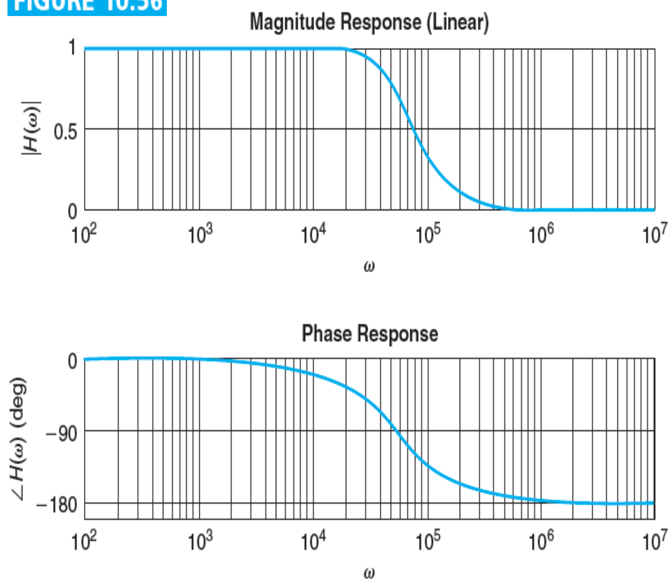


FIGURE 10.56



Parallel RLC HPF

- A parallel CRL circuit is shown in Figure 10.57.
- Nodal analysis yields

$$H(\omega)=\frac{j\omega C}{j\omega C+\frac{1}{R}+\frac{1}{j\omega L}}=\frac{(j\omega)^2 LC}{(j\omega)^2 LC+\frac{L}{R}j\omega+1}=\frac{-\omega^2}{-\omega^2+\frac{1}{RC}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\omega^2}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{\omega}{RC}\right)^2}},\quad \angle H(\omega)=\pi-\tan^{-1}\left(\frac{\frac{\omega}{RC}}{\frac{1}{LC}-\omega^2}\right)$$

- The corner frequency is $\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$
- This is a second order HPF.
- Figure 10.58 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.57

Parallel CRL circuit.

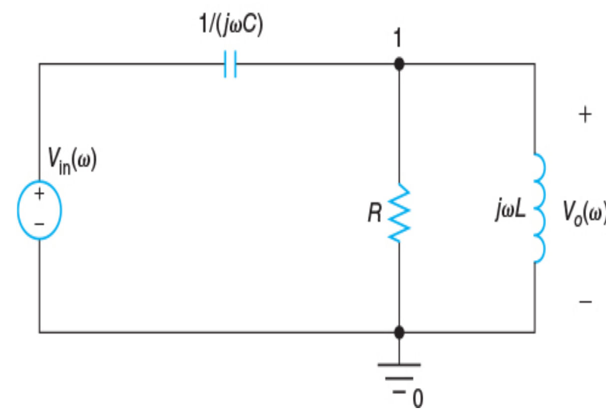
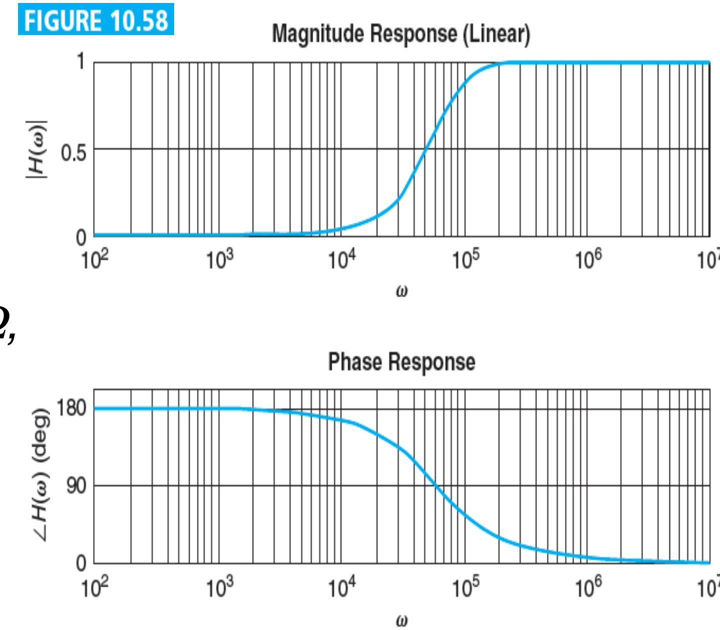


FIGURE 10.58



Parallel RLC BPF

- A parallel RCL circuit is shown in Figure 10.59.
- Nodal analysis yields

$$H(\omega)=\frac{\frac{1}{R}}{j\omega C+\frac{1}{R}+\frac{1}{j\omega L}}=\frac{\frac{L}{R}j\omega}{(j\omega)^2 LC+\frac{L}{R}j\omega+1}=\frac{\frac{1}{RC}j\omega}{-\omega^2+\frac{1}{RC}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\frac{1}{RC}\omega}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{1}{RC}\omega\right)^2}},\ \angle H(\omega)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\frac{1}{RC}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The resonant frequency is $\omega_0=\frac{1}{\sqrt{LC}}\quad rad/s$
- This is a second order BPF.
- Figure 10.60 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.59

Parallel RCL circuit.

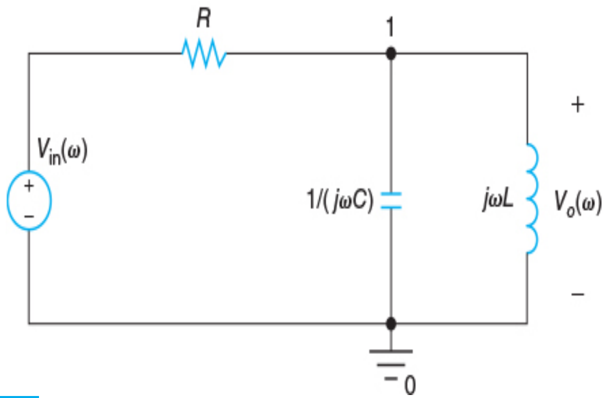
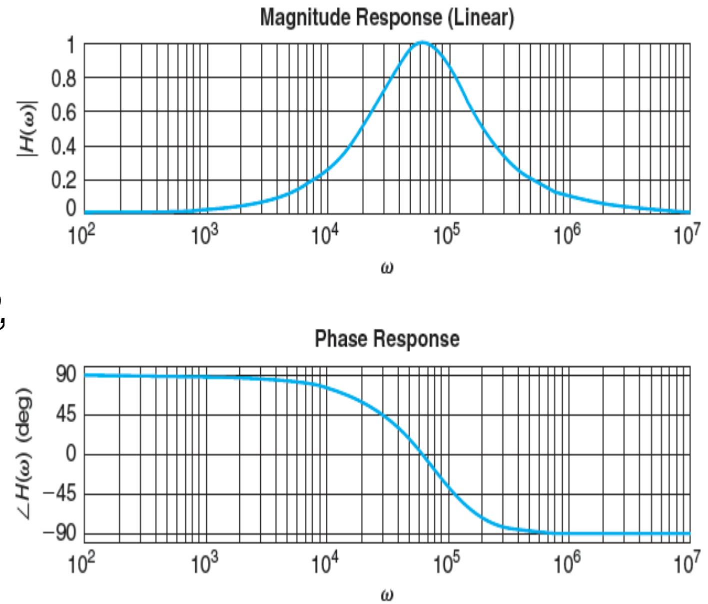


FIGURE 10.60



Parallel RLC BPF (Continued)

- **Lower** *3-dB* **cutoff frequency:**

$$\omega_1 = -\frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- **Upper** *3-dB* **cutoff frequency:**

$$\omega_2 = \frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L}+1}$$

- **3-dB bandwidth:** $\omega_{3dB} = \omega_2 - \omega_1 = \dfrac{1}{RC}$

- **For** *R = 2 kΩ, L = 50 mH, C = 5 nF,*

  ω_1 = *46,332.4958 rad/s,* ω_2 = *86,332.4958 rad/s,* $\omega_{3dB} = \omega_2 - \omega_1$ = *40,000 rad/s*

Parallel RLC BSF

- A parallel LCR circuit is shown in Figure 10.61.
- Nodal analysis yields

$$H(\omega)=\frac{j\omega C+\frac{1}{j\omega L}}{j\omega C+\frac{1}{R}+\frac{1}{j\omega L}}=\frac{(j\omega)^2 LC+1}{(j\omega)^2 LC+\frac{L}{R}j\omega+1}=\frac{-\omega^2+\frac{1}{LC}}{-\omega^2+\frac{1}{RC}j\omega+\frac{1}{LC}}$$

- The magnitude and phase responses are given by

$$|H(\omega)|=\frac{\left|-\omega^2+\frac{1}{LC}\right|}{\sqrt{\left(\omega^2-\frac{1}{LC}\right)^2+\left(\frac{1}{RC}\omega\right)^2}},\ \angle H(\omega)=-\tan^{-1}\left(\frac{\frac{1}{RC}\omega}{\frac{1}{LC}-\omega^2}\right)$$

- The null frequency is $\omega_0=\frac{1}{\sqrt{LC}}\ rad/s$
- This is a second order BSF.
- Figure 10.62 shows $|H(\omega)|$ and $\angle H(\omega)$ for $R = 2\ k\Omega$, $L = 50\ mH$, $C = 5\ nF$. $\omega_0 = 63245.55\ rad/s$.

FIGURE 10.61

Parallel LCR circuit.

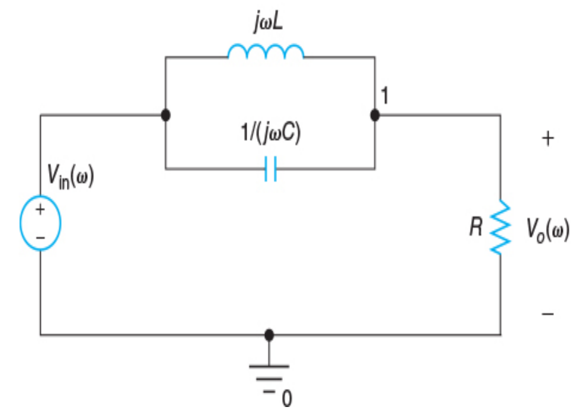
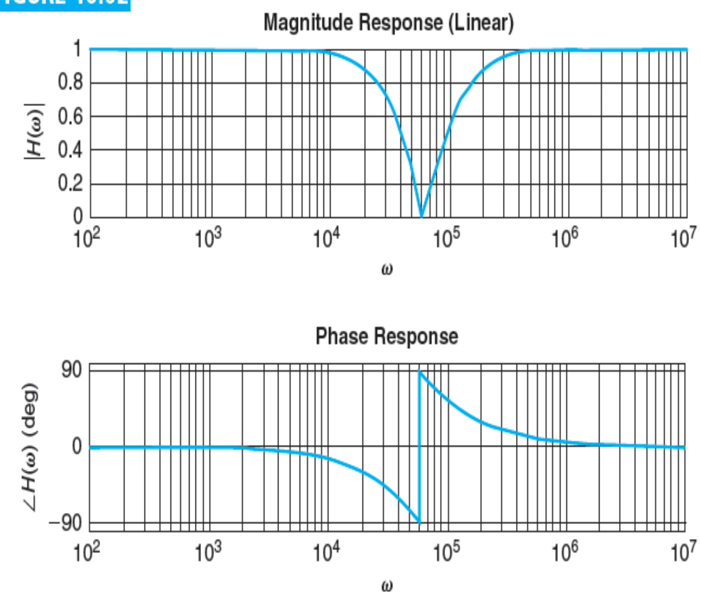


FIGURE 10.62



Parallel RLC BSF (Continued)

- **Lower** *3-dB* **cutoff frequency:**

$$\omega_1 = -\frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L} + 1}$$

- **Upper** *3-dB* **cutoff frequency:**

$$\omega_1 = \frac{1}{2RC} + \frac{1}{2RC}\sqrt{\frac{4R^2C}{L} + 1}$$

- *3-dB* **bandwidth of null:** $\omega_{3dB} = \omega_2 - \omega_1 = \frac{1}{RC}$

- **For** *R = 2 kΩ, L = 50 mH, C = 5 nF,*

  ω_1 = 46,332.4958 rad/s, ω_2 = 86,332.4958 rad/s, $\omega_{3dB} = \omega_2 - \omega_1$ = 40,000 rad/s

EXAMPLE 10.11

- **Find the transfer function for the circuit shown in Figure 10.63, and state the type of filter (LPF, HPF, BPF, BSF).**

- **Node b:** $\dfrac{V_o - V_a}{j\omega\times 1} + \dfrac{V_o}{1} = 0 \Rightarrow V_o - V_a + j\omega V_o = 0 \Rightarrow V_a = (j\omega+1)V_o \quad (1)$

- **Node a:** $\dfrac{V_a - V_{in}}{1} + V_a j\omega 0.2 + \dfrac{V_a - V_o}{j\omega\times 1} = 0 \Rightarrow j\omega V_a - j\omega V_{in} + (j\omega)^2 V_a 0.2 + V_a - V_o = 0 \quad (2)$

- **Substitute (1) into (2):** $\left[(j\omega)^2 0.2 + j\omega + 1\right]\left(j\omega+1\right)V_o - V_o = j\omega V_{in}$
- **Rearrangement yields** $\left[(j\omega)^3 0.2 + (j\omega)^2 + j\omega + (j\omega)^2 0.2 + j\omega + 1\right]V_o - V_o = j\omega V_{in} \quad (3)$

- **From Equation (3), we obtain**

$$H(\omega) = \frac{V_o}{V_{in}} = \frac{j\omega}{(j\omega)^3 0.2 + (j\omega)^2 + j\omega + (j\omega)^2 0.2 + j\omega}$$

$$H(\omega) = \frac{1}{0.2(j\omega)^2 + 1.2 j\omega + 2} = \frac{5}{(j\omega)^2 + 6 j\omega + 10}$$

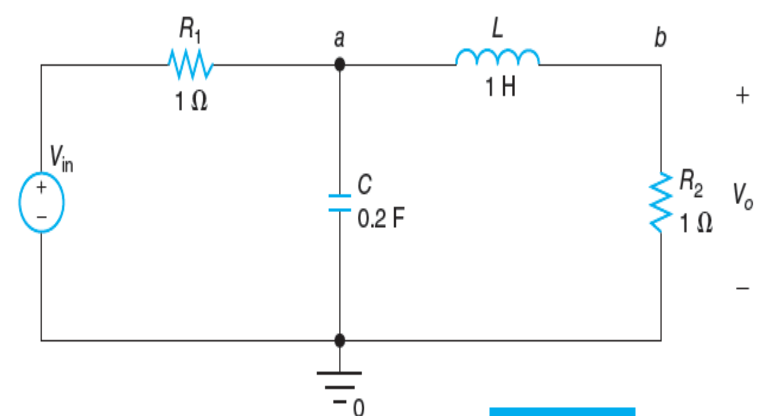


FIGURE 10.63

EXAMPLE 10.12

- **Find the transfer function for the circuit shown in Figure 10.65.**
- **Sum the currents leaving node a:**

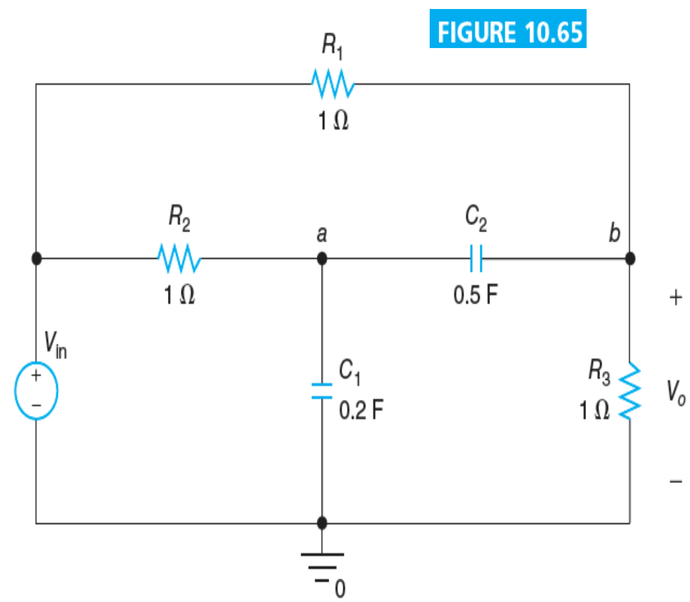
$$\frac{V_a - V_{in}}{1} + V_a j\omega 0.2 + \left(V_a - V_o\right) j\omega 0.5 = 0 \Rightarrow (0.7 j\omega + 1)V_a - 0.5 j\omega V_o = V_{in} \quad (1)$$

- **Sum the currents leaving node b:**

$$(V_o - V_a)0.5j\omega + V_o + (V_o - V_{in}) = 0 \Rightarrow -0.5j\omega V_a + (0.5j\omega + 2)V_o = V_{in} \quad (2)$$

- **Applying Cramer's rule on Equations (1) and (2), we obtain**

$$V_o = \frac{\begin{vmatrix} 0.7 j\omega + 1 & 1 \\ -0.5 j\omega & 1 \end{vmatrix}}{\begin{vmatrix} 0.7 j\omega + 1 & -0.5 j\omega \\ -0.5 j\omega & 0.5 j\omega + 2 \end{vmatrix}} V_{in} = \frac{1.2 j\omega + 1}{0.1(j\omega)^2 + 1.9 j\omega + 2} V_{in}$$



- **The transfer function is given by**

$$H(\omega) = \frac{V_o}{V_{in}} = \frac{12 j\omega + 10}{(j\omega)^2 + 19 j\omega + 20}$$

- **At** $\omega = 0$, $H(\omega) = 0.5$. **At** $\omega = \infty$, $H(\omega) = 0$. **This is an LPF.**

EXAMPLE 10.13

- **Find the transfer function for the circuit shown in Figure 10.67.**
- **Application of voltage divider rule yields**

$$V_b = V_o \times \frac{R_2}{R_2 + \frac{1}{j\omega C_2}} = V_o \times \frac{j\omega C_2 R_2}{j\omega C_2 R_2 + 1} = V_o \times \frac{j\omega}{j\omega + \frac{1}{R_2 C_2}}$$

$$V_a = V_{in} \times \frac{\frac{1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}} = V_{in} \times \frac{1}{j\omega C_1 R_1 + 1} = V_{in} \times \frac{\frac{1}{R_1 C_1}}{j\omega + \frac{1}{R_1 C_1}}$$

- **Since $V_a = V_b$, we have**

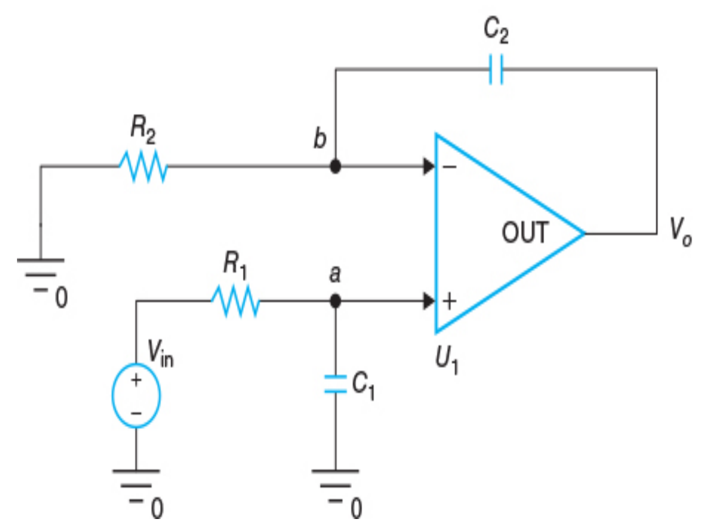
$$V_o \times \frac{j\omega}{j\omega + \frac{1}{R_2 C_2}} = V_{in} \times \frac{\frac{1}{R_1 C_1}}{j\omega + \frac{1}{R_1 C_1}}$$

- **Thus**

$$H(\omega) = \frac{V_o}{V_{in}} = \frac{j\omega + \frac{1}{R_2 C_2}}{j\omega} \times \frac{\frac{1}{R_1 C_1}}{j\omega + \frac{1}{R_1 C_1}} = \frac{1}{R_1 C_1} \times \frac{j\omega + \frac{1}{R_2 C_2}}{j\omega\left(j\omega + \frac{1}{R_1 C_1}\right)}$$

FIGURE 10.67

Circuit for EXAMPLE 10.13.



Summary

- **Transform a circuit to frequency domain by transforming capacitors by impedances, $1/(j\omega C)$, inductors by impedances, $j\omega L$, without specifying values of ω, and designating the input and output of the circuit as a function of ω. The transfer function $H(\omega)$ is defined as the ratio of the output to input.**
- **A filter is a device that passes certain frequencies and blocks other frequencies. Common types of filters are lowpass filter (LPF), highpass filter (HPF), bandpass filter (BPF), and bandstop filter (BSF). The filters that cannot be realized are called ideal filters.**
- **In practical filters, the gain in the passband cannot be one for all frequencies, and the gain in the stopband cannot be zero for all frequencies. Also, the transitions from passband to stopband and stopband to passband are gradual.**
- **A simple first order LPF can be implemented in RC circuit or LR circuit. A simple first order HPF can be implemented in CR circuit or RL circuit.**
- **The second order filters (LPF, HPF, BPF, BSF) can be implemented in series RLC circuit or parallel RLC circuit.**