Circuit Analysis and Design

Academic year 2019/2020 – Semester 1 – Presentation 2

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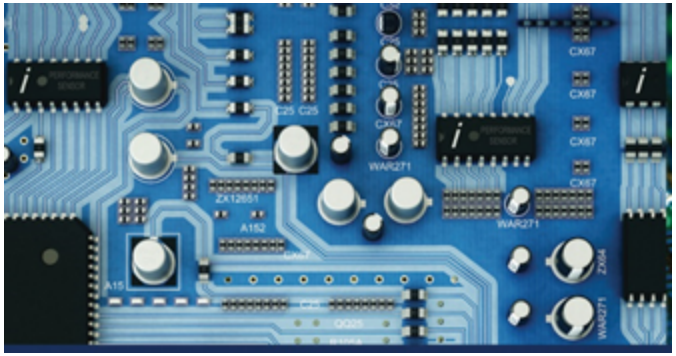
"A good student never steal or cheat"

Agenda



- Review of previous lecture
- Independent sources
- Dependent sources
- Elementary signals
- Summary

Independent Sources

- A voltage source with voltage V_s provides a constant potential difference to the circuit connected between the positive terminal and the negative terminal. The circuit notations for voltage source are shown in Figure 1.6.
- If a positive charge Δq is moved from the negative terminal to the positive terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$.
- If a negative charge with magnitude Δq is moved from the positive terminal

  to the negative terminal through the voltage source, the potential energy of the charge is increased by $\Delta q V_s$.
- A current source with current I_s provides a constant current of I_s amperes to the circuit connected to the two terminals. The circuit notation for current source is shown in Figure 1.7.

FIGURE 1.6

Circuit symbols for voltage sources.

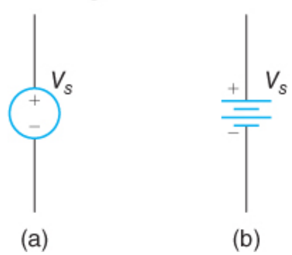
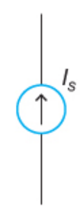


FIGURE 1.7

A circuit symbol for the current source.



DC Sources and AC Sources

- If the voltage from the voltage source is constant with time, the voltage source is called the direct current (dc) voltage source. Likewise, if the current from the current source is constant with time, the current source is called the direct current (dc) current source.
- If the voltage from the voltage source is a sinusoid as shown in Figure 1.8, the voltage source is called alternating current (ac) voltage source. Likewise, if the current from the current source is a sinusoid, the current source is called alternating current (ac) current source.
- A circuit notation for ac voltage source and ac current source are shown in Figure 1.9.

FIGURE 1.8

Plot of a cosine wave with period T, amplitude V_m, and phase zero.

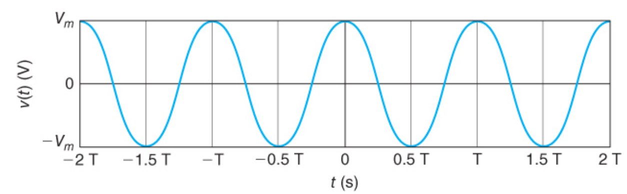
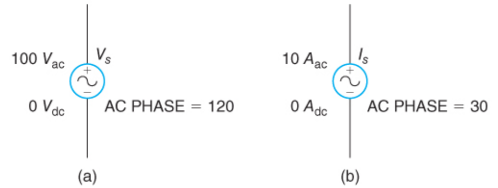


FIGURE 1.9

Circuit symbols for (a) ac voltage source; (b) ac current source.



From Electric Circuits by James S. Kang (Cenange Learning)

Equivalent Voltage Source and Equivalent Current Source

- When dc voltage sources are connected in series, they can be combined into single equivalent dc voltage source as shown in Figure 1.10.
- $V_3 = V_1 + V_2 = 4.5\text{ V} + 7.5\text{ V} = 12\text{ V}$
- When dc current sources are connected in parallel, they can be combined into single equivalent dc current source as shown in Figure 1.11.
- $I_3 = I_1 + I_2 = 3\text{ A} + 5\text{ A} = 8\text{ A}$

FIGURE 1.10

An equivalent voltage source.

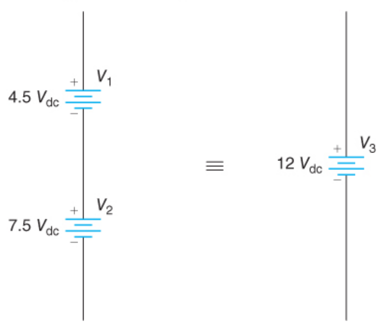
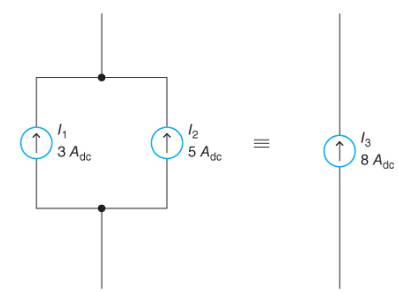


FIGURE 1.11

An equivalent current source.

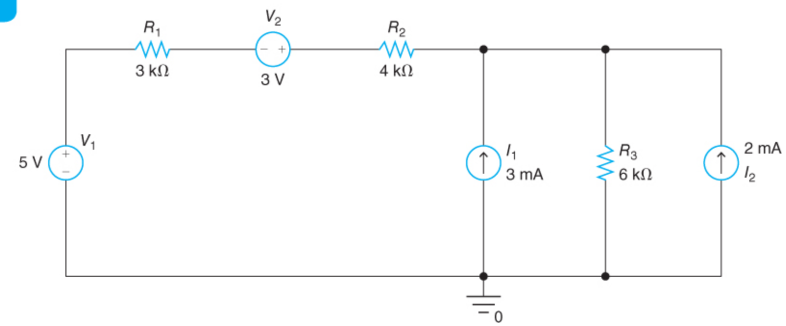


EXAMPLE 1.4

- Redraw the circuit shown in Figure 1.12 with one voltage source and one current source without affecting the voltages across and currents through the resistors in the circuit.

FIGURE 1.12

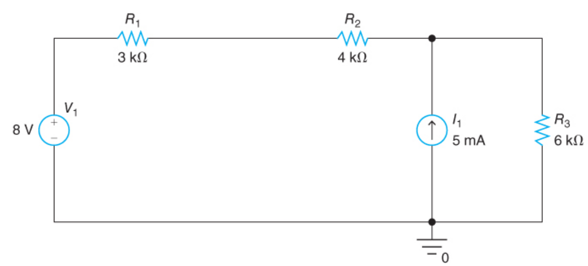
Circuit for EXAMPLE 1.4.



- $V_3 = V_1 + V_2 = 5\text{ V} + 3\text{ V} = 8\text{ V}$
- $I_3 = I_1 + I_2 = 3\text{ mA} + 2\text{ mA} = 5\text{ mA}$

FIGURE 1.13

A circuit with one current source and one voltage source.



- The equivalent circuit with one voltage source and one current source is shown in Figure 1.13.

Sinusoidal Signal

- An ac voltage waveform can be represented as $v(t) = V_m \cos\left(\frac{2\pi t}{T} + \phi\right)$ V
- V_m = peak amplitude (V), T = period (s), ϕ = phase (rad or deg).
- f = 1/T = frequency (Hz). $v(t) = V_m \cos(2\pi ft + \phi)$ V
- $\omega = 2\pi f = 2\pi/T$ = angular velocity (rad/s). $v(t) = V_m \cos(\omega t + \phi)$ V
- The ac current waveform can be written as

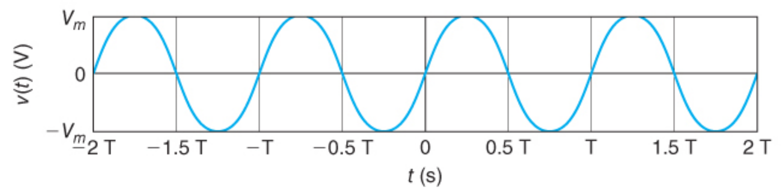
  $i(t) = I_m \cos(2\pi t/T + \phi) = I_m \cos(2\pi ft + \phi) = I_m \cos(\omega t + \phi)$ A
- If the cosine wave shown in Figure 1.8 is shifted to the right by T/4, we have

$$v(t) = V_m \cos\left[2\pi\left(t - T/4\right)/T\right] = V_m \cos\left(\frac{2\pi}{T}t - \frac{\pi}{2}\right) = V_m \sin\left(\frac{2\pi}{T}t\right) = V_m \sin\left(2\pi ft\right) = V_m \sin\left(\omega t\right)$$

- The sine wave is shown in Figure 1.16.

FIGURE 1.16

Plot of a sine wave with period T and amplitude V_m.



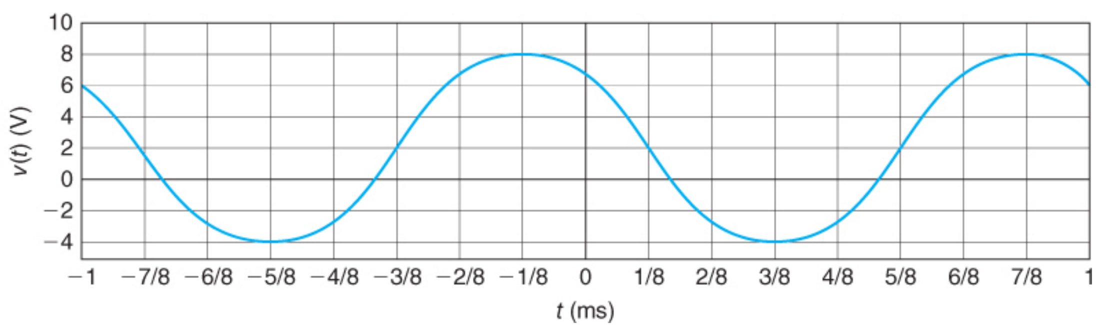
EXAMPLE 1.5

- Find the equation of the sinusoidal signal shown in Figure 1.17.
- $T = 1$ ms, $f = 1/T = 1000$ Hz $= 1$ kHz, $\omega = 2\pi f = 6283.1853$ rad/s
- Peak-to-peak amplitude $= V_{p\text{-}p} = 8 - (-4) = 12$ V, peak amplitude $= V_m = V_{p\text{-}p}/2 = 6$ V
- DC offset = average amplitude $= V_{dc} = [8 + (-4)]/2 = 2$ V
- The cosine wave is shifted to the left by T/8 ms, which is $\pi/4$ rad $= 45^o$. Therefore, the equation is given by:

  $v(t) = 2 + 6\cos(2\pi 1000t + 45^o)$ V

FIGURE 1.17

A sinusoid for EXAMPLE 1.5.



Dependent Sources

- The voltage or current on the dependent sources depend solely on the controlling voltage or controlling current. The dependent sources are used to model integrated circuit (IC) devices.
- Depending on whether the dependent source is a voltage source or a current source and whether the dependent source is controlled by a voltage or a current, there are four different dependent sources:

  Voltage-Controlled Voltage Source (VCVS): $v_d = k_v\, v_c$

  Voltage-Controlled Current Source (VCCS): $i_d = g_m\, v_c$

  Current-Controlled Voltage Source (CCVS): $v_d = r_m\, i_c$

  Current-Controlled Current Source (CCCS): $i_d = k_i\, i_c$
- Figures 1.19 - 1.22 show the circuit symbols.

FIGURE 1.19

Circuit symbol for VCVS.

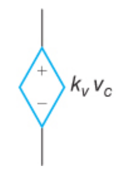


FIGURE 1.20

Circuit symbol for VCCS.

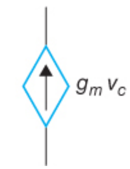


FIGURE 1.21

Circuit symbol for CCVS.

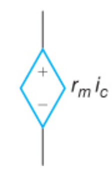
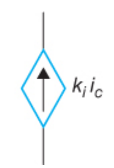


FIGURE 1.22

Circuit symbol for CCCS.



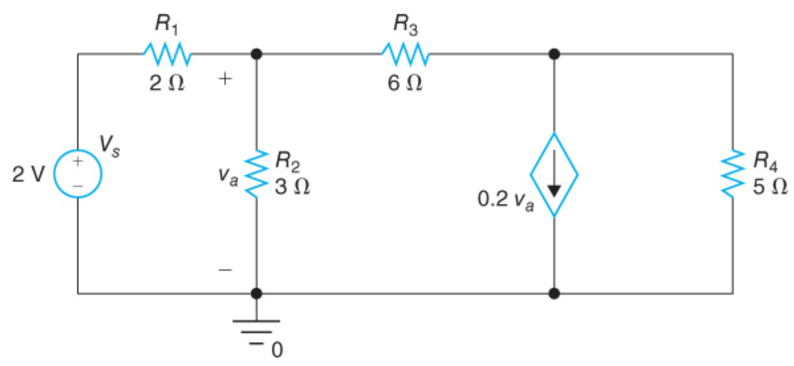
EXAMPLE 1.6

- In the circuit shown in Figure 1.23, the controlling voltage, which is the voltage across R_2, is $v_a = 0.9851$ V. Find the controlled current through the VCCS.
- The current through the VCCS in the direction indicated in Figure 1.23 (↓) is

  $0.2\ v_a = 0.2\ (\text{A/V}) \times 0.9851\ \text{V} = 0.1970\ \text{A}$

FIGURE 1.23

Circuit for EXAMPLE 1.6.



Dirac Delta Function

- A rectangular pulse with height $1/\tau$ and width τ is shown in Figure 1.25. The pulse is centered at $-\tau/2$ and the area of the pulse is one. The rectangular pulse can be written as

$$f(t) = \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau}\right)$$

- If the pulse width τ is decreased to zero, the height of the pulse is increased to infinity while maintaining the area at one. The limiting form of a rectangular pulse shown in Figure 1.25 as $\tau \to 0$ is defined as Dirac delta function (or delta function) and is denoted by $\delta(t)$:

$$\delta(t) = \lim_{\tau \to 0} \frac{1}{\tau} rect\left(\frac{t + \frac{\tau}{2}}{\tau}\right)$$

FIGURE 1.25

A rectangular pulse.

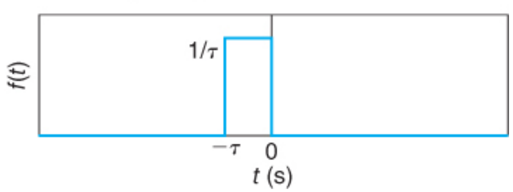
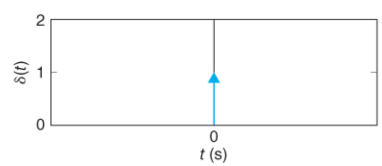


FIGURE 1.26

Symbol for the Dirac delta function.



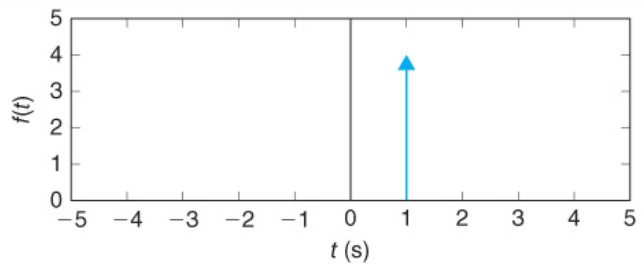
EXAMPLE 1.7

- Plot $f(t) = 4\,\delta(t - 1)$.

- The Dirac delta function is located at $t = 1$ and has area of 4. The signal $f(t)$ is shown in Figure 1.27.

FIGURE 1.27

Plot of $f(t)$.



Sifting Property

- When a continuous signal f(t) is multiplied by δ(t – a) and integrated from – ∞ to ∞, we obtain f(a), that is,

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$$

- This result is called the sifting property of the delta function because it sifts out a single value of f(t), f(a), at the location of the delta function (t = a). To prove the sifting property, we replace δ(t – a) by

$$\delta(t-a) = \lim_{\tau \to 0} \frac{1}{\tau} rect\left(\frac{t-a+\tau/2}{\tau}\right)$$

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = \lim_{\tau \to 0} \frac{1}{\tau} \int_{-\infty}^{\infty} f(t) rect\left(\frac{t-a+\tau/2}{\tau}\right) dt = \lim_{\tau \to 0} \frac{1}{\tau} \int_{a-\tau}^{a} f(t) rect\left(\frac{t-a+\tau/2}{\tau}\right) dt$$

- As τ → 0, f(t) → f(a) for (a – τ) < t < a. Thus, the integral becomes

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = \lim_{\tau \to 0} \frac{1}{\tau} \int_{a-\tau}^{a} f(a) \times 1\, dt = \lim_{\tau \to 0} \frac{1}{\tau} f(a)\tau = f(a)$$

Step Function

- The unit step function u(t) is the integral of the Dirac delta function δ(t). If a rectangular pulse f(t) = (1/τ) rect[(t + τ/2)/τ] is integrated, we obtain

$$\int_{-\infty}^{t} f(\lambda)d\lambda = \frac{1}{\tau}\int_{-\infty}^{t} rect\left(\frac{\lambda+\frac{\tau}{2}}{\tau}\right)d\lambda = \begin{cases} 0, & t < -\tau \\ \frac{t}{\tau}+1, & -\tau \le t < 0 \\ 1, & 0 \le t \end{cases}$$

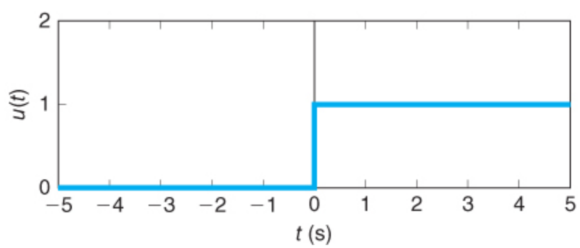
- The unit step function is defined as the limiting form of this equation. In the limit as τ → 0, this equation becomes

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 \le t \end{cases}$$

- Notice that at t = 0, u(t) = 1. The unit step function is shown in Figure 1.29.

FIGURE 1.29

A unit step function.

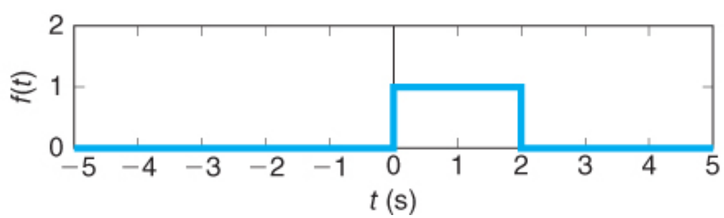


EXAMPLE 1.8

- Plot $f(t) = u(t) – u(t – 2)$.
- Notice that $u(t) = 1$ for $t \geq 0$ and zero for $t < 0$, and $u(t – 2) = 1$ for $t \geq 2$ and zero for $t < 2$. Thus, $u(t) – u(t – 2) = 0$ for $t \geq 2$, and $u(t) – u(t – 2) = 1$ for $0 \leq t < 2$, and zero for $t < 0$. The signal $f(t)$ is shown in Figure 1.30.

FIGURE 1.30

Plot of $f(t)$.



Ramp Function

- A unit ramp function is defined by
  r(t) = t u(t)
- The unit ramp function is shown in Figure 1.32.
- The unit ramp function is the integral of the unit step function:

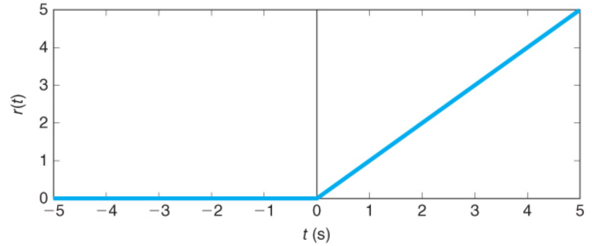
$$r(t) = \int_{-\infty}^{t} u(\lambda)\, d\lambda$$

- The derivative of the unit ramp function is the unit step function:

$$u(t) = \frac{dr(t)}{dt}$$

FIGURE 1.32

A unit ramp function.

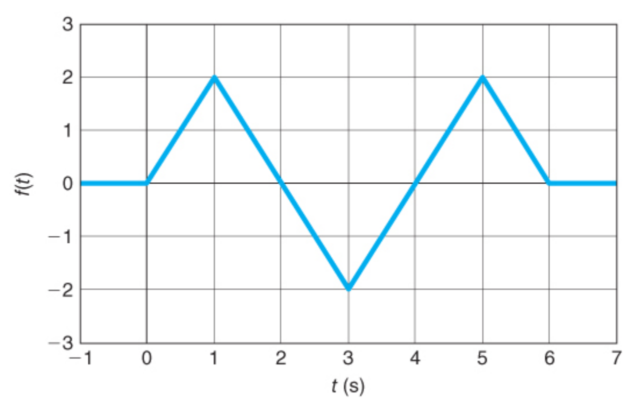


EXAMPLE 1.9

- Plot $f(t) = 2tu(t) - 4(t-1)u(t-1) + 4(t-3)u(t-3) - 4(t-5)u(t-5) + 2(t-6)u(t-6)$.
- For $t < 0$, $f(t) = 0$.
- For $0 \le t < 1$, $f(t)$ is a linear line with slope of 2.
- For $1 \le t < 3$, $f(t)$ is a linear line with slope of –2.
- For $3 \le t < 5$, $f(t)$ is a linear line with slope of 2.
- For $5 \le t < 6$, $f(t)$ is a linear line with slope of –2.
- For $6 \le t$, $f(t) = 0$.

FIGURE 1.33

Waveform $f(t)$.

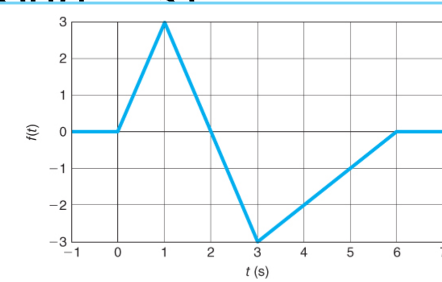


EXAMPLE 1.10

- Find the equation of the waveform shown in Figure 1.35.
- For $t < 0$, $f(t) = 0$.
- For $0 \le t < 1$, $f(t)$ is a linear line with slope of 3. Thus, $f(t) = 3tu(t)$.
- For $1 \le t < 3$, $f(t)$ is a linear line with slope of -3. To change the slope from 3 to -3, we need to add $-6(t - 1)u(t - 1)$. At this point, we have $f(t) = 3tu(t) - 6(t - 1)u(t - 1)$.
- For $3 \le t < 6$, $f(t)$ is a linear line with slope of 1. To change the slope from -3 to 1, we need to add $4(t - 3)u(t - 3)$. At this point, we have $f(t) = 3tu(t) - 6(t - 1)u(t - 1) + 4(t - 3)u(t - 3)$.

FIGURE 1.35

Waveform $f(t)$.



- For $6 \le t$, $f(t) = 0$. To change the slope from 1 to 0, we need to add $-(t - 6)u(t - 6)$.
- The final equation is given by

$$f(t) = 3tu(t) - 6(t - 1)u(t - 1) + 4(t - 3)u(t - 3) - (t - 6)u(t - 6).$$

Exponential Decay

- A signal that decays exponentially can be written as $f(t) = e^{-at} u(t)$, $a > 0$.
- The signal $f(t)$ for $a = 0.5$ is shown in Figure 1.37.
- A damped cosine and damped sine can be written respectively as

  $f(t) = e^{-at}\cos(bt)u(t)$, $a > 0$

  $f(t) = e^{-at}\sin(bt)u(t)$, $a > 0$
- A damped cosine signal is shown in Figure 1.38 for $a = 0.5$ and $b = 4$.

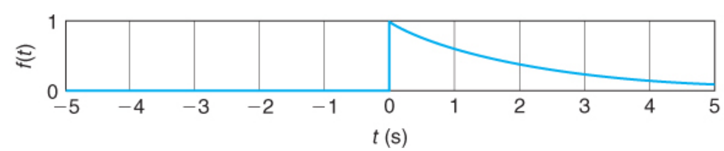


FIGURE 1.37

$f(t) = e^{-at}u(t), a = 0.5.$

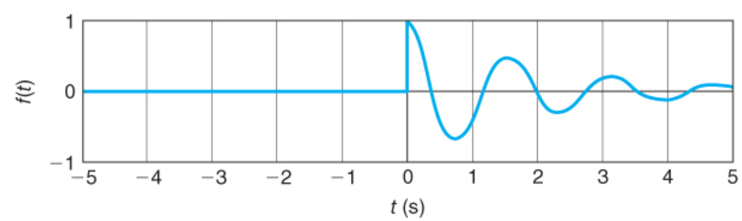


FIGURE 1.38

A damped cosine signal.

Rectangular Pulse and Triangular Pulse

- A rectangular pulse with amplitude A pulse width τ is shown in Figure 1.39. The center of the pulse is at t = 0.
- The rectangular pulse shown in Figure 1.39 is denoted by

$$f(t) = A rect\left(\frac{t}{\tau}\right)$$

- A triangular pulse with amplitude A and base 2τ is shown in Figure 1.42. The center of the pulse is at t = 0.
- The triangular pulse shown in Figure 1.42 is denoted by $f(t) = A\ tri\left(\frac{t}{\tau}\right)$

FIGURE 1.39

A rectangular pulse.

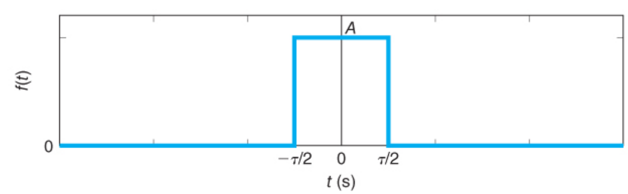
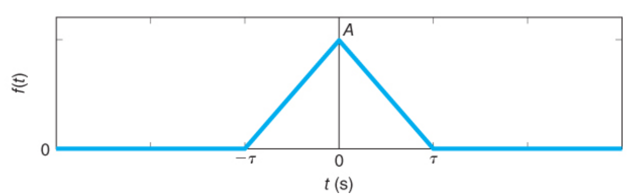


FIGURE 1.42

A triangular pulse.



EXAMPLE 1.11

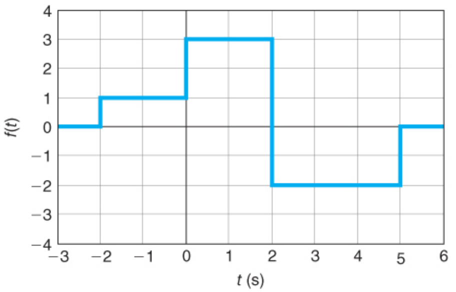
- Plot

$$f(t) = rect\left(\frac{t+1}{2}\right) + 3rect\left(\frac{t-1}{2}\right) - 2rect\left(\frac{t-3.5}{3}\right)$$

- The first rectangle is centered at t = –1 and has a height of 1 and width of 2. The second rectangle is centered at t = 1 and has a height of 3 and width of 2. The third rectangle is centered at t = 3.5 and has a height of –2 and width of 3. The waveform f(t) is shown in Figure 1.40.

FIGURE 1.40

Waveform $f(t)$.

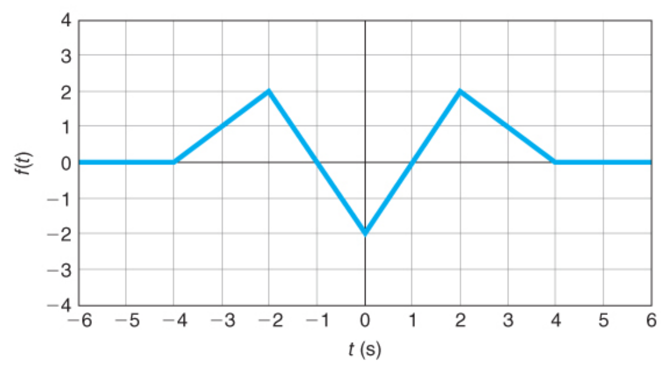


EXAMPLE 1.12

- Plot $$f(t) = 2\,tri\left(\frac{t+2}{2}\right) - 2\,tri\left(\frac{t}{2}\right) + 2\,tri\left(\frac{t-2}{2}\right)$$
- The first triangle is centered at t = –2 and has a height of 2 and base of 4. The second triangle is centered at t = 0 and has a height of -2 and base of 4. The third triangle is centered at t = 2 and has a height of 2 and base of 4. The waveform f(t) is shown in Figure 1.43.

FIGURE 1.43

Waveform $f(t)$.



Summary

- The seven base units of the International System of Units (SI) along with derived units relevant to electrical and computer engineering are presented.
- The electric field E is a force per unit charge.
- When electric field E is integrated along the path against the field, we get work (force × displacement) done per unit charge.
- The potential difference between points A (final) and B (initial) is defined as the work done per unit charge against the force: $v_{AB} = v_A - v_B = dw_{AB}/d_q$ (J/C)
- The current is defined as the rate of change of charge: i(t) = dq(t)/dt
- The power is the product of current and voltage: p(t) = i(t)v(t)
- The energy is the integral of power: $w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda$
- Power is the derivative of energy: p(t) = dw(t)/dt
- Four types of dependent sources: VCVS, VCCS, CCVS, CCCS
- Elementary signals: Dirac delta, step, ramp, exponential, rectangular pulse, triangular pulse.
- What will we study in next lecture