电子科技大学
格拉斯哥学院
Glasgow College, UESTC

Circuit Analysis and Design

Academic year 2019/2020 – Semester 1 – Presentation 6

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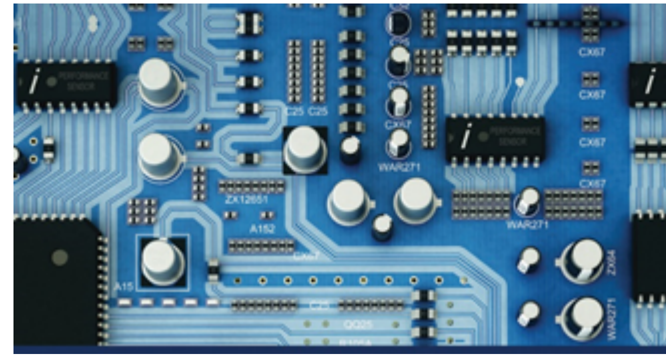
"A good student never steal or cheat"

Agenda

- Review of previous lecture
- Mesh analysis
- Supermesh
- Summary



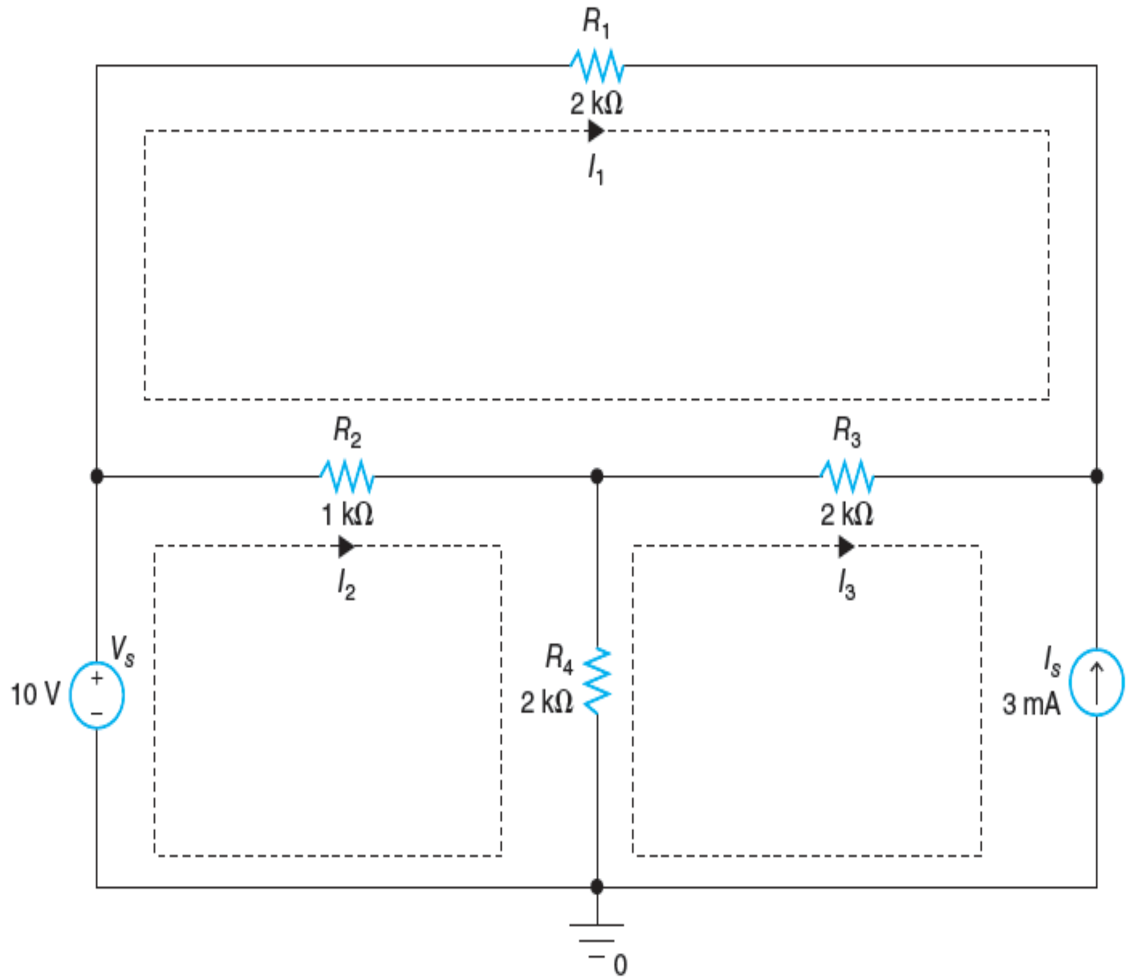
Mesh Analysis

- A mesh is a loop that does not contain any other loops.
- For each mesh, define a mesh current as the current flowing around the mesh in the clockwise direction.
- If a mesh contains a current source, the mesh current is the same as the current from the current source if they point in the same direction. If the mesh current is negative, the physical current flows in the opposite direction to the mesh current.
- If there is a branch in a mesh that is shared by another mesh, the physical current flowing through the branch is the difference in mesh currents sharing the branch.

Mesh Analysis

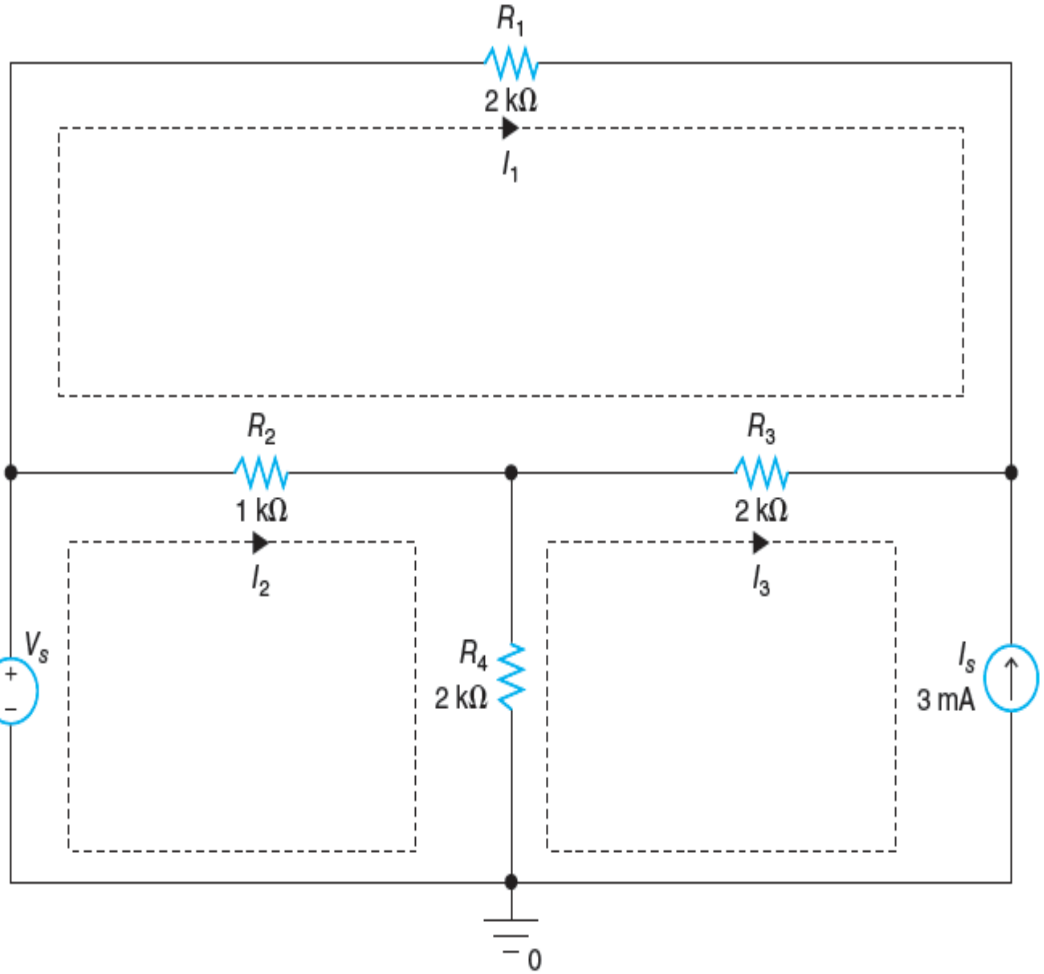
- The mesh analysis is based on KVL:
  - The sum of voltage drops around a mesh is zero.
  - For each mesh, sum the voltage drops around the mesh in the clockwise direction and set that equal to zero.
  - The voltage drop across a resistor is the product of the resistance and the net current (physical current) through the resistor.
  - If the resistor is not shared by another mesh, the voltage drop is the product of the resistance and the mesh current.
  - If the resistor is shared by another mesh, the voltage drop is the product of the resistance and the difference in mesh currents.
  - The voltage drop across a voltage source V_s from positive terminal to negative terminal is V_s, and $-V_s$ from negative terminal to positive terminal.

Mesh Analysis



- There are three meshes
- The mesh currents are I_1, I_2, and I_3.
- Notice that mesh current I
flows in the opposite direc
to I_s. Thus, $I_3 = – I_s = – 3$ n
- Since I_3 is known, no need
to write a mesh equation f
mesh 3.
- Summing the voltage drops around mesh 1 in the clockwise direction starting from the left terminal of R_1, we obtain
$2000I_1 + 2000(I_1 – I_3) + 1000(I_1 – I_2) = 0$
- Divide by 1k ➔ $2I_1 + 2(I_1 – I_3) + (I_1 – I_2) = 0 \Rightarrow 5I_1 – I_2 = – 0.006$ (1)

Mesh Analysis



- Summing the voltage drops around mesh 2 in the clockwise direction starting from the negative terminal of V_s, we obtain

  $-10 + 1000(I_2 - I_1) + 2000(I_2 - I_3) = 0$

- Divide by 1000 ➔

  $-0.01 + (I_2 - I_1) + 2(I_2 - I_3) = 0 \Rightarrow$

  $-I_1 + 3I_2 = 0.01 - 0.006 \Rightarrow$

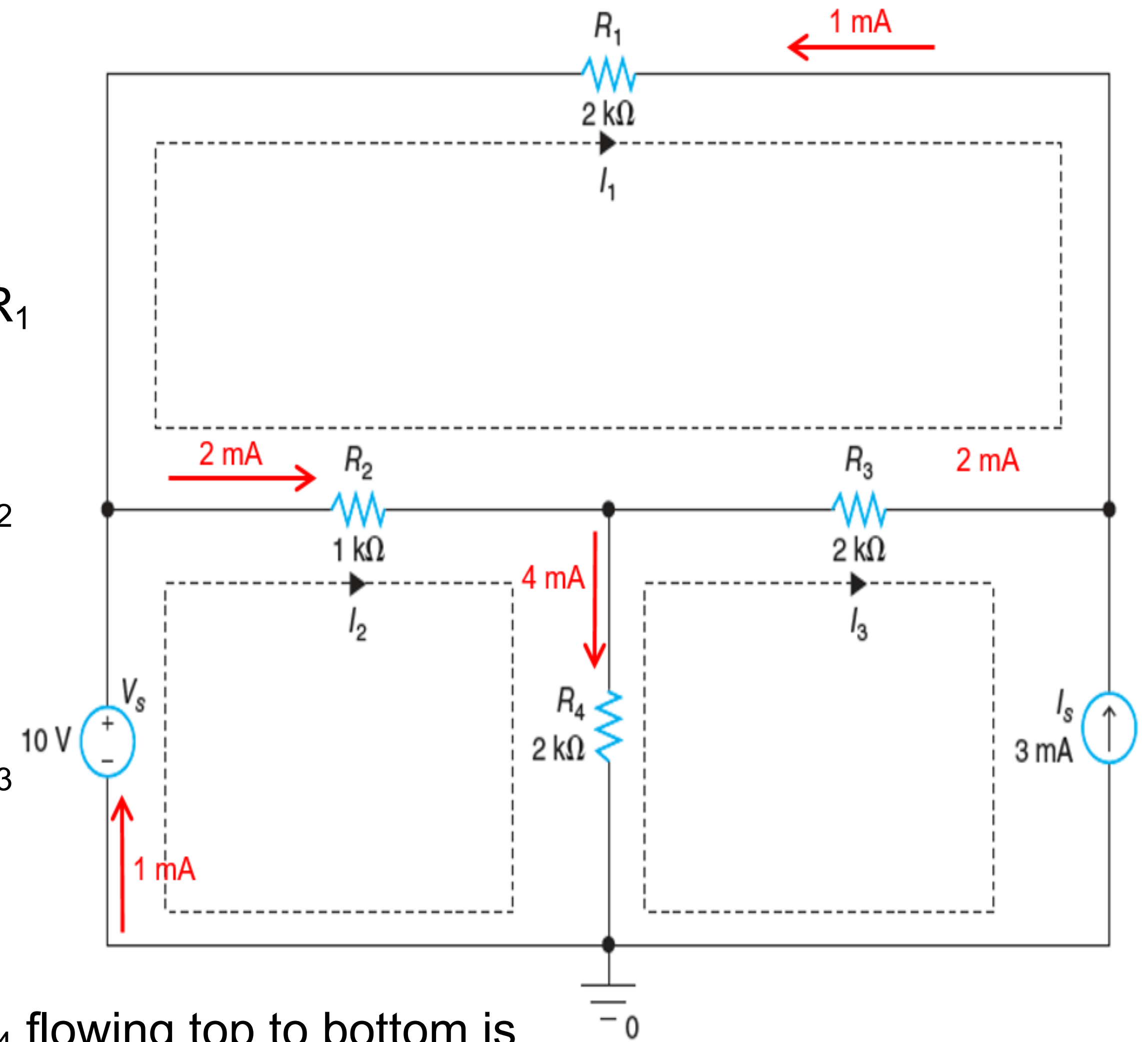
  $-I_1 + 3I_2 = 0.004 \qquad (2)$

- Multiply (2) by 5 ➔ $-5I_1 + 15I_2 = 0.02 \qquad (3)$

- Add $(5I_1 - I_2 = -0.006)(1)$ and (3) ➔ $14I_2 = 0.014 \Rightarrow I_2 = 0.001 \text{ A} = 1 \text{ mA}$

- From Equation (2): $I_1 = 3I_2 - 0.004 \Rightarrow I_1 = -0.001 \text{ A} = -1\text{mA}$

Mesh Analysis



- Since $I_1 = -1$ mA, the physical current of 1 mA flows through R_1 from right to left.

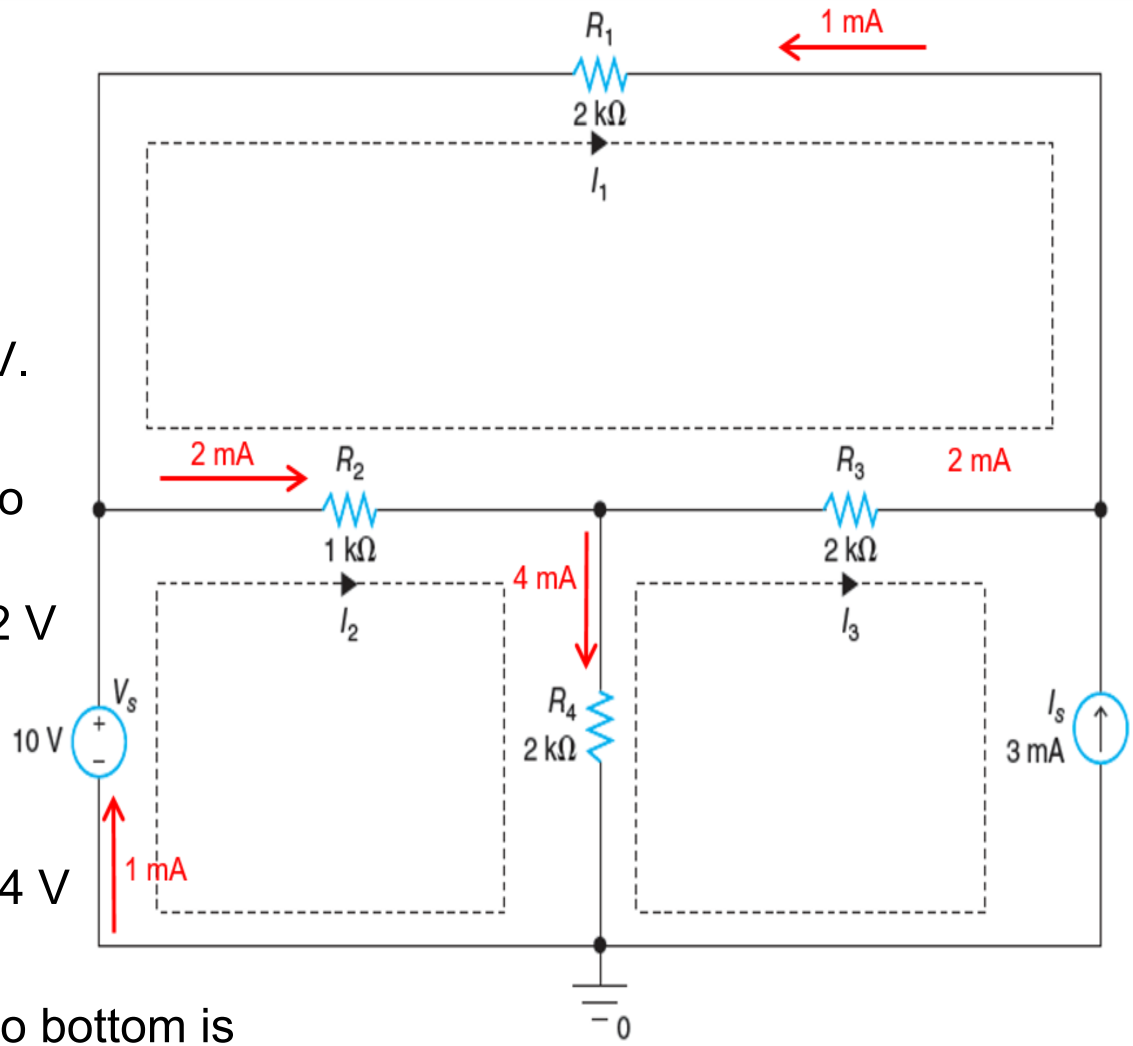
- The physical current through R_2 from left to right is given by
  $I_2 - I_1 = 1 - (-1) = 2$ mA

- The physical current through R_3 from right to left is
  $I_1 - I_3 = -1 - (-3) = 2$ mA

- The physical current through R_4 flowing top to bottom is
  $I_2 - I_3 = 1 - (-3) = 4$ mA)

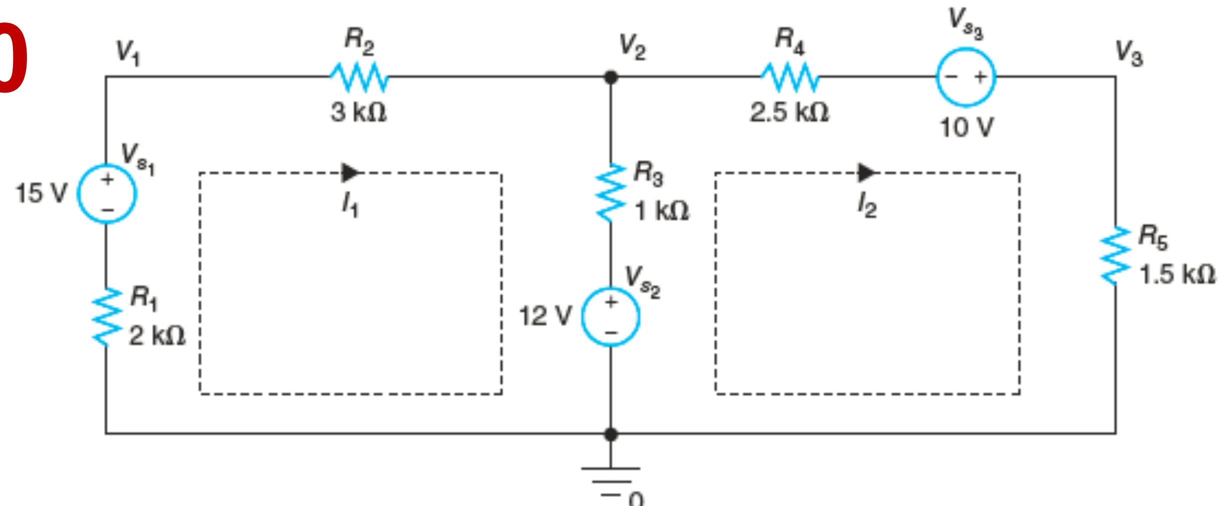
- KCL at a node 2 ➔ $I_2 - I_1 + I_1 - I_3 = I_2 - I_3 \Rightarrow 2 + 2 = 4$ mA

Mesh Analysis



- The voltage across R_1 from right to left is $V_{R1} = R_1(-I_1) = 2000\times0.001 = 2$ V.
- The voltage across R_2 from left to right is $V_{R2} = R_2(I_2 - I_1)= 1000\times0.002 = 2$ V
- The voltage across R_3 from right to left is $V_{R3} = R_3(I_1 - I_3) = 2000\times0.002 = 4$ V
- The voltage across R_4 from top to bottom is $V_{R4} = R_4(I_2 - I_3) = 2000\times0.004 = 8$ V
- The voltage across the current source is $V_{R3} + V_{R4} = 4\text{ V} + 8\text{ V} = 12$ V

EXAMPLE 3.10



- Find I_1, I_2, V_1, V_2, V_3.

- Sum the voltage drops around mesh 1:

$$2000I_1 - 15 + 3000I_1 + 1000(I_1 - I_2) + 12 = 0 \Rightarrow 6000I_1 - 1000I_2 = 3$$

- Divide by 1000: $6I_1 - I_2 = 0.003$ (1)

- Sum the voltage drops around mesh 2:
  $-12 + 1000(I_2 - I_1) + 2500I_2 - 10 + 1500I_2 = 0 \Rightarrow -1000I_1 + 5000I_2 = 22$

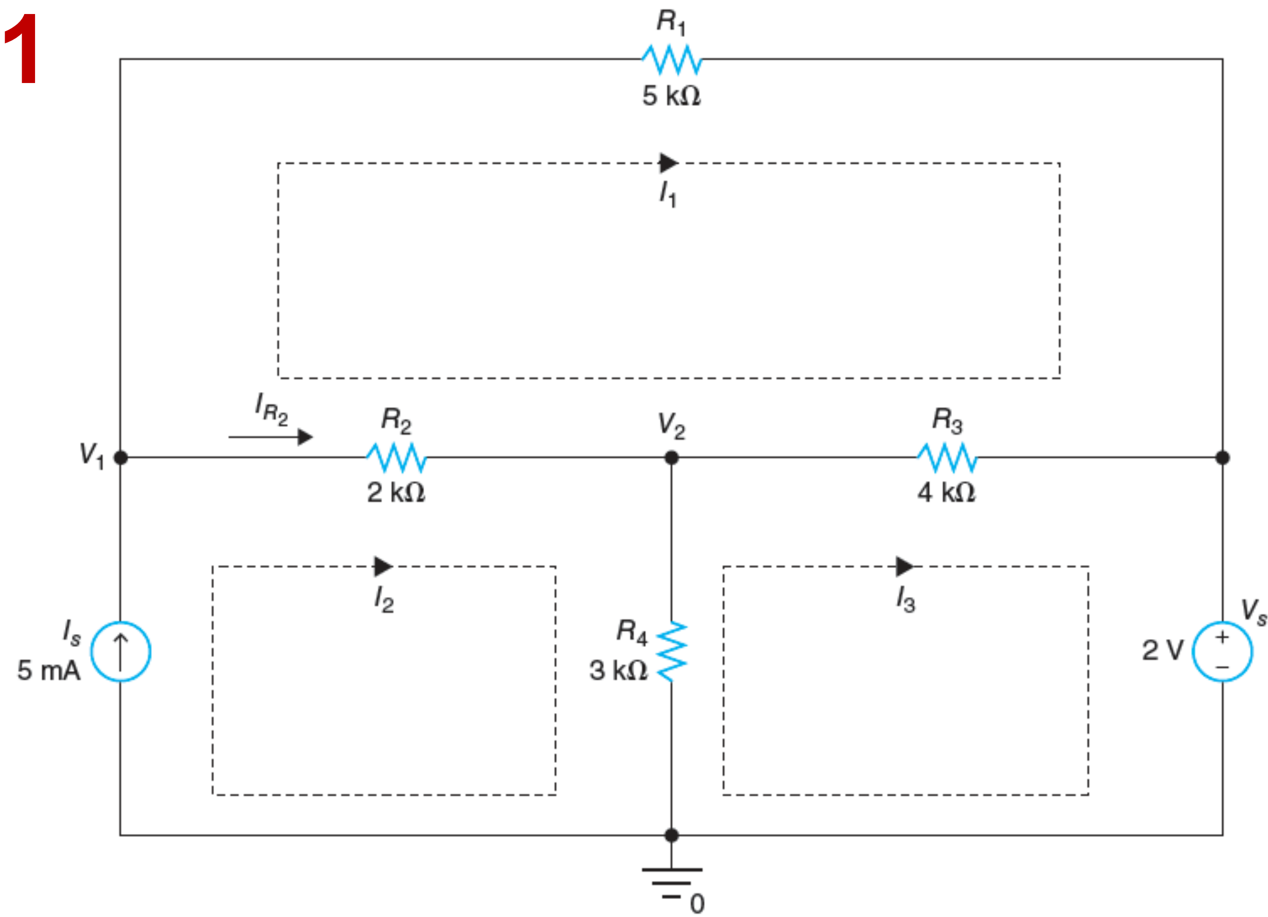
- Divide by 1000 ➔ $-I_1 + 5I_2 = 0.022$ (2)

- Multiply (1) by 5 ➔ $30I_1 - 5I_2 = 0.015$ (3)

- (2) + (3): $29I_1 = 0.037$ ➔ $I_1 = 1.2759$ mA

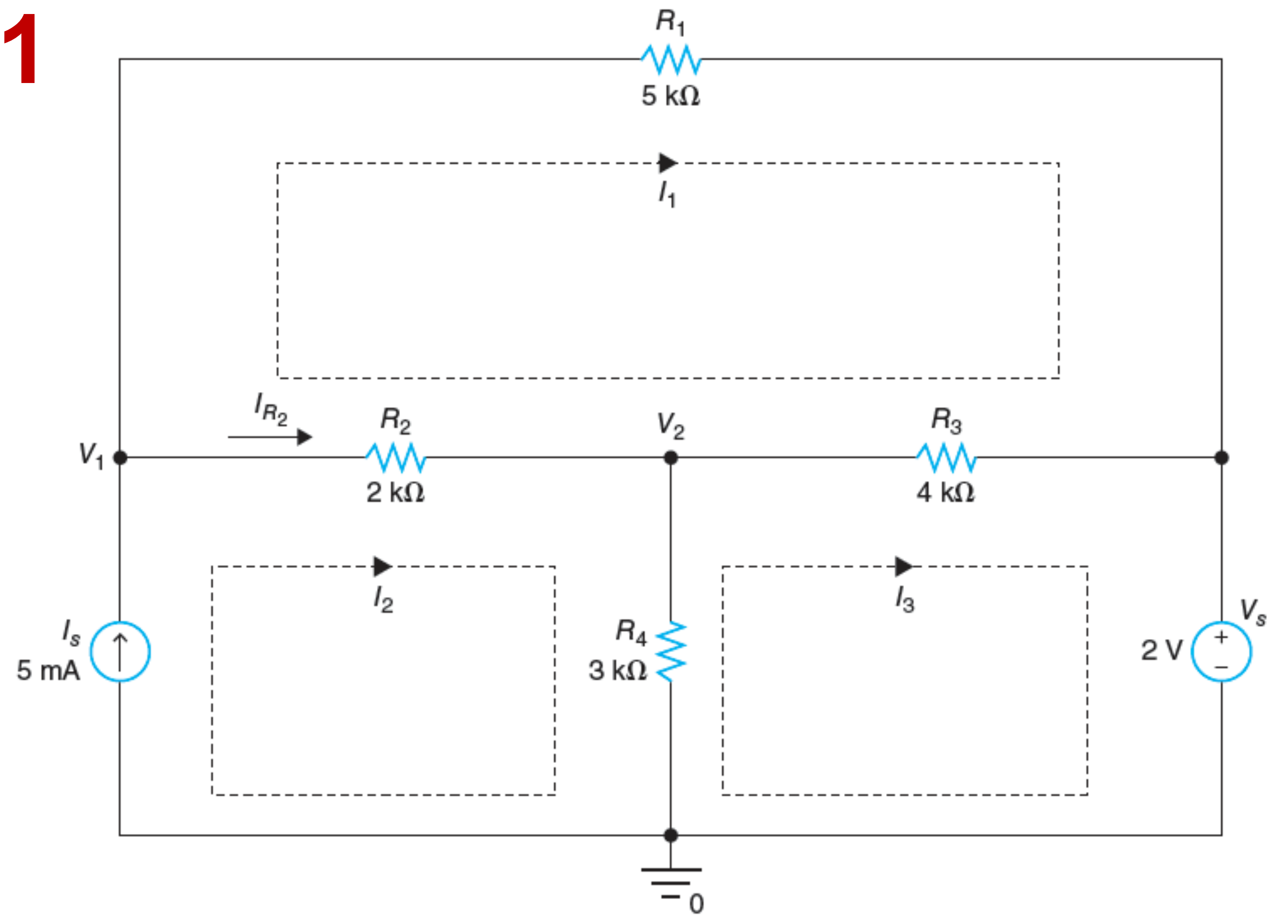
- From (1) ➔ $I_2 = 6I_1 - 0.003 = 4.6552$ mA

EXAMPLE 3.11



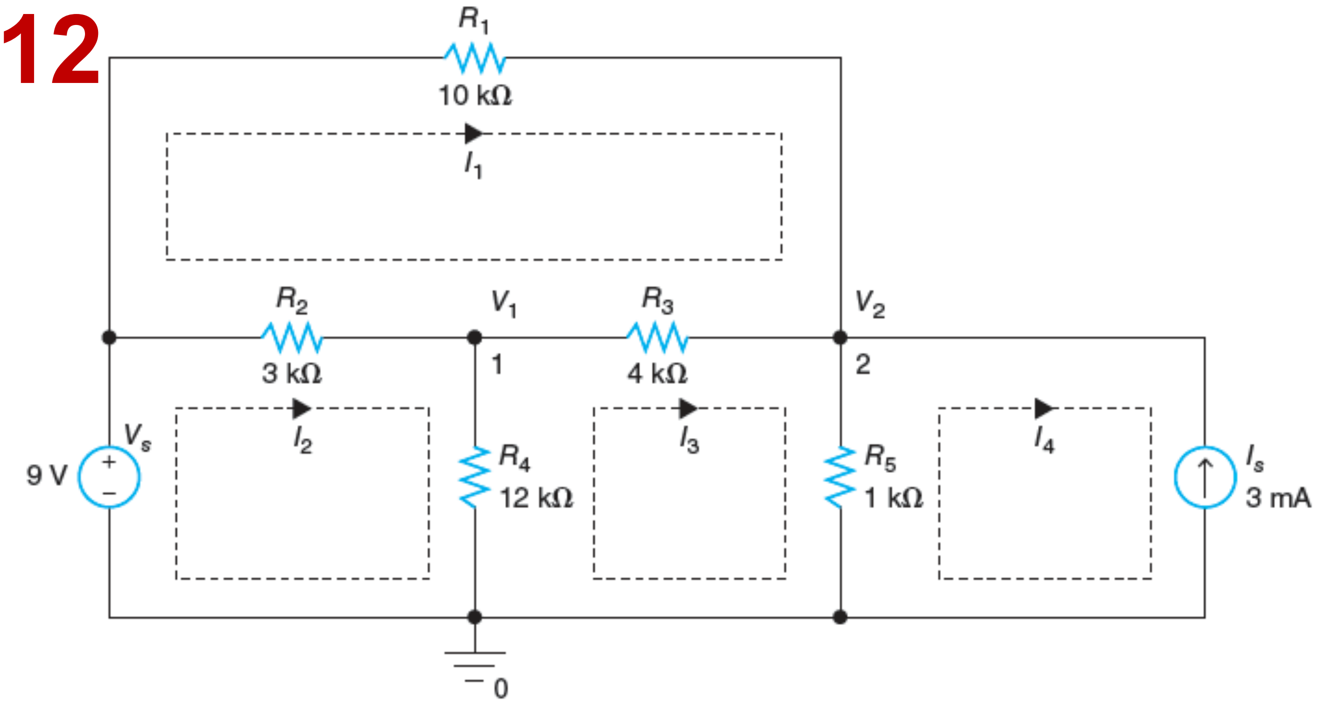
- Find I_1, I_2, I_3, V_1, V_2, I_{R2}
- $I_2 = I_s = 5$ mA
- Sum the voltage drops around mesh 1: $5000I_1 + 4000(I_1 – I_3) + 2000(I_1 – I_2) = 0 \Rightarrow$ $11000I_1 – 4000I_3 = 2000I_2$
- Divide by 1000 ➔ $11I_1 – 4I_3 = 0.01$ (1)
- Sum the voltage drops around mesh 3 ➔ $3000(I_3 – I_2) + 4000(I_3 – I_1) + 2 = 0 \Rightarrow$
  $– 4000I_1 + 7000I_3 = – 2 + 15 \Rightarrow – 4I_1 + 7I_3 = 0.013$ (2)
- Multiply (1) by 7 ➔ $77I_1 – 28I_3 = 0.07$ (3)
- Multiply (2) by 4 ➔ $– 16I_1 + 28I_3 = 0.052$ (4)
- Add (3) and (4) ➔ $61I_1 = 0.122 \Rightarrow I_1 = 0.002$ A = 2 mA
- From (1) ➔ $I_3 = (11I_1 – 0.01)/4 = 0.003$ A = 3 mA

EXAMPLE 3.11



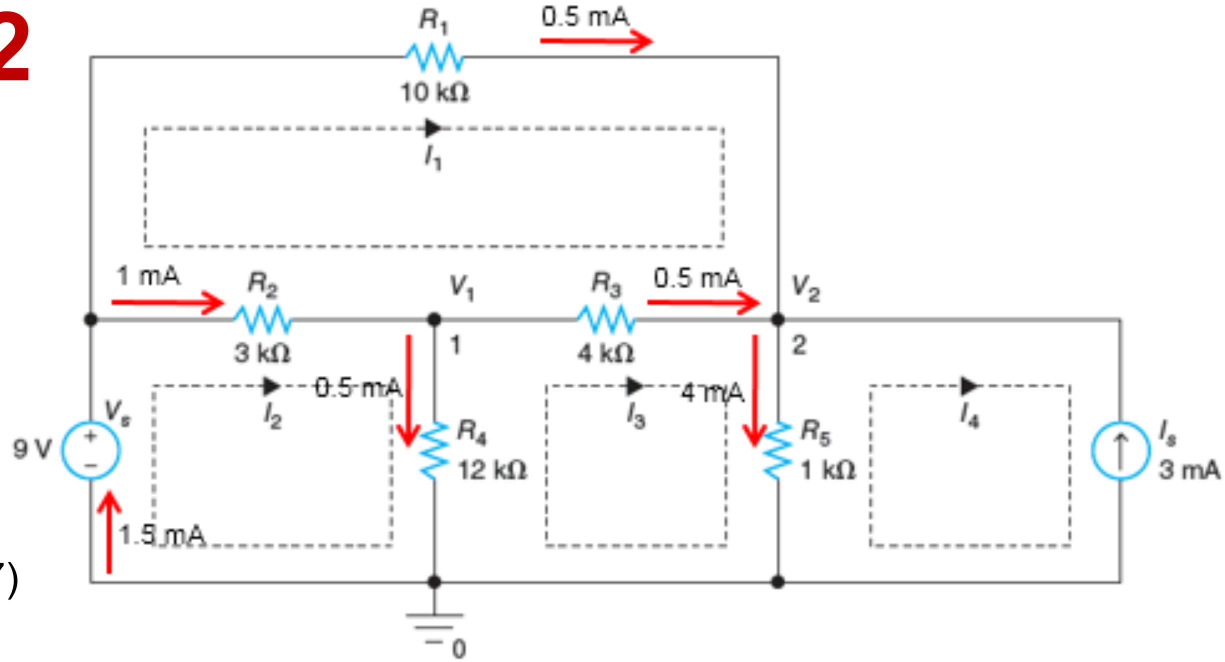
- $V_1 = V_s + R_1 I_1 = 2 + 5000 \times 0.002 = 12$ V
- $V_2 = R_4(I_2 - I_3) = 3000 \times (0.005 - 0.003) = 6$ V
- $I_{R2} = I_2 - I_1 = 5$ mA – 2 mA = 3 mA

EXAMPLE 3.12



- Find I_1, I_2, I_3, I_4, V_1, V_2, and currents through branches
- $I_4 = -I_s = -3$ mA
- Sum the voltage drops around mesh 1 ➔ $10000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0$ $\Rightarrow 17000I_1 - 3000I_2 - 4000I_3 = 0 \Rightarrow$ $\mathbf{17I_1 - 3I_2 - 4I_3 = 0}$ **(1)**
- Sum the voltage drops around mesh 2 ➔ $-9 + 3000(I_2 - I_1) + 12000(I_2 - I_3) = 0 \Rightarrow$ $-3000I_1 + 15000I_2 - 12000I_3 = 9 \Rightarrow$ $\mathbf{-3I_1 + 15I_2 - 12I_3 = 0.009}$ **(2)**
- Sum the voltage drops around mesh 3 ➔ $12000(I_3 - I_2) + 4000(I_3 - I_1) + 1000(I_3 - I_4) = 0$ $-4000I_1 - 12000I_2 + 17000I_3 = -3$ ➔ $\mathbf{-4I_1 - 12I_2 + 17I_3 = -0.003}$ **(3)**
- Multiply (1) by 5 ➔ $85I_1 - 15I_2 - 20I_3 = 0$ (4)
- (2)+(4): $82I_1 - 32I_3 = 0.009$ (5)

EXAMPLE 3.12



- $17I_1 - 3I_2 - 4I_3 = 0$ (1)
- $-3I_1 + 15I_2 - 12I_3 = 0.009$ (2)
- $-4I_1 - 12I_2 + 17I_3 = -0.003$ (3)
- $82I_1 - 32I_3 = 0.009$ (5)

- Multiply (1) by 4 ➔ $68I_1 - 12I_2 - 16I_3 = 0$ (6)

- (6) – (3) ➔ $72I_1 - 33I_3 = 0.003$ (7)

- Multiply (5) by 33 ➔ $2706I_1 - 1056I_3 = 0.297$ (8)
- Multiply (7) by 32 ➔ $2304I_1 - 1056I_3 = 0.096$ (7)

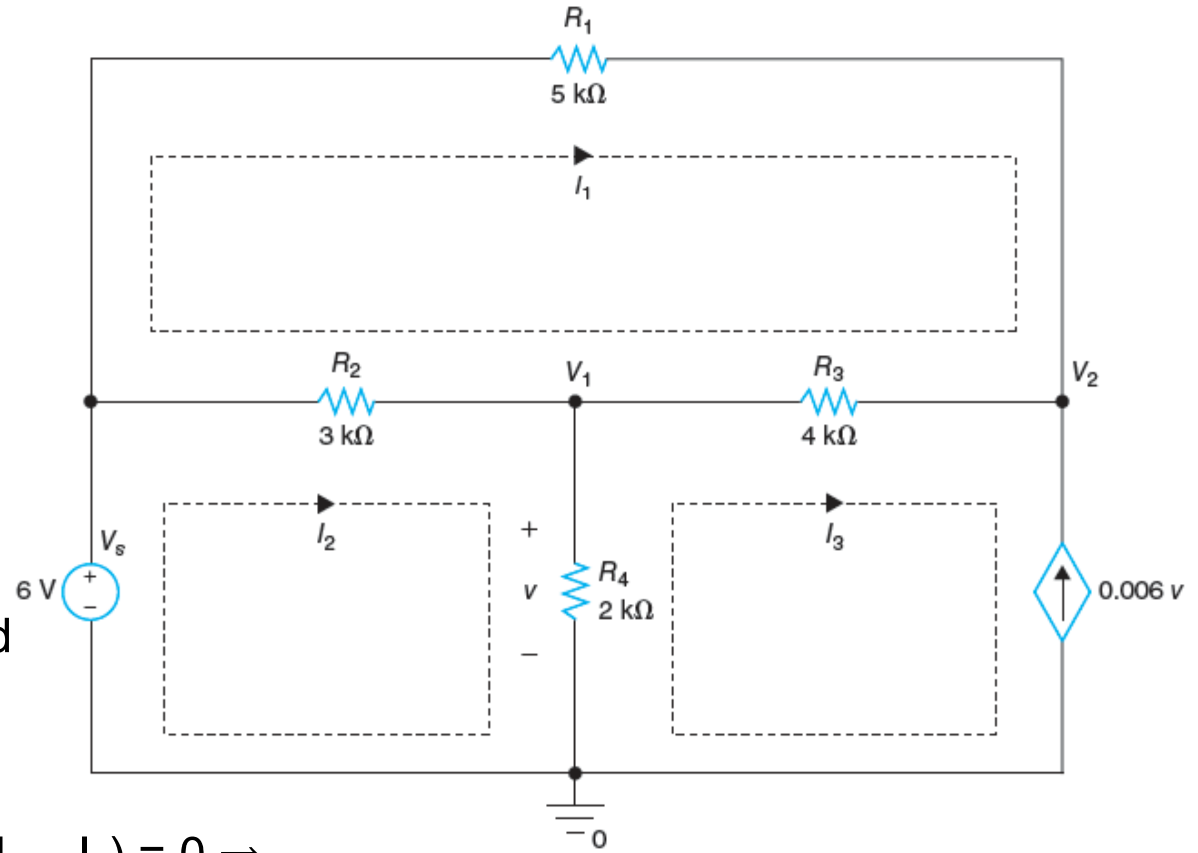
- (8) – (7): $402I_1 = 0.201 \Rightarrow I_1 = 0.0005\ \text{A} = 0.5\ \text{mA}$

- From (5)➔ $I_3 = (82I_1 - 0.009)/32 = 1\ \text{mA}$
- From (1)➔ $I_2 = (17I_1 - 4I_3)/3 = 1.5\ \text{mA}$

- $V_1 = R_4(I_2 - I_3) = 12000 \times 0.0005 = 6\ \text{V}$
- $V_2 = R_5(I_3 - I_4) = 1000 \times 0.004 = 4\ \text{V}$

- $I_{R1}\ (\rightarrow) = I_1 = 0.5\ \text{mA}$, $I_{R2}\ (\rightarrow) = I_2 - I_1 = 1\ \text{mA}$
- $I_{R3}\ (\rightarrow) = I_3 - I_1 = 0.5\ \text{mA}$
- $I_{R4}\ (\downarrow) = I_2 - I_3 = 0.5\ \text{mA}$, $I_{R5}\ (\downarrow) = I_3 - I_4 = 4\ \text{mA}$

EXAMPLE 3.13



- Find I_1, I_2, I_3, V_1, V_2, and powers

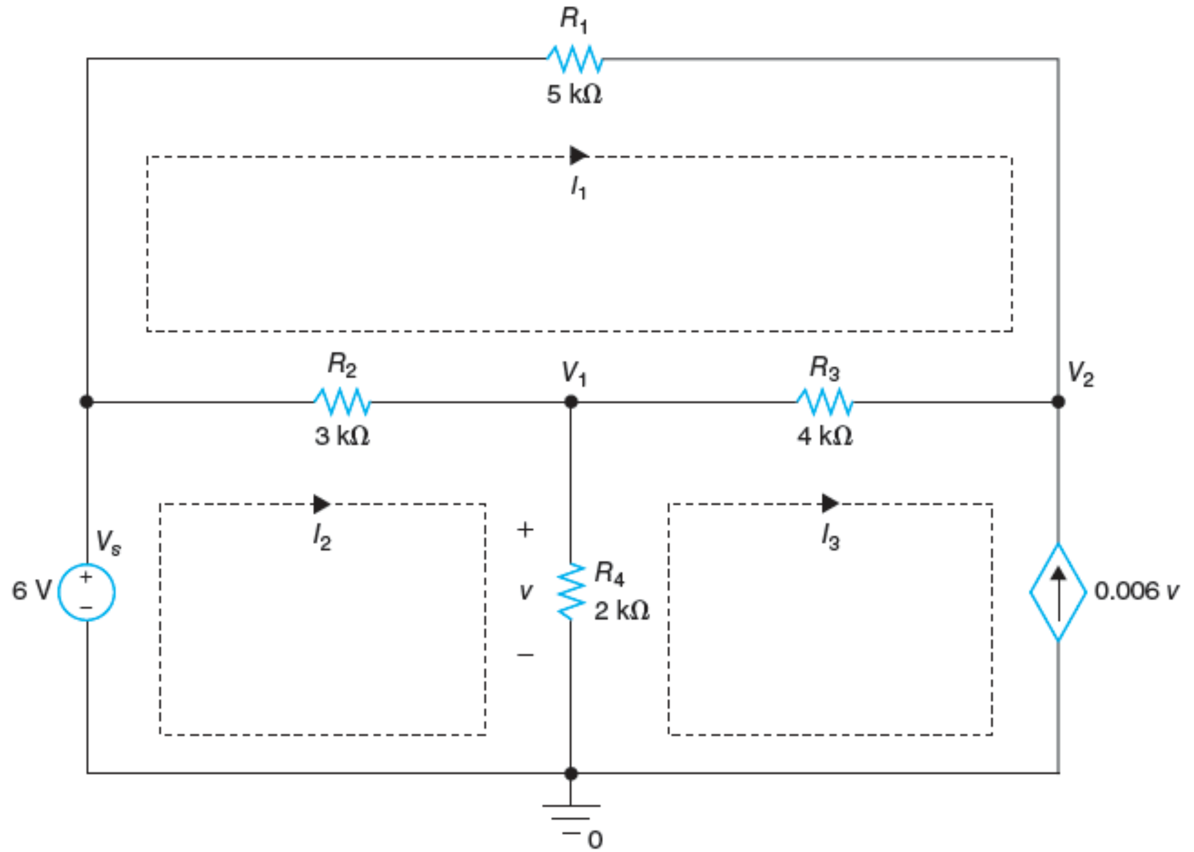
- $V_1 = 2000(I_2 - I_3)$,
- $I_3 = -0.006$
- $V_1 = -12(I_2 - I_3)$
- $12I_2 = 11I_3 \Rightarrow$ $I_3 = (12/11)I_2 = 1.09091I_2$

- Sum the voltage drops around mesh 1 ➔

- $5000I_1 + 4000(I_1 - I_3) + 3000(I_1 - I_2) = 0 \Rightarrow$ $12000I_1 - 3000I_2 - 4000I_3 = 0 \Rightarrow 12I_1 - 3I_2 - 4I_3 = 0 \Rightarrow 12I_1 - 3I_2 - 4(12/11)I_2 = 0 \Rightarrow$ $132I_1 - 81I_2 = 0 \Rightarrow I_2 = (132/81)\, I_1$ (1)
- Sum the voltage drops around mesh 2 ➔ $-6 + 3000(I_2 - I_1) + 2000(I_2 - I_3) = 0 \Rightarrow$ $-3I_1 + 5I_2 - 2I_3 = 0.006 \Rightarrow -33I_1 + 55I_2 - 24I_2 = 0.066 \Rightarrow -33I_1 + 31I_2 = 0.066$ (2)
- Add (1) and (2) ➔ $-33I_1 + 31(132/81)\, I_1 = 0.006$

- $I_1 = 3.767442$ mA, $I_2 = 6.139535$ mA
- $I_3 = 6.69767$mA, $V_1 = R_4(I_2 - I_3) = -1.11628$ V
- $V_2 = V_1 + R_3(I_1 - I_3) = -12.83721$ V

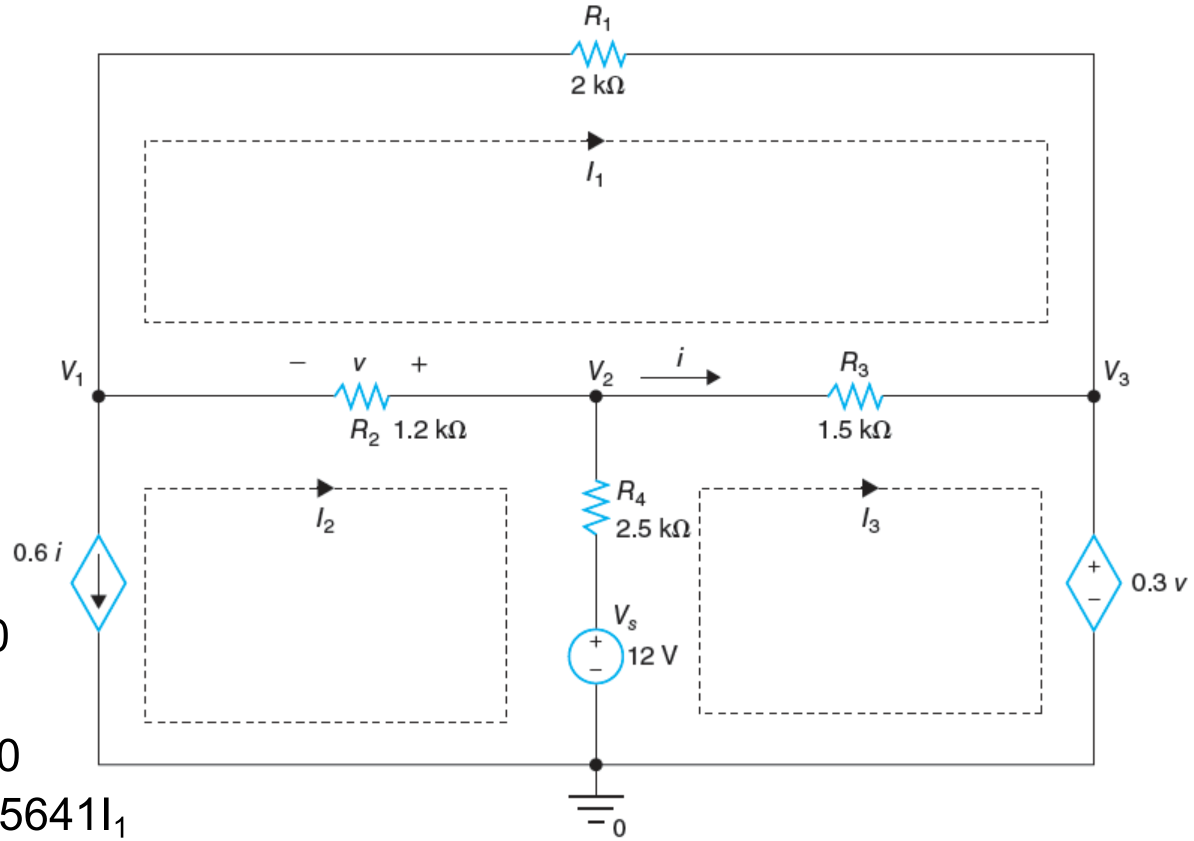
EXAMPLE 3.13



- $P_{R1} = (I_1)^2 R_1 = 70.9681$ mW
- $P_{R2} = (I_1 - I_2)^2 R_2 = 16.8805$ mW
- $P_{R3} = (I_1 - I_3)^2 R_3 = 34.3451$ mW
- $P_{R4} = (I_2 - I_3)^2 R_4 = 0.6230$ mW
- $P_{Vs} = - I_2 V_s = - 36.83721$ mW
- $P_{VCCS} = I_3 V_2 = - 85.97945$ mW
- $P_{R1} + P_{R2} + P_{R3} + P_{R4} + P_{Vs} + P_{VCCS} = 0$

EXAMPLE 3.14

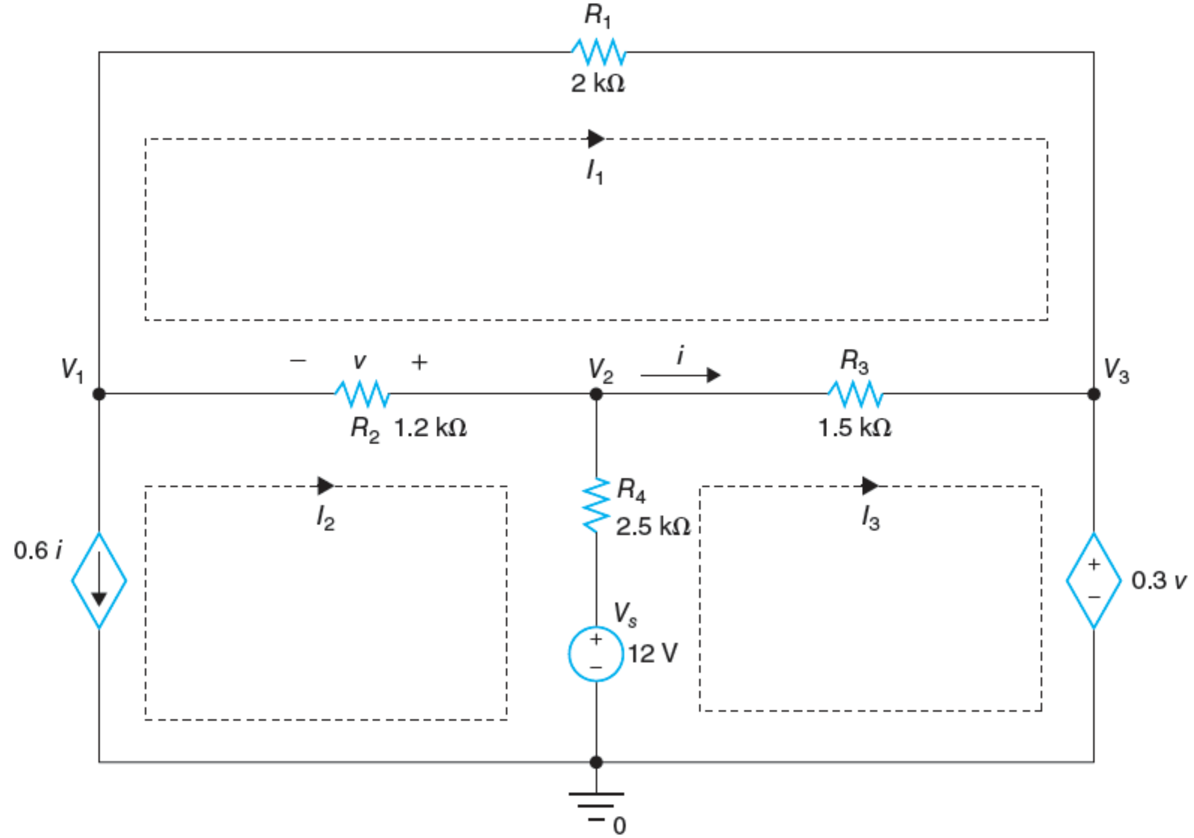
- Find I_1, I_2, I_3, V_1, V_2, V_3
- $I_2 = -0.6i = -0.6(I_3 - I_1)$



- Mesh 1 ➔
  $2I_1 + 1.5(I_1 - I_3) + 1.2(I_1 - I_2) = 0$
  $4.7I_1 - 1.2I_2 - 1.5I_3 = 0$
  $4.7I_1 + 0.72I_3 - 0.72I_1 - 1.5I_3 = 0$
  $3.98I_1 - 0.78I_3 = 0 \Rightarrow I_3 = 5.1025641I_1$
  $I_2 = -0.6I_3 + 0.6I_1 = -2.46153846I_1$

- Mesh 2 ➔
  $-12 + 2500(I_3 - I_2) + 1500(I_3 - I_1)$
  $+ 0.3\times1200(I_1 - I_2) = 0 \Rightarrow$
  $-1140I_1 - 2860I_2 + 4000I_3 = 12$
  $-1.14I_1 - 2.86I_2 + 4I_3 = 0.012$
  $-1.14I_1 - 2.86(-2.46153846I_1) + 4(5.1025641I_1) = 0.012$

EXAMPLE 3.14



- $I_1 = 456.0959\ \mu A$
- $I_2 = -2.46153846 I_1 = -1.1227\ mA$
- $I_3 = 5.1025641 I_1 = 2.3273\ mA$
- $V_2 = V_s - R_4(I_3 - I_2) = 3.3751\ V$
- $V_1 = V_2 - R_2(I_1 - I_2) = 1.4806\ V$
- $V_3 = V_1 - R_1 I_1 = 0.5684\ V$

Supermesh

- If there is a current source that is a common branch between two different meshes, we do not know the voltage drop across the current source.
- Let the unknown voltage across the current source be *v.*
- Write the mesh equation for each mesh and add the two equations to remove the unknown voltage *v.*
- The voltage *v* is removed when added because in one equation, the voltage drop across the current source is *v* and in the other equation, the voltage drop across the current source is -*v.*

Supermesh

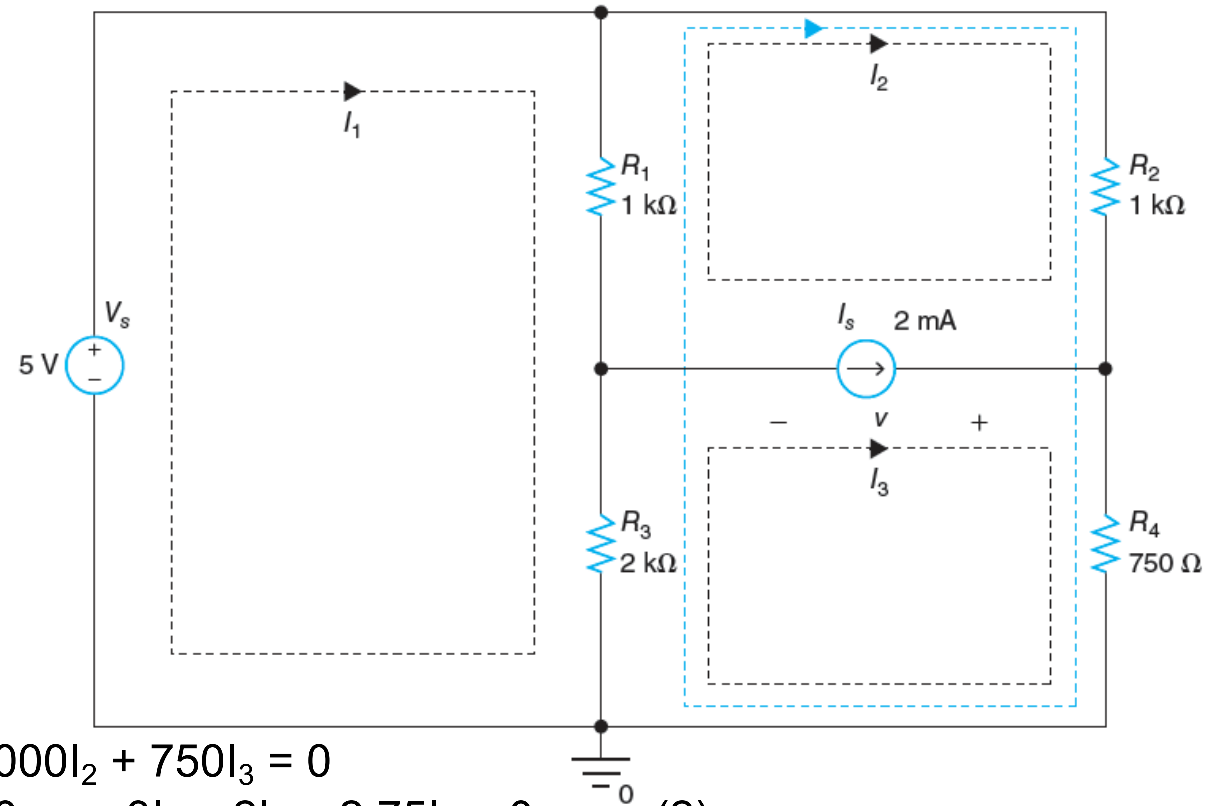
- Alternatively, the sum of the two equations can be obtained directly by defining a supermesh consisting of the two meshes, excluding the current source.

- When the voltage drops around the supermesh are added, we get the same equation that we obtain by adding the two equations.

- The extra equation needed to find the mesh currents is obtained by representing the current of the current source by the difference of the two mesh currents.

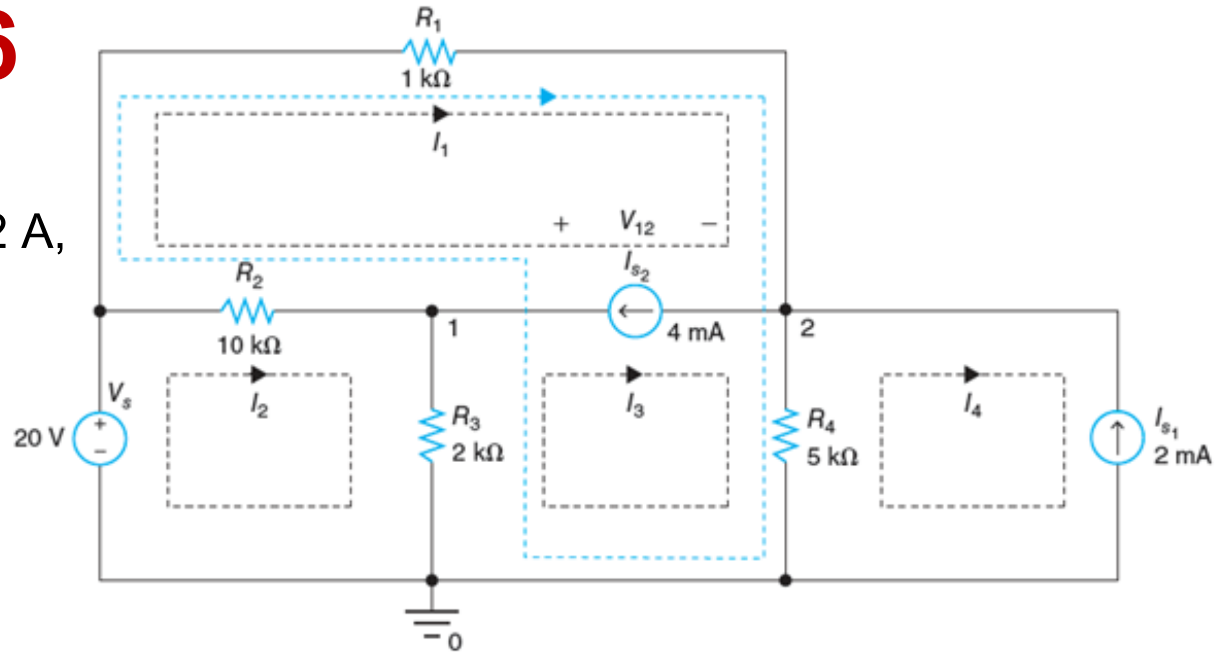
- The current of the current source is obtained by subtracting the mesh current pointing in the opposite direction from the mesh current pointing in the same direction as the current source.

EXAMPLE 3.15



- Find I_1, I_2, I_3, v
- $I_3 - I_2 = I_s = 2$ mA=0.002 A
  $\Rightarrow I_3 = I_2 + 0.002$ (1)
- Sum the voltage drops around supermesh➔
- $2000(I_3 - I_1) + 1000(I_2 - I_1) + 1000I_2 + 750I_3 = 0$
  $-3000I_1 + 2000I_2 + 2750I_3 = 0 \Rightarrow -3I_1 + 2I_2 + 2.75I_3 = 0$ (2)
- (1) to (2) ➔ $-3I_1 + 4.75I_2 = -0.0055$ (3)
- Sum the voltage drops around mesh 1➔ $-5 + 1000(I_1 - I_2) + 2000(I_1 - I_3) = 0$
  $3000I_1 - 1000I_2 - 2000I_3 = 5$
  $3I_1 - I_2 - 2I_3 = 0.005$
  $3I_1 - I_2 - 2(I_2 + 0.002) = 0.005$
  $3I_1 - 3I_2 = 0.009$ (4)
- (3) + (4) ➔ $1.75I_2 = 0.0035 \Rightarrow I_2 = 2$ mA
- $I_1 = (3I_2 + 0.009)/3 = 5$ mA, $I_3 = 4$ mA
- $v = R_4I_3 - R_3(I_1 - I_3) = 1$ V

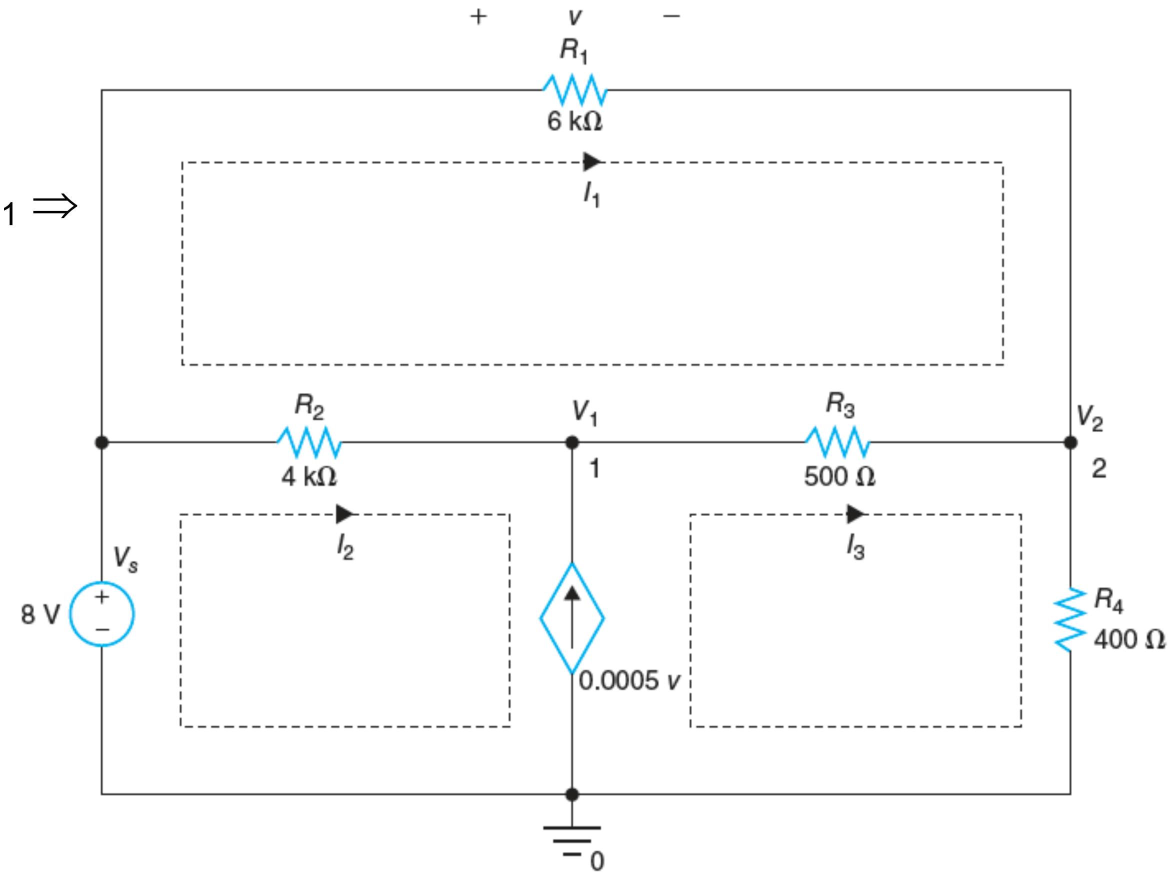
EXAMPLE 3.16



- Find I_1, I_2, I_3, I_4.
- $I_4 = -I_{S1} = -2$ mA $= -0.002$ A,
  $I_1 - I_3 = I_{S2} = 0.004 \Rightarrow$
  $I_3 = I_1 - 0.004$ (1)
- Supermesh➔
  $1000I_1 + 5000(I_3 - I_4) + 2000(I_3 - I_2) + 10000(I_1 - I_2) = 0$
  $11000I_1 - 12000I_2 + 7000I_3 = -10 \Rightarrow 18000I_1 - 12000I_2 = 28 - 10 = 18 \Rightarrow$
  $18I_1 - 12I_2 = 0.018$ ➔ $3I_1 - 2I_2 = 0.003$ (2)
- Mesh 2 ➔ $-20+10000(I_2 - I_1)+2000(I_2-I_3) = 0$ ➔ $-10000I_1 + 12000I_2 - 2000I_3 = 20$
  $-12000I_1 + 12000I_2 = 20 - 8$ ➔ $-2I_1 + 2I_2 = 0.002$ (3)
- (2)+(3) ➔ $I_1 = 5$ mA, $I_3 = 1$ mA
- From (3) ➔ $I_2 = I_1 + 0.001 = 6$ mA

EXAMPLE 3.17

- Find I_1, I_2, I_3, V_1, V_2.
- $I_3 - I_2 = 0.0005v = 0.0005 \times 6000 I_1 \Rightarrow$
  $I_3 = 3I_1 + I_2 \quad (1)$



Supermesh ➔

$$-8 + 4000(I_2 - I_1) + 500(I_3 - I_1) + 400I_3 = 0$$
$$-4500I_1 + 4000I_2 + 900I_3 = 8 \Rightarrow$$
$$-4500I_1 + 4000I_2 + 900(3I_1 + I_2) = 8$$
$$-4500I_1 + 4000I_2 + 900(3I_1 + I_2) = 8$$
$$-1800I_1 + 4900I_2 = 8$$
$$-1.8I_1 + 4.9I_2 = 0.008 \qquad (2)$$

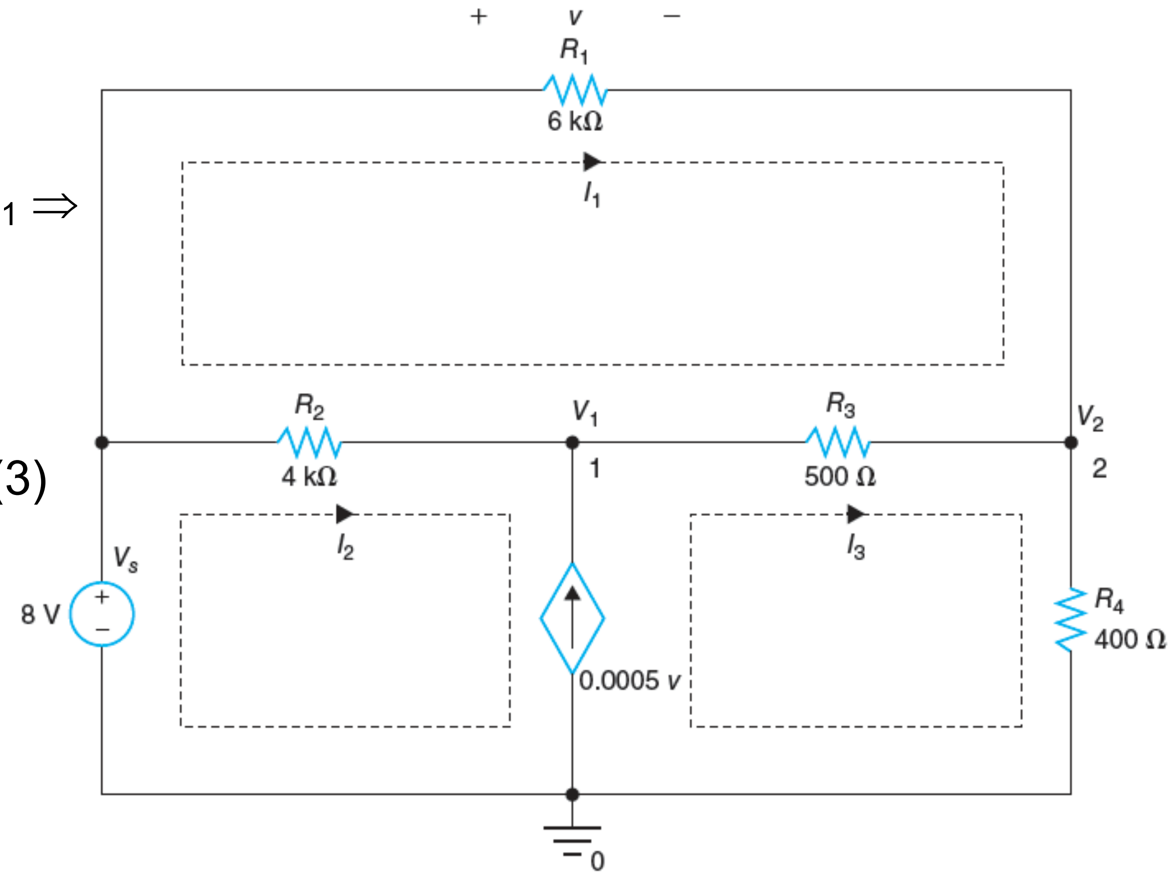
Mesh 1 ➔

$$6000I_1 + 500(I_1 - I_3) + 4000(I_1 - I_2) = 0$$
$$10500I_1 - 4000I_2 - 500I_3 = 0$$
$$10500I_1 - 4000I_2 - 500(3I_1 + I_2) = 0$$
$$9000I_1 - 4500I_2 = 0 \Rightarrow I_2 = 2I_1 \ (3)$$

EXAMPLE 3.17

- Find I_1, I_2, I_3, V_1, V_2.
- $I_3 - I_2 = 0.0005v = 0.0005 \times 6000 I_1 \Rightarrow$
  $I_3 = 3I_1 + I_2$ (1)

- $-1.8I_1 + 4.9I_2 = 0.008$ (2)
- $9000I_1 - 4500I_2 = 0 \Rightarrow I_2 = 2I_1$ (3)



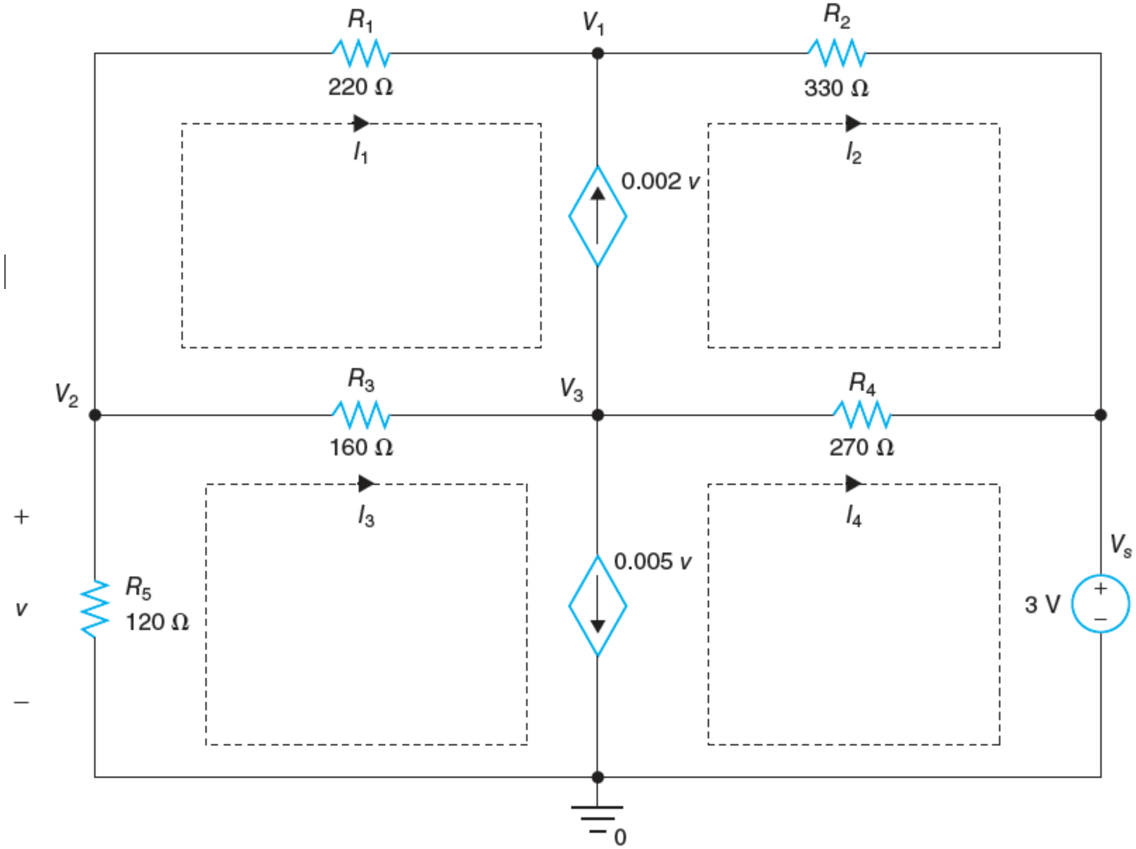
- (3) to (2) ➔ $-1.8I_1 + 9.8I_1 = 0.008 \Rightarrow I_1 = 1$ mA

- $I_2 = 2$ mA, $I_3 = 5$ mA, $V_2 = R_4 I_3 = 2$ V

- $V_1 = V_s - R_2(I_2 - I_1) = 4$ V

EXAMPLE 3.18

- Find I_1, I_2, I_3, I_4, V_1, V_2, V_3
- $I_2 - I_1 = 0.002(-120I_3) = -0.24I_3$
- $I_3 - I_4 = 0.005(-120I_3) \Rightarrow I_4 = 1.6I$



Supermesh (top):

$220I_1 + 330I_2 + 270(I_2 - I_4) + 160(I_1 - I_3) = 0$

$380I_1 + 600I_2 - 160I_3 - 270I_4 = 0$

$380I_1 + 600(I_1 - 0.24I_3) - 160I_3 - 270(1.6I_3) = 0$

$980I_1 - 736I_3 = 0 \Rightarrow I_3 = 1.3315I_1$

Supermesh (bottom):

$120I_3 + 160(I_3 - I_1) + 270(I_4 - I_2) + 3 = 0$

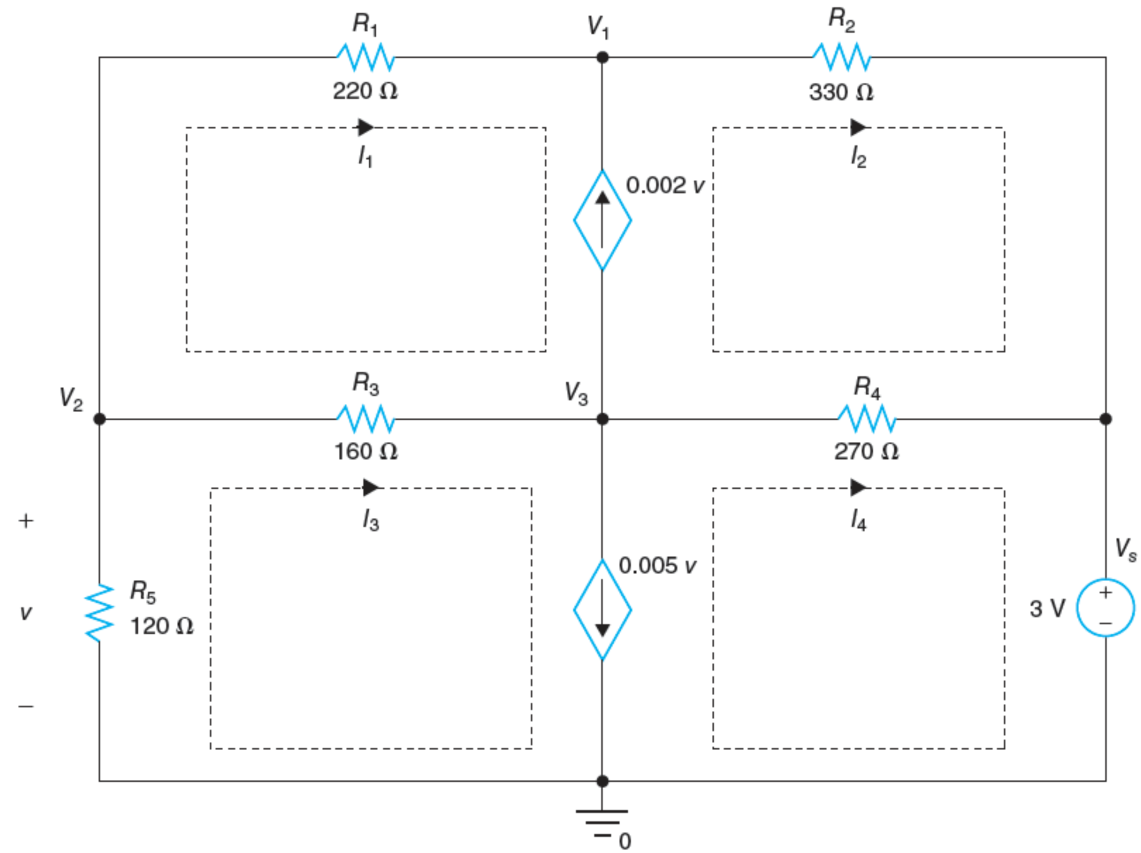
$-160I_1 - 270I_2 + 280I_3 + 270I_4 = -3$

$-160I_1 - 270(I_1 - 0.24I_3) + 280I_3 + 270(1.6I_3) = -3$

$-430I_1 + 776.8I_3 = -3$

$-430I_1 + 776.8(1.3315I_1) = -3$

EXAMPLE 3.18



- $I_1 = -\,4.9642$ mA
- $I_3 = 1.33152174 I_1 = -\,6.60995$ mA
- $I_2 = I_1 - 0.24 I_3 = -\,3.3778$ mA
- $I_4 = 1.6 I_3 = -\,10.57592$ mA
- $V_1 = V_s + R_2 I_2 = 1.8853$ V
- $V_2 = V_1 + R_1 I_1 = 0.7932$ V
- $V_3 = V_s - R_4(I_2 - I_4) = 1.0565$ V

Summary

- Nodal analysis is a method of finding all the unknown node voltages of a circuit.
- The method is based on Kirchhoff's current law (KCL): The sum of the currents leaving a node is zero.
- For each node whose voltage is unknown, we can write a node-voltage equation by summing the currents leaving the node.
- This is tantamount to writing KCL at each node. The currents leaving the node through resistors can be found by applying Ohm's law.
- A solution to the node voltages is obtained by solving the set of node-voltage equations.
- Once all the node voltages are computed, the current in each branch can be found using Ohm's law.
- If there is a voltage source in a circuit between two nodes whose voltages are unknown, we do not know the current through the voltage source, and it is not possible to write the node equations for the two nodes that include the voltage source. In this case, combine the two nodes to form a supernode. We can then write the node equation for this supernode.
- One additional equation, commonly referred to as a constraint equation relating the two node voltages, can be obtained by representing the voltage source as a potential drop or as a potential rise between the two nodes.

Summary (Continued)

- A mesh is a loop that does not contain any other loops. For each mesh, define a mesh current as the current flowing around the mesh in the clockwise direction.
- The mesh analysis is based on KVL: The sum of voltage drops around a mesh is zero. For each mesh, sum the voltage drops around the mesh in the clockwise direction and set that equal to zero. The voltage drop across a resistor is the product of the resistance and the net current (physical current) through the resistor. If the resistor is not shared by another mesh, the voltage drop is the product of the resistance and the mesh current. If the resistor is shared by another mesh, the voltage drop is the product of the resistance and the difference in mesh currents.
- If there is a current source that is a common branch between two different meshes, we do not know the voltage drop across the current source. Define a supermesh consisting of the two meshes, excluding the current source. When the voltage drops around the supermesh are added, we get the same equation that we obtain by adding the two equations for two meshes.
- The extra equation needed to find the mesh currents is obtained by representing the current of the current source by the difference of the two mesh currents. The current of the current source is obtained by subtracting the mesh current pointing in the opposite direction from the mesh current pointing in the same direction as the current source.
- What we will study next