POWER ENGINEERING

#03 THREE-PHASE AC POWER SYSTEMS (1)

Semester 1 – 2021/2022

University of Glasgow
电子科技大学
University of Electronic Science and Technology of China

Outline

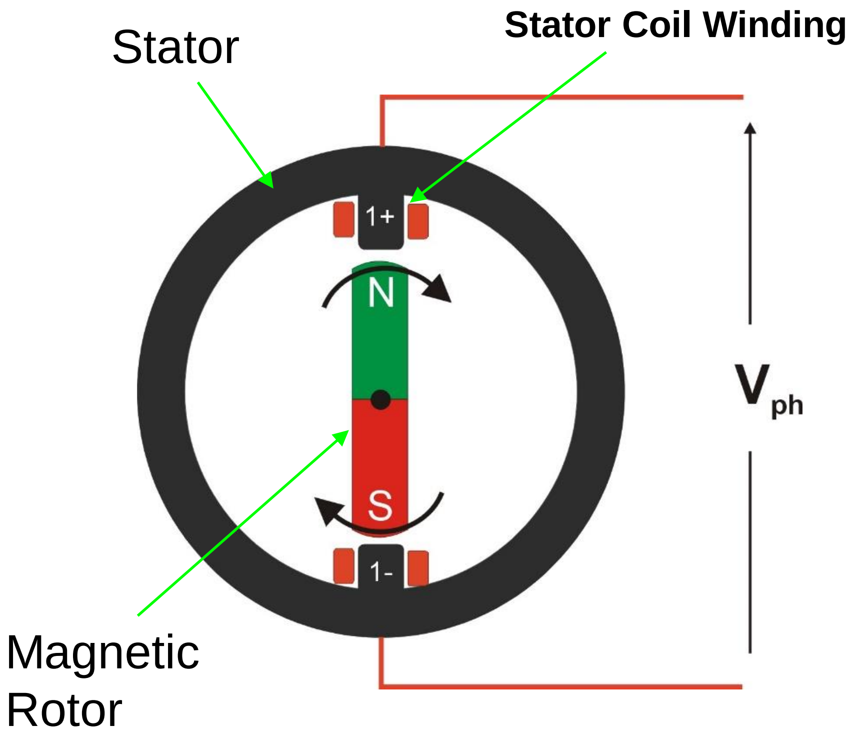
- Benefits of three-phase power systems
- 3-phase generator: phase voltages and line voltages
- 3 Phase Transmission Lines
- **Balanced STAR** connected 3 phase RCL load
  - 3-phase 3-wire system
  - 3-phase 4-wire systems
- **Balanced DELTA** connected 3 phase RCL load
- Power Measurement
  - 3 Wattmeter Method
  - 2 Wattmeter Method
- Modern Digital Sampling Power Meters

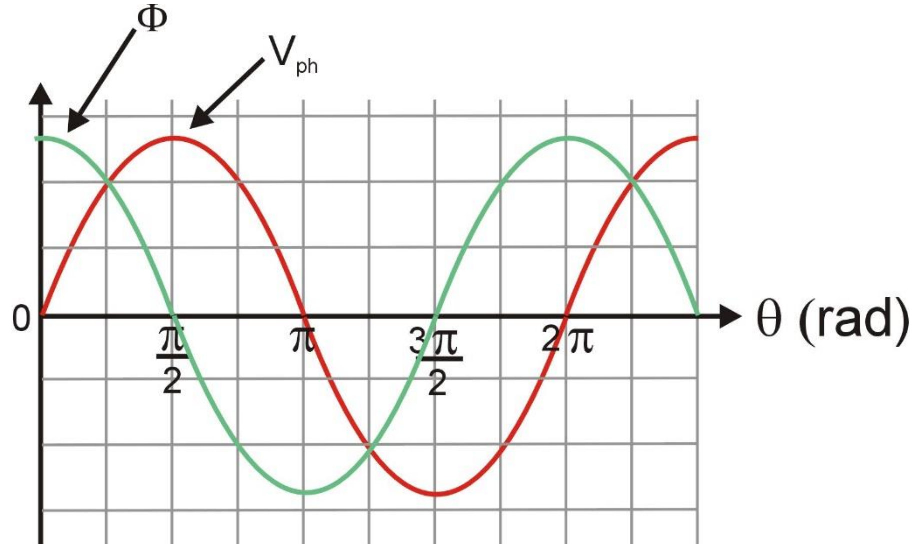
BENEFITS OF THREE-PHASE POWER SYSTEMS

Three-phase power systems are the de facto standard in the industry.

- The ‘workhorse’ of industry; the 3 phase induction motor requires a 3 phase AC power supply
- A balanced 3 phase system will lead to **constant instantaneous power demand** on the generator – results in a smoother running generator
- A 3 phase generator has far better energy density compared to a single phase machine, hence it is smaller for a given output power (see next slides)
- The weight of the conductors and other components in a three-phase system is much lower than in a single-phase system delivering the same amount of power.

Single Phase AC Generator



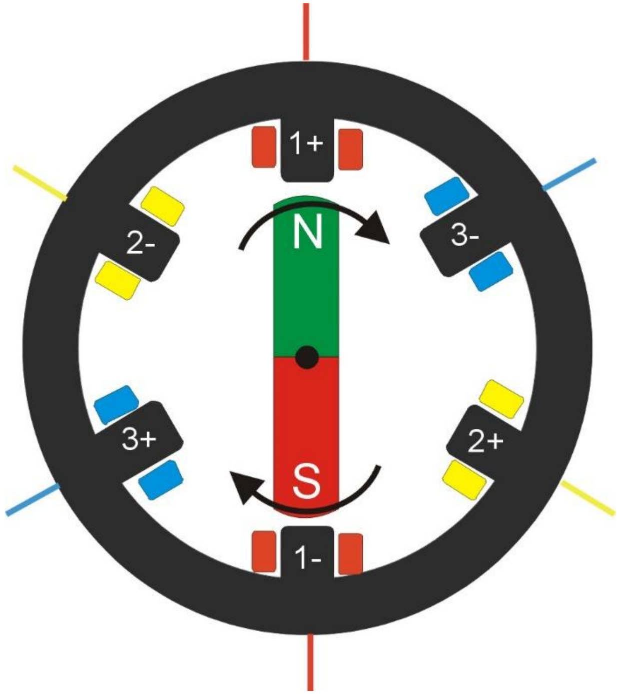


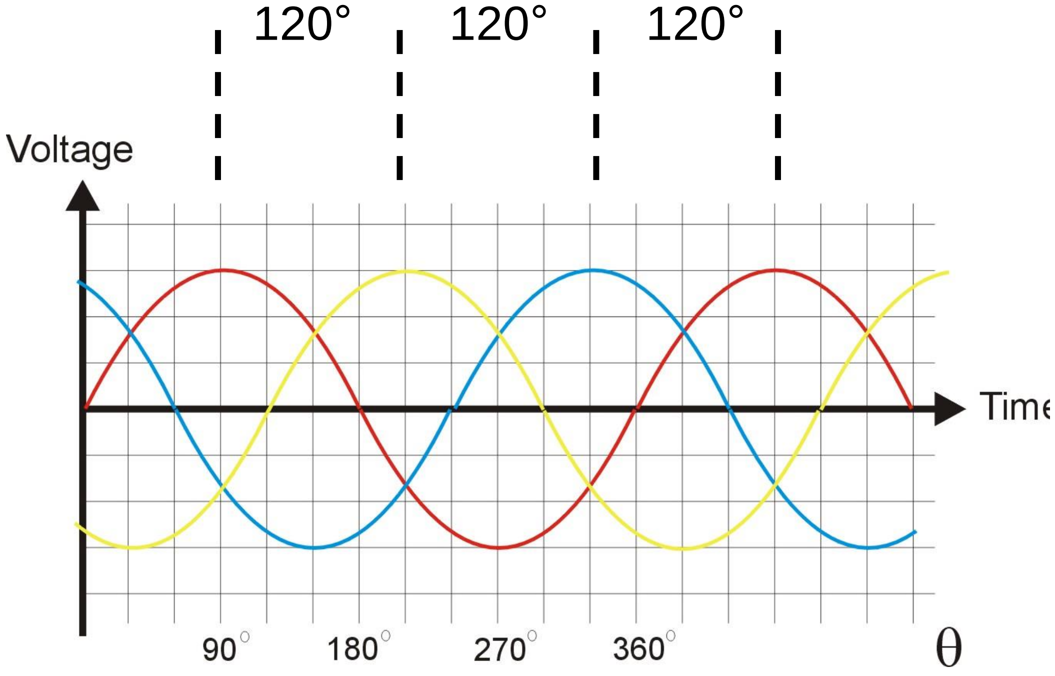
Voltage (V_{ph}) induced in Stator Winding Coils (1+, 1-) :

$$\boxed{v_{ph} = \frac{N \cdot d\phi}{dt}}$$

where *N* is the number of the stator coils turns and ϕ is the rotor magnet flux.

THREE PHASE AC GENERATOR

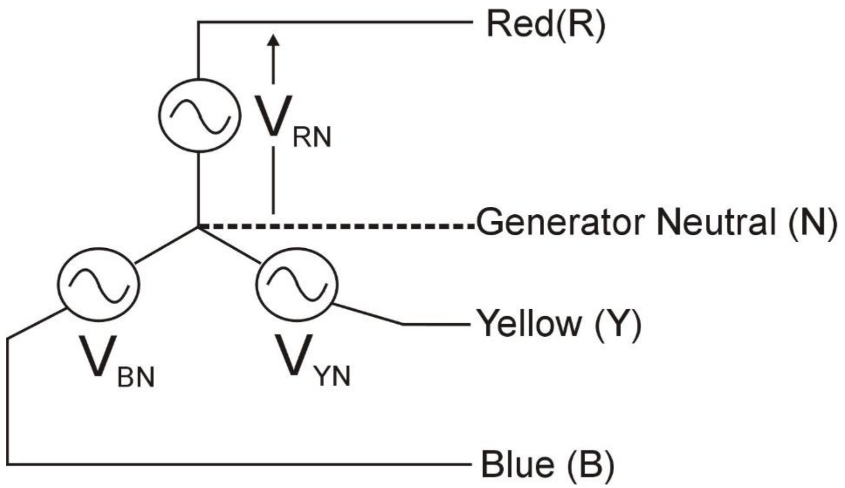




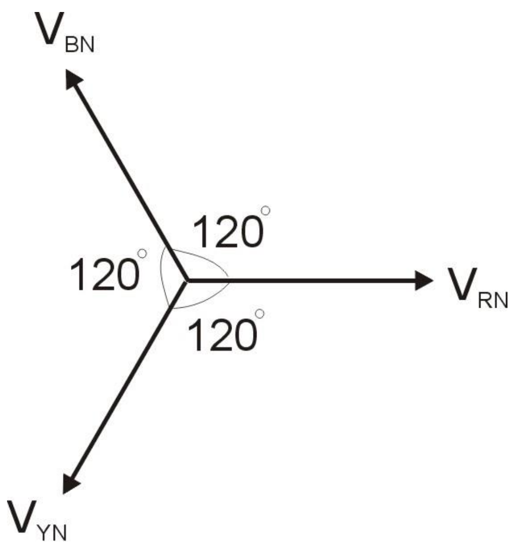
Now have 3 Phase windings on the Stator 120° apart

Countries use different conventions for naming the 3 PHASE voltages. We will adopt the (old!) UK convention of RED, YELLOW & BLUE PHASES

3-Phase AC Generator: Phase Voltages



Phase Voltages



Phasor Diagram

$$V_{RN} = V_{pk} \cos \theta$$

$$V_{YN} = V_{pk} \cos\left(\theta - 2\pi / 3\right)$$

$$V_{BN} = V_{pk} \cos\left(\theta + 2\pi / 3\right)$$

$$V_{RN} + V_{YN} + V_{BN} = 0$$

Complex Form (Polar):

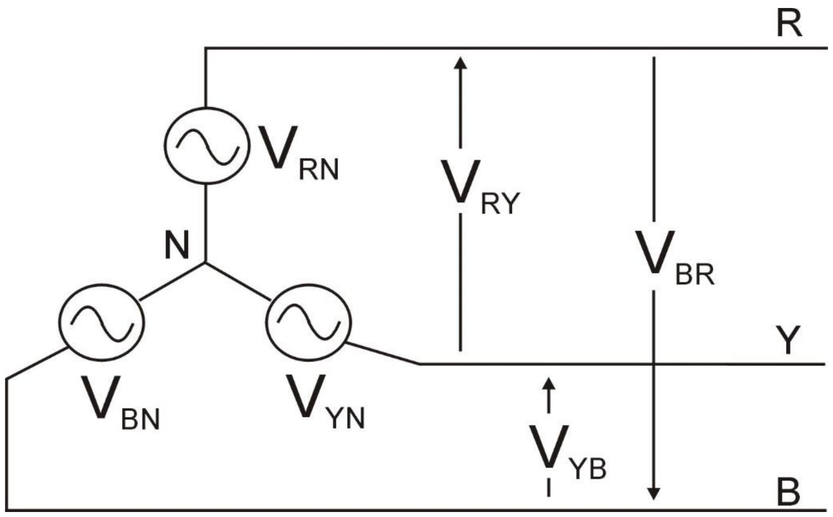
$$V_{RN} = V_{ph} \angle 0^o$$

$$V_{YN} = V_{ph} \angle -120^o$$

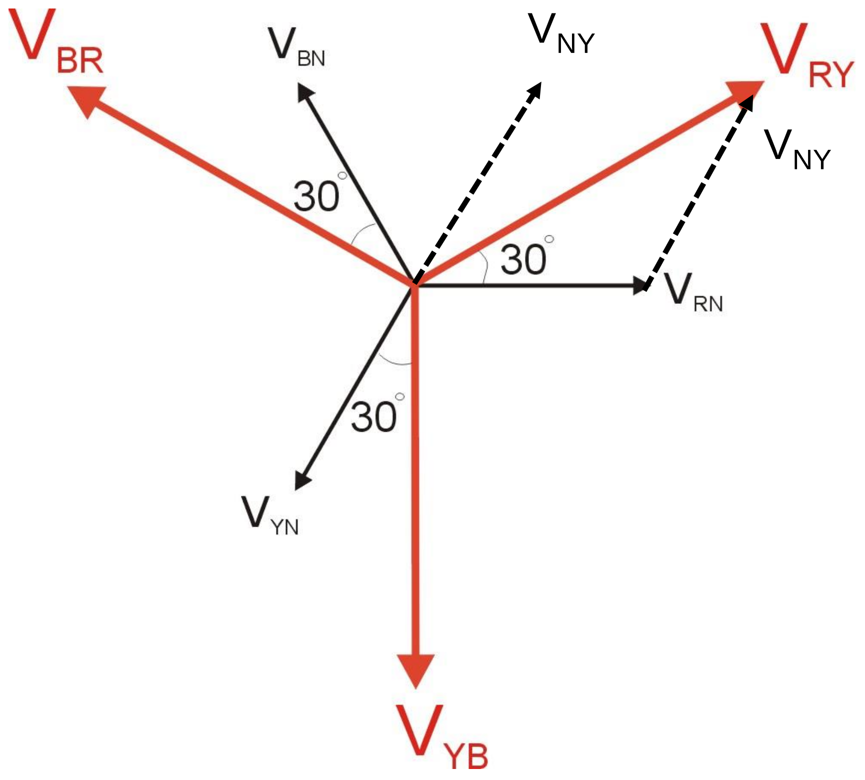
$$V_{BN} = V_{ph} \angle 120^o$$

Note: all 3 Phase Voltages have the same rms magnitude V_{ph}

3-Phase AC Generator: Line Voltages



Phase and Line Voltages



Phasor Diagram

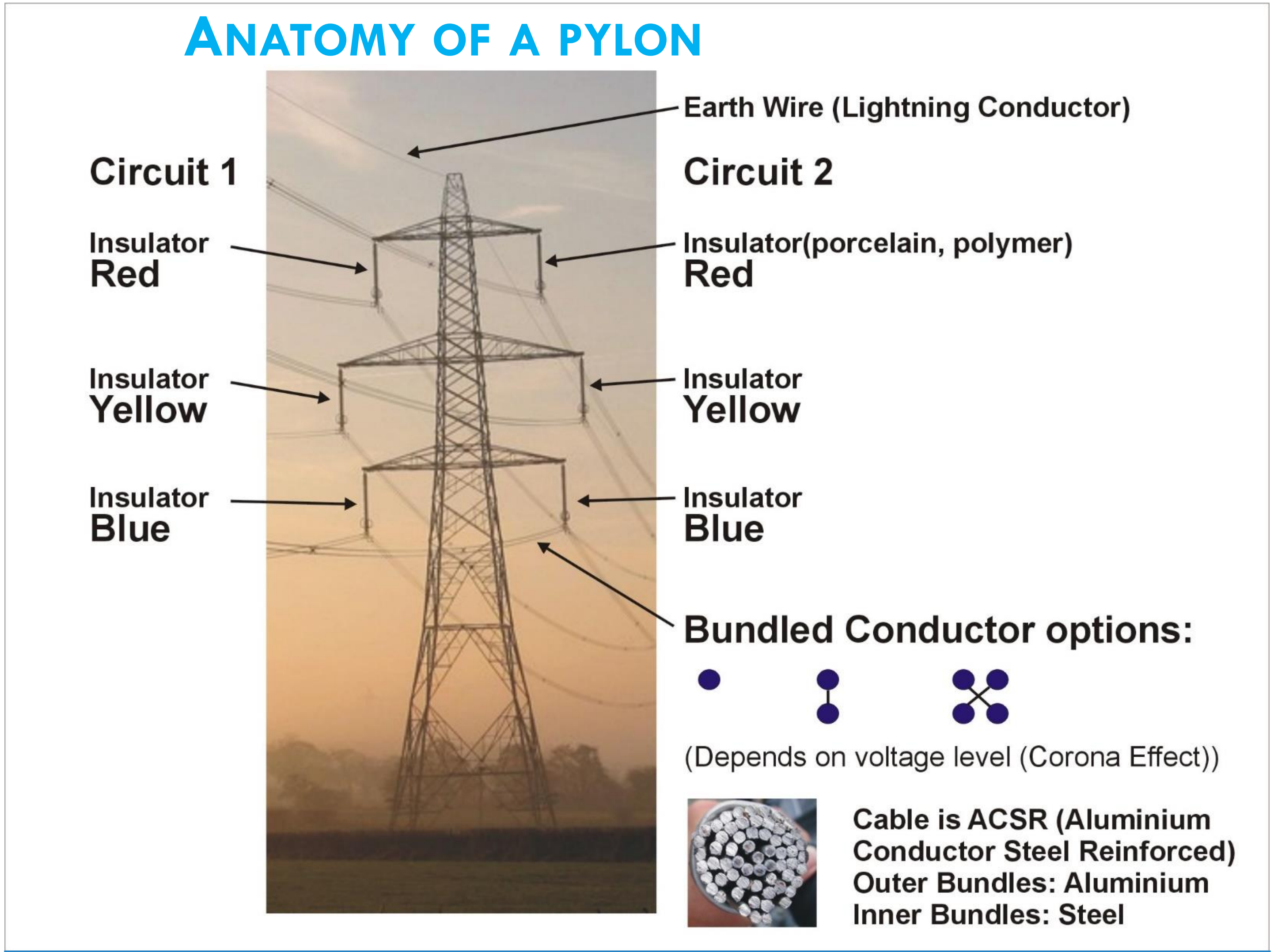
$$V_{RY}=V_{RN}+V_{NY}=V_{RN}-V_{YN}$$

$$V_{BR}=V_{RN}+V_{NB}=V_{RN}-V_{BN}$$

$$V_{YB}=V_{YN}+V_{NB}=V_{YN}-V_{BN}$$

$$V_{RY}=\sqrt{3}\cdot V_{ph}\angle 30^o \quad V_{BR}=\sqrt{3}\cdot V_{ph}\angle 150^o \quad V_{YB}=\sqrt{3}\cdot V_{ph}\angle -90^o$$

3-Phase Transmission Lines



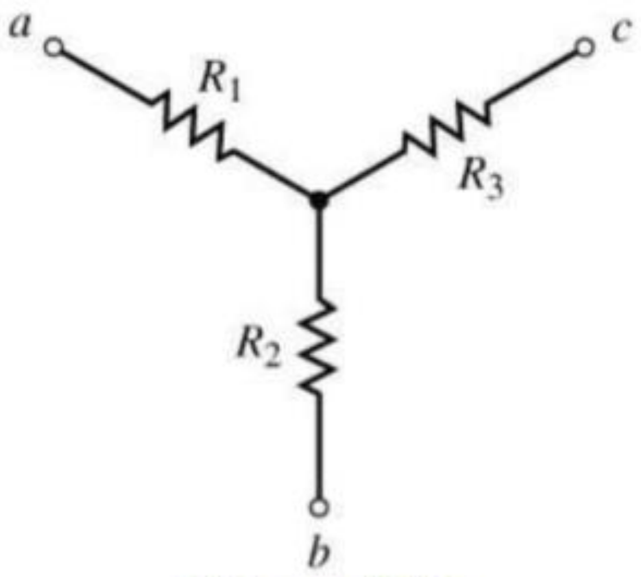
If you want to find out more about pylons then check out the Pylon Appreciation Society at **http://www.pylons.org/**

3-PHASE TRANSMISSION LINES

Some interesting facts!

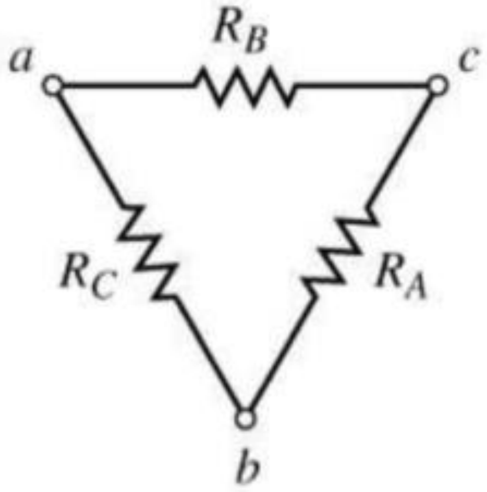
- The ‘standard’ UK pylon was chosen by Sir Reginald Blomfield (a leading architect) in 1928
- The tallest pylon in the world is in China. The Yangtze River crossing pylon is 346.5m high
- The Beauly-Denny transmission line upgrade in Scotland consists of 600 towers with an average height of 53m
- The UK National Grid is made up of 440kV, 275kV, 132kV, 110kV, 33kV and 11kV transmission lines

- AC three-phase generators can be connected in multiple configurations. The first we will discuss is the **Y-Connected Generator**…



Wye (Y)

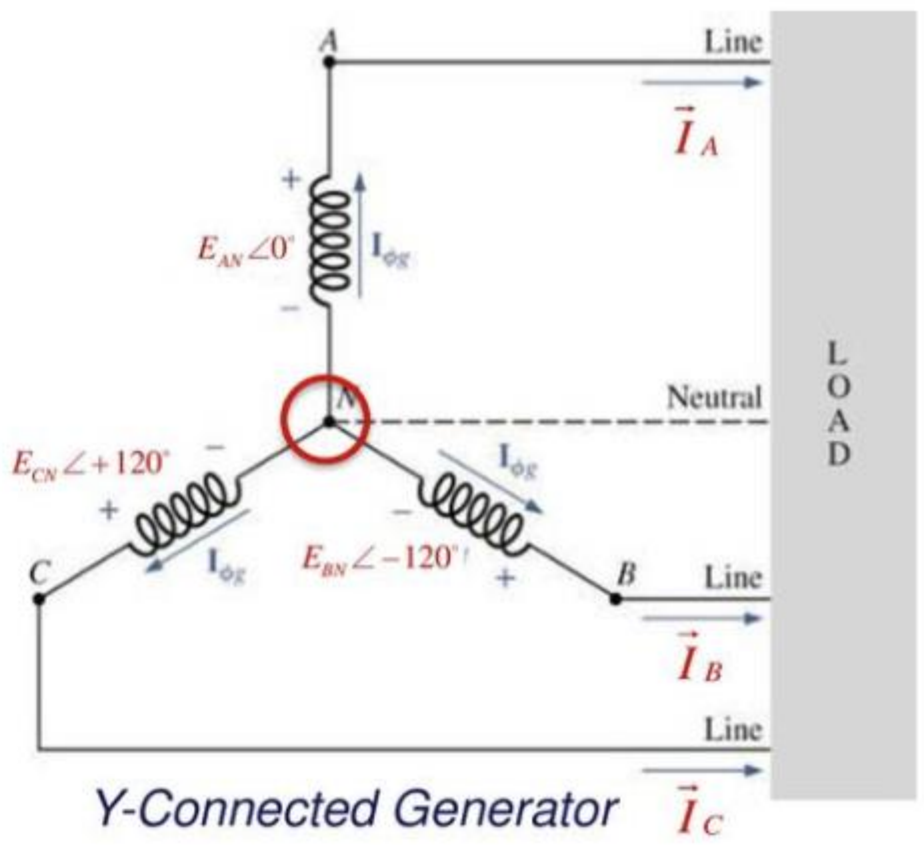
Has a grounding advantage for varying loads



Delta (Δ)

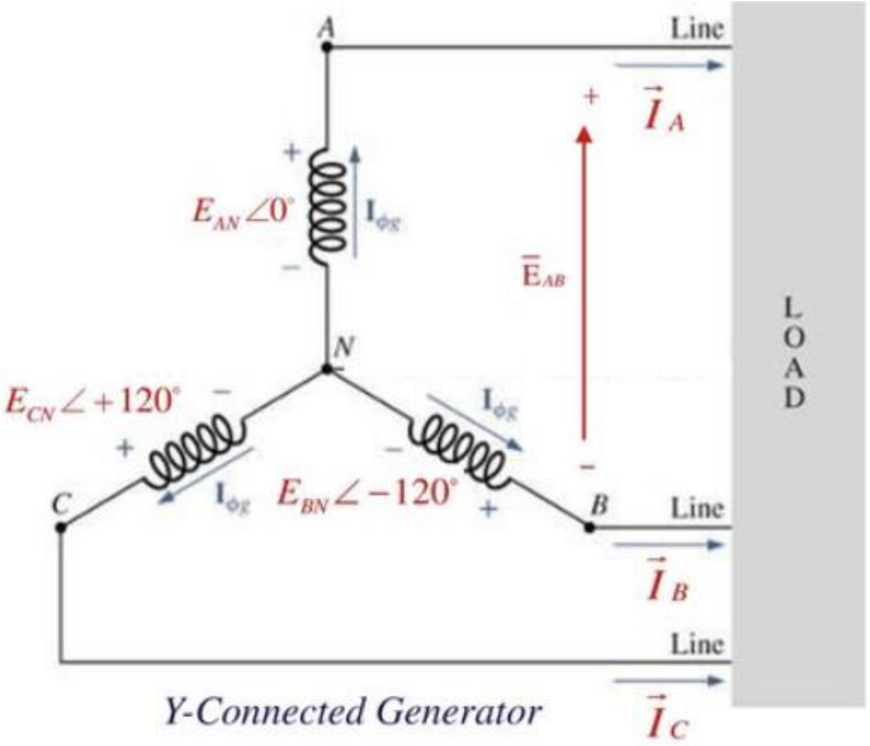
Advantage for dedicated loads where grounding not as important

- If the three phase terminals are connected together at ***N***, the generator is referred to as a **Y-connected three-phase generator.**
- Note that the negative (-) terminals are connected together at ***N***, *which is the neutral.*
- The three phase Y generator is connected to the load via the three lines labeled with a corresponding phasor current (I_A, I_B and I_C).



Y-Connected Generator

- However, the phase for $\boldsymbol{E_{AN}}$ is rarely fixed at zero. So for any $\boldsymbol{\Theta}$ relationship it can be shown that for a balanced Y-Connected Generator the magnitude of line-to-line voltage is 1.732 ($\sqrt{3}$) times the magnitude of the phase voltage.
- Line to line voltage leads the phase voltage by 30°.

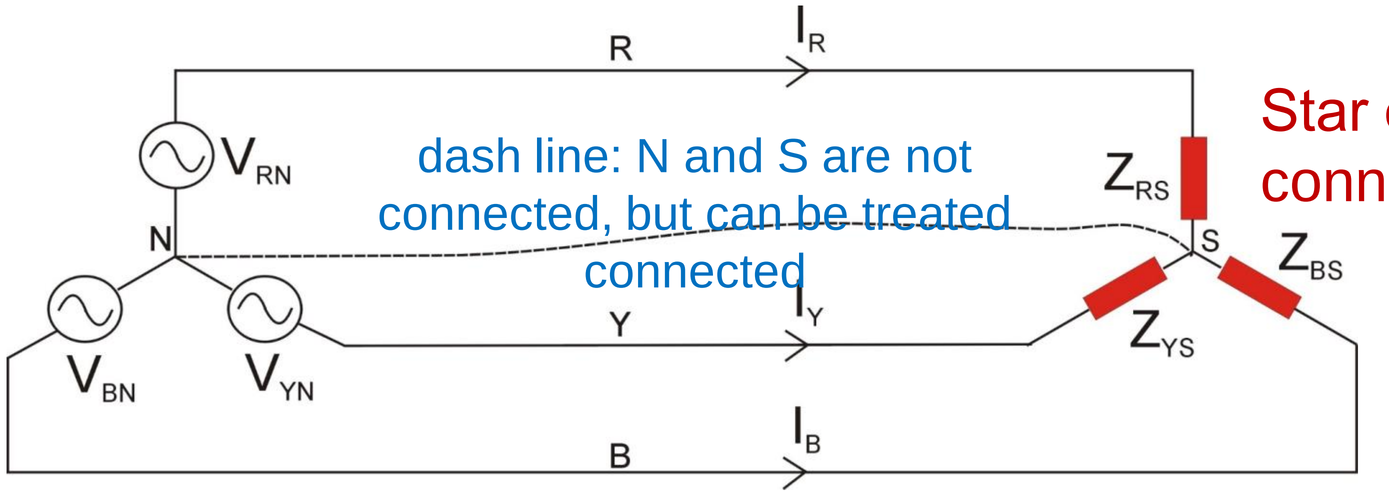


Y-Connected Generator

Phase Voltages		Line Voltages
$\vec{\mathbf{E}}_{AN} = E\angle(\theta + 0^\circ)$	$\Longrightarrow$	$\bar{\mathbf{E}}_{AB} = \sqrt{3}E\angle(30^\circ + \theta)$
$\vec{\mathbf{E}}_{BN} = E\angle(\theta - 120^\circ)$	$\Longrightarrow$	$\bar{\mathbf{E}}_{BC} = \sqrt{3}E\angle(-90^\circ + \theta)$
$\vec{\mathbf{E}}_{CN} = E\angle(\theta + 120^\circ)$	$\Longrightarrow$	$\bar{\mathbf{E}}_{CA} = \sqrt{3}E\angle(150^\circ + \theta)$

BALANCED STAR connected load:

3-PHASE 3-WIRE SYSTEM



Star or "Y" connected load

1. **'Balanced'** means that the load Impedances are equal:
   $Z_{RS} = Z_{YS} = Z_{BS}$

2. I_R, I_Y and I_B are termed the **LINE** currents, and **are 120° apart and equal in magnitude**.

3. If the load is balanced, then the load **STAR** point (**S**) is at the same voltage as the Generator Neutral (**N**): $V_{SN} = 0$

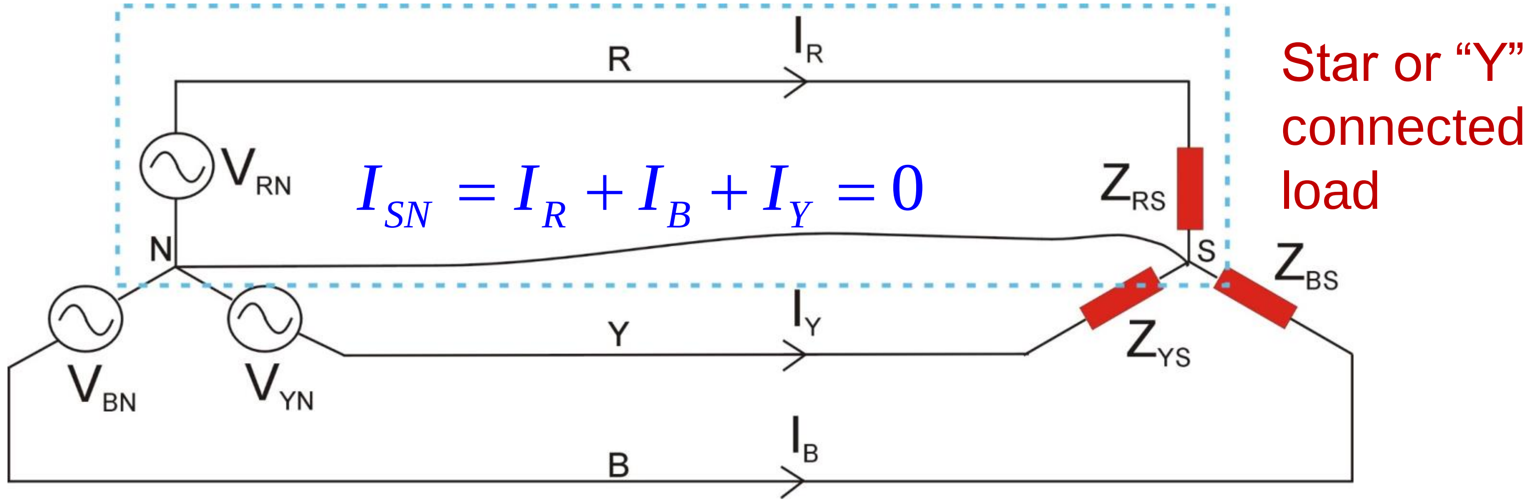
$$I_R = \frac{V_{RN}}{Z_{RS}} = I_L \angle -\phi$$

$$I_B = \frac{V_{BN}}{Z_{BS}} = I_L \angle -\phi + 120°$$

$$I_Y = \frac{V_{YN}}{Z_{YS}} = I_L \angle -\phi - 120°$$

Balanced STAR connected load:

3-Phase 4-Wire System



1. The balanced 3-Phase 4-Wire System can be decomposed into three separated Single-Phase Systems:

$$I_R = \frac{V_{RN}}{Z_{RS}} = I_L \angle -\phi$$

$$I_B = \frac{V_{BN}}{Z_{BS}} = I_L \angle -\phi + 120°$$

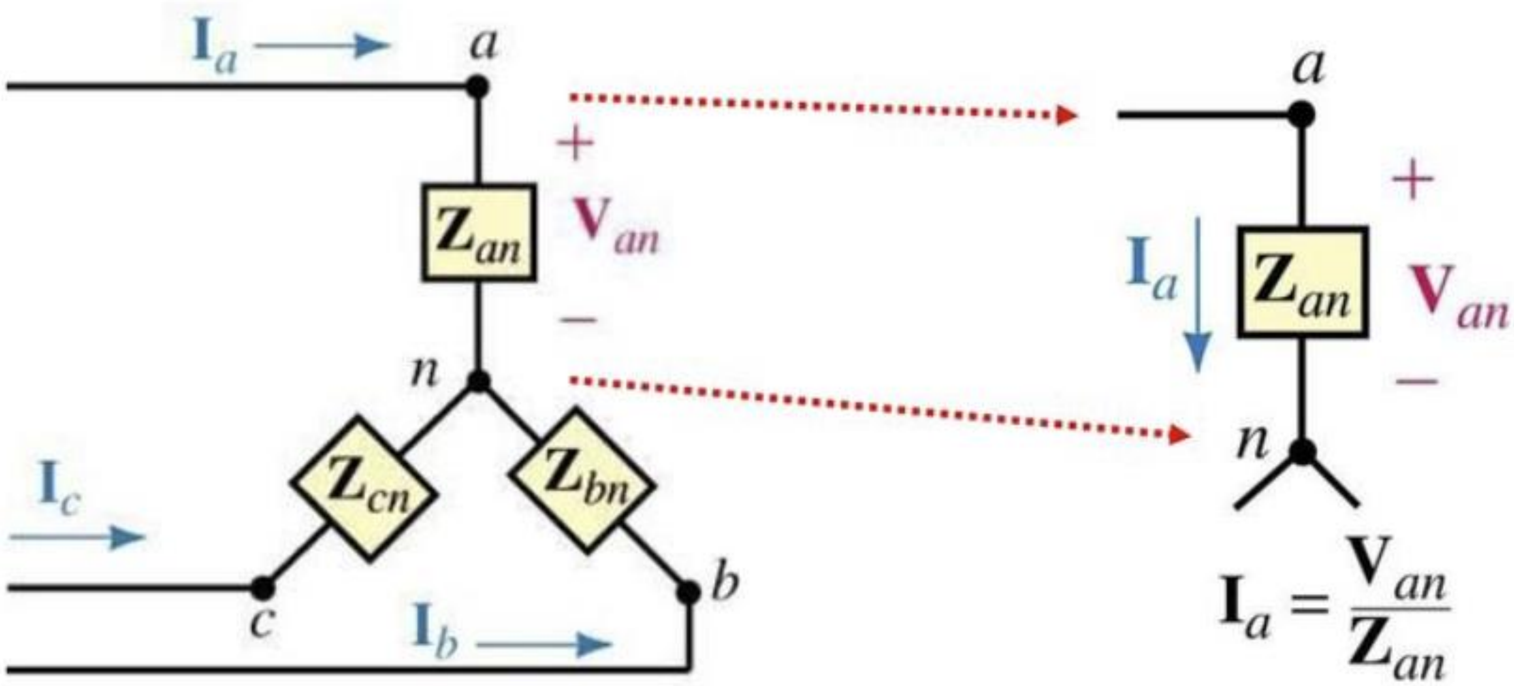
$$I_Y = \frac{V_{YN}}{Z_{YS}} = I_L \angle -\phi - 120°$$

2. **I_R, I_Y** and **I_B** are **120° apart and equal in magnitude**, and

$$I_{SN} = I_R + I_B + I_Y = 0$$

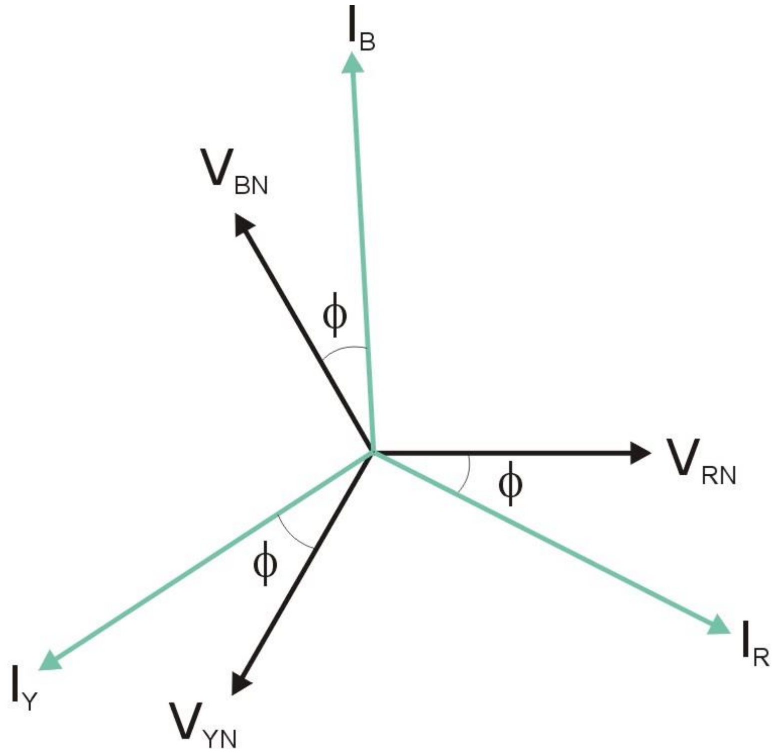
- For Y loads, line current and phase current are the same.

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_{an}} \qquad \mathbf{I}_b = I\angle(\theta - 120°) \qquad \mathbf{I}_c = I\angle(\theta + 120°)$$



Line current **Phase current**

BALANCED STAR connected load



Phasor Diagram

Power calculations for each phase :

$$S = |V_{RN}| \cdot |I_R| = V_{ph} \cdot I_L$$

$$P = |V_{RN}| \cdot |I_R| \cdot \cos\phi = V_{ph} \cdot I_L \cdot \cos\phi$$

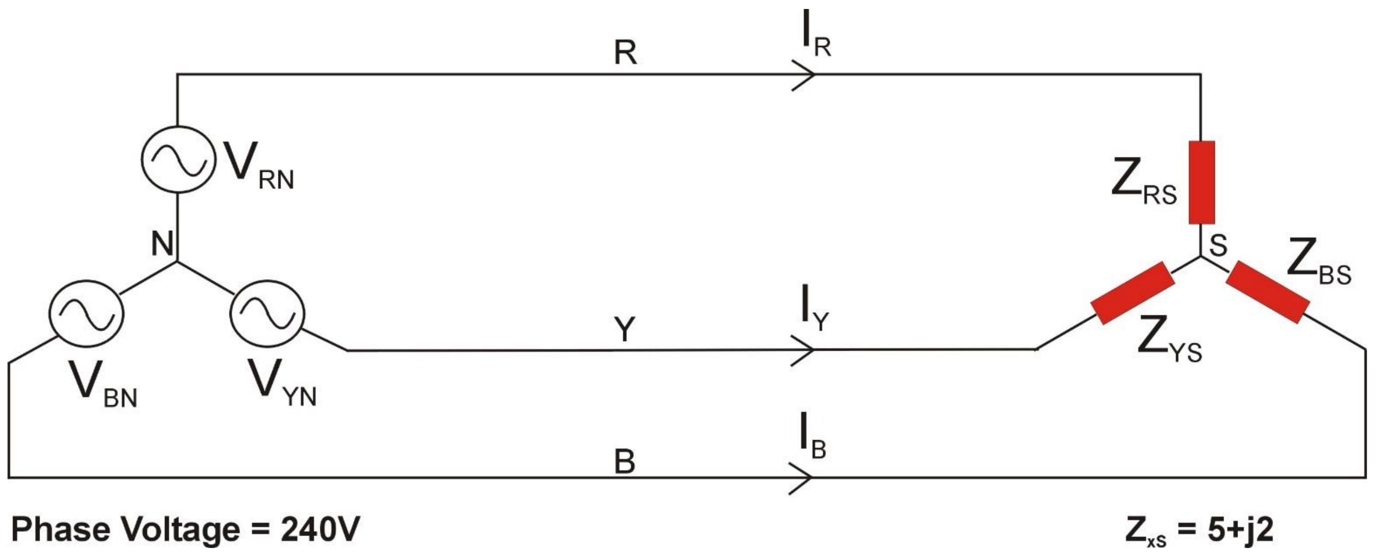
$$Q = |V_{RN}| \cdot |I_R| \cdot \sin\phi = V_{ph} \cdot I_L \cdot \sin\phi$$

where V_{ph} is the rms magnitude of the phase voltage, and I_L is the rms line current and Φ is the angle between them

Total Real Power in 3-phase system:

$$P_T = 3.|V_{ph}|.|I_L|.\cos\Phi$$

An Example of Balanced STAR connected load



For the balanced 3-phase 3-wire system, determine the following:

1. The magnitude of the line voltages
2. The line currents $\mathbf{I_R}$, $\mathbf{I_Y}$ and $\mathbf{I_B}$
3. The phasor diagram showing all line currents and phase voltages
4. The TOTAL real power supplied by the 3 phase supply

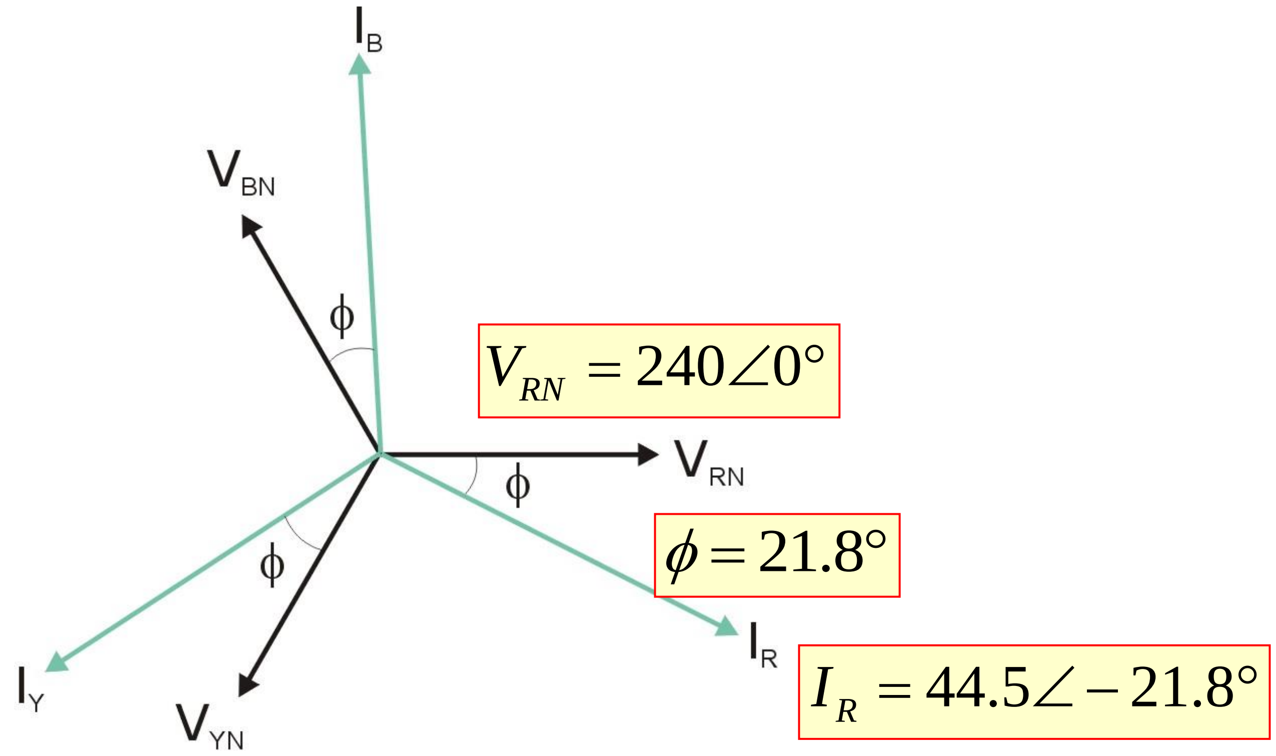
Solution: 1. $V_L = \sqrt{3}V_{ph} = 240\sqrt{3} = 415.7V$

2. $I_R = V_{RN} / (5 + j2) = 240\angle 0° / 5.39\angle 21.8° = 44.5\angle -21.8°$

$I_B = 44.5\angle 98.2°$

$I_R = 44.5\angle -141.8°$

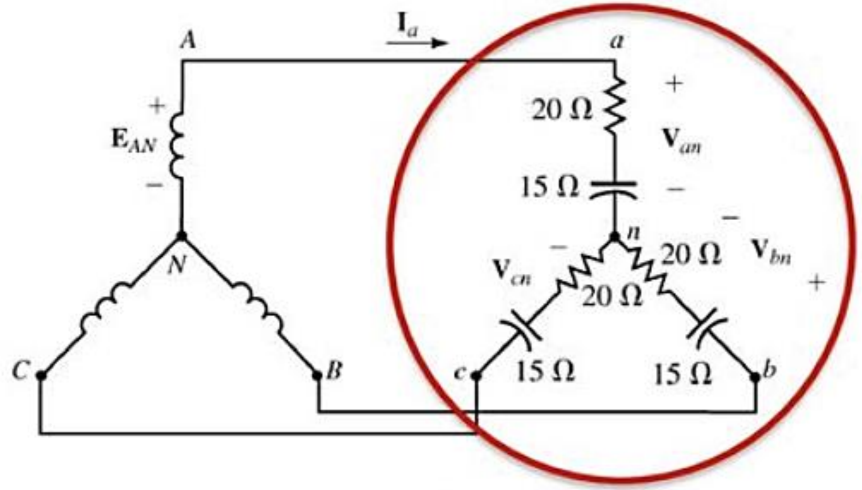
3.



4. $P_{total} = 3V_{ph}I_{ph}\cos\phi = 3I_R^2R = 3\times(44.5)^2\times 5 = 29703.75W$

AN EXAMPLE OF BALANCED STAR CONNECTED LOAD

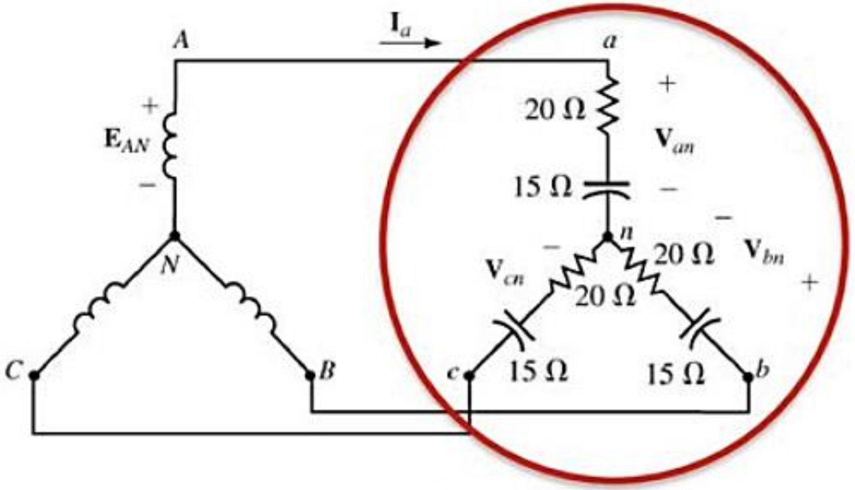
For the load depicted below, $\mathbf{E}_{AB} = 208\angle 0°$ V . Find the phase voltages and line voltages and currents (remember that for a Y system, the phase and line currents are the same).



Notice the balanced load

AN EXAMPLE OF BALANCED STAR CONNECTED LOAD

For the load depicted below, $\mathbf{E}_{AB} = 208\angle 0°\text{ V}$. Find the phase voltages and line voltages and currents (remember that for a Y system, the phase and line currents are the same).



Notice the balanced load

Line Voltages:

$$\bar{E}_{AB} = 208\angle 0^\circ V$$

$$\bar{E}_{BC} = 208\angle -120^\circ V$$

$$\bar{E}_{CA} = 208\angle 120^\circ V$$

Phase Voltages:

$$E_{AN} = \frac{E_{AB}}{\sqrt{3}\angle 30°} = \frac{208\angle 0°V}{\sqrt{3}\angle 30°} = 120\angle -30°V$$

$$E_{BN} = E_{AN}\angle(\theta - 120°) = 120\angle(-30° - 120°) = 120\angle -150°V$$

$$E_{CN} = E_{AN}\angle(\theta + 120°) = 120\angle(-30° + 120°) = 120\angle 90°V$$

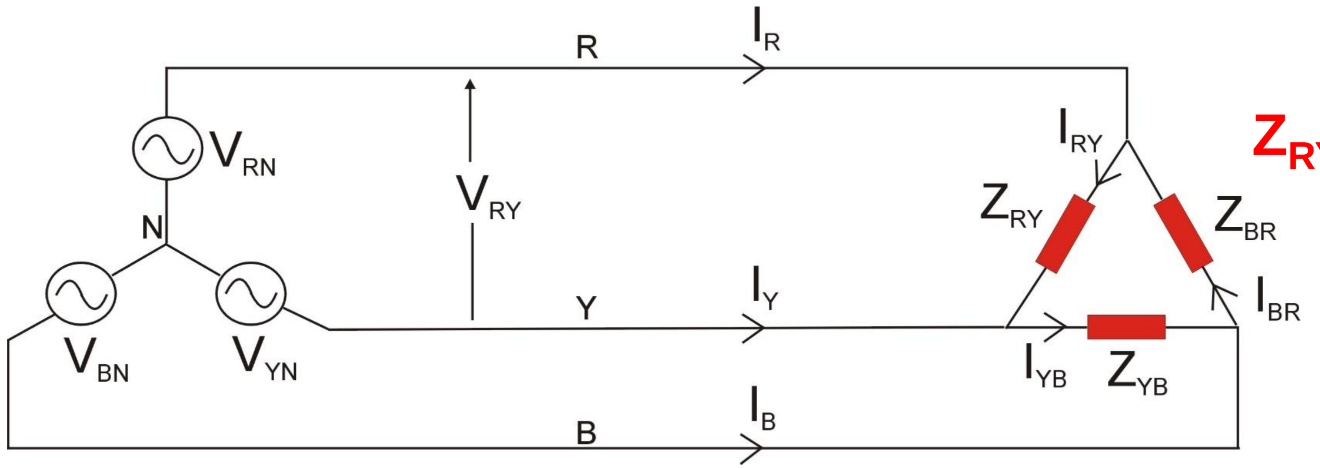
Phase/Line Currents:

$$\mathbf{I}_a = \frac{E_{an}}{\mathbf{Z}_{an}} = \frac{208\angle 0°}{20 - j15} = 4.8\angle 7°A$$

$$\mathbf{I}_b = I_a\angle(\theta - 120°) = 4.8\angle(7° - 120°) = 4.8\angle 113°A$$

$$\mathbf{I}_c = I_a\angle(\theta + 120°) = 4.8\angle(7° + 120°) = 4.8\angle 127°A$$

BALANCED DELTA CONNECTED 3 PHASE LOAD



$Z_{RY} = Z_{YB} = Z_{BR}$

Load phase currents I_{RY}, I_{YB}, I_{BR} and line currents I_R, I_Y, I_B :

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = I_{ph}\angle-\phi$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = I_{ph}\angle(-\phi-120^o)$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = I_{ph}\angle(-\phi+120^o)$$

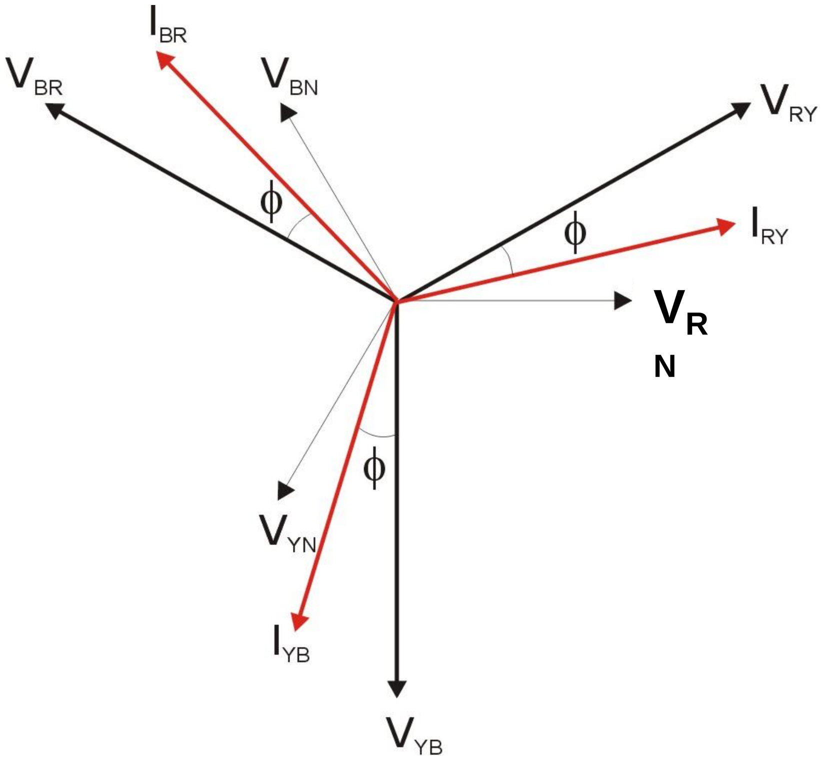
$$I_R = I_{RY} - I_{BR} = \sqrt{3}\cdot I_{ph}\angle-\phi-30°$$

$$I_Y = I_{YB} - I_{RY} = \sqrt{3}\cdot I_{ph}\angle-\phi-150°$$

$$I_B = I_{BR} - I_{YB} = \sqrt{3}\cdot I_{ph}\angle-\phi+90°$$

where all the currents are **phasors**

Balanced DELTA connected 3 Phase Load



Phasor Diagram

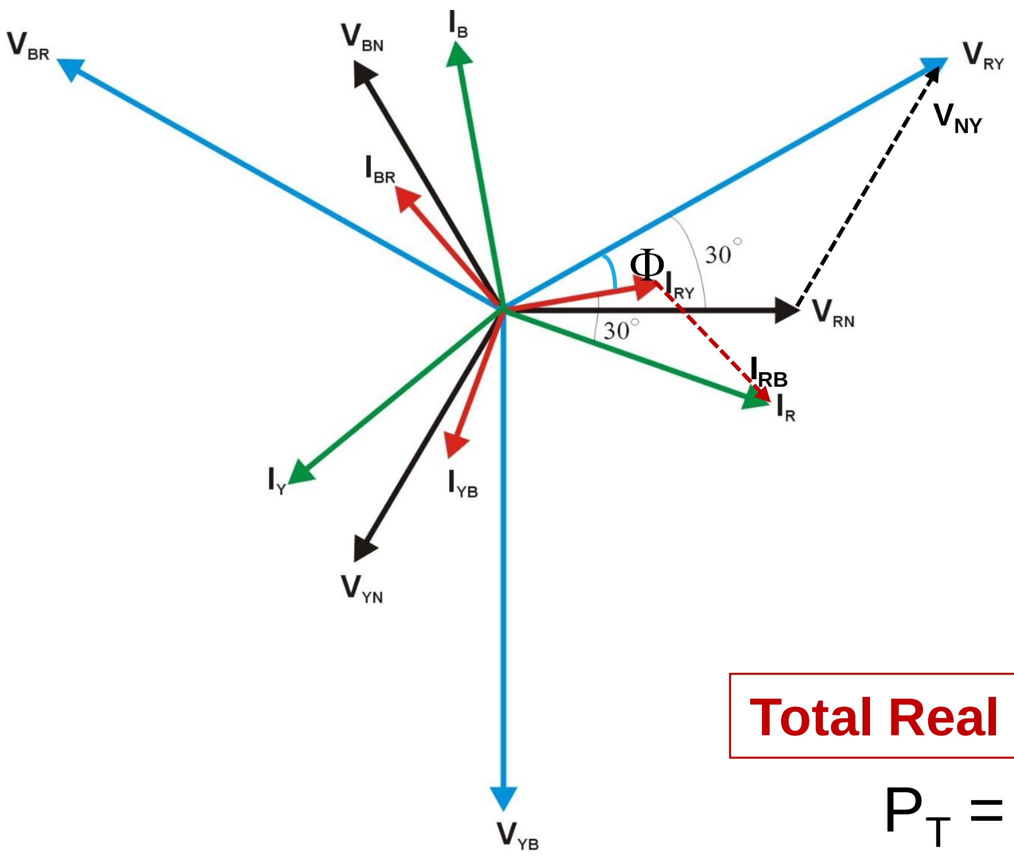
Power calculations for any phase:

$$S = |V_{RY}| \cdot |I_{RY}| = V_L \cdot I_{ph}$$
$$P = |V_{RY}| \cdot |I_{RY}| \cdot \cos\phi = V_L \cdot I_{ph} \cdot \cos\phi$$
$$Q = |V_{RY}| \cdot |I_{RY}| \cdot \sin\phi = V_L \cdot I_{ph} \cdot \sin\phi$$

where V_L is the rms magnitude of the line voltage, and I_{ph} is the rms load phase current and Φ is the angle between them

Balanced DELTA connected 3 Phase Load

Phasor Diagram



Notes:

1. **Line Voltages (eg V_{RY}) are √3x Phase Voltages (eg V_{RN}) and LEAD the phase voltages by 30º**

2. **Line Currents (eg I_R) are √3x Load Phase Currents (eg I_{RY}) and LAG the phase currents by 30º**

Total Real Power in 3-phase system:

$$P_T = 3.|V_L|.|I_{ph}|.\cos\Phi$$

$$= 3.|V_{ph}|.|I_L|.\cos\Phi$$

AN EXAMPLE OF BALANCED STAR CONNECTED LOAD

In the balanced 3-phase system of **Fig. Q.2**, determine the RMS phase and line voltages and currents in the resistors, the average power dissipated per phase and the total average power dissipated.

$$v_a(t) = 167.9\cos(62.8t) \quad \text{(time-domain representation)}$$

$$R_a = R_b = R_c = 30\Omega$$

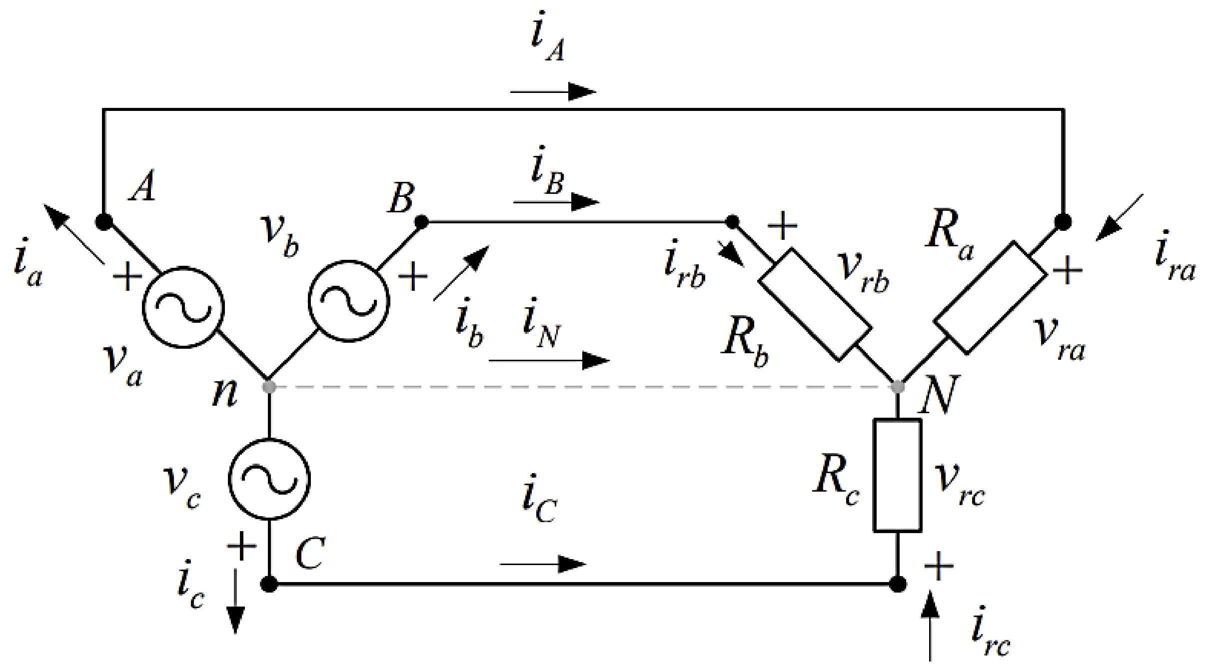
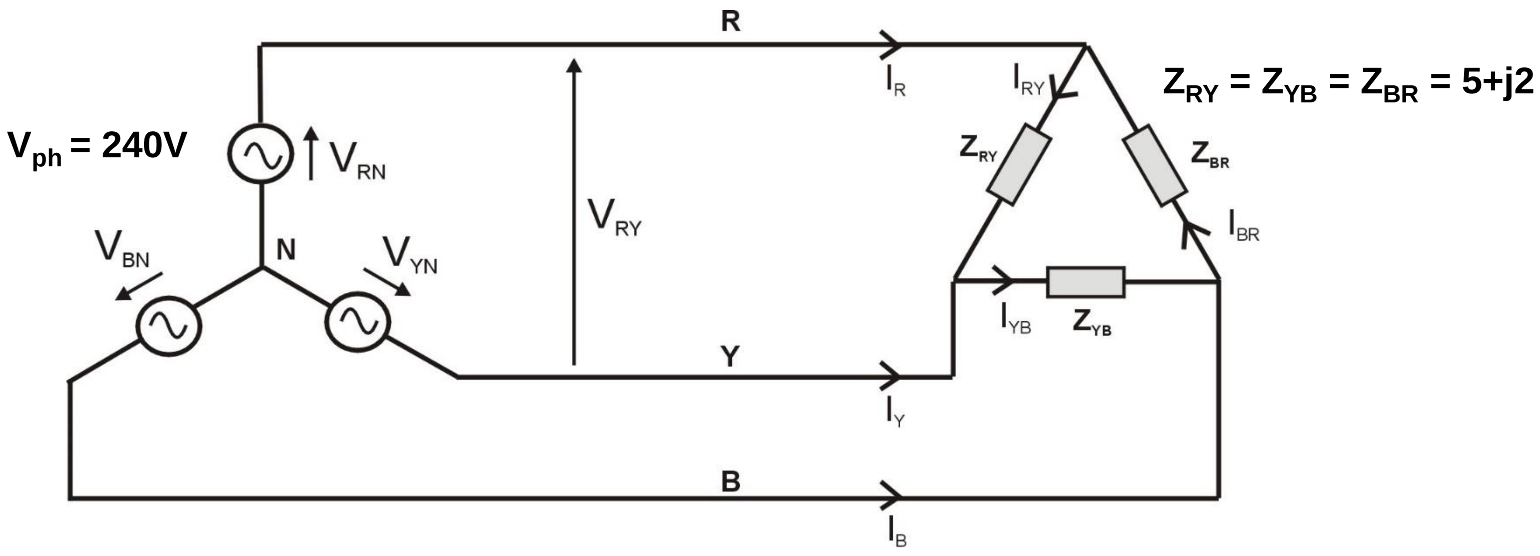


Fig. Q.2. A simple 3-phase, 3- or 4-wire system. (Wye – Wye Circuit).

AN EXAMPLE OF BALANCED DELTA CONNECTED LOAD



For the balanced 3 phase Delta connected load determine the following:

1. The magnitude of the line voltages
2. The line currents I_R, I_Y and I_B
3. The phasor diagram showing all currents and voltages
4. The TOTAL real power supplied by the 3 phase supply

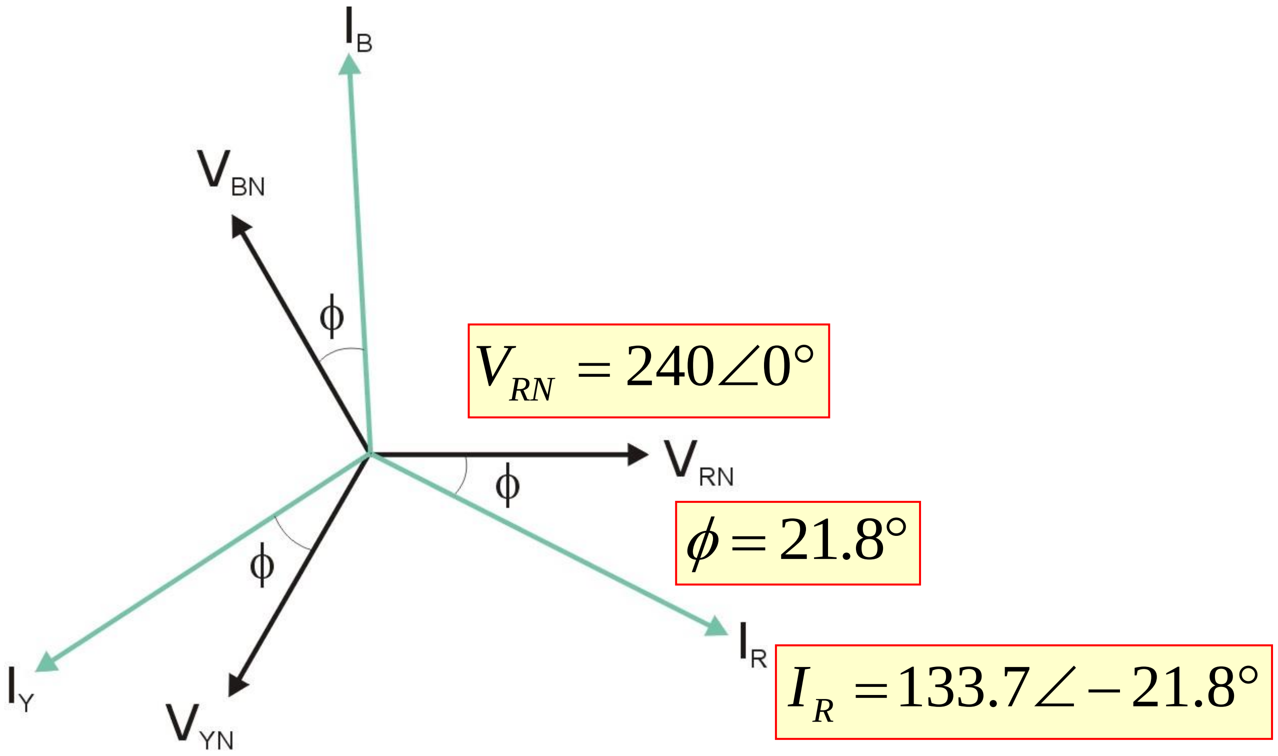
Solution: 1. $V_L = \sqrt{3}V_{ph} = 240\sqrt{3} = 415.7V$

2. $V_{RY} = \sqrt{3}V_{RN}\angle 30°,$

$I_{RY} = V_{RY} / Z_{RY} = 77.2\angle 8.2°, I_R = \sqrt{3}I_{RY}\angle -30° = 133.7\angle -21.8°$

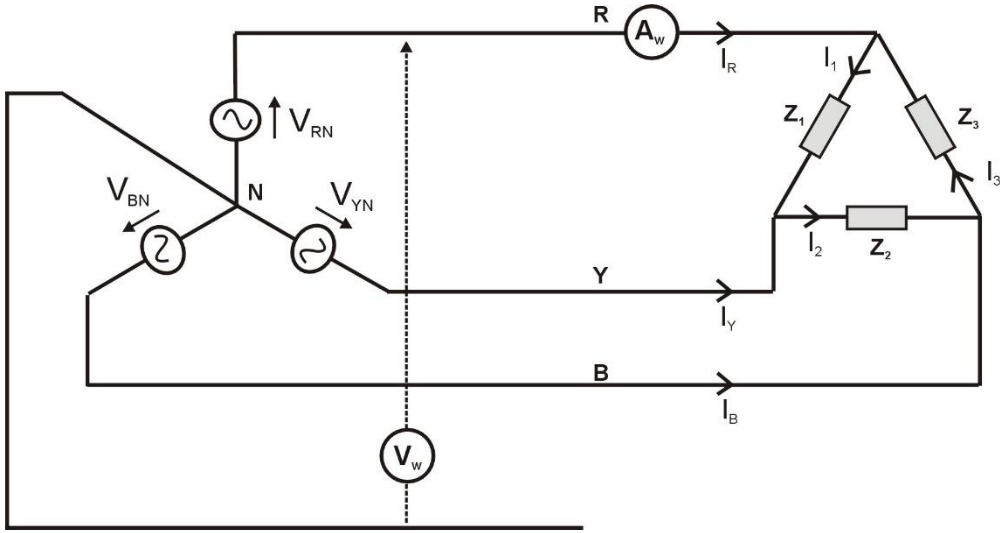
$I_B = 133.7\angle 98.2°, I_Y = 133.7\angle -141.8°$

3.



4. $P_{total} = 3V_{ph}I_{ph}\cos\phi = 3I_{RY}^2 R = 3\times(77.2)^2 \times 5 = 89398\text{W}$

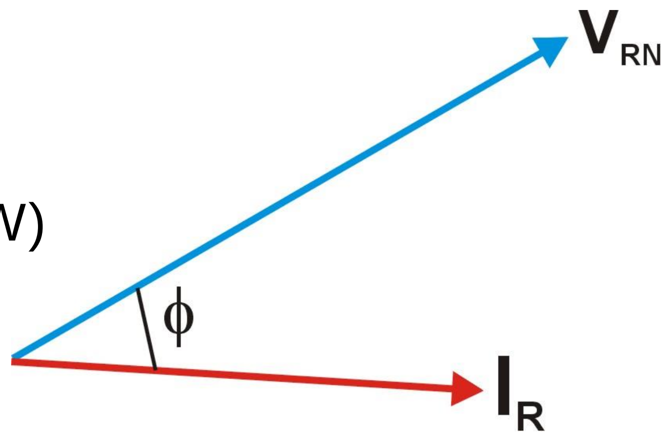
Power Measurements in 3 Phase Systems



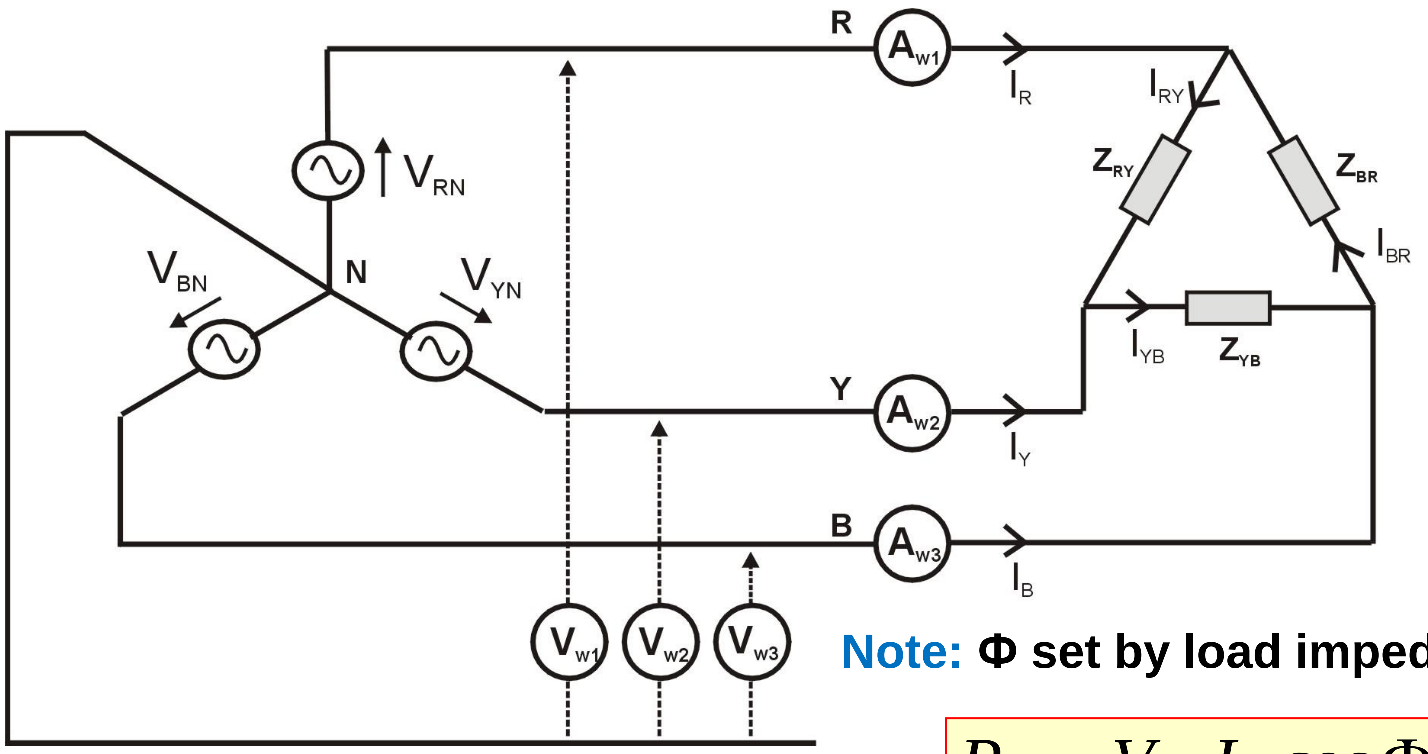
‘Classic’ Wattmeter Measures:

- Voltage (V)
- Current (A)
- Power Factor

→ Real Power **P** (W)



POWER MEASUREMENT: 3 WATTMETER METHOD



Note: **Φ set by load impedance**

Wattmeter 1: V_{W1}, A_{W1}, & Φ_{W1}

Wattmeter 2: V_{W2}, A_{W2}, & Φ_{W2}

Wattmeter 3: V_{W3}, A_{W3} & Φ_{W3}

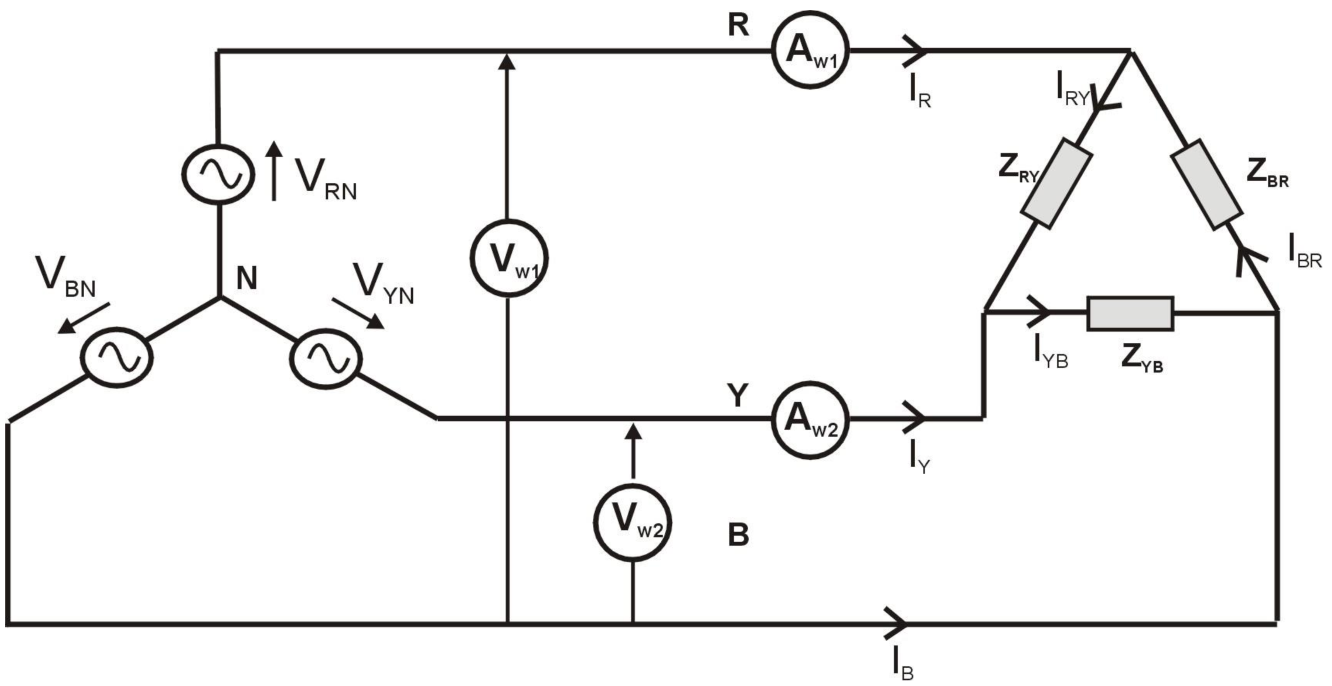
$$P_{W1} = V_{RN} I_R \cos\Phi_1$$

$$P_{W2} = V_{YN} I_Y \cos\Phi_2$$

$$P_{W2} = V_{BN} I_B \cos\Phi_3$$

$$P_{Total} = P_{W1} + P_{W2} + P_{W3}$$

Power Measurement: 2 Wattmeter Method



$P_{W1} = |V_{RB}|.|I_R|.\cos\Phi_{W1}$ **Where Φ_{W1} = angle between V_{RB} and I_R**

$P_{W2} = |V_{YB}|.|I_Y|.\cos\Phi_{W2}$ **Where Φ_{W2} = angle between V_{YB} and I_Y**

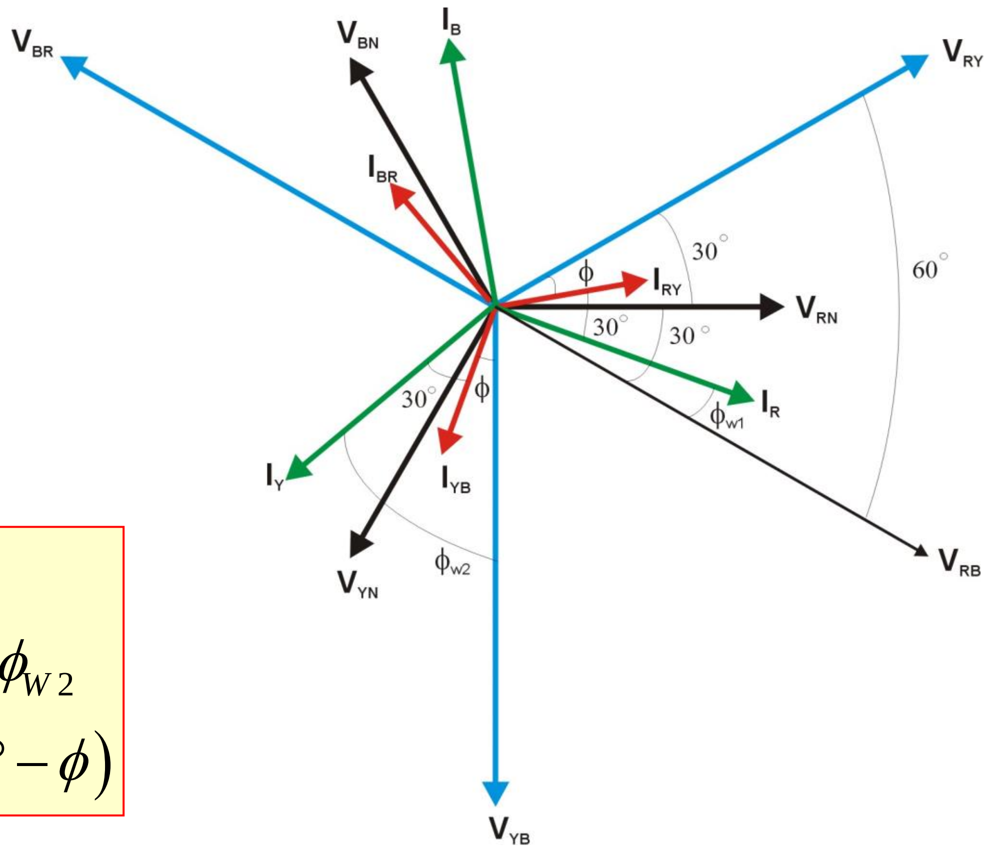
$$P_{Total} = P_{W1} + P_{W2} = |V_{RB}| \cdot |I_R| \cdot \cos\phi_{W1} + |V_{YB}| \cdot |I_Y| \cdot \cos\phi_{W2}$$

AN EXAMPLE OF 2 WATTMETER METHOD

For a balanced DELTA connected load, from Phasor Diagram,

$$\Phi + 30^o + \Phi_{W1} = 60^o$$

$$\Phi_{W2} = 30^o + \Phi$$

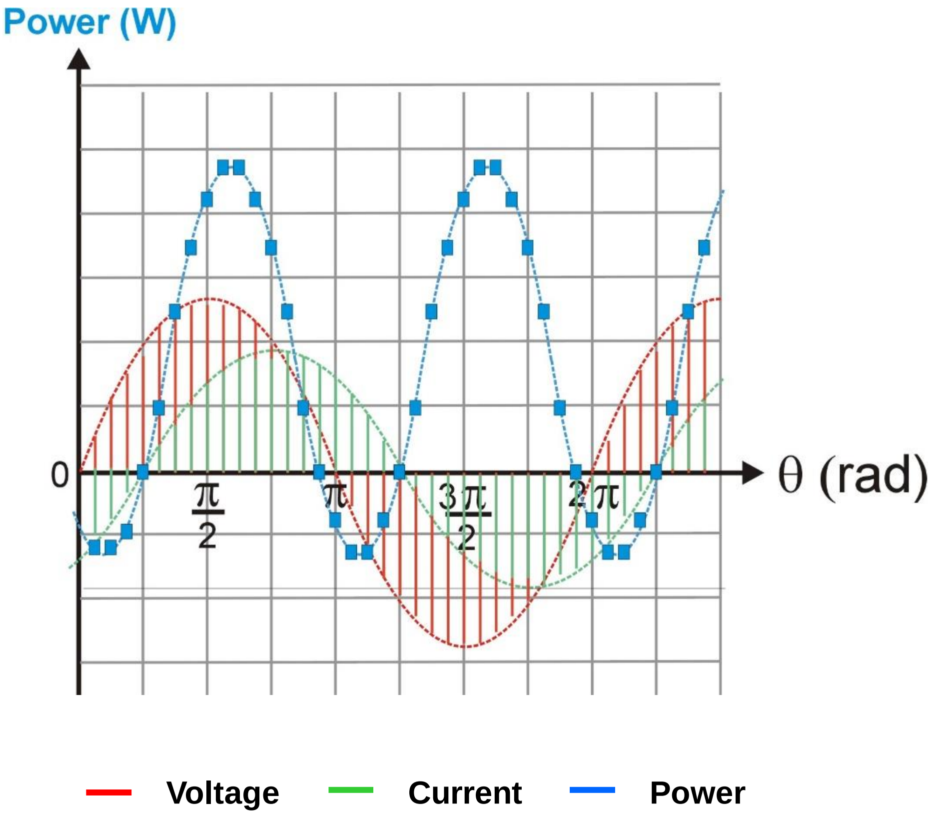


$$\begin{aligned}P_{Total} &= P_{W1} + P_{W2} \\ &= |V_{RB}| \cdot |I_R| \cdot \cos\phi_{W1} + |V_{YB}| \cdot |I_Y| \cdot \cos\phi_{W2} \\ &= V_L I_L \cos(30^\circ - \phi) + V_L I_L \cos(30^\circ - \phi)\end{aligned}$$

$$\begin{aligned}P_{Total} &= V_L I_L \left(\frac{\sqrt{3}}{2} \cos\phi + \frac{1}{2} \sin\phi \right) + V_L I_L \left(\frac{\sqrt{3}}{2} \cos\phi - \frac{1}{2} \sin\phi \right) \\ &= \sqrt{3} V_L I_L \cos\phi = 3 V_{ph} I_L \cos\phi = 3 V_L I_{ph} \cos\phi\end{aligned}$$

Modern Digital Sampling Power Meters/Analysers

Manufacturers:
Voltech
Fluke
Yokagawa



$$Power(W) = \frac{1}{N}\sum_{0}^{N} v.i$$

where N is the number of samples in a period

#03 Three-Phase AC Power Systems (1)