UESTC4004
Digital Communications

Channel Coding

Refreshing the contents!

- Define Source Coding.

$\angle 5$

QPSK

Capacity

Shahon-Fano source coding.

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加冗余 抗噪声 失真

Channel Coding

- Adding redundancy to improve immunity against channel noise and distortions. The purpose is to correct or at least detect the errors introduced due to channel in our information signal.
- Type of error control
  - **Error detection and retransmission** e.g., Automatic Repeat Request (ARQ)
  - **Error correction cording,** e.g., Convolutional Coding

错误控制.

Terminal Connectivity

- Simplex
- Half duplex
- Full duplex

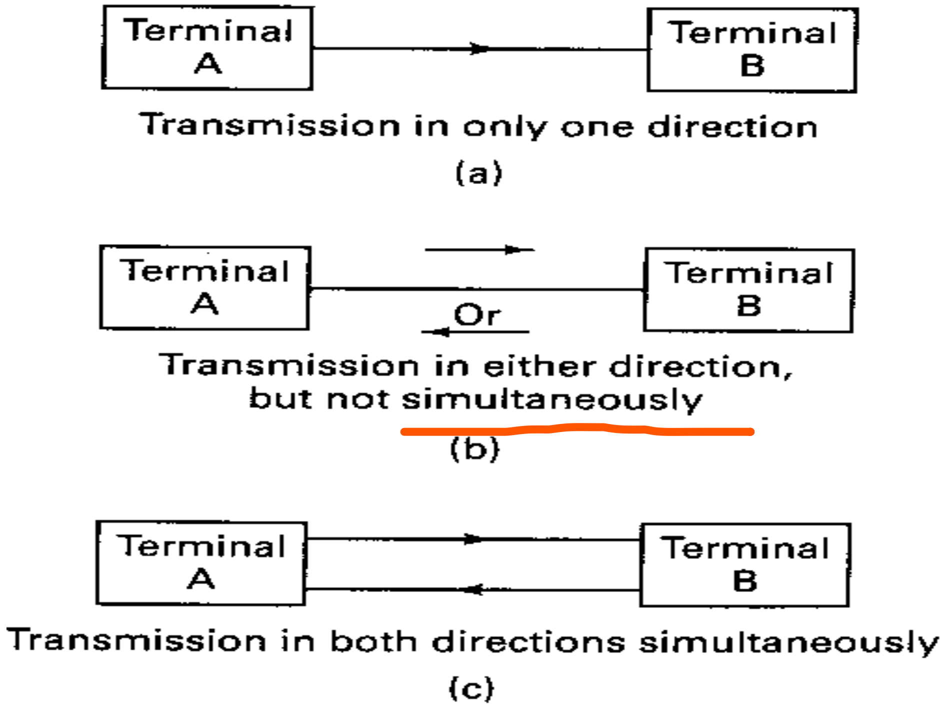

Figure: Terminal connectivity classifications (a) Simplex (b) Half-duplex (c) Full-duplex

Automatic Repeat Request (ARQ)

- ARQ is much simpler than FEC and needs no redundancy. 无冗余

- ARQ is sometimes not possible if

  - A reverse channel is not available

  - The retransmission strategy is not conveniently implemented

  - The expected number of errors, without corrections, would require excessive retransmissions

3种ARQ

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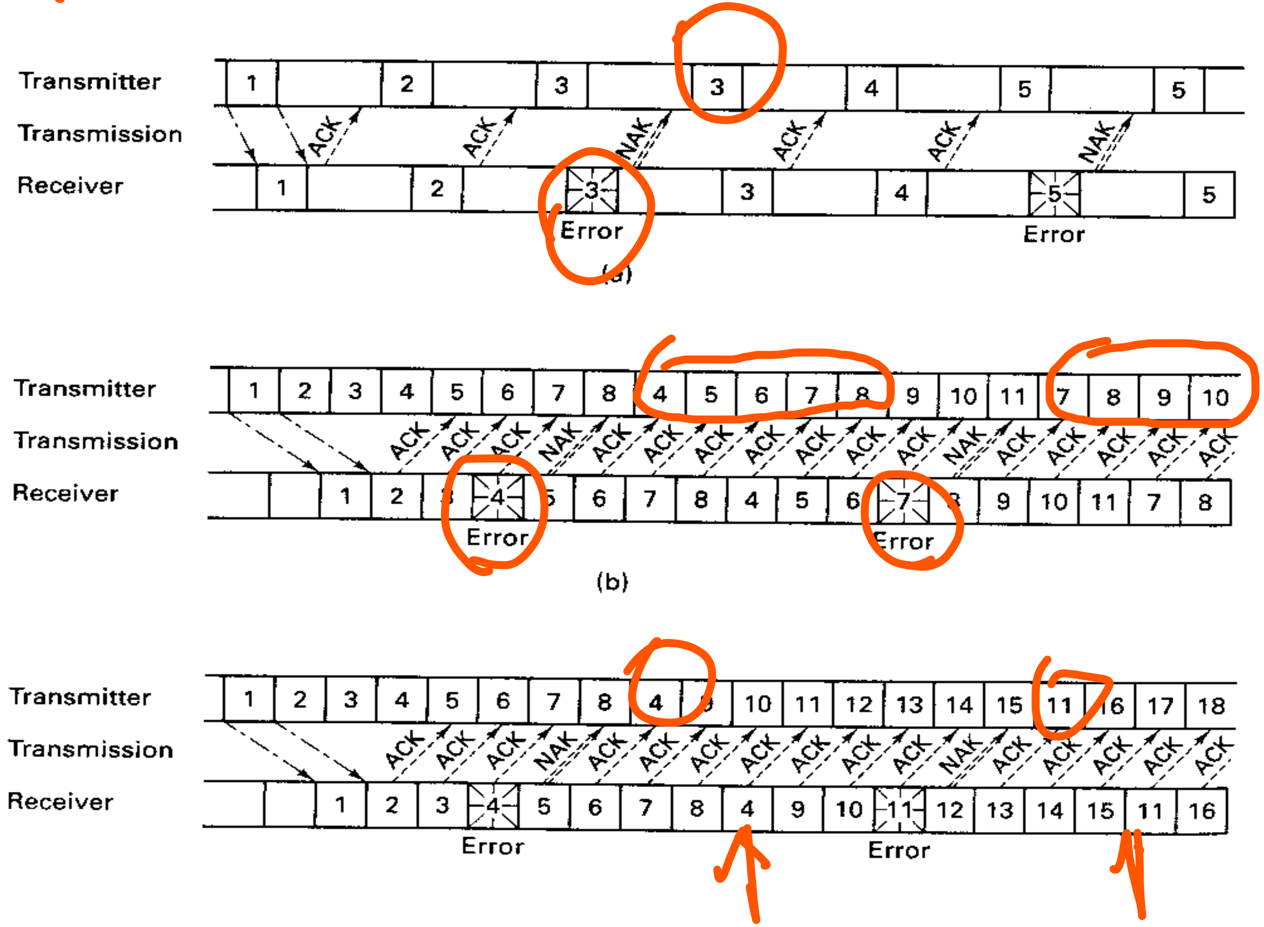


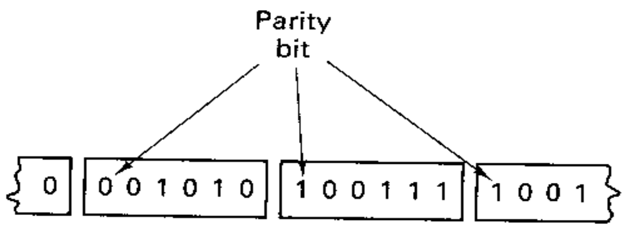
Figure 6.7: Automatic Repeat Request (ARQ) (a) Stop and wait ARQ (b) Continuous ARQ with pullback (c) Continuous ARQ with selective repeat

Parity-Check Codes

Single-parity-Check Code

奇偶校验.

- Parity bit check is the simplest example of channel coding.



(a)

- Single parity check code can only detect odd number of errors,
- However, it can not correct any number of errors

Rectangular Code

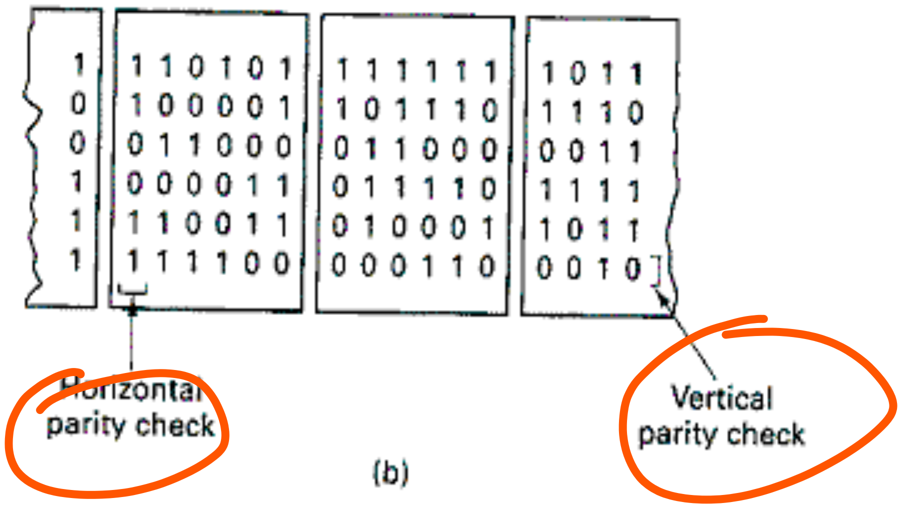


Figure 6.8: Parity checks for parallel structure

- Rectangular code can not only detect a single error, but it can also correct it.

Question

- Can rectangular code detect a double error?

Error Correction Coding

错误修正

- Block codes
- Convolutional codes
- Turbo codes

x3

Why Use Error-Correction Coding

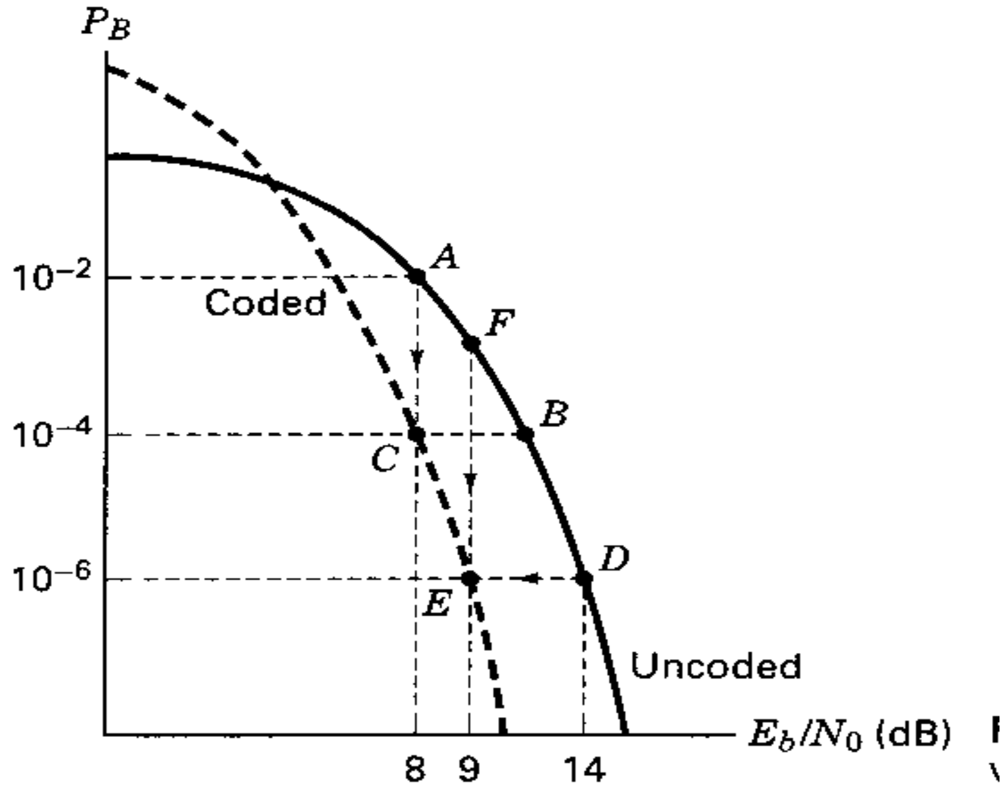


Figure 6.9:Comparison of typical coded versus uncoded error performance

Question

- As we move from point B of the uncoded curve to point C on the coded curve, what gain is achieved?

$$P_B(B) = P_B(C)$$

Channel Models

Discrete Memoryless Channel (DMC)

- The outcomes relate to the current input only and depends on the current probability for each independent transmission

Binary Symmetric Channel

- The conditional probability for transmission (1|0) and receiving (0|1) is symmetric

$$\boxed{\begin{aligned} P(0\,|\,1) &= P(1\,|\,0) = p \\ P(1\,|\,1) &= P(0\,|\,0) = 1-p \end{aligned}}$$

Linear block Codes

P14

Vector Spaces

- The set of all binary n-tuple is called a vector space over the binary field of 0 and 1

Vector Subspaces

- A subset S of the vector space is called a subspace if
  - The all-zeros vector is in S
  - The sum of any two vectors in S is also in S (Closure property/Linear property) e.g. { 0000 0101 1010 1111 }

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Linear block code

A (6,3) Linear Block Code Examples

Message Vector	Codeword
000	000000
100	110100
010	011010
110	101110
001	101001
101	011101
011	110011
111	000111

Table: Assignment of Codewords to Messages

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code位数

$n > k \leftarrow$ message 数

Code Rate and Redundancy

code bits
{channel symbol

- In case of block codes, encoder transforms each *k*-bit data block into a larger block of *n*-bits called code bits or channel symbol
- The *(n-k)*-bits added to each data block are are called redundant bits, parity bits or check bits
- They carry no new information
- Ratio of redundant bits to data bits: $(n-k)/k$ is called redundancy of code
- Ratio of data bits to total bits, k/n is called code rate

冗余: $\frac{n-k}{k}$

码率 $\frac{k}{n}$

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Generator Matrix

生成矩阵

- If k is large, a lookup table implementation of the encoder becomes prohibitive
- Let the set of 2^k codewords {U} be described as:

$$\boxed{\mathbf{U}=m_1\mathbf{V}_1+m_2\mathbf{V}_2+\ldots..+m_k\mathbf{V}_k}$$

m_k是单个比特

$m = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_k \end{bmatrix}^T$

- In general, generator matrix can be defined by the following k x n array:

$$G=\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix}=\begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & & & \\ v_{k1} & v_{k2} & \cdots & v_{kn} \end{bmatrix} \quad (6.24)$$

U, V_k是横向量

- Generation of codeword U:

$$\boxed{\mathbf{U}=\mathbf{mG}} \quad (6.25)$$

生成矩阵

编码

信息

Example:

- Let the generator matrix be:

$$G = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad (6.26)$$

- Generate Codeword U4 for the fourth message vector 1 1 0 in Table 6.1

二进制计算

U_4 $=[1\ 1\ 0]\ G = V_1 + V_2 + 0 * V_3$

$= 110100 + 011010 + 000000$

$= 101110$ (Codeword for the message vector 110)

1 1 0 1 0 0
0 1 1 0 1 0

1 0 1 1 1 0

△ 不进位 → 加起就成

Systematic Linear Block codes

- A systematic (n,k) linear block code is a mapping a k-dimensional message vector to an n-dimensional code word such that part of the sequence has k message digits and remaining (n-k) are parity digits
- A systematic linear code will have a generator matrix

$$G = \left[P \;\vdots\; I_k \right] = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1,(n-k)} & 1 & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & p_{2,(n-k)} & 0 & 1 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{k,(n-k)} & 0 & 0 & \cdots & 1 \end{bmatrix} \qquad (6.27)$$

- Combining (6.26) and (6.27):

$$u_1, u_2, \ldots u_n = [m_1, m_2, \ldots m_k] \times \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1,(n-k)} & 1 & 0 & \cdots & 0 \\ p_{21} & p_{22} & \cdots & p_{2,(n-k)} & 0 & 1 & \cdots & 0 \\ \vdots & & & & & & & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{k,(n-k)} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

k

n-k　　k

n-k+k=n

计算u: $u=mG$ 的变体:

① 单mess: $u_k=[x\ x\ x \cdots x]\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_k \end{bmatrix}$

② 用奇偶校验: $u=[m_1 \cdots m_k][P|I_k]$

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Example

- For (6,3) code example in sec.6.4.3, the codewords can be described as:

$$U=[m_1,m_2,m_3]\begin{bmatrix} \underbrace{\begin{matrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}}_{P} & \vdots & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_3} \end{bmatrix} \begin{matrix} v_1 \\ v_2 \\ v_3 \end{matrix} \tag{6.30}$$

注意向量计算用的模2

$$=\underbrace{m_1+m_3}_{u_1},\underbrace{m_1+m_2}_{u_2},\underbrace{m_2+m_3}_{u_3},\underbrace{m_1}_{u_4},\underbrace{m_2}_{u_5},\underbrace{m_3}_{u_6} \tag{6.31}$$

u= 奇偶校验+信息

编码

P矩阵组成

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Question

- Consider a (7, 4) code whose generator matrix is

7-4=3 parity ch-b.

$$G = \begin{array}{c} \begin{bmatrix} 1\,1\,0 & 1\,0\,0\,0 \\ 0\,1\,1 & 0\,1\,0\,0 \\ 1\,1\,1 & 0\,0\,1\,0 \\ 1\,0\,1 & 0\,0\,0\,1 \end{bmatrix} \end{array} \begin{array}{l} v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}$$

1) What is the code redundancy and code rate?

2) Find the code word corresponding to message 1110.

1). $\frac{n-k}{k} = \frac{7-4}{4} = 0.75$

2). $v(1110) = m(1110)\,G = [1\,1\,1\,0]\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$

$= v_1 + v_2 + v_3 + 0 \cdot v_4$

$= 0101110$