DIGITAL IMAGE PROCESSING

MORPHOLOGICAL IMAGE PROCESSING

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2022-11-15

OUTLINE

- Overview
- Preliminaries
- Binary morphology
  - Erosion and dilation
  - Opening and closing
  - The Hit-or-Miss transformation
  - Basic morphological algorithms
- Gray-scale morphology

MORPHOLOGICAL IMAGE PROCESSING

- Morphology
- Mathematical morphology
  - Dates back to the research work of Euler(19c) and Minkowski(20c)
  - Matheron, *Random Sets and Integral Geometry*, 1975

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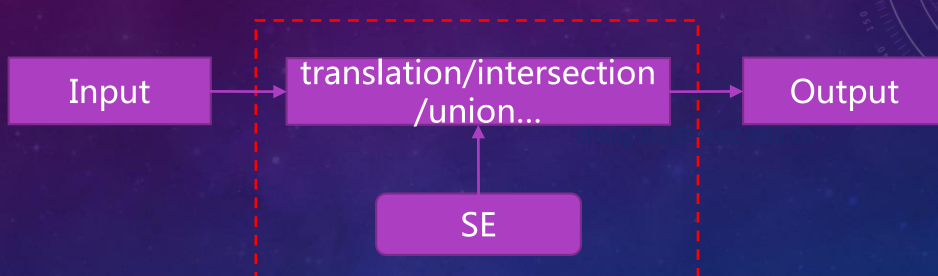
MORPHOLOGICAL IMAGE PROCESSING

- Set theory
  - Image → Set
  - Spatial operations → Set operations
- Structuring Element(SE)
  - SE: small sets or subimages used to probe an image under study for properties of interest

MORPHOLOGICAL IMAGE PROCESSING



- SE v.s. Filter Mask，Logical operation v.s. Convolution(multiplication & addition)

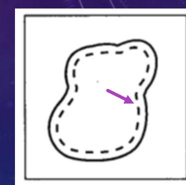
MORPHOLOGICAL IMAGE PROCESSING

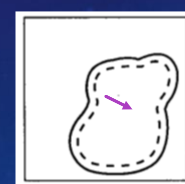
- translation :

$$B_z = \left\{ \vec{x} \mid \vec{x} = \vec{b} + \vec{z},\ for\ \vec{b} \in B \right\}$$

$$\vec{z} = (z_1, z_2)$$

$$(x, y) \quad -> \quad (x + z_1, y + z_2)$$



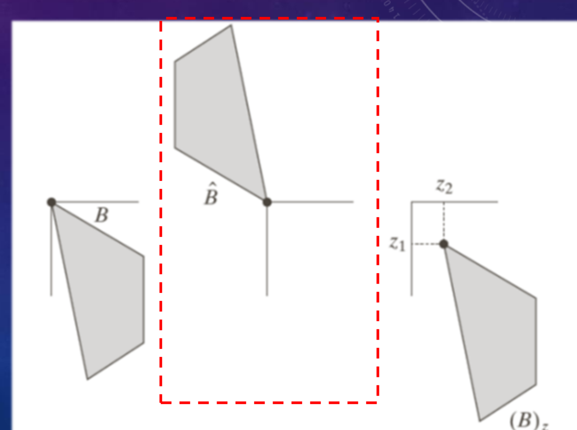


MORPHOLOGICAL IMAGE PROCESSING

- reflection :

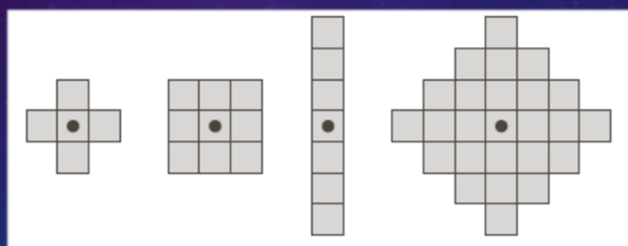
$$\hat{B} = \left\{ \vec{w} \mid \vec{w} = -\vec{b},\ for\ \vec{b} \in B \right\}$$

$$(x, y) \quad -> \quad (-x_1, -y)$$



MORPHOLOGICAL IMAGE PROCESSING

- Origin of SE :



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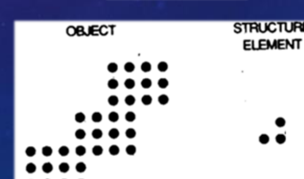
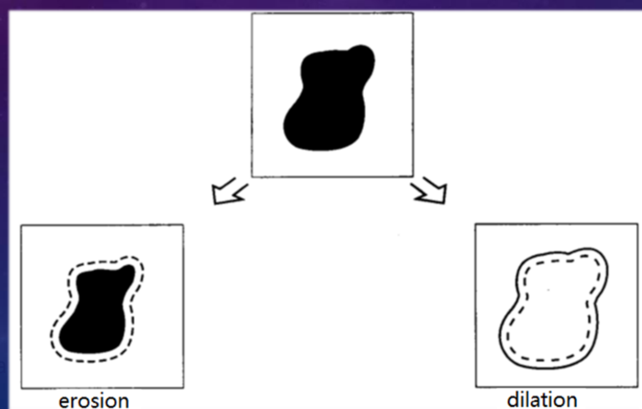


MORPHOLOGICAL IMAGE PROCESSING

- Erosion and dilation



MORPHOLOGICAL IMAGE PROCESSING

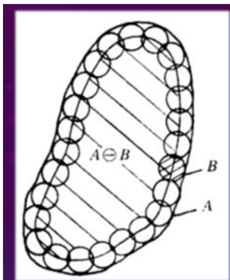
- Erosion :

$$E = A \Theta B = \{\vec{z} \mid B_z \subseteq A\}$$

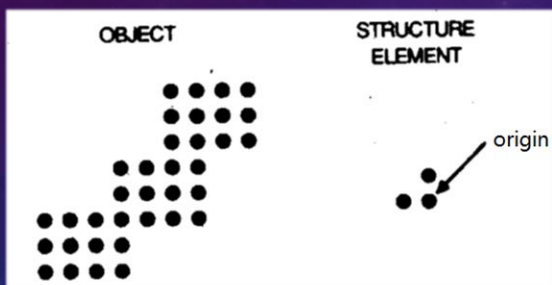
Equivalently,

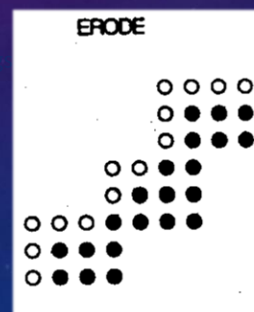
$$A \Theta B = \{\vec{z} \mid B_z \cap A^C = \varnothing\}$$

- Erosion shrinks or thins objects in a binary image. Image details smaller than the SE are filtered from the image.



MORPHOLOGICAL IMAGE PROCESSING





MORPHOLOGICAL IMAGE PROCESSING

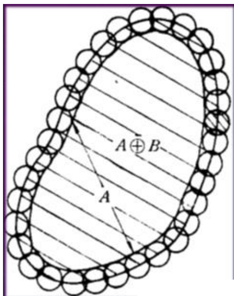
- Dilation :

$$D = A \oplus B = \left\{ \vec{z} \mid \hat{B}_z \cap A \neq \varnothing \right\}$$

Equivalently,

$$A \oplus B = \left\{ \vec{z} \mid (\hat{B}_z \cap A) \subseteq A \right\}$$

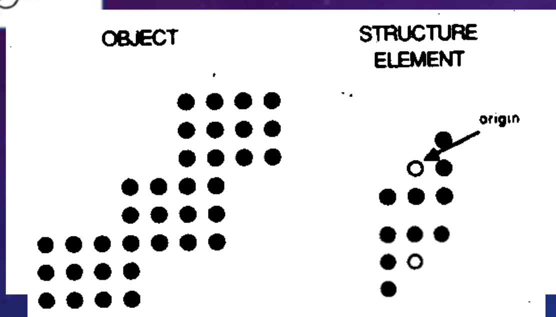
- Dilation grows or thickens objects in a binary image. Gaps narrower than the SE are bridged.

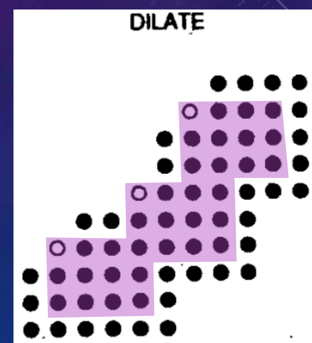


MORPHOLOGICAL IMAGE PROCESSING

$$D = A \oplus B = \left\{ \vec{z} \mid \hat{B}_z \cap A \neq \varnothing \right\}$$

$$A \oplus B = \left\{ \vec{z} \mid (\hat{B}_z \cap A) \subseteq A \right\}$$





MORPHOLOGICAL IMAGE PROCESSING

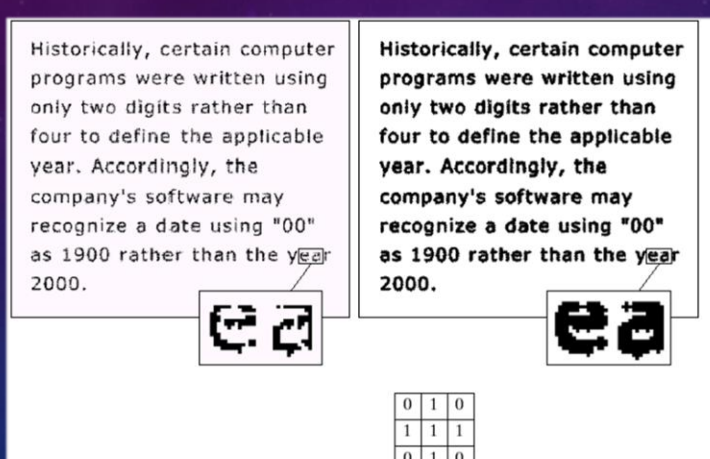
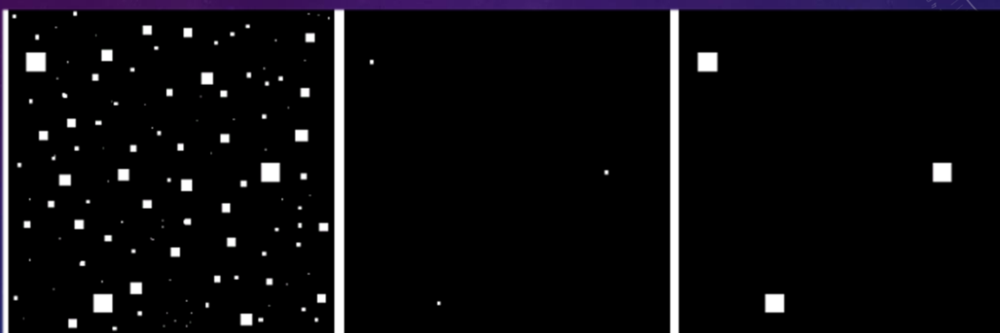


FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

MORPHOLOGICAL IMAGE PROCESSING



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

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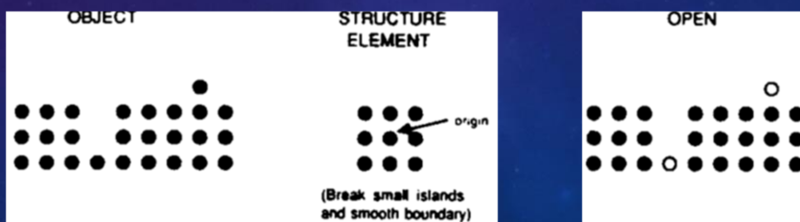
MORPHOLOGICAL IMAGE PROCESSING

- Opening :

$$A \circ B = (A \Theta B) \oplus B$$

- Opening smoothes the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions



MORPHOLOGICAL IMAGE PROCESSING

$$E = A\Theta B = \{\vec{z} \mid B_z \subseteq A\} \qquad D = A \oplus B = \{\vec{z} \mid \hat{B}_z \cap A \neq \varnothing\}$$

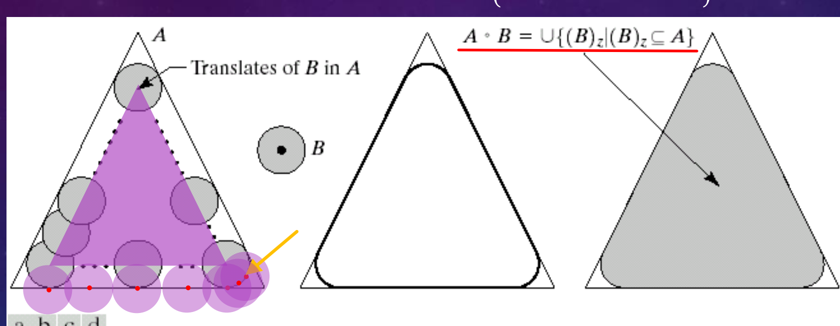


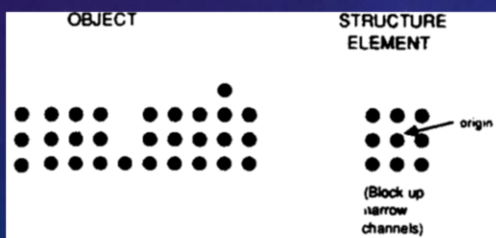
FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

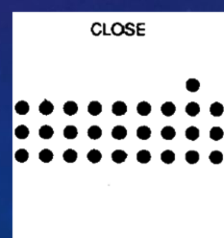
MORPHOLOGICAL IMAGE PROCESSING

- Closing :

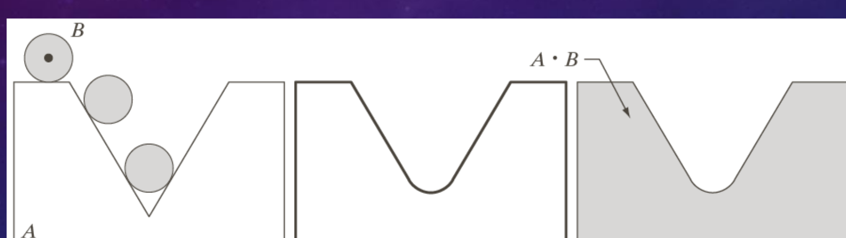
$$A \bullet B = (A \oplus B) \Theta B$$

- Closing also tends to smooth sections of contours, fuses narrow breaks and long thin gulfs, eliminates small holes and fills gaps in the contour





MORPHOLOGICAL IMAGE PROCESSING



a b c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

a
b c
d e
f g
h i

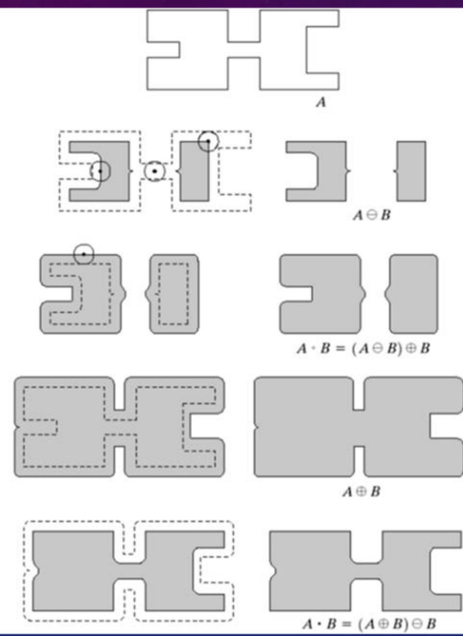
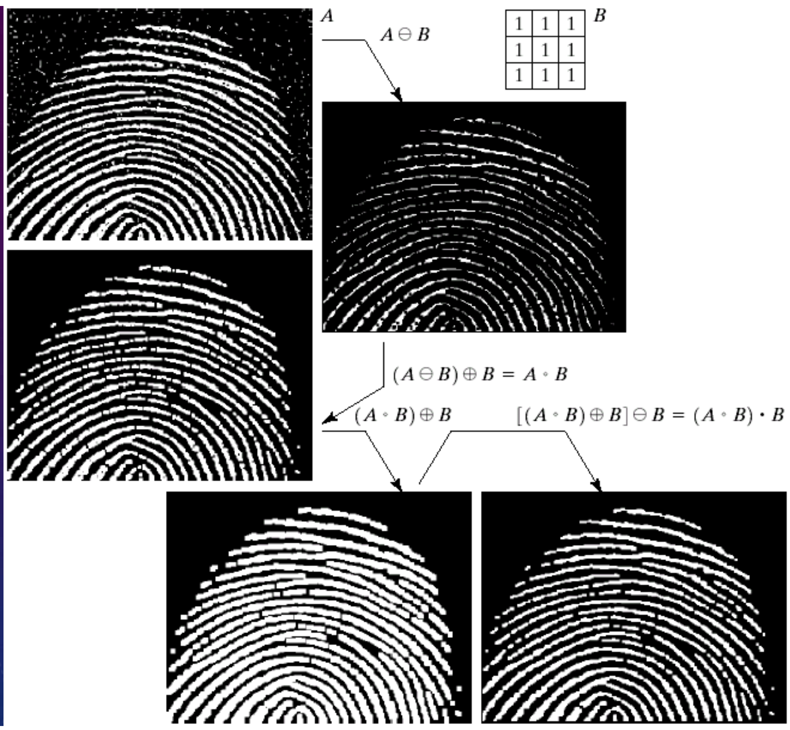


FIGURE 9.10 Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

MORPHOLOGICAL IMAGE PROCESSING

- Property of opening :
  1 $A \circ B$ is a subset of A
  2 If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$
  3 $A \circ B \circ B = A \circ B$
- Property of closing :
  1 A is a subset of $A \bullet B$
  2 If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$
  3 $A \bullet B \bullet B = A \bullet B$



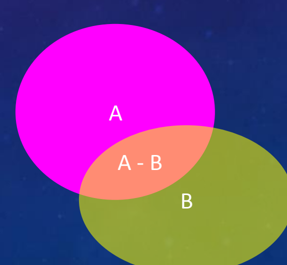
a b
c
d
e f

FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

MORPHOLOGICAL IMAGE PROCESSING

- Question : Can you extract the boundary of the input binary object by means of erosion/dilation/opening/closing?
- Difference:

$$A - B = \{\vec{w} \in A, \vec{w} \notin B\} = A \cap B^C$$



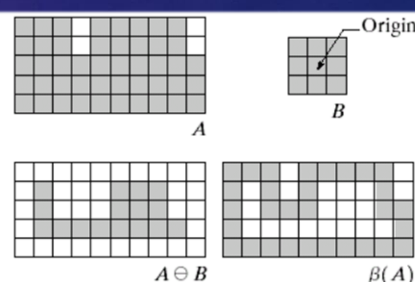
MORPHOLOGICAL IMAGE PROCESSING

- Boundary extraction：

$$Edge(A) = A - (A\Theta B)$$
$$Edge(A) = (A \oplus B) - A$$

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



MORPHOLOGICAL IMAGE PROCESSING

- Commutative Property

$$A \oplus B = B \oplus A$$

- Associative Property

$$(A \oplus B_1) \oplus B_2 = A \oplus (B_1 \oplus B_2)$$
$$(A\Theta B_1)\Theta B_2 = A\Theta(B_1 \oplus B_2)$$

- Distributive Property

$$(A_1 \cap A_2)\Theta B = (A_1\Theta B) \cap (A_2\Theta B)$$
$$A\Theta(B_1 \cup B_2) = (A\Theta B_1) \cap (A\Theta B_2)$$

- Duality

$$A^C \oplus \hat{B} = (A\Theta B)^C$$
$$A^C \Theta \hat{B} = (A \oplus B)^C$$

MORPHOLOGICAL IMAGE PROCESSING

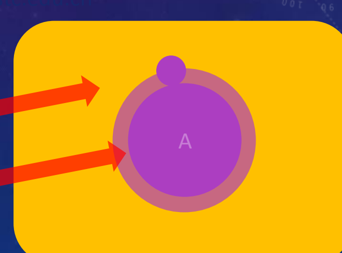
- Commutative Property $A \oplus B = B \oplus A$
- Associative Property
  $$(A \oplus B_1) \oplus B_2 = A \oplus (B_1 \oplus B_2)$$
  $$(A \Theta B_1) \Theta B_2 = A \Theta (B_1 \oplus B_2)$$
- Distributive Property
  $$(A_1 \cap A_2) \Theta B = (A_1 \Theta B) \cap (A_2 \Theta B)$$
  $$A \Theta (B_1 \cup B_2) = (A \Theta B_1) \cap (A \Theta B_2)$$
- Duality
  $$A^C \oplus \hat{B} = (A \Theta B)^C$$
  $$A^C \Theta \hat{B} = (A \oplus B)^C$$

MORPHOLOGICAL IMAGE PROCESSING

- Proof $A^C \Theta \hat{B} = (A \oplus B)^C$ $\quad A^C \oplus \hat{B} = (A \Theta B)^C$

$$E = A \Theta B = \{\vec{z} \mid B_z \subseteq A\}$$
$$A \Theta B = \{\vec{z} \mid B_z \cap A^C = \varnothing\}$$
$$D = A \oplus B = \{\vec{z} \mid \hat{B}_z \cap A \neq \varnothing\}$$

$$A^C \Theta \hat{B} = \{z \mid \hat{B}_z \subseteq A^C\}$$
$$= \{z \mid \hat{B}_z \cap A = \varnothing\}$$
$$= \{z \mid \hat{B}_z \cap A \neq \varnothing\}^C$$
$$= (A \oplus B)^C$$



MORPHOLOGICAL IMAGE PROCESSING

- SE decomposition
  - Decompose complex SE into simple SEs according to:

$$A\Theta(B_1 \oplus B_2) = (A\Theta B_1)\Theta B_2$$

  - E.g. Hexagonal element SE

MORPHOLOGICAL IMAGE PROCESSING