OUTLINE

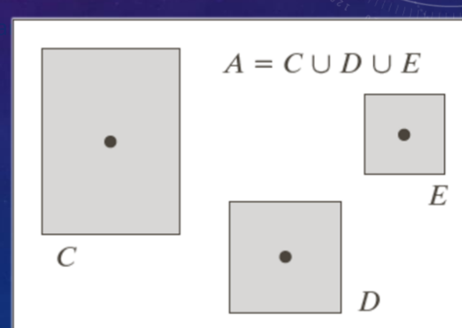
- Overview
- Preliminaries
- Binary morphology
  - Erosion and dilation
  - Opening and closing
  - The Hit-or-Miss transformation
  - Basic morphological algorithms
- Gray-scale morphology

HMT（HIT OR MISS TRANSFORMATION）

- To find the location of certain object
- Template matching with： $T=(T_1,T_2)$

  T_1 ——foreground； T_2 ——background
- The match of T in A contains all the points at which, simultaneously, T_1 found a match in A and T_2 found a match in A^C. Hit points of HMT are marked as 1，miss points are marked as 0

$A®T$

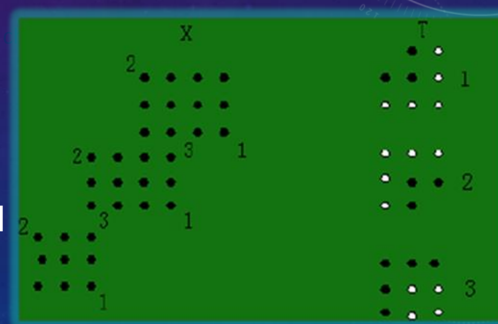


HMT（HIT OR MISS TRANSFORMATION）

- To find the location of certain object
- Template matching with： $T = (T_1, T_2)$

  T_1 ——foreground； T_2 ——background
- The match of T in X contains all the points at which, simultaneously, T_1 found a match in X and T_2 found a match in X^C. Hit points of HMT are marked as 1，miss points are marked as 0

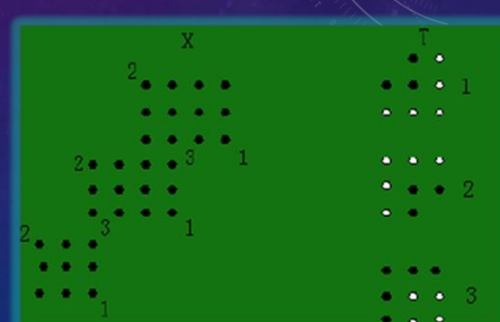
$X \circledR T$



HMT（HIT OR MISS TRANSFORMATION）

- If $T_2 = \varnothing$ ： $A \circledR T = A \Theta T_1$
- Generally：

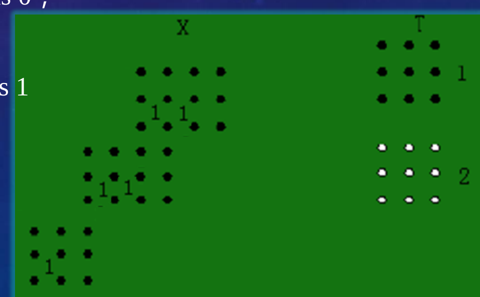
$$A \circledR T = \left\{ x \mid T_1 \subseteq A, T_2 \subseteq A^C \right\}$$
$$= (A \Theta T_1) \bigcap (A^C \Theta T_2)$$
$$= (A \Theta T_1) \bigcap (A \oplus \hat{T}_2)^C$$
$$= (A \Theta T_1) - (A \oplus \hat{T}_2)$$



$set\ difference: A - B = \left\{ x \mid x \in A, x \notin B \right\} = A \cap B^C$

HMT（HIT OR MISS TRANSFORMATION）

- HMT combination can be used to realize any logic operation for any structuring element
- E.g. ：
  - Erosion：mark hit $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ points as 1，miss points as 0；
  - Dilation：mark hit $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ points as 0，miss points as 1



OUTLINE

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MORPHOLOGICAL IMAGE PROCESSING

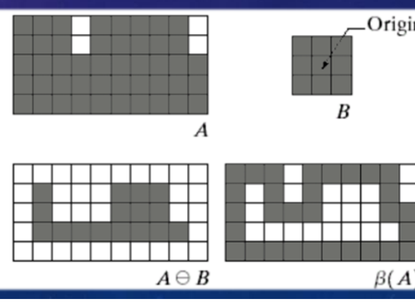
- Boundary extraction :

$$Edge(A) = A - (A\Theta B)$$

$$Edge(A) = (A \oplus B) - A$$

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.



MORPHOLOGICAL IMAGE PROCESSING

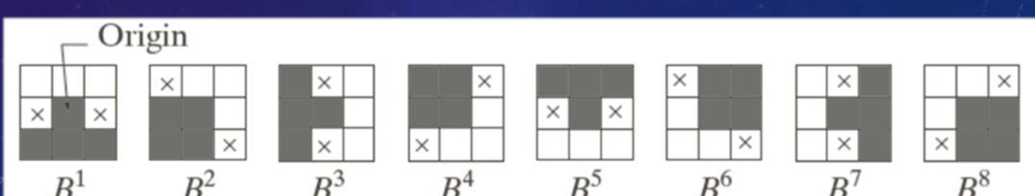
- Thinning :

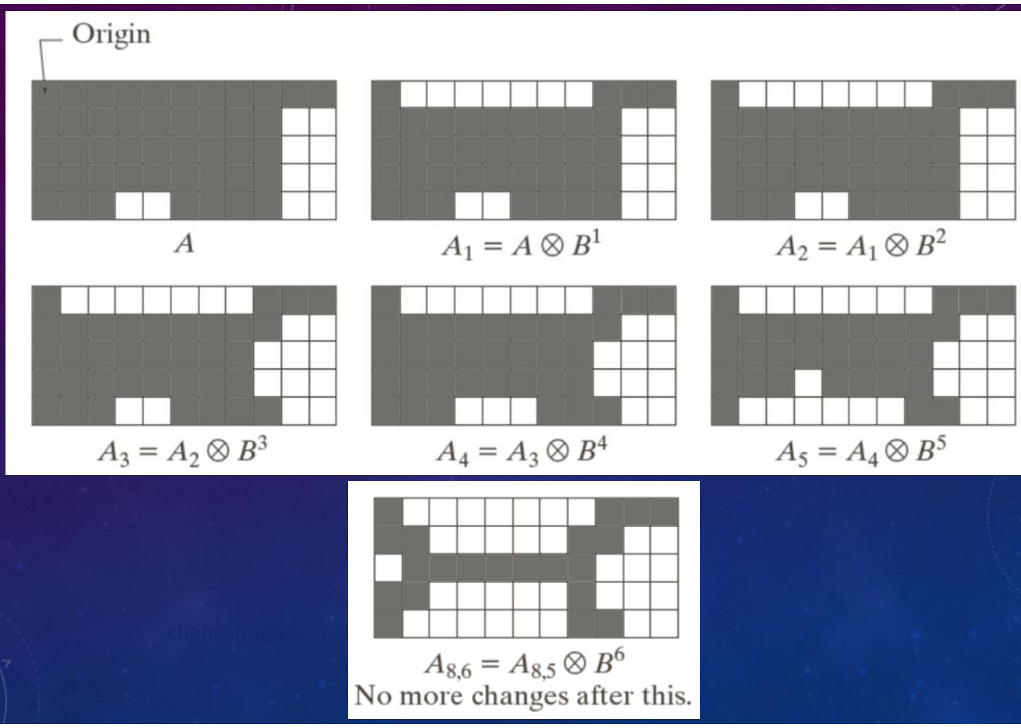
$$A \otimes B = A - A \circledR B$$

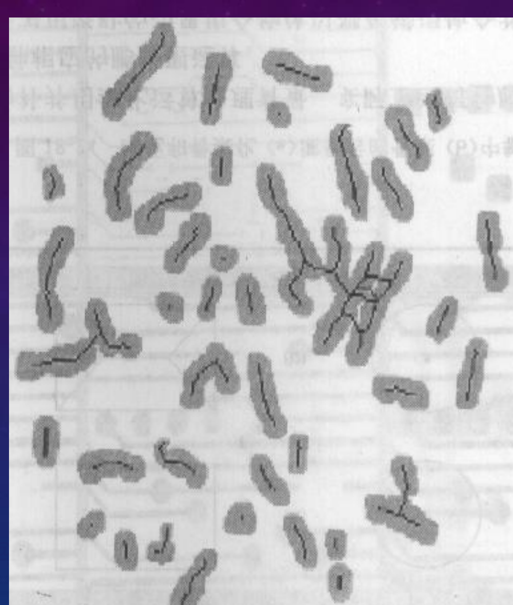
$$= A \cap (A \circledR B)^C$$

$$A \otimes \{B\} = ((((A \otimes B^1) \otimes B^2)\ldots) \otimes B^n)$$

$$where\ B^i\ is\ a\ rotated\ version\ of\ B^{i-1}$$




Origin
A
$A_1 = A \otimes B^1$
$A_2 = A_1 \otimes B^2$
$A_3 = A_2 \otimes B^3$
$A_4 = A_3 \otimes B^4$
$A_5 = A_4 \otimes B^5$
$A_{8,6} = A_{8,5} \otimes B^6$
No more changes after this.



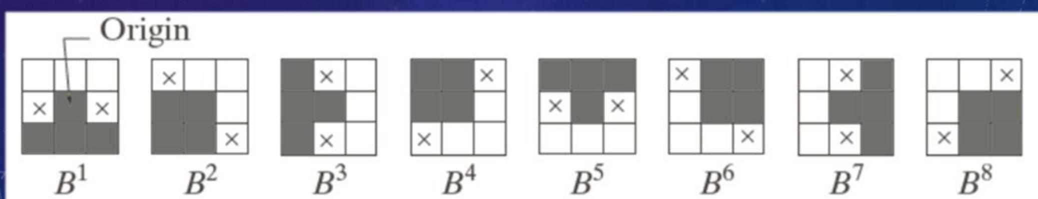
MORPHOLOGICAL IMAGE PROCESSING

- Thickening : $A \odot B = A \cup A \circledR B$

$$A \odot \{B\} = ((((A \odot B^1) \odot B^2) \ldots) \odot B^n)$$

$$A^C \otimes B = (A \odot B)^C$$

where B^i *is a rotated version of* B^{i-1}



MORPHOLOGICAL IMAGE PROCESSING

- Thickening :

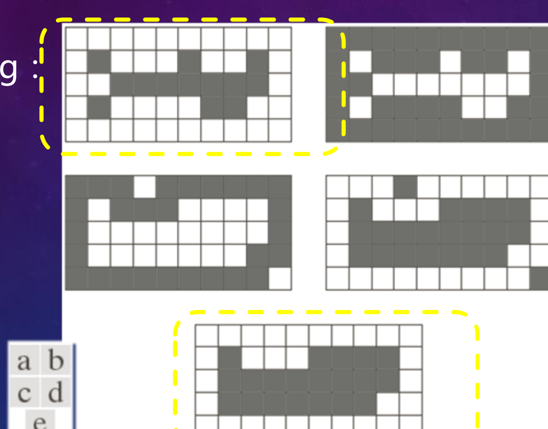
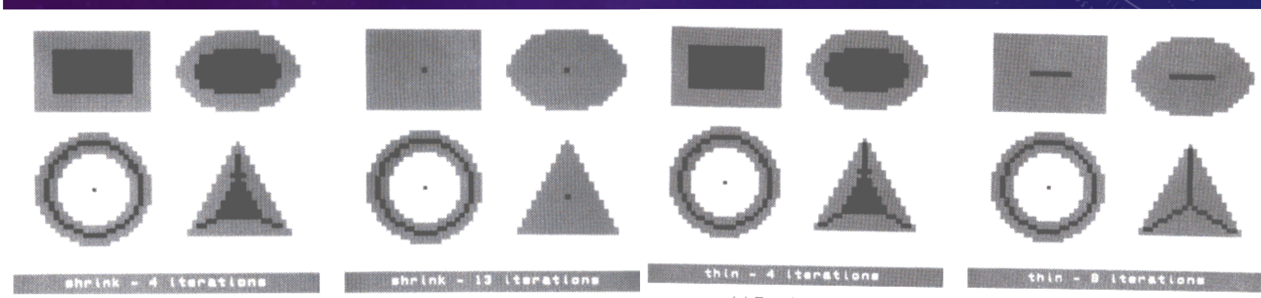


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

MORPHOLOGICAL IMAGE PROCESSING

- Shrinking
  - Shrink. Erase black pixels such that an object without holes erodes to a single pixel at or near its center of mass, and an object with holes erodes to a connected ring lying midway between each hole and its nearest outer boundary

MORPHOLOGICAL IMAGE PROCESSING



(*a*) Four iterations (*b*) Thirteen iterations

FIGURE 14.3-2. Shrinking of a binary image.

(*a*) Four iterations (*b*) Eight iterations

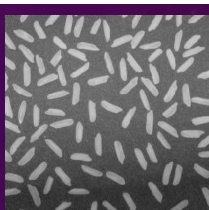
FIGURE 14.3-3. Thinning of a binary image.

Shrinking v.s. Thinning

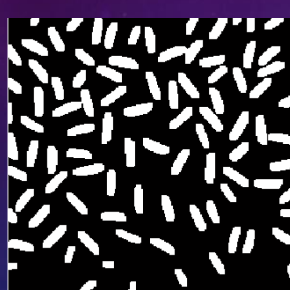
MORPHOLOGICAL IMAGE PROCESSING

- Shrinking
  - Shrinking. Erase black pixels such that an object without holes erodes to a single pixel at or near its center of mass, and an object with holes erodes to a connected ring lying midway between each hole and its nearest outer boundary
  - Thinning. Erase black pixels such that an object without holes erodes to a minimally connected stroke located equidistant from its nearest outer boundaries, and an object with holes erodes to a minimally connected ring midway between each hole and its nearest outer boundary

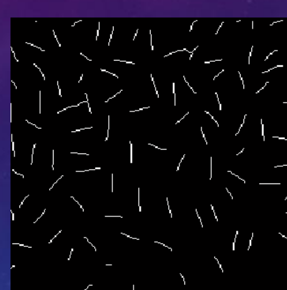
MORPHOLOGICAL IMAGE PROCESSING



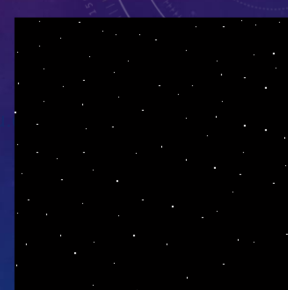
BILEVEL THRESHOLDING



OPENING

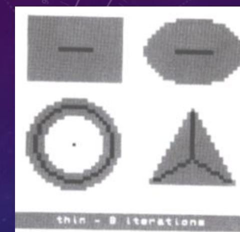


THINNING



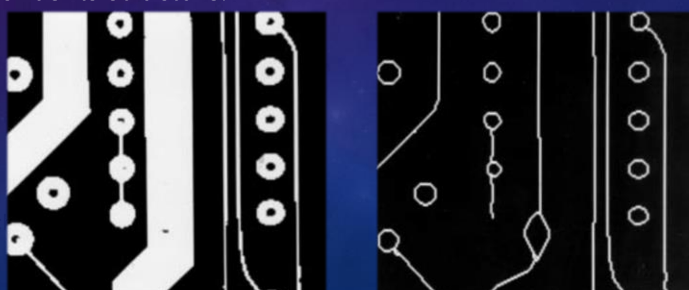
SHRINKING

MORPHOLOGICAL IMAGE PROCESSING



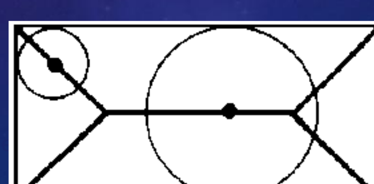
(b) Eight iterations

- Skeletonizing :
  - A skeleton or stick figure representation of an object can be used to describe its structure.



MORPHOLOGICAL IMAGE PROCESSING

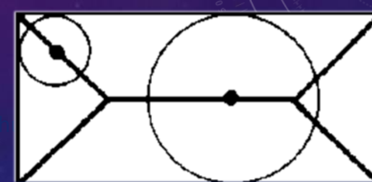
- Skeletonizing :
  - Skeletonizing technique is also called *medial axis transformation* that produces a unique skeleton for a given object.
  - An intuitive explanation of the medial axis transformation is based on the *prairie fire* analogy



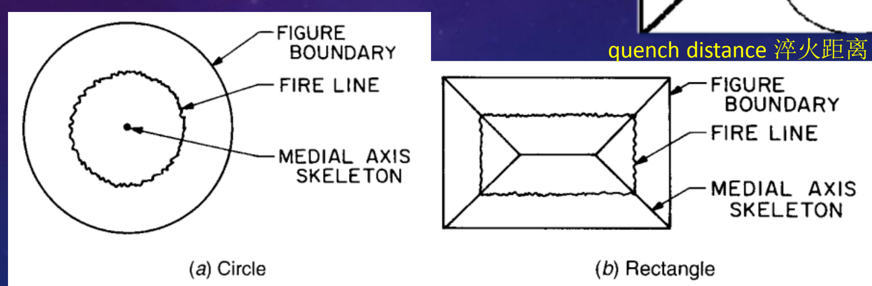
MORPHOLOGICAL IMAGE PROCESSING

- Skeletonizing：
  - the *prairie fire* analogy

circumscribing



quench distance 淬火距离



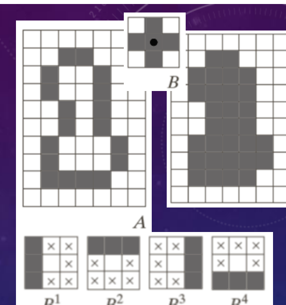
(*a*) Circle (*b*) Rectangle

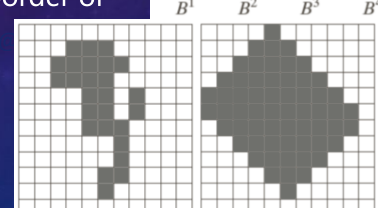
quench point 淬熄点

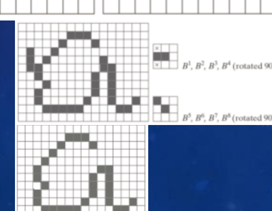
medial axis transformation

MORPHOLOGICAL IMAGE PROCESSING

- Hole filling：
  - A background region surrounded by a connected border of foreground pixels
- Convex hull：
  - The smallest convex set containing the given set
- Pruning：
  - The morphological technique for *cleaning up* parasitic components







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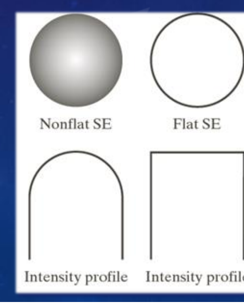
MORPHOLOGICAL IMAGE PROCESSING

- In gray-scale morphology，the input and output value $f(i,j)$ can be any value other than 0 and 1，thus cannot be represented by set expression
- The gray-scale morphology is an extension based on binary image morphology
  - **Or(a,b)   -> max(a,b)**
  - **And(a,b) -> min(a,b)**

MORPHOLOGICAL IMAGE PROCESSING

- There are two kinds of structuring elements in gray-scale morphology: *nonflat* and *flat*
- They are used as probes to examine a given image for specific properties



MORPHOLOGICAL IMAGE PROCESSING

- Image : $A(x,y)$, SE : $B(x,y)$, with domain of definition D_A and D_B respectively
- Erosion :

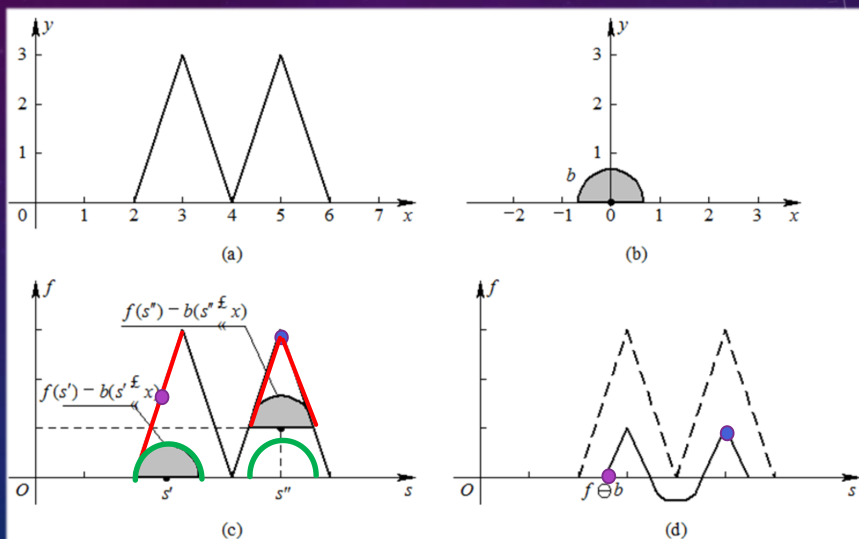
$$E = A\Theta B = \{z \mid B_z \subseteq A\} = \{\vec{z} \mid B + \vec{z} \subseteq A\}$$

$$E(z_1, z_2) = (A\Theta B)_z = \min_{z_1+i, z_2+j \in D_A, i, j \in D_B} [A(z_1+i, z_2+j) \cap B(i,j)]$$

- Generalized erosion :

$$E(z_1, z_2) = (A\Theta B)_z = \min_{z_1+i, z_2+j \in D_A, i, j \in D_B} [A(z_1+i, z_2+j) - B(i,j)]$$

MORPHOLOGICAL IMAGE PROCESSING



MORPHOLOGICAL IMAGE PROCESSING

- Image : $A(x,y)$, SE : $B(x,y)$, with domain of definition D_A and D_B respectively
- Erosion :

$$E = A \Theta B = \{z \mid B_z \subseteq A\} = \{\vec{z} \mid B + \vec{z} \subseteq A\}$$

$$E(z_1, z_2) = (A \Theta B)_z = \min_{z_1+i, z_2+j \in D_A, i, j \in D_B} [A(z_1 + i, z_2 + j) \cap B(i, j)]$$

- Dilation:

$$D = A \oplus B = \{z \mid B_z \cap A \neq \varnothing\} = \{\vec{z} \mid B + \vec{z} \cap A \neq \varnothing\}$$

$$D(z_1, z_2) = (A \oplus B)_z = \max_{i, j \in D_B} [A(z_1 + i, z_2 + j) \cap B(i, j)]$$

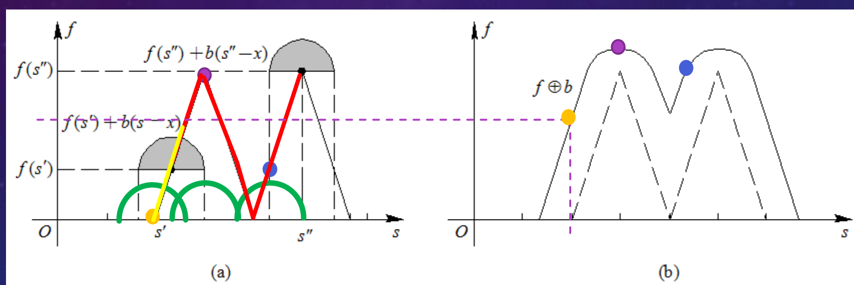
- Generalized erosion :

$$E(z_1, z_2) = (A \Theta B)_z = \min_{z_1+i, z_2+j \in D_A, i, j \in D_B} [A(z_1 + i, z_2 + j) - B(i, j)]$$

- Generalized dilation:

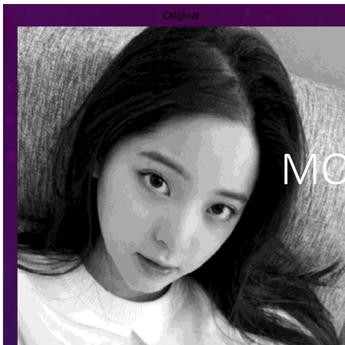
$$D(z_1, z_2) = (A \oplus B)_z = \max_{z_1+i, z_2+j \in D_A, i, j \in D_B} [A(z_1 + i, z_2 + j) + B(i, j)]$$

MORPHOLOGICAL IMAGE PROCESSING

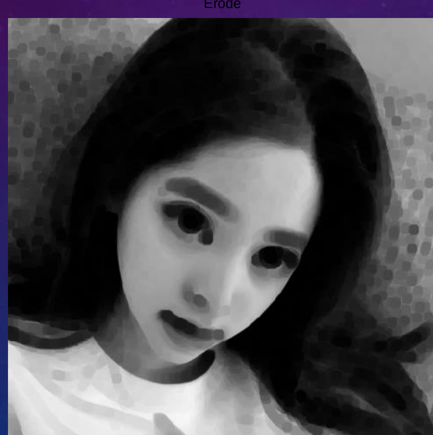


MORPHOLOGICAL IMAGE PROCESSING

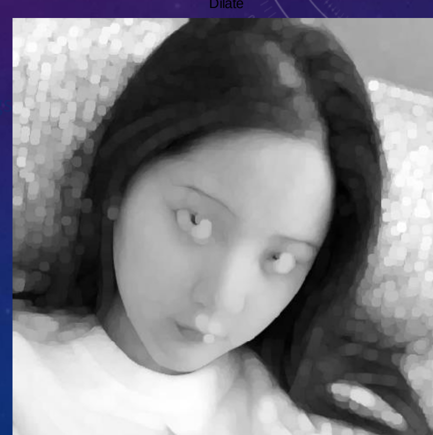
Original



Erode



Dilate



MORPHOLOGICAL IMAGE PROCESSING

- Opening & closing :

$$A \circ B = (A \ominus B) \oplus B$$

$$A \bullet B = (A \oplus B) \ominus B$$

- The expressions have the same form as their binary counterparts
- Opening is used to remove small, bright details, while closing is used to attenuate dark features

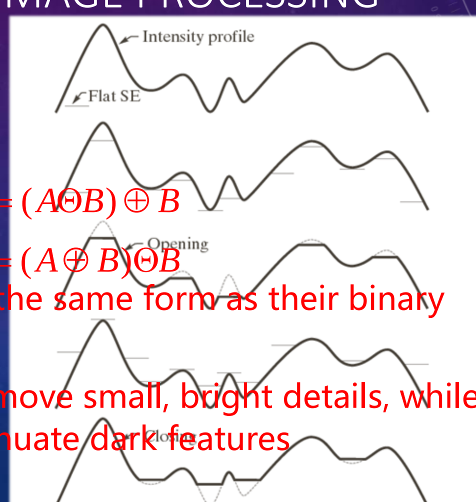
MORPHOLOGICAL IMAGE PROCESSING

- Opening & closing :

$$A \circ B = (A \ominus B) \oplus B$$

$$A \bullet B = (A \oplus B) \ominus B$$

- The expressions have the same form as their binary counterparts
- Opening is used to remove small, bright details, while closing is used to attenuate dark features



a
b
c
d
e

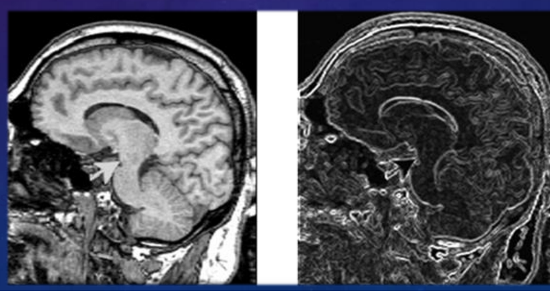
FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

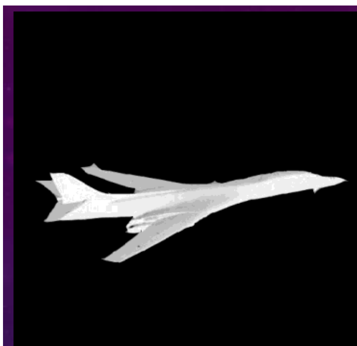
MORPHOLOGICAL IMAGE PROCESSING

- Morphological gradient :

$$Grad(f) = (f \oplus g) - (f \Theta g)$$

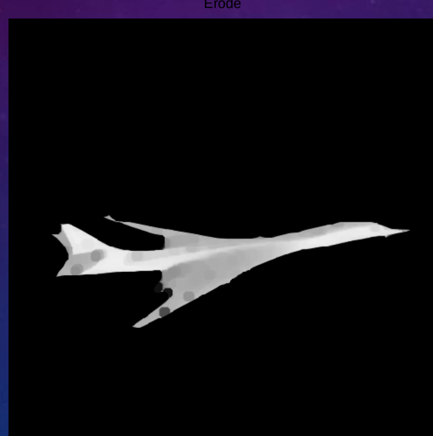
  - Derivative-like (gradient) effect



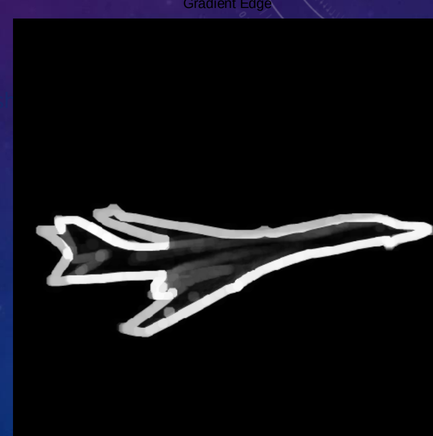


HOLOGICAL IMAGE PROCESSING

Erode



Gradient Edge



TOP HAT TRANSFORM

- f minus its opening:

$$THT(f) = f - (f \circ g)$$

g *is a structuring element*

- THT is used for light objects on a dark background

BOTTOM HAT TRANSFORM

- The closing of f minus f:

$$BHT(f) = (f \bullet g) - f$$

g *is a structuring element*

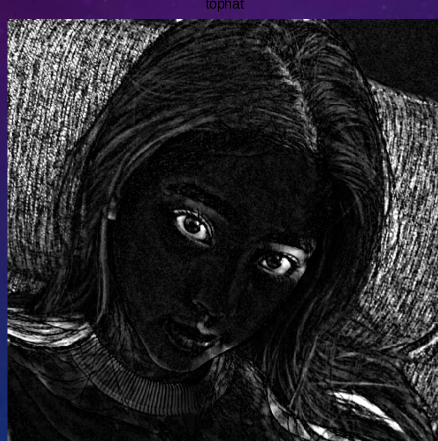
- BHT is used for dark objects on a light background

TOP-HAT AND BOTTOM-HAT TRANSFORMATIONS

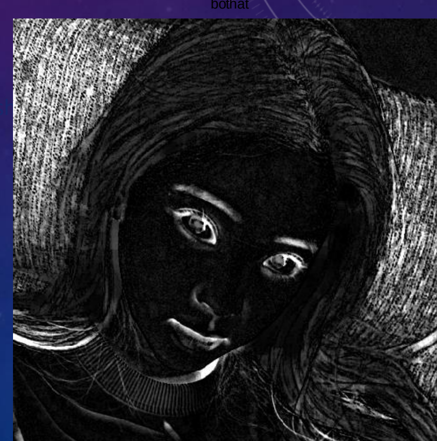

(a)house (b) Top-Hat (b) Bot-hat

AND BOTTOM-HAT TRANSFORMATIONS

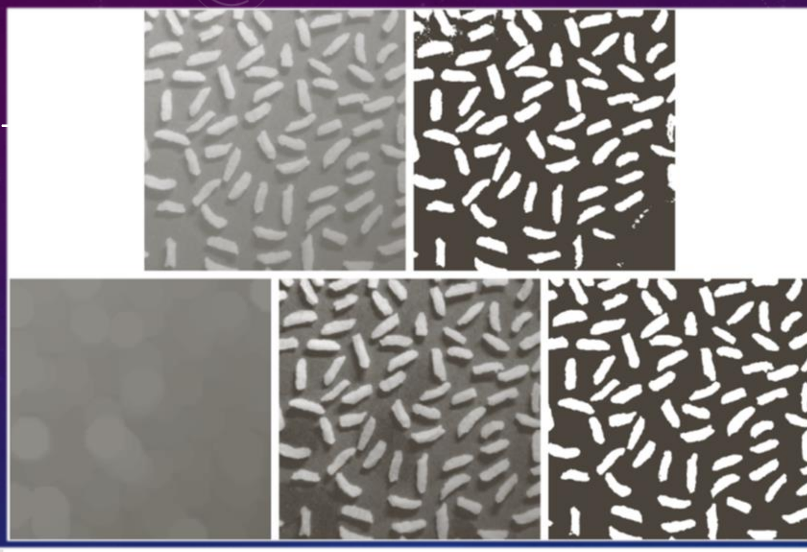
tophat



bothat



TOP- NS



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

SUMMARY

- Overview
- Preliminaries (translation/reflection)
- Binary morphology
  - Erosion and dilation, Opening and closing, The Hit-or-Miss transformation, Basic morphological algorithms(thinning/thickening/skeletonizing/shrink/…)
- Gray-scale morphology
  - Erosion and dilation, Opening and closing, Morphological gradient, Top-hat and bottom-hat transformations