Example Sheet Transformer Efficiency and Three Phase Transformers

2nd November 2021

Q1

1. A resistive load of 1.6Ω is connected across the secondary terminals of a 10kV/400V transformer. If Rc = 50KΩ, Xm = 10KΩ, R1eq = 50Ω and X1eq = 80Ω calculate:
   a. Percentage Voltage regulation
   b. Efficiency
   c. Secondary load resistance such that the transformer operates at its maximum efficiency point.

Note: You should have recognized that this question is an extension from Q2 of the previous example sheet

Determine the % Voltage Regulation and Efficiency of the transformer.

From Q2 of previous example sheet the secondary load voltage (V_{sl}) was calculated to be 380 V. We also calculated the iron (2 kW) and the copper losses (4512 kW). So:

$$\text{Voltage Regulation} = \frac{V_{soc} - V_{sl}}{V_{soc}} \times 100\% = \frac{400 - 380}{400} \times 100\% = 5\%$$

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Output Power + Losses}} \times 100\% = \frac{V_s I_s \cos\phi}{V_s I_s \cos\phi + P_{1req} + P_{Cu}} \times 100\%$$

$$= \frac{380 \times 237.5 \times \cos 0}{380 \times 237.5 \times \cos 0 + 2000 + 4512} \times 100\% = 93.3\%$$

Determine the % Voltage Regulation and Efficiency of the transformer.

From Q2 of previous example sheet the secondary load voltage (V_{sl}) was calculated to be 380 V. We also calculated the iron (2 kW) and the copper losses (4.512 kW). So:

$$\text{Voltage Regulation} = \frac{V_{soc} - V_{sl}}{V_{soc}} \times 100\% = \frac{400-380}{400} \times 100\% = 5\%$$

$$\text{Efficiency} = \frac{\text{Output Power}}{\text{Output Power + Losses}} \times 100\% = \frac{V_s I_s \cos\phi}{V_s I_s \cos\phi + P_{1req} + P_{Cu}} \times 100\%$$

$$= \frac{380 \times 237.5 \times \cos 0}{380 \times 237.5 \times \cos 0 + 2000 + 4512} \times 100\% = 93.3\%$$

Note: PF=1 because V_{SL} and I_{SL} have same phase.

Determine the necessary secondary load resistance such that the transformer operates at its maximum efficiency point.

The maximum efficiency is when the copper losses equal the iron losses. The copper losses are load dependent but the iron losses are not.

So we want:

Copper Losses = Iron Losses = 2000 W

$$\therefore I_1^{\,2} R_{1eqv} = 2000$$

$$I_1^2 = \frac{2000}{50} = 40$$

$$I_1 = \sqrt{40} = 6.32\ \text{A}$$

$$|V_p| = |I_1||Z_T| = |I_1||Z_{1s}^{'} + 50 + j80|$$

$$|Z_T| = \frac{10000}{6.32} = 1582.3$$

$$\therefore |Z_T|^2 = 2503605 = (Z_{1s}^{'} + 50)^2 + 80^2$$

$$So: Z_{1s}^{'\,2} + 100Z_{1s}^{'} + 2500 + 6400 = 2503605$$

Solving this quadratic equation:

$$Z_{1s}^{'} = 1530\ \Omega$$

Actual Secondary Load Resistance:

$$Z_L = \left(\frac{n_s}{n_p}\right)^2 . Z_{1s}^{'} = \left(\frac{1}{25}\right)^2 \times 1530 = 2.45\ \Omega$$

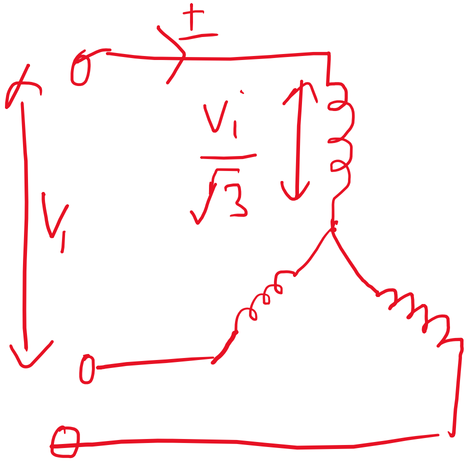
A three-phase transformer has 400 primary turns and 32 turns on the secondary winding. If the supply voltage is 3 kV determine the secondary line voltage with no load when the windings are connected:

$\boxed{N = \frac{N_s}{N_p}}$

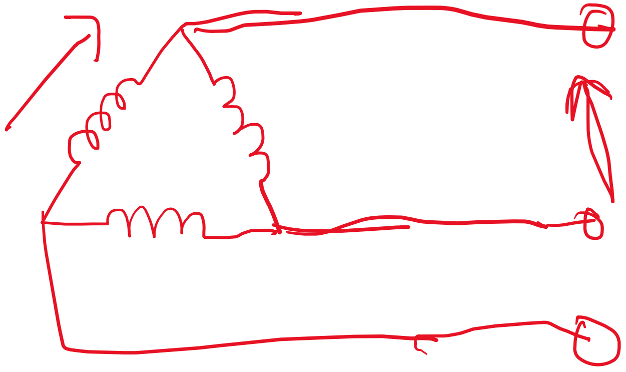
For Δ $V_p = V_L$

$I_L = \sqrt{3} I_{ph}$

Star-Delta



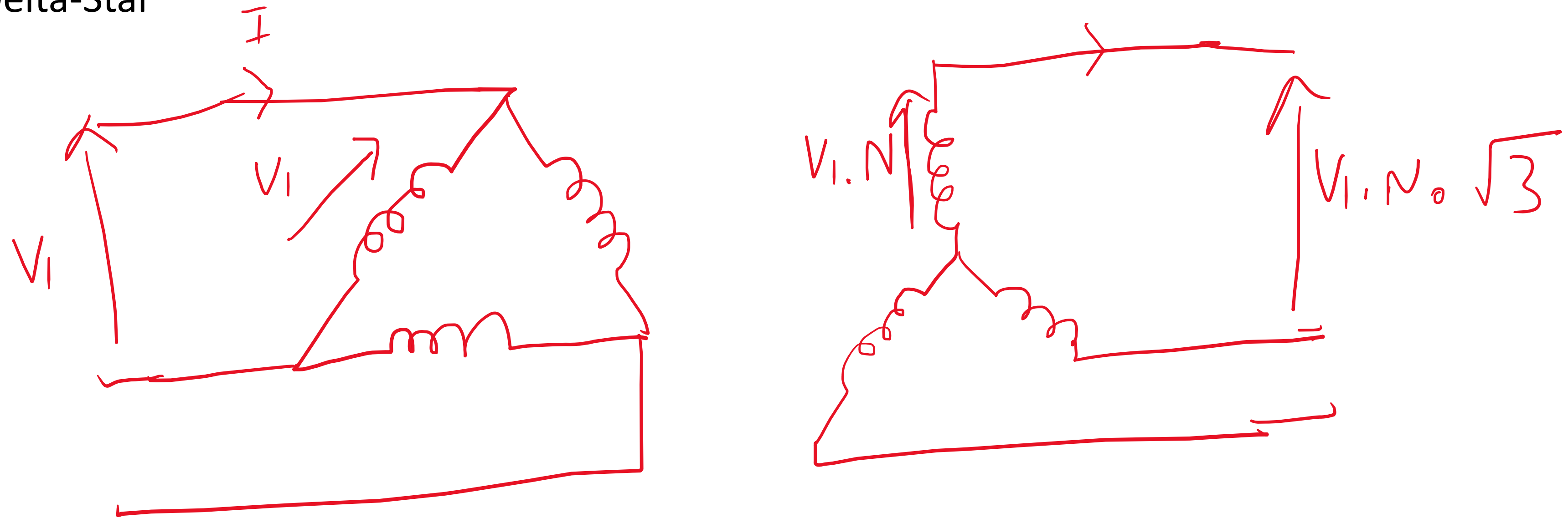
$\frac{I}{N}$



$\frac{V_1 \cdot N}{\sqrt{3}}$

Primary phase voltage = 3000/1.73 = 1734 V
Secondary phase voltage = 1734 × 32/400 = 139 V = secondary line voltage

Delta-Star



Primary phase voltage = 3000 V

Secondary phase voltage = 3000 × 32/400 = 240 V

Secondary line voltage = 240 × 1.73 = 415.2 V

$$N = \frac{N_s}{N_p}$$

Q3 The primary and secondary windings of a 500 kVA transformer have resistances of 0.4 Ω and 0.0021 Ω respectively. The primary and secondary voltages are 11 kV and 400 V, respectively. The core loss is 3 kW and the power factor of the load is 0.85.

The full load secondary current is 500000/400 = 1250 A

The full-load primary current is ≈ 500000/11000 = 45.5 A

The full load primary and secondary copper losses are then:

Primary: $45.5^2 \times 0.4 = 828.1\,\mathrm{W}$

Secondary: $1250^2 \times 0.0021 = 3281.25\,\mathrm{W}$

The Efficiency on Full Load:

The total copper losses on full load will be 4109.35 W

The total loss will be 7109.35 W (4109.35+3000) $\approx$ 7.1 kW

The output power will be 500 x 0.85 = 425 kW

The input power = Output power + losses = 425 + 7.1 = 432.1 kW

So Efficiency $= \left(1 - \frac{7.1}{432.1}\right) = 0.984$ (per unit) = 98.4%

Efficiency on Half Load (212.5 kW):

The copper losses vary as the square of the current so:

Total copper losses on half load are $4109.35 \times 0.5^2 = 1027.34$ W

Iron loss is constant at 3kW

So total half load loss is 4027.34 W ≈4.027 kW

$$\text{Efficiency on half load} = \left(1 - \frac{4.027}{216.527}\right) = 0.9814 \text{ per unit} = 98.14\%$$

* [HALF LOAD MEANS HALF CURRENT $\left(\frac{I_S}{2}\right)$]