Power Electronics

Revision of past exam questions

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Figure Q3 shows a load whose current is switched periodically with a MOSFET. It also shows a plot of the current $i_{\text{load}}(t)$ through the load for one cycle of period T. The switch may be assumed to behave ideally.

(a) Calculate the value of the duty cycle D for this waveform. [1]

(b) Calculate the average value of the current through the load. [4]

(c) Calculate the rms value of the current through the load. [6]

(d) Calculate the average power absorbed by the load. [3]

(e) Calculate the average conduction loss in the MOSFET, taking the channel resistance to be $R_{\text{DS(on)}} = 20\,\text{m}\Omega$. [4]

(f) Calculate the average switching loss in the MOSFET, taking the switching times to be $T_{\text{on}} = T_{\text{off}} = 20\,\text{ns}$. [3]

(g) Explain how the conduction and switching losses would change if the frequency of the waveform were increased by a factor of 10, assuming that the duty cycle and the shape of the current remained the same. [4]

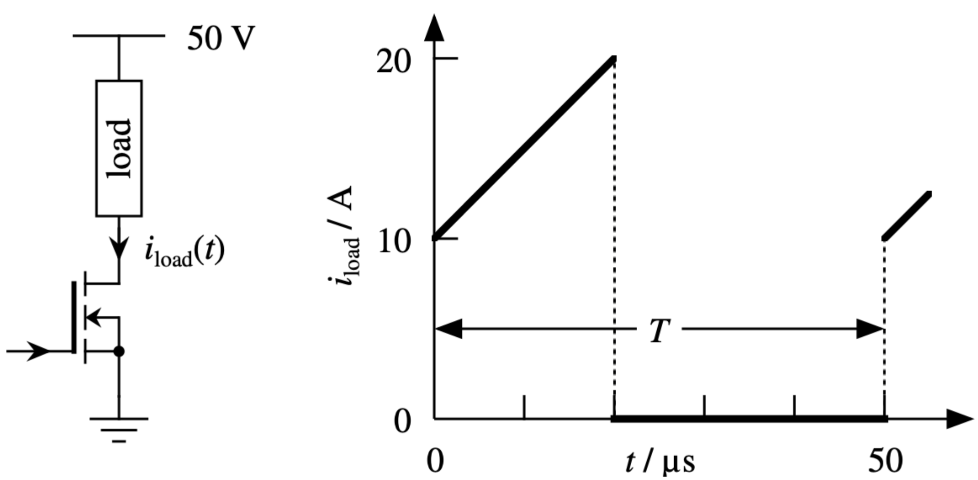


Figure Q3.

(a) Reading from the graph, $T_{\text{on}} = 20\,\mu\text{s}$ and $T = 50\,\mu\text{s}$ so $D = 20/50 = 0.4$.

(b) Geometrically, the area under $i_{\text{load}}(t)$ for times between 0 and $20\,\mu\text{s}$ is $(15\,\text{A}) \times (20\,\mu\text{s}) = 300\,\text{A}\,\mu\text{s}$. The period is $50\,\mu\text{s}$ and dividing the two gives

$$I_{\text{ave}} = \frac{300\,\text{A}\,\mu\text{s}}{50\,\mu\text{s}} = 6.0\,\text{A}. \tag{9}$$

We need an expression for $i(t)$. Measure i in A and t in μs. The function is clearly a straight line of the general form $i = mt + c$. It passes through 10 A at $t = 0$ so $i = mt + 10$. It also passes through 20 A at $t = 20$ and substituting these values gives $20 = 20m + 10$ so $10 = 20m$ and $m = 0.5$. Therefore $i = 0.5t + 10$.

(c) The rms value of the current *must* be calculated with an integral. The general expression is

$$I_{\text{rms}}^2 = \frac{1}{T}\int_0^T [i(t)]^2\,\mathrm{d}t\,. \tag{15}$$

Using the form of $i(t)$ derived above gives

$$\begin{aligned}
I_{\text{rms}}^2 &= \frac{1}{50}\int_0^{20}\left(\frac{t}{2}+10\right)^2\mathrm{d}t && (16)\\
&= \frac{1}{50}\int_0^{20}\left(\frac{t^2}{4}+10t+100\right)\mathrm{d}t && (17)\\
&= \frac{1}{50}\left[\frac{t^3}{12}+\frac{10t^2}{2}+100t\right]_0^{20} && (18)\\
&= \frac{1}{50}\left[\frac{8000}{12}+\frac{4000}{2}+2000\right] && (19)\\
&= \frac{1}{50}\frac{14000}{3} = \frac{280}{3} = 93.3\,\text{A}^2 && (20)
\end{aligned}$$

Taking the square root gives $I_{\text{rms}} = 9.66\,\text{A}$.
[There is no short cut, definitely not a factor of $\sqrt{2}$; you can't avoid the integral. Make sure that you square $i(t)$ correctly.]

(d) We found I_{ave} above so $P_{\text{ave}} = V_{\text{supply}} I_{\text{ave}} = (50\,\text{V}) \times (6.0\,\text{A}) = 300\,\text{W}$.

(e) The channel of a MOSFET behaves as a resistance so the average power loss is $P_{\text{cond}} = R_{\text{DS(on)}} I_{\text{rms}}^2 = (20\,\text{m}\Omega) \times (9.66\,\text{A})^2 = 1.87\,\text{W}$.

(f) Use the expression provided for the average switching loss in the MOSFET.

$$\begin{aligned}
P_{\text{switching}} &= \frac{f_{\text{switching}} V_{\text{DS(off)}}}{2}(T_{\text{on}} I_{\text{on}} + T_{\text{off}} I_{\text{off}}) && (26)\\
&= \frac{(20\,\text{kHz}) \times (50\,\text{V})}{2}[(20\,\text{ns}) \times (10\,\text{A}) + (20\,\text{ns}) \times (20\,\text{A})] && (27)\\
&= 0.3\,\text{W}. && (28)
\end{aligned}$$

(g) Conduction losses would stay the same if the frequency of the waveform were changed because they depend only on the RMS value of current. This depends on the shape of the current waveform, which remains the same.
Switching losses are proportional to frequency so they would rise by a factor of 10 to 3.0 W.
[Explanations are required, not just the numbers.]

Figure Q4 shows a rectifier and smoothing capacitor feeding a resistive load. The output of the transformer is specified as 12.0 V RMS at 50 Hz. You may assume that all components are perfect and neglect the voltage drop across the diodes.

(a) What type of rectifier circuit is this? List two advantages and two disadvantages of this particular circuit. [4]

(b) Sketch and label the waveform of the voltage $v_{\text{load}}(t)$ across the load that would be seen if there were no smoothing capacitor. [2]

(c) What would be the peak and rms values of $v_{\text{load}}(t)$ under these conditions? [2]

(d) Sketch and label the waveform of the voltage $v_{\text{load}}(t)$ across the load assuming that the smoothing capacitor has a very large capacitance. [2]

(e) What would be the peak and rms values of $v_{\text{load}}(t)$ under these conditions? [2]

(f) The load has a resistance of 100 Ω and the ripple voltage across it must not exceed 0.5 V peak-to-peak. Calculate the value of smoothing capacitance required. You may assume that the capacitor provides current to the load for each complete cycle of the rectified voltage (in other words it is recharged instantaneously). [5]

(g) Sketch and label the waveform of the current $i_{\text{rect}}(t)$ drawn from the rectifier by the load and smoothing capacitor in the previous part of the question. Do not attempt to calculate exact values but show estimates of the scales for time and current. [4]

(h) Suppose that the ripple voltage had to be reduced further. Suggest two ways in which this could be done and comment on their impact on the behaviour of the circuit. [4]

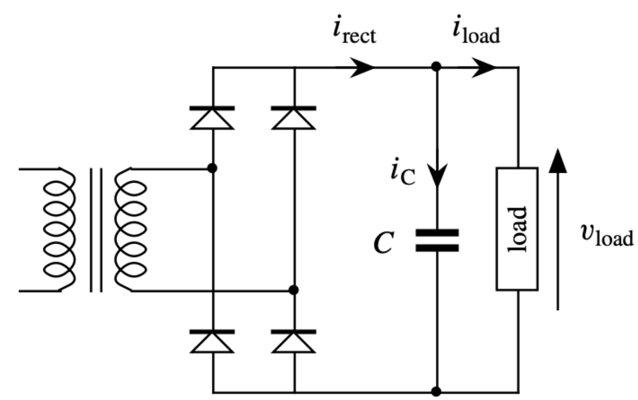


Figure Q4.

(a) Draw the symbols of any three typical fully-controllable power switches, clearly labeling their names, and briefly compare their maximum power ratings and switching frequency. [6]

(b) List and briefly describe at least four advantages of power electronic conversion over other power conversion ways (e.g. transformers) [4]

(c) Draw an unpolarized voltage (i.e. turn-off) snubber circuit for the power switches, and briefly describe how the circuit operates. [4]

(d) For an inductor load L=1mH, the steady-state inductor current waveform is shown in Figure Q1,

1) Calculate the average current and RMS current for the inductor. [3]

2) Derive and sketch the corresponding inductor voltage, labelling the axes clearly. [3]

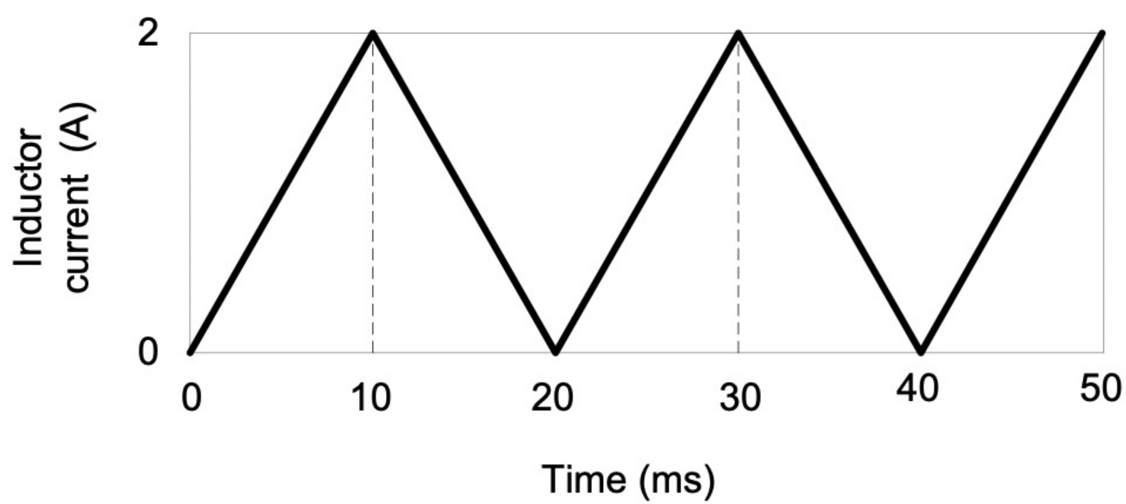
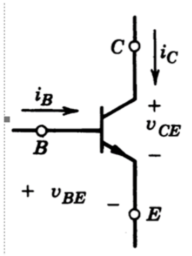
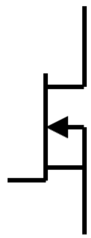


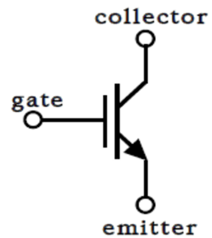
Figure Q1

Solution:

(a) [6]







GTR MOSFET IGBT

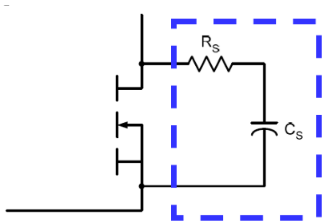
Power: IGBT (middle-high), MOSFET (low-middle), GTR (small-middle)

Frequency: MOSFET (high, up to 1MHz), IIGBT (up to 100kHz), GTR (up to10kHz)

(b) [4]

1. Efficient: typically in excess of 90% and up to 98% for large systems
2. Flexible: DC-DC, DC-AC, AC-DC, AC-AC
3. High power density, cost-effective: light and small, cheap
4. High performance conditioning: fast, accurate, robust
5. Static and quiet: no mechanical rotation
6. Reliable: no failures over semiconductor device lifetime
7. Switching Frequency: up to 1MHz
8. Power Level: controlled power levels from milli-watts (e.g. portable appliances) through to giga-watts (e.g. high voltage dc transmission).

(c)



When the switch opens, the capacitor Cs charges, the dv/dt across the switch is limited.

(d)

[3]

$$I_{avg} = \frac{1}{0.01}\int_0^{0.01}(2/0.01)t\,dt = \frac{200\times 0.5\times t^2}{0.01}\Bigg|_0^{0.01} = 1$$

$$I_{rms} = \sqrt{\frac{1}{0.01}\int_0^{0.01}[(2/0.01)]^2 t^2 dt} = \sqrt{100\int_0^{0.01}40000t^2 dt} = \sqrt{4\times 10^6\int_0^{0.01}t^2 dt} = \sqrt{\frac{4}{3}}$$

(e) [3]

$$v_L = L\frac{di_L}{dt} = 0.001\frac{di_L}{dt} = \begin{cases} 0.2\text{V} & 0<t<0.01 \\ -0.2\text{V} & 0.01<t<0.02 \end{cases}$$

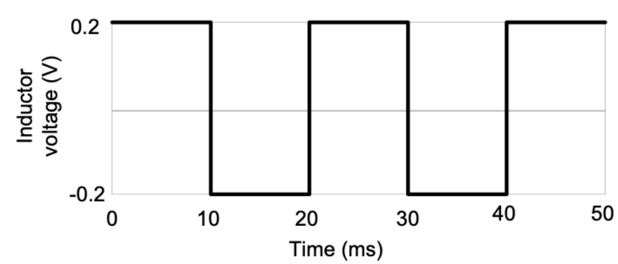


Figure Q3 shows a load whose current is switched periodically with a MOSFET. It also shows a plot of the current $i_{load}(t)$ through the load for one cycle of period T. The switch may be assumed to behave ideally.

(a) Calculate the value of the duty cycle D for this waveform. [1]

(b) Calculate the average value of the current through the load. [3]

(c) Calculate the RMS value of the current through the load. [6]

(d) Calculate the average power absorbed by the load. [3]

(e) Calculate the average conduction loss in the MOSFET, taking the channel resistance to be $R_{DS(on)} = 50\text{m}\Omega$. [3]

(f) Calculate the average switching loss in the MOSFET, taking both the switch-on and switch-off times to be $T_{on} = 20$ ns and $T_{off} = 30$ ns respectively. [3]

(g) Assume the MOSFET is bolted to a heatsink of thermal resistance θ_{SA}=1.5°C/W, the thermal resistance θ_{CS} of the contact between the MOSFET case and the heatsink is 0.4°C/W, the MOSFET's junction-to-case thermal resistance θ_{JC} is 1°C/W and the MOSFET is operating in free air with an ambient temperature of 25°C, draw the thermal circuit diagram of the MOSFET and calculate the heatsink temperature, case temperature and junction temperature. [6]

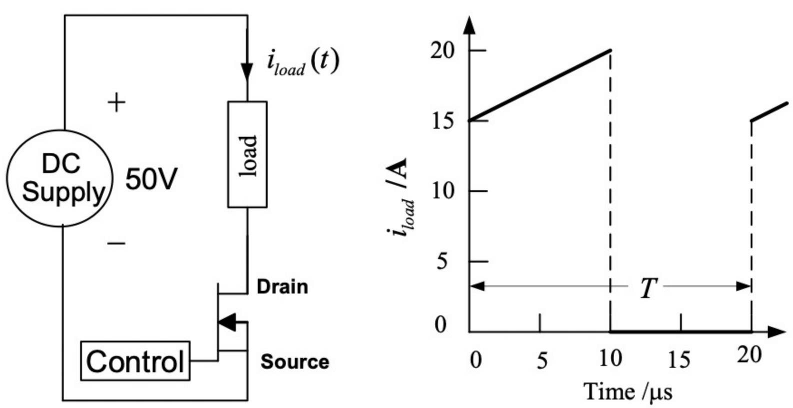


Figure Q3

(a) $D = 10/20 = 0.5$

(b) $I_{avg} = \dfrac{1}{2\times10^{-5}} = \left[\int_{0}^{1\times10^{-5}} \left(15+(20-15)t/\left(1\times10^{-5}\right)\right)dt + 0\right] = 8.75\text{A}$

(c) $I_{rms} = \sqrt{\dfrac{1}{2\times10^{-5}} \int_{0}^{1\times10^{-5}} \left(15+5/\left(1\times10^{-5}\right)t\right)^2 dt}$

$$= \sqrt{\frac{1}{2\times10^{-5}} \int_{0}^{1\times10^{-5}} \left(225+\frac{150t}{1\times10^{-5}}+\frac{25t^2}{\left(1\times10^{-5}\right)^2}\right)dt}$$

$$= \sqrt{\frac{225\times10^{-5}+150\times\left(1\times10^{-5}\right)/2+25\times\left(1\times10^{-5}\right)/3}{2\times10^{-5}}}$$

$$= \sqrt{(225+75+25/3)/2} = 12.4\text{A}$$

(d) $P_{avg} = \dfrac{1}{2\times10^{-5}} \int_{0}^{1\times10^{-5}} vidt = \dfrac{1}{2\times10^{-5}} \left[\int_{0}^{1\times10^{-5}} 50idt + 0\right]$

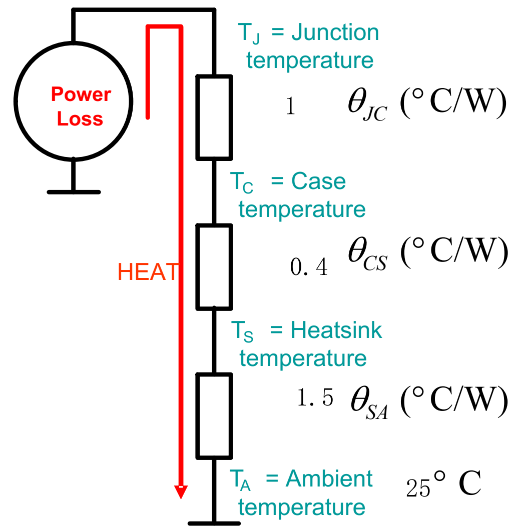
$$= \frac{50}{2\times10^{-5}} \left[\int_{0}^{1\times10^{-5}} idt + 0\right] = 50I_{avg} = 437.5\text{W}$$

(e) $P_{conduction} = I_{rms}^2 R = (12.4)^2 \times 50\times10^{-3} = 7.688\text{W}$

(f) $P_{switching} = P_{on} + P_{off} = \dfrac{1}{20\times10^{-6}}\left(\dfrac{1}{2}\times50\times15\times20\times10^{-9} + \dfrac{1}{2}\times50\times20\times30\times10^{-9}\right)$

$$= \frac{1}{20\times10^{-6}}\left(7500\times10^{-9}+15000\times10^{-9}\right) = 1.125\text{W}$$

(g)



$$P_{loss} = P_{conduction} + P_{switching} = 1.125 + 7.668 = 8.813\text{W}$$

$$T_S = 25 + 8.813\times1.5 = 38.22$$

$$T_C = T_S + 8.813\times0.4 = 41.75$$

$$T_J = T_C + 8.813\times1 = 50.56$$

Q4 A switched-mode buck-boost converter is required to deliver 12.0V at 500mA from a 5.0V supply. It runs in continuous conduction mode at 50 kHz. The circuit is shown in figure Q4 and has a 100 μH inductor. You may assume that all components are ideal and that the input and output are smoothed so effectively that their voltages may be treated as constant.

(a) Calculate the duty cycle required for the converter. [1]

(b) Calculate the average input current from the supply. [2]

(c) What are the voltages across the inductor during charging and discharging? [2]

(d) Calculate the values of di_L/dt during charging and discharging. [2]

(e) What is the value of the average current through the inductor? [3]

(f) What are the minimum and maximum values of $i_L(t)$ [4]

(g) Sketch $i_L(t)$, $i_{in}(t)$, and $v_L(t)$. An accurate scale drawing is not required but you must show the numerical values at all important points. [6]

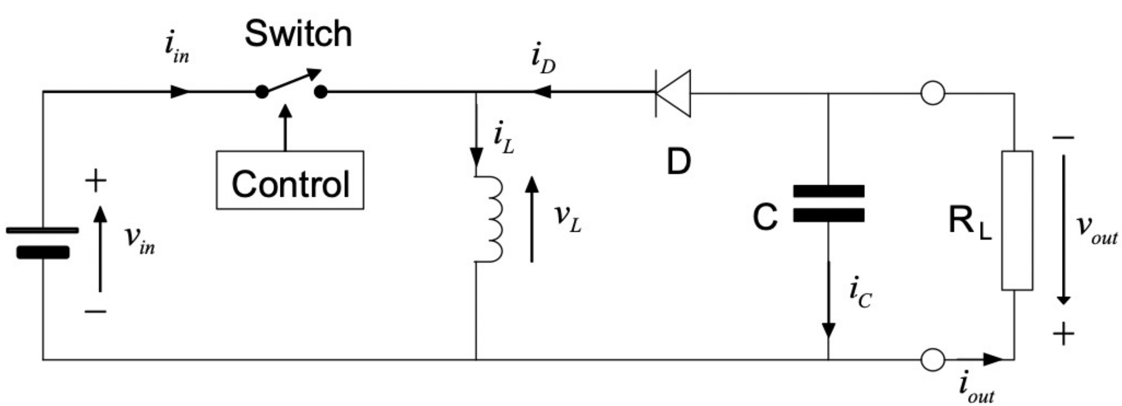


Figure Q4

(a) $$V_{out} = \frac{\Phi}{1-\Phi} V_{In} \Rightarrow 12 = \frac{\Phi}{1-\Phi} \times 5 \Rightarrow \Phi = 0.706$$

(b) $$V_{out} I_{out} = V_{In} I_{in} \Rightarrow I_{in} = \frac{V_{out}}{V_{In}} \times I_{out} \Rightarrow I_{in} = 2.4 \times 0.5 = 1.2\text{A}$$

(c) & (d)

$$\text{Charging: } V_L = V_{in} = 5\text{V} \;,\; \frac{di_L}{dt} = \frac{v_L}{L} = \frac{5}{100\times10^{-6}} = 5\times10^{4}\,\text{A}/s$$

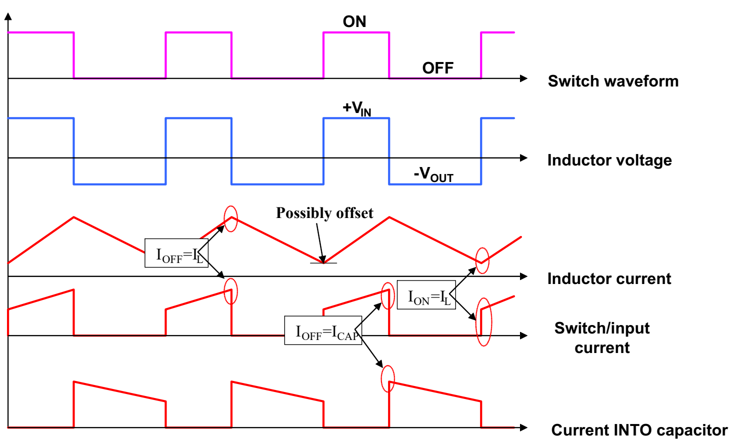
$$\text{Discharging: } V_L = -V_{out} = -12\text{V} \,,\; \frac{di_L}{dt} = \frac{v_L}{L} = \frac{12}{100\times10^{-6}} = 1.2\times10^{5}\,\text{A}/s$$

(e) $$I_L = \frac{1}{1-\Phi} I_{out} = \frac{1}{1-0.706} \times 0.5 = 1.7$$

(f) $$I_{L\max} = I_L + \frac{1}{2}\left(5\times10^4 \times \frac{1}{50000} \times 0.706\right) = 1.7 + 0.353 = 2.053\text{A}$$

$$I_{L\min} = I_L - \frac{1}{2}\left(5\times10^4 \times \frac{1}{50000} \times 0.706\right) = 1.7 - 0.353 = 1.347\text{A}$$

(g)



(b)
(b)

Q5 Figure Q5 shows a single-phase PWM inverter with its bipolar PWM drive circuit, where the carrier frequency $f_{carrier}$ is much higher than the control signal frequency f, the control signal voltage $v_{control} = m_a \sin(2\pi f t)$, and the peak values of the triangle carrier signal are 1 and -1.

(a) If $0 < m_a \leq 1$, determine the output voltage v_o of the single-phase inverter. We assume the high frequency harmonic components in the voltage v_o are neglected. [6]

(b) What is the 'square-wave' operation mode of the inverter? Indicating its benefits and disadvantages. [4]

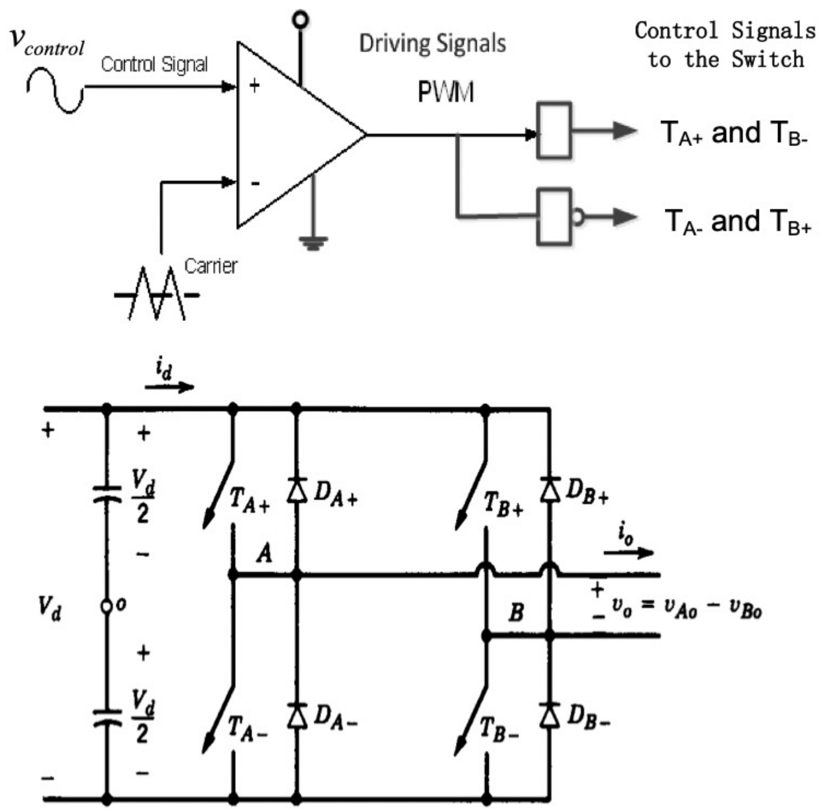


Figure Q5

(a) $v_{out} = \frac{v_{control}}{\hat{v}_{carrier}} \times v_d = v_d m_a \sin(2\pi ft)$ *[6]*

(b) If $m_a >> 1$*, overmodulation, the RMS value of output voltage reach its maximum value, however, unwanted harmonics distortions is very serious (may lead torque ripples of motor), switching frequency is the lowest [4]*

Q3 A transistor is mounted on a heatsink, with the thermal resistances being given in the table below.

(a) Draw the thermal circuit diagram for the transistor, indicating the names of every thermal resistance clearly. [4]

(b) Calculate the heatsink temperature, case temperature and junction temperature for the transistor if the ambient temperature is 25°C and the heatsink thermal resistance is 1°C/W? [6]

Name	Transistor
Junction-Case resistance	θ_{JC}=0.2°C/W
Case-Sink resistance	θ_{CS}=0.1°C/W
Power loss heat	60W

Table Q3

Figure Q3 shows a load whose current is switched periodically with a MOSFET. It also shows a plot of the current $i_{load}(t)$ through the load for one cycle of period T. The switch may be assumed to behave ideally.

(a) Calculate the duty cycle D and average value for this load current waveform. [4]

(b) Calculate the average power consumption by the load. [3]

(d) Calculate the average conduction loss in the MOSFET, taking the channel resistance to be $R_{DS(on)} = 20\text{m}\Omega$. [4]

(e) Calculate the average switching loss in the MOSFET, taking both the switching-on and switching-off times to be $T_{on} = T_{off} = 20$ ns. [4]

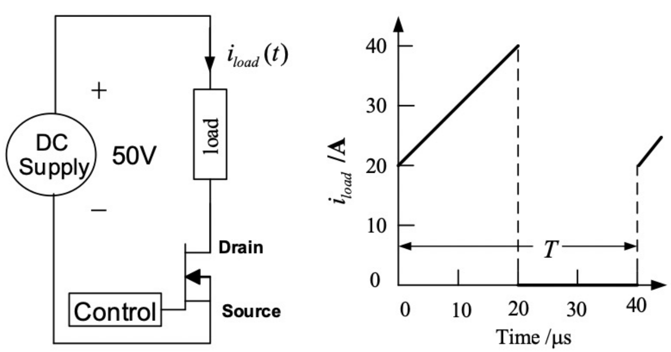
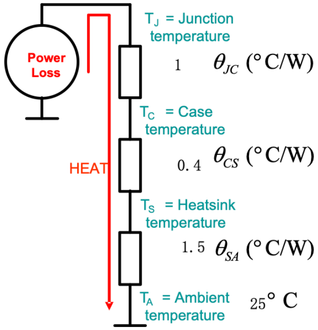


Figure Q3

(a) (b)



$$P_{loss} = 60\text{W}$$
$$T_S = 25 + 60\times 1 = 85°C$$
$$T_C = T_S + 60\times 0.1 = 91°C$$
$$T_J = T_C + 60\times 0.2 = 107°C$$

(a) $D = 20/40 = 0.5$

(b) $$I_{avg} = \frac{1}{2\times 10^{-5}} = \left[\int_0^{1\times 10^{-5}} \left(20 + (40-15)t/\left(2\times 10^{-5}\right)\right)dt + 0\right] = 15\text{A}$$

$$I_{rms} = \sqrt{\frac{1}{4\times 10^{-5}} \int_0^{2\times 10^{-5}} \left(20 + 20/\left(2\times 10^{-5}\right)t\right)^2 dt}$$

$$= \sqrt{\frac{1}{4\times 10^{-5}} \int_0^{2\times 10^{-5}} \left(400 + \frac{200t}{1\times 10^{-5}} + \frac{100t^2}{\left(1\times 10^{-5}\right)^2}\right)dt}$$

$$= \sqrt{(200 + 100 + 50/3)/2} = 17.8\text{A}$$

(d) $$\text{P}_{load} = \frac{1}{40\times 10^{-6}} \int_0^{20\times 10^{-6}} 50 i dt = I_{avg} * 50 = 750W$$

(e) $$\text{P}_{conduction} = I_{rms}^2 R_{DS(on)} = 6.33W$$

$$P_{switching} = \frac{1}{40\times 10^{-6}} \frac{1}{2} 50\left(20\times 20\times 10^{-9} + 40\times 20\times 10^{-9}\right) = 0.75W$$

A switched-mode boost converter is required to deliver 15.0V at 1A from a 5.0V supply. It runs in continuous conduction mode at 50 kHz. The circuit is shown in Figure Q4-1 and has a 100 µH inductor. You may assume that all components are ideal and that the input and output are smoothed so effectively that their voltages may be treated as constant.

(a) Calculate the duty cycle required for the converter. [2]

(b) Calculate the average input current from the supply. [2]

(c) Calculate the inductor voltages during charging and discharging of the inductor respectively [4]

(e) Calculate the minimum and maximum values of $i_L(t)$ [4]

(f) Sketch $i_L(t)$ and $v_L(t)$. An accurate scale drawing is not required but you must show the numerical values at all important points. [4]

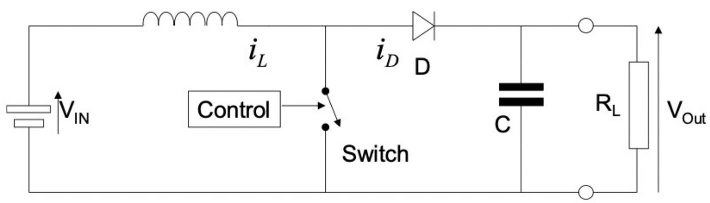


Figure Q4-1

Figure Q4-2 shows a single-phase PWM converter with its bipolar PWM drive circuit, where and dc bus voltage V_d=100V, the carrier frequency $f_{carrier}$ is much higher than the control signal frequency f, the control signal $v_{control} = m_a \sin(2\pi ft)$, and the peak values of the triangle carrier signal are 1 and -1.

(g) When $m_a = 0.5$, determine the output voltage v_o of the single-phase converter. We assume the high frequency harmonic components in the voltage v_o is neglected. [3]

(h) When $m_a >> 1$, what will happen to the converter? Indicating its benefits and disadvantages. [3]

(i) If the control signal is changed to $v_{control} = 0.6$V, determine the average output voltage v_o of the single-phase converter. We assume the high frequency harmonic components in the voltage v_o is neglected. [3]

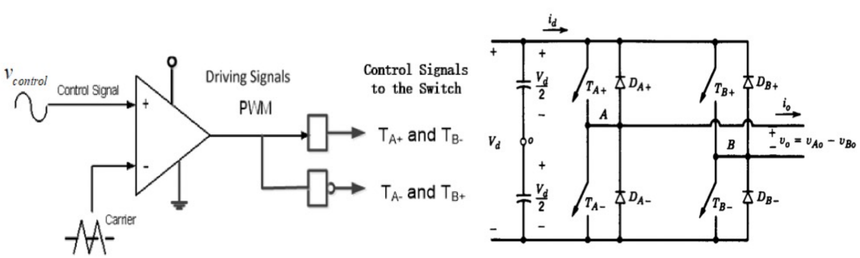


Figure Q4-2

(a)

$$V_{out}=\frac{1}{1-\Phi}V_{In}\Rightarrow 15=\frac{1}{1-\Phi}\times 5\Rightarrow \Phi=0.667$$

(b)

$$V_{out}I_{out}=V_{In}I_{in}\Rightarrow I_{in}=\frac{V_{out}}{V_{In}}\times I_{out}\Rightarrow I_{in}=3\text{A}$$

(c) and (d)

Charging: $V_L=V_{in}=5\text{V}$, $\frac{di_L}{dt}=\frac{v_L}{L}=\frac{5}{100\times 10^{-6}}=5\times 10^{4}\,\text{A}/s$

Discharging: $V_L=-10\text{V}$, $\frac{di_L}{dt}=\frac{v_L}{L}=\frac{10}{100\times 10^{-6}}=1.0\times 10^{5}\,\text{A}/s$

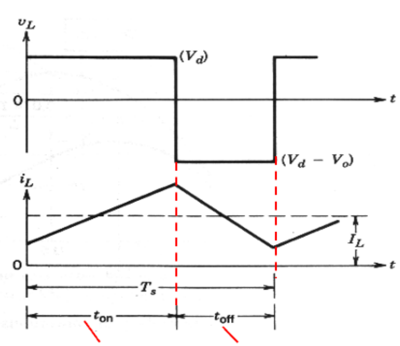
(e)

$$I_L=\frac{1}{1-\Phi}I_{out}=\frac{1}{1-0.667}\times 1=3A$$

$$I_{L\max}=I_L+\frac{1}{2}\left(5\times 10^{4}\times\frac{1}{50000}\times 0.667\right)=3+0.353=2.053\text{A}$$

$$I_{L\min}=I_L-\frac{1}{2}\left(5\times 10^{4}\times\frac{1}{50000}\times 0.667\right)=3-0.353=2.647\text{A}$$

(f)



(g) $v_{out} = \frac{v_{control}}{\hat{v}_{carrier}} \times v_d = 50\sin\left(2\pi ft\right)$ *[3]*

(h) *If $m_a >> 1$, overmodulation, the RMS value of output voltage reach its maximum value, however, unwanted harmonics distortions is very serious (may lead torque ripples of motor), switching frequency is the lowest [3]*

(i) *Vo=60V*

Best of Luck!

Thank you…