DIGITAL IMAGE PROCESSING

IMAGE DIGITALIZATION

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2022-9-23

IMAGE DIGITALIZATION

- Image quality assessm-ent
- Represe-ntation
- Digitalization
  - Sampling
  - Quantization

REPRESENTATION

- Digitalization
  - Sampling

$$f(x,y) \longrightarrow f(m,n)$$

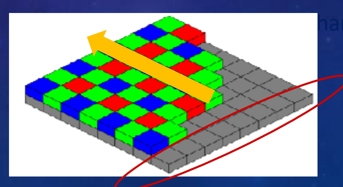
  - Quantization

$$f(x,y) \longrightarrow I(x,y)$$

$$\boldsymbol{f(x,y,t)} = \int_0^{\infty} f(x,y,\lambda,t)\, V(\lambda)\, d\lambda$$

$$f(x,y) \longrightarrow I(m,n)$$

- Sensor, quantizer



REPRESENTATION

pix depth

Full-color/True-color image

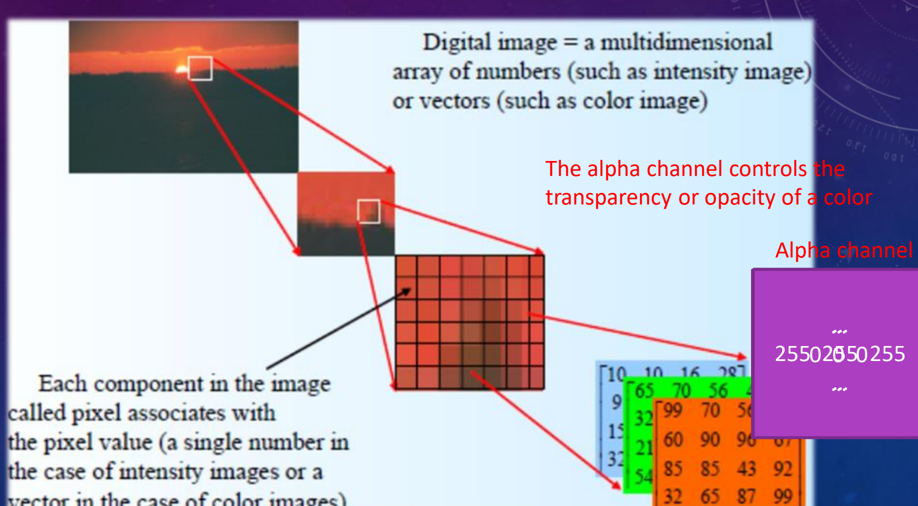
Color Image representation

$(2^8)^3$=16,777,216

ARGB image

The alpha channel controls the transparency or opacity of a color

Alpha channel



REPRESENTATION

- A gray image of size $M \times N$, gray scale level G, with M, N, G satisfying:

$$M=2^m,\quad N=2^n,\quad G=2^k$$

  - How many BYTES are needed to store this image?

$$2^{m+n-3}k$$

k-bit image ------ 2^k intensity levels

IMAGE QUALITY ASSESSMENT

- Fidelity, processed image v.s. original image
- Intelligibility, information conveyed

IMAGE QUALITY ASSESSMENT

- Subjective way

score	Quality
6	**Excellent**
5	**Fine**
4	**Passable**
3	**Marginal**
2	**Inferior**
1	**Unusable**

IMAGE QUALITY ASSESSMENT

- Objective way

For analog images，normalized cross correlation (归一化互相关) function K

$$K = \frac{\int_{-Lx}^{Lx}\int_{-Ly}^{Ly} f(x,y)\hat{f}(x,y)dxdy}{\int_{-Lx}^{Lx}\int_{-Ly}^{Ly} f^2(x,y)dxdy}$$

IMAGE QUALITY ASSESSMENT

- Objective way

For $M\times N$ digital images，*root-mean-square error* (均方根误差)and *mean-square signal-to-noise ratio* (均方信噪比) are often used

original　　processed

$$e_{rms}=\sqrt{\frac{1}{MN}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left\{Q[f(i,j)]-Q[\hat{f}(i,j)]\right\}^2}$$

Preprocessing function

$$Q(f)=K_1\log_b[K_2+K_3 f(i,j)]$$

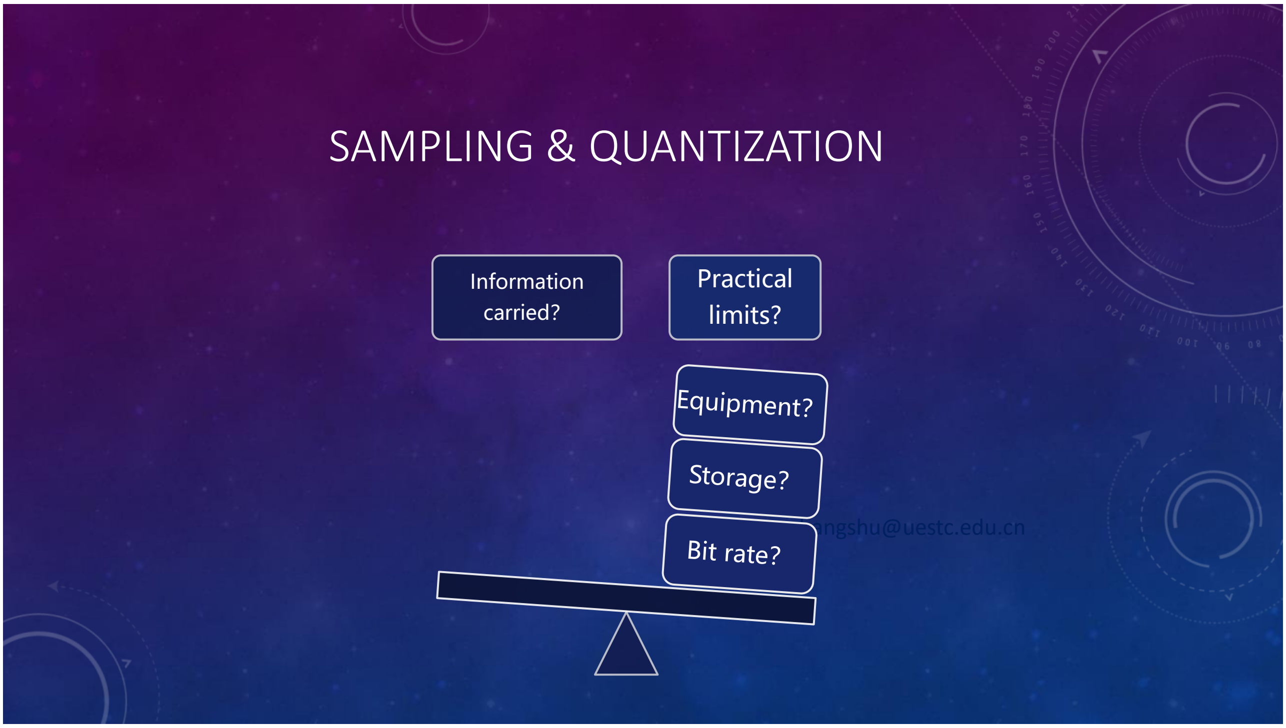
IMAGE QUALITY ASSESSMENT

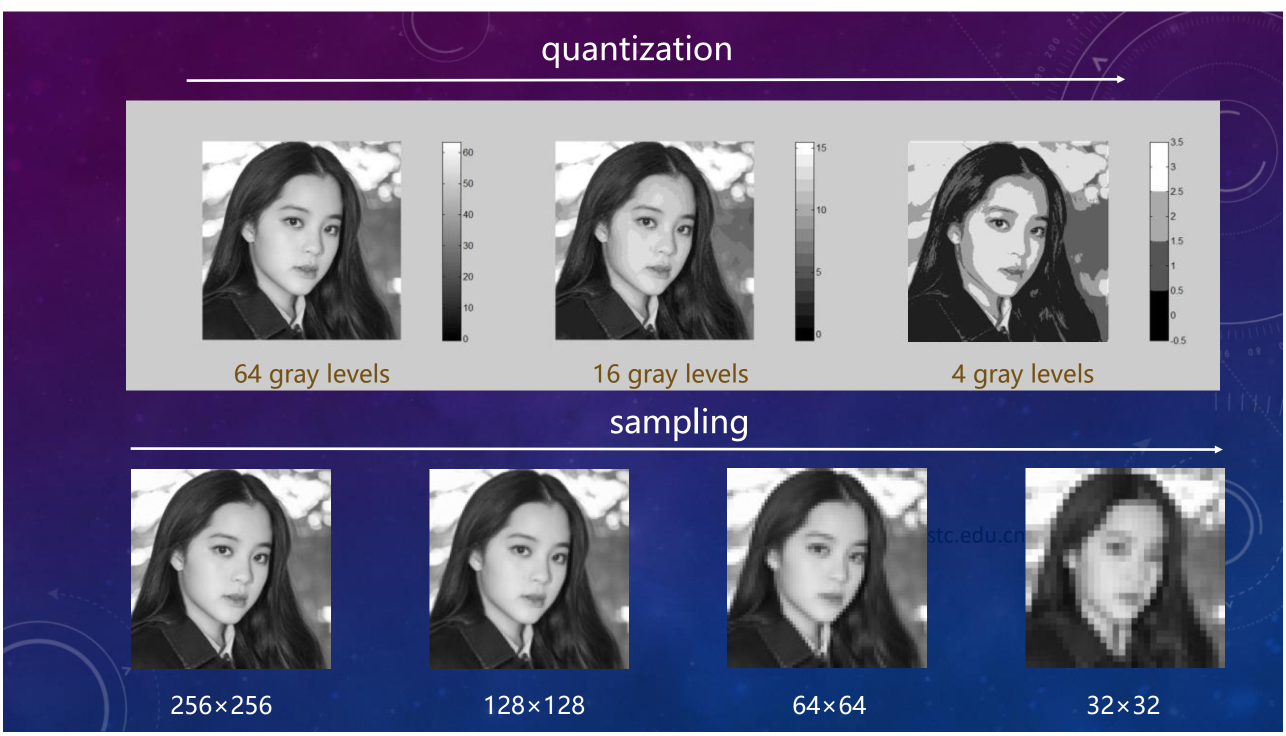
- Objective way

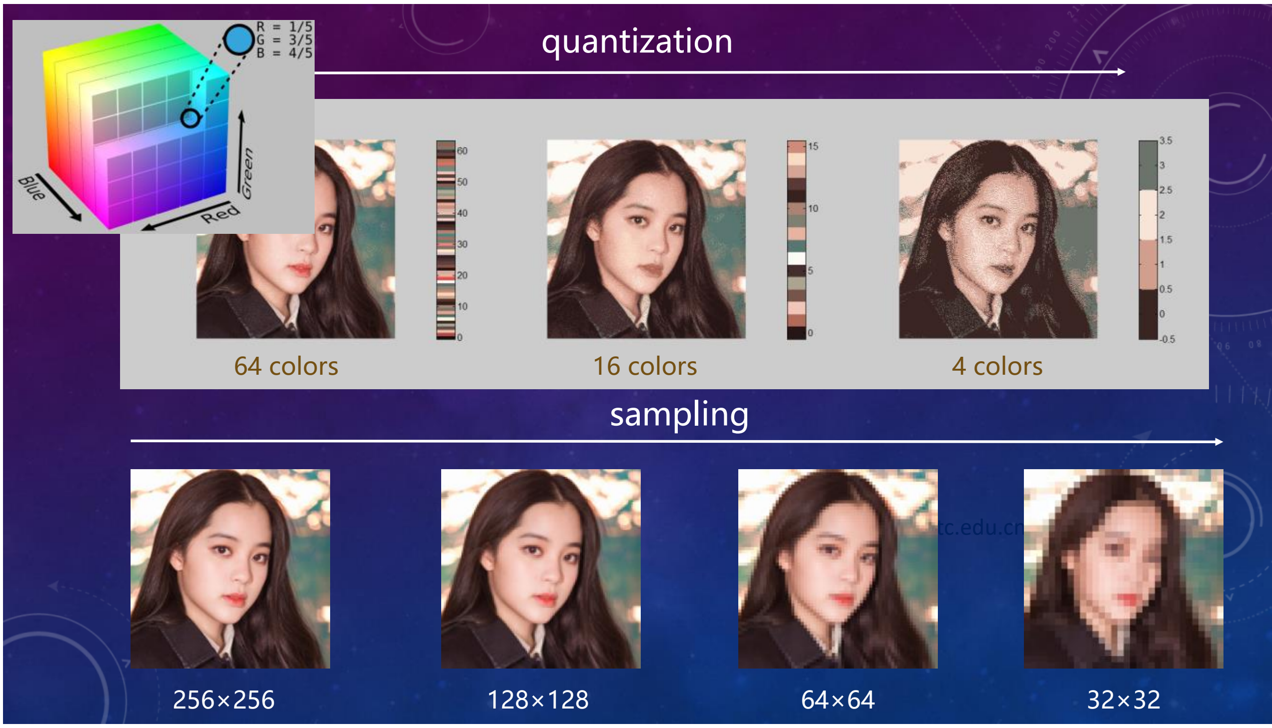
For $M\times N$ digital images，*root-mean-square error* (均方根误差)and *mean-square signal-to-noise ratio* (均方信噪比) are often used

$$SNR_{ms}=\frac{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}Q\left[\hat{f}(i,j)\right]^2}{\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\left\{Q\left[\hat{f}(i,j)\right]-Q\left[f(i,j)\right]\right\}^2}$$

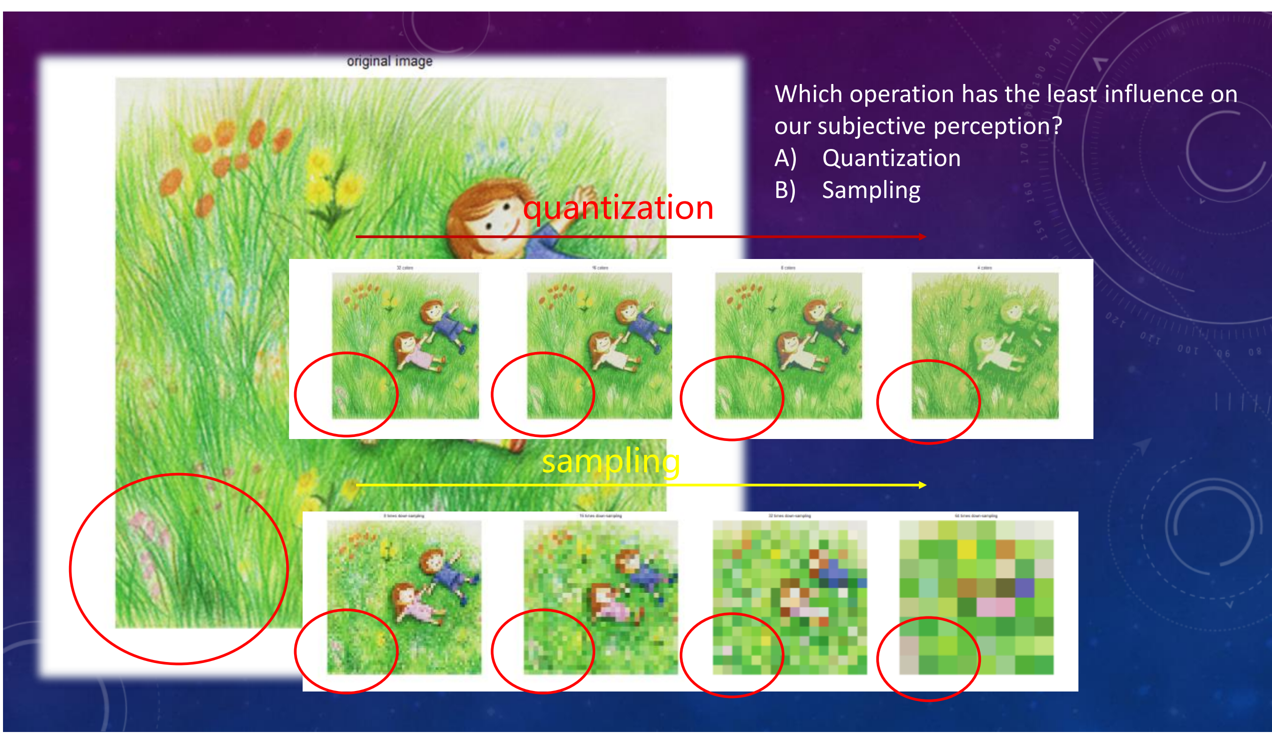
$$SNR_{rms}=\sqrt{SNR_{ms}}$$


SAMPLING & QUANTIZATION
Information carried?
Practical limits?
Equipment?
Storage?
Bit rate?

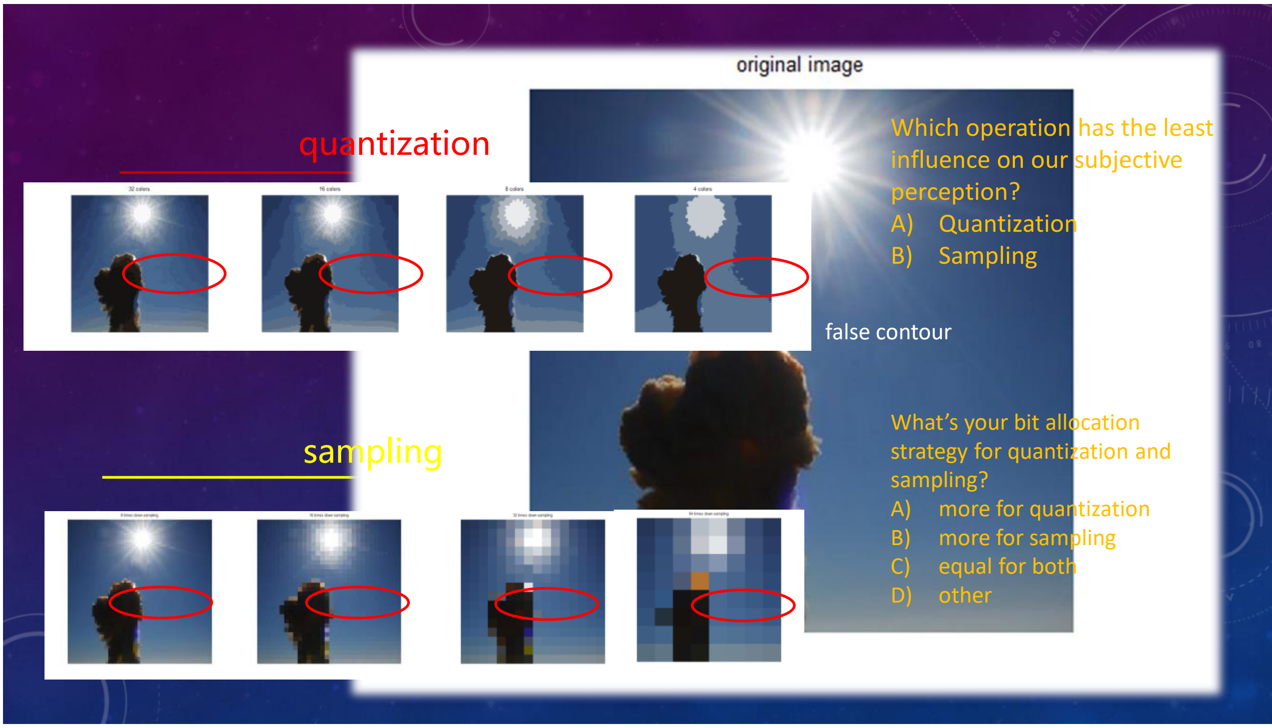

quantization
64 gray levels
16 gray levels
4 gray levels
sampling
256×256
128×128
64×64
32×32

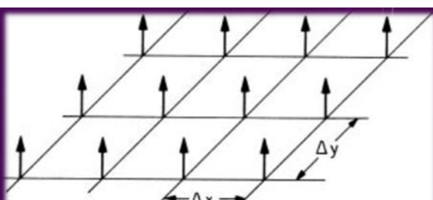


quantization
R = 1/5
G = 3/5
B = 4/5
Blue
Red
Green
64 colors
16 colors
4 colors
sampling
256×256
128×128
64×64
32×32



original image
Which operation has the least influence on our subjective perception?
A) Quantization
B) Sampling
quantization
sampling





2D SAMPLING

$f(x,y)$

$\times$ $comb(x,y,\Delta x,\Delta y) = \sum_m \sum_n \delta(x - m\Delta x, y - n\Delta y)$

$f_s(x,y) = \sum_m \sum_n f(m\Delta x, n\Delta y)\delta(x - m\Delta x, y - n\Delta y)$

Spatical domain

$F(u,v)$

$\otimes$ $COMB(u,v) = \frac{1}{\Delta x \Delta y} \sum_k \sum_l \delta(u - k\frac{1}{\Delta x}, v - l\frac{1}{\Delta y})$

$F_s(u,v) = \frac{1}{\Delta x \Delta y} \sum_k \sum_l F(u - k\frac{1}{\Delta x}, v - l\frac{1}{\Delta y})$

Frequency domain

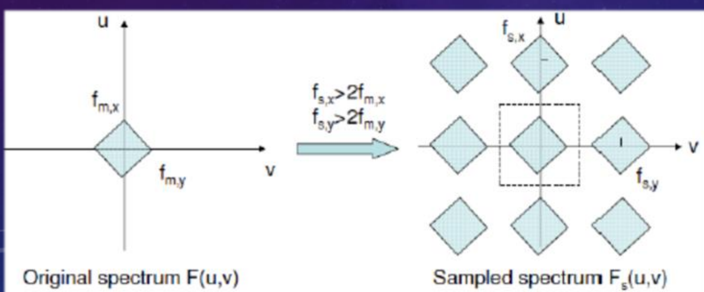
Nyquist criterion

2D SAMPLING

$$F(u,v)=\begin{cases}F(u,v) & |u|< f_{mx},|v|\le f_{my}\\ 0 & |u|> f_{mx},|v|> f_{my}\end{cases}$$

$$F(u,v)$$

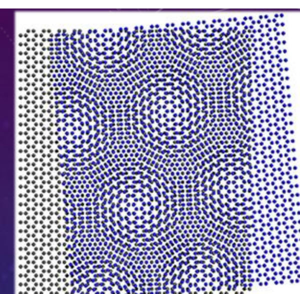
$$\otimes\ COMB(u,v)=\frac{1}{\Delta x\Delta y}\sum_k\sum_l\delta(u-k\frac{1}{\Delta x},v-l\frac{1}{\Delta y})$$



$$F_s(u,v)=$$

$$\frac{1}{\Delta x\Delta y}\sum_k\sum_l F(u-k\frac{1}{\Delta x},v-l\frac{1}{\Delta y})$$

oversampled



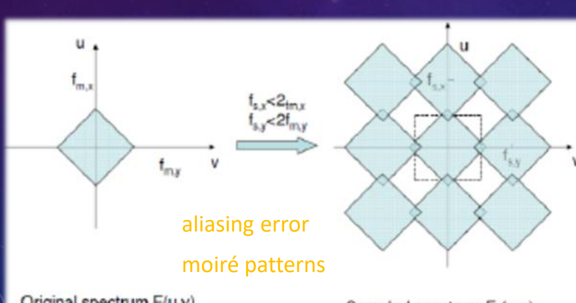
Nyquist criterion

2D SAMPLING

$$F(u,v)=\begin{cases}F(u,v) & |u|< f_{mx},|v|\le f_{my}\\ 0 & |u|> f_{mx},|v|> f_{my}\end{cases}$$

$$F(u,v)$$

$$\otimes\ COMB(u,v)=\frac{1}{\Delta x\Delta y}\sum_k\sum_l\delta(u-k\frac{1}{\Delta x},v-l\frac{1}{\Delta y})$$



aliasing error

moiré patterns

$$F_s(u,v)=$$

$$\frac{1}{\Delta x\Delta y}\sum_k\sum_l F(u-k\frac{1}{\Delta x},v-l\frac{1}{\Delta y})$$

undersampled

2D SAMPLING

$$f_s(x,y) = \sum_m \sum_n f(m\Delta x, n\Delta y)\delta(x - m\Delta x, y - n\Delta y)$$

$$F_s(u,v) = \frac{1}{\Delta x \Delta y} \sum_k \sum_l F(u - k\frac{1}{\Delta x}, v - l\frac{1}{\Delta y})$$

$\otimes$

$\times$

$$h(x,y) = \sin c(f_{sx}x)\sin c(f_{sy}y)$$

$$H(u,v) = \begin{cases} \Delta x \Delta y, & |u| < \frac{1}{2} f_{sx}, |v| \le \frac{1}{2} f_{sy} \\ 0 & otherwise \end{cases}$$

$$f_{sx} < 2 f_{mx}, \quad f_{sy} > 2 f_{my}$$

$$f_r(x,y) = \sum_m \sum_n f(m,n) Sinc(xf_{sx} - m) Sinc(yf_{sy} - n)$$

$$F_r(u,v)$$

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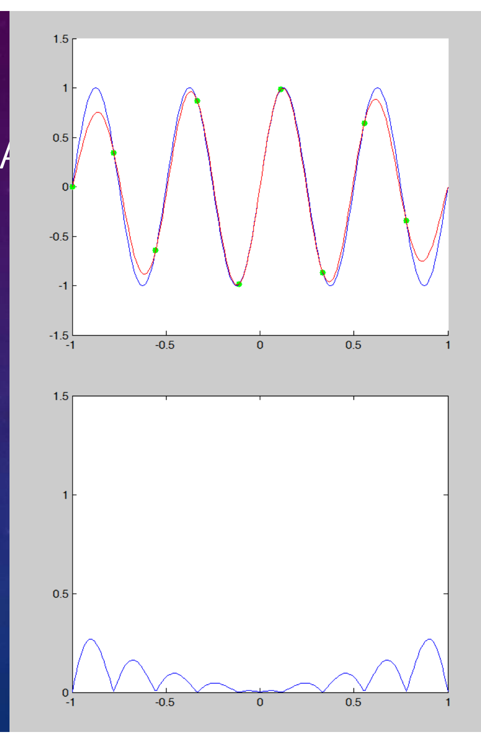
```
%aliasing one D
clear
close all
N = 9;
a = -1;b = 1;
mfreqx = 2;
stepx = (b-a)/N;
dsstep = 0.01;

disp('freq x max : ')
mfreqx
disp('freq x sampling : ')
1/stepx
e = 0.000001;
x = a:stepx:b-e;
y = zeros(1,N);

for i = 1:N
    y(i) = sin(2*pi*(mfreqx*x(i)));
end
scatter(x,y,'go','filled');
hold on;
dsx = a:dsstep:b;
dsy = sin(2*pi*(2*dsx));
plot(dsx,dsy)
```



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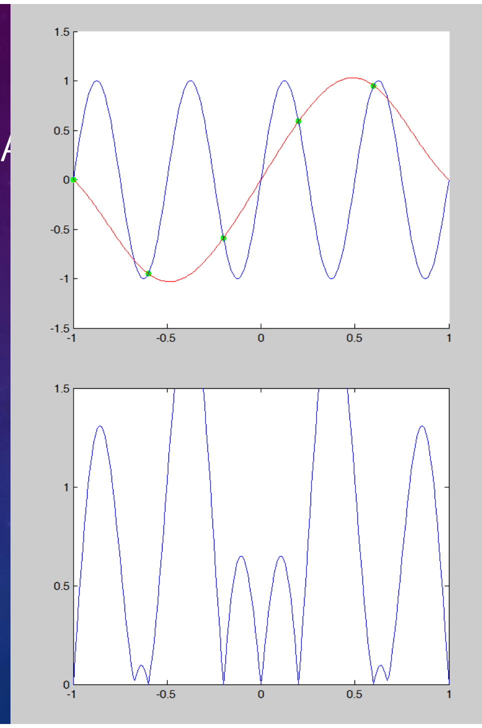
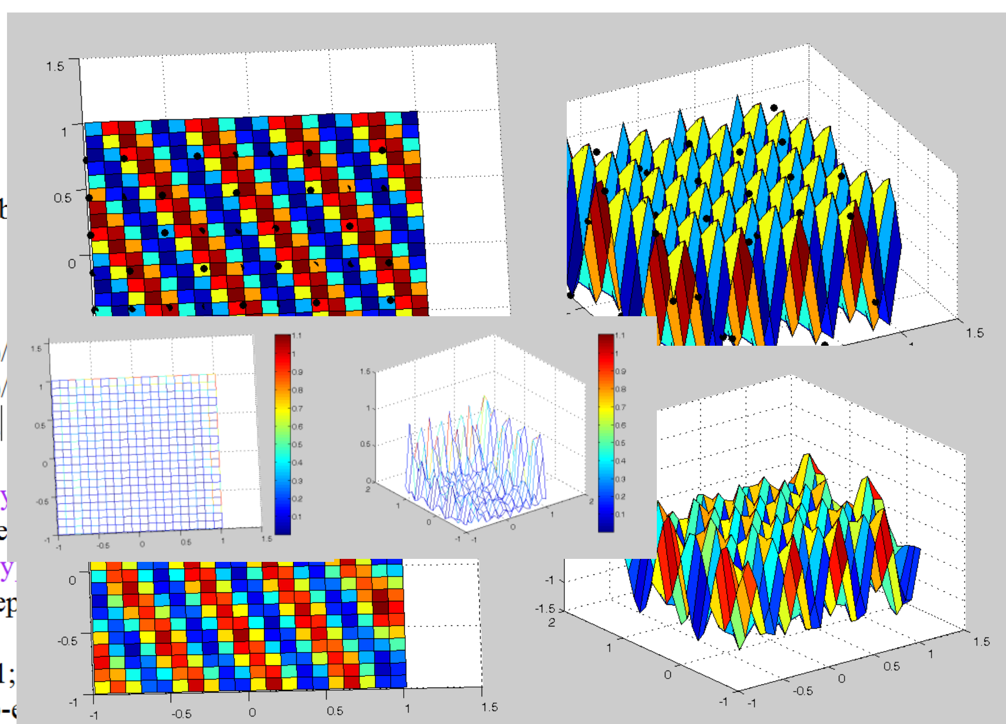
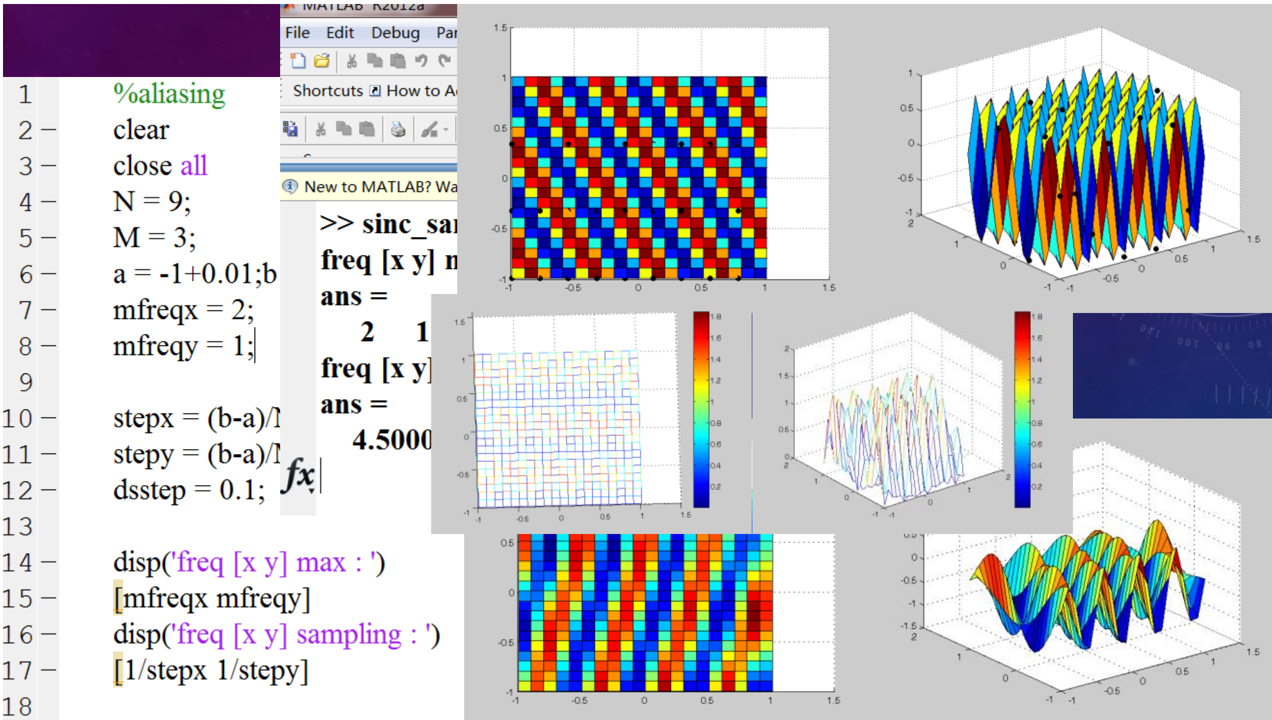
File Edit Text Go Cell Tools Debug Desktop Window Help

```
%aliasing one D
clear
close all
N = 5;
a = -1;b = 1;
mfreqx = 2;
stepx = (b-a)/N;
dsstep = 0.01;

disp('freq x max : ')
mfreqx
disp('freq x sampling : ')
1/stepx
e = 0.000001;
x = a:stepx:b-e;
y = zeros(1,N);

for i = 1:N
    y(i) = sin(2*pi*(mfreqx*x(i)));
end
scatter(x,y,'go','filled');
```



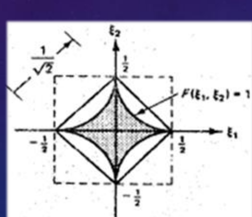
```
%aliasing
clear
close all
N = 9;
M = 7;
a = -1+0.01;b
mfreqx = 2;
mfreqy = 1;

stepx = (b-a)/
stepy = (b-a)/
dsstep = 0.1;

disp('freq [x y
[mfreqx mfre
disp('freq [x y
[1/stepx 1/step

e = 0.000001;
x = a:stepx:b-e
```

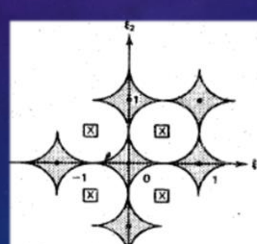


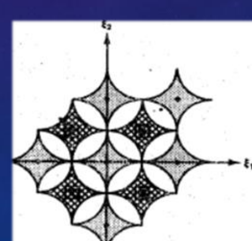


SAMPLING EFFICIENCY

- When the nonzero area of the signal spectrum has a square shape , rectangular sampling is the most efficient way
- Can you find a different sampling strategy more efficient for other spectral shapes?
  - Rhombus(菱形)



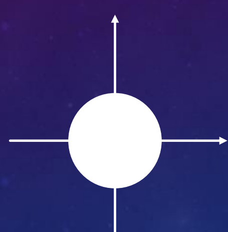


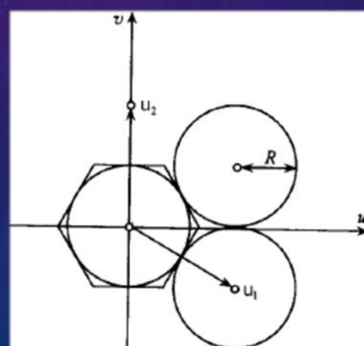


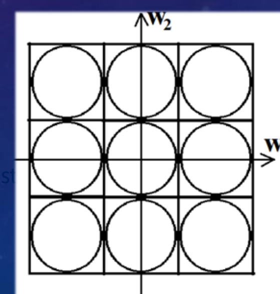
region of support

SAMPLING EFFECIENCY

- Can you find a different sampling strategy more efficient for other spectral shapes?
  - Disk







2D SAMPLING-RECIPROCAL LATTICES

- Sampling periodically with equal intervals in any two directions in space domain, the corresponding spectrum also repeats periodically with equal intervals in two frequency directions
- Given $\vec{v}_1$, $\vec{v}_2$ are *primitive vectors* which lie in different directions, the position vector can be written as:

$$\vec{r}_{mn} = m\vec{v}_1 + n\vec{v}_2 \qquad m,n = 0, \pm 1, \cdots$$

$$V = \begin{bmatrix} T_1 & 0 \\ 0 & T_2 \end{bmatrix}$$ sampling matrix

- Spectrum repeats in direction:

$$\vec{w}_{kl} = k\vec{u}_1 + l\vec{u}_2 \qquad k,l = 0, \pm 1, \cdots$$

periodicity matrix $$U = \begin{bmatrix} \frac{2\pi}{T_1} & 0 \\ 0 & \frac{2\pi}{T_2} \end{bmatrix}$$

- satisfying

$$\vec{v}_i \cdot \vec{u}_j = \begin{cases} 1, & i = j \\ 0 & i \neq j \end{cases}$$

$$U^T V = 2\pi I$$