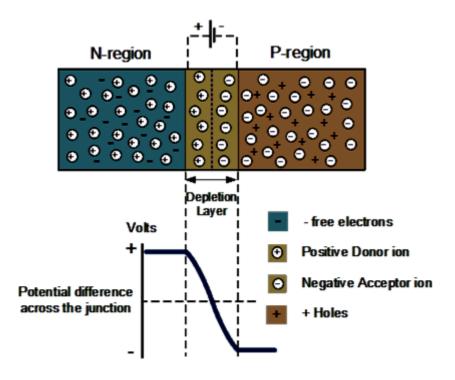




# Bipolar Junction Transistors (BJTS

3002 Electronic Devices









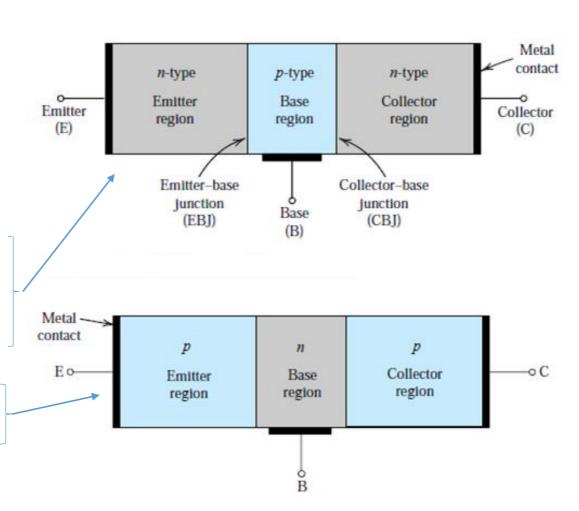
# Bipolar Junction Transistor

Invention of BJT in 1948 at Bell Labs led to electronics changing the way we work, play, and live.

By 2009, the MOSFET was undoubtedly the most widely used electronic device. CMOS technology became the technology of choice in the design of integrated circuits.

In an npn transistor, the BJT consists of three semiconductor regions: the emitter region (n type), the base region (p type), and the collector region (n type).

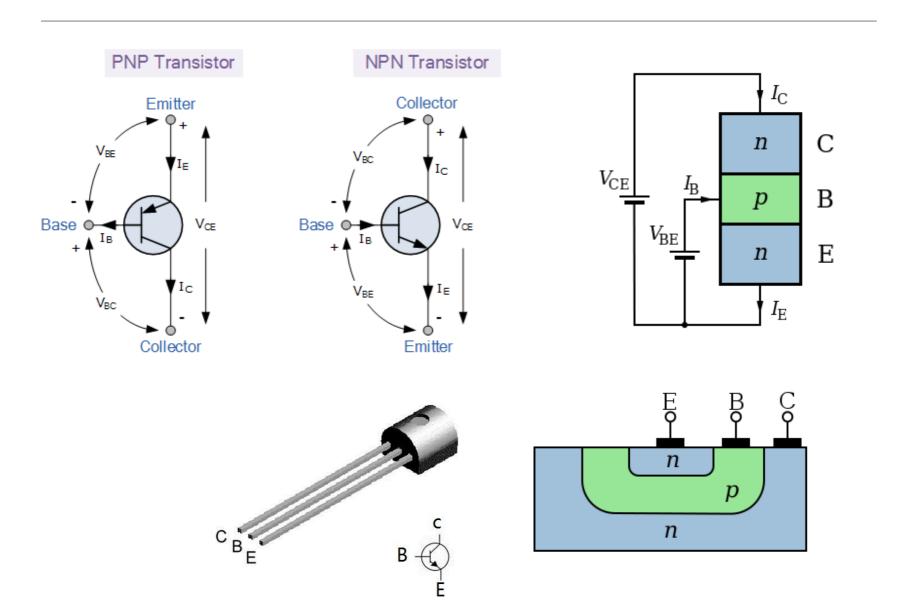
A pnp transistor has a p-type emitter, an n-type base, and a p-type collector.







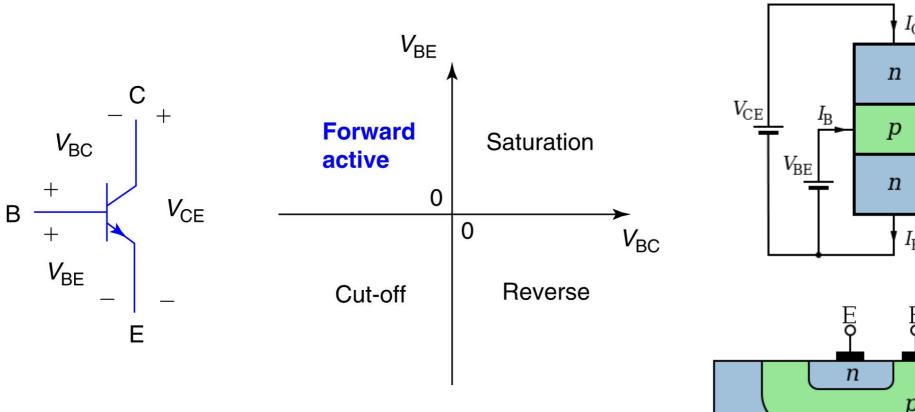
# **BJT Modes of Operation**

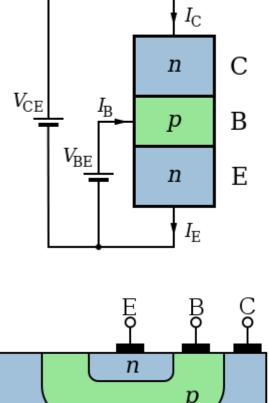






# **BJT Modes of Operation**



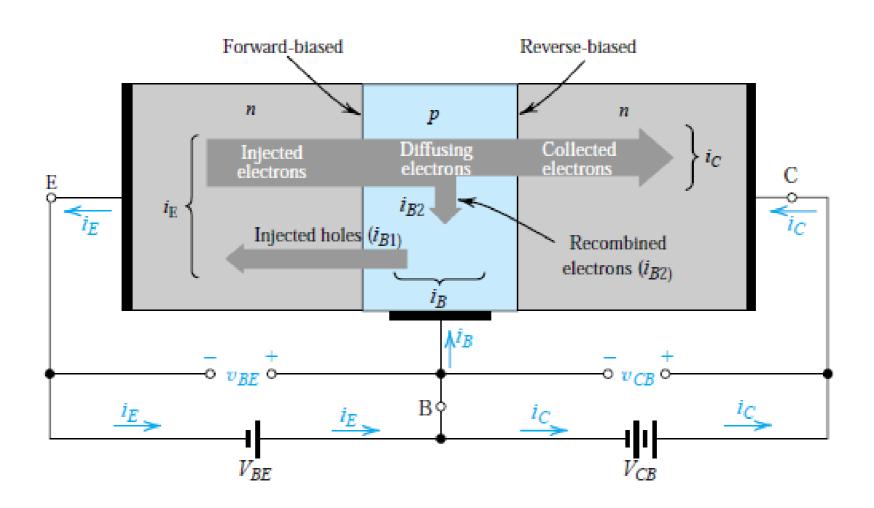


n





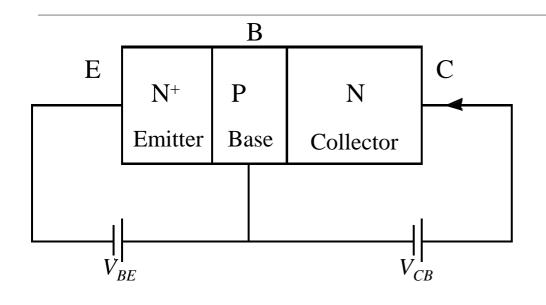
# Active Mode npn-BJT

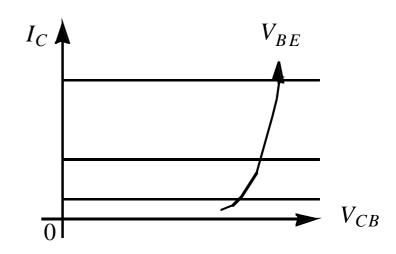


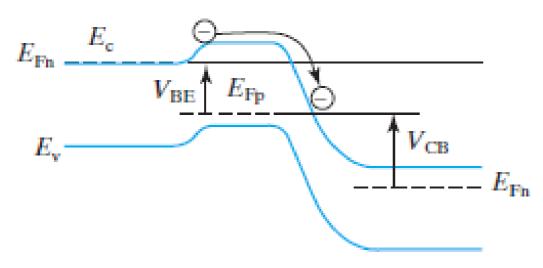




# Active Mode npn-BJT





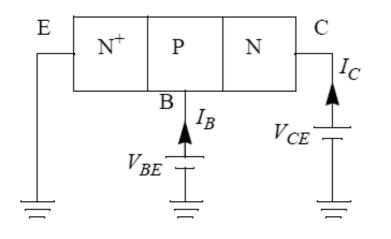


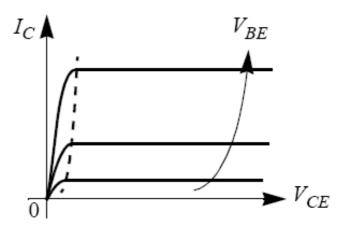
 $I_C$  is an exponential function of forward  $V_{BE}$  and independent of reverse  $V_{CB}$ .

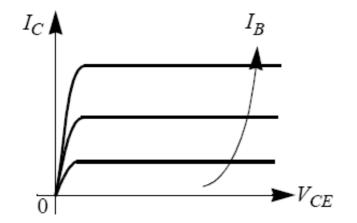


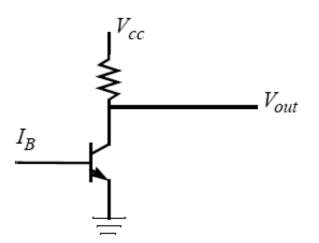
### Common-Emitter Configuration













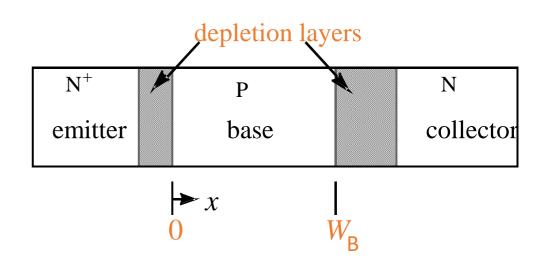
### Collector Current



$$L_B = \sqrt{\tau_B D_B}$$

 $\tau_R$ : base recombination lifetime

 $D_B$ : base minority carrier (electron) diffusion constant



### Boundary conditions:

$$n'(0) = n_{B0} (e^{qV_{BE}/kT} - 1)$$

$$n'(W_R) = n_{R0} (e^{qV_{BC}/kT} - 1) \approx -n_{R0} \approx 0$$

$$I_C = \left| A_E q D_B \frac{dn}{dx} \right|$$

$$= A_E q \frac{D_B}{W_B} \frac{n_{iB}^2}{N_B} (e^{qV_{BE}/kT} - 1)$$
ductor Devices for

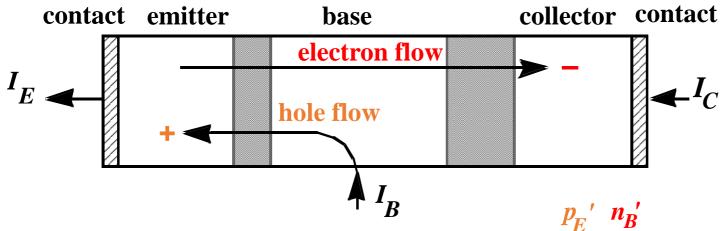
$$I_C = I_S (e^{qV_{BE}/kT} - 1)$$



#### Base Current

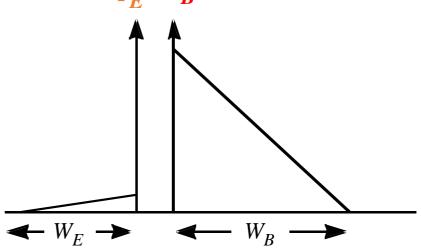


Some holes are injected from the P-type base into the N<sup>+</sup> emitter. The holes are provided by the base current,  $I_B$ .



For a uniform emitter,

$$I_B = A_E q \frac{D_E n_{iE}^2}{W_E N_E} (e^{qV_{BE}/kT} - 1)$$





#### Current Gain



### Common-emitter current gain, $\beta_F$ :

$$\beta_F \equiv \frac{I_C}{I_B}$$

### Common-base current gain:

$$\begin{split} I_C &= \alpha_F I_E \\ \alpha_F &\equiv \frac{I_C}{I_E} = \frac{I_C}{I_B + I_C} = \frac{I_C / I_B}{1 + I_C / I_B} = \frac{\beta_F}{1 + \beta_F} \end{split}$$

It can be shown that 
$$\beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

$$\beta_{F} = \frac{G_{E}}{G_{B}} = \frac{D_{B}W_{E}N_{E}n_{iB}^{2}}{D_{E}W_{B}N_{B}n_{iE}^{2}}$$

How can  $\beta_F$  be maximized?





#### EXAMPLE: Current Gain

A BJT has  $I_C = 1$  mA and  $I_B = 10$   $\mu$ A. What are  $I_E$ ,  $\beta_F$  and  $\alpha_F$ ?

Solution:





#### EXAMPLE: Current Gain

A BJT has  $I_C = 1$  mA and  $I_B = 10$   $\mu$ A. What are  $I_E$ ,  $\beta_E$  and  $\alpha_E$ ?

#### Solution:

$$I_E = I_C + I_B = 1 \text{ mA} + 10 \text{ }\mu\text{A} = 1.01 \text{ mA}$$
  
 $\beta_F = I_C / I_B = 1 \text{ mA} / 10 \text{ }\mu\text{A} = 100$   
 $\alpha_F = I_C / I_E = 1 \text{ mA} / 1.01 \text{ mA} = 0.9901$ 

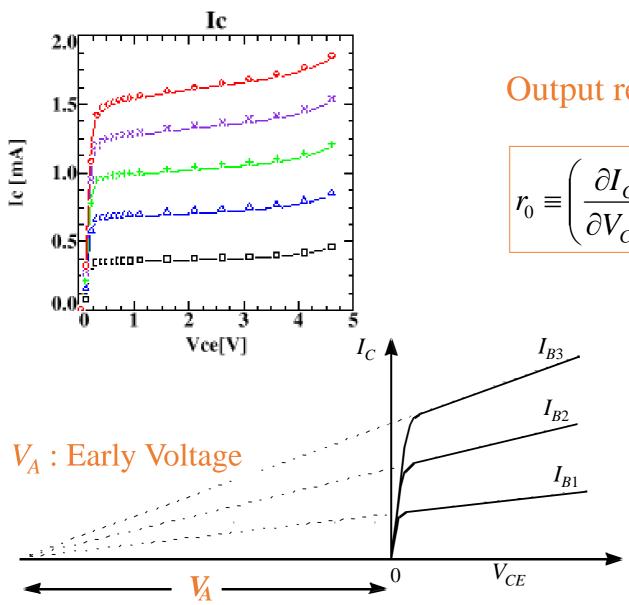
We can confirm

$$\alpha_F = \frac{\beta_F}{1 + \beta_F}$$
 and  $\beta_F = \frac{\alpha_F}{1 - \alpha_F}$ 



### Base-Width Modulation





#### Output resistance:

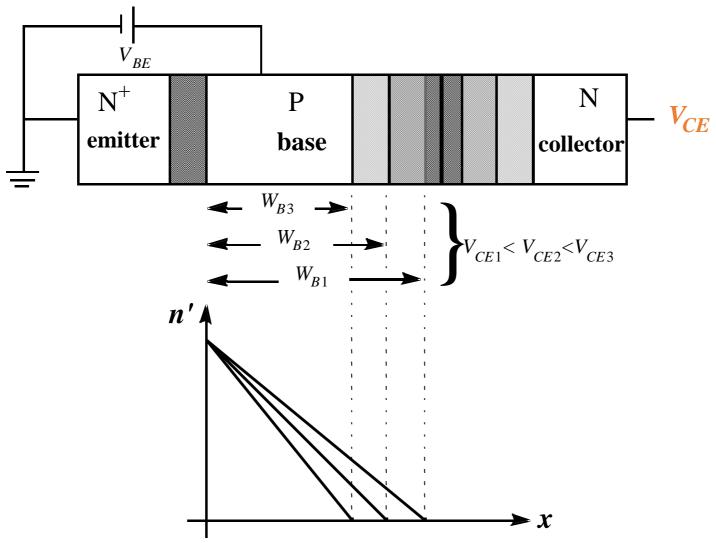
$$r_0 \equiv \left(\frac{\partial I_C}{\partial V_{CE}}\right)^{-1} = \frac{V_A}{I_C}$$

Large  $V_A$  (large  $r_o$ ) is desirable for a large voltage gain





### Base-Width Modulation by Collector Voltage



How can we reduce the base-width modulation effect?

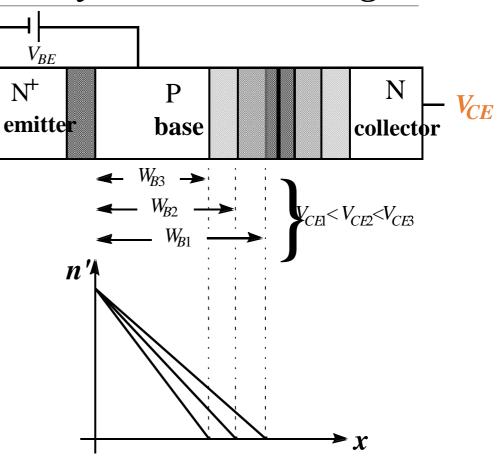




### Base-Width Modulation by Collector Voltage

The base-width modulation effect is reduced if we

- (A) Increase the base width,
- (B) Increase the base doping concentration,  $N_B$ , or
- (C) Decrease the collector doping concentration,  $N_C$ .

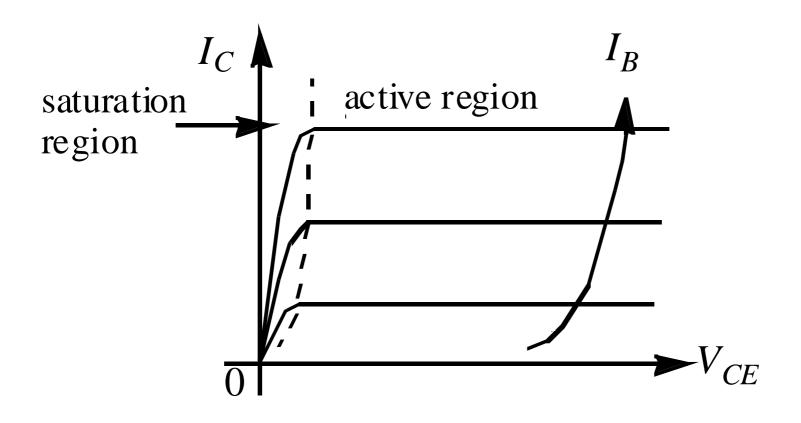


Which of the above is the most acceptable action?



#### Ebers-Moll Model





The Ebers-Moll model describes both the active and the saturation regions of BJT operation.



#### Ebers-Moll Model

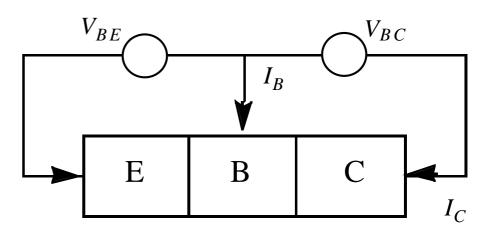


 $I_C$  is driven by two two forces,  $V_{BE}$  and  $V_{BC}$ .

When only  $V_{RE}$  is present:

$$I_C = I_S(e^{qV_{BE}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_F} (e^{qV_{BE}/kT} - 1)$$



Now reverse the roles of emitter and collector.

When only  $V_{BC}$  is present:

$$I_E = I_S(e^{qV_{BC}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_R} (e^{qV_{BC}/kT} - 1)$$

$$\beta_R$$
: reverse current gain

$$\beta_F$$
: forward current gain

$$I_C = -I_E - I_B = -I_S (1 + \frac{1}{\beta_R}) (e^{qV_{BC}/kT} - 1)$$



#### Ebers-Moll Model



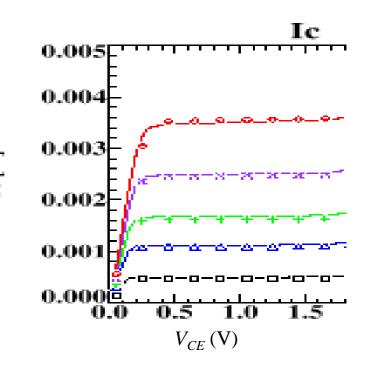
In general, both  $V_{BE}$  and  $V_{BC}$  are present:

$$I_C = I_S (e^{qV_{BE}/kT} - 1) - I_S (1 + \frac{1}{\beta_R})(e^{qV_{BC}/kT} - 1)$$

$$I_{B} = \frac{I_{S}}{\beta_{F}} (e^{qV_{BE}/kT} - 1) + \frac{I_{S}}{\beta_{F}} (e^{qV_{BC}/kT} - 1)$$

In saturation, the BC junction becomes forward-biased, too.

 $V_{BC}$  causes a lot of holes to be injected into the collector. This uses up much of  $I_B$ . As a result,  $I_C$  drops.





### Transit Time and Charge Storage



When the BE junction is forward-biased, excess holes are stored in the emitter, the base, and even in the depletion layers.  $Q_F$  is all the stored excess hole charge

$$\tau_F \equiv \frac{Q_F}{I_C}$$

 $\tau_F$  is difficult to be predicted accurately but can be measured.

 $\tau_F$  determines the high-frequency limit of BJT operation.



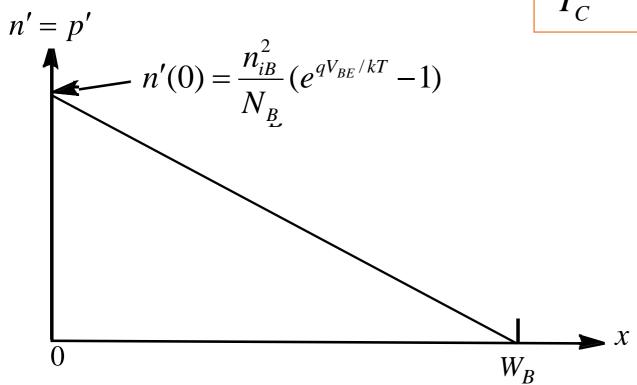
# of Glasgow Base Charge Storage and Base Transit Time



Let's analyze the excess hole charge and transit time in the base only.

$$Q_{FB} = qA_E n'(0)W_B / 2$$

$$\frac{Q_{FB}}{I_C} \equiv \tau_{FB} = \frac{W_B^2}{2D_B}$$









What is  $\tau_{FB}$  if  $W_B = 70$  nm and  $D_B = 10$  cm<sup>2</sup>/s?

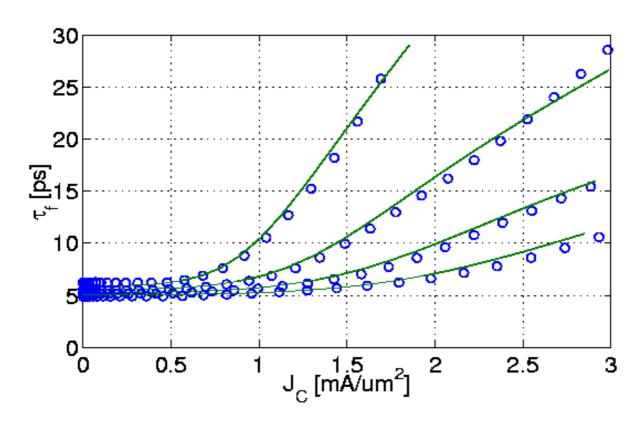
Answer:



### **Emitter-to-Collector Transit Time and Kirk Effect**



- To reduce the total transit time, emitter and depletion layers must be thin, too.
- Kirk effect or base widening: At high  $I_C$  the base widens into the collector. Wider base means larger  $\tau_F$ .



Top to bottom:  $V_{CE} = 0.5 \text{ V}, 0.8 \text{ V}, 1.5 \text{ V}, 3 \text{ V}.$ 

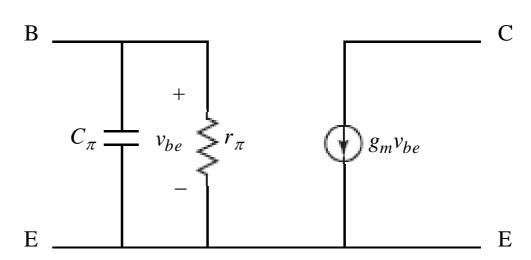






$$I_C = I_S e^{qV_{BE}/kT}$$

#### Transconductance:



$$g_{m} \equiv \frac{dI_{C}}{dV_{BE}} = \frac{d}{dV_{BE}} (I_{S}e^{qV_{BE}/kT})$$
$$= \frac{q}{kT}I_{S}e^{qV_{BE}/kT} = I_{C}/(kT/q)$$

$$g_m = I_C / (kT/q)$$

At 300 K, for example,  $g_m = I_C/26$ mV.

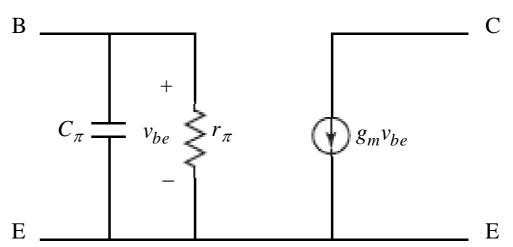






$$\frac{1}{r_{\pi}} = \frac{dI_B}{dV_{BE}} = \frac{1}{\beta_F} \frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta_F}$$

$$r_{\pi} = \beta_F / g_m$$



$$C_{\pi} = \frac{dQ_F}{dV_{BE}} = \frac{d}{dV_{BE}} \tau_F I_C = \tau_F g_m$$

This is the charge-storage capacitance, better known as the *diffusion* capacitance.

Add the depletion-layer capacitance,  $C_{dBE}$ :

$$C_{\pi} = \tau_F g_m + C_{dBE}$$





### **EXAMPLE: Small-Signal Model Parameters**

A BJT is biased at  $I_C = 1$  mA and  $V_{CE} = 3$  V.  $\beta_F = 90$ ,  $\tau_F = 5$  ps, and T = 300 K. Find (a)  $g_m$ , (b)  $r_\pi$ , (c)  $C_\pi$ .

#### Solution:

(a)

(b)

(c)



### Summary



The base-emitter junction is usually forward-biased while the base-collector is reverse-biased.  $V_{BE}$  determines the collector current,  $I_{C}$ .

$$I_{C} = A_{E} \frac{q n_{i}^{2}}{G_{B}} (e^{q V_{BE}/kT} - 1)$$

$$G_B \equiv \int_0^{W_B} \frac{n_i^2}{n_{iB}^2} \frac{p}{D_B} dx$$

•  $G_B$  is the base Gummel number, which represents all the subtleties of BJT design that affect  $I_C$ .



### **Summary**



The base (input) current,  $I_B$ , is related to  $I_C$  by the common-emitter current gain,  $\beta_F$ . This can be related to the common-base current gain,  $\alpha_F$ .

 $\alpha_F = \frac{I_C}{I_E} = \frac{\beta_F}{1 + \beta_F}$ 

In an npn BJT, an emitter is efficient if the emitter current is mostly the useful electron current injected into the base with little useless hole current (the base current). The **emitter efficiency** is defined as:

$$\gamma_E = \frac{I_E - I_B}{I_E} = \frac{I_C}{I_C + I_B} = \frac{1}{1 + D_E W_B N_B n_{IE}^2 / D_B W_E N_E n_{IB}^2}$$

Base-width modulation by  $V_{CB}$  results in a significant slope of the  $I_C$  vs.  $V_{CE}$  curve in the active region (known as the Early effect).



### **Summary**



Due to the forward bias  $V_{BE}$ , a BJT stores a certain amount of excess carrier charge  $Q_F$  which is proportional to  $I_C$ .

$$Q_F \equiv I_C \tau_F$$

 $\tau_F$  is the forward transit time. If no excess carriers are stored outside the base, then

$$au_F = au_{FB} = rac{W_B^2}{2D_B}$$
 , the base transit time.

• The charge-control model first calculates  $Q_F(t)$  from  $I_B(t)$  and then calculates  $I_C(t)$ .

$$\frac{dQ_F}{dt} = I_B(t) - \frac{Q_F}{\tau_F \beta_F}$$

$$I_C(t) = Q_F(t) / \tau_F$$







The small-signal models employ parameters such as transconductance,

$$g_m \equiv \frac{dI_C}{dV_{BE}} = I_C / \frac{kT}{q}$$

input capacitance,

$$C_{\pi} = \frac{dQ_F}{dV_{BE}} = \tau_F g_m$$

and input resistance.

$$r_{\pi} = \frac{dV_{BE}}{dI_{B}} = \beta_{F} / g_{m}$$





**EXAMPLE**: A  $P^+N$  junction has  $N_a=10^{20}$  cm<sup>-3</sup> and  $N_d=10^{17}$ cm<sup>-3</sup>. What is a) its built in potential, b) $W_{dep}$ , c) $x_N$ , and d)  $x_P$ ?

#### Solution:

*a*)

*b*)

c)

d)





### **EXAMPLE:** Carrier Injection

A PN junction has  $N_a=10^{19} {\rm cm}^{-3}$  and  $N_d=10^{16} {\rm cm}^{-3}$ . The applied voltage is 0.6 V.

**Question**: What are the minority carrier concentrations at the depletion-region edges?

Solution:

Question: What are the excess minority carrier concentrations?

Solution:





#### EXAMPLE: Emitter Bandgap Narrowing and SiGe Base

Assume  $D_B = 3D_E$ ,  $W_E = 3W_B$ ,  $N_B = 10^{18}$  cm<sup>-3</sup>, and  $n_{iB}^2 = n_i^2$ . What is  $\beta_F$  for (a)  $N_E = 10^{19}$  cm<sup>-3</sup> and  $\Delta E_{gE} \approx 50$  meV; (b)  $N_E = 10^{20}$  cm<sup>-3</sup> and  $\Delta E_{gE} \approx 95$  meV; (c)  $N_E = 10^{20}$  cm<sup>-3</sup> and a SiGe base with  $\Delta E_{gB} = 60$  meV?

(a) At 
$$N_E = 10^{19} \text{ cm}^{-3}$$
,  $\Delta E_{gE} \approx 50 \text{ meV}$ ,

(b) At 
$$N_E = 10^{20} \, \text{cm}^{-3}$$
,  $\Delta E_{gE} \approx 95 \, \text{meV}$ 

(c)





#### EXAMPLE: Emitter Bandgap Narrowing and SiGe Base

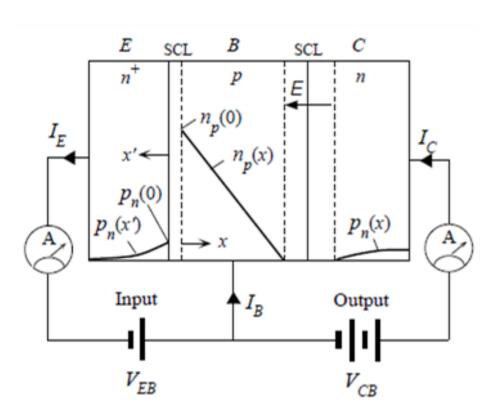
Assume  $D_B = 3D_E$ ,  $W_E = 3W_B$ ,  $N_B = 10^{18}$  cm<sup>-3</sup>, and  $n_{iB}^2 = n_i^2$ . What is  $\beta_F$  for (a)  $N_E = 10^{19}$  cm<sup>-3</sup>, (b)  $N_E = 10^{20}$  cm<sup>-3</sup>, and (c)  $N_E = 10^{20}$  cm<sup>-3</sup> and a SiGe base with  $\Delta E_{gB} = 60$  meV?

(a) At 
$$N_E = 10^{19} \text{ cm}^{-3}$$
,  $\Delta E_{gE} \approx 50 \text{ meV}$ ,  
 $n_{iE}^2 = n_i^2 e^{\Delta E_{gE}/kT} = n_i^2 e^{50 \text{ meV}/26 \text{ meV}} = n_i^2 e^{1.92} = 6.8 n_i^2$   
 $\beta_F = \frac{D_B W_E}{D_E W_B} \cdot \frac{N_E n_i^2}{N_B n_{iE}^2} = \frac{9 \cdot 10^{19} \cdot n_i^2}{10^{18} \cdot 6.8 n_i^2} = 13$ 

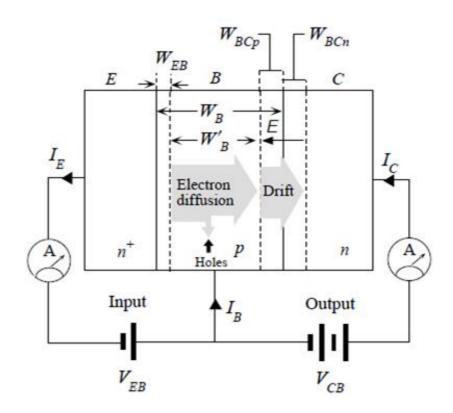
(c) 
$$n_{iB}^2 = n_i^2 e^{\Delta E_{gB}/kT} = n_i^2 e^{60 \text{ meV}/26 \text{ meV}} = 10 n_i^2$$
  $\beta_F = 237$ 







Minority carrier concentration profiles in the emitter, base and collector of an *npn* BJT. Depletion regions marked as **SCL** (Space Charge Layer). *E* is the electric field in the collector junction SCL.



Depletion regions in an *npn* transistor operated in the common base (CB) configuration.





Consider an idealized silicon npn bipolar transistor with the properties listed below in Table 1. The base region has a relatively uniform doping. The emitter and collector donor concentrations are mean values. The cross-sectional area is  $100 \ \mu m \times 100 \ \mu m$ . The transistor is biased to operate in the normal active mode. The base-emitter forward bias voltage is  $0.6 \ V$  and the reverse bias base-collector voltage is  $18 \ V$ .

**Table 1**Properties of an *npn* bipolar transistor.

Emitter width	Emitter doping	Hole lifetime in emitter	Base width	Base doping	Electron lifetime in base	Collector doping
10 µm	$2 \times 10^{18} \text{ cm}^{-3}$	10 ns	4 μm	$1 \times 10^{16} \text{ cm}^{-3}$	400 ns	$1 \times 10^{16} \text{ cm}^{-3}$

- (a) Calculate the depletion layer width extending from the collector into the base and also from the emitter into the base. What is the width of the neutral base region?
- (b) Calculate  $\alpha$  and hence  $\beta$  for this transistor assuming unity emitter injection efficiency. How do  $\alpha$  and  $\beta$  change with  $V_{CB}$ ?
- (c) What is the emitter injection efficiency and what are α and β taking into account the emitter injection efficiency is not unity?





the base-collector junction  $V_r = V_{BC} + V_o \approx V_{BC}$ . Thus, the depletion layer  $W_{BC}$  at the base-collector junction is given by;

$$W_{BC} \approx \left[ \frac{2\varepsilon (N_a + N_d) V_{BC}}{e N_a N_d} \right]^{1/2}$$

i.e.

$$W_{BC} = \left[ \frac{2(8.854 \times 10^{-12} \text{ F m}^{-1})(11.9)(10^{22} \text{ m}^{-3} + 10^{22} \text{ m}^{-3})(18 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{22} \text{ m}^{-3})(10^{22} \text{ m}^{-3})} \right]^{1/2}$$

$$W_{BC} = 2.18 \times 10^{-6} \text{ m or } 2.18 \,\mu\text{m}$$

Only a portion of  $W_{BC}$  is in the base side. Suppose that  $W_{BCp}$  and  $W_{BCn}$  are the depletion widths in the base and collector sides of the SCL respectively. Since the total charge on the p and n-sides of the SCL must be the same

$$N_a W_{BCp} = N_d W_{BCn}$$

and since

$$W_{BC} = W_{BCp} + W_{BCn}$$

we can find  $W_{BC_p}$ ,

$$W_{BCp} = \frac{N_d}{N_a + N_d} W_{BC} = \frac{10^{16}}{10^{16} + 10^{16}} (2.17 \text{ } \mu\text{m}) = 1.09$$

Since  $N_d(E) >> N_a(B)$ , the depletion layer width  $W_{EB}$  is almost totally in the p-side (in the base). With forward bias,  $V_{EB} = 0.6$  V across the emitter-base junction,  $W_{EB}$  is given by

$$W_{EB} = \left[\frac{2\varepsilon(V_o - V_{EB})}{eN_a}\right]^{1/2}$$





We first need to calculate the built-in voltage  $V_o$  between the emitter and base,

$$V_o = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.0259 \text{ V}) \ln \left[ \frac{(2 \times 10^{18} \text{ cm}^{-3})(1 \times 10^{16} \text{ cm}^{-3})}{(1.5 \times 10^{10} \text{ cm}^{-3})^2} \right]$$

i.e. 
$$V_{o} = 0.830 \text{ V}$$

Then, 
$$W_{EB} = \left[ \frac{2\varepsilon (V_o - V_{EB})}{eN_a} \right]^{1/2}$$

i.e. 
$$W_{EB} = \left[ \frac{2(8.854 \times 10^{-12} \text{ F m}^{-1})(11.9)(0.830 \text{ V} \pm 0.6 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{22} \text{ m}^{-3})} \right]^{1/2}$$

or 
$$W_{EB} = 1.74 \times 10^{-7} \text{ m or } 0.174 \,\mu\text{m}$$

Notice that due to the forward bias across the EB junction,  $W_{EB}$  is an order of magnitude smaller than  $W_{BCp}$ .

If  $W_B$  is the base width between emitter and collector metallurgical junctions, then the width  $W'_B$  of the neutral region in the base between the depletion regions is given by,

$$W'_{B} = W_{B} - W_{BCp} - W_{EB}$$
 so that 
$$W'_{B} = 4 \ \mu m - 1.09 \ \mu m - 0.174 \ \mu m = 2.74 \ \mu m$$





(b) The electron drift mobility  $\mu_e$  in the base is determined by the dopant (acceptor) concentration here. For  $N_a = 1 \times 10^{16}$  cm<sup>-3</sup>,  $\mu_e = 1250$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup> and the diffusion coefficient  $D_e$  from the Einstein relationship is,

$$D_e = kT\mu_e/e = (0.02585 \text{ V})(1250 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}) = 3.23 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$$

The electron diffusion length  $L_e$  in the base is

$$L_e = (D_e \tau_e)^{1/2} = [(3.23 \times 10^{-3} \text{ m}^2 \text{ s}^{-1})(400 \times 10^{-9} \text{ s})]^{1/2}$$

i.e. 
$$L_e = 36.0 \times 10^{-6} \text{ m} (= 36.0 \, \mu\text{m})$$

In order to calculate  $\alpha$ , first we need to find the transit (diffusion) time  $\tau$ , through the base

$$\tau_t = \frac{W_B^{\prime 2}}{2D_e} = \frac{(2.74 \times 10^{-6} \text{ m})^2}{2(3.23 \times 10^{-3} \text{ m}^2 \text{ s}^{-1})} = 1.161 \times 10^{-9} \text{ s}$$

If we assume unity injection ( $\gamma = 1$ ), then  $\alpha = \alpha_B$ , base transport factor:

$$\alpha = \alpha_{\rm B} = 1 - \frac{\text{Transit (diffusion) time across base}}{\text{Minority carrier recombination time in base}} = 1 - \frac{\tau_{\rm t}}{\tau_{\rm e}}$$

i.e. 
$$\alpha = 1 - (1.161 \text{ ns})/(400 \text{ ns}) = 0.99710$$





The current gain  $\beta$  is

$$\beta = \frac{\alpha}{1 - \alpha} = \frac{0.9971}{1 - 0.9971} = 343$$

(c) The hole drift mobility  $\mu_h$  in the emitter is determined by the dopant (donor) concentration here. For  $N_d = 2 \times 10^{18}$  cm<sup>-3</sup>,  $\mu_h \approx 100$  cm<sup>2</sup> V<sup>-1</sup> s<sup>-1</sup> and the diffusion coefficient  $D_h$  from the Einstein relationship is,

$$D_h = kT\mu_h/e = (0.02585 \text{ V})(100 \times 10^{-4} \text{ m}^2 \text{ V}^{-1} \text{ s}^{-1}) = 2.59 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$$

The hole diffusion length  $L_h$  in the base is

$$L_h = (D_h \tau_h)^{1/2} = [(2.59 \times 10^{-4} \text{ m}^2 \text{ s}^{-1})(10 \times 10^{-9} \text{ s})]^{1/2}$$

$$L_h = 1.61 \times 10^{-6} \ (= 1.61 \ \mu \text{m})$$

Thus the hole diffusion length is much shorter than the emitter width.

The emitter current is given by electron diffusion in the base and hole diffusion in the emitter so that

$$I_E = I_{E(\text{electron})} + I_{E(\text{hole})}$$

where for electrons diffusing in the base,

$$I_{E(\text{electron})} = I_{soe} \exp\left(\frac{eV_{EB}}{kT}\right); \qquad I_{soe} = \frac{eAD_e n_i^2}{N_o W_B}$$



i.e.



# BJT npn Solved Example

and holes diffusing in the emitter,

$$I_{E(\text{hole})} = I_{soh} \exp\left(\frac{eV_{EB}}{kT}\right);$$
  $I_{soh} = \frac{eAD_h n_i^2}{N_d L_h}$ 

where we used  $L_h$  instead of  $W_E$  because  $W_E >> L_h$  (emitter width is 10  $\mu$ m and hole diffusion length is 1.62  $\mu$ m)

Substituting the values we find,

$$I_{soe} = \frac{(1.601 \times 10^{-19} \,\mathrm{C})(1 \times 10^{-8} \ \mathrm{m}^2)(3.23 \times 10^{-3} \ \mathrm{m}^2 \ \mathrm{s}^{-1})(1.5 \times 10^{16} \ \mathrm{m}^{-3})^2}{(1 \times 10^{22} \ \mathrm{m}^{-3})(2.74 \times 10^{-6} \ \mathrm{m})}$$

i.e. 
$$I_{soe} = 4.267 \times 10^{-14} \text{ A or } 42.67 \text{ fA}$$

and 
$$I_{soh} = \frac{(1.601 \times 10^{-19} \,\mathrm{C})(1 \times 10^{-8} \,\mathrm{m}^2)(2.59 \times 10^{-4} \,\mathrm{m}^2 \,\mathrm{s}^{-1})(1.5 \times 10^{16} \,\mathrm{m}^{-3})^2}{(2 \times 10^{24} \,\mathrm{m}^{-3})(1.61 \times 10^{-6} \,\mathrm{m})}$$

i.e. 
$$I_{soh} = 2.93 \times 10^{-17} \text{ A or } 0.0293 \text{ fA}$$

The emitter injection efficiency is the fraction of the emitter current that is due to minority carriers injected into the base; those that diffuse across the base towards the collector.

$$\gamma = \frac{I_{E(\text{electron})}}{I_{E(\text{electron})} + I_{E(\text{hole})}} = \frac{I_{soe}}{I_{soe} + I_{soh}}$$

$$\gamma = \frac{4.267 \times 10^{-14} \text{ A}}{4.267 \times 10^{-14} \text{ A} + 2.93 \times 10^{-17} \text{ A}} = 0.99931$$

The current gains, taking into account the emitter injection efficiency, are





$$\alpha = \gamma \alpha_{\rm B} = (0.99931)(0.9971) = 0.99641$$

and

$$\beta = \alpha/(1-\alpha) = 0.99641/(1-0.99641) = 278$$

(d) The emitter current with  $V_{EB} = 0.6 \text{ V}$  is

$$I_E = (I_{soe} + I_{soh}) \exp(eV_{EB}/kT)$$

$$I_E = (4.267 \times 10^{-14} \text{ A} + 2.93 \times 10^{-17} \text{ A}) \exp[(0.6 \text{ V})/(0.2585 \text{ V})]$$

$$I_E = 5.13 \times 10^{-4} \text{ A or } 0.513 \text{ mA}$$

The collector current is determined by those minority carriers in the base that reach the collector junction. Only  $\gamma_{I_E}$  of  $I_E$  is injected into the base as minority carriers and only a fraction  $\alpha_{I_E}$  make it to the collector,

$$I_C = \alpha_B \gamma I_E = \alpha I_E = (0.99641)(0.513 \text{ mA}) = 0.511 \text{ mA}$$

The base current is given by,

$$I_B = I_C/\beta = (0.511 \text{ mA})/278 = 1.83 \times 10^{-3} \text{ mA} = 1.83 \text{ } \mu\text{A}$$