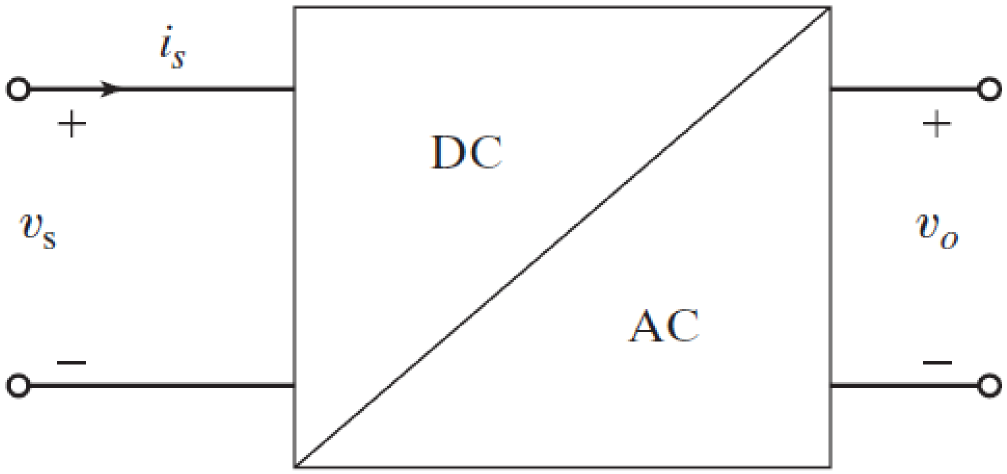
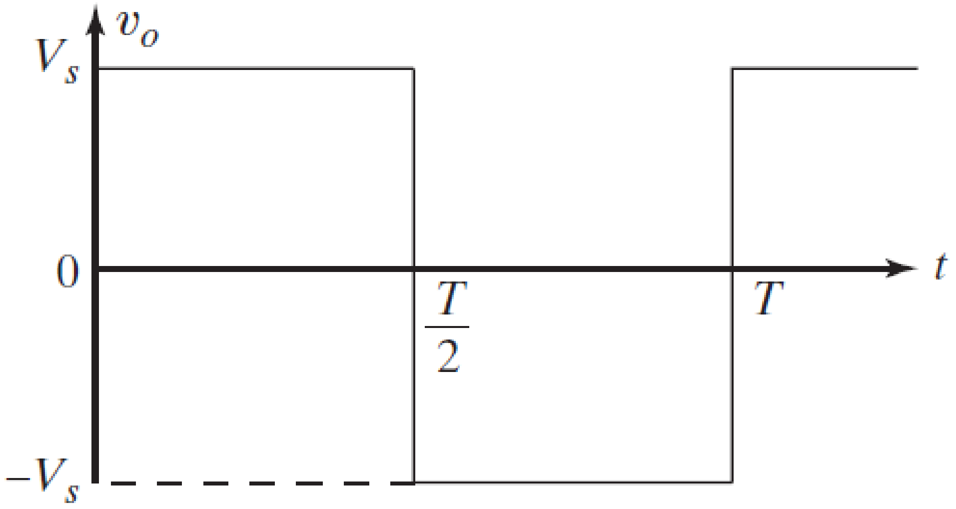
DC-AC Inverters – Fundamental Concepts

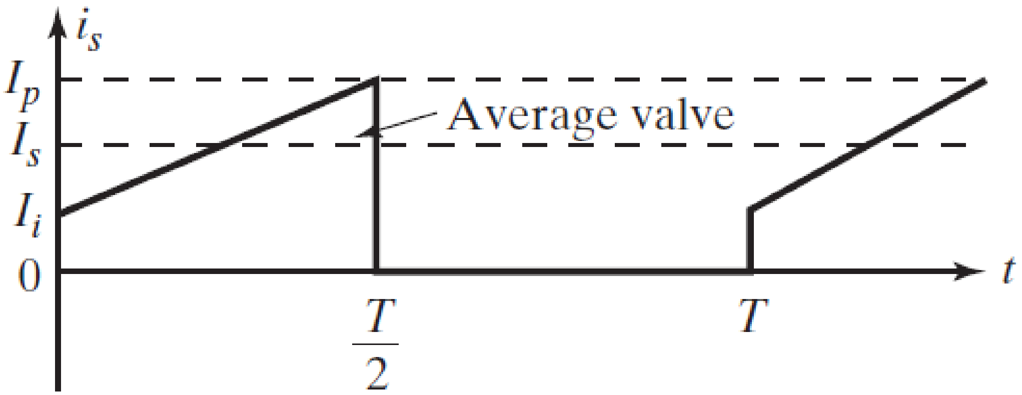
- DC-AC Converters are known as inverters
- Role is to convert a DC signal to AC
- Ideally, output should be sinusoidal.
  - In reality, they are non-sinusoidal and contain harmonics
    - This is fine for low and medium power applications
- Divided into two main types
  - Single Phase
  - Three Phase
- Semiconductor devices typically used



(a) Block diagram



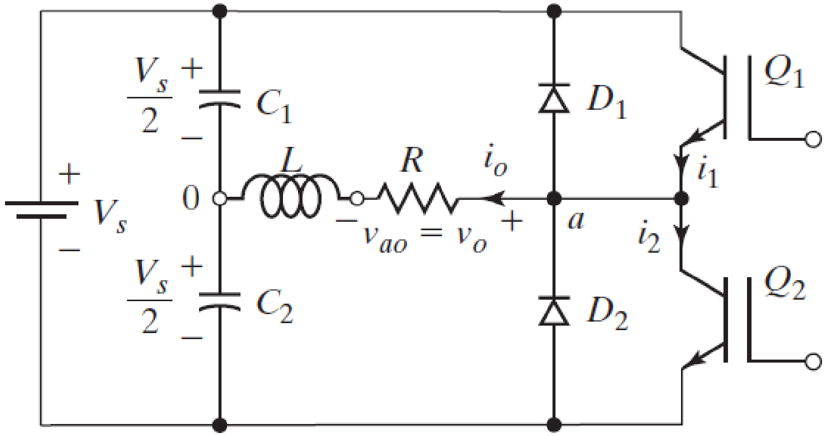
(b) Output voltage



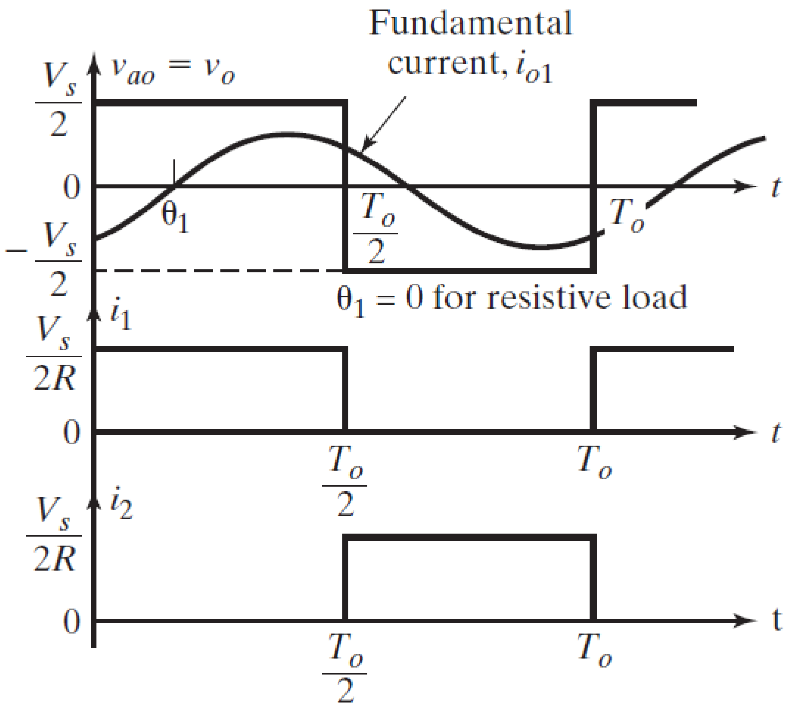
(c) Input current

Important Performance Parameters

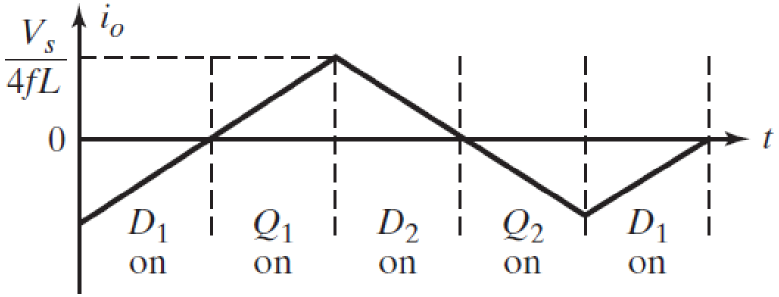
- Power Output: $I_o V_o cos\theta = I_0^2 R$, where I_0 and V_0 are the rms load voltage and current. θ is the angle of the load impedance.
- Input Power of Inverter: $P_s = I_s V_s$, where I_s and V_s are the average i/p current and voltage.
- Total Harmonic Distortion: $\frac{1}{V_{o1}}\left(\Sigma_0^\infty V_{0n}^2\right)^{1/2}$, where V_{01} is rms value of fundamental component and V_{0n} is rms value of nth harmonic component.



(a) Circuit



(b) Waveforms with resistive load



(c) Load current with highly inductive load

Parameter Equations

The root-mean-square (rms) output voltage can be found from

$$V_o = \left(\frac{2}{T_0} \int_0^{T_0/2} \frac{V_s^2}{4}\, dt \right)^{1/2} = \frac{V_s}{2}$$

The instantaneous output voltage can be expressed in Fourier series as

$$v_o = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos(n\omega t) + b_n \sin(n\omega t) \right)$$

Due to the quarter-wave symmetry along the x-axis, both a_0 and a_n are zero. We get b_n as

$$b_n = \frac{1}{\pi} \left[\int_{\frac{-\pi}{2}}^{0} \frac{-V_s}{2} \sin(n\omega t)\, d(\omega t) + \int_0^{\frac{\pi}{2}} \frac{V_s}{2} \sin(n\omega t)\, d(\omega t) \right] = \frac{2V_s}{n\pi}$$

which gives the instantaneous output voltage v_o as

$$v_0 = \sum_{n=1,3,5,\ldots}^{\infty} \frac{2V_s}{n\pi} \sin n\omega t$$
$$= 0 \qquad \text{for } n = 2, 4, \ldots$$

$$V_{o1} = \frac{2V_s}{\sqrt{2}\pi} = 0.45 V_s$$

Dc supply current. Assuming a lossless inverter, the average power absorbed by the load must be equal to the average power supplied by the dc source. Thus, we can write

$$\int_0^T v_s(t)\, i_s(t)\, dt = \int_0^T v_o(t)\, i_o(t)\, dt$$

where T is the period of the ac output voltage. For an inductive load and a relatively high switching frequency, the load current i_o is nearly sinusoidal; therefore, only the fundamental component of the ac output voltage provides power to the load. Because the dc supply voltage remains constant $v_s(t) = V_s$, we can write

$$\int_0^T i_s(t)\, dt = \frac{1}{V_s} \int_0^T \sqrt{2} V_{o1} \sin(\omega t) \sqrt{2} I_o \sin(\omega t - \theta_1)\, dt = T I_s$$

where V_{o1} is the fundamental rms output voltage;
I_o is the rms load current;
θ_1 is the load angle at the fundamental frequency.
Thus, the dc supply current I_s can be simplified to

$$I_s = \frac{V_{o1}}{V_s} I_o \cos(\theta_1)$$

Example

Finding the Parameters of the Single-Phase Half-Bridge Inverter

The single-phase half-bridge inverter in slide 5 has a resistive load of *R* = 2.4 Ω and the dc input voltage is V_s = 48 V. Determine:

(a) the rms output voltage at the fundamental frequency V_{01},

(b) the output power P_0,

(c) the average and peak currents of each transistor,

(d) the peak reverse blocking voltage $\mathrm{V_{BR}}$ of each transistor,

(e) the average supply current I_s,

(f) the THD,

Solutions

V_s = 48 V and *R* = 2.4 Ω.

a. V_{01} = 0.45 * 48 = 21.6 V.

b. V_0 = V_s/2 = 48/2 = 24 V. The output power P_0 = V_0^2/*R* = 24^2/2.4= 240 W.

c. The peak transistor current *Ip* = 24/2.4 = 10 A. Because each transistor conducts for 50% duty cycle, the average current of each transistor is *IQ* = 0.5 * 10 = 5 A.

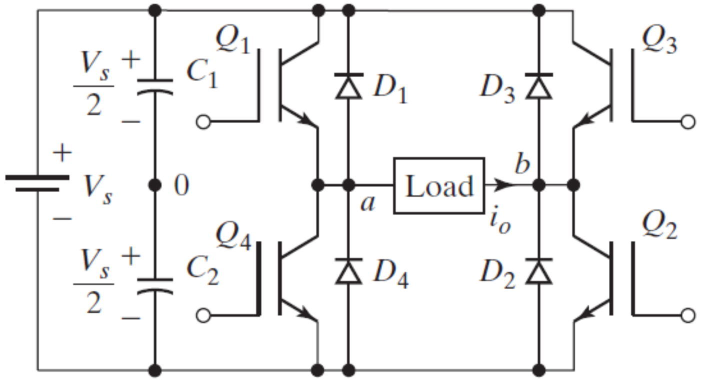
d. The peak reverse blocking voltage $\mathrm{V_{BR}}$ = 2 * 24 = 48 V.

e. The average supply current: $I_s = P_0/V_s$ = 240/48 = 5 A.

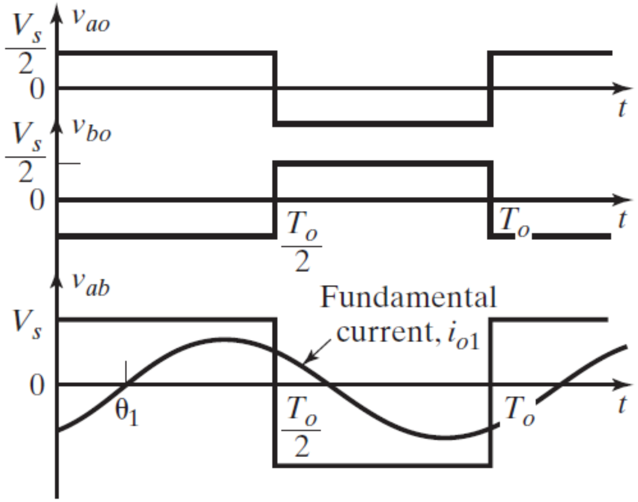
f. V_{01} = 0.45V_s and the rms harmonic voltage V_h=0.2176V_s ,

THD = 10.2176V_s/10.45V_s = 48.34.

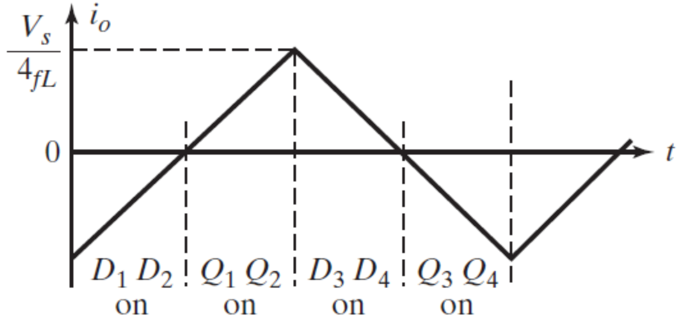
Single Phase Bridge Inverter



(a) Circuit



(b) Waveforms



(c) Load current with highly inductive load

Switch States

Switch States for a Single-Phase Full-Bridge Voltage-Source Inverter						
State	State No.	Switch State*	v_{ao}	v_{bo}	v_o	Components Conducting
S_1 and S_2 are on and S_4 and S_3 are off	1	10	$V_S/2$	$-V_S/2$	V_S	S_1 and S_2 if $i_o > 0$ D_1 and D_2 if $i_o < 0$
S_4 and S_3 are on and S_1 and S_2 are off	2	01	$-V_S/2$	$V_S/2$	$-V_S$	D_4 and D_3 if $i_o > 0$ S_4 and S_3 if $i_o < 0$
S_1 and S_3 are on and S_4 and S_2 are off	3	11	$V_S/2$	$V_S/2$	0	S_1 and D_3 if $i_o > 0$ D_1 and S_3 if $i_o < 0$
S_4 and S_2 are on and S_1 and S_3 are off	4	00	$-V_S/2$	$-V_S/2$	0	D_4 and S_2 if $i_o > 0$ S_4 and D_2 if $i_o < 0$
S_1, S_2, S_3, and S_4 are all off	5	off	$-V_S/2$ $V_S/2$	$V_S/2$ $-V_S/2$	$-V_S$ V_S	D_4 and D_3 if $i_o > 0$ D_1 and D_2 if $i_o < 0$

Parameter Equations

The rms output voltage can be found from

$$V_o = \left(\frac{2}{T_0} \int_0^{T_0/2} V_s^2 \, dt \right)^{1/2} = V_s$$

$$v_o = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi} \sin n\omega t$$

$$V_{o1} = \frac{4V_s}{\sqrt{2}\pi} = 0.90V_s$$

$$i_0 = \sum_{n=1,3,5,\ldots}^{\infty} \frac{4V_s}{n\pi\sqrt{R^2 + (n\omega L)^2}} \sin(n\omega t - \theta_n)$$

where $\theta_n = \tan^{-1}(n\omega L/R)$.

$$v_s(t) i_s(t) = v_o(t) i_o(t)$$

$$i_s(t) = \frac{1}{V_s} \sqrt{2} V_{o1} \sin(\omega t) \sqrt{2} I_o \sin(\omega t - \theta_1)$$

$$i_s(t) = \frac{V_{o1}}{V_s} I_o \cos(\theta_1) - \frac{V_{o1}}{V_s} I_o \cos(2\omega t - \theta_1)$$