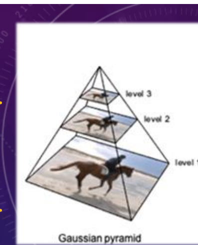
mean pyramid

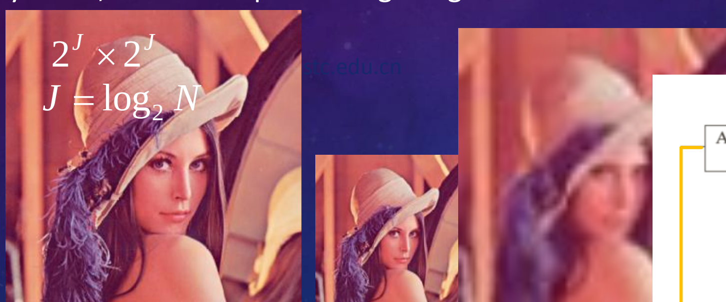
subsampling pyramid

IMAGE TRANSFORMS – DWT



- Image pyramid

A collection of decreasing resolution images arranged in the shape of a pyramid, used for representing images at more than one resolution.



Gaussian pyramid

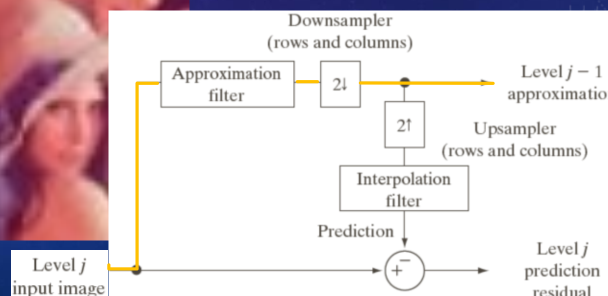
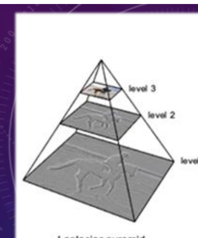
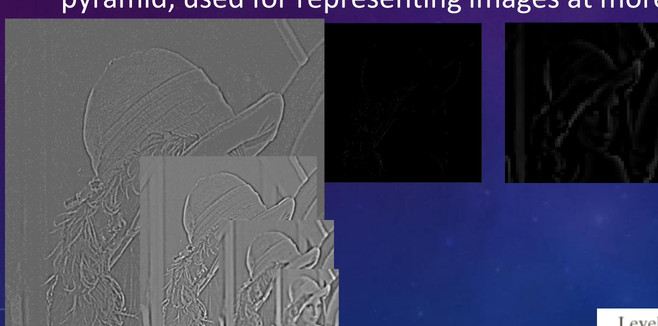


IMAGE TRANSFORMS – DWT



- Image pyramid

A collection of decreasing resolution images arranged in the shape of a pyramid, used for representing images at more than one resolution.



Laplacian pyramid

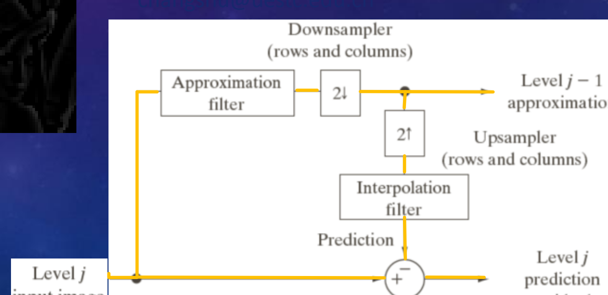
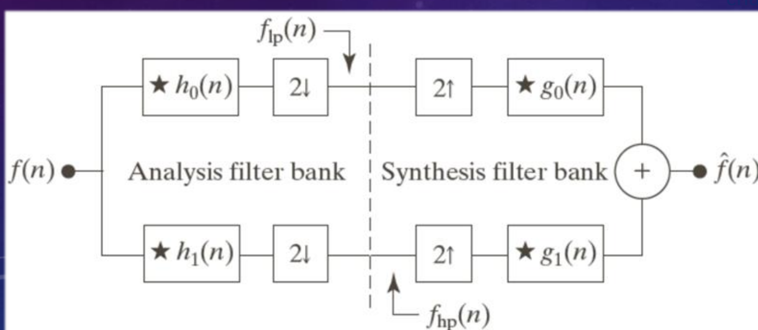


IMAGE TRANSFORMS – DWT

- Subband coding

An image is decomposed into a set of bandlimited components, called subbands. The subbands can be reassembled to reconstruct the original image without error



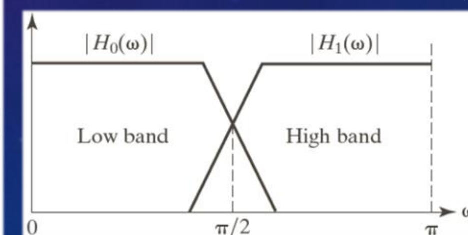
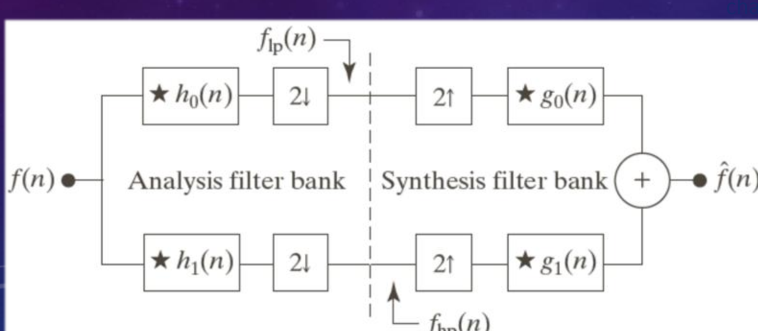


IMAGE TRANSFORMS – DWT

- Subband coding

An image is decomposed into a set of bandlimited components, called subbands. The subbands can be reassembled to reconstruct the original image without error



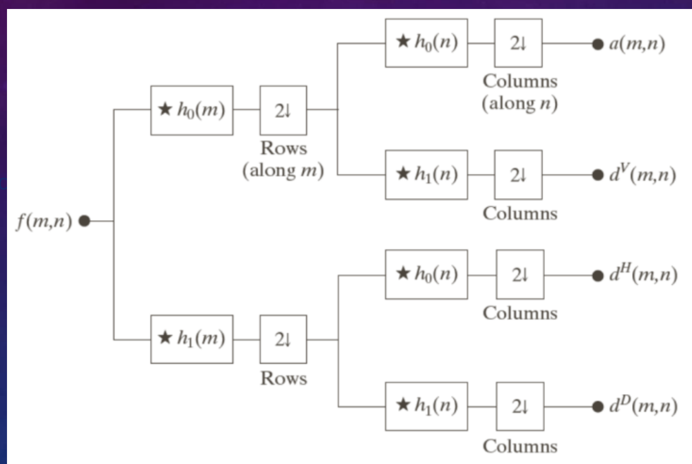
$$g_0(n) = (-1)^n h_1(n)$$
$$g_1(n) = (-1)^{n+1} h_0(n)$$

or

$$g_0(n) = (-1)^{n+1} h_1(n)$$
$$g_1(n) = (-1)^n h_0(n)$$

cross-modulated

IMAGE TRANSFORMS – DWT



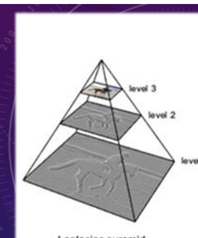
approximation

vertical detail

horizontal detail

diagonal detail

IMAGE TRANSFORMS – DWT

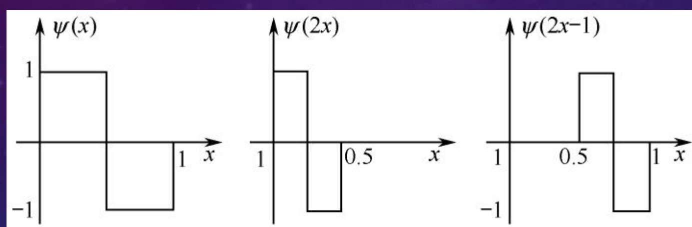


- In mathematics, a wavelet series is a representation of a square-integrable (real or complex-valued) function by a certain orthonormal series generated by a wavelet:

$$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi\left(\frac{x-b}{a}\right)$$

  - *Mother wavelet* $\psi(x)$ has a finite-length or fast-decaying oscillating waveform
  - a and b are scaling and translation parameters respectively

IMAGE TRANSFORMS – DWT



- Translation and scaling of haar wavelet

- The basis functions of Haar transform are the oldest and simplest known orthonormal wavelets

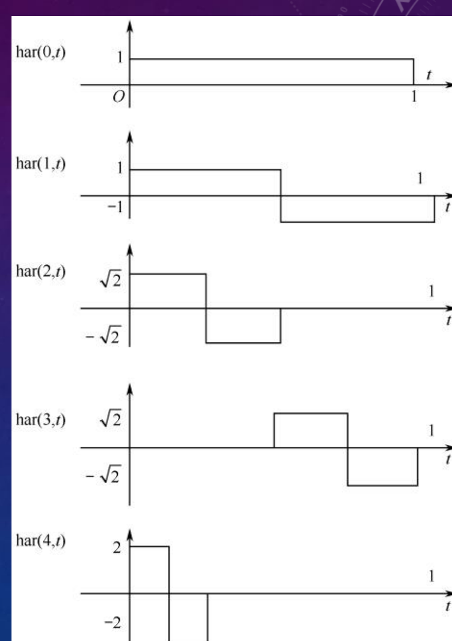
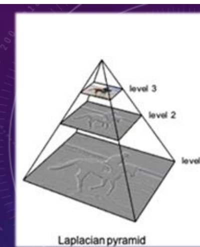


IMAGE TRANSFORMS – DWT CWT



- Forward CWT :

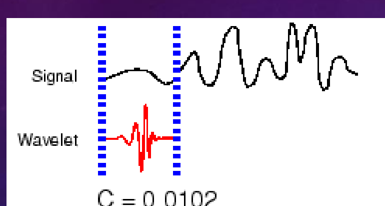
$$W_f(a,b) = \langle f, \psi_{a,b}(x) \rangle = |a|^{-\frac{1}{2}} \int_R f(x) \psi^* \left(\frac{x-b}{a} \right) \mathrm{d}x$$

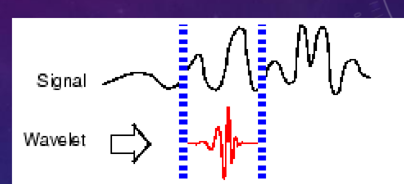
- Inverse CWT :

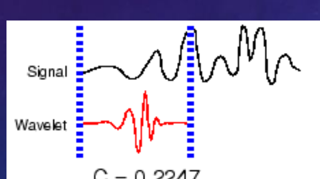
$$f(x) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_f(a,b) \psi_{a,b}(x) \frac{\mathrm{d}a}{a^2} \mathrm{d}b \qquad C_\psi < \infty$$

$$C_\psi = \int_{-\infty}^{\infty} \frac{|\Psi(\mu)|^2}{|\mu|} d\mu$$

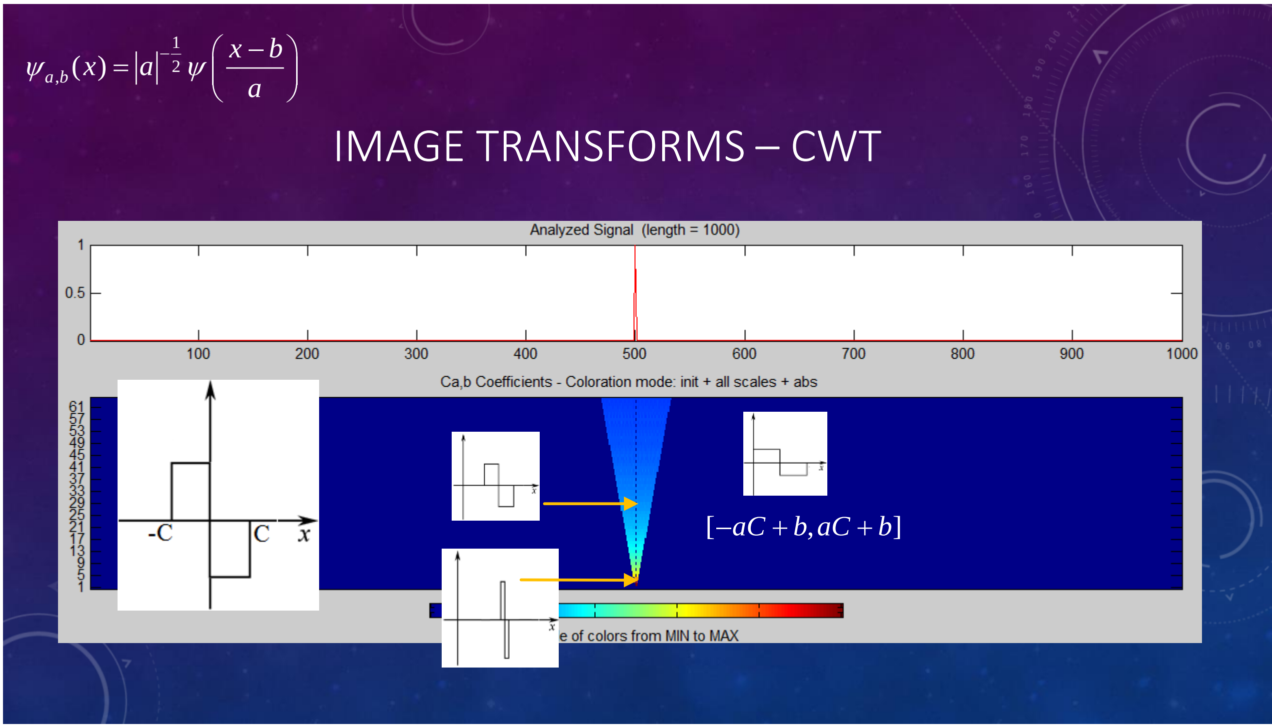
IMAGE TRANSFORMS – DWT







- Continuous Wavelet Transform



$\psi_{a,b}(x) = |a|^{-\frac{1}{2}} \psi\left(\frac{x-b}{a}\right)$
IMAGE TRANSFORMS – CWT
Analyzed Signal (length = 1000)
1
0.5
0
100 200 300 400 500 600 700 800 900 1000
Ca,b Coefficients - Coloration mode: init + all scales + abs
61 57 53 49 45 41 37 33 29 25 21 17 13 9 5 1
-C
C
x
$[-aC+b, aC+b]$
e of colors from MIN to MAX

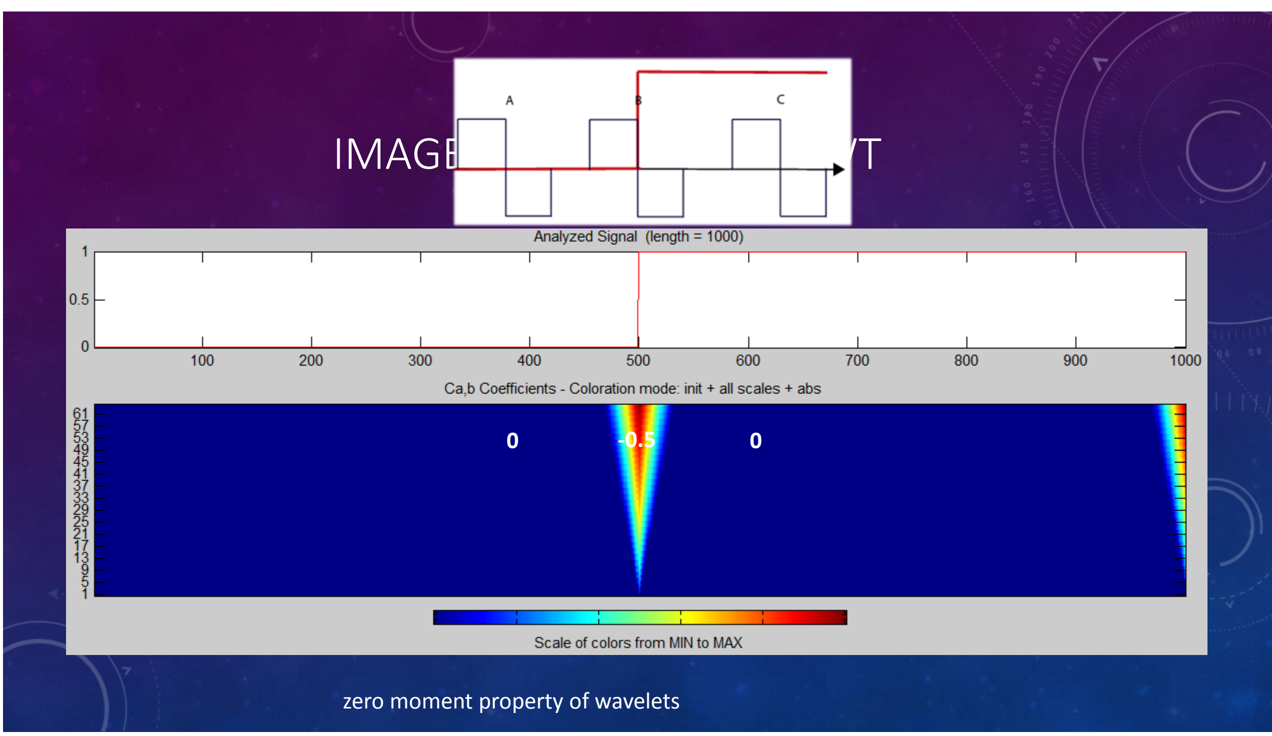


IMAGE
T
A
B
C
Analyzed Signal (length = 1000)
1
0.5
0
100 200 300 400 500 600 700 800 900 1000
Ca,b Coefficients - Coloration mode: init + all scales + abs
61 57 53 49 45 41 37 33 29 25 21 17 13 9 5 1
0
-0.5
0
Scale of colors from MIN to MAX
zero moment property of wavelets

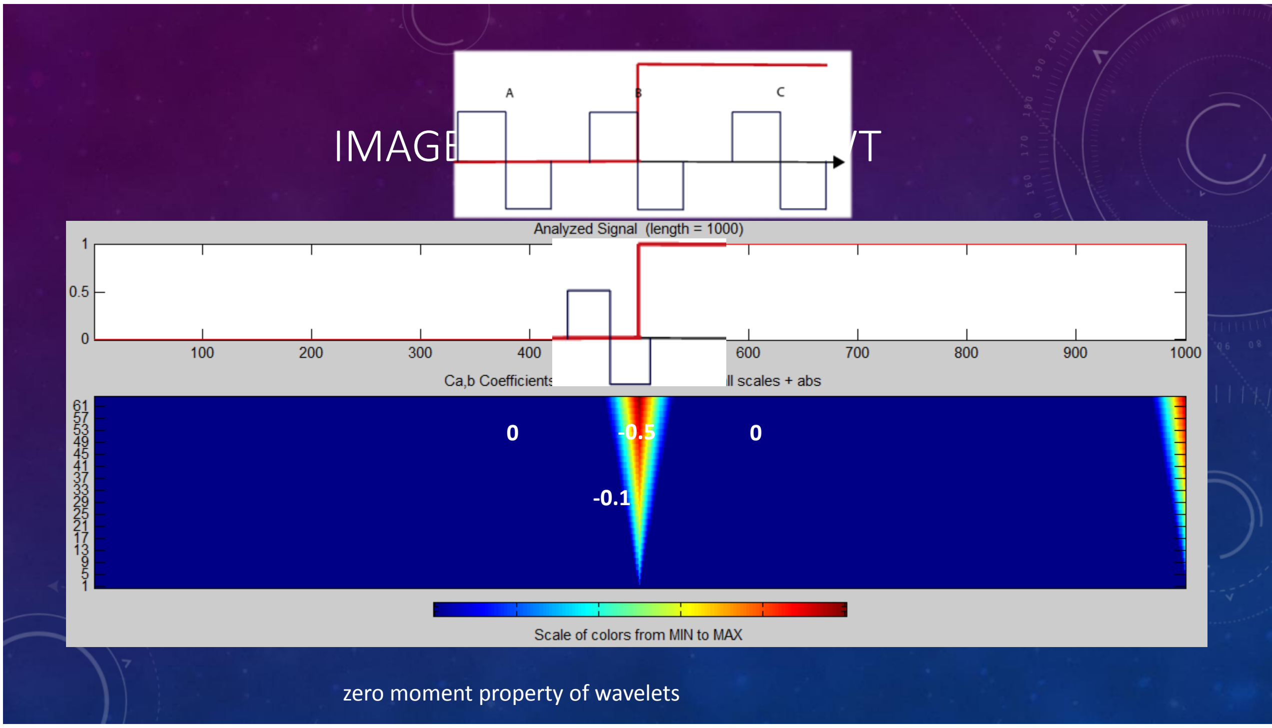


IMAGE
T
A
B
C
Analyzed Signal (length = 1000)
Ca,b Coefficients
ll scales + abs
0
-0.5
0
-0.1
Scale of colors from MIN to MAX
zero moment property of wavelets

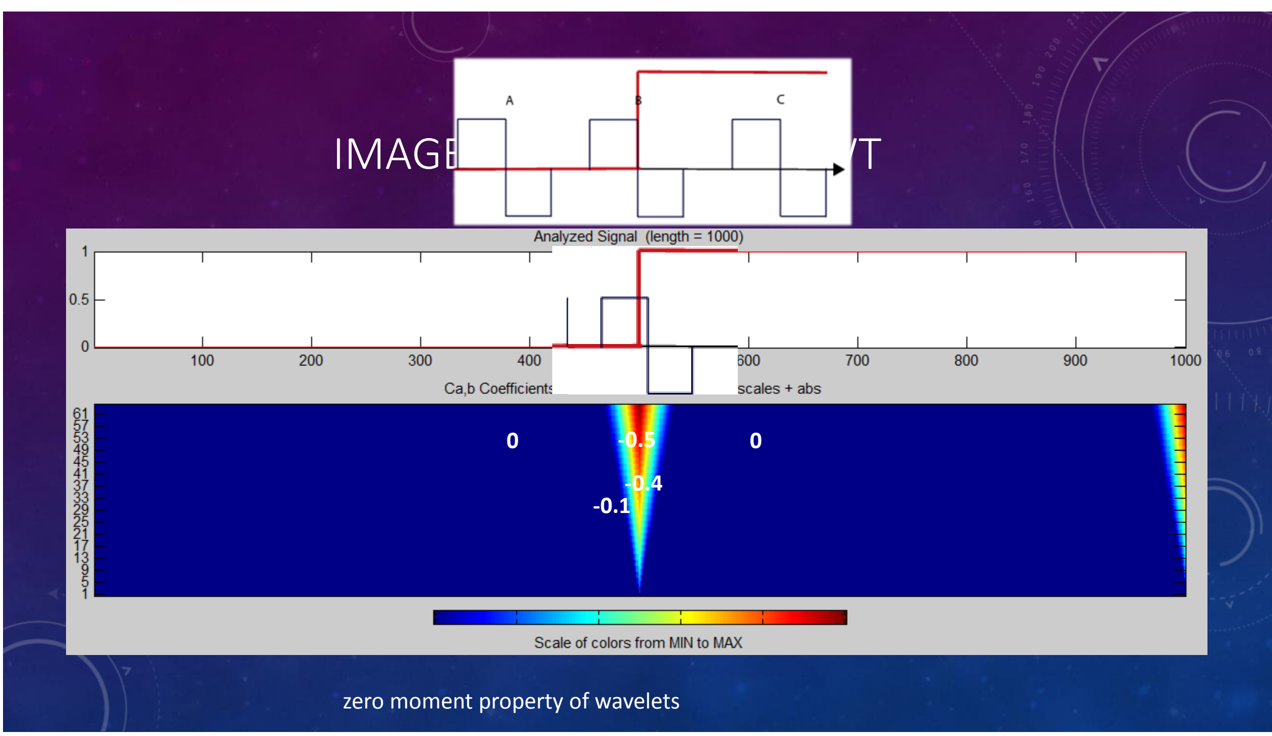
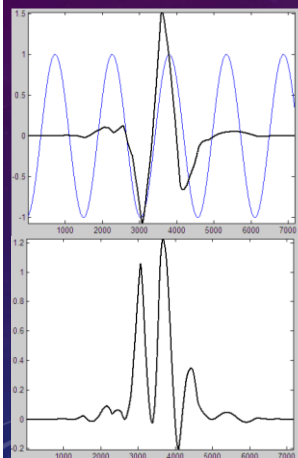


IMAGE
T
A
B
C
Analyzed Signal (length = 1000)
Ca,b Coefficients
scales + abs
0
-0.5
0
-0.4
-0.1
Scale of colors from MIN to MAX
zero moment property of wavelets

zero moment property of wavelets

IMAGE TRANSFORMS – CWT

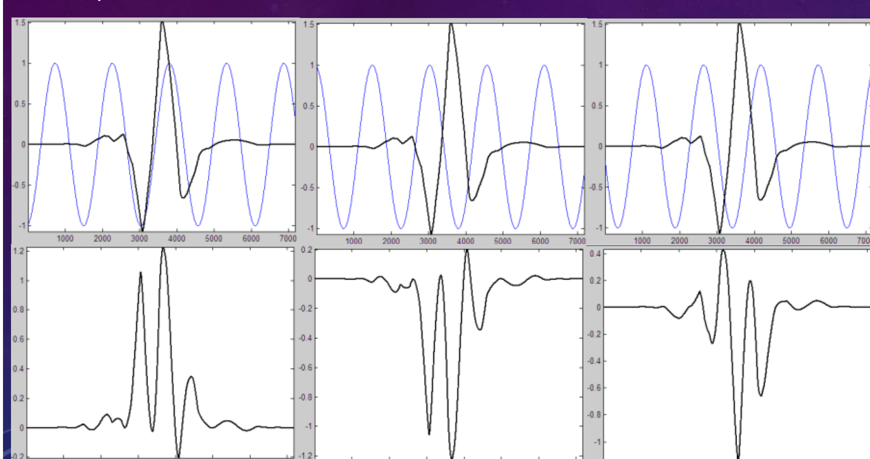
Symlets 4

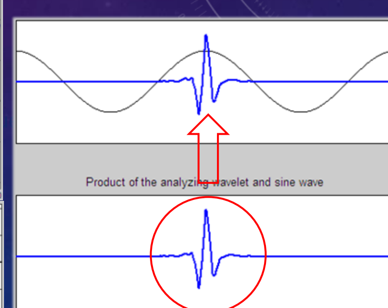


in phase同相

IMAGE TRANSFORMS – CWT

Symlets 4





in phase同相

out of phase反相

IMAGE TRANSFORMS – CWT

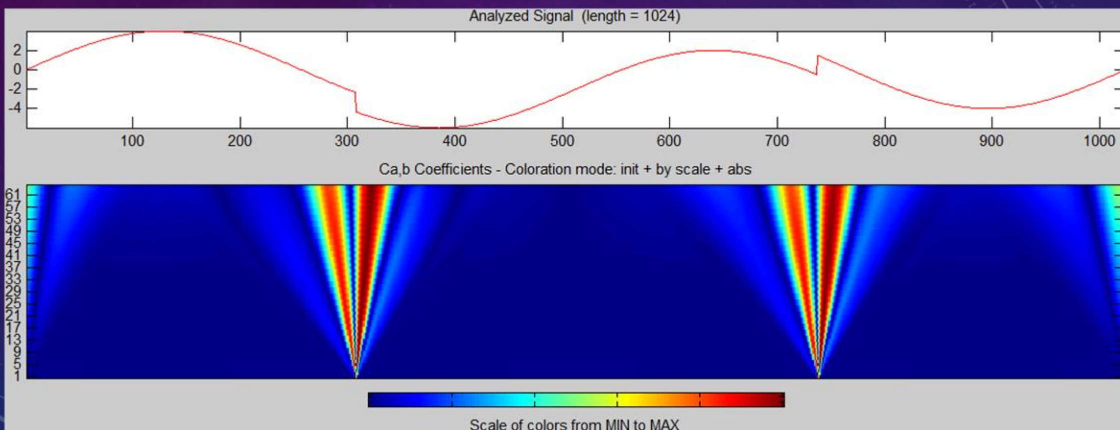


IMAGE TRANSFORMS – CWT

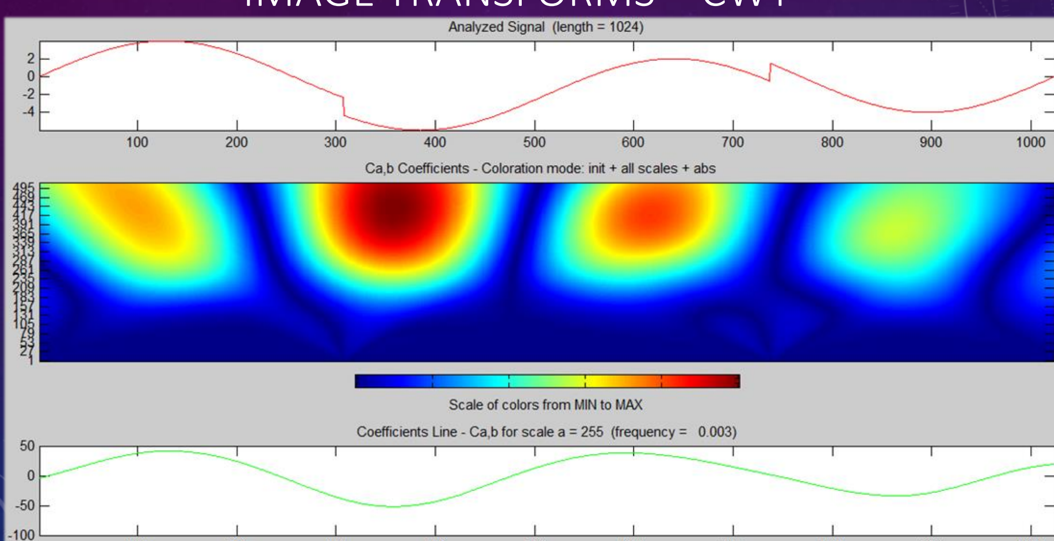
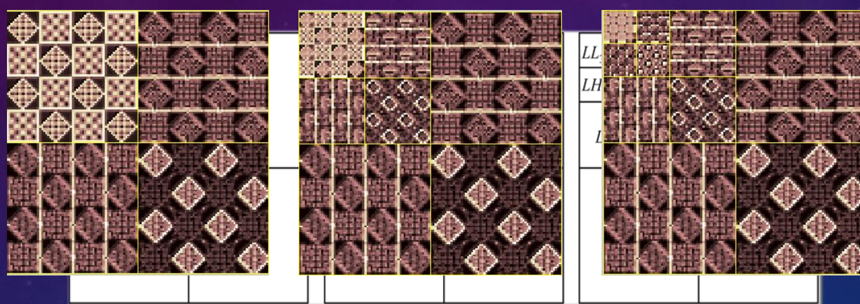


IMAGE TRANSFORMS – CWT



(a) single-scale (b) two-scale (c) three-scale

- The decomposition of separable 2D DWT

SUMMARY

- In some cases, image processing tasks are best formulated in a transform domain
- The transformation kernels are generally orthogonal
- By decorrelating the original image, efficient compression could be achieved
- Different transforms have different basis functions