

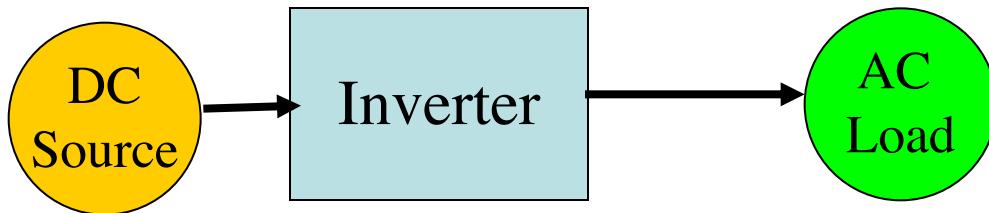


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# Inverters



# Inverters Basics



- Inverters are DC to AC converters
- Voltage source inverters synthesise an AC voltage from a DC supply voltage

<https://www.youtube.com/watch?v=qVeERT4nyz8>

- Inverters operate by controlling the duty cycle of the power devices.
  - Inverters can generate:
    - Single-phase AC supply
    - Three-phase AC supply
  - The basic building block is the two ‘Bridge’ circuit
- <https://www.youtube.com/watch?v=QNyoak45MSY>

# Inverters



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An inverter changes dc voltage to an ac voltage. For smaller renewable energy systems, the output voltage is usually the same as provided by the power company.

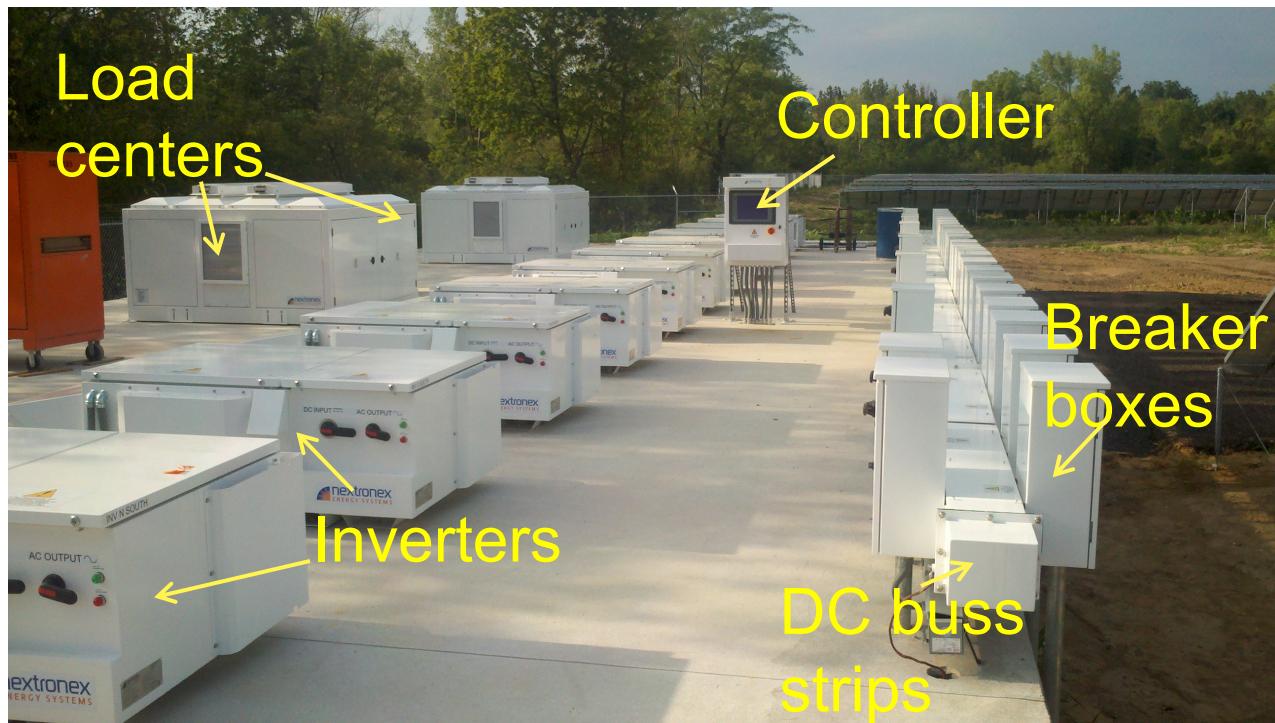
Typical inverters for small systems are illustrated. These can range from simple low power inverters that power isolated loads to grid tie units capable of sending excess power to the grid.



# Inverters



Large grid tied inverters are used in commercial and utility systems. Large inverters reduce the installation time and do not require a special transformer. A modular approach is another method to connect an array and gives flexibility to add or subtract capability as conditions change.

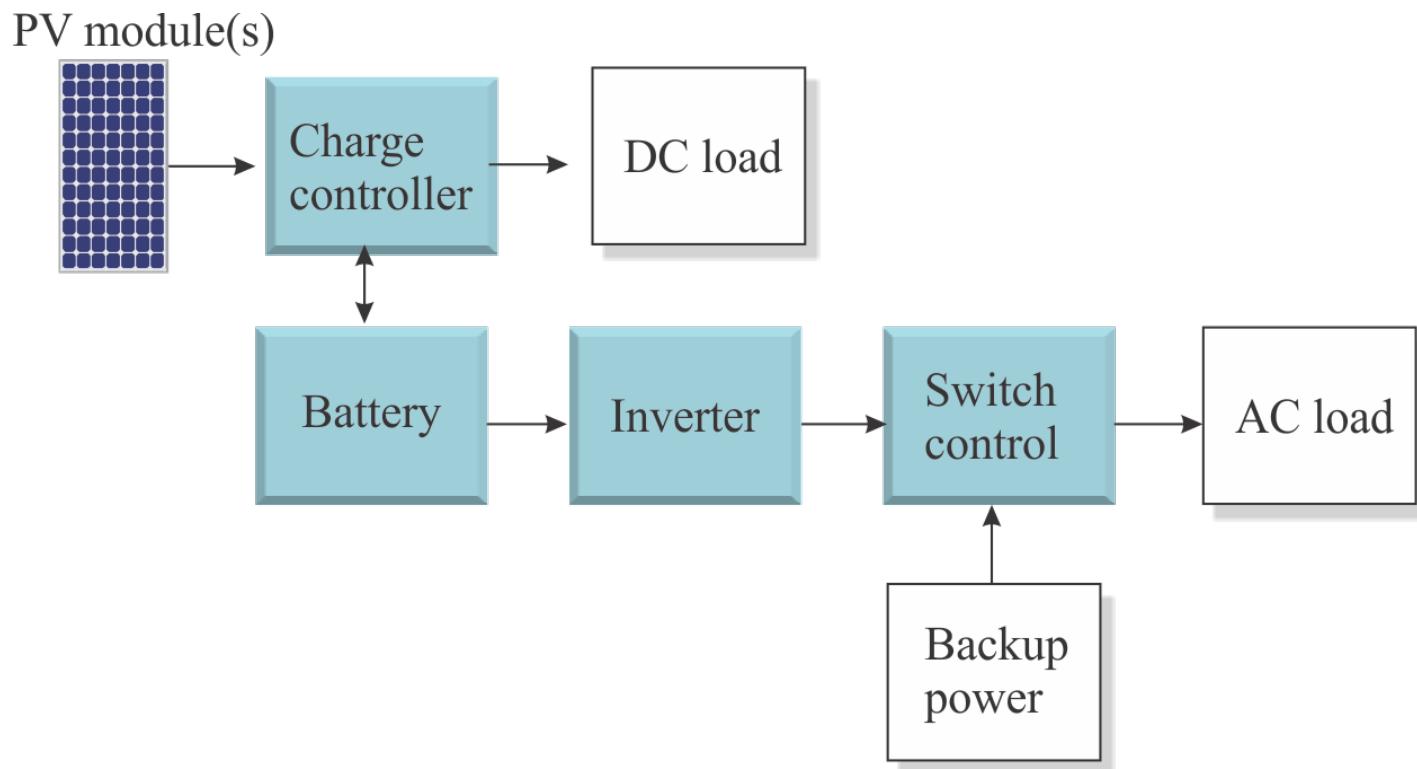


Source: courtesy of Nextronex

# Inverters



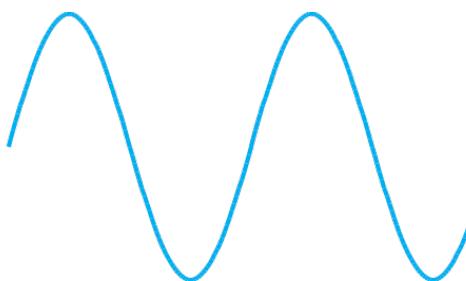
## Off Grid Inverter



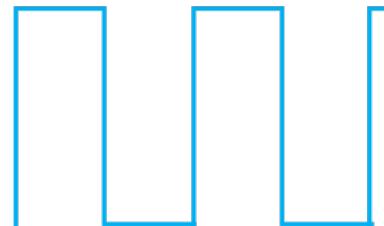
# Inverters



Grid-tie inverters must produce a low-distortion sine wave for the grid; if an inverter is not connected to the grid, another waveform can be used. Low-end inverters may produce a square wave or a modified sine wave, but these are not suitable for all loads.



(a) Sine wave



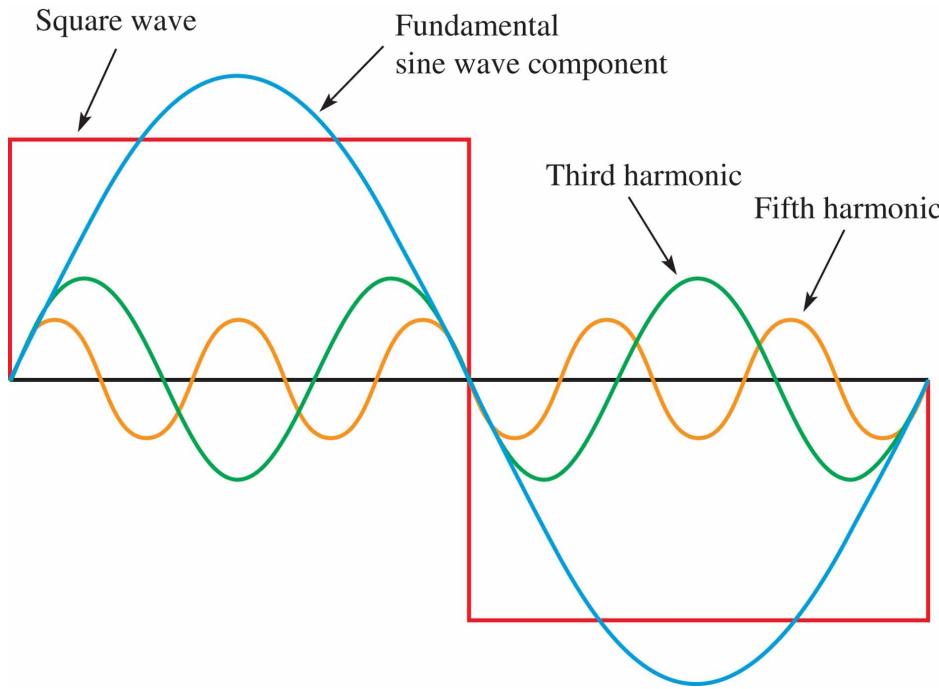
(b) square wave



(c) Modified sine wave



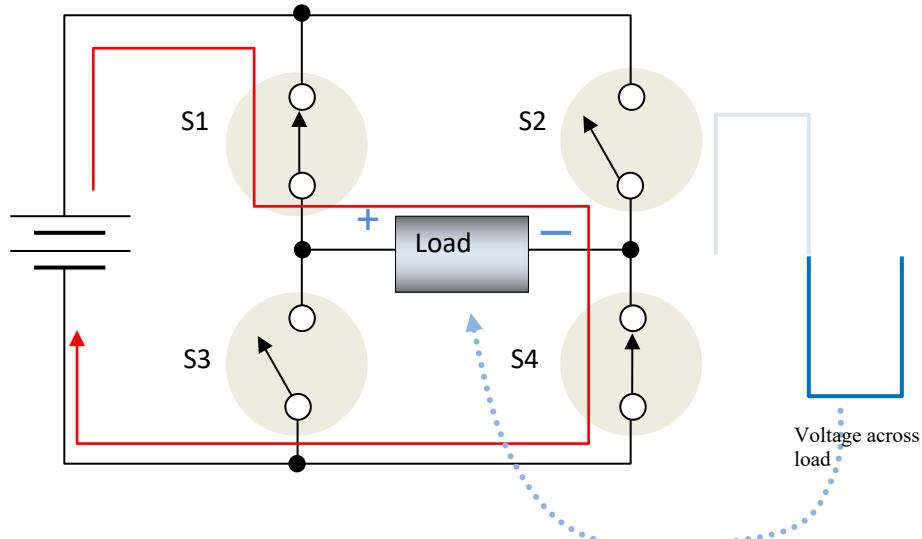
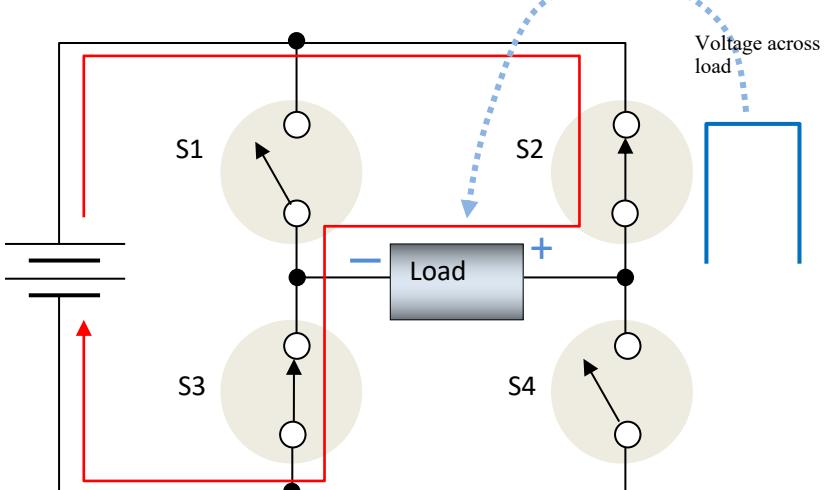
A square wave is simple to generate from dc but consists of a fundamental sine wave and multiples of the fundamental frequency called harmonics. It is the harmonics that cause difficulty with some loads.



# Inverters



An H-bridge can produce a basic bipolar square wave. A variation of this circuit is used with other circuitry to produce a sine wave using pulse-width modulation (PWM).

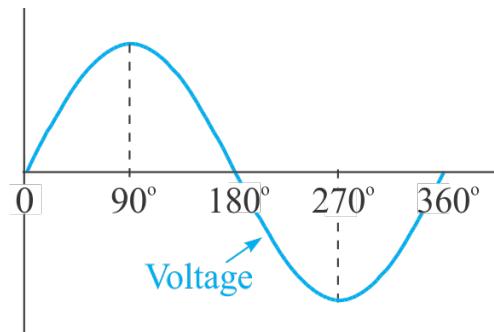


# Inverter Functions

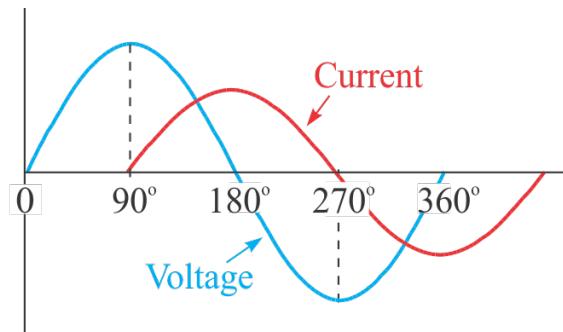


When a reactive load is connected to a source, the voltage and current shift in phase, which causes a reduction in the true power that can be delivered. To avoid this, some inverters can correct for power factor.

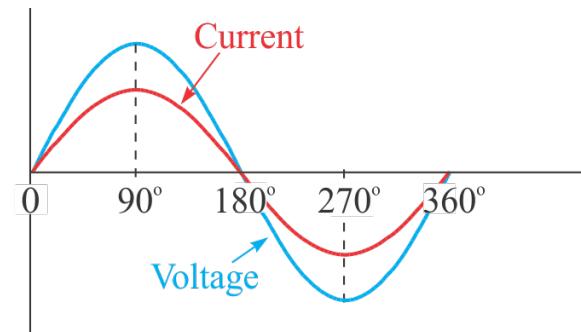
$$PF = \cos(\theta)$$



power factor = 0



power factor = 1

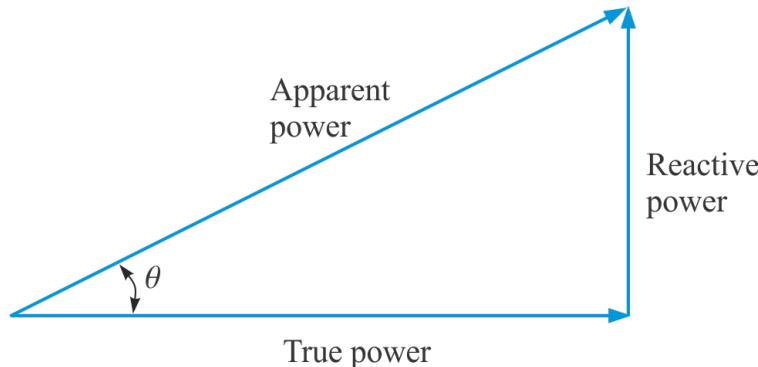


# Inverter Functions

The power relationship between true power, reactive power, and apparent power can be illustrated with a right triangle.

## Example

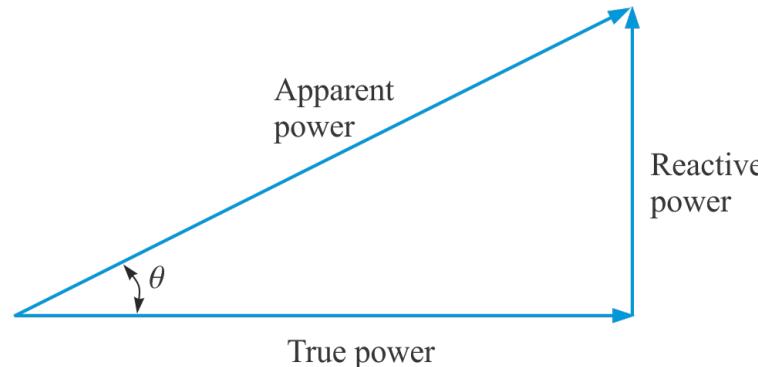
- What is the apparent power if the true power is 60 W and the reactive power is 30 VAR?
- What is the phase angle?



# Inverter Functions



The power relationship between true power, reactive power, and apparent power can be illustrated with a right triangle.



## Example

- What is the apparent power if the true power is 60 W and the reactive power is 30 VAR?
- What is the phase angle?

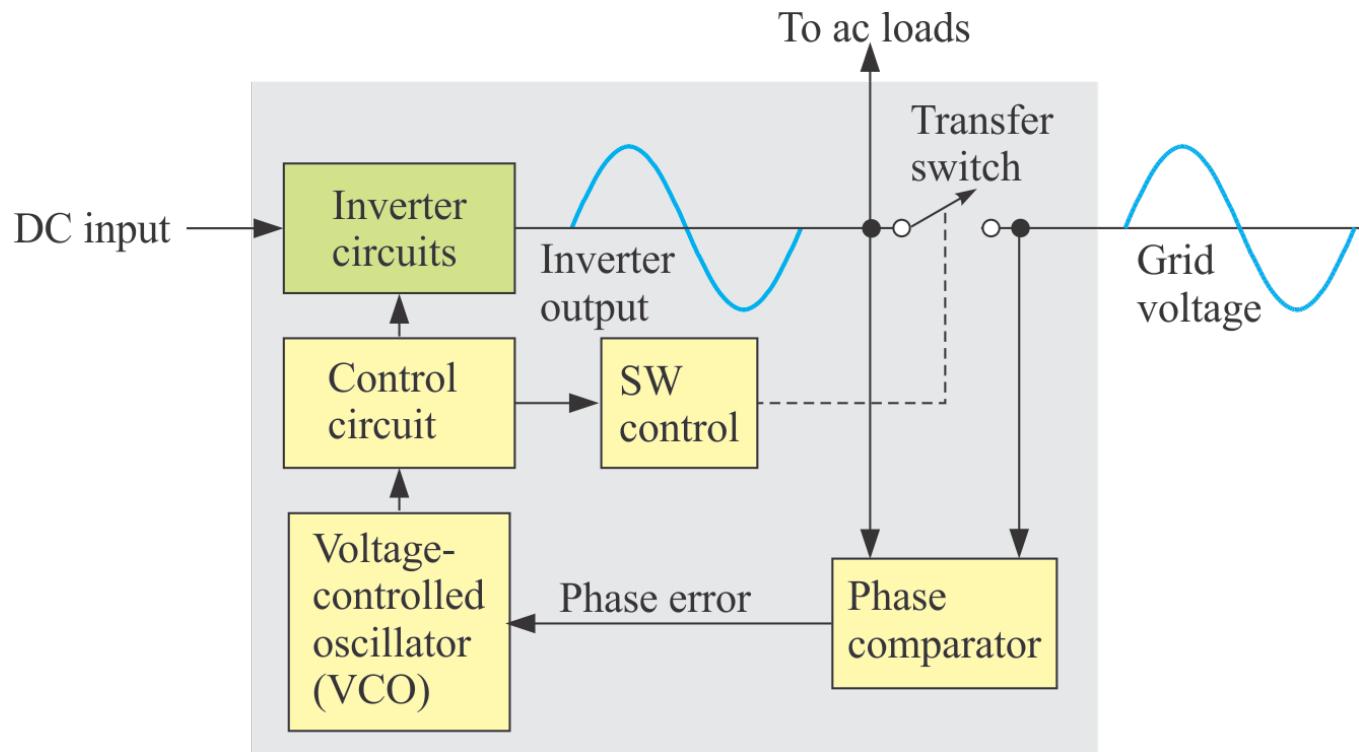
## Solution

$$(a) P_a = \sqrt{P_{\text{true}}^2 + P_r^2} = \sqrt{(60 \text{ W})^2 + (30 \text{ VAR})^2} = 67.1 \text{ VA}$$

$$(b) \theta = \cos^{-1} \left( \frac{\text{True power}}{\text{Apparent power}} \right) = \cos^{-1} \left( \frac{60 \text{ W}}{67.1 \text{ VAR}} \right) = 26.5^\circ$$

# Inverter Functions

A grid-tie inverter must synchronize its output voltage with the grid voltage in terms of frequency, phase and amplitude. A phase-locked loop can synchronize the inverter:



Anti-islanding is a protective feature that detects when there is a power outage and disconnects from the grid with a **transfer switch** when it happens.

## Passive detection

- Detects a sudden change in frequency, voltage, phase, or power

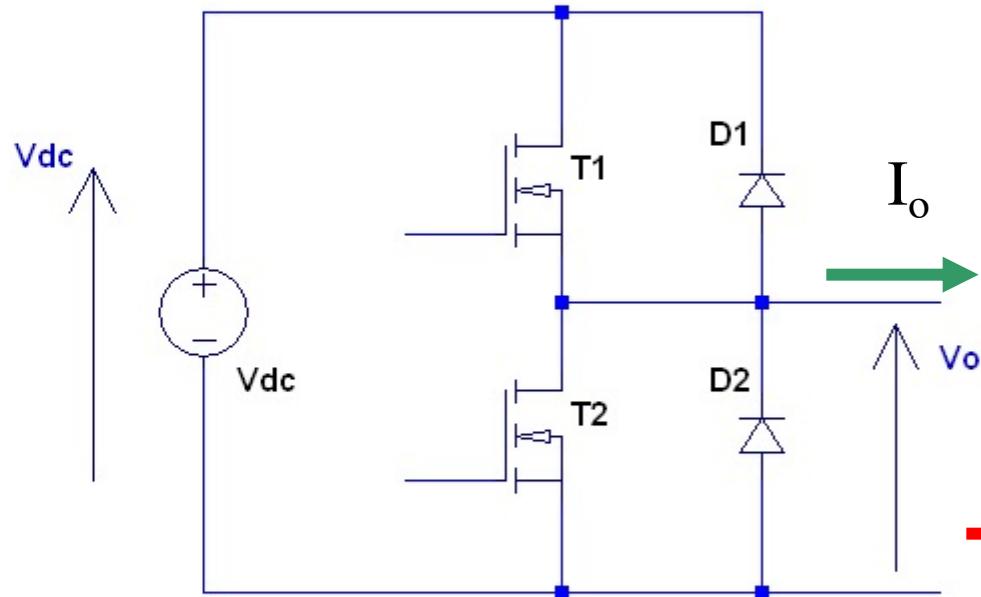
## Active detection

- Looks for a response to an injected signal

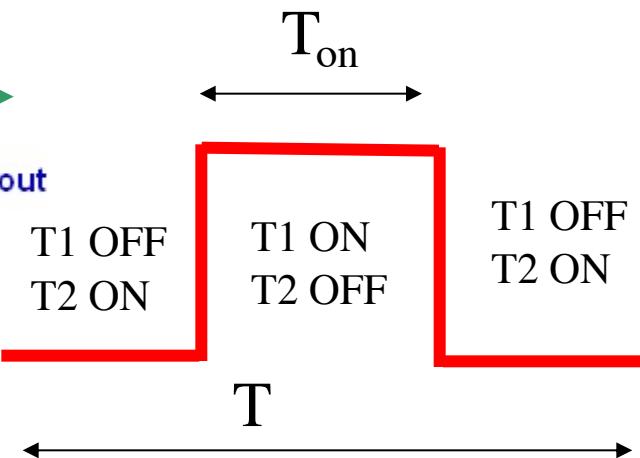
## Utility notification

- Utility company issues a signal to the inverter to stop generating power

# Bridge Circuit



$$V_{out} = V_{dc} \frac{t_{1,on}}{T_s}$$

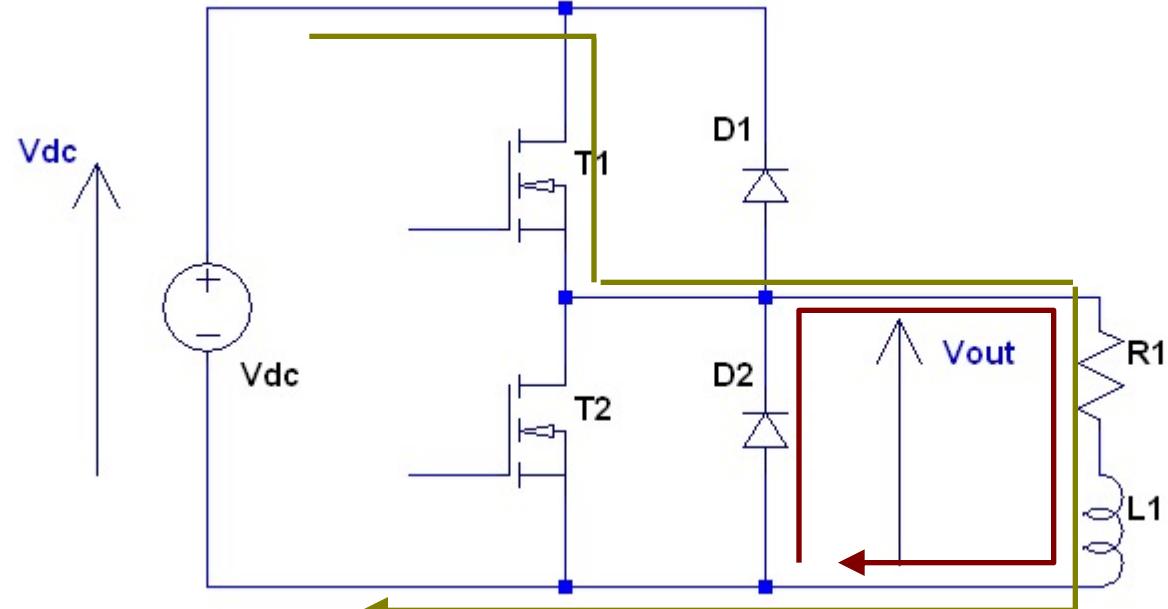


$I_o > 0$   $T_1$  and  $D_2$  are active

$I_o < 0$   $D_1$  and  $T_2$  are active

# Current Paths

- Two switches with freewheel diodes provides uni-directional voltage and bi-directional current control.
- Only when T1 is ON is energy supplied from the source.
- When T2 ON, a zero voltage loop is applied.
- With positive current flow →

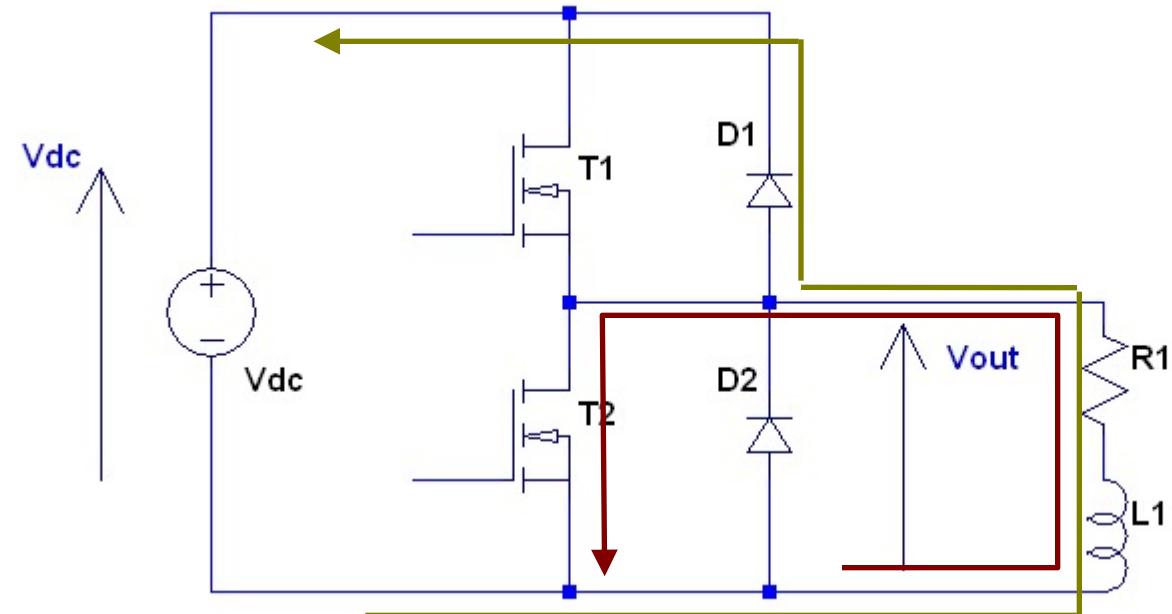


← Current path if T2 ON, or T1 and T2 OFF

← Current path if T1 ON

# Current Paths

- When T1 is ON (or T1 and T2 OFF) energy has to be **absorbed** by the source.
- When T2 ON, a zero voltage loop is applied.
- With negative current flow →



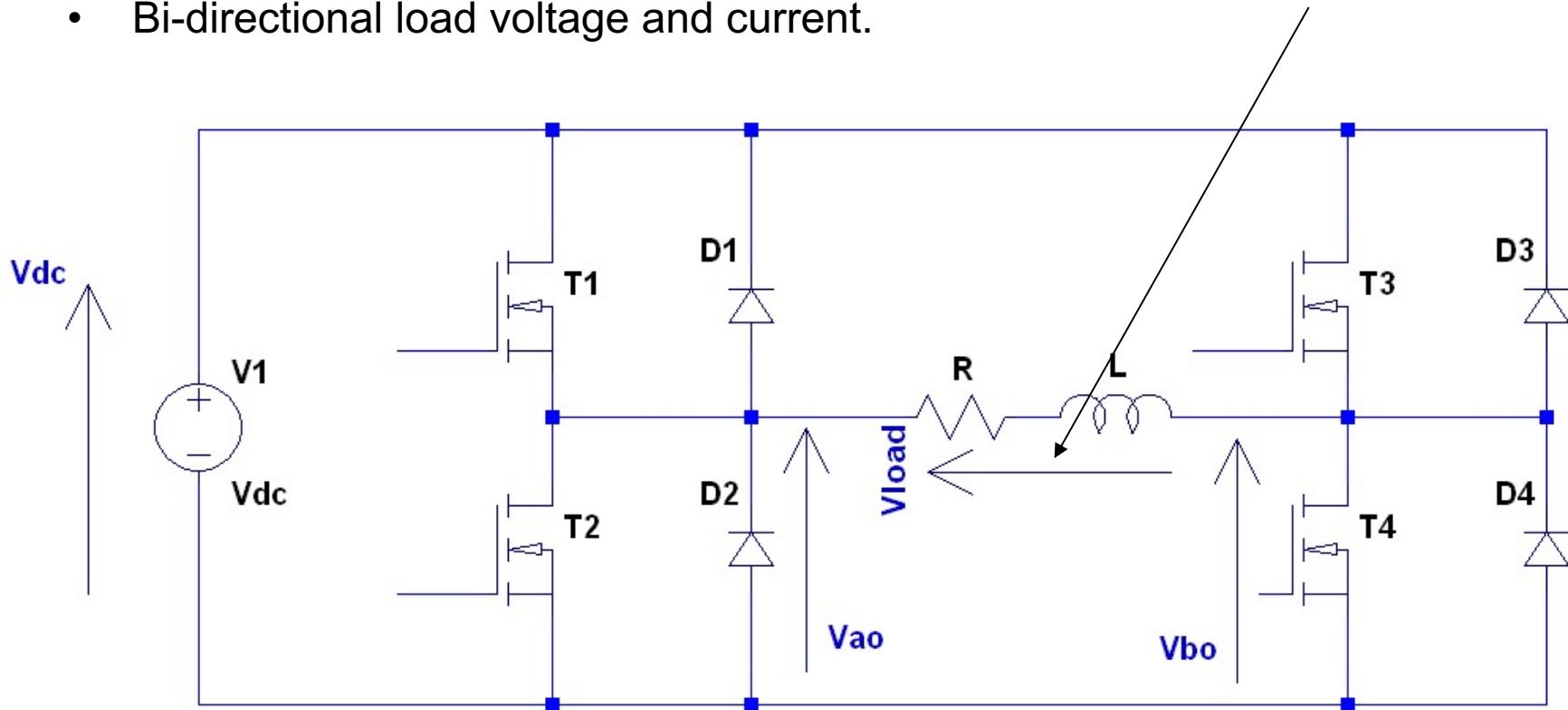
← Current path if T2 ON

← Current path if T1 ON, or T1 and T2 OFF

# Single-phase H-bridge

- Two inverter legs connected in parallel.
- Bi-directional load voltage and current.

$$V_{load} = V_a - V_b$$



# Single Phase AC Output

# Basic AC Waveform Generation

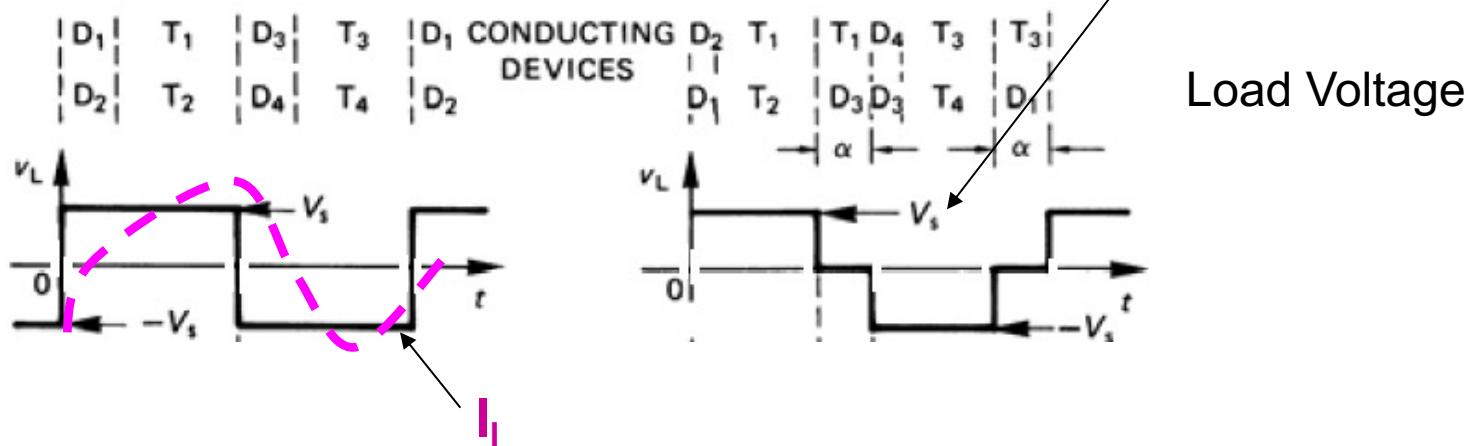
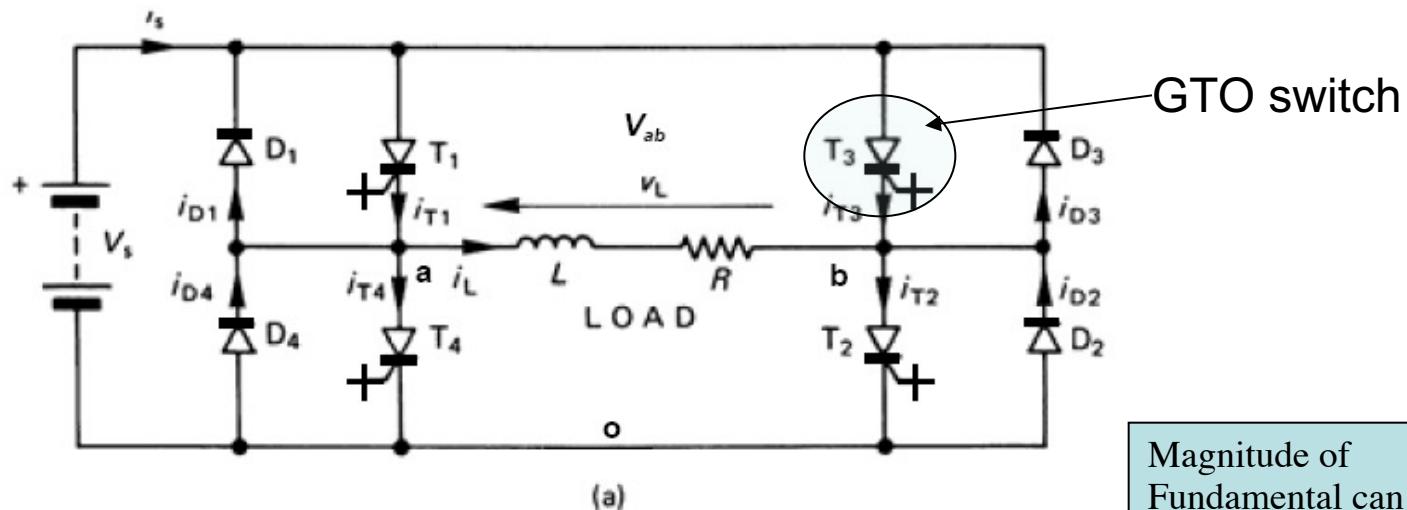
A simple AC waveform may be generated by switching the bridge legs at the fundamental frequency.

The converter gives square wave output at the desired power frequency.

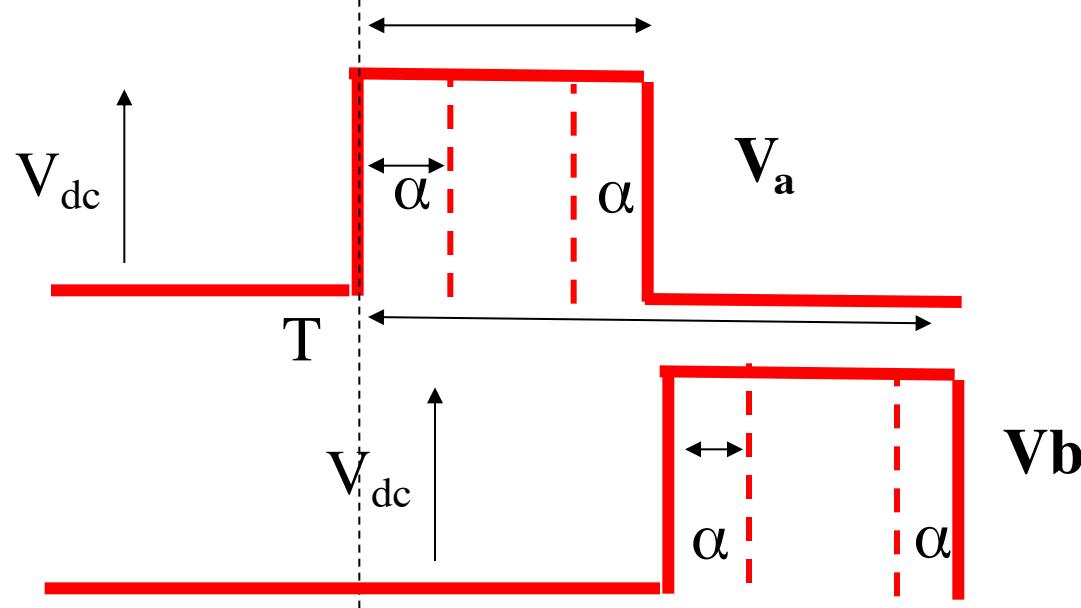
The maximum output corresponds to a  $-V_{dc}$  to  $+V_{dc}$  square-wave.

It is possible to vary the magnitude of the fundamental component of the AC output by introducing zero output voltage regions where  $V_a = V_b$

# Single Phase H-Bridge Inverter

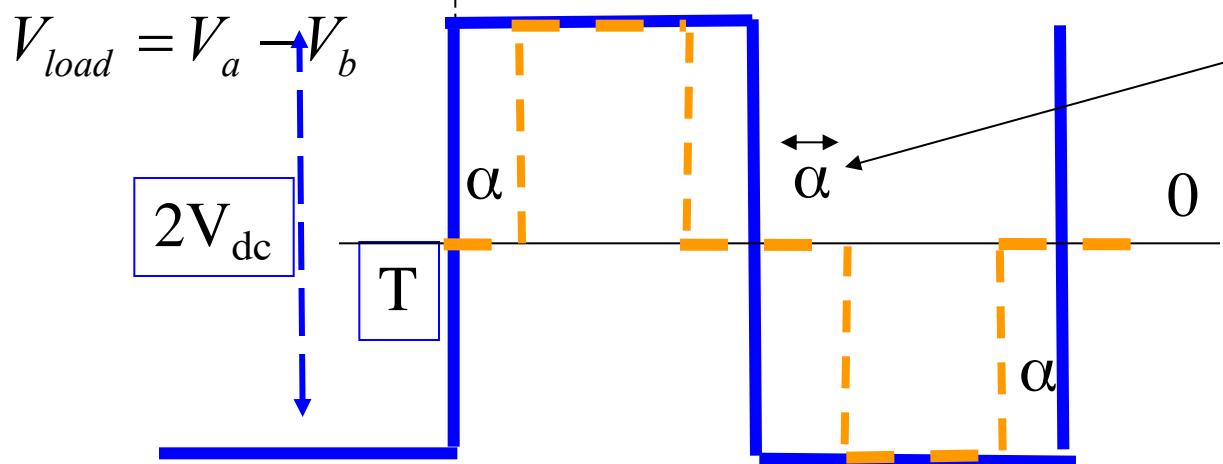


# Load Voltage



The common DC Component  $a_0$  is cancelled.

The load sees a fundamental of:  
 $4V_{dc}/\pi$



Fundamental magnitude may be controlled by a delay angle:  $\alpha$



# Voltage Harmonics

The simplest modulation technique is to use a square wave approximation with devices switched at the fundamental frequency.

This has the problem of a large low frequency harmonic content.

Fourier series of single phase gives:

$$a_0 = \frac{V_s}{2} \quad \text{DC component}$$

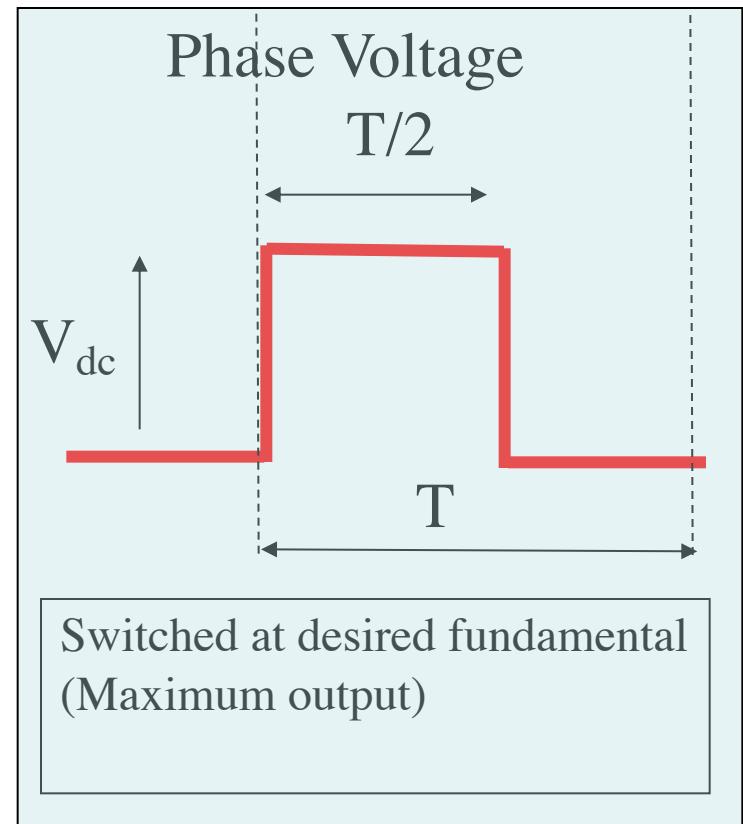
$$a_n = \frac{1}{\pi} \int_0^\pi V_a \sin n\theta d\theta$$

$$0 < \theta < \pi \Rightarrow V_a = V_{dc} \quad \pi < \theta < 2\pi \Rightarrow V_a = 0$$

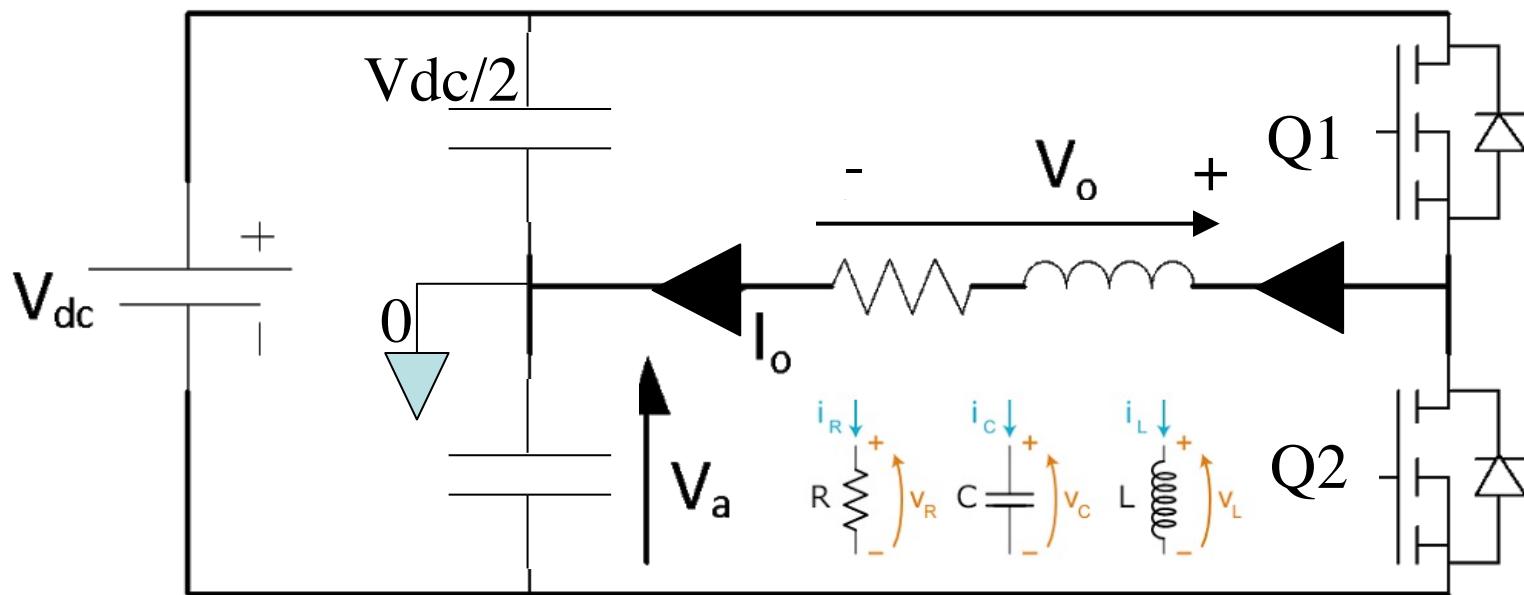
$$a_n = \frac{V_{dc}}{\pi} \left[ \frac{-\cos \theta n}{n} \right]_0^\pi \quad \text{Harmonics}$$

$$|a_n| = \frac{2V_{dc}}{n\pi} \quad n : \text{odd}$$

Integral in angular base where  $\theta = 2\pi^*(t/T)$

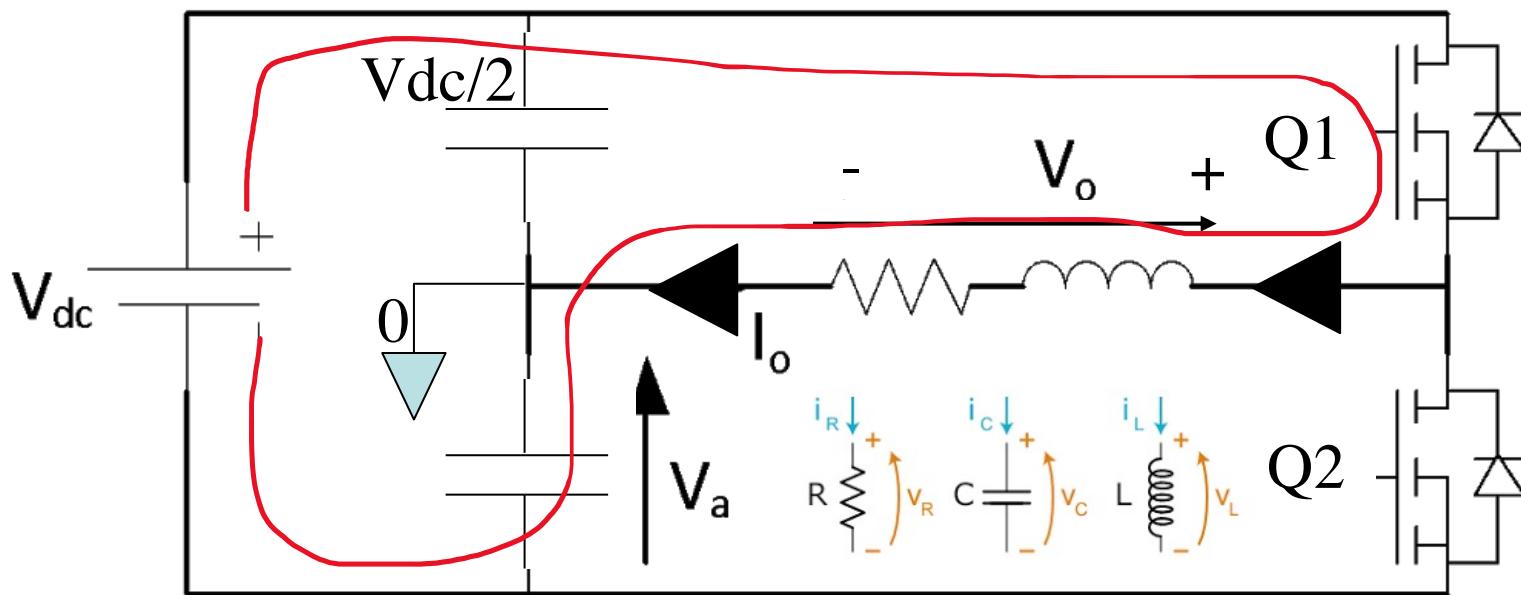


# Switching States – Half Bridge



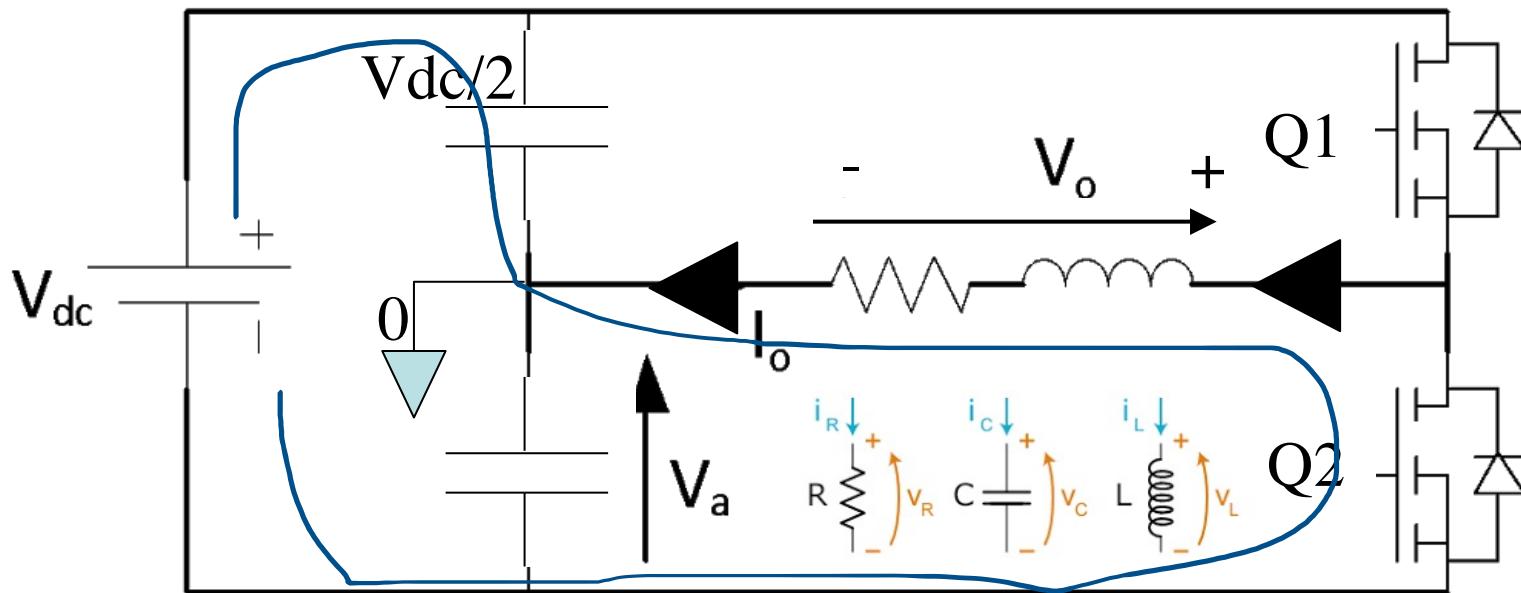
| <b>Q1</b> | <b>Q2</b> | <b><math>V_o</math></b> |
|-----------|-----------|-------------------------|
| 0         | 0         |                         |
| 0         | 1         |                         |
| 1         | 0         |                         |
| 1         | 1         |                         |

# Switching States – Half Bridge

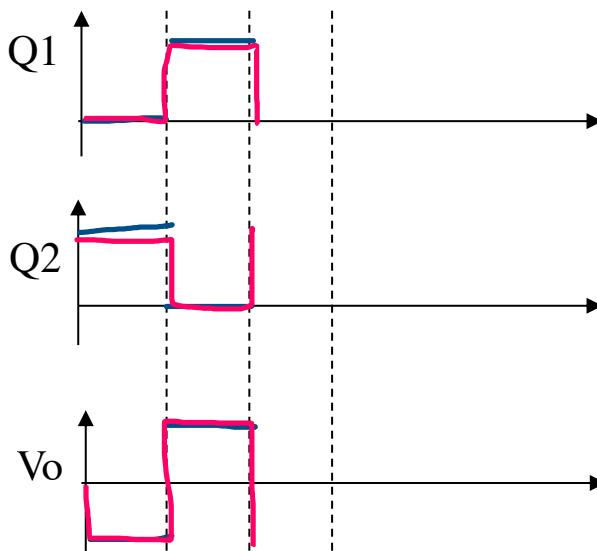


| <b>Q1</b> | <b>Q2</b> | <b><math>V_o</math></b> |
|-----------|-----------|-------------------------|
| 0         | 0         |                         |
| 0         | 1         |                         |
| 1         | 0         | $V_{dc}/2$              |
| 1         | 1         |                         |

# Switching States – Half Bridge

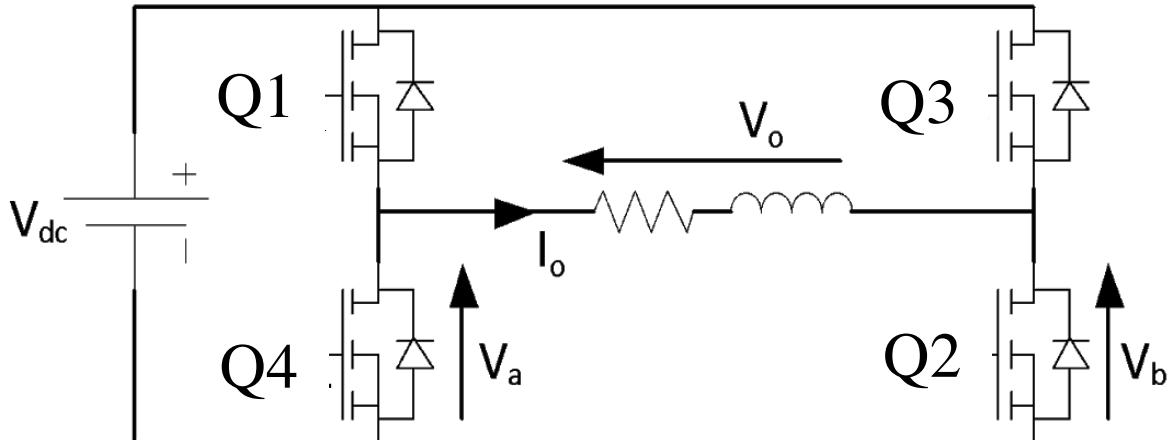


| <b>Q1</b> | <b>Q2</b> | <b><math>V_o</math></b> |
|-----------|-----------|-------------------------|
| 0         | 0         | X                       |
| 0         | 1         | $-V_{dc}/2$             |
| 1         | 0         | $V_{dc}/2$              |
| 1         | 1         | X                       |

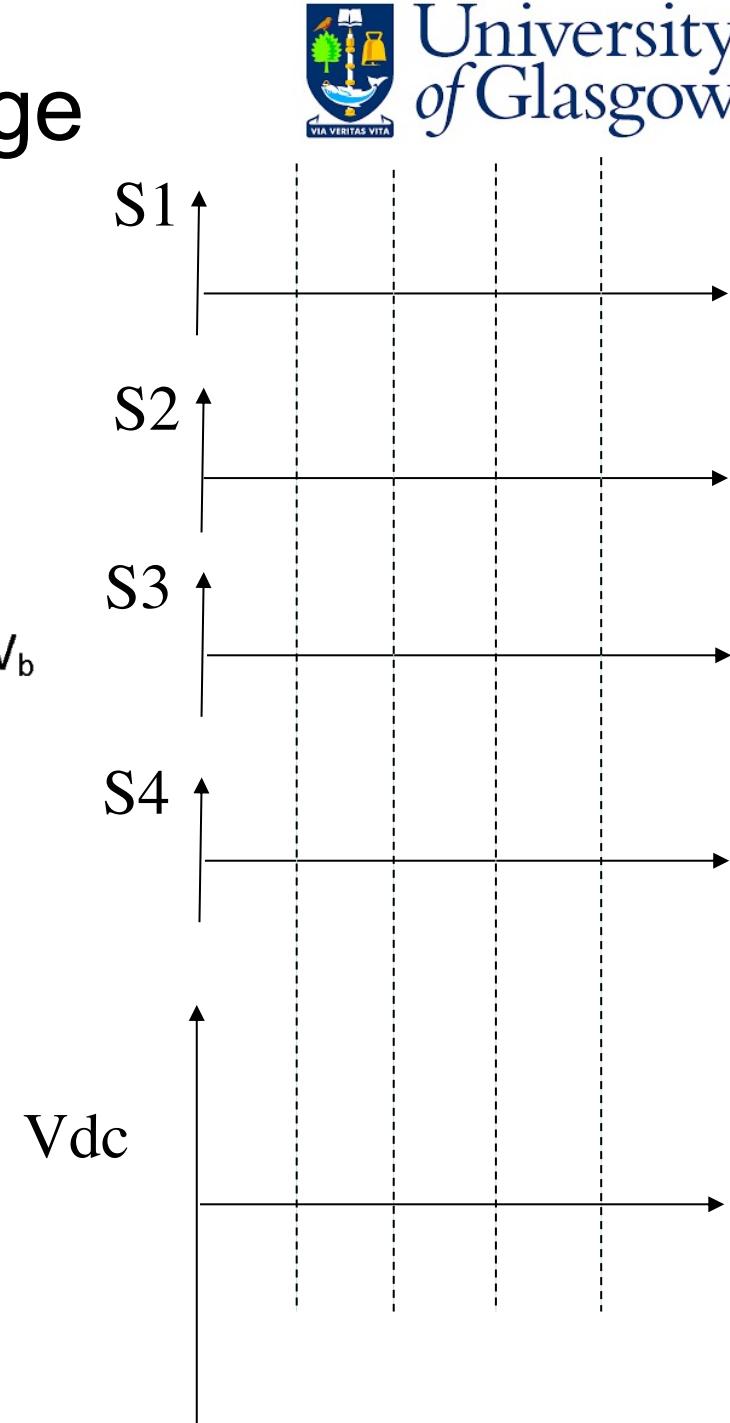




# Switching States – Full Bridge

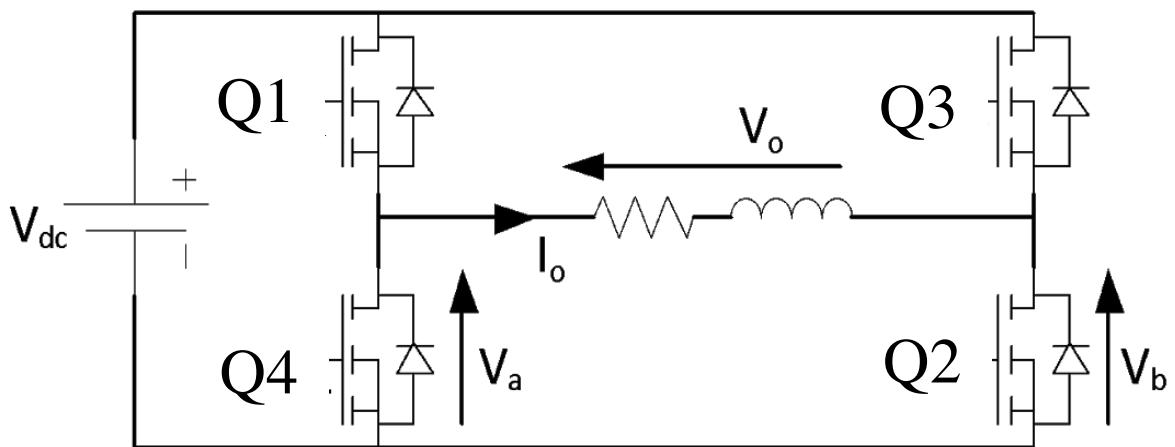


| Q1 | Q2 | Q3 | Q4 | $v_o$ |
|----|----|----|----|-------|
| 1  |    |    |    |       |
| 1  |    |    |    |       |
| 0  |    |    |    |       |
| 0  |    |    |    |       |

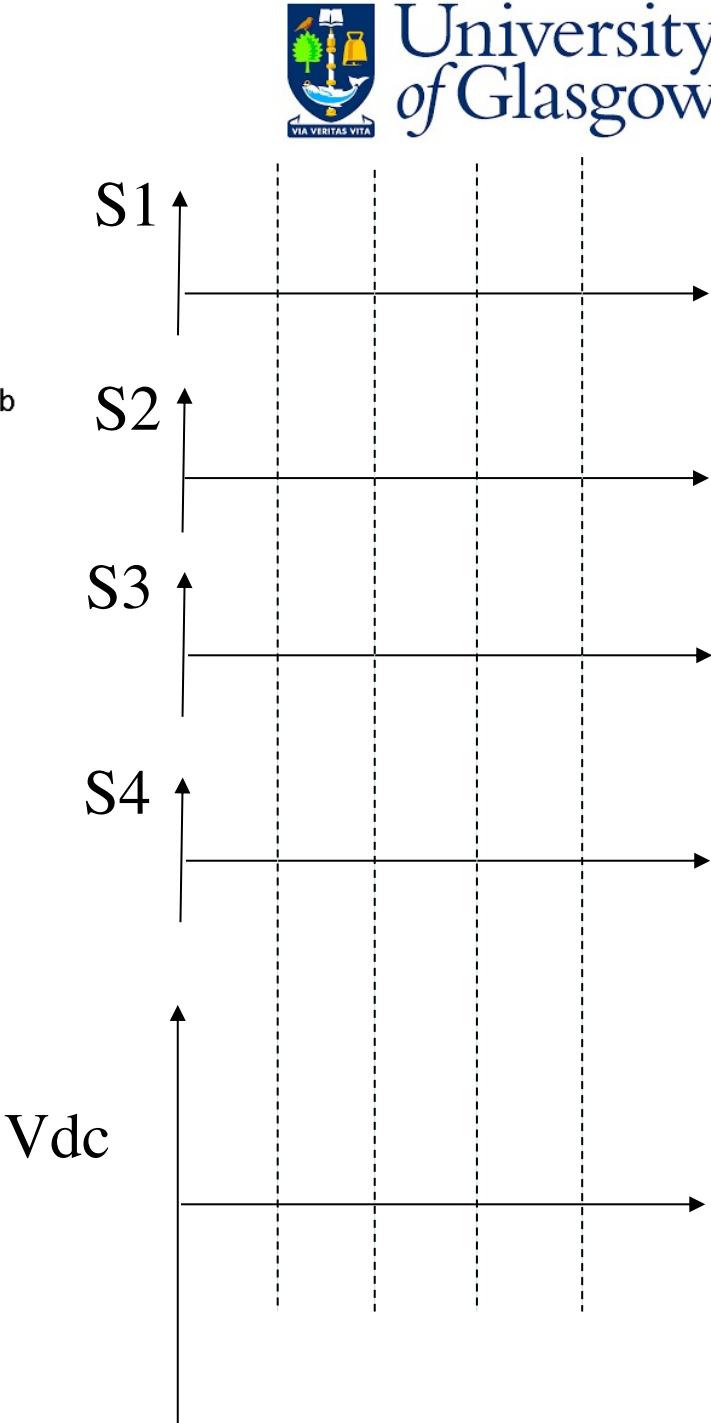


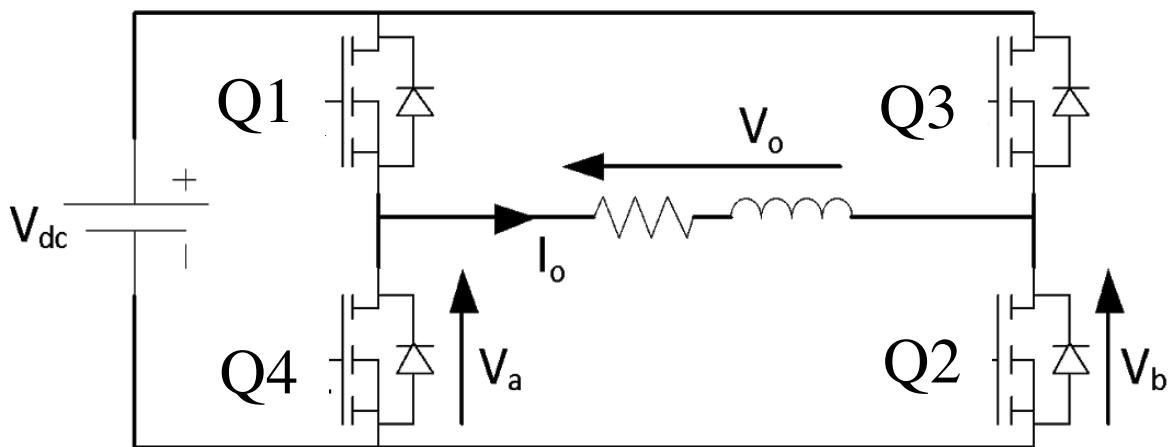


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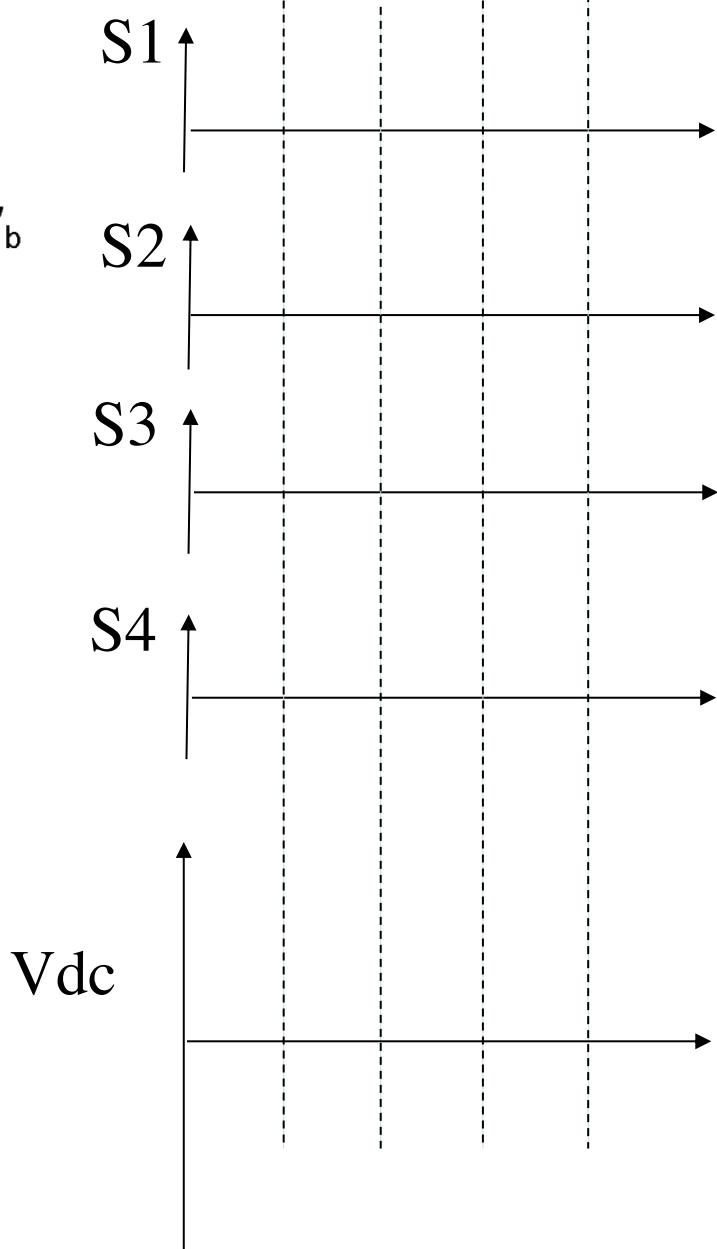


| Q1 | Q2 | Q3 | Q4 | $v_o$ |
|----|----|----|----|-------|
| 1  |    |    |    | 0     |
| 1  |    |    |    |       |
| 0  |    |    |    |       |
| 0  |    |    |    |       |



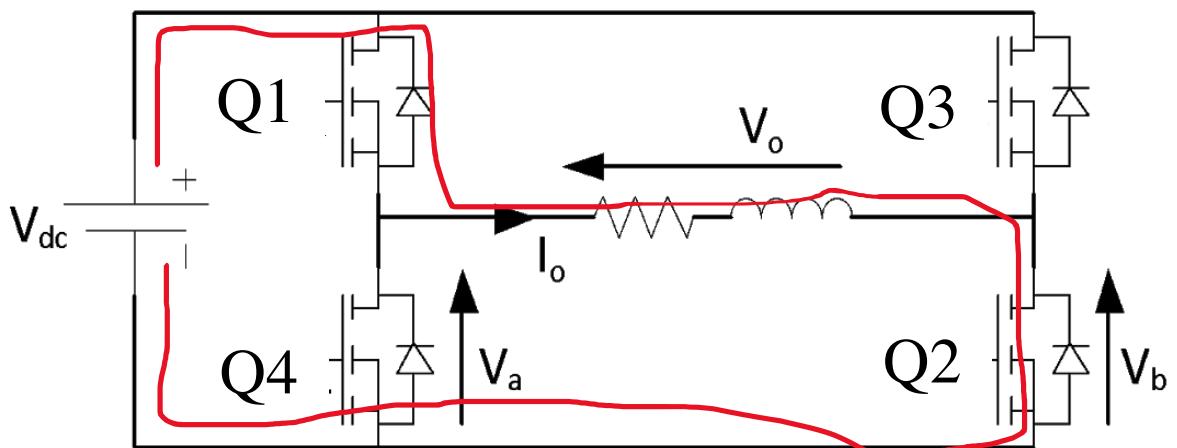


| Q1 | Q2 | Q3 | Q4 | $v_o$ |
|----|----|----|----|-------|
| 1  | 1  | 0  | 0  |       |
| 1  |    |    |    |       |
| 0  |    |    |    |       |
| 0  |    |    |    |       |

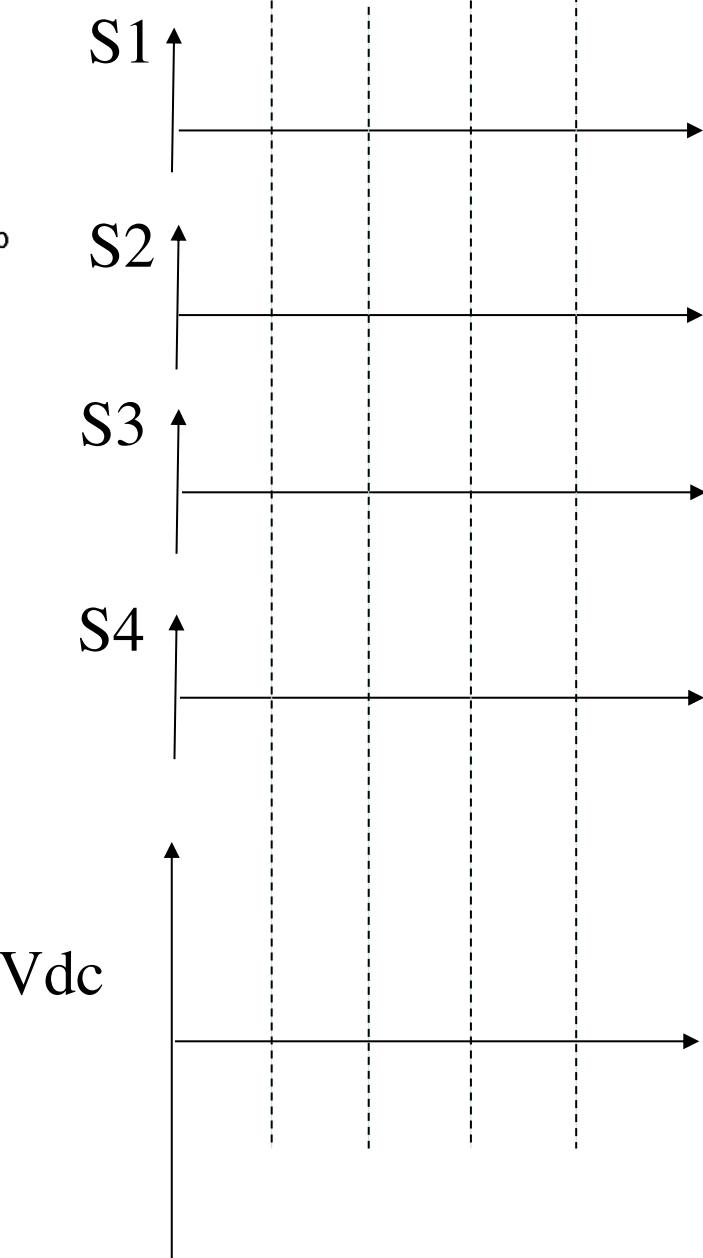


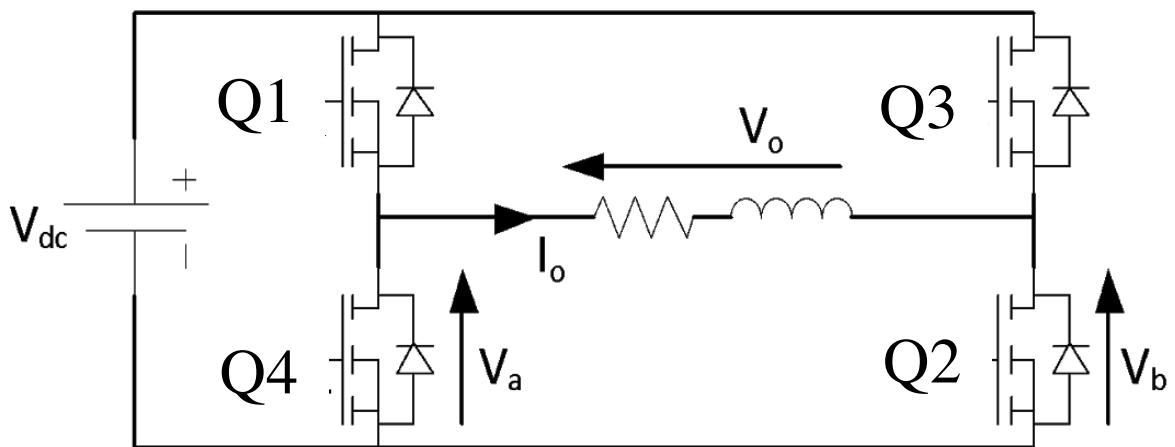


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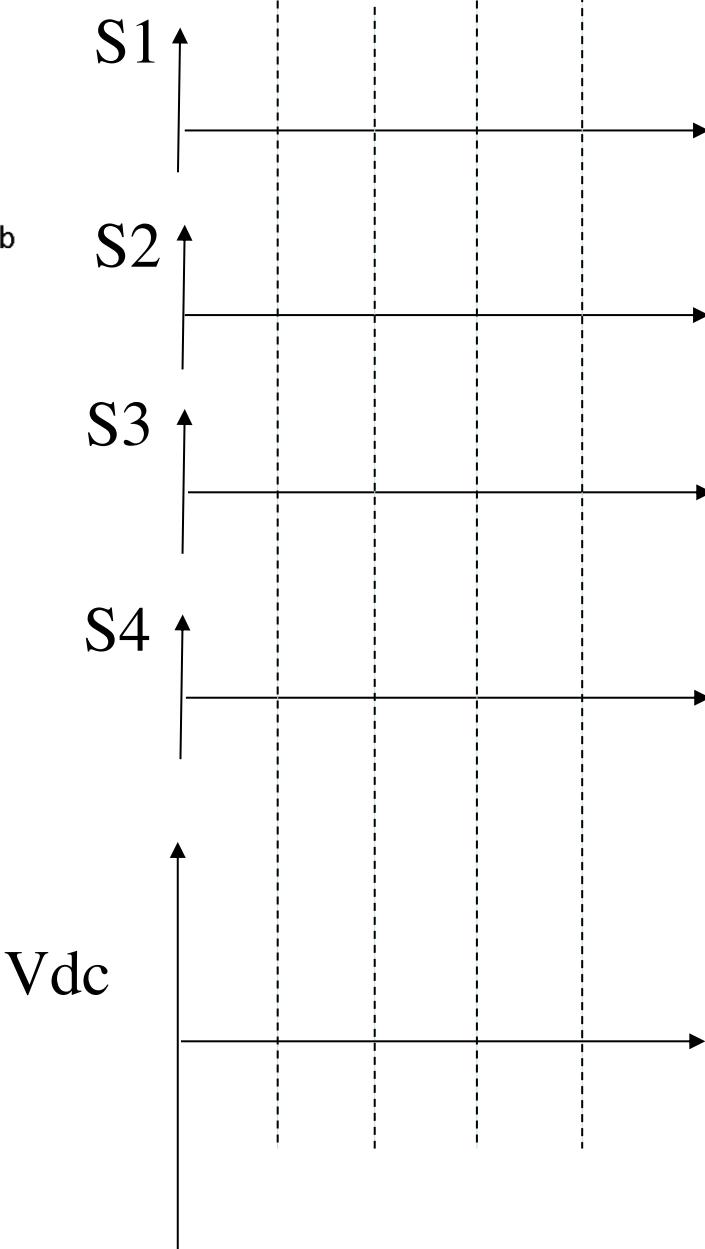


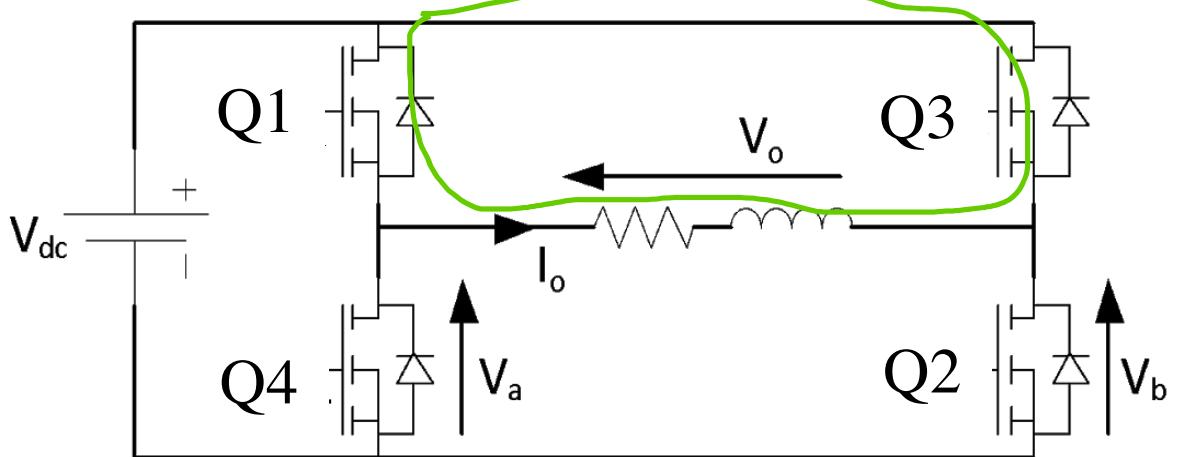
| Q1 | Q2 | Q3 | Q4 | $v_o$     |
|----|----|----|----|-----------|
| 1  | 1  | 0  | 0  | $-V_{dc}$ |
| 1  |    |    |    |           |
| 0  |    |    |    |           |
| 0  |    |    |    |           |



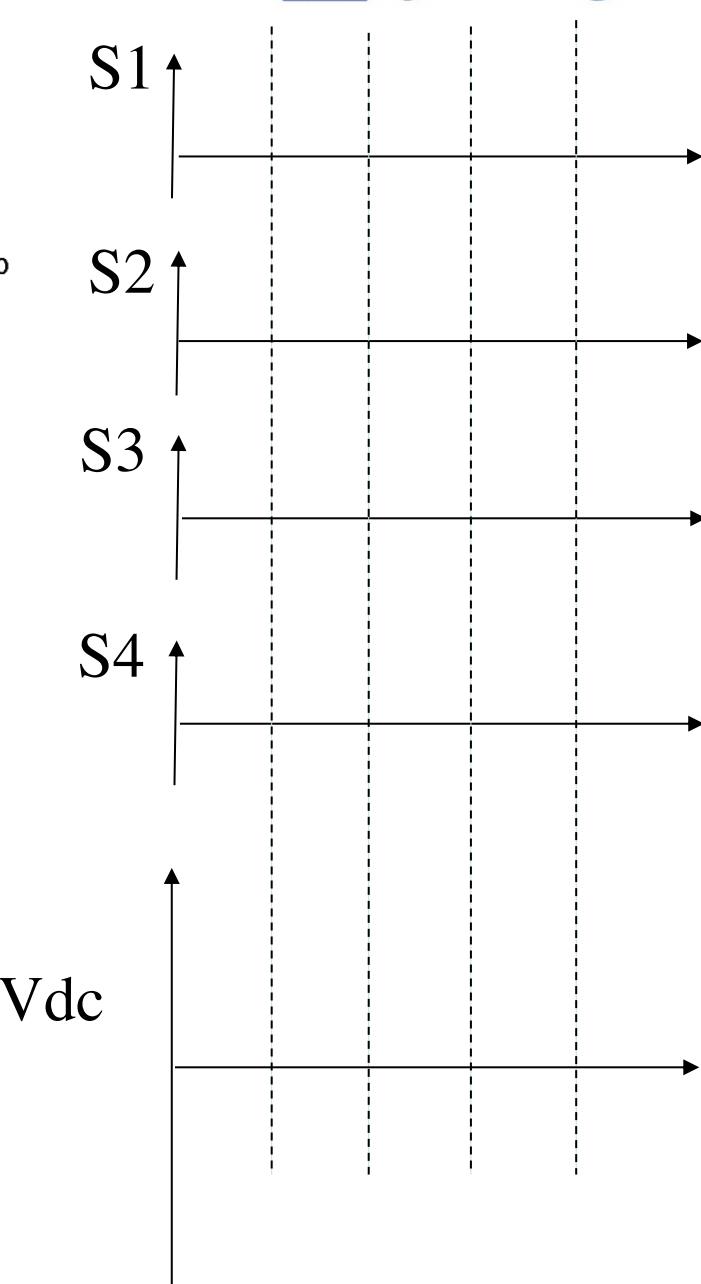


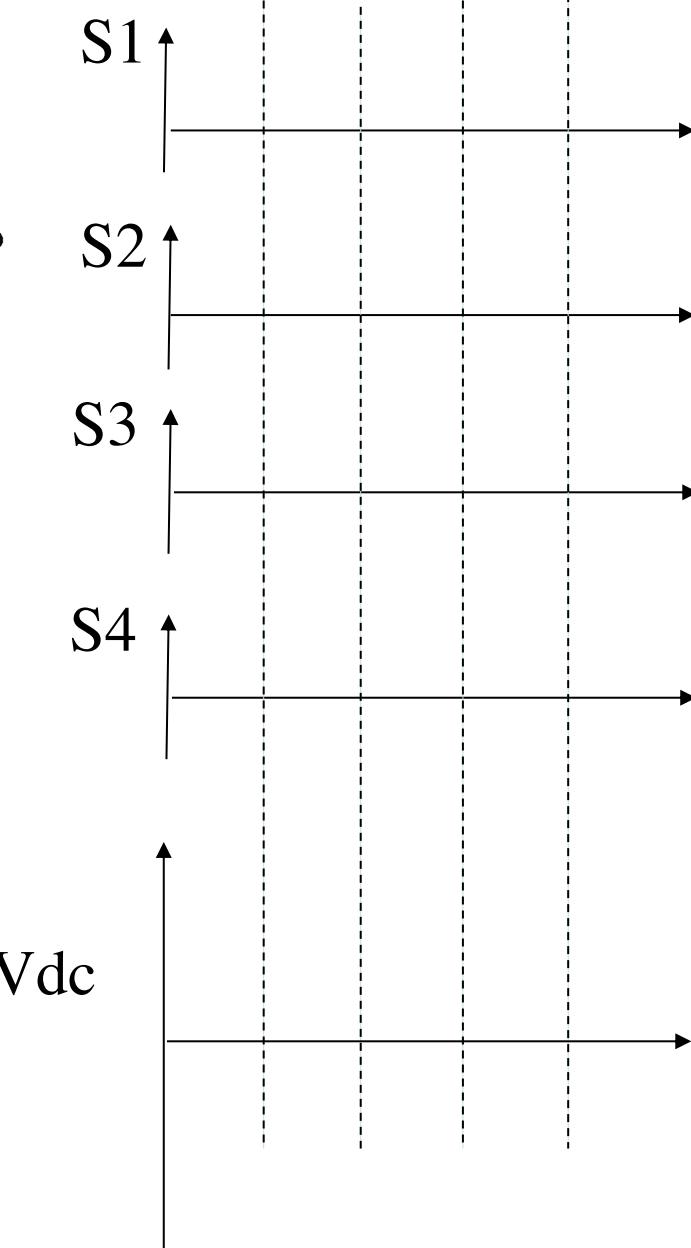
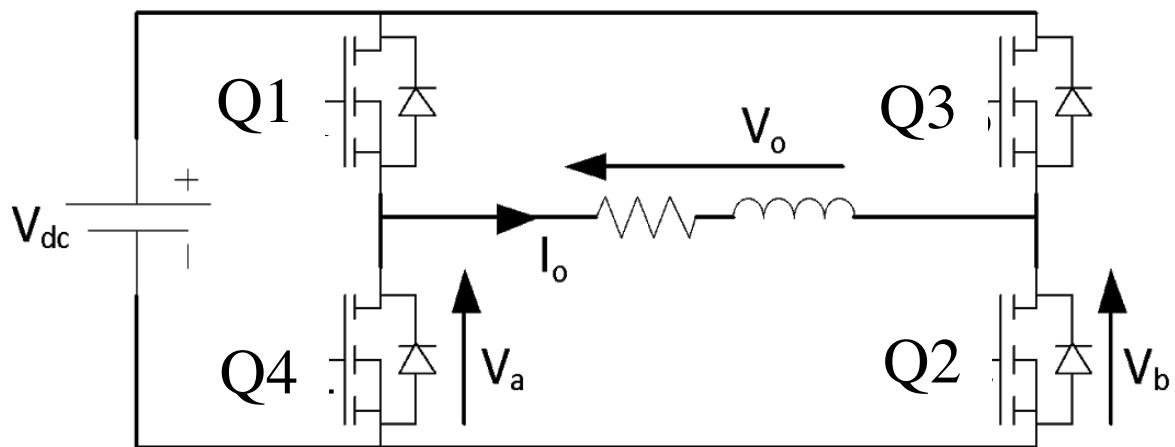
| Q1 | Q2 | Q3 | Q4       | $v_o$     |
|----|----|----|----------|-----------|
| 1  | 1  | 0  | 0        | $-V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) |           |
| 0  |    |    |          |           |
| 0  |    |    |          |           |



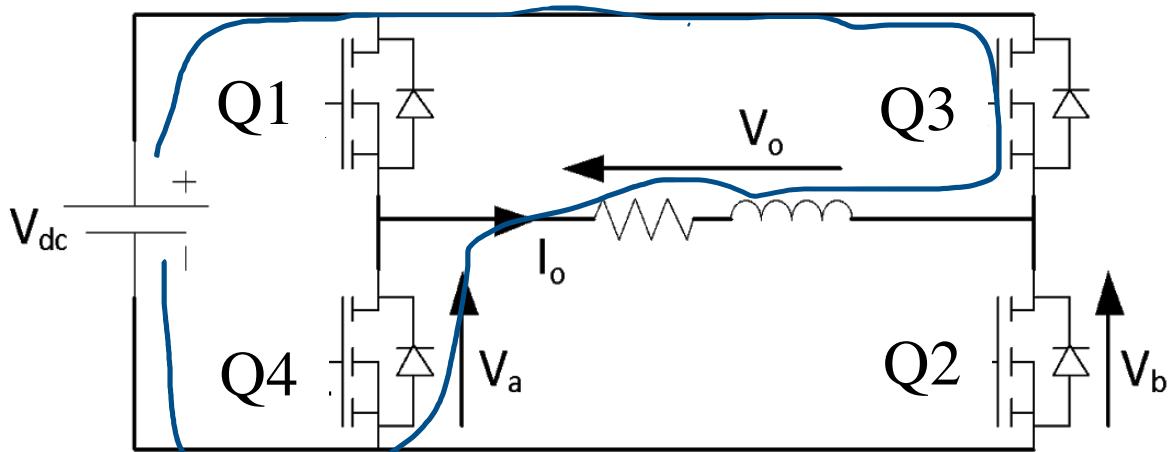


| Q1 | Q2 | Q3 | Q4       | $v_o$     |
|----|----|----|----------|-----------|
| 1  | 1  | 0  | 0        | $-V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) | 0         |
| 0  |    |    |          |           |
| 0  |    |    |          |           |

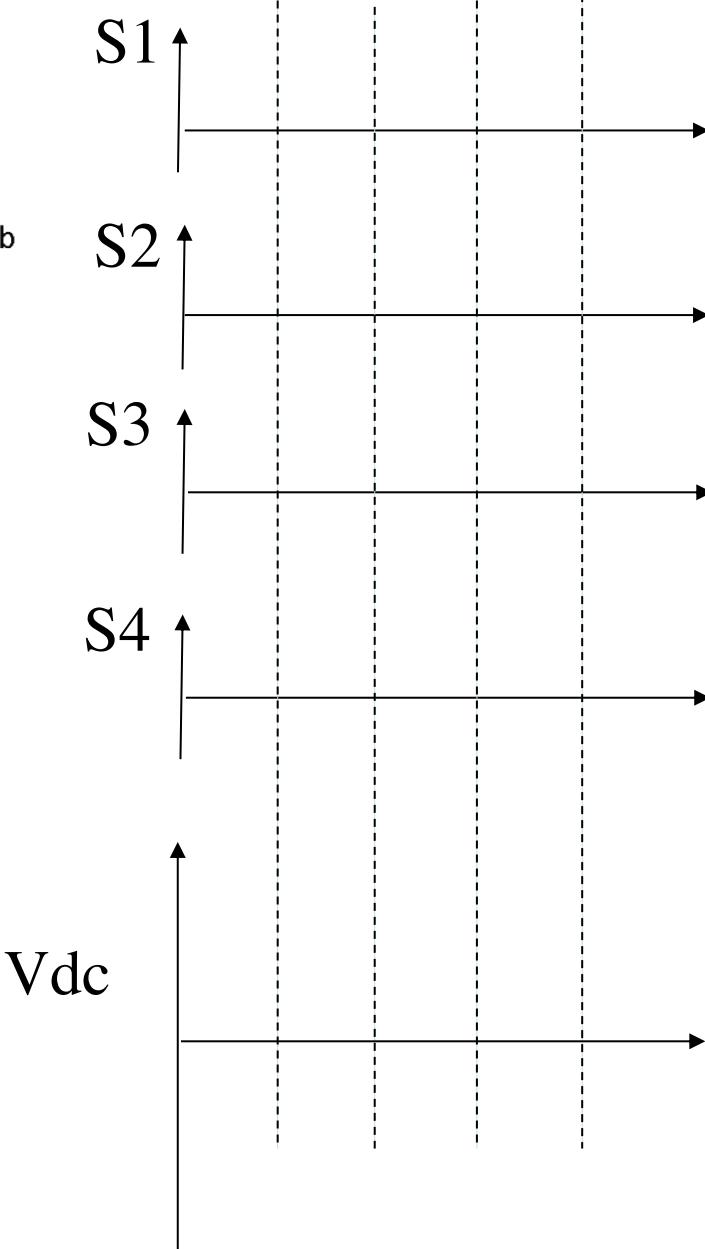


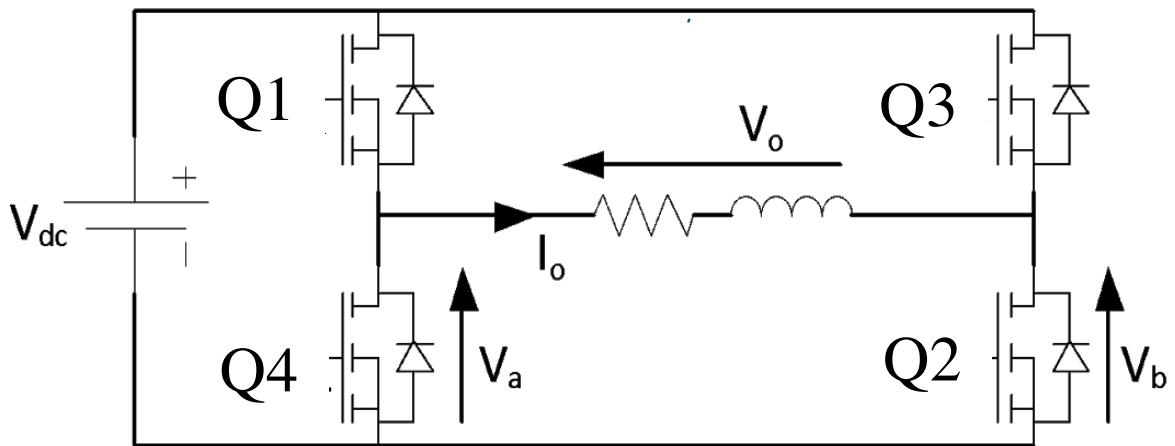


| Q1 | Q2 | Q3 | Q4       | $v_o$     |
|----|----|----|----------|-----------|
| 1  | 1  | 0  | 0        | $-V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) | 0         |
| 0  | 0  | 1  | 1        |           |
| 0  |    |    |          |           |

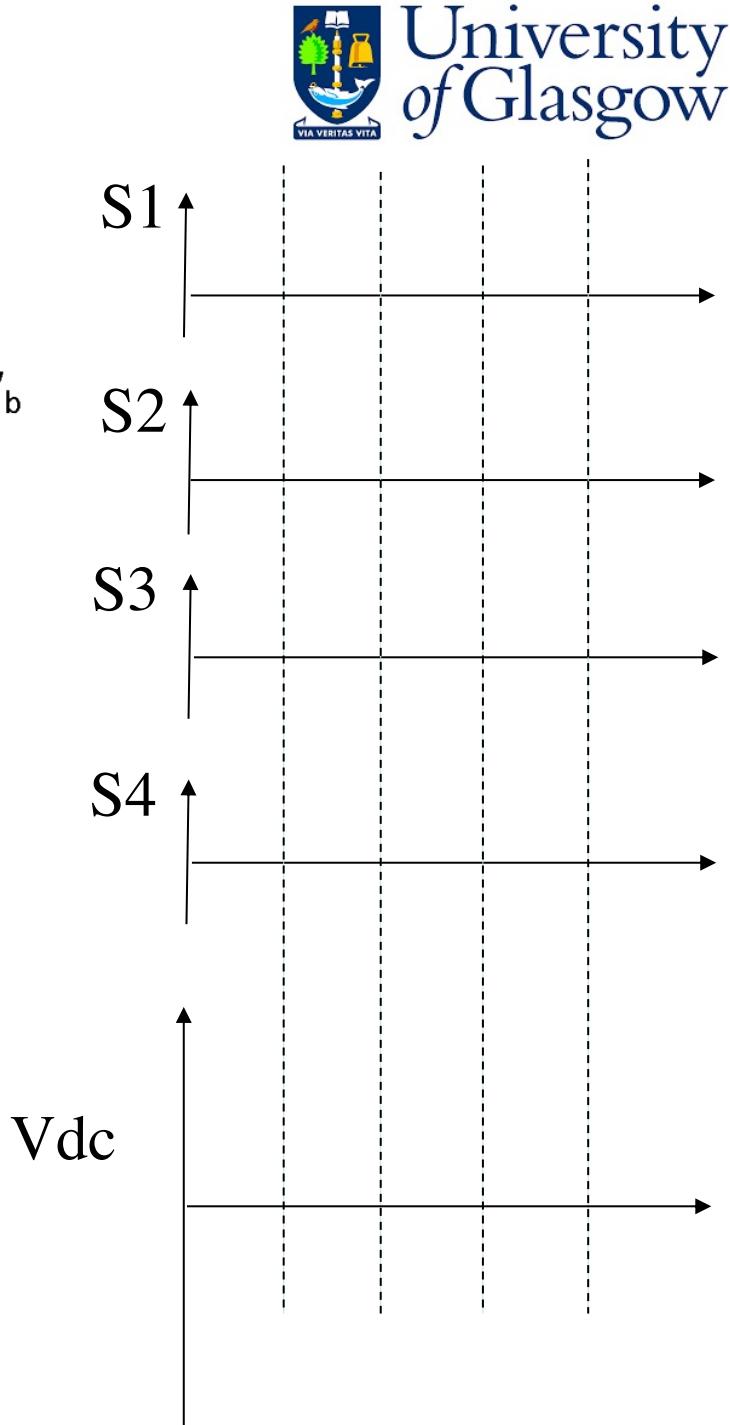


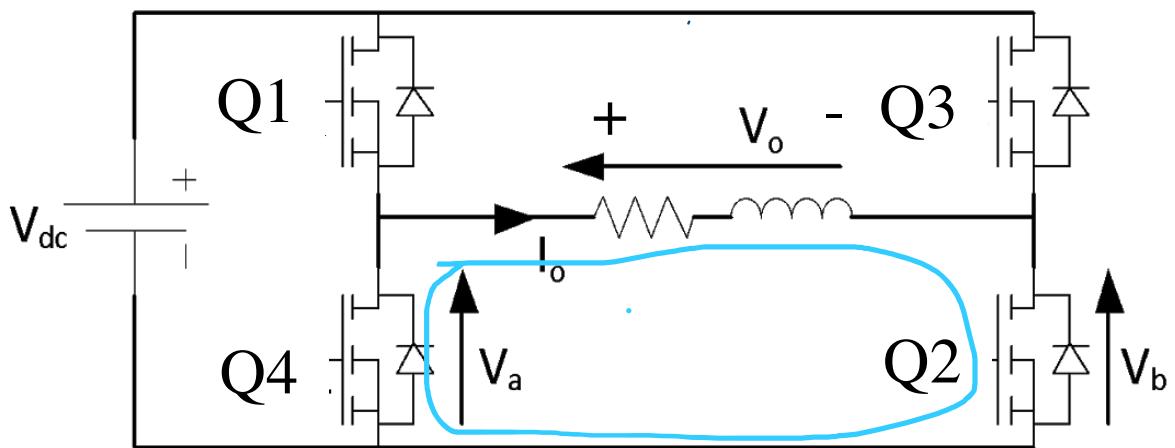
| Q1 | Q2 | Q3 | Q4       | $v_o$      |
|----|----|----|----------|------------|
| 1  | 1  | 0  | 0        | - $V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) | 0          |
| 0  | 0  | 1  | 1        | $V_{dc}$   |
| 0  |    |    |          |            |



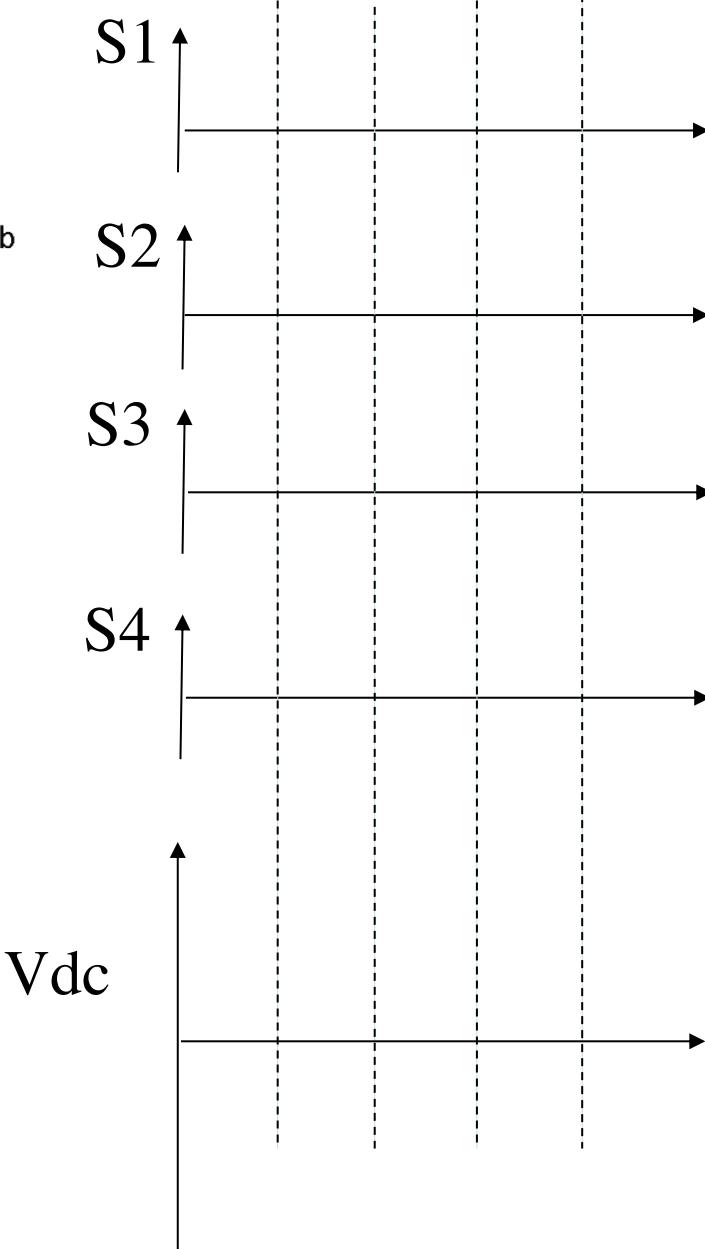


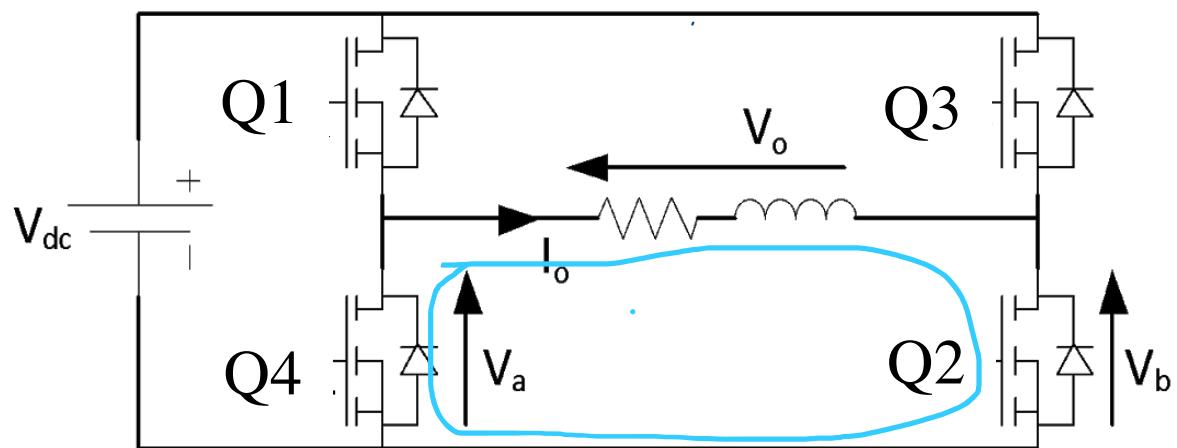
| Q1 | Q2 | Q3 | Q4       | Vo   |
|----|----|----|----------|------|
| 1  | 1  | 0  | 0        | -Vdc |
| 1  | 0  | 1  | 0 (comp) | 0    |
| 0  | 0  | 1  | 1        | Vdc  |
| 0  | 1  | 0  | 1        | 0    |



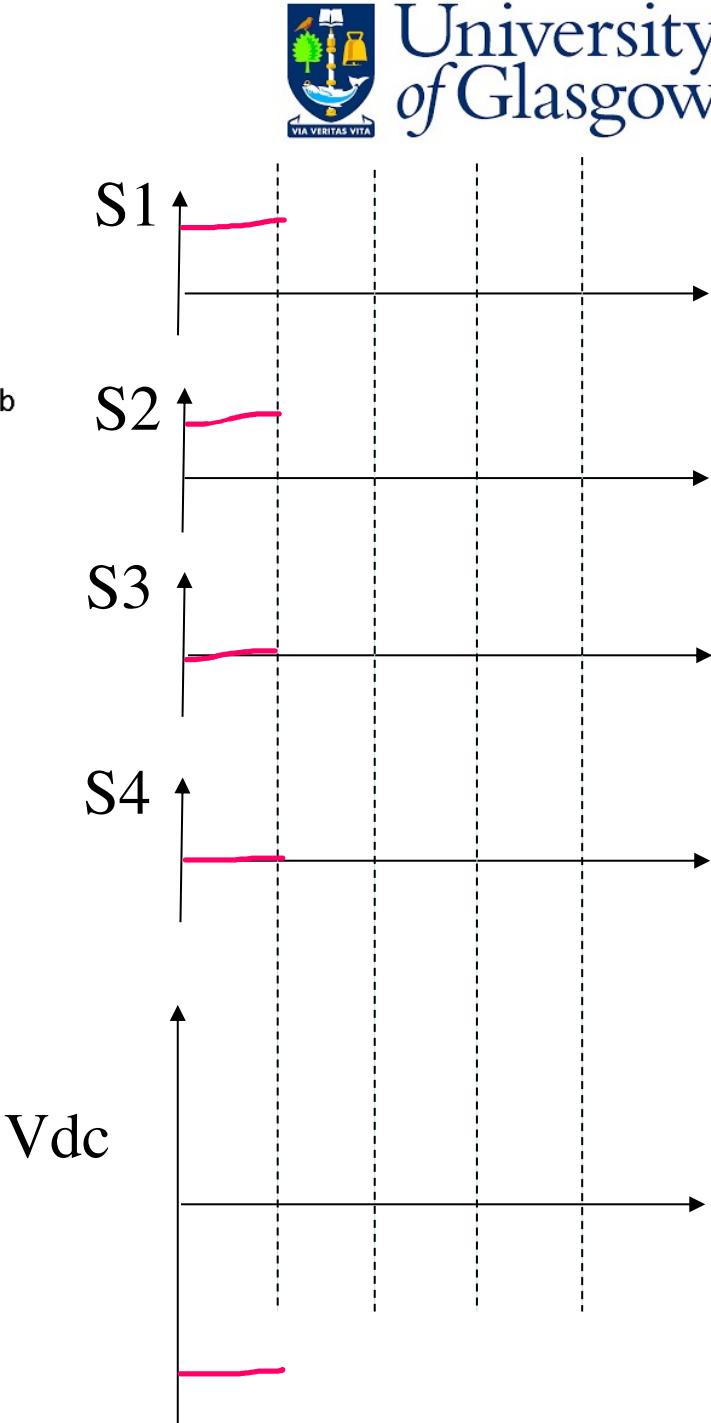


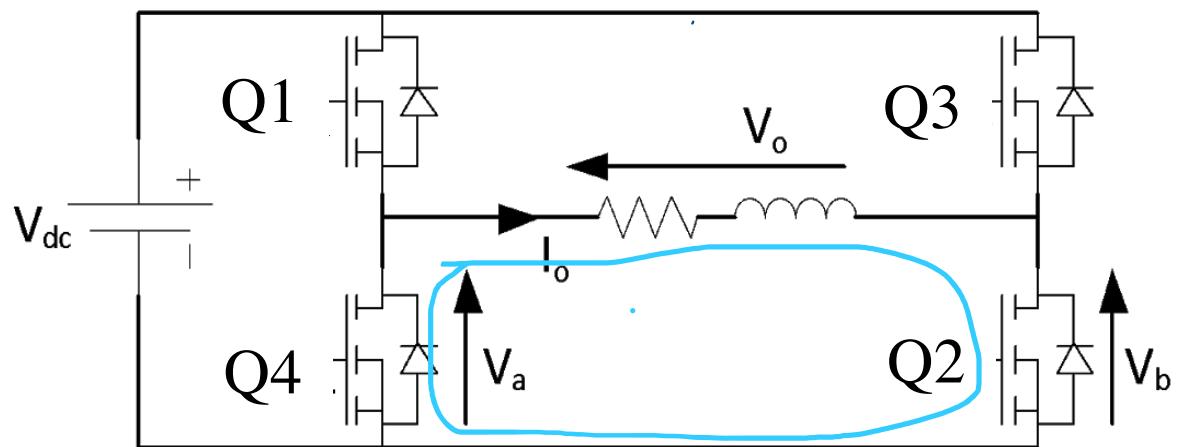
| Q1 | Q2 | Q3 | Q4       | v <sub>o</sub> |
|----|----|----|----------|----------------|
| 1  | 1  | 0  | 0        | -Vdc           |
| 1  | 0  | 1  | 0 (comp) | 0              |
| 0  | 0  | 1  | 1        | Vdc            |
| 0  | 1  | 0  | 1        | 0              |



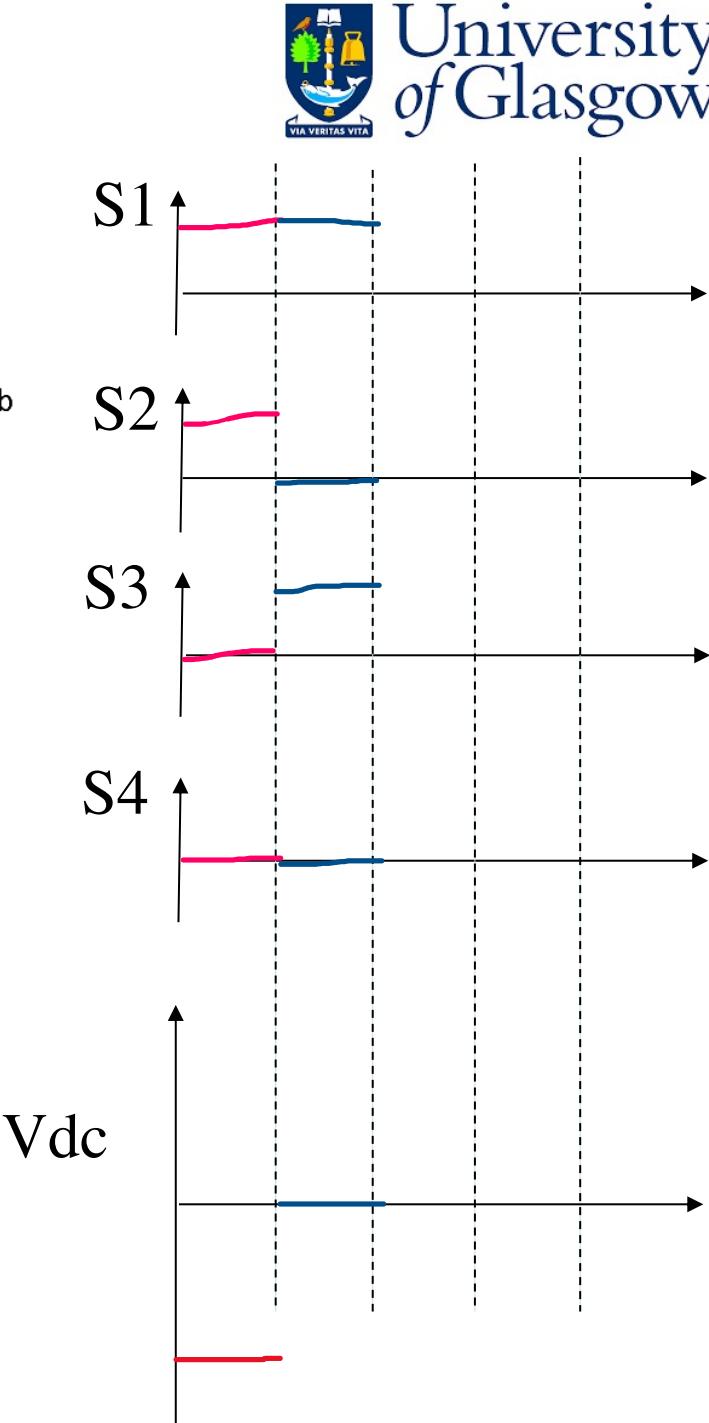


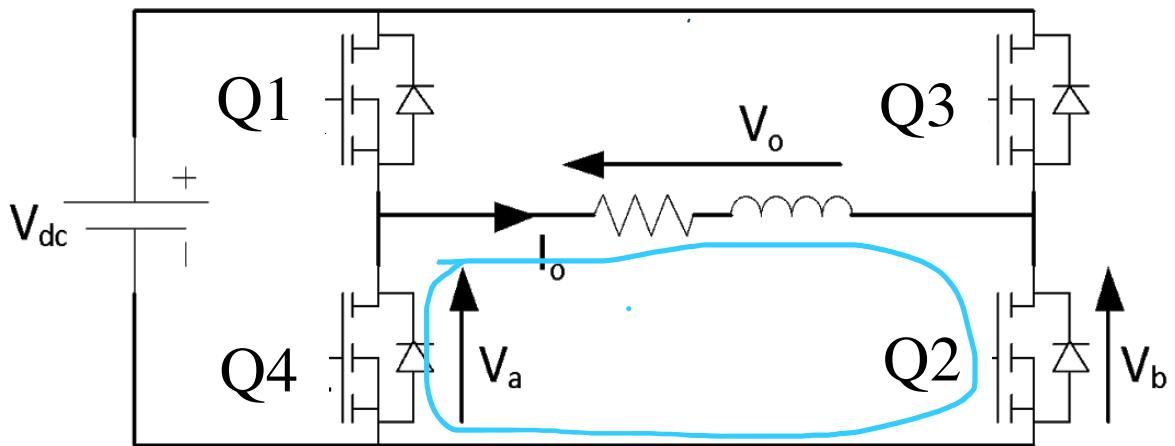
| Q1 | Q2 | Q3 | Q4       | $v_o$      |
|----|----|----|----------|------------|
| 1  | 1  | 0  | 0        | - $V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) | 0          |
| 0  | 0  | 1  | 1        | $V_{dc}$   |
| 0  | 1  | 0  | 1        | 0          |



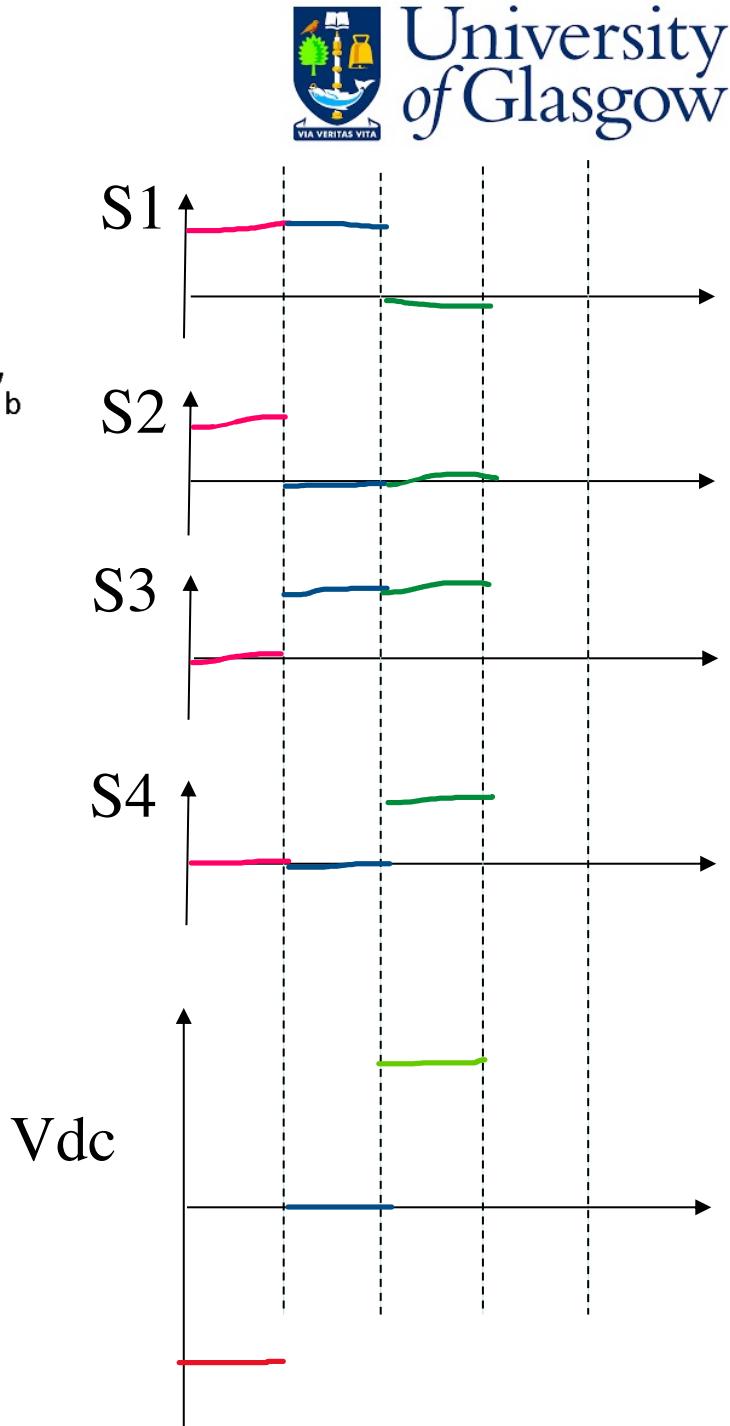


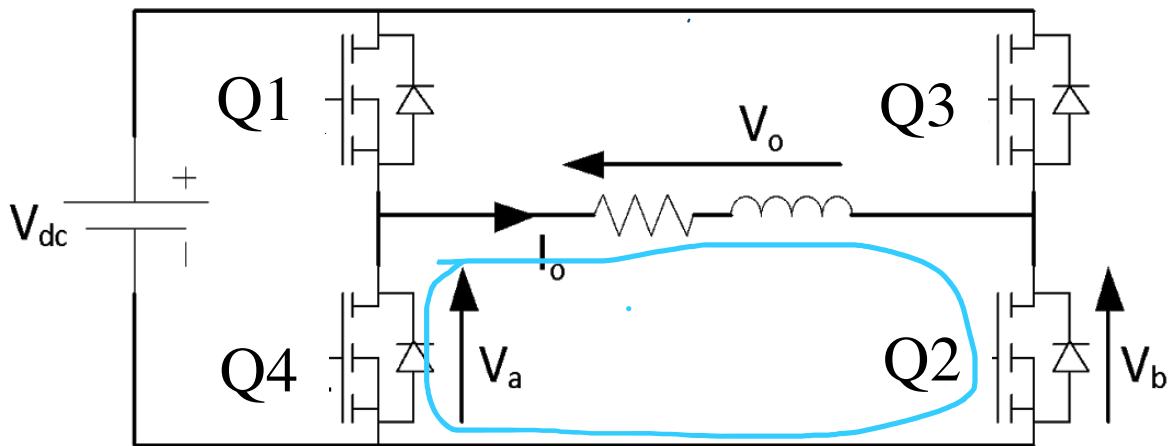
| Q1 | Q2 | Q3 | Q4       | $v_o$      |
|----|----|----|----------|------------|
| 1  | 1  | 0  | 0        | - $V_{dc}$ |
| 1  | 0  | 1  | 0 (comp) | 0          |
| 0  | 0  | 1  | 1        | $V_{dc}$   |
| 0  | 1  | 0  | 1        | 0          |



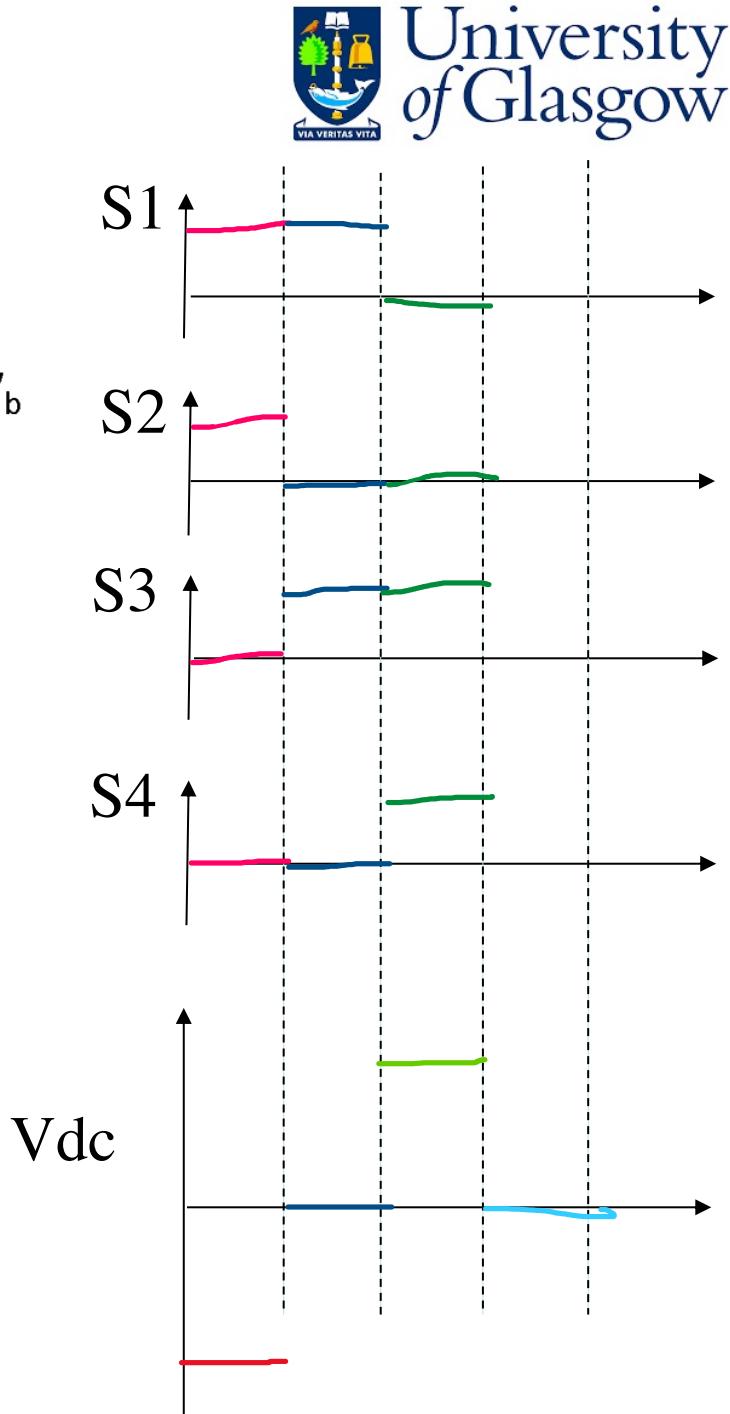


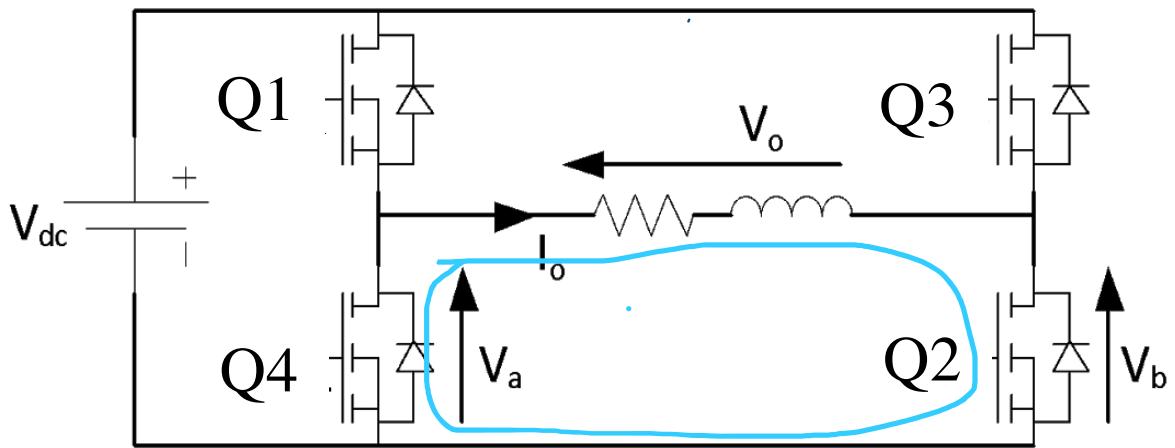
| Q1 | Q2 | Q3 | Q4       | Vo   |
|----|----|----|----------|------|
| 1  | 1  | 0  | 0        | -Vdc |
| 1  | 0  | 1  | 0 (comp) | 0    |
| 0  | 0  | 1  | 1        | +Vdc |
| 0  | 1  | 0  | 1        | 0    |



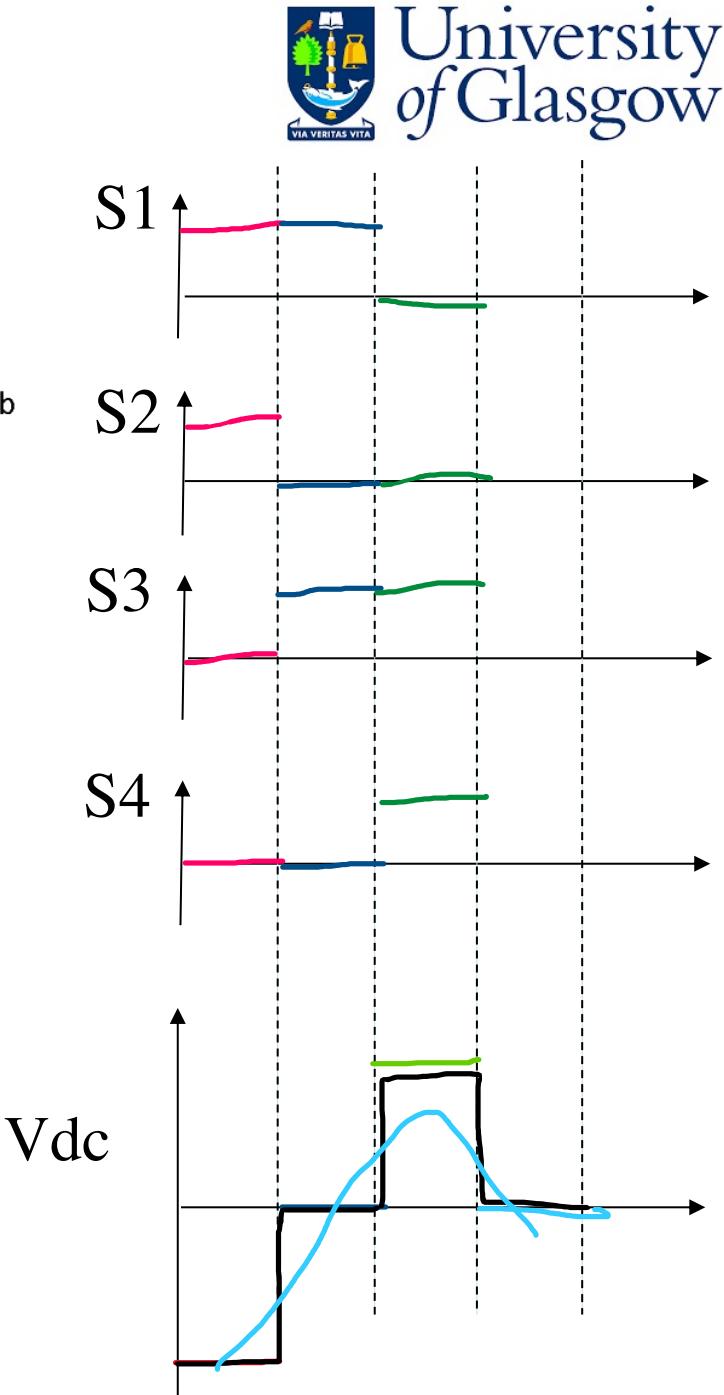


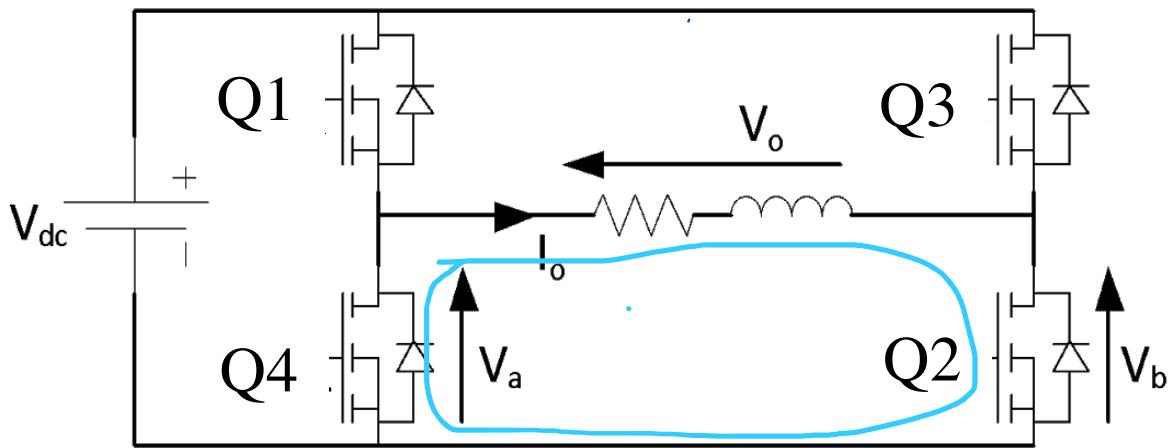
| Q1 | Q2 | Q3 | Q4       | Vo   |
|----|----|----|----------|------|
| 1  | 1  | 0  | 0        | -Vdc |
| 1  | 0  | 1  | 0 (comp) | 0    |
| 0  | 0  | 1  | 1        | +Vdc |
| 0  | 1  | 0  | 1        | 0    |



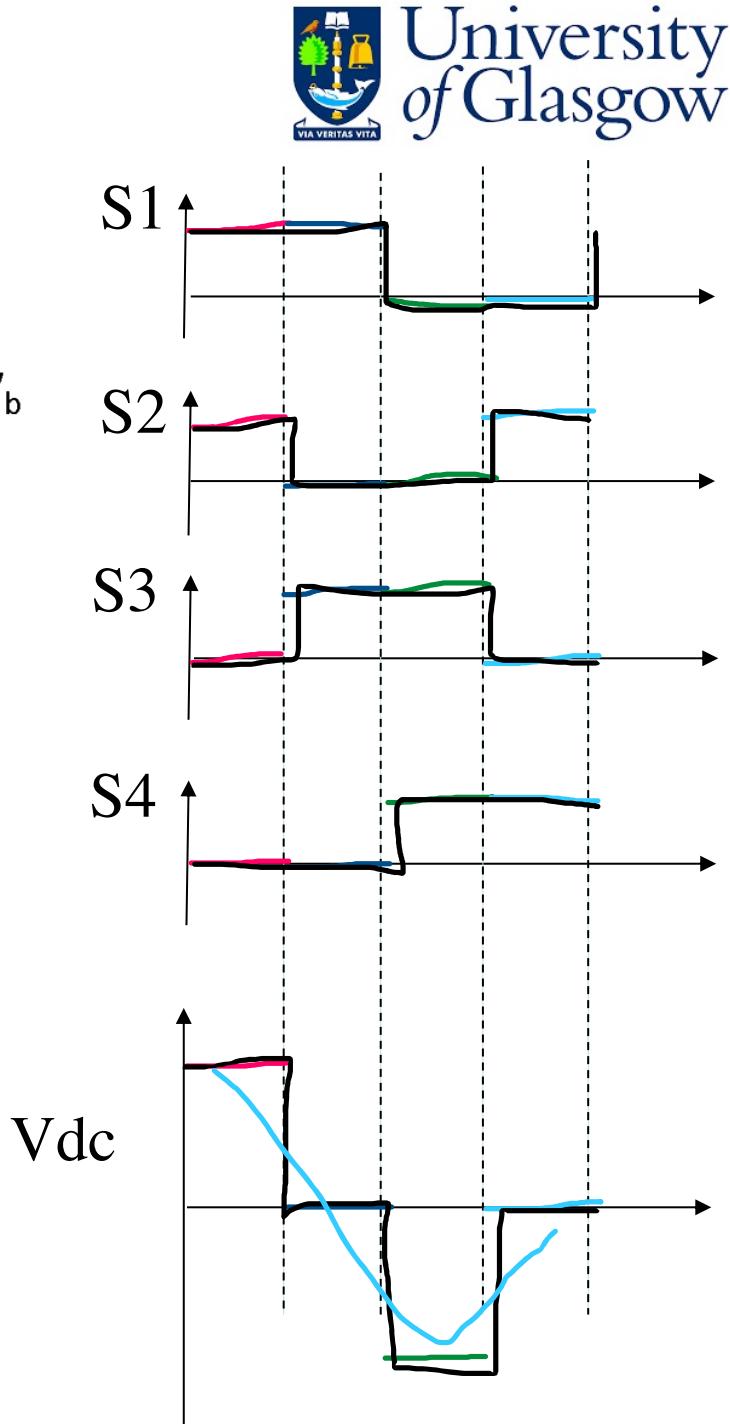


| Q1 | Q2 | Q3 | Q4       | Vo   |
|----|----|----|----------|------|
| 1  | 1  | 0  | 0        | -Vdc |
| 1  | 0  | 1  | 0 (comp) | 0    |
| 0  | 0  | 1  | 1        | +Vdc |
| 0  | 1  | 0  | 1        | 0    |





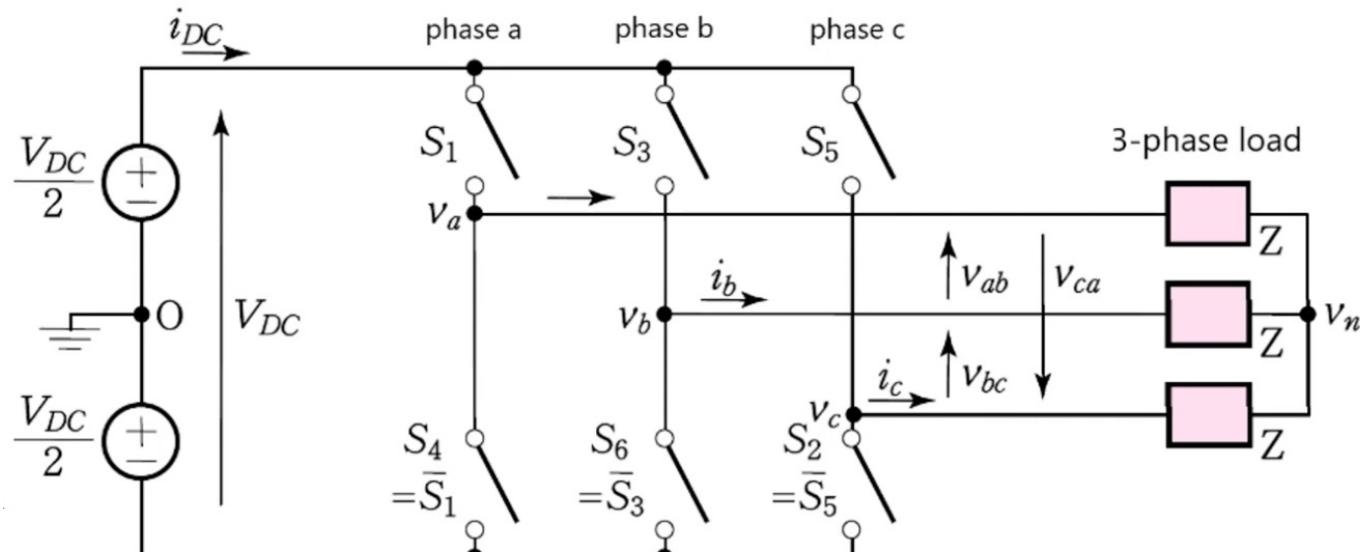
| Q1 | Q2 | Q3 | Q4       | Vo   |
|----|----|----|----------|------|
| 1  | 1  | 0  | 0        | Vdc  |
| 1  | 0  | 1  | 0 (comp) | 0    |
| 0  | 0  | 1  | 1        | -Vdc |
| 0  | 1  | 0  | 1        | 0    |



# Three Phase Output

Three Phase Output –  
Balanced  
waveform at the load  
(sequence shifted by 120  
degrees)

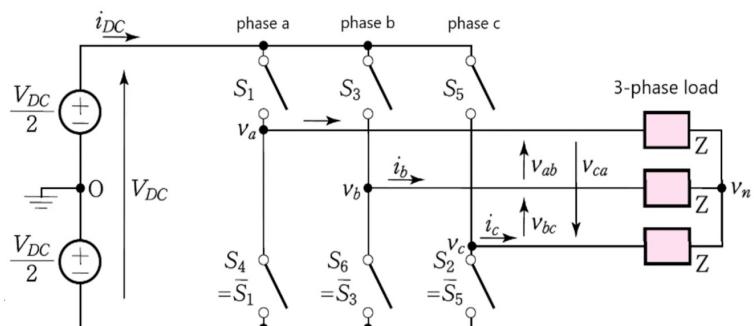
# Three Phase Inverters – Half Bridge



$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$



$$v_{ab} = v_{an} - v_{bn}$$

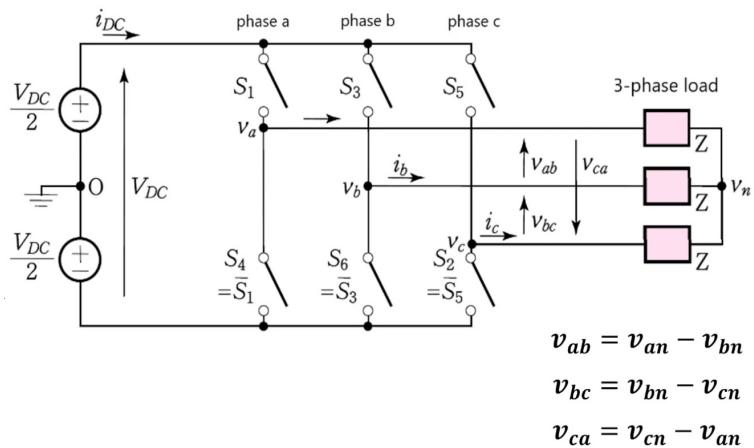
$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

3 half bridges (phases a, b and c)

Line to line voltage (V<sub>ab</sub>, V<sub>bc</sub>, V<sub>ca</sub>)

Line to neutral voltage (v<sub>an</sub>, v<sub>bn</sub>,..etc)



3 half bridges (phases a, b and c)

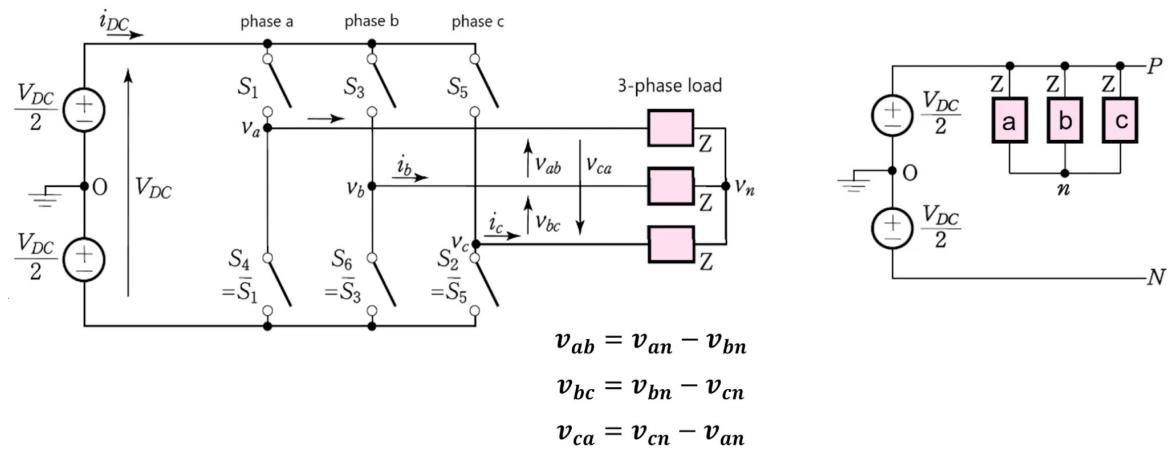
Line to line voltage (V<sub>ab</sub>, V<sub>bc</sub>, V<sub>ca</sub>)

Line to neutral voltage (v<sub>an</sub>, v<sub>bn</sub>,..etc)

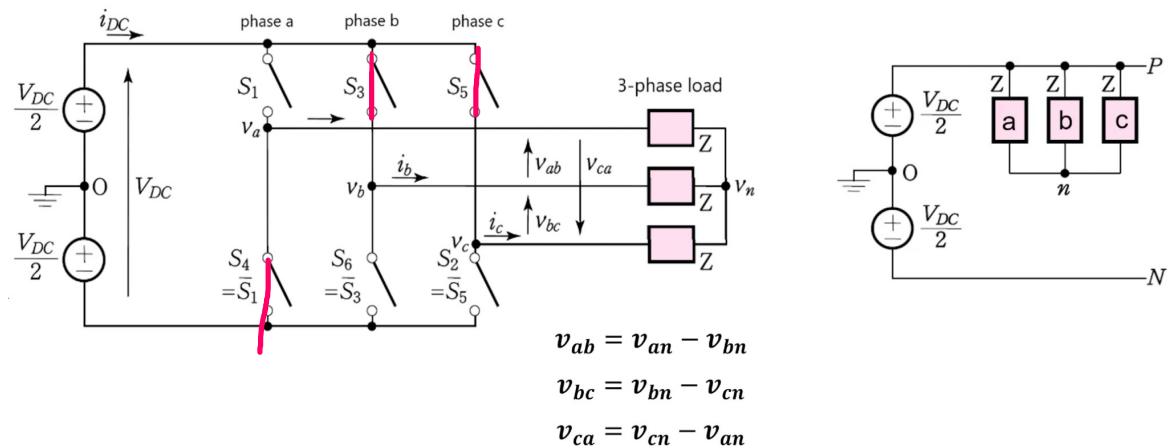
| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

“+” = indicates switch is “closed”

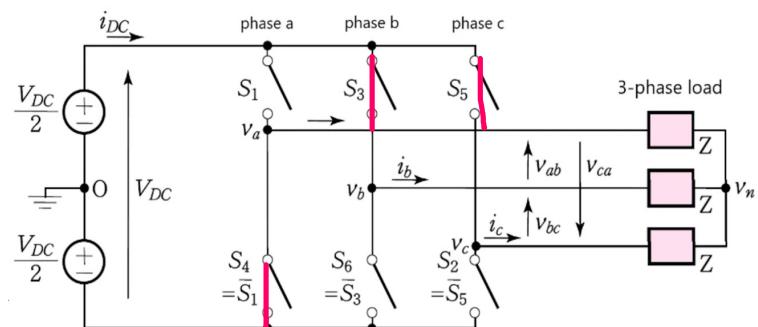
“-” = indicates complimentary switch is “closed”



| <b>Va</b> | <b>Vb</b> | <b>Vc</b> |
|-----------|-----------|-----------|
| +         | +         | +         |
| -         | +         | +         |
| +         | -         | +         |
| +         | +         | -         |
| -         | -         | +         |
| +         | -         | -         |
| -         | +         | -         |
| -         | -         | -         |



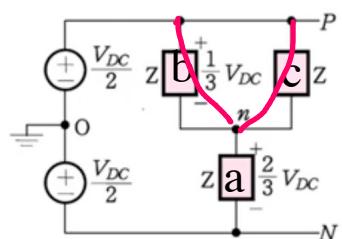
| <b>Va</b> | <b>Vb</b> | <b>Vc</b> |
|-----------|-----------|-----------|
| +         | +         | +         |
| -         | +         | +         |
| +         | -         | +         |
| +         | +         | -         |
| -         | -         | +         |
| +         | -         | -         |
| -         | +         | -         |
| -         | -         | -         |



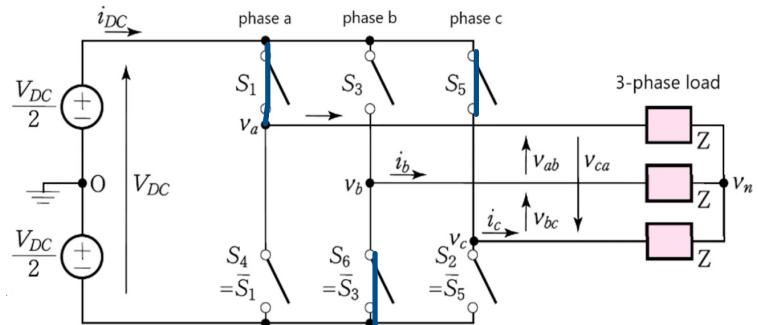
$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$



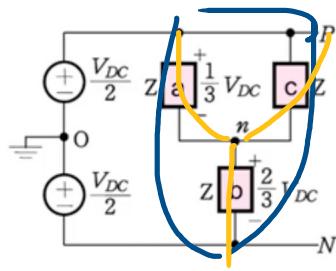
| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |



$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

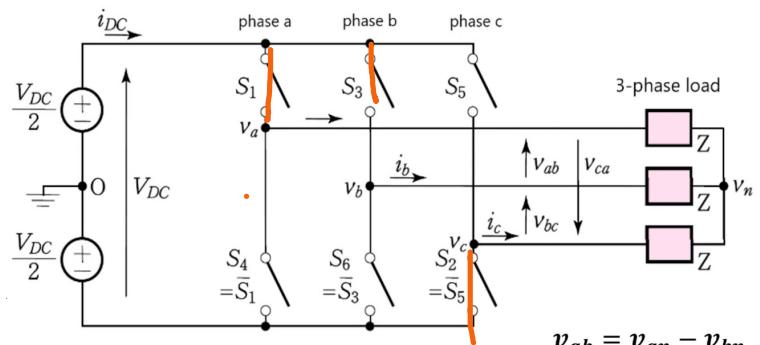
$$v_{ca} = v_{cn} - v_{an}$$



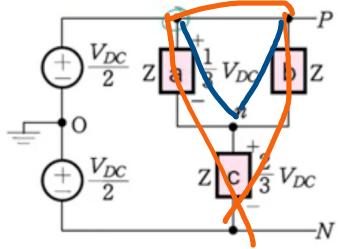
| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

| Vab  | Vbc  | Vca |
|------|------|-----|
| 0    | 0    | 0   |
| -Vdc | 0    | Vdc |
| Vdc  | -Vdc | 0   |

| Van     | Vbn     | Vcn    |
|---------|---------|--------|
| 0       | 0       | 0      |
| -2/3Vdc | 1/3Vdc  | 1/3Vdc |
| 1/3Vdc  | -2/3Vdc | 1/3Vdc |
|         |         |        |
|         |         |        |
|         |         |        |
|         |         |        |
|         |         |        |



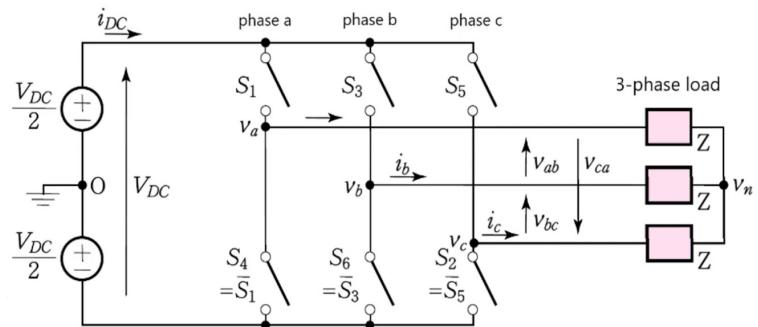
$$\begin{aligned}v_{ab} &= v_{an} - v_{bn} \\v_{bc} &= v_{bn} - v_{cn} \\v_{ca} &= v_{cn} - v_{an}\end{aligned}$$



| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

| Vab  | Vbc  | Vca  |
|------|------|------|
| 0    | 0    | 0    |
| -Vdc | 0    | Vdc  |
| Vdc  | -Vdc | 0    |
| 0    | Vdc  | -Vdc |

| Van     | Vbn     | Vcn     |
|---------|---------|---------|
| 0       | 0       | 0       |
| -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| 1/3Vdc  | 1/3Vdc  | -2/3Vdc |



$$v_{ab} = v_{an} - v_{bn}$$

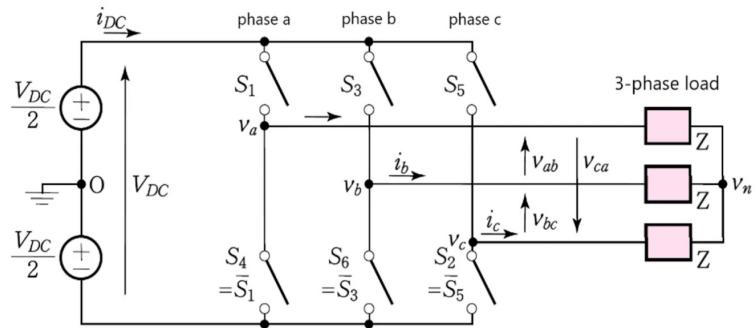
$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

| Vab  | Vbc  | Vca  |
|------|------|------|
| 0    | 0    | 0    |
| -Vdc | 0    | Vdc  |
| Vdc  | -Vdc | 0    |
| 0    | Vdc  | -Vdc |
| 0    | -Vdc | Vdc  |
| Vdc  | 0    | -Vdc |
| -Vdc | Vdc  | 0    |

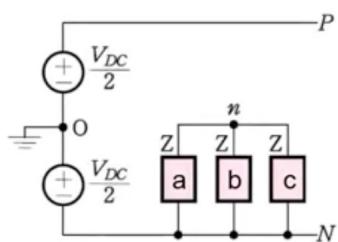
| Van     | Vbn     | Vcn     |
|---------|---------|---------|
| 0       | 0       | 0       |
| -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| 1/3Vdc  | 1/3Vdc  | -2/3Vdc |
| -1/3Vdc | -1/3Vdc | 2/3Vdc  |
| 2/3Vdc  | -1/3Vdc | -1/3Vdc |
| -1/3Vdc | 2/3Vdc  | -1/3Vdc |



$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

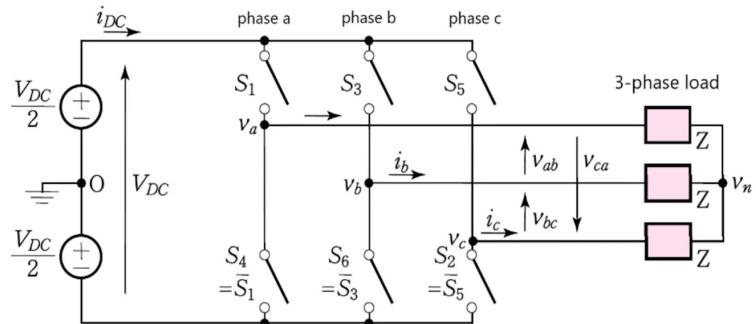
$$v_{ca} = v_{cn} - v_{an}$$



| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

| Vab  | Vbc  | Vca  |
|------|------|------|
| 0    | 0    | 0    |
| -Vdc | 0    | Vdc  |
| Vdc  | -Vdc | 0    |
| 0    | Vdc  | -Vdc |
| 0    | -Vdc | Vdc  |
| Vdc  | 0    | -Vdc |
| -Vdc | Vdc  | 0    |
| 0    | 0    | 0    |

| Van     | Vbn     | Vcn     |
|---------|---------|---------|
| 0       | 0       | 0       |
| -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| 1/3Vdc  | 1/3Vdc  | -2/3Vdc |
| -1/3Vdc | -1/3Vdc | 2/3Vdc  |
| 2/3Vdc  | -1/3Vdc | -1/3Vdc |
| -1/3Vdc | 2/3Vdc  | -1/3Vdc |
| 0       | 0       | 0       |

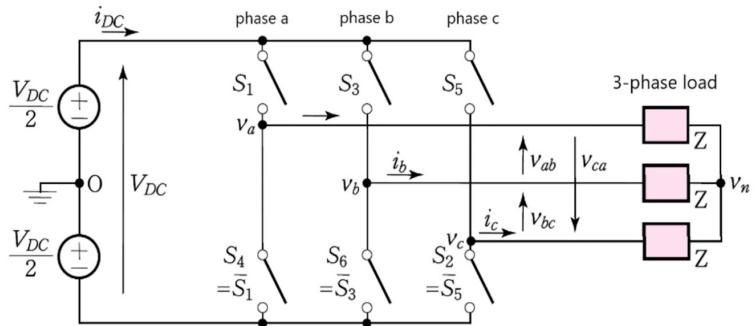


$$\begin{aligned}v_{ab} &= v_{an} - v_{bn} \\v_{bc} &= v_{bn} - v_{cn} \\v_{ca} &= v_{cn} - v_{an}\end{aligned}$$

-Vdc, 0 and Vdc

| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |

| Vab  | Vbc  | Vca  | Van     | Vbn     | Vcn     |
|------|------|------|---------|---------|---------|
| 0    | 0    | 0    | 0       | 0       | 0       |
| -Vdc | 0    | Vdc  | -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| Vdc  | -Vdc | 0    | 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| 0    | Vdc  | -Vdc | 1/3Vdc  | 1/3Vdc  | -2/3Vdc |
| 0    | -Vdc | Vdc  | -1/3Vdc | -1/3Vdc | 2/3Vdc  |
| Vdc  | 0    | -Vdc | 2/3Vdc  | -1/3Vdc | -1/3Vdc |
| -Vdc | Vdc  | 0    | -1/3Vdc | 2/3Vdc  | -1/3Vdc |
| 0    | 0    | 0    | 0       | 0       | 0       |



$$v_{ab} = v_{an} - v_{bn}$$

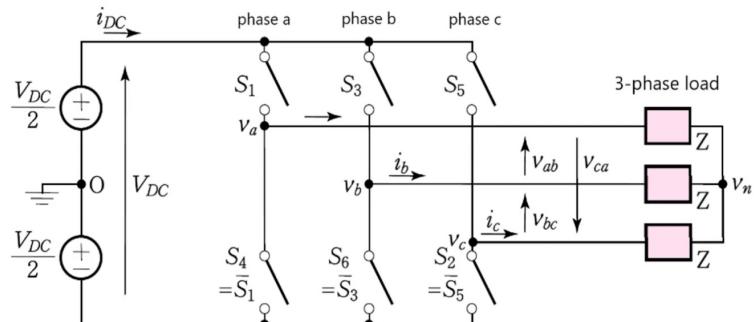
$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

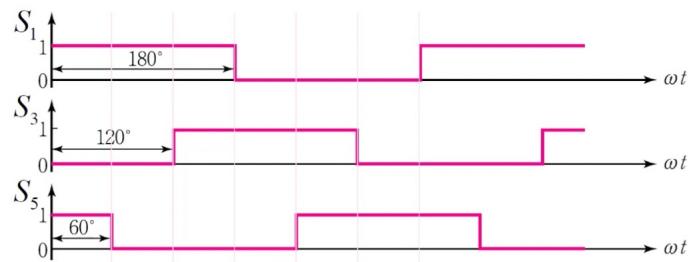
$1/3V_{dc}$ ,  $2/3V_{dc}$ ,  $-1/3V_{dc}$ ,  
and  $-2/3V_{dc}$



| Va | Vb | Vc | Vab  | Vbc  | Vca  | Van     | Vbn     | Vcn     |
|----|----|----|------|------|------|---------|---------|---------|
| +  | +  | +  | 0    | 0    | 0    | 0       | 0       | 0       |
| -  | +  | +  | -Vdc | 0    | Vdc  | -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| +  | -  | +  | Vdc  | -Vdc | 0    | 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| +  | +  | -  | 0    | Vdc  | -Vdc | 1/3Vdc  | 1/3Vdc  | -2/3Vdc |
| -  | -  | +  | 0    | -Vdc | Vdc  | -1/3Vdc | -1/3Vdc | 2/3Vdc  |
| +  | -  | -  | Vdc  | 0    | -Vdc | 2/3Vdc  | -1/3Vdc | -1/3Vdc |
| -  | +  | -  | -Vdc | Vdc  | 0    | -1/3Vdc | 2/3Vdc  | -1/3Vdc |
| -  | -  | -  | 0    | 0    | 0    | 0       | 0       | 0       |



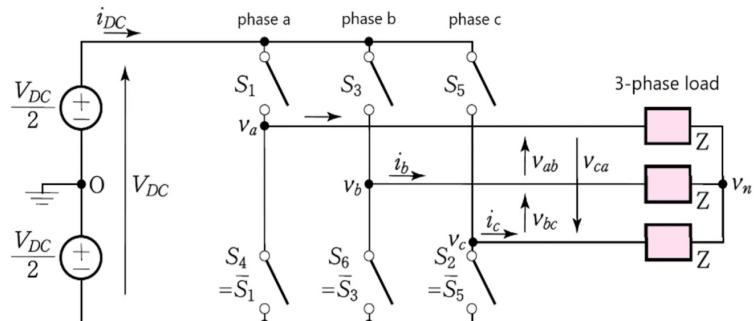
$$\begin{aligned}v_{ab} &= v_{an} - v_{bn} \\v_{bc} &= v_{bn} - v_{cn} \\v_{ca} &= v_{cn} - v_{an}\end{aligned}$$



Only need 3 switches (half bridge)

Notice that 1 TP is 360 degrees. Each time division is 60 degrees

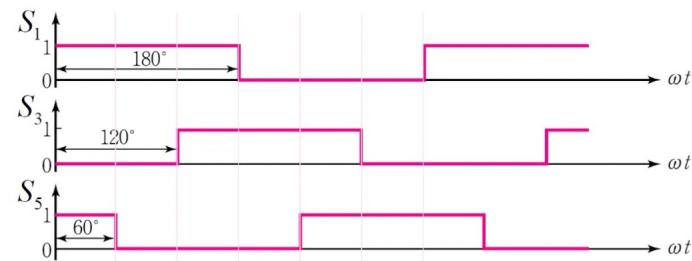
| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |



$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

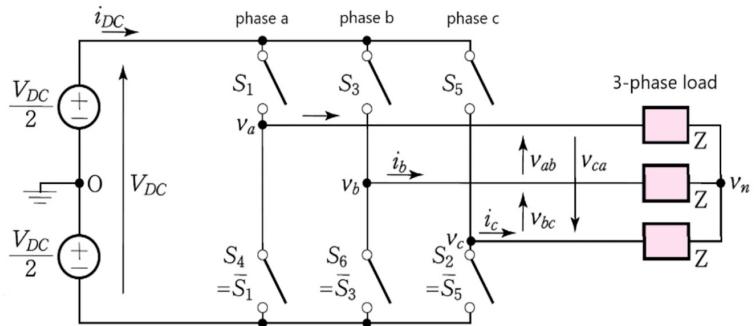


Only need 3 switches (half bridge)

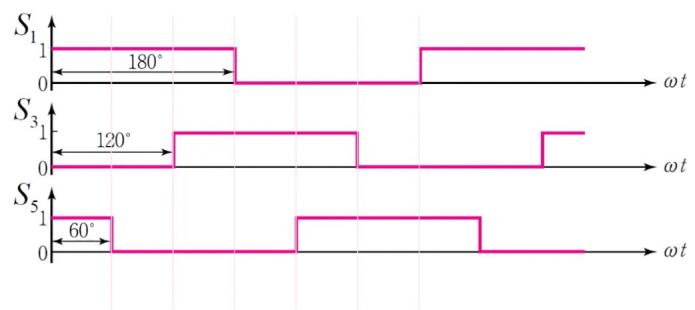
Notice that 1 TP is 360 degrees. Each time division is 60 degrees

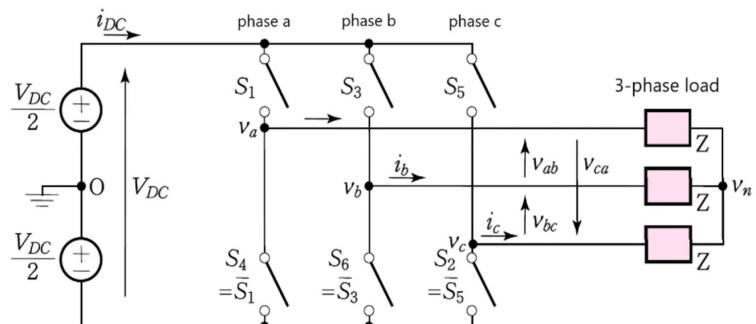
S1 is on for half the time.

| Va | Vb | Vc |
|----|----|----|
| +  | +  | +  |
| -  | +  | +  |
| +  | -  | +  |
| +  | +  | -  |
| -  | -  | +  |
| +  | -  | -  |
| -  | +  | -  |
| -  | -  | -  |



| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
|     | -  | +  | +  |
| 1   | +  | -  | +  |
|     | +  | +  | -  |
|     | -  | -  | +  |
|     | +  | -  | -  |
|     | -  | +  | -  |
|     | -  | -  | -  |



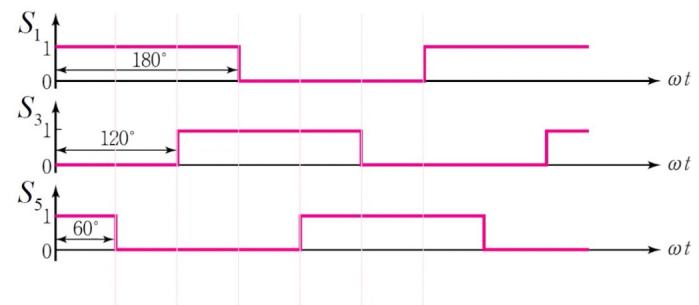


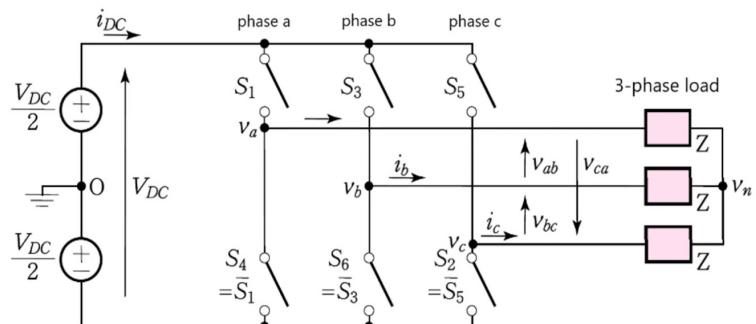
$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
|     | -  | +  | +  |
| 1   | +  | -  | +  |
|     | +  | +  | -  |
|     | -  | -  | +  |
| 2   | +  | -  | -  |
|     | -  | +  | -  |
|     | -  | -  | -  |



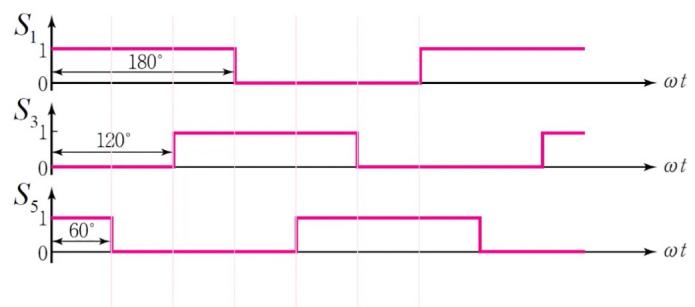


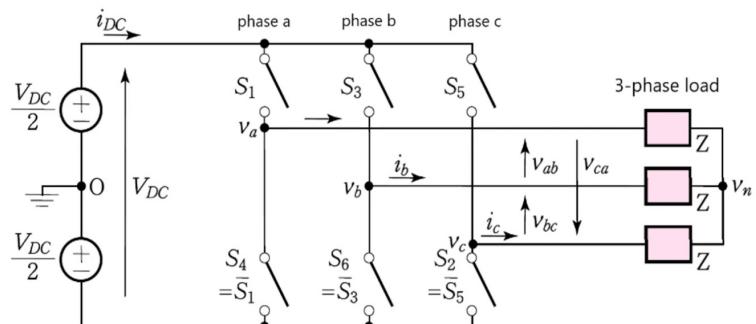
$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
|     | -  | +  | +  |
| 1   | +  | -  | +  |
| 3   | +  | +  | -  |
|     | -  | -  | +  |
| 2   | +  | -  | -  |
|     | -  | +  | -  |
|     | -  | -  | -  |



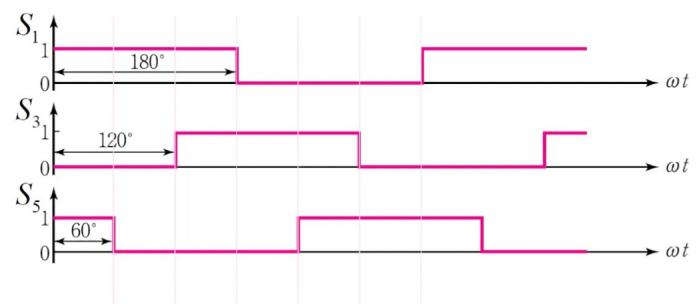


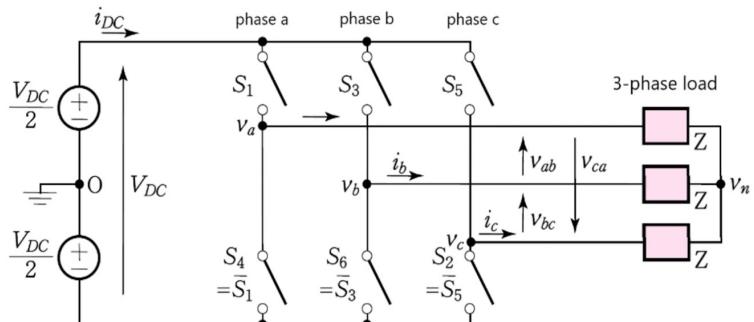
$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
|     | -  | +  | +  |
| 1   | +  | -  | +  |
| 3   | +  | +  | -  |
|     | -  | -  | +  |
| 2   | +  | -  | -  |
| 4   | -  | +  | -  |
|     | -  | -  | -  |

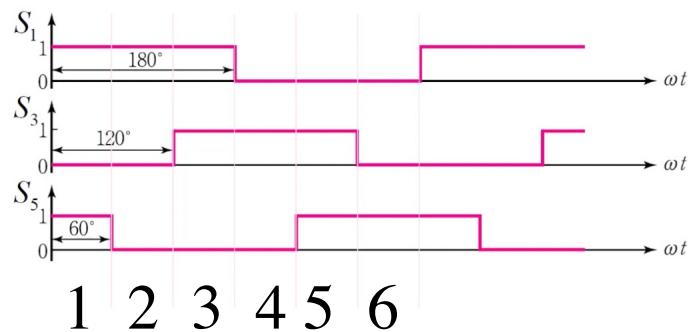




$$v_{ab} = v_{an} - v_{bn}$$

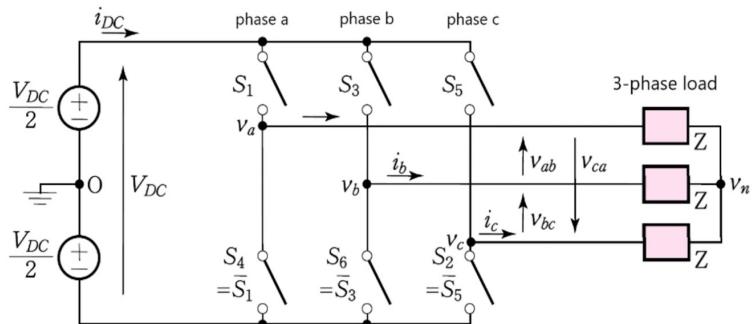
$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$



| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
| 5   | -  | +  | +  |
| 1   | +  | -  | +  |
| 3   | +  | +  | -  |
| 6   | -  | -  | +  |
| 2   | +  | -  | -  |
| 4   | -  | +  | -  |
|     | -  | -  | -  |

| Vab  | Vbc  | Vca  | Van     | Vbn     | Vcn     |
|------|------|------|---------|---------|---------|
| 0    | 0    | 0    | 0       | 0       | 0       |
| -Vdc | 0    | Vdc  | -2/3Vdc | 1/3Vdc  | 1/3Vdc  |
| Vdc  | -Vdc | 0    | 1/3Vdc  | -2/3Vdc | 1/3Vdc  |
| 0    | Vdc  | -Vdc | 1/3Vdc  | 1/3Vdc  | -2/3Vdc |
| 0    | -Vdc | Vdc  | -1/3Vdc | -1/3Vdc | 2/3Vdc  |
| Vdc  | 0    | -Vdc | 2/3Vdc  | -1/3Vdc | -1/3Vdc |
| -Vdc | Vdc  | 0    | -1/3Vdc | 2/3Vdc  | -1/3Vdc |
| 0    | 0    | 0    | 0       | 0       | 0       |



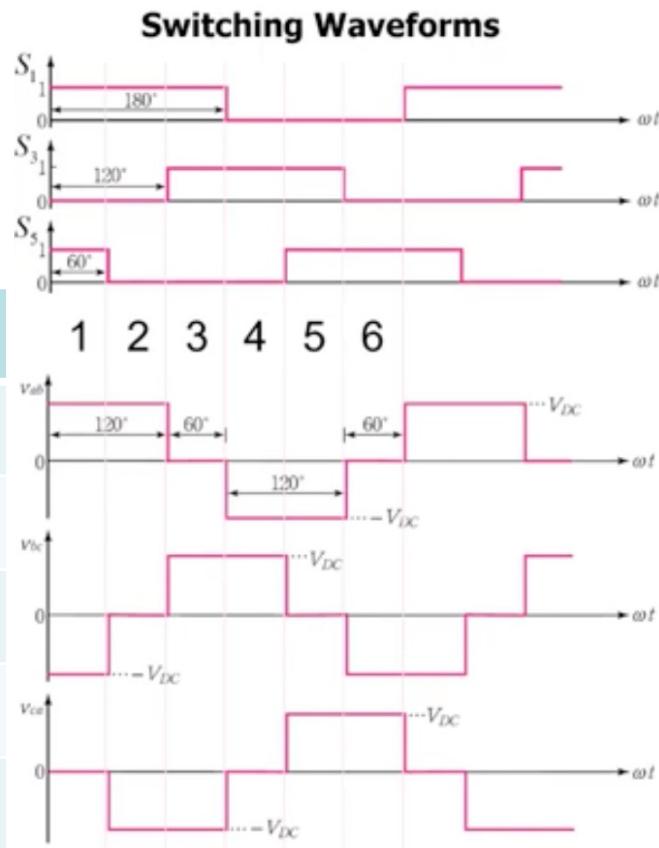
$$v_{ab} = v_{an} - v_{bn}$$

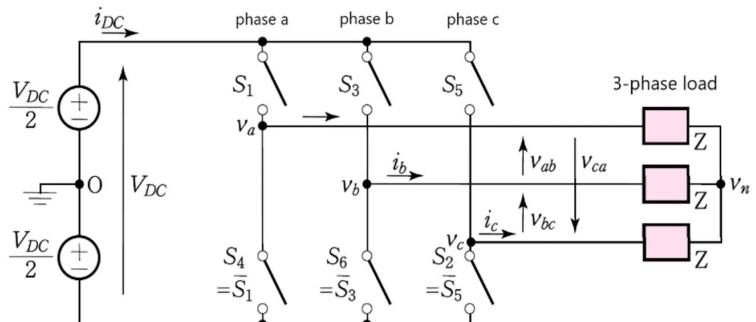
$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
| 5   | -  | +  | +  |
| 1   | +  | -  | +  |
| 3   | +  | +  | -  |
| 6   | -  | -  | +  |
| 2   | +  | -  | -  |
| 4   | -  | +  | -  |
|     | -  | -  | -  |

|   | Vab  | Vbc  | Vca  |
|---|------|------|------|
|   | 0    | 0    | 0    |
| 5 | -Vdc | 0    | Vdc  |
| 1 | Vdc  | -Vdc | 0    |
| 3 | 0    | Vdc  | -Vdc |
| 6 | 0    | -Vdc | Vdc  |
| 2 | Vdc  | 0    | -Vdc |
| 4 | -Vdc | Vdc  | 0    |
|   | 0    | 0    | 0    |





$$v_{ab} = v_{an} - v_{bn}$$

$$v_{bc} = v_{bn} - v_{cn}$$

$$v_{ca} = v_{cn} - v_{an}$$

| Seq | Va | Vb | Vc |
|-----|----|----|----|
|     | +  | +  | +  |
| 5   | -  | +  | +  |
| 1   | +  | -  | +  |
| 3   | +  | +  | -  |
| 6   | -  | -  | +  |
| 2   | +  | -  | -  |
| 4   | -  | +  | -  |
|     | -  | -  | -  |

|   | Vab              | Vbc              | Vca              |
|---|------------------|------------------|------------------|
|   | 0                | 0                | 0                |
| 5 | -V <sub>DC</sub> | 0                | V <sub>DC</sub>  |
| 1 | V <sub>DC</sub>  | -V <sub>DC</sub> | 0                |
| 3 | 0                | V <sub>DC</sub>  | -V <sub>DC</sub> |
| 6 | 0                | -V <sub>DC</sub> | V <sub>DC</sub>  |
| 2 | V <sub>DC</sub>  | 0                | -V <sub>DC</sub> |
| 4 | -V <sub>DC</sub> | V <sub>DC</sub>  | 0                |
|   | 0                | 0                | 0                |

