Circuit Analysis and Design

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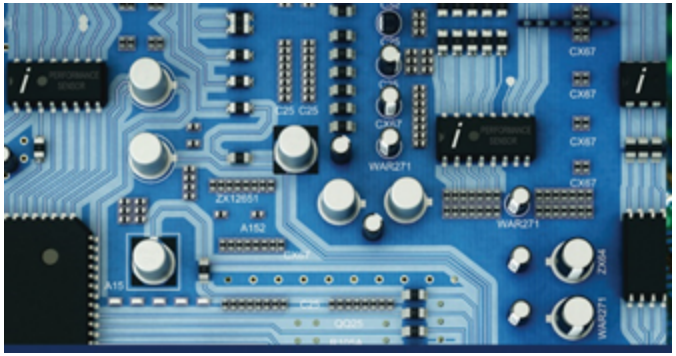
"A good student never steal or cheat"

Agenda



- Definition of node, branch, path, loop, and mesh
- Resistor
- Ohm's law
- Kirchhoff's current law (KCL)
- Kirchhoff's voltage law (KVL)
- Equivalent resistance of series connection of resistors
- Summary

Introduction

- Nodes, branches, loops, and meshes are defined in this chapter.
- The equation of resistance of a conductor is expressed as a function of conductivity (or resistivity), and the dimension of the conductor.
- Ohm's law is introduced.
- Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL) are presented in this chapter.
- Finding the equivalent resistance of series and parallel connection of resistors are discussed.
- The voltage divider rule and the current divider rule are introduced.
- If a circuit contains resistors in wye (Y) shape, it can be changed to delta (Δ) shape. On the other hand, if a circuit contains resistors in delta (Δ) shape, it can be changed to wye (Y) shape. The transformation from wye-to-delta and delta-to-wye may make it easier to simplify the circuit.

Definition of Circuit, Node, Loop, and Mesh

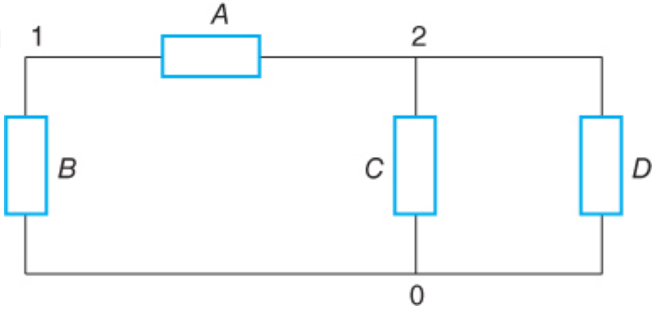
- A circuit is an interconnection of elements, which can be voltage sources, current sources, resistors, capacitors, inductors, coupled coils, transformers, op amps, etc.
- A node is a point in a circuit where two or more elements are joined.
- A simple node is a node that connects two elements.
- An essential node is a node that connects three or more elements.
- A path in a circuit is a series of connected elements from a node to another node that does not go to the same node more than once.
- A branch is a path in a circuit consisting of a single element.
- A loop of a circuit is a closed path starting from a node and returning to the same node.
- A mesh is a loop that does not contain another loop inside it.
- The ground node where the voltage is at ground level is usually taken to be the reference node.
- The voltage of a node measured with respect to a reference node is called node voltage.
- The current through a mesh is called mesh current.

EXAMPLE 2.1

- Find all the nodes, loops, and meshes for the circuit shown in Figure 2.1.
- There are three nodes (labeled as 0, 1, 2). Node 1 is a simple node and nodes 2 and 0 are essential nodes.
- There are three loops in the circuit shown in Figure 2.1. The three loops are
  0-B-1-A-2-D-0
  0-B-1-A-2-C-0
  0-C-2-D-0
- There are two meshes in the circuit shown in Figure 2.1. The two meshes are
  0-B-1-A-2-C-0
  0-C-2-D-0
- The loop 0-B-1-A-2-D-0 contains two meshes 0-B-1-A-2-C-0 and 0-C-2-D-0.

FIGURE 2.1

Circuit for EXAMPLE 2.1.



Resistor

- A resistor is a circuit component that regulates the flow of current.
- The resistance of a resistor measures its ability to limit the current. When the resistance value is large, the amount of current flow through the resistor is small. On the other hand, if the resistance value is small, the amount of current flow through the resistor is large. The resistance value of a resistor is determined by the conductivity (or resistivity) of the material used to make it, as well as its dimensions.
- Low-power resistors can be made from carbon composition material made of fine

  granulated graphite mixed with clay. For high power, wire-wound resistors can be used. The

  wire-wound resistors are constructed by twisting a wire made of

  nichrome or similar material around a ceramic core.
- The circuit symbol for a resistor is shown in Figure 2.3.

FIGURE 2.3

Circuit symbol for a resistor.

Resistance

- The current density is defined as the amount of current through the unit area. If A is the cross-sectional area of a wire that carries a constant current I, the current density is given by

  J = I/A
- It can be shown that the current density is proportional to the electric field intensity:

  J = σE

  where σ is the conductivity of the material.
- Let l be the length of the wire and V be the potential difference (voltage) between the ends of the wire. The potential difference generates a constant electric field E inside the conductor. The potential difference V is related to the electric field through

  V = El
- Substituting E = V/l and J = I/A into J = σE, we obtain I/A = σV//l. Thus, V = [l/(σA)]I. The resistance is defined as

$$R = \frac{\ell}{\sigma A}$$

Resistance (Continued)

- The resistance is proportional to the length of the wire and inversely proportional to the cross-sectional area of the wire and conductivity of the material.
- The resistivity ρ of the material is the inverse of the conductivity:

  $\rho = 1/\sigma$
- In terms of the resistivity, the resistance is given by

$$R = \frac{\rho \ell}{A}$$

- The resistance is proportional to the length of the wire and resistivity, and inversely proportional to the cross-sectional area of the wire.
- The unit for resistance is ohm (Ω).
- In terms of resistance R, Equation $V = [\ell/(\sigma A)]I$ becomes

  $V = RI$ (Ohm's law)
- Find the resistance of a wire with radius 1 mm, length 10 m, conductivity 5×10^4 S/m.

  $R = 10/(\pi \times 0.001^2 \times 5 \times 10^4) = 63.662\ \Omega$.

Ohm's Law (V = RI, I = V/R, R = V/I)

- The voltage-current relation of a resistor with resistance R is given by

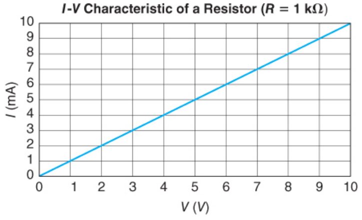
  V = RI

  where V is the voltage across the resistor, and I is the current through the resistor.
- The voltage across a resistor is proportional to the current through the resistor. The proportionality constant in this linear relation is the resistance R. For the given current I, the voltage across the resistor increases as R increases.
- I = V/R (I-V characteristic of a resistor)

  The current through the resistor is proportional to the voltage across the resistor. The proportionality constant in this linear relation is the conductance defined by G = 1/R. For the given voltage V, the current through the resistor decreases as R increases. The slope of the I-V characteristic is the conductance. For Figure 2.5, G = 1 mA/1 V = 0.001 S. The unit for conductance is siemens.

FIGURE 2.5

I-V characteristic of a resistor.



Ohm's Law (R=V/I) and Power absorbed by Resistor

- The resistance R of a resistor is given by

  $R = V/I$

  The resistance of a resistor is the ratio of voltage to current.
- Power absorbed by a resistor is given by

  $P = IV = VI$ (W)

  The power absorbed by a resistor is the product of the current through the resistor and the voltage across the resistor.
- Substituting $V = IR$ into $P = IV$, we get

  $P = I^2R$ (W)

  The power absorbed by a resistor is the product of the square of the current through the resistor and the resistance value.
- Substituting $I = V/R$ into $P = IV$, we get

  $P = V^2/R$ (W)

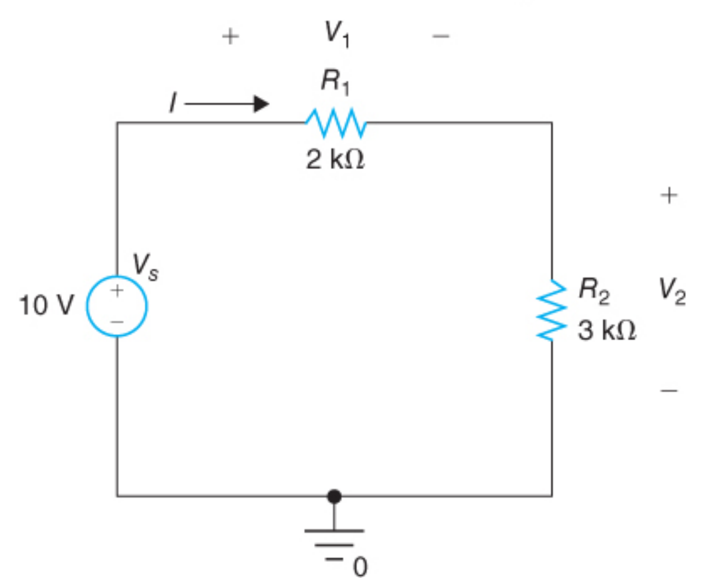
  The power absorbed by a resistor is the ratio of the square of the voltage across the resistor to the resistance value.

Circuit with Two Resistors and a Voltage Source

- Given I = 2 mA, find V_1, V_2, and powers.
- $V_1 = R_1 \times I = 2000 \times 0.002 = 4$ V (Ohm's law)
- $V_2 = R_2 \times I = 3000 \times 0.002 = 6$ V (Ohm's law)
- $P_{R1} = I \times V_1 = 0.002 \times 4 = 0.008$ W = 8 mW
- $P_{R2} = I \times V_2 = 0.002 \times 6 = 0.012$ W = 12 mW
- $P_{Vs} = – I \times V_s = -0.002 \times 10 = – 0.02$ W = -20 mW
- Power absorbed by R_1 and R_2 = 20 mW
- Power released by V_s = 20 mW
- Power absorbed = Power released

FIGURE 2.6

Circuit with two resistors and a voltage source.

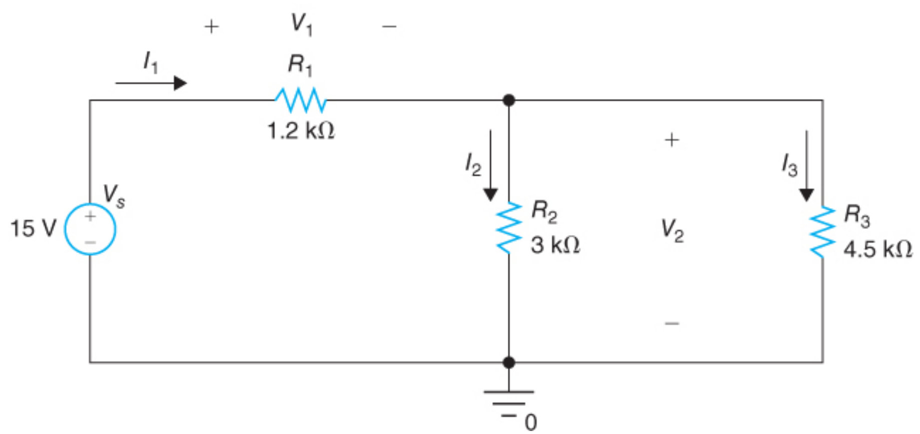


EXAMPLE 2.2

- Given $V_2 = 9$ V, find I_2, I_3, V_1, I_1, and powers in the circuit shown in Figure 2.7.
- $I_2 = V_2/R_2 = 9\text{ V}/3\text{ k}\Omega = 3\text{ mA}$ (Ohm's law), $I_3 = V_2/R_3 = 9\text{ V}/4.5\text{ k}\Omega = 2\text{ mA}$ (Ohm's law)
- $V_1 = V_s - V_2 = 15\text{ V} - 9\text{ V} = 6\text{ V}$, $I_1 = V_1/R_1 = 6\text{ V}/1.2\text{ k}\Omega = 5\text{ mA}$
- $P_{R1} = I_1V_1 = 30\text{ mW}$, $P_{R2} = I_2V_2 = 27\text{ mW}$, $P_{R3} = I_3V_2 = 18\text{ mW}$, $P_{Vs} = -I_1V_s = -75\text{ mW}$
- $P_{R1} + P_{R2} + P_{R3} + P_{Vs} = 0$

FIGURE 2.7

Circuit for EXAMPLE 2.2.



Kirchhoff's Current Law (KCL)

- The sum of currents entering a node equals the sum of currents leaving the same node.

  All currents are positive. A node is a point in a circuit where two or more elements are connected. It is part of wires that interconnect elements. A node cannot store or destroy electric charges. What comes into a node must leave the same node. The number of charges entering a node per second must equal the number of changes leaving the same node per second.
- The sum of currents leaving a node is zero.

  At least one of the currents leaving the node must be negative (meaning that the current actually enters the node).
- The sum of currents entering a node is zero.

  At least one of the currents entering the node must be negative (meaning that the current actually leaves the node).

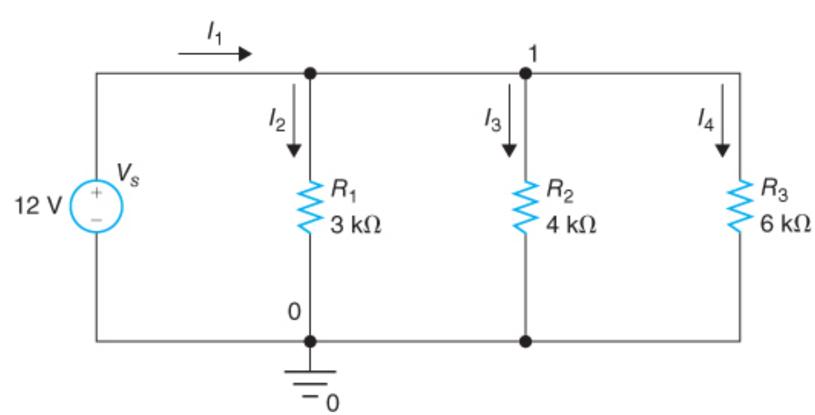
Circuit with Three Resistors and a Voltage Source

- $I_2 = V_s/R_1 = 12\ V/3\ k\Omega = 4\ mA$, $I_3 = V_s/R_2 = 12\ V/4\ k\Omega = 3\ mA$, $I_4 = V_s/R_3 = 12\ V/6\ k\Omega = 2\ mA$
- $I_1 = I_2 + I_3 + I_4 = 4\ mA + 3\ mA + 2\ mA = 9\ mA$
- Sum of currents entering node 1 = I_1 = 9 mA

  Sum of currents leaving node 1 = $I_2 + I_3 + I_4$ = 9 mA
- Sum of all currents leaving node 1 = $- I_1 + I_2 + I_3 + I_4 = - 9\ mA + 4\ mA + 3\ mA + 2\ mA = 0$
- Sum of all currents entering node 1 = $I_1 - I_2 - I_3 - I_4 = 9\ mA - 4\ mA - 3\ mA - 2\ mA = 0$

FIGURE 2.9

Circuit with three resistors and a voltage source.

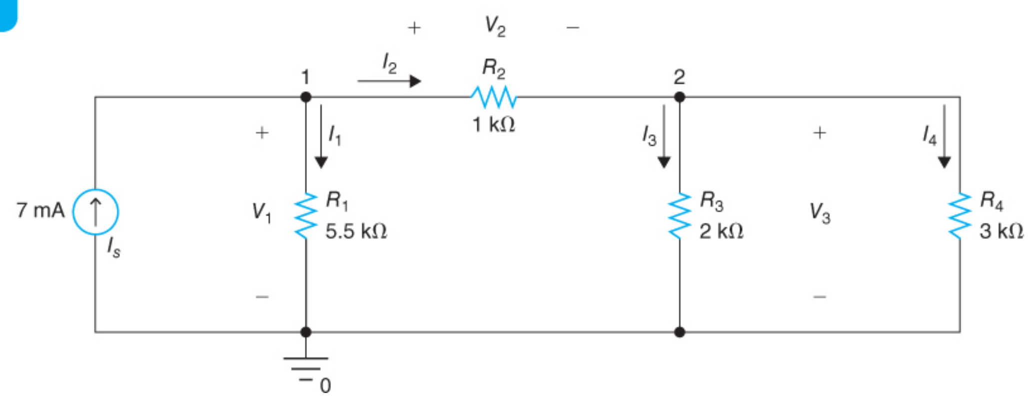


EXAMPLE 2.3

- Given I_3 = 3 mA, find V_3, I_4, I_2, V_2, I_1, and V_1 in the circuit shown in Figure 2.11.
- $V_3 = R_3 I_3 = 2000 \times 0.003 = 6$ V, $I_4 = V_3/R_4 = 6/3000 = 2 \times 10^{-3}$ A = 2 mA
- $I_2 = I_3 + I_4 = 3$ mA + 2 mA = 5 mA
- $V_2 = R_2 I_2 = 1000 \times 0.005 = 5$ V
- $I_1 = I_s - I_2 = 7$ mA – 5 mA = 2 mA
- $V_1 = R_1 I_1 = 5500 \times 0.002 = 11$ V

FIGURE 2.11

Circuit for EXAMPLE 2.3.



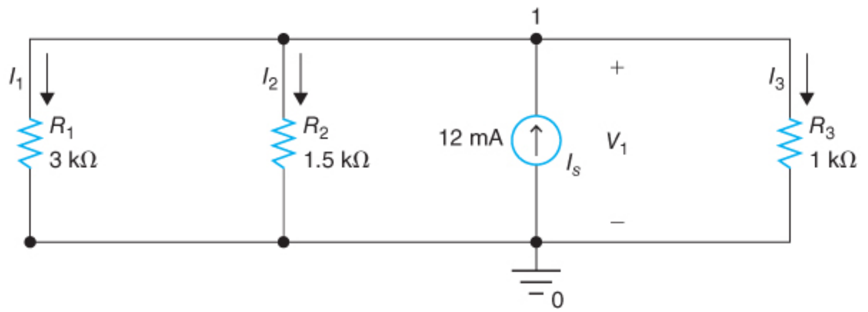
EXAMPLE 2.4

- Find V_1, I_1, I_2, and I_3 in the circuit shown in Figure 2.13.
- KCL: $I_s = I_1 + I_2 + I_3 = V_1/R_1 + V_1/R_2 + V_1/R_3 = V_1(1/R_1 + 1/R_2 + 1/R_3)$
- $I_1 = V_1/R_1 = 2$ mA, $I_2 = V_1/R_2 = 4$ mA, $I_3 = V_1/R_3 = 6$ mA

$$V_1 = \frac{I_s}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{0.012}{\frac{1}{3000}+\frac{1}{1500}+\frac{1}{1000}} = \frac{3000\times 0.012}{\frac{3000}{3000}+\frac{3000}{1500}+\frac{3000}{1000}} = \frac{36}{6} = 6\,\mathrm{V}$$

FIGURE 2.13

Circuit for EXAMPLE 2.4.



EXAMPLE 2.5

- Given $I_1 = 3$ A, $I_3 = 10$ A, and $I_6 = -8$ A, find I_2, I_4, and I_5 in the circuit shown in Figure 2.15.
- KCL at node 1: $I_2 + I_3 = I_1$

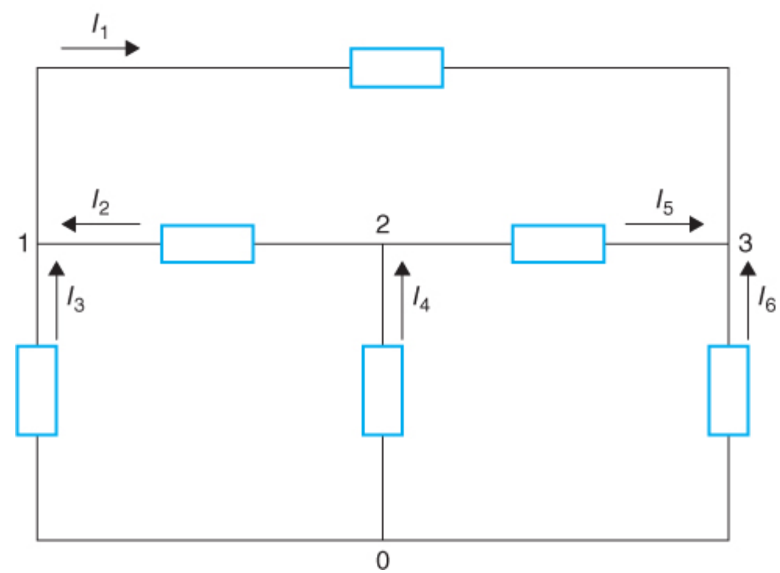
  $I_2 = I_1 - I_3 = 3\text{ A} - 10\text{ A} = -7\text{ A}$
- KCL at node 3: $I_1 + I_5 + I_6 = 0$

  $I_5 = -I_1 - I_6 = -3\text{ A} - (-8\text{ A}) = 5\text{ A}$
- KCL at node 2: $I_4 = I_2 + I_5$

  $I_4 = I_2 + I_5 = -7\text{ A} + 5\text{ A} = -2\text{ A}$

FIGURE 2.15

Circuit for EXAMPLE 2.5.



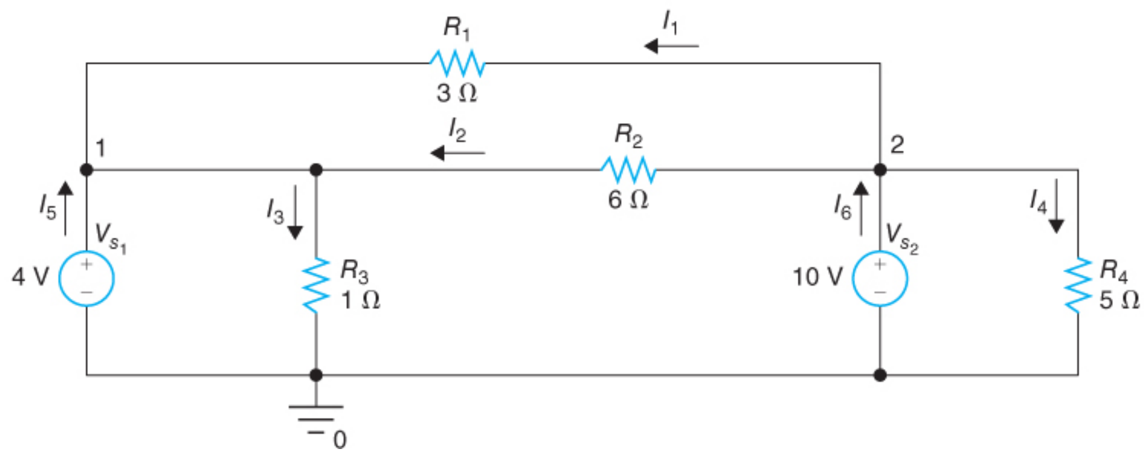
EXAMPLE 2.6

- Find I_1, I_2, I_3, I_4, I_5, and I_6 in the circuit shown in Figure 2.18.
- Ohm's law: $I_1 = (V_{s2} - V_{s1})/R_1 = 6\ \text{V}/3\ \Omega = 2\ \text{A}$, $I_2 = (V_{s2} - V_{s1})/R_2 = 6\ \text{V}/6\ \Omega = 1\ \text{A}$,

  $I_3 = V_{s1}/R_3 = 4\ \text{V}/1\ \Omega = 4\ \text{A}$, $I_4 = V_{s2}/R_4 = 10\ \text{V}/5\ \Omega = 2\ \text{A}$
- KCL at node 1: $I_5 = - I_1 - I_2 + I_3 = - 2\ \text{A} - 1\ \text{A} + 4\ \text{A} = 1\ \text{A}$
- KCL at node 2: $I_6 = I_1 + I_2 + I_4 = 2\ \text{A} + 1\ \text{A} + 2\ \text{A} = 5\ \text{A}$

FIGURE 2.18

Circuit for EXAMPLE 2.6.



Kirchhoff's Voltage Law (KVL)

- The sum of voltage drops around a loop equals the sum of voltage rises of the same loop.

  The voltage of a node must be unique, and the voltage for any node cannot have two different values.
- The sum of voltage drops around a loop is zero.

  At least one of the voltage drops around the loop must be negative (meaning that the voltage actually rises on the branch).
- The sum of voltage rises around a loop is zero.

  at least one of the voltage rises around the loop must be negative (meaning that the voltage actually drops on the branch) for this statement to be true.
- Since a mesh is also a loop, the KVL applies to mesh as well.

Circuit with Three Resistors and a Voltage Source

- Ohm's law: $V_1 = R_1I$, $V_2 = R_2I$, $V_3 = R_3I$
- According to KVL, the sum of voltage drops around the mesh in the clockwise direction is zero:
  $-V_s + R_1I + R_2I + R_3I = 0$

$$I = \frac{V_s}{R_1 + R_2 + R_3} = \frac{9}{4000 + 6000 + 8000} \mathrm{A} = 0.5\,\mathrm{mA}$$

- Ohm's law: $V_1 = R_1I = 4000 \times 0.0005\ \mathrm{V} = 2\ \mathrm{V}$
  $V_2 = R_2I = 6000 \times 0.0005\ \mathrm{V} = 3\ \mathrm{V}$
  $V_3 = R_3I = 8000 \times 0.0005\ \mathrm{V} = 4\ \mathrm{V}$

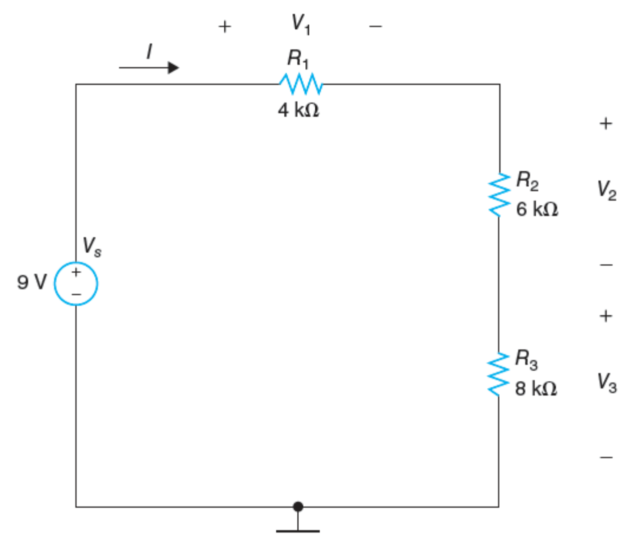


FIGURE 2.20

EXAMPLE 2.7

- Given V_2 = 6 V, find I_2, I_3, I_4, V_4, I_1, V_1, V_s in the circuit shown in Figure 2.21.
- Ohm's law: $I_2 = V_2/R_2 = 6/3000$ A = 2 mA

  $I_3 = V_2/R_3 = 6/4000$ A = 1.5 mA
- KCL: $I_1 = I_4 = I_2 + I_3$ = 2 mA + 1.5 mA = 3.5 mA
- Ohm's law: $V_4 = R_4I_4 = 1000 \times 0.0035$ V = 3.5 V

  $V_1 = R_1I_1 = 600 \times 0.0035$ V = 2.1 V
- KVL: $-V_s + V_1 + V_2 + V_4 = 0$

  $V_s = V_1 + V_2 + V_4$ = 2.1 V + 6 V + 3.5 V = 11.6 V

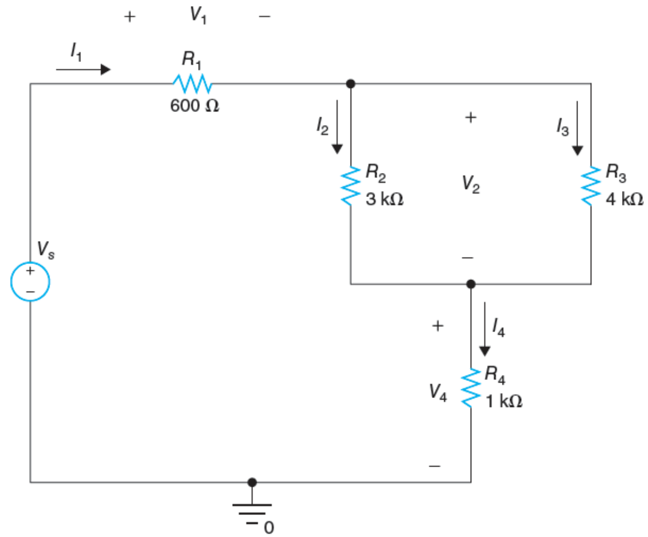


FIGURE 2.21

EXAMPLE 2.8

- Given $V_1 = 6$ V, $V_5 = 5$ V, $V_6 = 3$ V, and $V_7 = 7$ V, find V_2, V_3, V_4, and V_8 in the circuit shown in Figure 2.23.
- KVL around the mesh in the lower left:
  $-V_6 + V_4 + V_7 = 0$, $V_4 = V_6 - V_7 = 3\text{ V} - 7\text{ V} = -4\text{ V}$
- KVL around the mesh in the lower right:
  $-V_7 - V_5 + V_8 = 0$,
  $V_8 = V_5 + V_7 = 5\text{ V} + 7\text{ V} = 12\text{ V}$
- KVL around the mesh in the upper left:
  $-V_1 + V_3 - V_4 = 0$, $V_3 = V_1 + V_4 = 6\text{ V} - 4\text{ V} = 2\text{ V}$
- KVL around the mesh in the upper right:
  $-V_3 + V_2 + V_5 = 0$
  $V_2 = V_3 - V_5 = 2\text{ V} - 5\text{ V} = -3\text{ V}$

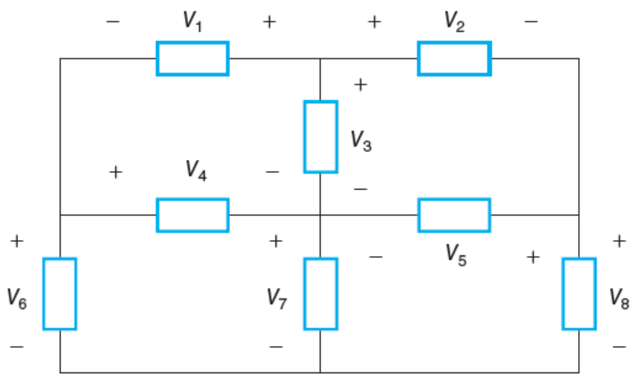
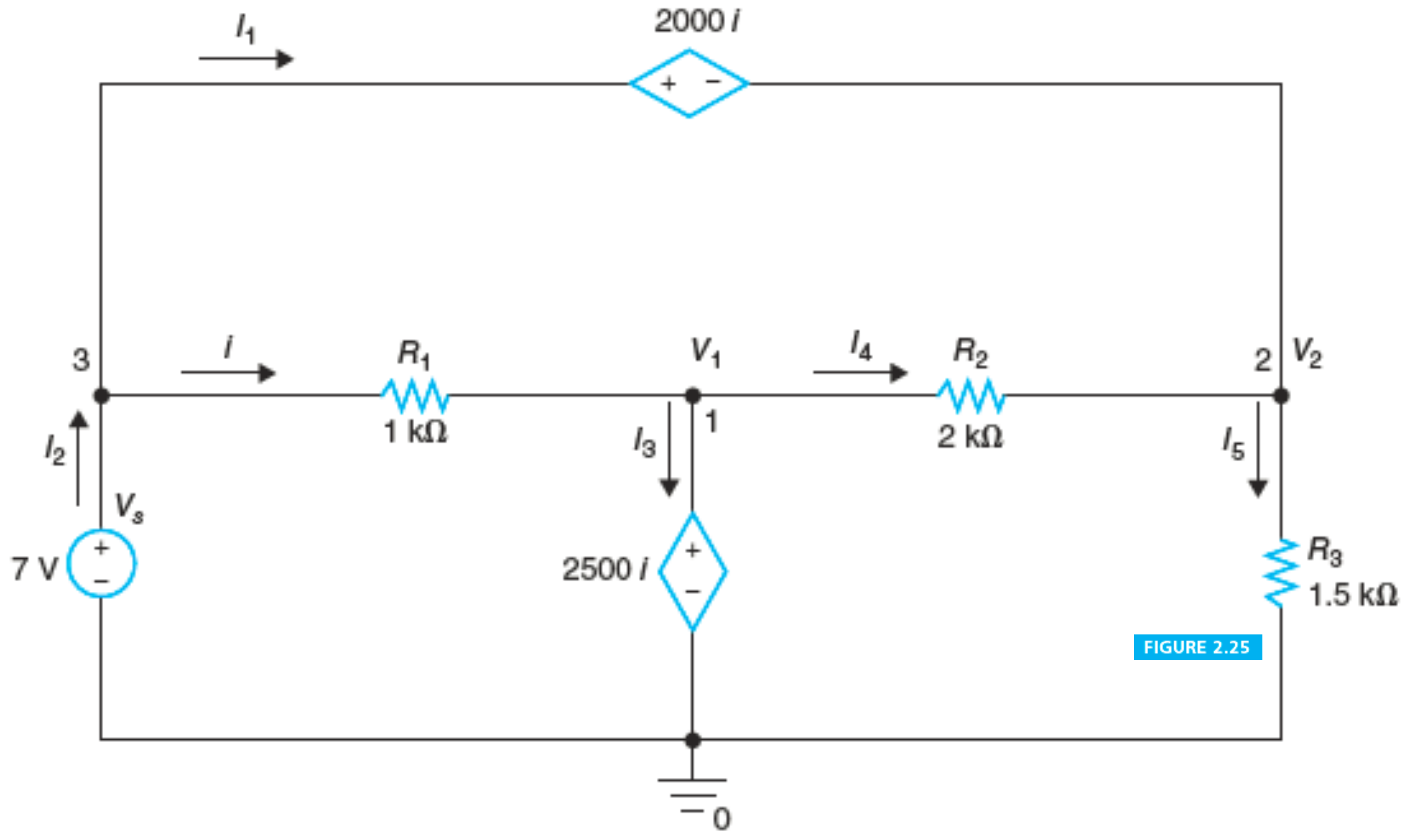


FIGURE 2.23

- Find i, V_1, V_2, I_2, I_3, I_4, I_5 in the circuit shown in Figure 2.25.



EXAMPLE 2.9

- Find i, V_1, V_2, I_2, I_3, I_4, I_5 in the circuit shown in Figure 2.25.
- KVL around the mesh in the lower left: $-V_s + R_1 i + 2500i = 0$ or $-7 + 1000i + 2500i = 0$

  $i = 7/3500$ A $= 1/500$ A $= 2$ mA, $V_1 = 2500i = 2500 \times 0.002$ V $= 5$ V
- KVL around the outside loop:

  $-V_s + 2000i + V_2 = 0$

  $V_2 = V_s - 2000i = 7 - 2000 \times 0.002 = 3$
- Ohm's law:

  $I_4 = (V_1 - V_2)/R_2 = 2\text{ V}/2000\ \Omega = 1$ mA

  $I_5 = V_2/R_3 = 3\text{ V}/1500\ \Omega = 2$ mA
- KCL at node 1: $I_3 = i - I_4 = 1$ mA
- KCL at node 2: $I_1 = I_5 - I_4 = 1$ mA
- KCL at node 3: $I_2 = I_1 + i = 3$ mA

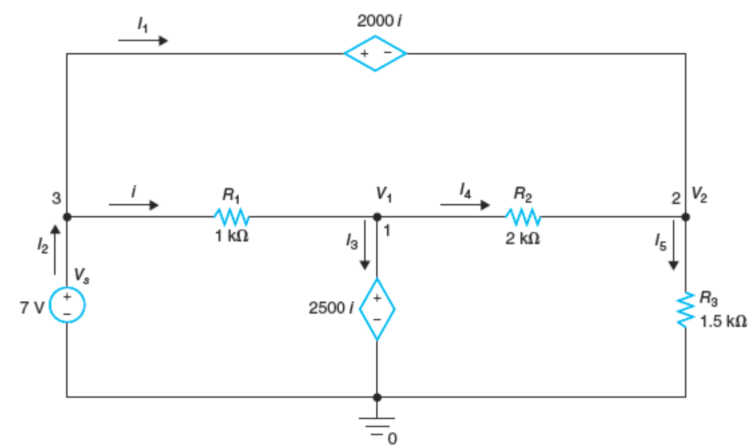


FIGURE 2.25

Equivalent Resistance of Series Connection of Resistors

- Two resistors with resistances R_1 and R_2 are connected in series in Fig.2.27(a).
- I = current through R_1 and R_2, V_1 = voltage across R_1, V_2 = voltage across R_2
- Ohm's law: $V_1 = R_1 I$, $V_2 = R_2 I$
- KVL: $-V + V_1 + V_2 = 0$

  $V = V_1 + V_2 = R_1 I + R_2 I = (R_1 + R_2)I = R_{eq} I$

  where R_{eq} is the equivalent resistance of the series connection of R_1 and R_2.
- The circuit shown in Fig.2.27(a) can be replaced by the circuit shown in Fig.2.27(b).
- If n resistors with resistances R_1, R_2, …, R_n are connected in series, the equivalent resistance is given by

  $R_{eq} = R_1 + R_2 + \ldots + R_n$

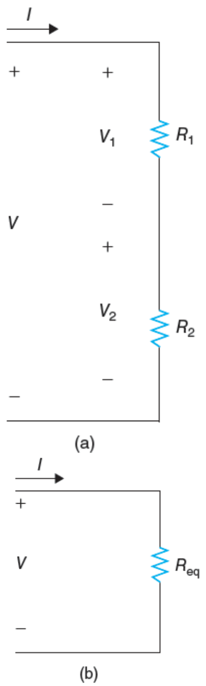


FIGURE 2.27

Circuit with Series Connection of Resistors

- The equivalent resistance of the series connection of R_1 and R_2 in Fig.2.29 is given by

  $R_{eq} = R_1 + R_2 = 25\ k\Omega$
- When R_1 and R_2 are replaced by R_{eq}, we obtain the circuit shown in Figure 2.30.
- Ohm's law:

  $I = V_s/R_{eq} = 5\ V/25\ k\Omega = 0.2\ mA$
- Ohm's law:

  $V_1 = R_1I = 10\ k\Omega \times 0.2\ mA = 2\ V$

  $V_2 = R_2I = 15\ k\Omega \times 0.2\ mA = 3\ V$

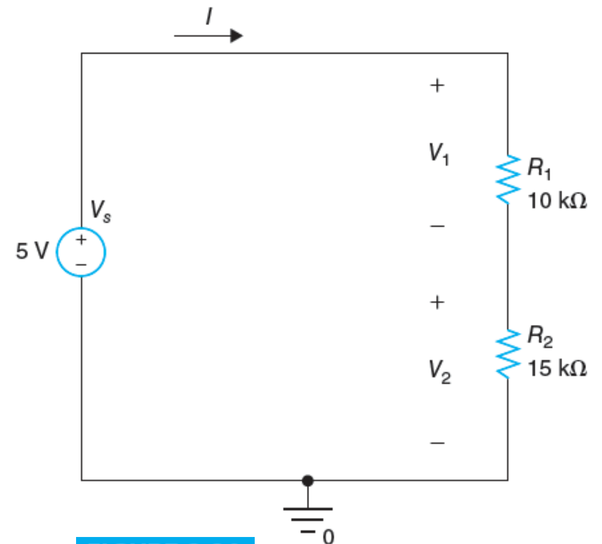


FIGURE 2.29

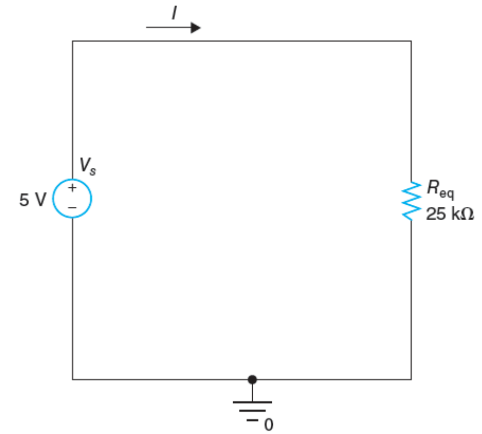


FIGURE 2.30

EXAMPLE 2.10

- Given I_3 = 750 μA, find V_a, I_2, I_4, I_1, V_s in the circuit shown in Figure 2.31.

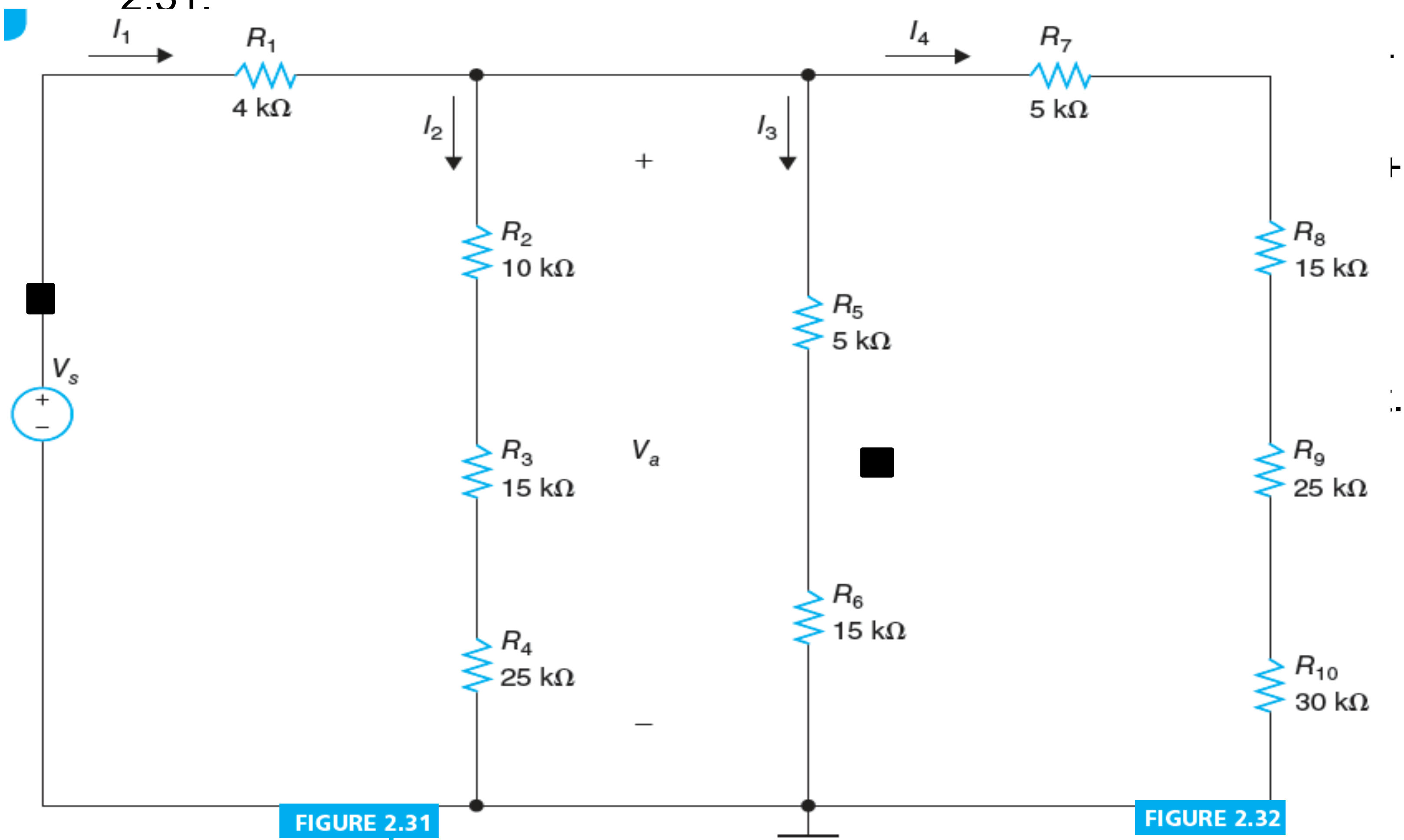
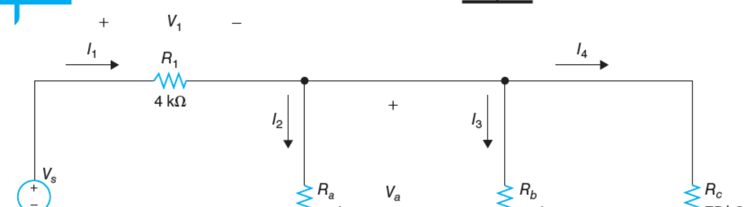


FIGURE 2.31

FIGURE 2.32



EXAMPLE 2.10

- Given I_3 = 750 μA, find V_a, I_2, I_4, I_1, V_s in the circuit shown in Figure 2.31.
- $R_a = R_2 + R_3 + R_4 = 50\ k\Omega$, $R_b = R_5 + R_6 = 20\ k\Omega$, $R_c = R_7 + R_8 + R_9 + R_{10} = 75\ k\Omega$
- $V_a = R_b I_3 = 15\ V$, $I_2 = V_a/R_a = 0.3\ mA$, $I_4 = V_a/R_c = 0.2\ mA$, $I_1 = I_2 + I_3 + I_4 = 1.25\ mA$

$V_1 = R_1 I_1 = 5\ V$, $V_s = V_1 + V_a = 20\ V$

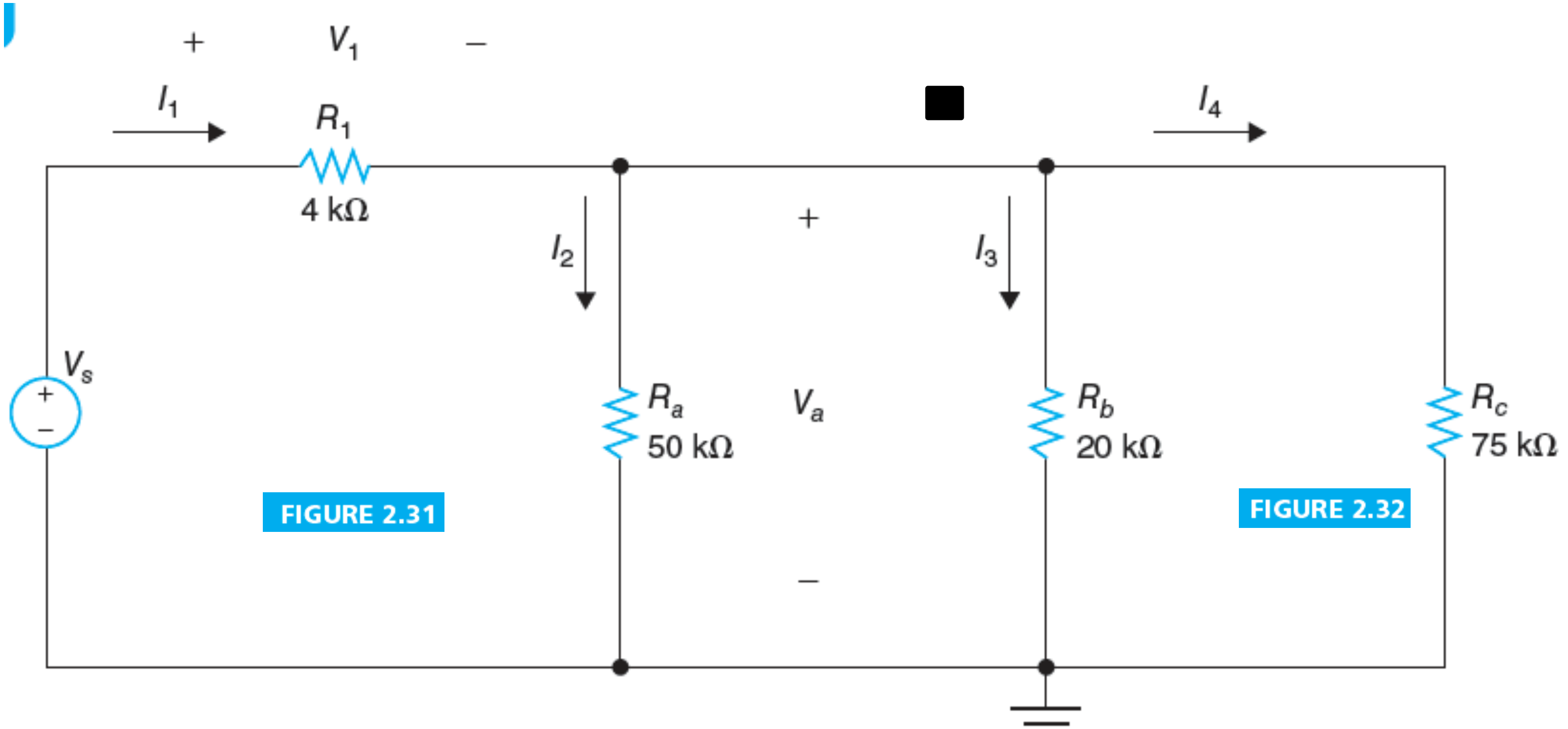


FIGURE 2.31

FIGURE 2.32

Summary

- Definition of node, branch, path, loop, and mesh
- Resistor, Ohm’s law, KCL and KVL
- Equivalent resistance of series connection of resistors
- What will we study in next lecture.