IMAGE RESTORATION

changshu@uestc.edu.cn

- Constrained least square filtering

$$\hat{\boldsymbol{F}} = \frac{\boldsymbol{G}}{\boldsymbol{H}} \qquad \hat{F} = \left[\frac{H^*}{|H|^2 + \gamma[S_n / S_f]} \right] G \qquad \hat{F} = \left[\frac{H^*}{|H|^2 + \gamma |P|^2} \right] G$$

1. Observation
2. Experimentation
3. Mathematical modeling

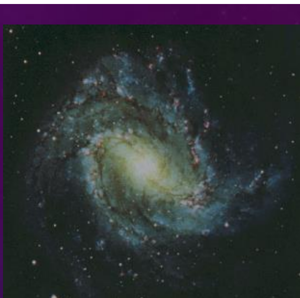
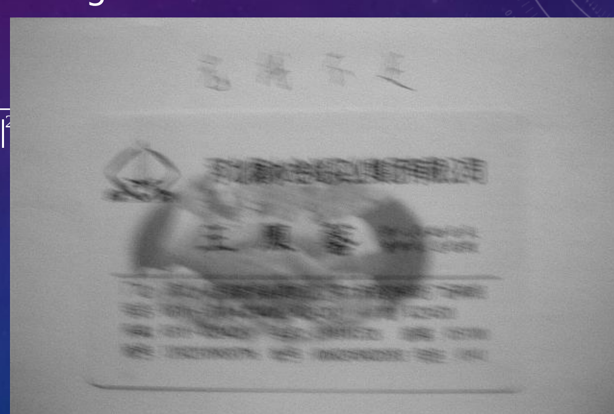


IMAGE RESTORATION

- l least square filtering

$$\hat{\boldsymbol{F}} = \frac{\boldsymbol{G}}{\boldsymbol{H}} \qquad \hat{F} = \left[\frac{}{|H|^2} \right.$$

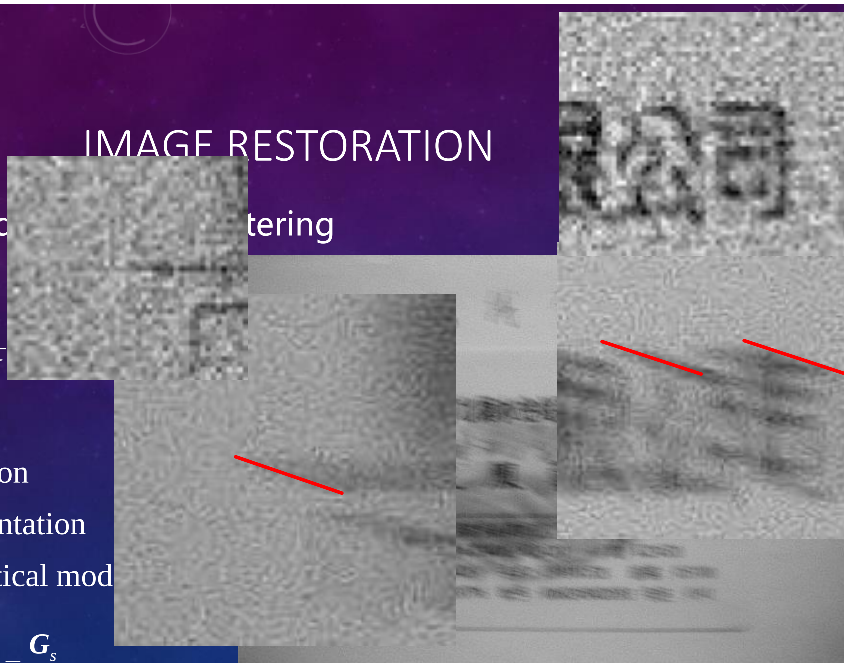


1. Observation
2. Experimentation
3. Mathematical modeling

IMAGE RESTORATION

- Constrained tering

$$\hat{F} = \frac{G}{H}$$

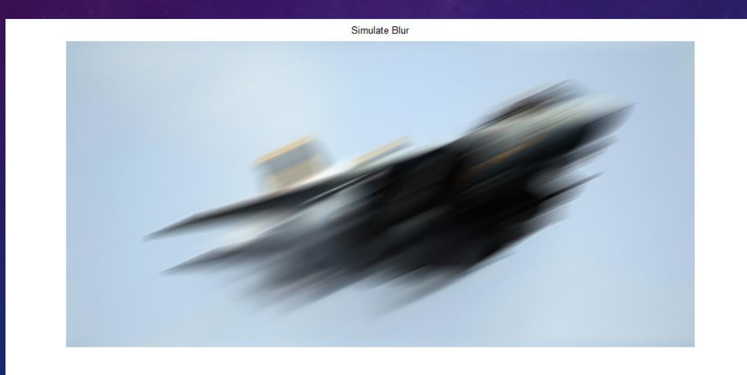


1. Observation
2. Experimentation
3. Mathematical mod

$$H_s = \frac{G_s}{\hat{F}_s}$$

BLUR CAUSED BY UNIFORM LINEAR MOTION

3. Mathematical modeling



$$H(u,v)$$

$$e^{-k(u^2+v^2)^{\frac{5}{6}}}$$

$$k = 0.0025$$

$$h(x,y) = \frac{1}{V^2T} rect(x/VT)$$

BLUR CAUSED BY UNIFORM LINEAR MOTION

Suppose an image $f(x,y)$ undergoes planar motion and that $x_0(t)$ and $y_0(t)$ are the time-varying components of motion in the x- and y-directions, respectively, T is the duration of the exposure, it follows that :

$$g(x,y)=\int_0^T f[x-x_0(t),y-y_0(t)]dt$$

The FT of g is :

$$\begin{aligned}G(u,v)&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\exp[-j2\pi(ux+vy)]dxdy\\&=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x-x_0(t),y-y_0(t)]\exp[-j2\pi(ux+vy)]dxdy\right]dt\\&=F(u,v)\underbrace{\int_0^T \exp\{-j2\pi[ux_0(t)+vy_0(t)]\}dt}\rightarrow H(u,v)\end{aligned}$$

BLUR CAUSED BY UNIFORM LINEAR MOTION

changshu@uestc.edu.cn

$$G(u,v)=H(u,v)F(u,v)$$

$$H(u,v)=\frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

$$x_0(t)=at/T$$

$$y_0(t)=bt/T$$

IMAGE RESTORATION

- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering
- Constrained least squares filtering
- Geometric image transformation

GEOMETRIC IMAGE MODIFICATION

- Image translation, scaling, rotation
- Generalized linear geometrical transformation
- Affine transformation
- Perspective transformation

GEOMETRIC IMAGE MODIFICATION

- Linear transformations

$$T(\vec{x}) = A\vec{x} \qquad R^n \to R^m$$

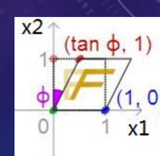
- Non-linear transformations $\qquad R^n \to R^{n+1}$
  - Affine transformations
  - Perspective transformations
- Why matrices
  - Consistent format
  - Easy concatenating

GEOMETRIC IMAGE MODIFICATION

changshu@uestc.edu.cn

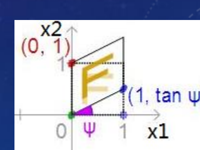
- Shear parallel to the x1 axis

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & k_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y_1 = x_1 + k_2 x_2$$



- Shear parallel to the x2 axis

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ k_1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y_2 = k_1 x_1 + x_2$$



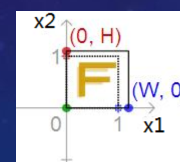
GEOMETRIC IMAGE MODIFICATION

- Scale the x1 axis

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y_1 = s_1 x_1$$

- Scale the x2 axis

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad y_2 = s_2 x_2$$

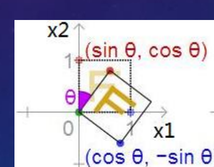


$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} s_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

GEOMETRIC IMAGE MODIFICATION

- Clockwise rotation

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$



- Counter-clockwise rotation

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

GEOMETRIC IMAGE MODIFICATION

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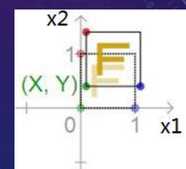
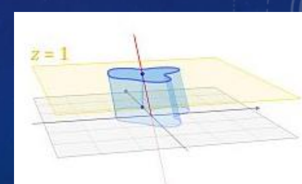
- Translation

$$y_1 = x_1 + b_1 \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
$$y_2 = x_2 + b_2$$



- Augmented matrix - homogeneous coordinates

$$\begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$



GEOMETRIC IMAGE MODIFICATION

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- Augmented matrix

$$\begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & k_2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

GEOMETRIC IMAGE MODIFICATION

changshu@uestc.edu.cn

- Affine transformations
  - translation+scaling+rotation+reflection+shear+…
  - every linear transformation is affine, not every affine transformation is linear

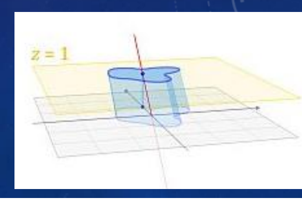
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

- linear transformation followed by a translation

$$\vec{y} = Affine(\vec{x}) = A\vec{x} + \vec{b} \qquad \vec{y} = M\vec{t} = \begin{bmatrix} A & \vec{b} \end{bmatrix} \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix}$$

- Augmented matrix representation

$$\begin{bmatrix} \vec{y} \\ 1 \end{bmatrix} = \begin{bmatrix} A & \vec{b} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix}$$



GEOMETRIC IMAGE MODIFICATION

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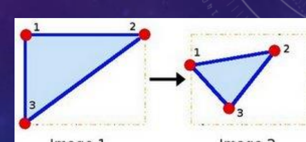
- Affine transformations properties
  - Point line (colinearity) plane
  - Ratios of vectors alone a line
  - parallel lines
  - barycenters



$$f\left(\sum_{i \in I} \lambda_i a_i\right) = \sum_{i \in I} \lambda_i f(a_i)$$

$$\{(a_i, \lambda_i)\}_{i \in I}$$

$$\sum_{i \in I} \lambda_i = 1$$

  - Invertible transformation

$$f:\ A \to B \qquad \begin{bmatrix} \vec{x} \\ 1 \end{bmatrix} = \begin{bmatrix} A^{-1} & -A^{-1}\vec{b} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{y} \\ 1 \end{bmatrix}$$

  - all triangles are related to one another by affine transformations

GEOMETRIC IMAGE MODIFICATION

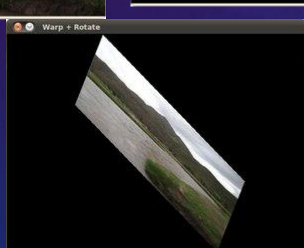
- Augmented matrix

$$
\begin{bmatrix} y_1 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\Delta x_1 \\ 0 & 1 & -\Delta x_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \Delta x_1 \\ 0 & 1 & \Delta x_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} 1 & 0 & -\Delta x_1 \\ 0 & 1 & -\Delta x_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & \Delta x_1 \cos\theta + \Delta x_2 \sin\theta \\ -\sin\theta & \cos\theta & -\Delta x_1 \sin\theta + \Delta x_2 \cos\theta \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} \cos\theta & \sin\theta & \Delta x_1 \cos\theta + \Delta x_2 \sin\theta - \Delta x_1 \\ -\sin\theta & \cos\theta & -\Delta x_1 \sin\theta + \Delta x_2 \cos\theta - \Delta x_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}
$$

GEOMETRIC IMAGE MODIFICATION



SUMMARY

- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering
- Constrained least squares filtering
- Geometric image modification

IMAGE RESTORATION

Image inpainting

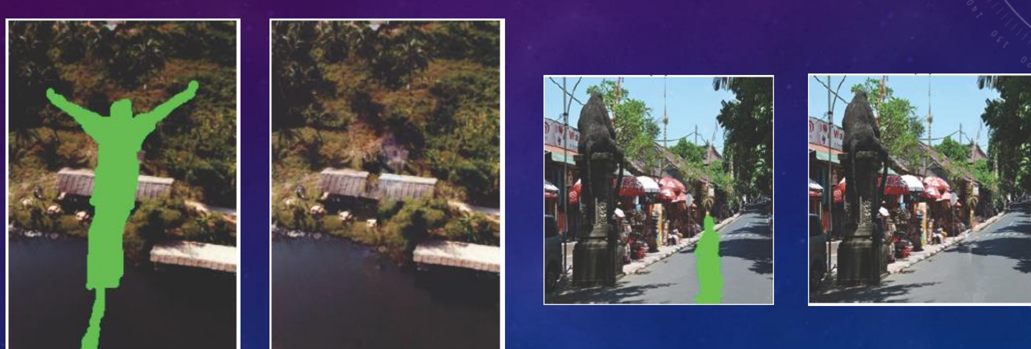


IMAGE RESTORATION

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Image inpainting

IMAGE RESTORATION

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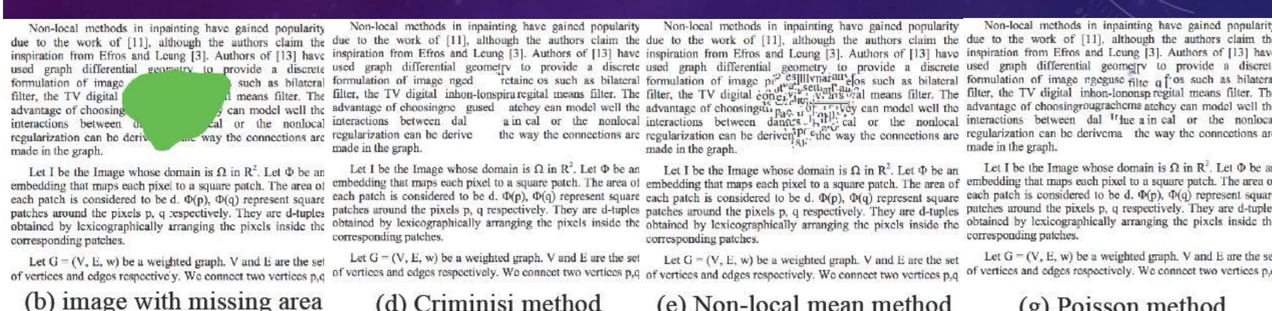
Image inpainting



Non-local methods in inpainting have gained popularity due to the work of [11], although the authors claim the inspiration from Efros and Leung [3]. Authors of [13] have used graph differential geometry to provide a discrete formulation of image [illegible] such as bilateral filter, the TV digital [illegible] means filter. The advantage of choosing [illegible] can model well the interactions between [illegible] cal or the nonlocal regularization can be deriv[illegible] the way the connections are made in the graph.

Let I be the Image whose domain is Ω in R^2. Let Φ be an embedding that maps each pixel to a square patch. The area of each patch is considered to be d. $\Phi(p)$, $\Phi(q)$ represent square patches around the pixels p, q respectively. They are d-tuples obtained by lexicographically arranging the pixels inside the corresponding patches.

Let G = (V, E, w) be a weighted graph. V and E are the set of vertices and edges respectively. We connect two vertices p,q

(b) image with missing area (d) Criminisi method (e) Non-local mean method (g) Poisson method