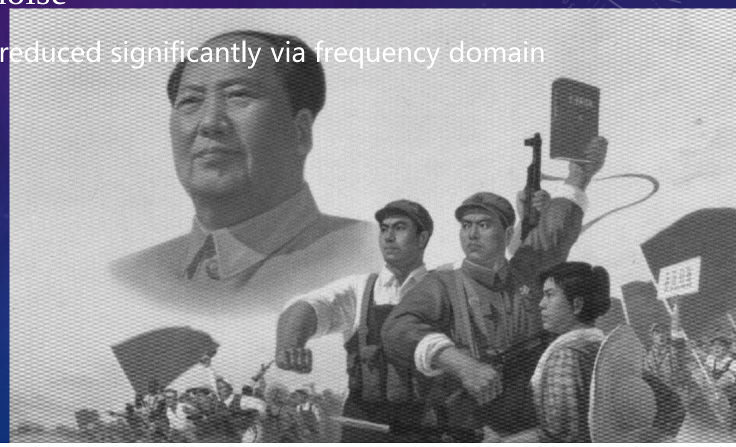
RESTORATION IN THE PRESENCE OF NOISE ONLY

$g(x,y) = f(x,y) + n(x,y)$ $\qquad$ $G(u,v) = F(u,v) + N(u,v)$

- Removal of periodic noise
  - Periodic noise can be reduced significantly via frequency domain filtering.



RESTORATION IN THE PRESENCE OF NOISE ONLY

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$g(x,y) = f(x,y) + n(x,y)$ $\qquad$ $G(u,v) = F(u,v) + N(u,v)$

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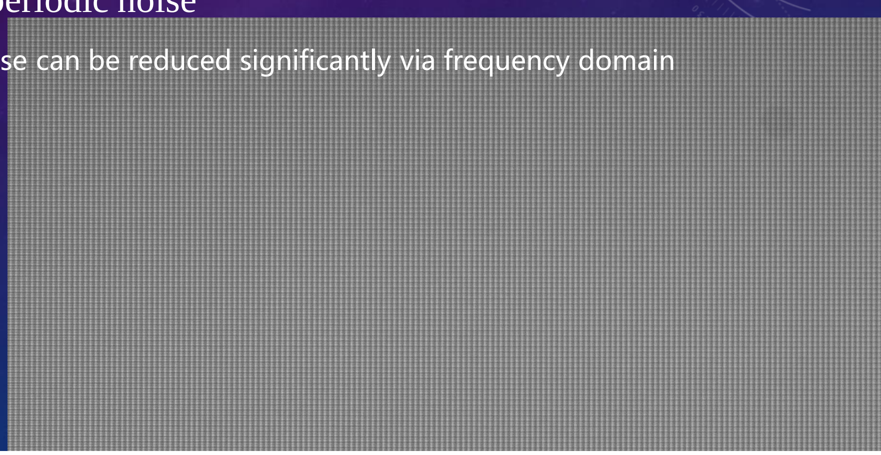

RESTORATION IN THE PRESENCE OF NOISE ONLY

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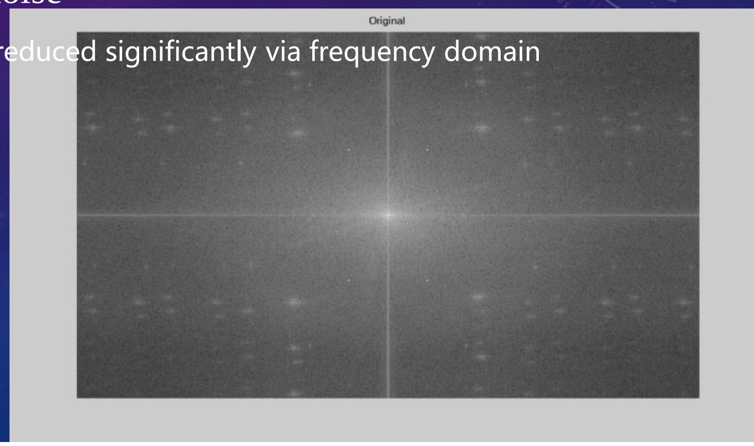
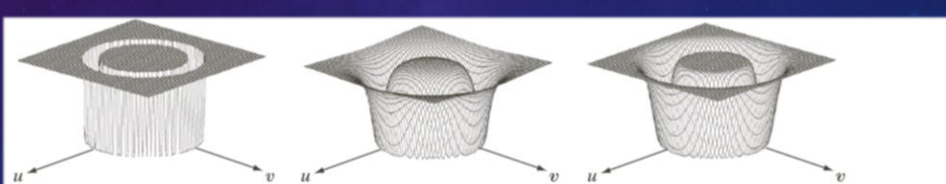


Band Reject

RESTORATION IN THE PRESENCE OF NOISE ONLY

$g(x,y) = f(x,y) + n(x,y)$ $\qquad$ $G(u,v) = F(u,v) + N(u,v)$

- Removal of periodic noise
  - Periodic noise can be reduced significantly via frequency domain filtering.



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

Band Reject
Notch Reject

RESTORATION IN THE PRESENCE OF NOISE ONLY

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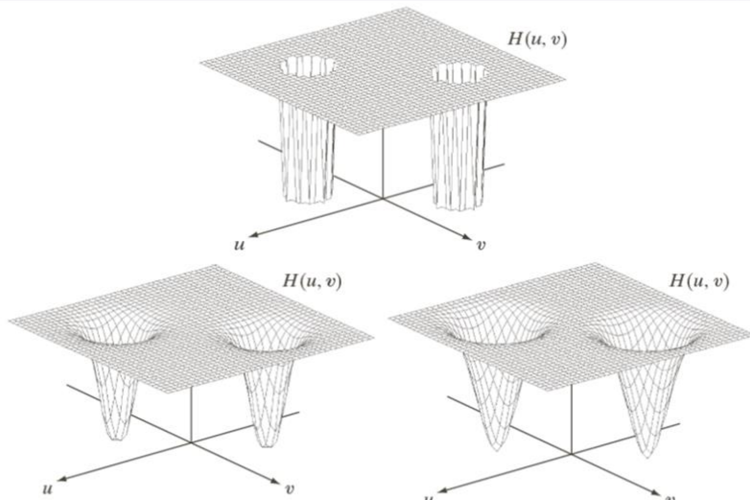
a
b c

- **FIGURE 5.18** Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



RESTORATION IN THE PRESENCE OF NOISE ONLY

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$g(x,y) = f(x,y) + n(x,y)$ $\qquad$ $G(u,v) = F(u,v) + N(u,v)$

- Removal of periodic noise
  - Periodic noise can be reduced significantly via frequency domain filtering.
- Noise PDF estimation
  - The intensity values in the noise component may be considered random variables characterized by a probability density function.

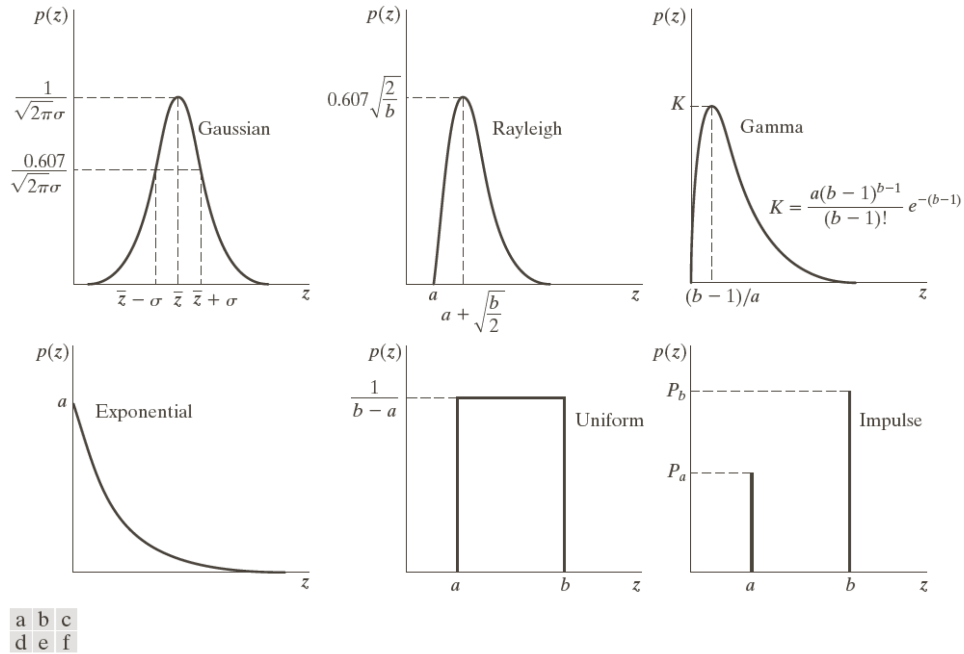

FIGURE 5.2 Some important probability density functions.

RESTORATION IN THE PRESENCE OF NOISE ONLY

$$g(x,y) = f(x,y) + n(x,y) \qquad G(u,v) = F(u,v) + N(u,v)$$

- Removal of periodic noise
  - Periodic noise can be reduced significantly via frequency domain filtering.
  - The parameters of periodic noise can be estimated by inspection of the Fourier spectrum of the image
- Noise PDF estimation
  - The intensity values in the noise component may be considered random variables characterized by a probability density function.
  - The parameters of the PDF can be estimated from small patches of reasonably constant background intensity

IMAGE RESTORATION

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- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering
- Constrained least squares filtering
- Geometric image transformation

INVERSE FILTERING

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- Unconstrained image restoration
  - Given the degradation function *H*, we compute an estimate of the transform of the original image by dividing the transform of the degraded image by the degradation function:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$G(u,v) = H(u,v)F(u,v) + N(u,v)$

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

IMAGE TRANSFORMS – DFT2

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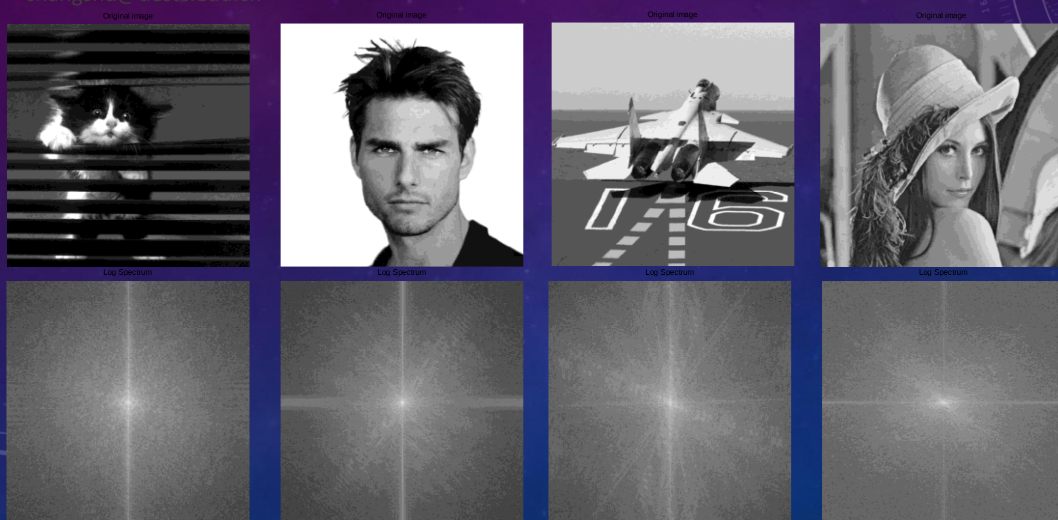

$$\hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

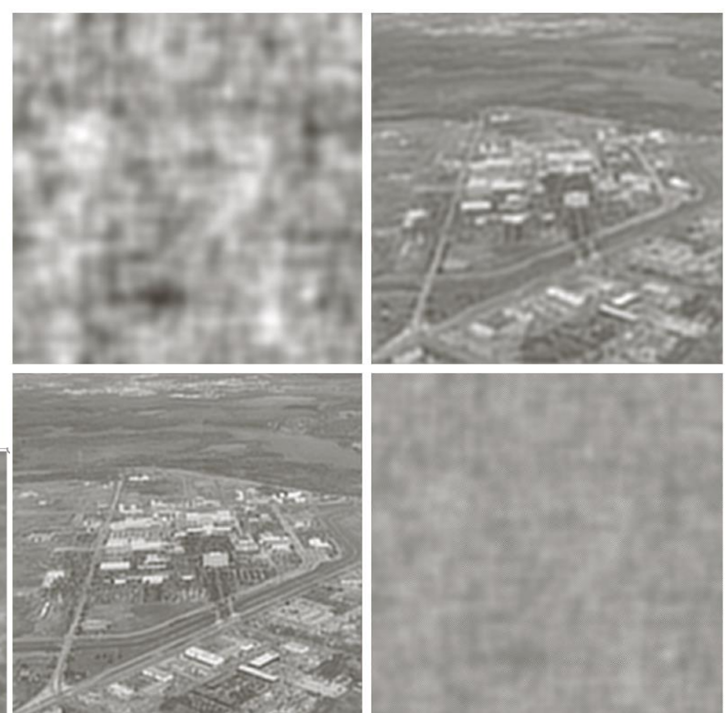

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

IMAGE RESTORATION

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- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering/minimum mean square error filtering
- Constrained least squares filtering
- Geometric image transformation

WIENER FILTERING

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- Error measure：

$$e^2 = E\left[(f - \hat{f})^2\right]$$

- Solution：

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma[S_n(u,v)/S_f(u,v)]}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma[S_n(u,v)/S_f(u,v)]}\right]G(u,v)$$

power spectrum of the noise

power spectrum of the undegraded image

WIENER FILTERING

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- SNR :

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|F(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|N(u,v)|^2} = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|S_f(u,v)|^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}|S_n(u,v)|^2}$$

$$SNR = \frac{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\hat{f}(x,y)^2}{\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}\left[f(x,y)-\hat{f}(x,y)\right]^2}$$

WIENER FILTERING

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$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\cdot\frac{|H(u,v)|^2}{|H(u,v)|^2+\gamma[S_n(u,v)/S_f(u,v)]}\right]G(u,v)$$

$$\Downarrow$$

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\cdot\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,v)$$

R FILTERING

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

IMAGE RESTORATION

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FIG. WIENER FILTERING

（a）Motion blurred image with Gauss noise （b）the restored image by WF

IMAGE RESTORATION

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- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering
- Constrained least squares filtering
- Geometric image transformation

IMAGE RESTORATION

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- Constrained least square filtering

$$\min_{\hat{f}}\left\{\hat{f}^T C^T C\hat{f}\right\} = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 \hat{f}(x,y)\right]^2 \qquad \min_{\hat{f}}\left\|\boldsymbol{Q}\hat{\boldsymbol{f}}\right\|^2$$

$$s.t. \quad \left\|\boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}}\right\|^2 = \left\|\boldsymbol{n}\right\|^2$$

IMAGE RESTORATION

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- Solution : $\hat{f}_e = \left([\hbar_e]^T[\hbar_e] + \alpha[C_e]^T[C_e]\right)^{-1}[\hbar_e]^T g_e$
- Diagonalize by DFT $[\hbar_e]\vec{W}_k = \lambda_h(k)\vec{W}_k$ $[W]^{-1}[h_e][W] = [\Lambda_h]$

  $[W]^* \hat{f}_e = F_e$ $[W]^* g_e = G_e$
- Diagonalize the circulant matrix : $E = W^{-1}CW$

$$W^{-1}\hat{f}_e = \left(D^*D + \alpha E^*E\right)^{-1} D^*W^{-1}g_e$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma\left|p_e(u,v)\right|^2}\right]G(u,v)$$

IMAGE RESTORATION

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- Second order derivative :

$$f(i+1) - 2f(i) + f(i-1)$$

- In 1D case :

$$\min_f\left\{f^TC^TCf\right\} = \min_f\left\{\sum_{i=1}^{N}\left[f(i+1) - 2f(i) + f(i-1)\right]^2\right\}$$

$$C = \begin{bmatrix} 1 & & & & & \\ -2 & 1 & & & & \\ 1 & -2 & \cdots & 1 & & \\ & 1 & \cdots & -2 & 1 \\ & & & 1 & -2 \\ & & & & 1 \end{bmatrix}^T$$

IMAGE RESTORATION

- 2D Laplacian operator in matrix form:

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_0 & \boldsymbol{C}_{M-1} & \boldsymbol{C}_{M-2} & \cdots & \boldsymbol{C}_1 \\ \boldsymbol{C}_1 & \boldsymbol{C}_0 & \boldsymbol{C}_{M-1} & \cdots & \boldsymbol{C}_2 \\ \boldsymbol{C}_2 & \boldsymbol{C}_1 & \boldsymbol{C}_0 & \cdots & \boldsymbol{C}_3 \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{C}_{M-1} & \boldsymbol{C}_{M-2} & \boldsymbol{C}_{M-3} & \cdots & \boldsymbol{C}_0 \end{bmatrix}$$

$$\boldsymbol{C}_j = \begin{bmatrix} p_e(j,0) & p_e(j,N-1) & p_e(j,N-2) & \cdots p_e(j,1) \\ p_e(j,1) & p_e(j,0) & p_e(j,N-1) & \cdots p_e(j,2) \\ \vdots & \vdots & \vdots & \vdots \\ p_e(j,N-1) & p_e(j,N-2) & p_e(j,N-3) & \cdots p_e(j,0) \end{bmatrix}$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |p_e(u,v)|^2} \right] G(u,v) \qquad s.t. \quad \left\| \boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}} \right\|^2 = \|\boldsymbol{n}\|^2$$

IMAGE RESTORATION

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- Since convolution is employed to obtain f, C and g should be padded:

$$g_e(i,j) = \begin{cases} g(i,j) & 0 \le i, j \le M-1 \\ 0 & M \le i, j \le p-1 \end{cases}$$

$$p_e(i,j) = \begin{cases} p(i,j) & 0 \le i, j \le J-1 \\ 0 & J \le i, j \le p-1 \end{cases} \qquad p(i,j) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

IMAGE RESTORATION

$$\hat{F} = \left[\frac{H^*}{|H|^2 + \gamma |P|^2} \right] G$$

- Resisual:

$$r = (g - H\hat{f}) \qquad \phi(\gamma) = r^T r = \|r\|^2$$

1. Specify an initial value of γ
2. Compute $\phi(\gamma)$
3. Stop if the following equation is satisfied:

$$\|r\|^2 = \|n\|^2 \pm a$$

otherwise return to step 2 after

increasing γ if $\|r\|^2 < \|n\|^2 - a$

or decreasing γ if $\|r\|^2 > \|n\|^2 + a$

and use the new value of γ to compute $\phi(\gamma)$

IMAGE RESTORATION

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- Constrained least square filtering

$$\boldsymbol{R}(u,v) = \boldsymbol{G}(u,v) - \boldsymbol{H}(u,v)\hat{\boldsymbol{F}}(u,v)$$

$$\|r\|^2 = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} r^2(x,y)$$

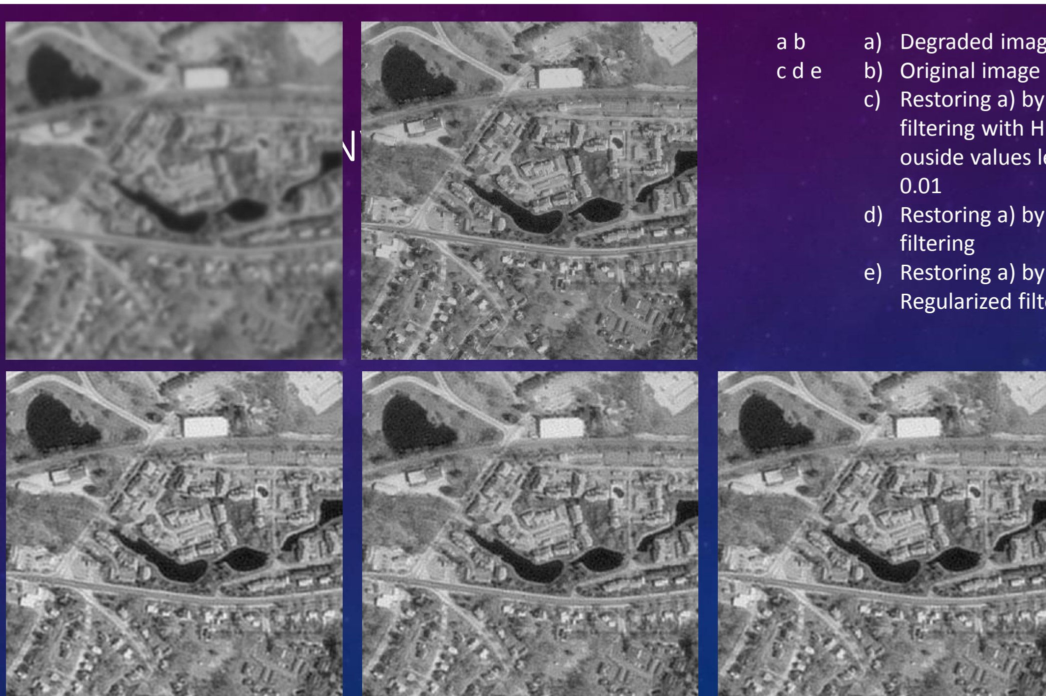
variance of the noise $\sigma_n^2 = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}[n(x,y) - m_n]^2$ $\qquad m_n = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} n(x,y)$

$$\|n\|^2 = MN(\sigma_n^2 + m_n^2)$$

(1,1) image corrupted by motion blur and additive noise. (1,2) IF. (1,3) WF. (1,4) CLSF.

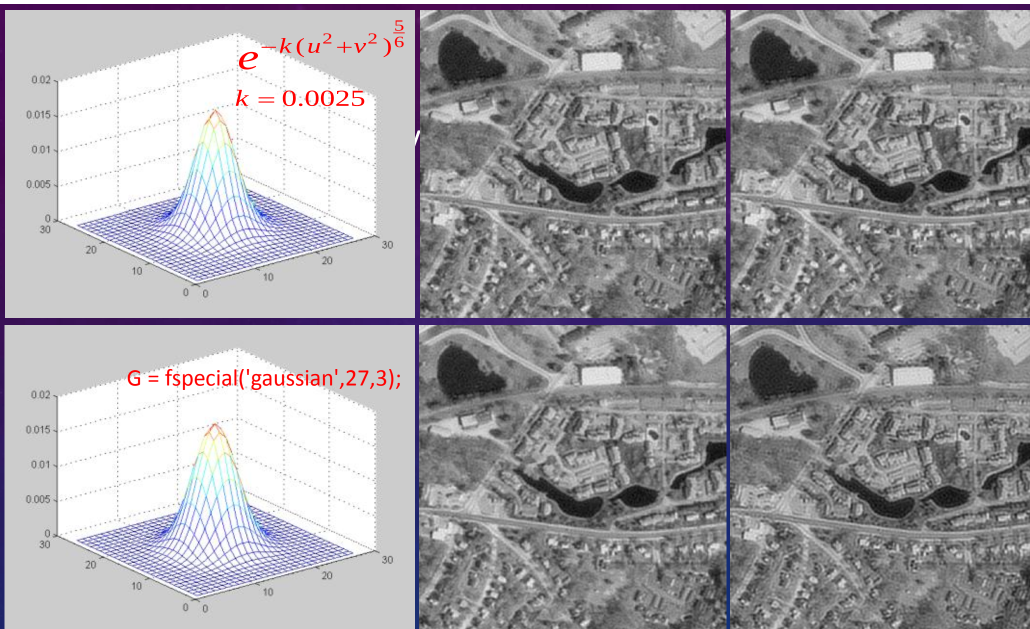
(2,1:4) same sequences, but with noise variance one order of magnitude less.

(3,1:4) same sequences, but noise variance reduced by five orders of magnitude from (1,1).



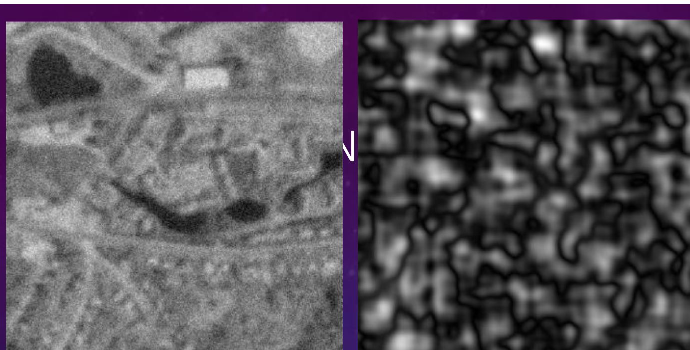
a b
c d e

a) Degraded image
b) Original image
c) Restoring a) by inverse filtering with H cut off ouside values less than 0.01
d) Restoring a) by Wiener filtering
e) Restoring a) by Regularized filtering



a b c b) Restoration of Wiener filtering with psf from a). c) Restoration of regularized filtering with psf from a)

d e f e) Restoration of Wiener filtering with psf from d). f) Restoration of regularized filtering with psf from d)



a b
c d e

a) Degraded image
b) Restoring a) by inverse filtering using full filter
c) Restoring a) by inverse filtering with H cut off outside values less than 0.3
d) Restoring a) by Wiener filtering
e) Restoring a) by Regularized filtering