UESTC3001 Dynamics & Control
Lecture 2

Basics of Control System Analysis

Dr Kelum Gamage kelum.gamage@glasgow.ac.uk

Associate Professor (Senior Lecturer)
School of Engineering, University of Glasgow, UK

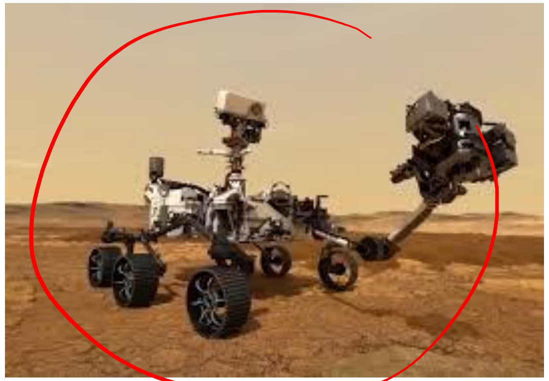
Outline

- Introduction to Control Systems
- Block Diagram Representation

Introduction to Control System

Desired response

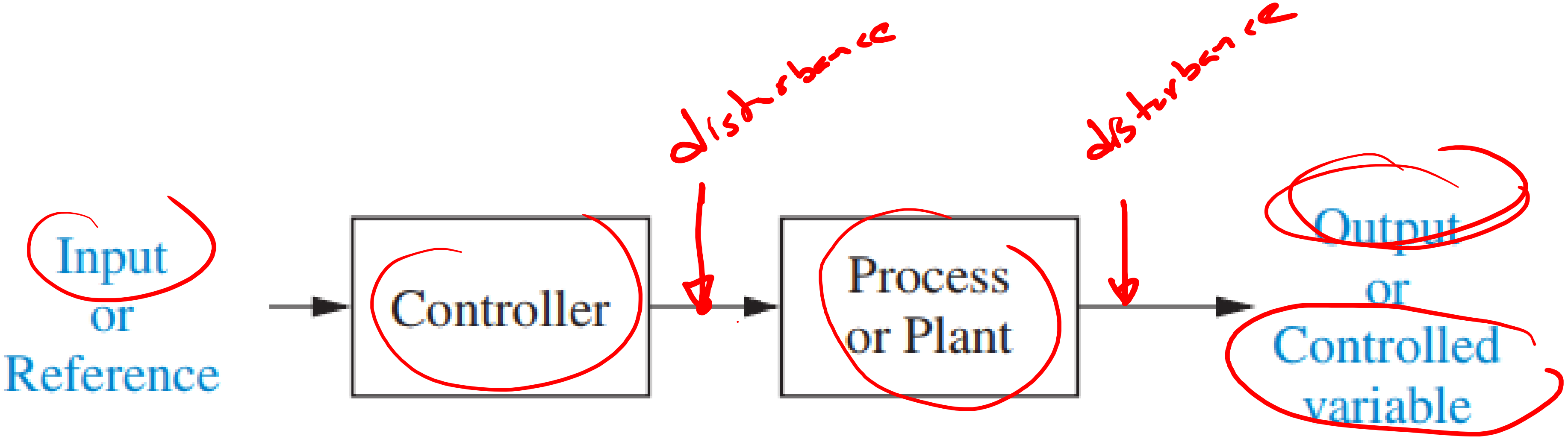
actual response



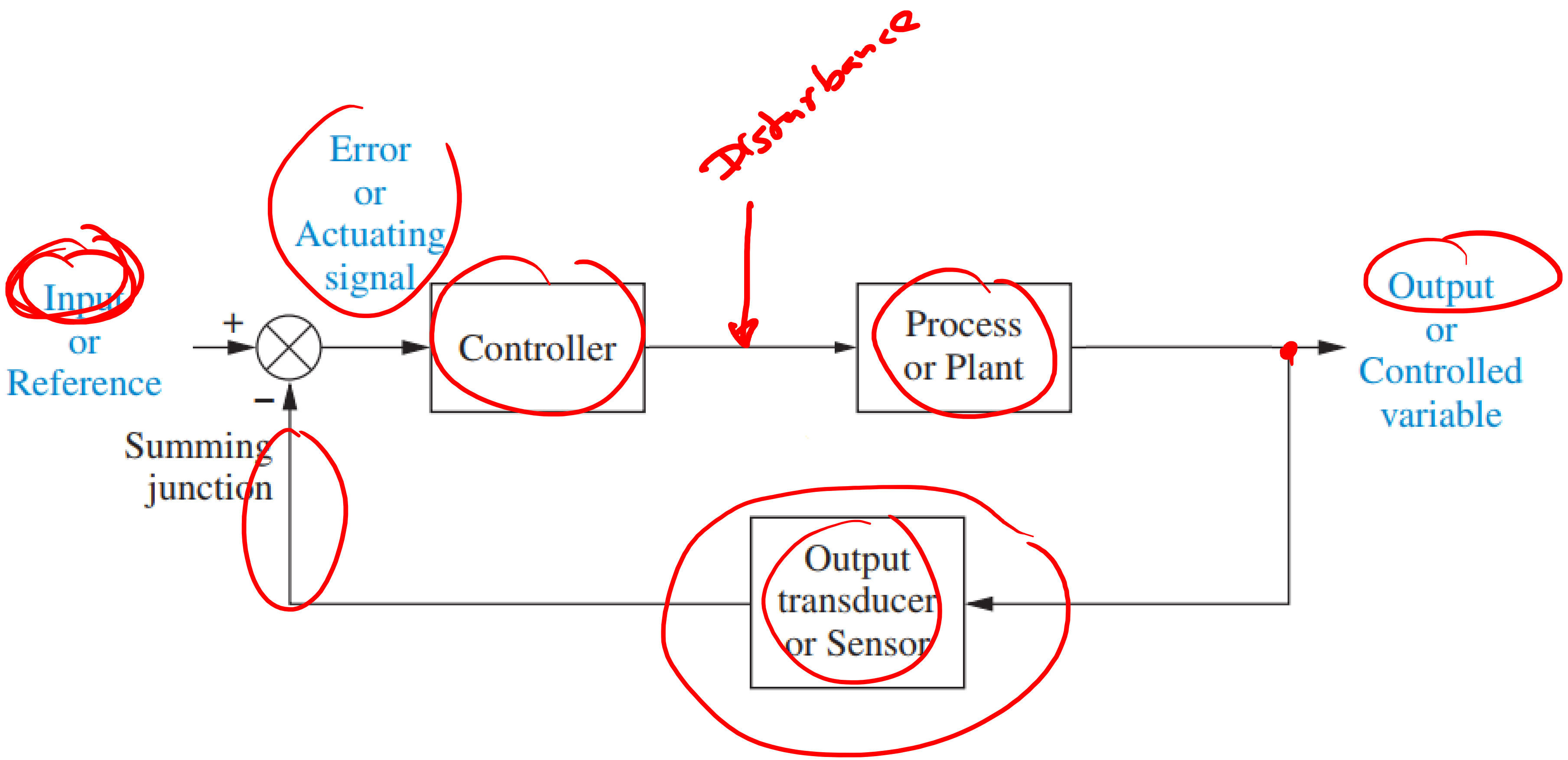
Advantages of Control Systems

- Power amplification
- Remote control
- Convenience of input form
- Compensation for disturbances

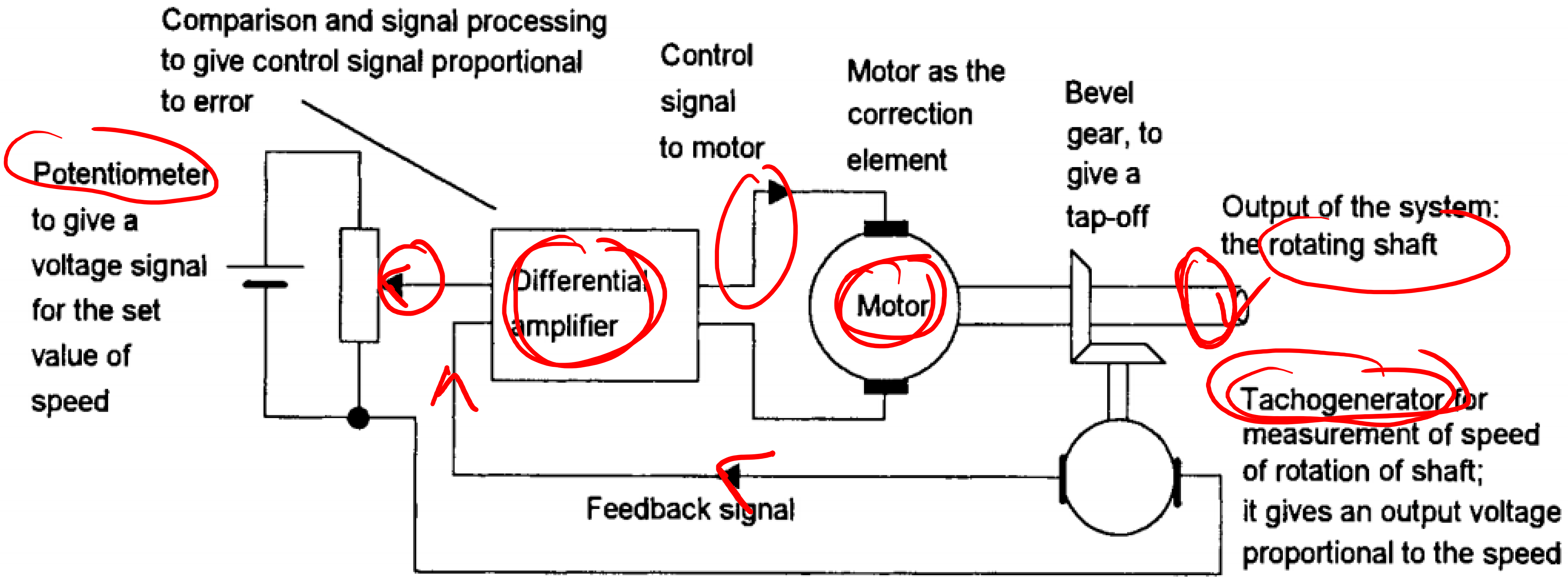
Open-loop Control

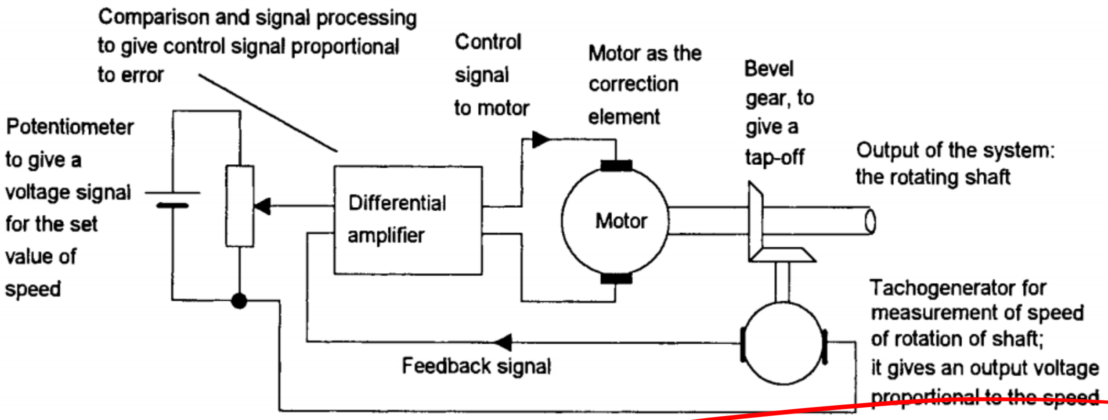


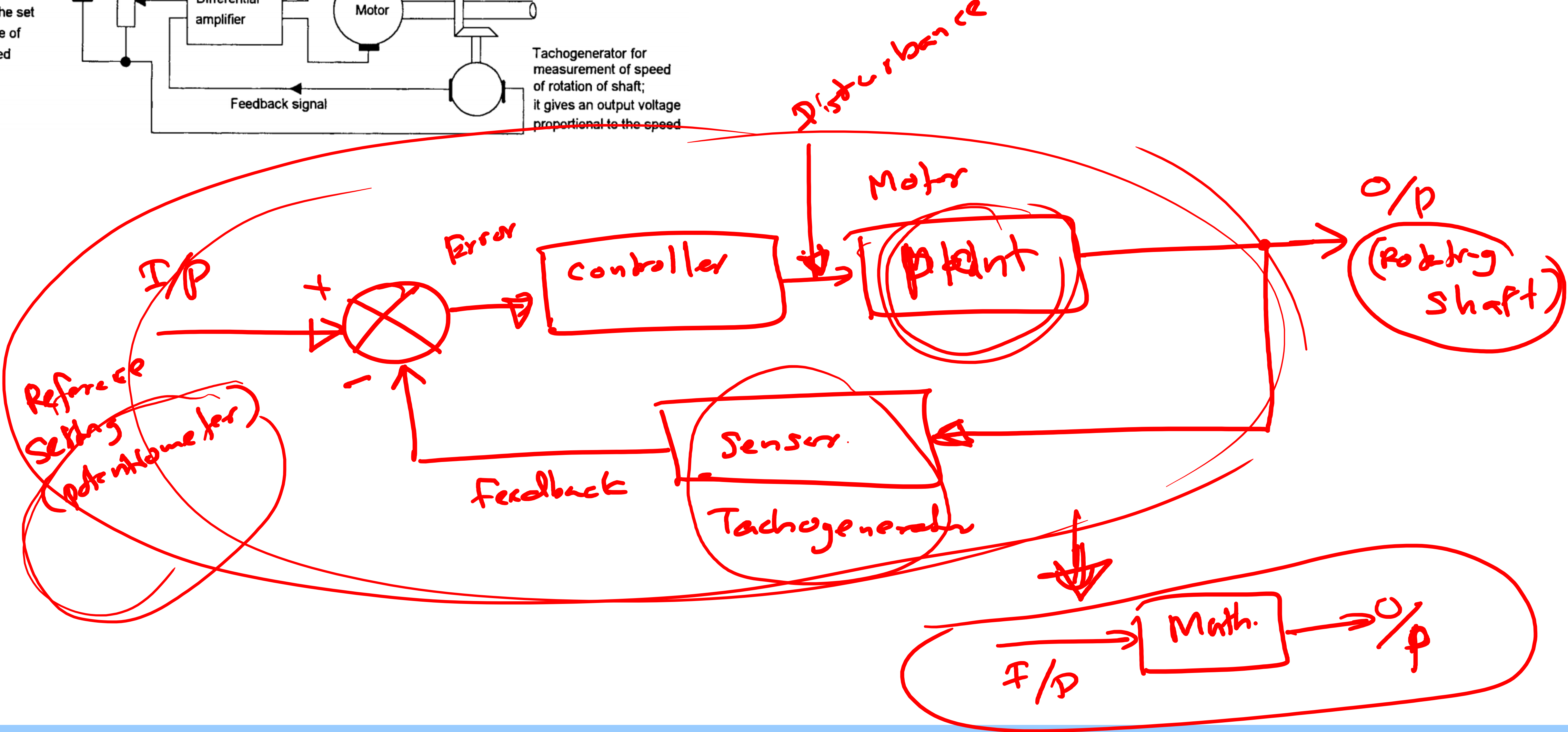
Closed-loop Control



Example
Control of the speed of rotation of a motor shaft




Comparison and signal processing to give control signal proportional to error
Control signal to motor
Motor as the correction element
Bevel gear, to give a tap-off
Output of the system: the rotating shaft
Potentiometer to give a voltage signal for the set value of speed
Differential amplifier
Motor
Tachogenerator for measurement of speed of rotation of shaft; it gives an output voltage proportional to the speed
Feedback signal


Disturbance
Motor
O/P
I/P
Error
+
-
controller
plant
(Rotating shaft)
Reference setting (potentiometer)
Sensor
Feedback
Tachogenerator
I/P
Math.
O/P

The Design Process

Step 1: Determine a physical system and specifications from the requirements.

→ **Step 2:** Draw a functional block diagram.

→ **Step 3:** Transform the physical system into a schematic.

→ **Step 4:** Use the schematic to obtain a block diagram, signal-flow diagram, or state-space representation.

→ **Step 5:** If multiple blocks, reduce the block diagram to a single block or closed-loop system.

→ **Step 6:** Analyze, design, and test to see that requirements and specifications are met.

→ ?

I/P → Mathematical Description → O/P

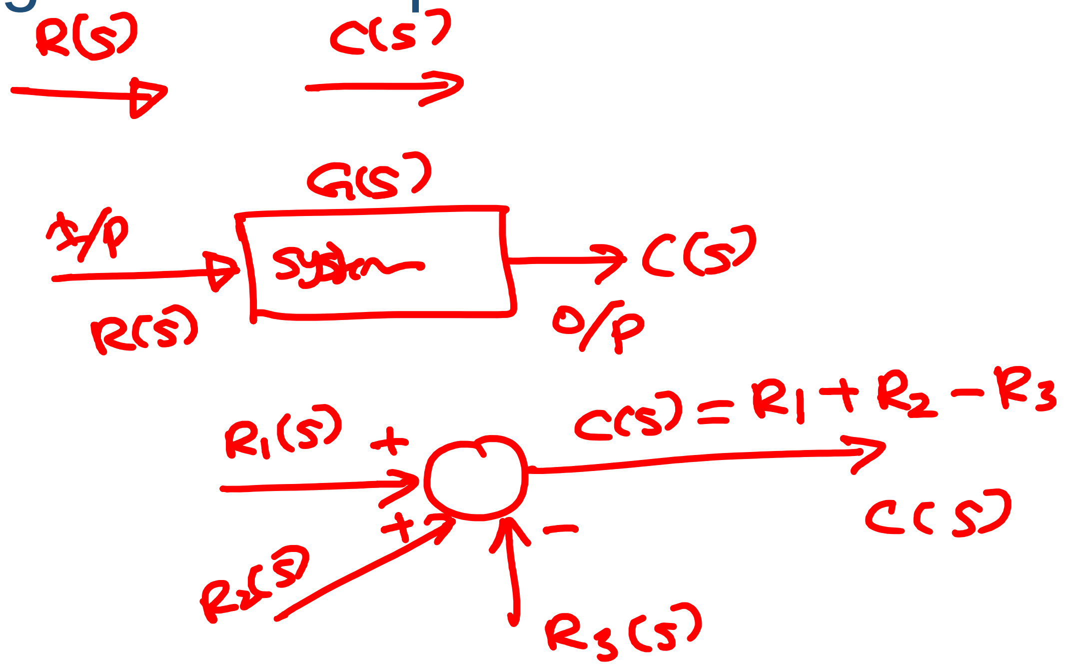
Introduction to Block Diagrams

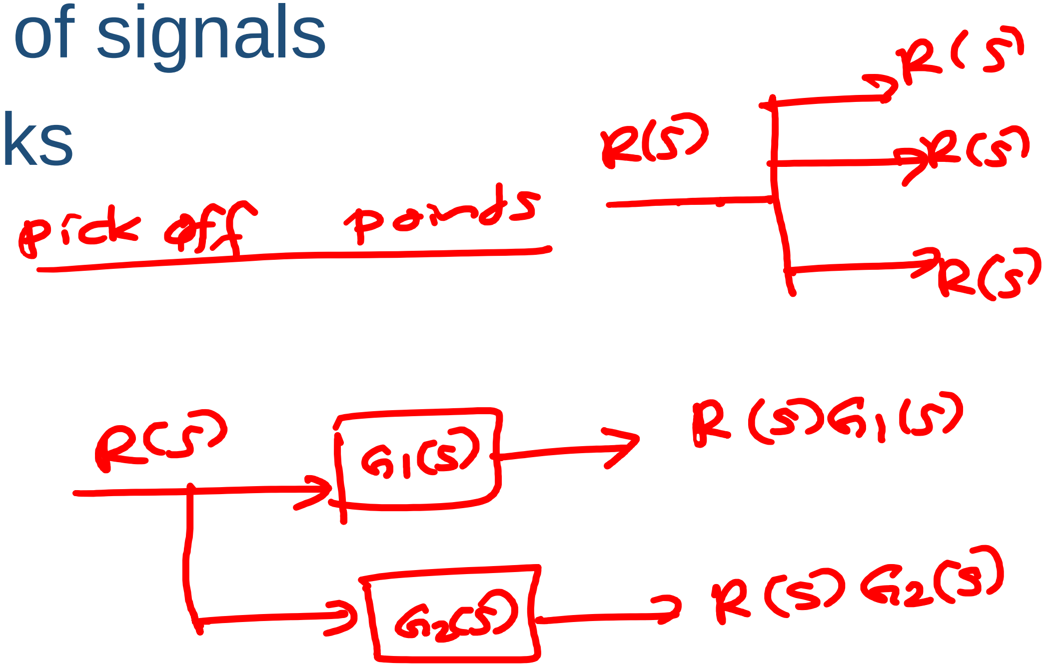
- Use for frequency-domain analysis and design
- Graphical representation of the interconnections between the components of the system and the flow of signals
- Diagram is composed of functional blocks

Signal

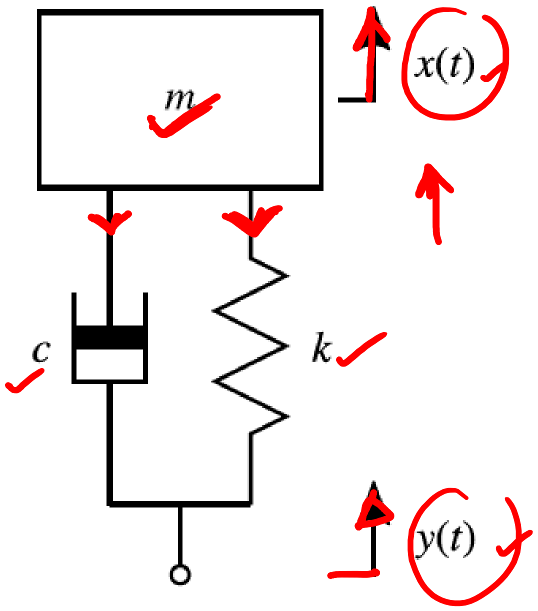
System

Summing junction





Example:
Block Diagram Representation of Control Systems



m ↑ $F = m\ddot{x}$

$$-c(\dot{x}-\dot{y}) - k(x-y) = m\ddot{x}$$

$$c\dot{y} + ky = m\ddot{x} + c\dot{x} + kx$$

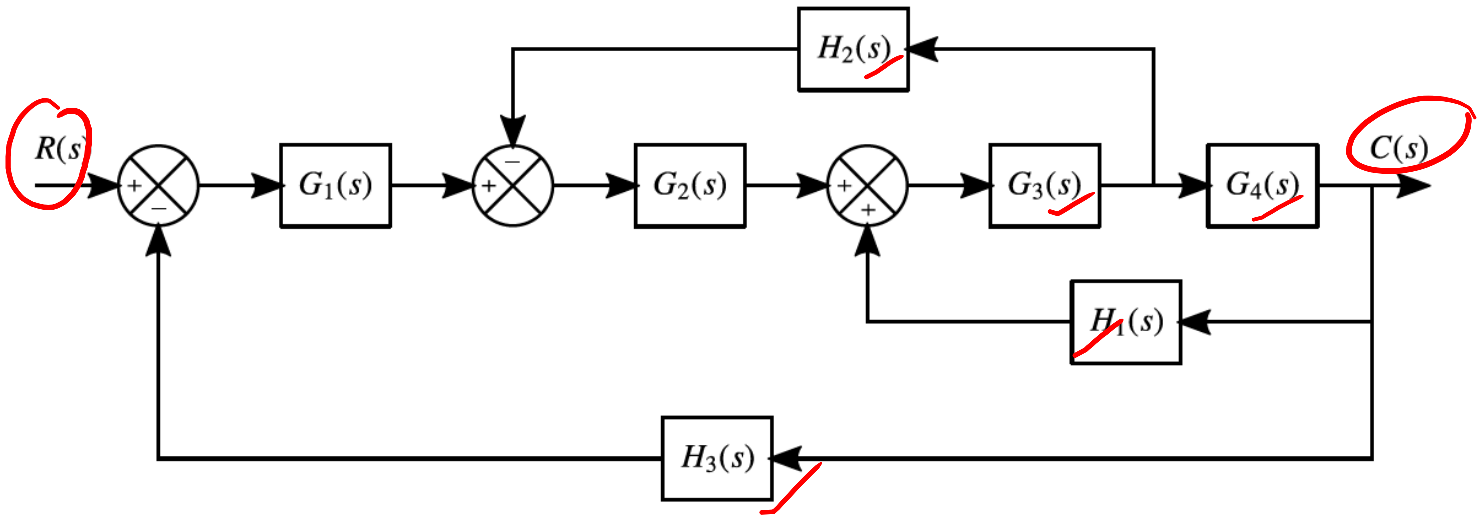
$$\mathcal{L} \Rightarrow \quad csY(s) + kY(s) = ms^2X(s) + csX(s) + kX(s)$$

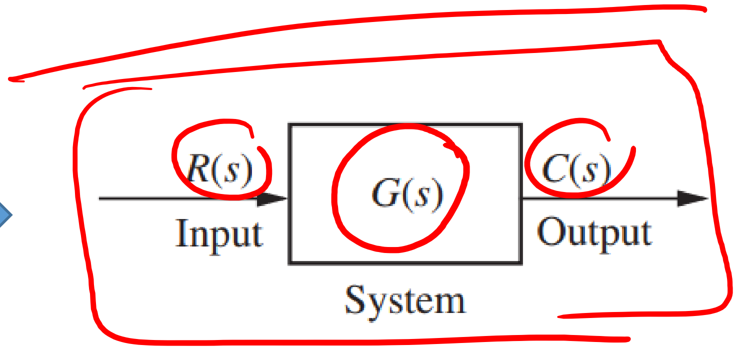
$$Y(s)\{cs+k\} = X(s)\{ms^2+cs+k\}$$

$$\frac{X(s)}{Y(s)} = \frac{cs+k}{ms^2+cs+k}$$

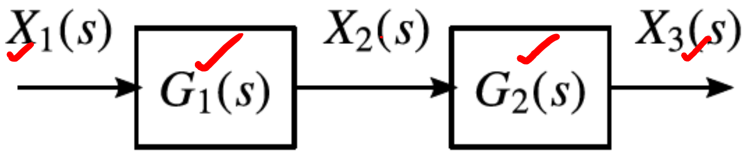
$$Y(s) \longrightarrow \boxed{\frac{cs+k}{ms^2+cs+k}} \longrightarrow X(s)$$

Block Diagram Reduction





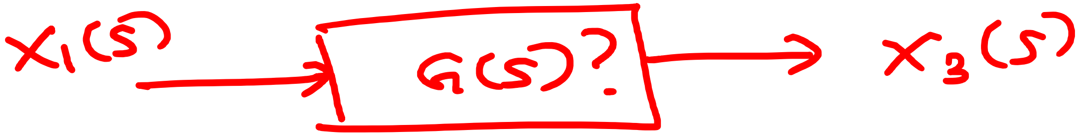
1: Cascaded Blocks

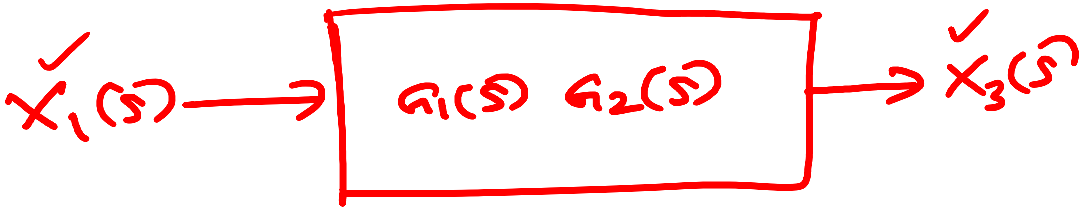


$X_2(s) = X_1(s)\, G_1(s)$

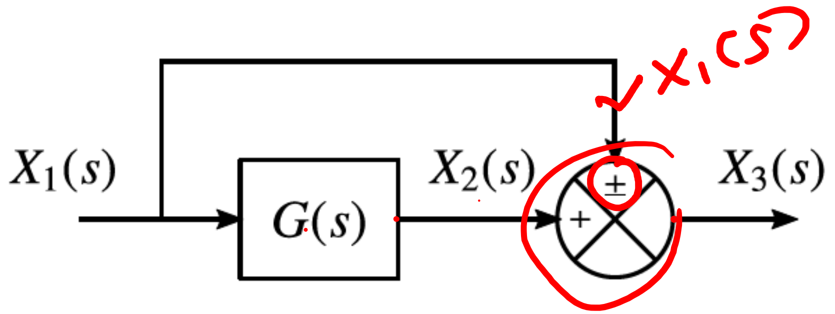
$X_3(s) = X_2(s)\, G_2(s)$

$X_3(s) = X_1(s)\, G_1(s)\, G_2(s)$





2: Summing Two Signals

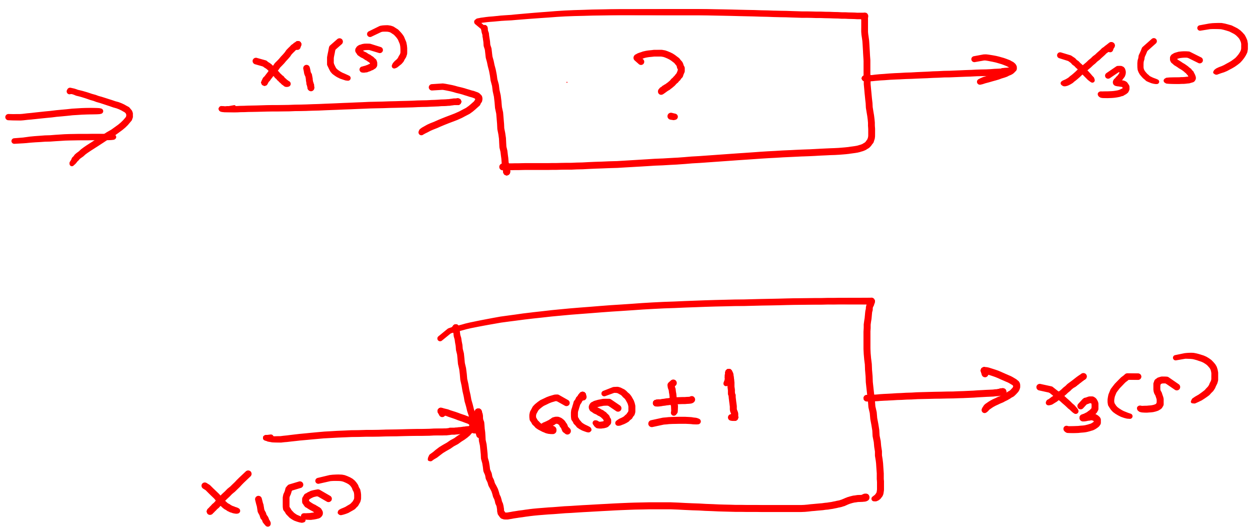


$$X_2(s) = X_1(s)G(s)$$

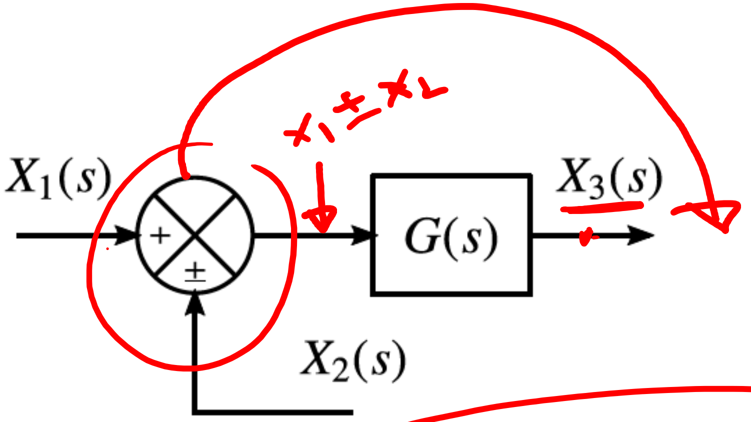
$$X_3(s) = \pm X_1(s) \oplus X_2(s)$$

$$X_3(s) = \pm X_1(s) + X_1(s)G(s)$$

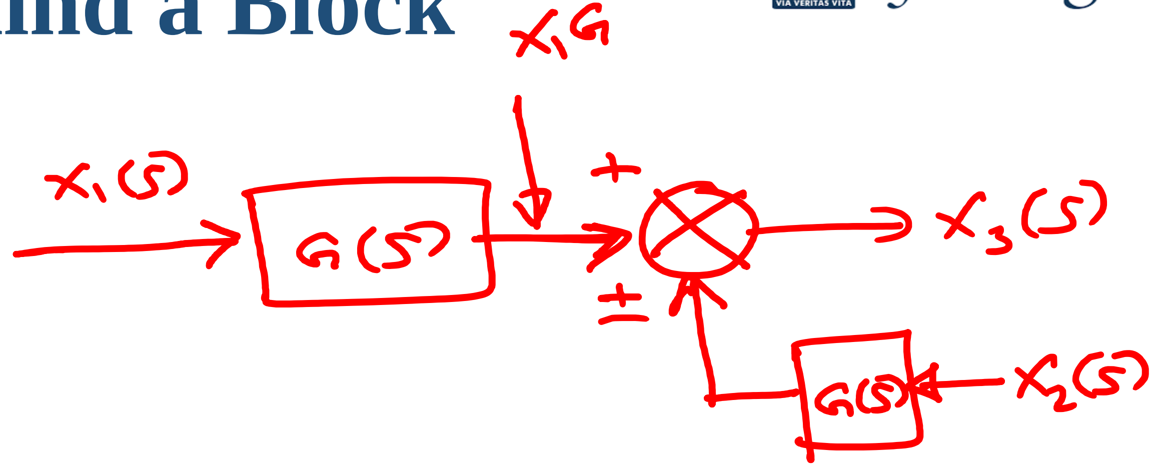
$$X_3(s) = X_1(s)\{G(s) \pm 1\}$$



3: Moving a Summing Point Behind a Block



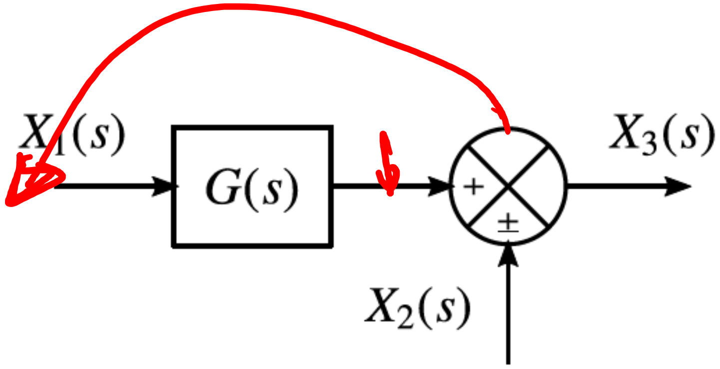
$\Rightarrow$

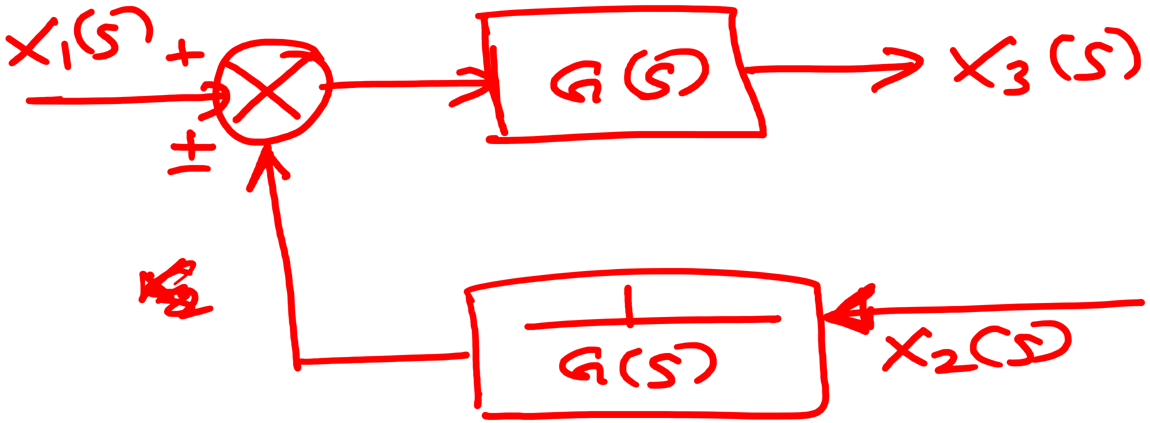


$$x_3(s) = (x_1 \pm x_2(s))\,G(s)$$

$$x_3(s) = x_1(s)\,G(s) \pm x_2(s)\,G(s)$$

4: Moving a Summing Point Ahead of a Block

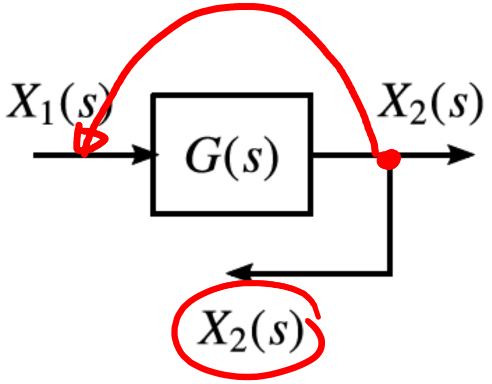




$$X_3(s) = X_1(s)G(s) \pm X_2(s)$$

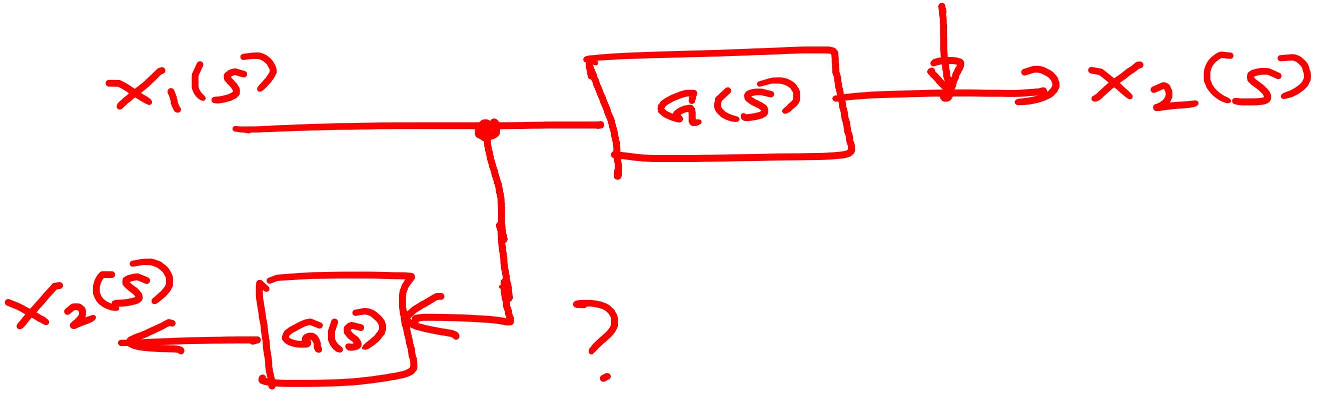
$$X_3(s) = X_1(s)G(s) \pm \left(\frac{X_2(s)}{G(s)}\right)G(s)$$

5: Moving a Branch Point Ahead of a Block

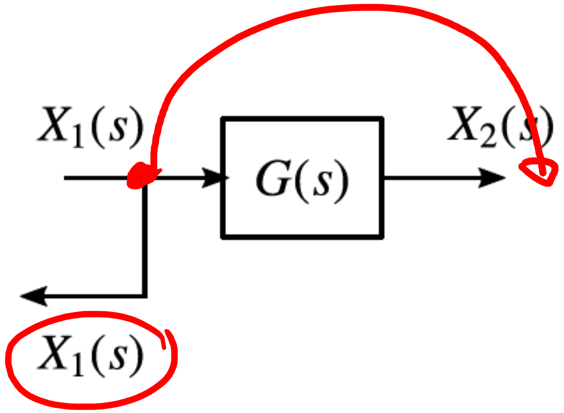


$X_2(s) = X_1(s)\,G(s)$

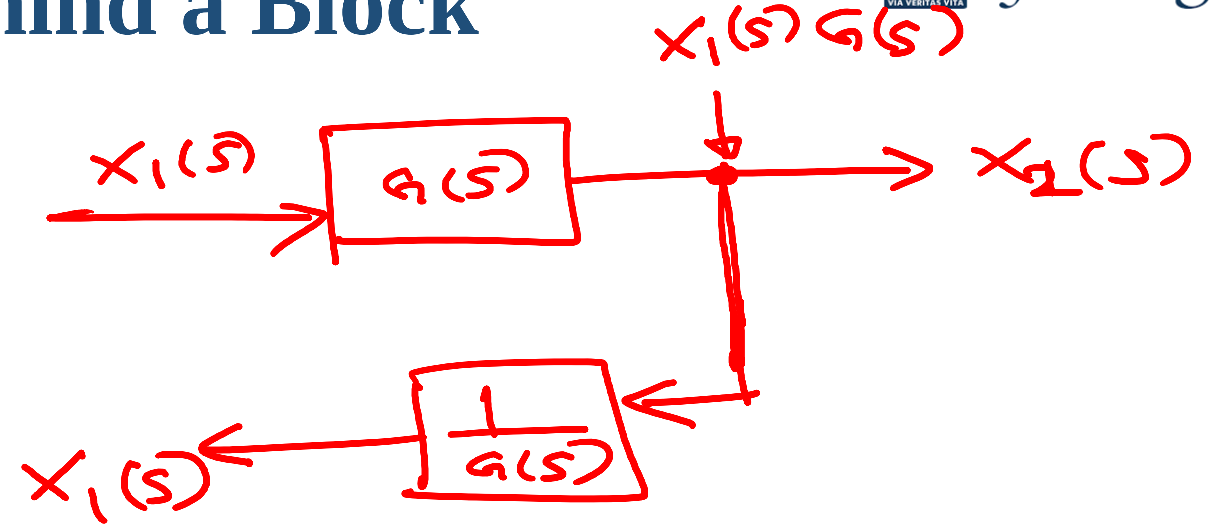
$X_2(s) = X_1(s)\,G(s)$



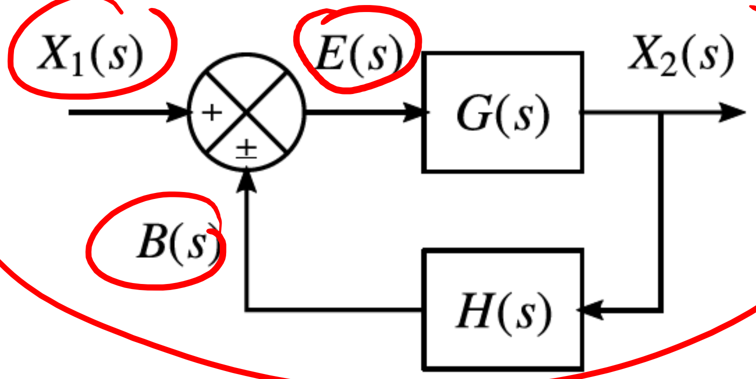
6: Moving a Branch Point Behind a Block



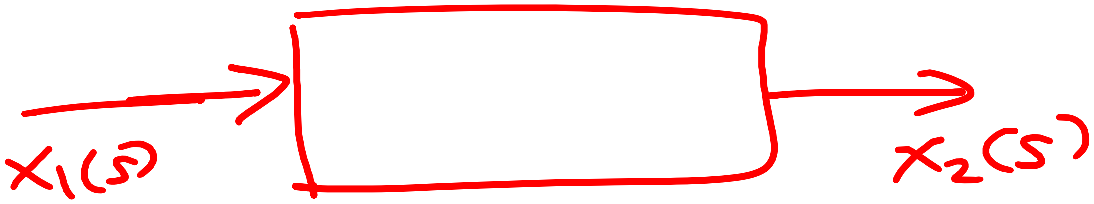
$X_2(s) = X_1(s)G(s)$



7: Eliminating a Feedback Loop



$\Rightarrow$



$$E(s) = X_1(s) \pm B(s)$$

$$X_2(s) = E(s)G(s) = [X_1(s) \pm B(s)]\,G(s)$$

$$B(s) = X_2(s)H(s)$$

$$X_2(s) = [X_1(s) \pm X_2(s)H(s)]\,G(s)$$

$$X_2(s)[1 \pm G(s)H(s)] = X_1(s)G(s)$$

$$\boxed{\frac{X_2(s)}{X_1(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

Summary

- Overview to Control Systems
- Block Diagram Representation of Control Systems
- Block Diagram Reduction
- Block Diagram Reduction Rules

Reference:

-Control Systems Engineering, 7th Edition, N.S. Nise

-UESTC3001 2019/20 Notes, J. Le Kernec