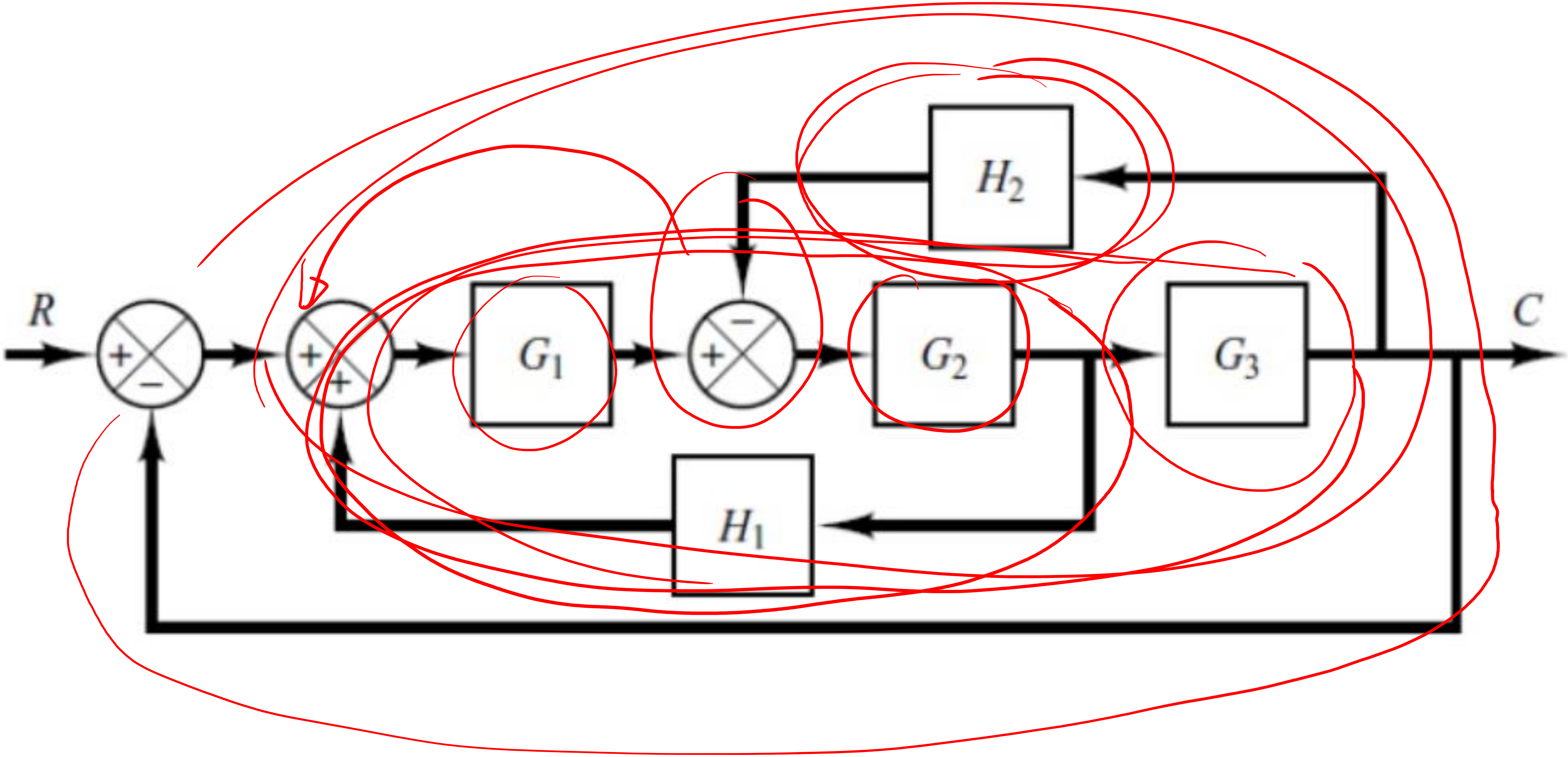
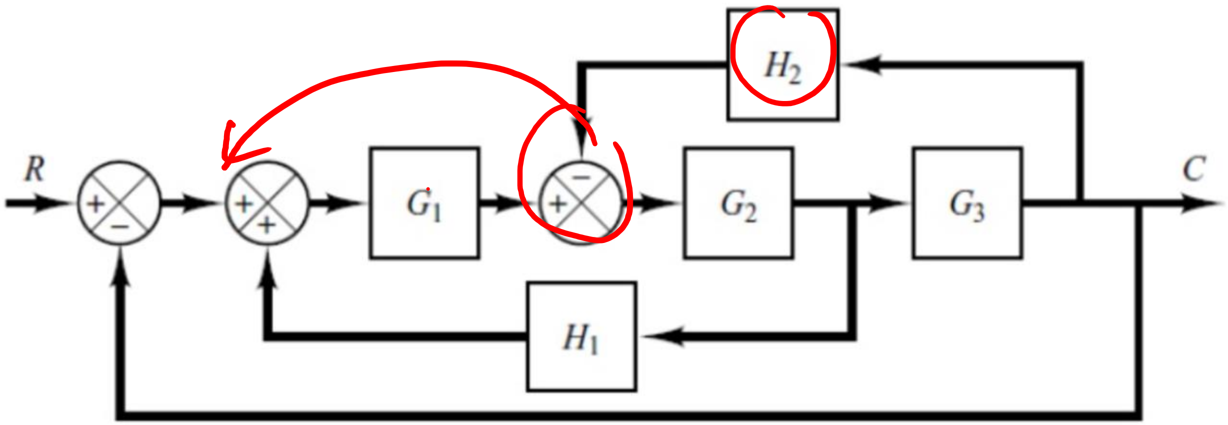
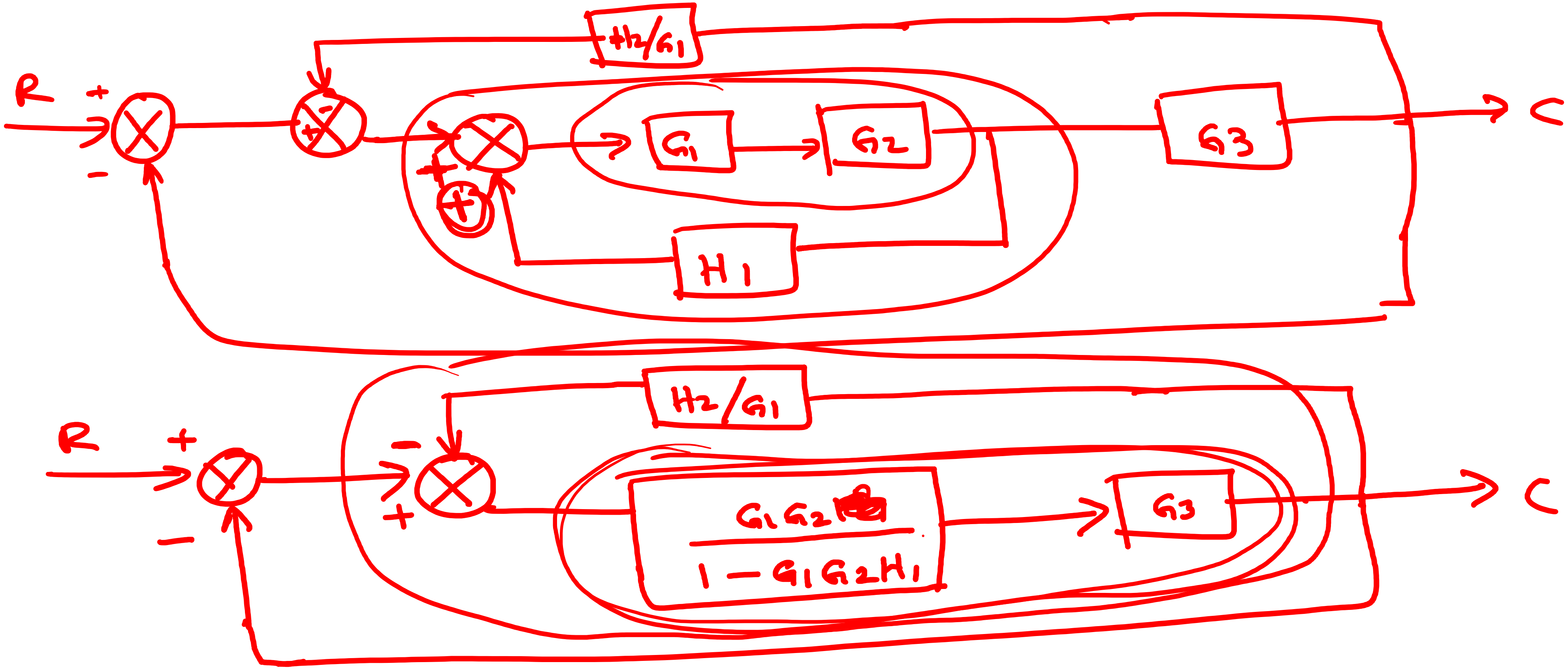
Q1 (a) Using block diagram reduction techniques, reduce the block diagram shown in figure to a single block. [18 marks]

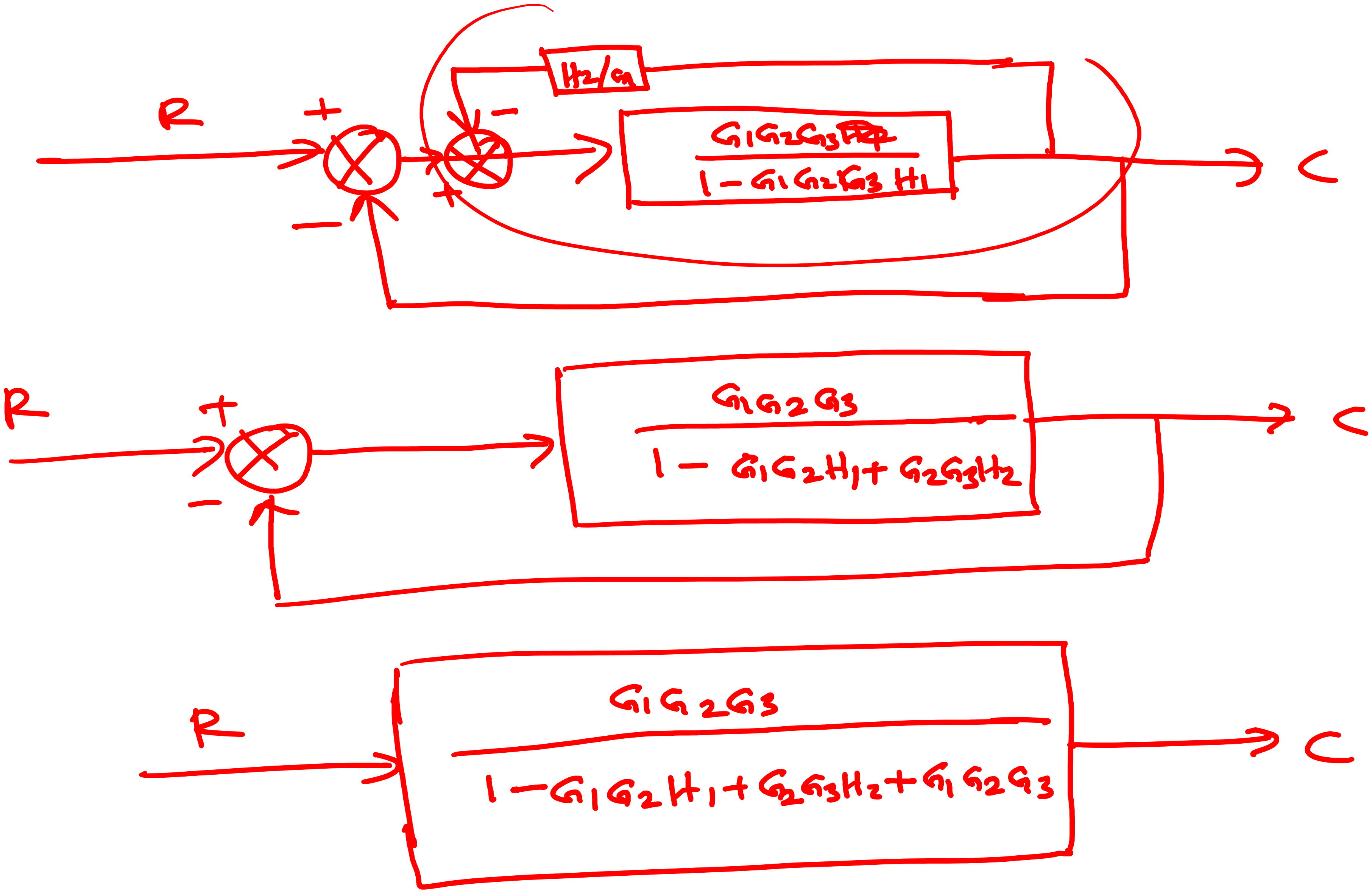




$$\frac{G_1 G_2}{1 + G_1 G_2}$$



$$\frac{G_1 G_2 G_3}{1 - G_1 G_2 H_1}$$


R
C
H2/G1
G1G2G3
1−G1G2G3 H1
R
C
G1G2G3
1 − G1G2H1+ G2G3H2
R
C
G1G2G3
1−G1G2H1+G2G3H2+G1G2G3

(b) Use Routh's Criterion to determine whether the system stable stable: $s^4 + 2s^3 + 3s^2 + 4s + 5 = 0$ [7 marks]

s^4 :	1	3	5
s^3 :	2	4	0
s^2 :	1	5	
s^1 :	-6		
s^0 :	5		

+ − +

$$-\frac{\begin{vmatrix}1 & 3\\ 2 & 4\end{vmatrix}}{2}$$

$$-\frac{\begin{vmatrix}1 & 5\\ 2 & 0\end{vmatrix}}{2} = 5$$

$$-\frac{\begin{vmatrix}2 & 4\\ 1 & 5\end{vmatrix}}{1} = -6$$

$$-\frac{\begin{vmatrix}1 & 5\\ -6 & 0\end{vmatrix}}{-6} = 5$$

$\Rightarrow$ system is unstable

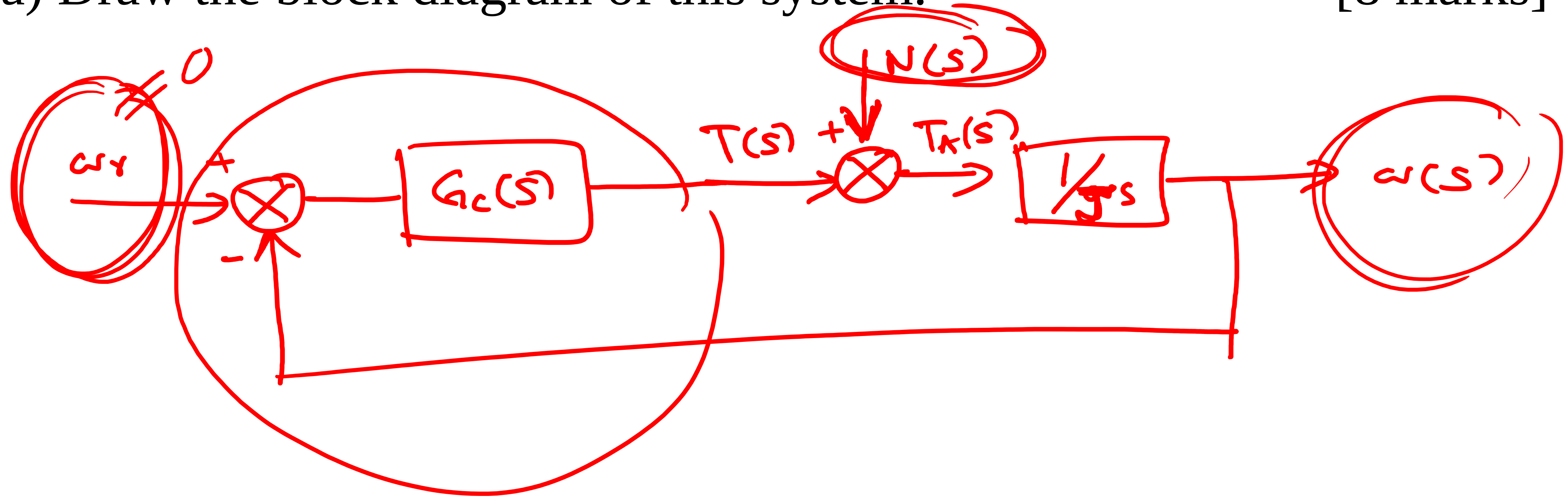
$\Rightarrow$ (2) sign changes $\Rightarrow$ 2 roots with (+)ve real parts.

Q2 A cooling fan's angular speed ω can be related to the applied torque $T_A(t)$ by the relationship:

$$* \quad \boxed{J\dot{\omega} = T_A(t)} \Longrightarrow \boxed{J s\,\omega(s) = T_A(s) \qquad \omega(s) = \frac{T_A(s)}{Js}}$$

where J is the moment of inertia of the fan. The applied torque TA(t) is the difference between the driving torque T(t), supplied by a motor, and a disturbance torque N(t), due to changes in the local airstream. It is desirable to keep the fan running at a constant speed ωr and so a feedback control system is used to counter the disturbances N(t). The controller has transfer function Gc(s) which operates on the error between the actual fan speed ω and the required speed ωr. The controller (via an electric servo motor) supplies the driving torque T(t).

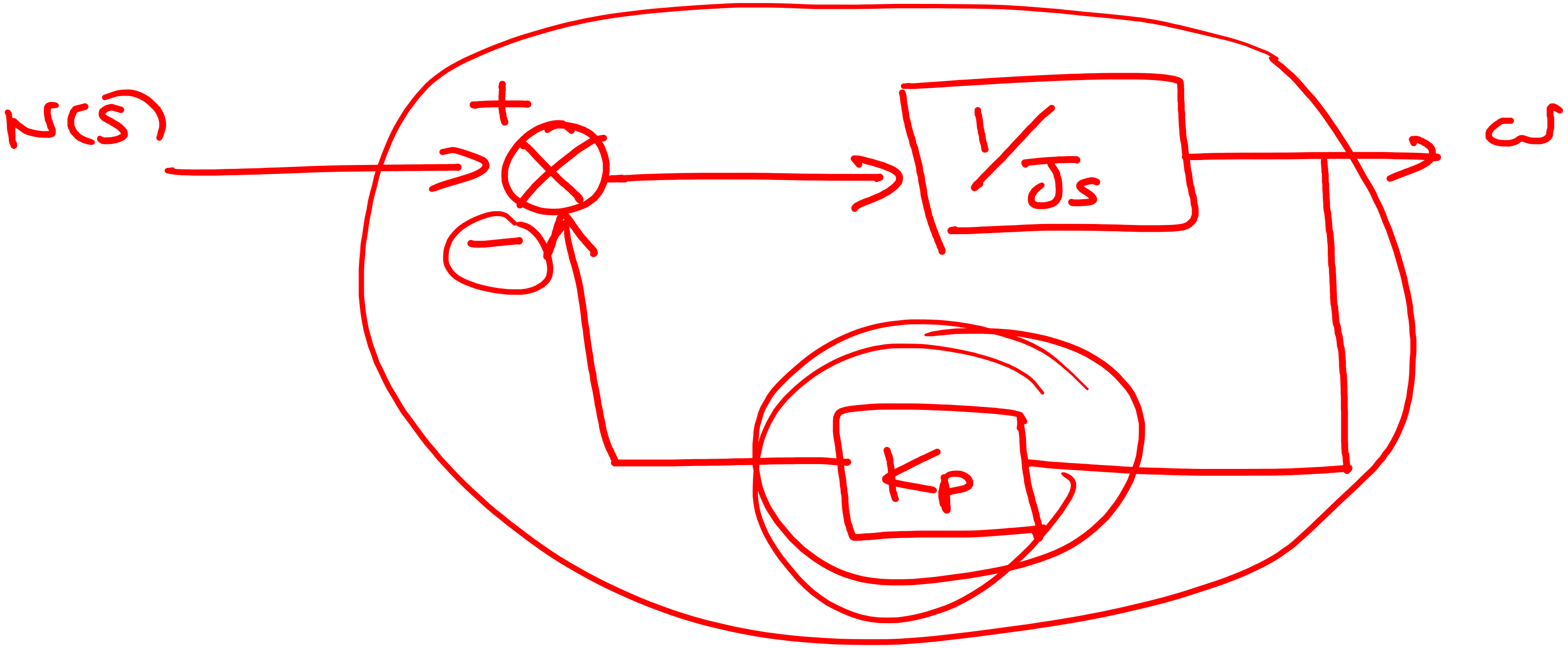
(a) Draw the block diagram of this system. [8 marks]



(b) If proportional control with gain K_p is used, derive an expression for the steady state error due to a unit step in the disturbance. [8 marks]

$\omega_r = 0$

$N(s) = 1/s$



$$\frac{G_1 G_2}{1 + G_1 G_2}$$

$$\frac{\omega(s)}{N(s)} = \frac{1/\cancel{Js}}{1 + (1/\cancel{Js}) \times K_p} = \frac{1}{Js + K_p}$$

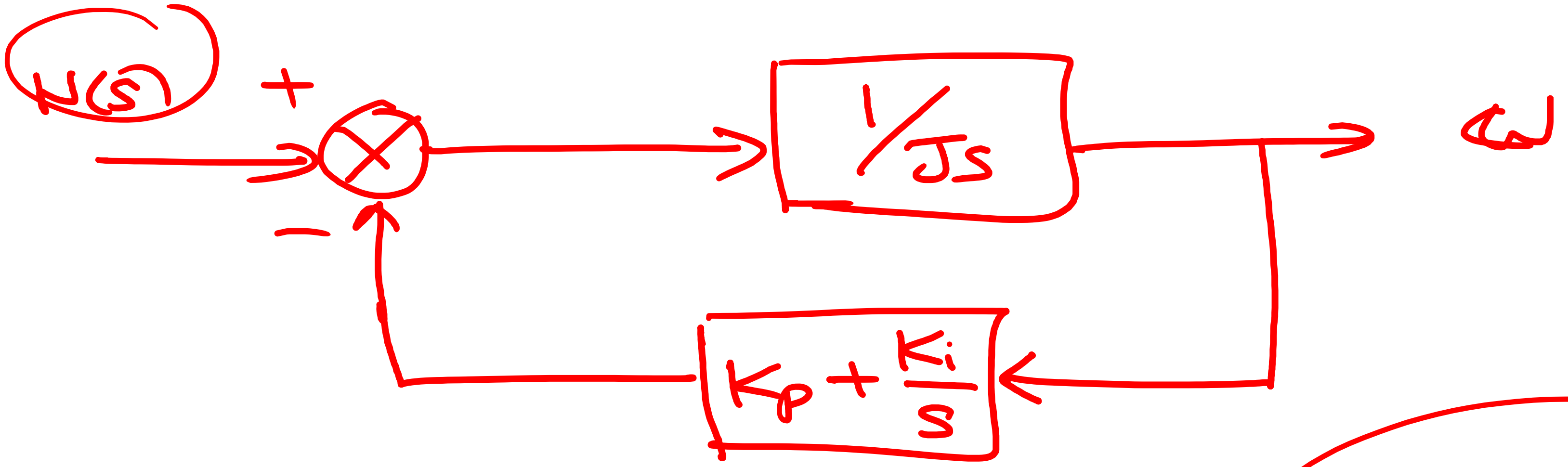
$$N(s) = \frac{1}{s} \Longrightarrow \omega(s) = \left(\frac{1}{s} \times \left(\frac{1}{Js + K_p}\right)\right)$$

$$\omega_{ss} = \lim_{s \to 0} s\,\omega(s) = \lim_{s \to 0} \cancel{s}\left(\frac{1}{\cancel{s}} \times \left(\frac{1}{Js + K_p}\right)\right)$$

$$0$$

$$= \frac{1}{K_p}$$

(c) Show that is integral action of gain Ki is added to the proportional action then the steady state error can be removed. [9 marks]



$$\frac{\omega(s)}{N(s)} = \frac{1/Js}{1+(1/Js)(K_p + K_i/s)} = \left(\frac{s}{Js^2 + K_p s + K_i}\right)$$

$N(s) = \frac{1}{s}$

$$\omega(s) = \left(\frac{1}{s}\right) \times \underbrace{\left(\qquad \right)}$$

$$\omega_{ss} = \lim_{s \to 0} s\,\omega(s) = \lim_{s \to 0} \cancel{s} \times \frac{1}{\cancel{s}} \times \frac{\overset{\to 0}{s}}{\underbrace{Js^2 + K_p s}_{\to 0} + K_i}$$

$$= \frac{0}{K_i} = 0$$