Transformer Equivalent Circuit

Practical Transformer

As we saw previously a Transformer is not 100% efficient, losses and flux leakages need to be considered. We will use these losses to produce an equivalent circuit of a practical transformer.

Winding Resistance.

The copper wires which make up the Primary and Secondary windings have an associated resistance. We can represent this on an equivalent circuit with the resistances R_P and R_S.

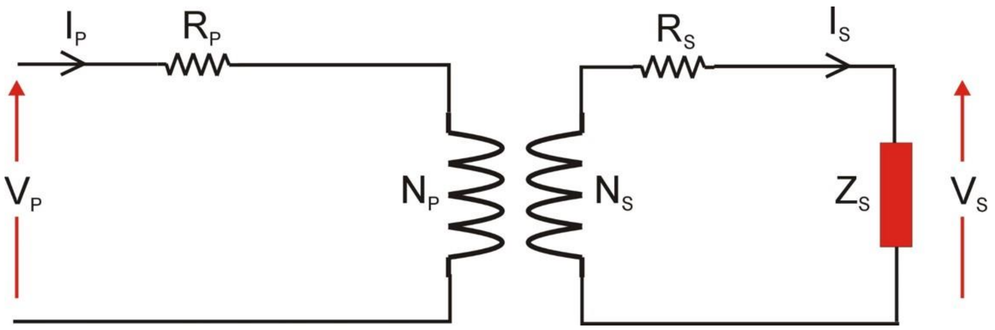

Figure 1. Representation of winding resistance in the equivalent circuit.

Magnetizing Current

In a practical transformer, made up of an Iron Core with a finite permeability μ_r, we need a current I_M to establish the Flux in the Core. Given that this current is associated with a Magnetic component the circuit element (X_M) is purely Inductive. The Magnetising current I_M flows even when the secondary winding is Open Circuit.

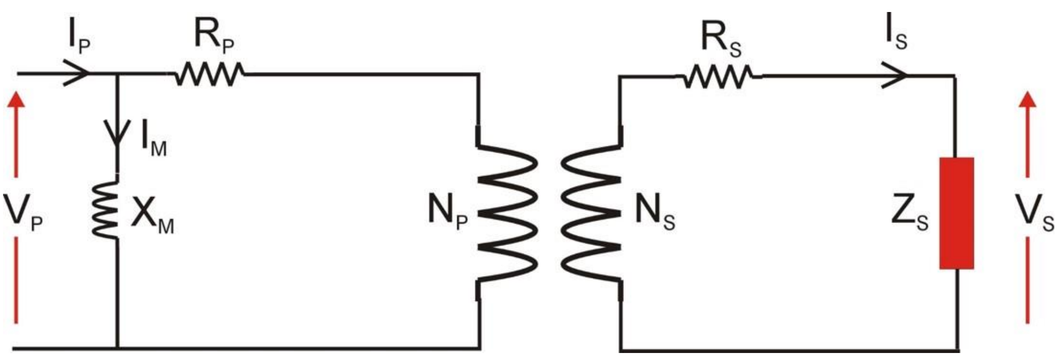

Figure 2. Inductive element X_M to represent magnetizing current in the equivalent circuit.

Leakage Inductance

In a practical transformer the flux is not totally confined to the iron core. This "leakage flux" is represented by inductive circuit elements (X_P>0 and X_S>0). They are in series with the winding resistances, as these elements are associated with load currents.

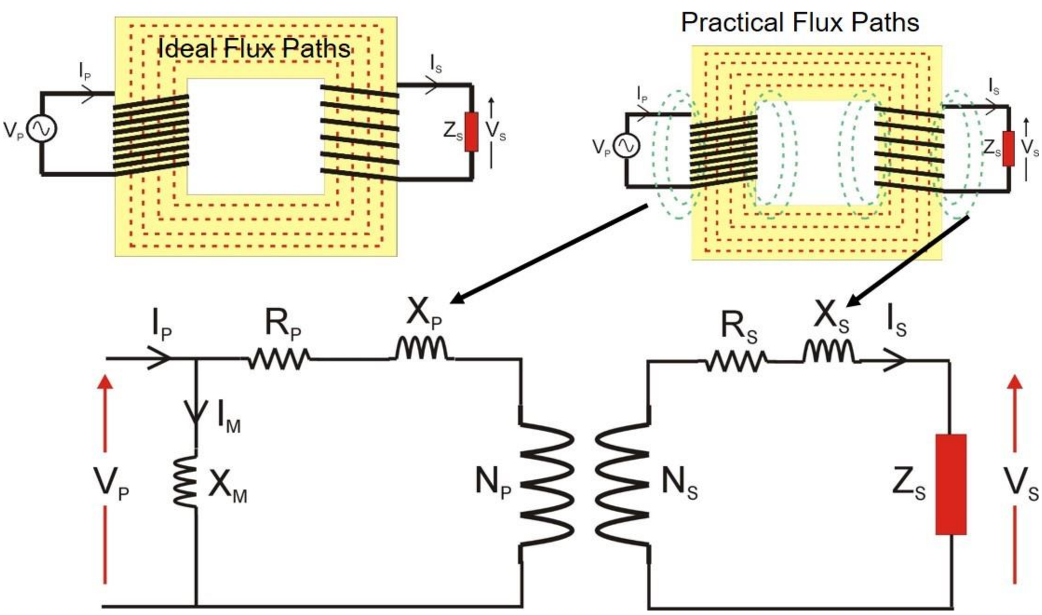

Figure 3. Representation of leakage flux in the equivalent circuit.

Iron Loss

In addition to the winding copper losses (I^2R), a transformer has an additional power loss associated with the Core – this is termed the IRON LOSS. The loss is associated with Eddy currents induced in the Iron core and with hysteresis loss. Since this is a REAL power loss (W) causing heat, and the fact that this loss is proportional to V_P^2, the component (R_C) representing this is resistive and in parallel with V_P.

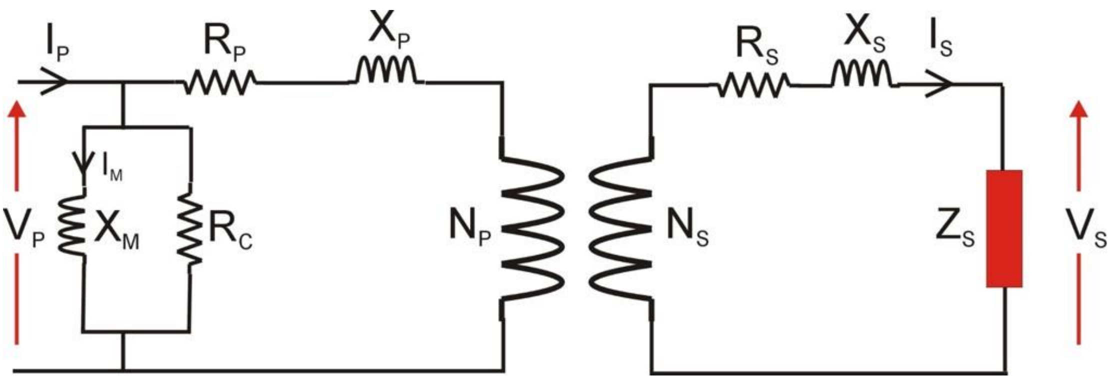

Figure 4. Inclusion of R_c in equivalent circuit to account for losses associated with the core.

Equivalent Circuit: Refer to Primary

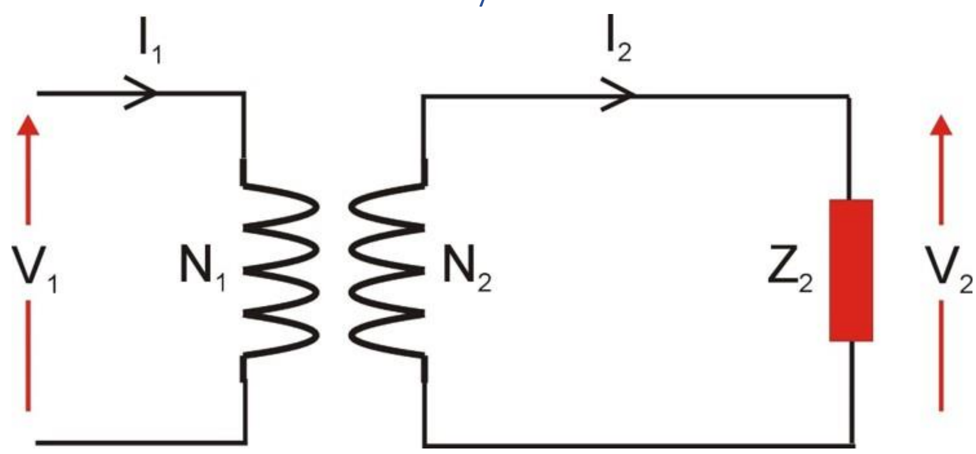

Figure 5. Load on secondary side of a prefect transformer

We now have an equivalent circuit with an ideal transformer. We will now rearrange the placement of the components and eliminate the Ideal Transformer element. This makes analysis a bit easier. Consider an impedance on the secondary side of the transformer (Figure 5):

$$Z_2 = \frac{V_2}{I_2} \tag{1}$$

From the ideal transformer equation:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \tag{2}$$

Substituting for V_2 and I_2:

$$Z_2 = \left(\frac{N_2}{N_1}\right)^2 \frac{V_1}{I_1} \tag{3}$$

So:

$$Z_1 = \frac{V_1}{I_1} = Z_2 \left(\frac{N_1}{N_2}\right)^2 \tag{4}$$

Using Equation (4) we can now refer all the components on the secondary side to the primary side as depicted in Figure 6. A final equivalent circuit referred to the primary is shown in Figure 7.

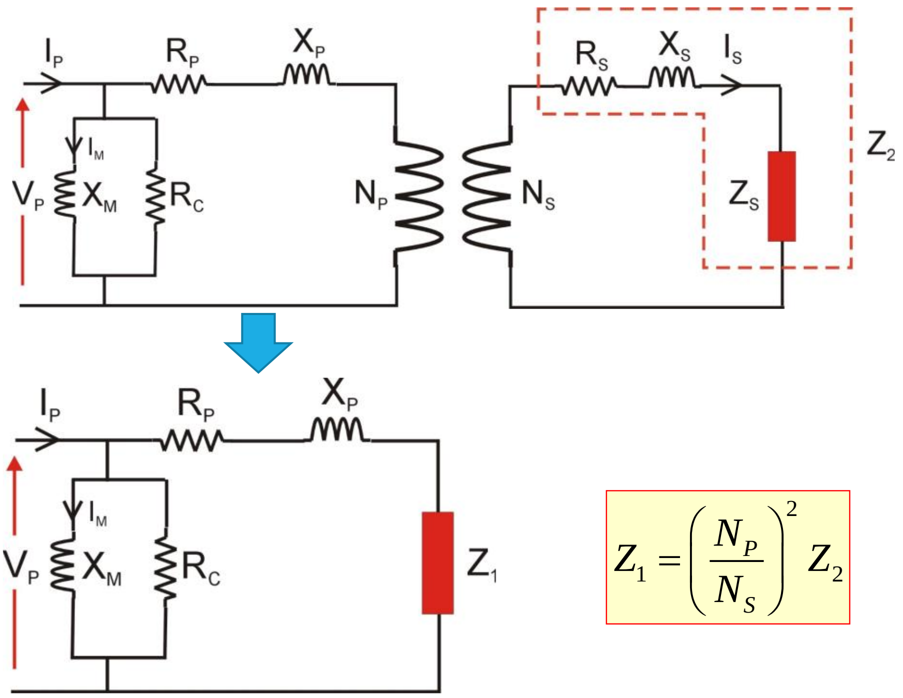

Figure 6. Refer to primary side of transformer

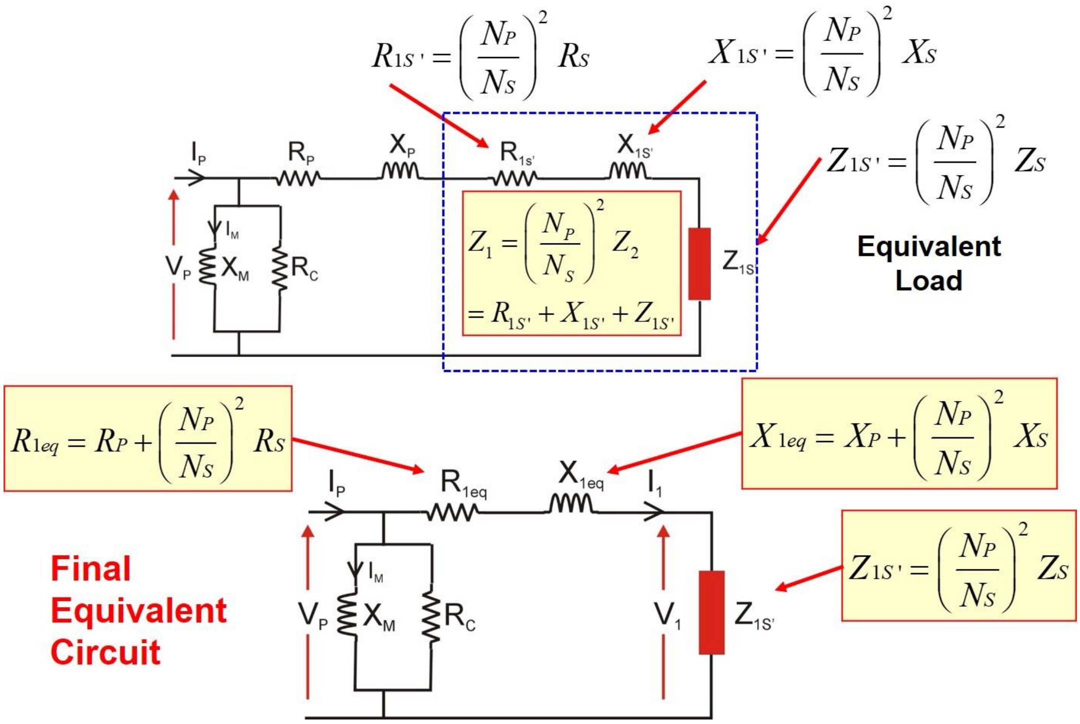

Figure 7. Final equivalent circuit referred to primary.

Open and Short Circuit Tests

Open and short circuit tests are performed on a transformer to determine the equivalent Circuit. The power required for open circuit tests and short circuit tests on a transformer is equal to the power loss occurring in the transformer and is therefore, relatively low.

Test 1: Open Circuit Test

In an open circuit test the secondary winding is left open circuit. This enables us to determine R_c and X_m and the turns ratio.

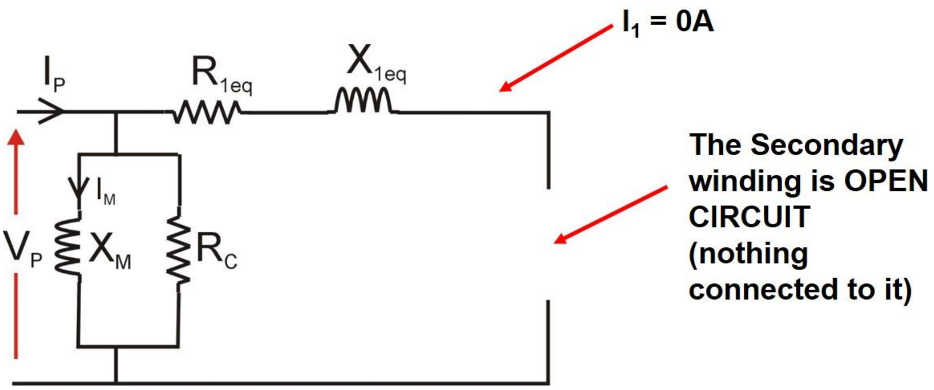

Figure 8. Secondary winding open circuit

In the open circuit test, Figure 9, we measure the primary voltage, V_p, the primary current, I_p, the primary real power, P_P, and the secondary open circuit voltage, V_{soc}. From these measurements we can find the core losses and the turns ratio from:

$$Turns\_Ratio = \frac{V_P}{V_{soc}} \tag{5}$$

$$R_C = \frac{V_P^2}{P_P} \tag{6}$$

$$X_M = \frac{V_P^2}{Q_P} \tag{7}$$

Where $Q_P = \sqrt{(S_p^2 - P_P^2)}$ and $S_P = V_P I_P$

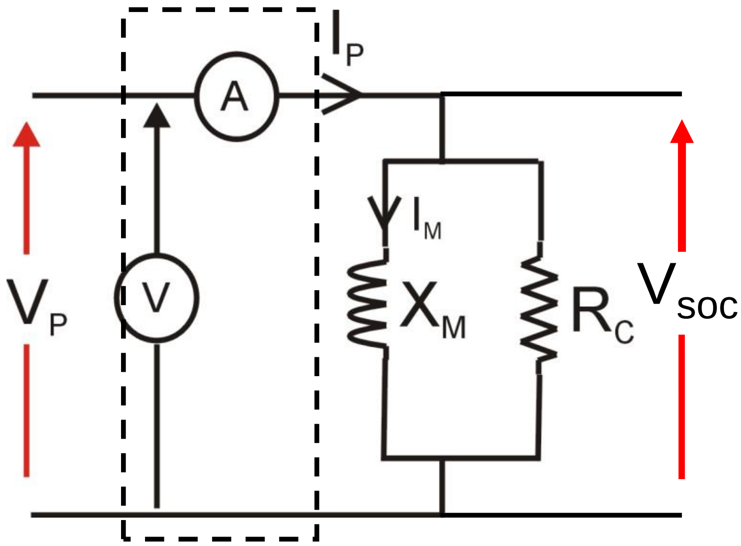

Figure 9. Open circuit test on a single-phase transformer

Test 2: Short Circuit Test

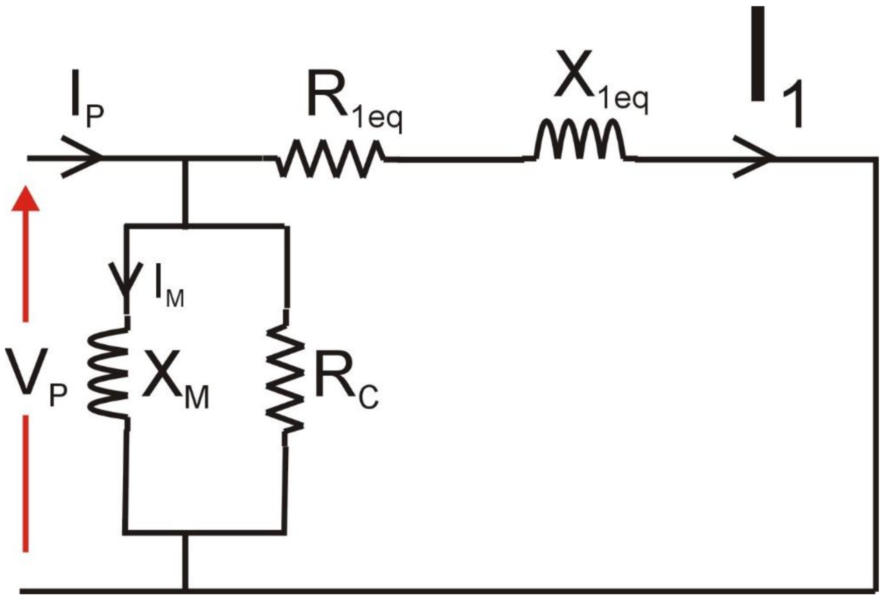

Figure 10. Secondary winding output short circuit.

The Secondary Winding is Short Circuited (Z_S=0), and as a result I_1 flows through R_{1eq} and X_{1eq}. Typically the current flowing through the parallel network formed by X_m and R_c is much less than I_1, so $I_1 \approx I_P$. We can simplify the equivalent circuit to determine **X_{1eq}, R_{1eq}** (Figure 11). Since the secondary winding is short circuit, this test requires a very small primary voltage.

In the short circuit test we measure primary power, V_p, primary current, I_p, and primary real power, W. From this we can calculate R_{1eq} and X_{1eq}:

$$R_{1eq} = \frac{P_P}{I_p^2} \tag{8}$$

$$X_{1eq} = \frac{Q_P}{I_p^2} \tag{9}$$

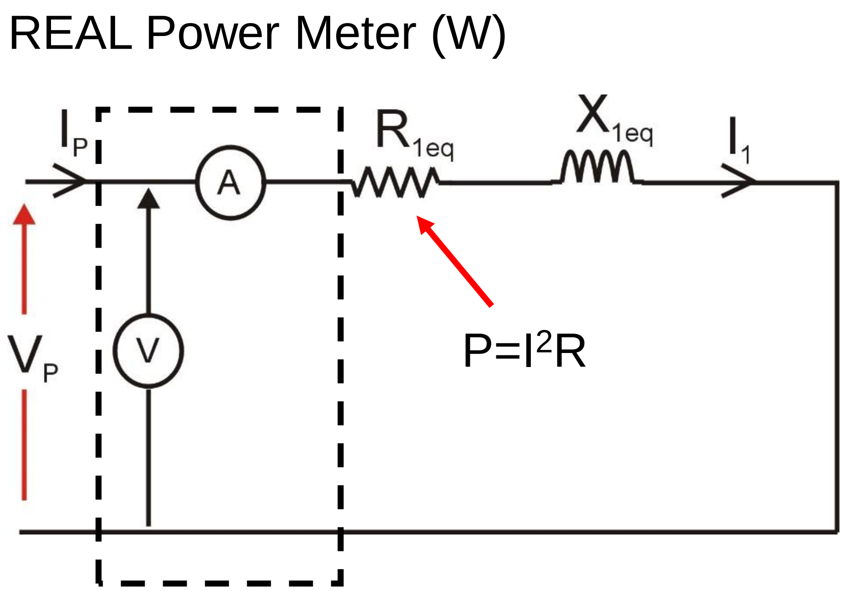

Figure 11. Equivalent circuit for short circuit test

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