POWER ENGINEERING

#02 SINGLE-PHASE AC POWER SYSTEMS

Semester 1 – 2021/2022

University of Glasgow

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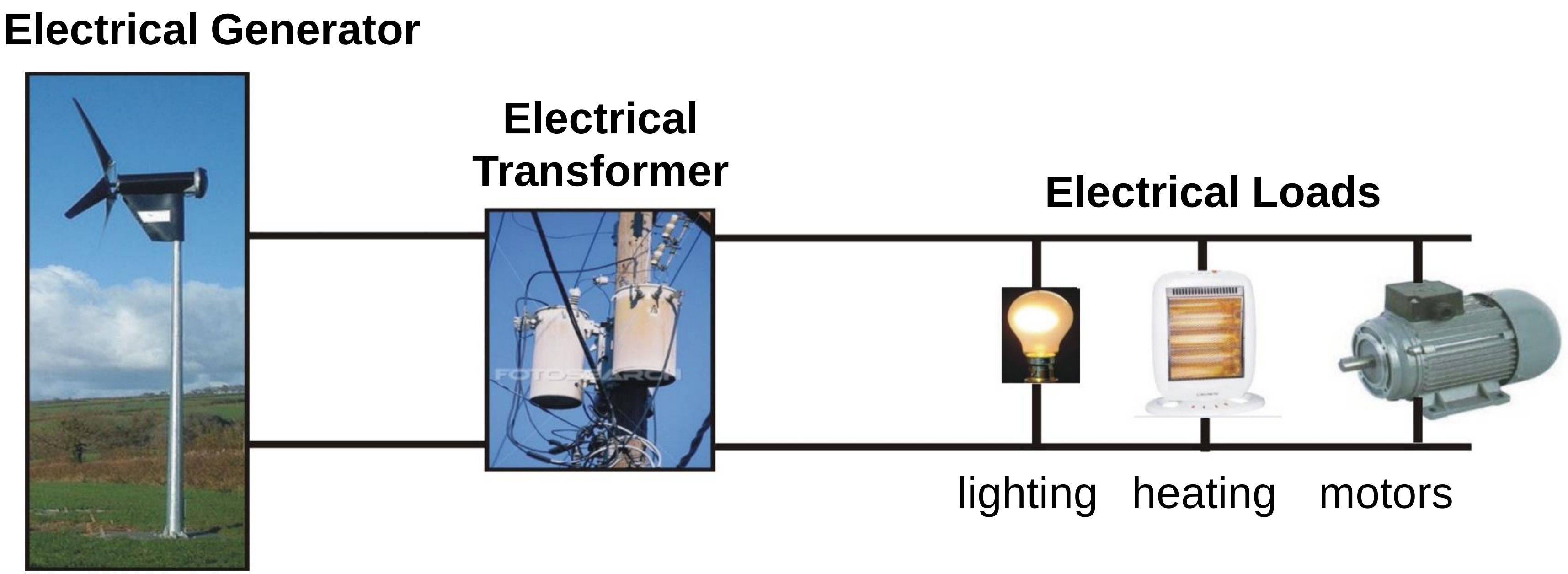
Outline

- **RLC Components**
- **AC Voltage/Current Source**
  - Instantaneous Voltage/Current
  - Average and RMS Voltage/Current
- **AC Power**
  - Instantaneous Power
  - Average Power
- **Power Factor**
- **Voltage/Current Phasors and Phasor Diagram**
- **Component Impedance**
- **Power Triangle**
  - Apparent Power
  - Real Power
  - Reactive Power
- **Power Efficiency**

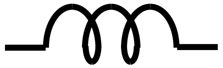
A SIMPLE ELECTRICAL POWER SYSTEM



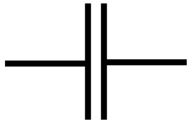
- Each **component** can be represented by **an equivalent circuit** made up of a combination of simple electrical components: **voltage source, resistance, inductance and capacitance**
- With the equivalent circuit we can determine component voltages and currents, and then from this determine **POWER** related properties such as **Real Power, Apparent Power, System Efficiency and Power Factor**

Basic Passive Elements LCR

resistance (R) / conductance (G)
unit: ohm (Ω) / siemens (S)
Energy Dissipation Element !!!



inductance (L)
unit: henry (H)
Energy Storage Element



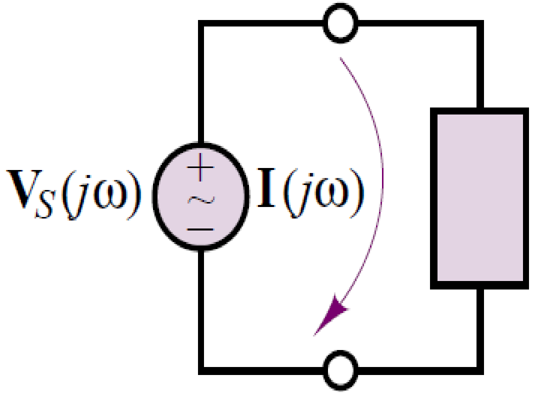
capacitance (C)
unit: farad (F)
Energy Storage Element

Voltage-Current Relationship in Time Domain

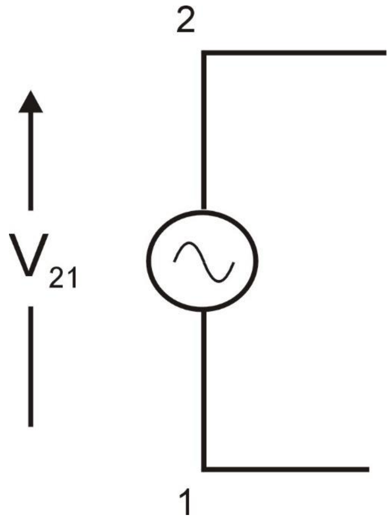
	DC	AC
Resistor Ohm's Law	$V = RI$	$v(t) = Ri(t)$
Inductor		$v(t) = L\dfrac{di(t)}{dt}$
Capacitor		$v(t) = \dfrac{1}{C}\int i(t)dt$

Behaviors of LCR in time domain

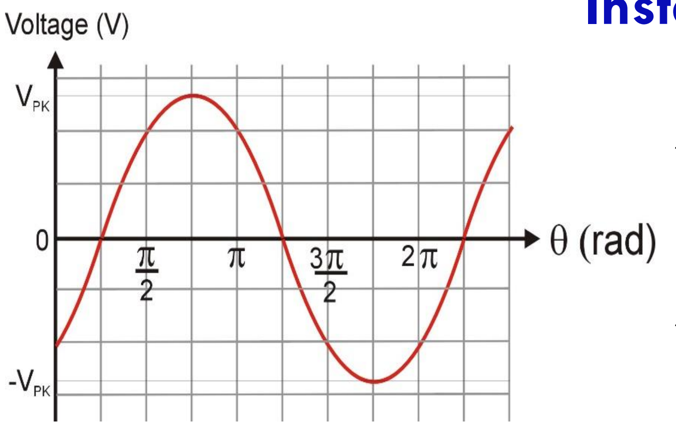
STEADY-STATE AC CIRCUIT



- Circuit in ***Steady State***;
- ***Excitation Source V_s or I*** is a sinusoidal function with constant *amplitude* and constant *frequency*.
- ***DC Circuit*** is a special case of **steady-state AC Circuit** in the case of frequency ω=0



Ideal Voltage Source



Instantaneous voltage

$$v(\theta) = V_{PK} \sin(\theta - \frac{\pi}{4})$$

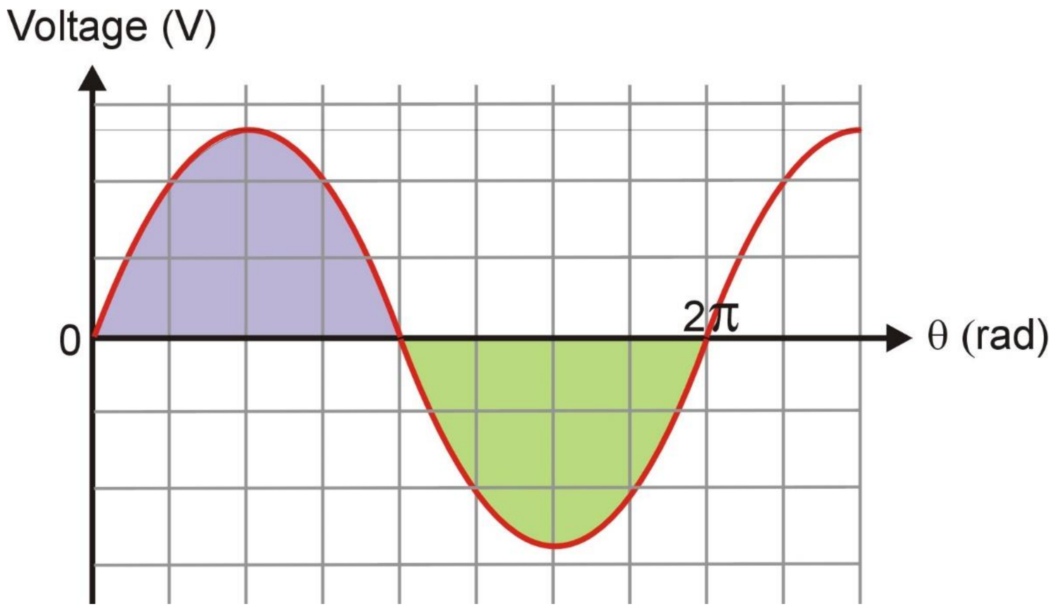
$$v(t) = V_{PK} \sin(\omega t - \frac{\pi}{4})$$

AVERAGE VALUE

A general periodic function $y(t)$, with period T, has an average or mean value Y_{av} given by:

$$\boxed{Y_{av} = \frac{1}{T}\int_0^T y(t)dt}$$

The mean value of a sine or cosine function is 0.



$$V_{av} = \frac{1}{2\pi}\int_0^{2\pi} V_{pk}\sin\theta d\theta$$

$$V_{av} = \frac{V_{pk}}{2\pi}\left[-\cos\theta\right]_0^{2\pi}$$

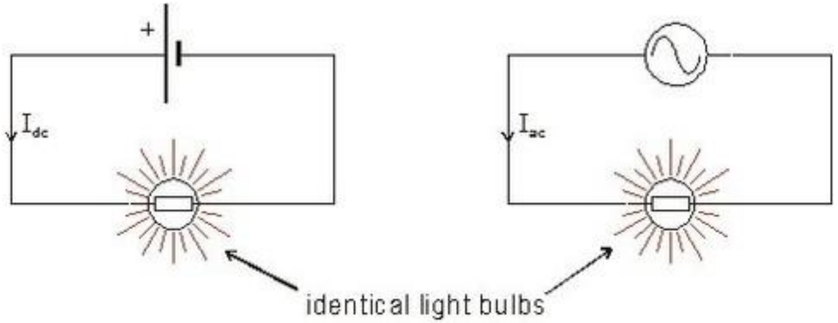
$$V_{av} = \frac{V_{pk}}{2\pi}\left[-1+1\right]$$

$$V_{av} = 0$$

Effective or RMS Value

The Root Mean Square (RMS) or Effective value of a general periodic function $y(t)$, with period T, has an effective value:

$$\boxed{Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2\,dt}}$$



The mean value of sine/cosine is 0, can not be used to measure the power of an AC circuit. The RMS value is also referred to as the "heating" value since a current passing through a pure resistor results in power being dissipated. RMS value of the AC current I_{ac} is **equivalent** to the DC current I_{dc} passing through a resistor, to produce same heat !!!

RMS Value & Power Dissipation

Power Dissipation in resistors

$$p_{dc} = \frac{V_{dc}^2}{R} = I_{dc}^2 R$$

$$p_{av} = \frac{1}{T}\int_0^T v(t)i(t)dt = \frac{1}{T}\int_0^T \frac{v(t)^2}{R}dt$$

$$= \frac{1}{R}\left(\underbrace{\sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}}_{rms}\right)^2 = \frac{v_{rms}^2}{R}$$

$$= R\left(\underbrace{\sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}}_{rms}\right)^2 = i_{rms}^2 R$$

Instantaneous Power

$$\boxed{p(t) = v(t)i(t)}$$

Average Power

$$\boxed{p_{av} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt}$$

RMS voltage →

$$\boxed{v_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2 dt}}$$

RMS Current →

$$\boxed{i_{rms} = \sqrt{\frac{1}{T}\int_0^T i(t)^2 dt}}$$

RMS value of a Sinusoidal Waveform

$$V_{rms} = \sqrt{\left[\frac{1}{\tau}\int_0^{\tau} v^2(\theta)d\theta\right]} = \sqrt{\left[\frac{1}{2\pi}\int_0^{2\pi} V_{pk}{}^2 Sin^2\theta.d\theta\right]}$$

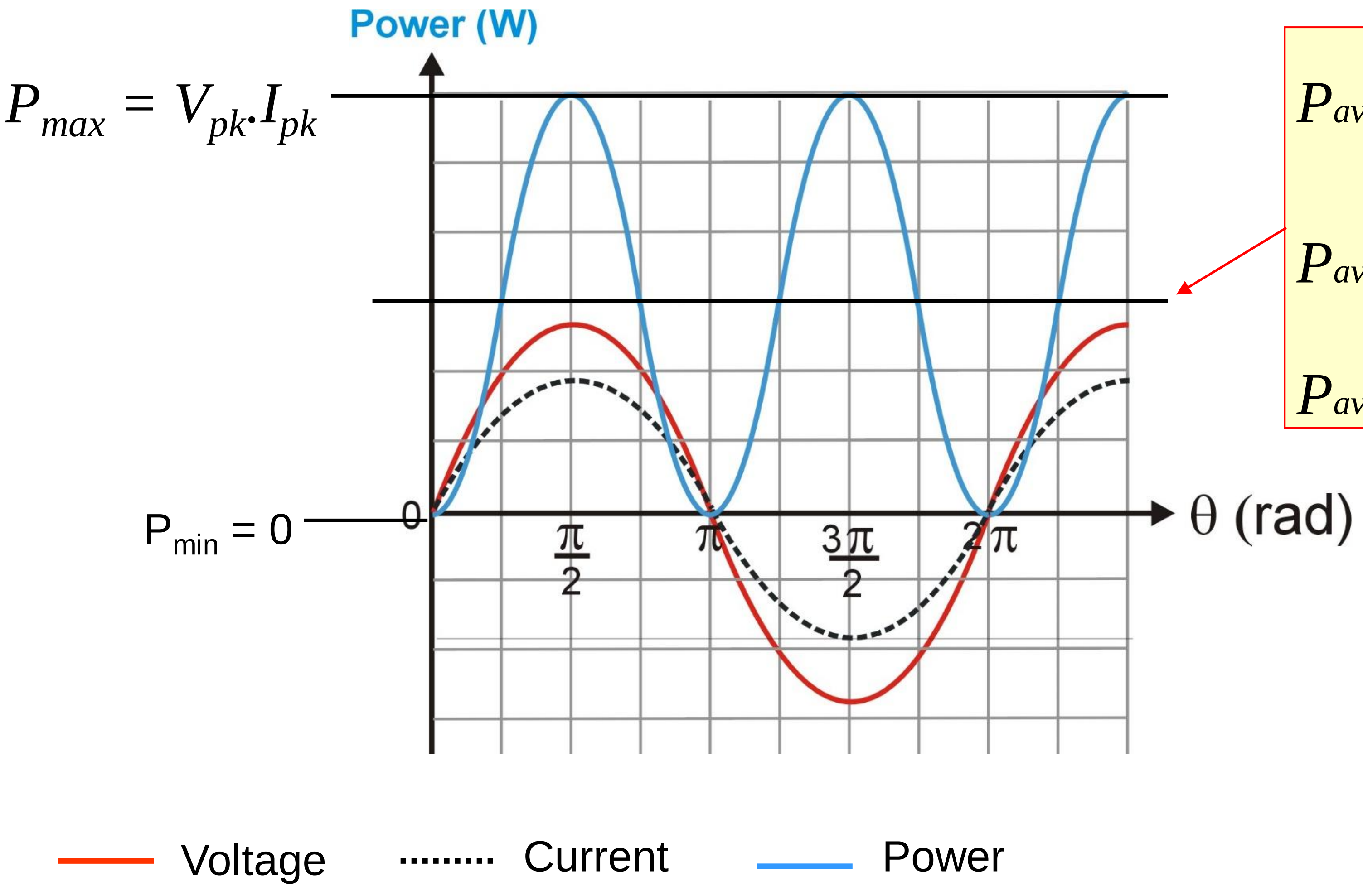
$$= \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\int_0^{2\pi}\left[\frac{1}{2}\left(1-Cos2\theta\right)\right]d\theta\right]}$$

$$V_{rms} = \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\left[\frac{1}{2}\left(\theta-\frac{1}{2}Sin2\theta\right)\right]_0^{2\pi}\right]} = \sqrt{\left[\frac{V_{pk}{}^2}{2\pi}\left[\frac{2\pi}{2}-0-0+0\right]\right]} = \frac{|V_{pk}|}{\sqrt{2}}$$

$$\boxed{V_{rms} = \frac{|V_{pk}|}{\sqrt{2}}}$$

Resistance (Power Dissipation > 0)

Instantaneous Power: $p(t) = v(t) \times i(t)$

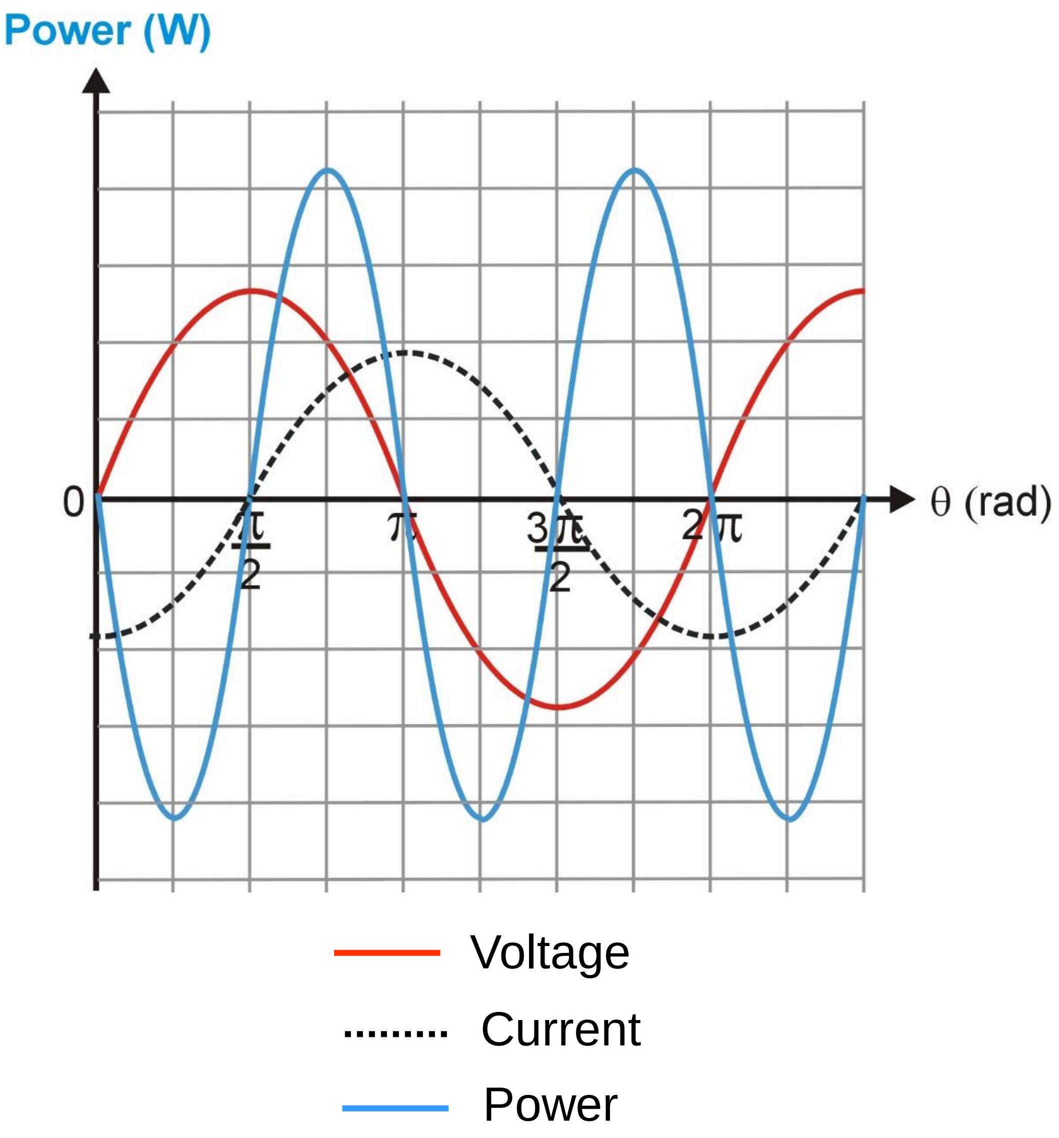


$$P_{av} = \frac{V_{pk}.I_{pk}}{2}$$

$$P_{av} = \frac{V_{pk}}{\sqrt{2}} \cdot \frac{I_{pk}}{\sqrt{2}}$$

$$P_{av} = V_{rms}.I_{rms}$$

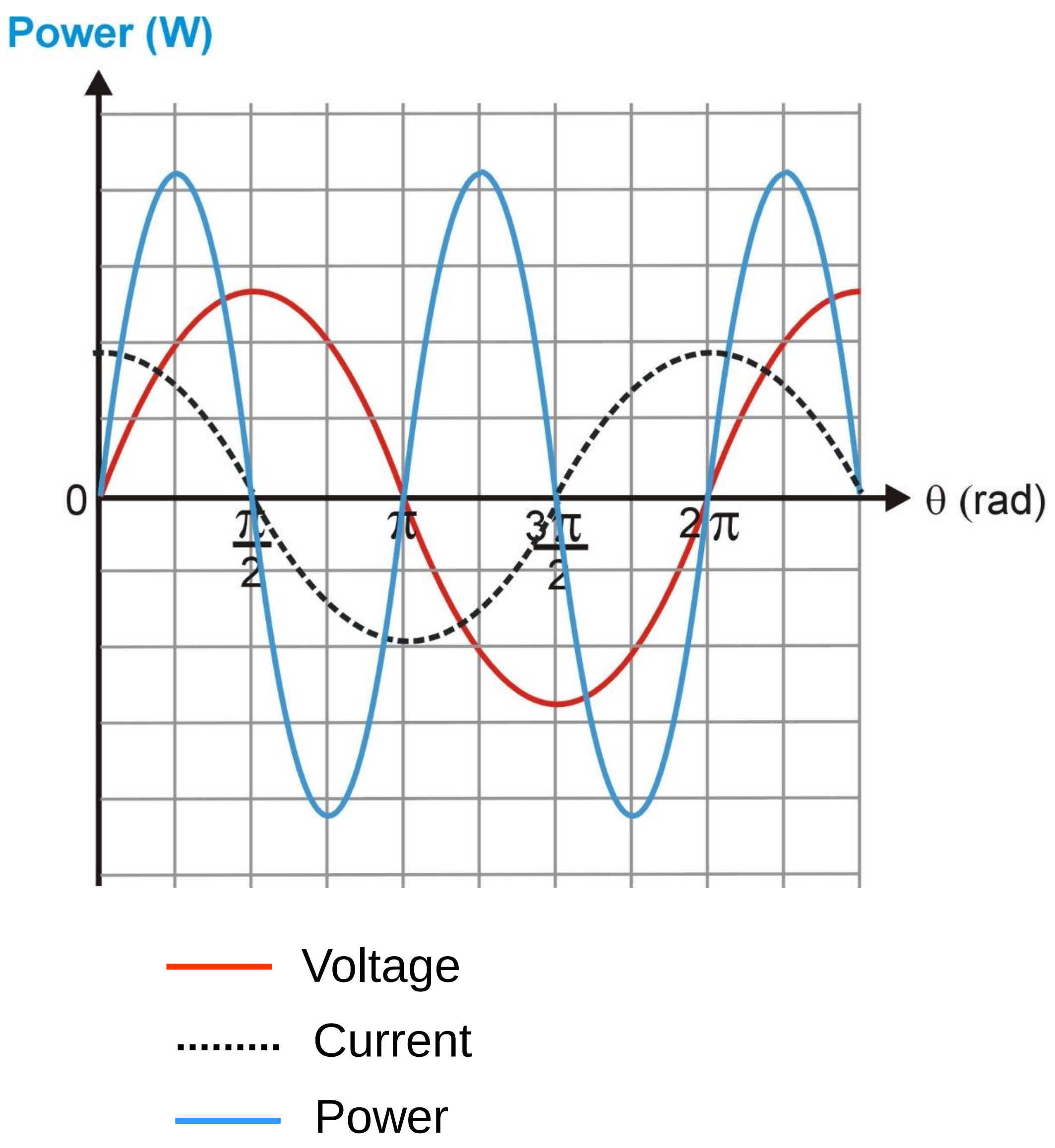
Inductance (Power Dissipation = 0)



$$P_{av} = 0$$

The power simply oscillates between the inductor and the supply at twice the supply frequency

Capacitance (Power Dissipation = 0)

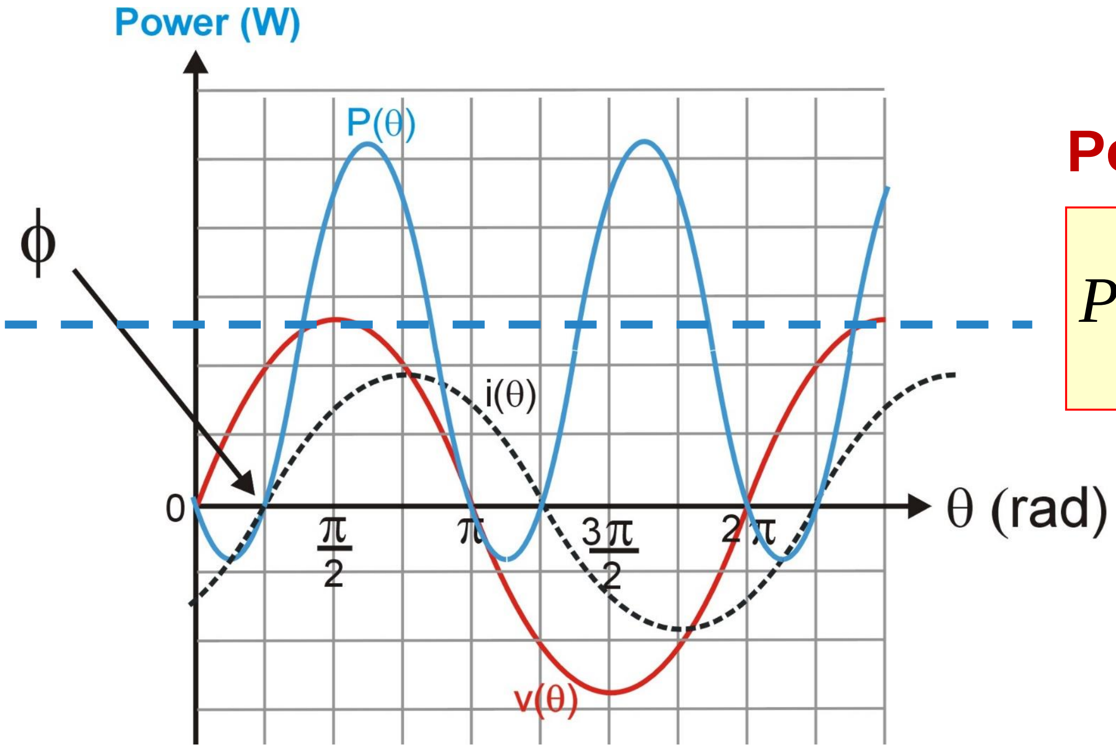


$$P_{av} = 0$$

The power simply oscillates between the capacitor and the supply at twice the supply frequency

Complex Load (combination of R, L and C)

A general expression for average power where **the currents lags (or leads) the voltage** by **angle ϕ**



Power Dissipation

$$P_{av} = \frac{1}{\tau}\int_0^{\tau} v(\theta)\cdot i(\theta-\phi)\,d\theta$$

Complex Load (combination of R, L and C)

$$P_{av} = \frac{1}{\pi}\int_0^{\pi}\left(V_{pk}\cdot\sin\theta\right)\cdot\left(I_{pk}\cdot\sin\left(\theta-\phi\right)\right)d\theta$$

$$= \frac{V_{pk}\cdot I_{pk}}{\pi}\left[\int_0^{\pi}\left(\sin^2\theta\cdot\cos\phi-\sin\theta\cdot\cos\theta\cdot\sin\phi\right)d\theta\right]$$

$$= \underbrace{\left[\frac{V_{pk}\cdot I_{pk}\cdot\cos\phi}{\pi}\int_0^{\pi}\sin^2\theta d\theta\right]}_{V_{rms}\cdot I_{rms}\cdot\cos\phi} - \underbrace{\left[\frac{V_{pk}\cdot I_{pk}\cdot\sin\phi}{\pi}\int_0^{\pi}\left(\sin\theta\cdot\cos\theta\right)d\theta\right]}_{=0}$$

$$= V_{rms}\cdot I_{rms}\cdot\cos\phi$$

Power Dissipation

$$\boxed{P_{av} = V_{rms}\cdot I_{rms}\cdot\cos\phi}$$

POWER FACTOR

The average power dissipated by an AC load is dependent on the cosine of the angle of the impedance, the term $\cos\Phi$ is referred to as the power factor (**pf**).:

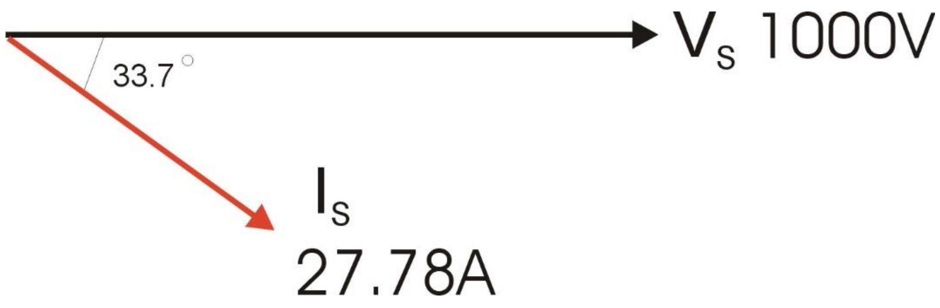
$$\boxed{\mathbf{pf} = \cos\phi}$$

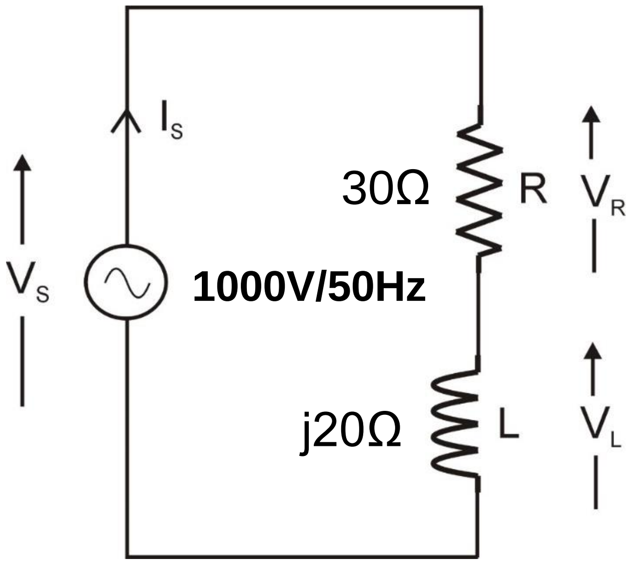
Note: The **power factor** is equal to **0 for a purely inductive or capacitive load**; and is equal to **1 for a purely resistive load**; in every other case,

$$0 < \mathbf{pf} < 1$$

Power factor is dimensionless, a measure of how effectively the load draws the real power.

An Example





$$I_S = \frac{V_S}{(30 + j20)\Omega} = \frac{1000\angle 0°}{36.1\angle 33.7°}$$

$$= 27.78\angle -33.7°$$

$$P_{av} = V_{rms} \cdot I_{rms} \cdot \cos\phi = I_{rms}^2 R$$

$$P_{av} = 1000 \times 27.78 \times \cos(33.7^o)$$

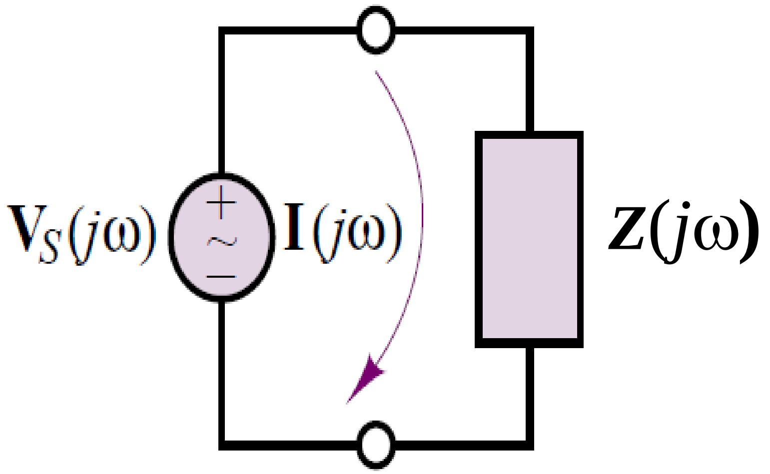
$$P_{av} = 23112W$$

$$\text{pf} = \cos(33.7^o) \approx 0.832$$

Complex Ohm's Law

Phasor and **Impedance** are introduced to simplify AC circuit analysis, without complicated calculus (differential equations) of trigonometric function.

Voltage and Current **Phasors**



Component **Impedance**

$$Z(j\omega) = \frac{V_S(j\omega)}{I(j\omega)}$$

PHASOR

Phasor *is only used to express* ***sinusoidal voltage*** *or* ***current*** *variables with complex number for mathematical convenience.* ***No Real Physical Significance.***

Any sinusoidal signal may be represented in one of two ways:

- Instantaneous form in the time domain:

$$v(t) = V_m \cos(\omega t + \varphi_1); \quad i(t) = I_m \cos(\omega t + \varphi_2)$$

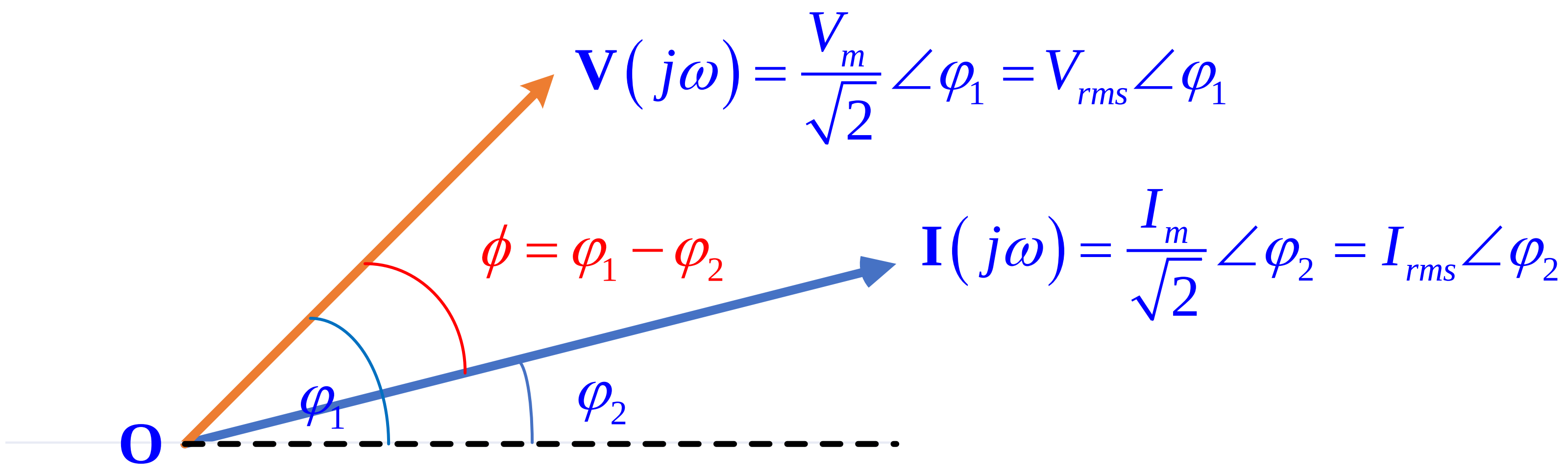
- Phasor form:

RMS value

$$\mathbf{V}(j\omega) = \frac{V_m}{\sqrt{2}} e^{j\varphi_1} = \frac{V_m}{\sqrt{2}} \angle \varphi_1; \quad \mathbf{I}(j\omega) = \frac{I_m}{\sqrt{2}} e^{j\varphi_2} = \frac{I_m}{\sqrt{2}} \angle \varphi_2$$

Note: $j\omega$ in the notation $\mathbf{V}(j\omega)$, indicating $e^{j\omega t}$ dependence of the phasor.

PHASOR DIAGRAM



IMPEDANCE

$$\boxed{\mathbf{Z}(j\omega) = \frac{\mathbf{V}(j\omega)}{\mathbf{I}(j\omega)} = \frac{V_{rms}}{I_{rms}}\angle\phi = R + jX = |Z|(\cos\phi + j\sin\phi)}$$

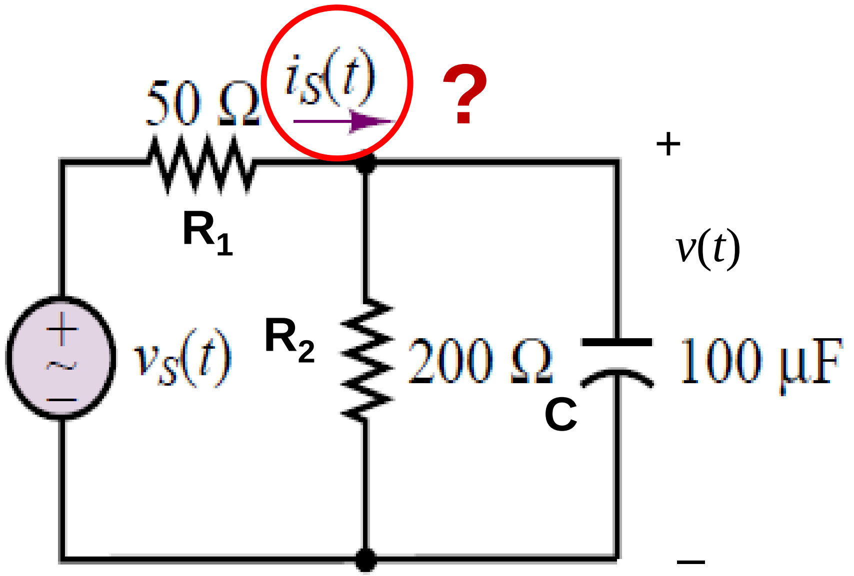
IMPEDANCE

Component	Voltage/Current Phase Relationship	Complex Impedance (Cartesian)	Complex Impedance (Polar)
Resistance	**Voltage and Current are in phase**	$Z_R = R + j0$	$Z_R = R\angle 0^o$
Inductance	**Current lags the Voltage by 90º**	$Z_L = 0 + j\omega L$	$Z_L = \omega L \angle 90^o$
Capacitance	**Current leads the Voltage by 90º**	$Z_C = 0 - j/(\omega C)$	$Z_C = \frac{1}{\omega C}\angle -90^o$

Note: The REACTANCE of an Inductance $X_L = \omega L$

The REACTANCE of a capacitance $X_C = 1/(\omega C)$

An Example of AC Circuit Analysis



$v_S(t) = 10\cos(100t)$

1. Source Phasor

$$v_S(t) = 10\cos(100t)$$

$$\omega = 100\,\mathbf{rad/t}$$

Time Domain

$$\mathbf{V_s}(j\omega) = \left(10/\sqrt{2}\right)\angle 0°$$

2. Impedance

$$Z_{R1} = 50\Omega$$

$$Z_{R2} = 200\Omega$$

$$Z_C = \frac{1}{j\omega C} = -j100\Omega$$

3. Ohm's Law

$$\mathbf{I}_S = \frac{\mathbf{V_s}}{Z_{R1} + Z_{R2} \parallel Z_C} = \frac{\mathbf{V_s}}{R_1 + \dfrac{R_2}{1 + j\omega C R_2}}$$

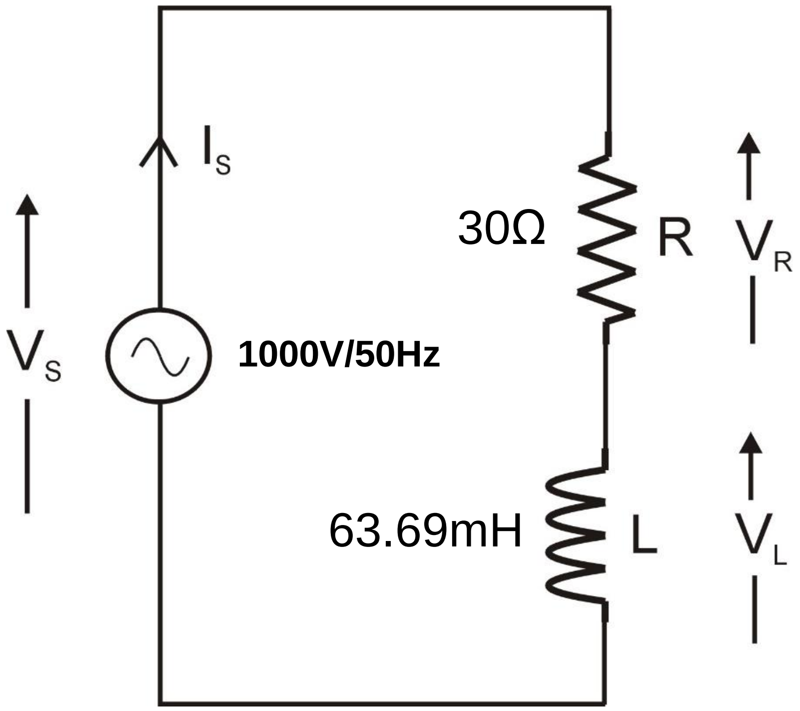
$$= \left(0.083/\sqrt{2}\right)\angle 0.727\,\mathbf{A}$$

4. Instantaneous value

$$i_s(t) = 0.083\ \cos(100t + 0.727)\ \mathrm{A}$$

A PRACTICE QUESTION

Determine the values of $\mathbf{V_R}$ and $\mathbf{V_L}$



Note: 1000V is RMS voltage!

Power Triangle

$$\mathbf{S} = \tilde{\mathbf{V}}\tilde{\mathbf{I}}^*$$ Complex Power

* **Conjugate**

$$\mathbf{S} = V_{rms} I_{rms} \cos\phi + jV_{rms} I_{rms} \sin\phi$$

$$= P + jQ$$

Apparent power :

$$S = \sqrt{P^2 + Q^2} = V_{rms} I_{rms}$$

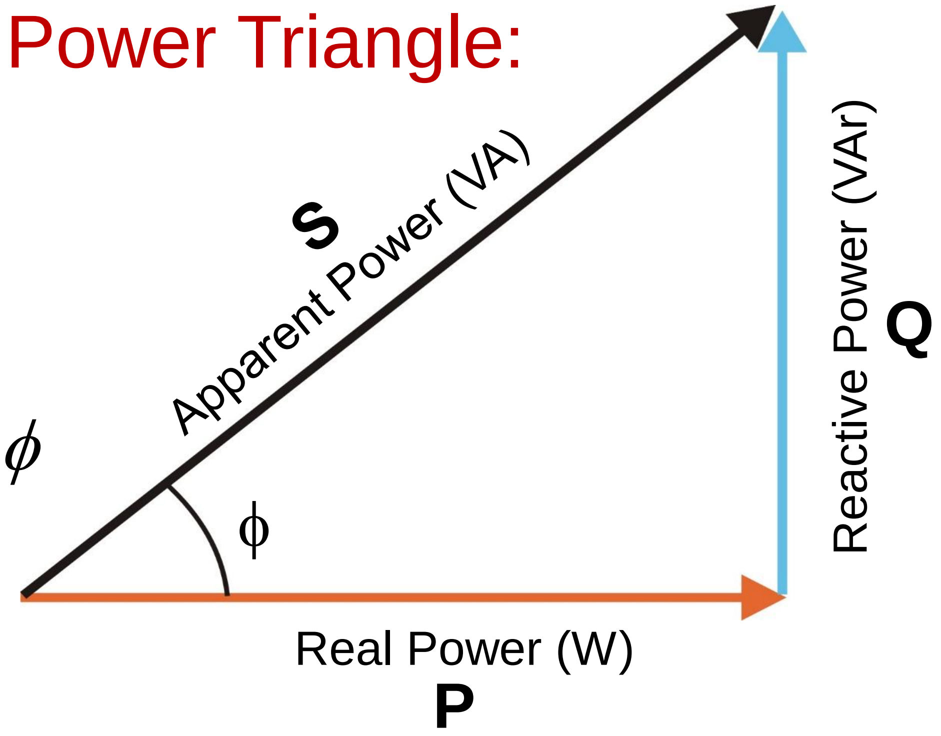
Real (average) power :

$$P = P_{av} = V_{rms} I_{rms} \cos\phi = S\cos\phi$$

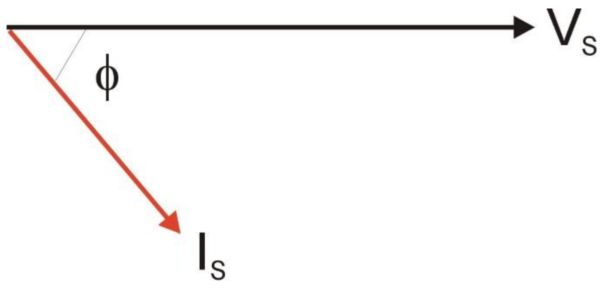
Reactive power :

$$Q = V_{rms} I_{rms} \sin\phi = S\sin\phi$$

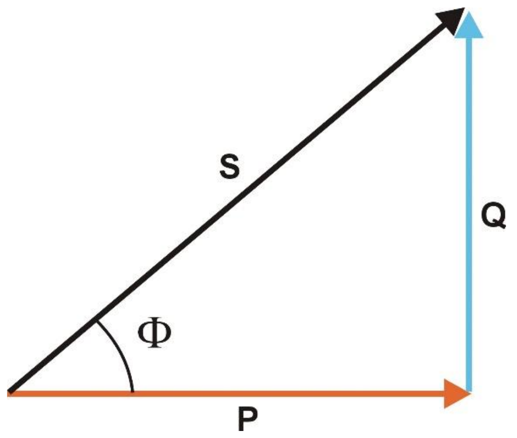
Power Triangle:



INDUCTIVE LOAD

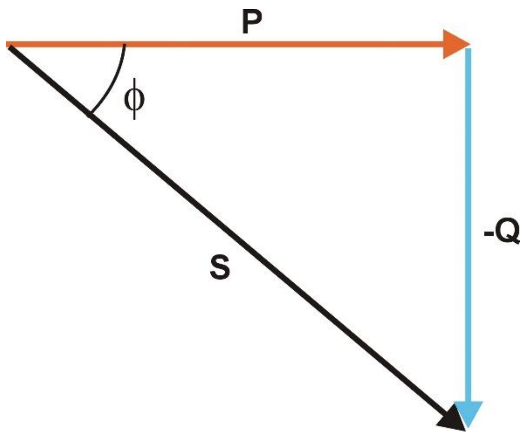


If the Current **LAGs** the voltage by angle ϕ (indicating that the load is inductive + resistive) then Reactive Power (Q) is deemed to be Positive and is termed **ABSORBING VAr's**

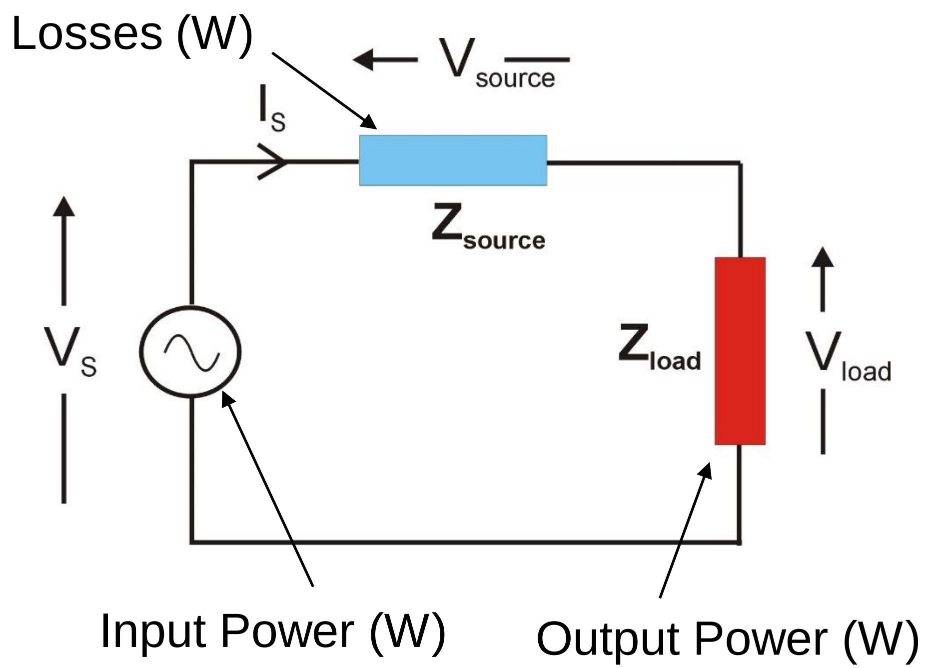


CAPACITIVE LOAD

If the Current **LEADs** the voltage by angle ϕ (indicating that the load is capacitive + resistive) then Reactive Power (Q) is deemed to be Negative and is termed **GENERATING VAr's**



Power Efficiency in EPS



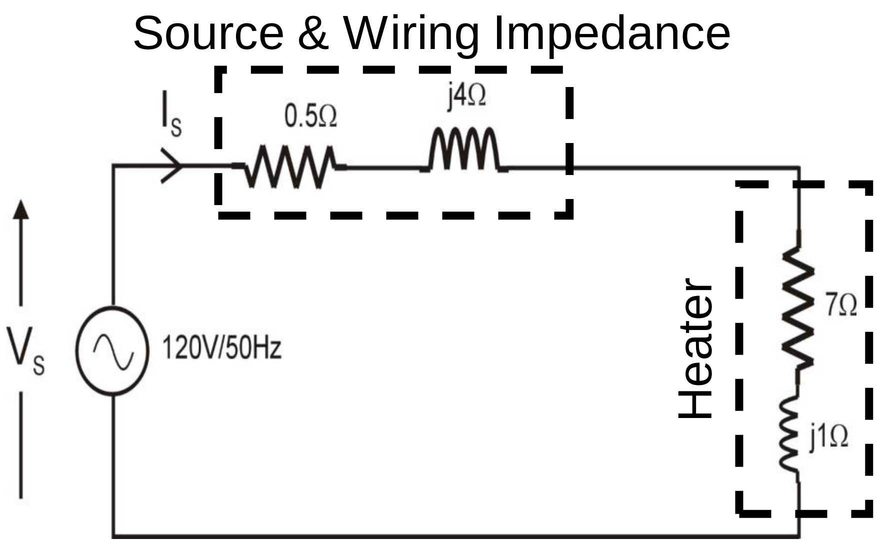
$$\text{Efficiency} = \frac{\text{Output Power (W)}}{\text{Input Power (W)}} \times 100\%$$

$$= \frac{\text{Output Power (W)}}{\text{Output Power (W)} + \text{Losses (W)}} \times 100\%$$

Note that Input & Output Power are **REAL Powers** (W)

AN EXAMPLE

The heating system for a remote monitoring station consists of a diesel generator which outputs 120V/50Hz connected to a Heater:



1. The **total circuit impedance Z_T**
2. The supply current $\mathbf{I_S}$
3. A phasor diagram indicating $\mathbf{V_S}$ and $\mathbf{I_S}$
4. **Apparent Power**, **Real Po**wer, **Reactive Power** and **Power Factor** at the power supply
5. The Heater output power
6. **Power Efficiency** of the heating system

Consider:

$$V = V_{rms}\angle\theta_V$$

Where θ_V is the phase angle of the voltage, and

$$I = I_{rms}\angle\theta_I$$

Where θ_i is the phase angle of the current.

We could say that:

$$S = VI = V_{rms}\angle\theta_V \times I_{rms}\angle\theta_I = V_{rms}I_{rms}\angle(\theta_V + \theta_I)$$

Let's now do the same calculation with the complex conjugate of I (I*)

$$S = VI^* = V_{rms}\angle\theta_V \times I_{rms}\angle-\theta_I = V_{rms}I_{rms}\angle(\theta_V - \theta_I)$$

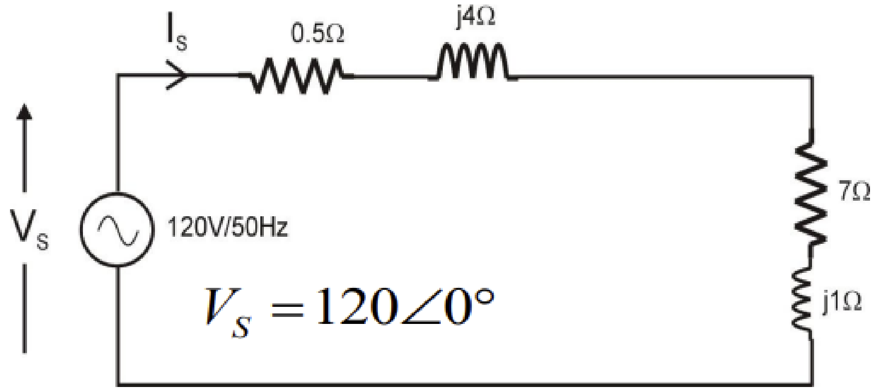
Which is correct ?

P is the real part of S, which is S.cos(θ), where θ is the difference in phase angle between V and I (θ_V-θ_I).

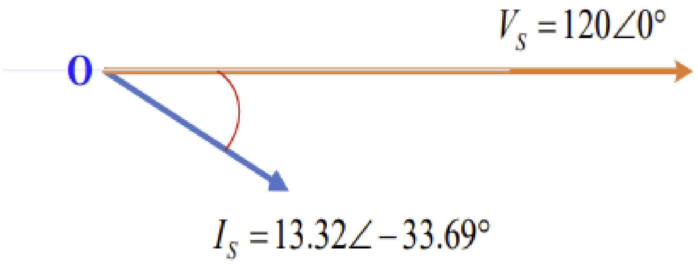
1. $S=VI^*$ will give the correct real part.
2. $S=VI$ will give the wrong real part.
   - Therefore to calculate the correct Real Power we must use $S=VI^*$. [You could use the complex conjugate of V but it is convention to use I*]

Hence, when we calculate Q we use I* and the angle in the power triangle will be positive for a load which has inductive reactance. (The opposite to the power factor angle)

Solution to Example Question



3.



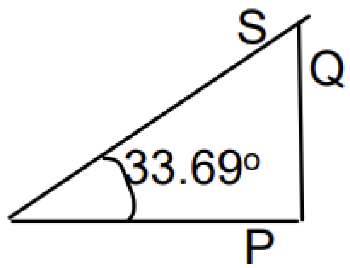
1. $Z_T = (0.5 + j4 + 7 + j1)\Omega = 7.5\Omega + j5\Omega = 9.01\angle 33.69°$

2. $I_S = V_S / Z_T = 120\angle 0° / 9.01\angle 33.69° = 13.32\angle -33.69°$

4. $S = V_{Srms} \times I^*_{Srms} = 120 \times 13.32 \approx 1598\,\mathbf{VA}$

$P = S\cos(33.69°) = I_S^2 \times$ 0.832 =1329.6 **W**

$Q = S\sin(33.69°) = I_S^2 \times$ 0.555 =98.42 **Var**



5. $P_{out} = I_S^2 \times 7 \approx$ 1242 **W**

6. $\eta = P_{out} / P =$ 93.4%

#02 Single-Phase AC Power Systems