DIGITAL IMAGE PROCESSING

IMAGE RESTORATION

CHANG SHU

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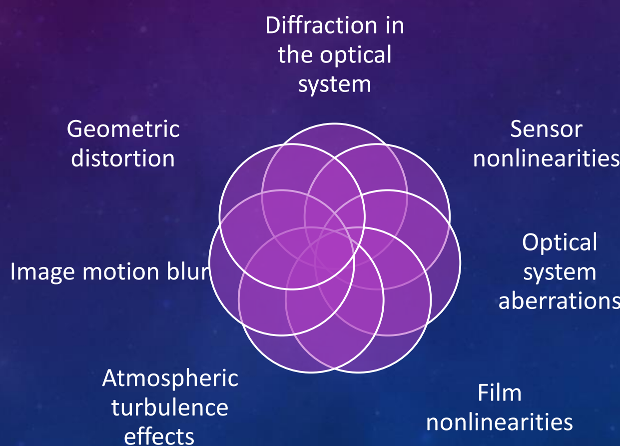
IMAGE RESTORATION

changshu@uestc.edu.cn

- Image degradation
- General image restoration models
- Inverse filtering
- Wiener filtering
- Constrained least squares filtering
- Geometric image transformation

IMAGE RESTORATION



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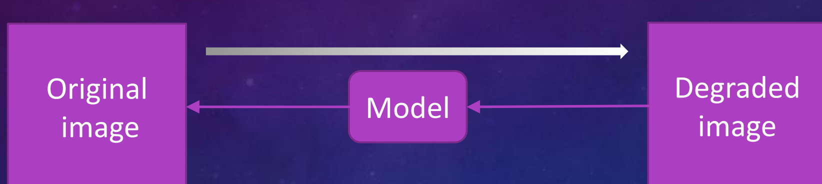
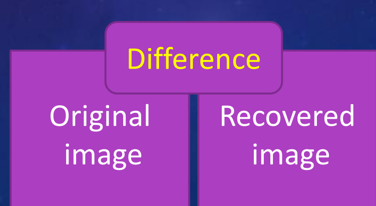
- Goal



- Evaluation



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- Traditional
  - Stable, linear position-invariant, priori knowledge of the signal and noise
- Modern
  - Non-stable(Kalman filter), nonlinear(ANN), no priori knowledge(blind image restoration)

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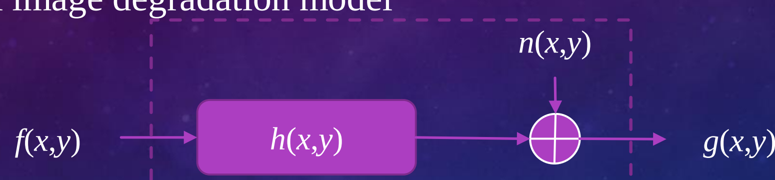
- General image degradation model



- Degraded output image in frequency domain

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- Degraded output image in spatial domain

$$g(x,y) = h(x,y) \otimes f(x,y) + n(x,y)$$

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- Given $g(x, y)$, some knowledge about the degradation function H and some knowledge about the additive noise term $n(x, y)$， the objective of restoration is to obtain an estimate $\hat{f}(x, y)$ of the original image

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- Property of degradation function H
  - linear
    - Additivity
    - Homogeneity

$$H[k_1 f_1(x,y) + k_2 f_2(x,y)] = k_1 H\left[f_1(x,y)\right] + k_2 H\left[f_2(x,y)\right]$$

  - Position invariant

$$H\left[f(x-a, y-b)\right] = g(x-a, y-b)$$

$$g(x,y) = H\left[f(x,y)\right]$$

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$$g(x,y)=H[f(x,y)]=H\left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha d\beta\right]$$

$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} H\left[f(\alpha,\beta)\delta(x-\alpha,y-\beta)\right]d\alpha d\beta$$ additivity property

$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)H[\delta(x-\alpha,y-\beta)]d\alpha d\beta$$ homogeneity property

$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)d\alpha d\beta$$

- $h(x,\alpha, y,\beta)$ is called the *impulse response* of H, commonly referred to as the point spread function(PSF)
- The last line is called the superposition integral of the first kind
- H is completely characterized by its impulse response

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$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)H[\delta(x-\alpha,y-\beta)]d\alpha d\beta$$ homogeneity property

$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta$$ position invariant

$$g(x,y)=f(x,y)\otimes h(x,y)+n(x,y)$$

$$=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta+n(x,y)$$

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$$g=Hf+n$$

- $f(x, y)$ and $h(x, y)$ are of size $A\times B$, $C\times D$ respectively, by zero padding, they are extended to $M\times N$($M=A+C-1$, $N=B+D-1$) periodical functions:

$$f_e(x,y)=\begin{cases} f(x,y) & 0\le x\le A-1, 0\le y\le B-1 \\ 0 & \text{otherwise} \end{cases}$$

$$h_e(x,y)=\begin{cases} h(x,y) & 0\le x\le C-1, 0\le y\le D-1 \\ 0 & \text{otherwise} \end{cases}$$

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$$g_e(x,y)=\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f_e(m,n)h_e(x-m,y-n)+n_e(x,y)$$

$$g=Hf+n$$

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- 2D Block Circulant Matrix

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_0 & \boldsymbol{H}_{M-1} & \boldsymbol{H}_{M-2} & \cdots & \boldsymbol{H}_1 \\ \boldsymbol{H}_1 & \boldsymbol{H}_0 & \boldsymbol{H}_{M-1} & \cdots & \boldsymbol{H}_2 \\ \boldsymbol{H}_2 & \boldsymbol{H}_1 & \boldsymbol{H}_0 & \cdots & \boldsymbol{H}_3 \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{H}_{M-1} & \boldsymbol{H}_{M-2} & \boldsymbol{H}_{M-3} & \cdots & \boldsymbol{H}_0 \end{bmatrix}$$

$$\boldsymbol{H}_j = \begin{bmatrix} h_e(j,0) & h_e(j,N-1) & h_e(j,N-2) & \cdots h_e(j,1) \\ h_e(j,1) & h_e(j,0) & h_e(j,N-1) & \cdots h_e(j,2) \\ \vdots & \vdots & \vdots & \vdots \\ h_e(j,N-1) & h_e(j,N-2) & h_e(j,N-3) & \cdots h_e(j,0) \end{bmatrix}$$

$$H = WDW^{-1}$$