电子科技大学
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Circuit Analysis and Design

Academic year 2019/2020 – Semester 1 – Presentation 9

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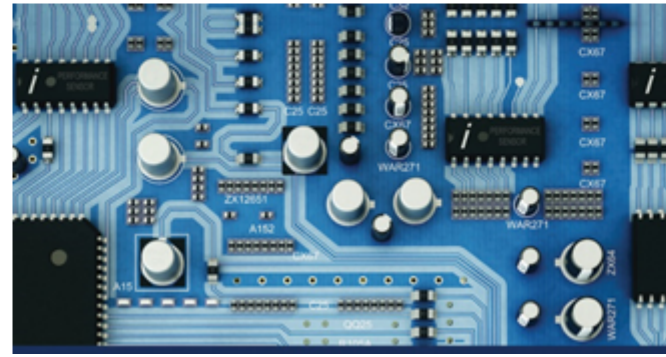
"A good student never steal or cheat"

Agenda



- Capacitors
- Series and parallel connection of capacitors
- Inductors
- Series and parallel connection of inductors

Introduction

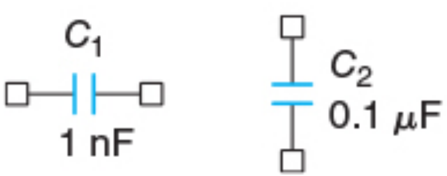
- Unlike resistors, capacitors and inductors are passive elements that can store and release energy.
- The capacitance of a capacitor is the amount of charge the capacitor can hold for the given voltage.
- The energy is stored in the form of electric field from the positive plate to the negative plate. The amount of energy stored on the capacitor depends on the capacitance and the voltage across the capacitor plates.
- The current through the capacitor is given by the product of the capacitance and the time rate of change of the voltage across the capacitor plates.
- The inductance of an inductor is the measure of the inductor to generate magnetic flux for the given change of current.
- The energy is stored in the form of magnetic field. The amount of energy stored in the inductor depends on the inductance and the current through the inductor.
- The voltage across the inductor is given by the product of the inductance and the time rate of change of the current through the inductor.

Capacitors

- A capacitor is a passive element that can store and release energy.
- A parallel plate capacitor has two plates filled by dielectric material in between. The positive plate holds positive charges, and the negative plate holds negative charges. The capacitance of a capacitor is the amount of charge that the capacitor can hold for the given voltage. The energy is stored in the form of an electric field from the positive plate to the negative plate. The amount of energy stored on the capacitor depends on the capacitance and the voltage across the capacitor plates.
- The symbol for capacitors is shown in Figure 6.1.

FIGURE 6.1

Symbol for capacitors.



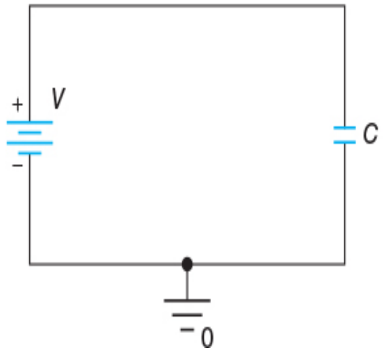
Capacitance

- When a capacitor is connected to a voltage source such as a battery, as shown in Figure 6.2, positive charges accumulate on the plate connected to the positive terminal of the battery, and negative charges accumulate on the plate connected to the negative terminal of the battery.
- The amount of charge Q (in coulombs) on the plates is proportional to the voltage V (in volts) of the voltage source. Let C be the proportionality constant in this linear relation. Then we have $Q = CV$
- The constant C is called capacitance, measured in farads (F*).*

  The capacitance C can be written as $C = Q/V$
- The capacitance is defined as the ratio of the charge stored to the potential difference.

FIGURE 6.2

A capacitor connected to a battery.



Capacitance (Continued)

- The capacitance is the amount of charge that can be stored for 1 V of potential difference (voltage). If the voltage is time varying, the charge can be written as $Q = C\,v(t)$
- Common capacitors used in electronic circuits have small capacitance values in the microfarad (μF), nanofarad (nF), and picofarad (pF) range.

  1 μF = 10^{-6}F, 1 nF = 10^{-9}F, 1 pF = 10^{-12}F
- Capacitor Markings

  101 = 10×10^{1} pF = 100 pF = 0.1 nF = 0.0001 μF

  102 = 10×10^{2} pF = 1000 pF = 1 nF = 0.001 μF

  103 = 10×10^{3} pF = 10,000 pF = 10 nF = 0.01 μF

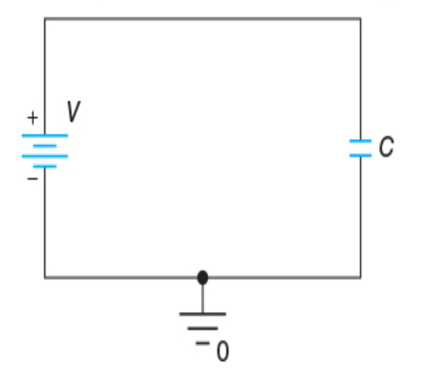
  104 = 10×10^{4} pF = 100,000 pF = 100 nF = 0.1 μF

  223 = 22×10^{3} pF = 22,000 pF = 22 nF = 0.022 μF

  474 = 47×10^{4} pF = 470,000 pF = 470 nF = 0.47 μF

FIGURE 6.2

A capacitor connected to a battery.



Current-Voltage Relation of a Capacitor

- Let v(t) be the voltage across a capacitor and i(t) be the current through the capacitor, as shown in Figure 6.5.
- Since the current is defined as the time rate of change of the charge, we have

$$i(t) = \frac{dQ(t)}{dt} \quad (1)$$

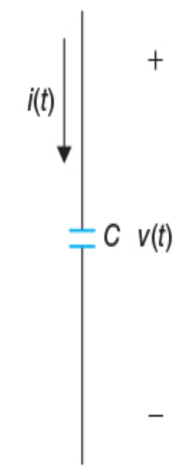
- Since Q(t) = Cv(t), Equation (1) becomes

$$i(t) = C\frac{dv(t)}{dt} \quad (2)$$

- The current through the capacitor is proportional to the time rate of change to the voltage across the capacitor. When a dc voltage is applied to a capacitor, the current through the capacitor is zero in the steady state [dv(t)/dt = 0]. The capacitor acts as an open circuit to dc input in the steady state.

FIGURE 6.5

Voltage across and current through a capacitor.



Current-Voltage Relation of a Capacitor (Continued)

- **If Equation (2) is integrated, we obtain**

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\lambda)d\lambda = v(0) + \frac{1}{C}\int_{0}^{t} i(\lambda)d\lambda$$

where v(0) is the voltage across the capacitor at t = 0.

- **The instantaneous power on the capacitor is given by**

$$p(t) = v(t)i(t) = v(t)C\frac{dv(t)}{dt} = Cv(t)\frac{dv(t)}{dt}$$

- **The energy stored in the capacitor at time t is given by**

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda = C\int_{-\infty}^{t} v(\lambda)\frac{dv(\lambda)}{d\lambda}d\lambda = C\int_{-\infty}^{t} v(\lambda)dv(\lambda) = \frac{1}{2}Cv^2(t)$$

- **If** v(t) = V **(constant),** w = 0.5CV^2**.**

EXAMPLE 6.2

- The voltage across a capacitor with capacitance of 100 μF is given by

$$v(t) = \begin{cases} 50t, & 0 \le t < 1 \\ -50t + 100, & 1 \le t < 3 \\ 50t - 200, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{V}$$

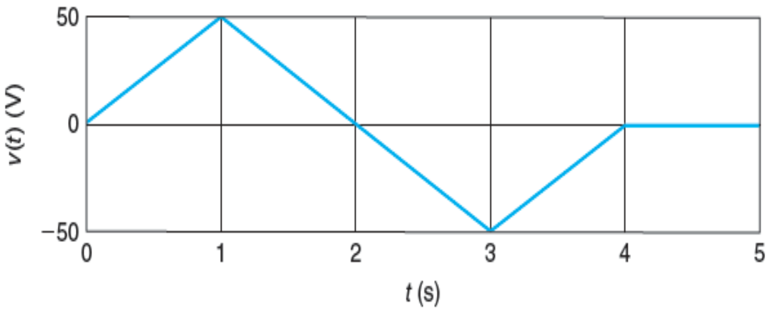
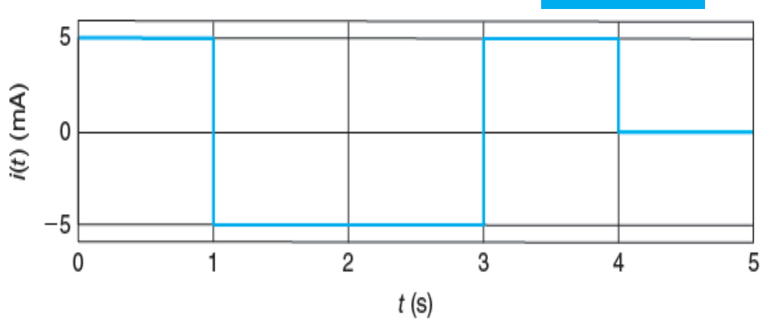


FIGURE 6.6

- The current through the capacitor is given by

$$i(t) = C\frac{dv(t)}{dt} = \begin{cases} 100 \times 10^{-6} \times 50, & 0 \le t < 1 \\ 100 \times 10^{-6} \times (-50), & 1 \le t < 3 \\ 100 \times 10^{-6} \times 50, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{A} = \begin{cases} 5, & 0 \le t < 1 \\ -5, & 1 \le t < 3 \\ 5, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{mA}$$

FIGURE 6.7



- v(t) is shown in Figure 6.6 and i(t) is shown in Figure 6.7.

EXAMPLE 6.2 (Continued)

- The instantaneous power absorbed (p(t) > 0) or released (p(t) < 0) is given by

$$p(t) = v(t)i(t) = \begin{cases} 0.25t, & 0 \le t < 1 \\ 0.25t - 0.5, & 1 \le t < 3 \\ 0.25t - 1, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ W}$$

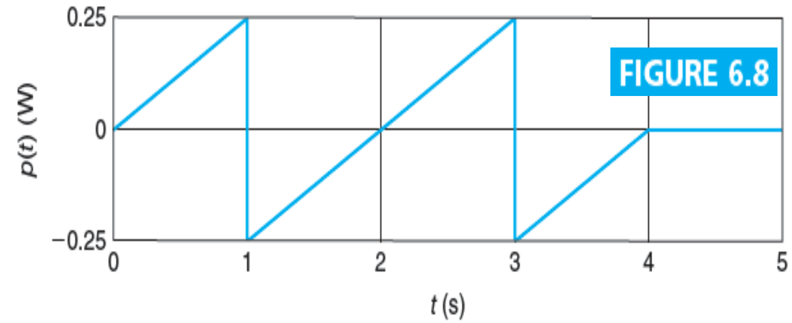


FIGURE 6.8

- The energy stored in the capacitor is given by

$$w(t) = \frac{1}{2}Cv^2(t) = \begin{cases} 0.125t^2, & 0 \le t < 1 \\ 0.125(t-2)^2, & 1 \le t < 3 \\ 0.125(t-4)^2, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ J}$$

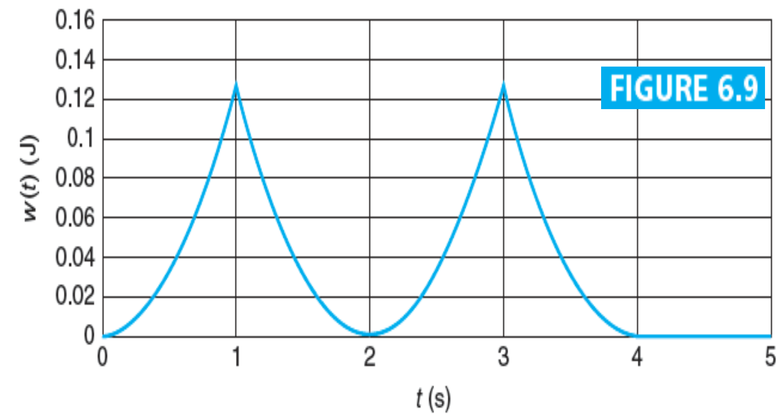


FIGURE 6.9

- The energy stored in the capacitor increases during intervals 0 < t < 1 s and 2 s ≤ t < 3 s and the energy stored in the capacitor decreases during intervals 1 s ≤ t < 2 s and 3 s ≤ t < 4 s. p(t) in Figure 6.8 and w(t) in Figure 6.9.

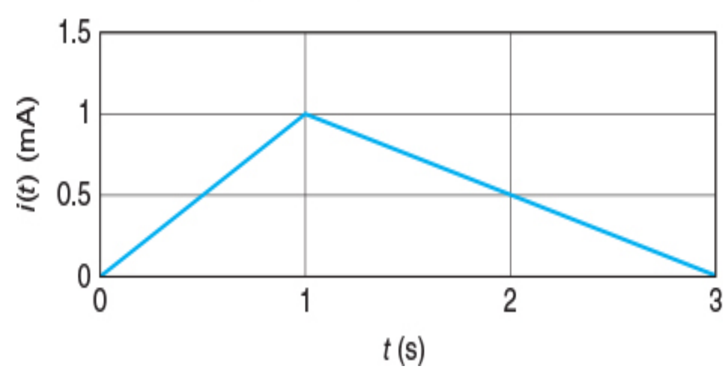
EXAMPLE 6.3

- The current through a capacitor with capacitance 100 μF is shown in Figure 6.11. Find the voltage, power, and energy on the capacitor.
- The current through a capacitor is given by

$$i(t)=\begin{cases} t, & 0\le t<1 \\ -\dfrac{t}{2}+\dfrac{3}{2}, & 1\le t<3 \text{ mA} \\ 0, & \text{otherwise} \end{cases}$$

FIGURE 6.11

The current through the capacitor.



- The voltage across the capacitor is given by

$$v(t)=\frac{1}{C}\int_0^t i(\lambda)d\lambda=\begin{cases} \dfrac{10^{-3}}{100\times10^{-6}}\displaystyle\int_0^t \lambda d\lambda = 5t^2,\ 0\le t<1 \\ 5+\dfrac{10^{-3}}{100\times10^{-6}}\displaystyle\int_1^t\left(\frac{-\lambda}{2}+\frac{3}{2}\right)d\lambda = 5+10\left[\frac{-\lambda^2}{4}+\frac{3}{2}\lambda\right]_1^t = -2.5(t-3)^2+15,\ 1\le t<3 \\ 15,\ 3\le t \end{cases} \text{V}$$

EXAMPLE 6.3 (Continued)

- The power on the capacitor is given by

$$p(t) = v(t)i(t) = \begin{cases} t^3/200, \ 0 \le t < 1 \\ [(t-3)(t^2-6t+3)]/800, \ 1 \le t < 3 \text{ W} \\ 0, \text{ otherwise} \end{cases}$$

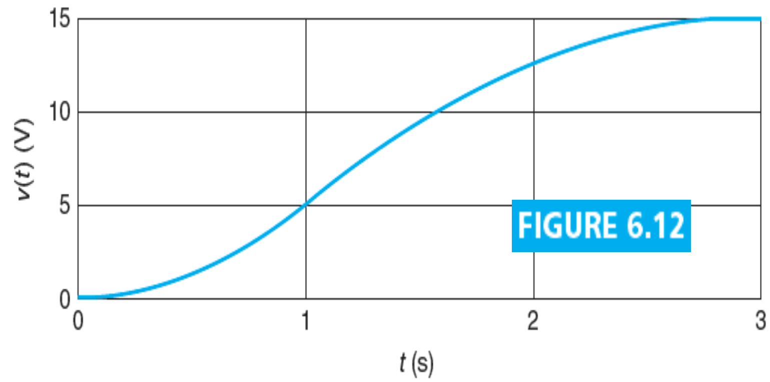


FIGURE 6.12

- The energy stored on the capacitor is given by

$$w(t) = \frac{1}{2}Cv^2(t) = \begin{cases} t^4/800, \ 0 \le t < 1 \\ [(t-3)^2-6]^2/3200, \ 1 \le t < 3 \text{ J} \\ 0.01125, \ 3 \le \text{t} \end{cases}$$

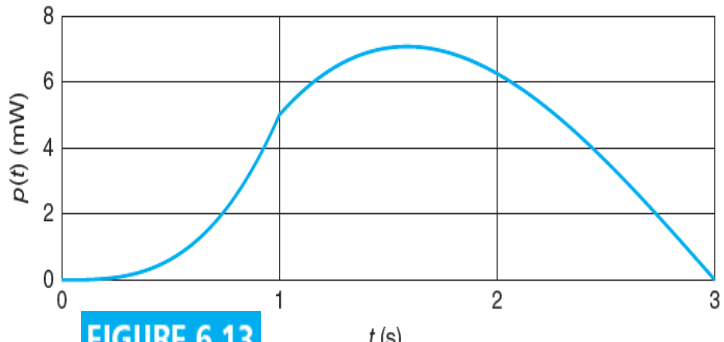


FIGURE 6.13

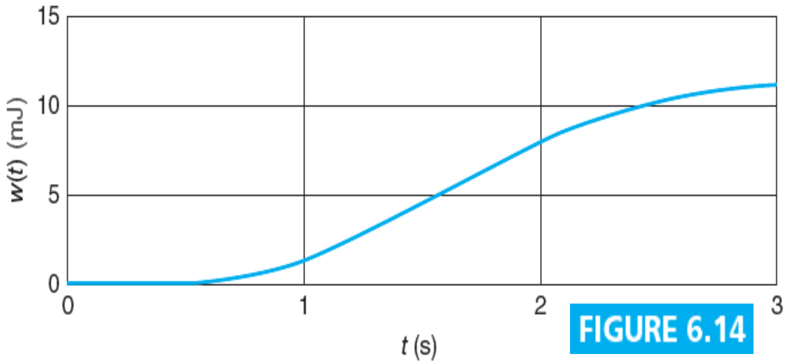


FIGURE 6.14

Sinusoidal Input to a Capacitor

- A sinusoidal voltage, v(t) = cos(2π10t) is applied to a capacitor with capacitance 0.01 F as shown in Figure 6.15.
- The current through the capacitor is given by

$$i(t) = C\frac{dv(t)}{dt} = 0.01\times(-1)\times(2\pi 10)\times\sin(2\pi 10t)$$
$$= -0.6283\sin(2\pi 10t) = 0.6283\cos(2\pi 10t + 90^{o})\,\mathrm{A}$$

- The phase of current is 90°, compared to 0° for the voltage. The current leads the voltage by 90°.
- The current crosses zero T/4 s earlier than voltage as shown in Figure 6.16. T is a period given by 0.1 s. T/4 s is equivalent to 360°/4 = 90°

FIGURE 6.15

Sinusoidal input to a capacitor.

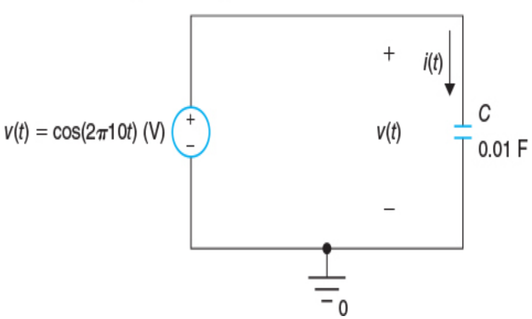
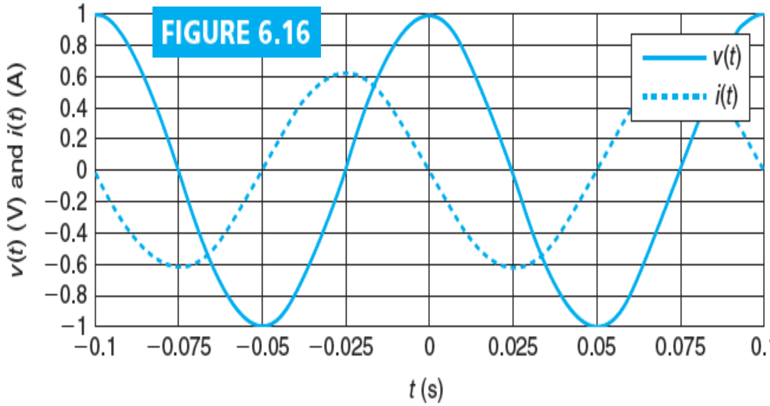


FIGURE 6.16



Sinusoidal Input to a Capacitor (Continued)

- **A sinusoidal voltage,** $v(t) = V_m \cos(2\pi ft)$ **is applied to a capacitor with capacitance C as shown in Figure 6.17.**
- **The current through the capacitor is given by**

$$i(t) = C\frac{dv(t)}{dt} = CV_m \times (-1) \times (2\pi f) \times \sin(2\pi ft) = -CV_m 2\pi f \times \sin(2\pi ft)$$

- **The amplitude of the current is proportional to the frequency of the voltage applied. As the frequency decreases, the amplitude decreases as shown in Figure 6.18.**
- **For a dc voltage (f = 0), the current through the capacitor is zero in the steady state.**
- **The capacitor acts as an open circuit for a dc voltage, and acts as a short circuit for high frequency voltage.**

FIGURE 6.17

Sinusoidal input to a capacitor.

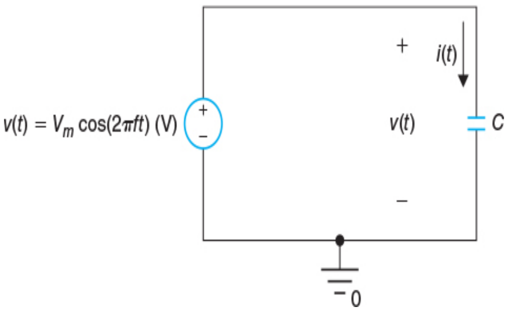
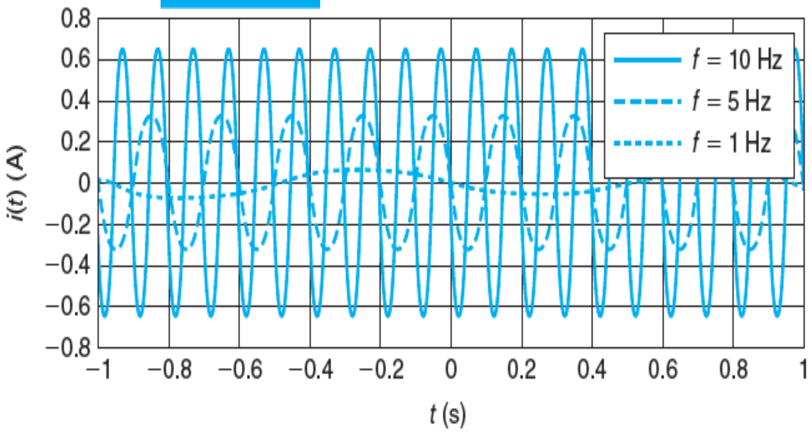


FIGURE 6.18



Series Connection of Capacitors

- **Figure 6.19(a) shows n capacitors connected in series. The current through the n capacitors is i(t). Let $v_1(t)$ be the voltage across C_1, $v_2(t)$ be the voltage across C_2, . . ., $v_n(t)$ be the voltage across C_n, and v(t) be the voltage across all n capacitors. Then,** $v(t) = v_1(t) + v_2(t) + \ldots + v_n(t)$ (1)
- **Substitution of the i-v relation to Equation (1) yields**

$$v_k(t) = \frac{1}{C_k}\int_{-\infty}^{t} i(\lambda)d\lambda,\ 1 \le k \le n$$

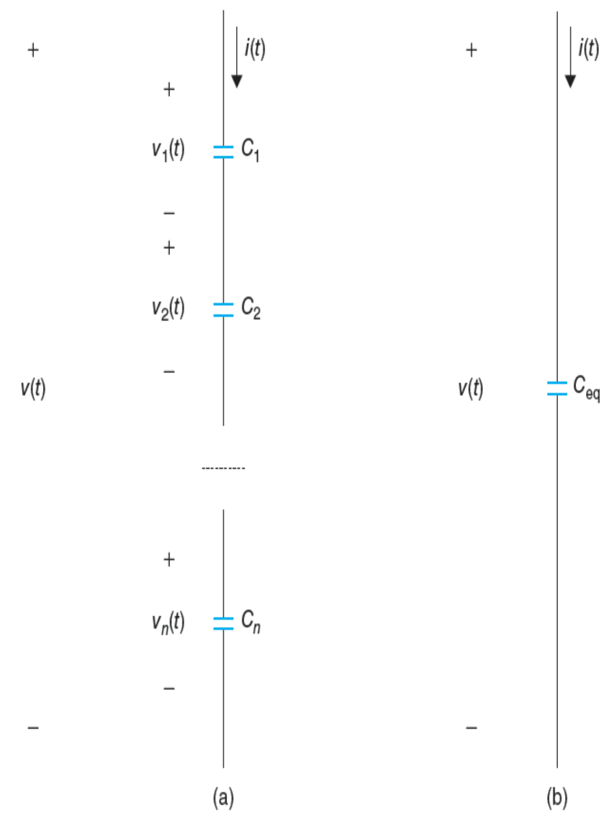
$$v(t) = \frac{1}{C_1}\int_{-\infty}^{t} i(\lambda)d\lambda + \frac{1}{C_2}\int_{-\infty}^{t} i(\lambda)d\lambda + \ldots + \frac{1}{C_n}\int_{-\infty}^{t} i(\lambda)d\lambda$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}\right)\int_{-\infty}^{t} i(\lambda)d\lambda = \frac{1}{C_{eq}}\int_{-\infty}^{t} i(\lambda)d\lambda$$

where C_{eq} is

$$C_{eq} = \frac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \ldots + \dfrac{1}{C_n}}$$

FIGURE 6.19



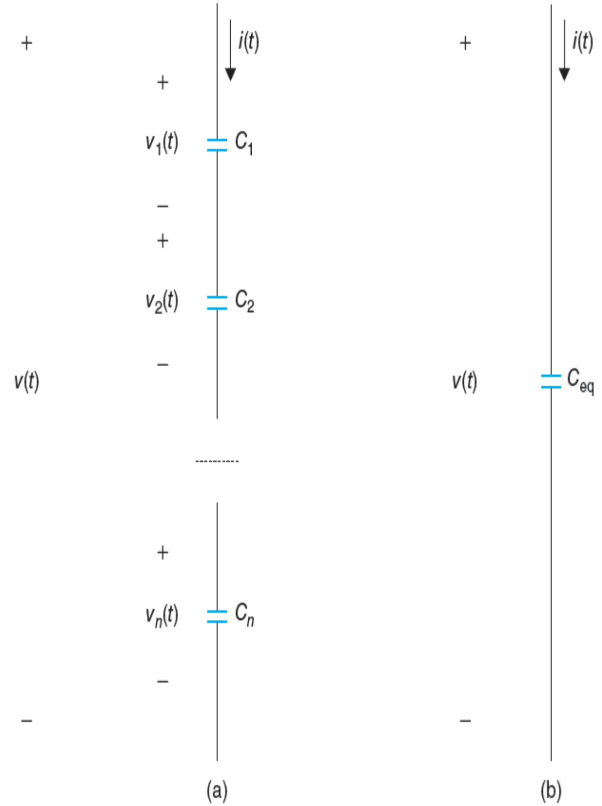
Series Connection of Capacitors (Continued)

- Notice that the equation for the equivalent capacitance of the series connected capacitors is similar to the equivalent resistance of parallel connected resistors.
- The equivalent capacitance of two capacitors with capacitances C_1 and C_2 connected in series is given by

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

- The equivalent capacitance of three capacitors with capacitances C_1, C_2, and C_3 connected in series is given by

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3}$$



Parallel Connection of Capacitors

- **Figure 6.20(a) shows n capacitors connected in parallel. The voltage across the n capacitor is v(t). Let $i_1(t)$ be the current through C_1, $i_2(t)$ be the current through C_2, . . ., $i_n(t)$ be the current through C_n, and i(t) be the current through all n capacitors. Then,** $i(t) = i_1(t) + i_2(t) + \ldots + i_n(t)$ (1)
- **Substitution of the i-v relation to Equation (1) yields**

$$i_k(t) = C_k \frac{dv(t)}{dt}, \quad 1 \le k \le n$$

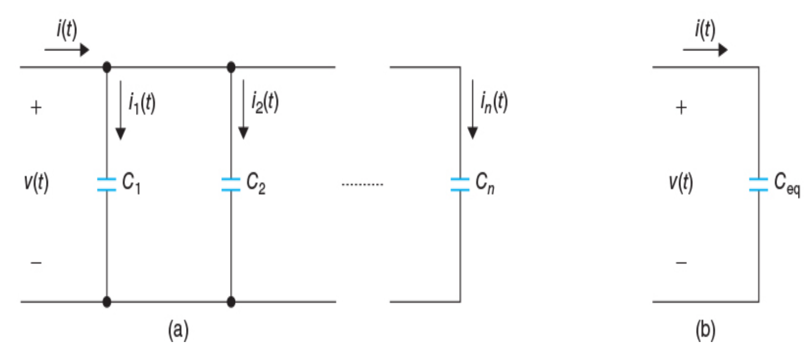
$$i(t) = i_1(t) + i_2(t) + \ldots + i_n(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + \ldots + C_n \frac{dv(t)}{dt} = \left(C_1 + C_2 + \ldots + C_n\right) \frac{dv(t)}{dt} = C_{eq} \frac{dv(t)}{dt}$$

where the equivalent capacitance, C_{eq}, is given by

$C_{eq} = C_1 + C_2 + \ldots + C_n$

FIGURE 6.20

(a) Parallel connection of capacitors. (b) Equivalent capacitor.



EXAMPLE 6.4

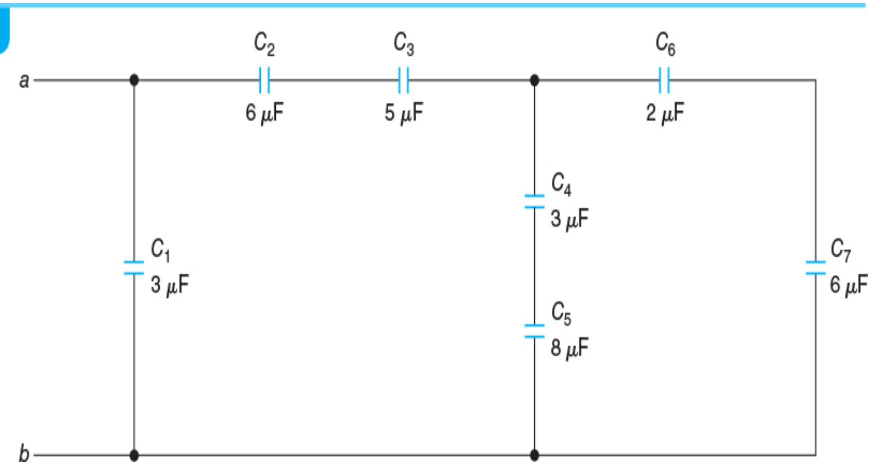
- **Find the equivalent capacitance between a and b for the circuit shown in Figure 6.21.**
- **C_8 = Equivalent capacitance of C_6 and C_7 = 2×6/8 μF = 12/8 μF = 1.5 μF.**
- **C_9 = Equivalent capacitance of C_4 and C_5 = 3×8/11 μF = 24/11 μF = 2.1818 μF.**
- **C_{10} = Equivalent capacitance of C_8 and C_9 = $C_8 + C_9$ = 3.6818 μF.**
- **C_{11} = Equivalent capacitance of C_2, C_3, and C_{10}**

$$C_{11} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_{11}}} = \frac{1}{\frac{1}{6} + \frac{1}{5} + \frac{1}{3.6818}} \mu\text{F} = 1.5667\,\mu\text{F}$$

- **The equivalent capacitance is $C_{eq} = C_1 + C_{11}$ = 4.5667 μF.**

FIGURE 6.21

Circuit for EXAMPLE 6.4.



EXAMPLE 6.5

- You have three capacitors with capacitance values of $C_1 = 0.1\ \mu F$, $C_2 = 0.22\ \mu F$, and $C_3 = 0.47\ \mu F$, respectively. List all the capacitance values that you can get from these three capacitors.
- a. Use one capacitor: $0.1\ \mu F$, $0.22\ \mu F$, $0.47\ \mu F$
- b. Use two capacitors:

  Parallel: $0.1\ \mu F + 0.22\ \mu F = 0.32\ \mu F$, $0.1\ \mu F + 0.47\ \mu F = 0.57\ \mu F$, $0.22\ \mu F + 0.47\ \mu F = 0.69\ \mu F$

  Series: $C_1C_2/(C_1 + C_2) = 0.06875\ \mu F$, $C_1C_3/(C_1 + C_3) = 0.0825\ \mu F$, $C_2C_3/(C_2 + C_3) = 0.1499\ \mu F$
- c. Use three capacitors: All three parallel: $C_1 + C_2 + C_3 = 0.79\ \mu F$

  All three series: $C_1C_2C_3/(C_1C_2 + C_1C_3 + C_2C_3) = 0.06\ \mu F$

  Two parallel, series with third: $0.32 \times 0.47/(0.32 + 0.47)\ \mu F = 0.1904\ \mu F$

  $0.57 \times 0.22/(0.57 + 0.22)\ \mu F = 0.1587\ \mu F$, $0.69 \times 0.1/(0.69 + 0.1)\ \mu F = 0.0873\ \mu F$

  Two series, parallel with third: $0.06875\ \mu F + 0.47\ \mu F = 0.5387\ \mu F$,

  $0.0825\ \mu F + 0.22\ \mu F = 0.3025\ \mu F$, $0.1499\ \mu F + 0.1\ \mu F = 0.2499\ \mu F$

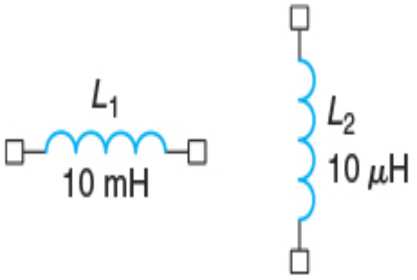
Inductors

•An inductor is a passive circuit element that can store energy in the form of a magnetic field. The circuit symbol for inductors is shown in Figure 6.26.

•A simple inductor consists of a solenoid, which is a wire wound in helix. The core can be filled with air or a ferromagnetic material. When current flows through an inductor, it introduces a magnetic field. According to Faraday's law, a changing magnetic field induces an electromotive force. The inductance of an inductor is the measure of the inductor to generate magnetic flux for the given change of current. The amount of energy stored in the inductor depends on the inductance and the current through the inductor.

FIGURE 6.26

Circuit symbol for inductors.



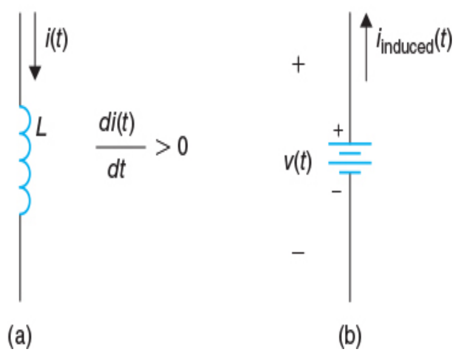
Current-Voltage Relation of an Inductor

- If an inductor is traversed in the direction of current, and the time rate of change of the current is positive, di(t)/dt > 0, as shown in Figure 6.32(a), according to Lenz's law, the current $i_{induced}(t)$ generated from the induced emf will flow in the opposite direction of i(t) to oppose the increase in i(t), as shown in Figure 6.32(b). To generate the induced current in the direction shown in Figure 6.32(b), the polarity of the induced voltage should be the one shown in Figure 6.32(b). Thus, the voltage v(t) is given by

$$v(t) = L\frac{di(t)}{dt} \quad (1)$$

FIGURE 6.32

Polarity of the voltage $v(t)$ for $\frac{di(t)}{dt} > 0$.



Current - Voltage Relation of an Inductor (Continued)

- **If Equation (1) is integrated, we obtain**

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\lambda)d\lambda = i(0) + \frac{1}{L}\int_{0}^{t} v(\lambda)d\lambda$$

where i(0) **is the current through the inductor at t = 0.**

- **The instantaneous power on the inductor is given by**

$$p(t) = i(t)v(t) = i(t)\left[L\frac{di(t)}{dt}\right] = Li(t)\frac{di(t)}{dt}$$

- **The energy stored in the inductor is given by**

$$w(t) = \int_{-\infty}^{t} p(\lambda)d\lambda = L\int_{-\infty}^{t} i(\lambda)\frac{i(\lambda)}{d\lambda}d\lambda = L\int_{-\infty}^{t} i(\lambda)i(\lambda) = \frac{1}{2}Li^2(t)$$

- **If the current through the inductor is constant,** i(t) = I**, the energy stored is** $W = 0.5LI^2$**.**

EXAMPLE 6.8

- The current through an inductor with inductance of 100 mH is given by

$$i(t) = \begin{cases} 20t, & 0 \le t < 1 \\ -40t + 60, & 1 \le t < 2 \\ 10t - 40, & 2 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ A}$$

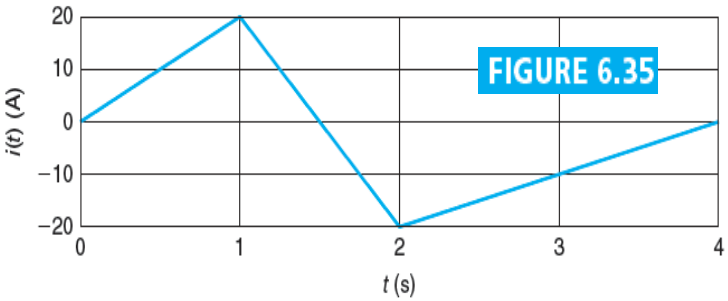


FIGURE 6.35

- The voltage across the inductor is given by

$$v(t) = L\frac{di(t)}{dt} = \begin{cases} 0.1 \times 20, & 0 \le t < 1 \\ 0.1 \times (-40), & 1 \le t < 2 \\ 0.1 \times 10, & 2 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{V} = \begin{cases} 2, & 0 \le t < 1 \\ -4, & 1 \le t < 2 \\ 1, & 2 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ V}$$

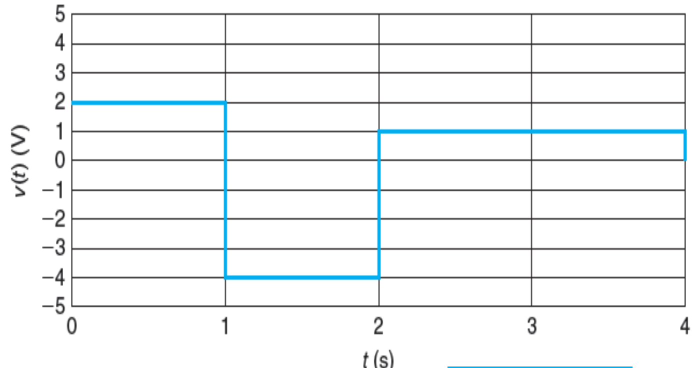


FIGURE 6.36

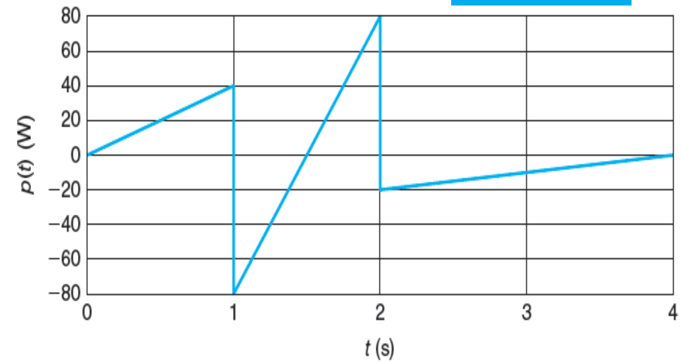
- The current i(t) is shown in Figure 6.35 and voltage v(t) is shown in Figure 6.36. The power p(t) is shown in Figure 6.37 and energy w(t) is shown in Figure 6.38.

EXAMPLE 6.8 (Continued)

- The instantaneous power absorbed (p(t) > 0) or released (p(t) < 0) is given by

$$p(t) = v(t)i(t) = \begin{cases} 40t, & 0 \le t < 1 \\ 160t - 240, & 1 \le t < 2 \\ 10t - 40, & 2 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ W}$$

FIGURE 6.37



- The energy stored in the inductor is given by

$$w(t) = \frac{1}{2}Li^2(t) = \begin{cases} 20t^2, & 0 \le t < 1 \\ 80(t-1.5)^2, & 1 \le t < 2 \\ 5(t-4)^2, & 2 \le t < 4 \\ 0, & \text{otherwise} \end{cases} \text{ J}$$

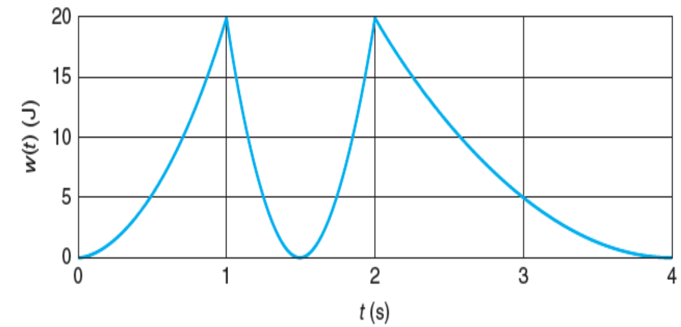


FIGURE 6.38

- The energy stored in the inductor increases during intervals 0 < t < 1 s and 1.5 s ≤ t < 2 s and the energy stored in the inductor decreases during intervals 1 s ≤ t < 1.5 s and 2 s ≤ t < 4 s.

EXAMPLE 6.9

- **A signal,** v(t) = cos(2π100t), **t ≥ 0, V is applied to an inductor with inductance of 100 mH.**
- **The current through the inductor is given by**

$$i(t) = \frac{1}{L}\int_0^t v(\lambda)d\lambda = \frac{1}{0.1}\int_0^t \cos(2\pi 100\lambda)d\lambda$$

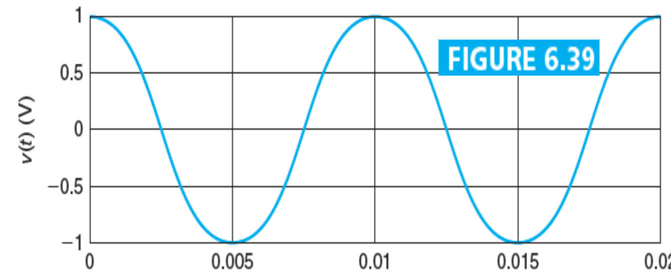
$$= \frac{10}{2\pi 100}\sin(2\pi 100t) = 0.01592\sin(2\pi 100t)\,\mathrm{A}$$

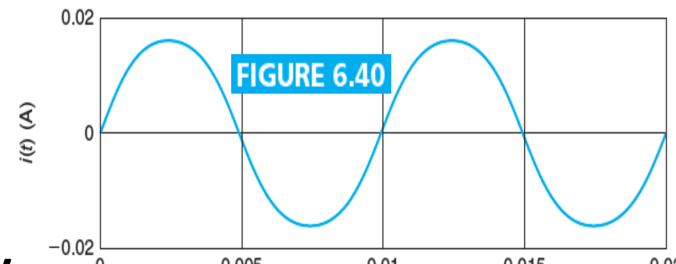
- **The instantaneous power of the inductor is given by**

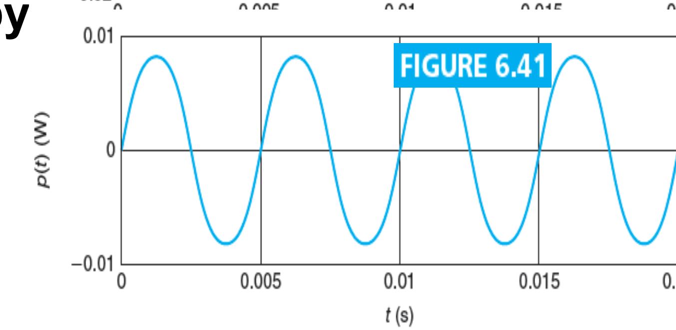
$$p(t) = v(t)i(t) = \frac{\sin(2\pi 200t)}{400\pi} = 0.007958\sin(2\pi 200t)\,\mathrm{W}$$

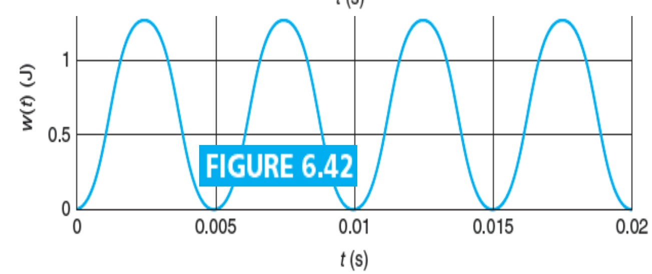
- **The energy stored in the inductor is given by**

$$w(t) = \frac{1}{2}Li^2(t) = 1.2665\times 10^{-5}\left[\sin(2\pi 100t)\right]^2\,\mathrm{J}$$









Sinusoidal Current to an Inductor

- A sinusoidal current, $i(t) = \cos(2\pi 10t)$ A is applied to an inductor with inductance 0.01 H as shown in Figure 6.43.
- The voltage across the inductor is given by

$$v(t) = L\frac{di(t)}{dt} = 0.01\times(-1)\times(2\pi 10)\times\sin(2\pi 10t)$$

$$= -0.6283\sin(2\pi 10t) = 0.6283\cos(2\pi 10t + 90^{o})\,\mathrm{V}$$

- The phase of voltage is 90°, compared to 0° for the current. The current lags the voltage by 90°.
- The current crosses zero T/4 s later than the voltage as shown in Figure 6.44. T is a period given by 0.1 s. T/4 s is equivalent to 360°/4 = 90°

FIGURE 6.43

An inductor with sinusoidal input.

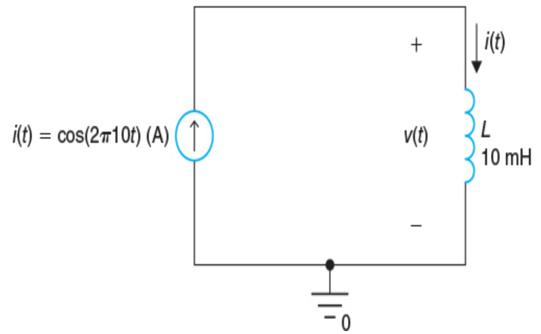
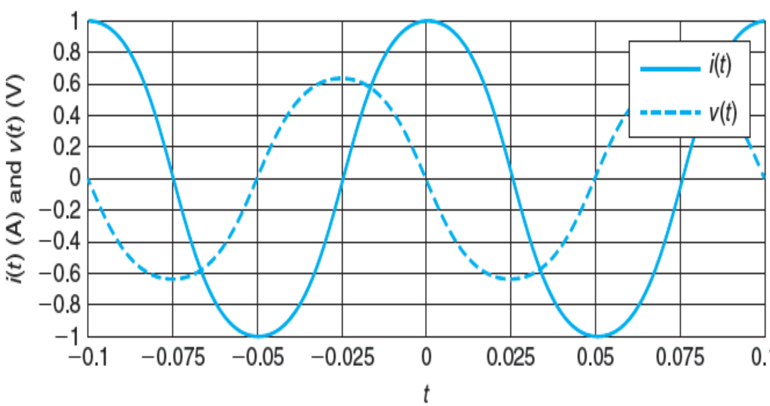


FIGURE 6.44



Sinusoidal Current to an Inductor (Continued)

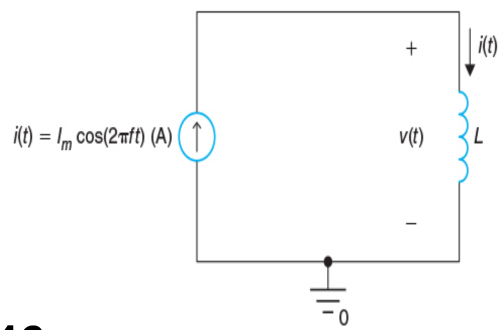
- A sinusoidal current, $i(t) = I_m \cos(2\pi ft)$ is applied to an inductor with inductance L as shown in Figure 6.45.
- The voltage across the inductor is given by

$$v(t) = L\frac{di(t)}{dt} = LI_m \times (-1) \times (2\pi f) \times \sin(2\pi ft) = -LI_m 2\pi f \times \sin(2\pi ft)$$

- The amplitude of the voltage is proportional to the frequency of the current applied. As the frequency decreases, the amplitude decreases as shown in Figure 6.46.
- For a dc current (f = 0), the voltage across the inductor is zero in the steady state.
- The inductor acts as a short circuit for a dc voltage, and acts as an open circuit for high frequency voltage.

FIGURE 6.45

An inductor with sinusoidal input.



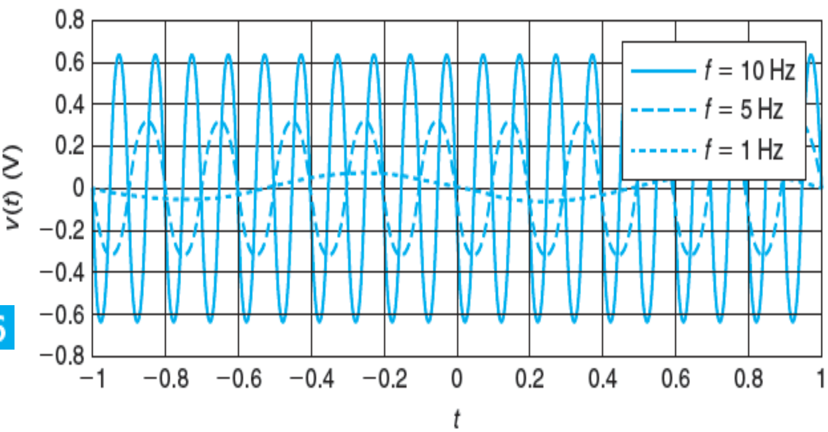


FIGURE 6.46

Series Connection of Inductors

- **Figure 6.47(a) shows n inductors connected in series. The current through the n inductors is i(t). Let $v_1(t)$ be the voltage across L_1, $v_2(t)$ be the voltage across L_2, . . ., $v_n(t)$ be the voltage across L_n, and v(t) be the voltage across all n inductors . Then,**

  $v(t) = v_1(t) + v_2(t) + \ldots + v_n(t)$ (1)

- **Substitution of the i-v relation** $v_k(t) = L_k \dfrac{di(t)}{dt}, \quad 1 \le k \le n$ **to Equation (1) yields**

$$v(t) = v_1(t) + v_2(t) + \ldots + v_n(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \ldots + L_n \frac{di(t)}{dt}$$

$$= (L_1 + L_2 + \ldots + L_n)\frac{di(t)}{dt} = L_{eq} \frac{di(t)}{dt}$$

where the equivalent inductance, L_{eq}, is given by

$L_{eq} = L_1 + L_2 + \ldots + L_n$

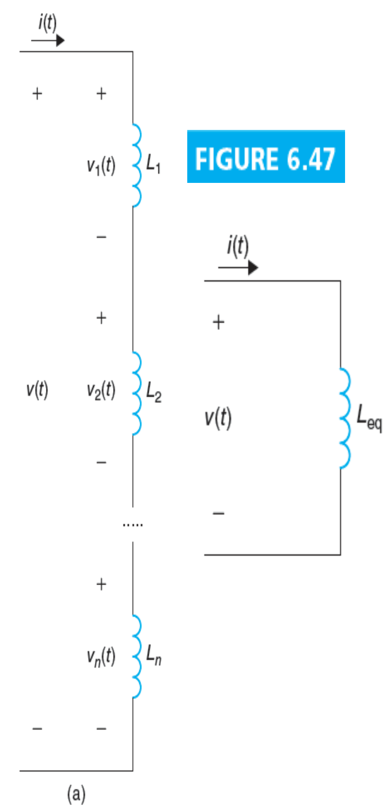


FIGURE 6.47

Parallel Connection of Inductors

•Figure 6.48(a) shows n inductors connected in parallel. The voltage across the n inductors is v(t). Let $i_1(t)$ be the current through L_1, $i_2(t)$ be the current through L_2, . . ., $i_n(t)$ be the current through L_n, and i(t) be the current through all n inductors. Then,

$$i(t) = i_1(t) + i_2(t) + \ldots + i_n(t) \quad (1)$$

•Substitution of the i-v relation $i_k(t) = \frac{1}{L_k}\int_{-\infty}^{t} v(\lambda)d\lambda,\ 1 \le k \le n$

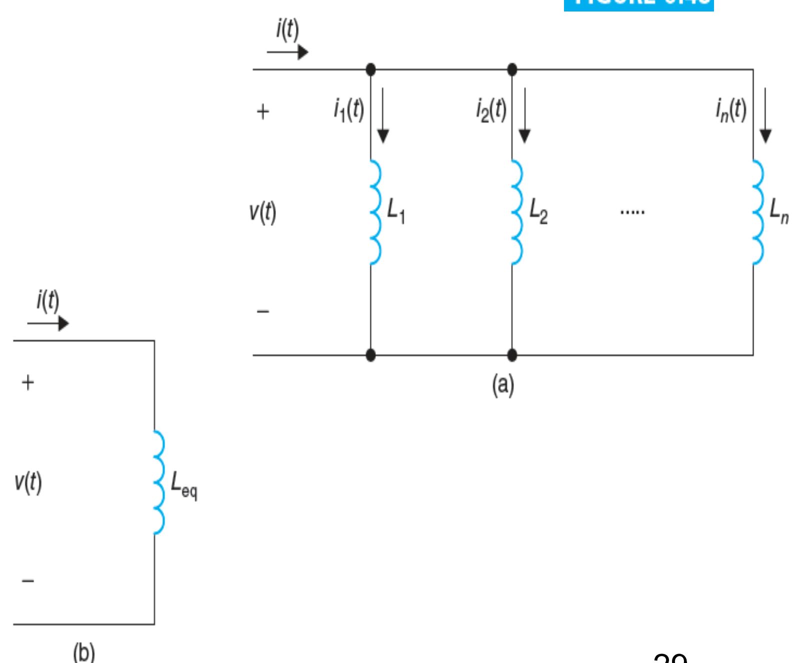
to Equation (1) yields

$$i(t) = \frac{1}{L_1}\int_{-\infty}^{t} v(\lambda)d\lambda + \frac{1}{L_2}\int_{-\infty}^{t} v(\lambda)d\lambda + \ldots + \frac{1}{L_n}\int_{-\infty}^{t} v(\lambda)d\lambda$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}\right)\int_{-\infty}^{t} v(\lambda)d\lambda = \frac{1}{L_{eq}}\int_{-\infty}^{t} v(\lambda)d\lambda$$

where L_{eq} is $$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}}$$

FIGURE 6.48



Parallel Connection of Inductors (Continued)

- **Notice that the equation for the equivalent inductance of the parallel connected inductors is similar to the equivalent resistance of parallel connected resistors.**
- **The equivalent inductance of two inductors with inductances L_1 and L_2 connected in parallel is given by**

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$

- **The equivalent inductance of three inductors with inductances L_1, L_2, and L_3 connected in parallel is given by**

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}} = \frac{L_1 L_2 L_3}{L_1 L_2 + L_1 L_3 + L_2 L_3}$$

EXAMPLE 6.10

- Find the equivalent inductance L_{eq} between *a* and *b* for the circuit shown in Figure 6.49.
- Let $L_a = L_4 \,||\, L_5 \,||\, L_6$. Then,

$$L_a = \frac{1}{\frac{1}{L_4}+\frac{1}{L_5}+\frac{1}{L_6}} = \frac{1}{\frac{1}{6}+\frac{1}{7}+\frac{1}{21}}\,\text{mH} = \frac{42}{\frac{42}{6}+\frac{42}{7}+\frac{42}{21}}\,\text{mH} = \frac{42}{7+6+2}\,\text{mH} = \frac{42}{15}\,\text{mH} = 2.8\,\text{mH}$$

- Let $L_b = L_3 + L_a$. Then,

  L_b = 3.2 mH + 2.8 mH = 6 mH
- Let $L_c = L_2 \,||\, L_b$. Then,

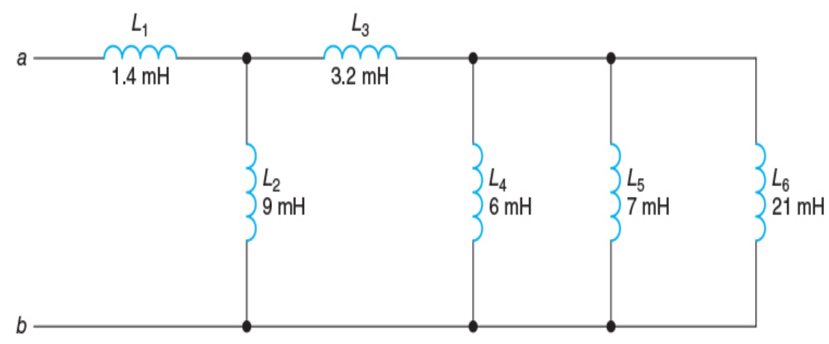
  $L_c = L_2L_b/(L_2 + L_b)$

  = 54/15 mH = 3.6 mH
- $L_{eq} = L_1 + L_c$ = 1.4 mH + 3.6 mH

  L_{eq} = 5 mH

FIGURE 6.49

Circuit for EXAMPLE 6.10.



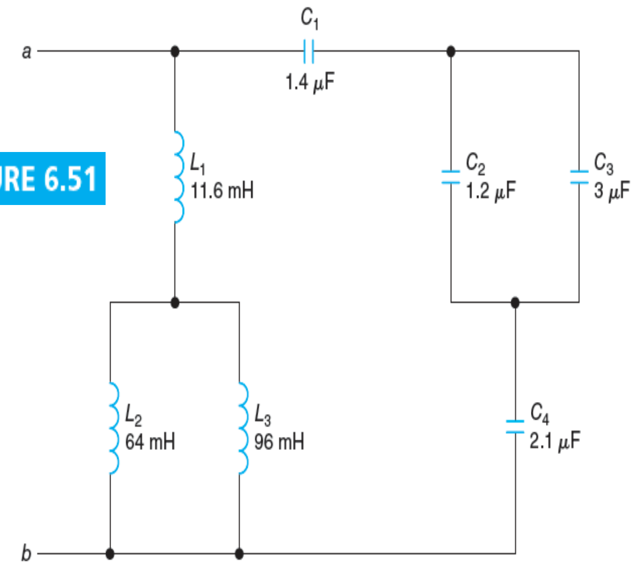
EXAMPLE 6.11

•For the circuit shown in Figure 6.51, find the equivalent circuit consisting of a single inductor and a single capacitor connected in parallel between *a and b.*

•Let $C_5 = C_2 + C_3 = 4.2\ \mu F$. The equivalent capacitance is

$$C_{eq} = \frac{1}{\frac{1}{C_1}+\frac{1}{C_5}+\frac{1}{C_4}} = \frac{1}{\frac{1}{1.4}+\frac{1}{4.2}+\frac{1}{2.1}}\mu\text{F} = \frac{4.2}{6}\mu\text{F} = 0.7\,\mu\text{F}$$

FIGURE 6.51



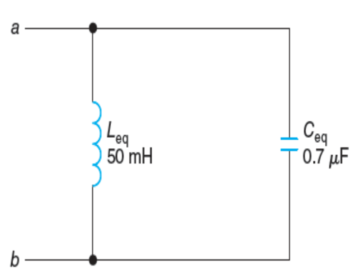
•Let $L_a = L_2 \parallel L_3$. Then,

$$L_a = \frac{L_2 \times L_3}{L_2 + L_3} = \frac{64\times 96}{64+96}\text{mH} = \frac{6144}{160}\text{mH} = 38.4\,\text{mH}$$

•$L_{eq} = L_1 + L_a = 11.6$ mH + 38.4 mH = 50 mH

•The circuit with L_{eq} and C_{eq} is shown in Figure 6.52.

FIGURE 6.52



Summary

•A capacitor is a passive element that can store and release energy. The energy is stored in the form of an electric field from the positive plate to the negative plate. The amount of energy stored on the capacitor depends on the capacitance and the voltage across the capacitor plates.

•In a parallel plate capacitor, the amount of charge Q (in coulombs) on the plates is proportional to the voltage V (in volts) across the plates. Let C be the proportionality constant in this linear relation. Then we have Q = CV (C = Q/V, V = Q/C).

•The current through a capacitor is proportional to the time rate of change of the voltage across the capacitor.

$$i(t) = C\frac{dv(t)}{dt}$$

•The voltage across a capacitor is given by $v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\lambda)d\lambda$

•The energy stored in the capacitor is given by $w(t) = \frac{1}{2}Cv^2(t)$

Summary (Continued)

- **The equivalent capacitance of two capacitors with capacitances C_1 and C_2 connected in series is given by**

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} = \frac{C_1 C_2}{C_1 + C_2}$$

- **The equivalent capacitance of n capacitors with capacitances C_1, C_2, …, C_n connected in series is given by**

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}}$$

- **The equivalent capacitance of n capacitors with capacitances C_1, C_2, …, C_n connected in parallel is given by**

  $C_n = C_1 + C_2 + \ldots + C_n$

Summary (Continued)

- **An inductor is a passive circuit element that can store energy in the form of a magnetic field.**
- **The voltage – current relation of an inductor is given by**

$$v(t) = L\frac{di(t)}{dt}$$

- **The current can be represented as**

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\lambda)d\lambda$$

- **The energy stored in the inductor is given by**

$$w(t) = \frac{1}{2}Li^2(t)$$

Summary (Continued)

- **If n inductors are connected in series, the equivalent inductance, L_{eq}, is given by** $L_{eq} = L_1 + L_2 + \ldots + L_n$

- **If n inductors are connected in parallel, the equivalent inductance, L_{eq}, is given by**

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_n}}$$

- **If two inductors are connected in parallel, the equivalent inductance, L_{eq}, is given by**

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} = \frac{L_1 L_2}{L_1 + L_2}$$