UESTC3001 Dynamics & Control
Lecture 5

Characteristics and Performance of Feedback Control Systems – I

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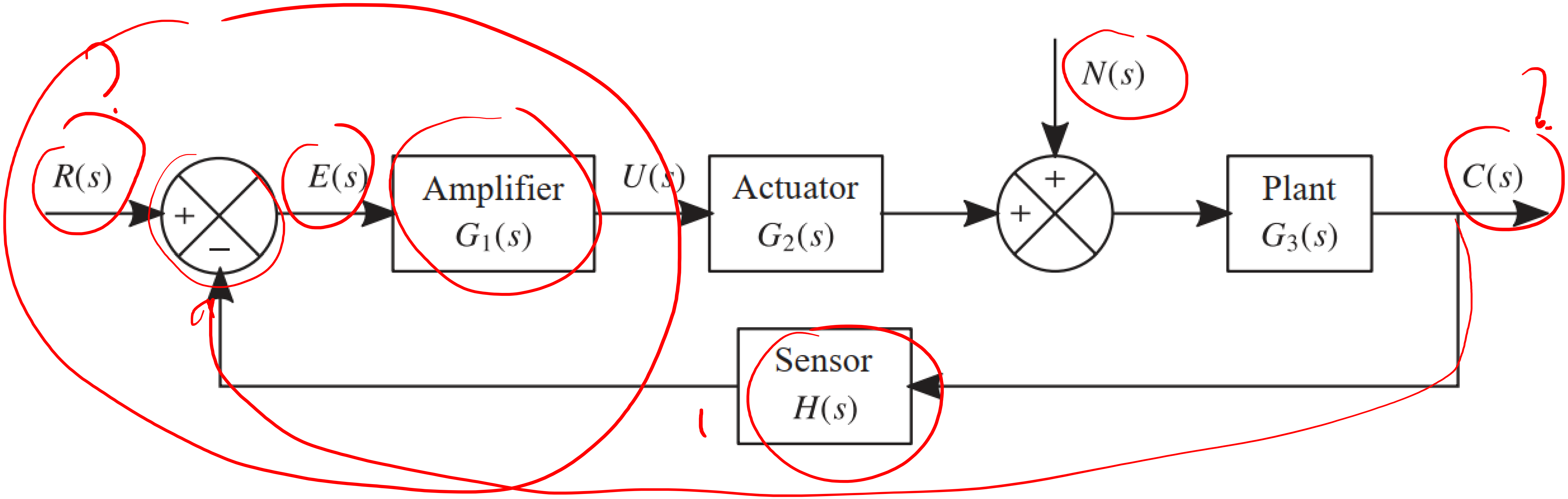
Outline

- Proportional Control, Derivative Control, Integral Control
- Proportional plus Integral Control, Proportional + Derivative Control, Proportional + Integral + Derivative Control
- Proportional Control of a First-Order/Second-order System and Effect on a First-Order/Second-order System
- Proportional + Derivative Control of a First-Order System and Effect on a First-Order System

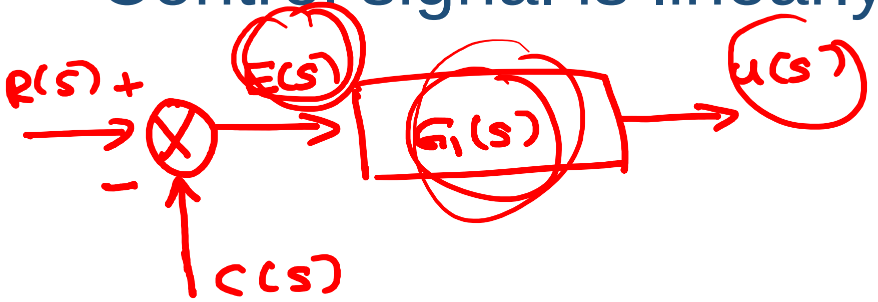
Basic Control Actions

- Controller compares actual O/P with desired O/P
- Produce a control signal to reduce the deviation



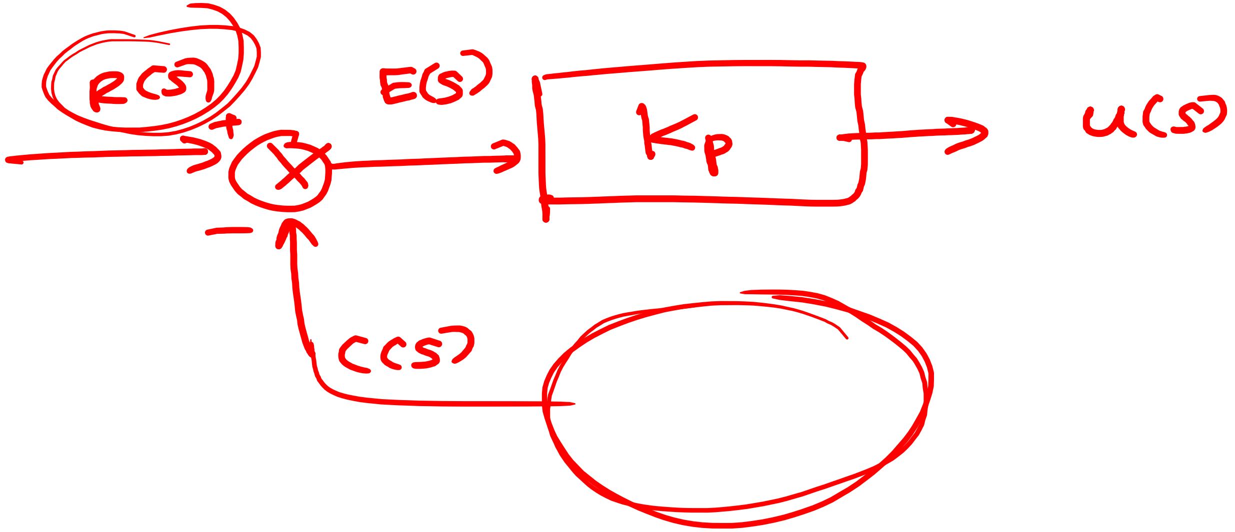
Proportional Control (P)

- Control signal is linearly proportional to the system error



$$u(t) \propto e(t) \Rightarrow u(t) = K_p e(t)$$

$$U(s) = K_p E(s) \Rightarrow \frac{U(s)}{E(s)} = G_1(s) = K_p$$



Derivative Control (D)

$u(t) \propto \frac{de(t)}{dt}$

- Improve C/L system stability, speed up the transient response etc.
- Control signal is proportional to the derivative of the system error

$$u(t) = K_d \frac{de(t)}{dt} \Rightarrow U(s) = K_d\, s\, E(s) \Rightarrow \frac{U(s)}{E(s)} = G_1(s) = K_d\, s$$

$\Rightarrow 0$

- Usually augmented by proportional control
- Tends to amplify noise
- Introduced into the feedback path to eliminate response to I/P

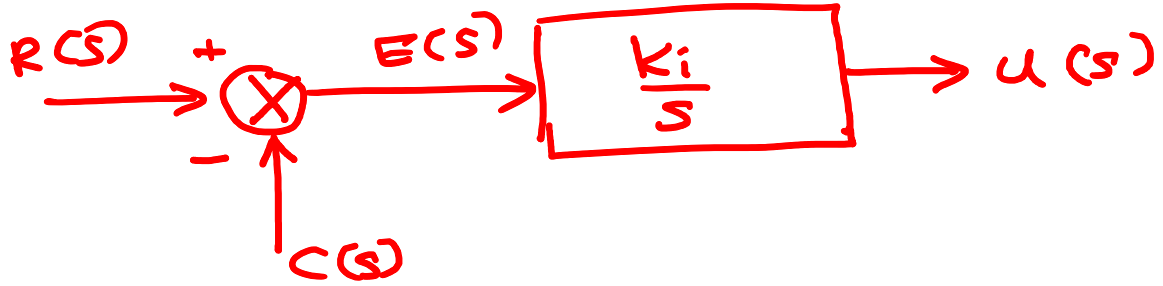
Integral Control (I)

$$u(t) \propto \int_0^t e(t)\,dt$$

- Control signal is proportional to the integral of the system error

$$u(t) = K_i \int_0^t e(t)\,dt \Rightarrow U(s) = K_i \frac{1}{s} E(s) \Rightarrow \frac{U(s)}{E(s)} = G(s) = \frac{K_i}{s} \rightarrow (s=0) \rightarrow \infty$$

- Minimize steady-state error; output response to disturbances
- Superior performance in the steady state
- Constant disturbances can be cancelled with zero error



Proportional plus Derivative Control (PD Control)

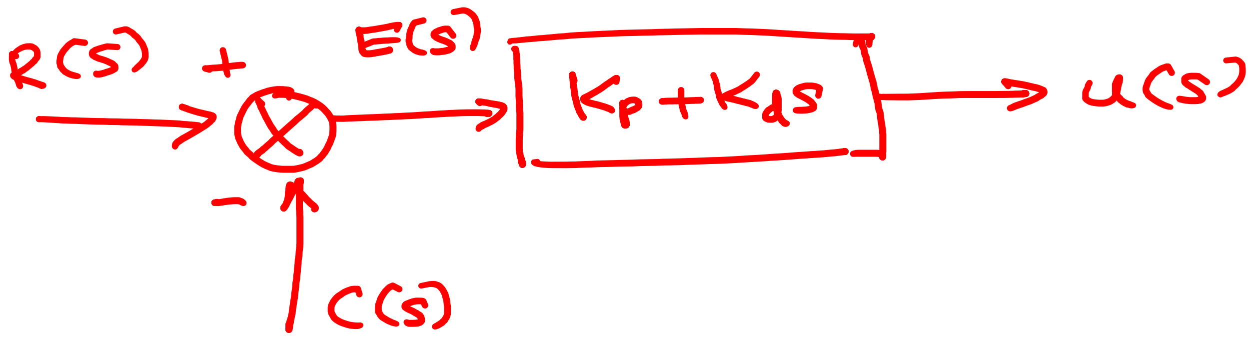
- Derivative action may be added to control action

$$\boxed{u(t) = K_p e(t) + K_d \frac{de(t)}{dt}}$$

$$\Rightarrow U(s) = K_p E(s) + K_d s\, E(s)$$

$$\frac{U(s)}{E(s)} = G_1(s) = K_p + K_d s$$

- Derivative action speed the effect of the proportional action



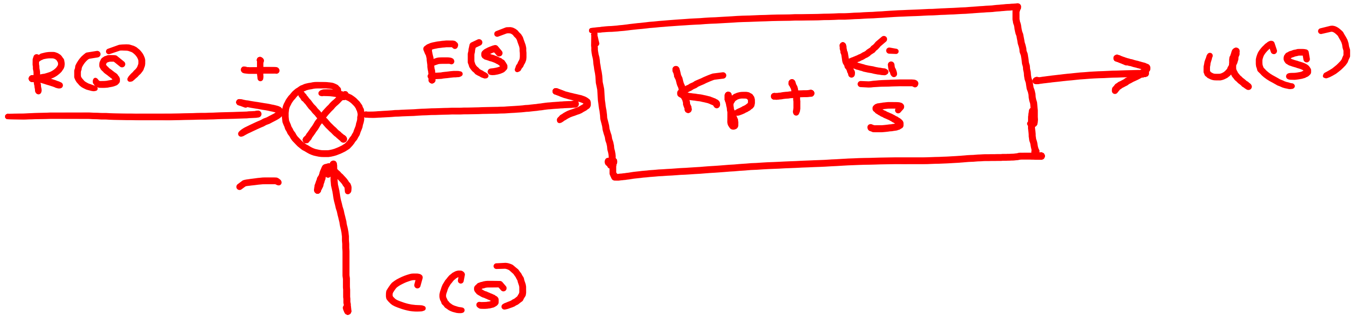
Proportional plus Integral Control (PI Control)

- Proportional action adds a steady offset to a system's response. This may be reduced by adding integral action.

$$\boxed{u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt}$$

$$\Rightarrow u(s) = K_p E(s) + K_i \frac{1}{s} E(s)$$

$$\frac{u(s)}{E(s)} = K_p + \frac{K_i}{s}$$



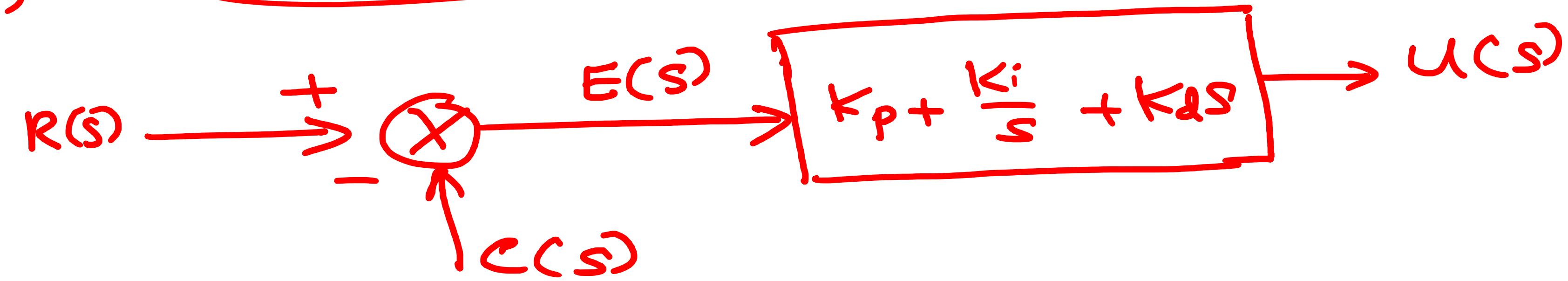
Proportional plus Integral plus Derivative Control (PID Control)

- Putting all the three terms together results in PID

$$u(t) = K_p e(t) + K_i \int_0^t e(t)\,dt + K_d \frac{de(t)}{dt}$$

$$U(s) = K_p E(s) + K_i \frac{1}{s} E(s) + K_d s E(s)$$

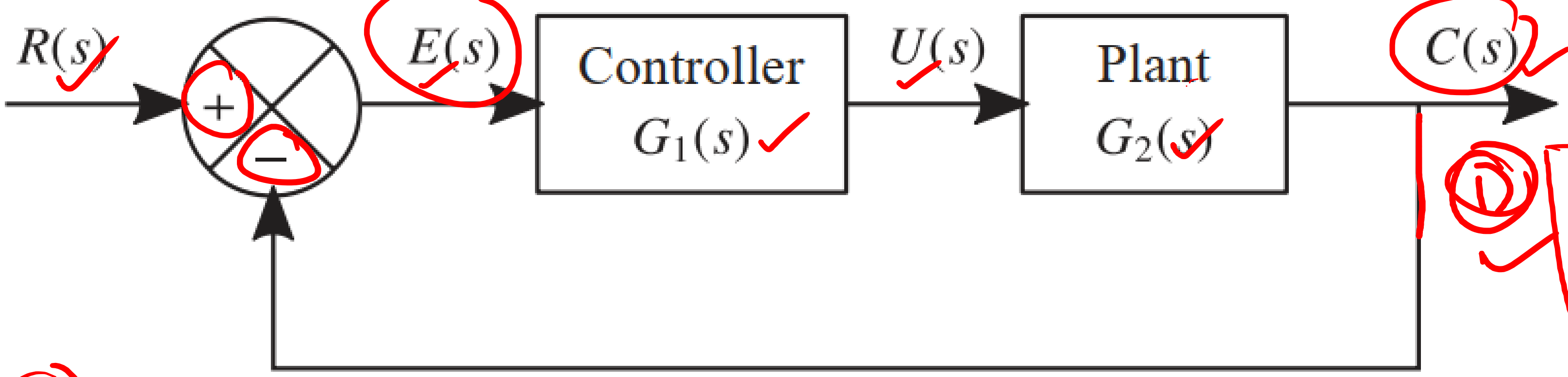
$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s = G(s)$$



Effect of Control Actions

$N(s) \times \Rightarrow 0$

$H(s) = 1$



① $$C/L = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)}$$

$$\frac{C(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)}$$

② $$E(s) = R(s) - C(s)$$

$$\frac{E(s)}{R(s)} = 1 - \frac{C(s)}{R(s)}$$

$$\frac{E(s)}{R(s)} = 1 - \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)} = \frac{1 + \cancel{G_1(s)\,G_2(s)} - \cancel{G_1(s)\,G_2(s)}}{1 + G_1(s)\,G_2(s)}$$

$$\boxed{\frac{E(s)}{R(s)} = \frac{1}{1 + G_1(s)\,G_2(s)}}$$

Uncontrolled Open-Loop Response of a First-Order System

$$\boxed{\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}}$$

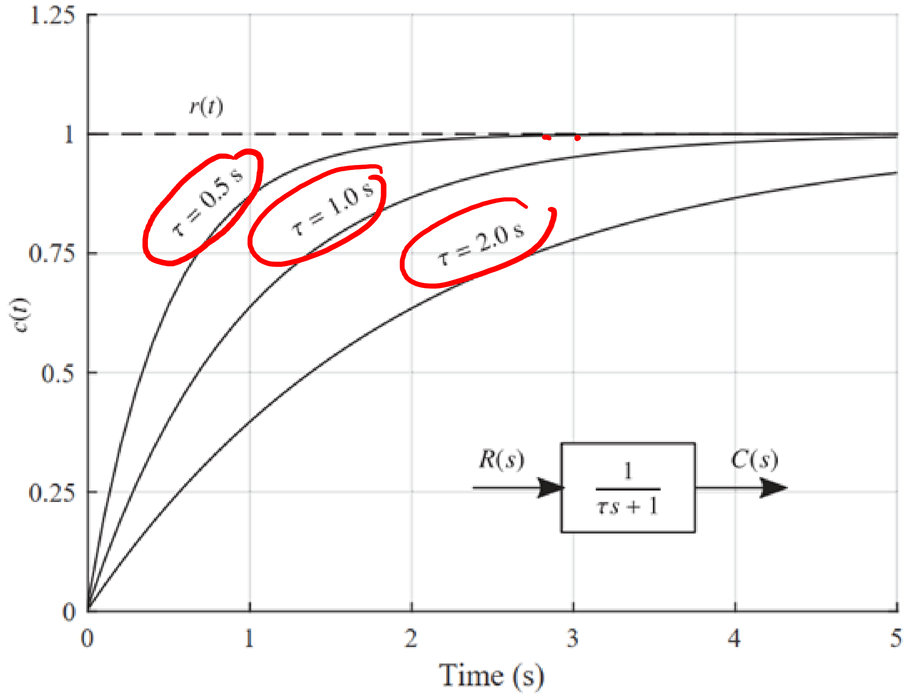
- E.g. find response for a unit step input

$R(s) = \frac{1}{s}$ $C(s)$? $\longrightarrow$ $c(t)$

$$C(s) = \frac{1}{s} \cdot \frac{1}{\tau s+1} = \frac{A}{s} + \frac{B}{s+1/\tau}, \quad A = 1, \quad B = -1$$

$$C(s) = \frac{1}{s} - \frac{1}{s+1/\tau}$$

$$c(t) = 1 - e^{-t/\tau}$$



Uncontrolled Open-Loop Response of a Second-Order System

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

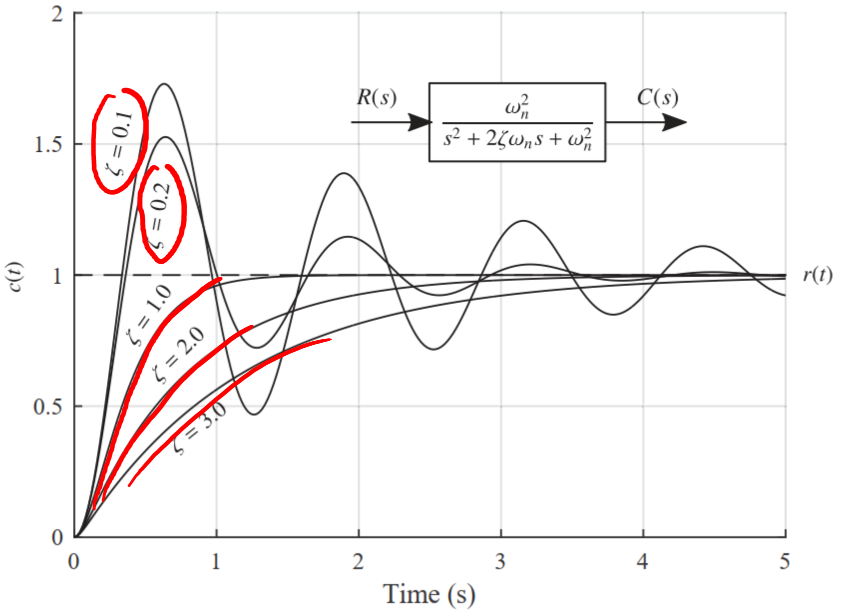
$c(t)$

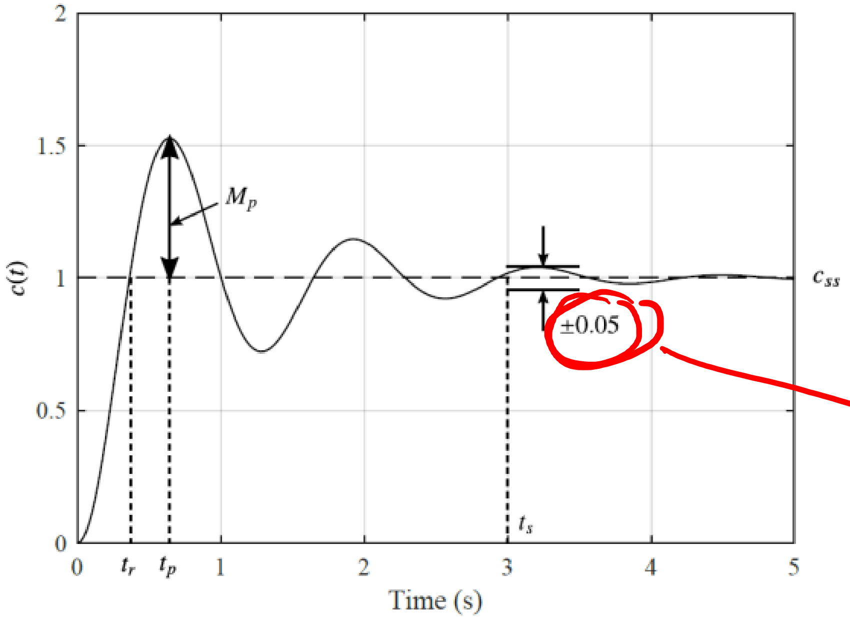
$C(s) = R(s) \times$

$R(s) = \frac{1}{s}$

$\downarrow$

$c(t)$





$$t_r = \frac{1}{\omega_d}\tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

$$t_p = \frac{\pi}{\omega_d}$$

$$M_p = 100e^{-\zeta\omega_n t_p}\%$$

$$t_s = \frac{3}{\zeta\omega_n}$$

$\pm 0.02 \Longrightarrow \boxed{t_s = \frac{4}{\zeta\omega_n}}$

Proportional Control of a First-Order System

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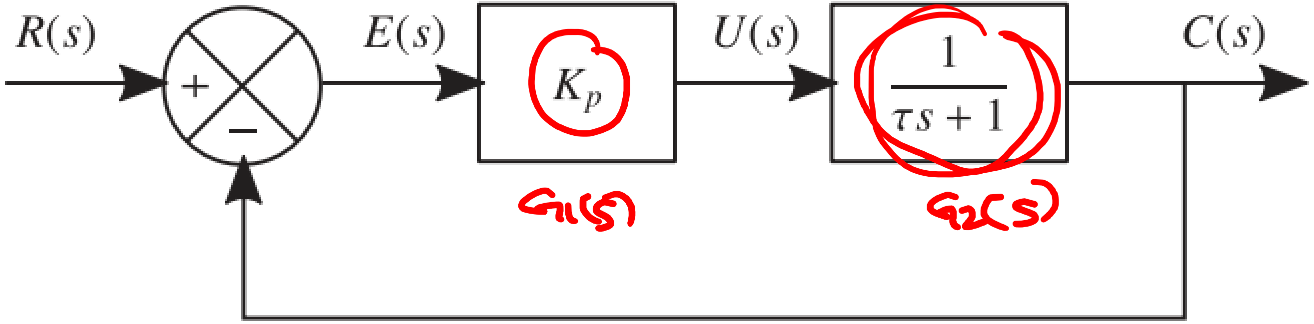


$\frac{C(s)}{R(s)}$ $\quad$ $\frac{E(s)}{R(s)}$

$R(s) = \frac{1}{s}$

$$\frac{C(s)}{R(s)} = \frac{G_1(s)\,G_2(s)}{1 + G_1(s)\,G_2(s)} = \frac{K_p/(\tau s+1)}{1 + K_p/(\tau s+1)} = \frac{K_p}{\tau s + 1 + K_p}$$

$$C(s) = \frac{1}{s} \times \frac{K_p}{\tau s + 1 + K_p} = \frac{K_p}{s(\tau s + 1 + K_p)} \rightarrow c(t)$$

$$C_{ss} = \lim_{s \to 0} s\,C(s) = \lim_{s \to 0} \cancel{s}\,\frac{K_p}{\cancel{s}(\tau s + 1 + K_p)} = \frac{K_p}{K_p + 1}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1+G_1(s)G_2(s)} = \frac{1}{1+\frac{K_p}{\tau s+1}} = \frac{\tau s+1}{\tau s+1+K_p}$$

$$R(s) = \frac{1}{s} \quad ; \quad E(s) = \frac{1}{s}\left(\frac{\tau s+1}{\tau s+1+K_p}\right)$$

$$E(s) = \frac{\tau s+1}{s(\tau s+1+K_p)}$$

$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{\tau s+1}{\cancel{s}(\tau s+1+K_p)} = \frac{1}{1+K_p}$$

$\to 0$

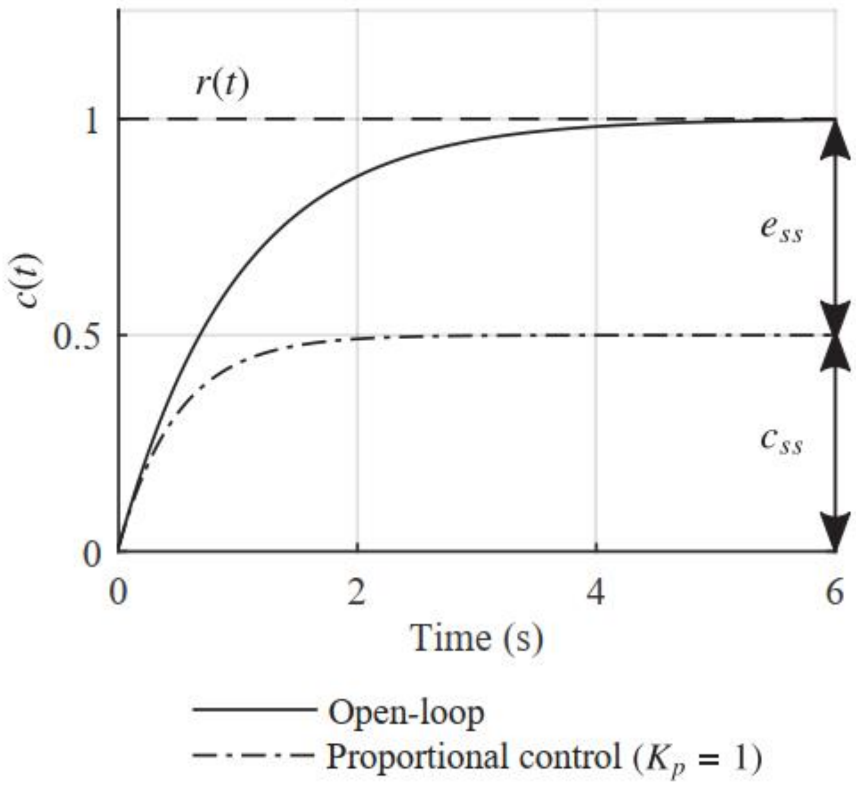
$\to 0$

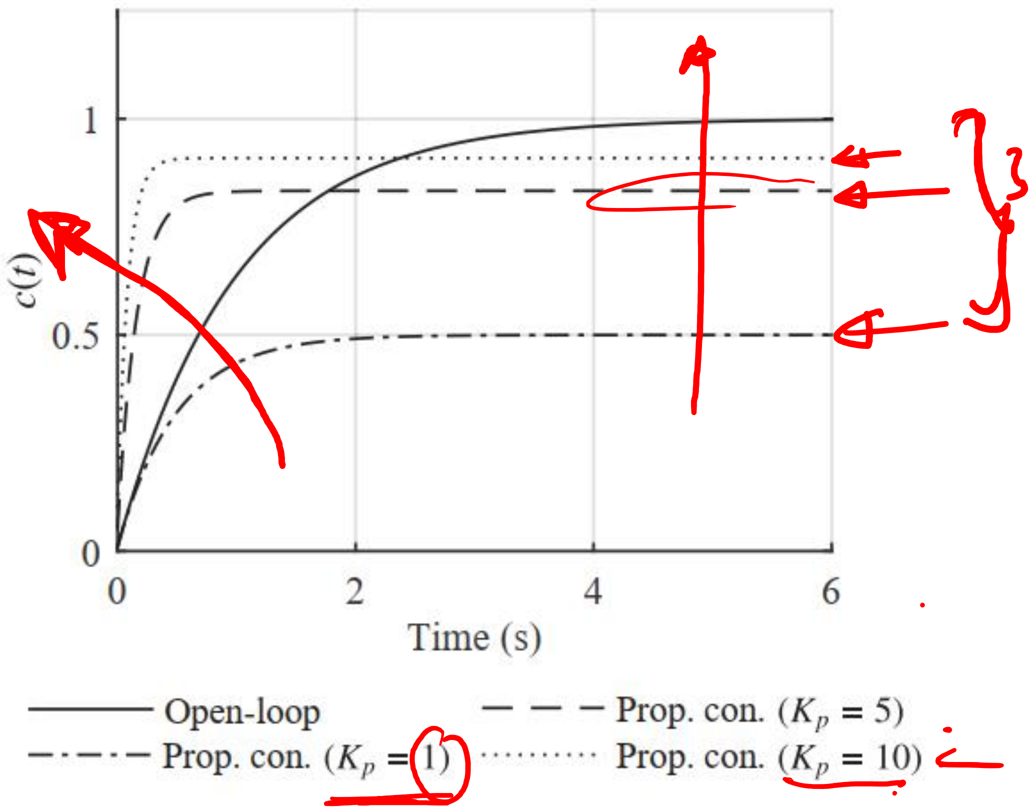
Effect of Proportional Control on a First-Order System

- Closed-loop response:

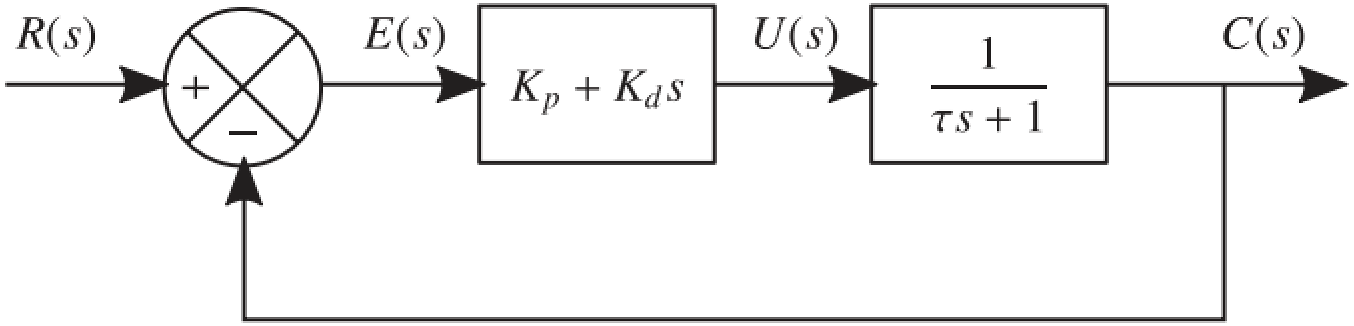
$$\boxed{c(t) = \frac{K_p}{K_p + 1}\left(1 - e^{-\frac{t}{\tau_{cl}}}\right)}$$

$$\boxed{\tau_{cl} = \frac{\tau}{K_p + 1}}$$





Proportional Plus Derivative Control of a First-Order System



$\frac{C(s)}{R(s)}$, $\frac{E(s)}{R(s)}$

C_{ss} E_{ss}

$R(s) = 1/s$

$$\frac{C(s)}{R(s)} = \frac{K_p + K_d s}{(\tau + K_d)s + 1 + K_p}$$

$$C(s) = \frac{K_p + K_d s}{s[(\tau + K_d)s + 1 + K_p]}$$

$$C_{ss} = \lim_{s \to 0} \cancel{s} \frac{K_p + K_d s}{\cancel{s}[(\tau + K_d)s + 1 + K_p]} = \frac{K_p}{K_p + 1}$$

0

$$\frac{E(s)}{R(s)} = \frac{\tau s + 1}{(\tau + K_d)s + 1 + K_p}$$

$$E(s) = \frac{\tau s + 1}{s\left[(\tau + K_d)s + 1 + K_p\right]}$$

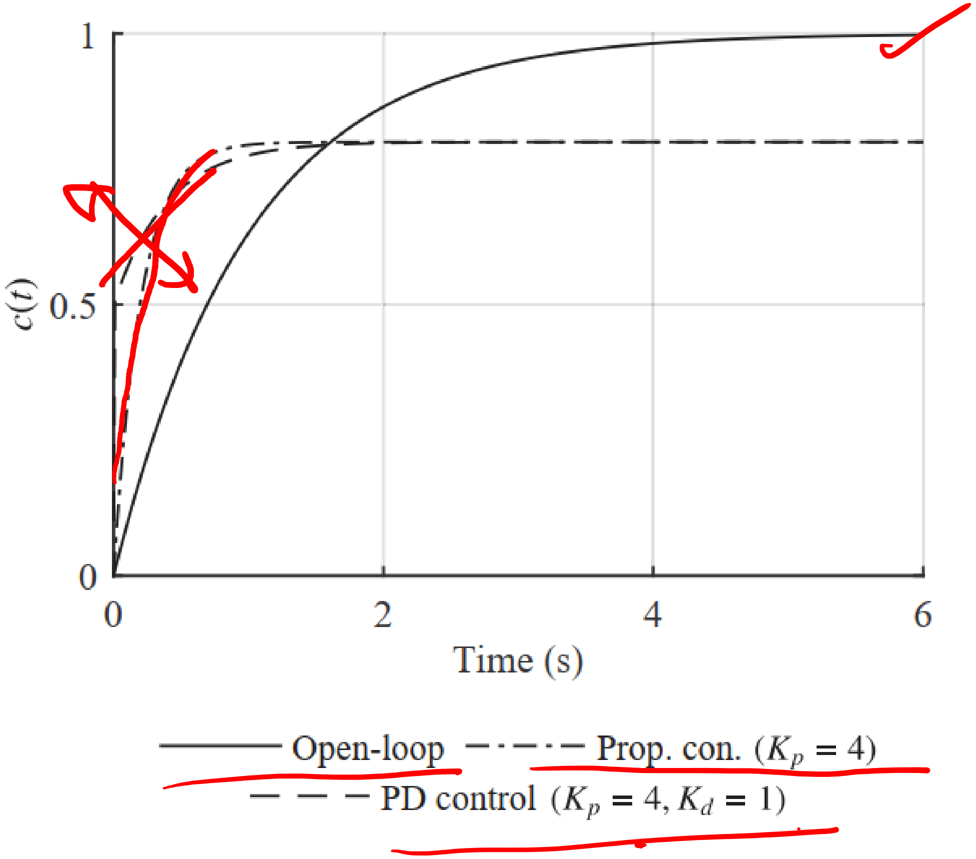
$$e_{ss} = \lim_{s \to 0} s\,E(s) = \lim_{s \to 0} \cancel{s}\,\frac{\overset{\to 0}{\tau s} + 1}{\cancel{s}\left[\underbrace{(\tau + K_d)s}_{\to 0} + 1 + K_p\right]}$$

$$e_{ss} = \frac{1}{K_p + 1}$$

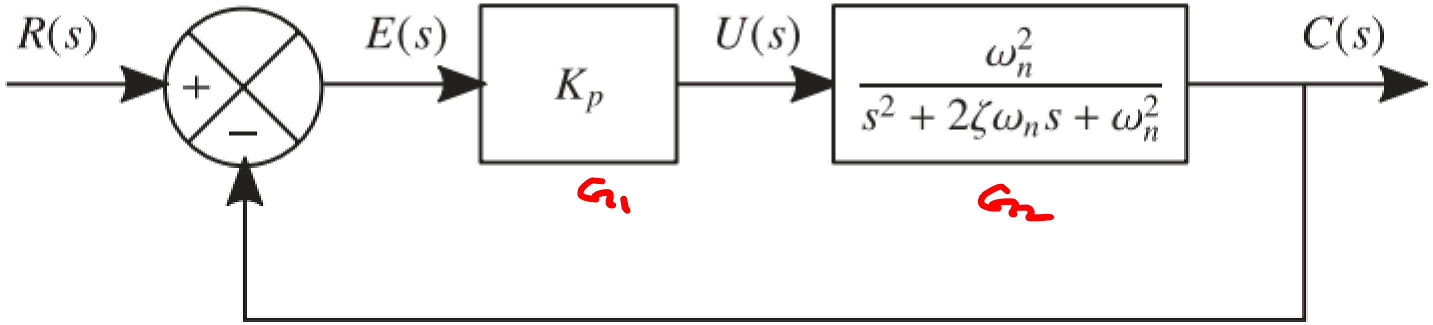
Effect of Proportional Plus Derivative Control on a First-Order System

- Closed-loop response: $c(t) = \frac{K_p}{K_p + 1}\left[1 - \frac{K_p\tau - K_d}{K_p(\tau + K_d)}e^{-\frac{t}{\tau_{cl}}}\right]$

$$\tau_{cl} = \frac{\tau + K_d}{K_p + 1}$$



Proportional Control of a Second-Order System



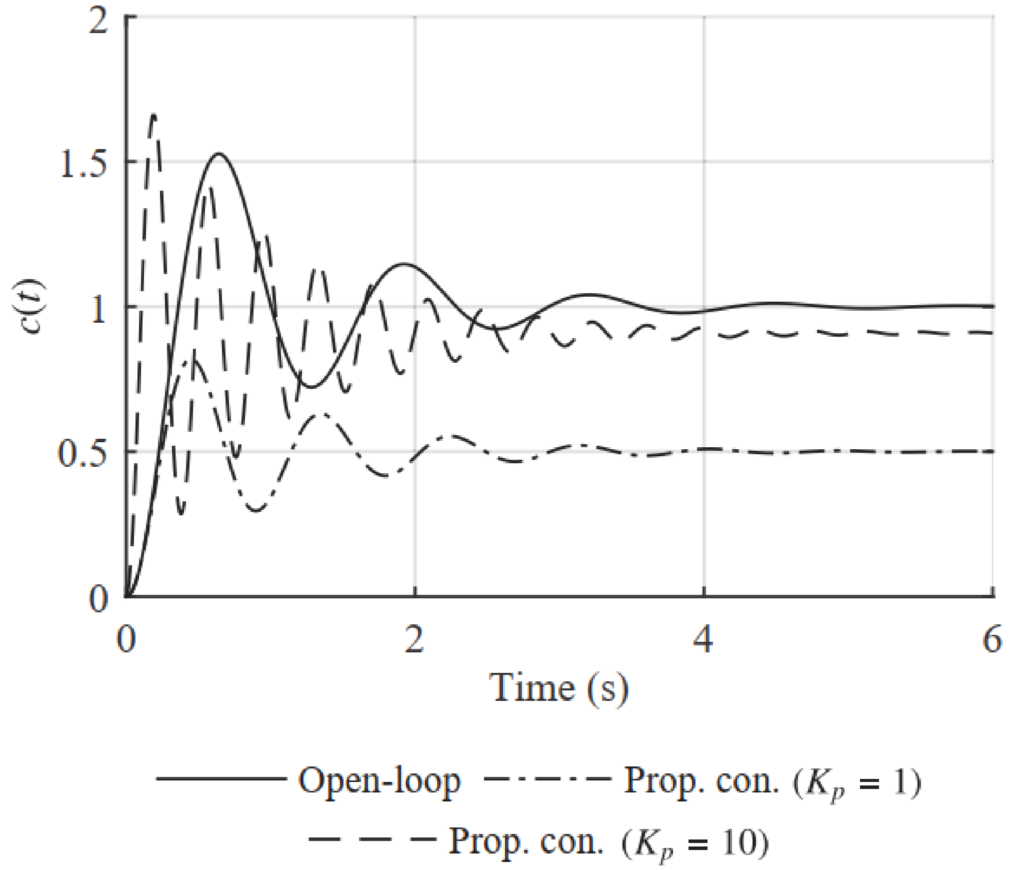
$C(s)$

$$c_{ss} = \frac{K_p}{K_p + 1}$$

$$e_{ss} = \frac{1}{K_p + 1}$$

Effect of Proportional Control on a Second-Order System

- Closed-loop response: $c(t) = \frac{K_p}{K_p + 1}\left[1 - e^{-\zeta\omega_n t}\left(\cos\omega_{d,cl}t + \frac{\zeta}{\sqrt{K_p + 1 - \zeta^2}}\sin\omega_{d,cl}\right)\right]$



$$\omega_{d,cl} = \omega_n\sqrt{K_p + 1 - \zeta^2}$$

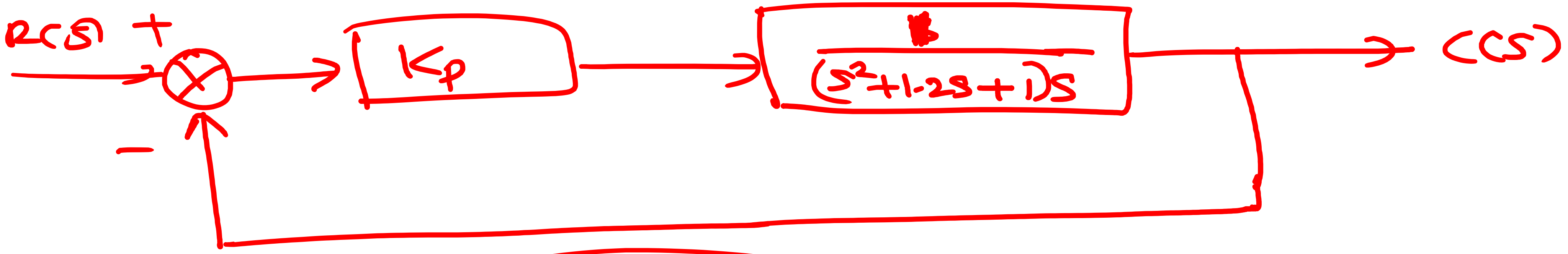
Exercise

A plant with transfer function *G(s)* is controlled by a controller of variable proportional gain K_p and unity negative feedback. Given

$$G(s) = \frac{1}{s\,(s^2 + 1.2s + 1)}$$

Show that the value of the proportional gain K_p has no influence on the steady state value of the response of the plant to a unit step input. What is the effect on the stability of the system of a negative value of K_p?

Investigate the stability of the system for the values: $K_p = 1; 1.5$



$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s^2+1.2s+1)+K_p}$$

→ ch eq

↓

poles

$$R(s) = \frac{1}{s} \implies C(s) = \frac{K_p}{s(s^3+1.2s^2+s+K_p)}$$

$$c_{ss} = \lim_{s\to 0} \cancel{s}\,\frac{K_p}{\cancel{s}(\underbrace{s^3+1.2s^2+s}_{0}+K_p)} = \frac{K_p}{K_p} = 1$$

Summary

- Proportional Control, Derivative Control, Integral Control
- PI Control, PD Control, PID Control
- Proportional Control of a First-Order/Second-order System and Effect on a First-Order/Second-order System
- PD Control of a First-Order System and Effect on a First-Order System

Reference:

-Control Systems Engineering, 7th Edition, N.S. Nise

-UESTC3001 2019/20 Notes, J. Le Kernec