Power Electronics

Lecture 2

Revision of Electric Circuit

电路知识复习

Please read pages 33-46 in Chapter 3 of the textbook

Electric Circuit （电路）

When electric charges （电荷）move in a wire（导线）, we say that an **electric current** (电流) flows in the wire. It's like the way a current of water flows in a river.

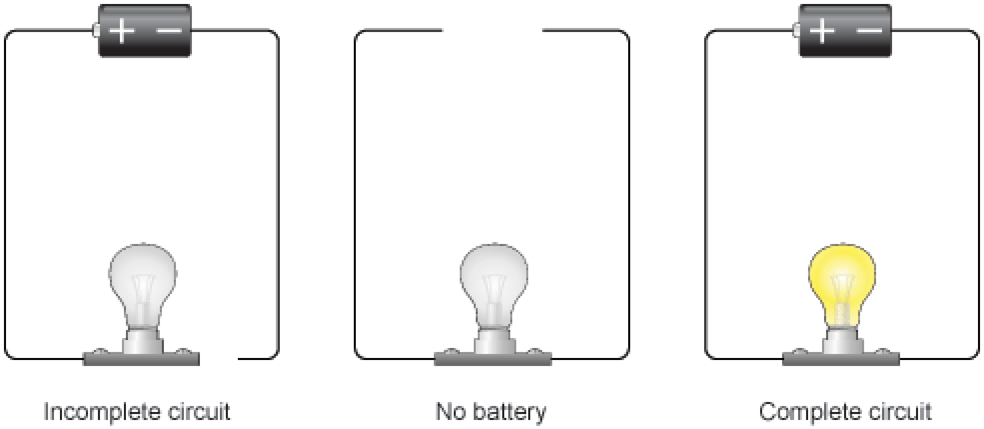
For an electric current to flow, we need two things:

• something to make the electricity flow, such as a battery or power pack

• a complete path （通路）for the current to flow in. This is called an electric circuit.

An electric current will not flow if we do not have a **power source** (电源) (a cell, battery or power pack). It also won‘t flow if the circuit is not **complete**(闭合). One end of the power source must be joined to the other end by the wires and components （元件） of the circuit.

- **power source**
- **Components**
- **wires**



The bulb will only light if there is a battery and a complete circuit

We usually add in a **switch** （开关）to the circuit, so that we can break （关断） the circuit and stop （阻断） the electric current when we want to.

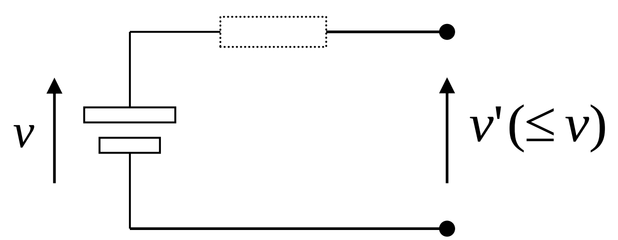
Potential Difference, the Volt (v) (电势差，电压)

The potential difference (p.d.), v, between two points is measured by the work （功） required to transfer unit charge from one point to another. The volt is the p.d. between two points when 1 joule (焦耳) of work is required to transfer 1 coulomb (库仑) of charge between the points.

1 volt = 1 joule / coulomb. 1伏特=1焦耳/库仑

If two points of an external circuit have a p.d. v then a charge q when passing between the two circuit points does an amount of work qv as it moves from the higher to the lower potential point.

An element such as a battery has an **Electromotive Force (EMF, 电动势)** if it does work on the charge moving through it. The charge receives electrical energy as it moves from the lower to the higher potential side. EMF is measured by the potential difference between the terminals when the battery is not delivering current (the “open circuit” voltage).



Basic Circuit Elements

These are some of the basic circuit elements.

•The element that only dissipates the energy is a pure **resistor(电阻)**.

•The element that only stores energy in a magnetic field is a pure **inductor(电感)**.

•The element that only stores energy in an electric field is a pure **capacitor(电容)**.

In general, one effect (作用/效应）predominates (主导) in any circuit element, but the others are there.

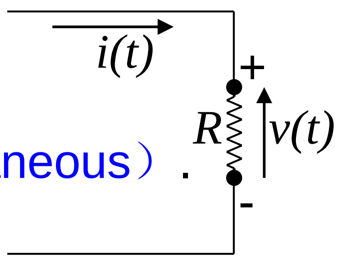
Resistance *R* 电阻值

The potential difference $v(t)$ across the terminals of a pure resistor is directly proportional to the current $i(t)$ in it. The constant of proportionality R is called the resistance of the resistor and is expressed in volts / amperes or ohms (Ω,欧姆).

$$\boxed{v(t) = Ri(t) \text{ and } i(t) = \frac{v(t)}{R}}$$

(**Ohm's Law**, 欧姆定律)

No restriction is placed on $v(t)$ and $i(t)$; they may be constant with respect to time, as in DC circuits, or they may be time varying functions, as in AC.



Conventions(约定):

时间的函数

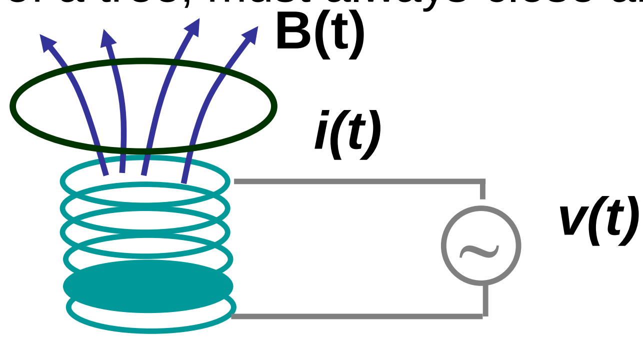
Lower case letters (v, i, p) indicate general functions of time（Instantaneous）.

Upper case letters (V, I, P) indicate time-invariant (DC) values.

Subscripts (V_m, I_m, P_m) are used to show peak, maximum, etc. values.

Inductance L, units of henrys (H) 电感值， 单位：亨利

It is common to conceptualise a magnetic field (磁场) as a set of flux lines磁力线. These, like the growth rings of a tree, must always close and can never cross.

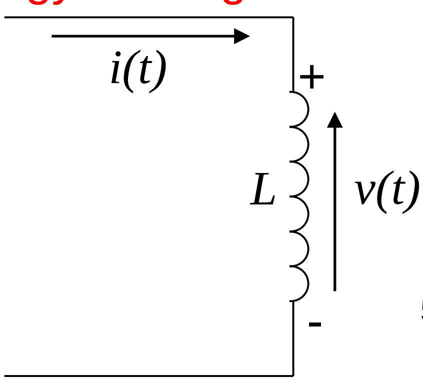


When the current in a circuit is changing, the magnetic flux 磁通 linking that circuit is changing too. This change in flux causes an emf v to be induced (感应) in the circuit. The induced emf v is proportional to the time rate of change of current if the permeability (磁导率) is constant. The constant of proportionality is called the self-inductance or *inductance* of the circuit.

$$\boxed{v(t) = L\frac{di}{dt} \quad \text{and} \quad i(t) = \frac{1}{L}\int v\,dt}$$

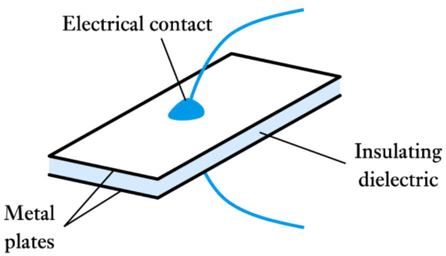
- Oppose any change in current
- Whereas resistors oppose current flow.
- Inductor store energy as magnetic field

When v is in volts and di/dt in amperes per second, L is in volt-seconds per ampere, or henrys (H). The self-inductance of a circuit is 1H if an emf of 1 volt is induced in it when the current changes at the rate of 1 ampere per second.

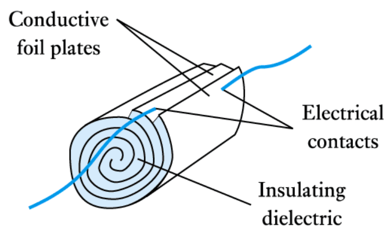


Capacitor

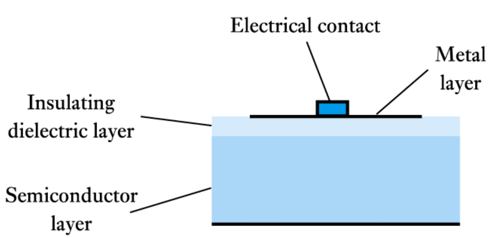
Capacitors consist of two conducting surfaces separated by an insulating （绝缘） layer called a **dielectric （电介质）**



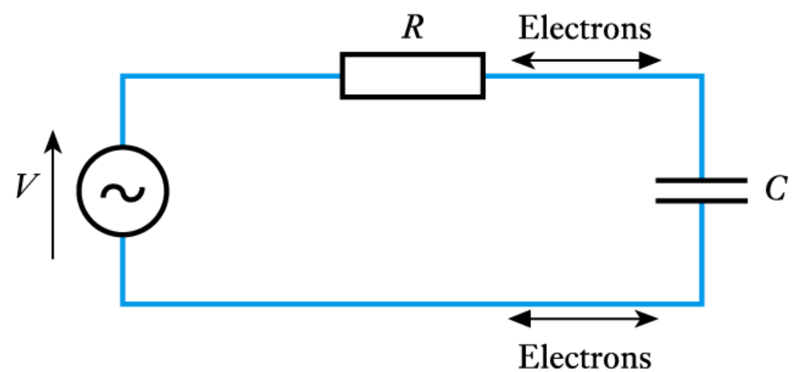
(a) A parallel-plate capacitor



(b) A cylindrical capacitor



(c) An integrated circuit capacitor



A constant current (e.g. DC) cannot flow through a capacitor. An alternating current (AC) can flow through the capacitor.

Capacitance C, units of farads 电容值，单位：法拉

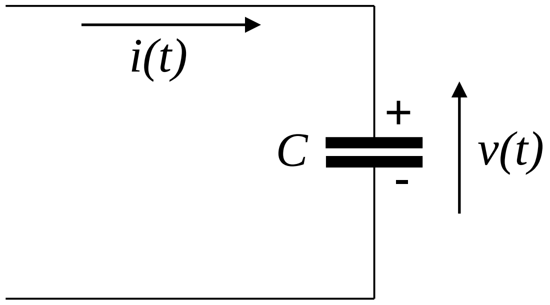
The potential difference v between the plates of a capacitor is proportional to the charge stored on the plates. The constant of proportionality, C, is called the *capacitance* of the capacitor.

$$\boxed{q(t) = Cv(t), \quad i = \frac{dq}{dt} = C\frac{dv}{dt}, \quad v(t) = \frac{1}{C}\int i dt}$$

When q is in coulombs and v is in volts, C is in coulombs per volt or *farads*. A capacitor has a capacitance of 1 farad if it requires 1 coulomb of charge per volt of potential difference between its plates.

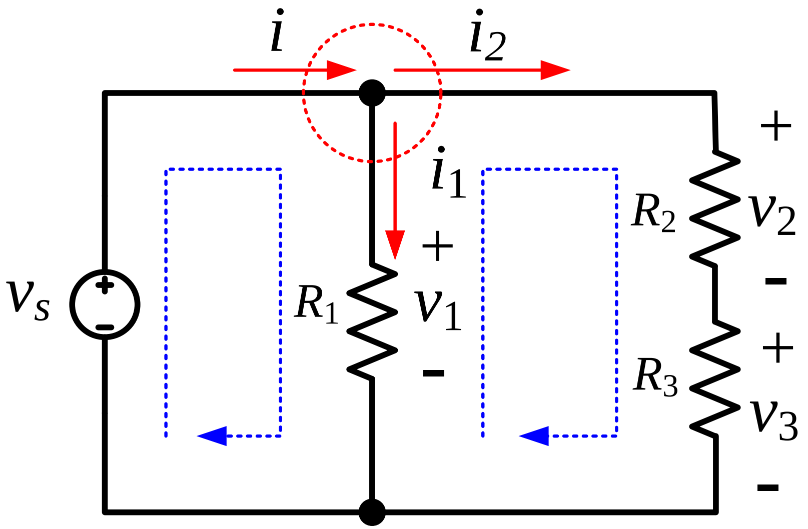
Multiples & submultiples			
peta (P)	10^{15}	femto (f)	10^{-15}
tera (T)	10^{12}	pico (p)	10^{-12}
giga (G)	10^{9}	nano (n)	10^{-9}
mega (M)	10^{6}	micro (u)	10^{-6}
kilo (K)	10^{3}	milli (m)	10^{-3}

- Stores energy in electric field
- Resist the voltage change
- Keeps the charge if open-circuited



Every year students fail the exam because of carelessness with multiples.

Kirchhoff's Circuit Laws
基尔霍夫电路定律



$$\text{KVL}: \quad \sum_k v_k = 0, \quad \textit{for any loop}$$

$$\text{KCL}: \quad \sum_k i_k = 0, \quad \textit{for any node}$$

Resistors in series and parallel.

* By putting resistors in series you just get a bigger resistor *

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

If the resistors are connected in parallel, **the Current gets divided**

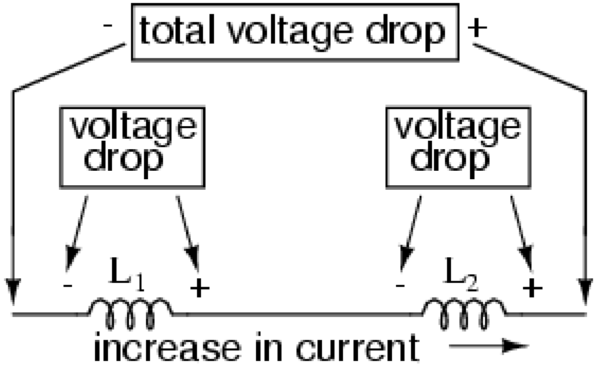
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots.}$$

* By putting resistors in parallel you always end up with a smaller resistor *

Inductors in series and parallel.

- The total inductance of several inductors in series is the sum of the individual inductances
- By putting inductors in series you just get a bigger inductor *

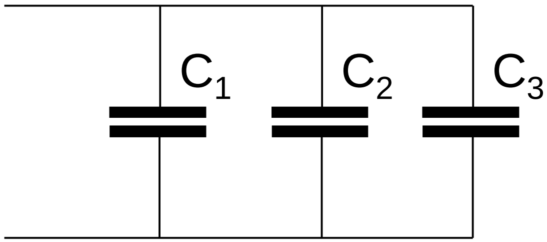
$$L_{eq} = L_1 + L_2 + L_3 + \ldots$$



Capacitors in series and parallel.

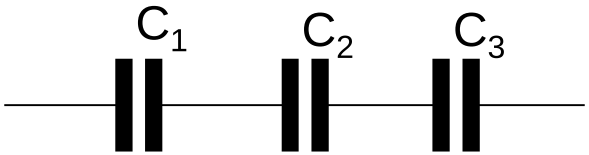
* By putting capacitors in parallel you just get a bigger capacitor *

$$C_{eq} = C_1 + C_2 + C_3 + ...$$



For capacitors in series, the formula is similar to that for resistors in parallel.

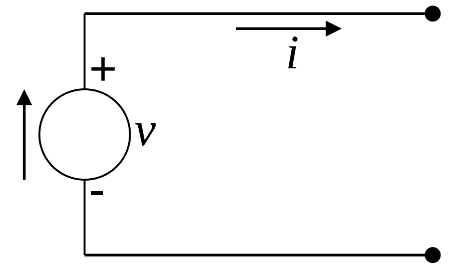
$$C_{total} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...}$$



Power (p) and Energy (w) 功率与能量

Instantaneous electrical power (瞬时功率) p is the product （乘积）of the impressed voltage v and resulting current i.

$$p \text{ (watts)} = v \text{ (volts)} \times i \text{ (amperes)}$$



Positive current, by definition, is in the direction of the arrow on the voltage source – it leaves by the + terminal（端）, as shown opposite.

When p has a positive value the source transfers energy to the circuit.

If power p is a periodic function of time t with period T, then:

平均功率 $$\boxed{\text{Average power } P = \frac{1}{T}\int_0^T p\,dt}$$

Since power p is the time rate of energy transfer,

$$\boxed{p = \frac{dW}{dt} \quad \text{and} \quad W = \int_{t_1}^{t_2} p\,dt}$$

where W is the energy transferred during the time interval.

Power and Energy in Resistors, Inductors and Capacitors.

For a resistor *R*, power $p = iv = i^2R = v^2/R$.
A resistor cannot store energy.

For an inductor *L*, power $p = iv = iL\dfrac{di}{dt}$

And the energy stored in the inductor is $\boxed{W = \int p dt = \int iL di = \frac{1}{2}Li^2}$

DANGER!!!

Charged inductor ($i \neq 0$) do not allowed to be **open**. $di/dt \rightarrow \infty$, $v \rightarrow \infty$ will damage circuit components, leads to overhigh discharging power p.

For a capacitor *C*, power $p = iv = vC\dfrac{dv}{dt}$

And the energy stored in the capacitor is $\boxed{W = \int p dt = \int Cv dv = \frac{1}{2}Cv^2}$

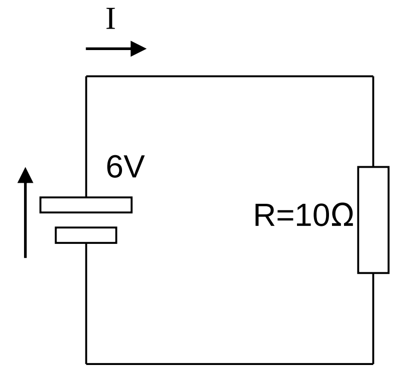
DANGER!!!

Charged capacitors ($v \neq 0$) do not allowed to be **short**. $dv/dt \rightarrow \infty$, $i \rightarrow \infty$ will damage the capacitor, leads to overhigh discharging power p.

In the case of the resistor there is an energy conversion process in that the power supplied is converted into heat. For the inductor and capacitor there is no conversion: the energy supplied is stored (in a magnetic or electric field respectively), and is available for return to the circuit if the conditions change.

Examples.

A resistor of value 10Ω is connected across a battery of 6V. What is the current that flows in the circuit, and what power is dissipated in the resistor?

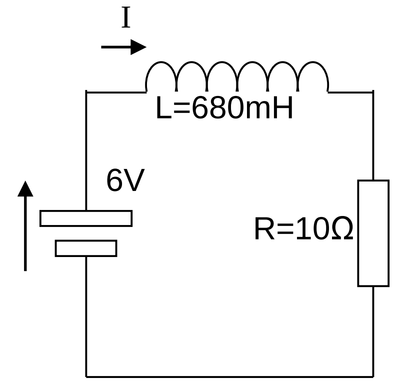


$$I = V/R = \frac{6}{10} = \underline{600\,mA}$$

$$P = \frac{V^2}{R} = I^2 R = VI$$

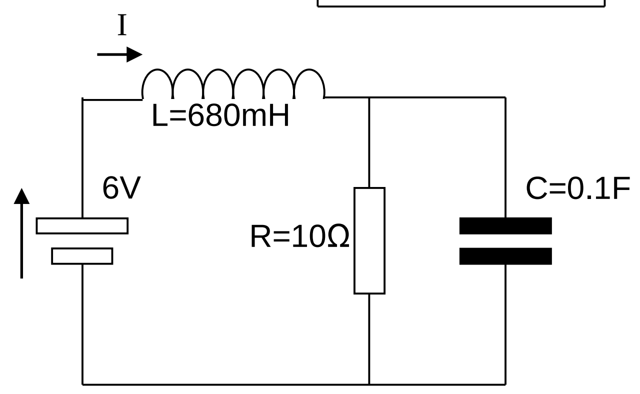
$$= \frac{6^2}{10} = 0.6^2 \times 10 = 6 \times 0.6 = \underline{3.6W}$$

An inductor of 680mH is added to the circuit as shown. What energy is stored in it?



$$E = \frac{1}{2}LI^2 = \frac{1}{2} \times 0.68 \times 0.6^2 = \underline{0.1224\,J}$$

A capacitor of 0.1F is added across the resistor, as shown. What energy is stored in the circuit now?

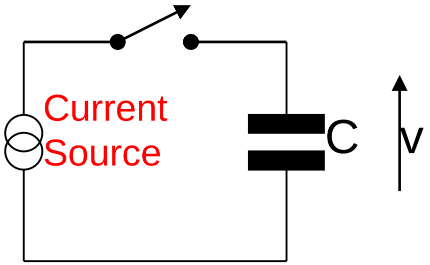


$$E = \frac{1}{2}CV^2 + \frac{1}{2}LI^2$$

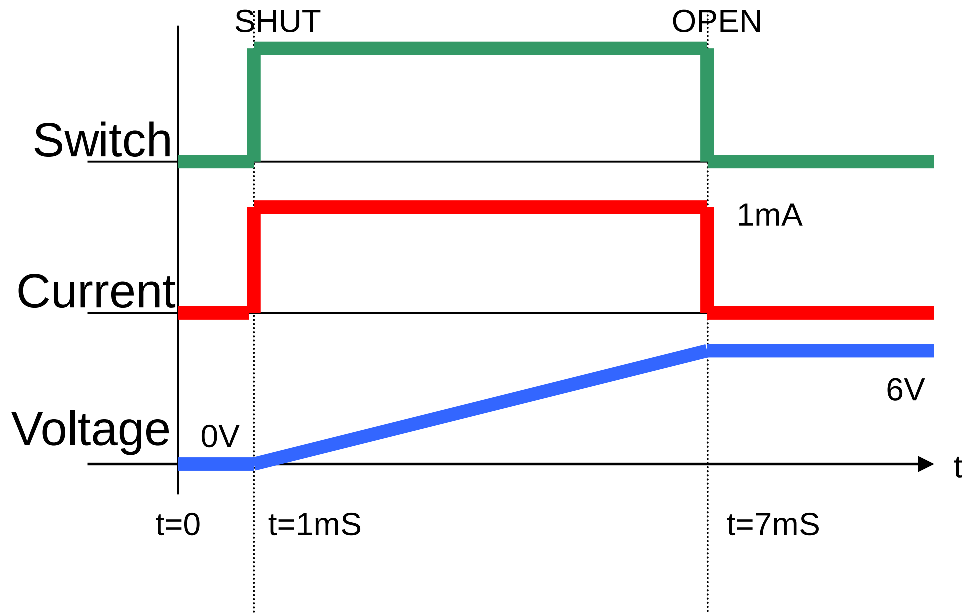
$$E = 0.1224\,J + \frac{1}{2} \times 0.1 \times 6^2 = \underline{1.9224\,J}$$

* Note that neither the inductor not the capacitor dissipate any power. *

Responses of voltage and current with time in inductors, capacitors and resistors.



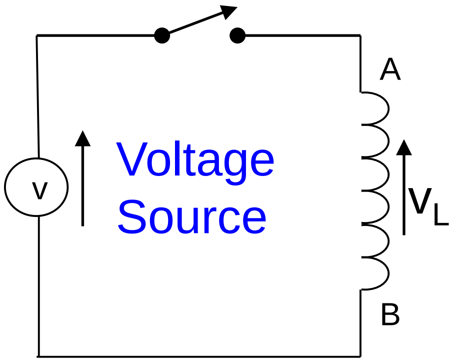
Note: I generally use red lines for current and blue lines for voltage on graphs.



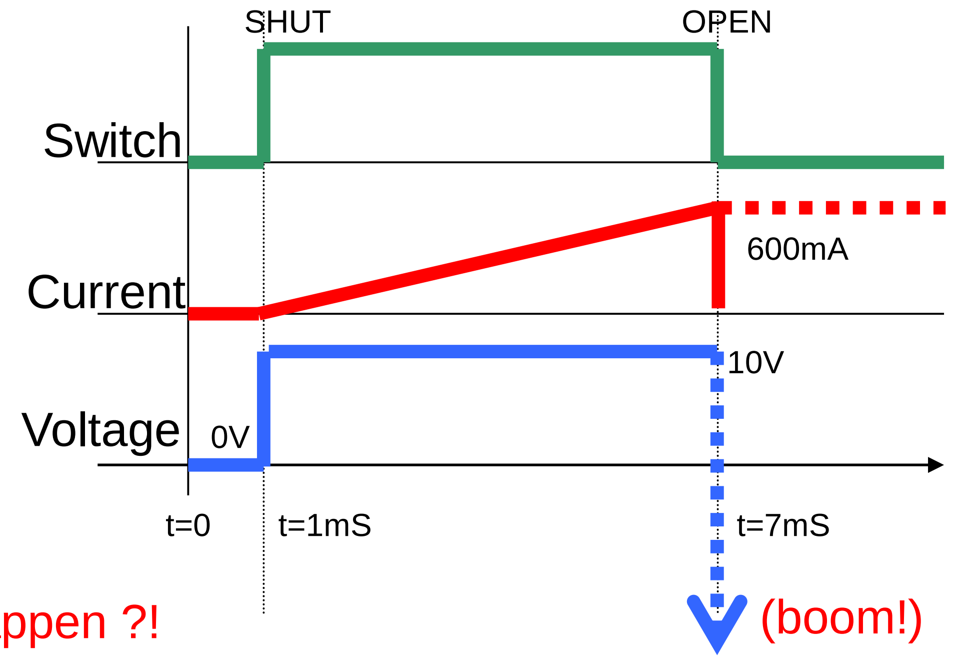
When the switch is closed at t=1ms, the current source （电流源) starts to charge the capacitor. The rate of change of voltage has a constant slope. The switch is opened at t = 7mS.
(This is how ramp generators work.)

$$I = 1mA$$
$$C = 1\mu F$$
$$\Delta v = 6V$$
$$\Delta t = 6mS$$
$$\frac{dv}{dt} = \frac{6}{0.006} = \underline{1000v/s}$$

For a capacitor, $I = C\frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{I}{C}$



v=10V,
L = 0.1H



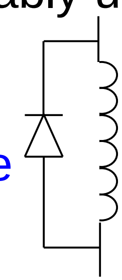
$$v = L\frac{dI}{dt} \Rightarrow \frac{dI}{dt} = \frac{V}{L}$$

$$\frac{dI}{dt} = 100\,A/s = \underline{100\,mA/mS}$$

What will happen ?!

At t=7mS the switch is **opened.** The inductor tries to keep 600mA flowing through it, from A to B, as it was the instant before the switch was opened. Hence terminal A goes negative relative to terminal B, probably to several hundred volts, before an arc is established between the switch contacts. If the switch is a mechanical device, the contacts could be burned. If the switch is a semiconductor (e.g. a transistor), this voltage spike may destroy it. Semiconductors often fail to a short-circuit, so this is just the start of the problems since current will now start to rise uncontrollably until another circuit failure occurs to interrupt the current.

We will see in a later lecture how to fix the problem with a diode in inverse parallel with the inductor.



<u>Notes about dI/dt and dV/dt</u>

In electrical circuits, the rise and fall time of signals is often the parameter the designer will use. For example, the cathodes of a Cathode Ray Tube will typically swing from +90V (fully "off") to +30V (fully "on") in as little as 3nS. The cathode itself is simply a small metal plate whose potential controls the flow of electrons in the tube. For electrical purposes, the cathode is modelled as a 10pF capacitor.

In this example the designer needs to know how much current is necessary to accomplish the voltage swing on the cathode in the required time, and it is simply:

$$I = C\frac{dV}{dt} = 10\times10^{-12}\times\frac{(90-30)}{3\times10^{-9}} = \underline{200\,mA}.$$

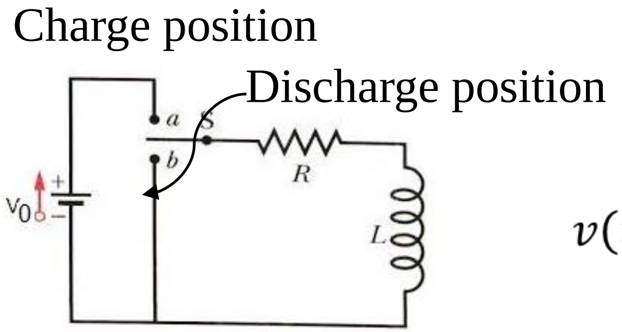
For this example,

$$\frac{dV}{dt} = 2{,}000{,}000{,}000 \text{ volts per second}$$

$$\frac{dI}{dt} = 66.7 \text{ million amps per second}$$

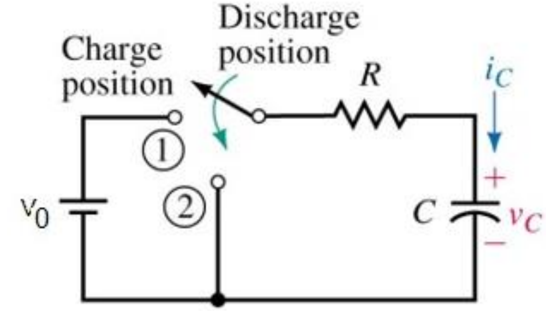
In summary, think of dI/dt and dV/dt not as "**horrible derivatives**" but as "rate of change of voltage and current".

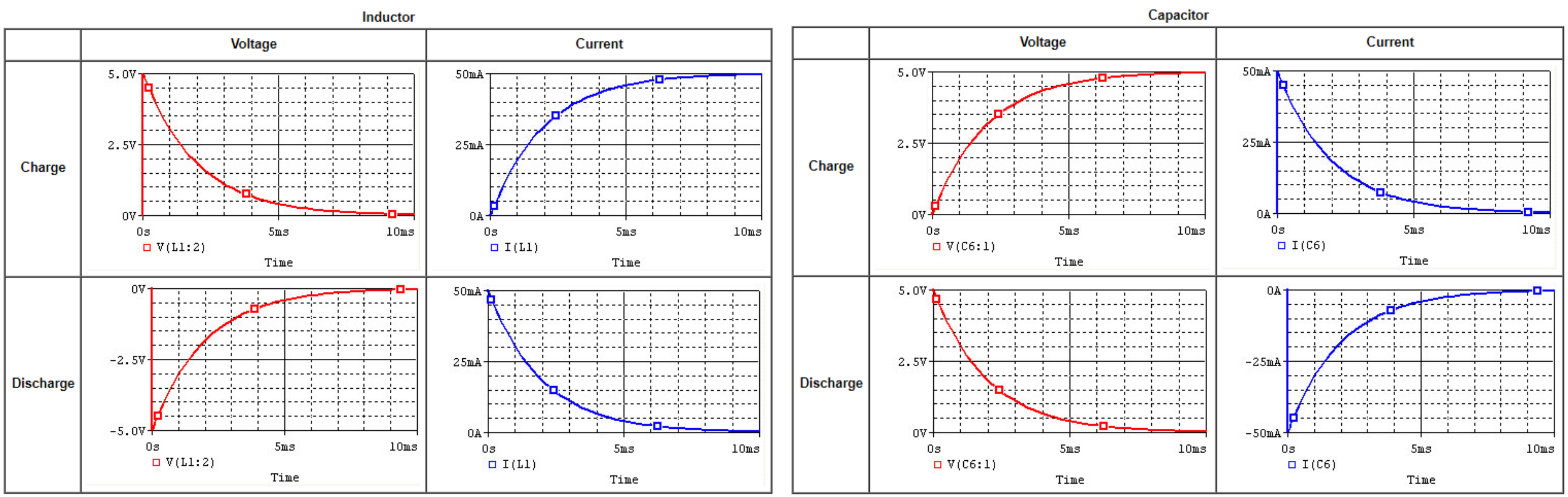
LR and RC Series Circuit charging and discharging



$$v(t) = L\frac{di}{dt}$$

$$i = C\frac{dv}{dt}$$





Exponential responses of capacitors and inductors

	Discharging	Charging	Time Constant
Capacitor	$v_C(t) = V_0 e^{-\frac{t}{RC}}$	$v_C(t) = V_0(1 - e^{-\frac{t}{RC}})$	RC
Inductor	$i_L(t) = I_0 e^{-\frac{R}{L}t}$	$i_L(t) = I_0(1 - e^{-\frac{R}{L}t})$	$\frac{L}{R}$

RMS and Other Measurements

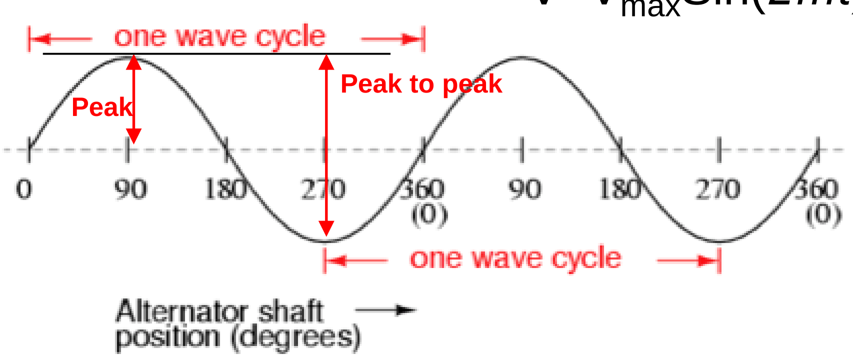
Many measures are used for the amplitude or magnitude of periodic waveforms. Each has a particular application.

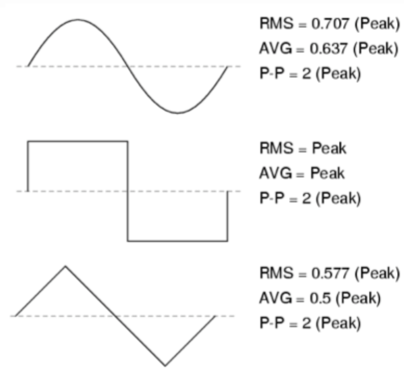
- **Peak value** – maximum positive (,or negative?) value
- **Peak-to-peak value** – range between minimum and maximum values
- **Average value** – the value that you would get if you ‘smoothed out’ the waveform
- **Root mean square (RMS) value** – essentially a measure of **power**

The last two typically require integrals to evaluate them.

- RMS values also arise in mechanical systems (vibrations for example)
- You may need to look back at Engineering Mathematics 1 for the integrals used to find RMS values
- Piecewise continuous functions are common in power electronic waves

$V = V_{max}\mathrm{Sin}(2\pi f t)$

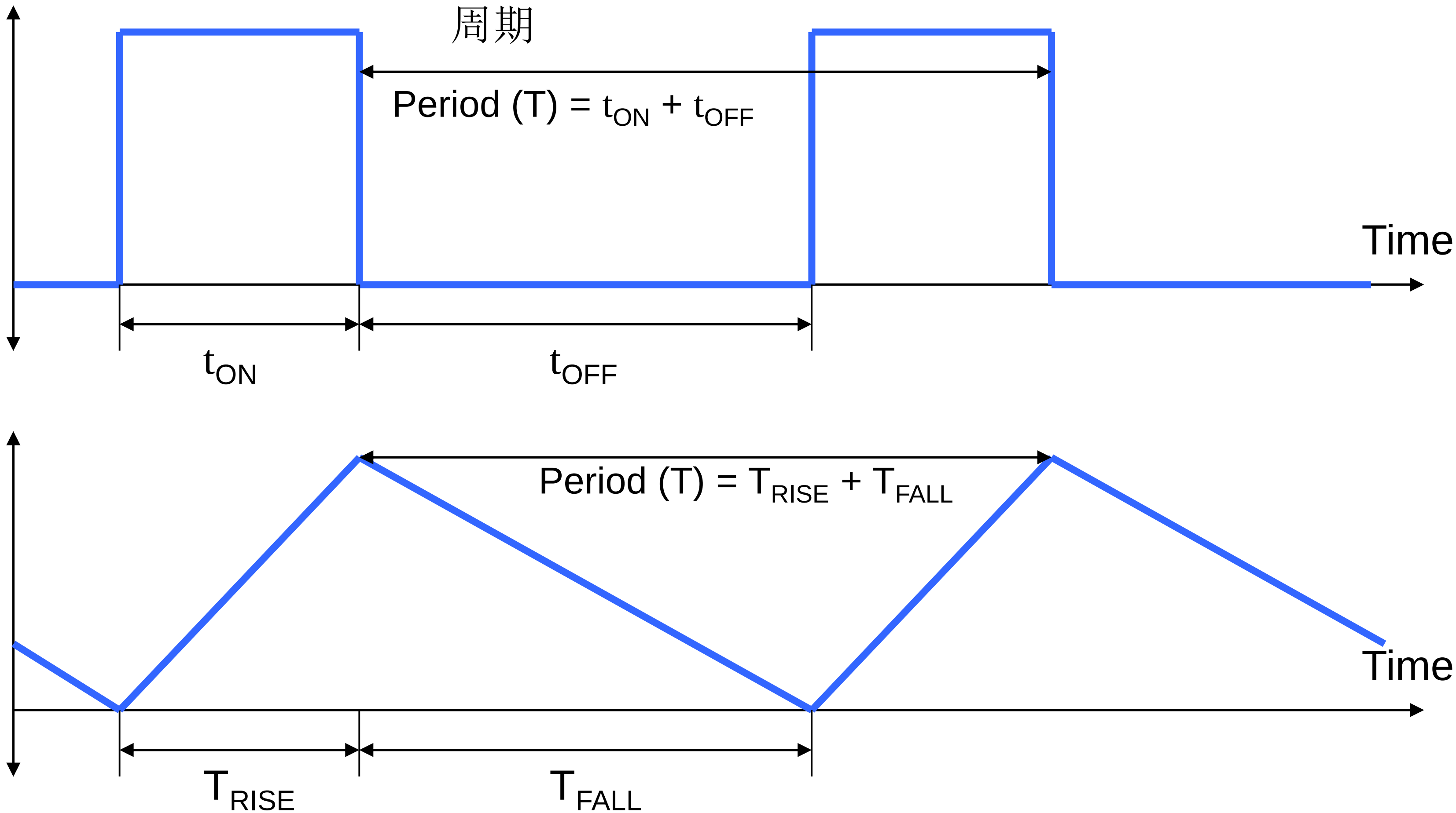




$$RMS = V_m/\sqrt{2}$$
$$AVG = 2V_m/\pi$$

$$RMS = V_m/\sqrt{3}$$
$$AVG = V_m/2$$

Repetitive Pulse Trains 重复脉冲序列



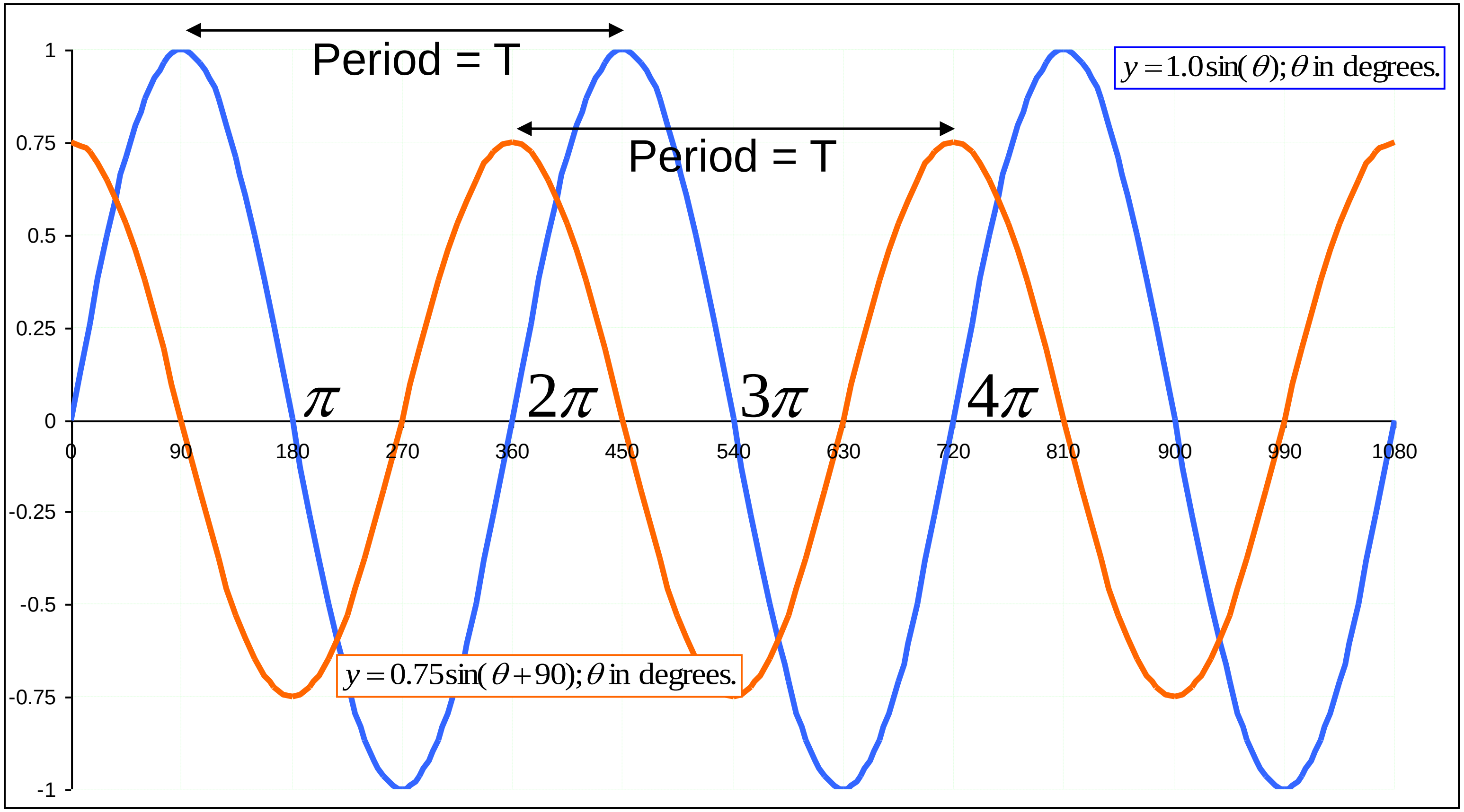
频率

$$Frequency(f) = \frac{1}{Period\ (\mathrm{T})}$$

占空比

$$\mathrm{Duty\ Cyle}\ \phi = \frac{\mathrm{t_{ON}}}{\mathrm{Period\ (T)}}$$

Periodic Waveforms



$$Frequency(f) = \frac{1}{Period}$$

$$\omega = 2\pi f$$

Peak value

Peak to Peak Value

Relation between average values of functions and sets of numbers

You know that the average (arithmetic mean) of a set of numbers is defined by

$$\langle X \rangle = \frac{1}{N}[x_1 + x_2 + \cdots + x_N] \qquad \text{Discrete values}$$

We are dealing with continuous, periodic functions (of time) so the sum is replaced by an integral:

$$\langle X \rangle = \frac{1}{T}\int_0^T x(t)dt$$

The integral can be over any period because the result must be the same, so you could use this if it were more convenient:

$$\langle X \rangle = \frac{1}{T}\int_{-T/2}^{T/2} x(t)dt$$

In statistics, you also need the mean squared value to calculate the variance,

$$\langle X^2 \rangle = \frac{1}{N}[(x_1)^2 + (x_2)^2 + \cdots + (x_N)^2]$$

The equivalent integral is

$$\langle X^2 \rangle = \frac{1}{T}\int_0^T [x(t)]^2 dt$$

The root mean square (RMS) value is equivalent to the standard deviation for a pure AC wave (no DC component, average value of zero).

<u>Average and Effective Values.</u>平均值与有效值

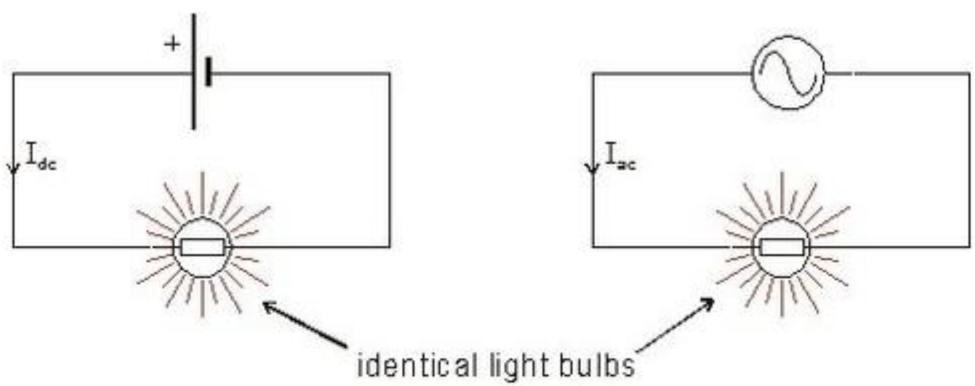
A general periodic function $y(t)$, with period T, has an average or mean value Y_{ave} given by:

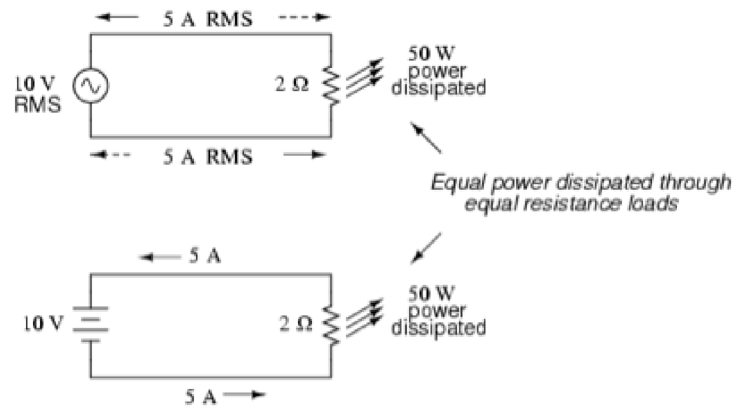
$$\boxed{Y_{av} = \frac{1}{T}\int_0^T y(t)dt}$$

The mean value (平均值) of sine and cosine functions *a* sin(wt) and *a* cos(wt) is 0.

Average value will let you know:

“How much amps of current do you need to pass through a resistor for same amount of time as you pass AC in it so that same amount of charge (电荷）gets transferred." (In case of pure AC, its always zero.)





If the bulbs light with the same **<u>brightness</u>** (that is, they are working at the same power), then it would be logical to regard the current $\boldsymbol{I_{ac}}$ as being <u>equivalent</u> to the current Idc.

Average and Effective Values.平均值与有效值

Why ?

The **Root Mean Square (RMS)** 均方根or **Effective value** of a general periodic function $y(t)$, with period T, has an effective value Y_{rms} given by:

$$\boxed{Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y(t)^2\,dt}}$$

The simple *mathematical average value* of I_{ac} (usually represented as sine and cosine functions *a* sin(ωt) and *a* cos(ωt)) is **equal to zero**. We therefore use a different method of finding an "average" or effective value of an alternating current (or voltage). This is why we need RMS values!!!

The RMS value of sine and cosine functions a sin(ωt) and a cos(ωt) is $\dfrac{a}{\sqrt{2}}$

The RMS value is sometimes also referred to as the "heating" value since a current passing through a pure resistor results in power being dissipated. RMS value of the AC current equivalent of the DC current which need to pass through a resistor, to produce same heat as the AC !!!

It is the convention to use RMS values for time varying voltages. Household mains voltage is $240V_{rms}$. The peak voltage of $240V_{rms}$ is $240\times\sqrt{2}$ = $340V_p$.

Definitions

Instantaneous Power:

$$p(t) = v(t)i(t)$$

Average Power:

$$p_{av} = \frac{1}{T}\int_0^T p(t)dt = \frac{1}{T}\int_0^T v(t)i(t)dt \rightarrow$$

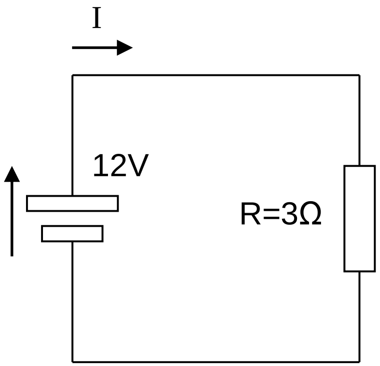
Dissipated power

For resistors

$$p_{av} = \frac{1}{T}\int_0^T vi dt = \frac{1}{T}\int_0^T \frac{v^2}{R}dt = \frac{1}{R}\left(\sqrt{\frac{1}{T}\int_0^T v^2 dt}\right)^2 = \frac{v_{rms}^2}{R} = i_{rms}^2 R$$

Example: RMS and Average Values

Consider the battery circuit below. What power is dissipated in the resistor?

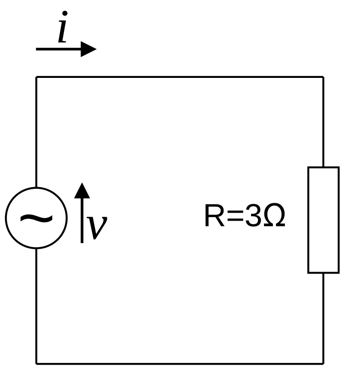


$$I = V/R = \frac{12}{3} = 4A$$

$$P = \frac{V^2}{R} = I^2 R = VI$$

$$= \frac{12^2}{3} = 4^2 \times 3 = 12 \times 4 = 48W$$

Now suppose the battery is replaced by a power supply that generates 12V AC.



Before thinking about power, what does a sine wave actually look like in "electrical" terms?

$$P = \frac{V_{rms}^2}{R} = I_{rms}^2 R = \frac{12^2}{3} = 4^2 \times 3 = 48W$$

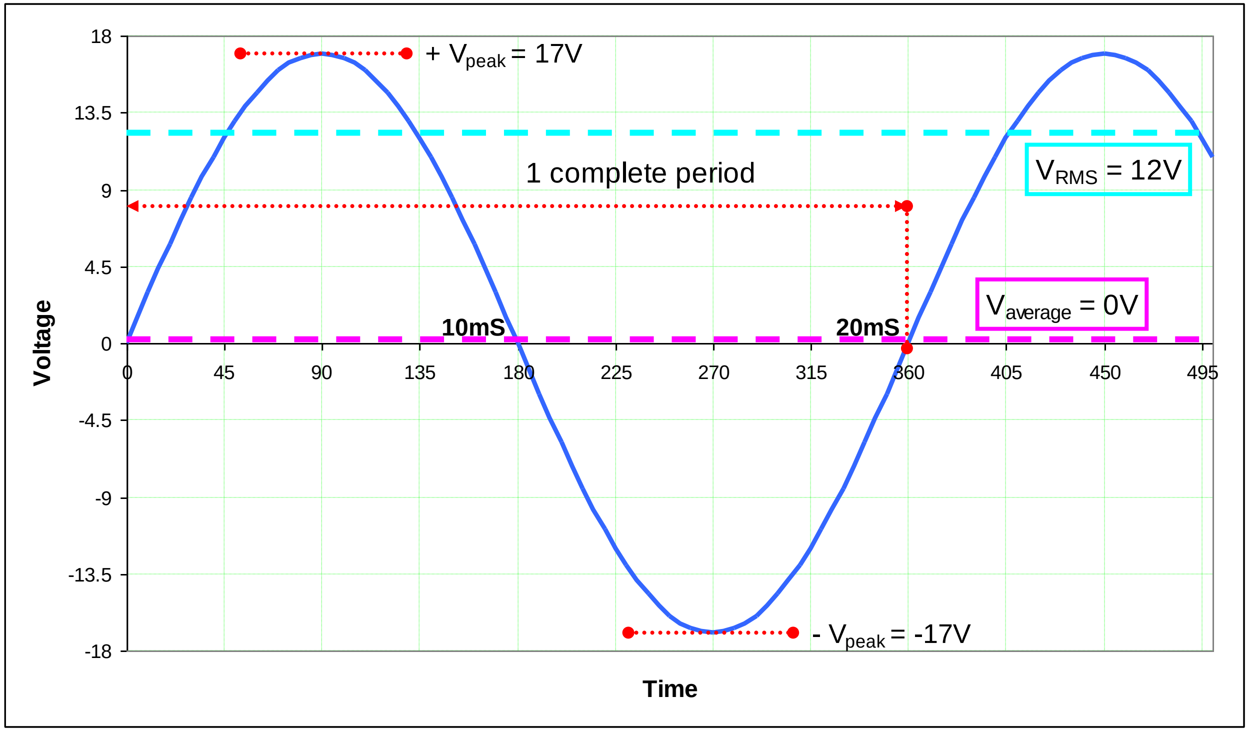
$$V_{rms} = 12 \qquad I_{rms} = 4$$

For Resistors, the RMS value is simply the value of the voltage (or current) equivalent to the DC value.

A sine (or cosine) waveform has a period of 2π radians and hence the RMS is:

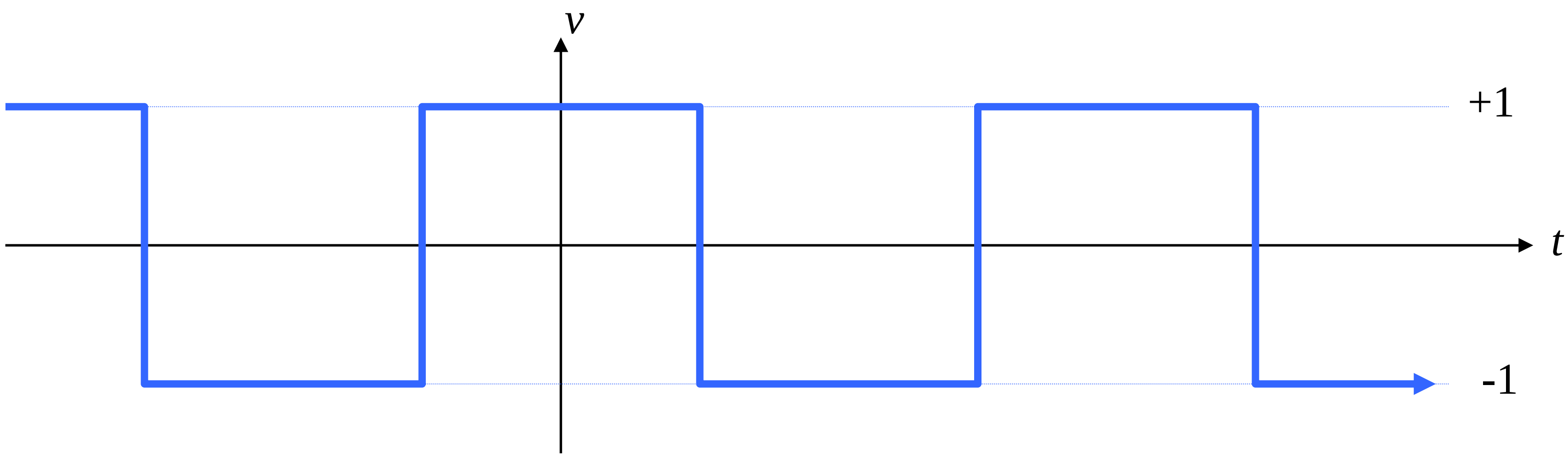
$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_{peak}^2 \cos^2(t)dt} = \sqrt{\frac{V_{peak}^2}{2\pi}\int_0^{2\pi} \frac{1}{2}\left[1+\cos(2t)\right]dt}$$

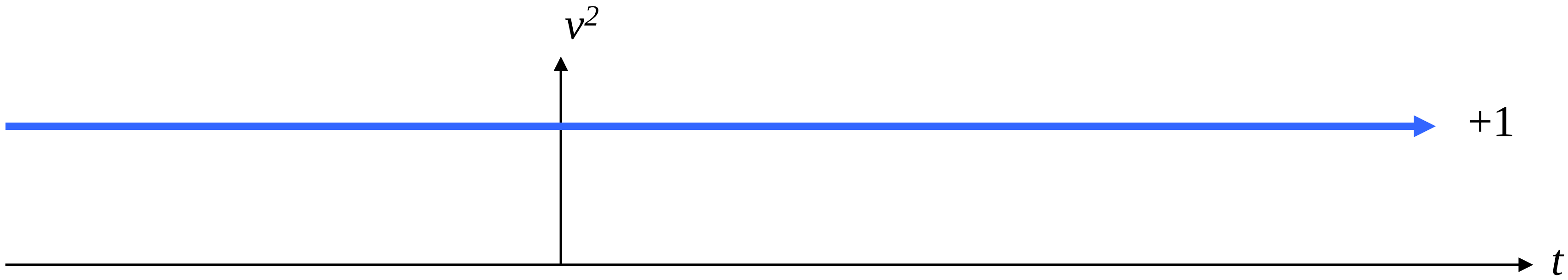
$$= V_{peak}\sqrt{\frac{1}{4\pi}\left[t\right]_0^{2\pi} + \left[\frac{1}{2}\sin(2t)\right]_0^{2\pi}} = V_{peak}\sqrt{\frac{2\pi}{4\pi}} = \frac{V_{peak}}{\sqrt{2}}$$



The average is 0V, the RMS is 12V, the peak is ±12√2V and the peak to peak is 24√2V.

Instead of a sine wave, suppose we have a square wave of amplitude $\pm 1V_{pp}$.

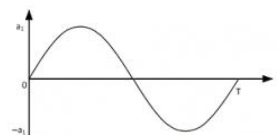
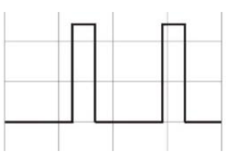
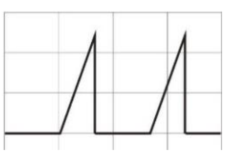


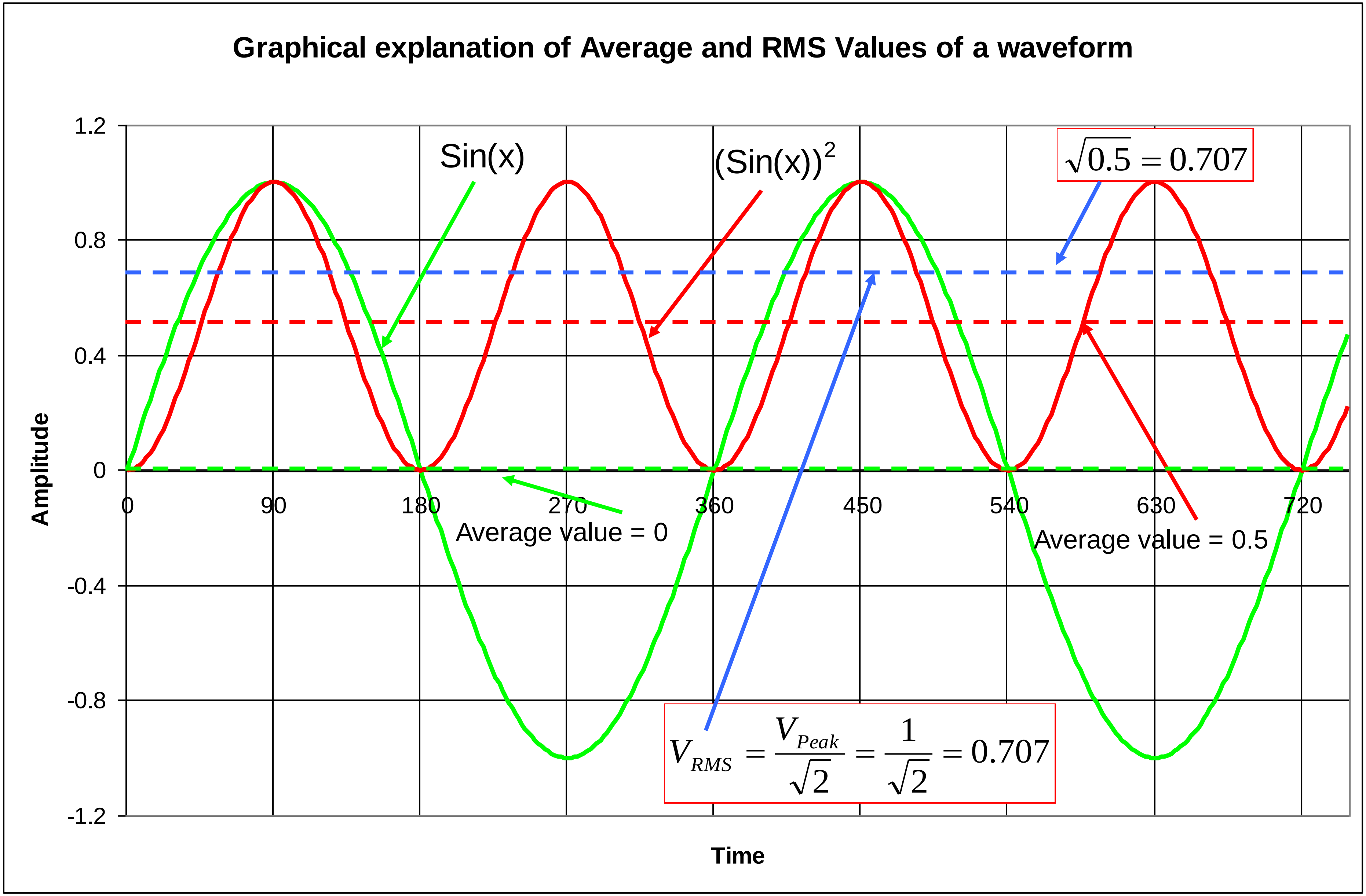


The mean squared voltage is $1V^2$ and $V_{RMS} = 1V$

The factor of is not universal: just sine waves (and special cases). It does not depend on the frequency, nor on the amplitude of the wave. In the general case **the RMS value depends on the shape of the periodic signal**.

RMS of common waveforms (a = peak amplitude)

Sine		$\frac{a}{\sqrt{2}}$
Square		a
Sawtooth		$\frac{a}{\sqrt{3}}$



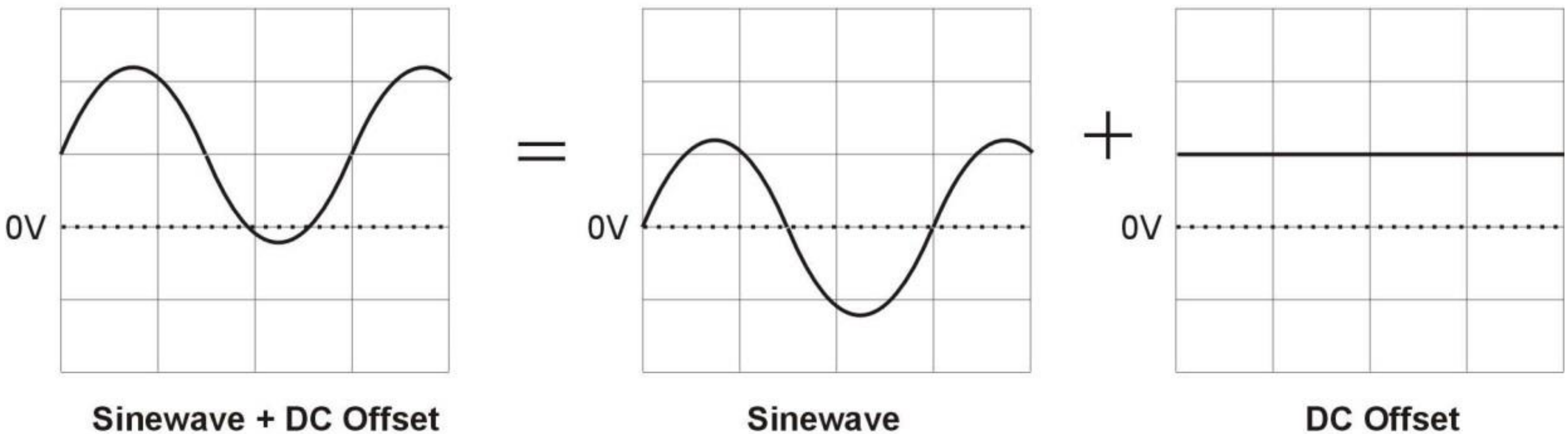
$$\boxed{V_{Peak \to Peak} \geq V_{Peak} \geq V_{RMS} \geq V_{Ave}}$$ Always!!!

Question:

Show that the RMS value of a sinusoidal waveform with peak amplitude A volts and DC offset B volts is given by:

$$V_{RMS} = \sqrt{\frac{A^2}{2} + B^2}$$

$$V_{TOTAL\,rms} = \sqrt{\left({V_{ACrms}}^2 + {V_{DCrms}}^2\right)}$$



$$\mathrm{A}\sin(\omega \mathrm{t}) + \mathrm{B} \qquad \mathrm{A}\sin(\omega \mathrm{t}) \qquad \mathrm{B}$$

Question:

Show that the RMS value of a sinusoidal waveform with peak amplitude A volts and DC offset B volts is given by:

$$V_{RMS} = \sqrt{\frac{A^2}{2} + B^2}$$

Solution:

$$Let\ \mathrm{v(t) = Asin(}\omega\mathrm{t) + B.\ Then}$$

$$\mathrm{v(t)}^2 = A^2 \sin^2(\omega t) + 2AB\sin(\omega t) + B^2$$

$$V_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} v(t)^2\,dt = \frac{1}{2\pi}\int_0^{2\pi}\left(A^2\sin^2(\omega t) + 2AB\sin(\omega t) + B^2\right)d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{A^2}{2}\left(1-\cos(2\omega t)\right) + 2AB\sin(\omega t) + B^2\right)d\omega t$$

$$= \frac{1}{2\pi}\left[\left[\frac{A^2\omega t}{2}\right]_0^{2\pi} - \left[\frac{A^2\omega t}{4}\sin(2\omega t)\right]_0^{2\pi} - \left[2AB\cos(\omega t)\right]_0^{2\pi} + \left[B^2\omega t\right]_0^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2\pi A^2}{2} + 2\pi B^2 - 2AB + 2AB\right] = \frac{\mathrm{A}^2}{2} + B^2$$

$$=> V_{RMS} = \sqrt{\frac{\mathrm{A}^2}{2} + B^2}$$

Form Factor （FF）波形因数

The ratio of the RMS to average value of a periodic function is called the Form Factor, F, of the function.

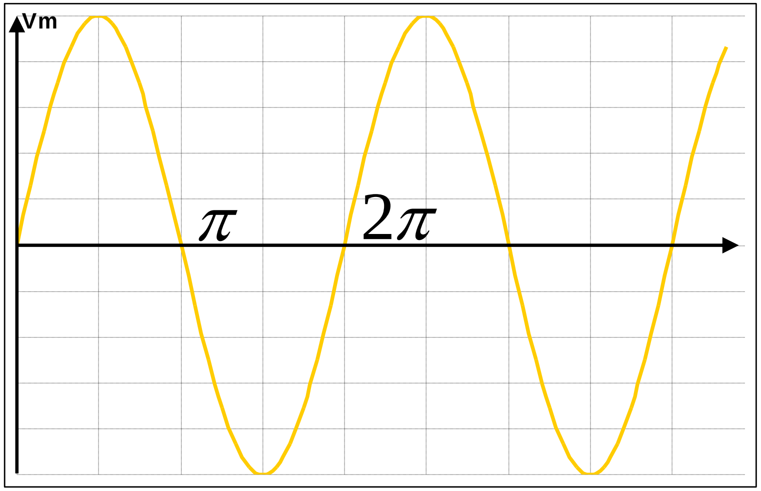
$$\textbf{Form Factor} = \frac{Y_{rms}}{Y_{ave}} = \frac{\sqrt{\frac{1}{T}\int_0^T |y(t)|^2\,dt}}{\frac{1}{T}\int_0^T |y(t)|\,dt}$$

Don’t forget the abs

- The form factor is important because the **heating effect** (and hence the temperature rise) in circuit components, transformer windings, etc. is proportional to the square of the RMS current. Measurement instruments also frequently assume a sinusoidal signal, and high form factor waveforms can lead to significant measurement errors.
- It identifies the ratio of the direct current of equal power relative to the given alternating current

Example: Calculate the form factor for a sinusoidal voltage waveform.

Let the voltage be $v = V_m \sin(\omega t)$



$$v_{ave} = \frac{1}{\pi}\int_0^{\pi} V_m \sin(\omega t) d(\omega t) = \frac{V_m}{\pi}\int_0^{\pi} \sin(\omega t) d(\omega t)$$

$$= \frac{V_m}{\pi}\left[-\cos(\omega t)\right]_0^{\pi} = -\frac{V_m}{\pi}\cos(\pi) + \frac{V_m}{\pi}\cos(0) = \frac{2V_m}{\pi}$$

$$= 0.64V_m$$

$$v_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi}\left[V_m \sin(\omega t)\right]^2 d(\omega t)} = \sqrt{\frac{V_m^{\,2}}{\pi}\int_0^{\pi}\left[\frac{1}{2}\left(1-\cos(2\omega t)\right)\right] d(\omega t)}$$

$$= \sqrt{\frac{V_m^{\,2}}{2\pi}\left[\omega t\right]_0^{\pi} - \frac{V_m^{\,2}}{2\pi}\left[\frac{1}{2}\sin(2\omega t)\right]_0^{\pi}} = \sqrt{\frac{V_m^{\,2}}{2}} = \frac{V_m}{\sqrt{2}} = 0.71V_m$$

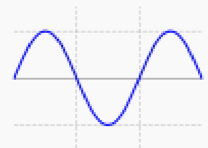
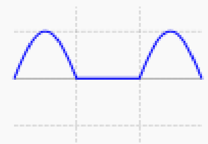
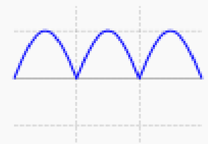
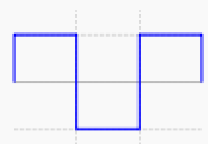
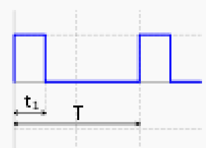
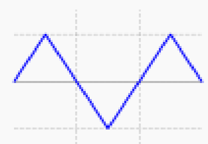
$$F = \frac{v_{rms}}{v_{ave}} = \frac{\dfrac{V_m}{\sqrt{2}}}{\dfrac{2V_m}{\pi}} = \frac{\dfrac{\pi V_m}{\sqrt{2}}}{2V_m} = \frac{\pi V_m}{2\sqrt{2}V_m} = \frac{\pi}{2\sqrt{2}} = 1.111$$

If you ever get a form factor less than one your maths is incorrect.

Typical Form Factor

Different waveforms have different ratio between its RMS value and its average value – ***Form Factor***

DC:
form factor=1

Waveform	Image	RMS	ARV	Form Factor
Sine wave		$\frac{a}{\sqrt{2}}$[2]	$a\frac{2}{\pi}$[2]	$\frac{\pi}{2\sqrt{2}} \approx 1.11072073$[3]
Half-wave rectified sine		$\frac{a}{2}$	$\frac{a}{\pi}$	$\frac{\pi}{2} \approx 1.5707963$
Full-wave rectified sine		$\frac{a}{\sqrt{2}}$	$a\frac{2}{\pi}$	$\frac{\pi}{2\sqrt{2}}$
Square wave, constant value		a	a	$\frac{a}{a} = 1$
Pulse wave		$a\sqrt{D}$[7]	aD	$\frac{1}{\sqrt{D}} = \sqrt{\frac{T}{\tau}}$
Triangle wave		$\frac{a}{\sqrt{3}}$[8]	$\frac{a}{2}$	$\frac{2}{\sqrt{3}} \approx 1.15470054$

Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x$$

$$\int \sin x\, dx = -\cos x + C$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\int \cos x\, dx = \sin x + C$$

$$\sin\alpha \cdot \cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha \cdot \sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

$$\cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$\sin\alpha \cdot \sin\beta = -\frac{1}{2}\left[\cos(\alpha+\beta) - \cos(\alpha-\beta)\right]$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos 2\alpha - \sin 2\alpha = 2\cos 2\alpha - 1 = 1 - 2\sin 2\alpha$$

Summary

- LCR Components
  - v-i Relationship
- Electrical Power Calculation
  - Instantaneous Value
  - Average Value
  - Effective / RMS Value
  - Form Factor
- Basic Parameters:
  - Frequency, Period, Peak, Peak-to-Peak