# Three-Phase Synchronous Electric Generators

An electrical generator will produce electrical power from mechanical power. We need a prime mover, which could be a steam or gas turbine, a wind turbine, an engine, or a water turbine (Figure 1).

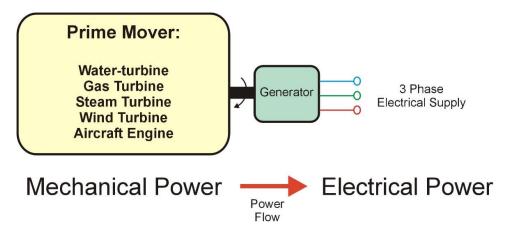


Figure 1. Electrical Generation

## A Basic Permanent Magnet Generator

Figure 2 depicts a permanent magnet, single phase generator. The rotor consists of a permanent magnet that is rotated by the prime mover. As the magnet rotates the flux will cut the stator winding. The flux linkage will have a sinusoidal waveform. Figure 3 shows the flux linkage as a function of the rotor position.

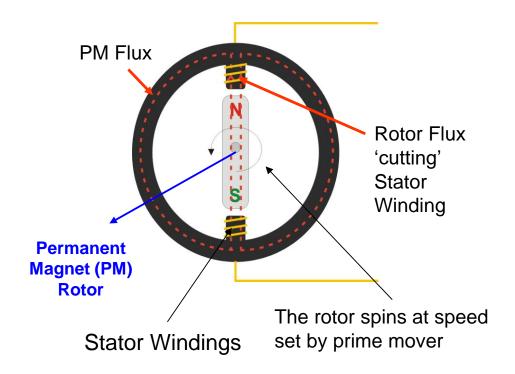


Figure 2. Permanent magnet, single phase generator

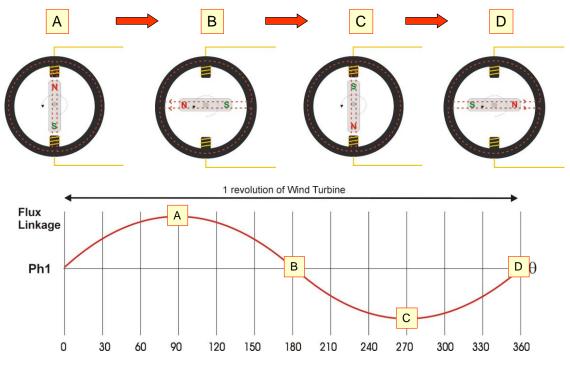


Figure 3. Flux Linkage versus rotor position

## Stator Open Circuit Voltage (V<sub>ph</sub>)

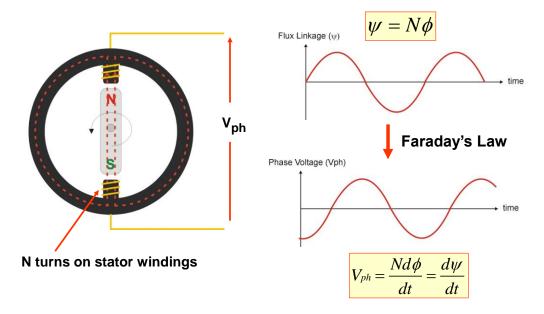
We first want to define what we mean by flux linkage. Consider a coil of N turns, where each turn forms a loop with exactly the same boundary (i.e. each loop is identical). Each turn will "link" the "same identical" flux,  $\phi$ , through N turns. Then we can say that the total flux linkage is:

$$\Psi = N\phi \tag{1}$$

We can then find the stator open circuit voltage:

$$V_{ph} = \frac{Nd\phi}{dt} = \frac{d\psi}{dt} \tag{2}$$

This is illustrated in Figure 4.



Note: Flux magnitude FIXED by Permanent Magnet

Figure 4. Open circuit stator voltage

We can see that the value of  $V_{ph}$  is determined by how fast the rotor rotates. Typically, we want a generator to operate at as high a voltage as possible, as this will limit  $I^2R$  losses for a given power output. From equation (2) we can either increase the number of turns on the stator windings, but

this is limited by I<sup>2</sup>R losses, or we can increase  $\int_{-\infty}^{\infty} dt$ . The result is that permanent magnet generators typically have a high number of magnet poles on the rotor.

Each passing of a north and south pole corresponds to a complete "cycle" of a magnet field oscillation so the constant of proportionality is P/120, where P is the number of magnetic rotor poles. Frequency of Generator Output Voltage Frequency ( $\mathbf{f}_s$ , Hz) as a function of generator speed ( $\mathbf{N}_r$ , rpm) and number of rotor poles ( $\mathbf{P}$ ) is therefore given as:

$$f_s = \frac{N_r \cdot P}{120} \tag{3}$$

Given the electrical frequency is proportional to the frequency of the rotor we call this a SYNCHRONOUS machine. The result is that both the Output Voltage magnitude and frequency are linearly proportional to generator speed.

## 3-Phase Permanent Magnet Generator

As the power density of a 3-phase machine is superior to its single phase equivalent 3 phase permanent magnet generators are common. Figure 5 depicts a 3-phase synchronous generator and its voltage waveform. There is a limit to how large a magnet you can install on the rotor.

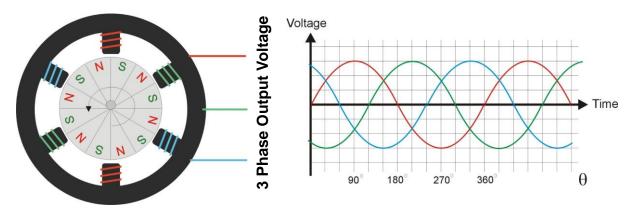


Figure 5. 3 phase synchronous machine output voltage waveform

#### Wound Field Synchronous Generators

The wound field synchronous generator is similar to the permanent magnet generator but with the permanent magnets replaced by electro-magnets on the rotor. DC Current is supplied to the rotor winding to produce rotor Magnetic Flux. Large synchronous generators are FIXED speed machines to give a fixed frequency (50Hz) Output Voltage.

Figure 6 shows the anatomy of a wound rotor synchronous generator. The rotor is still rotated by a prime mover, but a DC power supply needs to be connected to the windings to power the electromagnet. The prime mover rotates the shaft at a given speed. DC current ( $I_f$ ) in the Field Winding produces a rotating magnetic flux. This flux links with the 3 phase stator windings to produce 3 phase sinusoidal voltages from the 3 Stator (power) windings. These windings are connected to an electrical load.

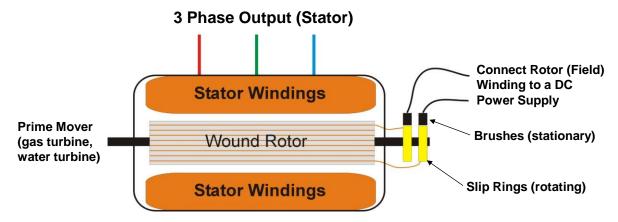


Figure 6. Anatomy of a wound rotor synchronous generator.

By changing the rotor field current we can change the rotor field intensity and rotor flux. Hence the 3-phase stator voltages  $(E_{ph})$  can also be controlled by the DC power supply (field current). However due to saturation of the iron laminations in the stator there is a limit to the stator voltage that can be obtained (Figure 7).

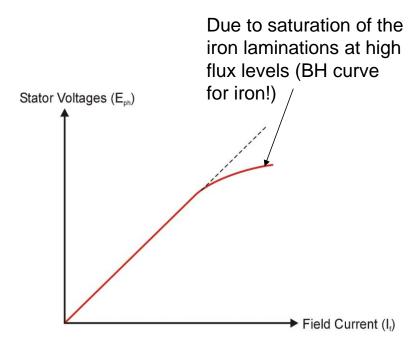


Figure 7. Changing stator voltages with field current.

#### Per Phase Equivalent Circuit

We now want to develop an equivalent circuit. First let's consider how the rotor and stator are configured. Referring to Figure 8, the DC Field Winding Current  $I_f$  creates a rotor magnetic field which induces three AC Excitation voltages  $E_{RN}$ ,  $E_{YN}$  and  $E_{BN}$ . The line currents  $I_R$ ,  $I_Y$  and  $I_B$  will flow if a 3-phase load is connected to the generator terminals.

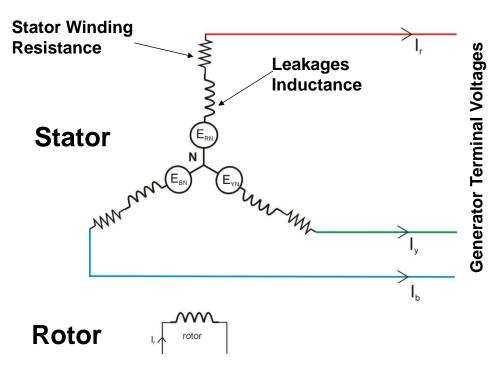


Figure 8. Depiction of rotor and Wye connected stator in 3-phase synchronous generator

The stator winding resistance is negligible and can in most practical cases be considered zero. We can then draw a per-phase equivalent circuit, as shown in Figure 9. It can be seen that:

$$\vec{E}_{ph} = \vec{V}_{ph} + jX_S \vec{I}_{ph} \tag{4}$$

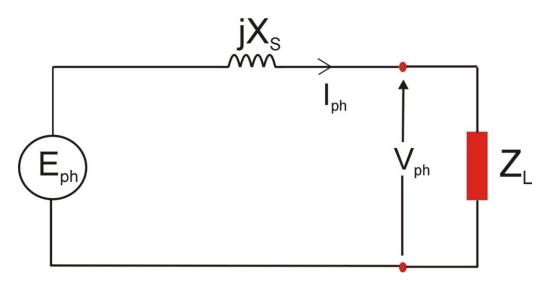


Figure 9. Per phase equivalent circuit.

Figure 10 shows a phaser diagram. The angle  $\Phi$ , between  $V_{ph}$  and  $I_{ph}$ , is the load Power Factor Angle. The Voltage across the Synchronous Reactance ( $V_{XS}$ ) leads the phase current  $I_{ph}$  by 90°, and the angle  $\delta$  between  $V_{ph}$  and  $E_{ph}$  is termed the **LOAD ANGLE**.

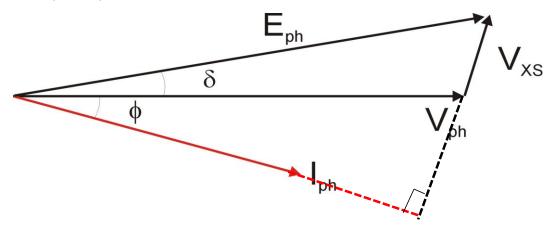


Figure 10. Typical per-phase phasor diagram.

## 3-phase phasor diagram

Lets now multiply all the voltage phasors in the phasor diagram by  $\frac{3.|V_{ph}|}{X_s}$ . The result is shown in Figure 11. It can be seen that on the right-hand side of the diagram is a power triangle showing the reactive, real and apparent powers. It can be seen that  $\sin(\delta) = \frac{X_s \cdot P(W)}{E_{ph} V_{ph}}$ 

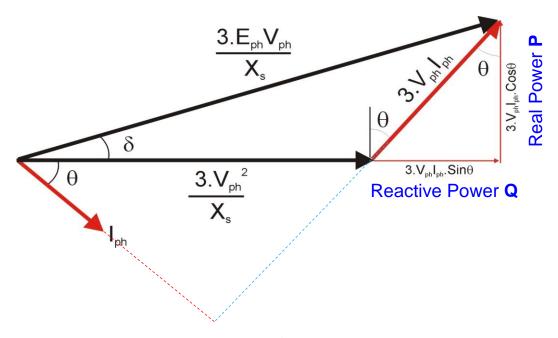


Figure 11. Phasor Diagram

## Bibliography

- 1. Theodore Wildi "Electrical Machines, Drives, and Power Systems" 2nd Edition, Prentice-Hall (1981). Chapter 16.
- 2. John Hindmarsh "Electrical Machines and their Applications" 4<sup>th</sup> Edition, Pergamon (1984). Chapter 8
- 3. Edward Hughes "Electrical Technology" 10th Edition, Pearson Education Limited (2008). Chapter 36.
- 4. Edward Hughes "Electrical Technology" 10th Edition, Pearson Education Limited (2008). Chapter 37.