Signals and Systems

Wang Lingfang (王玲芳)

(School of Optoelectronic Science and Engineering, UESTC)

Email：lf.wang@uestc.edu.cn
Tel: 18581848032

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Signals and Systems

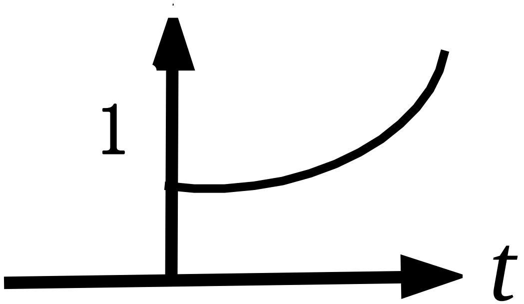
Chapter 9

The Laplace Transform

9. The Laplace Transform

(1) Fourier transform of this signal?

$$x(t) = e^{t}u(t)$$



→ The signal has no FT existed.

(2) FT can only be used to analysis of the **stable systems**.

9.1 The Laplace Transform

(1) Definition $x(t) \xleftrightarrow{L} X(s)$

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}dt \longrightarrow L\{x(t)\}$$

(A Function of Complex Variable)

$$s = \sigma + j\omega$$

9.1 The Laplace Transform

(2) Region of Convergence (ROC)

$$X(s) = \int_{-\infty}^{+\infty} x(t)e^{-(s=\sigma+j\omega)t}dt = \int_{-\infty}^{+\infty} \boxed{x(t)e^{-\sigma t}} e^{-j\omega t}dt$$

ROC: Range of Re{s} (or σ) for X(s) to converge

Representation:

A. Inequality

B. Region in S-plane

Example 1: $x(t) = e^{-at}u(t)$

$$X(s) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-st}dt = \int_{0}^{\infty} e^{-at}e^{-st}dt$$

$$= -\frac{1}{s+a}e^{-(s+a)t}\Big|_{0}^{\infty} = \frac{1}{s+a}, \quad \mathrm{Re}\{s\} > -a$$

or $$e^{-at}u(t) \xleftrightarrow{L} \frac{1}{s+a}, \mathrm{Re}\{s\} > -a$$

When a>0

$$X(j\omega) = X(s)\Big|_{s=j\omega} = X(0+j\omega) = \frac{1}{j\omega + a}$$

Example 2 $$x(t) = -e^{-at}u(-t)$$

In the same way as 9.1,we can get

$$X(s) = \frac{1}{s+a}, \quad \mathrm{Re}\{s\} < -a$$

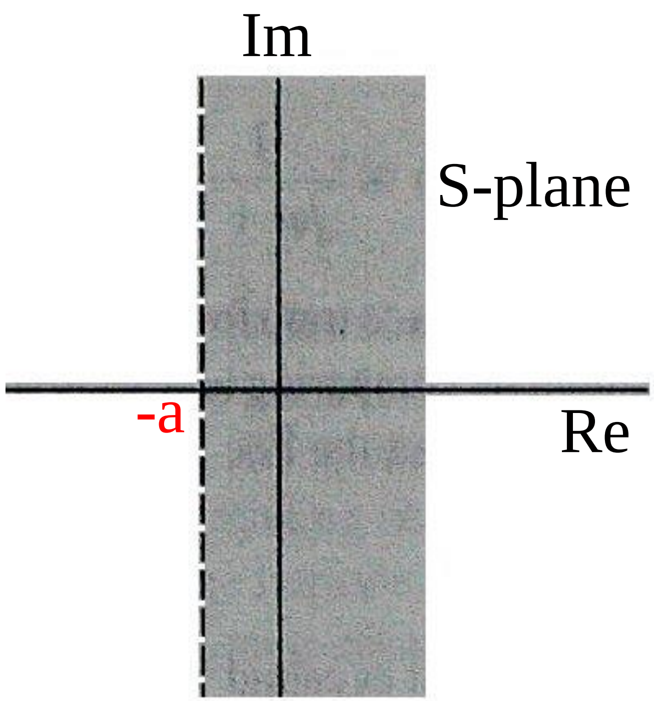
or $$-e^{-at}u(-t) \xleftrightarrow{L} \frac{1}{s+a}, \mathrm{Re}\{s\} < -a$$

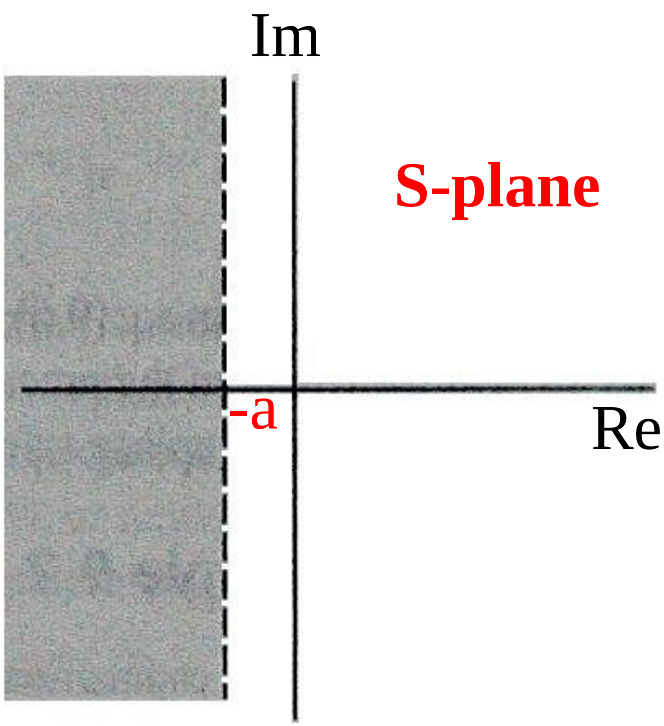
$$e^{-at}u(t) \xleftrightarrow{L} \frac{1}{s+a}, \mathrm{Re}\{s\} > -a$$

ROC of Example 1 and 2

$$x(t) = e^{-at}u(t)$$

$$x(t) = -e^{-at}u(-t)$$





The ROC of
a right-sided signal

The ROC of
a left-sided signal

Example 3 $$x(t) = 3e^{-2t}u(t) - 2e^{-t}u(t)$$

$$e^{-2t}u(t) \xleftrightarrow{L} \frac{1}{s+2}, \qquad \mathrm{Re}\{s\} > -2$$

$$e^{-t}u(t) \xleftrightarrow{L} \frac{1}{s+1}, \qquad \mathrm{Re}\{s\} > -1$$

$$X(s) = \frac{3}{s+2} - \frac{2}{s+1}, \qquad \mathrm{Re}\{s\} > -1$$

$$\therefore X(s) = \frac{s-1}{s^2+3s+2}, \qquad \mathrm{Re}\{s\} > -1$$

Example $x(t)=e^{-at}u(t) \longrightarrow X(s)=\dfrac{1}{s+a}, \mathrm{Re}\{s\}>-a$

(1) Let $a = 0$,

$$u(t) \xleftrightarrow{L} \frac{1}{s}, \qquad \mathrm{Re}\{s\}>0$$

(2) Let $a=\pm j\omega_0$

$$e^{-j\omega_0 t}u(t) \xleftrightarrow{L} \frac{1}{s+j\omega_0}, \qquad \mathrm{Re}\{s\}>0$$

$$e^{j\omega_0 t}u(t) \xleftrightarrow{L} \frac{1}{s-j\omega_0}, \qquad \mathrm{Re}\{s\}>0$$

$$e^{-j\omega_0 t}u(t) \xleftrightarrow{L} \frac{1}{s+j\omega_0} = \frac{s-j\omega_0}{s^2+\omega_0^2}, \operatorname{Re}\{s\} > 0$$

$$e^{j\omega_0 t}u(t) \xleftrightarrow{L} \frac{1}{s-j\omega_0} = \frac{s+j\omega_0}{s^2+\omega_0^2}, \operatorname{Re}\{s\} > 0$$

$$\cos\omega_0 t u(t) \xleftrightarrow{L} \frac{s}{s^2+\omega_0^2}, \quad \operatorname{Re}\{s\} > 0$$

$$\sin\omega_0 t u(t) \xleftrightarrow{L} \frac{\omega_0}{s^2+\omega_0^2}, \quad \operatorname{Re}\{s\} > 0$$

Example

$$\delta(t) \xleftrightarrow{L} 1 \qquad -\infty < \mathrm{Re}\{s\} < \infty$$

$$\because X(s) = \int_{-\infty}^{\infty} \delta(t) e^{-st} dt = 1$$

The ROC is the entire s-plane.

Generally, X(s) is a ratio of polynomials
In complex variable s

$$X(s) = \frac{N(s)}{D(s)},$$

$N(s)$ **---numerator**

$D(s)$ **---denominator**

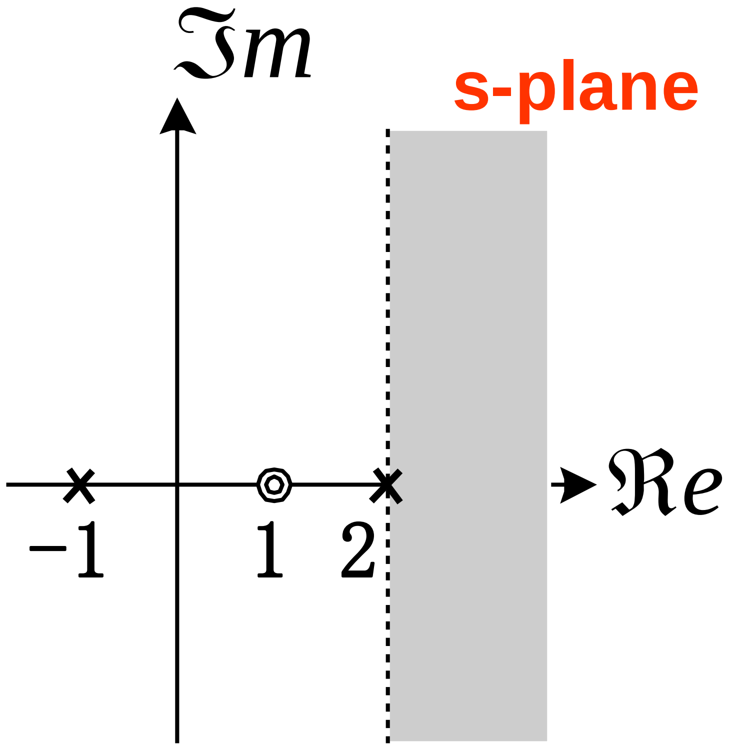
$$X(s) = \frac{(s-1)^2}{(s+1)(s-2)},$$ **in example 9.5**

poles--- the roots of $D(s)$

zeros--- the roots of $N(s)$

pole-zero plot and ROC of X(s) in s-plane

$$\mathrm{Re}\{s\} > 2$$



9.2 The Region of Convergence for Laplace Transform

Property1:

The ROC of $X(s)$ consists of strips parallel to $j\omega$-axis in the s-plane.

Property2:

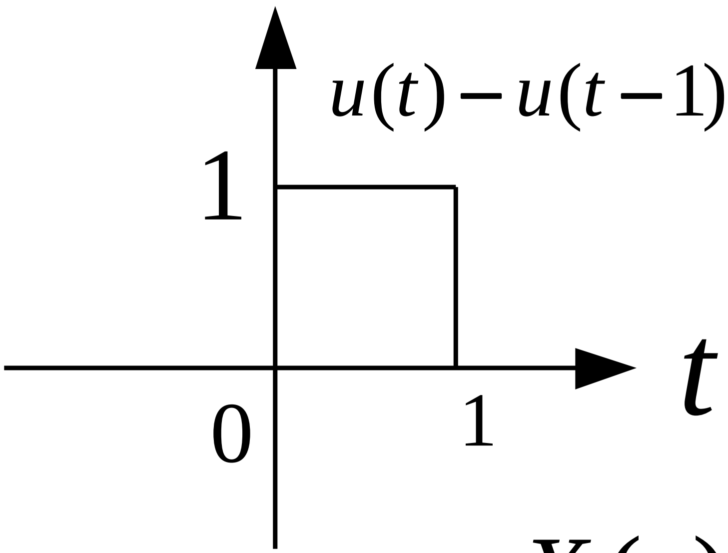
For rational Laplace transform, the ROC does not contain any poles.

Property3:

If $x(t)$ is of finite duration and is absolutely integrable, then the ROC is the entire s-plane

Example

$$u(t)-u(t-1) \xleftrightarrow{L} \frac{1}{s}[1-e^{-s}],$$



$$-\infty < \mathrm{Re}\{s\} < \infty$$

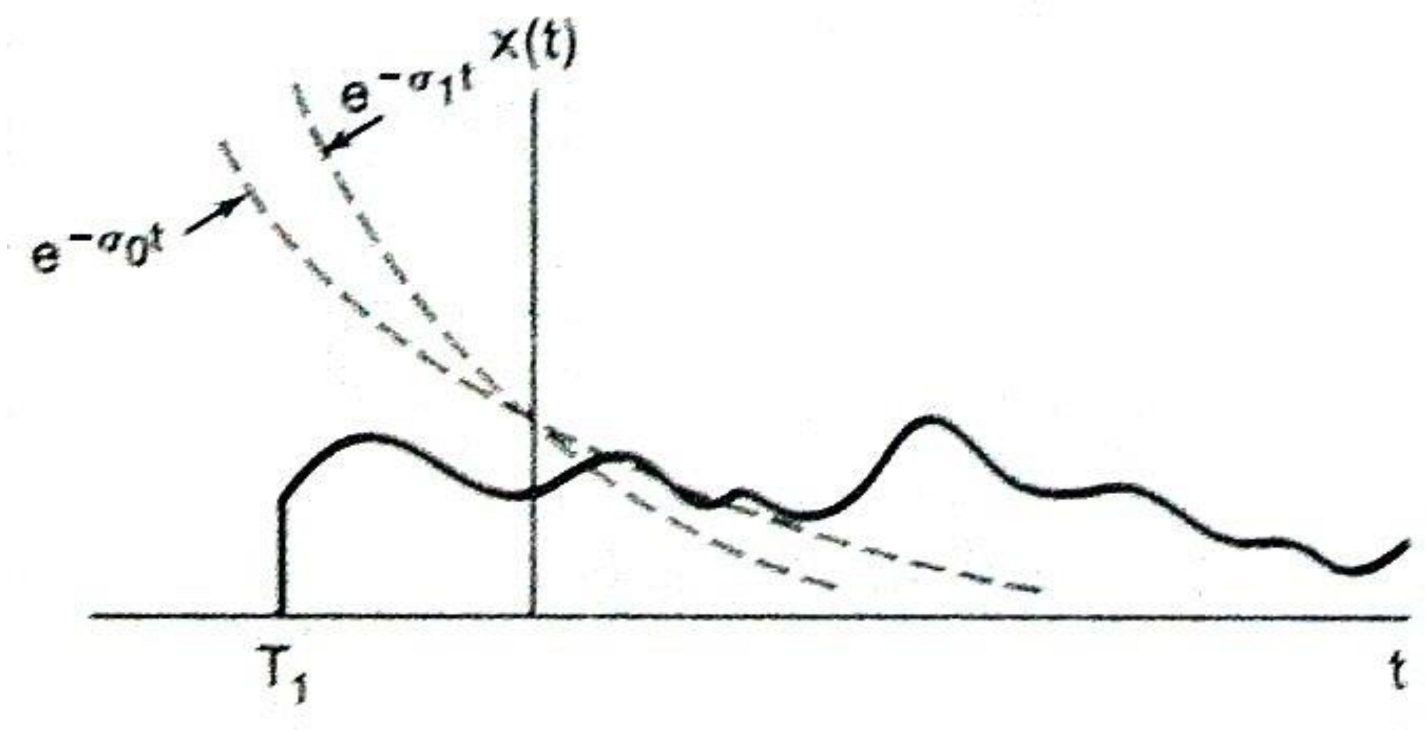
$$\because X(s) = \int_0^1 e^{-st}dt = \frac{1}{s}[1-e^{-s}],$$

It is convergent for any Re{s}.

The ROC is the entire s-plane.

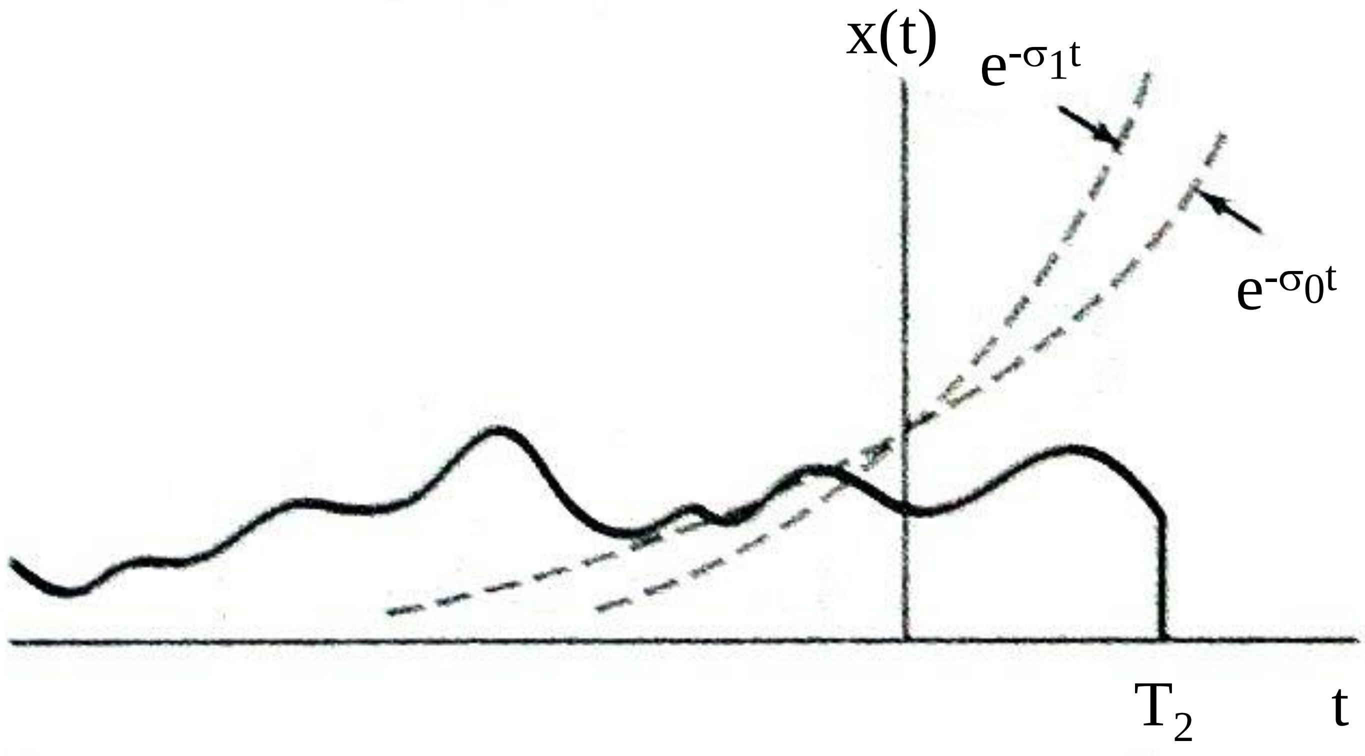
Property4:

If x(t) is right sided, and if the line $\mathrm{Re}\{s\}=\sigma_0$ is in the ROC, then all values of s for which $\mathrm{Re}\{s\}>\sigma_0$ will also in the ROC.



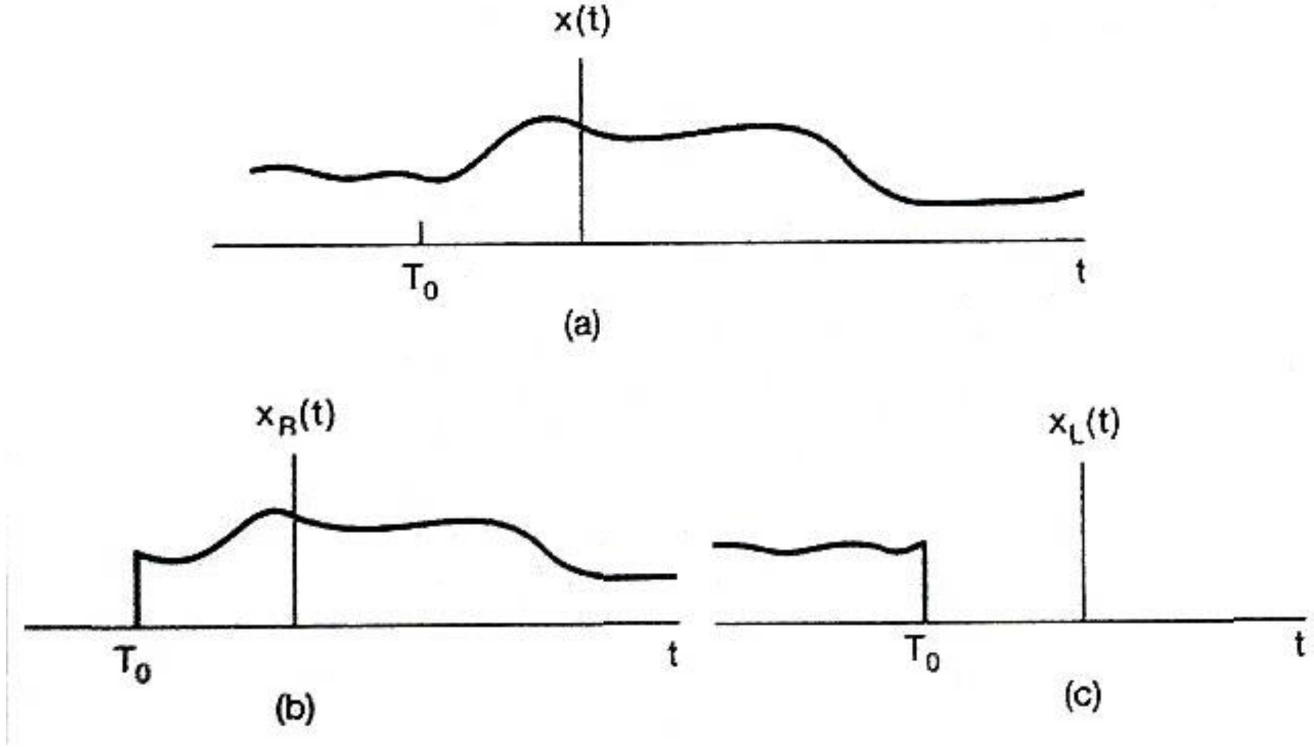
Property5:

If x(t) is left sided, and if the line Re{s}=σ_0 is in the ROC, then all values of s for which Re{s}<σ_0 will also in the ROC.

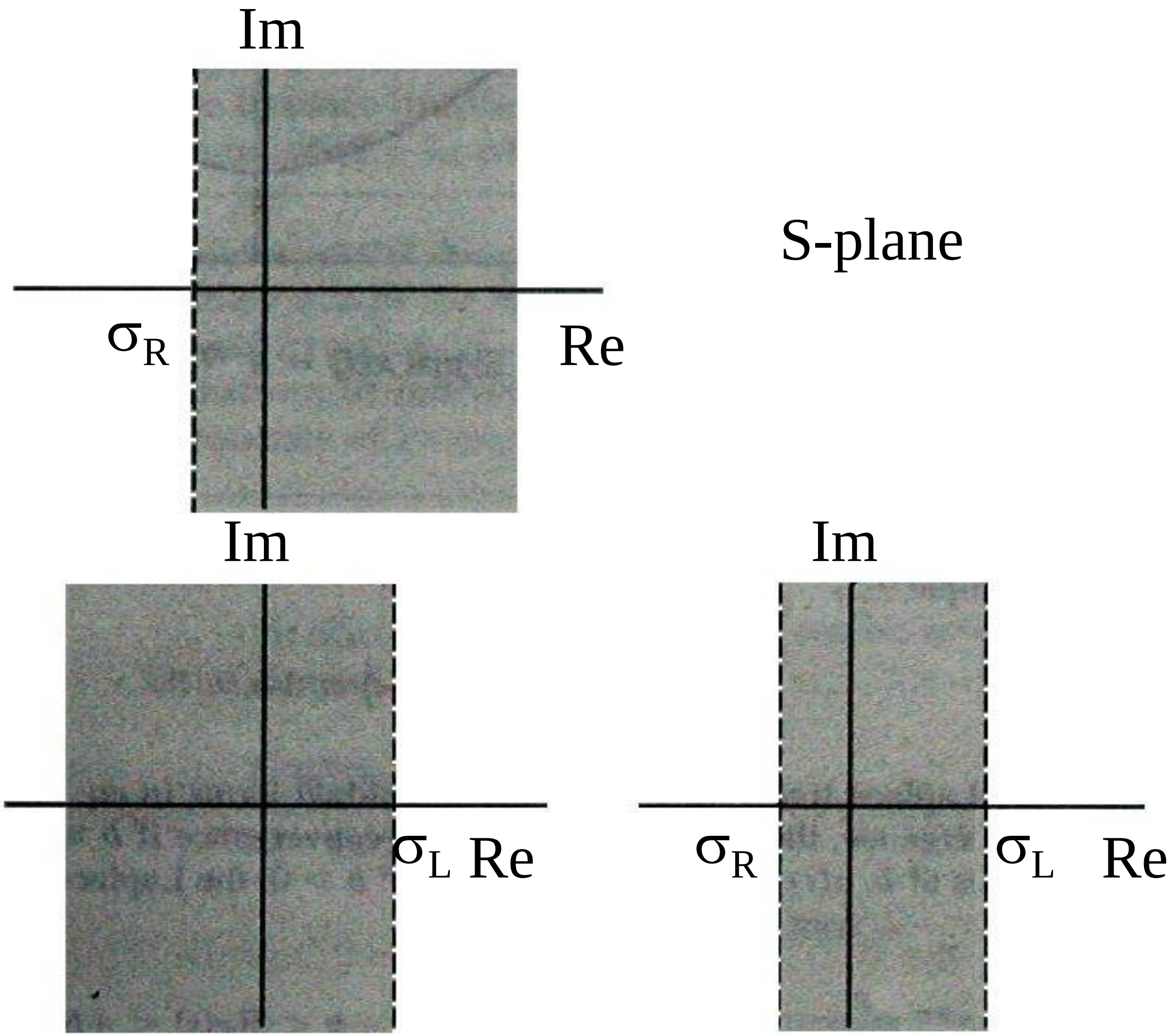


Property6:

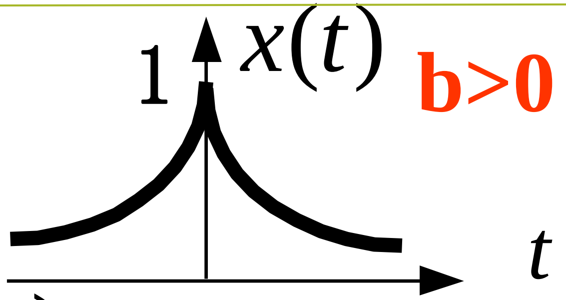
If $x(t)$ is two sided, and if the line $Re\{s\}=\sigma_0$ is in the ROC, then the ROC will consist of a strip in the s-plane that includes the line $Re\{s\}=\sigma_0$ .Normally, $\sigma_R < Re\{s\} < \sigma_L$. ($\sigma_R < \sigma_L$)



9 The Laplace Transform



Example 9.7 $x(t)=e^{-b|t|}$



b>0

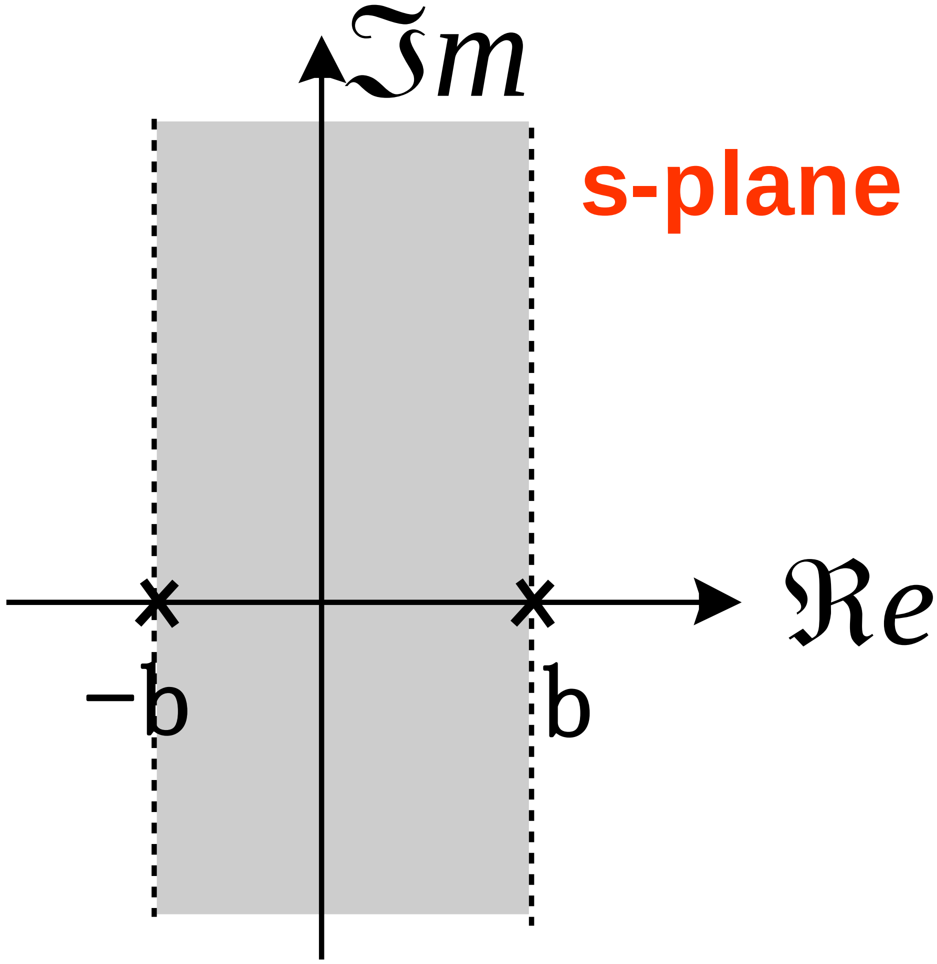
$$x(t)=e^{-bt}u(t)+e^{bt}u(-t)$$

$$e^{-bt}u(t)\xleftrightarrow{LT}\frac{1}{s+b},\qquad \mathrm{Re}\{s\}>-b$$

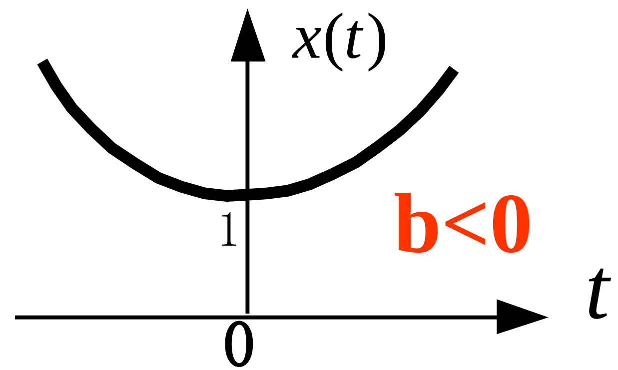
$$e^{bt}u(-t)\xleftrightarrow{LT}\frac{-1}{s-b},\qquad \mathrm{Re}\{s\}<b$$

$$X(s)=\frac{1}{s+b}+\frac{-1}{s-b}=\frac{-2b}{s^2-b^2},$$

ROC: $-b < \mathrm{Re}\{s\} < b$



If b<0,



X(s) is not existed.

Property7:

If the Laplace transform X(s) of x(t) is rational, then its ROC is bounded by poles or extends to infinity. In addition, no poles of X(s) are contained in the ROC.

Property8:

If the Laplace transform X(s) of x(t) is rational,

then if x(t) is right sided, the ROC is the region

in the s-plane to the right of the rightmost pole.

If x(t) is left sided, the ROC is the region

in the s-plane to the left of the leftmost pole.

Example 9.8 $X(s) = \dfrac{1}{(s+1)(s+2)},$

poles: s= -1, s= -2

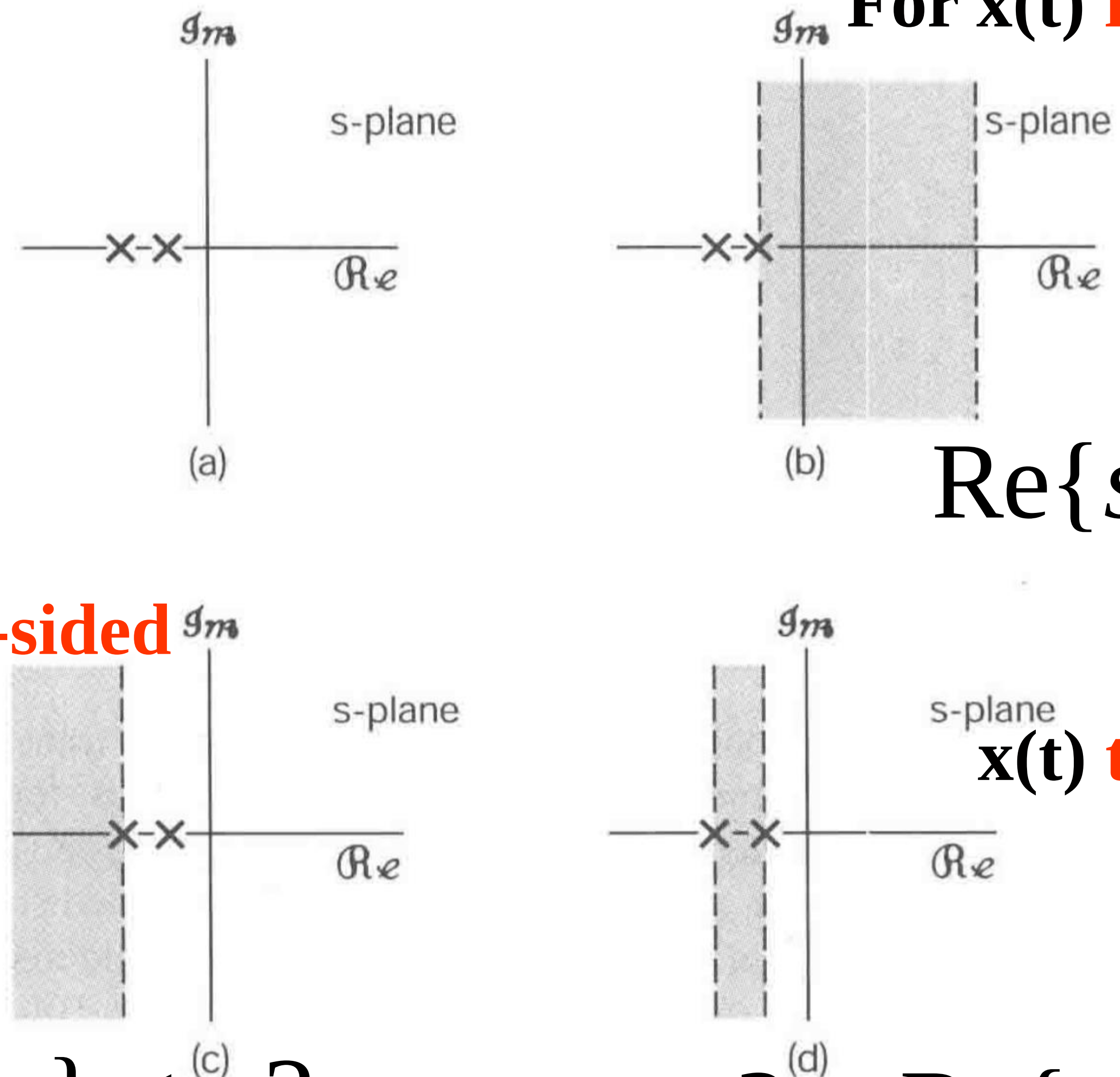
ROC1: $\mathrm{Re}\{s\} > -1$ **For x(t) right-sided**

ROC2: $-2 < \mathrm{Re}\{s\} < -1$ **For x(t) two-sided**

ROC3: $\mathrm{Re}\{s\} < -2$ **For x(t) left-sided**

ROC of X(s) could be one of the up three ROCs.

For x(t) **right-sided**



$$\mathrm{Re}\{s\} > -1$$

x(t) **left-sided**

$$\mathrm{Re}\{s\} < -2$$

x(t) **two-sided**

$$-2 < \mathrm{Re}\{s\} < -1$$

9.3 The Inverse Laplace Transform

$$X(s)=\int_{-\infty}^{+\infty}x(t)e^{-st}dt$$

$$X(\sigma+j\omega)=\int_{-\infty}^{+\infty}x(t)e^{-\sigma t}e^{-j\omega t}dt \ (\ s=\sigma+j\omega\ )$$

$$=F[x(t)e^{-\sigma t}]$$

$$x(t)e^{-\sigma t}=F^{-1}[X(\sigma+j\omega)]$$

$$=\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(\sigma+j\omega)e^{j\omega t}d\omega$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\sigma + j\omega) e^{(\sigma + j\omega)t} d\omega$$

So $$\boxed{x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds}$$

Inverse Laplace transform

The calculation for inverse Laplace transform:

(1) Integration of complex function by equation.

(2) Compute by Partial Fraction Expansion .

General form of X(s): $X(s) = \dfrac{N(s)}{D(s)}$,

$$X(s) = \frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2} + \cdots + \frac{A_n}{s-\lambda_n}$$

$$= \sum_{i=1}^{n} \frac{A_i}{s-\lambda_i}$$

Important transform pair:

$$\frac{1}{s-\lambda_i} \leftrightarrow \begin{cases} e^{\lambda_i t}u(t), \ left \quad pole \\ -e^{\lambda_i t}u(-t), \ right \quad pole \end{cases}$$

The inverse Laplace transform can be determined.

Example:

$$X(s)=\frac{1}{(s+1)(s+2)},\mathrm{Re}\{s\}>-1$$

Partial-fraction expansion

$$X(s)=\frac{1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$$

$$X(s)=\frac{1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}$$

$$A=[(s+1)X(s)]\big|_{s=-1}=1$$

$$B=[(s+2)X(s)]\big|_{s=-2}=-1$$

$$So,\ X(s)=\frac{1}{s+1}-\frac{1}{s+2}\quad ,\mathrm{Re}\{s\}>-1$$

$$e^{-t}u(t)\overset{L}{\leftrightarrow}\frac{1}{s+1},\mathrm{Re}\{s\}>-1$$

$$e^{-2t}u(t)\overset{L}{\leftrightarrow}\frac{1}{s+2},\mathrm{Re}\{s\}>-2$$

$$[e^{-t}-e^{-2t}]u(t)\overset{L}{\leftrightarrow}\frac{1}{(s+1)(s+2)},\mathrm{Re}\{s\}>-1$$

$$Example: X(s) = \frac{1}{s+1} - \frac{1}{s+2} \quad \textbf{\textit{If}}\ \mathbf{Re\{\textit{s}\} < -2}$$

$$-e^{-t}u(-t) \xleftrightarrow{L} \frac{1}{s+1}, \mathrm{Re}\{s\} < -1$$

$$-e^{-2t}u(-t) \xleftrightarrow{L} \frac{1}{s+2}, \mathrm{Re}\{s\} < -2$$

$$x(t) = [-e^{-t} + e^{-2t}]u(-t) \xleftrightarrow{L} \frac{1}{(s+1)(s+2)}, \mathrm{Re}\{s\} < -2$$

$$Example: X(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

If ROC: -2<Re{s} < -1

$$x(t) = -e^{-t}u(-t) - e^{-2t}u(t) \overset{L}{\leftrightarrow} \frac{1}{(s+1)(s+2)},$$

$$-2 < \mathrm{Re}\{s\} < -1$$

9.4 The Properties of Laplace Transform

9.4.1 Linearity

If $x_1(t) \xleftrightarrow{L} X_1(s)$, **ROC:R1**

$x_2(t) \xleftrightarrow{L} X_2(s)$, **ROC: R2**

Then

$$ax_1(t) + bx_2(t) \xleftrightarrow{L} aX_1(s) + bX_2(s),$$

R1 ∩R2

Note: (1) normally, common ROC.

(2) R1 ∩R2 may be larger than R1 or R2

9.4.2 Time Shifting

If $$x(t) \xleftrightarrow{L} X(s),$$ **ROC: R**

Then $$x(t-t_0) \xleftrightarrow{L} e^{-st_0}X(s),$$ **ROC:R**

9.4.3 Shifting in s-Domain

If $$x(t) \xleftrightarrow{L} X(s),$$ **ROC: R**

Then $$e^{s_0 t} x(t) \xleftrightarrow{L} X(s - s_0),$$

ROC: R+Re{s_0}

$$e^{j\omega_0 t} x(t) \xleftrightarrow{L} X(s - j\omega_0),$$ **ROC: R**

Example:

From *LT* pair

$$\sin \alpha t u(t) \xleftrightarrow{L} \frac{\alpha}{s^2+\alpha^2}, \qquad \mathrm{Re}\{s\} > 0$$

We can get

$$e^{-\beta t} \sin \alpha t u(t) \xleftrightarrow{L} \frac{\alpha}{(s+\beta)^2+\alpha^2},$$

$$\mathrm{Re}\{s\} > -\beta$$

Example $X(s) = \dfrac{e^{-(s+1)}}{(s+1)^2 + 2}, \quad \mathrm{Re}\{s\} > -1$

$$x(t) = ?$$

Solution:

(1) From LT pair

$$\sin\sqrt{2}t\,u(t) \xleftrightarrow{L} \frac{\sqrt{2}}{s^2+2}, \quad \mathrm{Re}\{s\} > 0$$

(2) Shifting in s-Domain

$$e^{-t}\sin\sqrt{2}t\,u(t) \xleftrightarrow{L} \frac{\sqrt{2}}{(s+1)^2+2}, \quad \mathrm{Re}\{s\} > -1$$

(3) Shifting in time-Domain

$$e^{-t}\sin\sqrt{2}(t-1)u(t-1) \xleftrightarrow{L} \frac{e^{-1}e^{-s}\sqrt{2}}{(s+1)^2+2},$$

$$\operatorname{Re}\{s\} > -1$$

$$\therefore x(t) = \frac{1}{\sqrt{2}}e^{-t}\sin\sqrt{2}(t-1)u(t-1)$$

You can get same x(t) by Shifting in time-Domain first.

Solution: #2

$$X(s) = \frac{e^{-(s+1)}}{(s+1)^2+2}, \quad \rightarrow \quad x(t) = e^{-t}x_1(t)$$

$$X_1(s) = e^{-s}\frac{1}{s^2+2}, \quad \rightarrow \quad x_1(t) = x_2(t-1)$$

$$X_2(s) = \frac{1}{s^2+2}, \quad \rightarrow \quad x_2(t) = \frac{\sqrt{2}}{2}\sin(\sqrt{2}t)u(t)$$

9.4.4 Time Scaling

If $x(t) \xleftrightarrow{L} X(s)$, R

Then $x(at) \xleftrightarrow{L} \frac{1}{|a|} X(\frac{s}{a})$, |a|R

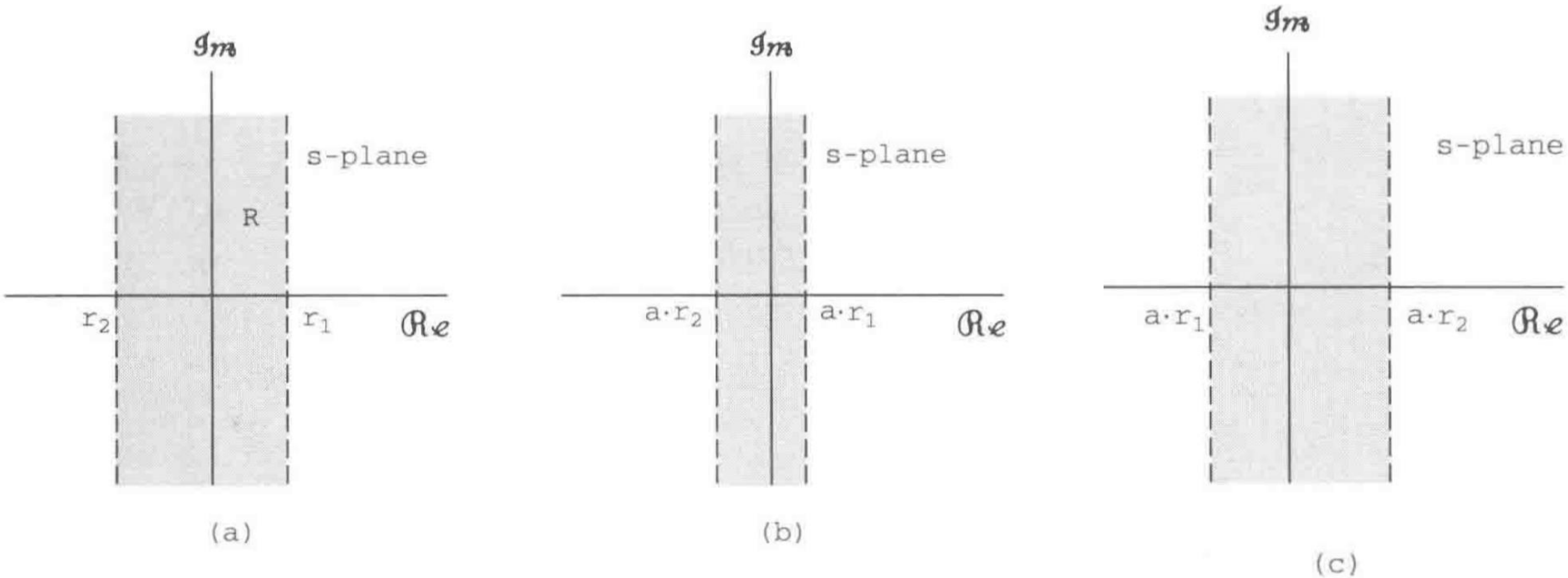
Note: $R:(r_2, r_1)$

$$aR:(ar_2, ar_1), a > 0$$

$$aR:(ar_1, ar_2), a < 0$$

Specially, $x(-t) \xleftrightarrow{L} X(-s)$, -R

9 The Laplace Transform



Effect of ROC of time scaling:

(a)ROC for $\mathbf{X(s)}$ (b)ROC for $\left(\frac{1}{|a|}X\left(\frac{s}{a}\right)\right)$, for $0 < a < 1$

(c) ROC for $\left(\frac{1}{|a|}X\left(\frac{s}{a}\right)\right)$, for $-1 < a < 0$

Example

From LT pair

$$e^{-at}u(t) \overset{L}{\longleftrightarrow} \frac{1}{s+a}, \quad \mathrm{Re}\{s\} > -a$$

$$e^{at}u(-t) \overset{L}{\longleftrightarrow} \frac{1}{-s+a}, \quad \mathrm{Re}\{s\} < a$$

9.4.5 Conjugation

If $x(t) \xleftrightarrow{L} X(s),$ **R**

Then $x^*(t) \xleftrightarrow{L} X^*(s^*),$ **R**

If $x(t)$ **is real** $X(s) = X^*(s^*)$

9.4.6 The Convolution Property

If $x_1(t) \xleftrightarrow{L} X_1(s),$ **R1**

$x_2(t) \xleftrightarrow{L} X_2(s),$ **R2**

Then $x_1(t) * x_2(t) \xleftrightarrow{L} X_1(s)X_2(s)$ **R1 ∩R2**

R1 ∩ R2 maybe larger than R1 or R2

For example

$$X_1(s)=\frac{s+1}{s+2},\mathrm{Re}\{s\}>-2$$

$$X_2(s)=\frac{s+2}{s+1},\mathrm{Re}\{s\}>-1$$

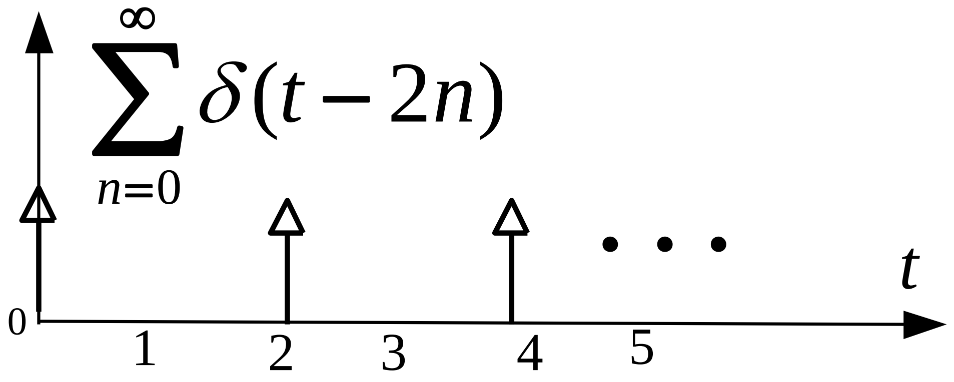
$$X_1(s)X_2(s)=1 \quad ROC:$$ **entire s-plane**

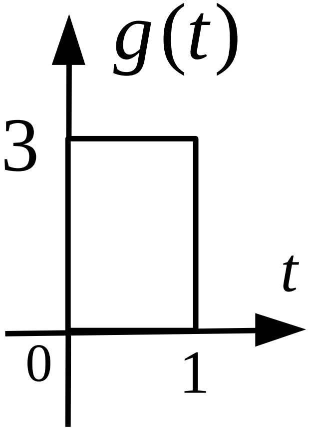
Note: if $R_1 \cap R_2 = \varnothing$, $X_1(s)X_2(s)$ does not exist.

Example $$x(t)=g(t)*\sum_{n=0}^{\infty}\delta(t-2n)$$

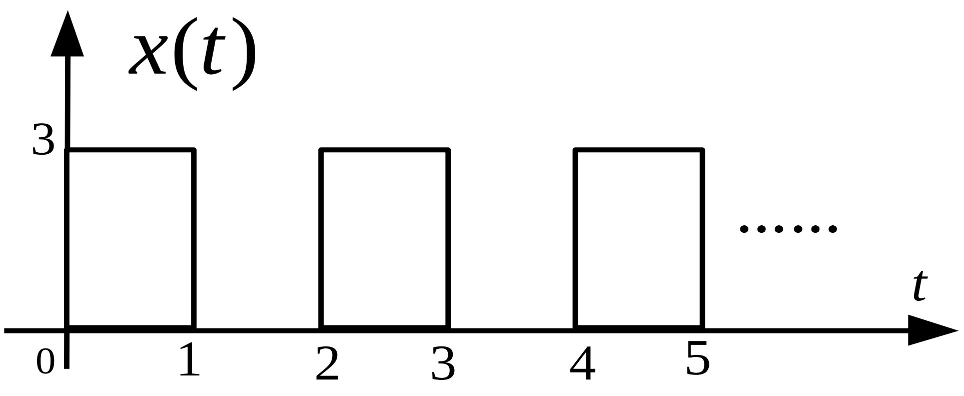
$$x(t) = g(t) * \sum_{n=0}^{\infty} \delta(t-2n)$$

(1)





So



where $g(t) = 3[u(t) - u(t-1)]$

$$G(s) = 3\left[\frac{(1-e^{-s})}{s}\right], \quad -\infty < \mathrm{Re}\{s\} < \infty$$

$$\sum_{n=0}^{\infty} \delta(t-2n) \xleftrightarrow{L} \sum_{n=0}^{\infty} e^{-2ns} = \frac{1}{1-e^{-2s}}$$

Re{s}>0

$$\therefore X(s) = G(s)\frac{1}{(1-e^{-2s})}$$

$$= 3[\frac{(1-e^{-s})}{s}]\frac{1}{(1-e^{-2s})}$$

$$= \frac{3}{s(1+e^{-s})},$$

$R_e\{s\}>0$

9.4.7 Differentiation in the time Domain

If $$x(t) \stackrel{L}{\longleftrightarrow} X(s), \quad \mathbf{R}$$

Then $$\frac{d}{dt}x(t) \stackrel{L}{\longleftrightarrow} sX(s),$$

Containing R

Example

$$u(t) \xleftrightarrow{L} \frac{1}{s}, \qquad \mathrm{Re}\{s\} > 0$$

$$\frac{d}{dt}u(t) = \delta(t) \xleftrightarrow{L} 1, \quad -\infty < \mathrm{Re}\{s\} < \infty$$

$$\delta'(t) \xleftrightarrow{L} s, \quad -\infty < \mathrm{Re}\{s\} < \infty$$

9.4.8 Differentiation in the s-Domain

If $$x(t) \xleftrightarrow{L} X(s),$$ **R**

Then $$-tx(t) \xleftrightarrow{L} \frac{d}{ds}X(s),$$ **R**

$$\{X(s) = \int_{-\infty}^{+\infty} x(t)e^{-st}\,dt$$

$$\frac{dX(s)}{\mathrm{ds}} = \int_{-\infty}^{+\infty} (-t)x(t)e^{-st}\,dt\}$$

Example **From LT pair**

$$e^{-at}u(t) \xleftrightarrow{L} \frac{1}{s+a}, \quad \mathrm{Re}\{s\} > -a$$

$$-te^{-at}u(t) \xleftrightarrow{L} \frac{d}{ds}[\frac{1}{s+a}] = \frac{-1}{(s+a)^2},$$

$$\mathrm{Re}\{s\} > -a$$

Example $x(t) = ?$

$$X(s) = \frac{2s^2 + 5s + 5}{(s+1)^2(s+2)}, \qquad \mathrm{Re}\{s\} > -1$$

By partial-fraction expansion

$$X(s) = \frac{2}{(s+1)^2} - \frac{1}{s+1} + \frac{3}{s+2},$$

$$\mathrm{Re}\{s\} > -1$$

$$X(s) = \frac{2}{(s+1)^2} - \frac{1}{s+1} + \frac{3}{s+2},$$

$$\mathrm{Re}\{s\} > -1$$

$$x(t) = [2te^{-t} - e^{-t} + 3e^{-2t}]u(t)$$

9.4.9 Integration in the time Domain

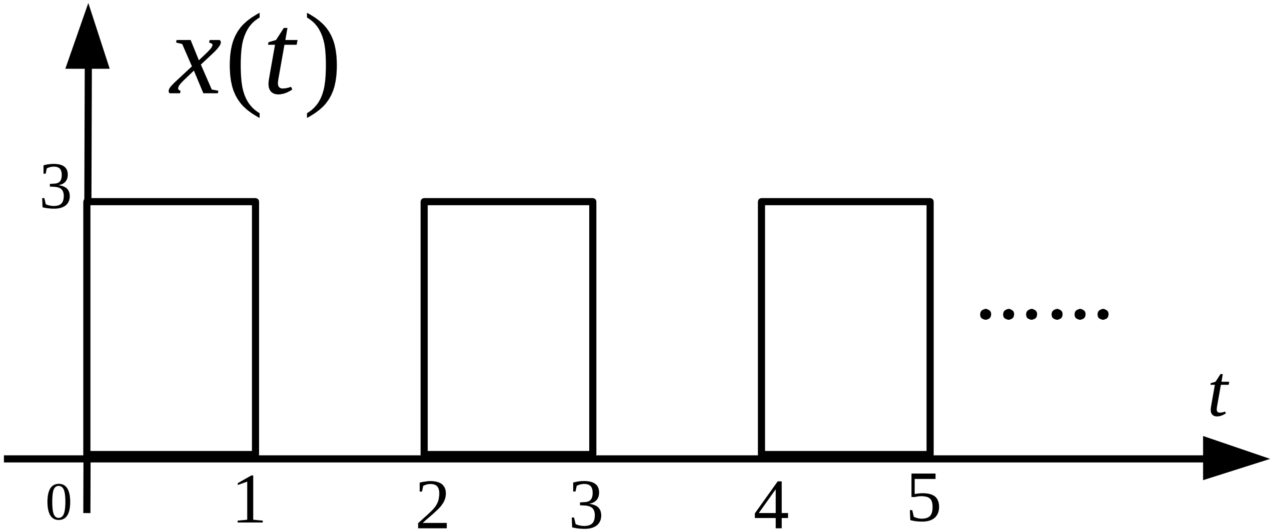
If $$x(t) \xleftrightarrow{L} X(s),$$ **R**

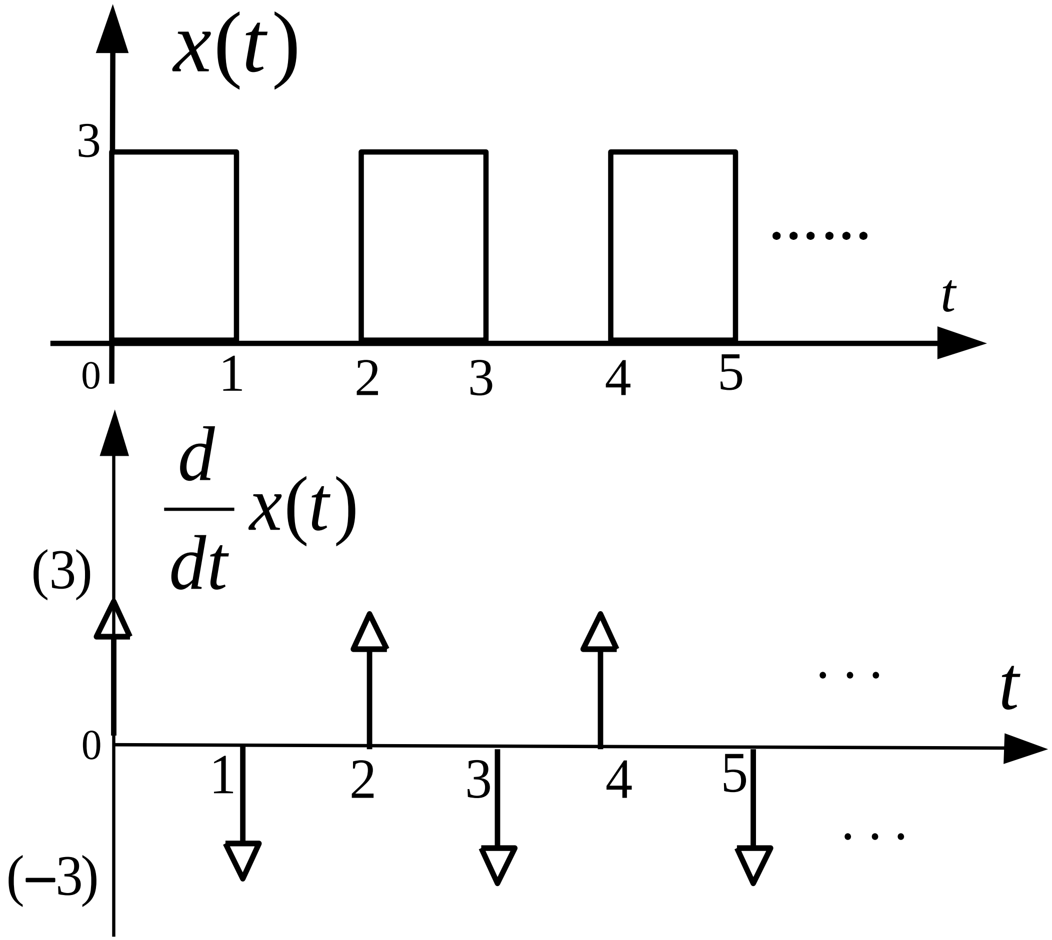
Then $$\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{L} \frac{1}{s}X(s),$$

R∩Re{s}>0

Example

$X(s)$ **=?**





So $$\frac{d}{dt}x(t)=3\sum_{n=0}^{\infty}\delta(t-2n)-3\sum_{n=0}^{\infty}\delta(t-2n-1)$$

$$\frac{d}{dt}x(t) = 3\sum_{n=0}^{\infty}\delta(t-2n) - 3\sum_{n=0}^{\infty}\delta(t-2n-1)$$

$$\Big\downarrow L \qquad\qquad \Big\downarrow L \qquad\qquad \Big\downarrow L$$

$$sX(s) = 3\sum_{n=0}^{\infty}e^{-2ns} - 3\sum_{n=0}^{\infty}e^{-(2n+1)s}$$

$$= 3[\sum_{n=0}^{\infty}e^{-2ns}](1-e^{-s})$$

$$= 3[1/(1-e^{-2s})](1-e^{-s})$$

$$\therefore X(s) = \frac{3}{s(1+e^{-s})}$$

$R_e\{s\}>0$

9.4.10 The Initial- and Final-Value Theorems

If $x(t) = 0,\ for \quad t < 0.$

Its *LT* $X(s),$ **ROC: $R_e\{s\} > \sigma_1$**

Then the Initial- Value Theorem

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

The Final-Value Theorem

$$\lim_{t\to\infty} x(t) = x(\infty) = \lim_{s\to 0} sX(s)$$

Example 9.16 Read by yourself!

Example

$$x(t) = e^{-2t}u(t) + e^{-t}\cos(3t)u(t)$$

$$X(s) = \frac{2s^2 + 5s + 12}{(s^2 + 2s + 10)(s + 2)} \qquad \mathrm{Re}\{s\} > -1$$

$$\left\{\begin{array}{l} x(0^+) = 1 + 1 = 2 \\[2em] x(0^+) = \lim\limits_{s \to \infty} sX(s) = 2 \end{array}\right.$$

Example:

Let

$$g(t) = x(t) + \alpha x(-t),$$

where

$$x(t) = \beta e^{-t} u(t)$$

and the Laplace transform of $g(t)$ is

$$G(s) = \frac{s}{s^2 - 1}, \qquad -1 < \Re e\{s\} < 1.$$

Determine the values of the constants α and β.

Solution:

We have $$X(s) = \frac{\beta}{s+1}, \mathrm{Re}\{s\} > -1$$

Also $$G(s) = X(s) + \alpha X(-s), -1 < \mathrm{Re}\{s\} < 1$$

Therefore, $$G(s) = \beta\left[\frac{1 - s + \alpha s + \alpha}{1 - s^2}\right]$$

Comparing with the given equation for G(s),

$$\alpha = -1, \beta = \frac{1}{2}$$

9.4.11 Table of LT Properties

TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	R
		$x_1(t)$	$X_1(s)$	R_1
		$x_2(t)$	$X_2(s)$	R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re e\{s\} > 0\}$

Initial- and Final-Value Theorems

9.5.10 If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher-order singularities at $t = 0$, then

$$x(0^+) = \lim_{s \to \infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and $x(t)$ has a finite limit as $t \longrightarrow \infty$, then

$$\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$$

9.5 Some LT Pairs

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\Re e\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\Re e\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re e\{s\} < 0$
6	$e^{-at}u(t)$	$\frac{1}{s+a}$	$\Re e\{s\} > -a$
7	$-e^{-at}u(-t)$	$\frac{1}{s+a}$	$\Re e\{s\} < -a$
8	$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(s+a)^n}$	$\Re e\{s\} > -a$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-at}u(-t)$	$\frac{1}{(s+a)^n}$	$\Re e\{s\} < -a$
10	$\delta(t-T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2+\omega_0^2}$	$\Re e\{s\} > 0$
13	$[e^{-at}\cos \omega_0 t]u(t)$	$\frac{s+a}{(s+a)^2+\omega_0^2}$	$\Re e\{s\} > -a$
14	$[e^{-at}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+a)^2+\omega_0^2}$	$\Re e\{s\} > -a$
15	$u_n(t) = \frac{d^n\delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re e\{s\} > 0$

9.6 Analysis and Characterization of LTI systems Using LT (including 9.4 9.7)

Consider an LTI system:

$x(t)$ → $h(t)$ → $y(t)=x(t)*h(t)$

$X(s)$ → $H(s)$ → $Y(s) = X(s)H(s)$

(e^{st}) → $(H(s)e^{st})$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) e^{st} ds$$

$$y(t) = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} X(s) H(s) e^{st} ds$$

$$H(s) = \frac{Y(s)}{X(s)}$$ (**system function**)

$$h(t) \xleftrightarrow{L} H(s)$$

9.6.1 Causality

(1)A causal system ⇒ $H(s)$, **ROC: right-half plane**

$$(h(t)=0,\ for \qquad t<0.)$$

($R_e\{s\} > \sigma_1$)

(2) For rational $H(s)=\dfrac{N(s)}{D(s)}$,

A causal system ⇔ **ROC: right-half plane To the right of rightmost pole(σ_1)**

equivalent

Example 9.17 $$h(t) = e^{-t}u(t)$$

Since h(t)=0 for t<0, so the system is causal

$$H(s) = \frac{1}{s+1}, \quad \mathrm{Re}\{s\} > -1$$

H(s) is rational and ROC is to the right of the rightmost pole, consistent with our statement.

Example 9.18

$$h(t) = e^{-|t|} \quad \xleftrightarrow{L} \quad H(s) = \frac{-2}{s^2 - 1},$$

$$-1 < \mathrm{Re}\{s\} < 1$$

H(s) is rational, but ROC is not to the right of the rightmost pole

So: the system is noncausal system

Example 9.19 $$H(s)=\frac{e^{s}}{s+1},\quad \mathrm{Re}\{s\}>-1$$

$$e^{-t}u(t)\xleftrightarrow{L}\frac{1}{s+1},\mathrm{Re}(s)>-1$$

$$e^{-(t+1)}u(t+1)\xleftrightarrow{L}\frac{e^{s}}{s+1},\mathrm{Re}(s)>-1$$

$$h(t)=e^{-(t+1)}u(t+1)$$

A noncausal system

9.6.2 Stability

(1)A stable system $\Leftrightarrow H(s)$, **ROC:**

($R_e\{s\}=0$) includes $j\omega$ -axis

(2) A causal and stable system with rational

$$H(s)=\frac{N(s)}{D(s)}, \Leftrightarrow$$

All poles lies in the left-half of s-plane

Example:

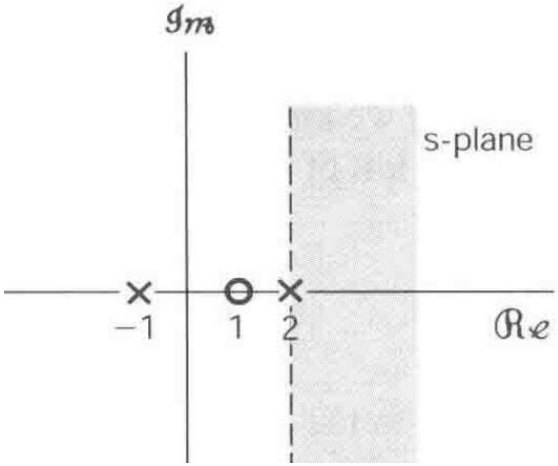
$$H(s) = \frac{s-1}{(s+1)(s-2)}$$

Impulse response: $h(t) = ?$

$$H(s) = \frac{A}{s+1} + \frac{B}{s-2} = \frac{2/3}{s+1} + \frac{1/3}{s-2}$$

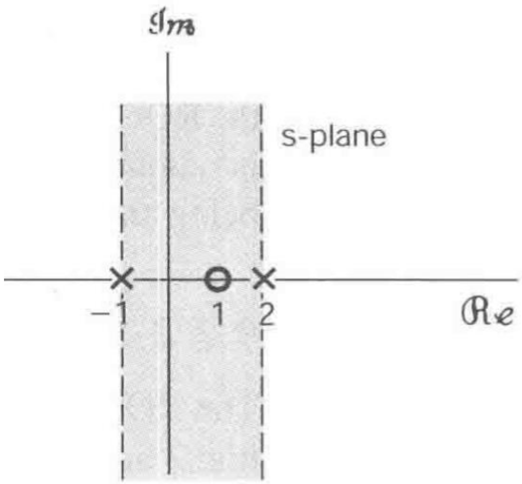
$$h(t) = \frac{2}{3}e^{-t}u(t) + \frac{1}{3}e^{2t}u(t)$$ **???**

$$H(s)=\frac{2/3}{s+1}+\frac{1/3}{s-2}$$



If the system is causal

$$h(t)=\frac{2}{3}e^{-t}u(t)+\frac{1}{3}e^{2t}u(t)$$

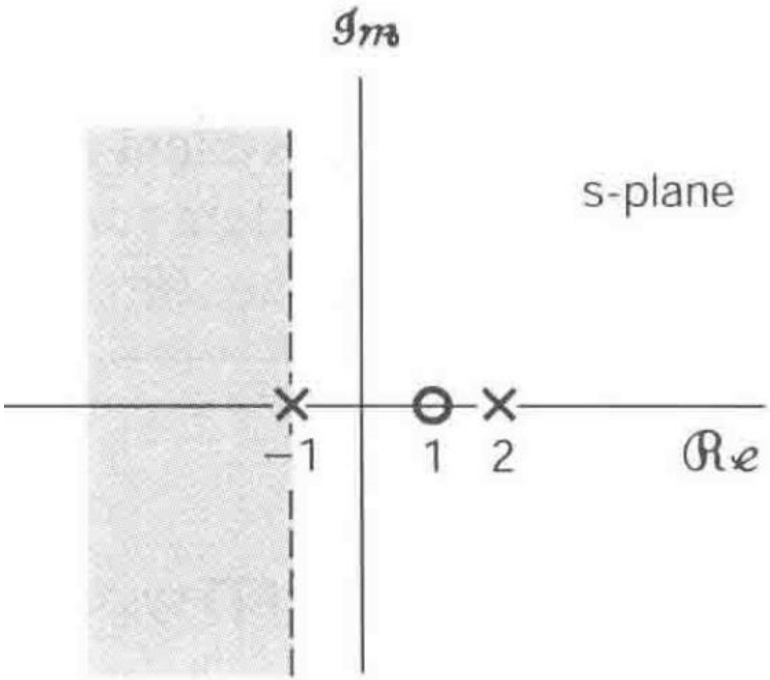


If the system is stable

$$h(t)=\frac{2}{3}e^{-t}u(t)-\frac{1}{3}e^{2t}u(-t)$$

If the system is anticausal and unstable

$$h(t) = -\frac{2}{3}e^{-t}u(-t) - \frac{1}{3}e^{2t}u(-t)$$



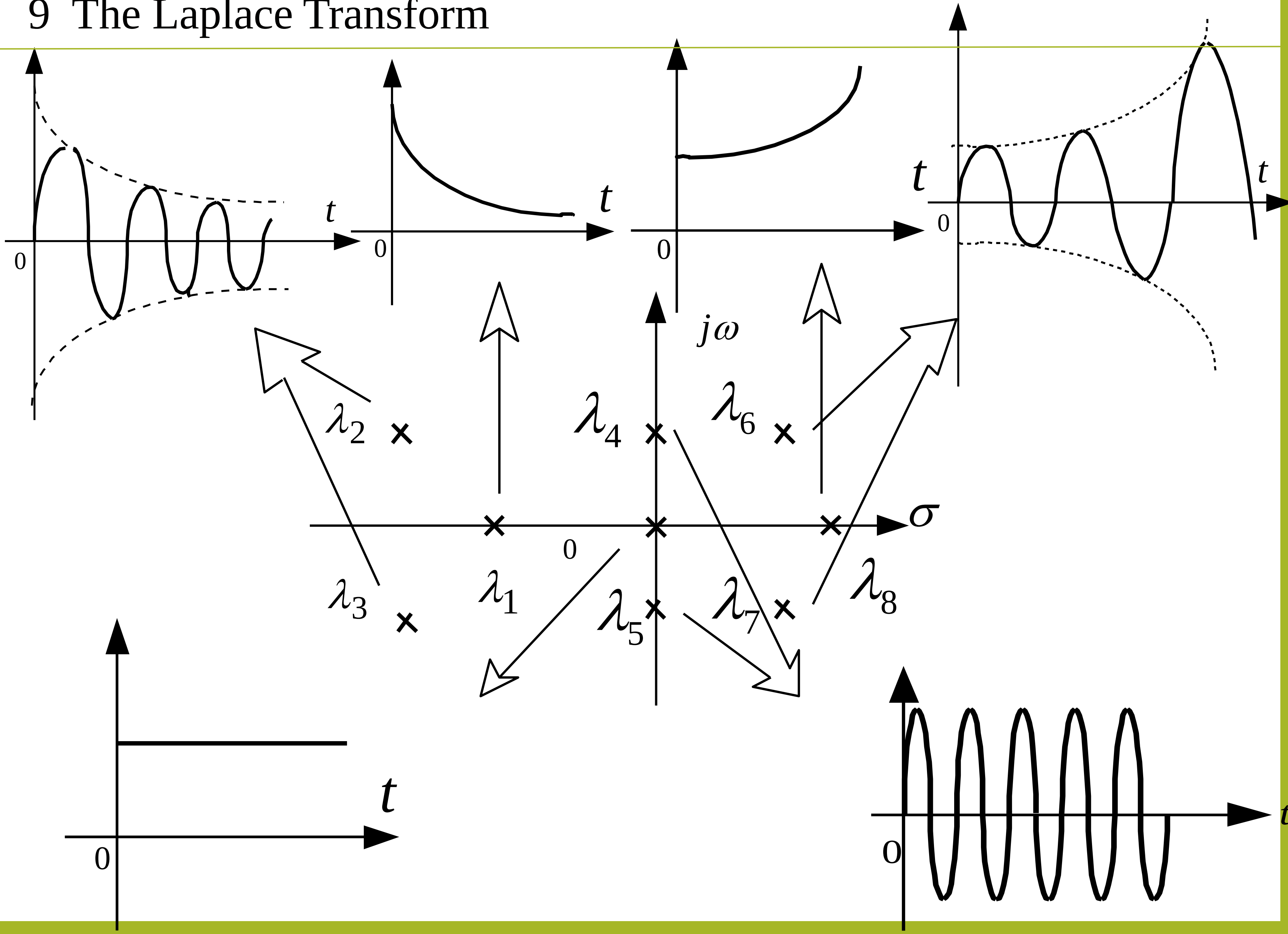
9.6.3 Pole-Zero Plot of $H(s)$ and Evaluation of Frequency Response

$$H(s) = \frac{N(s)}{D(s)}$$

$$H(s) = \frac{b_0 \prod_{i=1}^{M} (s - \gamma_i)}{\prod_{i=1}^{N} (s - \lambda_i)}$$

$$H(j\omega) = \frac{b_0 \prod_{i=1}^{M} (j\omega - \gamma_i)}{\prod_{i=1}^{N} (j\omega - \lambda_i)}$$

9 The Laplace Transform

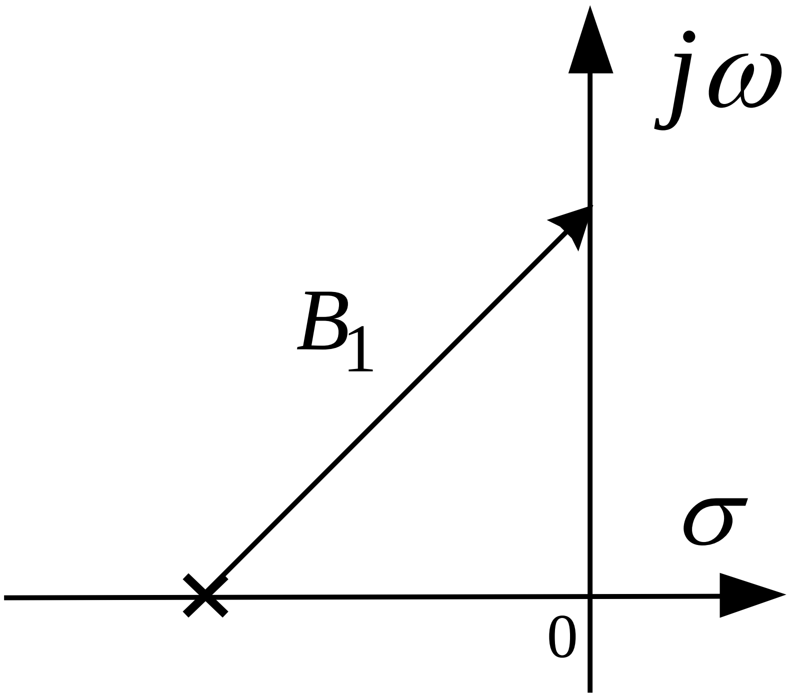


Example: A causal system

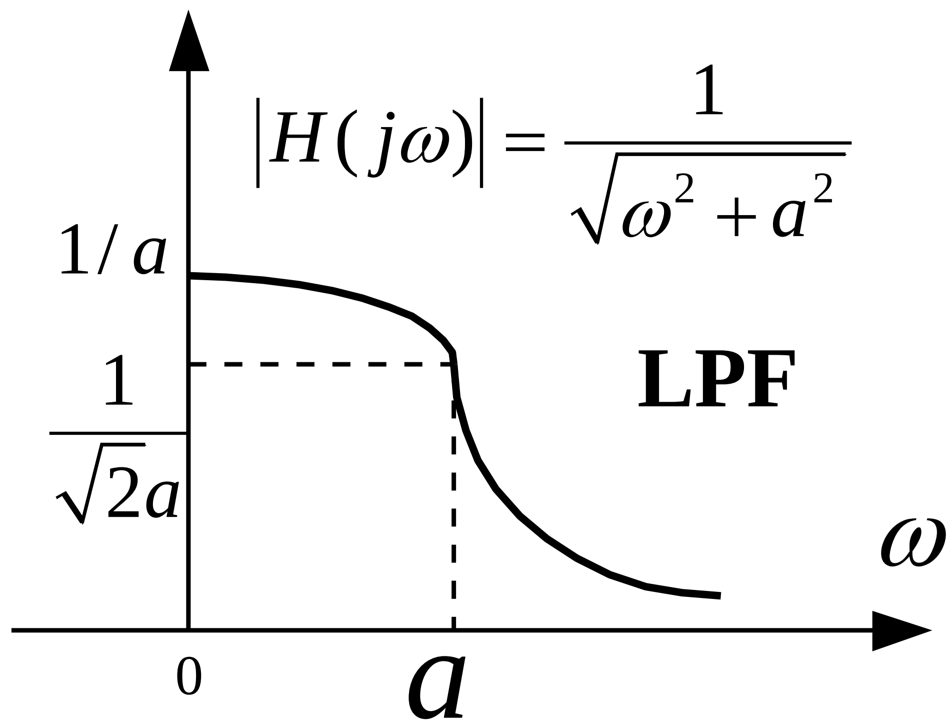
$$H(s)=\frac{1}{s+a}$$

$$(a>0)$$

$$H(j\omega)=\frac{1}{j\omega+a}$$



$$\left|H(j\omega)\right|=\frac{1}{\sqrt{\omega^2+a^2}}$$



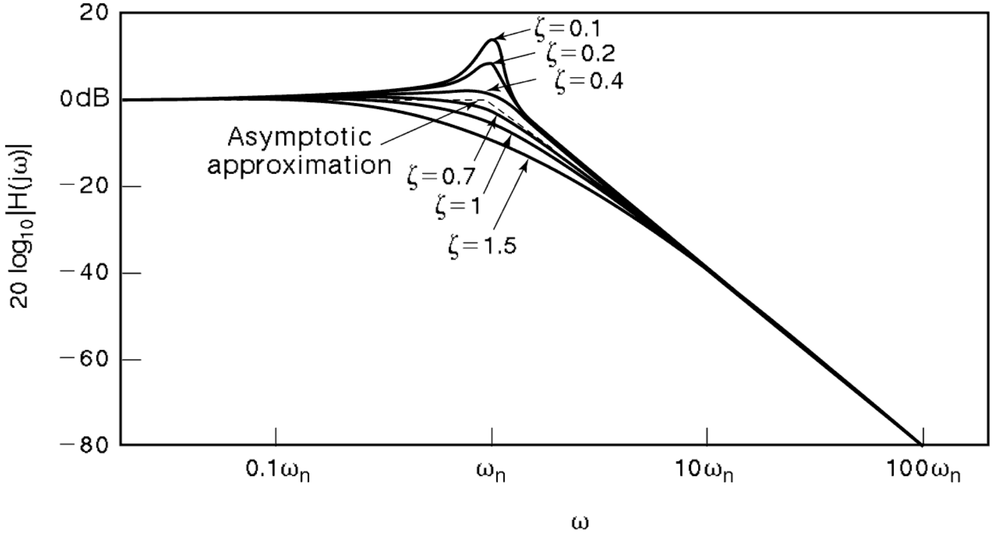
Example: Second-Order System

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \text{ROC } \Re e\{s\} > \Re e(\text{pole})$$

$0 < \zeta < 1 \quad \Rightarrow$ complex poles – *Under*damped

$\zeta = 1 \quad \Rightarrow$ double poles at $s = -\omega_n$ – *Critically* damped

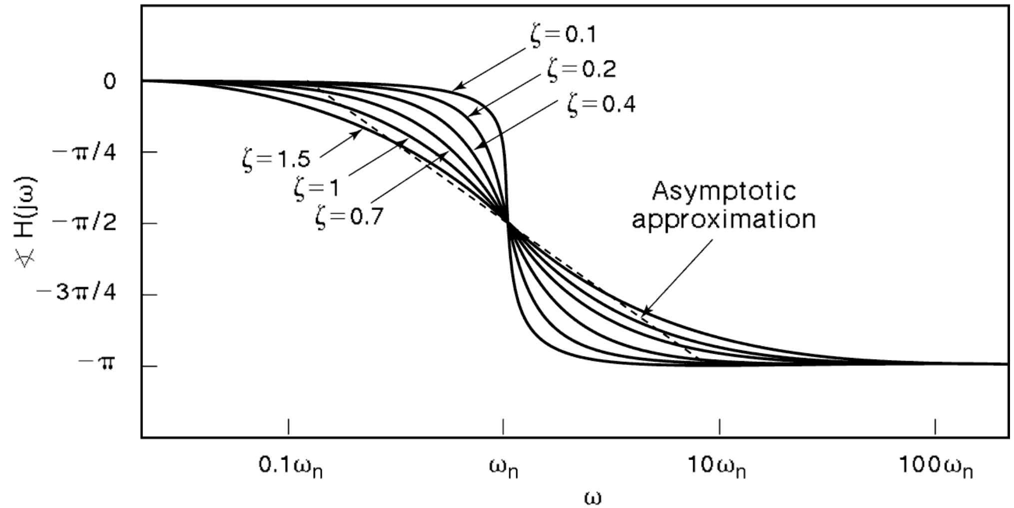
$\zeta > 1 \quad \Rightarrow$ 2 poles on negative real axis – *Over*damped



Top is flat when $\zeta = 1/\sqrt{2} = 0.707$

$\Rightarrow$ an LPF for $\omega < \omega_n$

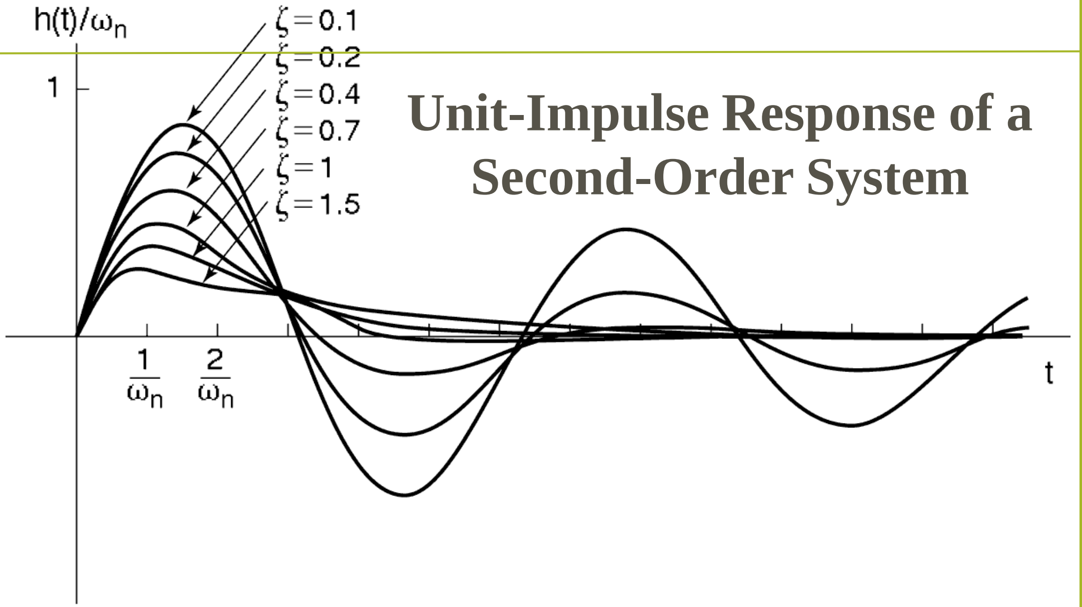
Bode Plot of a Second-Order System

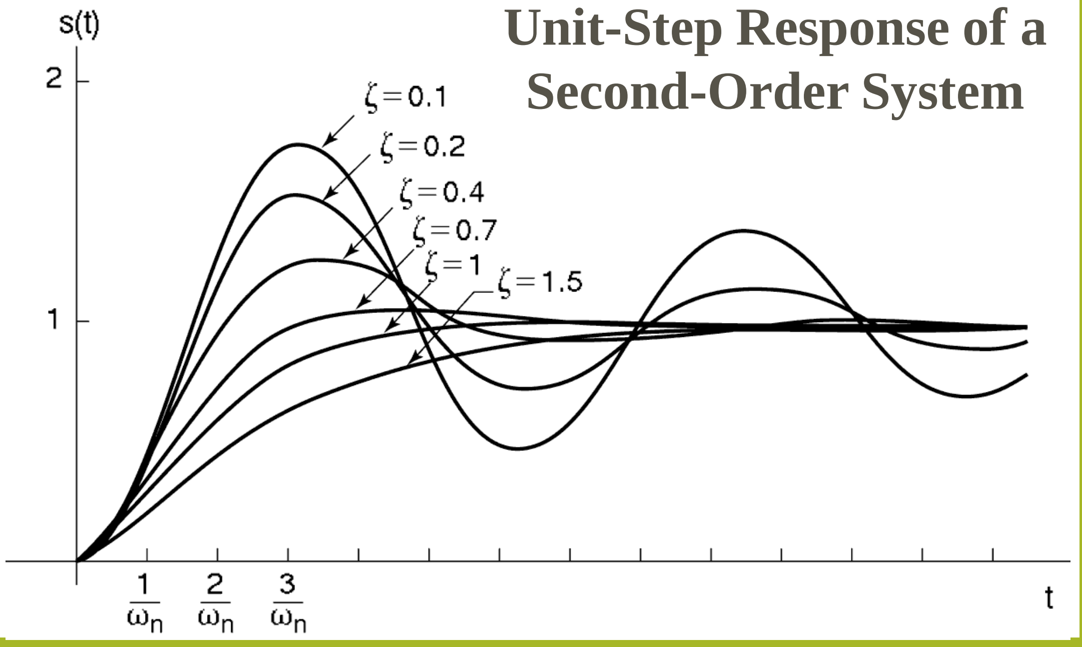


h(t)---No oscillations when

$\zeta \geq 1$

$\Rightarrow$ Critically (=) and over (>) damped.





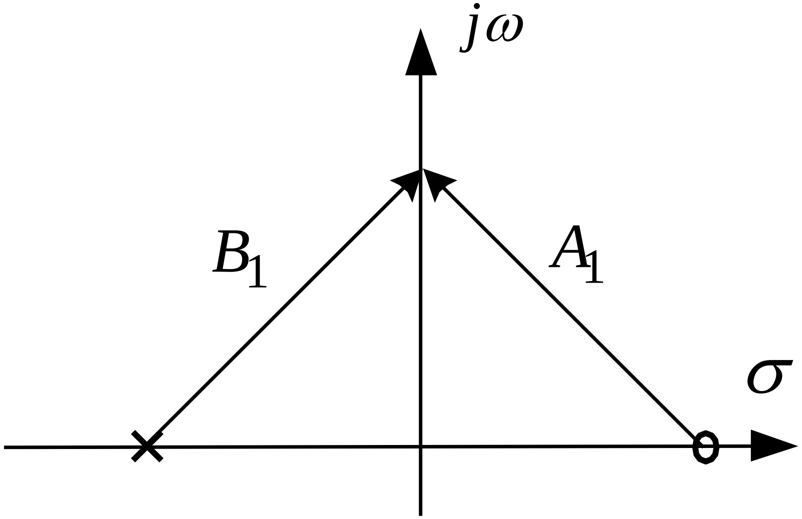
Read P677~681 by yourself!

Example: A causal system

$$H(s)=\frac{s-a}{s+a}\qquad (a>0)$$

$$H(j\omega)=\frac{j\omega-a}{j\omega+a}$$

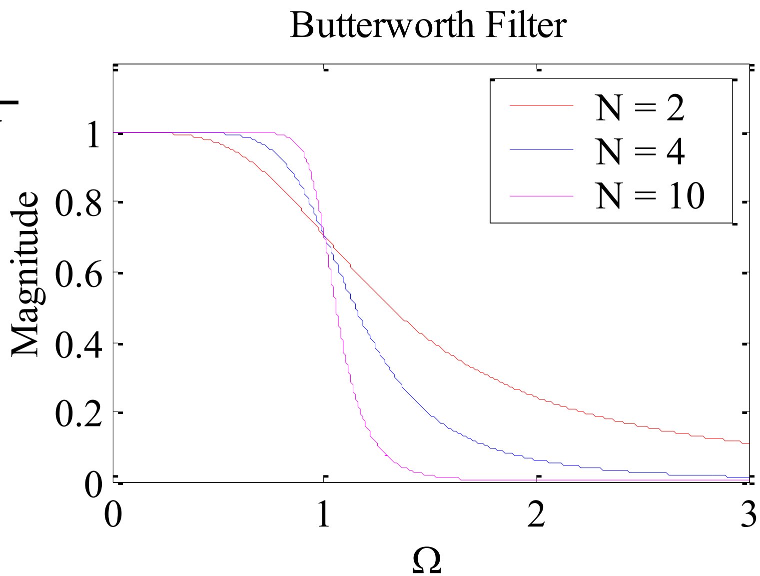
$$\left|H(j\omega)\right|=1$$



(All-pass system)

Example(9.7.5) *N-order* Butterworth Filter

$$\left|H(j\omega)\right|^2 = \frac{1}{1+(j\omega / j\omega_c)^{2N}}$$



$$\left|H(j\omega)\right|^2 = H(s)H(-s)\big|_{s=j\omega}$$

$$H(s) = \frac{\omega_c^N}{\prod_{p=1}^{N}(s-s_p)}$$

$$s_p = \omega_c e^{j\left(\frac{1}{2}+\frac{2p-1}{2N}\right)\pi}$$

$$p = 1, 2, \ldots, N$$

Read P703~706 by yourself !

9.6.4 LTI Systems Characterized by Linear Constant-Coefficient Differential Equations

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{dx^k(t)}{dt^k}$$

Laplace transform: $$\sum_{k=0}^{N} a_k s^k Y(s) = \sum_{k=0}^{M} b_k s^k X(s)$$

$$H(s) = \frac{\sum_{k=0}^{M} b_k s^k}{\sum_{k=0}^{N} a_k s^k} = \frac{Y(s)}{X(s)}$$ **(rational)**

Usually, a practical system is causal and stable.

Example

$$\frac{dy(t)}{dt} + 3y(t) = x(t)$$

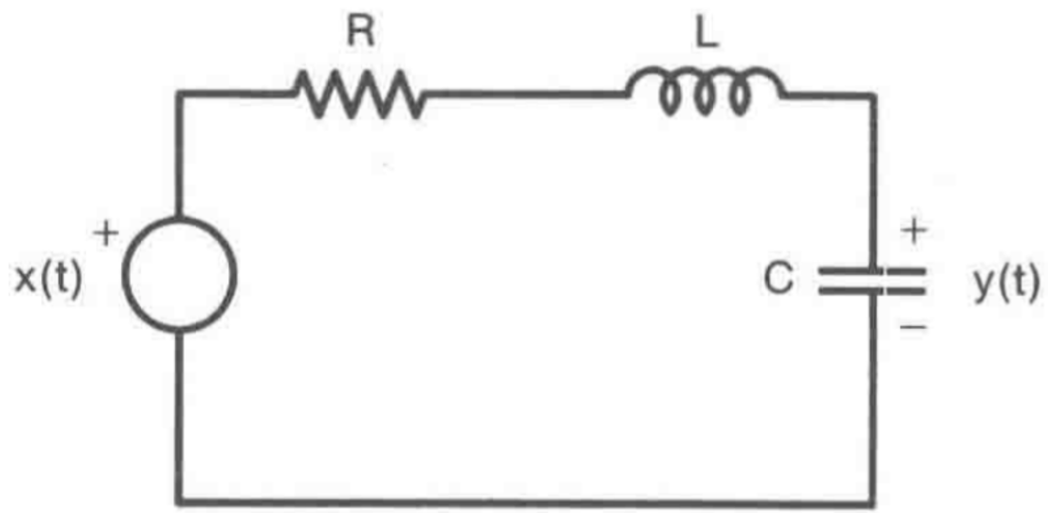
Applying Laplace transform to both sides:

$$sY(s) + 3Y(s) = X(s) \Rightarrow H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+3}$$

Causal system: Re{s}>-3 $h(t) = e^{-3t}u(t)$

Anticausal system: Re{s}<-3 $h(t) = -e^{-3t}u(-t)$

Example 9.24



In another way

$$RC\frac{dy(t)}{dt}+LC\frac{d^2y(t)}{dt^2}+y(t)=x(t)$$

$$H(s)=\frac{1/LC}{s^2+(R/L)s+(1/LC)}$$

9.6.5 Examples Relating System Behavior to the System Function

Example 9.25 **If input** $x(t) = e^{-3t}u(t)$

output $y(t) = [e^{-t} - e^{-2t}]u(t)$

$$X(s) = \frac{1}{s+3}, \quad \mathrm{Re}\{s\} > -3$$

$$Y(s) = \frac{1}{s+1} - \frac{1}{s+2} = \frac{1}{(s+1)(s+2)}, \quad \mathrm{Re}\{s\} > -1$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+3}{(s+1)(s+2)} = \frac{s+3}{s^2+3s+2},$$

Example: Suppose an LTI system:

1.causal,

2. $H(s)$ **is rational ,has only 2 poles, at** $\mathrm{s} = -2$ and $\mathrm{s} = -4$

3. If $x(t) = 1, then \quad y(t) = 0.$

4.The value of the impulse response at $t = 0^{+}$ **is 4.**

From 1 and 2,we can deduce

$$H(s) = \frac{p(s)}{(s+2)(s+4)} = \frac{p(s)}{s^2+6s+8},$$

From 3,

$$x(t) = 1 = e^{0t}, \Rightarrow y(t) = H(0)e^{0t} = 0 \quad .$$

$\therefore H(s)$ **has a zero at** $s = 0$

$$\therefore p(s) = sq(s)$$

From 4,

$$h(0^+) = \lim_{s\to\infty} sH(s) = \lim_{s\to\infty} \frac{s^2 q(s)}{s^2 + 6s + 8} = 4$$

$$\therefore q(s) = K = 4(\text{constant})$$

$$\therefore H(s) = \frac{4s}{(s+2)(s+4)}$$

Example: A real, causal and stable LTI system with $H(s)$ and frequency response

$$H(j\omega) = \mathrm{Re}(\omega) + j\,\mathrm{Im}(\omega)$$

Suppose $$\lim_{s\to\infty} sH(s) = K(\text{constant})$$

Show that $$\int_0^{\infty} \mathrm{Re}(\omega)d\omega = \frac{K\pi}{2}$$

Proof: **Impulse response $h(t)$ real, causal**

$$h(t) \xleftrightarrow{L} H(s)$$ **we can deduce**

at $t = 0^+$ $$h(0^+) = \lim_{s\to\infty} sH(s) = K$$

$$\frac{h(t)+h(-t)}{2} \xleftrightarrow{F} \mathrm{Re}(\omega)$$

$$\therefore \frac{h(0^{+})+h(0^{-})}{2} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{Re}(\omega)d\omega$$

$$h(0^{-}) = 0 \qquad \therefore \int_{-\infty}^{\infty}\mathrm{Re}(\omega)d\omega = K\pi$$

Because $h(t)$ **is real,**

$\mathrm{Re}(\omega)$ **is even.**

$$\therefore \int_{0}^{\infty}\mathrm{Re}(\omega)d\omega = \frac{K\pi}{2}$$

Example:

The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2},$$

Determine and sketch the response y(t) when the input is

$$x(t) = e^{-|t|}, -\infty < t < \infty$$

Solution:

Since $$x(t) = e^{-|t|} = e^{-t}u(t) + e^{t}u(-t)$$

$$X(s) = \frac{1}{s+1} - \frac{1}{s-1} = \frac{-2}{(s+1)(s-1)}, -1 < \operatorname{Re}\{s\} < 1$$

We are also given that

$$H(s) = \frac{s+1}{s^2+2s+2}$$

Since the poles of H(s) are at $-1 \pm j$, and since h(t) is causal, we may conclude that the ROC of H(s) is Re{s}>-1. Now,

$$Y(s) = H(s)X(s) = \frac{-2}{(s^2+2s+2)(s-1)}$$

The ROC of Y(s) will be the intersection of the ROCs of X(s) and H(s). This is -1<Re{s}<1

We may obtain the following partial fraction expansion for Y(s):

$$Y(s) = -\frac{2/5}{s-1} + \frac{2s/5+6/5}{s^2+2s+2}$$

We may rewrite this as:

$$Y(s) = -\frac{2/5}{s-1} + \frac{2}{5}\left[\frac{s+1}{(s+1)^2+1}\right] + \frac{4}{5}\left[\frac{1}{(s+1)^2+1}\right]$$

Noting that the ROC of Y(s) is -1<Re{s}<1 and using Table 9.2, we obtain

$$y(t) = \frac{2}{5}e^{t}u(-t) + \frac{2}{5}e^{-t}\cos t u(t) + \frac{4}{5}e^{-t}\sin t u(t)$$

Example:

Consider the LTI system (input x(t) and output y(t)) with the following information:

$$X(s) = \frac{s+2}{s-2}, and \quad x(t) = 0, t > 0$$

$$y(t) = -\frac{2}{3}e^{2t}u(-t) + \frac{1}{3}e^{-t}u(t)$$

(a) Determine H(s) and its region of convergence

(b) Determine h(t).

(c) Using the system function H(s) found in part (a), determine the output y(t) if the input is

$$x(t) = e^{3t}, -\infty < t < +\infty$$

Solution:

(a) Taking the Laplace transform of the signal y(t), we get

$$Y(s) = \frac{2/3}{s-2} + \frac{1/3}{s+1} = \frac{s}{(s-2)(s+1)},$$

The ROC is -1<Re{s}<2.

Also, note that since x(t) is a left-sided signal, the ROC for X(s) is Re{s}<2.

Now, $$H(s)=\frac{Y(s)}{X(s)}=\frac{s}{(s+2)(s+1)},$$

We know that the ROC of Y(s) has to be the intersection of the ROCs of X(s) and H(s).
This leads us to conclude that the ROC of H(s) is Re{s}>-1.

(b) The partial fraction expansion of H(s) is

$$H(s)=\frac{2}{s+2}-\frac{1}{s+1},\quad \textbf{Re\{s\}>-1}$$

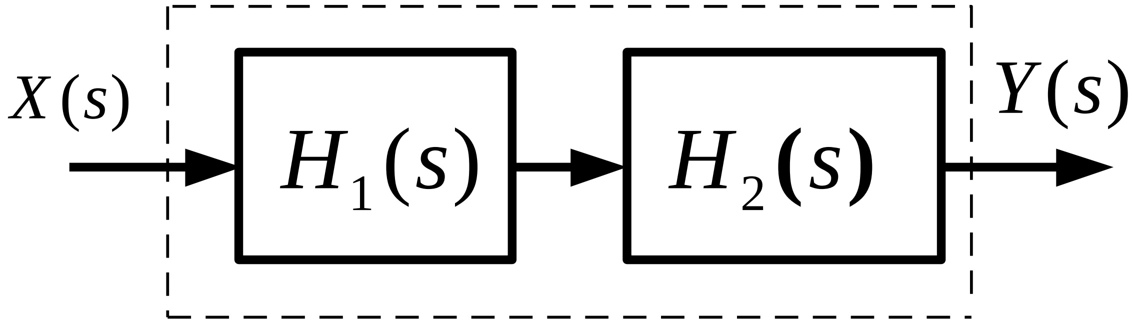
Therefore, $$h(t)=2e^{-2t}u(t)-e^{-t}u(t)$$

(c) e^{3t} **is an Eigenfunction of the LTI system. Therefore,**

$$y(t)=H(3)e^{3t}=\frac{3}{20}e^{3t}$$

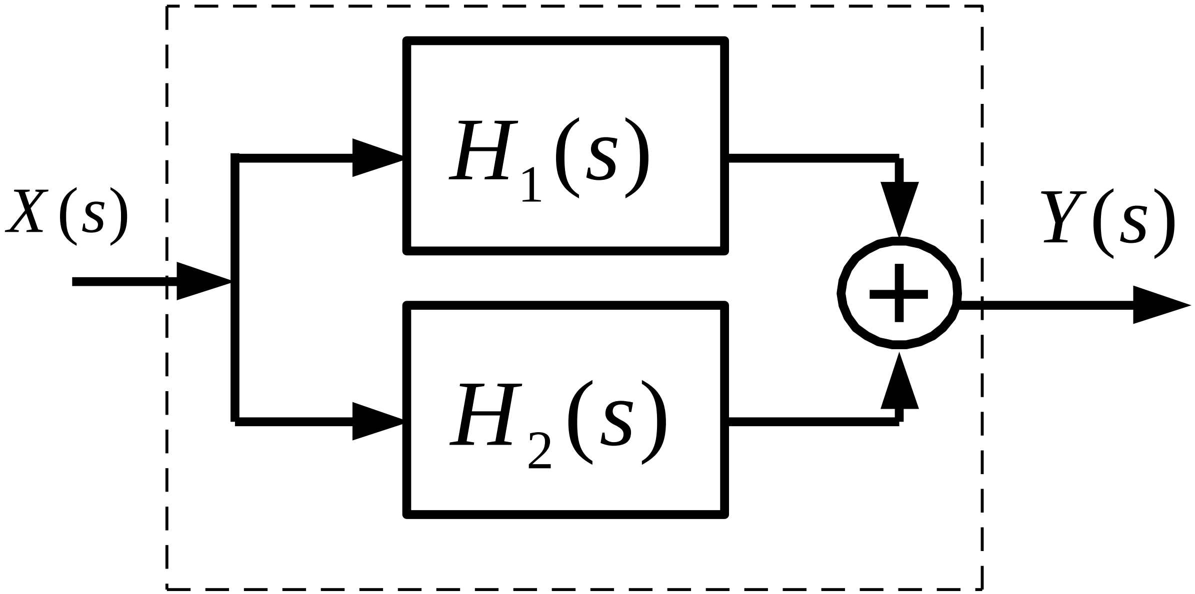
9.7 System Function Algebra and Block Diagram Representations (9.8)

9.7.1 System Functions for Interconnections of LTI Systems



$$H(s) = H_1(s)H_2(s)$$

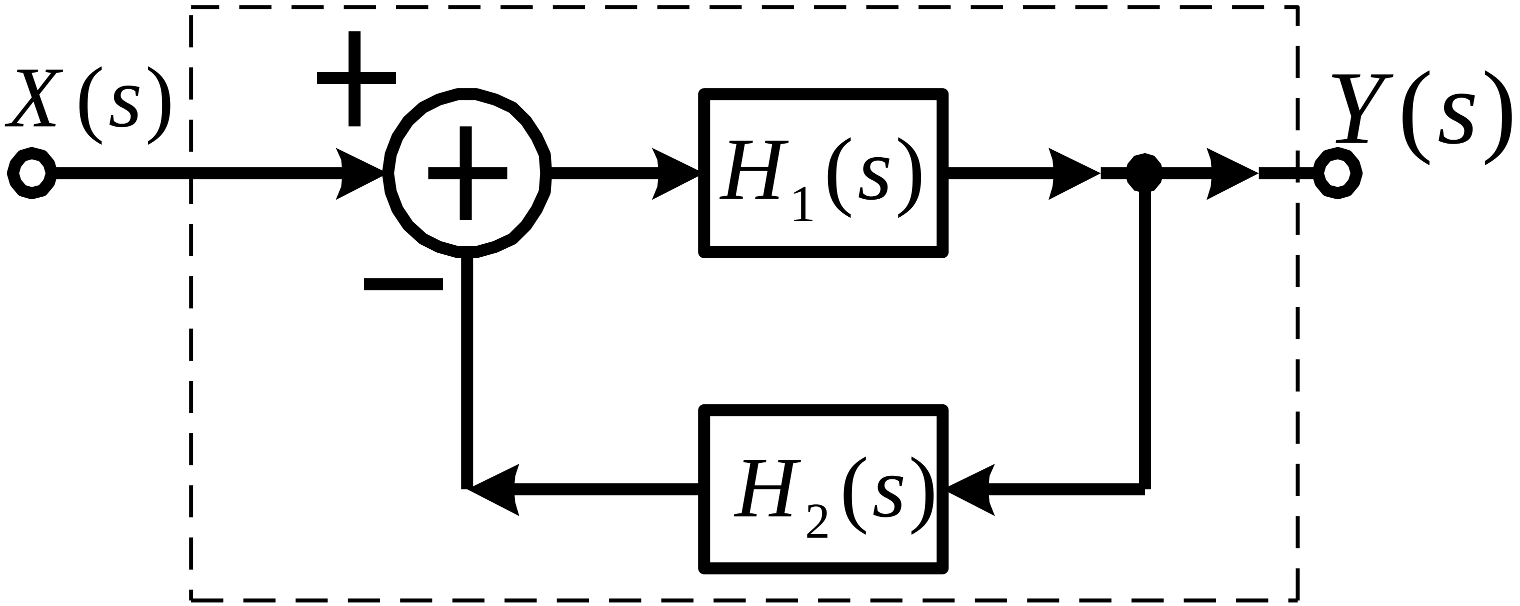
Series(cascade)



$$H(s) = H_1(s) + H_2(s)$$

Parallel

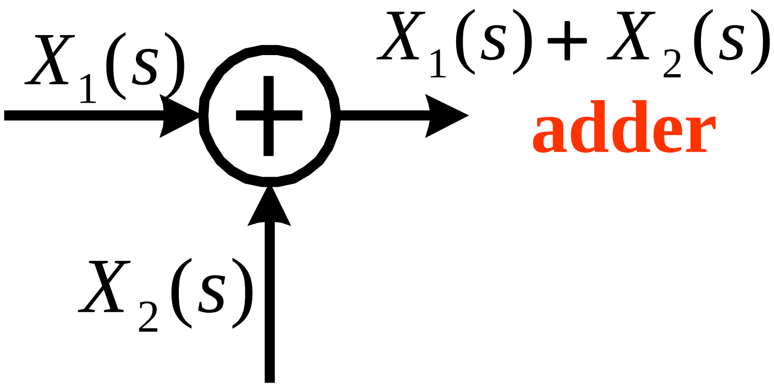
Feed-back



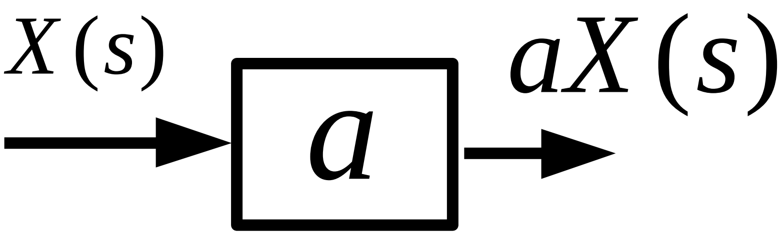
$$H(s) = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$

9.7.2 Block Diagram Representation for causal LTI Systems Described by Differential Equations and Rational System Functions

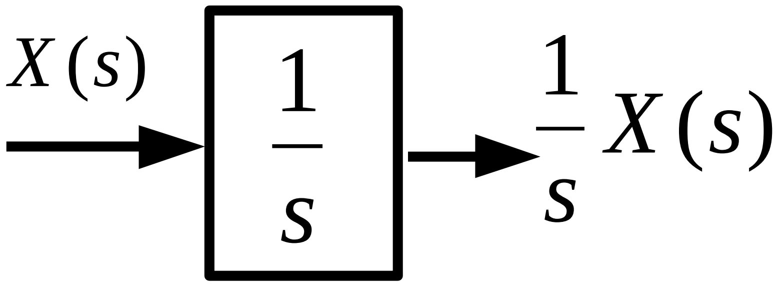
Basic elements:



adder



multiplication



integrator

Example 9.28

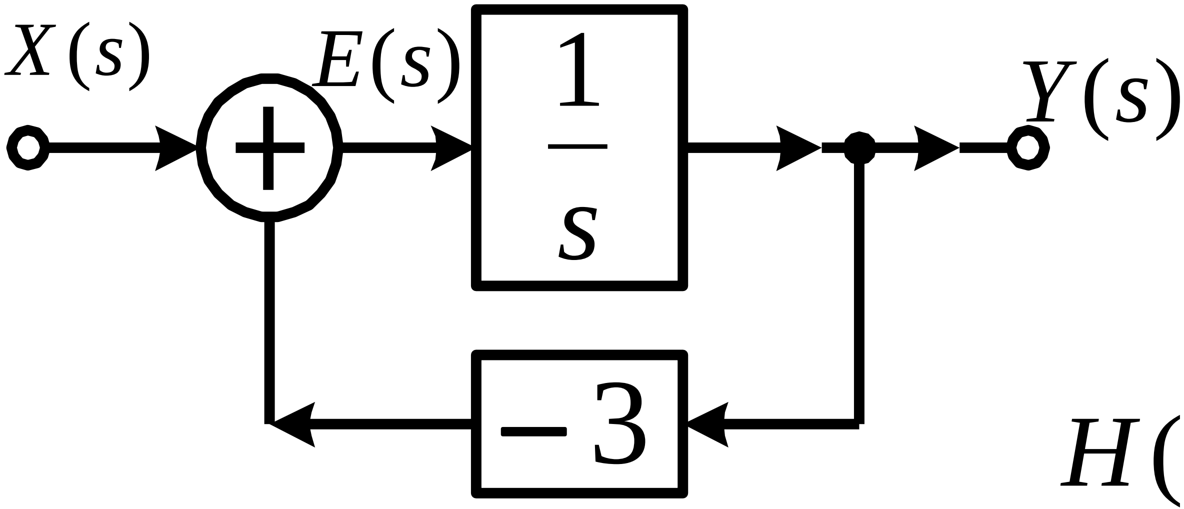
$$H(s)=\frac{1}{s+3}=\frac{Y(s)}{X(s)}$$

$$\frac{d}{dt}y(t)+3y(t)=x(t)$$

So $$sY(s)+3Y(s)=X(s)$$

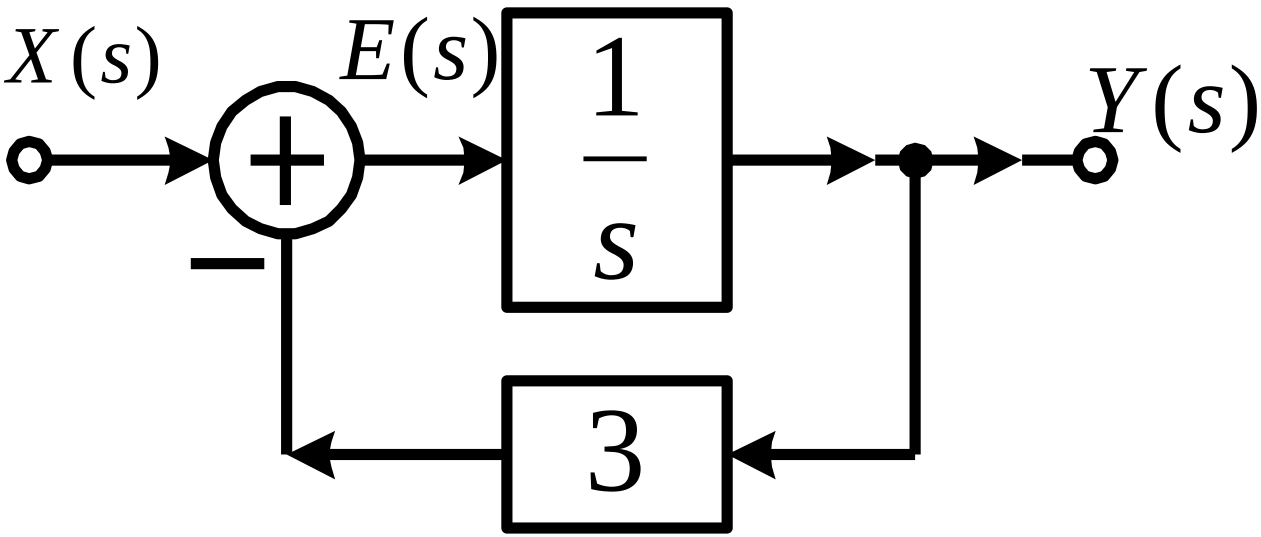
Let $$E(s)=sY(s) \qquad \frac{1}{s}E(s)=Y(s)$$

Then, $$E(s)=X(s)-3Y(s)$$



$$H(s) = \frac{1/s}{1+3/s}$$

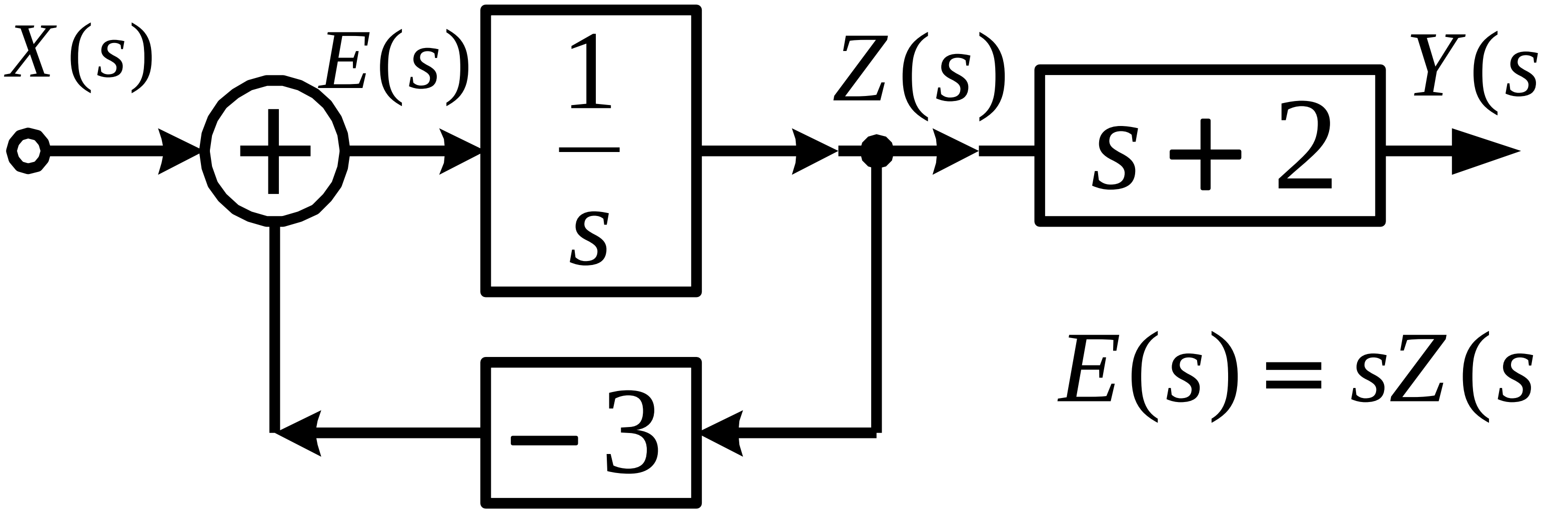
Equivalent representation



Example 9.29 $\frac{d}{dt}y(t)+3y(t)=\frac{d}{dt}x(t)+2x(t)$

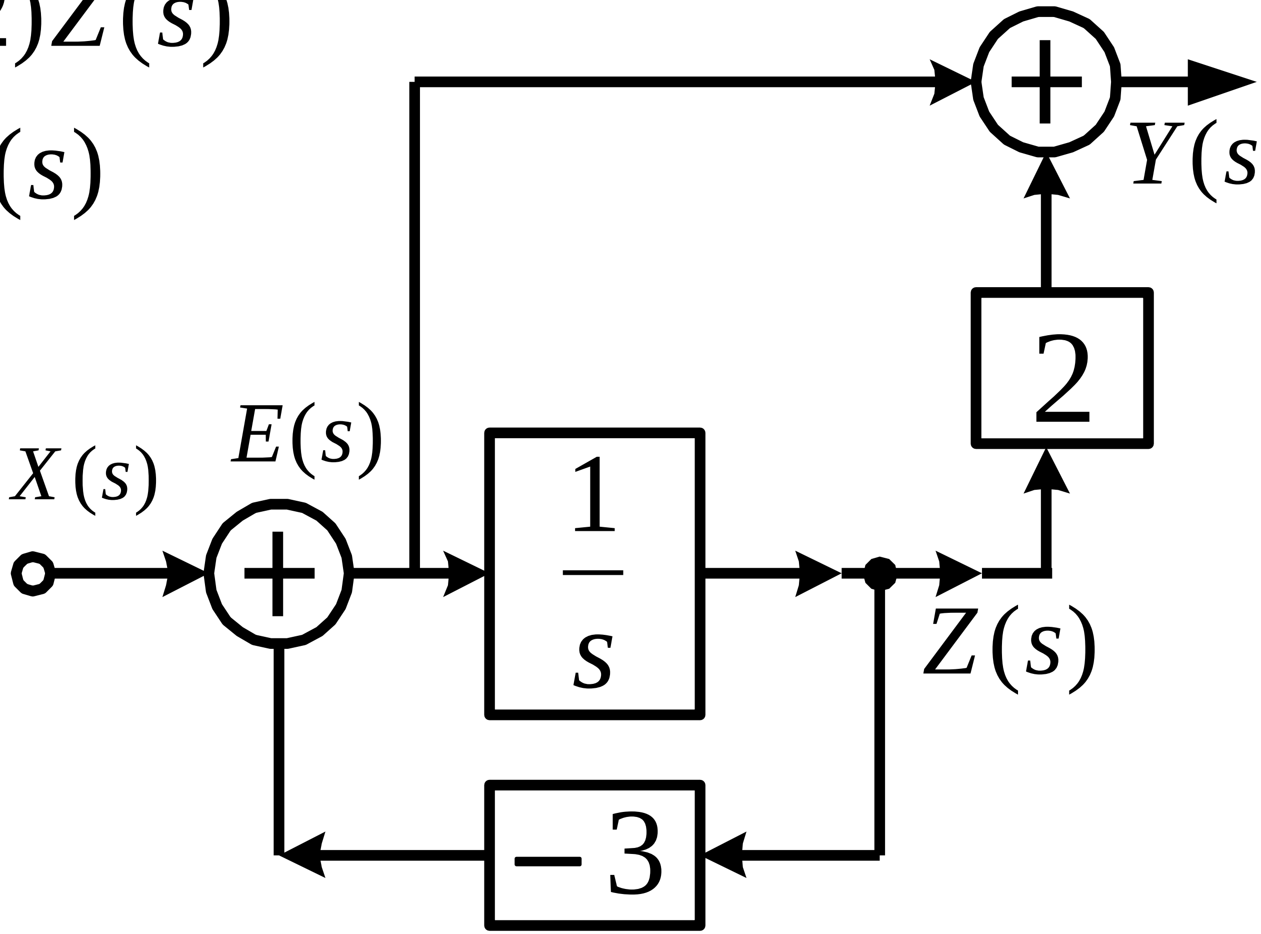
$$H(s)=\frac{s+2}{s+3}=(\frac{1}{s+3})(s+2)=\frac{Y(s)}{X(s)}$$

Let $Z(s)=\frac{1}{s+3}X(s)$ $\quad (s+2)Z(s)=Y(s)$



$$E(s)=sZ(s)$$

$$Y(s) = (s+2)Z(s)$$
$$= E(s) + 2Z(s)$$



canonic **form: the number of integrator =the order of differential equation**

Example 9.30 $$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=x(t)$$

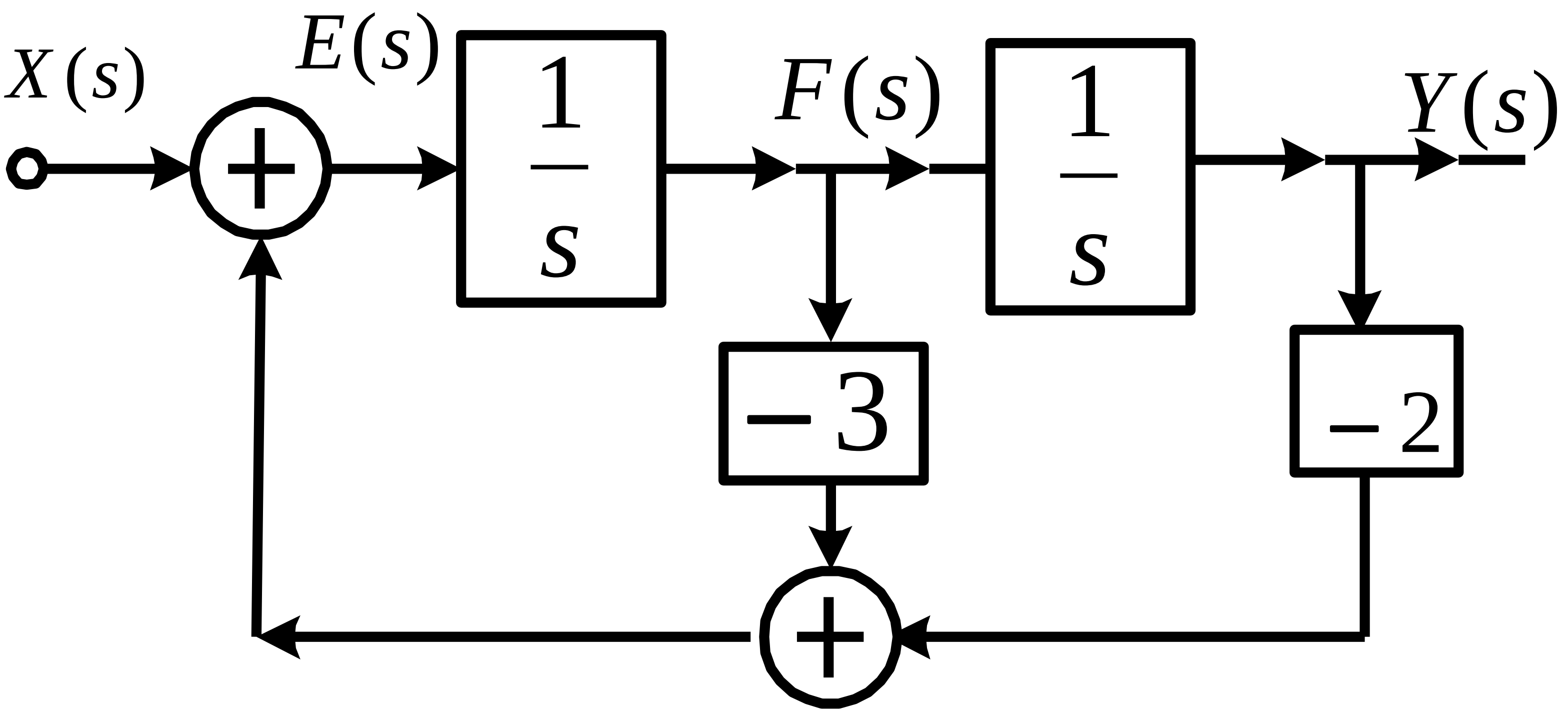
$$H(s)=\frac{1}{s^2+3s+2}=\frac{Y(s)}{X(s)}$$

(1) ***direct-form***

Let $$F(s)=sY(s)$$

$$E(s)=sF(s)=s^2Y(s)$$

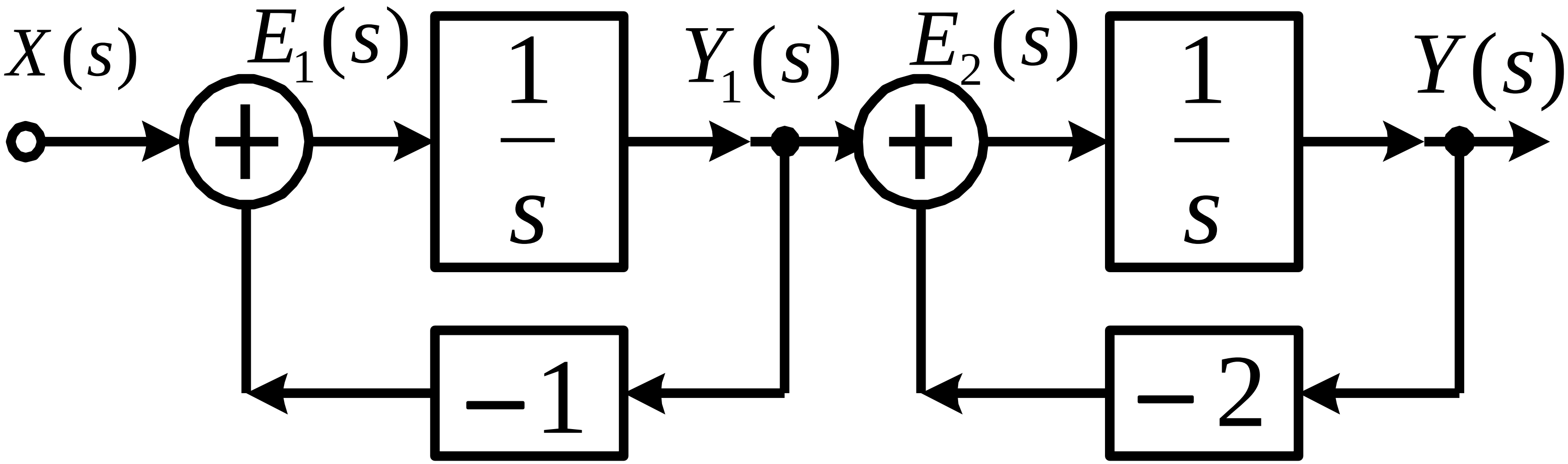
Then, $$E(s)=X(s)-3F(s)-2Y(s)$$



$$H(s) = \frac{1/s^2}{1 + 3/s + 2/s^2}$$

(2) ***cascade-form***

$$H(s)=\frac{1}{s^2+3s+2}=\frac{1}{(s+1)}\times\frac{1}{(s+2)}$$

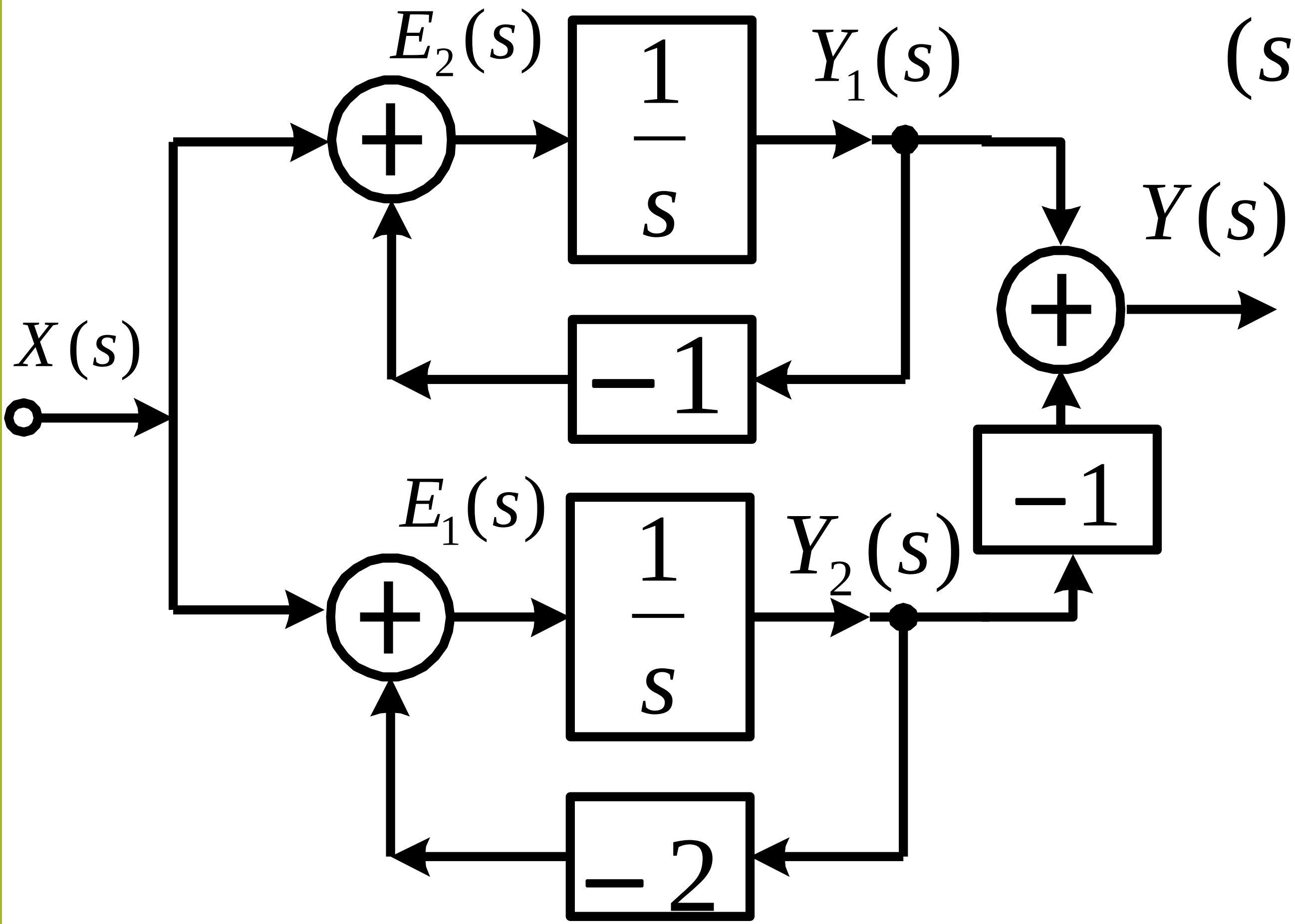


$$H(s) = \frac{1}{s^2 + 3s + 2}$$

(3) ***parallel-form***

$$= \frac{1}{(s+1)} - \frac{1}{(s+2)}$$

Partial Fraction Expansion



Example 9.31

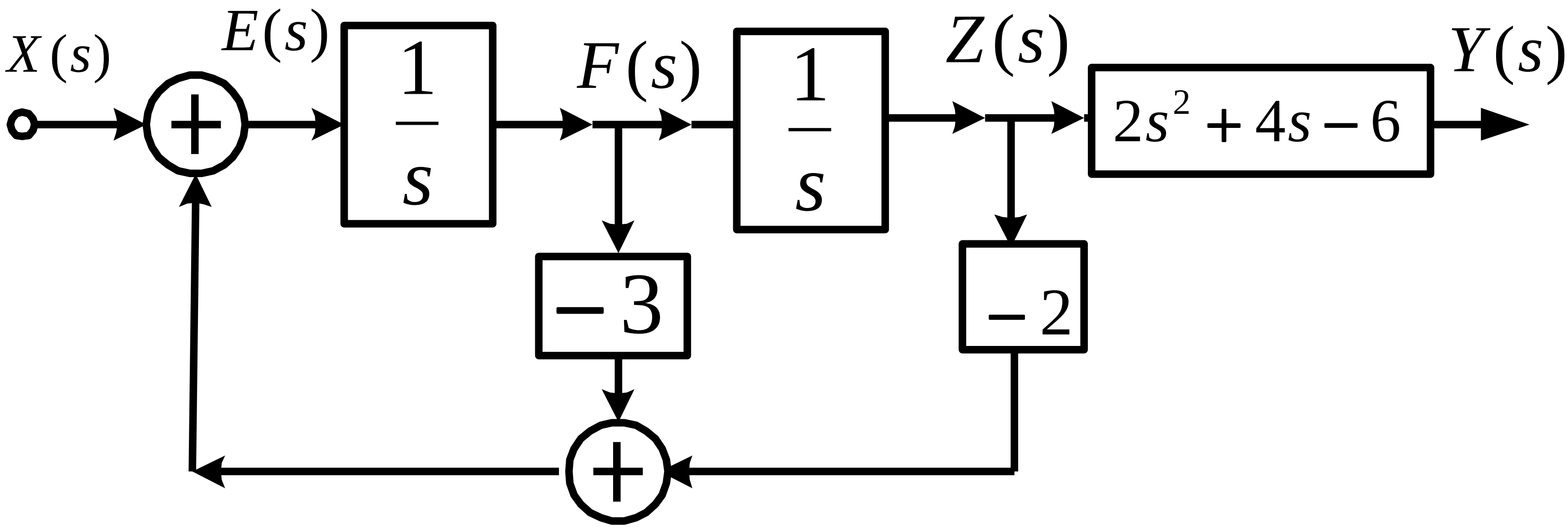
$$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=2\frac{d^2}{dt^2}x(t)+4\frac{d}{dt}x(t)-6x(t)$$

$$H(s)=\frac{2s^2+4s-6}{s^2+3s+2}$$

$$H(s)=(\frac{1}{s^2+3s+2})(2s^2+4s-6)=\frac{Y(s)}{X(s)}$$

Let $$Z(s)=\frac{1}{s^2+3s+2}X(s)$$

$$(2s^2+4s-6)Z(s)=Y(s)$$

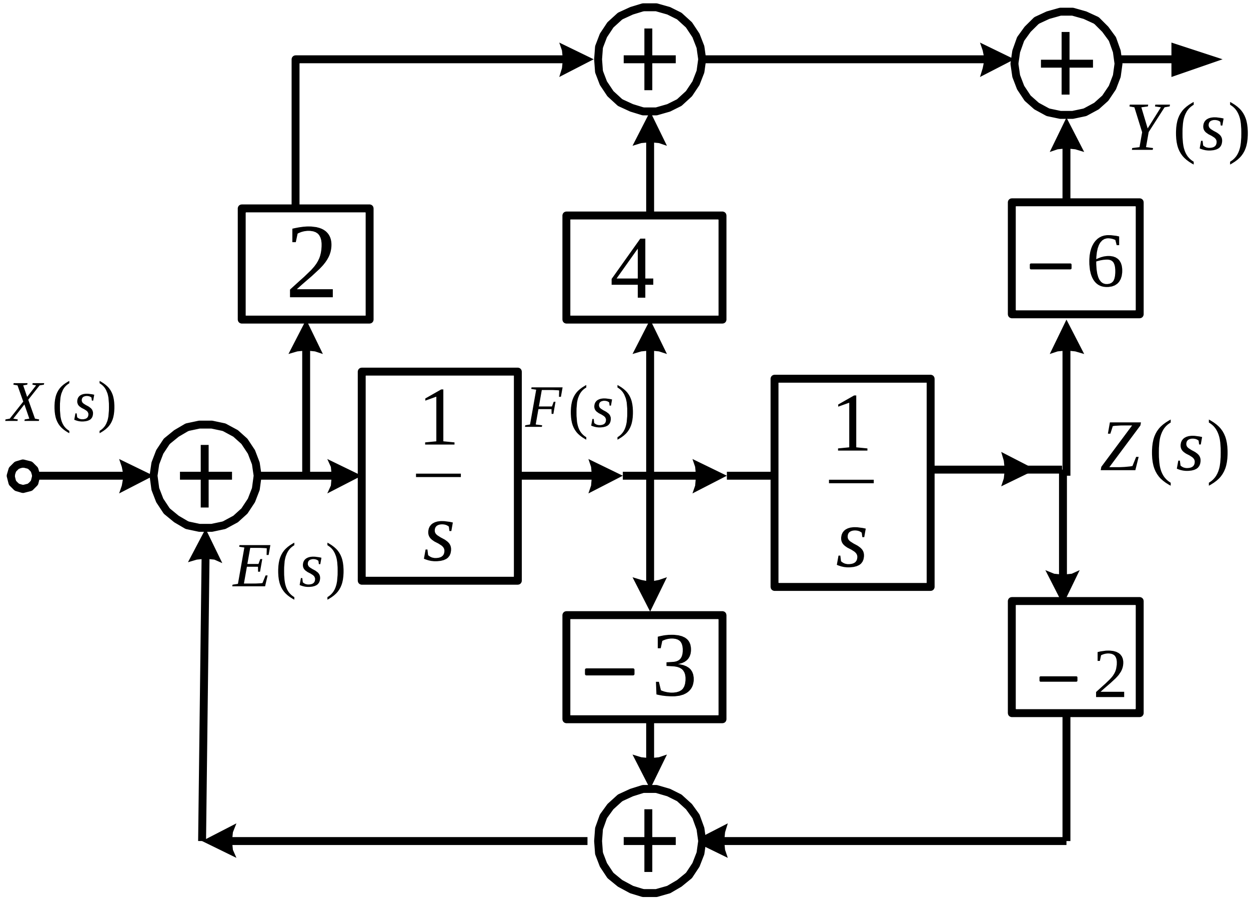


Let $$F(s) = sZ(s)$$

$$E(s) = sF(s) = s^2 Z(s)$$

Then, $$Y(s) = (2s^2 + 4s - 6)Z(s)$$

$$= 2E(s) + 4F(s) - 6Z(s)$$



$$H(s) = \frac{2 + 4/s - 6/s^2}{1 + 3/s + 2/s^2}$$

9.8 The Unilateral LT

$$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

or $x(t)u(t) \xleftrightarrow{L} X(s),$ $\mathrm{R_e}\{s\} > \sigma_1$

$$x(t) \xleftrightarrow{UL} X(s) = UL\{x(t)\}$$

The most properties of Unilateral *LT* are same as *LT*, except differentiation and integration in the time Domain.

Differentiation in the time Domain

$$x(t)u(t) \xleftrightarrow{L} X(s), \quad \text{Re}\{s\} > \sigma_1$$

$$[\frac{d}{dt}x(t)]u(t) \xleftrightarrow{L} sX(s) - x(0^-),$$

$$[\frac{d^2}{dt^2}x(t)]u(t) \xleftrightarrow{L} s^2X(s) - sx(0^-) - x'(0^-)$$

Integration in the time Domain (appended)

$$g(t) = [\int_{-\infty}^{t} x(\tau)d\tau]u(t) \xleftrightarrow{L} G(s) = \frac{X(s)}{s} + \frac{x(0^-)}{s}$$

Example: $$x(t) = e^{-a(t+1)}u(t+1)$$

The bilateral transform X(s) is

$$X(s) = \frac{e^{s}}{s+a}, \mathrm{Re}\{s\} > -a$$

The unilateral transform X(s) is

$$X(s) = \int_{0^-}^{\infty} e^{-a(t+1)}u(t+1)e^{-st}dt$$

$$= \int_{0^-}^{\infty} e^{-a}e^{-(s+a)t}dt = e^{-a}\frac{1}{s+a}, \mathrm{Re}\{s\} > -a$$

Applications of Unilateral LT

It is very useful to solving a differential equation with nonzero initial conditions

Example

$$\frac{d^2}{dt^2}y(t)+3\frac{d}{dt}y(t)+2y(t)=x(t)$$

$$y(0^-)=\beta,\quad y'(0^-)=\gamma,\quad x(t)=\alpha u(t)$$

$$y(t)=?,t>0$$

Using Unilateral LT: $y(t)u(t) \xleftrightarrow{L} Y(s)$

$$s^2Y(s) - \beta s - \gamma + 3sY(s) - 3\beta + 2Y(s) = \alpha / s$$

$$Y(s) = \frac{\beta(s+3)+\gamma}{(s+1)(s+2)} + \frac{\alpha}{s(s+1)(s+2)},$$

If $\alpha = 2, \beta = 3, \gamma = 5$

$$Y(s) = \frac{1}{s} - \frac{1}{s+1} + \frac{3}{s+2}$$

$$y(t) = [1 - e^{-t} + 3e^{-2t}]u(t),$$

$$\textit{for} \qquad t > 0$$

Exercise:

Consider the following system function H(s)

$$H(s)=\frac{1}{s^2+2s+2}$$

Draw the block diagram for H(s) of the second-order system.

Solution:

H(s) can be drawn directly from the form

The corresponding differential equation is as follows:

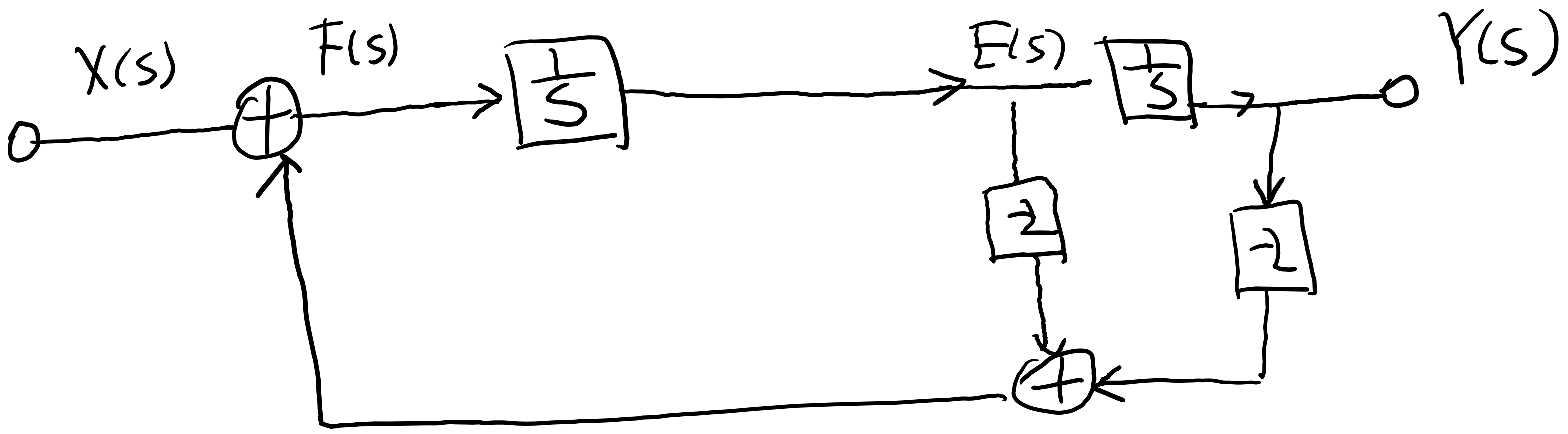
$$H(s) = \frac{1}{s^2 + 2s + 2} = \frac{Y(s)}{X(s)},$$

$$s^2Y(s) + 2sY(s) + 2Y(s) = X(s)$$

let ① $E(s) = sY(s) \Longrightarrow \frac{1}{s} E(s) = Y(s)$

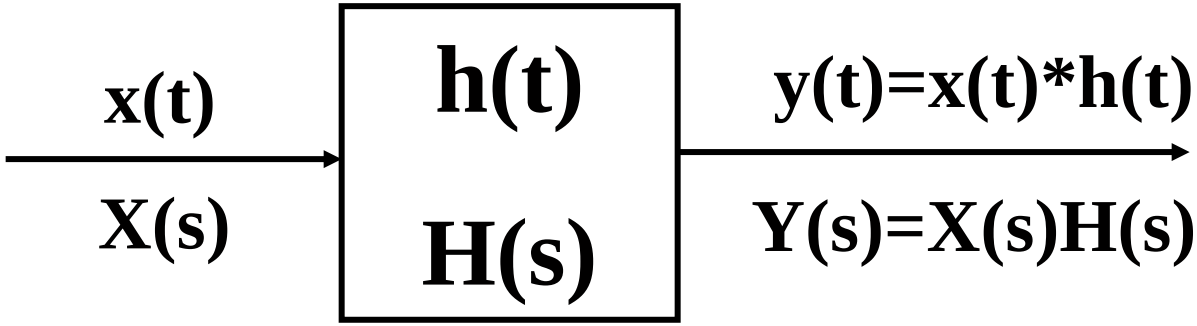
② $F(s) = s^2 Y(s) \Longrightarrow \frac{1}{s} F(s) = E(s)$

③ $F(s) + 2E(s) + 2Y(s) = X(s)$



Resume of Chapter 9

Key points of analysis:



Key points of caculation:

Properties and Basic LT Pairs(ROC)

Partial Fraction Expansion

Block Diagram Representation

Homework list for Chapter 9

2, 5, 7, 8, 9, **13**, 21(a, b, i, j) 22(a,b,c), **28**, 31, 32, 33, **34**, **35**