# 3-Phase AC Systems: Unbalanced Loads

## Introduction

We now look at what happens when the load impedances are not the same. When this occurs we have what is termed unbalanced (or general) load conditions.

## Unbalanced Delta Connected Load

In this section we consider the situation depicted in Figure 1.

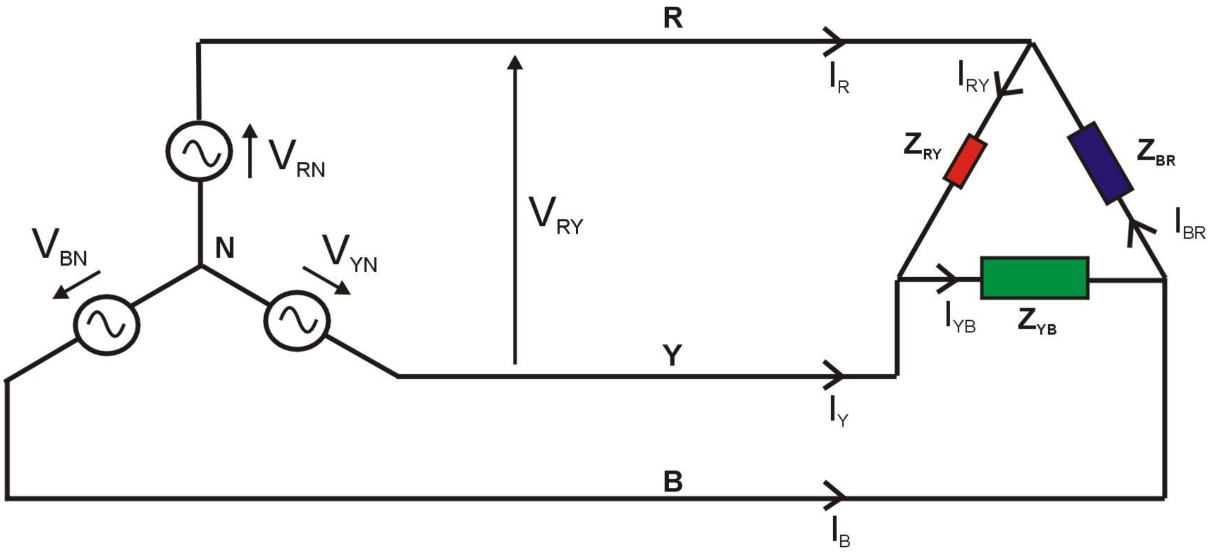



*Figure 1. Unbalanced Delta Connected Load*

We will analyse for the case where $Z_{RY} \neq Z_{YB} \neq Z_{BR}$.

### Example:

$$Z_{RY} = 10 + j20\ \Omega$$
$$Z_{YB} = 5\ \Omega$$
$$Z_{BR} = 15 - j5\ \Omega$$

The supply phase voltage is 6.6 kV.

We can find the current through each load (Hint: You may need to refer back to some earlier lecture notes)

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{\sqrt{3}\times 6600\angle 30^o}{10 + j20} = \frac{11432\angle 30^o}{22.4\angle 63.4^o} = 510.4\angle -33.4^o \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{\sqrt{3}\times 6600\angle -90^o}{5} = 2286\angle -90^o \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{\sqrt{3}\times 6600\angle 150^o}{15 - j5} = 723.5\angle 168.4^o \text{ A}$$

The main point here is that across each load is the respective line voltage (not phase voltage). You should realize this by careful consideration of figure 1.

Now we can find the line currents:

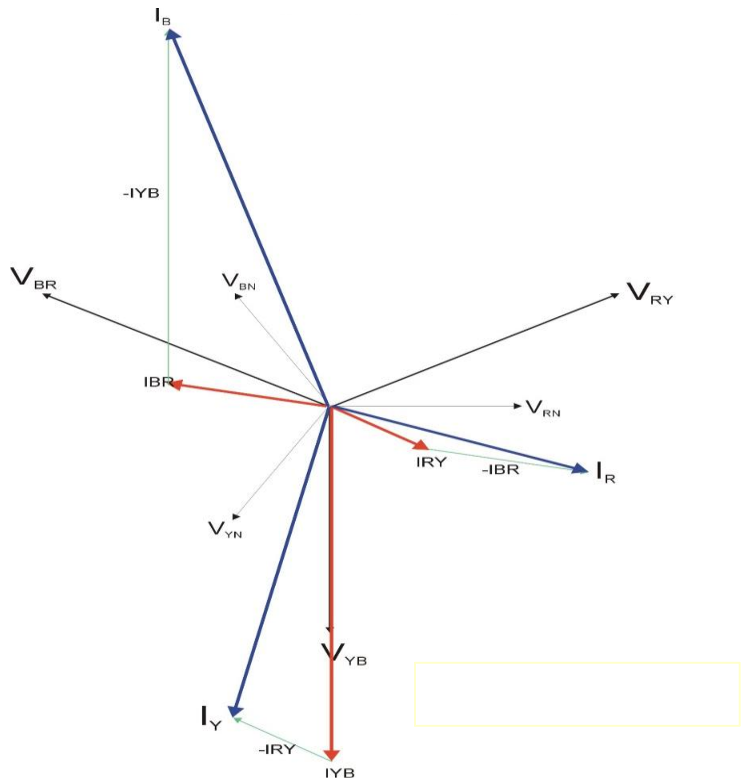
$$I_R = I_{RY} - I_{BR} = 510\angle -33.4 - 723\angle 168.4 = 425.77 - j280.745 + 708 - j145.38$$
$$= 1133.77 - j426.38 = 1211.2\angle -20.6^o \text{ A}$$

Without showing the full working (but you should do it):

$$I_Y = I_{YB} - I_{RY} = 2050\angle -102^o \text{ A}$$

$$I_B = I_{BR} - I_{YB} = 2532\angle -106^o \text{ A}$$

It is good practice, and you may find it useful, to draw a phasor diagram:




To calculate the total power the easiest, and best way, is to consider the current flowing through each resistor and summing, so:

$$P_{total} = I_{RY}^2 \times 10 + I_{YB}^2 \times 5 + I_{BR}^2 \times 15 = 510^2 \times 10 + 2280^2 \times 5 + 723^2 \times 15$$
$$= 2.6\times 10^6 + 26\times 10^6 + 7.8\times 10^6 = 36.4\times 10^6 = 36.4 \text{ MW}$$

## Unbalanced 3 Wire Star Connected Load

Figure 2 shows a 3 wire star connected load. It is 3 wire because there is no wire connecting the S and N points.

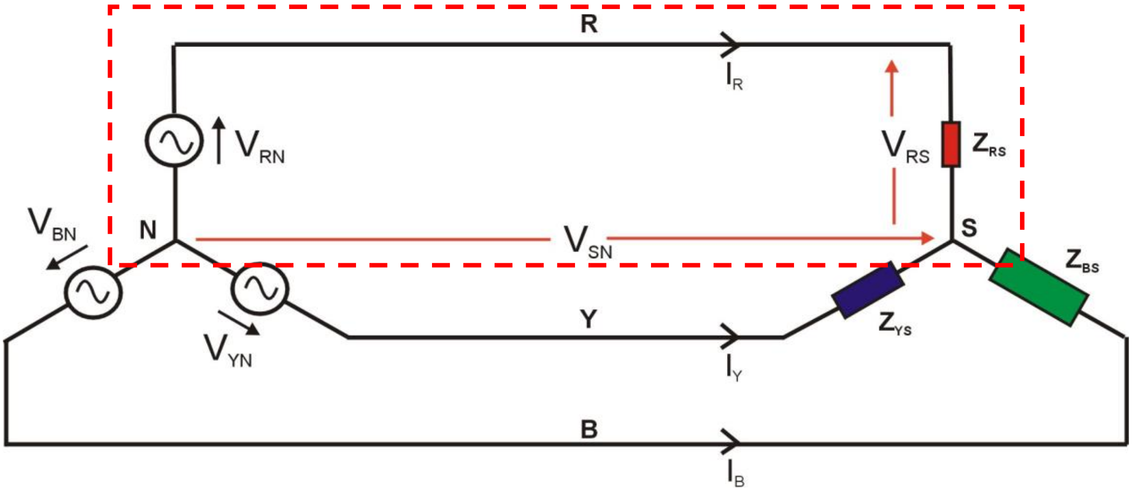



*Figure 2. 3 Wire Star Connected Load*

Since the load impedances are not equal the line currents will all be different. This means that $V_{sn}$ will not be zero. This will lead to the following conditions for the phase voltages:

$$V_{RN} = V_{RS} + V_{SN} \quad (1)$$

$$V_{YN} = V_{YS} + V_{SN} \quad (2)$$

$$V_{BN} = V_{BS} + V_{SN} \quad (3)$$

Generally speaking, 3 wire unbalanced loads are difficult to deal with. The easiest, and best way to deal with them is to convert the Y load to a delta load.

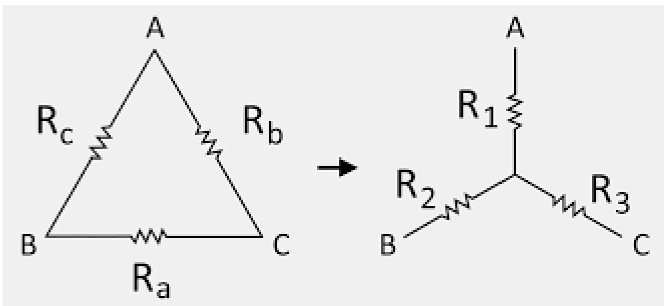



*Figure 3. Delta and Wye connection*

Referring to Figure 3 the Wye to delta conversion formulas are:

$$R_a = R_2 + R_3 + \left( \frac{R_2 \times R_3}{R_1} \right) \quad (4)$$

$$R_b = R_3 + R_1 + \left( \frac{R_3 \times R_1}{R_2} \right) \tag{5}$$

$$R_a = R_1 + R_2 + \left( \frac{R_1 \times R_2}{R_3} \right) \tag{6}$$

Now we can convert from a Y connected load to a delta connected load and solve the line currents as before. [We will do a detailed example in class].

Fortunately, we rarely have to deal with a 3-wire unbalanced Y load. It is avoided because the load phase voltages are no longer equal to the supply phase voltages. This can result in dangerous over-voltage conditions in one, or more, of the phases.

## Unbalanced 4 Wire Star Connected Load.

A solution to the over-voltage condition is to physically connect the generator neutral and the load star point, as depicted in Figure 4.

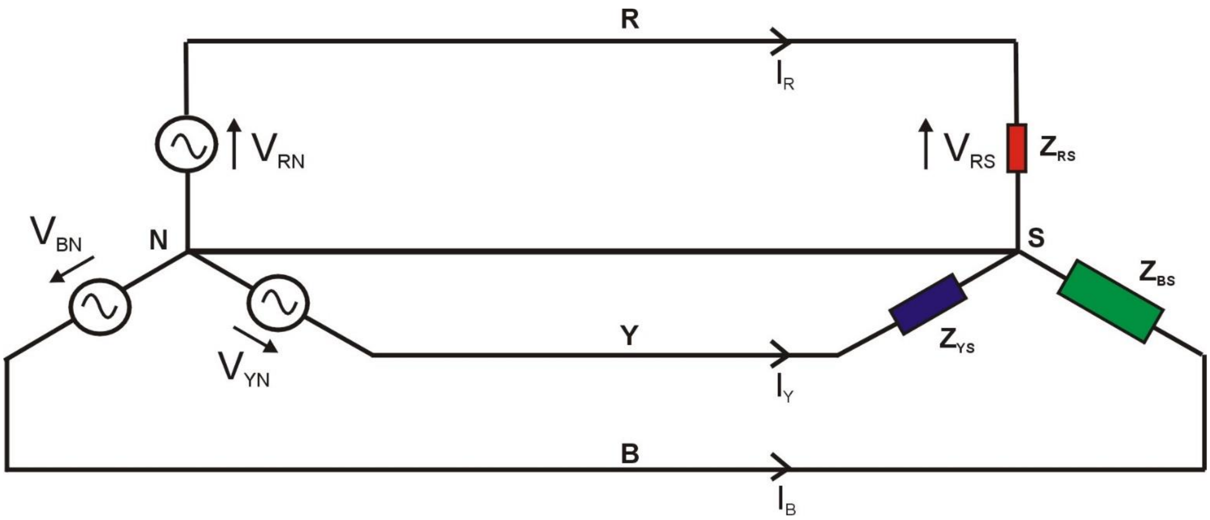
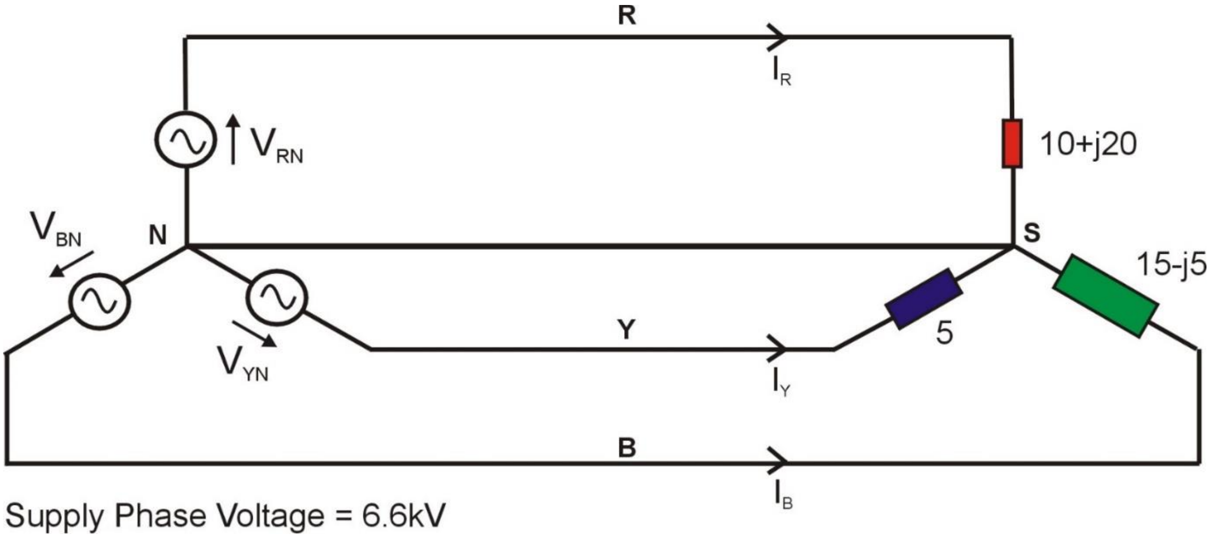



*Figure 4. 4 Wire Star Connected Load*

Example:




Find all the line currents, including the current flowing between **S** and **N**.

$$I_R = V_{RN} / Z_{RN} = 6600\angle 0° / (10 + j20) = 6600\angle 0° / 22.4\angle 63.4° = 295\angle -63.4° \text{ A}$$
$$I_Y = V_{YB} / Z_{YB} = 6600\angle -120° / 5 = 1320\angle -120° \text{ A}$$
$$I_B = V_{BR} / Z_{BR} = 6600\angle 120° / (15 - j5) \ = 6600\angle 120° / 15.8\angle -18.4° = 417\angle 138.4° \text{ A}$$

$$I_{SN} = I_R + I_Y + I_B = 295\angle -63.4^o + 1320\angle -120^o + 417\angle 138.4^o$$
$$= 132.1 - j263.8 - 660 - j1143 - 311 + j276.9 = -838.9 - j1130$$
$$= 1407\angle -126.6^o \text{ A}$$

# Bibliography


1. Edward Hughes "Electrical Technology" 10th Edition, Pearson Education Limited (2008). Chapter 33.
2. https://www.bu.edu.eg/portal/uploads/Engineering,%20Shoubra/Electrical%20Engineering/2460/crs-12142/Files/Cir2_Lect_10_UnBalanced%203phase%20systems.pdf
3. https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-061-introduction-to-electric-power-systems-spring-2011/readings/MIT6_061S11_ch3.pdf