Electronic System Design 3:
L9.3: Basic noise calculations

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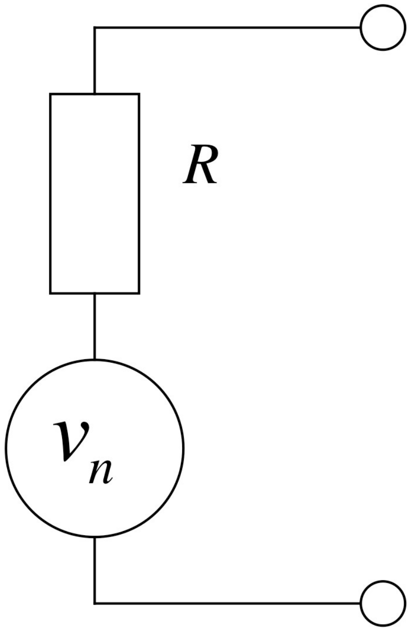
Thermal Noise: Nyquist Formula

Thermal noise appears as a voltage source in series with the resistor $v_n = \sqrt{4 \cdot k_B TRB}$ Often expressed as $v_n = \sqrt{4 \cdot k_B TR} \cdot \sqrt{B}$

Useful Rule-of-Thumb: 60Ω resistor @300K $\cong 1nV.\sqrt{B}$



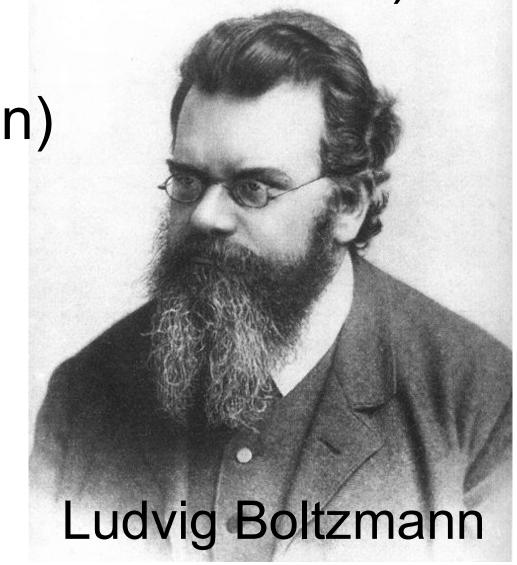
where

$k_B = 1.3806503 \cdot 10^{-23} J \cdot K$ (Boltzmann's constant)

T Absolute temperature (kelvin)

R Resistance, Ω

B Bandwidth (Hz)



Ludvig Boltzmann

Thermal Noise: Nyquist Formula (2)

$v_n = \sqrt{4k_B TR}$ is called the "Voltage Noise Spectral Density"

Units are "Volts per root Hertz": V $Hz^{-1/2}$

Note it doesn't matter what the frequency is, **just** the bandwidth

Can also express as a noise current: $i_n = \frac{v_n}{R} = \sqrt{4k_B T/R}$

"Current Noise Spectral Density", Amps per Root Hertz; A $Hz^{-1/2}$

- Noise increases with bandwidth
- Voltage Noise increases with R
- Current Noise decreases with R

=> Keep bandwidth low (subject to other constraints)

Thermal Noise:
Example: 1k Resistor

For a 1kΩ resistor in 1Hz bandwidth at room temperature (300K)

$$v_n = \sqrt{4k_B \cdot 300K \cdot 1000\Omega \cdot 1Hz} = 4.07nV(rms)$$

For a 1kΩ resistor over the audio band
(20Hz-20kHz = 19.98kHz bandwidth)

$$v_n = \sqrt{4k_B \cdot 300K \cdot 1000\Omega \cdot 19.98kHz} = 575nV(rms)$$

Noise current for a 1kΩ resistor over the audio band

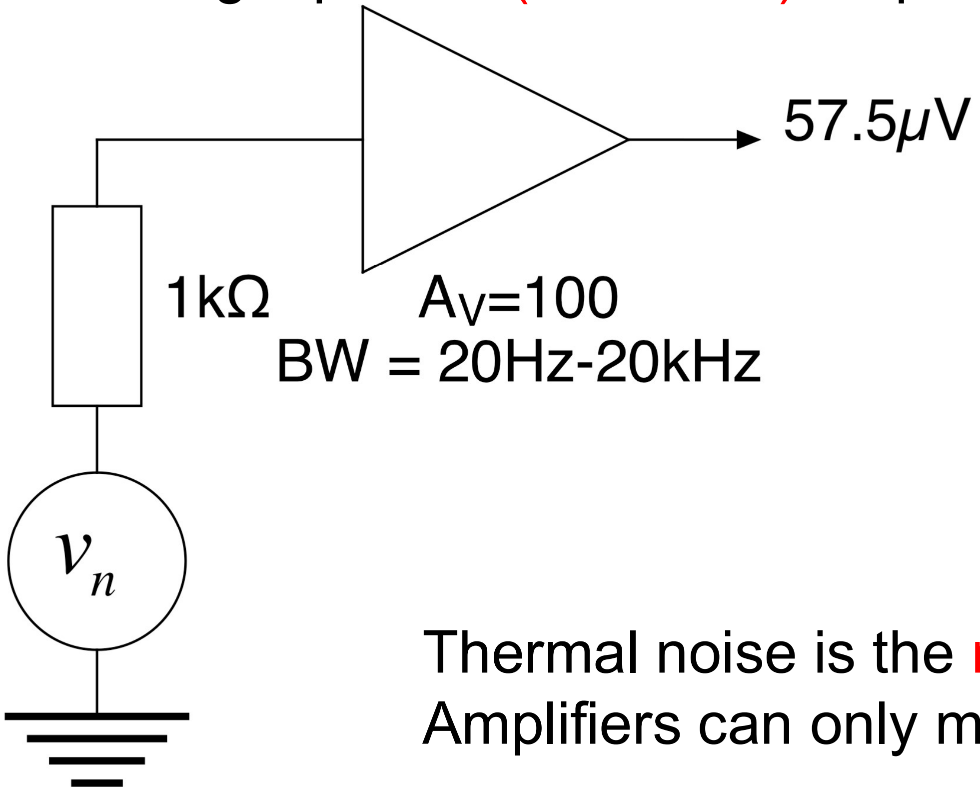
$$i_n = \sqrt{4k_B \cdot 300K / 1000\Omega \cdot 19.98kHz} = 575pA(rms)$$

Thermal noise isn't very big!

Thermal Noise models

Noise voltages and currents **behave <u>just like any other</u>** voltages and currents

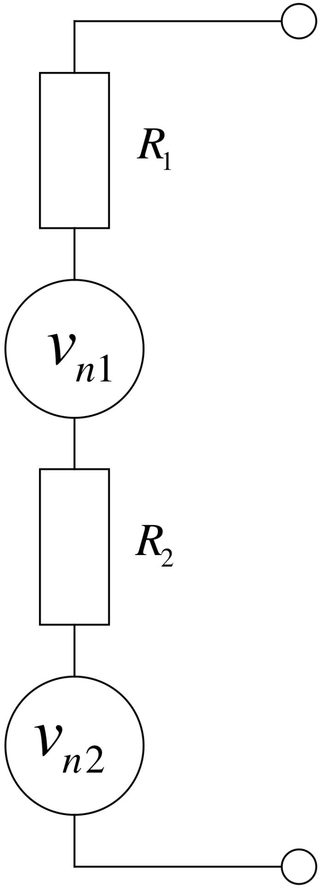
Assuming a perfect (noiseless) amplifier:



Note: noise voltage will change if amplifier bandwidth changes

Thermal noise is the minimum you can achieve: Amplifiers can only make it worse

Multiple Noise Sources:
<u>Two resistors in series</u>



Noise voltages are uncorrelated

Sometimes both noise voltages will be positive

Sometimes both noise voltages will be negative

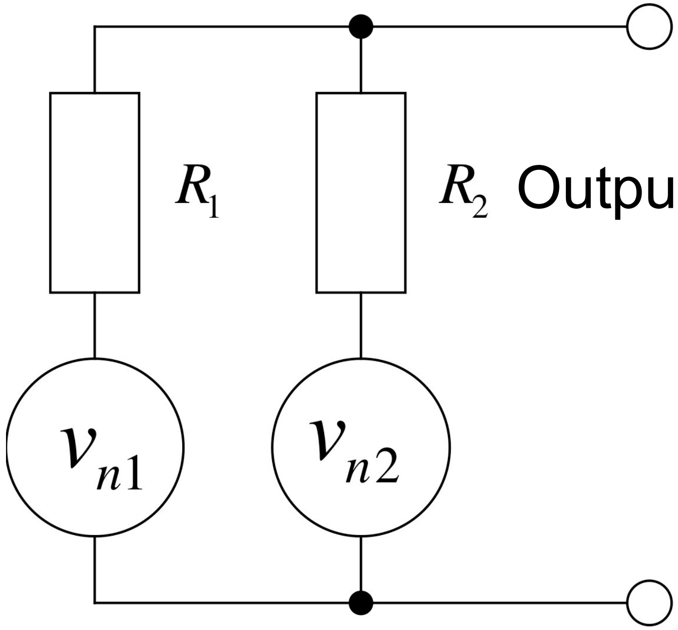
Sometimes voltages will tend to cancel

Average noise power adds:
Add square of RMS voltages & take square root

$$v_n(total) = \sqrt{v_{n1}^2 + v_{n2}^2} = \sqrt{\sqrt{4k_BTR_1B}^2 + \sqrt{4k_BTR_2B}^2}$$
$$= \sqrt{4k_BTR_1B + 4k_BTR_2B} = \sqrt{4k_BT(R_1 + R_2)B}$$

Same as thermal noise of 2 resistors in series
i.e. The noise from 2 x 10kΩ resistors is the same as 1 x 20kΩ resistor

Multiple Noise Sources: Two resistors in parallel



Using superposition & voltage divider rule:

Output due to $v_{n1} = v_{n1}\dfrac{R_2}{R_1+R_2}$, due to $v_{n2} = v_{n2}\dfrac{R_1}{R_1+R_2}$

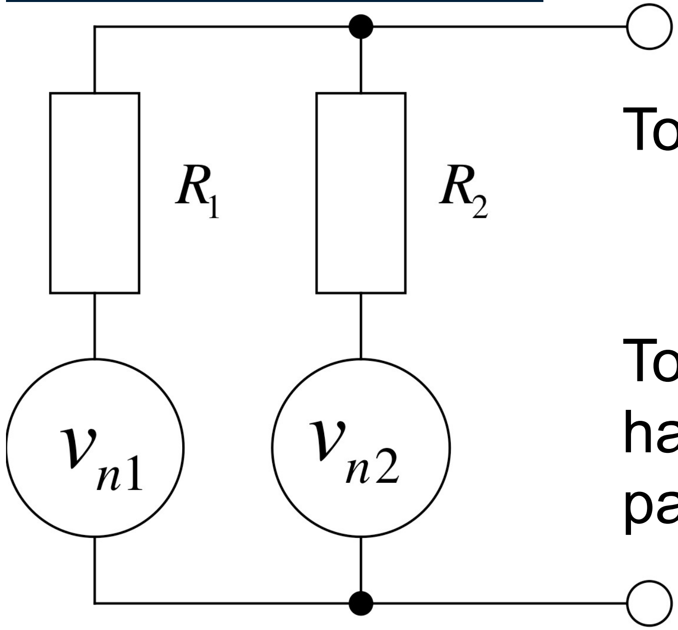
Square, add and take the square root:

Total output: $v_\Sigma = \sqrt{v_{n_1}^2\dfrac{R_2^2}{(R_1+R_2)^2} + v_{n_2}^2\dfrac{R_1^2}{(R_1+R_2)^2}}$

$$= \sqrt{\frac{1}{(R_1+R_2)^2}\left(v_{n_1}^2 R_2^2 + v_{n_2}^2 R_1^2\right)} = \sqrt{\frac{1}{(R_1+R_2)^2}\left(4k_BTR_1R_2^2 + 4k_BTR_2R_1^2\right)}$$

$$= \sqrt{\frac{4k_BT}{(R_1+R_2)^2}\left(R_1R_2\cdot R_2 + R_2R_1\cdot R_1\right)} = \sqrt{\frac{4k_BT}{(R_1+R_2)^2}\left(R_1R_2\cdot(R_1+R_2)\right)} = \sqrt{\left(4k_BT\frac{R_1R_2}{(R_1+R_2)}\right)}$$

Multiple Noise Sources (3)



Total output: $v_\Sigma = \sqrt{4k_B T \dfrac{R_1 R_2}{R_1 + R_2}} = \sqrt{4k_B T \left(R_1 // R_2\right)}$

Total output is just the same as one resistor having the same resistance as the two in parallel.

If the temperature of all resistors is the same, the thermal noise for a resistor must be the same however the resistor is made: If you could get a different thermal noise voltage from two 1kΩ resistors in series than you did from a single 2kΩ resistor you could run a power station from a resistor network: Perpetual motion!

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Thank you
谢谢

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