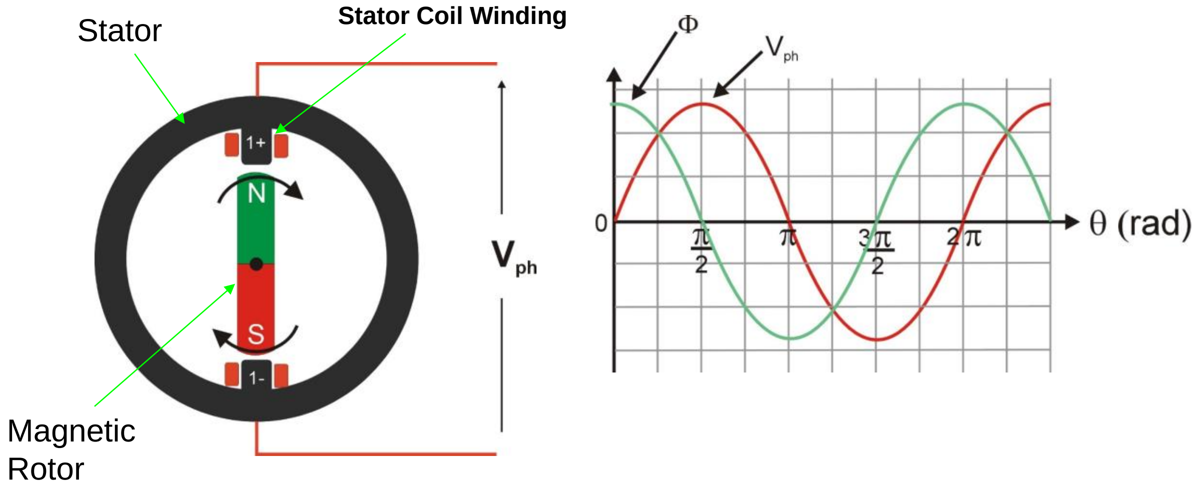
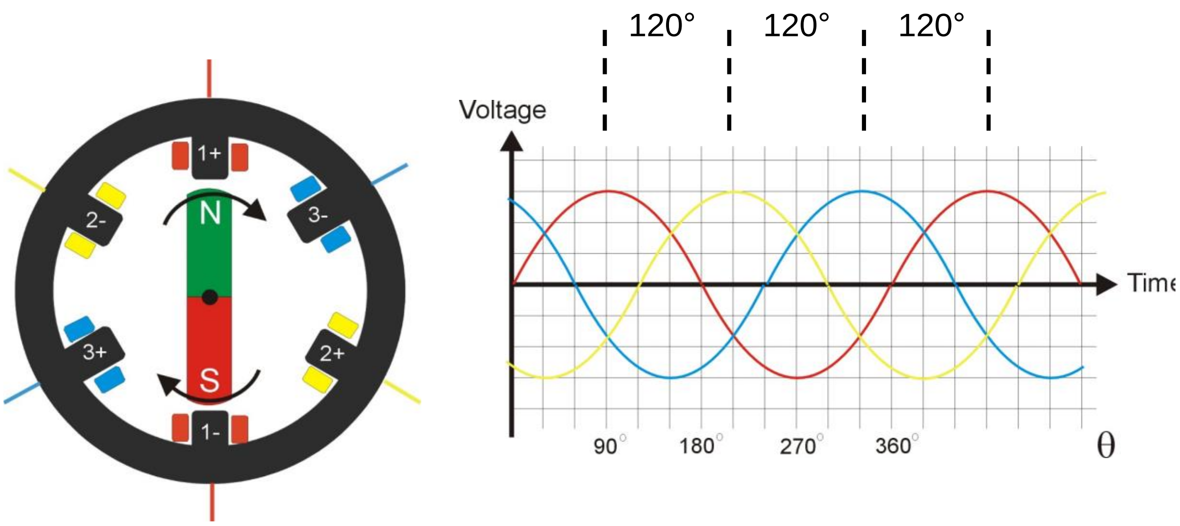
Introduction to Three-Phase Electrical Power

Three-phase electric power is a method of alternating current electric power generation, transmission, and distribution. It is a polyphase system and the most common method used by electrical grids worldwide to transfer power. It is also used to power large motors and other heavy loads. It is typically more economical than an equivalent 2-wire single-phase circuit because it requires less conductor material to transmit a given amount of power.

Figure 1a shows the generation of a single-phase waveform. This can be compared with Figure 1b which depicts the generation of three-phase AC. In Figure 1a V_{ph} is given by $V_{ph} = N\frac{d\phi}{dt}$, where N is the number of stator coil turns and ϕ is the rotor magnetic flux.



(a)



(b)

Figure 1. AC Generation (a) Single Phase (b) Three Phase.

From Figure 1(b) a three phase AC system can be considered as three single phase waveforms with a separation of 120° between each phase. Countries use different conventions for naming the 3 PHASE voltages but in this course we will adopt the old UK convention of RED, YELLOW & BLUE PHASES. What convention we adopt makes no difference to the theory.

Generation of 3-Phase Voltages

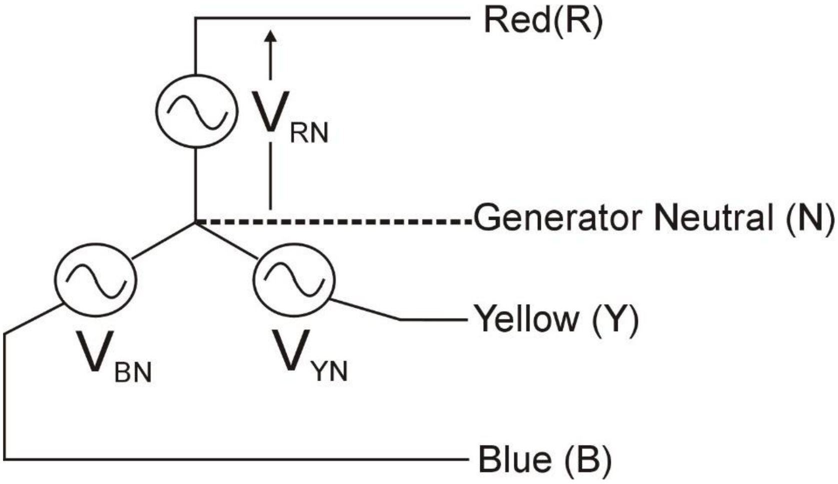


Figure 2. Generation of Phase Voltages.

Consider Figure 2 which considers the generated voltages of the three phases. We can write the phase voltages as:

$$V_{RN} = V_{pk} \cos\theta \tag{1}$$

$$V_{YN} = V_{pk} \cos\left(\theta - \frac{2\pi}{3}\right) \tag{2}$$

$$V_{BN} = V_{pk} \cos\left(\theta + \frac{2\pi}{3}\right) \tag{3}$$

It can be shown (and you should!) that $V_{RN} + V_{YN} + V_{BN} = 0$.

We can represent these **phase voltages** on a phasor diagram (Figure 3) and in phasor form:

$$V_{RN} = V_{PH} \angle 0^o \tag{4}$$

$$V_{YN} = V_{PH} \angle -120^o \tag{5}$$

$$V_{BN} = V_{PH} \angle 120^o \tag{6}$$

Note: V_{PH} is the rms value of V_{pk}. In phasor form the voltage is always a rms value.

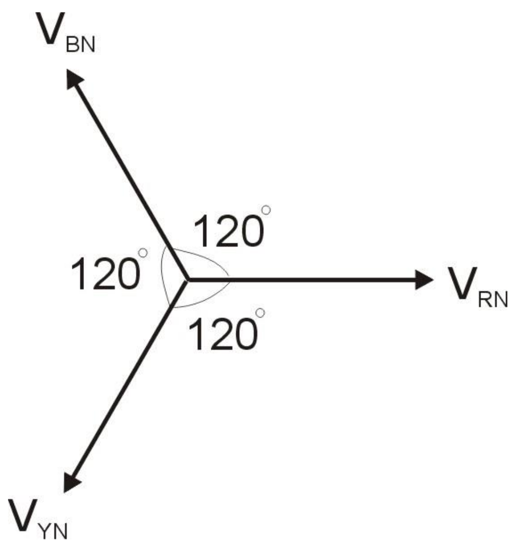


Figure 3. Phasor Diagram showing the phase difference and phase voltages of 3-phase generation.

Line Voltages

We have defined phase voltages, which are the voltages between each phase and the neutral point. Here we consider the potential difference between the different phase lines, which is termed **line voltage.** The phase and line voltages are depicted in Figure 4.

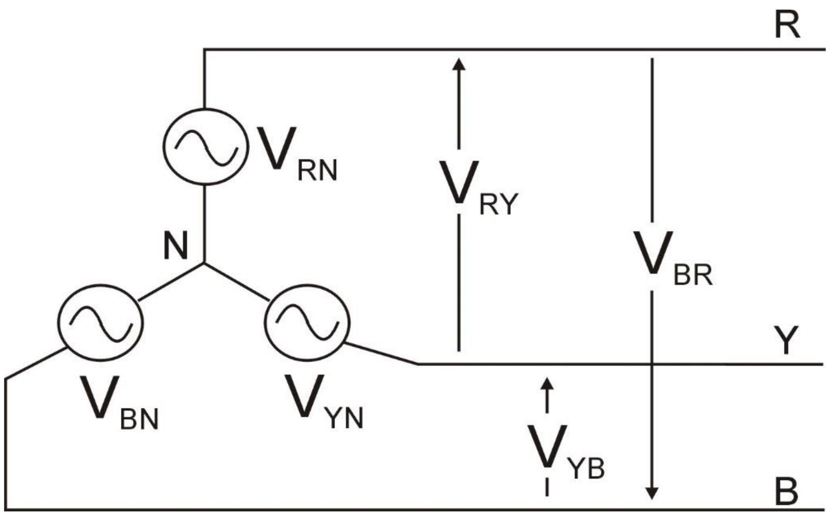


Figure 4. Phase and Line Voltages.

From Figure 4 it can be seen that:

$$V_{RY} = V_{RN} - V_{YN} \tag{7}$$

$$V_{BR} = V_{BN} - V_{RN} \tag{8}$$

$$V_{YB} = V_{YN} - V_{BN} \tag{9}$$

Considering Equation (7) then:

$$V_{RY} = V_{RN} - V_{YN} = V_{PH}(1 + j0) - V_{PH}\angle -120^o = V_{PH} - V_{PH}(-0.5 - 0.866)$$
$$= V_{PH}(1.5 + j0.866) = \sqrt{3}V_{PH}\angle 30^0 \text{ V} \quad (10)$$

Similarly:

$$\text{V}_{\text{BR}} = \sqrt{3} \cdot V_{ph}\angle 150^o \text{ V} \quad (11)$$

$$\text{V}_{YB} = \sqrt{3} \cdot V_{ph}\angle -90^o \text{ V} \quad (12)$$

A phasor diagram showing the line and phase voltages is shown in Figure 5.

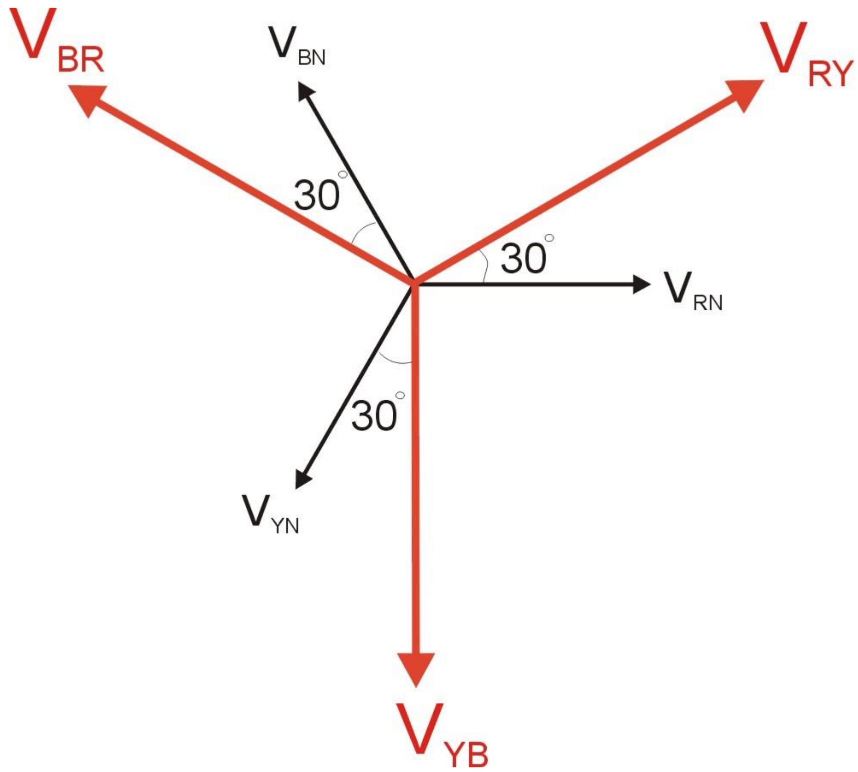


Figure 5. Phasor Diagram showing the line and phase voltages for 3-phase generation.

Balanced STAR Connected Load

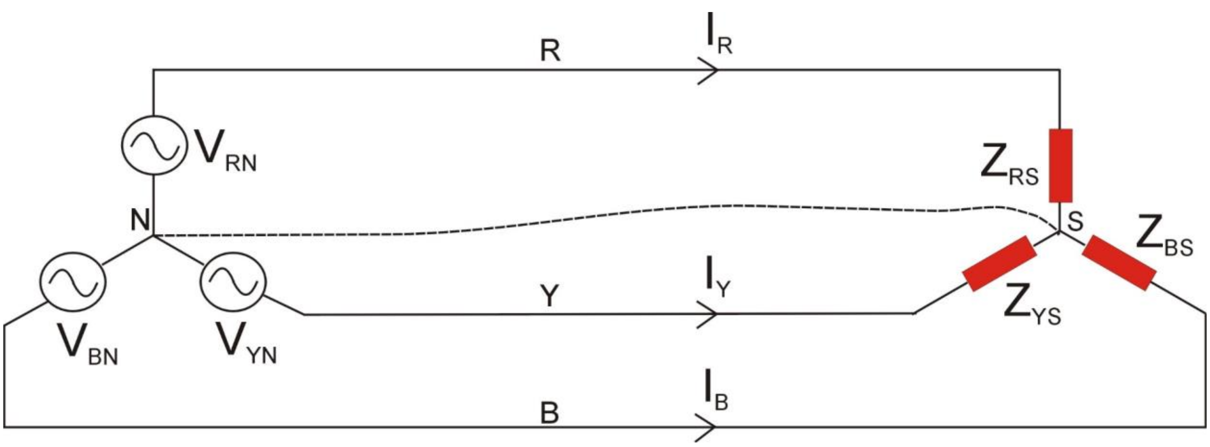


Figure 6. Star Connected Load

Figure 6 depicts a STAR connected load. If $Z_{RS} = Z_{BS} = Z_{YS}$ then we say that the load is balanced. In this instance we can evaluate three separate line currents which are separated in phase by 120° and equal in magnitude:

$$I_R = \frac{V_{RN}}{Z_{RS}} = I_L \angle -\phi \text{ A} \tag{13}$$

$$I_Y = \frac{V_{YN}}{Z_{YS}} = I_L \angle -\phi - 120^o \text{ A} \tag{14}$$

$$I_B = \frac{V_{BN}}{Z_{BS}} = I_L \angle -\phi + 120^o \text{ A} \tag{15}$$

The current between S and N will be zero since $I_{SN} = I_R + I_Y + I_B = I_L + 2I_L(-0.5) = 0$ and $V_{SN} = 0.$

Power Calculations for Balanced Star Connected Load

Figure 7 is a phasor diagram showing the phase voltage and line currents where ϕ is the phase angle between them. For each phase we can calculate the apparent, real and reactive power for each phase:

$$S = |V_{RN}||I_R| = V_{PH} I_L \tag{16}$$

$$P = |V_{RN}||I_R| \cos\phi = V_{PH} I_L \cos\phi \tag{17}$$

$$Q = |V_{RN}||I_R| \sin\phi = V_{PH} I_L \sin\phi \tag{18}$$

where $\boldsymbol{V_{ph}}$ is the rms magnitude of the phase voltage, $\boldsymbol{I_L}$ is the rms line current and $\boldsymbol{\Phi}$ is the angle between them.

The total power in a three-phase system is three times the single-phase real power:

$$P_T = 3V_{PH} I_L \cos\phi$$

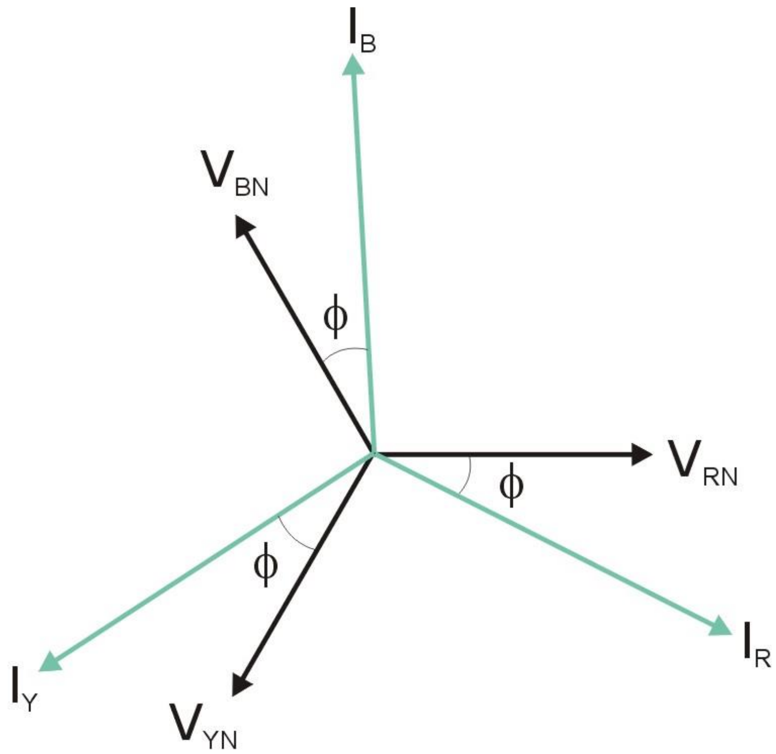


Figure 7. Phasor Diagram showing the phase voltages and line currents.

Bibliography

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3. Ahmed M. Hussein "The Basics of Three Phase Circuits for Beginners", https://electrical-engineering-portal.com/res/res5/The-basics-of-three-phase-circuits-for-beginners.pdf (Accessed online 18th August 2021).