Advanced Data Analysis in Python

p-Values

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472/502: p-Value

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- The smaller the p-value, the less likely the data comes from the null distribution

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- $p(H_0|D) = \frac{p(H_0)}{p(D)}p(D|H_0)$

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- Therefore, $p(M|C) = \frac{p(M)}{p(C)}p(C|M) = 0.38$
- Probability of correctly classifying an individual as militia member given they are a member is 0.95, yet the probability that a person is a militia member given a positive classification is 0.38

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- Interested in magnitude of effect; making binary decisions about the existence of an effect is not particularly informative

Further Issues

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- Asymmetry and accepting the null hypothesis: H_1 is held innocent until proven guilty, H_0 is held guilty until proven innocent

Pervasive Problem

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- These errors waste research efforts and misinform policy decisions

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- The researchers from the earlier, statistically significant, study found the exact same risk ratio of 1.2
- That study was simply more precise, with an interval spanning from 9% to 33% greater risk (p=0.0003)

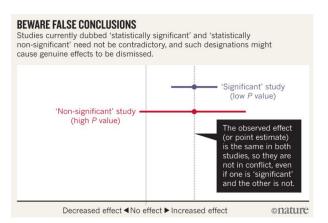
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- Reliance on thresholds of statistical significance can mislead us



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- "Deeply flawed or else ill-used by researchers" (Serlin and Lapsley 1993)
- "A terrible mistake, basically unsound, poor scientific strategy, and one of the worst things that ever happened in the history of psychology" (Meehl 1978)

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- "The significance test as it is currently used in the social sciences just does not work" (Hunter 1997)

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- The authors call for the entire concept of statistical significance to be abandoned

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- One advocate called it a "surgical strike against thoughtless testing of statistical significance" and "an opportunity to register your voice in favour of better scientific practices"

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- Meta-analysis: Offers attractive proposition that the accumulation of knowledge on some research question can be compared and combined in a single procedure

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- Why is the null hypothesis significance test pervasive?
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- Are there any alternatives not mentioned that you can think of?

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- This occurs even with the best of intentions

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- Whether a p-value is small or large, caution is warranted

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- An interval that contains the null value will often also contain non-null values of high practical importance

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- It is thus wrong to claim that an interval shows all possible values

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- Interpreting the point estimate, while acknowledging its uncertainty, will keep you from making false declarations of 'no difference,' and from making overconfident claims

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- It is based on the false idea that there is a 95% chance that the computed interval itself contains the true value, coupled with the vague feeling that this is a basis for a confident decision
- Interval estimates can perpetuate the problems of statistical significance when the dichotomization they impose is treated as a scientific standard

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- Inferences should be scientific, and that goes far beyond the merely statistical
- Factors such as background evidence, study design, data quality and understanding of underlying mechanisms are often more important than statistical measures such as *p*-values or intervals