UNIVERSITI MALAYSIA PERLIS

Peperiksaan Akhir Semester Kedua
Sidang Akademik 2018/2019

Jun 2019

ENT342 – Computational Fluid Dynamics
[Pengiraan Dinamik Bendalir]

Masa: 3 Jam

Please make sure that this question paper has **SEVEN (7)** printed pages including this front page before you start the examination.
*[Sila pastikan kertas soalan ini mengandungi **TUJUH (7)** muka surat yang bercetak termasuk muka hadapan sebelum anda memulakan peperiksaan ini.]*

This question paper has **FIVE (5)** questions. Answer **ALL** questions in **PART A** and any **TWO (2)** questions in **PART B**. Each question contributes 25 marks.
*[Kertas soalan ini mengandungi **LIMA (5)** soalan. Jawab **semua** soalan di **BAHAGIAN A** dan mana-mana **DUA (2)** soalan di **BAHAGIAN B**. Markah bagi setiap soalan adalah 25 markah.]*

Note: Tables and equations are given in the Appendix.
[Nota: Jadual dan persamaan diberi dalam Lampiran.]

Part A: Answer all questions

[Bahagian A: Jawab semua soalan]

Question 1

[Soalan 1]

The governing equations for inviscid compressible flow is the Euler equations. The equations can be derived from three conservations principle which are the mass conservation, momentum conservation and energy conservation. The principles can be easily demonstrated for one-dimensional flow. Answer the questions below regarding the one-dimensional Euler equations.

(a) Explain the three principle of conservations and their relations to the control volume analysis. Sketch a suitable control volume for your explanation.

[Jelaskan tiga prinsip keabadian itu dan hubungannya dalam analisa isipadu kawalan. Lakarkan rajah isipadu kawalan yang sesuai untuk penjelasan anda.]

(10 Marks /*Markah*)

(b) Using the conclusions from Q1(a), prove that the one-dimensional Euler equations can be written in the form below. Here, ρ, u, p, e are density, x-axis velocity, pressure and total energy, respectively.

[Dengan menggunakan kesimpulan daripada Q1(a),buktikan bahawa persamaan Euler satu dimensi boleh ditulis dalam bentuk dibawah. Disini, ρ, u, p, e adalah masing-masingnya ketumpatan, halaju pada paksi-x, tekanan dan tenaga keseluruhan.]

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} = 0$$

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u^2 \\ e \end{bmatrix}$$

$$\mathbf{E} = \begin{bmatrix} \rho u^2 \\ \rho u^2 + p \\ (e+p)u \end{bmatrix}$$

(10 Marks /*Markah*)

(c) There are four (4) unknown variables in the equations from Q1(b) but only three (3) equations, meaning that the system needs another equation to close it. The pressure p and total energy e are linked together by the state equation of ideal gas. Explain and derive the relation.

[Terdapat empat (4) anu dalam kesemua persamaan daripada Q1(b) tetapi cuma tiga (3) persamaan, bermakna sistem persamaan ini memerlukan satu lagi persamaan untuk dipenuhi. Tekanan p dan tenaga keseluruhan e adalah berkaitan dengan persamaan keadaan untuk gas unggul. Jelaskan dan terbitkan hubungan itu.]

(5 Marks /*Markah*)

Question 2

[Soalan 2]

The Marker-and-Cell (MAC) algorithm belongs to a group of solution algorithm called *pressure correction* method for incompressible Navier-Stokes equations. The incompressible Navier-Stokes equations written below in differential form where $\mathbf{u}$, p and ν are velocity vector, pressure and dynamic viscosity coefficient, respectively. The MAC algorithm closes the system equations by using the mass conservation equation or divergence equation to link the pressure with velocity gradients. Answer the questions below regarding the MAC algorithm.

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u},$$

(a) Construct the step-by-step of MAC algorithm using the incompressible Navier-Stokes equations given. Give the supporting mathematical argument for each step.

[Binakan langkah-langkah penyelesaian persamaan tidak mampat Navier-Stokes menggunakan algoritma MAC. Berikan sokongan matematik bagi setiap langkah-langkah tersebut.]

(10 Marks /*Markah*)

(b) The staggered grid configuration is very effective in removing checkered pressure field solution. For this reason, the MAC algorithm applied the staggered grid configuration in its early inception. Construct the staggered grid configuration for two-dimensional structure and propose how to discretize the differential terms $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial p}{\partial x}$ for the given grid.

[Konfigurasi grid tidak serentak sangat berkesan untuk menyahkan solusi medan tekanan berkotak. Oleh sebab itu, pada awal-awalnya algoritma MAC telah menggunakan konfigurasi grid tidak serentak ini. Binakan grid tidak serentak untuk struktur dua dimensi dan berikan cadangan bagaimana terma-terma pembezaan $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ dan $\frac{\partial p}{\partial x}$ boleh didiskretkan untuk grid tersebut.]

(10 Marks /*Markah*)

(c) Can the checkered pressure field solution be avoided using normal grid? Propose and justify your solution.

[Bolehkah solusi medan tekan berpetak dielak menggunakan grid normal? Cadangkan dan justifikasikan solusi anda.]

(5 Marks /*Markah*)

Part B: Answer TWO (2) questions ONLY

[Bahagian B: Jawab DUA (2) soalan SAHAJA]

Question 3

[Soalan 3]

The one-dimensional linear advection equation is the canonical example of transport phenomena. The equation is an excellent model equation to test the behavior of numerical method. The equation is given as below.

[Bahasa Melayu]

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0, \quad c > 0$$

(a) Construct the discrete equation of the linear advection equation using finite volume method. Use Euler time integration for temporal term and equally spaced control volume for the spatial term. Take flux approximation at the cell interface as the average value of neighboring cells.

[Binakan persamaan diskrit untuk persamaan adveksi linear menggunakan kaedah isipadu terhingga. Gunakan integrasi Euler untuk terma masa dan gunakan isipadu kawalan yang bersamaan saiznya untuk terma ruang. Ambilkan nilai purata antara sel yang berjiran untuk penghampiran flux di permukaan sel.]

(10 Marks /*Markah*)

(b) From the discrete equation in Q3(a), derive the numerical stability condition using complex Fourier analysis.

[Daripada persamaan diskrit Q3(a), terbitkan syarat kestabilan berangka menggunakan analysis kompleks Fourier]

(10 Marks /*Markah*)

(c) What is the conclusion from Q3(b)? Is the method stable? Propose a solution if the method is not stable.

[Apakah kesimpulan daripada Q3(b)? Adakah kaedah ini stabil? Cadangkan satu solusi jika kaedah ini tidak stabil.]

(5 Marks /*Markah*)

Question 4
[Soalan 4]

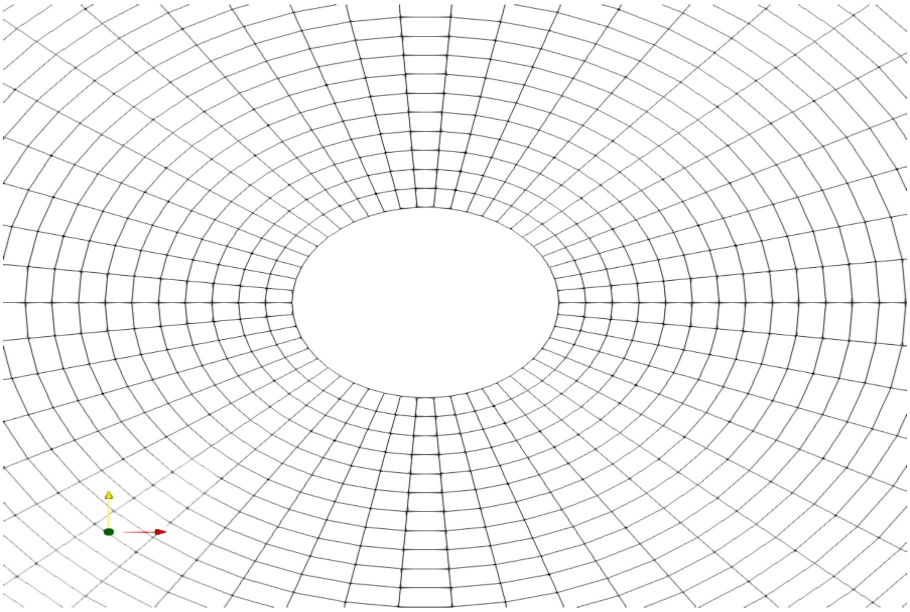

Figure 1
[Rajah 1]

The curvilinear boundary fitted grid as shown in **Figure 1** is generated analytically using the equations below with r_0 as the radius of the cylinder. Answer the questions below regarding coordinate transformation.

$$x(r,\theta) = (r_0 + r)\cos\theta$$
$$y(r,\theta) = (r_0 + r)\sin\theta$$

(a) For each coordinate, the metrics are given as x_r, y_r, x_θ and y_θ, where the subscript refers to partial derivatives. These metrics are required to transform any physical quantity derivative, for example u_x, into the computational domain derivatives of u_r and u_θ. Construct the projection of u_x into domain (r,θ) using the metrics and (u_r, u_θ).

[Untuk setiap koordinat, metrik-metrik diberikan sebagai x_r, y_r, x_θ dan y_θ, dimana subskrip merujuk kepada terbitan separa. Metrik-metrik ini diperlukan untuk mengubah apa-apa terbitan kuantiti fizik seperti u_x untuk terma terbitan di domain pengiraan u_r dan u_θ. Binakan unjuran u_x ke domain (r,θ) menggunakan metric-metric dan (u_r, u_θ).]

(10 Marks /*Markah*)

(b) **Table**1 shows a sample coordinate of a cell from the grid in **Figure**1. The Jacobian metric, **J** of any cell in the grid is given as the equation below. Calculate the area of the cell and interpret its relation to the Jacobian metric.

*[Jadual1 menunjukkan sampel koordinat satu sel daripada grid seperti di Rajah1. Metrik Jacobian **J** bagi mana-mana sel dalam grid diberikan seperti persamaan di bawah. Kirakan luas kawasan sel tersebut dan interpretasikan hubungannya dengan metric Jacobian.]*

$$\mathbf{J} = \det\begin{pmatrix} x_r & y_r \\ x_\theta & y_\theta \end{pmatrix} = x_r y_\theta - x_\theta y_r \tag{3}$$

Table 1

[Jadual 1]

x	**y**
0.927051	2.85317
0.562144	2.94686
0.988854	3.04338
0.59962	3.14332

(10 Marks /*Markah*)

(c) Sketch the cell in Q4(b) roughly and calculate the outward pointing unit normal vector on each of the cell's side.

[Lakarkan secara kasar sel daripada Q4(b) dan kirakan nilai unit vektor normal di setiap sisi sel.]

(5 Marks /*Markah*)

Question 5

[Soalan 5]

The time-dependent one-dimensional heat equation is a second-order partial differential equation. The second-order term infers that the solution propagation will be restricted to the order of $\mathcal{O}(1/\Delta h^2)$, where Δh is the mesh spacing. This put a severe stability restriction to explicit numerical scheme when solving this equation. Answer the questions referring to the time-dependent one-dimensional heat equation below.

[Persamaan haba satu-dimensi bersandar masa merupakan persamaan kebezaan separa darjah kedua. Terma darjah kedua bererti propagasi solusi disekat pada darjah $\mathcal{O}(1/\Delta h^2)$, dimana Δh adalah jarak antara grid. Ini menyebabkan penyekatan syarat kestabilan yang teruk bagi kaedah berangka tersurat ketika menyelesaikan persamaan ini. Jawab soalan-soalan berkaitan dengan persamaan haba satu-dimensi bersandar masa dibawah.]

$$\frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} \right)$$

(a) Construct the discretized equation for the heat equation using *forward* Euler time integration and centered difference scheme for the spatial term. Analyze the discrete equation to derive the numerical stability condition.

[Bina persamaan diskrit untuk persamaan haba itu menggunakan kaedah integrasi masa Euler kehadapan dan kaedah pembezaan pertengahan untuk terma ruang. Analisakan persamaan diskrit itu untuk menerbitkan syarat kestabilan berangka.]

(10 Marks /*Markah*)

(b) From the discrete equation obtained in Q5(a) modify the time integration scheme to *backward* Euler time integration. Analyze the discrete equation to derive the numerical stability condition. Justify the merit of using *backward* Euler time integration.

[Ubahsuaikan kaedah integrasi masa persamaan diskrit yang diperolehi oleh Q5(a) kepada kaedah integrasi masa Euler kemunduran. Analisakan persamaan ini untuk menerbitkan syarat kestabilan berangka. Justifikasikan merit menggunakan kaedah integrasi masa Euler kemunduran.]

(10 Marks /*Markah*)

(c) The discrete equation from Q5(b) contains more unknown at the next time step than at the previous time step. This condition necessitates the construction of simultaneous linear equations. Construct the matrix-vector form for the simultaneous linear equations.

[Persamaan diskrit daripada Q5(b) mengandungi lebih banyak terma di masa seterusnya daripada masa sebelumnya. Keadaan ini memerlukan pembinaan persamaan linear serentak. Bina persamaan linear serentak itu dalam bentuk matriks-vektor.]

(5 Marks /*Markah*)

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