ENT441 Computational Fluid Dynamics

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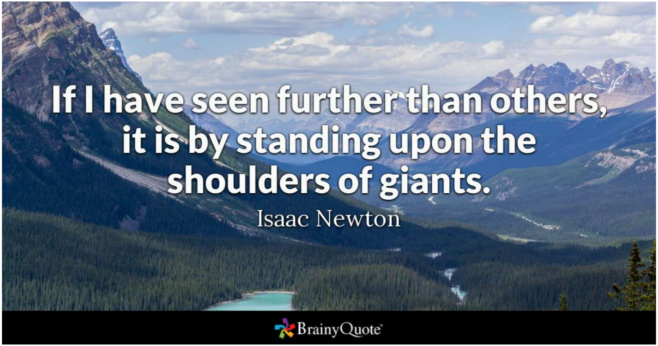
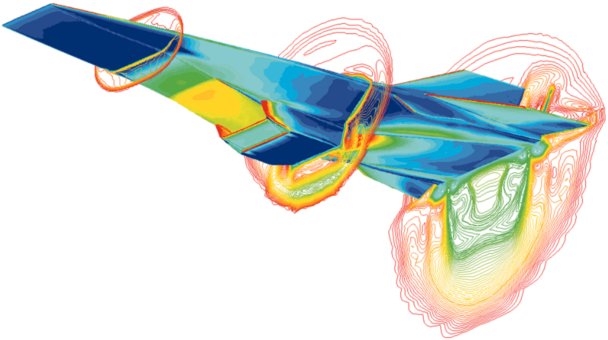
Why Computational Fluid Dynamics?



COLORFUL FLUID DYNAMICS



Overview

Advection equation 1

- Linear advection equation.

$$\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0, \qquad c \geq 0 \tag{1}$$

$$u\left(x,0\right) = u_0$$

Advection equation 2

- Initial condition and analytical solution at t=2.0000.

$$u_0 = \begin{cases} 1.0 & 2.0 \leqslant x \leqslant 4.0 \\ 0.0 & other \end{cases}$$

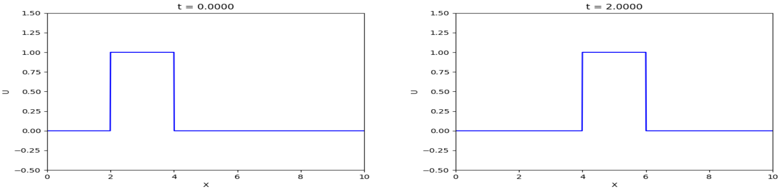


Figure: Left is at t=0.0000 and right is at t=2.0000

Advection equation 3

- Sample of upwind discretization in Python.

```
import numpy as np
import matplotlib.pyplot as plt
```

```
c = 1.0
alpha = c*dt/dx
for iter in range(itermax):

    for i in range(1,imax-1):
        iup = i - int(np.sign(c))
        un[i] = u[i] - alpha*( u[i] - u[iup] )

    #update, make sure its a copy; not the same object
    u[:] = un[:]
```

Diffusion/Heat equation 1

- Diffusion equation or heat equation.

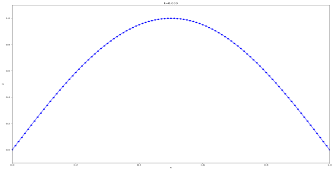
$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial x^2} \tag{2}$$

- The boundary condition

$$T(x,0) = \begin{cases} f(x) & \textit{Neumann BC} \\ const. & \textit{Dirichlet BC} \end{cases} \qquad \forall x \in [0, L] \tag{3}$$

Diffusion/Heat equation 2

- Diffusion/Heat equation in one-dimension



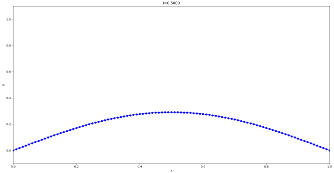


Figure: Left is at t=0.0000 and right is at t=0.5000

Diffusion/Heat equation 3

- Sample of the heat equation discretization in Python.

```
dt = np.float64(input("Enter dt, dx=%s (dt is
    propotional to dx*dx) "%dx ))
itermax = np.int64( 0.5/dt ) +1
print("Maximum iteration: ", itermax)
kappa = 0.25 #diffusion coefficient
alpha = kappa * dt /(dx**2)
steps = itermax/1000
for iter in range(itermax):
    for i in range(1,imax-1):
        un[i] = u[i] + alpha*( u[i-1] - 2.0*u[i] +
    u[i+1] )
    #update
    u[:] = un[:]
```

Poisson/Laplace equation 1

- Poisson equation/ Laplace equation.

$$-\frac{\partial^2 u}{\partial x^2} = \begin{cases} f(x) & \textit{Poisson equation} \\ 0 & \textit{Laplace equation} \end{cases} \qquad \forall x \in [0, L] \tag{4}$$

- The boundary condition

$$T(x,0) = \begin{cases} g(x) & \textit{Neumann BC} \\ const. & \textit{Dirichlet BC} \end{cases} \qquad \forall x \in [0, L] \tag{5}$$

Poisson/Laplace equation 2

- Poisson equation solution in one-dimension

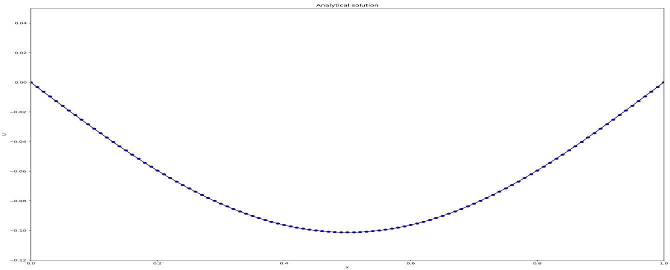


Figure: Poisson equation solution

Poisson/Laplace equation 3

- Sample of Poisson equation discretization in Python.

```
for iter in range(1,itermax):
    for i in range(1,imax-1):
        un[i] = u[i] + alpha*( u[i-1] - 2.0*u[i] + u[i+1] ) - dt*source[i]
```

- The source term refers to $f(x)$

Finite Difference Method

- The oldest approach to numerically solve differential equation.
- Regarding notation for 1D, U_i^n:

$$U_i^n \leftarrow \begin{cases} n & timestep\ or\ iteration \\ i & space\ index\ in\ x-axis \end{cases} \tag{6}$$

- For 2D, $U_{i,j}^n$:

$$U_{i,j}^n \leftarrow \begin{cases} n & timestep\ or\ iteration \\ i,j & index\ i\ for\ x-axis\ and\ j\ for\ y-axis \end{cases} \tag{7}$$

Finite Volume Method

- The most popular for computational fluid dynamics.
- Based on Gaussion divergence theorem.

$$\int_{\Omega} \nabla \cdot \mathbf{u} \, d\Omega = \oint_{S} \mathbf{n} \cdot \mathbf{u} \, dS \tag{8}$$

Standard template

- This is a standard template slide.
- Modify by adding items.

Thank You!

Questions?