Markov Chain Monte Carlo Monopoly Board Game Simulation

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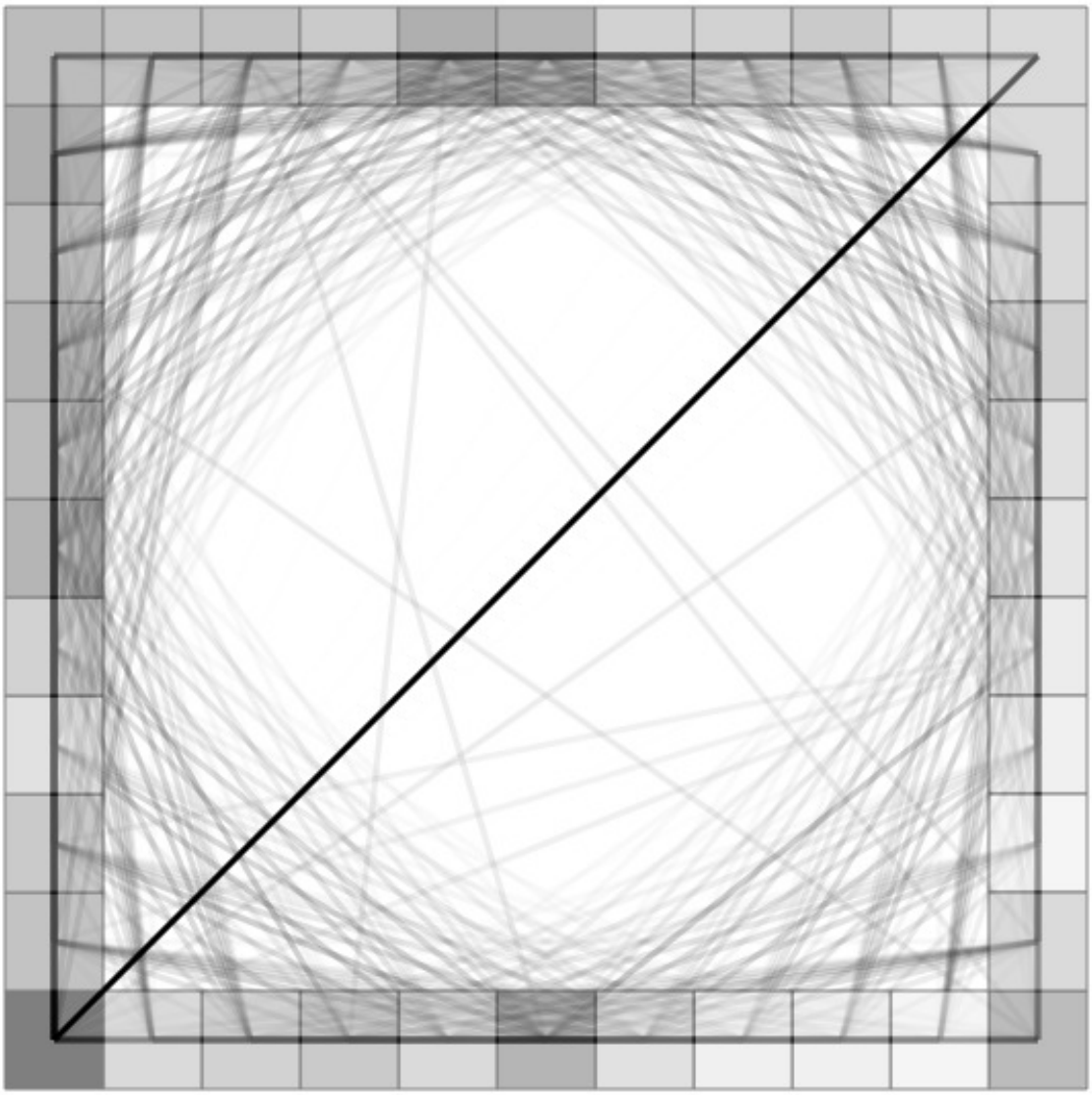
Abstract

This paper examines the probability of landing on tiles across a Monopoly board. Utilizing markov chain monte carlo simulations, movement across the board can be accurately predicted. Insights for property investments and game decisions can be determined from a mathematical foundation. This paper creates a simulation based on board game movement rules resulting in a board game probability distribution.



1 Introduction

Monopoly is a multiplayer board game where players move around a board buying, trading, and paying rent. Monopoly has different house rules and ending points by different households where no two games ever look the same and strategies may be different.

While player interactions differ greatly between games, they have no effect on a player's position on the board. The main component for movement is from two six sided dice with some additional factors including going to jail and chance cards. Since game movement is independent from player choices and dictated from randomization, a monte carlo simulation can be created to model the game and give board game positional distribution.

2 Methodology

This section describes the procedure created to accurately model a player's movement across a Monopoly board.

The game board was indexed *[0, 39]* where each integer represents a tile on the board. The board is a circle, where they go loop back to the start of the board once the position exceeds 39. Most tiles are properties which do not effect the location, the tiles of interest and which can move a player include:

- Chance cards *[7, 22, 36]*
- Community chest cards *[2, 17, 33]*
- Go to jail *[30]* & Jail *[10]*

2.1 Game Rules

In a Monopoly game there are a few factors which can affect a players location. These factors include:

1. Rolling dice
2. Chance & community chest cards
3. Going to jail

For each simulated round, these heuristics were implemented to simulate a real world turn.

2.1.1 Rolling Dice

At the beginning of each turn, a player rolls two 6-sided dice and moves forward the resulting sum. If the roll includes doubles, the player rolls again. Three doubles in a row results in the player being sent directly to jail *[10]*.

2.1.2 Chance & community chest cards

There are six tiles where the player picks up a card where an action occues including recieve money, pay money, and moving. The chance and community chest cards decks were created to replicate the real world options.[1] Once a player lands on a card tile, a card was randomly chosen without replacement from the respective deck. If the action on the card was a movement, the player's location was moved across the board.

[1] https://www.monopolyland.com/list-monopoly-chance-community-chest-cards/

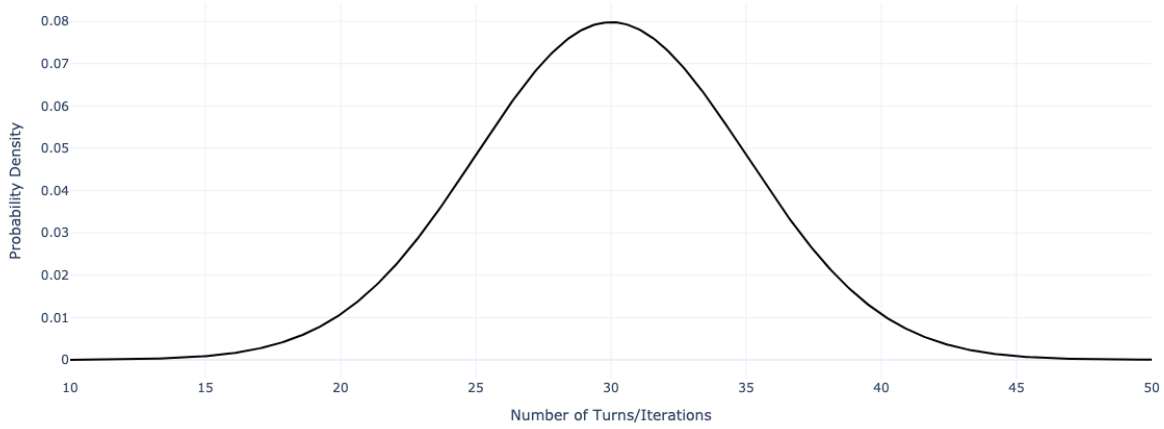


Figure 1: Markov Chain Number of Rounds Distribution

2.1.3 Going to jail

When the player lands on the "Go to Jail" tile *[30]*, the player automatically moves to Jail *[10]*.

2.2 Markov Chain Simulation

Markov chains are utilized to simulate a stochastic, random with known probability, process modelling steps where the next step is independent from the previous step. This works for the Monopoly process as a player's next move is independent at each turn. A game can be modelled from a set number of independent turns.

Monopoly can have different rules for ending including a time limit, until people get tired, or until there is only one person left with money and property. For this paper, a set number of rounds for a game will be set to an average game of 30 with a standard deviation of 5.[2]Figure 1 shows the sampled distribution for number of iterations for the markov chain to simulate a game.

This paper looks at the board probability, not just the final state. Therefore, a player's location was recorded at every movement throughout a simulated round.

2.3 Monte Carlo Simulation

Each game is a stochastic process, producing random steps and outcomes. While the distribution cannot be statistically analyzed directly, the underlying rules are known. A Monte Carlo simulation can provide insights into the distribution. The Monte Carlo method involves running a simulation a large number of times to track the possible paths. In this paper, 10,000 games were simulated.

3 Results

This section presents the results of the Markov Chain Monte Carlo simulation to provide insights on Monopoly board move-

[2]https://blog.ed.ted.com/2017/12/01/heres-how-to-win-at-monopoly-according-to-math-experts/

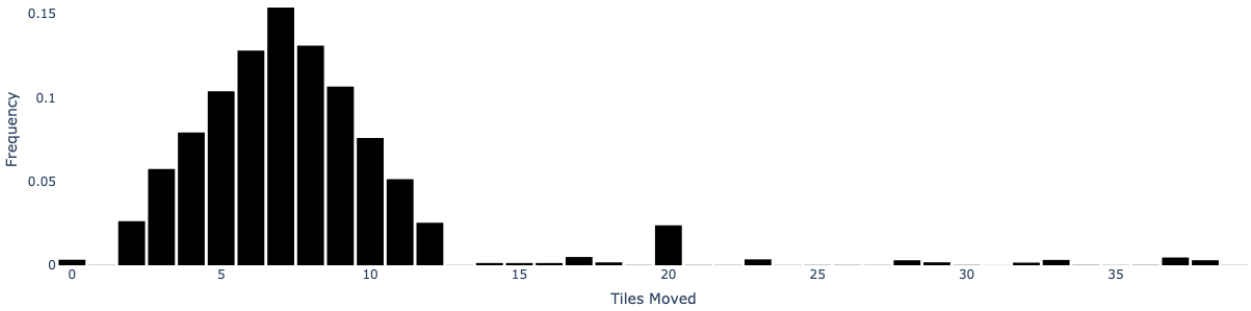


Figure 2: Tile Movement Distribution

ment.

Figure 2 shows the distribution of tiles moved in each round. The majority of moves are normally distributed around 7, which makes sense given that 7 is the average roll of two six-sided dice with a range between 2 to 12. The other movements originate from the going to jail and movement cards.

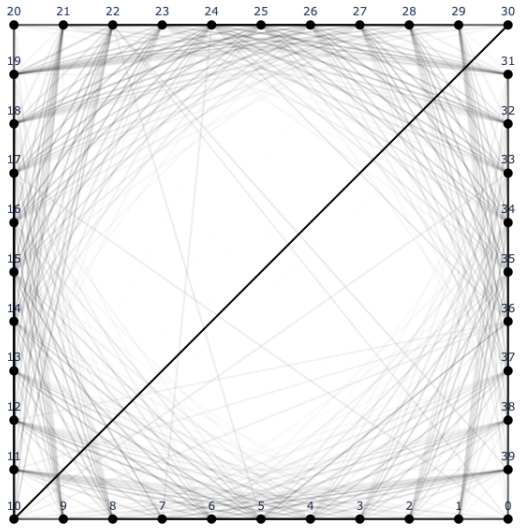


Figure 3: Board Movement

Figure 3 shows the movement a player can make across the board from dice rolls or actions such as going to jail or cards. The movement across the board appears relatively consisent, execpt go to jail and card tiles.

3.1 Tile Distribution

Aggregating every simulated game from monte carlo, a board distribution can be calculated.

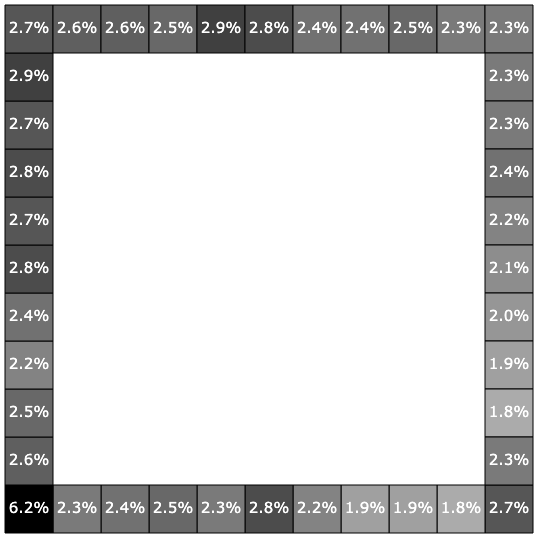


Figure 4: Board Probability

Figure 4 shows the probability of landing on each of the tiles. The first and last tiles near Go are the least likely to be landed on. The most likely are around the the second corner diagonally across from Go. This is due to the fact Jail, the most popular tile, is on the first corner and the average roll is 7.