MATH 390.4 Lecture 3

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*Recall

$$* = y = t(z) = f(x) + \delta = h^*(x) + \epsilon$$

where epsilon is "Error"

1 Supervised Learning

Supervised Learning: has 3 ingredients

1. Training Data:

$$D =< x, y >$$

$$X = [x1, x2, ..., xn], \mathbf{Y} = [y1, y2, ..., yn] xi \in \mathbf{X}, yi \in \mathbf{Y}$$

2. **H**: a candidate set of functions

3. **A**, an algorithm which takes in data **D** and set **H** and produces a model **g** where

$$\mathbf{g = A(D,H)}$$

Question: is

$$f \in \mathbf{H}$$

generally? **NO**

*However there is a

$$\mathbf{h^* \in H}$$

which is the closet possible model (function) to f

$$* = h^*(x) + (f(x) - h^*(x)) + (t(z) - f(x))$$

where

$$(f(x) - h^*(x)) = \delta$$

and

$$(t(z) - f(x)) = \epsilon$$

Just because h* $\in H$does not mean A will locate it. A will not be perfect and the value of ϵ will confuse A. This g $\neq h*$, g is the best A can do.

$$Y = g(x) + (h^*(x) - g(x)) + (f(x) - h^*(x)) + (t(z) - f(x))$$

where g(x) is your model

(h$^*(x) - g(x)) is your estimation error$

(f(x)-h$^*(x)) + (t(z) - f(x)) is your epsilon error where$

(t(z)-f(x)) is your delta error and

(f(x)-h$^*(x)) is your mis - specification error$

y* = g(x) where y* is the prediction of y in setting x

e = y-y* residual if x element of D (training data) otherwise they are unknown

How to reduce errors

1. Delta, ignorance error can be reduced by measuring more Xj's (features) of the units that contain information about Z.

2. Misspecification error can be reduced by expanded H to include more complicated functions

3. Estimation error can be reduced by increasing sample size

Example:

This is like 1. Of supervised learning Mortgages loan Y = 0,1 where 0 did not pay back and 1 paid back the loan P = 1 x1 is credit score X = [300,850]

$$D =< x, y >= [810, 390, 750. \ldots ...][1, 0, 1. \ldots ....]$$

This is data set of the relation of credit score and if they paid back the loan or not

This is like 2 in supervised learning trying to make H a set of candidate functions

$$\mathbb{1}$$

is the indicator function where

$$\mathbb{1}(w) = 1 \text{ if w} \in A$$

$$0 \text{ if w} \notin A$$

$$H = \mathbb{1}x >= \theta : \theta \in \Theta$$

$\theta =$ is a parameter sometimes denoted as β or w... and others.

$\Theta =$ is the set of all parameters

e.g.

$$g(x) = \mathbb{1}x >= 515.3$$

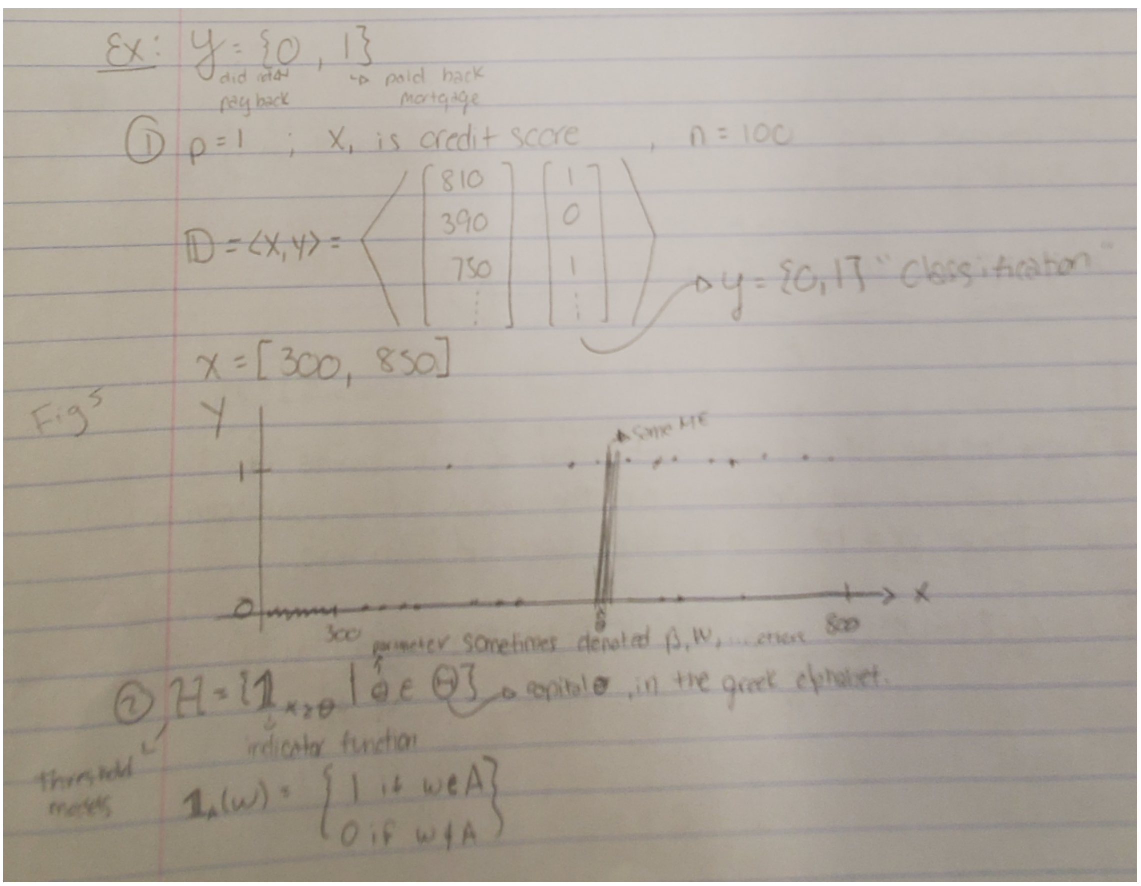
or

$$g(x) = \mathbb{1}x >= 407.9$$

3. This is 3 of supervised learning

Algorithm A is a estimation of theta $\sum_{i=1}^{n}$

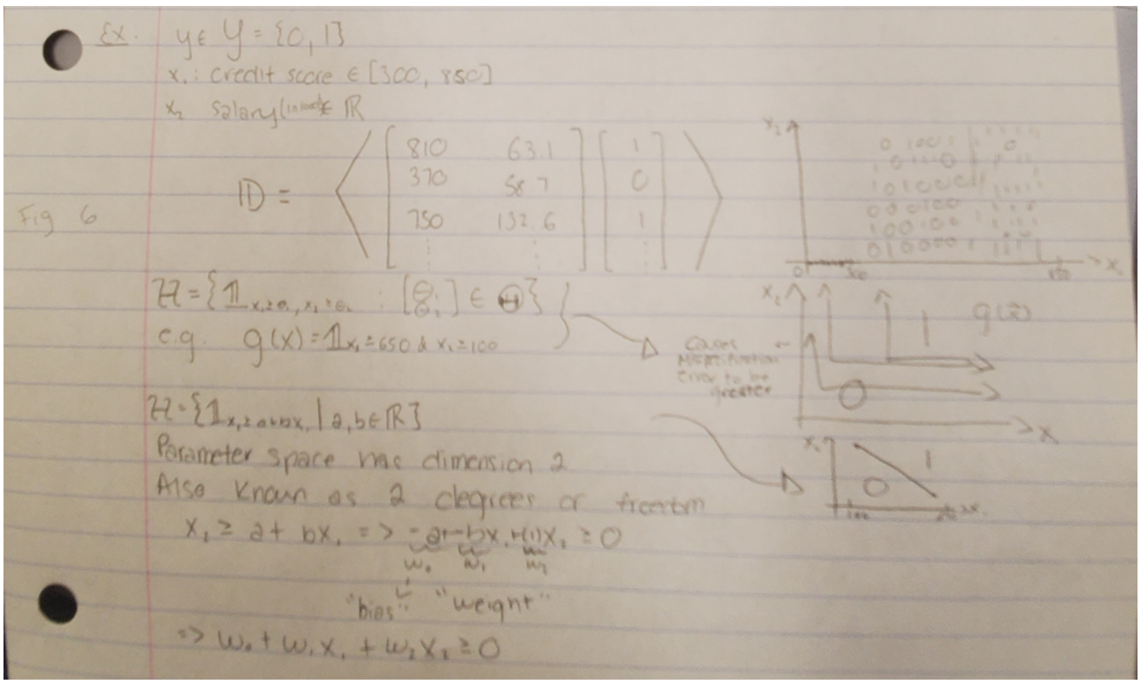
First lets define "Misclassification error" ME:= 1/n $\sum_{i=1}^{n} \mathbb{1}g(xi)! = yi$



$$\text{Accuracy } ACC := 1 - ME$$

A: will minimize ME over $\theta \in \Theta$ Where $\theta \in$ Unique $x's$

1/n $\sum_{i=1}^{n} |yi - yi'| = 1/n \sum ei^2$
$1/n \sum ei^2$ is the mean squarred error(MSE) where
$\sum ei^2$ is the sum squred error (SSE)



X is PH = 3(of columns in x) = $[\vec{i}X]$ redefine the matrix X by appending a column of 1's on the left

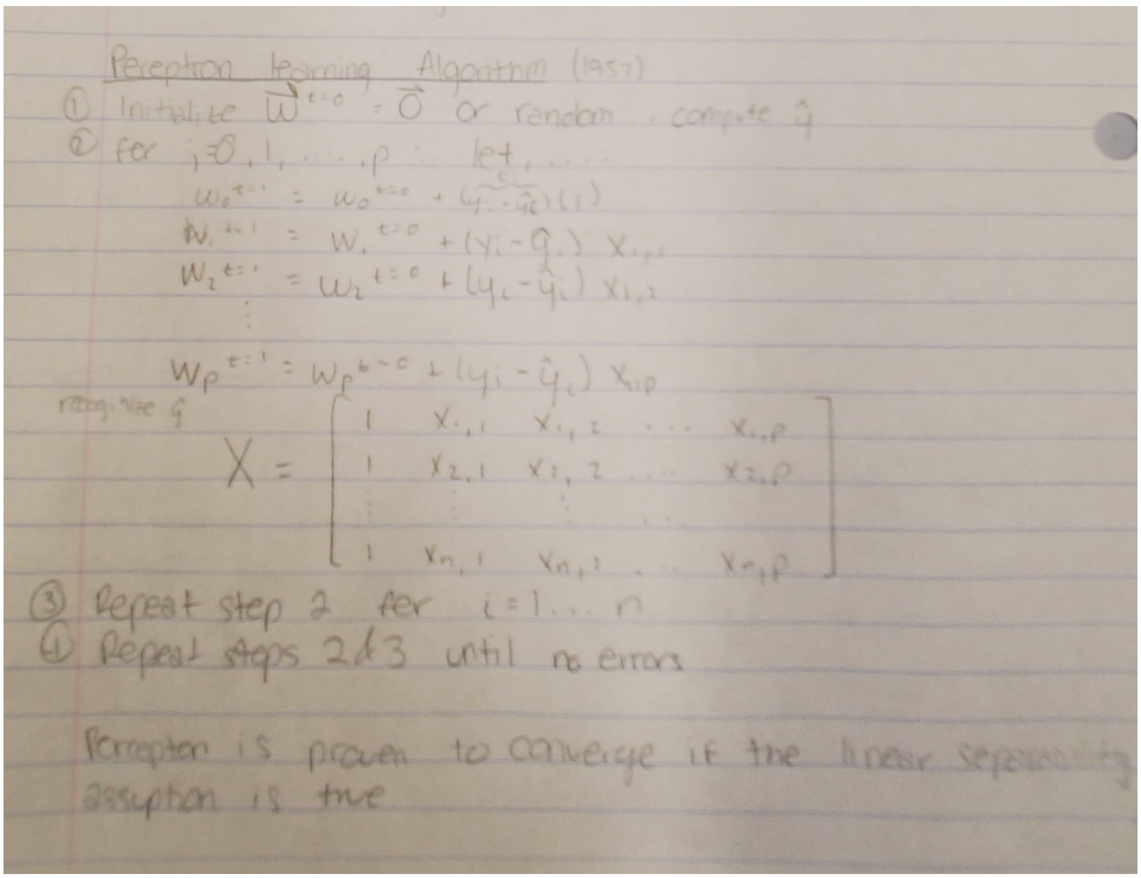
$\vec{x} = [1, x_1, x_2]$

$$H = \mathbb{1}\vec{w} * \vec{x} > 0 : \vec{W} \in R^3$$

This is an "over-parameterized" model, where each line has infinite $\vec{w}'s$ that specify it.
Need Algorithm A

$$\mathbf{g = A(D,H)}$$

Assume that 0's and 1's are linearly separable there exists w's such that $g(\vec{x})$ has no error



Perceptron Learning Algorithm (1957)

① Initialize $\vec{w}^{t=0} = \vec{0}$ or random, compute $\hat{y}$

② for $i = 0, 1, \ldots, p$ let, ...

$$w_0^{t=1} = w_0^{t=0} + (\overbrace{y_i - \hat{y}_i}^{e})(1)$$
$$w_1^{t=1} = w_1^{t=0} + (y_i - \hat{y}_i)\, x_{i,1}$$
$$w_2^{t=1} = w_2^{t=0} + (y_i - \hat{y}_i)\, x_{i,2}$$
$$\vdots$$
$$w_p^{t=1} = w_p^{t=0} + (y_i - \hat{y}_i)\, x_{i,p}$$

recognize $\hat{y}$

$$X = \begin{bmatrix} 1 & x_{1,1} & x_{1,2} & \cdots & x_{1,p} \\ 1 & x_{2,1} & x_{2,2} & \cdots & x_{2,p} \\ \vdots & & \vdots & \ddots & \\ 1 & x_{n,1} & x_{n,2} & \cdots & x_{n,p} \end{bmatrix}$$

③ Repeat step 2 for $i = 1 \ldots n$

④ Repeat steps 2 & 3 until no errors

Perceptron is proven to converge if the linear separability assumption is true