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Tuning strategy for dynamic matrix control with reduced horizons

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ABSTRACT

In Dynamic Matrix Control (DMC) algorithm, the control signal is computed optimally based on the process model. In effect, the DMC algorithm allows for obtaining a better quality of control than conventional controllers, especially for plants with large time delays. However, in spite of these advantages, there are still some difficulties that can appear in the implementation of DMC in local control loops. This is due to limitations of the computational resources in industrial devices (e.g., Programmable Logic Controllers). To overcome these difficulties, we propose a tuning strategy for the DMC algorithm with reduced horizons. It is shown that a reduction in the length of prediction and dynamic horizons can reduce the required memory in industrial controllers without degrading the quality of control.

1. Introduction

The advanced controllers that belong to a group of MPC (model predictive control) algorithms are attracting more and more attention in industry (see, e.g., [1,2]). The major reason is that the control action can be computed optimally based on the process model. At the same time, the constraints for the control signals and for the process variables can be easily incorporated in the control algorithm for both SISO (single input single output) and MIMO (multi input multi output) plants [3,4]. One of the first versions of the MPC algorithms was the DMC (dynamic matrix control) algorithm proposed in Cutler and Ramaker [5] for a chemical process. In comparison to the MPC algorithm, the DMC controller uses the step response of the plant, which can be approximated by the first order plus delay time (FOPDT) model, specified for a chosen operating point of the system. The mathematical model of the plant is used to predict its future outputs over a prediction horizon and the control signals are determined from the minimization of an objective function that includes the predicted data [6]. In effect, the DMC algorithm allows for obtaining a better quality of control (e.g., smaller overshoots or a shorter settling time) than the classical PI or PID controllers (see, e.g., [6]), especially for plants with large time delays. However, PID controllers are still dominant in local control loops, since predictive algorithms require more computational resources and memory in control devices [7–10]. Moreover, the available memory space can be greatly limited in industrial reality, when several control algorithms have to be coded on a single device, or when the created programs must follow company standards. For example, it may be necessary to implement additional function blocks with optional control algorithms, although only one control algorithm is used at a time. The implementation problems can also occur when the optimization task in the predictive algorithm has to be solved on-line at each sampling instant [7,9]. This issue was tackled by other authors for MPC controllers with the state-space representation of the plant. One approach is based on multi-parametric methods, for which the controller outputs are calculated off-line as functions of state variables (parameters). Then, the control signal is dependent on current state variables and a region of active or inactive constraints in the state space [11,12,9]. In effect, there is no need to solve the optimization task on-line, but the number of regions to be stored in the controller memory may grow exponentially in the prediction horizon [13]. The other approach uses Laguerre functions that allow using longer control horizons with a reduced computational complexity and less number of parameters to be stored in the controller memory, and can also be implemented by using multi-parametric techniques [13–15].

Another important issue, which makes the implementation of the DMC algorithm difficult, is the selection of tuning parameters, i.e., prediction horizon H_P, control horizon H_C, dynamic horizon H_D, move suppression coefficient λ and controller sampling time T_c. In practice, the controller parameters can be found by trial and error

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method [16–18], but this often results in a poor quality of control. Therefore, the selection of controller parameters and their influence on closed-loop system responses are widely discussed in the literature. One of the most representative works in this area is the paper by Shridhar and Cooper [19], which was the basis for other tuning procedures. The authors present easy-to-use analytical expressions for the controller parameters. The proposed tuning method was also extended for MIMO systems [20,21] and for integrating processes [22,23]. The other tuning methods that can be found in the literature are rather focused on specified tuning parameters, which have the most significant impact on the control system behavior [24–31], and the other parameters are usually determined according to [19].

Since the DMC algorithm uses a linear model of the plant, the mentioned tuning procedures are often based on the FOPDT model, making the tuning procedure easy to use by less experienced engineers. However, when implementing the DMC algorithm one should be aware that the prediction H_P, control H_C and dynamic H_D horizons have a strong influence on the size of matrices that must be stored in the controller memory, and thus, on the computational complexity. Especially, all the tuning procedures that are based on the well-known rules given by Shridhar and Cooper [19] may require more space in the memory of the controller. In effect, it may be difficult or even impossible to implement the DMC algorithm with additional adaptive mechanisms (see, e.g., [32,33]), quadratic programming solvers, or to implement software that follows the company standards in typical PLC units. Hence, the main goal of the paper is to propose tuning rules for the DMC controller that have two basic features:

- the control algorithm is easy to tune
- the DMC controller can be implemented for SISO systems in local control loops, in PLC units with low computational resources

The proposed tuning rules are tested with real and simulated benchmark plants and their effectiveness is compared with the results obtained for the tuning rules given in Shridhar and Cooper [19]. It is shown that a reduction in the length of prediction H_P and dynamic H_D horizons can reduce the required memory in PLC unit without degrading the quality of control in comparison to the results given in Shridhar and Cooper [19]. Moreover, it is shown that the settling time of the control system can be easily changed by an additional tuning parameter. In the remainder of the paper, the tuning procedures proposed by Shridhar and Cooper [19] will be referred to as the S-C method or S-C parameters.

The DMC algorithm was implemented in the analytical form and the implementation details are given in the next section. The proposed tuning rules are given in Section 3 and Section 4 presents their effectiveness based on the simulations and laboratory experiments. Finally, Section 5 concludes the paper.

2. DMC algorithm for SISO plants

The general idea of the DMC algorithm is to determine the future control increments Δu at the current time instant k by minimizing the following cost function J over the prediction horizon H_P:

$$J(k)=\sum_{p=1}^{H_P}\left(y^{sp}(k+p|k)-y(k+p|k)\right)^2+\lambda\sum_{p=0}^{H_C-1}\left(\Delta u(k+p|k)\right)^2 \qquad (1)$$

where: y^{sp} $(k+p|k)$, y $(k+p|k)$, Δu $(k+p|k)$ are the set point, controlled variable and control increment at time instant $k+p$ predicted at time instant k, respectively. The analytical form of the DMC controller for a SISO plant, which is a solution to the optimal problem (1), can be determined as follows [34].

Step 1 (collection of step response data)

Collect the step response data for a specified operating point of the system and fit the FOPDT model (2) by determining its parameters, i.e., the overall time constant T, delay time T_o and plant gain k_o:

$$K(s)=\frac{k_o e^{-sTo}}{sT+1} \qquad (2)$$

Step 2 (determination of tuning parameters)

The parameters of the FOPDT model (2) are used to tune the DMC algorithm, i.e., to determine the prediction horizon H_P, control horizon H_C, dynamic horizon H_D, move suppression coefficient λ and controller sampling time T_c. In Section 3, it is shown how to select these parameters.

Step 3 (determination of K^e parameter)

As shown in Fig. 1, the approximated controlled variable y is sampled every T_c seconds and its samples $g_i = g\,(iT_c)$ over the prediction horizon H_P are the entries of the system's dynamic matrix G:

$$G=\begin{bmatrix} g_3 & 0 & \cdots & 0\\ g_4 & g_3 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ g_{H_C+2} & g_{H_C+1} & \cdots & g_3\\ \vdots & \vdots & \ddots & \vdots\\ g_{H_P} & g_{H_P-1} & \cdots & g_{H_P-H_C+1}\end{bmatrix} \qquad (3)$$

In the presented case, we omit the first elements of the FOPDT response that are equal to zero, since they do not influence the controller output signal. Moreover, the resulting matrices have a reduced size. The number of the first elements $g_i = 0$ in the FOPDT response is equal to the window horizon H_w [3]:

$$H_w=floor\left(\frac{T_0}{T_c}+1\right) \qquad (4)$$

Then, calculate the matrix K:

$$K=K_0^{-1}\cdot G^T \qquad (5)$$

where:

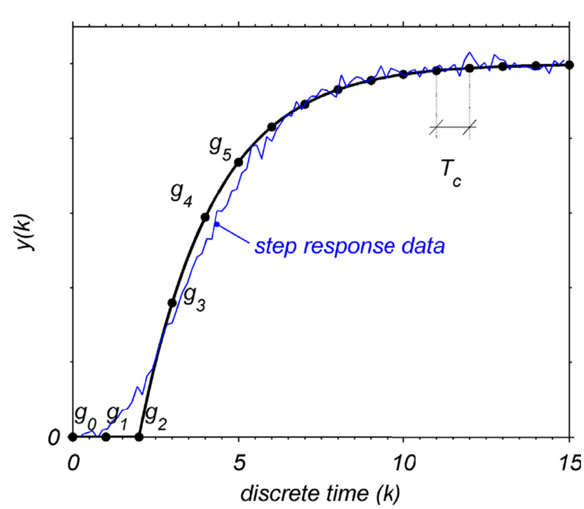

Fig. 1. Approximation of the step response data by the FOPDT model.

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$$K_0 = G^T G + \lambda I \tag{6}$$

and take the first row $K_1 = [K_{1,1}\ K_{1,2}\ \ldots\ K_{1,Hp-2}]$ of (5) to determine the parameter K^e:

$$K^e = \sum_{i=1}^{H_P-2} K_{1,i} \tag{7}$$

Step 4 (determination of K^U vector)

Taking into account the window horizon $H_w = 3$ (for the step response in Fig. 1), the remaining controller parameters can be obtained from matrix G^P, which is defined as follows:

$$G^P = \begin{bmatrix} g_4 - g_1 & g_5 - g_2 & \cdots & g_{HD+3} - g_{HD} \\ g_5 - g_1 & g_6 - g_2 & \cdots & g_{HD+4} - g_{HD} \\ \vdots & \vdots & \ddots & \vdots \\ g_{Hp+1} - g_1 & g_{Hp+2} - g_2 & \cdots & g_{HD+Hp} - g_{HD} \end{bmatrix} \tag{8}$$

The vector K^U is obtained by multiplying matrix (8) by the first row of matrix (5):

$$K^U = K_1 G^P \tag{9}$$

Hence, the optimal control increment Δu at time instant k is as follows [34]:

$$\Delta u(k) = K^e \cdot [y^{sp}(k) - y(k)] - \sum_{j=1}^{H_D} K_j^U \cdot \Delta u(k-j) \tag{10}$$

where the K_j^U parameters ($j = 1,2,\ldots,H_D$) are the elements of vector K^U. According to (10) the optimal control increments Δu must be stored in the H_D-element vector $\Delta\overline{u}$.

2.1. Control constraints and anti-windup mechanism

The analytical form of the control law equation (10) is obtained under the assumption that the controller output signal $u(k)$ is unconstrained. In fact, the control signal is always constrained and at time instant k:

$$u_{min} \le u(k) \le u_{max} \tag{11}$$

$$u(k) = u(k-1) + \Delta u(k) \tag{12}$$

where $u_{\min}$ and $u_{\max}$ are the lower and upper bounds, respectively. Hence, the control increments $\Delta u(k)$ in (10) must include the information on the constraints (11). In turn, to eliminate the saturation effect in the DMC algorithm, the control increment (10) must be compensated by a value of $\Delta u(k)$ that exceeds the lower or upper bounds (11), as shown in Tatjewski [34].

Although, the control increment (10) is determined by solving optimization problem (1), the control system performance is largely dependent on the controller parameters. Moreover, the size of vectors and matrices (5)–(9) and thus, computational complexity are strictly dependent on the length of controller horizons. Therefore, they must be carefully chosen, which can be done at the tuning stage.

3. The tuning procedure

The proposed tuning procedure is developed for a class of stable systems that can be approximated by the FOPDT model (2) at the specified operating point. Then, the controller parameters can be expressed by means of the parameter values in the FOPDT model. By reducing the length of controller horizons, it is possible to implement the DMC algorithm in low-cost industrial devices with low computational resources. Hence, for a given FOPDT model of the plant, the proposed controller parameters can be determined as follows.

As shown in Levine [35], the controller sampling time T_c can be selected according to various criteria, for example, based on rise time of the closed-loop system or for a known closed-loop bandwidth. However, in practice, a general rule of thumb is to select the controller sampling time T_c to be at least ten times smaller than the overall time constant T [19]:

$$T_c \le 0.1\ \mathrm{T} \tag{13}$$

The control horizon:

$$H_C = 2 \tag{14}$$

Further increase of this parameter does not significantly improve the control quality [36] and for $H_C = 1$, the control system may produce the oscillatory responses. The control horizon H_C has the influence on the size of matrix (6), which has H_C rows and H_C columns. Hence, assuming that $H_C = 2$, the matrix (6) can be easily inverted by using analytical expressions. There is no need to implement algorithms for computing the inverse matrix. According to the S-C tuning rules, the control horizon is an integer number between 1 and 6.

In turn, the prediction and dynamic horizons are usually determined as a sum of the overall time constants T and delay time T_0, divided by the controller sampling time T_c. According to the S-C rules, the prediction and dynamic horizons should be equal and determined for $5T + T_0$. For a step response of the plant, its output signal reaches about 99.3% of its final change after a time of $5T + T_0$. By using a similar formula, we propose that the prediction and dynamic horizons can be reduced without deteriorating the quality of closed-loop system.

Hence, the prediction horizon (rounded to the nearest integer) is defined as follows:

$$H_P = \frac{T}{T_c} + \frac{T_0}{T_c} \tag{15}$$

To obtain a smaller size of matrices (3) and (8), the prediction horizon should be as small as possible, but large enough to provide sufficient amount of the step response data to the cost function (1). Further increase of the H_P parameter results in a slower response of the control system [36] and unnecessarily increases the size of matrices that have to be stored in the controller memory. More significant influence on the settling time and stability of the closed-loop system has the move suppression coefficient λ [24].

The dynamic horizon (rounded to the nearest integer):

$$H_D = \frac{3T}{T_c} + \frac{T_0}{T_c} \tag{16}$$

For the systems described by the FOPDT model, the steady-state is obtained after about $(3\div5)\ T + T_0$, after a step change in the input signal. After a time $3T + T_0$, the output signal of the FOPDT model reaches 95% of the final change in the output signal. Hence, a sufficient information about the dynamical behavior of the plant can be obtained after a time $3T + T_0$. Further increase of the dynamic horizon H_D does not provide significant information about the plant dynamics, but results in a very large size of the matrix G^P and of the K^U vector that are necessary to calculate the optimal control increment $\Delta u(k)$.

The move suppression coefficient [37]:

$$\lambda = x \cdot k^2 H_P \tag{17}$$

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where $x \geq 0$ is an additional adjusting parameter. The greater the move suppression coefficient, the greater the penalty for the control increments Δu in the objective function (1). In [24] the authors have shown that for a plant described by (2) with uncertain parameters, the closed-loop system is stable provided that λ is sufficiently large. According to (17), the plant gain k has a decisive influence on λ. For $k \leq 1$, the controlled plant is less sensitive to control increments Δu, which means that the penalty factor (λ) can also be smaller. For $k > 1$, the penalty factor can be significantly greater, since it is proportional to the square of the plant gain k, as in the S-C tuning method. In turn, according to (15), the prediction horizon H_P contains the information about the delay time of the linear model (2) of the plant. Hence, greater delay times T_0 produce greater values of the penalty factor λ, resulting in more conservative closed-loop system responses. The inclusion of the delay term in λ can be especially effective for non-minimum phase or higher order plants as shown in results section. The additional parameter $x \geq 0$ allows for changing the control system behavior for a specified set of the controller parameters. For example, if the control system responses are too aggressive, the parameter x can be increased to produce responses without overshoots. As shown in Shridhar and Cooper [19] and Short [38], the parameter λ has also an influence on the condition number of matrix (6). Because in our case matrix (6) is a 2x2 matrix ($H_C = 2$), it is very easy to determine its condition number. By using (3) and (6) we have:

$$K_0 = G^T G + \lambda I_2 = \begin{bmatrix} \sum\limits_{i=3}^{H_P} g_i^2 + \lambda & \sum\limits_{i=4}^{H_P} g_i g_{i-1} \\ \sum\limits_{i=4}^{H_P} g_i g_{i-1} & \sum\limits_{i=3}^{H_P-1} g_i^2 + \lambda \end{bmatrix} \tag{18}$$

The matrix (18) is a symmetrical matrix, thus it has two real eigenvalues λ_1 and λ_2. Assuming that $|\lambda_2| \geq |\lambda_1| > 0$, the condition number of (18) can be determined as follows:

$$\kappa = \left|\frac{\lambda_2}{\lambda_1}\right| = \left|\frac{2\left(\sum\limits_{i=3}^{H_P-1} g_i^2 + \lambda\right) + g_{Hp}^2 + \sqrt{\Delta}}{2\left(\sum\limits_{i=3}^{H_P-1} g_i^2 + \lambda\right) + g_{Hp}^2 - \sqrt{\Delta}}\right| \tag{19}$$

where:

$$\Delta = g_{Hp}^4 + 4\left(\sum_{i=4}^{H_P} g_i g_{i-1}\right)^2 > 0 \tag{20}$$

Hence, the greater the parameter λ, the better-conditioned the matrix (6) is (κ tends to unity). In specific cases, the condition number of matrix (6) can be improved by increasing the parameter x. Shridhar and Cooper [19] determined the formula for λ based on the assumption that the condition number of matrix (6) should be equal to 500, which, in fact, may be higher as shown in Short [38]. Assuming that the controller sampling time is equal to $T_c = 0.1\ T$, it is possible to determine a minimum value of the parameter x, for which the condition number of (6) is not greater than proposed in the S-C rules:

$$x_{\min} = \frac{73}{5000} \cdot \frac{1}{1 + \frac{T_0}{T}} = \frac{0.0146}{1 + \frac{T_0}{T}} \leq x \tag{21}$$

As a result, the greater the parameter x, the better-conditioned the matrix (6) is and the more conservative step responses of the control system are. In the next section, we show that the parameter λ has a significant influence on the quality of control and for $x = x_{\min}$ the differences between the closed-loop responses obtained for the proposed and S-C tuning rules are negligible. Finally, the main differences between the tuning rules are presented in Table 1.

By using the proposed rules, it is possible to significantly reduce the size of matrices that are used in calculations of the DMC parameters for a specified operating point. Generally, the size of a float variable in PLC units is equal to 4 bytes. Because the controller matrices and vectors (e.g., G^P, K^u, K^e) contain the float variables, their size in the memory is strictly dependent on their dimensions. In turn, dimensions of matrices and vectors that are necessary to compute the controller parameters for a given operating point or the optimal control increment, are strictly dependent on the controller horizons. Table 2 shows the difference in the number of elements of matrices and vectors between the S-C and proposed tuning rules, assuming that the controller sampling time $T_c = 0.1\ T$ and the control horizon $H_C = 2$. The greatest influence has the dynamic horizon H_D, hence its reduction is the most preferable. For example, when the delay time in the FOPTD model is equal to zero ($T_0 = 0$), then the difference between the number of elements in G^P matrix for the S-C and proposed tuning parameters is equal to 2301 elements. This, in turn, is equal to (2301 elements) x (4 bytes) = 9.204 kB, which is the minimum amount of the saved memory, when using the proposed tuning rules in comparison to the S-C method. The amount of the saved memory can be substantially greater, because it is dependent on the ratio of delay time to the overall time constant (T_0/T) of the FOPDT model as shown in Fig. 2 and in Table 2.

In the case of numerical minimization of the objective function (1) at each sampling time T_c, the reduction of controller horizons may also lead to a lower complexity and memory requirements. The optimal control increment can be found by solving the quadratic programming task with linear constraints by using Hildreth's algorithm [39]. For m manipulating variables and longer control horizons, the Hessian matrix of the quadratic programming problem is a $mH_C\, x\, mH_C$ matrix and, as shown in Huyck et al. [7], the maximum size of Hessian matrix is limited by the size of data block associated with its function block in the PLC. Since in the proposed tuning strategy the control horizon is $H_C = 2$, the Hessian matrix for SISO plant ($m = 1$) is a 2x2 matrix and can be easily processed and stored in the controller memory. In the case of multi-parametric MPC, the number of regions determined by active and/or inactive constraints may grow exponentially as H_P increases [13]. Hence, the reduction of H_P may simplify the implementation of controller.

The next section presents the effectiveness of the proposed tuning strategy and the comparable result for the S-C method.

4. Results

The DMC algorithm with the proposed tuning parameters has been tested for benchmark plants (Table 3), defined in Shridhar and Cooper [19] and for two real laboratory plants, coded in more and less powerful PLC units. To compare and evaluate the effectiveness of the tuning rules, the following performance indices were calculated: ISE (Integral Square Error), ITAE (Integral of Time multiplied by the Absolute Error), absolute value of the maximum overshoot (or undershoot) M_p and the difference in required memory in kB for the matrices in Table 2. The indices ISE and ITAE were normalized to 0–1 by dividing each index by its maximum value obtained for the worst case.

4.1. Benchmark simulated plants

The benchmark plants were simulated based on the Unified Dynamics approach [40], which means that the state equations of

Table 1
The proposed and S-C tuning rules.

Proposed tuning rules	S-C tuning rules [19]
$T_c \le 0.1\,T$	$T_c \le 0.1\,T$
$H_C = 2$	$H_C = \{1, 2, \ldots, 6\}$
$H_P = \frac{T}{T_c} + \frac{T_0}{T_c}$	$H_P = \frac{5T}{T_c} + \frac{T_0}{T_c} + 1$
$H_D = \frac{3T}{T_c} + \frac{T_0}{T_c}$	$H_D = \frac{5T}{T_c} + \frac{T_0}{T_c} + 1$
$\lambda = x \cdot k^2 H_P$	$\lambda = f \cdot k^2$
$x_{\min} = \frac{0.0146}{1+\frac{T_0}{T}} \le x$	$f = \begin{cases} 0, & H_C = 1 \\ \frac{H_C}{500}\left(\frac{3.5T}{T_c} + 2 - \frac{H_c - 1}{2}\right), & H_C > 1 \end{cases}$

Table 2
A comparison between the number of elements in the controller matrices for the S-C and proposed tuning rules, for $T_c = 0.1\,T$ and $H_C = 2$.

The size of vectors and matrices	Differences in the number of elements of the matrices and vectors obtained for the S-C and proposed (reduced) tuning parameters
$\dim G = (H_P\text{-}H_w + 1) \times H_C$	$\dim G_{SC}\text{-}\dim G_{red.} = 82$
$\dim G^P = (H_P\text{-}H_w + 1) \times H_D$	$\dim G^P{}_{SC}\text{-}\dim G^P{}_{red.} = 2301 + 410\frac{T_0}{T}$
$\dim K^U = 1 \times H_D$	$\dim K^U{}_{SC}\text{-}\dim K^U{}_{red.} = 21$
$\dim K^e = 1 \times 1$	$\dim K^e{}_{SC}\text{-}\dim K^e{}_{red.} = 0$
$\dim K = H_C \times (H_P\text{-}H_w + 1)$	$\dim K_{SC}\text{-}\dim K_{red.} = 82$
$\dim K_0 = H_C \times H_C$	$\dim K_{0,SC}\text{-}\dim K_{0,red.} = 0$
$\dim \Delta\overline{u} = 1 \times H_D$	$\dim \Delta\overline{u}_{SC} - \dim \Delta\overline{u}_{red.} = 21$

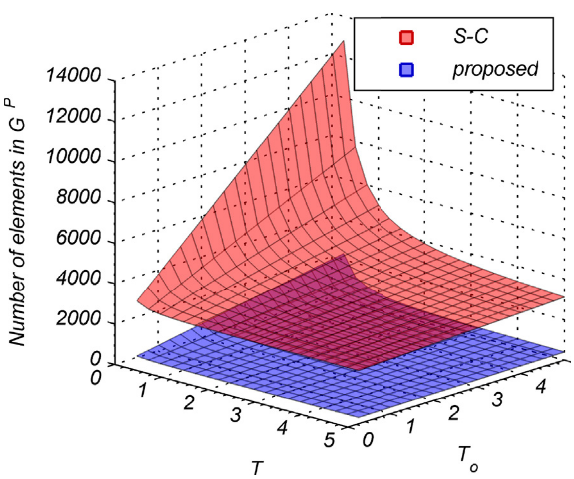

Fig. 2. The number of elements in G^P matrices for the proposed and S-C tuning rules as a function of T and T_0 parameters in the FOPDT model, and for the controller sampling time $T_c = 0.1\,T$. The G^P matrix in the S-C method contains at least 2301 more elements than the G^P matrix in the proposed tuning procedure.

Table 3
Benchmark plants and their FOPDT models.

Plant 1	Plant 2
$K_1(s) = \frac{(1-50s)e^{-10s}}{(100s+1)^2}$	$K_2(s) = \frac{e^{-10s}}{(50s+1)^4}$
$K_{1A}(s) = \frac{e^{-107.7s}}{154.1s+1}$	$K_{2A}(s) = \frac{e^{-101.7s}}{116.8s+1}$
$T_c = 15$ [s], $x_{\min} = 0.0086$	$T_c = 12$ [s], $x_{\min} = 0.0078$

the simulated plant were numerically solved with a small step time h to ensure numerical stability and accuracy. In turn, the controller output signal was determined every T_c seconds and the relation between h and T_c is as follows [40]:

$$h = \frac{T_c}{NIS} \tag{22}$$

where $NIS \in \mathbf{N}$ is the number of integration steps within the controller sampling time T_c. To tune the DMC algorithm, the controlled plants were approximated by the FOPDT model based on the least square method.

The simulation experiments were carried out for a step change in set point from 0.0 to 1.0 at $t = 0$ and in the presence of disturbances, i.e., by adding 0.1 to the plant output signal at $t = 1000$ [s]. The performance indices IAE, ITAE and maximum overshoots M_p were determined only for set point changes, for $t \in [0,1000]$ seconds.

For each benchmark plant, the controller was tuned according to the S-C and proposed rules for $x = 1.0$, $x = 0.1$, and by using (21), for $x = x_{\min}$ (Table 3). The results show that a decisive influence on the control system responses has the move suppression coefficient λ. For the proposed tuning method, an increase of the additional tuning parameter x results in longer settling times and slightly larger overshoots, but the smallest overshoots were obtained for $x = 0.1$ (Table 4). A decrease of the parameter x results in a more aggressive behavior of the control system and for $x = x_{\min}$, the step responses were very similar to the responses obtained for the S-C tuning parameters (Fig. 3).

As can be noticed, for the control system tuned according to the proposed rules, there is a small deviation from the set point in the process variable, which is caused by a change in the controller output signal, for example, at time instant $t = 678$ [s] for plant 1 or at $t = 558$ [s] for plant 2 (Fig. 3). This is a result of a smaller dynamic horizon in comparison to the H_D parameter proposed by Shridhar and Cooper [19]. In effect, the length of the vector K^U and of the vector $\Delta\overline{u}$ containing past control increments is also smaller. One of the greatest control increments Δu appears at the beginning of the step change in the set point value. According to (10), the optimal control increment $\Delta u(k)$ at time instant k is a weighted sum of past control increments Δu (k-j) and the K_j^U ($j = 1,2,\ldots,H_D$) parameters are the weighting factors. Moreover, from (8) one can notice that the absolute values of K_j^U parameters tend to zero as the index j increases. Hence, the greatest control increment Δu (k-j) is removed from the vector $\Delta\overline{u}$ after a time $H_D \cdot T_c$, which may lead to a noticeable change in the controller output signal (Fig. 3). The increase of the dynamic horizon H_D to the value ($H_D = 60$) proposed by Shridhar and Cooper [19] eliminates the fluctuations in the control variable as shown in Fig. 4b for plant 1, but significantly increases the size of the matrix G^P (Table 2). The effect of the reduced dynamic horizon ($H_{D,red.} = 38$) appears after about $H_{D,red.} \cdot T_c + T_o = 678$ s from the step change in set point (Fig. 4a).

In turn, an increase of the prediction horizon H_P in the proposed tuning method led to slower closed-loop system responses without overshoots.

The results obtained for benchmark plants show that the move suppression coefficient λ must be carefully chosen and the delay time of the FOPDT model must be taken into account. Since plant 1 is the non-minimum phase system and plant 2 is the fourth order system with relative degree equal to 4, the delay time T_0 in the FOPDT model is comparable to the overall time constant T. In effect, the FOPDT approximation is less accurate and the S-C parameters produce more aggressive responses with apparent oscillations in the controller output signal in comparison to the proposed tuning strategy for $x = 1.0$ (Fig. 3). It should be noted that the formula for the move suppression coefficient λ in the S-C tuning strategy does not include the delay time T_0. In the proposed tuning method, the parameter λ is dependent on the prediction horizon H_P, and thus on the delay time T_0. For example, for plant 1 and $x = 1.0$ we have $\lambda = 17.0$, while the S-C method gives $\lambda = 0.1498$. Hence, for a larger delay time in the

Table 4
Performance indices for benchmark plants.

Tuning method	Plant 1				Plant 2			
	$\frac{ISE}{ISE_{max}}$	$\frac{ITAE}{ITAE_{max}}$	M_p	Reduction in the memory consumption for the proposed tuning rules	$\frac{ISE}{ISE_{max}}$	$\frac{ITAE}{ITAE_{max}}$	M_p	Reduction in the memory consumption for the proposed tuning rules
Proposed ($x = 1.0$)	1.0	1.0	0.090	12.064 kB	1.0	1.0	0.089	10.880 kB
Proposed ($x = 0.1$)	0.74	0.51	0.055		0.73	0.58	0.081	
Proposed ($x = x_{min}$)	0.66	0.46	0.092		0.64	0.50	0.095	
S-C	0.65	0.41	0.104		0.63	0.54	0.100	

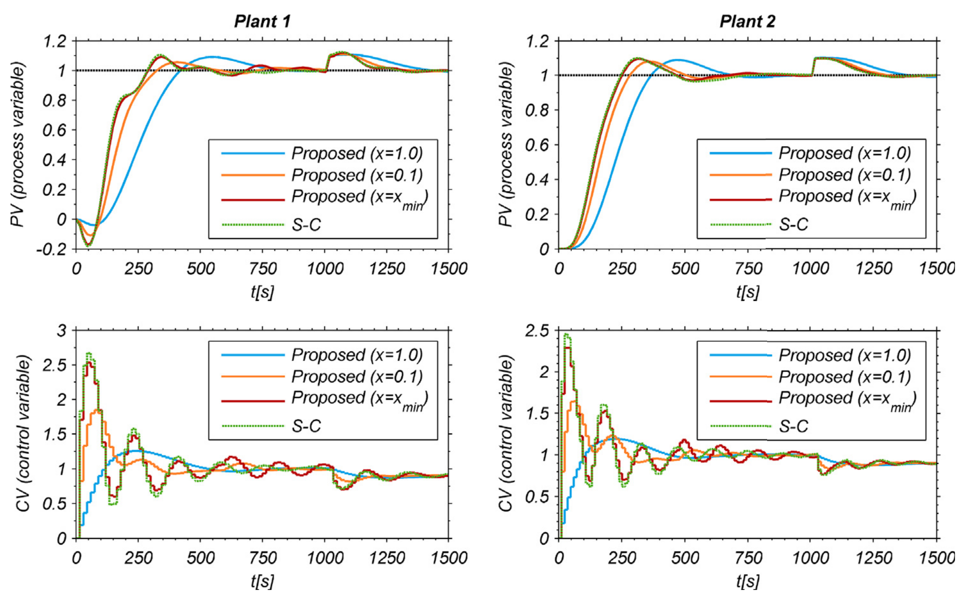

Fig. 3. Time courses of the process variable (PV) and the corresponding control variable (CV) in the control system with the proposed and S-C tuning parameters for benchmark plants.

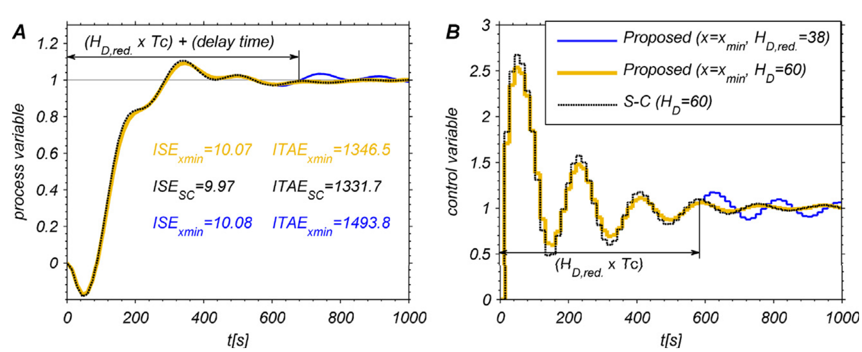

Fig. 4. Step responses of the closed-loop systems with the corresponding controller outputs for the reduced ($H_{D,red.} = 38$) and full length of dynamic horizons ($H_D = 60$) in plant 1.

FOPDT model, the S-C tuning strategy may result in a more oscillatory behavior of the control system, especially when the FOPDT model is less accurate. This justifies that the move suppression coefficient λ has to be dependent on the delay time T_0 and, for larger delays, the DMC control algorithm should be tuned with $x \geq 0.1$. In turn, in the presence of disturbances, the best results were obtained for the S-C and proposed tuning parameters with $x = x_{min}$ (Fig. 3).

In each case, the reduction in the length of controller horizons resulted in a substantial decrease of required memory in comparison to the S-C tuning method, by at least 10 kB. At the same time, the quality of the control system was not significantly deteriorated as shown in Table 4 for $x = x_{min}$.

4.2. Laboratory plant I - pneumatic system

The proposed tuning parameters were tested for the pneumatic system with the structure shown in Fig. 5. The steel tanks of the following volumes $V_1 = 5.0$ [L], $V_2 = 2.0$ [L] and $V_3 = 0.75$ [L] are connected in series and the manual valves in the output (that can be automatically switched) are to create a different pressure loss in the tanks.

Hence, pressure changes due to switching between the manual valves in the output are treated as a disturbance to the system. The

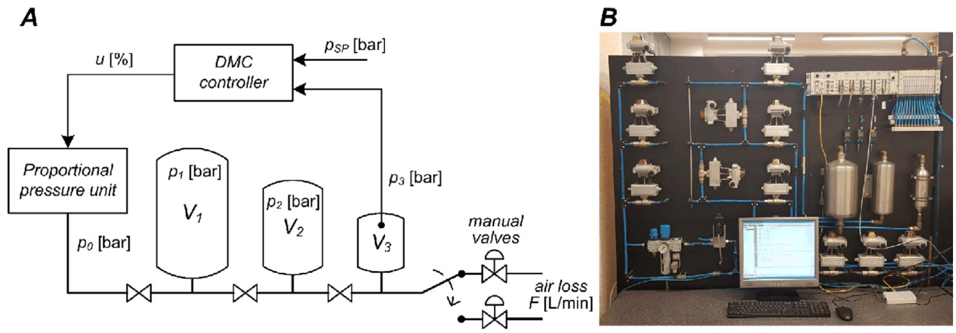

Fig. 5. Pressure control system: a) simplified scheme; b) real laboratory plant.

process variable is the pressure p_3 inside the third tank and the supply pressure p_0 is changed by the DMC algorithm using the proportional pressure unit (MPPES Festo) in the range 0–6 [bar]. The control algorithm has been implemented in the CPX-CEC Festo controller with 4 MB for user program and with Ethernet interface module for communication. The program code was created in CoDeSys v2.3 by using ST language and COPA-DATA Zenon 7 was used to create the application for visualization, data acquisition, and for controlling the on-off valves with electro-pneumatic actuators (VZBA Festo) between the tanks (Fig. 5a). The information on the valve opening is provided by sensor boxes on top of each of the on-off valves and can be used for visualization purposes. The additional on-off valves (Fig. 5b) allow for switching between the manual valves in the output (Fig. 5a) and for modification of the system structure, for example, one can study the system with one, two or three tanks connected in series. In the presented case, the dynamical behavior of the plant is similar to a third order system with non-oscillatory responses and can be described by the FOPDT model at the chosen operating point. The linear model was fitted to the step response at 1.5 [bar] for a step change in p_0 by 1.0 [bar]:

$$K_{N1}(s) = \frac{0.7e^{-2.92s}}{10.32s+1},\ T_c = 1.0[\mathrm{s}] \tag{23}$$

The parameter values in the linear model (23) were determined by using the least square method and the minimum value of the x parameter ($x_{\min} = 0.0114$) was determined by (21). To evaluate the control system performance, the ISE indices were calculated over the whole time horizon and the ITAE indices were calculated separately in five time intervals corresponding to the three set point changes and the switching between two valves at the output at time instants $t = 275$ [s] and $t = 350$ [s], as shown in Fig. 6a. Then, the ITAE is a sum of ITAE indices obtained for each time interval and divided by the maximum sum of ITAE indices obtained for the worst case (Table 5).

It can be easily noticed that the control system with the proposed tuning parameters for $x = x_{\min}$ results in very similar responses as in the control system tuned according to the S-C method (Fig. 6). The first differences between the control signals appear after about 34 s from a set point change, which corresponds to a shorter dynamic horizon in the proposed tuning parameters ($H_D = 34$ in comparison to $H_D = 55$ for the S-C strategy). Further increase of the x parameter results in less aggressive responses. However, the best rejection of disturbances (after switching between the manual valves in the output) was achieved for the S-C and proposed parameters with $x \in [x_{\min}, 0.1]$. By using the proposed parameters, it was possible to save about 11.46 kB of memory (Table 5), which is almost equal to the amount of memory required to store the matrices from Table 2 when using the S-C tuning method. Since the matrices in Table 2 contain float variables, the S-C strategy take up 13.428 kB of the memory in comparison to 1.964 kB for the proposed tuning rules.

Table 5
Performance indices for $K_{N1}(s)$.

Tuning method	$\frac{ISE}{ISE_{max}}$	$\frac{ITAE}{ITAE_{max}}$	Reduction in the memory consumption for the proposed tuning rules
Proposed ($x = 1.0$)	1.0	1.0	11.464 kB
Proposed ($x = 0.1$)	0.7844	0.7584	
Proposed ($x = x_{\min}$)	0.7421	0.7226	
S-C	0.7212	0.6437	

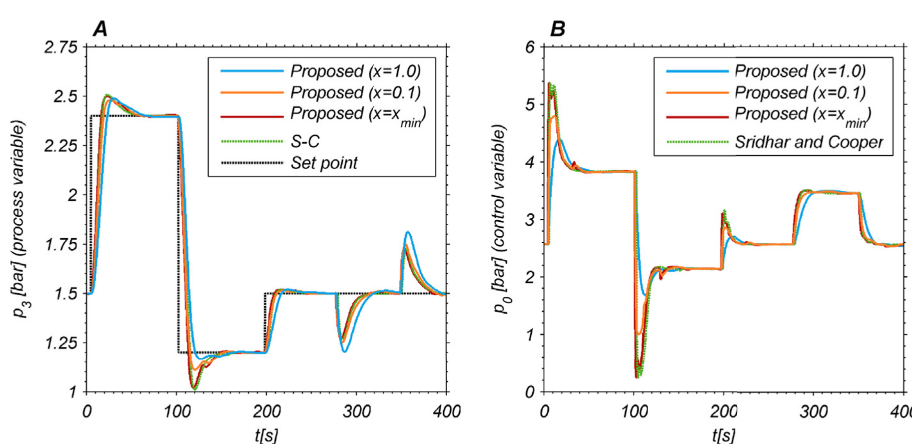

Fig. 6. The dynamical behavior of the control system: a) step responses to step changes in set point pressure; b) the corresponding control pressure p_0 (control variable).

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4.3. Laboratory plant II - heating plant (electric flow heater)

The controlled plant is the electric flow heater (a cylindrical container, filled with water of constant volume $V = 0.25$ [L]). The water of inlet temperature T_{in} is heated to the temperature T_{out} inside the flow heater and the temperatures are measured by PT100 sensors. The flow rate F is kept constant by the local PI controller and control valve with electro-pneumatic positioner (Fig. 7a). The heating power P_h of the electric heater is changed by the DMC controller using the PWM signal with a period of 10 s. As a result, the average heating power can be changed from 0 to $P_{hmax} = 5000$ [W], simply by changing the duty cycle of the PWM signal, which, in turn, is proportional to the controller output signal u. The control algorithm has been implemented in the PLC Simatic S7-300, 314C 2PN/DP that offers 192 kB of work memory for a program code, but the maximum size of each data block (DB) associated with its function block (FB) is only 64 kB. The program code has been created in Simatic Step 7 v5.5 Professional environment by using Instruction List (IL) language, since the generated code takes up less memory. When preparing a code in other language (e.g., LAD - ladder diagram), the Step7 software translates it into IL language before sending it to the PLC unit, thus generating redundant and useless code. In turn, the additional application for visualization and data acquisition was created in WinCC Flexible 2008. The control objective is to maintain the output temperature T_{out} at the desired level T_{SP} by changing the heating power P_h (Fig. 7). Any changes in the flow rate F are treated as a disturbance to the control system and, as shown in Klopot and Skupin [33], the FOPDT parameters may significantly vary depending on the flow rate F in comparison to changes in the heating power.

The flow heater is an example of a higher order plant with a time delay resulting from measuring the output temperature T_{out} at the end of a long pipe (Fig. 7). To tune the DMC algorithm, the FOPDT model was fitted to step response data collected for $T_{out} = 35$ [°C] at a constant flow rate $F = 1.0$ [L/min] and for a step change (5%) in the duty cycle of the PWM signal (i.e., a step change in the average heating power P_h):

$$K_{N2}(s) = \frac{0.68e^{-17.76s}}{37.65s + 1}, \; T_c = 3.7[\mathrm{s}] \tag{24}$$

The parameter values in the linear model (24) were determined by using the least square method and the minimum value of the x parameter ($x_{\min} = 0.0099$) was determined by (21). All the experimental data was collected for similar ambient temperature $T_{amb} \approx 19$ [°C] and for similar inlet water temperatures $T_{in} \approx 18$ [°C] within 2.5 h. The average temperature differences in T_{in} and T_{amb} were less than 0.5 [°C] during the experiments. Table 6 presents the performance indices obtained for the experimental data and calculated in the same way as for the pneumatic plant. The control system was also tested in the presence of disturbances, i.e., step changes in the flow rate F from 1.0 to 1.3 [L/min] and back from 1.3 to 1.0 [L/min]. To obtain step changes in the flow rate F, a constant bias was added to the local PI controller output (Fig. 7a) at $t = 800$ [s] and then subtracted at $t = 1050$ [s]. The average flow rate for all the experiments is represented by a dashed line in Fig. 8a.

As shown in Table 6 and in Fig. 8, the behavior of the control system is similar to the behavior observed for the simulated plants. The smallest overshoots were obtained for the proposed tuning parameters with $x = 0.1$. The output temperature tends to the set point value with a rate dependent on the parameter x (Fig. 8). In the presence of disturbances (step changes in the flow rate F) the best performance was obtained for a more aggressive control, i.e., for $x = x_{\min}$. As in the case of pneumatic system or benchmark plants, the closed-loop responses for $x = x_{\min}$ are very similar to the responses obtained for the S-C parameters (Fig. 8).

Moreover, the matrices and vectors for the proposed method require less memory (about 2.016 kB) in comparison to the S-C strategy (Table 6). Furthermore, by using the proposed rules, the controller parameters (K^e and K^u) could be calculated by the PLC unit twice as fast as in the S-C method. However, irrespective of the tuning method, the computation time was insignificant in comparison to the controller sampling time.

Table 6
Performance indices for $K_{N2}(s)$.

Tuning method	$\frac{\mathrm{ISE}}{\mathrm{ISE}_{\max}}$	$\frac{\mathrm{ITAE}}{\mathrm{ITAE}_{\max}}$	Reduction in the memory consumption for the proposed tuning rules
Proposed ($x = 1.0$)	1.0	1.0	11.392 kB
Proposed ($x = 0.1$)	0.7198	0.7042	
Proposed ($x = x_{\min}$)	0.6439	0.6759	
S-C	0.6611	0.7050	

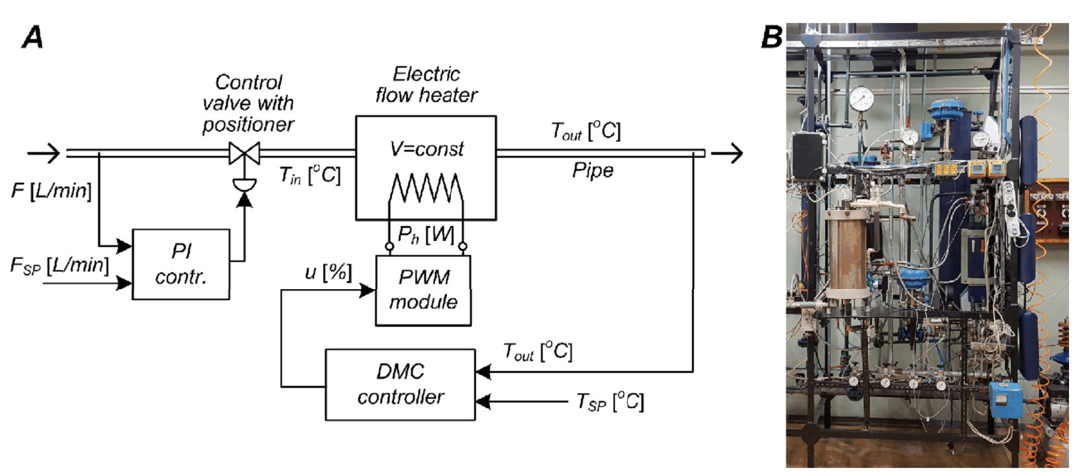

Fig. 7. Temperature control system: a) simplified scheme; b) real laboratory plant.

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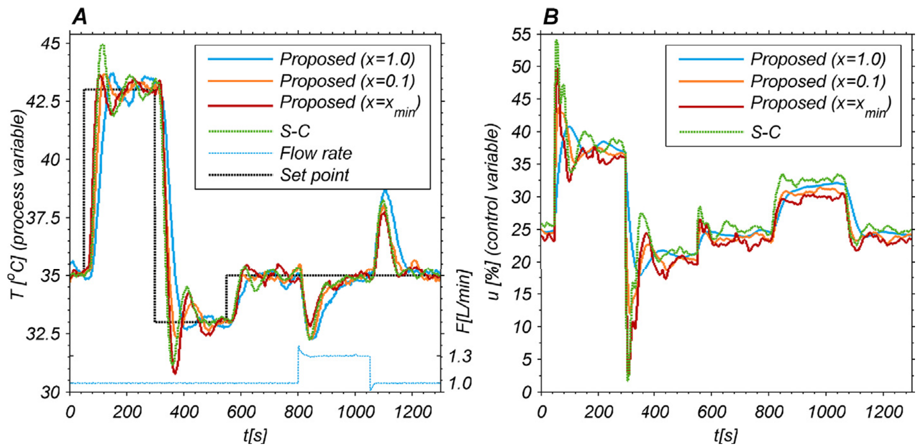

Fig. 8. The dynamical behavior of the control system: a) step responses to step changes in set point temperature and flow rate *F* (disturbance); b) the corresponding controller output signal *u*.

5. Concluding remarks

The proposed tuning method for the DMC algorithm has been tested with real and simulated plants, and compared with the results obtained for the S-C tuning strategy. By using the proposed method, it was possible to reduce the memory consumption in the controller device by at least 10 kB and thus to implement the DMC algorithm in local control loops, in devices with lower computational resources. The consumption of memory for the controller matrices (e.g., G, G^P, K^u) was several times smaller for the proposed tuning strategy in comparison the S-C method. The additional parameter $x \geq x_{\min}$ ensures that the matrix (6) is well-conditioned and simplifies the controller tuning procedure. For $x = x_{\min}$, the behavior of the control system is very similar to the behavior obtained for the S-C method, while at the same time, the length of prediction and dynamic horizons are substantially smaller. The simulations and real experiments show that the smallest overshoots could be obtained for the proposed tuning method with $x = 0.1$. For higher values of the parameter x, the control system responses are more conservative with a longer settling time. Moreover, the proposed formula for the move suppression coefficient λ includes the prediction horizon H_P, thus making it dependent on the delay time T_0 of the FOPDT model. In effect, the proposed tuning rules led to a more conservative behavior of the control system than the S-C method, especially for plants with large delay time T_0 in their FOPDT models. Finally, the reduction of controller horizons may simplify the implementation of quadratic programming solvers and multi-parametric predictive controllers. This possibility will be investigated in future work.

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