Performance Evaluation of a Cognitive Radio Network with Exponential and Truncated Usage Models

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***Abstract*—In this paper, we study the forced termination probability, blocking probability and throughput of secondary users with practical usage models, which include fixed, truncated exponential, truncated lognormal, truncated Pareto distributions. Moreover, we compare the results of these usage models with the well known exponential distribution model. We consider two system scenarios with/without spectrum handoff. For all distributions, introduction of spectrum handoff provides significant gains in forced termination probability and throughput. We found that the behaviour of all the truncated distributions is similar to exponential distribution. The results show that among all the distributions models, the secondary user connections with fixed length has the largest aggregate throughput, whereas the truncated Pareto has the smallest aggregate throughput. In general, as the mean duration of secondary user connections decreases, the throughput increases.**

***Index Terms*—Cognitive radio, forced termination probability, spectrum handoff, aggregate throughput**

I. Introduction

In recent years, there is a shortage of vacant (unlicensed) spectrum as a consequence of increased demand and rigid allocation policies. On the other hand, experimental studies [1] [2] have shown that the major portion of the licensed (primary) bands (channels) are under utilized. To ensure future growth of wireless services, it is necessary to increase the usage of these channels.

Cognitive radio [3] has emerged as a promising technology aimed to increase the efficiency of largely under utilized spectrum [4]. Specifically, cognitive radio enables unlicensed users in a secondary network to opportunistically build their transmission links in the vacant primary channels. However, when primary users reappear, the secondary users must immediately vacate these channels. Without additional means to continue in other vacant channels, these secondary user sessions are terminated prematurely. For the secondary network, forced termination is undesirable because it degrades quality of service (QoS) and throughput in the network. In order to reduce the adverse impacts, spectrum handoff is often used to allow the secondary users to move to other vacant primary channels. To ensure the availability of vacant channels, a number of vacant channels can be reserved at a cost of a small increase of network blocking probability. This technique is commonly referred as channel reservation.

In the analysis of a cognitive radio network, the exponential usage model is often assumed for both primary and secondary users because of its inherent Markov property [5] [6] [7]. For example, in [6], the blocking and forced termination probabilities and network throughput are analyzed using a continuous time Markov model. Independent exponential models are assumed for both primary and secondary data traffic streams. A more complete analysis and with minor extensions on the same topic is shown separately in [5] and [7]. For a general distribution usage model, the authors in [8] proposed random access schemes for the secondary users. The maximum achievable capacity of each of the schemes is calculated while concentrating on exponential and fixed packet lengths. Although exponential model is mathematically more tractable, it is less practical in real world applications. Field measurements [9] [10] [11] have shown that the duration of user connections is always finite and bounded. To capture this effect, truncated distributions are often used to model traffic in existing wireless networks [12] [13].

In this paper, using the practical traffic models with truncated distributions [14], we take another look at the blocking and forced termination probabilities and network throughput of a secondary network overlaying a primary network. The results are compared with the analysis results of ideal exponential models in the previous work [5] [7]. Two system setups are considered with/without spectrum handoff. In the first scenario, an arrival of a primary user results in immediate termination of the secondary users on the reclaimed channel. In the second scenario spectrum handoff allows the secondary users on the reclaimed channel by the primary user to move to other vacant channels.

The rest of the paper is organized as follows. In Section II we introduce the system model and key assumptions made throughout the paper. The brief overview of commonly used truncated distributions is provided in Section III. The mathematical expressions with baseline exponential model are summarized in Section IV. Numerical results are provided in Section V. Finally, the main findings are outlined in Section VI.

II. System Model

Fig. 1 shows the opportunistic spectrum access model assumed in this paper. We consider a primary network in which there are K available channels, each of which has a fixed bandwidth W_p. Each primary channel is subdivided into N subchannels with a bandwidth of $W_s = W_p/N$. There is a single group of secondary users S_A oppportunistically accessing the primary subchannels. Furthermore, each primary and secondary connection has the fixed bandwidth requirement W_p and W_s respectively. The system state is defined by integer pair (i, k) where i and k are the number of active primary and secondary user connections respectively. We assume that the arrivals of new connections from each of the primary and secondary user groups follow an independent Poisson process denoted by λ_P, λ_S respectively [15]. The service duration of primary connections is exponentially distributed with a mean rate μ_P [11]. We further assume that the service duration of secondary user connections follow fixed (FX), ideal exponential (IEX), truncated exponential (TEX), truncated pareto (TPA) and truncated lognormal (TLN) distribution with an average length m_S.

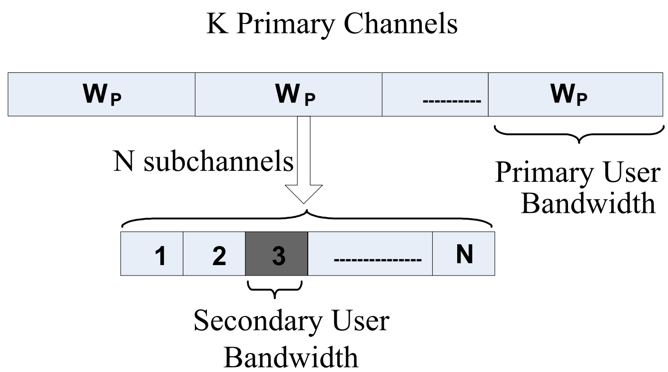

Fig. 1. Channel arrangements of primary and secondary user channels.

III. Traffic Distributions

In this section, we provide an overview of statistical distributions which are commonly used to characterize the service duration/length of incoming connections. It includes the TEX [14], TPA [16] and TLN [17] distributions. Normally, if $f(x)$ and $F(X)$ are the probability density function (PDF) and cumulative density function (CDF) of a random variable X respectively, then the corresponding truncated PDF of X with in interval (a, b) is given as [14]

$$f_X^{(a,b)}(x) = \begin{cases} \dfrac{f(x)}{F(b) - F(a)} & a < x \leq b \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Note that (1) denotes a valid PDF since the total area under $f_X^{(a,b)}(x)$ in the range of (a, b) is normalized to unity. Fig. 2 shows the ideal lognormal, Pareto and exponential distributions along with their truncated counterparts having a same mean value $m_S = 1.4285$ and truncation limits $(0.199, 8)$

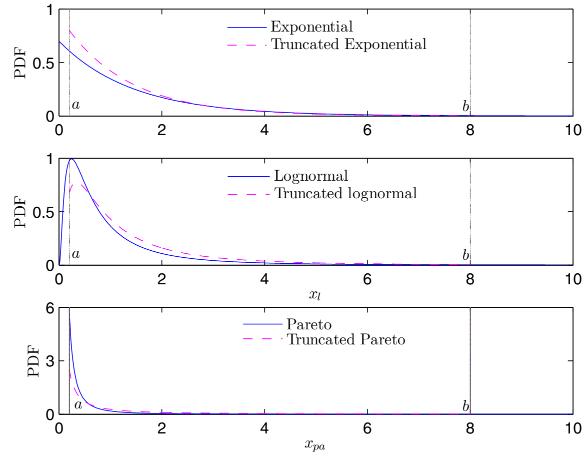

Fig. 2. Truncated and non-truncated probability distribution functions, Mean service duration $m_S = 1.4286$, lower truncation limit $a = 0.199$, upper truncation limit $b = 8$

A. Truncated Exponential Distribution

Exponential distribution is the most commonly used traffic model in theoretical analysis. However, truncated exponential distribution is more appropriate in modeling real world data and voice traffic in wired or wireless networks [18] [19] [20]. Using (1), truncated distribution $f_{X_E}^{(a,b)}(x_e)$ of exponential random variable X_E with a mean rate λ can be written as

$$f_{X_E}^{(a,b)}(x_e) = \frac{\lambda e^{-\lambda x_e}}{e^{-\lambda a} - e^{-\lambda b}} \quad a < x_e \leq b \quad (2)$$

B. Truncated Lognormal Distribution

Lognormal distribution belongs to a family of normal distribution and is frequently used to model the duration of voice calls in cellular networks [11]. On the other hand, truncated lognormal distribution often arise in packet based networks such as internet traffic [12] [19], video telephony [21]. Mathematically, truncated distribution of lognormal random variable X_L can be calculated using (1)

$$f_{X_L}^{(a,b)}(x_l) = \frac{e^{-\dfrac{(\ln(x_l) - \mu_{X_L})^2}{2\sigma_{X_L}^2}}}{x_l \sigma \sqrt{2\pi}(F_{X_L}(b) - F_{X_L}(a))} \quad a < x_l \leq b \quad (3)$$

where μ_{X_L} and σ_{X_L} is the mean and standard deviation of normally distributed random variable and F_{X_L} is the CDF of X_L [17].

C. Truncated Pareto Distribution

Pareto distribution is a power law distribution and has many applications in communication networks. Pareto distribution is often used to generate self similar traffic with long range dependence [22]. However, due to the fact that not all of its higher order moments are finite [16], truncated Pareto distribution is often used to study the packet size behaviour in steaming video [12], wireless internet [23] and 3G networks

[24]. Let X_{PA} be a Pareto distributed random variable, then its truncated density function can be written as

$$f_{X_{PA}}^{(c,b)}(x_{pa}) = \frac{ka^k x_{pa}^{-(k+1)}}{1 - \left(\dfrac{a}{b}\right)^k} \quad a \le x_{pa} \le b \tag{4}$$

where k is the shape and a is the threshold (minimum possible value of X_{PA}) parameter.

IV. Theoretical Analysis- Exponential Distribution

In this section, we briefly summarize the mathematical expressions for forced termination probability, blocking probability and throughput of secondary users with IEX service duration. Note that only final expressions are included and a complete mathematical analysis can be found in [5]

A. Basic System

In a basic system, no spectrum handoff is allowed. All the secondary user connections on the reclaimed channel by a primary user will be prematurely terminated, regardless vacant subchannels might be still available in other primary channels. Using continuous time Markov model, we first calculate the probabilities $P_\phi^b(i,k)$ for all possible states $\phi(i,k)$. Given the state probabilities, it is shown that the blocking probability of secondary users connection is given by

$$P_B^b = \sum_{i=0}^{NK}\sum_{k=0}^{K} \delta(i + Nk - NK) P_\phi^b(i,k) \tag{5}$$

and the forced termination probability given by

$$P_F^b = \frac{1}{(1 - P_B^b)} \sum_{i=0}^{NK}\sum_{k=0}^{K}\sum_{l=1}^{N} \frac{l}{\lambda_S} \gamma_{(i-l,k+1)}^{(i,k)} P_\phi^b(i,k) \tag{6}$$

where $\gamma_{(i-l,k+1)}^{(i,k)}$ [6][Eq. (1)] follows a hypergeometric distribution and is defined as the transition rate from state (i,k) to $(i-l,k+1)$ state on the arrival of a new primary connection. The integer l is the number of secondary connections on the reclaimed primary channel.

B. Basic System With Spectrum Handoff

As indicated previously, spectrum handoff allows the secondary connections on the reclaimed channel to move to another vacant subchannels. As such, the forced termination probability of an average secondary user is expected to be reduced, comparing to the basic system. Let P_B^h and P_F^h be the blocking and forced termination probabilities of the basic system with spectrum handoff respectively. It is shown that P_B^h and P_F^h are given by

$$P_B^h = \sum_{i=0}^{NK}\sum_{k=0}^{K} \delta(i + Nk - NK) P_\phi^h(i,k) \tag{7}$$

and,

$$P_F^h = \frac{1}{(1 - P_B^h)} \sum_{i=0}^{NK}\sum_{k=0}^{K}\sum_{l=1}^{N} \frac{l}{\lambda_S} \lambda_P P_\phi^h(i,k) \delta_f \tag{8}$$

where $\delta_f = \delta(i - (K - k - 1)N - l)\delta(i + Nk > (K - 1)N)$ and $P_\phi^h(i,k)$ is state probabilities of the basic system with spectrum handoff. Note that δ_f refers to two conditions. Firstly, the forced termination occurs only when $(i + Nk) > (K - 1)N$. Secondly, on arrival of new primary connection $(i-l)$ secondary connections fill the remaining $(K-k-1)N$ subchannels with the rest of l connections are terminated.

C. Throughput

Assuming that each user has the same normalized data rate $R = 1$ bps, the aggregate network throughput basic system with/without spectrum handoff can be calculated as

$$\begin{aligned} \rho^s &= (1 - P_B^s)(1 - P_F^s)\lambda_S \frac{\mu_S}{(1 - P_F^s)} \int_{x=0}^{\infty} x e^{\left(\dfrac{-\mu_S}{1 - P_F^s}\right)x} dx \\ &= (1 - P_B^s)(1 - P_F^s)^2 \theta_S \end{aligned} \tag{9}$$

where $\theta_S = (\lambda_S/\mu_S)$ is called the traffic intensity and $s \in \{b, h\}$ refers to basic system with/without spectrum handoff respectively. In (9), $(1 - P_B^s)(1 - P_F^s))\lambda_S$ is the connection completion rate and the remaining term is the average duration of completed connections.

V. Numerical Results

In this section, we investigate the forced termination probability $P_F^b(P_F^h)$, blocking probability $P_B^b(P_B^h)$, and throughput $\rho^b(\rho^h)$ of the secondary users. In all the examples, we set $K = 3, N = 6$, and $\theta_S = 8$. The lower and upper truncation limits (a, b) are same for all the probability distributions. The results are plotted against the mean service duration of secondary users m_S, whereas the duration of user connections follows FX, IEX, TEX, TPA and TLN distribution. To compare results, the values of m_S is kept same for all the probability distribution, by properly selecting the limits (a, b). MATLAB is used as a simulation tool. In simulation, we used the following definitions to calculate the P_F^s and P_B^s of secondary users respectively,

$$P_F^s = \lim_{T\to\infty} \frac{\text{Total No. of terminated connections in } [0,T]}{\text{Total No of } \textit{active} \text{ connections in } [0,T]}$$

$$P_B^s = \lim_{T\to\infty} \frac{\text{Total No. of blocked connections in } [0,T]}{\text{Total No. of connections in } [0,T]}$$

Assuming each user has the normalized data rate $R = 1$, the throughput ρ of secondary users is calculated as

$$\rho^s = \lim_{T\to\infty} \frac{\text{Total duration of completed connections in } [0,T]}{\text{T}} \tag{10}$$

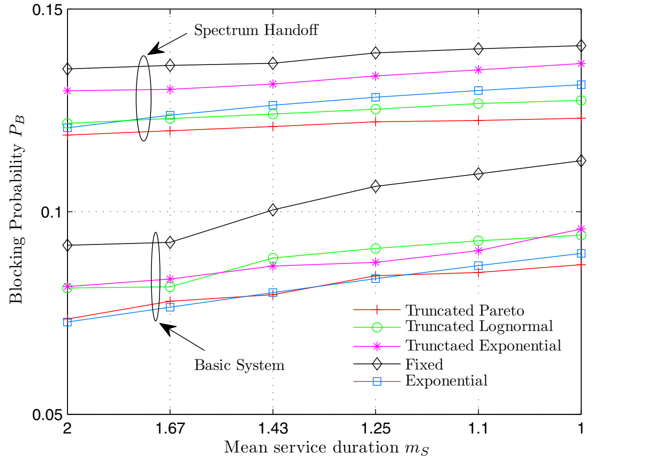

Fig. 3. Blocking probability $P_B^b(P_B^h)$ versus mean service duration m_S. Lower truncation limit $a = 0.19$, Upper truncation limit $b = 8$, $\theta_P = 1$ and $\theta_S = 8$

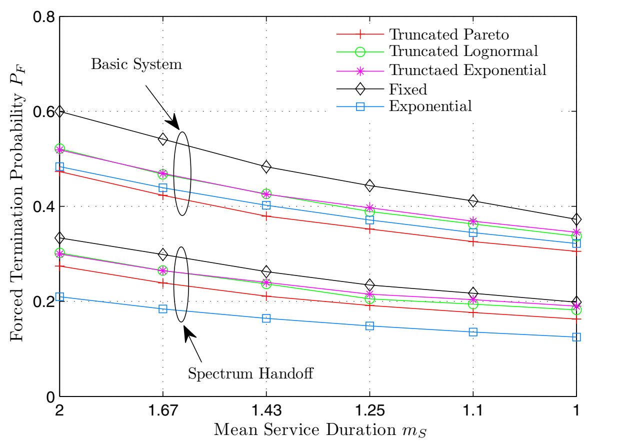

Fig. 4. Forced termination probability $P_F^b(P_F^h)$ versus mean service duration m_S. Lower truncation limit $a = 0.2$, Upper truncation limit $b = 8$, $\theta_P = 1$ and $\theta_S = 8$

In Fig. 3, $P_B^b(P_B^h)$ is plotted against the mean service duration of secondary users. In the figure, for fixed θ_S and θ_P, introduction of spectrum handoff and/or reduction of m_s results in lower values of $P_F^b(P_F^h)$ as shown in Fig.4. This implies an increased number of secondary user connections on the channel and subsequently higher blocking. However, $P_B^b(P_B^h)$ is constant when the $m_S \to 0$. Among all the distributions, FX length connections offers the maximum $P_B^b(P_B^h)$, whereas TPA has the minimum P_B^h.

Fig. 4 shows forced termination probability $P_F^b(P_F^h)$ of secondary users with FX, IEX, TEX, TPA and TLN distributed connection length. For all these distributions, $P_F^b(P_F^h)$ is an increasing function of mean service duration m_S. Introduction of spectrum handoff results in a significant improvement of P_F^b. For both systems, secondary users connection with FX duration has the maximum $P_F^b(P_F^h)$. In contrast to the case of spectrum handoff, P_F^b for TPA connection lengths is lower than IEX. This is because, in lower truncation limit a, the density of TPA is much higher than that of IEX (TEX) and the probability of termination increases rapidly with the connection length as shown in Fig. 5. In Fig. 4, $P_F^b(P_F^h)$ of TEX is higher than IEX because the minimum connection length of TEX is not zero.

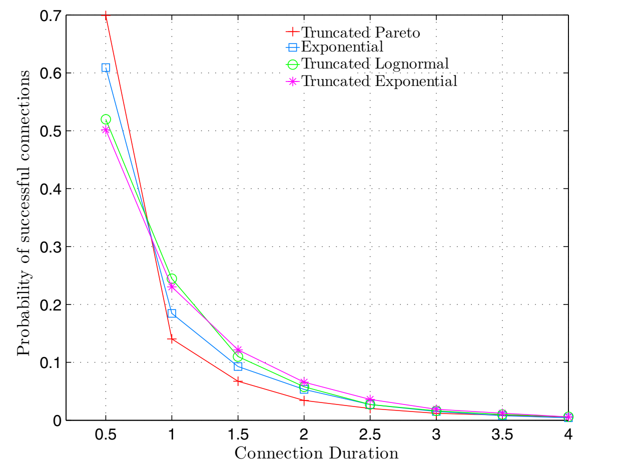

Fig. 5. Histogram: Probability of successful completion versus connection duration. Number of bins=17, bin size=0.5, mean service duration $m_S = 1.4286$, lower truncation limit $a = 0.2$, upper truncation limit $b = 8$, $\theta_P = 1$ and $\theta_S = 8$

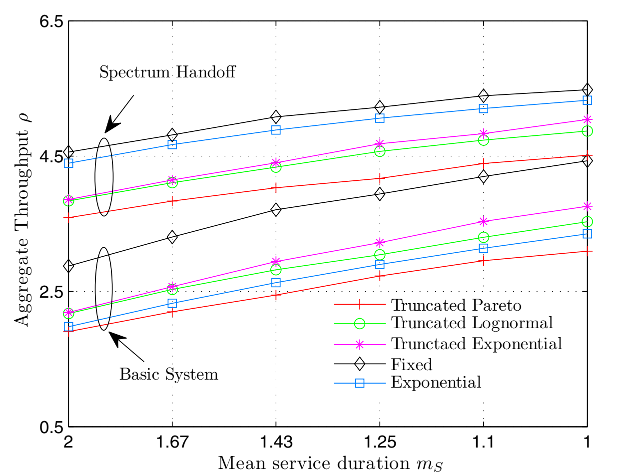

Fig. 6. Aggregate Throughput $\rho^b(\rho^h)$ versus mean service duration m_S. Lower truncation limit $a = 0.2$, Upper truncation limit $b = 8$, $\theta_P = 1$ and $\theta_S = 8$

Fig. 6 shows the throughput $\rho_b(\rho_h)$ of the secondary users with IEX, TEX, TPA, TLN and FX distributed connection length. The $\rho_b(\rho_h)$ is also monotonically decreasing function of m_S. For both system scenarios, FX length connections has the maximum $\rho_b(\rho_h)$. In spite the fact that the TPA has lowest P_F^b and P_B^b, the throughput ρ^b of TPA is even less than IEX. This is because in Fig. 5, the completion probability of TPA connections is lowest for large value of connection duration. This implies that relatively short length connections contribute to the throughput ρ^b of TPA. The ρ^b of IEX connection increases significantly with spectrum handoff because of less dominant P_F^h, where as the throughput $\rho^b(\rho^h)$ of TLN connections closely follow TEX.

VI. Conclusions and Future Work

Assuming a cognitive radio network model, we provide a numerical study of throughput, forced termination and blocking probabilities of secondary users with practical usage models. Furthermore, we also compare the results of these distribution to well known exponential model. We found that for a fixed traffic intensity, the maximum throughput can be achieved by using FX length secondary connections at the cost of large forced termination and blocking probabilities. On the contrary, secondary users with TPA connections length has the lowest forced termination and blocking probabilities and throughput. Spectrum handoff is effective in reducing forced termination probability for all truncated distributions. The exponential model can be used to predict the behaviour of all truncated distributions. The future direction is to mathematically analyze these distributions models in a cognitive radio network.

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