Optimal Power Control for Cognitive Radio Networks Under Coupled Interference Constraints: A Cooperative Game-Theoretic Perspective

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***Abstract*—Distributed power control is investigated for cognitive radio networks (CRNs) based on a cooperative game-theoretic framework. Taking into consideration both network efficiency and user fairness, a cooperative Nash bargaining power-control game (NBPCG) model is formulated, where interference power constraints (IPCs) are imposed to protect the primary users' (PUs') transmissions, and minimum signal-to-interference-plus-noise ratio (SINR) requirements are employed to provide reliable transmission opportunities to secondary cognitive users. An SINR-based utility function is designed for this game model, which not only reflects the spectrum efficiency of the CRN but also complies with all the axioms in the Nash theorem and, hence, facilitates efficient algorithmic development. The existence, uniqueness, and fairness of this game solution are proved analytically. To deal with the IPCs where the power-control decisions of all users are coupled, these IPCs are properly transformed into a pricing function in the objective utility. Accordingly, a Kalai–Smorodinsky (KS) bargaining solution and a Nash bargaining solution (NBS) are developed, which result in Pareto-optimal solutions to the NBPCG problem with different user-fairness policies. Theoretical analysis and simulations are provided to testify the effectiveness of the proposed cooperative game algorithms for efficient and fair power control in CRNs.**

***Index Terms*—Cognitive radio networks (CRNs), cooperative game theory, fairness, Kalai–Smorodinsky (KS) bargaining game, Nash bargaining game, power control.**

I. Introduction

THE RAPID growth of wireless services over the past decade has led to growing demand for radio spectrum, which is limited, valuable, and increasingly congested. On the other hand, recent spectrum-measurement campaigns have indicated that most of the licensed spectrum is underutilized [1], [2]. To deal with the dilemma between spectrum congestion and spectrum underutilization, cognitive radio (CR) technology has been proposed and advocated [3], [4]. CR technology promises to improve the network spectrum-utilization efficiency by allowing cognitive secondary users (SUs) to intelligently sense and opportunistically access those spectrum holes temporarily unused by license-holding primary users (PUs). There are two basic spectrum-sharing schemes for coexisting PU and SU networks: One is spectrum overlay, in which SUs detect and avoid overlapping with active PUs in frequency [9], [11], and the other is spectrum underlay, in which SUs spread its transmission over the wide spectrum to lower the transmit power density inflicted on overlapping PUs [12], [14], [15]. In addition, these two spectrum-sharing methods can be interwoven to collect the benefits of both [10]. In all these spectrum-sharing schemes, the primary goal of the CR network (CRN) is to provide transmission opportunities and minimum quality of service (QoS) for SUs and, at the same time, avoid causing harmful interference to PUs [9]. To this end, power control is essential for CRNs.

The topic of power control for infrastructure networks and ad hoc wireless networks has extensively been studied over the years [5]–[8] and most recently for CR networks [9]–[15]. This paper specifically studies the case of distributed power control, which is preferred for its low requirements on network infrastructure and computational complexity, as well as its scalability to network size. In addition to some heuristic techniques, game theory has widely been recognized as a powerful tool for distributed resource allocation and decision making in interactive multiuser systems.

Many power control games are noncooperative games, such as [5]–[8] and [10]–[14]. In these games, rational but selfish users maximize their individual utilities in a self-interested manner without being concerned with the impact of their strategies on other users. A typical solution to a noncooperative game is the Nash equilibrium solution (NES), which is an equilibrium point where each player has no chance to increase its utility by unilaterally deviating from this equilibrium. Unfortunately, the NES has been proven to be inefficient [21], which means that the achievable network-wide sum utility can be low compared with centralized optimization. There has been recent work that aims to improve the network-wide utility by introducing some pricing schemes [5], [6], [13]. Some forms of user cooperation are enforced in these schemes to implement the pricing policy, leading to improved network utility. While these schemes offer

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some remedies to the noncooperative game approach, they still leave room for improvement toward the global optimum.

In CRNs, secondary CR users are motivated to cooperate with one another to enhance their own transmission opportunities and achieve high spectrum efficiency. As a result, cooperative game theory is preferred for distributed power control. In fact, efficiency and fairness are two of the most concerned metrics for power control algorithms, both of which can be handled well by the cooperative game framework. The Pareto optimality guarantees the efficiency, and the proportional fairness property arising from the Nash axioms ensures user fairness [16]–[19]. For example, [16] investigates resource allocation using the Nash bargaining game and the coalition game in orthogonal frequency-division multiple-access networks, whereas [17] focuses on the issues of efficiency, fairness, and QoS guarantee using the framework of the Nash bargaining solution (NBS). The latter is also a focal topic of this paper, but we design a different objective function that is fully compliant with Nash axioms [20]; further, we develop different analytical methods and particularly employ the pricing technique as a powerful way to attain the NBS at low implementation complexity.

This paper addresses both system efficiency and user-fairness issues by making use of cooperative game theory. A cooperative Nash bargaining power control game (NBPCG) model is formulated, where interference power constraints (IPCs) are imposed to protect the PUs' transmissions, and minimum signal-to-interference-plus-noise ratio (SINR) requirements are employed to provide reliable transmission opportunities to secondary cognitive users. The power control decisions of all users are coupled in the IPCs, which complicates distributed optimization. To circumvent this difficulty, the IPCs are properly transformed into an extra pricing term in a properly constructed SINR-based utility function, which not only reflects the spectrum efficiency of the CRN but also complies with all the axioms in the Nash theorem and, hence, facilitates development of efficient distributed algorithms. The existence, uniqueness, and fairness of the solution to this game model are proved analytically. Accordingly, an iterative Nash bargaining algorithm is developed, which is shown to converge to a Pareto-optimal equilibrium for the cooperative game. The iterative implementation of the Nash bargaining algorithm is adaptive in nature, which can accommodate time-varying channel conditions. An alternative Kalai–Smorodinsky (KS) bargaining solution is also developed and proven to be Pareto optimal in terms of system efficiency with a different user-fairness policy that is quite desirable for a competitive multiuser network with a common network goal. In the KS bargaining solution (KSBS), the constrained vector-optimization problem in the NBPCG is transformed into a simple scalar search problem, which can conveniently be solved using a simple bisection method. Both the NBS and the KSBS are simple to implement and offer efficient and fair power-control decisions for all SUs in various interference environments.

The rest of this paper is organized as follows: Section II provides the system model and the problem formulation for multi-CR power control. Section III describes the basic framework for the cooperative NBPCG problem, including the basic concepts of the cooperative game, the Nash bargaining equilibrium, and

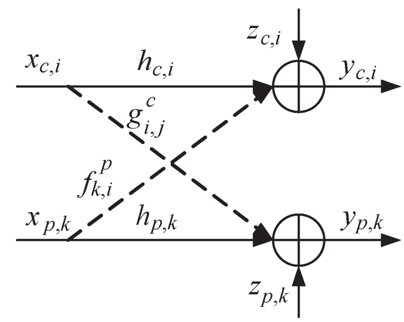


Fig. 1. System model for a spectrum underlay network.

the design of the utility function. Implementation algorithms for the NBPCG are also developed based on the concepts of KSBS and NBS, along with analytical proofs that show the uniqueness and convergence of the solutions. Corroborating simulations are provided in Section IV, followed by a summary in Section V.

II. System Model and Problem Formulation

A. System Model

Consider a spectrum-sharing scenario in which a primary network consisting of M active PUs coexists with a secondary cognitive network made of N SUs equipped with CRs in a spectrum underlay manner. SUs can simultaneously transmit with PUs but have to strictly control their transmit power to avoid harmful interference with PUs. Each user refers to a transceiver pair, including a transmitter and its intended receiver. When all these users simultaneously transmit, multiple-access interference may arise, as illustrated in Fig. 1. Solid lines indicate intended communication links with channel gains $h_{c,i}$ and $h_{p,k}$ for the ith SU link and kth PU link, respectively; meanwhile, dotted lines depict the interference links with channel gains $g^c_{i,k}$ and $f^p_{k,i}$, where superscripts ^{c} and ^{p} refer to the transmission from the corresponding cognitive SU and PU, respectively. Further, a cognitive SU i is subject to interference from other SUs as well with interference link gains $\{h^c_{j,i}\}_{j\neq i}$ (not depicted in Fig. 1). On the other hand, the primary network is well managed by some multiple-access protocol, such that it is justified to assume negligible interference among PUs themselves. Let $x_{c,i}$ and $x_{p,k}$ denote the transmitted signals from the SU i and the PU k, respectively. Their received signals $y_{c,i}$ and $y_{p,k}$ are given by

$$y_{c,i} = x_{c,i}h_{c,i} + \sum_{j=1,j\neq i}^{N} x_{c,j}h^c_{j,i} + \sum_{k=1}^{M} x_{p,k}f^p_{k,i} + z_{c,i} \quad (1)$$

$$y_{p,k} = x_{p,k}h_{p,k} + \sum_{i=1}^{N} x_{c,i}g^c_{i,k} + z_{p,k} \quad (2)$$

where $z_{c,i}$ and $z_{p,k}$ are the additive white Gaussian noise (AWGN) received at the SU i and the PU k, respectively.

B. IPCs for PU Protection

One of the most important features of the CRN is to allow multiple SUs to dynamically access spectrum holes to improve the overall network spectrum-utilization efficiency. In the coexistence scenario shown in Fig. 1, SUs' opportunistic transmissions give rise to interference with PUs. To quantify and manage the interference, proper IPCs shall be imposed to protect active PUs from harmful interference caused by SUs. To this end, we note that the maximum interference power tolerated by the kth PU can be calculated as $Z_k = BT_k$, where B is Boltzmann's constant, and T_k is a predefined threshold for the tolerable interference temperature by PU receivers. With a slight abuse of notation, we let $g_{i,k} = E\{|g^c_{i,k}|^2\}$ denote the channel power for the link between the ith SU's transmitter to the kth PU's receiver ($E\{\cdot\}$ denotes expectation). In addition, the transmit power of the SU i is defined as $p_i = E\{|x_{c,i}|^2\}$. To prevent the total interference power received by any PU from reaching a harmful level, the following IPC must be satisfied by the SUs [1], [5]:

$$\sum_{i=1}^{N} p_i g_{i,k} \leq Z_k, \qquad k = 1, \ldots, M. \tag{3}$$

Evidently, the IPCs indicate the admissible worst radio-frequency environment for the PUs. In this paper, we will formalize the power control problem for the CRN as a traditional optimization problem under the IPCs imposed for PU protection, in addition to the requirements on the minimum SINR for the SUs.

C. Interference Analysis

Under strict IPCs, SUs need to carefully adjust their transmit power to control the interference inflicted on PUs. Meanwhile, the PUs' emission also affects the transmission quality of cognitive SUs. It is hence essential to delineate the interference environments of both the PU and SU networks.

Let p_i represent the transmit power of the ith SU, and let $p_{p,k}$ represent that of the kth PU. Meanwhile, the channel powers corresponding to the link gains $h_{c,i}$, $h^c_{j,i}$, $f^p_{k,i}$, and $g^c_{i,j}$ in (1) and (2) are defined as $h_{i,i} = E\{|h_{c,i}|^2\}$, $h_{j,i} = E\{|h^c_{j,i}|^2\}$, $f_{k,i} = E\{|f^p_{k,i}|^2\}$, and $g_{i,j} = E\{|g^c_{i,j}|^2\}$, respectively. Accordingly, the SINR γ_i received at the SU i is given by

$$\gamma_i = \frac{p_i h_{i,i}}{\sum_{j=1, j\neq i}^{N} p_j h_{j,i} + \sum_{k=1}^{M} p_{p,k} f_{k,i} + \sigma^2_{c,i}} \tag{4}$$

where $\sigma^2_{c,i} := E\{|z_{c,i}|^2\}$ is the noise variance. It can equivalently be written as

$$\gamma_i = p_i h_{i,i} / I_i \tag{5}$$

where the total interference and noise I_i received by the SU i is defined as

$$I_i = \sum_{j=1, j\neq i}^{N} p_j h_{j,i} + \sum_{k=1}^{M} p_{p,k} f_{k,i} + \sigma^2_{c,i}. \tag{6}$$

The SINR expression in (4) describes the relationship between transmit powers and received interference. Our next goal is to formulate optimization problems for optimal and fair power control in the secondary SU network based on a cooperative game-theoretic framework.

D. Problem Formulation

For a secondary CRN seeking transmission opportunities, the network goal is to maximize certain network sum utility of all N SUs, where the utility functions $\{U_i\}_{i=1}^{N}$ closely reflect the spectrum-utilization efficiency. Meanwhile, the IPCs in (3) shall be imposed to limit the aggregated interference from SUs to ensure QoS for the PUs, and the total average transmit power of the SU network shall be constrained for practical implementation considerations. With these considerations, we formulate a power control optimization problem that decides the SUs' transmit powers $\mathbf{p} = (p_1, \ldots, p_N)$ as follows:

$$\max_{\mathbf{p}=(p_1,\ldots,p_N)} \quad \sum_{i=1}^{N} U_i(p_1, \ldots, p_N)$$

$$\text{subject to} \quad \sum_{i=1}^{N} p_i \leq p_{\max} \tag{7a}$$

$$\gamma_i \geq \gamma_{i,\min}, \qquad i = 1, \ldots, N \tag{7b}$$

$$\sum_{i=1}^{N} p_i g_{i,k} \leq Z_k, \qquad k = 1, \ldots, M. \tag{7c}$$

Here, the constraint (7a) limits the total transmit power of all SUs to be within an upper threshold $p_{\max}$, (7b) sets the minimum SINR threshold $\gamma_{i,\min}$ to ensure QoS for active SUs, and (7c) is the set of IPCs for provisioning the PUs' QoS. Notice that the power-control decisions of all users are coupled in the IPCs, which means that (7) is not in a separable form and, hence, is inconvenient to solve in a distributed manner [23]. To reach the globally optimal solution with distributed implementation, a cooperative game perspective is useful.

In (7), a major design consideration is the choice of the utility functions U_i, $i = 1, \ldots, N$. In macroeconomics theory, the utility function U_i can be regarded as the reward received by a player i during the bargaining process in a strategic interactive environment [8], [21], [24]. The reward U_i depends on not only the player's own strategy (i.e., the transmit power p_i) but on the opponents' transmission strategies (i.e., $\{p_j\}_{j=1, j\neq i}^{N}$) as well. There have been recent works in the CR literature that employ some capacity-related expressions for the utility function, predominantly for noncooperative power control games (NPCGs), see, e.g., [5] and [7]–[9]. Apart from these works, this paper seeks to design some proper utility function that not only reflects the network spectrum utilization efficiency but also facilitates the implementation of power control algorithms in terms of convexity and global convergence. Our design aims for cooperative games that attain highly efficient spectrum utilization with user fairness compared with noncooperative counterparts.

III. Nash Bargaining Power Control Game

This section develops an NBPCG model for CRNs. A novel utility function will be formulated for (7) based on the Nash theorem. Two solutions to the NBPCG model will be derived for different considerations in algorithm development. The NBS requires that the feasible strategy set be convex, and a fair and optimal solution directly arises from the convex optimization formulation. The alternative KSBS also optimally distributes the power resources in a Pareto-optimal sense, but the user fairness is imposed differently to ensure equal utility penalty for users with equal bargaining powers. In addition, it does not require convexity of the strategy set but needs proper reformulation of the original vector problem into a scalar search problem over the feasible utility set. The reformulated search problem enjoys low computation complexity in reaching the optimal solution to the original NBPCG problem. Both of these solutions are provided to demonstrate the fairness policies and the tradeoffs in problem reformulation and implementation simplicity. Comparisons of other aspects of the two solutions will be detailed in the following sections.

A. Utility Function Design

For a cooperative game, Nash proposed *Nash axioms* that specify the conditions for reaching Pareto-optimal NBSs [20]. The original Nash theorem is for the two-player case, and we use the *extended Nash theorem* for the multiplayer case [20], as in *Theorem 1*.

Theorem 1 (Extended Nash Theorem): A unique and fair NBS $\mathbf{p}^\star = (p_1^\star, \ldots, p_N^\star)$ can be obtained by maximizing a Nash product term as

$$\mathbf{p}^\star = \arg \max_{U_i \in S,\ U_i \geq U_{i,\min}, \forall i} \prod_{i=1}^{N} (U_i(\mathbf{p}) - U_{i,\min}). \tag{8}$$

Note that the extended Nash theorem is used under two constraints $U_i(\mathbf{p}) \in S$ and $U_i(\mathbf{p}) \geq U_{i,\min}$, where S represents the feasible utility set for all SUs, and $U_{i,\min}$ is the minimum utility requirement for the SU player i to satisfy its basic communication need. The utility space S, along with a disagreement point indicated by $(U_{1,\min}, \ldots, U_{N,\min})$, defines a bargaining problem [20], [21].

To render *Theorem 1* useful for our power control problem, a crucial step is the utility function design. The key is to find utility expressions that are not only physically meaningful for a CRN but mathematically attractive for ensuring global convergence to the NES [20] as well. In this paper, we adopt the SINR in (4) as the QoS metric for SU players and accordingly construct the utility function U_i in an SINR-related form. Based on Nash bargaining and KS bargaining concepts [19], [20], we use the minimum SINR thresholds $\{\gamma_{i,\min}\}_i$ in (7b) as the disagreement points for SUs, which reflects SUs' minimum QoS in terms of received SINR. Accordingly, the term $V_i(\mathbf{p}) := \gamma_i(\mathbf{p}) - \gamma_{i,\min}$ indicates the difference between the SINR obtained and the minimum SINR requirement for the SU i during the bargaining game. Apparently, $V_i(\mathbf{p})$ is a concave and injective function with respect to the argument $\mathbf{p}$. As such, we propose a desired utility function in *Lemma 1*.

Lemma 1: Define $U_i(\mathbf{p}) := \log(V_i(\mathbf{p})) = \log(\gamma_i(\mathbf{p}) - \gamma_{i,\min}),\ i = 1, \ldots, N$. These objective functions are strongly concave and injective and satisfy all the Nash axioms required by Theorem 1.

Proof: Letting $V(p) : A \to \Re_+$ be concave, then $U(p) = \log(V(p)) : \Re_+ \to \Re$ is also concave. If $V(p)$ is injective, then the $U(p)$ is strictly concave. Because $V_i(\mathbf{p})$ is concave and injective, $U_i(\mathbf{p})$ defined above meets all of the Nash axiom constraints. ■

Adopting the utility function designed in *Lemma 1*, we can expect to find unique and fair Nash bargaining equilibrium for the power-allocation vector $\mathbf{p}$ via *Theorem 1*. To make it convenient for ensuing equilibrium analysis, we specify the disagreement points in (8) to be $U_{i,min} = 0, \forall i$, which is consistent with [19]. We now formulate an *NBPCG model* for strategic cooperative game as follows:

$$\max_{\mathbf{p}=(p_1,\ldots,p_N)} \quad \left(\sum_{i=1}^{N} \log\left(\gamma_i(\mathbf{p}) - \gamma_{i,\min}\right) \right)$$

$$\text{subject to} \quad \sum_{i=1}^{N} p_i \leq p_{\max} \tag{9a}$$

$$\gamma_i \geq \gamma_{i,\min}, \qquad i = 1, \ldots, N \tag{9b}$$

$$\sum_{i=1}^{N} p_i g_{i,k} \leq Z_k, \qquad k = 1, \ldots, M. \tag{9c}$$

One of the advantages of the designed utility function in (9) is that it leads to user fairness. A widely adopted fairness metric is proportional fairness, which requires that $\prod_{i=1}^{N}(\gamma_i - \gamma_{i,\min})/\gamma_i \geq 0$ for the interested utilities $\{\gamma_i\}_i$ of all users [14]. The following theorem arises for the NBPCG model in (9).

Theorem 2: The new utility function adopted in (9) is designed based on the Nash theorem and meets the metric of proportional fairness.

Proof: From Lemma 1, it can be shown that the utility function takes a concave form. Let $\gamma_i^\star(\mathbf{p}^\star)$ denote the optimal utility of the cognitive SU i for the optimal power-allocation solution $\mathbf{p}^\star$ to (9). Since $\mathbf{p}^\star$ satisfies (9b), it holds that $\gamma_i^\star(\mathbf{p}^\star) - \gamma_{i,\min} \geq 0,\ \forall i$. Taking the first-order derivative of the objective function $\gamma_i^\star$ with respect to p_i, we obtain from (4) that $(1/p_i \ln 2)(\gamma_i^\star/(\gamma_i^\star - \gamma_{i,\min})) \geq 0$. Since $(1/p_i \ln 2) \geq 0$, we conclude that $\gamma_i^\star/(\gamma_i^\star - \gamma_{i,\min}) \geq 0\ \forall i$, which guarantees the proportional fairness condition $\prod_{i=1}^{N}(\gamma_i^\star - \gamma_{i,\min})/\gamma_i^\star \geq 0$. ■

From the proof, it is evident that proportional fairness is a special case of the NBS fairness described in the Nash theorem [20] when the minimum QoS requirement on the SINR is set to $\gamma_{i,\min} = 0\ \forall i$.

B. KS Bargain Solutions Using the Bisection Method

Having formulated the NBPCG model in (9), we now solve for the optimal power allocation $\mathbf{p}^\star$ based on the game approach. This section provides KSBSs to NBPCG based on a simple bisection method. For clarity, we first derive the KSBS in the absence of IPCs for PU protection. Then, we introduce the pricing concept used in economics to reformulate the NBPCG model into a viable form for handling the IPC.

1) Without IPCs: In the absence of the IPCs in (9c), the feasible utility set S in (8) for the SUs can be derived from (5), (9a) and (9b) as follows:

$$S = \left\{ (U_1, \ldots, U_N) \left| \sum_{i=1}^{N} \frac{I_i}{h_{i,i}} e^{U_i} \leq \tilde{p}_{\max} \right. \right\} \tag{10}$$

where $\tilde{p}_{\max} = p_{\max} - \sum_{i=1}^{N}(I_i/h_{i,i})\gamma_{i,\min}$.

Here, each term I_i is the total interference perceived by the SU i, which can be acquired at the receiver of SU i via interference estimation at the beginning of each power control interval without needing to acquire the powers $\{p_j\}_{j\neq i}$ of all the competing players. As a result, we suppose that I_i is known by SU i rather than being a function of the design parameters during power control. Accordingly, $\tilde{p}_{\max}$ can be computed once the channel parameters $\{h_{i,i}, I_i\}_i$ are acquired. Further, we can rewrite the total power constraint in (9b) as $p_i h_{i,i} \geq \gamma_{i,\min} I_i$, which suggests that the feasible power strategy always exists. To identify the KSBS, we also need to show that the feasible utility set S is nonempty, convex, closed, and bounded [20], which we prove in the following theorem.

Theorem 3: The feasible utility set S identified in (10) for the NBPCG model in (9) is nonempty, convex, closed, and bounded in the absence of the IPCs in (9c).

Proof: It is evident from (9a) and (9b) that S is nonempty, closed, and bounded. It is only left to show that S is convex. The set S is said to be convex if, for any two points $U^{(1)}, U^{(2)} \in S$ and all $\theta \in [0,1]$, the point $\theta U^{(1)} + (1-\theta)U^{(2)}$ is also in S [18]. For S in (10), we need to show that the following equation holds true:

$$\sum_{i=1}^{N} \frac{I_i}{h_{i,i}} e^{\theta U_i^{(1)} + (1-\theta)U_i^{(2)}} \leq \tilde{p}_{\max} \qquad \forall U^{(1)}, U^{(2)} \in S \text{ and } \theta \in [0,1]. \tag{11}$$

To show (11), let us define $f(\theta) = \sum_{i=1}^{N} f_i(\theta)$, where $f_i(\theta) = (I_i/h_{i,i}) e^{\theta U_i^{(1)} + (1-\theta)U_i^{(2)}}$. It is straightforward to show that the second-order derivative of $f(\theta)$ is not less than zero because

$$\frac{d^2 f_i(\theta)}{d\theta^2} = \frac{I_i}{h_{i,i}} e^{\theta U_i^{(1)} + (1-\theta)U_i^{(2)}} \left(U_i^{(1)} - U_i^{(2)}\right)^2 \geq 0 \qquad \forall i. \tag{12}$$

As a result, $f(\theta)$ is a convex function, and (11) holds. It is hence concluded that S is a convex set. ■

With *Theorem 3*, the KSBS becomes applicable to our NBPCG problem. As explained in [18]–[20] on a two-user utility graph, the KSBS can be found at the intersection point of the feasible utility set boundary and the line connecting the disagreement point $(\{U_{i,\min}\}_i)$ to the ideal point corresponding to desired maximum utilities $(\{U_{i,\max}\}_i)$. Both $U_{i,\min}$ and $U_{i,\max}$ are predefined based on user i's QoS requirements, e.g., rate QoS. Meanwhile, in the KS bargaining concept [18], each user decides its fairness factor α_i prior to the game, which reflects its attitude toward fairness. The fairness factors are normalized such that

$$\sum_{i=1}^{N} \alpha_i = 1, \qquad \alpha_i > 0 \quad \forall i.$$

Finding the intersection point in the KSBS amounts to solving for a scalar β that satisfies

$$U_i = \beta \alpha_i U_{i,\max}, \qquad i = 1, \ldots, N. \tag{13}$$

Substituting (13) into the feasible utility set in (10), the optimal $\beta^\star$ is given by

$$\beta^\star = \arg\max_{\beta} \left\{ \sum_{i=1}^{N} \frac{I_i}{h_{i,i}} e^{\beta \alpha_i U_{i,\max}} \leq \tilde{p}_{\max} \right\}. \tag{14}$$

At the optimal $\beta^\star$, the allocated power to each SU is

$$\begin{aligned}\mathbf{p}^\star : p^\star &= \frac{I_i}{h_{i,i}} \gamma_i(\beta^\star) = \frac{I_i}{h_{i,i}} \left(e^{U_i(\beta^\star)} + \gamma_{i,\min}\right) \\ &= \frac{I_i}{h_{i,i}} \left(e^{\beta^\star \alpha_i U_{i,\max}} + \gamma_{i,\min}\right) \qquad \forall i.\end{aligned} \tag{15}$$

At $\beta^\star$, it is evident from (14) that

$$\frac{\alpha_1 U_{1,\max}}{U_1^\star} = \cdots = \frac{\alpha_N U_{N,\max}}{U_N^\star}. \tag{16}$$

Viewing α_i as the bargaining power and $U_{1,\max}/U_i^\star$ as the utility penalty from the desired utility level $U_{i,\max}$, (16) suggests that the KSBS allocates resources such that the utility penalty is inversely proportional to the bargaining power of each player, and the penalty for all users will be the same if the bargaining powers are the same (i.e., setting $\alpha_i = 1/N \ \forall i$). This type of user-fairness policy is desirable for selfish users in competitive networks [19].

In addition to fairness, $\{p_i^\star\}_{i=1}^N$ in (15) is a Pareto-optimal solution to the original NBPCG problem in (9) when the IPCs in (9c) are omitted. The optimality is established in *Theorem 4*.

Theorem 4 (Model Equivalence Theorem): When $U_{i,\min} = 0 \ \forall i$, and in the absence of the IPCs in (9c), the power-control solution formalized in (14) and (15) is equivalent to the optimal solution to the original problem in (9).

Proof: First, we show that $\mathbf{p}^\star$ in (15) meets the feasibility constraints (9a) and (9b). The total power constraint in (9a) can conveniently be verified from (14). To verify (9b), we note that, at the disagreement point $U_{i,\min} = 0$, it holds that $U_{i,\min} = \log(\gamma_i - \gamma_{i,\min}) = 0$ by virtue of our utility function design. Hence, $\gamma_i = \gamma_{i,\min} + 1$, which obviously satisfies (9b).

Next, we show that the sum utility in (9) is maximized by (15). Substituting (13) into the objective function in (9), we have $\sum_i U_i(\mathbf{p}) = \beta \sum_i U_{i,\max}$, where $U_{i,\max}$ is the maximally achievable utility for each SU i. Meanwhile, β is maximized at $\beta^\star$ in (14). As a result, $\mathbf{p}^\star(\beta^\star)$ in (15) yields the maximum sum utility. ■

The result in *Theorem 4* is quite appealing because it allows us to solve the constrained optimization problem in the NBPCG model by simply solving for the scalar $\beta^\star$ in (14) and then making the optimal power control decision $\mathbf{p}^\star$ via (15). The optimal scalar $\beta^\star$ can be found by some simple search algorithm such as the Bisection method, which we will elaborate upon in Section III-B3.

2) With IPCs: When the IPCs in (9c) are present, it is difficult to establish the feasible utility set as in (10) for the no-IPC case due to the coupling of the power-allocation variables of all SUs in the IPCs. In addition, even if the feasible set can exactly be described, it is not necessarily a convex set, which would make it infeasible to solve for the globally optimal solution. To circumvent the difficulties, we adopt strategies in the ecomonics field by introducing a judiciously designed pricing function in the utility, which transforms the IPCs into some form that is amenable to optimal design [8]. The pricing technique has found a wide range of applications in the wireless communication field [6], [11].

In lieu of imposing the IPCs, our strategy for PU protection is to impose a proper price on the interference caused by the SUs' opportunistic transmission. When such a pricing function is introduced to the utility function, SUs need to balance between the rewards and prices of their power control decisions to maximize the net utility. As a result, SUs tend to avoid high prices, which in turn avoids harmful interference to the PUs. To this end, we define the following pricing function.

Definition 1 (Pricing Function): For each SU transmitting at power p_i $\forall i$, its pricing function $c(p_i)$ is defined by the total interference perceived by all PUs from this SU, namely, $c(p_i) = p_i \sum_{k=1}^{M} \psi_k$, where $\psi_k = \rho_k g_{i,k}$ represents the interference to the kth PU with respect to a unit increase in p_i, $g_{i,k}$ is the link power between the ith SU transmitter and the kth PU receiver, and ρ_k is an adjustable factor to track the PU's IPC change.

We assume that the link powers $\{g_{i,k}\}$ and, hence, $\{\psi_k\}$ are known, which may be acquired by means of pilot transmissions and feedback mechanisms [16]. We now define a net utility function in which the SINR reward $V_i(\mathbf{p}) = \gamma_i(\mathbf{p}) - \gamma_{i,\min}$ is offset by the interference price $c(p_i)$, as follows.

Definition 2 (Payoff Function): The payoff function is defined as $\varsigma_i(\mathbf{p}) = \log(V_i(\mathbf{p}) - c(p_i))$ $\forall i$, which reflects the net utility after pricing.

In the defined payoff function ς_i, an SU is encouraged to seek opportunistic spectrum access by increasing its SINR $V_i(\mathbf{p})$ but, through pricing, is discouraged from transmitting harmfully high powers that cause interference with the PU. The pricing strategy is effective for PU protection, which obviates the need for the IPCs. Bypassing the IPCs and imposing the interference pricing function, the NBPCG model in (9) can effectively be reformulated as

$$\max_{\mathbf{p}=(p_1,\ldots,p_N)} \quad \left(\sum_{i=1}^{N} \log\left(V_i(\mathbf{p}) - c(p_i)\right)\right)$$

$$\text{subject to} \quad \sum_{i=1}^{N} p_i \leq p_{\max} \tag{17a}$$

$$\gamma_i \geq \gamma_{i,\min}, \qquad i = 1, \ldots, N. \tag{17b}$$

A major advantage of the NBPCG formulation in (17) is that it provides PU protection and, at the same time, bypasses the IPCs, hence becoming convenient to analyze and solve, using the KSBS that we have developed in Section III-B1. Following our development therein, we can derive the feasible payoff set for (17) as

$$S = \left\{ (\varsigma_1, \ldots, \varsigma_N) \left| \sum_{i=1}^{N} \frac{I_i e^{\varsigma_i}}{h_{i,i} - I_i \sum_{k=1}^{M} \psi_k} \leq \tilde{p}_{\max} \right. \right\} \tag{18}$$

where

$$\tilde{p}_{\max} = p_{\max} - \sum_{i=1}^{N} \frac{I_i \gamma_{i,\min}}{h_{i,i} - I_i \sum_{k=1}^{M} \psi_k}.$$

Mimicking the proof for *Theorem 2*, we can show that S in (18) is a convex set. Then, similar to (13)–(15), the KSBS to the reformulated NBPCG in (17) can be found by searching for the optimal scalar $\beta^\star$ as

$$\beta^\star = \arg\max_{\beta} \left\{ \sum_{i=1}^{N} \frac{I_i e^{\beta \alpha_i \varsigma_{i,\max}}}{h_{i,i} - I_i \sum_{k=1}^{M} \psi_k} \leq \tilde{p}_{\max} \right\}. \tag{19}$$

Correspondingly, the optimal power allocation is

$$\mathbf{p}^\star : p_i^\star = \frac{I_i}{h_{i,i} - I_i \sum_{k=1}^{M} \psi_k} \left(e^{\beta^\star \alpha_i \varsigma_{i,\max}} + \gamma_{i,\min}\right) \qquad \forall i. \tag{20}$$

3) Implementation of the KSBSs: So far, we have analyzed the KSBSs for the NBPCG problem with and without IPC constraints. In both cases, we have transformed a constrained vector optimization problem into a simple line-search problem to identify the optimal scalar $\beta^\star$ in (14) and (19), respectively. Such a line search problem can be solved in a simple and effective manner by the well-studied bisection method, which we subsequently summarize as a simple implementation of our KSBSs.

Algorithm 1: Bisection Method for KSBSs

1) *Initialization*: Compute $\tilde{p}_{\max}$ in (10) or (18) based on acquired channel parameters. Set the lower bound $l = \beta_{\min}$ and upper bound $u = \beta_{\max}$ for β, and a small tolerance value $\xi > 0$.
2) *Iteration*: While $u - l > \xi$, repeat the following bisection search steps:
   a) Set $\beta = (l + u)/2$.
   b) Compute $\zeta_i = (I_i/h_{i,i})e^{\beta\alpha_i U_{i,\max}}$ for the no-IPC case in (14) or $\zeta_i = (I_i/h_{i,i} - I_i \sum_k \psi_k)e^{\beta\alpha_i\varsigma_{i,\max}}$ for the IPC case in (19).
   c) Check the feasibility of β in (14) or (19) by checking whether $\sum_i \zeta_i \leq \tilde{p}_{\max}$ holds.
   d) If feasible, let $l = x$; else, let $u = x$.
3) *Decision:* Let $\beta^\star = \beta$ after 2) converges, and compute the corresponding power control decision $\mathbf{p}^\star(\beta^\star)$ from (15) or (20).

In the foregoing procedure, Step 2c) requires user cooperation to check the feasibility of β and eventually reach the optimal $\beta^\star$, whereas the power-allocation decisions can be made locally.

C. NBS

For the reformulated NBPCG problem in (17) with PU protection, we offer an alternative solution by showing that it permits a Pareto-optimal and unique Nash equilibrium, giving rise to the NBS [23], [24].

1) Uniqueness of NBS for the NBPCG Model: Note that the feasible payoff set S in (18) has been shown to be a convex set. We now show the existence and uniqueness of the NBS to (17) using cooperative game theory [20], [21], [24].

Theorem 5 (Existence): There is at least one NBS to the NBPCG model in (17).

Proof: According to game theory, *Theorem 5* holds if and only if 1) the SUs' strategy space, which is defined as a

Cartesian product space in the form of $A = A_1 \times \cdots \times A_N$, is a nonempty, convex, and compact subset of some Euclidean space, and 2) the payoff functions are continuous and quasi-concave [21].

It is straightforward to show 1) for the feasible power strategy set defined by the constraints (17a) and (17b) [21]. To show 2), we note that the payoff function of the SU i, $\varsigma_i = \log(V_i - p_i \sum_{k=1}^{M} \rho_k g_{i,k})$ is continuous with respect to p_i, which allows us to take the second-order derivative of it. Defining $\vartheta_i = V_i - p_i \sum_{k=1}^{M} \rho_k g_{i,k}$, we have

$$\frac{\partial^2 V_i}{\partial^2 p_i} = -\left(\frac{\partial \gamma_i}{\partial p_i} - \sum_{k=1}^{M} \psi_k\right)^2 \vartheta_i^2 \leq 0.$$

As a result, the payoff function ς_i is continuous and quasi-concave. With both 1) and 2), the existence of the NBS is established. ∎

For the uniqueness of the Nash equilibrium in a cooperative game, it has been established that there is at most one NBS to the game if and only if the following four conditions are met [17], [22].

1) $S_i = \{p_i \in S,\ h(p_i) = \bar{p} - p_i \geq 0\}$ is nonempty, where $\bar{p}$ is the average power level.
2) There exists $p_i \in A_i$ that satisfies $h(p_i) \geq 0$.
3) The utility function ς_i of player i is continuous and quasi-concave.
4) The game model is diagonally strictly concave on its strategy set A, namely, for any $\mathbf{p}^{(0)} \neq \mathbf{p}^{(1)}$ with $\mathbf{p}^{(k)} = [p_1^{(k)}, \ldots, p_N^{(k)}]^T \in A$ for $k = 0, 1$, and for $\mathbf{t} = [t_1, \ldots, t_N]^T \geq \mathbf{0}$, the following inequality holds [22]:

$$\left(\mathbf{p}^{(0)} - \mathbf{p}^{(1)}\right)^T \mathbf{g}\left(\mathbf{p}^{(0)}, \mathbf{t}\right) + \left(\mathbf{p}^{(1)} - \mathbf{p}^{(0)}\right)^T \mathbf{g}\left(\mathbf{p}^{(1)}, \mathbf{t}\right) < 0$$

where the function $\mathbf{g}(\mathbf{p}, \mathbf{t})$ is defined as

$$\mathbf{g}(\mathbf{p}, \mathbf{t}) = \left[t_1 \frac{\partial \varsigma_1}{\partial p_1}, \ldots, t_N \frac{\partial \varsigma_N}{\partial p_N}\right]^T.$$

The following theorem addresses the uniqueness issue of the NBPCG game of interest.

Theorem 6 (Uniqueness): The NBPCG model in (17) satisfies all the foregoing conditions and, hence, permits a unique NBS.

Proof: Conditions 1 and 2 are met as direct results from the strategy space constraints (17a) and (17b). Condition 3 has been proved for *Theorem 5*. For Condition 4, one has

$$\begin{aligned}
&\left(\mathbf{p}^{(0)} - \mathbf{p}^{(1)}\right)^T \left[\mathbf{g}(\mathbf{p}^{(0)}, \mathbf{t}) - \mathbf{g}(\mathbf{p}^{(1)}, \mathbf{t})\right] \\
&= \left(\mathbf{p}^{(0)} - \mathbf{p}^{(1)}\right)^T \\
&\quad \times \left[t_1 \left(\frac{\partial \varsigma_1}{\partial p_1^{(0)}} - \frac{\partial \varsigma_1}{\partial p_1^{(1)}}\right), \ldots, t_N \left(\frac{\partial \varsigma_N}{\partial p_N^{(0)}} - \frac{\partial \varsigma_N}{\partial p_N^{(1)}}\right)\right]^T \\
&= \sum_{i=1}^{N} t_i \left(p_i^{(0)} - p_i^{(1)}\right) \left(\frac{\partial \varsigma_i}{\partial p_i^{(0)}} - \frac{\partial \varsigma_i}{\partial p_i^{(1)}}\right).
\end{aligned} \tag{21}$$

Let $\varepsilon_i = t_i(p_i^{(0)} - p_i^{(1)})(\partial \varsigma_i / \partial p_i^{(0)} - \partial \varsigma_i / \partial p_i^{(1)})$, where $t_i \geq 0$. Because of the concavity of the utility function, $\partial \varsigma_i / \partial p_i$ is monotonically decreasing with respect to p_i, we have $\partial \varsigma_i / \partial p_i^{(0)} - \partial \varsigma_i / \partial p_i^{(1)} > 0$ for $p_i^{(1)} > p_i^{(0)}$, and hence, $\varepsilon_i \leq 0$. Similarly, $\varepsilon_i \leq 0$ holds for $p_i^{(1)} < p_i^{(0)}$ as well. Since all conditions 1–4 are met, together with *Theorem 5*, we conclude that our NBPCG model has one unique NBS. ∎

2) Implementation of NBS: Having derived the existence and uniqueness of the Nash bargaining equilibrium, we now solve for this unique equilibrium by solving the constrained optimization formulation in (17) using the method of Lagrange multipliers [21]. Introducing Lagrange multipliers λ and $\boldsymbol{\mu} := (\mu_1, \ldots, \mu_N)$ for the multiple constraints, (17) can equivalently be solved by maximizing the following expression:

$$\begin{aligned}
\phi(\varsigma(\mathbf{p}), \lambda, \boldsymbol{\mu}) = &\sum_{i=1}^{N} \alpha_i \log\left(\mathrm{V}_i(\mathbf{p}) - p_i \sum_{k=1}^{M} \rho_k g_{i,k}\right) \\
&- \lambda\left(\sum_{i=1}^{N} p_i - p_{\max}\right) \\
&- \sum_{i=1}^{N} \mu_i (\gamma_{i,\min} - \gamma_i).
\end{aligned} \tag{22}$$

In the first line of (22), we have extended the original sum utility to the weighted sum utility with weights $\{\alpha_i\}_{i=1}^{N}$ for the N users. This is to reflect the different fairness policies used in NBSs. Similar to (16), $\{\alpha_i\}_i$ indicates the users' bargaining powers [18] but shows up differently from the KSBS [19].

Taking the first-order derivative of (22) with respect to p_i and setting it to zero, we reach the following optimal solution $p_i^\star$ as a function of the Lagrange multipliers:

$$p_i^\star = \frac{\gamma_{i,\min}}{\upsilon_i} + \frac{\alpha_i}{\lambda - \mu_i \frac{\partial \gamma_i}{\partial p_i}} \tag{23}$$

where $\upsilon_i = (\partial \gamma_i / \partial p_i) - \sum_{k=1}^{M} \psi_k$ represents the net SINR gain for an increase of a unit power level. We require that $\upsilon_i \geq 0$; otherwise, each SU player has no interest in increasing its power level. Note that when the channel conditions are near static and known as assumed before, we have $\partial \gamma_i / \partial p_i = h_{i,i}/I_i$, and hence, $\upsilon_i = (h_{i,i}/I_i) - \sum_{k=1}^{M} \psi_k > 0$ can be verified in advance. We adopt the first-derivative form $\partial \gamma_i / \partial p_i$ in (23) to accommodate time-varying channel conditions through a distributed adaptive implementation of (23), which is explained next.

Traditional algorithms for constrained optimization, e.g., the Karush-Kuhn-Tucker condition [21], can encounter the NP-hard problem. Here, we adopt the fixed-point method to derive an iterative procedure that updates the power control decisions and SINR gains through the following steps:

$$\upsilon_i^{(t)} = \left[\frac{\gamma_i^{(t)}}{p_i^{(t)}} - \sum_{k=1}^{M} \psi_k\right]_{\upsilon_i^{(t-1)}}^{+} \tag{24a}$$

$$p_i^{(t+1)} = \left[\frac{\gamma_{i,\min}}{\upsilon_i^{(t)}} + \frac{\alpha_i}{\lambda^{(t)} - \mu_i^{(t)} \frac{\gamma_i^{(t)}}{p_i^{(t)}}}\right]_0^{+} \tag{24b}$$

where $[x]_a^+ = x$ if $x > 0$, and $[x]_a^+ = a$ if $x \leq 0$. In this adaptive procedure, the dynamic $\gamma_i^{(t)}$ is locally measured at each CR i at t, and user cooperation is reflected through the selection of $\lambda^{(t)}$ that is common in all $p_i^{(t+1)}$ such that $\sum_i p_i^{(t+1)} \leq p_{\max}$. In this sense, (24b) gives rise to an iterative water-filling interpretation. The power allocation is adjusted according to a link-quality indicator γ_i/p_i as well as the bargaining power α_i and utility-satisfaction indicator $\gamma_{i,\min}/\upsilon_i$, and the water-filling level λ is an adjustable factor to comply with the total power constraint.

The multipliers $\lambda^{(t)}$ and $\{\mu_i^{(t)}\}_{i=1}^N$ need to carefully be chosen to ensure fast convergence. A simple yet effective way to choose these multipliers is to employ the subgradient method as follows:

$$\mu_i^{(t)} = \left[\mu_i^{(t-1)} - c_t\left(\gamma_i^{(t)} - \gamma_{i,\min}\right)\right]_0^+$$

$$\lambda^{(t)} = \left[\lambda^{(t-1)} - c_t\left(p_{\max} - \sum_{i=1}^N p_i^{(t)}\right)\right]_0^+ \qquad (25)$$

where c_t is a small step size. Apparently, $\mu_i^{(t)}$ is locally updated, whereas $\lambda^{(t)}$ is updated through cooperation. The steps in (24) and (25) implement NBS through the stochastic subgradient technique, which is known to converge under mild conditions. The overall iterative procedure is explained in detail as Algorithm 2. The convergence behavior of this procedure will be testified via simulations.

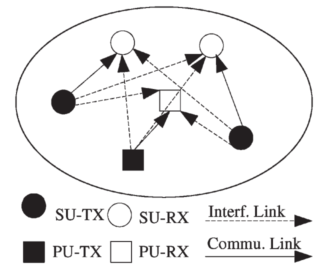
Algorithm 2: Nash Bargaining for Optimal Power Control

1) *Initialization:* Initialize the user-specific parameters $\gamma_{i,\min}$ and α_i, $i = 1, \ldots, N$; initialize the multipliers $\lambda^{(0)}$ and $\{\mu_i^{(0)}\}_i$ to some large values.
2) *Iteration:*
   a) For slow-varying channels, measure $\gamma_i^{(t)}$; for static channels, $\gamma_t^{(t)}/p_i^{(t)} = h_{i,i}/I_i, \forall i$.
   b) Update the power levels $p_i^{(t)}$ using (24).
   c) Adjust the multipliers $\lambda^{(t)}$ and $\mu_i^{(t)}$ using (25).
3) *Decision:* After 2) converges, the power control decisions $p_i^{(t)}$ reach the optimal NBS.

In the foregoing procedure, Step 2c) requires user cooperation to compute the multiplier λ for the total power constraint, whereas the power-allocation decisions can be made locally.

IV. Numerical Results

A simulation scenario with one PU pair and two SU pairs is depicted in Fig. 2. For all the communication links and interference links, the channel gains are generated as $cd_{i,j}^{-\tau}$ for transmitter i and receiver j, where $c = 0.097$ is constant, $\tau = 4$ is the fading exponent, and the transmitter–receiver distances $d_{i,j}$ of all links are listed in Fig. 2, as in [8]. The ambient noise for both PUs and SUs is AWGN with zero mean and variance $\sigma^2 = 10^{-9} W$.



T \ R	SU_1	SU_2	PU
SU_1	150	300	400
SU_2	300	200	350
PU	400	350	100

Fig. 2. Simulation scenario: one PU pair and two SU pairs. Solid lines and dotted lines represent communication links and interference links, respectively. The distances of all the links are listed in the table on the right.

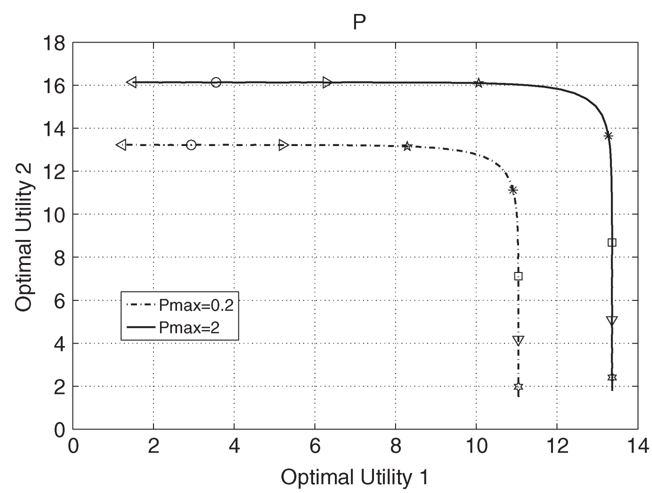


Fig. 3. Optimal utilities achieved by the KSBS for $p_{\max} = 0.2$ and 2, respectively. The markers correspond to different fairness factors $(\alpha_1, 1 - \alpha_1)$ for (SU 1, SU 2), with $\alpha_1 = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7,$ and 0.8 going from the upper left to the lower right on each curve.

A. KSBS Without IPC

First, we testify the KSBS for the NBPCG model without the IPC. The minimum SINR requirements of the two SUs are 10 and 11 dB, respectively. The bisection method summarized in Section III-B3 is used to search for the optimal $\beta^\star$ and further obtain the optimal power control decisions and maximum utilities for the SUs. Fig. 3 depicts the achieved optimal utility for various values of the fairness factors $(\alpha_1, \alpha_2 = 1 - \alpha_1)$. The following observations arise.

1) The optimal utility of one SU, e.g., SU 1, increases as its fairness factor (α_1) increases but saturates at certain value when α_1 exceeds a threshold. Meanwhile, the optimal utility of its opponent SU 2 varies little at small α_1 but drastically drops when the utility of SU 1 saturates at large α_1. Apparently, it is not desired to set the fairness factors to be strongly favoring one player over the other.
2) The sum utility of both SUs peaks when the fairness factors are balanced between the two users and decreases when α_1 is too small or too large.
3) When the total power budget $P_{\max}$ is larger, the resulting utilities are higher for both SUs because the feasible strategy set for power control has a larger size.

Apparently, a fair and optimal power policy can be obtained by adjusting the fairness factors. Take $P_{\max} = 0.2$ for example. When $(\alpha_1, \alpha_2) = (0.55, 0.45)$, the maximum utilities for

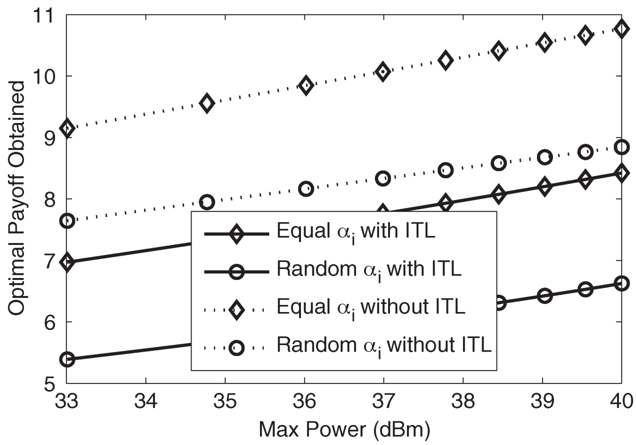


Fig. 4. Optimal payoff based on the KSBS.

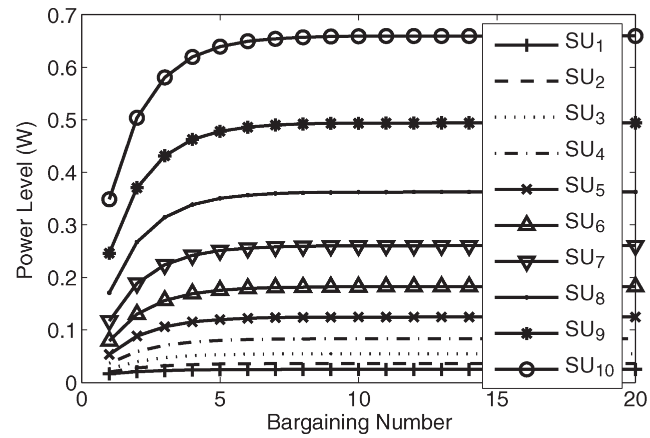


Fig. 5. Convergence behavior of the Nash bargaining algorithm.

SU 1 and SU 2 are 10.4085 and 12.4672, respectively. When $(\alpha_1, \alpha_2) = (0.3, 0.7)$, the achieved utilities become 4.7319 and 13.2248, respectively, favoring SU 2 over SU 1.

B. KS Bargain Solution for NBPCG With IPCs

In the section, we investigate the KSBS for the NBPCG model with IPC considerations. There are SUs randomly distributed within a circle. The PU receiver is located at the center of the circle. The other simulation parameters are the same as before. Fig. 4 shows that the payoff based on the KSBS increases as the maximum total power $P_{\max}$ increases. In addition, adopting equal fairness factors for all SUs results in higher payoff than randomly selected fairness factors. These observations corroborate the analysis and simulation results for the two-SU case.

Fig. 4 also sheds light on the effect of the IPCs. It can be observed that the optimal payoff without the IPCs is greater than the case with the IPC requirement. When the fairness factor is randomly selected and the IPC is not considered, the achieved payoff is quite close to the case in which the fairness factor is equally selected and the IPC is imposed. The best payoff is obtained when the fairness factor is equally selected and the IPC is not considered, and *vice versa*.

C. NBS to the NBPCG Model

This section evaluates our proposed NBS to the NBPCG problem. The bargain powers $\{\alpha_i\}$ of all users in (22) are set to be equal. Fig. 5 testifies the convergence of the proposed algorithm. It apparently converges fast to the maximum utility after five to ten iterations. Certainly, the choice of the Lagrange multipliers is crucial to the convergence behavior. We initialize with $\lambda^{(0)} = 10$ and $\mu_i^{(0)} = 10^{+16}\ \forall i$, since the feasible power requirements are quite harsh.

The proposed NBS for the NBPCG is compared with a couple of benchmark algorithms for power control: an NPCG, e.g., in [7], and the SINR balancing power control (SBPC) algorithm, as in (26). The NPCG in [7] chooses the optimal power level for each user based on a minimum cost function. The SBPC algorithm is a traditional power-control algorithm that distributively and iteratively searches for the suboptimal power levels updated from the tth iteration to the $(t+1)$th iteration based on the SINR (or interference) measurements $\gamma_i^{(t)}$ as follows:

$$p_i^{(t+1)} = p_i^{(t)} \frac{\gamma_{\text{target}}}{\gamma_i^{(t)}}, \qquad i = 1, \ldots, N. \tag{26}$$

Here, we set $\gamma_{\text{target}} = \gamma_{i,\min} = 9$ dB $\forall i$. Obviously, the SBPC only seeks to reach the target SINR via iterative power control without necessarily reaching the maximized SINR values of the NBPCG model. In addition, the SBPC might encounter difficulties when the spectrum opportunities offered to the SU network are too small to provide the targeted minimum SINRs for all SUs. Evidently, our NBPCG formulation is more suitable for CR networks with flexibility in handling both plentiful and merger transmission resources.

Fig. 6(a) compares the consumed powers of three power control algorithms: 1) SBPC; 2) NPCG; and 3) the proposed NBPCG implemented by the NBS. It turns out that the NPCG approach consumes the most power due to the SUs' self-interested noncooperative behavior in the game process. For example, when one of the SU players cannot reach or maintain its minimum SINR, it resorts to the only means of increasing its transmission power, as do other SU players in a similar situation. As a result, large mutual interference arises among these selfish SU players. For the NBPCG approach, on the other hand, SU players can perceive the interference environment well and accordingly make the most appropriate transmission power-adjustment decision, given the circumstances. Overall, the SBPC consumes the least power; but as shown in Fig. 6(b), it is a conservative strategy that cannot fully utilize the power resources to maximize the achieved utility. Further, it is shown in Fig. 6(b) that the NPCG approach does not provide fairness among the multiple SUs. SUs with close transceivers enjoy high utility, whereas those SUs with faraway transceivers do not attain the basic communication needs in terms of SINR $\gamma_{i,\min}$, although they might consume high transmit power. The NBPCG approach, on the other hand, not only maintains fairness among multiple SUs but also guarantees the minimum SINR requirements of all SUs under diverse channel-fading conditions and interference environments.

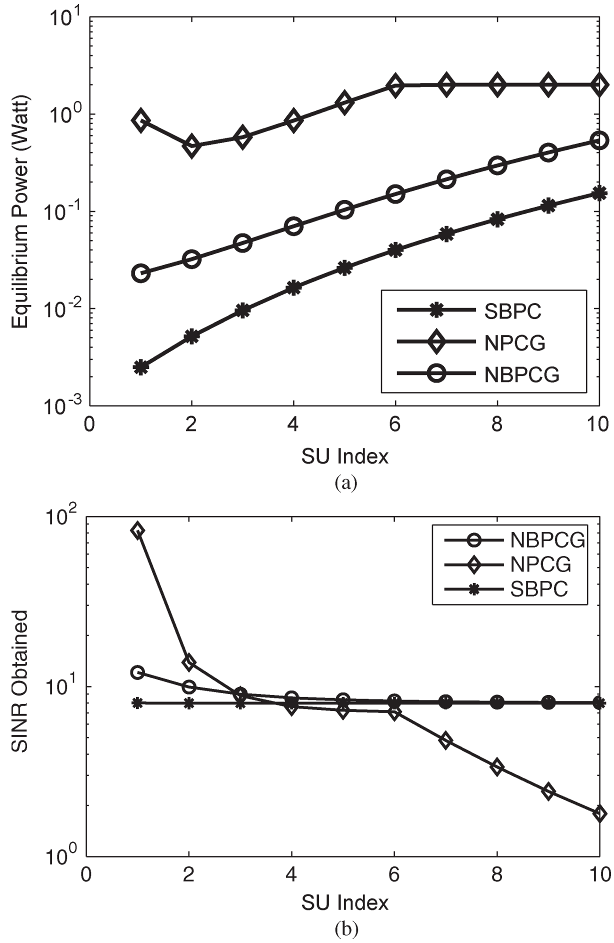


Fig. 6. Comparisons of various power control schemes. (a) Equilibrium power levels. (b) Achieved optimal SINR values.

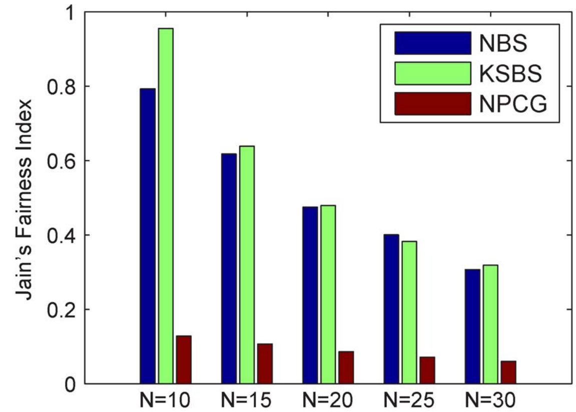


Fig. 7. Jain's fairness index for various games. KSBS and NBS for the cooperative NBPCG and the Nash equilibrium for the NPCG.

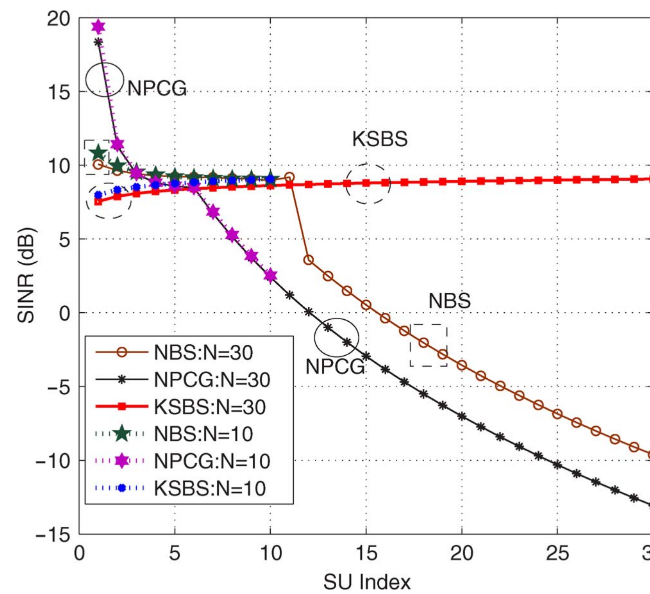


Fig. 8. Comparison of spectrum utilization efficiency in terms of the achieved SINRs of SUs, which are depicted in descending order from the highest SINR attained to the lowest. For $N = 10$, the curves stop at $N = 10$.

D. Performance Comparisons of Fairness and SINR Efficiency

This section compares the proposed KSBS and NBS for the cooperative NBPCG. The KSBS explicitly relies on user-defined fairness factors $\{\alpha_i\}_i$ as bargain powers and imposes equal utility penalties for users with equal bargain powers, as in (16), whereas in the NBS, the bargain powers α_i serve as utility weights as in (22), and fairness is implicitly realized by setting constraints on the minimum SINRs $\{\gamma_{i,\min}\}$ for all users. To compare both using a common fairness metric, we adopt the well-known Jain's fairness index defined as

$$J(x_1, x_2, \ldots, x_N) = \frac{\left(\sum_{i=1}^{N} x_i\right)^2}{N\sum_{i=1}^{N} x_i^2} \tag{27}$$

where x_i, $i = 1, \ldots, N$ is the SINR obtained by the SUs, and N is the total number of SUs in the game. As a benchmark, we also compare with the NES of the noncooperative NPCG algorithm.

Jain's fairness index for various values of N is depicted for the three game solutions in Fig. 7. It is confirmed that the cooperative schemes (NBS and KSBS) are indeed fairer than the noncooperative game (NPCG). There is a considerable performance gap between these two families of games. As N increases, Jain's fairness index decreases accordingly. This is because the mutual interference among these N spectrum-sharing SUs becomes increasingly severe.

Finally, we compare the spectrum-utilization efficiency of these game schemes in terms of the achieved SINR after convergence, for a total of $N = 10$ or $N = 30$ SUs (see Fig. 8). As N increases, some SUs will end up with a smaller SINR due to interference from other competing SUs, but the total SINR still increases since the user diversity order increases. When the NPCG is employed, the sixth SU cannot maintain the SINR threshold well, transitioning to a drop in SINR performance. For NBS, NBPCG maintains the SINR level until the 12th SU, whereas the KSBS provides good SINR performance for all SUs. Their behaviors are consistent with the achieved fairness metrics depicted in Fig. 7.

V. Summary

In this paper, we have formulated a cooperative NBPCG model for CR networks. In the model, the transmission quality of both PUs and SUs is respectively considered and modeled by the IPCs for PU protection and the minimum SINR requirements for the SUs. An SINR-based utility function is designed to comply with all the axioms in the Nash theorem, which guarantees the uniqueness and proportional fairness of the game equilibrium. The coupled IPCs are properly transformed to a pricing function to facilitate distributed implementations. Reflecting different user fairness policies, two Pareto-optimal algorithms are developed for the NBPCG model, including the KSBS and the NBS. The adaptive implementation of NBS offers tracking capability for slow-varying channel conditions. On the other hand, the user-fairness policy in KSBS imposes equal utility penalties to all users with the same bargaining power, which is desirable for competitive CRNs.

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