Resource Pricing with Primary Service Guarantees in Cognitive Radio Networks: A Stackelberg Game Approach

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***Abstract*—Cognitive Radio (CR) is envisioned as one of the most promising technologies to solve the problem of spectrum scarcity. A critical problem that limits the development of CR technology is the absence of a mechanism efficient enough to allocate under-utilized spectrum resource from primary users (PUs) to multiple secondary users (SUs) [1], [2]. Toward this end, we adopt "Game Theory" [3], [4] as our modeling tool. Spectrum users can be modeled as selfish, rational players in this spectrum selling game with PUs as spectrum sellers and SUs as buyers. Considering that PUs always know more about the "market" than SUs and marking prices should always be ahead of purchasing, we use a *Stackelberg Game* to analyze the pricing and allocation process of PUs and SUs under the circumstance of information asymmetry. A parameter *I* is later introduced to measure the negative impact from SUs to PUs. If given a predefined value of *I*, a feasible pricing region can be found to guarantee the primary service. Finally, a contract between PUs and SUs is proposed as a counterpart of the information asymmetry and a utility increase of SUs is observed when the contract is valid.**

I. Introduction

Currently, wireless communication networks suffer from the scarcity in spectrum resource and inefficiency in spectrum usage. Cognitive radio has been viewed as a novel approach for improving the utilization of spectrum resources. Secondary users (SUs) may access and use the channels when they are idle from primary users (PUs). In CR networks, increased spectrum utilization is achieved through spectrum sharing. A good spectrum sharing mechanism should be efficient enough and in a distributed way since a centralized control agency may be hard to realize.

In this paper, we view spectrum sharing as a spectrum trading process, i.e., spectrum pricing and purchasing, in which a market is formed by allowing the trading of spectrum between SUs and PUs. Spectrum trading gives SUs a chance to transmit and increases the total revenue of PUs at the same time. As the seller of spectrum, a PU first marks the price of a unit of spectrum according to the quality of spectrum and the demand from SUs. Then a SU decides on how much spectrum to buy after observing the price. These factors are incorporated into the utility functions of PUs and SUs. In reality, PUs that are often referred to as primary service providers may know more about the market than SUs. In order to depict this information asymmetry, the spectrum trading process is modeled as a *Stackelberg Game* [6], [4] in which sellers actually control the outcome of the game since they know more. Furthermore, the market is modeled as an oligopoly one in which sellers are so few compared with buyers that the action of any one of them will materially affect prices and has a measurable impact on peers.

What PUs and SUs can gain when they enter the market is defined respectively by the profit and utility function. Intuitively, the profit function should be the revenue from both primary and secondary service minus the cost of leasing the spectrum to SUs. PUs mark the prices of spectrum aiming at maximizing their profits. SUs then decide on their demands of spectrum according to the marked price and channel quality. Since we assume that PUs know the utility function of SUs, PUs are actually leaders and SUs are followers in this game. A leader selects a strategy first, and then the follower chooses its own according to the strategy it observes from the leader. These strategies can be obtained using backward induction which starts from the demand of followers and ends at the strategy of the leaders [4].

As there are multiple PUs, so SUs can buy a portion of spectrum from several different PUs at the same time. As there are more than one spectrum provider, PUs will compete with each other in pricing process to get higher revenues. Eventually, such competition will reach an equilibrium point that everybody's strategy is the optimal one considering others'. This is referred to as a *Bertrand Game* [3], [4]. Then, the service guarantee of PUs is taken into account [1], [13]. We introduce a parameter ***I*** to indicate the negative impact from SUs and this should be kept under a certain threshold to guarantee the successful transmission of PUs. Finally, we discuss about a contract [5] that could be made between PUs and SUs that the PUs will act in a unselfish way to help SUs gain higher utility and get reward from SUs.

The rest of the paper is organized as follows. We discuss related research work in section II. The network model and a pricing process is presented in section III. In section IV, we address the constraint of negative impacts on the outcome of the game. A contract is proposed in section V aiming at obtaining higher profit for SUs. Various simulation results are presented in section VI to validate the theoretical results in previous sections.

II. Background and Related Work

CR is considered as a novel approach for improving spectrum utility. Recently, the application of Game theory in CR networks became a hot topic. In [10], the authors studied the

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spectrum sharing problem in the context of 802.11 networks. Adjacent APs must use different channels in order to avoid interference. The concept of bargain is used in decreasing Price of Anarchy. In [14], the problem of pricing mechanism between the operator and the service users is formulated as a multi-unit sealed-bid auction. The concept of collusion is used in [8] to drive the solution from Nash equilibrium to social optimal. Machine Learning is used in market-based resource management in [9], which deploys reinforcement learning to steer the operation of CR towards the greater network good. In [12], the authors discuss the joint power and rate control problem. The QoS constraint is also addressed in this paper. The total interference power caused by SUs to each PUs must not exceed a predefined threshold to guarantee the successful transmission of PUs. To the best of our knowledge, most papers in CR area focus on the simplified scenario of one SU instead of multiple SUs. And metrics that guarantee the primary service are seldomly found in existing research work. This paper introduces a parameter which represents the level of negative impact from SUs and a contract is also proposed aiming at higher social gain.

III. Network Model and Spectrum Pricing

A. Underlying Assumptions

In our network model, there are M primary service providers as licensed users and the owners of spectrum. There are N SUs who rent idle spectrum whenever there is an opportunity. M is a relatively small number compared to N. But, this strategy changes once a PU has a substantial impact of the whole "market". Spectra owned by PUs are heterogeneous which means that they have different channel qualities and substitutability factors. It is assumed that PUs acquire the utility functions of SUs through many times of spectrum selling. As selfish entities, they will rationally choose the prices that lead to their own maximal revenues. In the *Stackelberg Game*, PUs are modeled as the leading players and SUs as the followers. In the first stage of the game, the strategy space for the leading player is the price vector, denoted here as $\mathscr{P}(p_1, ..., p_i, ..., p_n)$. In the second stage, SUs make their decisions after observing the strategies of PUs which is denoted as the demand vector $\mathscr{D}(\mathbf{D}_1(\mathscr{P}), ..., \mathbf{D}_i(\mathscr{P}), ...\mathbf{D}_m(\mathscr{P}))$ where $\mathbf{D}_i(\mathscr{P})$ is the demand of the i^{th} SUs in the form of $\mathbf{D}_i(d_{i1}, ..., d_{ij}, ..., d_{in})(\mathscr{P})$. And d_{ij} denotes the demand of the i^{th} SUs from the j^{th} PUs.

B. Transmission Model

Here, we adopt a widely used transmission model. Transmission efficiency k is used to denote the channel quality. PUs and SUs may have different transmission efficiencies when using the same channel, so $k^{(s)}$ denotes the parameter for SUs and $k^{(p)}$ is that for PUs. With adaptive modulation, the transmission rate can be dynamically adjusted based on the channel quality. Therefore, k can be obtained as follows:

$$k = \log_2(1 + K\gamma), \text{ where } K = \frac{1.5}{\ln(\frac{0.2}{BER^{tar}})} \tag{1}$$

Where γ is the signal-noise-ratio at the receiver and BER^{tar} is the target bit-error-rate.

C. Utility Function of SUs

Since the number of SUs is more than that of PUs and spectra are heterogeneous, we employ a commonly used utility function in oligopoly, differentiated market [6] to quantify the utility gain of SUs when they enter the network. Spectrum substitutability is taken into account as a parameter ν:

- $\nu_i = 0$ means that the i^{th} SU can not switch among different channels.
- $\nu_i \in (0, 1)$ means that the i^{th} SU can switch between different channels but there will be a certain cost for doing so.
- $\nu_i = 1$ means that i^{th} SU can switch among different channels freely, which is the ideal case for CR networks.

Actually, when $\nu_i = 0$, it is no longer a case of CR since SUs can not switch between channels. $\nu = 1$ is the other extreme case that will not happen in reality either. Thus, we will only consider $\nu_i \in [0.1, 0.6]$ for the simulation of section VI. Here, we present the utility function for SUs as follows:

$$\mathscr{U}_i(\mathbf{b}_i) = \sum_{j=1}^{N} b_{ij} k_{ij}^{(s)} - 1/2(\sum_{j=1}^{N} b_{ij}^2 + 2\nu_i \sum_{j \neq k}^{N} b_{ij} b_{ik}) - \sum_{j=1}^{N} p_j b_{ij} \tag{2}$$

Where $\mathscr{U}_i(\mathbf{b}_i)$ denotes the utility function of the i^{th} SU. $\mathbf{b}_i = (b_{i1}, ..., b_{ij}, ..., b_{in})$ is the spectrum sharing vector where b_{ij} is the spectrum shared by the i^{th} SU with the j^{th} PU. ν_i denotes the spectrum substitutability of the i^{th} SU, and also note that $k_{ij}^{(s)}$ is the spectral efficiency of wireless transmission by a i^{th} SU using spectrum owned by the j^{th} PU. Since the utility function is concave and quadratic, it is able to depict the satisfaction saturation of SUs. The quadratic also guarantees that we can get the bandwidth demand by differentiating the function which leads to a set of linear functions.

Note that utility is a function of the prices marked by PUs in the first stage. Every SU chooses the optimal strategy $\mathbf{b}_i^*$ to maximize its utility. This utility-maximal strategy is computed by solving the first-order condition of the function above, i.e., $\frac{\partial \mathscr{U}_i(\mathbf{b}_i)}{\partial b_{ij}} = 0$. Solving the set of linear functions, we get the spectrum demand of the i^{th} SU from the j^{th} PU as:

$$d_{ij}(\mathscr{P}) = \frac{(k_{ij}^{(s)} - p_j)(\nu_i(N-2)+1) - \nu_i \sum_{j \neq k}(k_{ik}^{(s)} - p_k)}{(1-\nu)(\nu(N-1)+1)} \tag{3}$$

Summing up the demand from every SU, we get the total demand for the j^{th} PU as:

$$\mathbf{D}_j(\mathscr{P}) = \sum_{i=1}^{M} \frac{(k_{ij}^{(s)} - p_j)(\nu_i(N-2)+1) - \nu_i \sum_{j \neq k}(k_{ik}^{(s)} - p_k)}{(1-\nu)(\nu(N-1)+1)} \tag{4}$$

D. Profit of PUs

The profit of a PU equals to the revenue from both primary and secondary service minus the cost due to spectrum sharing.

We model the cost of PUs as the service degradation caused by sharing spectrum with SUs [12].

$$\mathscr{C}_j(b_j) = c_2 M_j \left(B_j^{req} - k_j^{(p)} \frac{W_j - b_j}{M_j} \right)^2 \quad (5)$$

In Eqn. (5), $\mathscr{C}_j(b_i)$ denotes the cost of PU j due to sharing bandwidth b_i with SUs. M_i is the number of primary connections and B_j^{req} denotes the required bandwidth of one primary connection. $k_i^{(p)}$ is the spectral efficiency of wireless transmission for PUi. W_j is the total bandwidth of PUj.

Since we have already computed the bandwidth demand $\mathbf{D}_j(\mathscr{P})$, b_j should be substituted by the demand. Note here that the spectrum demand could exceed the total bandwidth W_j possessed by PUj. So, the modified cost function should be:

$$\mathscr{C}_j(b_j) = c_2 M_j (B_j^{req} - k_j^{(p)} \frac{W_j - \max(\mathbf{D}_j(\mathscr{P}), W_j)}{M_j})^2 \quad (6)$$

The revenues from primary and secondary services are $\mathscr{R}_j^1 = c_1 M_j$ and $\mathscr{R}_j^2 = p_j \mathbf{D}_j(\mathscr{P})$, respectively. Thus, we can write the profit of the j^{th} PU as follows:

$$\begin{aligned}\mathcal{P}_j(\mathscr{P}) &= p_j \mathbf{D}_j(\mathscr{P}) + c_1 M_j - \\ &c_2 M_j \left(B_j^{req} - k_j^{(p)} \frac{W_j - \max(\mathbf{D}_j(\mathscr{P}), W_j)}{M_j} \right)^2\end{aligned} \quad (7)$$

E. Bertrand Game Model for Pricing

At first, the primary profit formula is a function of demand $\mathbf{D}_j$ and service price $\mathscr{P}$. Since $\mathbf{D}_j(\mathscr{P})$ is a function of $\mathscr{P}$, the final primary profit function purely depends on the spectrum prices vector. But, there are multiple PUs and they will compete with each other on revenue. Every PU marks its price according to the prices of other service providers in order to maximize its gain. At last, the prices will reach the Nash Equilibrium that everyone's price is the best choice considering the prices of others. The Nash Equilibrium can be computed by solving the first order condition of the game, i.e., $\frac{\partial P_j(\mathscr{P})}{\partial p_j} = 0$. Intuitively, we know that the higher the price, the lower the demand which is the case of a real market. The equilibrium point can be obtained given all the parameters we need, but it may become very hard when the number of PUs and SUs increase.

We introduce the following notations to simplify the solution. If we let

$$d_{ij}^1(\mathscr{P}_{-j}) = \frac{k_{ij}^{(s)}(\nu_i(N-2)+1) - \nu_i \sum_{j \neq k}(k_{ik}^{(s)} - p_k)}{(1-\nu_i)(\nu_i(N-1)+1)}$$

$$d_{ij}^2 = \frac{\nu_i(N-2)+1}{(1-\nu_i)(\nu_i(N-1)+1)}$$

Then $d_{ij}(\mathscr{P})$ defined in (3) can be rewritten as $d_{ij}(\mathscr{P}) = d_{ij}^1(\mathscr{P}_{-j}) - d_{ij}^2 p_j$, and the total demand from PU j is $\mathbf{D}_j(\mathscr{P}) = \sum_{i=1}^M d_{ij}^1(\mathscr{P}_{-j}) - p_j \sum_{i=1}^M d_{ij}^2$. Every PU will choose the optimal strategy considering those of other players in order to maximize its profit and the selfish act will finally drive this Bertrand Game to a Nash Equilibrium. So the solution is given by the first order condition which is also the best response of the j^{th} PU given other PUs' prices.

As mentioned earlier, PUs may not have enough spectrum to meet the demand from SUs. So there are two situations here. When PUs can meet the demand of SUs, we have:

$$\begin{aligned}&2c_2 k_j^p \sum_{i=1}^M d_{ij}^2 \left(B_j^{req} - k_j^{(p)} \frac{W_j - (\sum_{i=1}^M d_{ij}^1(\mathscr{P}_{-j}) - p_j \sum_{i=1}^M d_{ij}^2)}{M_i} \right) \\ &+ \sum_{i=1}^M d_{ij}^1(\mathscr{P}_{-j}) - 2p_j \sum_{i=1}^M d_{ij}^2 = 0\end{aligned} \quad (8)$$

When PUs can not meet the demand of SUs, we have:

$$\sum_{i=1}^M d_{ij}^1(\mathscr{P}_{-j}) - 2p_j \sum_{i=1}^M d_{ij}^2 = 0 \quad (9)$$

IV. Negative Impact from SUs

In this section, we discuss the Nash Equilibrium with a constraint because of the primary service guarantee. In CR networks, primary service agents provide spectrum to SUs in order to gain higher spectrum utilization and total revenue. But the leasing of spectrum holes should not threaten the operation of PUs. In the game that we presented above, PUs can lease as much spectrum as they have if such act brings about a higher total revenue. But, the primary service PU offers will fail in this case. So, we need to keep the negative impact from SUs under a perdefined threshold. We have already defined a parameter B^{req} as the bandwidth required to guarantee the successful transmission of primary service. When SUs enter the network, the spectrum available to PUs is decreased by $\mathcal{D}_j$. Specifically, the negative impact from SUs means that the degree of spectrum scarcity of PUs after they lease spectrum out. We can formulate the threat from SUs as $(B_j^{req} - k_j^{(p)} \frac{W_j - \max(\mathbf{D}_j(\mathscr{P}), W_j)}{M_j})^2$.

Because there are multiple SUs, average threat from all SUs is taken into account. We introduce a parameter, namely $\boldsymbol{I}$, to measure the negative impact from SUs to PUs. If constraint of $\boldsymbol{I}$ is too tight, the spectrum utilization will not be efficient enough. But if it is too loose, the primary transmission will be greatly affected by SUs.

After considering the negative impact from SUs, the equilibrium problem with service guarantee constraint is

$$\begin{aligned}&\frac{\partial P_j(\mathscr{P})}{\partial p_j} = 0 \\ &\text{with } \sum_{j=1}^N \left(B_j^{req} - k_j^{(p)} \frac{W_j - \max(\mathbf{D}_j(\mathscr{P}), W_j)}{M_j} \right)^2 \leq N\ (\boldsymbol{I})\end{aligned} \quad (10)$$

Note that $\boldsymbol{I}$ here does not mean the real interference from SUs to PUs. It is a parameter that indicates the impact from SUs which causes the spectrum scarcity of PUs.

Let $\mathscr{P}^c = (p_1^c, p_2^c, ..., p_n^c)$ denote the critical price vector which is the solution of the game when the negative impact reaches NI. When $N = 3$, geometrically, the critical situation corresponds to a surface in the Euclidean space. But when N is over 3, the first equation in Eqn. (10) indicates a super plane in Euclidean space $\mathbb{R}^N$. The region between this super plane and the first quadrant is the feasible space for the Nash Equilibrium point. Figure 1 shows the possible situation of three PUs. The three plains in the Figure 1 correspond to the best response functions of different PUs, respectively. The Nash Equilibrium is indicated by the intersection point of these three planes.

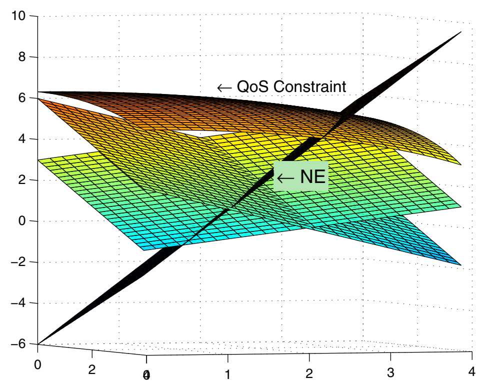

Fig. 1. constraint curve and Nash equilibrium point

V. Contract Between PUs and SUs

A. Information Asymmetric

The role of PUs and SUs are not symmetric in a *Stackelberg Game*. PUs know more about the resource market than SUs. When we get the outcome of the game by solving the first order condition of multiple PUs, which are: $\frac{\partial P_j(\mathscr{P})}{\partial p_j} = 0$, we implicitly assume that PUs know the utility functions of SUs. This is quite reasonable in the real market since PUs who are resource owners that lease spectrum to all SUs can learn from every transaction. They may observe the behavior of various SUs to get their utility functions. In this way, SUs are so passive that the price and profit are actually controlled by PUs. Naturally, PUs will selfishly set the price towards their own good which is the outcome concluded from previous sections.

B. Using Contract Theory to Gain More Profit

In an information asymmetric market, a contract can be made to obtain higher social well-being. Actually, PUs can set prices which are different from those derived from the Nash Equilibrium and as a result, help SUs gain more utility. The problem lies in that the new prices are not a stable point and no PU is willing to stay in that point, so a contract between PUs and SUs have to be made to ensure that PUs will not deviate because of selfishness. The contract can be that after SUs have gained more utility, they reward PUs with other utility such as money or helping PUs transmit in a cooperative way. Here, we only assume that there exists such a contract rather than going through the details of the contract. With a valid contract, the prices will be made towards the highest utility of SUs. Knowing the correlation between spectrum demand and the price, the utility function of SUs becomes:

$$\mathscr{U}_i(\mathbf{D}_i(\mathscr{P})) = \sum_{j=1}^{N} d_{ij}k_{ij}^{(s)} - 1/2\left(\sum_{j=1}^{N} d_{ij}^2 + 2\nu_i \sum_{j\neq k} d_{ij}d_{ik}\right) - \sum_{j=1}^{N} p_j d_{ij} \tag{11}$$

Considering the rule of the contract, the prices are determined by solving the first order condition of the SUs' utility function shown below instead of that of PUs', i.e., $\frac{1}{p_j}(\sum_{i=1}^{N} \mathscr{U}_i(\mathbf{D}_i(\mathscr{P}))) = 0$.

VI. Simulation and Discussion

A. Simulation Parameters

In the following simulation results, we will show various results based on the assumption that there are two PUs and one SUs co-existing in a CR environment. The target bit-error-rate is 10^{-4}. The constant parameter c_1 and c_2 in the cost function is set to be 2. The required bandwidth of a primary connection is 2MHz. The channel quality for primary service is set to be 8dB. Considering that the channel quality for secondary service sometimes can exceed that of PUs', so it set to be between 5dB and 20dB. Since it is impossible for SUs to switch freely among different channels or just stay in one channel, the channel substitutability ν is between 0.1 to 0.6.

B. Best Response Function and Nash Equilibrium

Given the best response functions of users, the Nash Equilibrium point is just the intersection of all these best response functions. Figure 2 shows the Nash Equilibrium point with different channel substitutabilities. Figure 3 shows the Nash Equilibrium point with different channel qualities. Combining Figures 2 and 3, we obverse that the slope of the best response function is determined by channel substitutability. When the substitutability is high, the equilibrium prices are relatively low because SUs rely less on any of the two channels. The position of the function is determined by channel quality. Specifically, the better the quality is, the higher the prices are, which is intuitive. We can also see from the figures that the best responses of PU 1 and 2 are inverse to each other, since the parameters of these two channels are totally the same.

C. Primary Service Guarantee

Here, we illustrate the simplest case of two PUs and two SUs (Figure 1). Two straight lines in Figure 4 indicate the best response functions of user 1 and 2, respectively. The point where they intersect with each other is the Nash Equilibrium point. The quadratic curves in Figure 4 are the interference constraint curves which are given by Eqn. (10) with different interference parameter $\boldsymbol{I}$. We can see from Figure 4 that the Nash Equilibrium point of the game is not a feasible solution when $\boldsymbol{I}$ is 200. But after we increase this parameter to 250, the Nash Equilibrium is a feasible solution.

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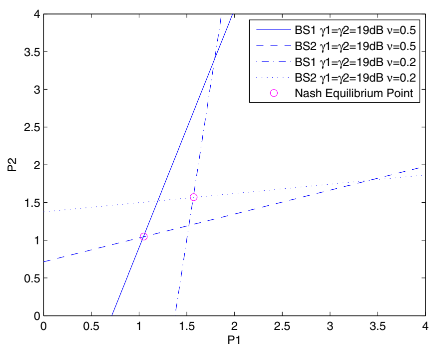

Fig. 2. Best Response Function with Different Channel Substitutability

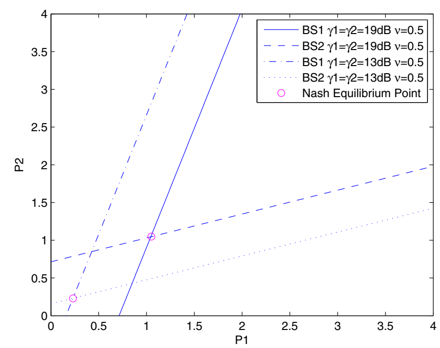

Fig. 3. Best Response Function with Different Channel Quality

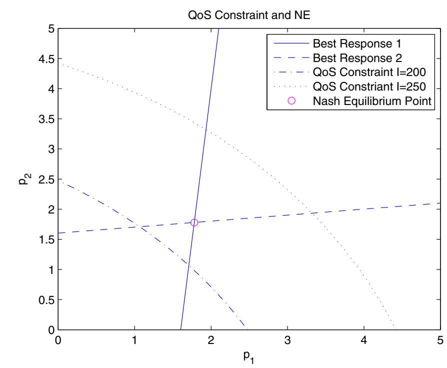

Fig. 4. Different QoS Constraint on Nash Equilibrium

D. *Contract and the Increase in Utility*

Contract between PUs and SUs can increase the utility of SUs. Figure 5 shows this increase. The red (top) line which means the utility obtained with a contract is above the blue (bottom) line which is the case when no contract is used. This indicates that a contract is very effective in increasing the utility of SUs. PUs sacrifice their own interests driving the outcome of the game towards the well-being of SUs. Then SUs can refund part of the increase to PUs which motivates PUs to follow the contract instead of deviating from it.

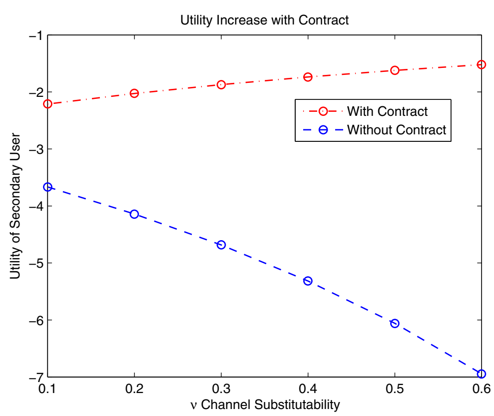

Fig. 5. Utility Increase of SUs with Contract

VII. Conclusion

In this paper, we investigated the resource allocation problem in CR networks. We used the *Stackelberg Game* to model and solve the pricing process of PUs. The situations of both enough and scarce spectra are addressed in this paper. Then, we presented the strategy to get the Nash Equilibrium point and show that strategy by simulation. Primary service guarantee is a very important issue in CR networks since the spectrum sharing with SUs should not threaten the transmission of PUs. In addition, we have introduced a parameter $\boldsymbol{I}$ which quantifies the negative impact from SUs to PUs, thus primary service can be guaranteed by keeping this parameter below a predefined threshold.

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