Optimal Fractional Frequency Reuse (FFR) and Resource Allocation in Multiuser OFDMA System

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***Abstract*—In this paper we determine the optimal Fractional Frequency Reuse (FFR) and resource allocation in OFDMA system. Since the users at the cell edge are more exposed to inter-cell interference therefore each cell is partitioned into two regions; inner region and outer region. We determine the optimal FFR factor for the outer region, bandwidth assigned to each region and subcarrier and power allocation to all the users in the cell. The problem is formulated as sum-power minimization problem subject to minimum rate constraints in both the regions. This is a mixed linear integer programing problem which is relaxed into a convex optimization problem. We develop an efficient algorithm by using Lagrange dual decomposition theory at reasonable computational cost.**

I. Introduction

Scarce bandwidth, limited power resources and intercell interference require intelligent resource allocation strategies for multiuser mobile networks. Orthogonal Frequency Division Multiple Access (OFDMA) is a promising physical layer multiple access technique which greatly aids in overcoming these challenges. It has been adopted for both uplink and downlink air interfaces of WiMAX fixed and mobile standards, i.e IEEE802.16d and IEEE802.16e respectively [1], [2]. It has also been adopted by Third Generation Partnership Project (3GPP) LTE (Long Term Evolution) downlink air interface [3]. OFDMA is based on OFDM technique and it divides the wideband frequency selective channel into a set of narrow band channels. These subchannels are orthogonal to each other and Inter Symbol Interference (ISI) is mitigated by adding a cyclic prefix. OFDMA has thus multi-carrier nature and provides enormous opportunities for dynamic resource allocation strategies [4]-[7]. In multiuser system different users are located at varying distances from the Base Station (BS) and have varying channel conditions on the subcarriers. Therefore, dynamic allocation of subcarriers allow for efficient exploitation of multiuser diversity in the system [4]. Moreover, power can also be dynamically allocated according to waterfilling principle over the inverse of channel gains [5].

In order to attain user satisfaction, minimum Quality-of-Service (QoS) guarantees should be provided to all the users. In this paper QoS requirements are in the form minimum bit rates. However, in providing these minimum bit rates to all the users, system throughput has to be sacrificed because the users with bad channel conditions even when they are transmitting on their best subcarriers consume lot of power. Normally if the frequency is universally reused in every cell then the users at the cell edge experience bad channel conditions and suffer from higher levels of intercell interference. This interference results from the use of same subcarriers in the adjacent cell in the same time slot. In order to reduce intercell interference three methods are currently being considered in IEEE802.16e [2] which are, 1) Inter-cell-interference randomization, 2) Inter-cell-interference cancellation and 3) Inter-cell-interference co-ordination/avoidance. Inter-cell-interference randomization by using cell scrambling or interleaving does not improve the performance of cell edge user since average interference level is not reduced while Interference cancellation can be achieved at the cost of huge computational complexity and specific receiver capabilities. Therefore, the best solution is Inter-cell-interference co-ordination/avoidance such as Fractional Frequency Reuse (FFR). In FFR the cell edge users operate with only a fraction of the total bandwidth while the users near the BS operate with the whole bandwidth. In this way the cell is effectively divided into an inner and an outer region. The users in the outer region now experience greater path loss but no inter cell interference while the users in the inner region experience intercell interference due to universal frequency reuse. In the literature, it is commonly known to use a frequency reuse factor equal to 3 for the outer region. This value is not issued from optimization analyses but from simulation and implementation issues.

In this paper we consider the margin adaptive resource allocation problem where the objective is sum-power minimization subject to minimum data rate constraints of all the users in both the regions. By carrying out this optimization we aim to determine, 1) the optimal FFR factor for the outer region, 2) bandwidth assignment for each region 3) optimal subcarrier allocation for each user and 4) optimal power allocation for each user. This problem is a combinatorial mixed linear integer programming problem with huge computational complexity. We convert the problem into a standard convex optimization problem and develop an efficient algorithm to find the optimal solution at reasonable computational cost.

The rest of the paper is organized as follows. In section II we provide the state of the art. The optimization problem is formulated in section III. We solve the problem and develop the algorithm in section IV. Simulation results are presented in section V while the paper is concluded in section VI.

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II. Related Work

Fractional Frequency Reuse (FFR) is based on the idea of applying a frequency reuse of one in areas close to the base station, and a higher reuse in areas closer to the cell border. This idea was first proposed for GSM networks (e.g. [8]) and has been adopted in the WiMAX forum [9], but also in the course of the 3GPP Long Term Evolution (LTE) standardization, e.g., in [10] and [11], where the focus lies on practically implementable algorithms. Several variations of such a scheme are possible. The proposals that tightly couple resource management and interference coordination fall into two categories, namely soft frequency reuse discussed in [10] and partial frequency reuse originally proposed in [12]. In [12], the reuse 1 and reuse 3 areas are on disjoint frequency bands, while [10] and [11] use the full set of available resources in the reuse 1 areas and one third of the same resources in the reuse 3 areas. Variations are also possible with respect to the transmit power level in each of the areas. In [10], the reuse 1 areas are covered with a reduced power level, while in [11] the transmit power of interfering base stations is reduced. In [13], the average cell capacity under different frequency reuse schemes is estimated in various scenarios. In [14], the authors investigated a global interference coordination scheme with beamforming antennas and full system knowledge in a dynamic 802.16e-system. Despite the fact that such a global scheme is not realizable, it provides an important reference for future distributed solutions. In [15]-[16], the authors study the impact of limited coordination between base stations. They use the full set of resources for the reuse 1 areas and one third of the same resources for the mobiles in the reuse 3 areas. The power is not controlled as part of the interference coordination, but in the course of the burst profile management.

In this paper we develop the problem of resource management and interference avoidance as an optimization problem. The optimal solution of this optimization problem determines the optimal FFR factor for the outer region, the bandwidth assigned to each region and the subcarrier and power assignment to all the users in the two regions.

III. System Model and Problem Formulation

We consider a time slotted multiuser downlink OFDMA system with K users in a given cell. The data transmission is on time frame basis and each frame consists of D OFDM symbols. User channels remain constant for the duration of a frame but may change from one frame to another. We assume that perfect Channel State Information (CSI) is available at the BS. Each user demands a minimum data rate which we denote by R^k_{min}. We assume that each cell is partitioned into two regions which we call the inner and the outer regions. We denote the inner region by $\mathcal{I}$ and the outer region by $\mathcal{O}$. The users in the inner region experience intercell interference from the neighboring cell because of the universal frequency reuse in this region. Let I be the average value of intercell interference experienced by the users in any given cell from any other cell. Let L be the total number of cells in the network, then due to universal frequency reuse the channel gain to noise ratio (GNR) of user k on subcarrier f in the inner region is given by,

$$g^I(k,f) = \frac{|h(k,f)|^2}{(\sigma^2 B) + (L-1)I} \tag{1}$$

where, $|h(k,f)|$ denotes the channel coefficient of user k on subcarrier f after Fast Fourier Transform (FFT), σ^2 is the power spectral density (PSD) of white noise and B denotes the bandwidth of a subcarrier. We assume that $M > 1$ is the FFR factor for the outer region. The GNR of user k on subcarrier f in the outer region is,

$$g^O(k,f) = \frac{|h(k,f)|^2}{(\sigma^2 B) + (\frac{L}{M}-1)I} \tag{2}$$

The interference from all those cells using a different subcarrier set in their outer regions is canceled. Let N be the total number of subcarriers in the system then,

$$N = N_I + MN_O \tag{3}$$

where, N_I and N_O are the subcarriers for the inner and the outer regions respectively. Thus the subcarriers in each cell for a given frequency reuse factor M becomes,

$$F = N_I + N_O \tag{4}$$

Let $\mathbb{I}(k,f)$ be the subcarrier allocation index which is set to 1 or 0 depending on whether the subcarrier is allocated to a specific user or not. Let $p(k,f)$ be the power allocated to user k on subcarrier f. Then, the sum-power minimization or the margin adaptive optimization problem in any cell can be written as,

$$\min \sum_{k=1}^{K}\sum_{f=1}^{F} p(k,f) \tag{5}$$

subject to the constraints,

$$\sum_{f=1}^{N_I} \mathbb{I}(k,f) r(k,f) \geq R^k_{min}, \quad k \tag{6}$$

$$\sum_{f=1}^{N_O} \mathbb{I}(k,f) r(k,f) \geq R^k_{min}, \quad k \in \mathcal{O} \tag{7}$$

where, $r(k,f)$ is the data rate allocated to user k on subcarrier f. Constraints (6) and (7) demand that the data rates allocated to the users should be greater than their minimum demanded rates. The optimal solution of this problem determines the FFR factor M for the outer region, percentage of bandwidth allocated to each region (N_I and N_O), subcarrier allocation index $\mathbb{I}(k,f), \forall f$ and the power allocation $p(k,f), \forall k, f$. However, this problem is a mixed linear integer programming problem. The integer variables are M, N_I, N_O and $\mathbb{I}(k,f)$ while $p(k,f)$ and $r(k,f)$ are the continuous optimization variables. Due to the presence of the integer constraints this problem has a combinatorial nature. Such type of problems have exponential computational complexity and are not too easy to solve. In the next section we propose the optimal solution for this problem.

IV. Resource Allocation Algorithm

We develop the optimal solution to our problem by relaxing the integer constraints. The integer variable $\mathbb{I}(k,f), \forall f$ arise from the OFDMA constraint which requires that each subcarrier be uniquely assigned to one and only one user. This constraint can be relaxed if users are allowed to share the subcarriers without destroying the OFDMA nature of the problem. Since time is an orthogonal dimension relative to the frequency and user channels remain constant for the duration of a time slot we allow subcarrier sharing in time. We introduce a time sharing factor $\gamma(k,f) \in [0,1]$ for kth user on subcarrier f. During a frame user k is allowed to transmit on subcarrier f in $D\gamma(k,f)$ OFDM symbols. This assumption on time sharing of subcarriers introduces the following constraint in the problem (5),

$$\sum_{k=1}^{K} \gamma(k,f) \leq 1 \quad \forall f \tag{8}$$

As a result of time sharing, the data rate achieved by user k on subcarrier f becomes,[1],

$$r(k,f) = \gamma(k,f) \log \Big(1 + p(k,f)g(k,f)\Big) \tag{9}$$

where, $g(k,f) = g^I(k,f)$ if $k \in \mathcal{I}$ and $g(k,f) = g^O(k,f)$ if $k \in \mathcal{O}$. Eq (9) is neither convex nor concave. We define $o(k,f) = \gamma(k,f)p(k,f)$, as the average power allocated to user k on subcarrier f. With this change of variable we have,

$$r(k,f) = \gamma(k,f) \log \left(1 + \frac{o(k,f)g(k,f)}{\gamma(k,f)}\right) \tag{10}$$

which is a concave function and it can be verified from its Hessian which is negative semi-definite when $\gamma(k,f) \geq 0$ and $o(k,f) \geq 0$. The optimization problem (5) now becomes,

$$\min \sum_{k=1}^{K} \sum_{f=1}^{F} o(k,f) \tag{11}$$

subject to the following constraints,

$$\sum_{f=1}^{N_I} \gamma(k,f) \log \left(1 + \frac{o(k,f)g^I(k,f)}{\gamma(k,f)}\right) \geq R_{min}^k, k \in \mathcal{I} \tag{12}$$

$$\sum_{f=1}^{N_O} \gamma(k,f) \log \left(1 + \frac{o(k,f)g^O(k,f)}{\gamma(k,f)}\right) \geq R_{min}^k, k \in \mathcal{O} \tag{13}$$

$$\sum_{k=1}^{K} \gamma(k,f) \leq 1 \quad \forall f \tag{14}$$

This problem still contains the integer variables N_I and M. Since the number of subcarriers in a network is limited this optimization problem can be solved by decoupling the integer optimization from the convex continuous optimization. In other words, for each Integer N_I value, the problem given by (11) is merely a classical convex optimization problem which can be solved by using convex optimization theory. By considering all the possible $N_I \leq N$ values for a given frequency reuse factor M, the optimal solution can be found. Moreover, for a given value of N_I, the problem can be separated into two independent margin adaptive problems for the two regions. This separation can be achieved because each user can be located in any one region at any given time. We assume that the users at the boundary of the two regions are exclusively assigned to only one region. Let, K_I be the number of users in the inner region and K_O be the number of users in the outer region such that $K_I + K_O = K$, then the two sub-problems become,

$$\sum_{k=1}^{K_I} \sum_{f=1}^{N_I} o(k,f) \tag{15}$$

subject to,

$$\sum_{f=1}^{N_I} \gamma(k,f) \log \left(1 + \frac{o(k,f)g^I(k,f)}{\gamma(k,f)}\right) \geq R_{min}^k, k \in \mathcal{I} \tag{16}$$

$$\sum_{k=1}^{K_I} \gamma(k,f) \leq 1, \quad \forall N_I \tag{17}$$

and for the outer region the problem is,

$$\sum_{k=1}^{K_O} \sum_{f=1}^{N_O} o(k,f) \tag{18}$$

subject to,

$$\sum_{f=1}^{N_O} \gamma(k,f) \log \left(1 + \frac{o(k,f)g^O(k,f)}{\gamma(k,f)}\right) \geq R_{min}^k, k \in \mathcal{O} \tag{19}$$

$$\sum_{k=1}^{K_O} \gamma(k,f) \leq 1, \quad \forall N_O \tag{20}$$

Problems (15) and (18) are convex optimization problems with convex objective and constraints. These problems can be solved by using convex optimization theory [17], [18]. We demonstrate the solution for problem (15). Let, $\left\{\psi_f^I\right\}_{f=1,\ldots,N_I}$ and $\left\{\delta_k^I\right\}_{k=1,\ldots,K_I}$ be the lagrange multipliers associated with the constraints (16) and (17) respectively, then the Lagrangian can be written as,

$$\mathcal{L}\{o(k,f), \gamma(k,f)\} = \sum_{k=1}^{K_I} \sum_{f=1}^{N_I} o(k,f) + \sum_{k=1}^{K_I} \delta_k^I \Bigg(\sum_{f=1}^{N_I} \gamma(k,f)$$

$$\log \left(1 + \frac{o(k,f)g^I(k,f)}{\gamma(k,f)}\right) - R_{min}^k \Bigg) \Bigg) -$$

$$\sum_{f=1}^{N_I} \psi_f^I \Bigg(\sum_{k=1}^{K_I} \gamma(k,f) - 1 \Bigg)$$

[1]Data rates are expressed in nats for analytical convenience.

Since the duality gap is zero we use Lagrange dual decomposition theory to solve this problem [17], [18]. By solving the appropriate Lagrange KKT conditions we get,

$$p(k,f) = \Big(\delta_k^I - \frac{1}{g^I(k,f)}\Big)^+ \tag{21}$$

$$\delta_k^I\Big(\big(\log(\delta_k^I g^I(k,f))\big)^+ - \big(1 - \frac{1}{\delta_k^I g^I(k,f)}\big)^+\Big) = \psi_f^I \tag{22}$$

Similarly for the outer region we get,

$$p(k,f) = \Big(\delta_k^O - \frac{1}{g^O(k,f)}\Big)^+ \tag{23}$$

$$\delta_k^O\Big(\big(\log(\delta_k^O g^O(k,f))\big)^+ - \big(1 - \frac{1}{\delta_k^O g^O(k,f)}\big)^+\Big) = \psi_f^O \tag{24}$$

where, $\{\psi_f^O\}_{f=1,\ldots,N_O}$ and $\{\delta_k^O\}_{k=1,\ldots,K_O}$ are the lagrange multipliers associated with the constraints (19) and (20) respectively. For each value of reuse factor M we have the following algorithm which is given in Table I. In this algorithm, since each user in the inner and the outer region should get at least one subcarrier to satisfy its minimum rate constraint, therefore the initial and terminal values of N_I are set accordingly. In the inner loop, steps (2-a)-(2-g) determine the subcarrier and power allocation for the users in the inner region. The water level δ_k^I of a user is increased till its data rate is achieved and subcarriers and power is accordingly allocated. This process is repeated for all the users till the minimum rate constraints are achieved. The subcarriers are allocated back and forth between different users till convergence and power is allocated according to the waterfilling principle. The solution is of OFDMA type since the subcarrier is always allocated to the best user. In step 3 of this algorithm the same process is repeated for the outer region. Finally for each value of N_I, we get the total power assigned which we denote by P_{margin}. The optimal solution is the one with the minimum power utilization.

TABLE I
RESOURCE ALLOCATION ALGORITHM

Initialization: $i = 0, \quad P = \infty$
Outer loop:
    For, $N_I = K_I, \ldots, (N - MK_O)$
    $N_O = \frac{N-N_I}{M}$
    $i = i + 1$
A) Inner Loop:
1) Initialization:
    $\delta_k^I = \min_{k,f} \frac{1}{g^I(k,f)}, \quad k \in \mathcal{I}, \quad f \in N_I,$
    $\delta_k^O = \min_{k,f} \frac{1}{g^O(k,f)}, \quad k \in \mathcal{O}, \quad f \in N_O,$
    $\phi(k,f) = 0, \forall k, f, \gamma(k,f) = 0, \quad \forall k, f,$
    $\Omega^I = \{\Omega_1^I, \ldots, \Omega_{K_I}^I\} = 0, \quad k \in \mathcal{I},$
    $\Omega^O = \{\Omega_1^O, \ldots, \Omega_{K_O}^O\} = 0, \quad k \in \mathcal{O},$
    $\mathbf{R}^I = \{R_{min}^1, \ldots, R_{min}^{K_I}\}, \quad k \in \mathcal{I},$
    $\mathbf{R}^O = \{R_{min}^1, \ldots, R_{min}^{K_O}\}, \quad k \in \mathcal{O}$
2) While $\Omega^I < \mathbf{R}^I$
  a) For $k = 1, \ldots, K_I$
    b) While $\Omega_k^I < R_{min}^k$
      c) Increase, $\delta_k^I = \delta_k^I + \Delta_m$.
      d) Compute on all $f \in N_I$, $\phi(k,f) =$
        $\delta_k^I\Big(\big(\log(\delta_k^I g^I(k,f))\big)^+ - \big(1 - \frac{1}{\delta_k^I g^I(k,f)}\big)^+\Big)$
      e) If $\phi(k,f)$ is maximum
          $\gamma(k,f) = 1$
        else
          $\gamma(k,f) = 0.$
      f) $\Omega_k^I = \sum_{f=1}^{N_I} \gamma(k,f)\Big(\log\big(\delta_k^I g^I(k,f)\big)\Big)^+$
      g) $P_{margin}^I(i) = \sum_{k=1}^{K_I}\sum_{f=1}^{N_I} \gamma(k,f)p(k,f)$
End While
3) While $\Omega^O < \mathbf{R}^O$
  a) For $k = 1, \ldots, K_O$
      b) Repeat steps (2-b)-(2-f) by replacing δ_k^I, N_I, $g^I(k,f)$ and Ω_k^I by δ_k^O, N_O, $g^O(k,f)$ and Ω_k^O respectively.
      c) $P_{margin}^O(i) = \sum_{k=1}^{K_O}\sum_{f=1}^{N_O} \gamma(k,f)p(k,f)$
End While
B) $P_{margin}(i) = P_{margin}^I(i) + P_{margin}^O(i)$
C) If $P_{margin}(i) < P$, retain the current solution and discard the previous solution.
D) $P = P_{margin}(i)$
End For

V. Simulation Results

We consider a time slotted OFDMA system with 50 subcarriers. The bandwidth of each subcarrier is assumed to be 375KHz. Time is divided into slots and duration of each Transmission Time Interval (TTI) is 1ms. The scenario assumed is urban canyon macro which exists in dense urban areas served by macro-cells. We consider a frequency selective Rayleigh fading channel with exponential delay profile. Path losses are calculated according to Cost-Hata Model [19]. The power spectral density of noise is assumed to be -174dbm/Hz. We assume that there are 18 cells in the network and each cell has a radius of 1Km. The radius of the interior region is assumed to be 2/3 of the maximum cell radius i.e. 667m. We assume that there are 10 users in the cell which are uniformly distributed. Furthermore we assume that 5 users are located in the inner region and 5 in the outer region.

We consider a scenario where all the users demand the same minimum rate. In fig (1) we plot the total power consumed by the users in the two regions for different values of the demanded minimum rate constraints. We plot curves for three different frequency reuse factors for the outer region ($M = 2, 3, 4$). It is evident from the plot that the minimum power is consumed for $M = 3$. For the optimal value of the reuse factor, we determine the number of subcarriers allocated to each region. The optimal value of N_I is 17 which means that 17 subcarriers are universally reused in the inner region while the rest are reserved for the outer region. The number of subcarriers in each cell is then 28.

VI. Conclusion

In this paper we studied the problem of optimal Fraction Frequency Reuse (FFR) and resource allocation in OFDMA system. The cell is divided into inner and outer regions with different set of subcarriers in both the regions. The problem is formulated as a constrained optimization problem. The mixed linear integer programming problem is relaxed and converted into a convex optimization problem. We develop an efficient algorithm to determine the optimal FFR factor for

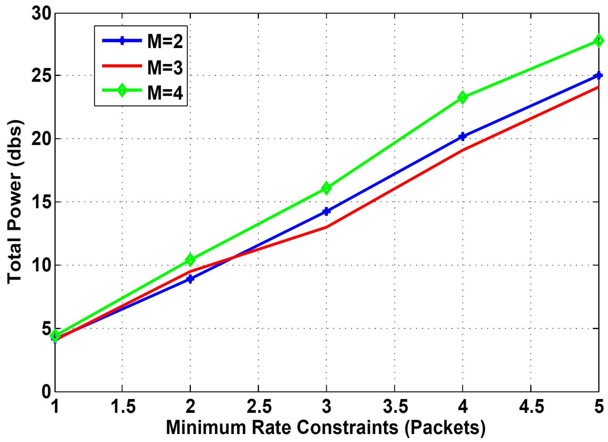

Fig. 1. Total Power vs Minimum Rate Constraints for various FFR factors for the outer region. Cell Radius = 1Km, $K_I = K_O = 5$ users and $N = 50$ subcarriers

the outer region, bandwidth assigned to each region, subcarrier allocation and power allocation to all the users in the cell.

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