Decentralized Inter-Cell Interference Coordination by Autonomous Spectral Reuse Decisions

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***Abstract*— Future wireless packet switched cellular networks will require dense frequency reuse to achieve high capacity. At the same time, measures are required which limit the interference on the frequency carriers. It is assumed that central entities performing the task of interference coordination with global knowledge should be avoided. Rather, distributed algorithms are sought for. To this end, we propose decentralized resource allocation algorithms that enable base stations to select a pool of favorable resources with low interference based on local knowledge only. The actual user-level resource allocation from that pool will then be performed by fast schedulers operating on the preselected resources within each cell. We analyze and evaluate the proposed resource selection algorithms by introducing a simplified wireless network model and applying methods from game theory. Proving the existence of Nash equilibria shows that stable resource allocations can be reached by selfish agents. In addition to that, we perform simulations to determine the speed of convergence and the resulting equilibrium interference levels. By comparing these to an optimal global solution, which is derived by solving an integer linear program, we are able to quantify the efficiency loss of the distributed game approach. It turns out that even though the distributed game results are sub-optimal, the low degree of system complexity and the inherent adaptability make the decentralized approach promising especially for dynamic scenarios.**

I. Introduction

In all well designed cellular systems, the inter-cell (co-channel) interference is the decisive factor ultimately limiting the overall system capacity given a fixed amount of frequency spectrum. Reducing inter-cell interference by interference management techniques is therefore one way to improve system capacity in cellular systems. A classical approach to reduce interference (in the following, “interference” will always be inter-cell interference) employed, e.g. in 2G GSM systems, is frequency planning. With frequency planning the set of available (frequency) resources is not fully re-used in every cell (“full reuse”) so that not all or even none of the neighboring cells re-use the same resource.

On the one hand, a low re-use allows a higher re-use distance between co-channel cells so that interfering signals experience a higher pathloss. This improves the signal to interference and noise ratio (SINR) yielding a higher channel capacity given a certain amount of spectrum. On the other hand, restricting the frequency resources available to each cell reduces the amount of spectrum available and thus the system capacity per cell. Trading off interference level against available spectrum, a system operator aims at maximizing its total system capacity by using some allocation scheme as discussed in the literature, cf. [1] or [2].

Future cellular systems like WiMAX 802.16m or 3GPP Long Term Evolution (LTE) will employ OFDMA (or SC-FDMA for the uplink) as the multiple access scheme. While aiming at a full frequency reuse, some inter-cell coordination mechanisms that limit interference especially for cell-edge users will still be necessary in these systems. One recently discussed compromise between a full re-use and classical frequency planning scheme is a fractional frequency reuse scheme [3]. This scheme allows for full reuse of most resources for users located in the cell centers while edge users are served with resources that are not used in neighboring cells. It therefore offers a way to trade-off full reuse with classical frequency planning.

Common to most frequency allocation schemes is the central control of the resource coordination process by some central entity like, e.g., a radio network controller (RNC). In contrast to this, we will consider a completely decentralized coordination process in this paper. The result of the envisioned coordination process in each cell will be a pool of resources that is not too highly interfered and which can then be used for fast scheduling by the base station.

By assuming that each base station independently aims at maximizing its utility (choose resources with low interference) we are able to model the distributed decision making as a game in the sense of non-cooperative game theory. This will allow us to use solution concepts like the Nash equilibrium to show that the distributed coordination process will iteratively reach a stable state.

There are multiple motivations for doing so. First, the global resource allocation problem is inherently complex meaning that it not only needs global knowledge (for which a lot of signaling is required) but is only computationally tractable using optimization heuristics that may not achieve the global optimum. So in line with recent research on algorithmic game theory [4], it seems plausible to examine if delegating the coordination to selfishly acting cells involves a too high *price of anarchy* in terms of optimality loss. Second, designers of future systems like LTE want to abolish radio network controllers meaning there will be no natural centralized optimization entities that could force base stations to adhere to some frequency plan. To reduce the complexity of network engineering when setting up and extending the radio access network, decentralized coordination performed by the base stations shall allow a plug-and-play operation of the radio resource management through autonomous self-coordination of radio resources. Third, distributed algorithms can be very simple and would therefore allow a much faster adaptation, e.g. to varying traffic demands, than centralized approaches.

The vast majority of literature proposing game theoretic approaches in wireless communications has focused on ad-hoc and/or cognitive radio settings in which, for example, unlicensed secondary users exploit unused "white spaces" in the spectrum allocation of a primary licensed user. More similar to our approach, the authors of [5] and [6] let terminals in an ad-hoc setting choose their frequency channel to transmit on based on the interference level. Contrary to that, in this paper we assume a cellular setting in which base stations aim at choosing a subset of the total resources that is least interfered. That way, they perform an allocation in a distributed fashion that is classically performed centrally by the operator.

To the best of our knowledge, the efficiency loss in comparison to central optimization (the *price of anarchy*) or conventional allocation methods in a cellular setting has not yet been discussed in the literature. However, literature in *algorithmic game theory* [4] considers similar approaches most notably in the area of *selfish routing* [7].

The remainder of the paper is organized as follows. In Section II we introduce both the system as well as our game model. Then, in Section III we prove that under certain conditions the distributed coordination converges to a stable state. In Section IV we compare the efficiency of the distributed allocation to an optimal allocation. Finally, we discuss the results and give an outlook suggesting further research.

II. Modeling Interference Coordination as a Non-Cooperative Game

The interference coordination mechanism presented in this paper is part of a hierarchical resource management process. On a longer time scale, it achieves interference avoidance by restricting the use of certain resources in neighboring cells. To this end, the cells are allocated a set of resources that the actual user/packet scheduling process is allowed to use. On the allocated resource subset the cell's scheduler may perform fast scheduling deciding the mapping of user packets to resources based on instantaneous channel conditions.

In the following, we will first introduce the assumptions for modeling the wireless system. Then, we will give a formal description of the distributed resource allocation process as a normal form game from non-cooperative game theory.

A. Cellular System Model

We assume a simplified model of a cellular system comprising n cells each having the same set of $1 \dots m$ orthogonal resources, e.g., OFDMA chunks or frequency channels. A resource allocation by some cell consists of choosing a subset of resources to be used in that cell. We denote the choice

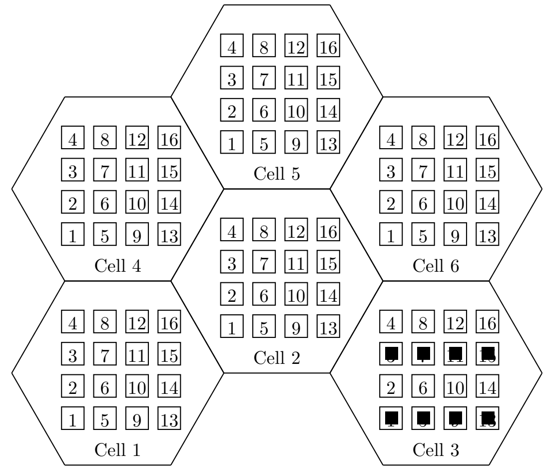

Fig. 1: Example scenario with 6 cells and 16 channels each. In cell 3 eight channels have been marked as allocated by that cell.

in cell i as the m-dimensional binary vector $\mathbf{x_i} \in \mathbb{B}^{\mathbf{m}}$ with $x_{i,k} = 1$ indicating that resource k is used in cell i. In Fig. 1 an exemplary scenario consisting of 6 cells each having 16 resources is shown where cell 3 is restricted to a resource allocation $\mathbf{x_3} = (1,0,1,0,1,0,1,0,1,0,1,0,1,0,1,0) \in \mathbb{B}^{16}$.

We will neither consider different geographic positions of individual users nor distinguish between uplink or downlink scenarios. Instead, it is only assumed that the usage of a resource in cell i causes interference in all other cells $j \neq i$ depending on the attenuation between cells i and j. We will denote the impact the utilization of resource k in cell i has on the same resource k in cell j by $h_{i,j}$. The resource allocation $\mathbf{x_{-i}}$ of all cells other than cell i is defined as $\mathbf{x_{-i}} = (\mathbf{x_1}, \mathbf{x_2}, \dots, \mathbf{x_{i-1}}, \mathbf{x_{i+1}}, \dots, \mathbf{x_n})$. Using this notation, we can express the total interference $I_{i,k}$ cell i receives on resource k as:

$$I_{i,k}(\mathbf{x_{-i}}) = \sum_{j \in N \setminus \{i\}} h_{i,j} x_{j,k} \tag{1}$$

Note that we assume that the interference impact is constant over all resources and symmetric between cells (that is, $h_{i,j} = h_{j,i}$). No power control, other than on/off with equal power levels, is assumed. Not having the ability to use more fine-grained power levels does not have to significantly impact the system performance as shown in [8].

B. Non Cooperative Game Model

A game in the context of game theory models a decision situation in which multiple decision makers face a situation of strategic interdependence when deciding their action to take. Here, the base stations are the decision makers or *players* of the game. Their *action* is to choose an allocation vector $\mathbf{x_i} \in \mathbb{B}^m$ which maximizes their *utility* U. In general, a finite *normal form game* Γ is denoted by a tuple $\Gamma = (N, (X_i)_{i \in N}, (U_i)_{i \in N})$ where N is the finite set of players and $(X_i)_{i \in N}$ and $(U_i)_{i \in N}$

denote possibly player-specific (finite) action sets and utility functions, respectively [9].

For our purposes, we assume that each cell demands a certain number D_i of resources. The *action set* X_i is therefore cell-specific because only allocation vectors (*actions*) $\mathbf{x_i}$ with $\sum_{k=1}^{m} x_{i,k} = D_i$ are allowed in cell i. We expect each player to choose a deterministic action, a so-called *pure strategy*.

The utility function $U(\mathbf{x_i}, \mathbf{x_{-i}})$ maps the player's own action $\mathbf{x_i}$ as well as the actions $\mathbf{x_{-i}}$ of all other players to some real value. As a cell is interested in choosing the resources that are least interfered, the cell's utility that is to be maximized can be defined as the negative sum of the total interference received on its own selected resources.

$$U_i(\mathbf{x_i}, \mathbf{x_{-i}}) = -\sum_{k=1}^{m} x_{i,k}\, I_{i,k} \tag{2}$$

$$= -\sum_{k=1}^{m} \left(x_{i,k} \sum_{j \in N \setminus \{i\}} h_{i,j} x_{j,k} \right) \tag{3}$$

The definition of the utility function (3) completes the formal definition of the normal form game. This game falls into the class of *anti-coordination games* because players will try to coordinate their resource allocations in a way that they use different resources.

Classical game theory distinguishes between *static games* that are only played once and *extensive games* that are played repeatedly. In *extensive games* the players know that they will face the same game over and over again so that they can choose their actions in a way that maximizes their possibly discounted utility in the long run. Phenomena like cooperation, revenge and threats can occur.

For our purposes, we also assume that the above game is played multiple times but that this is done mainly to find or *learn* a good solution [10]. In contrast to many classical games, e.g. the famous prisoners' dilemma, the players are unable to deduce a good solution from introspection (knowledge of the players' action sets and utility functions) only. The reason is that because apart from the interference level all resources are equal so that for combinatorial reasons there are generally a vast number of equally good solutions that only differ in the resource numbering. The players therefore have to learn which particular set of resources is heavily used by other players to avoid these resources.

The most important solution concept in game theory is the notion of a *Nash equilibrium* which will be introduced in the next section.

III. Convergence of Decentralized Coordination

If the resource allocation in a cellular system is left to autonomously deciding base stations, one central question is whether the distributed allocation process will stabilize and reach a steady state. Especially, conditions in which neighboring cells constantly flip-flop in lockstep between two resources A and B might occur if they decide at the same time with the knowledge of the interference level resulting from their previous decisions.

To show that the distributed coordination process will reach a steady state under certain conditions, we will first show the existence of a *Nash equilibrium* before we examine the preconditions that are necessary for the coordination process to arrive at such a steady state.

A. Nash equilibrium as a stable state

A *Nash equilibrium* action profile $(\mathbf{x_i}^*, \mathbf{x}^*_{\mathbf{-i}})$ describes a steady state in which no player can gain a higher utility by deviating from his chosen action $\mathbf{x}^*_{\mathbf{i}}$ as long as the other players stick to their actions $\mathbf{x}^*_{\mathbf{-i}}$. Formally, at a *Nash equilibrium* $U_i(\mathbf{x}^*_{\mathbf{i}}, \mathbf{x}^*_{\mathbf{-i}}) \geq U_i(\mathbf{x_i}, \mathbf{x}^*_{\mathbf{-i}})$ for all players $i \in N$ and for all $\mathbf{x_i} \in X_i$.

It is natural to assume that a base station i would only want to change its resource allocation from $\mathbf{x_i}$ to $\mathbf{x}'_{\mathbf{i}}$ if the total interference is reduced that way, i.e. if $U(\mathbf{x}'_{\mathbf{i}}, \mathbf{x_{-i}}) > U(\mathbf{x_i}, \mathbf{x_{-i}})$. Therefore, the *Nash equilibrium* resource allocation will be stable once it is reached. In our game we only want to allow deterministically chosen actions, so-called *pure strategies*. In contrast to *mixed strategies* where players choose a random action according to some probability distribution, games are not guaranteed to have a Nash equilibrium in pure strategies. Next, we will show that our game belongs to the class of *potential games* which are guaranteed to always possess Nash equilibria that will be reached after a finite number of game rounds.

B. Potential games

Potential games form a special class of normal form games where the unilateral change of one player's action $\mathbf{x_i}$ to $\mathbf{x}'_{\mathbf{i}}$ results in a change of his utility that is paralleled by a change of a so-called *potential function* $\Phi(\mathbf{x_i}, \mathbf{x_{-i}})$. If the change in player i's utility exactly equals the change in the global potential function: $U_i(\mathbf{x}'_{\mathbf{i}}, \mathbf{x_{-i}}) - U_i(\mathbf{x_i}, \mathbf{x_{-i}}) = \Phi(\mathbf{x}'_{\mathbf{i}}, \mathbf{x_{-i}}) - \Phi(\mathbf{x_i}, \mathbf{x_{-i}})$ the potential is called an *exact potential*.

The game model as defined in Section II-B above admits an exact potential function Φ as given in (4) that equals the total interference summed over all interfering links:

$$\Phi(\mathbf{x_i}, \mathbf{x_{-i}}) = -\sum_{i=1}^{n} \sum_{j=1}^{i} \sum_{k=1}^{m} h_{i,j} x_{i,k} x_{j,k} \tag{4}$$

To verify that this is indeed an exact potential, consider a cell i that unilaterally switches one of its previously allocated resources from resource k to resource l because $I_{i,l} < I_{i,k}$. By not using resource k anymore, the total interference $I_{i,k}(\mathbf{x_{-i}}) = \sum_{j \in N \setminus \{i\}} h_{i,j} x_{j,k}$ that it received from all other cells $j \neq i$ exactly equals the sum of the interference improvements all other cells using the same resource experience because cell i refrains from using resource k in the future. At the same time, the lower interference level $I_{i,l}$ that cell i now experiences exactly equals the sum of the additional interference it adds to all other cells using the same resource l.

C. Finite improvement path property of potential games

All potential games have the *finite improvement path* property [11] which both serves to prove the existence of a Nash equilibrium and to indicate how a Nash equilibrium might be reached by a set of players. To understand this, we assume that all players act sequentially choosing their actions one at a time after being able to observe the outcome in the form of interference resulting from the other players' resource allocation actions. Then, if a player i has a *better response* $\mathbf{x}'_\mathbf{i}$ that yields a higher utility (lower total interference) it can increase its utility by $U_i(\mathbf{x}'_\mathbf{i}, \mathbf{x}_{-\mathbf{i}}) - U_i(\mathbf{x}_\mathbf{i}, \mathbf{x}_{-\mathbf{i}})$ which exactly equals the corresponding increase in the game's potential. Note that we assumed all other players' allocations $\mathbf{x}_{-\mathbf{i}}$ to remain constant while player i adapts. Now, if some other player j performs a better response to the new interference situation, he will further increase the global potential Φ while improving his own utility. As the number of players as well as the size of their action sets are finite, the sequential improvement process has to reach a state in which no player is able to improve his utility by unilaterally changing his action from $\mathbf{x}_\mathbf{i}$ to $\mathbf{x}'_\mathbf{i}$ assuming all other actions as constant. This is the definition of a Nash equilibrium.

The selfish behavior of the players never makes them adapt a resource allocation yielding a lower utility so that the potential function is monotonically increasing in each round. This effectively prevents circles in the distributed allocation process that twice reach an allocation profile that is not an equilibrium.

D. Sequential deterministic decisions

In order to benefit from the above convergence result, the base stations as the players of the resource allocation game only have to implement a very simple resource allocation algorithm as shown below.

Algorithm 1 DETERMINISTICBESTRESPONSE

newResources $= \emptyset$
for $k = 1$ to m **do**
  $I_{i,k} =$ MEASUREINTERFERENCEONRESOURCE(k)
end for
sortedList $=$ SORTBYINCREASINGVALUE$(I_{i,k})$
for $l = 1$ to D_i **do**
  newResources $=$ newResources $\cup$ **sortedList**$[l]$
end for
return newResources

Only local information regarding the interference levels on all resources is needed and the computation complexity is linear in m as only the D_i currently best resources have to be identified. The algorithm behaves deterministically but requires that only one player at a time adjusts to the current interference conditions once they have been observed by measurements.

Provable convergence is a very desirable property but the required strictly sequential adaptation is problematic for two reasons. First, a suitable serialization mechanism would be required to prevent two base stations from adapting simultaneously. Second, the adaptation process would become too slow for larger cellular networks.

E. Simultaneous probabilistic decisions

To overcome the scalability problems of strictly sequential adaptations, we have implemented a second algorithm that allows for simultaneous decision making by introducing two probabilistic features. First, the algorithm does not strictly select the D_i currently best resources as new candidates, but compiles a set of candidates by drawing candidate resources with a certain probability according to the resource's utility, cf. line 7 of the algorithm below. Second, if there is a better resource among the selected candidates than currently used, it switches to that resource only with probability $1/p$ (cf. line 13); we have experimentally found $p = 10$ to be a good value. The rationale behind this is to prevent neighboring cells, which will have similar interference conditions, to switch simultaneously to the same resource thus causing high mutual interference.

Algorithm 2 PROBABILISTICBETTERRESPONSE

1: newResources $= \emptyset$
2: **for** $k = 1$ to m **do**
3:   $I_{i,k} =$ MEASUREINTERFERENCEONRESOURCE(k)
4: **end for**
5: cand $= \emptyset$
6: **for** $j = 1$ to D_i **do**
7:   $cand = cand \cup \{k\}$ where k is chosen from $rest = \{1, \ldots, m\} \setminus cand$ with a prob. proportional to $\frac{1 - I_{i,k}}{\sum_{l \in rest}(1 - I_{i,l})}$
8: **end for**
9: **for** old $\in$ currentChannels **do**
10:   resourceSwitched=False
11:   **for** new $\in$ cand **do**
12:     **if** $I_{i,\text{new}} < I_{i,\text{old}}$ **then**
13:       with probability $1/p$:
14:       newResources $=$ newResources $\cup$ new
15:       cand $=$ cand$\setminus\{$new$\}$
16:       resourceSwitched = True
17:     **end if**
18:   **end for**
19:   **if** not resourceSwitched **then**
20:     newResources $=$ newResources $\cup$ old
21:   **end if**
22: **end for**
23: **return** newResources

The algorithms of the presented distributed scheme are executed in parallel in all n cells in the system. Note that the time complexity of both algorithms is only dependent on the number of channels m but not on the number of cells n in the system. Both can easily be implemented to execute in time polynomial in m. In contrast, the time complexity of one single centralized algorithm would also depend on the number of cells n. Deriving an optimum solution in time polynomial in n would appear infeasible because these kind of optimization problems are known to be NP-complete.

The sampling according to an interference dependent probability was inspired by [6] which applies results from *selfish routing* to distributed channel selection in a wireless ad-hoc setting. The difference is that in [6] the authors propose to sample a higher interfered resource with higher probability because they take higher utilization as a sign of a better resource quality. This makes no sense in our context because apart from the interference level, all resources are assumed equal. In [5] the authors also propose to conduct a Bernoulli trial with some probability p before an agent adapts its allocation.

Of course, the probabilistic element prohibits proving general convergence because the Bernoulli trial might fail indefinitely long. But as we will see in Section IV, the probabilistic better response algorithm performs well and exhibits better convergence properties in larger scenarios.

IV. Performance of Decentralized Coordination

In order to examine the convergence properties of the proposed adaptation algorithms and to quantify the efficiency loss compared to a centralized optimization, we have performed simulations.

A. Simulation scenario

For the simulations we employ the cellular system model presented in Section II-A and shown in Fig. 1. The radio parameters for a typical sub-urban radio cell are derived from [12]; they are listed in Table I. The actual values serve to establish the mutual interference levels generated by reusing resources in different cells. Specifically, the interference impact $h_{i,j}$ is obtained by subtracting the pathloss $L(d_{i,j})$, based on the distance $d_{i,j}$ between cells i and j, from the transmit power P_{Tx} in dBm: $h_{i,j} = P_{Tx}[dBm] - L(d_{i,j})$. No power control is assumed.

TABLE I: Overview of radio simulation parameters [12]

Parameter	Value
Carrier frequency	900 MHz
System spectrum	10 MHz
Frequency channels	25
Channel bandwidth Δ_f	400 kHz
Total BS transmit power	43 dBm
Transmit power per channel	29 dBm
Pathloss model	L = $120.9 + 37.6 \log_{10}(R/[km])$
Cell radius	450 m

The distributed coordination process starts at a random resource allocation. In each cell $D_i = 8$ out of 25 resources have to be selected.

B. Speed of convergence

In Fig. 2 the development of the system-wide average interference level on an allocated resource is shown over 250 iterations in a system of 36 cells starting from 25 randomly selected initial resource allocations. During one iteration, only one cell in the case of the sequential deterministic decision scheme, or all cells in the case of the simultaneous probabilistic decision scheme, are allowed to adapt their resource allocations in reaction to the interference level resulting from the previous iteration.

The average interference under the sequential deterministic adaptation scheme decreases monotonically as it is directly related to the potential (4) of the game. Under the simultaneous probabilistic scheme, temporary increases of the average interference are possible, e.g. if two neighboring cells happen to switch to the same previously uninterfered resource. The adaptations eventually reach a steady state indicated by $\times$ marks when a Nash equilibrium is reached.

We first note that both schemes converge starting from all 25 different initial allocations. The simultaneous prob. scheme is able to almost reach its final interference level after only 25 iterations whereas the sequential scheme needs significantly more iterations. The reason for that is that after 25 iterations not all of the 36 cells in the scenario have been able to adapt. In addition to converging faster, Fig. 2 also suggests that the probabilistic scheme achieves slightly better equilibrium levels. Although the Nash equilibria are reached only after about 200 iterations, the distributed allocation schemes need significantly less iterations to realize most of the total interference reduction of about 3 dB as compared to the initial random allocation.

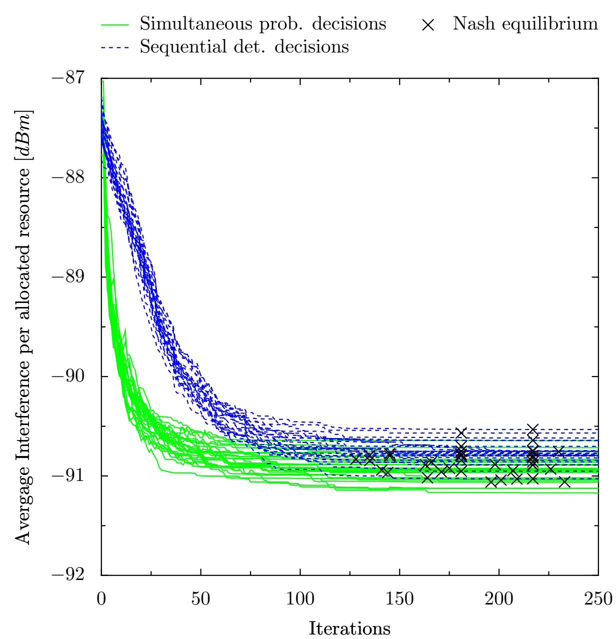

Fig. 2: Comparison of the convergence speed and equilibrium interference levels using the sequential deterministic best response and the simultaneous probabilistic better response algorithms starting from 25 different random initial resource allocations. Scenario consisting of 36 cells with $D_i = 8$ out of 25 resources to be chosen in every cell.

C. Efficiency loss due to game solution

Now we want to quantify the efficiency loss that an operator would suffer if he decides to employ a distributed scheme instead of a centralized optimization. The different interference levels at which the Nash equilibria were reached in Fig. 2 already indicate that no single efficiency loss value can be specified. In algorithmic game theory, the utility of the worst possible Nash equilibrium is compared to the social optimum to derive the "price of anarchy" [7]. We will determine an experimental bound for the "price of anarchy" and will also examine the distribution of the utilities of Nash equilibria.

1) Optimal solution as a benchmark: To derive an optimal allocation in our assumed scenario, we employ a classical optimization approach. By stating the interference coordination problem as an *integer linear program (ILP)* we are able to use standard optimization software like *ILOG's Cplex* [13].

The ILP is given in equations (5) to (7). The objective again is to minimize the global total interference (5) under the constraints that the external demands D_i are fulfilled in every cell. The pair constraint in (7) allows to state the problem as an *integer* LP by avoiding the multiplication of the decision variables $x_{i,k}$ and $x_{j,k}$.

$$\text{minimize} \quad \sum_{i \in N} \sum_{k \in K} \sum_{j \in N \setminus \{i\}} h_{i,j}\, p_{i,j,k} \tag{5}$$

$$\text{subject to} \quad \sum_{k \in K} x_{i,k} = D_i \quad \forall i \in N \tag{6}$$

$$p_{i,j,k} \geq x_{i,k} + x_{j,k} - 1 \tag{7}$$

$$\text{optimization variables} \quad x_{i,j},\, p_{i,j,k} \in \mathbb{B}$$

$$\text{constants} \quad h_{i,j} \in \mathbb{R}$$

The solution of the above ILP boils down to an exhaustive search using a *branch-and-cut* process. As the time complexity grows exponentially in the scenario size, we were only able to obtain an optimal solution for a scenario consisting of 11 cells where 8 out of 25 resources had to be selected in each cell.

2) Quantifying the efficiency loss: After obtaining a social optimum interference level of -95.69 dBm from solving the ILP, we have run 300 iterations of both the sequential deterministic as well as the simultaneous probabilistic scheme starting from 500 random resource allocations in the 11 cell scenario. The results are summarized in Table II.

First, we observe that after 300 iterations all instances of the sequential scheme have reached a Nash equilibrium and that only 2 instances of the simultaneous probabilistic scheme have not yet finally converged. As the scenario is small with only 11 cells, the sequential algorithm has a speed advantage, reaching a Nash equilibrium after a median number of 34 instead of 93 iterations.

Second, we observe that the distribution of the Nash equilibrium solution quality as represented by the worst and best interference levels do not differ significantly. In Fig. 3 we therefore only visualize the histogram of the Nash equilibrium distribution for the simultaneous probabilistic scheme.

TABLE II: Efficiency of Nash equilibria

Parameter	**Round robin with det. best-response**	**Simultaneous with prob. better-response**
Total games played	500	500
Nash equilibria reached	500	498
Iterations needed (median)	34	93
Best Nash equilibrium	-94.62 dBm	-94.26 dBm
Worst Nash equilibrium	-92.23 dBm	-92.31 dBm
Median Nash equilibrium	-92.96 dBm	-93.45 dBm
Social optimum	-95.69 dBm	
Min. Price of anarchy	**3.46 dB**	**3.38 dB**

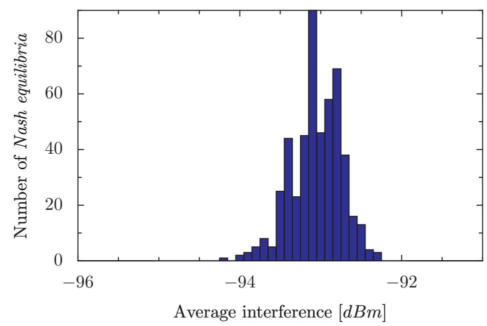

Fig. 3: Histogram showing the distribution of interference levels for Nash equilibria reached in 500 runs of the simultaneous prob. decision scheme with 300 iterations. Scenario consists of 11 cells, $D_i = 8$ out of 25 resources.

Now we compare the *social optimum* a central planner could achieve to the interference values for the worst *Nash equilibria* that we found experimentally. The difference of about 3.4 dB between the worst Nash equilibrium and the social optimum means that the distributed resource allocation by selfish agents could – in the worst case – carry an opportunity cost of more than 3.4 dB compared to the global optimum. However, equilibrium levels that are only about 1 dB short of the optimum are possible and the median values of -92.96 dBm and -93.45 dBm give a better impression of an average performance.

Still, the gap to the optimal solution seems significant. After all, a discrepancy between the optimization and Nash equilibrium solutions is not surprising because they achieve a different notion of fairness. The first approach might limit the utility of some cells in order to allow for a higher total utility while the latter reaches an equilibrium in which no cell could unilaterally improve its utility.

In practice, finding an exact optimum will not be feasible as it would not only require complete information but also proves to be computationally complex. So a better comparison would be to contrast the values derived here with the performance of classical channel assignment techniques as presented, for example, in [2] or [1].

In general, the presented decentralized interference coordination technique promises to be highly adaptive to varying traffic demands and user mobility in a cellular system. The presented approach reduces the complexity of network engineering as no central controller or signaling is needed. It would also allow for a plug-and-play operation of the radio resource management accommodating even devices like home nodeBs.

V. Conclusion

In this paper, we have introduced a decentralized resource allocation scheme aiming at interference coordination among cells in future wireless networks. Assuming a simplified cellular system, we were able to prove the convergence of the distributed allocation scheme to a Nash equilibrium by using methods from game theory. We have provided two different allocation algorithms designed for guaranteed and fast convergence in big scenarios, respectively. The performance of the proposed schemes was analyzed by simulations with respect to the convergence speed as well as the efficiency loss compared to a centralized optimization.

These results are very encouraging as they show that interference coordination can indeed be achieved decentrally in the base stations by executing simple adaptation algorithms based on local information only. The efficiency loss is partly offset by a higher degree of flexibility as well as lower setup and hardware costs.

Additional future work should detail the cellular scenario, accommodating for different positions of the mobile users. Considering mobile scenarios is expected to show the performance advantage of the presented fully distributed coordination scheme compared to centrally controlled or fixed schemes. Also, pricing schemes or other means to improve the achieved equilibria levels should be investigated. Finally, the benefit of allowing explicit direct communication between base stations should be considered as future systems are expected to provide such interfaces.

Acknowledgment

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