Modeling CSMA/CA in VANET

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Abstract. In this paper, we propose a simple theoretical model to compute the maximum spatial reuse feasible in a VANET. We focus on the ad hoc mode of the IEEE 802.11p standard. Our model offers simple and closed formulae on the maximum number of simultaneous transmitters, and on the distribution of the distance between them. It leads to an accurate upper bound on the maximum capacity. In order to validate our approach, results from the analytical models are compared to simulations performed with the network simulator NS-3. We take into account different traffic distributions (traffic of vehicles) and we study the impact of this traffic on capacity.

1 Introduction

In recent years, Inter-Vehicle Communication (IVC) has become an intense research area, as part of Intelligent Transportation Systems. It assumes that all or a subset of the vehicles is equipped with radio devices, enabling communication between them. Although classical 802.11 can be used for IVC, specific technologies such as IEEE 802.11p [1] (also referred to as Wireless Access in Vehicular Environments, WAVE) have been standardized to support these communications. This standard includes data exchanges between vehicles (ad hoc mode) and between infrastructure and vehicles. When the ad hoc mode is used, the network formed by the vehicles is called a Vehicular Ad hoc NETwork (VANET).

VANET can be used by two families of applications. The first family is user oriented. In this case the VANET may be used to advertise restaurants, gas stations, traffic condition, etc. But the most important applications are related to road safety. Information on road conditions, speed, traffic or alert messages (signalling an accident) may be exchanged in the VANET allowing drivers to anticipate dangerous situations [2]. Data from embedded sensors may also be exchanged in order to increase the perception of the environment. This helps drivers to make appropriate decisions, as it increases the information available on road conditions and traffic situations. The amount of data which can be exchanged between vehicles is thus crucial. Design of these applications has to take into account the limited capacity of the VANET to control the quantity of information which can be sent to other vehicles. In such networks, capacity is mainly limited by the 802.11p spatial reuse. As channels are shared by all the nodes, only a subset of nodes, sufficiently far from each other, can emit at the same time.

In this paper, we evaluate the maximum spatial reuse of the 802.11p technology. Our approach can be presented through a simple example. Let us consider the vehicles depicted in Figure 1. We suppose that we are in a saturated case where all these

K. Al-Begain, D. Fiems, and J.-M. Vincent (Eds.): ASMTA 2012, LNCS 7314, pp. 91–105, 2012.

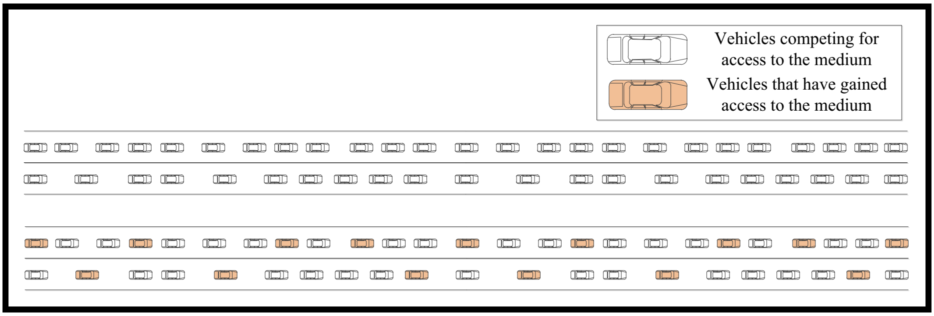


Fig. 1. Example of concurrent transmissions: the 802.11p MAC layer (CSMA/CA) set the rules to access the medium. Only orange vehicles are allowed to transmit frames at the same time.

vehicles wish to send a frame. The MAC layer of the 802.11p standard will select a subset of vehicles which will be allowed to transmit their frames (they are colored in orange in the figure). It selects vehicles in such a way that distances between concurrent transmitters is sufficiently great to avoid interference between the transmissions. At the same time, the capacity is directly related to these distances as they limit the number of simultaneous transmitters. This paper aims to propose a simple model to evaluate the distribution of these distances. We propose a Markovian model where locations of transmitting nodes are built recursively according to the rules used by the 802.11 MAC layer. The equilibrium distribution of this Markov chain allows us to deduce the mean intensity of the concurrent transmitters, i.e. the mean number of transmitting nodes per kilometer. Also, it leads to an estimate of the capacity. The capacity is defined here as the maximum number of frames per second that the network is able to send. Unlike classical approaches dealing with the asymptotic behavior of the capacity, our approach offers accurate estimates of this capacity. Results from the analytical model are then compared to simulations performed with the network simulator NS-3 [3]. We take into account different traffic scenarios (traffic of vehicles). The first scenario assumes that the distance between vehicles is constant and the second one uses a traffic simulator to emulate drivers' behavior on a highway. The combination of NS-3 and the traffic simulator allows us to obtain simulations that are as realistic as possible.

The paper is organized as follows. In Section 2 we present the technological context of this study. Section 3 overviews related works dealing with capacity of ad hoc networks and VANET. Our contributions with regard to the existing approaches are highlighted in the same section. The models are presented in Section 4. Theoretical estimations of the capacity and simulation results are compared in Section 5. We conclude in Section 6.

2 CSMA/CA in 802.11p

The IEEE 802.11p spectrum is composed of six service channels and one control channel. The control channel will be used for broadcast communications dedicated to high priority data and management frames, especially for safety communications. It should be the privileged channel used to disseminate messages from safety applications. The service channels can be used for safety and service applications, broadcast and unicast communications. The MAC layer in 802.11p is similar to the IEEE 802.11e Quality of Service extension. Application messages are categorized into one of four different queues depending on their level of priority. Each queue uses the classical CSMA/CA (Carrier Sense Multiple Access/Congestion Avoidance) mechanism to access the medium, but CSMA/CA parameters (backoff, etc.) are different from one queue to another in order to favour frames with high priority. In CSMA/CA, a candidate transmitter senses the channel before effectively transmitting. Depending on the channel state, idle or busy, the transmission is started or postponed. *Clear Channel Assessment* (CCA) depends on the MAC protocol and the terminal settings. For the CSMA/CA protocols used in IEEE 802.11, CCA is performed according to one of these three methods.

1. CCA Mode 1: *Energy above threshold*. CCA shall report a busy medium upon detecting any energy above the Energy Detection (ED) threshold. In this case, the channel occupancy is related to the total interference level.
2. CCA Mode 2: *Carrier sense only*. CCA shall report a busy medium only upon the detection of a signal compliant with its own standard, i.e. same physical layer (PHY) characteristics, such as modulation or spreading. Note that depending on threshold values, this signal may be above or below the ED threshold.
3. CCA Mode 3: *Carrier sense with energy above threshold*. CCA shall report a busy medium using a logical combination (e.g. AND or OR) of Detection of a compliant signal AND/OR Energy above the ED threshold.

The CCA mechanism ensures that there is a minimal distance between simultaneous transmitters (except when a collision occurs). If the receiver is in the transmitter radio range, it guarantees a low interference level at the receiver location. Also, it limits the number of simultaneous transmitters in a given area, and thus the number of frames that can be sent per second. Therefore, there is a direct relationship between the spatial reuse imposed by the CCA mechanism and the network capacity.

3 Related Works

A theoretical bound on the capacity of ad hoc networks was initially investigated in [4] where the authors prove that, in a network of n nodes, a capacity of $\Omega\left(\frac{1}{\sqrt{n \cdot \log n}}\right)$ is feasible. In [5], the authors improved this bound and proved that an asymptotic capacity of $\Omega\left(\frac{1}{\sqrt{n}}\right)$ is feasible. In these two articles, the capacity is reached by means of a particular transmission scheduling and routing scheme. In [6] and [7], more realistic link models have been used, both leading to a maximum asymptotic capacity of $O\left(\frac{1}{n}\right)$. In particular, the authors of [7] have shown that when there is a non-zero probability of erroneous frame reception, the cumulative impact of packet losses over intermediate links results in a lower capacity. Finally, it is shown in [5], that when the path-loss function is bounded, the capacity is also $O\left(\frac{1}{n}\right)$. However these last two results also suppose particular transmission scheduling and routing schemes. Moreover, all these studies deal with the asymptotic behavior of the capacity with regard to the number of nodes and do not propose precise estimates of this capacity.

On the other hand, in CSMA/CA based wireless networks, the transmission scheduling is distributed and asynchronous. It is not planned in advance and depends on the link conditions, interference, etc. at the time a node wants to emit its frame. The number of simultaneous transmitters is thus closely related to the CSMA/CA mechanism which limits the spatial reuse of the channel. The total number of frames sent in the whole network is thus bounded by a constant C whatever the number of nodes and the type of routing schemes. This constant has been evaluated in [8]. Therefore, CSMA/CA multi hop wireless networks would offer a capacity of $O\left(\frac{1}{n}\right)$.

However all these studies focus on networks where nodes are distributed on the plane or in a 2-dimensional observation window. VANETs have very different topologies as the vehicles/nodes are distributed along roads and highways. Radio range of the nodes (about 700 meters with 802.11p in rural environment) being much greater than the road width, we can consider that the topology is distributed on a line rather than in a 2 dimensional space. Lines, grids or topologies composed of a set of lines (to model streets in a city) are thus more appropriate to model VANET topologies.

In [9,10], the authors propose a bound on VANET capacity. They show that when nodes are at constant intervals or exponentially distributed along a line, the capacity is $\Omega\left(\frac{1}{n}\right)$ and $\Omega\left(\frac{1}{n \cdot ln(n)}\right)$ in downtown (city) grids. But it is also an asymptotic bound. Moreover, physical and MAC layers are unrealistic, radio ranges are constant and the same for all the nodes, interference is not taken into account and they assume a perfect transmission scheduling between the nodes. Thus, this bound cannot be applied to 802.11p networks.

In [11], the broadcast capacity of a VANET is estimated. The idea is similar to this paper; an estimation of the number of simultaneous transmitters is proposed. But this evaluation is based on numerical evaluation only, using integer programming.

The contributions of this paper are as follow. We propose two simple models to evaluate the maximum capacity of VANET. The first one, presented in Section 4.1, estimates the number of simultaneous transmitters for the CCA mode 2 of the 802.11. It is based on a existing mathematical model known as *the packing problem*. Since the extension of this model is not tractable for the CCA mode 1, we propose instead a Markovian approach. It is presented in Section 4.2. For this Markov chain, we deduce the transmitter intensity and the mean capacity. Also, we are able to compute the exact distribution of the distance between transmitters. To validate our approach, the theoretical results are compared to realistic simulations performed with NS-3. They focus on the CCA mode 1. Simulations show that our approach is suitable for evaluating the maximum capacity of VANET precisely. It gives precise estimates of CSMA/CA performances, rather than just the asymptotic behaviors, and can consequently be used as a dimensioning or parametrizing tool.

4 Modeling CCA Mode 1 and 2

4.1 Model for CCA Mode 2

When CCA mode 2 is used, the medium is assumed to be busy when a 802.11p frame is detected. This corresponds to cases where the node sensing the medium is at a distance where the signal from the transmitter is detected and compliant to the 802.11 standard. In this case, this approach is rather sensitive to the highest interfering signal rather than the overall interference level. A simple model consists of considering that the maximum distance at which a 802.11 frame is detected is constant. Let R be this distance. The medium is then busy if there is a transmitting node located at a distance less than R. With this model, the problem about the maximum number of simultaneous transmitters comes down to the following question: how many segment with size $2 \cdot R$ can we put in a certain interval $[a, b]$ under the constraint that the centers of these segment cannot be covered by another segment? The answer is simple. If we consider that the first point is located at a, we just have to set a segment at a distance R from the previous one until reaching b. But in a VANET, underlying transmitters are randomly distributed on the line, and transmitters are chosen randomly (it depends on the applications, backoffs, etc.). A more appropriate model consists in placing the segments randomly in $[a, b]$. The first segment is placed uniformly in $[a, b]$. Then, we place the second segment uniformly into all points x of $[a, b]$ such that a segment at x does not cover the center of the previous segment, and so on. The process terminates when there are no gaps in $[a, b]$ large enough to host another segment. This model is referred to as *the packing problem*. A rigorous analysis [12] shows that the mean number of segments divided by the interval length $(b - a)$ tends to a constant $c \approx 0.7476$ when $(b - a) \to +\infty$. The number of simultaneous transmitters with CCA mode 2 can then be estimated as $(b - a)\frac{c}{2 \cdot R}$ for $b - a$ large enough.

4.2 A Markovian Approach for CCA Mode 1

For CCA mode 1, where the sum of signals from all the current transmitters (i.e. Interference) is taken into account, assumptions about radio environment are required to model the signal strengths received from the current transmitters. Interference at a node located at x is generally considered as the sum of all interfering signals:

$$I(x) = \sum_{x_i \in \Phi} l(\|x_i - x\|) \tag{1}$$

where Φ is the set of concurrent transmitters, $\|x_i - x\|$ is the Euclidian distance between the nodes at x and x_i, and $l(.)$ is the path-loss function describing the received signal strength as a function of the distance. The medium is considered idle for a node at x if $I(x) < \theta$ where θ is the Energy Threshold (ED). In this case, the node at x can transmit its frame and becomes a transmitter. An approach similar to the packing problem could be considered in this case. For a given interval $[a, b]$, we sequentially add points uniformly distributed in all points x of $[a, b]$ such that $I(x) < \theta$. But this classical packing approach does not seem tractable. Therefore, we propose a tractable model, based on a Markov chain, to represent transmitters' location.

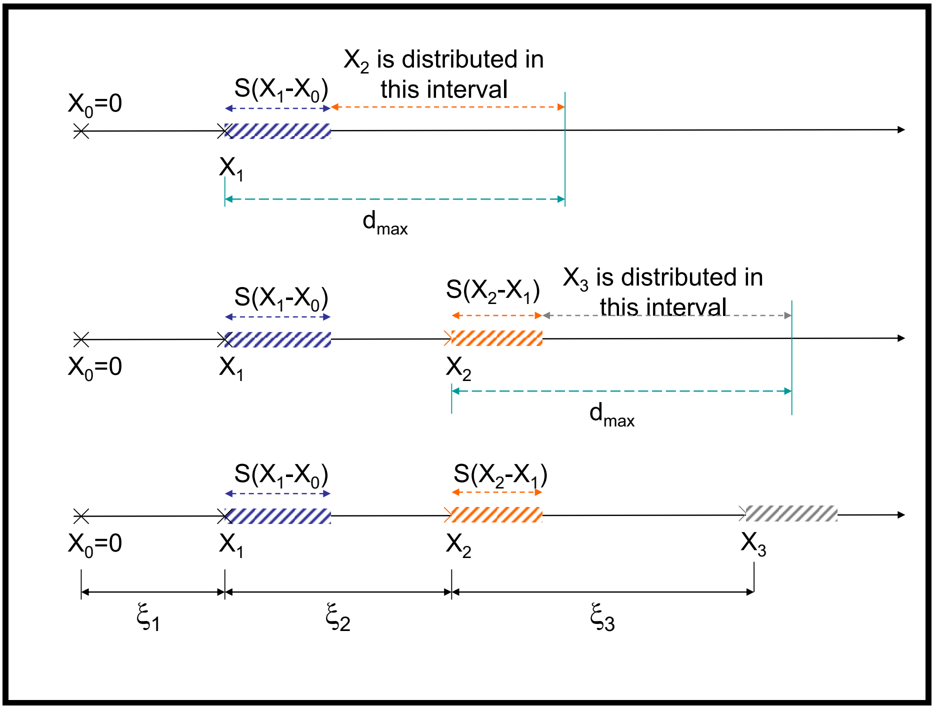


Fig. 2. Notations used in the model. The figure shows how the points X_2 and X_3 are distributed.

This model aims to evaluate the maximum number of simultaneous transmitters in a CSMA/CA network using CCA mode 1. First, we present the different assumptions on the path-loss function and Interference. Then, we define the intervals where the random variables of the Markov chain take their values. In the last paragraph, we present the transition density function and the main results (in Theorem 1).

a) Assumptions. We assume that the medium is detected idle for a node at $X \in I\!R^+$ if:

$$I(X) < \theta \tag{2}$$

where $I(X)$ is the interference at X and θ is the ED threshold (CCA mode 1). Here, $I(X)$ is defined as:

$$I(X) = l(X - L) + l(R - X) \tag{3}$$

where L and R are the locations of the two closest transmitters from X, the closest ones on the left and on the right. Function $l(.)$ is the path-loss function. In our model, Interference is thus computed from the signal strength of the two closest interferers. For the parameters of 802.11p technology, this model is very similar to a model where Interference from all the transmitters is taken into account. Indeed, as there is a significant distance between two successive transmitting nodes (due to the CCA mechanism), Interference generated by distant interferers is negligible with regard to the closest ones (in 802.11p and in a rural environment, the second interferer in a given direction will be at least 1 km away from the first one).

We assume that the path-loss function verifies the following conditions:

- $l(.)$ is continuous,
- $l(.)$ is a decreasing function,
- $l(0) > \theta$, where θ is a positive constant (the ED threshold),
- $\lim_{d\to+\infty} l(u) = 0$,
- there exists $u \in I\!R^+$ such that $l(u) > \theta$ and $l(v)$ is strictly decreasing and differentiable for all $v \in [u, +\infty)$.

These conditions hold for path-loss functions with the form: $l(u) = P_T\ min(1, c/u^\alpha)$ where P_T is the transmitting power (with $P_T > \theta$), c and d are two positive constants ($c > 0$ and $\alpha > 2.0$).

b) State space of the Markov chain. The chain is denoted $(X_n)_{n\in I\!N}$ with $X_{n-1} < X_n$. It represents the simultaneous transmitters of a CSMA/CA network and consists in a sequence of random points distributed on the line. Since all these transmitters/points have detected the medium idle, Interference at each point X_n must be less than the CCA threshold θ:

$$I(X_n) < \theta \ \ \forall n \geq 0$$

There is thus a minimal distance between the points of the process. We define a function $S(.)$ to describe this distance. According to equation (3) and the CCA condition, $S(u)$ is defined as the solution of

$$l(u) + l(S(u)) = \theta \tag{4}$$

where u corresponds to the distance between the two previous transmitters. X_n is thus distributed in $[X_{n-1} + S(X_{n-2} - X_{n-1}), +\infty]$.

A second assumption allows us to bound this interval. Since we are trying to estimate the maximum number of simultaneous transmitters, we shall distribute the points in such a way that it is not possible to add more points which could detect the medium idle. Consequently, the distance between transmitters must be bound by a maximal distance in order to prevent the presence of intermediate transmitters. Let d_{max} be this distance, it is solution of

$$2 \cdot l\left(\frac{d_{max}}{2}\right) = \theta \tag{5}$$

Thus, each point X_n ($n > 1$) belongs to the interval $[X_{n-1}+S(X_{n-1}-X_{n-2}), X_{n-1}+d_{max}]$. Distances between the transmitters are denoted $\xi_i = X_i - X_{i-1}$.

c) Building the point process. The point process is built as follows. The first two transmitters are located at $X_0 = 0$ and at X_1 with $X_1 \leq d_{max}$ almost surely. Assumptions about the distribution of X_1 are given in the theorem below.

The other points are built recursively. The location of a transmitter X_n ($n > 1$) is distributed in $[X_{n-1} + S(X_{n-1} - X_{n-2}), X_{n-1} + d_{max}]$. For convenience, we consider the sequence $\xi_n = X_n - X_{n-1}$ rather than X_n. ξ_n ($n > 1$) is thus distributed in $[S(\xi_{n-1}), d_{max}]$. It is possible to consider a different distribution on this interval leading to a different density of transmitters. As we do not know a priori the distribution of the distance between the transmitters, we have considered different distributions. In this paper, only the most accurate distribution, which has been determined by simulations, is presented. This distribution is the linear distribution in $[S(\xi_{n-1}), d_{max}]$. By linear distribution we mean an affine function, positive in $[S(\xi_{n-1}), d_{max}]$, null at d_{max}, and such that its integral on $[S(\xi_{n-1}), d_{max}]$ is 1. The pdf $f_{\xi_n|\xi_{n-1}}(.)$ of $\xi_n = X_n - X_{n-1}$ given $\xi_{n-1} = X_{n-1} - X_{n-2}$ is then:

$$f_{\xi_n|\xi_{n-1}=s}(u) = \Big(\frac{-2}{(d_{max} - S(s))^2}u + \frac{2d_{max}}{(d_{max} - S(s))^2}\Big)1_{u\in[S(s),d_{max}]} \tag{6}$$

where $1_{u\in[S(s),d_{max}]}$ is the indicator function, equals to 1 if $u \in [S(s), d_{max}]$ and 0 otherwise. The sequence $(\xi_n)_{n\geq 0}$ is thus a Markov chain which takes its values in the continuous state space $[S(d_{max}), d_{max}]$. In Figure 2, we present an example of this point process and the different notations. The stationary distribution of this Markov chain is given in the following theorem:

Theorem 1. *The process $(\xi_n)_{n\geq 0}$ defined in this Section is a Markov chain. The stationary distribution of ξ_n is $\pi(s)$ with:*

$$\pi(s) = a \cdot (d_{max} - s)(d_{max} - S(s))^2 1_{s\in[S(d_{max}),d_{max}]} \tag{7}$$

where a is a normalizing factor. The chain $(\xi_n)_{n>0}$ converges in total variation to the distribution $\pi(s)$ for all initial distribution of ξ_1 in $[S(d_{max}), d_{max}]$. If ξ_1 follows the stationary distribution $\pi(.)$ then ξ_n follows the distribution $\pi(.)$ for all n with $n > 0$.

The proof of this theorem is given in the appendix. In the following, we assume that ξ_1 follows the distribution $\pi(.)$. The intensity λ of the point process $(X_n)_{n\in\mathbb{N}}$, i.e. the mean number of point per unit length, is then given by:

$$\lambda = \frac{1}{\mathbb{E}[\xi_1]} = \left(\int_{S(d_{max})}^{d_{max}} s\pi(s)ds\right)^{-1} \tag{8}$$

The inverse of this intensity λ is the mean distance between two consecutive transmitters. Hence, the number of simultaneous transmitters over a road with length d will be $\lambda \times d$. Consequently, the capacity which is defined as the mean number of frames sent per second in the network can be estimated as:

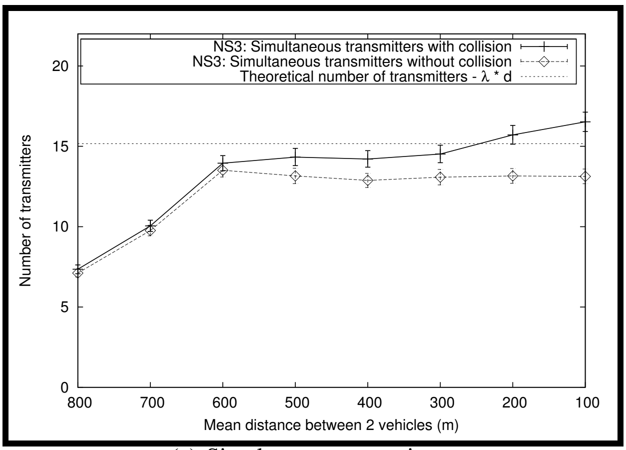
$$Capacity(d) = \frac{\lambda \times d}{T} \tag{9}$$

where λ is the intensity given by equation (8), d is the length of the road and T is the mean time to transmit a frame. This time takes into account the DIFS, the time to transmit the frame, the SIFS and the acknowledgement. We could wonder if it is pertinent to consider the number of transmitted frames rather than the number of received frames for the capacity. In practice, the ED threshold is significantly less than the signal strength required for correct reception. Therefore, when the transmitters respect the CCA rules, Interference does not disturb reception and the number of transmitted frames corresponds to the number of received ones. This will be validated by simulations in the next Section. Our simulations have shown that the only time, frames are not received properly is when collisions occur, i.e. when the CCA rules are not respected.

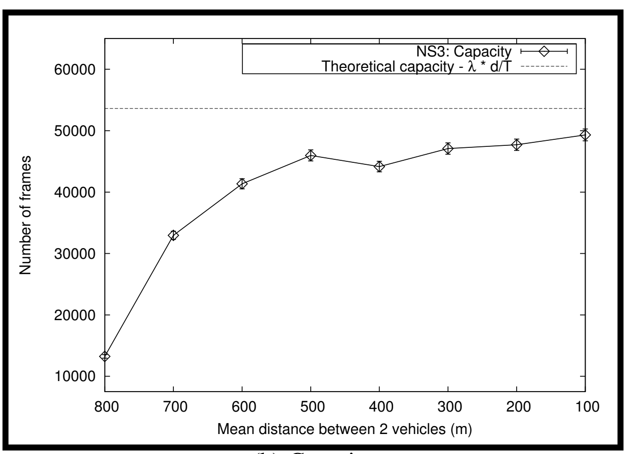
5 Simulations

In this Section, we compare the theoretical evaluation of the capacity to simulations performed with the network simulator NS-3 [3]. In the theoretical model, we consider the path-loss function used in NS-3. We compute for this path-loss the corresponding functions $S(.)$, $\pi(.)$, and the different constants (d_{max}, λ, T, etc.). We compute for all the simulations a confidence interval of 95%. For the simulations, all of the nodes transmit frames to a neighbor with a constant bit rate. All parameters are given in Table 1 and are set according to the IEEE 802.11p standard.

For vehicle locations, we take into account two scenarios: a scenario where the distances between vehicles are constant, and a scenario where vehicle locations are

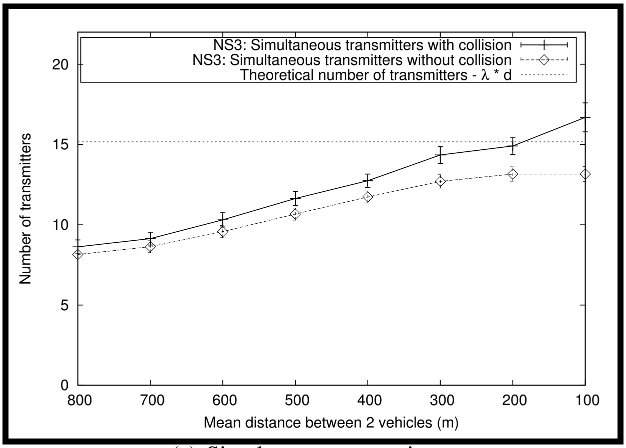


(a) Simultaneous transmitters

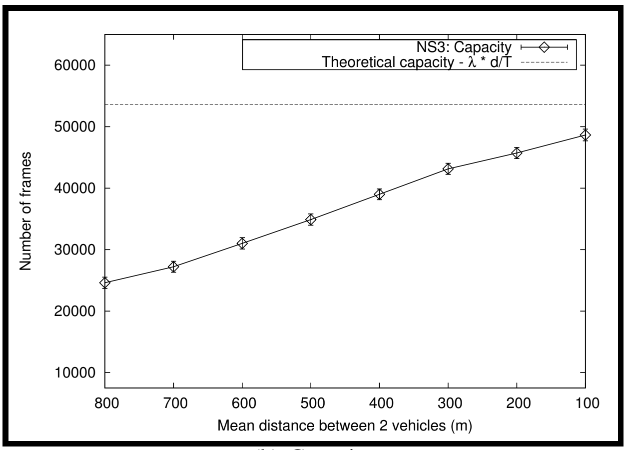


(b) Capacity

Fig. 3. Mean number of simultaneous transmitters and capacity for constant inter-distances



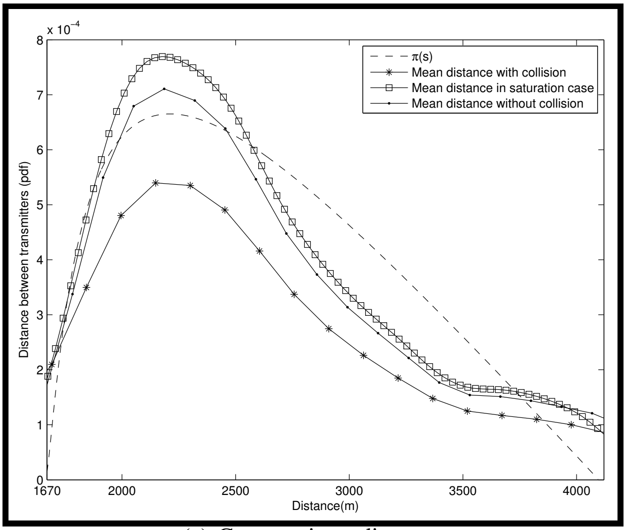
(a) Simultaneous transmitters



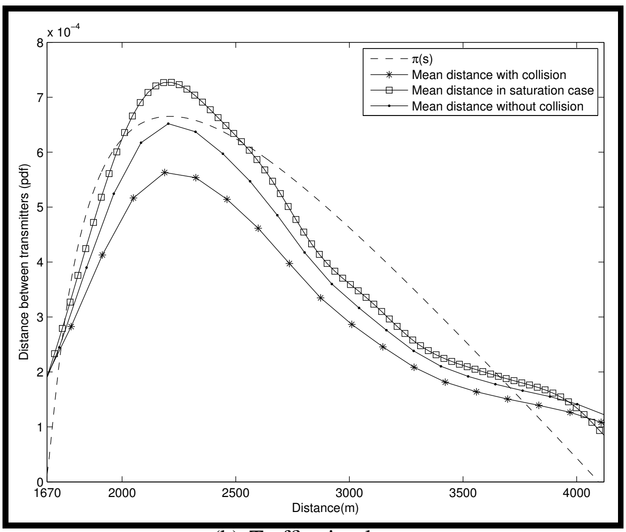
(b) Capacity

Fig. 4. Mean number of simultaneous transmitters and capacity for the traffic simulator

obtained from a realistic traffic simulator. This traffic simulator allows us to faithfully emulate driver behavior. On a highway, driver behavior is limited to accelerating, braking and changing lanes. We assume that there is no off-ramp on the section of highway. A desired speed is associated with each vehicle. It corresponds to the speed that the driver would reach if he was alone in his lane. If the driver is alone (the downstream vehicle is sufficiently far), he adapts his acceleration to reach his desired speed (free flow regime). If he is not alone, he adapts his acceleration to the vehicles around (car following regime). He can also change lanes if the conditions of another lane seem better. All these decisions are functions of traffic condition (speed and distance) and random variables used to introduce a different behavior for each vehicle. This kind of simulation is called micro simulation [13], and the model we used which has been tuned and validated with regard to real data collected on a highway is presented in detail in [14].



(a) Constant inter-distance.



(b) Traffic simulator.

Fig. 5. Distribution of the distances between concurrent transmitters

With the traffic simulator, we simulated a road/highway of 50 km with 2 lanes. The desired speed of the vehicles follows a Normal distribution with mean 120 km/h and standard deviation $\sigma = 10$. The distance shown on the x-axis in the figures corresponds to the mean distance between two successive vehicles.

a) Intensity and capacity results. In Figures 3 and 4 we plotted the mean number of transmitters and the capacity. The different figures correspond to the two kind of traffic: constant inter-distance and trajectories generated by the traffic simulator. It is worth

Table 1. Simulation parameters

Theoretical and NS-3 Parameters	Numerical Values
IEEE 802.11std	802.11p - CCH channel
Path-loss function	$l(d) = P_t \cdot min\left(1, \frac{10^{-4.5677}}{d^3}\right)$
CCA mode	CCA mode 1
ED Threshold (θ)	-82 dBm
Emission power P_t	43 dBm
Number of samples per point	100
Length of the packet	1024 bytes
Duration of the simulation	4 sec
$S(u)$	$\left(2.29 \times 10^{-10} - u^{-3}\right)^{\frac{-1}{3}}$
d_{max}	4120 m
λ	0.379×10^{-3}
DIFS	34 μs
Road length (d)	50 km
SIFS	16 μs

noting that the two traffic distributions (constant and traffic simulator) do not impact the results. This counter intuitive result is explained by the fact that the radio range and detection distance of the 802.11p technology are really greater than the mean distance between nodes. Comments are thus the same for these two traffic scenarios. When we processed the results from the NS-3 simulator, we distinguished transmitters provoking a collision and the ones respecting the CCA rules. When we do not take into account collisions, the theoretical model gives an accurate bound on both intensity and capacity. For the capacity, the difference is only 4% for 10 veh/km (distance between vehicles=100 meters) in Figure 3(b). The theoretical bound is thus approached even for very low density traffic as 10 veh/km corresponds to very sparse traffic. It was difficult to increase this density as the simulated highway is 50km (we already have 500 vehicles when the density is 10 veh/km). When we consider all the transmitters, the transmitters' intensity obtained by simulations exceeds the theoretical one. This is caused by transmitters provoking collisions, which by definition does not respect the CCA rules.

b) Distribution of the distance between transmitters. In Figure 5, we plotted the distributions of the distance between transmitters obtained with NS-3, and the distribution π. The abscissa is $[S(d_{max}), d_{max}]$. The simulated highway is 50 km with 2 lanes and 10 vehicles per kilometer in average. We collected distances between transmitters from 100 samples. For each sample we collected the distances between the transmitters and we plotted the corresponding empirical probability density function. The shape of the distribution for the transmitters without collisions fits very well with the stationary distribution $\pi(.)$. Nevertheless, we can observe a small difference when the function is decreasing. This difference is caused by samples greater than d_{max}. Indeed, it is very difficult to reach the absolute saturation of the network, where the medium is busy at every location, all the time. Therefore, sometimes there are regions where the medium is idle. Even if we simulated an important CBR for each source, nodes do not try to access the medium all the time because they are in the backoff procedure, they have nothing to send, etc. However if we consider only samples less than d_{max}, we obtain the curve in Figure 5(b). This allows us to estimate the distribution in the saturated case since we neglect the network parts where the medium is idle. It appears that it fits with the theoretical distribution $\pi(.)$ closely. If we compute the mean value of these samples, we obtain a mean inter-distance equal to 2.7 km corresponding to the mean inter-distance proposed in our model (2.64 km). It empirically proves that the theoretical model corresponds to a case where the CCA rule is respected by all the nodes (no collisions), and where the medium is spatially busy. Even if these conditions are not feasible in practice, the proposed Markovian approach still offers accurate bounds on the number of transmitters and capacity of VANET.

6 Conclusion

The particular topology of VANET, where nodes are distributed along a line, allows us to derive a simple model based on the Markov chain. It models distances between concurrent transmitters. Comparisons to realistic simulations show that the model is accurate and that it is quite independent of the traffic distribution. The theoretical intensity of the number of transmitters offers a very good upper bound on capacity, i.e. on the maximum number of frames that can be transmitted per second and per unit length. Our model can be used to tune the CSMA/CA parameters in order to optimize the capacity. Also, the distribution of the distance between two transmitters can be combined to elaborate radio models to evaluate Interference, Bit or Frame Error Rates. In this paper, the path-loss function does not take into account the multipath and fading properties of wireless link. We are currently working on an extension of this model to take into account more elaborate wireless models.

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Proof. Proof of Theorem 1. First, we prove that if the initial distribution of the Markov chain (the distribution of ξ_1) is π, ξ_n follows the distribution π for all $n > 0$. It suffices to show that π is the stationary distribution for this chain. We need to prove that

$$\pi(s) = \int_{S(d_{max})}^{d_{max}} f_{\xi_n|\xi_{n-1}=y}(s)\pi(y)dy \tag{10}$$

with $\pi(s) = a\left(d_{max} - S(s)\right)^2\left(d_{max} - s\right)$ and $f_{\xi_n|\xi_{n-1}=y}(s)$ given by equation 6.

We get,

$$\begin{aligned}
&\int_{S(d_{max})}^{d_{max}} f_{\xi_n|\xi_{n-1}=y}(s)\pi(y)dy \\
&= \int_{S(d_{max})}^{d_{max}} \left(\frac{-2}{(d_{max} - S(y))^2}s + \frac{2d_{max}}{(d_{max} - S(y))^2}\right) && (11)\\
&\times 1_{s\in[S(y),d_{max}]}a(d_{max} - y)(d_{max} - S(y))^2 dy && (12)\\
&= 2a(d_{max} - s)\int_{S^{-1}(s)}^{d_{max}} (d_{max} - y)dy && (13)\\
&= a\left(d_{max} - s\right)\left(d_{max} - S^{-1}(s)\right)^2 && (14)
\end{aligned}$$

where $S^{-1}(.)$ is the inverse function of $S(.)$. This function exists since due to the properties of the function $l(.)$, $S(u)$ is bijective, differentiable and strictly decreasing in $[S(d_{max}), d_{max}]$. To conclude, note that $S^{-1}(x) = S(x)$.

$$\begin{aligned}
&a\left(d_{max} - s\right)\left(d_{max} - S^{-1}(s)\right)^2 \\
&= a\left(d_{max} - s\right)\left(d_{max} - S(s)\right)^2 = \pi(s) && (15)
\end{aligned}$$

Also, we prove that ξ_n converges in total variation (it implies convergence in distribution) to π for any initial distribution of ξ_1 in $(S(d_{max}), d_{max}]$. We apply the Theorem 1 in [15] to prove this convergence. Since we have proved that π was the stationary distribution, it suffices to prove that the kernel P of this Markov chain is strongly

$\pi-$irreducible, i.e. $\forall x \in (S(d_{max}), d_{max}]$ and $A \subset [S(d_{max}), d_{max}]$ with $\pi(A) > 0$, there is a positive integer n_{xA} such that $P^n(x, A) > 0 \ \forall n \geq n_{xA}$. In our case, $\pi(A) > 0$ with $A \subset [S(d_{max}), d_{max}]$ is equivalent to $\nu(A) > 0$ where $\nu(.)$ is the Lebesgue measure in $\mathbb{R}^+$. The kernel P describes the transition probabilities, in our case it is formally defined as:

$$P(x, A) = \int_A f_{\xi_2|\xi_1=x}(y)dy \tag{16}$$

with $A \subset [S(d_{max}), d_{max}]$. $P^n(.,.)$ is the distribution of ξ_n $(n > 1)$ given ξ_1. It may be defined recursively:

$$P^n(x, A) = \int_{S(d_{max})}^{d_{max}} P(x, dy)P^{n-1}(y, A) \tag{17}$$

First, note that if $P^m(x, A) > 0$ with $m > 0$, $P^n(x, A) > 0 \ \forall n \geq m$. It can be easily proved by recurrence: Since $P^m(x, A) > 0 \ \forall y \in [S(d_{max}), d_{max}]$ and $P(x, dy) = f_{\xi_2|\xi_1=x}(y)dy$ with $f_{\xi_2|\xi_1=x}(y) > 0 \ \forall y \in [S(x), d_{max}]$, $P^{m+1}(x, A)$ expressed as

$$P^{m+1}(x, A) = \int_{S(d_{max})}^{d_{max}} P(x, dy)P^m(y, A) \tag{18}$$

will be positive if $\nu([S(x), d_{max}]) > 0$, in other words if $x > S(d_{max})$. We prove now that $P^2(x, A)$ for all $x \in [S(x), d_{max}]$ and $A \subset [S(x), d_{max}]$ with $\nu(A) > 0$. n_{xA} can thus be chosen equal to 2. Let $a = min\{u, u \in A\}$,

$$P^2(x, A) = \int_{S(d_{max})}^{d_{max}} P(y, A)f_{\xi_2|\xi_1=x}(y)dy \tag{19}$$

$$\geq \int_{S(min(x,a))}^{d_{max}} P(y, A)f_{\xi_2|\xi_1=x}(y)dy \tag{20}$$

$$> 0$$

Indeed, $P(y, A) > 0$ and $f_{\xi_2|\xi_1=x}(y) > 0$ for all y in $[S(min(x, a)), d_{max}]$. Equation (20) is thus positive when $\nu([S(min(x, a)), d_{max}]) > 0$, i.e. when $x > S(d_{max})$. This proves that the Markov chain is strongly $\pi-$irreducible, and thus μP^n converges in total variation to π when $n \to +\infty$ for any initial distribution μ in $(S(d_{max}), d_{max}]$.