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Auction Mechanisms for Distributed Spectrum Sharing

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Dynamic Spectrum Sharing

- Current command-and-control model is out of date.
  - Only licensees can operate in a licensed band.
  - Rigidity; patterns of over- and under-allocation
- Advances in radio technologies enables flexible use:
  - Software-defined radio → Cognitive radio
- Usage models being considered:
  - Exclusive use (ownership) model
  - Commons model

Interference Temperature

- An interference criterion proposed by the FCC .
- Definition: RF power measured at the receiver antenna per unit bandwidth.
  - Different from the traditional criterion on the transmit power.
- For fixed total bandwidth:
  - Maximum interference temperature constraint

$$\Updownarrow$$

Maximum total received power constraint

Problem Statement

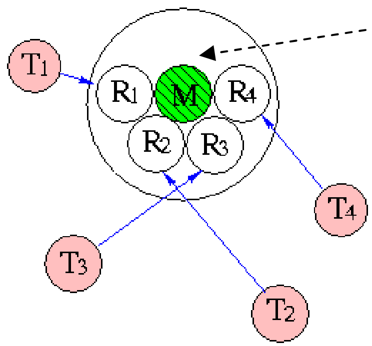
- Exclusive use model
  - Agency (or service provider) owns a single block of spectrum with bandwidth B.
  - M secondary (spread spectrum) users share spectrum with the spectrum owner (primary user).
  - Total received power (interference temperature) constraint at a single measurement point.
  - No transmission power constraints.

- How should the spectrum owner allocate resource (received power) among secondary users?
  - Objectives: efficiency, fairness, scalability
  - Approach: auction mechanisms

Related Work

- Auction mechanisms for network resource allocation:
  - Johari, Titsiklis (2004), Alpcan, Basar (2003)
  - Sun, Zheng, Modiano (2003)
- CDMA uplink power control:
  - Saraydar, Mandayam, Goodman (2002)
  - Shroff, Xiao, Chong (2001)
  - Heikkinen (2002)
- We consider auction mechanisms for allocating a constrained resource (total received power) among users with mutual interference.

System Models



Co-located receivers

(discussed in WiOpt'04)

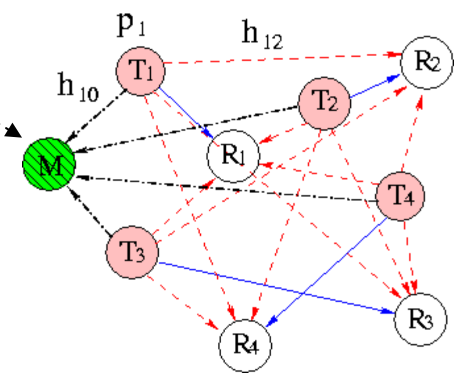
Measurement Point

T_i: transmitter

R_i: receiver

p_i: transmission power

h_{ij}: channel gain



Non-collocated receivers

- The measurement point corresponds to the primary user.
- User *i*'s received power $p_i h_{i0}$: the power received at the measurement point from user *i*.
- Interference temperature constraint $\sum_{i=1}^{M} p_i h_{i0} \le P$
- User *i*'s SINR $\gamma_i = \dfrac{p_i h_{ii}}{n_0 + \frac{1}{B}\left(\sum_{j \neq i} p_j h_{ji}\right)}$ (at user *i*'s receiver)

Utility Functions

- User *i* receives utility $U_i(\gamma_i) = U(\theta_i; \gamma_i)$
  - Strictly concave, increasing in SINR γ_i with $\lim_{\gamma_i \to \infty} U_i'(\gamma_i) = 0$
  - User-dependent priority parameter θ_i
  - Examples:
    - log utility: $\theta_i \ln(\gamma_i)$
    - Rate utility: $\theta_i \ln(1 + \gamma_i)$
- "Fair" allocation: resource allocation are based on utilities and not on network topology.
- Socially optimal allocation achieves $\max_{\{p_i\}} \sum_{i=1}^{M} U_i(\gamma_i)$
  - Typically non-convex due to interference.
- How to allocate the divisible resource (received power) when utility functions are private information?

Auction Mechanisms

- Manager (auctioneer) specifies mechanism (set of rules) for allocating resource.
- Users (bidders) submit bids for the resource.
- Manager determines allocations and payments.

Vickrey-Clarke-Groves (VCG) Auction

- Channel gains are known *a priori*.
- Users asked to submit utility functions $\{U_i(\gamma_i)\}$.
- The manager
  - Computes $U_{\max} = \max_{\{p_i\}} \sum_{i=1}^{M} U_i(\gamma_i)$ and allocate $\{p_i\}$ accordingly.
  - For each user *i*, computes $U_{\max/i} = \max_{\{p_j\}/p_i} \sum_{j \neq i} U_j(\gamma_j)$.
- Each user *i* pays $U_{\max} - U_{\max/i}$.
- Achieves socially optimal solution.
- Disadvantages:
  - Large information exchange.
  - Computational complexity.

Share Auction

- Mechanism for allocating a perfectly divisible good.
- The manager announces a reserve bid $\beta > 0$ and a unit price $\pi > 0$.
  - β ensures unique outcome.
  - π is for unit SINR or received power.
- Users submit one-dimensional bids $b_i \geq 0, i = 1, 2, ..., M$
- Manager allocates received powers proportional to bids:

$$p_i h_{i0} = \frac{b_i}{\sum_{j=1}^{M} b_i + \beta} P$$

Pricing Schemes

- SINR auction (pricing for QoS):
  user *i*'s payment = $\pi^s \gamma_i$
- Power auction (pricing for interference):
  user *i*'s payment = $\pi^p p_i h_{i0}$
- Payments are generally not the same as bids.
- Pricing improves auction outcome (total utility).

Nash Equilibrium (NE)

- Users are playing a non-cooperative game
  - Strategy = bid
  - Payoff = utility - payment
- User *i*'s best response (bid):

$$\mathcal{B}(\underbrace{b_{-i}}_{\text{Other users' bids}}) = \arg\max_{b_i} \Big(\underbrace{U_i(\gamma_i(b_i; b_{-i})) - \pi^s \gamma_i(b_i; b_{-i})}_{\text{Payoff}} \Big) \quad \text{(SINR)}$$

- A set of bids $\vec{b}^* = \{b_i^*\}_i$ is an NE if

$$b_i^* = \mathcal{B}(b_{-i}^*) \quad \text{for each } i$$

  - Fixed point for all users' best response functions.

Information and Convergence

- One-shot game with complete information
  - All utility functions, channel gains are known to all users.
  - The NE bids can be computed by each user.
- Iterative algorithm with partial information
  - Utility functions are private information.
  - Limited (local) channel knowledge.
  - Distributed bid updates converges to NE.

SINR Auction: Uniqueness of NE

- Prop. The SINR auction has a unique NE if and only if $\pi^s > \pi^s_{th}$ (threshold price).

Proof (main idea):

Best response (matrix form)

$$\mathcal{B}\left(\vec{\boldsymbol{b}}\right) = \mathbf{K}\vec{\boldsymbol{b}} + \vec{\boldsymbol{k}}_0\beta \qquad (1)$$

depend on $\{\theta_i\}$ and $\{h_{ij}\}$

$\rightarrow$

NE is a fixed point

$$\vec{\boldsymbol{b}}^* = \mathbf{K}\vec{\boldsymbol{b}}^* + \vec{\boldsymbol{k}}_0\beta \qquad (2)$$

use Perron-Frobenius theorem

SINR Auction: Properties of NE

- As $\pi^s \to \pi^s_{th}$, the system usage

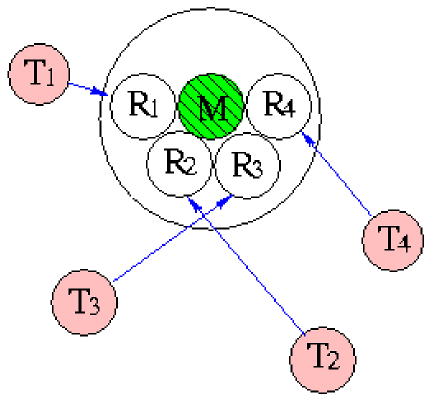
$$\eta = \frac{\sum_{i=1}^{M} p_i^* h_{i0}}{P} \to 1$$

i.e., there is no loss from the reserve bid β.

- The SINR allocations are independent on the network topology:
  - With log utility functions, users' SINR are weighted max-min fair w.r.t. $\{\theta_i\}$:

$$\gamma_i^* = \frac{\theta_i}{\pi^s}$$

Power Auction: Properties of NE (WiOpt'04)



Co-located receivers

- SINR $\gamma_i(p_i h_{i0}) = \dfrac{p_i h_{i0}}{n_0 + \frac{1}{B}(P - p_i h_{i0})}$
- $\dfrac{\left|U_i''(\gamma_i)\right|}{U_i'(\gamma_i)}(\gamma_i + B) > 2, \ \forall \gamma_i \in [0, P/n_0]$ $(\heartsuit)$

- Prop. The total utility at the power auction NE approaches the socially optimal solution if condition $(\heartsuit)$ is satisfied, and π^p approaches a threshold price π^p_{th}.

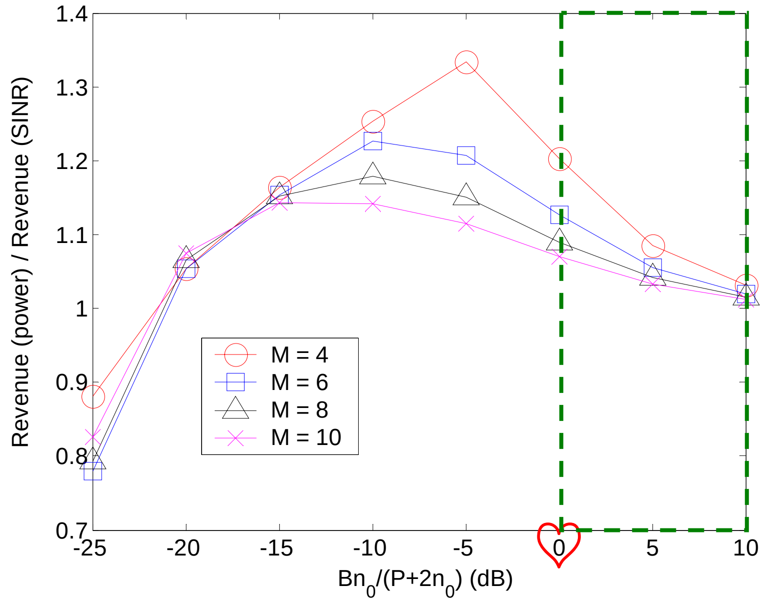
Revenue Comparison

- Revenue = total payments collected from the users
  - SINR auction: $R^s = \pi^s \sum_{i=1}^{M} \gamma_i$
  - Power auction: $R^p = \pi^p \sum_{i=1}^{M} p_i h_{i0}$

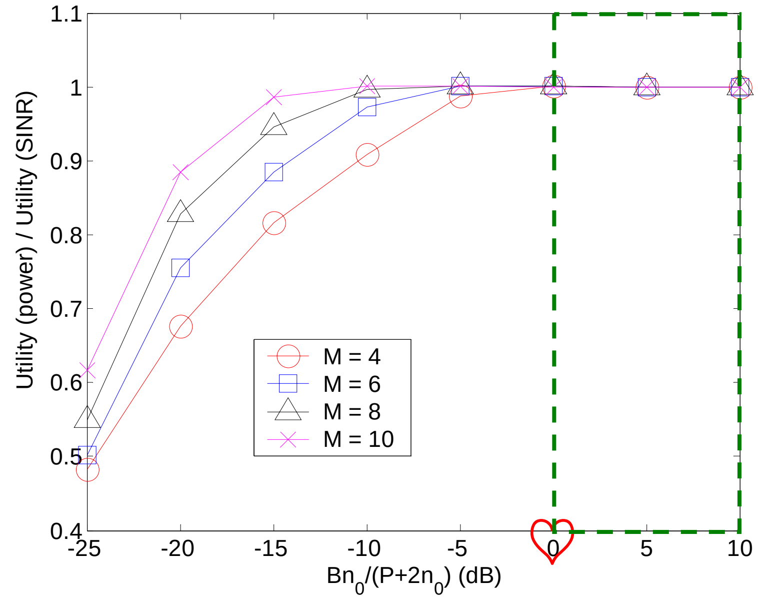
- Prop. With co-located receivers, if users have log utility functions, then $R^p > R^s$, and $R^p/R^s \to 1$ as $M \to \infty$.

- Same results hold if users have the same utility function $U(\gamma_i)$, and η is the same in both auctions.

Numerical Comparisons



(a) Revenue Comparison



(b) Utility Comparison

- Rate utility $\theta_i \ln(1+\gamma_i)$, θ_i uniformly distributed in [1,100].
- Condition ($\heartsuit$) is satisfied when $Bn_0/(P+2n_0) > 0$ dB.

Myopic Bid Update

- With SINR auction, log utilities functions:
  - Update with full information (at each time slot *t*)

$$\vec{\boldsymbol{b}}^{(t)} = \mathbf{K}\vec{\boldsymbol{b}}^{(t-1)} + \boldsymbol{k}_0\beta \qquad (1)$$

  - Update with limited information (equivalent to the above):

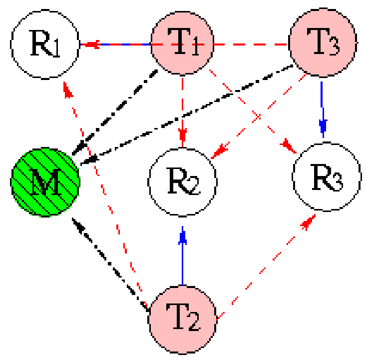
$$b_i^{(t)} = \frac{\frac{\theta_i}{\pi^s} - \gamma_i^{(t-1)}\varphi_i}{\gamma_i^{(t-1)} - \gamma_i^{(t-1)}\varphi_i} b_i^{(t-1)}, \quad \forall i \qquad (2)$$

  where

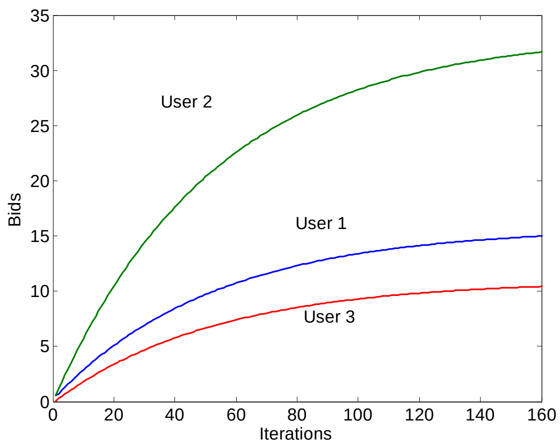
$$\varphi_i = n_0\theta_i / \left(h_{ii}/h_{i0}P\pi^s\right)$$

  - Only need to measure channel gains h_{ii}/h_{i0} and SINR $\gamma_i^{(t-1)}$.
  - Global and geometric convergences to the unique NE.
- Similar argument applies to general utility functions.

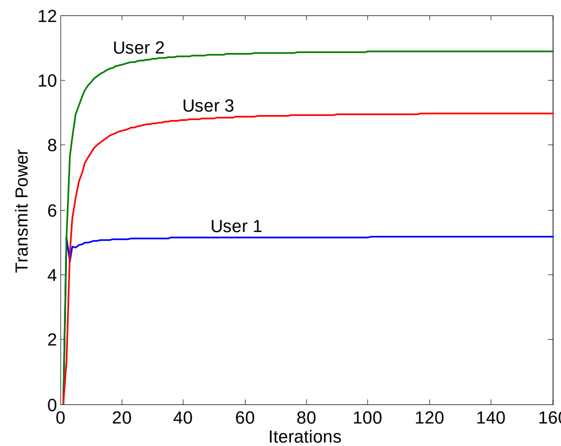
Convergence Example



- SINR auction
- Same log utility function
  $\Rightarrow$ Same SINRs at the NE



(a) Convergence of bids



(b) Convergence of transmit power

Conclusions

- Auction mechanisms can be applied to distributed spectrum sharing with non-collocated receivers.
- Properties of the Nash Equilibrium
  - SINR auction with log utility: weighted max-min fair
  - Power auction: socially optimal for large BW (♡).
  - Power auction generates more revenue.
- Distributed myopic bid updating algorithm
  - Requires only local information.
  - Global and geometric convergence.

Thank You!