Dynamic Resource Allocation in Multi-Service OFDMA Systems with Dynamic Queue Control

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***Abstract*—We consider the problem of resource allocation in downlink OFDMA systems for multi service and unknown environment. Due to users' mobility and intercell interference, the base station cannot predict neither the Signal to Noise Ratio (SNR) of each user in future time slots nor their probability distribution functions. In addition, the traffic is bursty in general with unknown arrival. The probability distribution functions of the SNR, channel state and traffic arrival/density are then unknown. Achieving a multi service Quality of Service (QoS) while optimizing the performance of the system (e.g. total throughput) is a hard and interesting task since it depends on the unknown future traffic and SNR values. In this paper we solve this problem by modeling the multiuser queuing system as a discrete time linear dynamic system. We develop a robust H^∞ controller to regulate the queues of different users. The queues and Packet Drop Rates (PDR) are controlled by proposing a minimum data rate according to the demanded service type of each user. The data rate vector proposed by the controller is then fed as a constraint to an instantaneous resource allocation framework. This instantaneous problem is formulated as a convex optimization problem for instantaneous subcarrier and power allocation decisions. Simulation results show small delays and better fairness among users.**

***Index Terms*—Resource management issues (WIN-RSMG), scheduling and queuing protocols (WIN-SQUE), system level optimization (WIN-SYLO), cross-layer design (WIN-CLRD).**

I. Introduction

PROVIDING QoS guarantees to wireless users with different QoS requirements is a challenging but important aspect of future resource allocation schemes. Wireless Channels are time varying and dynamic in nature and a wide range of phenomenon like path loss, shadowing, fast fading and frequency selection occur. In a multi-user environment dynamic allocation of system resources results in far higher performance compared to the static allocation of the very same resources.

Orthogonal Frequency Division Multiple Access (OFDMA) is a physical layer multiple access technique. In OFDMA the total frequency band is divided into orthogonal sub-channels. Different subcarriers can be assigned to different users and there is a greater flexibility in subcarrier assignments depending on system requirements. In wireless systems, OFDMA provides immunity from Inter Symbol Interference (ISI) and its multi-carrier nature provides enormous opportunities for dynamic resource allocation strategies [1]-[5]. Due to its superior performance OFDMA has been adopted for both uplink and downlink air interfaces of WiMAX fixed and mobile standards [6], [7]. 3rd Generation Partnership Project (3GPP) LTE (Long Term Evaluation) systems have also adopted OFDMA as their choice technology for the downlink [8].

A. Motivation and contributions

This paper addresses the problem of resource allocation in OFDMA systems for users having various QoS characterized by various delay constraints. QoS requirements of different users are achieved by making scheduling decisions over multiple time slots. It is obvious that the decisions made during a given time slot have a strong impact on the future backlog and PDR as well as future scheduling decisions. If the information about the future events (i.e. future channel conditions, future interference, traffic, etc.) is available, one can make much more informed scheduling decisions which can result in increased system throughput.

Unfortunately in wireless networks, the traffic is bursty in general with unpredictable arrivals and density. It is known that the assumptions on the Probability Distribution Function (PDF) of the traffic (e.g. Poisson,...) do not hold in practice. Besides, the distribution function of the future Signal to Noise and Interference Ratios (SINR) is hard to predict. Even if one assumes that the channel fast fading follows a given distribution function, the users'mobility have an impact on the average channel strength (e.g. attenuation) which makes the distribution function of the channel states unknown. In addition, the SINR depends on the future interference exerted by the other cells which in turn depends on the environment (radio conditions, traffic, users position, interference, etc...) of these cells. The PDF of users' SINR is then hard to predict. This lack of information has a severe impact on system performance. QoS provision without performance degradation thus becomes a very ambitious and challenging task.

In this paper we consider the problem of scheduling and resource allocation for users with long term QoS constraints (delay constraints) in a context where the PDFs of future traffic arrivals and densities as well as future channel states (and SINR) are unpredictable. In order to solve this problem, we first model the multi-user queuing system as a linear discrete dynamic system. The delay requirements of different service types are converted into virtual target queue lengths. We assume that the data packets exceeding these virtual targets fail to achieve their delay deadlines and are then dropped. User's queue length regulation around the virtual target queue length is then equivalent to controlling the PDR which not

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only achieves the QoS requirements but also achieves fairness among users in terms of performance. To regulate the queue lengths over multiple time slots in the context of unpredictable environment, we develop a robust controller/regulator based on H^{∞} technique [9], [10]. Our controller minimizes the variation of queue size around the virtual target queue length by deriving an instantaneous output rate every time slot that should be achieved by the physical allocation. In order to achieve the instantaneous data rate vector proposed by the controller, we develop an instantaneous power and subcarrier optimization framework that considers the instantaneous rates proposed by the controller as target rates (or minimum rates) to achieve. The objective of this problem is throughput / sum-rate maximization. This problem is formulated as a convex optimization problem and solved using KKT conditions and saddle point techniques. Due to the presence of maximum power constraint, feasibility problem should be studied. We call the instantaneous resource optimization problem feasible if all the target rates can be attained with maximum power constraint. The non-feasiblity in any time slot can be detected by solving a margin adaptive problem [11]. When the problem is non-feasible, the target rates proposed by the controller are higher than the actual instantaneous physical layer capacity. In this case we develop an algorithm which reduces some of these minimum rates while ensuring fairness among users.

B. State of the art

Dynamic resource allocation in multi-service downlink OFDMA systems requires scheduling, subcarrier allocation and power control. In [12]-[15] Proportional Fair (PF), Exponential Rule (Exp-rule) and Minimum Longest Weighted Delay First (M-LWDF) schedulers are proposed. The PF scheduler exploits the Channel State Information (CSI) while Exp-Rule and M-LWDF schedulers are channel and queue aware schedulers. These schedulers assume equal power allocation on all the subcarriers and they were developed for Time Division Multiple Access (TDMA) systems. Neither of these schedulers guarantee QoS requirements of the users nor they consider the impact of current scheduling decisions on future time slots. Their extension to multi-carrier systems like OFDMA result in further sub-optimality in addition to the absence of power control. The authors in [16] consider resource allocation problem in Gaussian broadcast channel with ISI. They allocate power and subcarriers in order to achieve proportional fairness among users. However the developed algorithms cannot be used to accommodate multiple services with varying QoS constraints. In order to handle multiple service types, long term utility optimization approach has been proposed in [17]-[19]. In this technique, the choice of utility function has strong impact on system performance. In general, utility functions which represent the optimal level of user satisfaction are hard to find. Moreover, this approach was also developed for TDMA systems [20] without power control, therefore to achieve a certain utility function in the presence of power control is not yet known.

In [21], [22] the authors maximize the long term throughput by assuming the knowledge of future channel state distribution. However the proposed techniques cannot be used when this distribution is not available. In [23] the authors consider packet scheduling with strict delay constraints and derive robust energy efficient scheduler which does not require future statistics of input arrival rate. This scheduler is developed for AWGN channel and cannot be extended to time varying multi-carrier channels. In [24] the authors propose Maximum Throughput Load Balancing (MTLB) policy in order to minimize average queue backlog in OFDMA systems. The proposed policy achieves the minimum average number of packets waiting in the system at any time when the channel follows a simple ON/OFF model. However this policy cannot be extended to more realistic channel models because it is not possible to simultaneously satisfy the Maximum Throughput and Load Balancing conditions. Some throughput has to be sacrifised in order to bring the system into a more balanced state to cater for future events. Moreover MLTB policy cannot accomodate multiple service types since different strategies have to be adopted for delay tolerant and delay sensitive traffics.

The rest of the paper is organized as follows. In section II system model and queue model is described. Optimization problem is formulated in section III. Dynamic Queue Control Problem is developed in section IV while H^{∞} controller is derived in section V. Resource allocation algorithm is discussed in section VI. Simulation results are presented in section VII while the paper is concluded in section VIII.

Notation: Throughout this paper we use uppercase boldface letters for matrices and vectors; the conjugate transpose of matrix $\mathbf{Q}$ is denoted by $\mathbf{Q}'$; $||.||^2_{\mathbf{Q}}$ denotes the squared-Euclidean norm in an appropriate finite dimensional vector space weighted by matrix $\mathbf{Q}$; $\mathbf{diag}(.)_K$ and $\mathbf{I}_K$ represents the diagonal and the Identity matrix of dimensions $K \times K$ respectively; the spectral radius of a matrix is denoted by $\overline{\rho}(.)$; Φ denotes an empty set; the expected value of a function is denoted by $E[.]$ while $(a)^+ := \max(a, 0)$.

II. System Model and Queue Model

A. System Model

We consider an OFDMA system with K users and F subcarriers. We assume that the total transmit power from the Base Station (BS) is constrained to P_{max}. Time is divided into discrete slots and during each time slot a data frame consisting of D OFDM symbols is transmitted. User channels remain constant for the duration of a time slot but may change from one time slot to another. The Channel gain to Interference and Noise Ratio (CINR) of user k on subcarrier f during time slot t is given by,

$$g^t_{k,f} = \frac{|h^t_{k,f}|^2}{\sigma^2 B + I^t_{k,f}}$$

where $h^t_{k,f}$ denotes the channel coefficient of user k on subcarrier f after fast fourier transform, σ^2 is the power spectral density of white noise, B denotes the bandwidth of a subcarrier and $I^t_{k,f}$ is the interference received by user k on subcarrier f due to transmissions in the neighboring cells. The channel coefficients $h^t_{k,f}$ vary due to fast fading and path loss attenuations. We assume that perfect value of CINR is available at the BS only for current time slot t. The CINR values for future time slots $t+i$, $\forall i \geq 1$ as well as their PDFs are assumed to be unknown. The non-availability of channel statistics is a very practical constraint due to the presence of the interference term in the SINR expression and variations in the path loss attenuations caused by user mobility.

B. Queue Model

We assume that each user maintains a separate queue at the BS. The queue length of user k evolves according to,

$$q_k^{t+1} = \left(q_k^t + f_k^t - \mu_k^t\right)^+ \tag{1}$$

where f_k^t is the input arrival rate during time slot t and μ_k^t is the actual departure rate from the queue after subcarrier and power allocation. We assume that packets arrive at the start of every time slot t and the distribution of future arrival rates is unknown. This constraint on the distribution of future input arrivals arise from practical considerations since the traffic arrives in bursts and has a variable bit rate. The input arrival rate and the delay requirements of the incoming packets depend on the service type. The departure rate from the queue depends on the resource allocation policy as well as the wireless channel conditions which are time varying in nature. Our aim is to control/regulate the queue state process of all the users in such a way that the service demands of the data in these queues is achieved. Moreover, the developed scheme should be robust to the variations in the arrival and departure processes.

C. Virtual Target Queue Length and QoS Constraint

Different services can be differentiated according to their QoS requirements on delay e.g, voice transmission and video/audio streaming services are delay sensitive whereas email and file transfer services are delay tolerant. Let $\overline{d}_k$ be the average delay requirement of the service and $\overline{q}_k = F(\overline{d}_k)$ be the target queue length of user k. Note that $F(.)$ is a function of the target delay that we want to achieve for user k. If the average input arrival rates denoted by $\overline{f}_k$ are known then using Little's law we note that $F(\overline{d}_k) = \overline{d}_k \overline{f}_k$. Other functions of delay can also be constructed. In this way we convert and differentiate different services according to their corresponding target queue lengths. Since the target queue length depends on the function $F(.)$ hence this is in fact a virtual target because changing this function will change the target queue length. We assume that the number of packets exceeding these virtual target queue lengths fail to achieve their delay deadline and are dropped. We introduce the following long term QoS constraint to regulate the user queues,

$$\mathbf{E}_{f_k^t,\mu_k^t}\left[\mathcal{U}(q_k^t, \overline{q}_k)\right] \leq \overline{\beta}_k \quad ,\forall k \tag{2}$$

where $\mathcal{U}(.)$ is some known function of queue state process and $\overline{\beta}_k$ is the target value to achieve for each user k. The nature of $\mathcal{U}(.)$ dictates the strictness with which the delay constraints of the packets are achieved. This function can have linear, exponential or some other dependence on the queue state process. In the subsequent analysis we will consider a linear function of queue state process which makes the target values equal to the virtual target queue lengths. Thus the QoS constraint takes the following form,

$$\lim_{T\to\infty} \frac{1}{T}\mathbf{E}_{f_k^t,\mu_k^t}\left[\sum_{t=1}^{T} q_k^t\right] \leq \overline{q}_k \quad ,\forall k \tag{3}$$

This simple function is chosen for the ease of analysis in latter sections. However the method developed in this paper is more general and can be used for any other function as well. We can define some target fairness level a_k for each user in terms of its virtual target queue length,

$$a_k = \frac{1/\overline{q}_k}{\sum_{k=1}^{K} 1/\overline{q}_k} \tag{4}$$

The choice of a_k is again not restricted since any other value of target fairness level can also be used. Target fairness level can even be a constant specified in advance for each and every user depending on its service type.

III. Optimization Problem Formulation and Our Approach

A. Optimization Problem Formulation

We consider an OFDMA system where a subcarrier is allocated to only one user. Let $\mathcal{F}_k^t$ denote the subcarriers of user k during time slot t. According to Shannon's Capacity formula, the data rate achieved on the allocated subcarrier set during time slot t is given as[1],

$$R_k^t = \sum_{f\in\mathcal{F}_k^t} \log\left(1 + p_{k,f}^t g_{k,f}^t\right) \quad nats/s/Hz \tag{5}$$

where $p_{k,f}^t$ is the power allocated to user k on subcarrier f during time slot t. This optimization problem is a combinatorial problem due to the fact that users cannot share the same subcarrier. There are K^F possible subcarrier allocations and the optimal solution will require a search across all these combinations. This problem can however be avoided by relaxing the exclusive subcarrier assignment constraint and using the familiar notion of time sharing of each subcarrier by different users [1],[5]. Let us introduce $\gamma_{k,f}^t \in [0,1]$ as the time sharing factor for user k on subcarrier f during time slot t. Thus during a time slot t user k is allowed to transmit on subcarrier f for $\gamma_{k,f}^t D$ OFDM symbols. This is possible from resource allocation point of view because we have assumed that users wireless channels remains constant in each time slot. The assumption on subcarrier sharing introduces the following constraint,

$$\sum_{k=1}^{K} \gamma_{k,f}^t \leq 1 \quad ,\forall t, f \tag{6}$$

As a result of time sharing, data rate achieved by user k on subcarrier f during time slot t becomes, $R_{k,f}^t = \gamma_{k,f}^t \log(1 + p_{k,f}^t g_{k,f}^t)$. This function is neither convex nor concave. Therefore we define, $o_{k,f}^t = \gamma_{k,f}^t p_{k,f}^t$ as the average power allocated to user k on subcarrier f during time slot t. With this change of variable we have,

$$R_{k,f}^t(o_{k,f}^t, \gamma_{k,f}^t, g_{k,f}^t) = \gamma_{k,f}^t \log\left(1 + \frac{o_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right) \tag{7}$$

[1]The data rates are in fact spectral efficiencies and they are expressed in nats for analytical convenience.

Eq (7) represents a concave function since its Hessian is negative semi-definite when $\gamma_{k,f}^t \geq 0$ and $o_{k,f}^t \geq 0$. Now with $R_{k,f}^t(.)$ as defined in (7), we can write our optimization problem as,

$$(\mathcal{P}_1)\begin{cases}\max_{o_{k,f}^t,\gamma_{k,f}^t} \mathbf{E}_{g_{k,f}^t}\left[\sum_{k=1}^{K}\sum_{f=1}^{F} R_{k,f}^t(o_{k,f}^t,\gamma_{k,f}^t,g_{k,f}^t)\right]\\ \text{subject to, } \sum_{k=1}^{K}\sum_{f=1}^{F} o_{k,f}^t \leq P_{max} \quad ,\forall t \text{ (A1)}\\ \sum_{k=1}^{K}\gamma_{k,f}^t \leq 1 \quad ,\forall t,f \quad \text{(A2)}\\ \lim_{T\to\infty}\frac{1}{T}\mathbf{E}_{f_k^t,\mu_k^t}\left[\sum_{t=1}^{T} q_k^t\right] \leq \overline{q}_k, \forall k\text{(A3)}\end{cases}$$

The objective of problem $(\mathcal{P}_1)$ is the long term throughput maximization or system capacity. Constraints (A1) and (A2) are the instantaneous or per time slot constraints. Constraint (A1) demands that the total transmit power in any time slot t should always be less than the maximum power available at the BS. Constraint (A2) is the time sharing constraint on the subcarriers. Constraint (A3) is the QoS constraint of the users. This constraint demands that the queue length of each user should be regulated around its virtual target queue length. Without this constraint the problem can be solved by adopting the instantaneous output rates of the users according to the instantaneous wireless channel conditions. The sum-rate is maximized when each subcarrier is allocated to the user with the best CINR value for that subcarrier during each time slot t [25], [26]. However, the presence of QoS constraint in our problem restricts such a solution. It should be noted that the optimal solution to the non-convex problem $(\mathcal{P}_1)$ with unknown distribution of future input arrivals and future channel states is not known in the literature. Since the distributions of future input arrivals and channel states are unknown we adopt a two step approach described in the next subsection.

B. Proposed Approach

Since the QoS constraint (A3) has to be achieved over multiple time slots we split the problem into two sub-problems. We develop a controller/regulator which regulates the queues around the target queue lengths by proposing an instantaneous data rate $\tilde{R}_k^t$ for each queue in each time slot. The instantaneous output rate vector proposed by the controller is then used as an additional constraint in the following instantaneous sum-rate maximization problem;

$$(\mathcal{P}_2)\begin{cases}\max_{o_{k,f}^t,\gamma_{k,f}^t} \sum_{k=1}^{K}\sum_{f=1}^{F} R_{k,f}^t(o_{k,f}^t,\gamma_{k,f}^t,g_{k,f}^t)\\ \text{subject to, } \sum_{k=1}^{K}\sum_{f=1}^{F} o_{k,f}^t \leq P_{max} \quad \text{(B1)}\\ \sum_{k=1}^{K}\gamma_{k,f}^t \leq 1 \quad ,\forall f \quad \text{(B2)}\\ \sum_{f=1}^{F} R_{k,f}^t(o_{k,f}^t,\gamma_{k,f}^t,g_{k,f}^t) \geq \tilde{R}_k^t \text{ ,}\forall k\text{(B3)}\end{cases}$$

(B3) is the instantaneous data rate constraint. This is a convex optimization problem and it has to be solved in each time slot t. If the data rates achieved by the instantaneous resource allocation algorithm are equal to or greater then the proposed data rates $\tilde{R}_k^t$, $\forall k$, then QoS constraint (A3) is satisfied. However if in any time slot these data rates cannot be achieved due to bad channel conditions and power limitations then this error has to be fed back to the controller which compensates for this loss in the next time slots. Hence time diversity in the wireless channel is exploited and constraint (A3) is achieved by tracking the controller decisions. Since we maximize the instantaneous sum-rate in $\mathcal{P}_2$ according to the regulated output rates therefore the long term objective in $\mathcal{P}_1$ is also maximized. This justifies the two step approach. This approach is detailed in Fig. 1. An H^∞ controller (which will be explained later) derives an output data rate $\tilde{R}_k^t$ for each user at each time slot t. This data rate is fed to the resource allocation block for subcarrier and power assignment. A set of subcarriers $\mathcal{F}_k^t$ is allocated to user k and a data rate vector $\overline{\mu} = \{\mu_k^t, \forall k\}$ is achieved according to the physical channel conditions. The allocation decisions are sent to the users via separate control channels which allow the users to recover their data. The achieved data rate vector $\overline{\mu}$ is sent back to the controller in order to track its decisions according to the changing wireless conditions. In the next section two sections we develop the Queue Control Problem and propose an H^∞ controller.

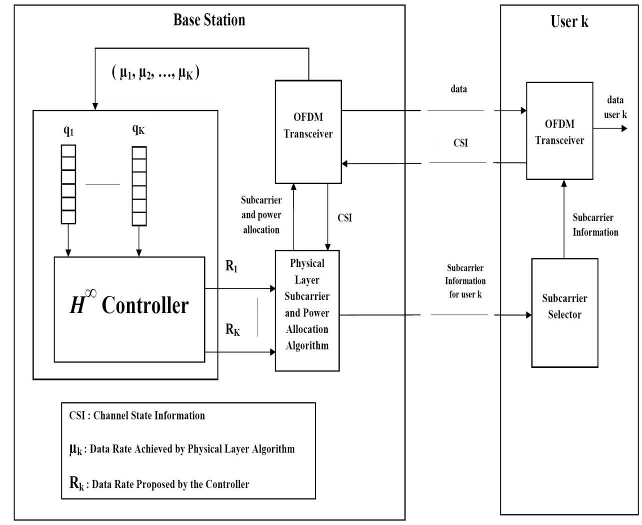

Fig. 1. System model.

IV. Dynamic Queue Control Problem

A. State Equation

Before developing the controller we need to appropriately model our problem as a queue control problem. The objective is to write the queue dynamics as a linear state space equation and then to introduce an appropriate control variable u that adapts this queue in such a way to minimize a given cost function. The cost function is defined in such a way that minimizing the cost function is equivalent to achieve the long term constraint (A3) of our original problem. Please notice that in order to ensure the existence of an optimal solution (optimal controller), the dynamic problem should have a standard form. In the following, we will define then new variables and make some changes in order to obtain a standard linear dynamic problem.

First of all, since the actual achieved data rates μ_k^t differ from the data rates $\tilde{R}_k^t$ proposed by the controller, we introduce $\tilde{R}_k^t$ into the queue dynamics in (1) as,

$$q_k^{t+1} = \left(q_k^t + f_k^t - \tilde{R}_k^t + \left[\tilde{R}_k^t - \mu_k^t\right]\right)^+ \quad (8)$$

where $\left(\tilde{R}_k^t - \mu_k^t\right)$ is now the tracking error which represents the difference between the proposed/controlled rate and the actual achieved rate after the instantaneous subcarrier and power allocation. We then define a control u_k^t as,

$$u_k^t = \left(a_k^t - a_k\right)C \tag{9}$$

a_k^t is the instantaneous fairness level of user k at time t (given by $a_k^t = \tilde{R}_k^t/C$) where C is the average capacity of the underlying physical channel and a_k is the target fairness level of each user k. Note that, in standard linear control problems, control u_k^t should be defined such that $lim_{t\to\infty} u_k^t = 0$ or the total norm $\sum_{t=1}^{\infty} \|u_k^t\|^2$ is minimized. This will result in our case in minimizing the difference between the instantaneous fairness level and the target fairness level, in other words our controller will lead us to achieve a given target fairness level. This target fairness level is defined according to the QoS of all the users in the system. Therefore, (8) becomes,

$$q_k^{t+1} = \left(q_k^t - u_k^t + f_k^t - a_k C + \left[\tilde{R}_k^t - \mu_k^t\right]\right)^+ \tag{10}$$

Now, let us group all the variables with unknown probability distribution function (i.e. the variables that affect our queue in an unpredictable way) in one variable denoted by λ_k^t. Then, $\lambda_k^t = f_k^t - a_k C + [\tilde{R}_k^t - \mu_k^t]$ and

$$q_k^{t+1} = \left(q_k^t - u_k^t + \lambda_k^t\right)^+ \tag{11}$$

λ_k^t represents now the unpredictable disturbance or noise in the system dynamics. The probability distribution of λ_k^t is unknown since it depends on the combined effect of tracking error as well as the variations in the input arrival rate. Finally, in order to get a standard state space equation, we define a new state variable,

$$x_k^t := q_k^t - \overline{q}_k \tag{12}$$

which measures the instantaneous deviation of user queue length from its virtual target queue length. In fact, the state variable should be defined such that the controller will minimize the total (or average) norm of the state variable i.e. $lim_{t\to\infty} x_k^t = 0$ or the total norm $\sum_{t=1}^{\infty} \|x_k^t\|^2$ (one can notice that this equivalent to achieve our constraint (A3) as we will explain in the next subsection when we define the cost function). In terms of state variable x_k^t, (11) can now be written as,

$$x_k^{t+1} = x_k^t - u_k^t + \lambda_k^t \tag{13}$$

Since there are K users in the system, we define, $\mathbf{X}^t = [x_1^t, \ldots, x_K^t]'$, $\mathbf{U}^t = [u_1^t, \ldots, u_K^t]'$ and $\Gamma^t = [\lambda_1^t, \ldots, \lambda_K^t]'$ and write the system state as,

$$\mathbf{X}^{t+1} = \mathbf{A}\mathbf{X}^t + \mathbf{B}\mathbf{U}^t + \mathbf{D}\Gamma^t \tag{14}$$

where $\mathbf{A} = \mathbf{I}_K$, $\mathbf{B} = -\mathbf{I}_K$ and $\mathbf{D} = \mathbf{I}_K$. Equation (14) represents a discrete time linear dynamic system affected by disturbance/noise with unknown distribution. The non-availability of the pdf of λ_k^t makes it impossible to predict the evolution of queue state dynamics. However, it is possible to control/regulate such systems to minimize a particular objective/cost function by using appropriate control theoretic tools.

B. Quadratic Cost Function

After modeling our system as a standard discrete time linear state space model, we define in this section a quadratic cost function. This cost function should be defined in such a way (as explained in the previous subsection) that if it is minimized then constraint (A3) is achieved and the tracking error between the proposed rate by the controller and the achieved rate is thus reduced. Therefore, we define the following quadratic cost function,

$$J = \lim_{T\to\infty} \frac{1}{T}\mathbf{E}\left[\sum_{t=1}^{T}\left(||\mathbf{X}^t||^2_{\mathbf{W}} + ||\mathbf{U}^t||^2_{\mathbf{V}}\right)\right] \tag{15}$$

where, $\mathbf{V} = \mathbf{diag}(\alpha, \ldots, \alpha)_K$ and $\mathbf{W} = \mathbf{I}_K$ are the weighting matrices. We define,

$$\alpha = \exp\left\{\rho \sum_{k=1}^{K} (\tilde{R}_k^t - \mu_k^t)\right\}$$

where ρ is a scaling constant. The objective is to minimize this cost function J. One can alternatively write the instantaneous objective in (15) as,

$$\mathbf{L} = ||\mathbf{X}^t||^2_{\mathbf{W}} + ||\mathbf{U}^t||^2_{\mathbf{V}} \tag{16}$$

There are two terms in our cost function:

- The first term (that contains our state vector $\mathbf{X}^t$) represents the penalty for deviating from the target queue length. Since target queue length depends on the service type therefore, this term directly affects the QoS requirements of the users. One can notice that minimizing this first term will result in achieving constraint (A3) of our original problem.
- The second term contains the control vector $\mathbf{U}^t$ and represents the penalty for deviating from the target fairness level (defined according to the QoS of all users in the system). Therefore, minimizing this second term will result in achieving the target fairness level. In addition, this second term contains the variable α in the weighting matrix which depends on the scaled difference between the proposed rate by the controller and the achieved data rates by the users. This term tracks the error in the controller decisions according to the changing wireless conditions at the physical layer. For example, if the instantaneous system throughput is less than the sum of data rates obtained by the controller, α increases in the cost function. Therefore, the control action $\mathbf{U}^t$ decrease the outputs (i.e. the proposed data rates) in the next iteration. Similarly, when the actual physical layer capacity is higher than the controlled/proposed data rates, then α decreases. In this case, $\mathbf{U}^t$ can either increase or decrease depending on the queue lengths. In other words, since enough physical layer capacity is available, the control $\mathbf{U}^t$ can increase or decrease the data rates of the users depending on the QoS constraints and the queue lengths of the users.

V. H^∞ CONTROLLER

A. H^∞ controller

H^∞ controller is a robust controller that can be used to drive a linear dynamic system affected by unknown disturbances to minimize a certain cost function. In order to develop an H^∞ controller for our problem with state equation (14) and quadratic cost function (15) a proper scaling of the objective function is required (in fact in standard H^∞ problem, the weight matrix $\mathbf{V}$ of the control vector should be the Identity matrix $\mathbf{I}_k$). Therefore, we define the scaling matrix $\mathbf{S}$ as,

$$\mathbf{S} = \mathbf{diag}\Big(\frac{1}{\sqrt{\alpha}}, \dots, \frac{1}{\sqrt{\alpha}}\Big)_K \tag{17}$$

We define also a control vector $\mathbf{U}_\mathbf{h}^t$ such that $\mathbf{U}^t = \mathbf{U}_\mathbf{h}^t\mathbf{S}$. With this scaling the state equation (14) becomes,

$$\mathbf{X}^{t+1} = \mathbf{A}\mathbf{X}^t + \mathbf{B_h}\mathbf{U}_\mathbf{h}^t + \mathbf{D}\Gamma^t \tag{18}$$

where, $\mathbf{B_h} = \mathbf{BS}$, while the objective function (16) transforms into standard form [9] i.e,

$$\overline{\mathbf{L}} = ||\mathbf{X}^t||^2_\mathbf{W} + ||\mathbf{U}_\mathbf{h}^t||^2_{\mathbf{I}_K} \tag{19}$$

The last step in our modeling, before solving our problem using H^∞, is to define the observation of our state (measurement equation). Two cases are possible: perfect and imperfect state. Perfect state means that once the controller makes a decision u, the state variable can be measured exactly by the system (in our case this means that the base station can know exactly the state X or the queue state q). Imperfect state means that the state is erroneous or known with noise. In our problem, we consider the more general case i.e. imperfect state. In fact, the controller makes a decision u or R and send it to the physical layer that decides to transmit rate μ. However, due to feedback delay and channel estimation errors, the channel state information used in the allocation is not known perfectly by the base station. This means that the packets transmitted to the users may not be received correctly and should be retransmitted again. The users inform the base station (with a given feedback delay greater than 2 timeslots in general), using an acknowledgment (non acknowledgment), the reception or not of the packets. This means that the base station (and therefore) the controller cannot know exactly the queue state before receiving this feedback from the users (notice also that the feedback is noisy and may generate some errors and then imperfection). Therefore, at time t, the controller cannot determine exactly the state x_k^t. Consequently, we write the observation/measurement equation of our system using the general form,

$$\mathbf{Y}^t = \mathbf{C}_3\mathbf{X}^t + \mathbf{D}_3\Gamma^t \tag{20}$$

where, $\mathbf{C}_3 = \mathbf{I}_K$ and $\mathbf{D}_3 = \mathbf{I}_K$.

The problem is now in standard form (state equation, cost function and measurement equation) and can be solved using H^∞ control theory. The H^∞ controller adopted in this section is the robust controller discussed in [9] and based on zero sum game theory. To solve the problem, we introduce the following H^∞ control criterion (15) [9]-[10],

$$J_\pi = \lim_{T\to\infty}\Big\{\sum_{t=0}^{T}\Big(||\mathbf{X}^t||^2_\mathbf{W} + ||\mathbf{U}_\mathbf{h}^t||^2_{\mathbf{I}_K} - \pi^2||\Gamma^t||^2\Big)\Big\} \tag{21}$$

where, π is the level of attenuation. The approach adopted here is to solve the problem using minimax optimization technique where the cost function is minimized over maximum unknown disturbance. This problem can be viewed as a kernel of a two player game. Player 1 is the controller $\mathbf{U}$ which tries to minimize the cost while player 2 aims to spoil the controller strategy by choosing a maximum disturbance Γ. One can refer to [9] for more general class of discrete-time zero-sum games, with various information patterns, where sufficient conditions for the existence of a saddle point are provided when the information pattern is perfect and imperfect state. The solution is then,

$$\min_{\mathbf{U}}\max_{\widetilde{\Gamma}} J_\pi^T = \overline{\mathbf{L}}_0^T - \pi^2\widetilde{\Gamma} \tag{22}$$

$$\text{where,}\quad \overline{\mathbf{L}}_0^T = \sum_{t=0}^{T}\Big(||\mathbf{X}^t||^2_\mathbf{W} + ||\mathbf{U}_\mathbf{h}^t||^2_{\mathbf{I}_K}\Big) \tag{23}$$

$$\text{and,}\quad \widetilde{\Gamma} = \sum_{t=0}^{T}||\Gamma^t||^2 \tag{24}$$

The H^∞ controller can now be obtained according to the following theorem [9];

Theorem: Let $s_1 = \frac{1}{\alpha} - \frac{1}{\pi^2}$ and $s_2 = 1 - \frac{1}{\pi^2}$. There exists a state feedback H^∞ controller $\mathbf{U}^t$ such that,

$$\mathbf{U}^t = \mathbf{U}_\mathbf{h}^t\mathbf{S} = (\frac{1}{\alpha})(\mathbf{M} - \mathbf{I})\hat{\mathbf{X}}^t \tag{25}$$

where Matrices $\mathbf{M}$ and Σ and state estimate vector $\hat{\mathbf{X}}^t$ are defined/given as,

1) $\mathbf{M}$ is a minimal non-negative definite solution to the following algebraic Riccati equation,

$$\mathbf{M} = \mathbf{I} + \mathbf{M}(\mathbf{I} + s_1\mathbf{M})^{-1} \tag{26}$$

$$\text{s.t.,}\quad \pi^2\mathbf{I} - \mathbf{M} > 0 \tag{27}$$

2) Σ is a minimal non-negative definite solution to the following algebraic Riccati equation,

$$\Sigma = \mathbf{I} + \Sigma(\mathbf{I} + s_2\Sigma)^{-1} \tag{28}$$

$$\text{s.t.,}\quad \pi^2\mathbf{I} - \Sigma > 0 \tag{29}$$

3) and the state estimate $\hat{\mathbf{X}}^t$ is generated by,

$$\hat{\mathbf{X}}^t = (\mathbf{I} + \Sigma - \pi^{-2}\Sigma\mathbf{M})^{-1}(\tilde{\mathbf{X}}^t + \Sigma\mathbf{Y}^t) \tag{30}$$

$$\text{and,}\quad \tilde{\mathbf{X}}^{t+1} = \tilde{\mathbf{X}}^t - \frac{1}{\alpha}\mathbf{U}^t + \Sigma(\mathbf{I} + s_2\Sigma)^{-1}(\mathbf{Y}^t - s_2\tilde{\mathbf{X}}^t) \tag{31}$$

In addition the following constraint on the spectral radius of $\Sigma\mathbf{M}$, i.e. $\overline{\rho}(\Sigma\mathbf{M}) < \pi^2$, should also be satisfied.

The above theorem allows us to find the optimal control $\mathbf{U}^t$ at each time. Using the measurement $\mathbf{Y}^t$ at time t and equations (31) and (30) we estimate the state vectors $\tilde{\mathbf{X}}^{t+1}$ and $\hat{\mathbf{X}}^{t+1}$. Then, after finding Σ and $\mathbf{M}$ using the Riccati equations and the constraints described above, one can feed Σ, $\mathbf{M}$ and $\hat{\mathbf{X}}^{t+1}$ into equation (25) and find the control $\mathbf{U}^{t+1}$ at time $t+1$ and so on and so forth for all timeslots. Then, once we know the control $\mathbf{U}^t$ at each time t, the minimum data rate $\tilde{R}_k^t$ proposed by the controller for user k at time t is given as (using the definition of the control u_k^t in (9)),

$$\tilde{R}_k^t = u_k^t + a_kC \tag{32}$$

These data rates are fed to the instantaneous physical resource allocation problem ($\mathcal{P}_2$) as explained before in section III-B.

The above theorem states the existence of an optimal control with a set of constraints on π^2 and matrices Σ and $\mathbf{M}$. However, it does not specify how to find π numerically in such a way that the H^∞ controller converges to the optimal solution. In the next subsection, we discuss this issue and provide an algorithm that finds the optimal value of π^2 which satisfies all the required conditions.

B. Algorithm to find π and Convergence

Searching the optimal solution among all possible values of π is a hard and complicated task. The main difficulty lies in the fact that the sign indefinite nature of $s_1 = \frac{1}{\alpha} - \frac{1}{\pi^2}$ and $s_2 = 1 - \frac{1}{\pi^2}$ in the Riccati equations (26) and (28) makes it quite difficult to test numerically whether or not a solution to the H^∞ problem exists. In this paper, we provide an algorithm that finds the optimal π at reduced complexity (see the complexity in the next subsection). The algorithm is given in Table I and is inspired from the computation of $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norms in [27]. First of all, one can notice that to minimize J_π^T in (22) as much as possible (in order to minimize the cost function and thus achieve constraint (A3)), π should be as small as possible. Therefore, we start with a small value of π and increase it until all constraints (defined in the above theorem) are satisfied. However, small values of π may not guarantee a stable solution to our H^∞ control. To overcome the problem of the sign indefinite nature of $s_1 = \frac{1}{\alpha} - \frac{1}{\pi^2}$ and $s_2 = 1 - \frac{1}{\pi^2}$ in the Riccati equations (26) and (28), we construct the associate Hamiltonian matrices of these two Riccati equations, namely $\mathbf{H}_1$ and $\mathbf{H}_2$: $\mathbf{H}_1 = \begin{bmatrix} \mathbf{I} & -s_1\mathbf{I} \\ -\mathbf{I} & -\mathbf{I} \end{bmatrix}$ and $\mathbf{H}_2 = \begin{bmatrix} \mathbf{I} & -s_2\mathbf{I} \\ -\mathbf{I} & -\mathbf{I} \end{bmatrix}$.
A known result from control theory states that if a Hamiltonian matrix has no eigenvalues on the imaginary axis, then the solution to the associate Riccati equation is stable (see [10], [27] for more details). Therefore, if the eigenvalues of $\mathbf{H}_1$ and $\mathbf{H}_2$ have no imaginary part, then the solution to the two associate Riccati equations (26) and (28) are stable. If in addition the constraints defined in the H^∞ theorem (in subsection V-A) are satisfied, the H^∞ controller is optimal. This means that instead of solving the Riccati equations for all possible values of π, we can use the test on $\mathbf{H}_1$ and $\mathbf{H}_2$ without losing stability or optimality. This proves that for finite π our algorithm will always converge to the optimal solution.

To complete the proof of convergence, let us consider the case when the optimal value of π is very high ($\pi \to \infty$). This may arise for example when the traffic arrival is very high (tend to infinity). If we set $\pi^{-2} = 0$ in the H^∞ equations (i.e. 25 to 31), the result reduces to that of Linear Quadratic (LQ) control problem. LQ control problem always has a solution [28]. Hence we expect that for sufficiently large value of π^2 the Riccati equations (26) and (28) will always have a solution and the algorithm described in Table I will always converge.

C. Complexity Analysis of π^2-Algorithm

The algorithm presented in Table I is an iterative algorithm. In each iteration of this algorithm there are three eigenvalue decompositions (two for the Hamiltonian matrices and one to find the spectral radius of ΣM), the solution of two matrix Riccati equations and three comparisons. Let $\mathcal{C}_K$ be the complexity of all these mathematical operations when there are K users in the system. Let $\mathcal{I}_K$ be the number of iterations required to find the optimal value $\pi*^2$. Then the total complexity of this algorithm is $\mathcal{I}_K\mathcal{C}_K$. The mathematical operations can be easily implemented without much complexity and the number of iterations depend on different system parameters which can only be found by simulations. The simulation results, presented in section VII, show clearly that the number of iterations needed to find the optimal value of π is very small and our algorithm has thus a very small complexity.

TABLE I
π^2 ALGORITHM

1. Start with small value of $\pi^2 > 0$.
2. Increase π^2 in small steps of ζ.
3. Formulate the two Hamiltonian matrices corresponding to the Riccati equations (26) and (28), $\mathbf{H}_1 = \begin{bmatrix} \mathbf{I} & -s_1\mathbf{I} \\ -\mathbf{I} & -\mathbf{I} \end{bmatrix}$ and $\mathbf{H}_2 = \begin{bmatrix} \mathbf{I} & -s_2\mathbf{I} \\ -\mathbf{I} & -\mathbf{I} \end{bmatrix}$
4. Find the eigenvalues of $\mathbf{H}_1$ and $\mathbf{H}_2$.
5. IF any eigenvalue of $\mathbf{H}_1$ and $\mathbf{H}_2$ has imaginary part then go to step 2 ELSE for this value of π^2 solve the corresponding Riccati equations (26) and (28) to find the values of $\mathbf{M}$ and Σ.
6. IF $\mathbf{M} < 0$ or $\Sigma < 0$, go to step 2 ELSE Test conditions (27) and (29).
7. IF (27) and (29) are not satisfied go to step 2 ELSE Check the spectral radius of $\Sigma\mathbf{M}$.
8. IF $\overline{\rho}(\Sigma\mathbf{M}) > \pi^2$, go to step 2 ELSE $\pi*^2$ is the optimal value and terminate the algorithm.

VI. INSTANTANEOUS RESOURCE ALLOCATION ALGORITHM

In this section we discuss the instantaneous subcarrier and power allocation problem $\mathcal{P}_2$ developed in section III-B.

$$(\mathcal{P}_2)\begin{cases} \max_{o_{k,f}^t,\gamma_{k,f}^t} \ \sum_{k=1}^K\sum_{f=1}^F R_{k,f}^t(o_{k,f}^t,\gamma_{k,f}^t,g_{k,f}^t) \\ \text{subject to, } \sum_{k=1}^K\sum_{f=1}^F o_{k,f}^t \le P_{max} \quad \text{(B1)} \\ \qquad \sum_{k=1}^K \gamma_{k,f}^t \le 1 \quad ,\forall f \quad \text{(B2)} \\ \qquad \sum_{f=1}^F R_{k,f}^t(o_{k,f}^t,\gamma_{k,f}^t,g_{k,f}^t) \ge \tilde{R}_k^t \ ,\forall k \text{(B3)} \end{cases}$$

This is a convex optimization problem with linear and convex differentiable constraints. We can solve this problem by using Lagrange Dual Decomposition theory [29]. Let η, $\{\psi_f\}_{f=1}^F$ and $\{\delta_k\}_{k=1}^K$, be the Lagrange multipliers associated with constraints (B1), (B2) and (B3) respectively. The Lagrangian of the problem is,

$$\mathcal{L}\{o_{k,f}^t,\gamma_{k,f}^t,\eta,\delta_k,\psi_f\} = \sum_{k=1}^K\sum_{f=1}^F \gamma_{k,f}^t \log\left(1+\frac{o_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right) + \sum_{k=1}^K \delta_k\left\{\sum_{f=1}^F \gamma_{k,f}^t \log\left(1+\frac{o_{k,f}^t g_{k,f}^t}{\gamma_{k,f}^t}\right) - \tilde{R}_k^t\right\} - \sum_{f=1}^F \psi_f\left\{\sum_{k=1}^K \gamma_{k,f}^t - 1\right\} - \eta\left\{\sum_{k=1}^K\sum_{f=1}^F o_{k,f}^t - P_{max}\right\} \tag{33}$$

Since the objective and the constraint functions are convex differentiable, therefore, the duality gap is zero. The solution can be obtained by using the primal-dual technique. In this technique we obtain the saddle point of the min-max optimization problem. The existence of the saddle point and hence the solution is guaranteed due to convexity. The maximization problem is the Lagrange dual problem which maximizes,

$$\mathcal{D}\{\eta, \delta_k\} = \max_{\{o_{k,f}^t, \gamma_{k,f}^t\} s.t (\mathrm{B2})} \mathcal{L}\{o_{k,f}^t, \gamma_{k,f}^t, \eta, \delta_k, \psi_f\} \quad (34)$$

while the minimization problem is the dual problem of $\mathcal{P}_2$,

$$\min_{\eta, \delta_k} \mathcal{D}\{\eta, \delta_k\} \quad (35)$$

Lagrange dual problem (34) can be readily decomposed into KF subproblems. The solution can then be obtained by solving the appropriate Karush-Kuhn-Tucker (KKT) conditions which are sufficient for global optimality. KKT conditions can be obtained by setting, $\frac{\partial \mathcal{L}\{.\}}{\partial o_{k,f}^t} = 0$ and $\frac{\partial \mathcal{L}\{.\}}{\partial \gamma_{k,f}^t} = 0$. From these conditions we obtain,

$$p_{k,f}^t = \Big(\frac{1+\delta_k}{\eta} - \frac{1}{g_{k,f}^t}\Big)^+ \quad (36)$$

$$(1+\delta_k)\Bigg(\Big(\log\big(\frac{(1+\delta_k)g_{k,f}^t}{\eta}\big)\Big)^+ - \Big(1 - \frac{\eta}{(1+\delta_k)g_{k,f}^t}\Big)^+\Bigg) = \psi_f \quad (37)$$

Thus for optimal values of Lagrange multipliers, the instantaneous power allocation is of multiuser waterfilling type. The subcarrier is allocated to the user for which the value of ψ_f is maximum. The values of Lagrange multipliers are obtained by the dual problem which results in iterative updation of these multipliers by using standard sub-gradient projection method. The algorithms for this problem (in case of feasibility) have been developed in an earlier paper [11]. In this paper we use the optimal algorithm developed in [11] for the feasible case. This algorithm achieves the rate and power constraints simultaneously. This algorithm consists of an inner loop and an outer loop. The outer loop starts with a small value of $\eta > 0$ and increments it in small steps. For each value of η, the inner loop allocates the power and subcarriers to the users according to their data rate constraints. This process is repeated till all the constraints are satisfied. For completeness we produce this algorithm from [11] in Table II.

A. Feasibility and Convergence

The problem $\mathcal{P}_2$ is considered feasible if all the minimum rate constraints (B3) can be satisfied within the given amount of power. From standard convex optimization theory, if the problem is feasible saddle point always exists for the corresponding min-max optimization problem. The saddle point is then the optimal solution of the problem [29]. Thus the convergence of the resource allocation algorithm is always guaranteed in case of feasiblity. The iterative algorithm will then optimally allocate subcarriers and powers to different users and will converge to the optimal (minimum) values of the corresponding Lagrange multipliers. On the contrary if the problem is non-feasible, the data rates proposed by the controller exceed the instantaneous capacity at the physical layer. In this situation, the saddle point does not exist (it lies outside the region defined by the constraint set). Thus it is impossible to attain the given set of data rates with the given amount of power. The optimization problem will not converge and the Lagrange multipliers will grow unbounded. The non-feasibility can thus be detected by the unbounded growth of the corresponding lagrange multipliers once the algorithm in Table II is executed. We can also detect non-feasibility by solving a margin adaptive algorithm as in [11] before executing the optimization algorithm in Table II.

TABLE II
INSTANTANEOUS RESOURCE ALLOCATION ALGORITHM

1) Initialization: $\eta = 0$
Outer loop:
2) While $P_{total} < P_{max}$
$\eta = \eta + \Delta_j$
Inner loop:
1) Initialization: $\delta_k = \min_{k,f} \frac{1}{g_{k,f}^t}, \forall k, \phi_{k,f} = 0, \forall k, f,$
$\gamma_{k,f}^t = 0, \forall k, f, \Theta = \{\theta_1, \ldots, \theta_K\} = 0.$
2) Repeat till all the rate constraints are achieved.
3a) Repeat till k^{th} user rate constraint is achieved.
3b) Increase waterlevel of user k in small steps, $\delta_k = \delta_k + \Delta_m$.
3c) On all the subcarriers compute, $\phi_{k,f} =$
$(1+\delta_k)\Bigg(\Big(\log\big(\frac{(1+\delta_k)g_{k,f}^t}{\eta}\big)\Big)^+ - \Big(1 - \frac{\eta}{(1+\delta_k)g_{k,f}^t}\Big)^+\Bigg)$
3d) Allocate subcarrier to this user if $\phi_{k,f}$ is maximum and set $\gamma_{k,f}^t = 1$ otherwise set $\gamma_{k,f}^t = 0$.
4) Compute the achieved data rate according to,
$\theta_k = \sum_{f=1}^F \gamma_{k,f}^t \Big(\log\big(\frac{(1+\delta_k)g_{k,f}^t}{\eta}\big)\Big)^+$
5) Compute the total power,
$P_{total} = \sum_{k=1}^K \sum_{f=1}^F \gamma_{k,f}^t \Big(\frac{1+\delta_k}{\eta} - \frac{1}{g_{k,f}^t}\Big)^+$

B. Non-Feasibility

In case of non-feasibility we need an algorithm to decrease the data rates of some of the users. Since state variable x_k^t measures the deviation of current queue length of the user from its target queue length, therefore, we decrease the demanded data rate of the user with minimum value of x_k^t. We adopt this criterion to ensure fairness among users in terms of achieved data rates when there is non-feasibility in the problem. In case, there are more than one users with the same minimum value of x_k^t, we select the user k' which consumes the maximum amount of power to achieve its data rate. Thus decreasing the data rate of user k' will result in maximum performance improvement and minimum packet drops. We then use the instantaneous resource allocation algorithm given in Table II for subcarrier and power allocation. We also compute the total power consumed in achieving the new data rates. This process is repeated and the data rate of the worst user is decreased in each iteration 'till all the proposed data rates are achieved with the given amount of power. This algorithm is presented in Table III. The number of iterations of this algorithm depends on the step-size ν. If this value is optimally determined then the algorithm converges relatively quickly in just few iterations.

VII. Simulation Results

We consider a downlink OFDMA system with 10 users and 24 subcarriers. We assume that the bandwidth of each subcarrier is 375 KHz (i.e. total bandwidth=9MHz) and BS has a peak power constraint of 43 dBm. We consider a

TABLE III
ALGORITHM FOR NON-FEASIBLE CASE

While $P_{total} > P_{max}$
1) Find the set of users Π such that $\Pi = \min x_k^t$.
2) Find the worst user k' such that, $k' = \max_{k \in \Pi} \frac{P_k^t}{R_k^t}$.
3) Decrease the data rate of user k' i.e. $\tilde{R}_k^t = \tilde{R}_k^t - \nu$.
4) Use the algorithm in Table II for subcarrier and power allocation and find the new value of P_{total}.
End

frequency selective Rayleigh fading channel with exponential delay profile. The power spectral density of noise is -174 dBm/Hz. Time is divided into slots and duration of each Transmission Time Interval (TTI) is 1ms. Without any loss of generality we assume that packets are generated according to poisson distribution with packet size of 1Kbits. The users are uniformly distributed in a cell of radius 500m. Path losses are calculated according to Cost-Hata Model [30]. The users experience different path loss due to varying distances from the BS which greatly impact their ability to achieve different rates. We perform the simulations for different scenarios / snapshots. In each snapshot, the distances of the users from the BS remain fixed but the channel is variable with time. We simulate a given snapshot for 10000 time slots and then we average the results over 1000 scenarios. In all the snapshots we however assume that the distance of user 1 and user 10 remain fixed. User 1 is assumed to be the best user in the system and is assumed to be located at a distance of 50m from the BS. Similarly user 10 is the worst user in the system and is assumed to be located at the cell edge in all the simulated scenarios.

We have dynamic power control in our resource allocation algorithm. Therefore in order to compare the performance of our controller which is acting here as a scheduler we need some algorithms with power control. The traditional PF and M-LWDF schedulers assume equal power on all the subcarriers. However it is possible to achieve dynamic power control in these algorithms by first calculating the scheduling weights and then using a weighted sum rate maximization algorithm. This gives us power control in PF and M-LWDF schedulers which then gives us enough base for comparison with our controller. In figure 2 we assume that the average demanded delay constraint of each user is 10 TTI and we plot the mean delay achieved by all the users in the systems versus the input arrival rate. As the input arrival rates increase the average delay achieved by the system also increases for all three schemes. The performance of PF scheduler is the worst because it is not designed to handle delay constrained traffic. The performance of M-LWDF is better than the PF scheduler but it can only guarantee the mean demanded delay till 6 Packets/TTI/User and for input arrival rates higher than this value the achieved delay is greater than the demanded delay. It is evident from the figure that our algorithm outperforms both these algorithms and can guarantee the mean delay for input arrival rates greater than 9 Packets/TTI/User. Since we have incorporated power control in the PF and M-LWDF scheduling rules we can safely say that this superior performance results from the use of our controller along with the instantaneous resource allocation algorithm.

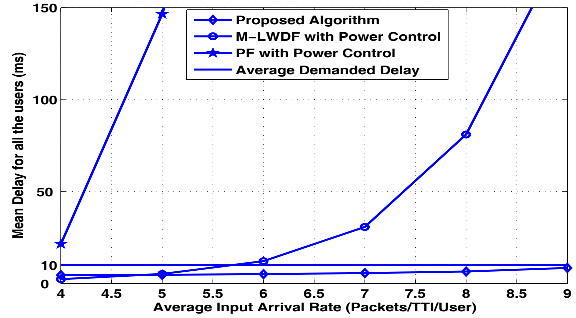

Fig. 2. Average achieved delay for all the users in the system vs average input arrival rate. Target delay deadline = 10 TTI.

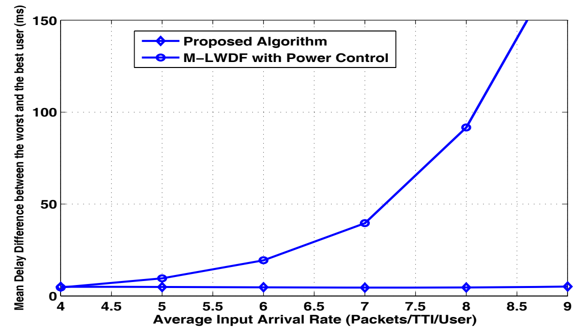

Fig. 3. Mean difference in delay achieved by the worst and the best user in the system. Best user location = 50 m from BS, worst user location = 500 m from BS.

In order to see the impact on the achieved delays of different users which are located at varying distances from the BS we plot figure 3. In this figure we compare the performance of the best and the worst user in the system by plotting the difference between the mean delays achieved by these two users. The average achieved delay of the worst user is always greater than that for the best user due to the difference between the attenuations caused by the corresponding path losses. However, this difference is not huge and is almost 5 ms for the plotted input arrival rates. The superior performance of our approach compared to the M-LWDF algorithm is due to the fact that the rates proposed by the controller are according to the delays experienced by the individual users in the system. Thus the worst user is compensated by the controller and a higher data rate is proposed for this user compared to the best user in most of the time slots. Moreover, if there is not enough capacity at the physical layer to satisfy the demanded rates then again in the non-feasible algorithm the worst user is given preference over the best user. The demand of higher data rate for the worst user is taken care of by the algorithm for the non-feasible case in the resource allocation algorithm.

In figure 4 we plot the outage probability of the system for different input arrival rates. We declare an outage when the achieved delay of any user is greater than its target delay requirement. In this case the queue length of the user is greater than its virtual target queue length and there are packet drops. Again we assume the target delay of 10 ms for each user.

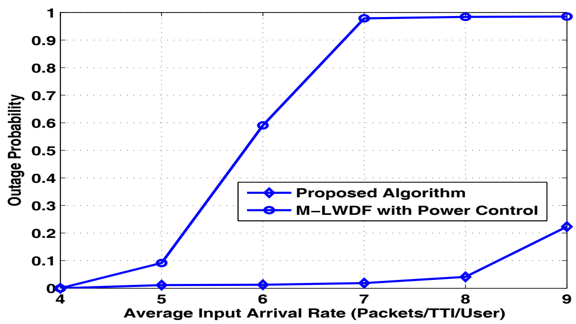

Fig. 4. Outage probability vs average input arrival rate. Target delay deadline = 10 TTI.

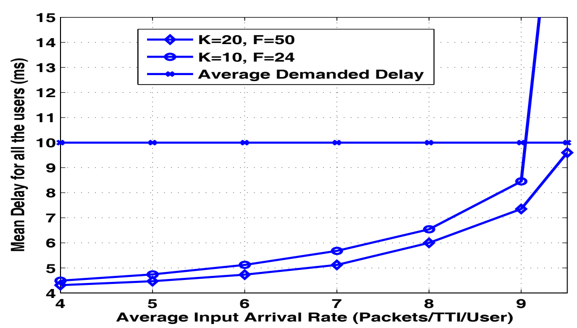

Fig. 5. Performance comparison for different number of uses (K) and subcarriers (F). Target delay deadline = 10 TTI.

Although at 6 Packets/TTI/User, the M-LWDF scheduler is able to achieve a mean delay of almost 10 TTI for the system but in terms of outage its performance is very bad. There is almost 60% probability that the demanded delay is not gauranteed for any one user in the system. Similarly when the input arrival rate is further increased, there is more than 95% probability of delay deadline violations for the users in the system. Our algorithm has negligible outages/packet drops till 8 Packets/TTI/User. However as the input arrival rate further increases, the outage probability jumps to almost 20% for the input arrival rate of 9 Packets/TTI/User. This increase in outage results from the fact that there is not enough capacity at the physcial layer to satisfy any higher input arrival rates.

In figure 5 we plot the performance of our approach in terms of mean delay achieved by all the users in the system for different number of users and subcarriers in the system. We assume that the demanded delay constraint is again 10 TTI. When we increase the number of uses to 20 and that of subcarriers to 50, the mean delay achieved by the system is less compared to the other case. Moreover, the system can deliver a mean delay of less than 10 ms for input arrival rates of almost 9.5 Packets/TTI/user. This superior performance results from exploitation of multi-user diversity and increased bandwidth of the system.

In table IV we present the number of iterations required to converge to the optimal value of π^2 for different number of users in the system. The simulations are carried out for $\zeta = 10^{-1}$, $\rho = 10^{-2}$, $F = 24$ subcarriers and mean arrival rate of 9 Packets/TTI/User. We can see that the algorithm converges to the optimal value of $\pi*^2$ in very few iterations and as the

TABLE IV
OPTIMAL VALUE OF $\pi*^2$ AND COMPLEXITY COMPARISON FOR DIFFERENT NUMBER OF USERS IN THE SYSTEM. $\zeta = 10^{-1}, \rho = 10^{-2}, F = 24$ SUBCARRIERS, $P_{max} = 43dBm$, MEAN ARRIVAL RATE = 9 PACKETS/TTI/USER

Users	$\pi*^2$	Iterations
2	3.0575	12.76
4	2.7943	12.54
6	2.6564	12.05
8	2.549	11.92
10	2.437	11.77

number of users increase there is no significant impact on the number of iterations. Thus the complexity of the controller is very low and can easily be implemented in practical systems.

VIII. CONCLUSION

In this paper we develop a two step solution to the dynamic resource allocation problem in multi-service downlink OFDMA systems in unknown environment. We solve the problem without assuming any knowlege of pdfs of future SINR and pakcet arrivals. We propose a robust H^∞ controller which does not require any such information. The controller is used to derive an instantaneous data rate for each user depending on its demanded service type. The controller can track the queue variations and it can efficiently control the PDR of different users. The proposed data rate is then fed as a constraint to the instantaneous resource optimization problem for subcarrier and power allocations. Simulation results show better fairness, smaller delays and efficient exploitation of time diversity in the system compared to other available approaches.

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