

firstPI

May 8, 2021

1 First order PI

```
[1]: import numpy as np
import sympy as sym
import matplotlib.pyplot as plt

sym.init_printing()
```

```
[2]: z, Wn, Kg, Kp, tau, Ti = sym.symbols('zeta omega_n K_G K_p tau T_i')
```

```
[3]: eq1 = sym.Eq(2*z*Wn, (1+Kg*Kp)/tau);
eq2 = sym.Eq(Wn**2, (Kg*Kp)/(tau*Ti));
sym.solve((eq1, eq2), (Kp, Ti))
```

```
[3]:  $\left\{ K_p: \frac{2\omega_n\tau\zeta - 1}{K_G}, T_i: \frac{2\zeta}{\omega_n} - \frac{1}{\omega_n^2\tau} \right\}$ 
```

1.1 Example

```
[4]: import control as ct
from control.matlab import *
import numpy as np
```

```
[5]: # System
num = [1]
den = [1, 0.375]
G = ct.tf(num, den)
print('Process: ')
print(G)

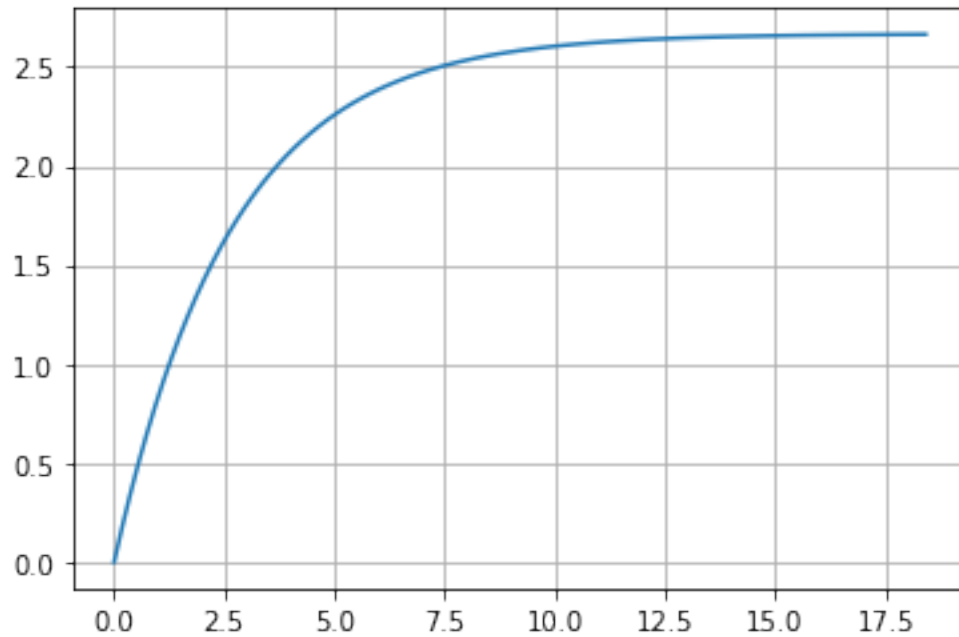
t, out = ct.step_response(G)
print('Step response:')
plt.plot(t, out)
plt.grid()
plt.show()

print('Step info: ')
stepinfo(G)
```

Process:

$$\frac{1}{s + 0.375}$$

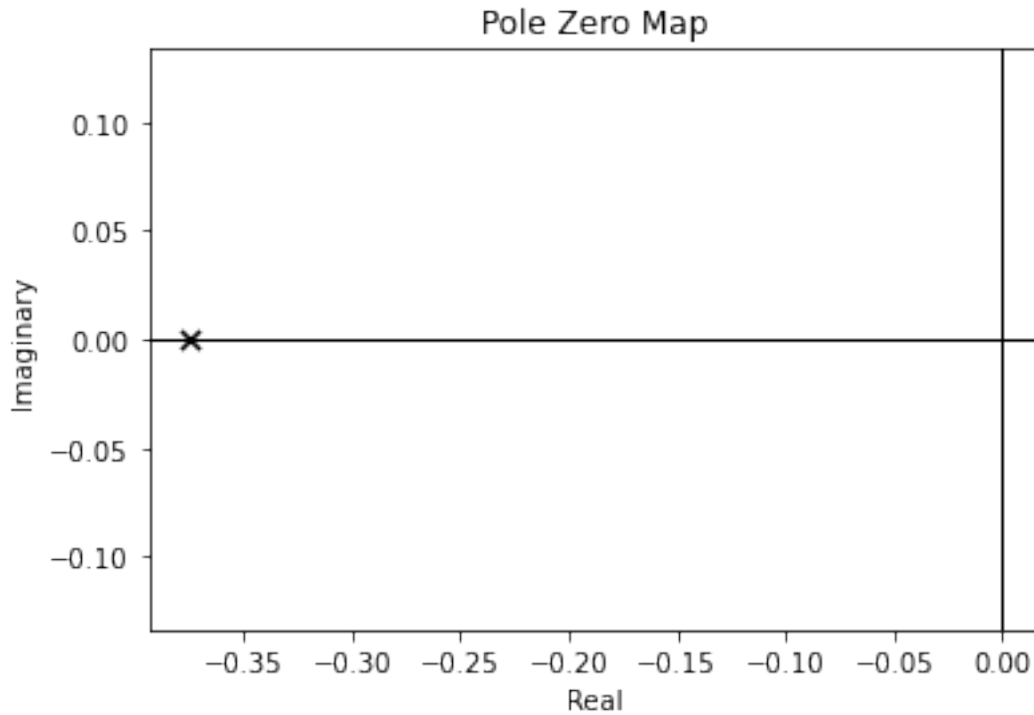
Step response:



Step info:

```
[5]: {'RiseTime': 5.954159432388643,  
      'SettlingTime': 10.60584648894227,  
      'SettlingMin': 2.4179724408311363,  
      'SettlingMax': 2.6639999999999991,  
      'Overshoot': 0,  
      'Undershoot': 0,  
      'Peak': 2.6639999999999991,  
      'PeakTime': 18.420680743952364,  
      'SteadyStateValue': 2.6666666666666665}
```

```
[6]: # Root locus  
      ct.pzmap(G);
```



```
[7]: # Data
      ts = 8
      Mp = 15
```

```
[8]: # Requirements
      z = np.sqrt(np.square(np.log(Mp/100))/(np.pi**2 + np.square(np.log(Mp/100))))
      Wn = 4/(z*ts)
      tau = 1/den[1]
      Kg = num[0]/den[1]

      print('z= ', z)
      print('Wn= ', Wn)
```

```
z= 0.5169308662051555
Wn= 0.9672473297455677
```

```
[9]: # Desired characteristic equation and poles
      print('delta = s^2 + {a}s + {b:1.4f}'.format(a=2*z*Wn,b=Wn**2))
      print('s = {a} + j{b:1.4f}'.format(a=-z*Wn, b=Wn*np.sqrt(1-z**2)))
```

```
delta = s^2 + 1.0s + 0.9356
s = -0.5 + j0.828
```

```
[10]: # PI controller
Kp = (2*z*Wn*tau-1)/Kg
Ti = (Kp*Kg)/(tau*Wn**2)

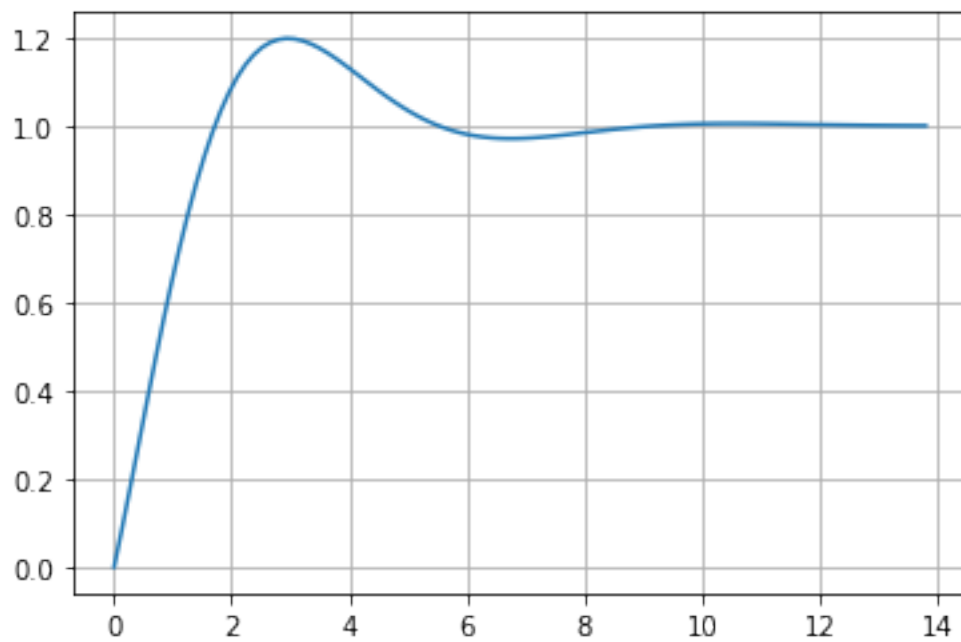
Gc = ct.tf([Kp*Ti, Kp],[Ti, 0])
print(Gc)
```

0.4175 s + 0.625

0.668 s

```
[11]: # PI response
sys = ct.feedback(Gc*G)
t, out = ct.step_response(sys)
plt.plot(t,out)
plt.grid()
plt.show()

print('Step info: ')
stepinfo(sys)
```

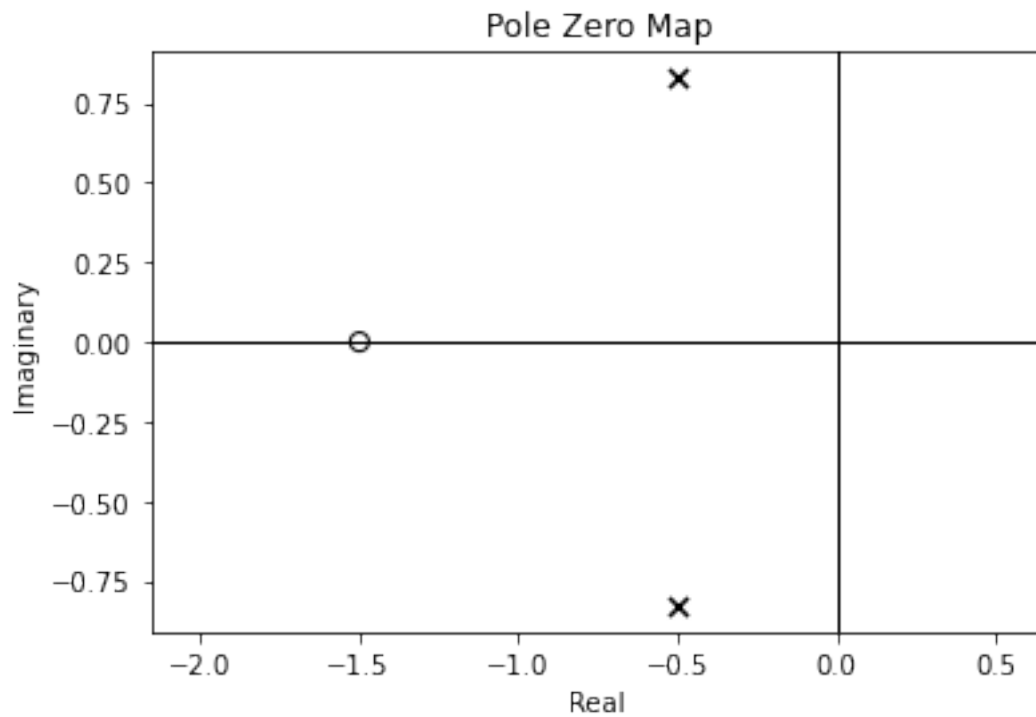


Step info:

```
[11]: {'RiseTime': 1.2559555052694795,  
      'SettlingTime': 7.814834255010095,  
      'SettlingMin': 0.9298750106558097,  
      'SettlingMax': 1.197286847173987,  
      'Overshoot': 19.728684717398703,  
      'Undershoot': 0,  
      'Peak': 1.197286847173987,  
      'PeakTime': 2.9305628456287858,  
      'SteadyStateValue': 1.0}
```

```
[12]: ct.pzmap(sys);  
      print('Poles: ',ct.pole(sys))
```

Poles: $[-0.5+0.82798997j \ -0.5-0.82798997j]$



```
[ ]:
```