

# secondPID

May 8, 2021

## 1 Second order PID

```
[55]: import numpy as np
import sympy as sym
import control as ct
from control.matlab import *
import matplotlib.pyplot as plt

sym.init_printing()
```

```
[56]: z, Wn, Kp, Ti, Td, B, a0, a1, b0= sym.symbols('zeta omega_n K_p T_i T_d beta_
↪a_0 a_1 b_0')
```

```
[57]: eq1 = sym.Eq((2*B*z**2 + 1)*Wn**2, a0+Kp*b0);
eq2 = sym.Eq(B*z*Wn**3, (Kp*b0)/Ti);
eq3 = sym.Eq((B + 2)*z*Wn, Kp*Td*b0 + a1)
sym.solve((eq1, eq2, eq3),(Ti, Td))
```

```
[57]:  $\left\{ T_d: \frac{-a_1 + \beta\omega_n\zeta + 2\omega_n\zeta}{K_p b_0}, T_i: \frac{K_p b_0}{\beta\omega_n^3\zeta} \right\}$ 
```

```
[127]: # System
num = [29870]
den = [1, 414.7, 33610]
G = ct.tf(num,den)
print('Process:')
print(G)

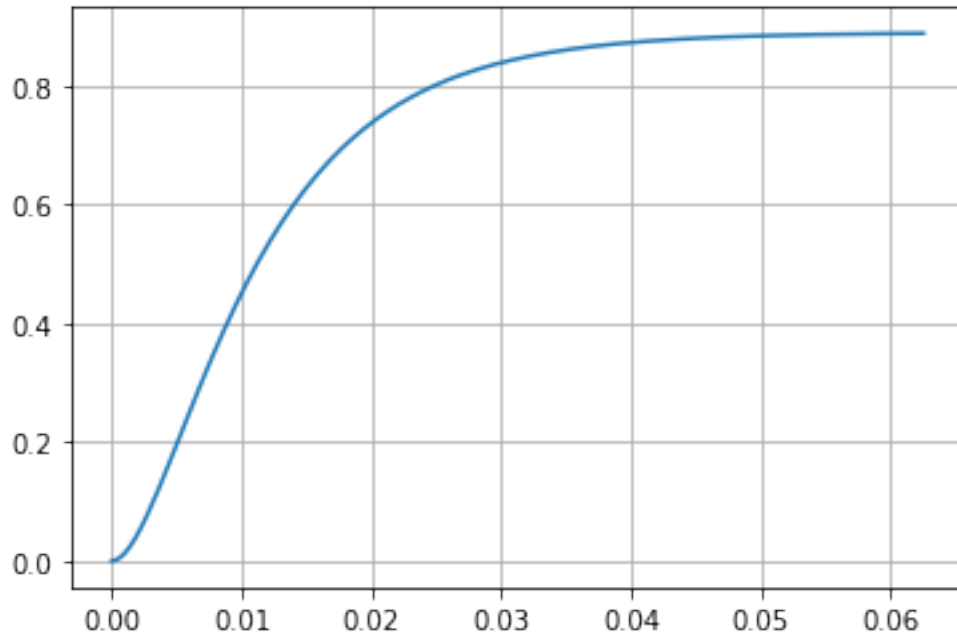
t, out = ct.step_response(G)
print('Step response:')
plt.plot(t, out)
plt.grid()
plt.show()

print('Step info:')
stepinfo(G)
```

Process:

$$\frac{2.987e+04}{s^2 + 414.7 s + 3.361e+04}$$

Step response:



Step info:

```
[127]: {'RiseTime': 0.0217480341475997,
        'SettlingTime': 0.039871395937266116,
        'SettlingMin': 0.8000456798025747,
        'SettlingMax': 0.8873280912347106,
        'Overshoot': 0,
        'Undershoot': 0,
        'Peak': 0.8873280912347106,
        'PeakTime': 0.06252559817434913,
        'SteadyStateValue': 0.8887235941684023}
```

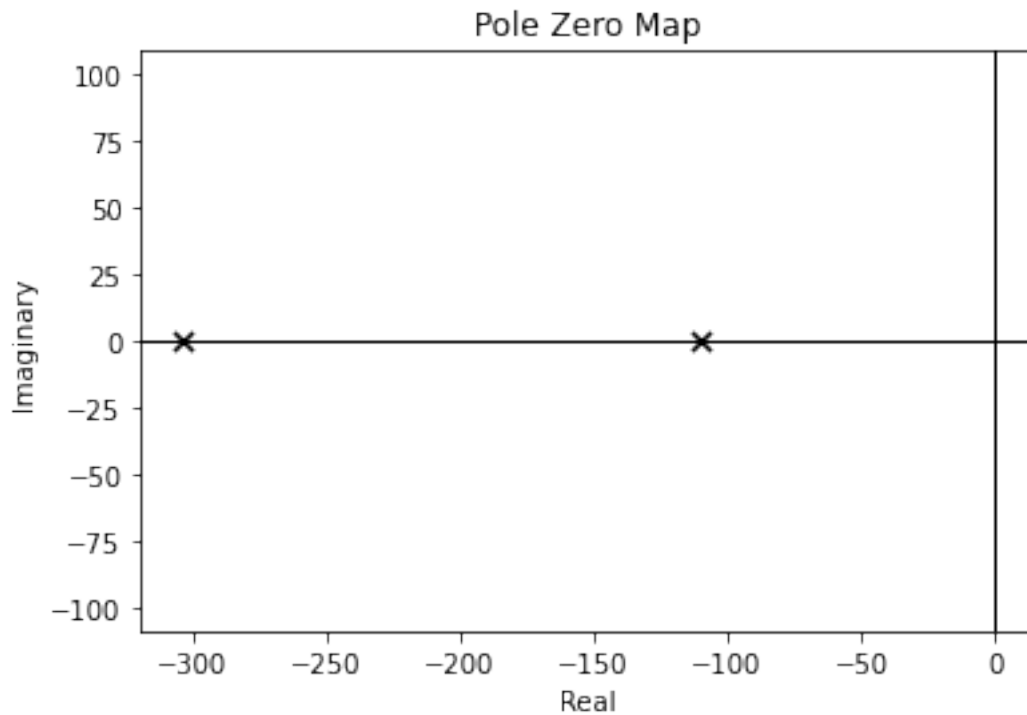
```
[128]: # Damping and natural frequency
ct.damp(G);
```

-----Eigenvalue-----	Damping---	Frequency_
-304.2	1	304.2
-110.5	1	110.5

```
[129]: # Root locus
print('Poles: ', ct.pole(G))
```

```
ct.pzmap(G);
```

Poles: [-304.22116444 -110.47883556]



```
[140]: # Data
ts = 0.03
Mp = 25
B = 10

b0 = float(num[0])
a1 = float(den[1])
a0 = float(den[2])

[141]: # Requirements
z = np.sqrt(np.square(np.log(Mp/100))/(np.pi**2 + np.square(np.log(Mp/100))))
Wn = 3/(z*ts)

print('z= ', z)
print('Wn= ', Wn)

z= 0.40371275194342066
Wn= 247.7008702811913
```

```
[142]: # Desired characteristic equation and poles
print('delta = s^2 + {a}s + {b:1.4f}'.format(a=2*z*Wn,b=Wn**2))
print('s = {a} + j{b:1.4f}'.format(a=-z*Wn, b=Wn*np.sqrt(1-z**2)))
```

```
delta = s^2 + 200.0s + 61355.7211
s = -100.0 + j226.6
```

```
[143]: # PI controller
Kp = ((2*B*z**2 + 1)*Wn**2 - a0)/b0
Ti = (Kp*b0)/(B*z*Wn**3)
Td = (((B + 2)*z*Wn) - a1)/(Kp*b0)

print(f'Kp: {Kp}, Ti: {Ti}, Td: {Td}')

Gc = ct.tf([Kp*Td*Ti, Kp*Ti, Kp],[Ti, 0])
print('PID-control system:')
print(Gc)
```

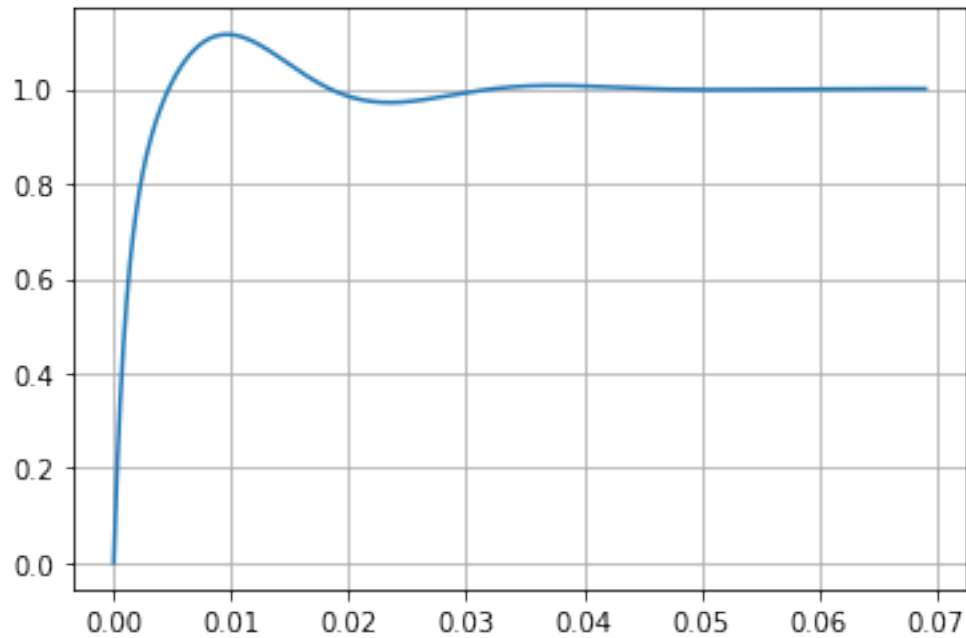
```
Kp: 7.624563814464665, Ti: 0.0037118905444139035, Td: 0.0034481438161639523
PID-control system:
```

```
9.759e-05 s^2 + 0.0283 s + 7.625
-----
0.003712 s
```

```
[144]: # PI response
sys = ct.feedback(Gc*G)
t, out = ct.step_response(sys)
print('Step response:')
plt.plot(t,out)
plt.grid()
plt.show()

print('Step info:')
stepinfo(sys)
```

Step response:



Step info:

```
[144]: {'RiseTime': 0.0031775674283317803,
        'SettlingTime': 0.027216555799189596,
        'SettlingMin': 0.9097901730555858,
        'SettlingMax': 1.1149079307048033,
        'Overshoot': 11.490793070480333,
        'Undershoot': 0,
        'Peak': 1.1149079307048033,
        'PeakTime': 0.009670857390574984,
        'SteadyStateValue': 1.0}
```

```
[145]: ct.pzmap(sys);
        print('Poles: ',ct.pole(sys))
```

```
Poles: [-1000. +0.j          -100.+226.61800709j  -100.-226.61800709j]
```

