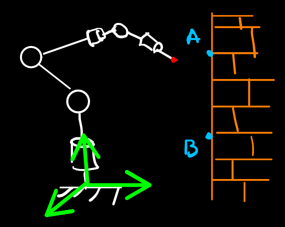
Robot Scara: Cinemática inversa

- Dada una pose cartesiana deseada, indicar los valores articulares necesarios

pose = posición + orientación



E. cartesiano

Pose

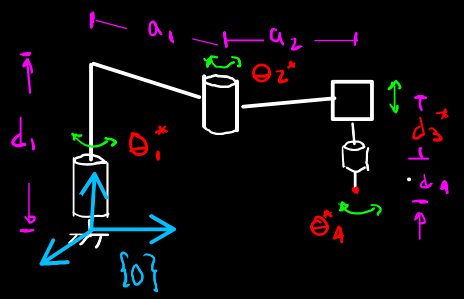
$$\bar{O} = \begin{bmatrix} O_x \\ O_y \\ O_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix}$$

E. articular

Variables articulares

$$\bar{q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$

- Ecuaciones de c. inversa del robot Scara.



- Articulación 3

$\rightarrow d_3 = d_1 - d_4 - O_z$

- Articulación 2

$$D = \frac{O_x^2 + O_y^2 - a_1^2 - a_2^2}{2 a_1 a_2}$$

$$\rightarrow \theta_2 = \operatorname{atan2}\left[\frac{\pm\sqrt{1-D^2}}{D}\right]$$

- Articulación 1:

$$\rightarrow \theta_1 = \operatorname{atan2}\left[\frac{O_y}{O_x}\right] - \operatorname{atan2}\left[\frac{a_2 \sin(\theta_2)}{a_1 + a_2\cos(\theta_2)}\right]$$

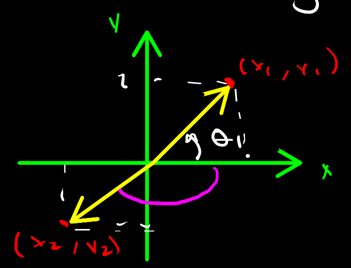
- Articulación 4:

$$\rightarrow \theta_4 = \theta_1 + \theta_2 - \alpha$$

α = orientación deseada

O_x, O_y, O_z = posiciones deseadas

Nota: Arco tangente de 2 argumentos



$\theta_1 = \operatorname{atan}\left[\frac{y_1}{x_1}\right]$

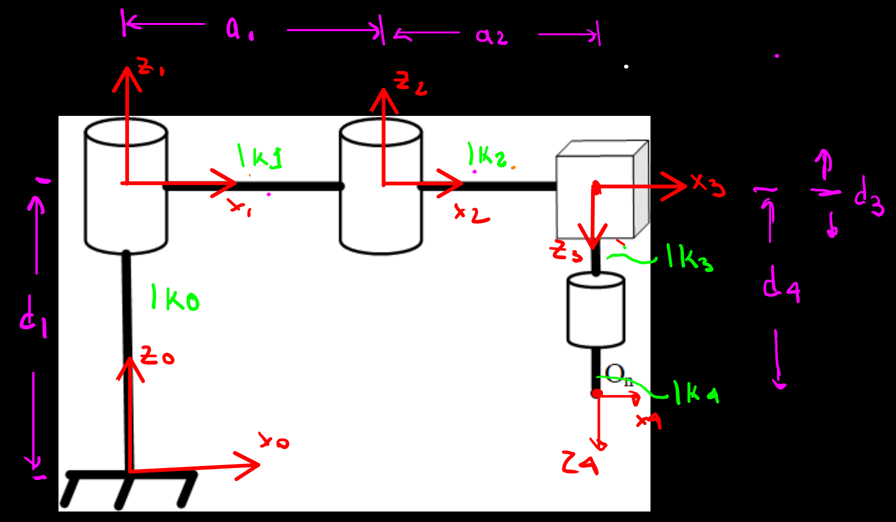
Si $x_1 = 2$, $y_1 = 2$: $\theta_1 = \operatorname{atan}\left[\frac{2}{2}\right] = \operatorname{atan} 1 = \pi/4$

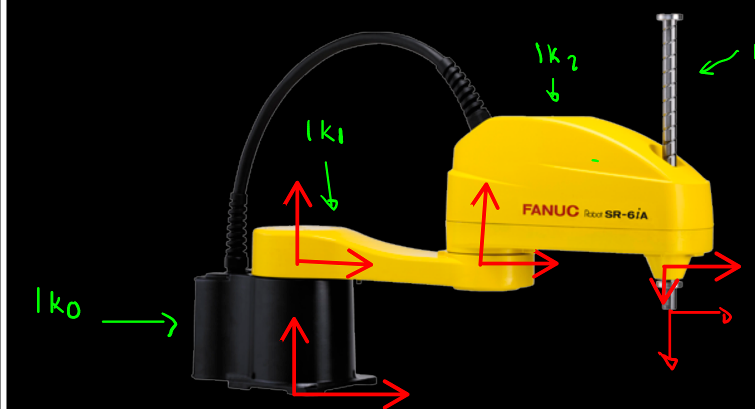
Si $x_2 = -2$, $y_2 = -2$: $\theta_2 = \operatorname{atan}\left[\frac{-2}{-2}\right] = \operatorname{atan}[1] = \pi/4$

URDF: Unified Robot Description Format

- Formato para describir robots y realizar simulaciones
- Se enfoca principalmente en sus eslabones y articulaciones
- Universal \ Unificado: Empleado como estándar en múltiples simuladores
- Se basa en XML (Extensible Markup Language)
- URDF no es Denavit-Hartenberg

Ejemplo: Scara





2 gdl → Se descompone en 2 eslabones:

1 prismática

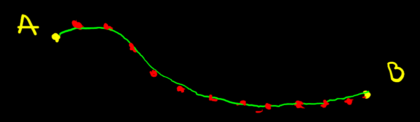
1 revoluta

Trajectory planning

- A function is needed in order to achieve smooth trajectories from the initial position to the last one.
- Simple functions, such as polynomia, are used in order to interpolate between sets in the joint and in the cartesian spaces.

- Concepts:

  Path - Set of position coordinates that the robot needs to follow to reach an objective



  Trajectories: Path + schedule. Set of positions, velocities and accelerations

Cubic polynomia:

- Based on 4 constrains:

Position:

$\theta_0 = \theta(0)$

$\theta_f = \theta(t_f)$

Velocity:

$\dot{\theta}_0 = \dot{\theta}(0)$

$\dot{\theta}_f = \dot{\theta}(t_f)$

- Coeficients of the polynomia

$$c_0 = \theta_0$$
$$c_1 = \dot{\theta}_0$$
$$c_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{\dot{\theta}_f + 2\dot{\theta}_0}{t_f}$$
$$c_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

- Polynomia

$\rightarrow \theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$

$\rightarrow \dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$

$\rightarrow \ddot{\theta}(t) = 2c_2 + 6c_3 t$

Example: For a rotational joint write a cubic polynomial that describes the movement from $\theta_0 = 15^\circ$ to $\theta_f = 75^\circ$ in 3 seconds. Consider initial and final velocities as 0. Compute the trajectory for 2 s.

Data:

$\theta_0 = 15^\circ$ $\quad \dot{\theta}_0 = 0$ $\quad t_f = 3s$

$\theta_f = 75^\circ$ $\quad \dot{\theta}_f = 0$

Coefficients:

$$c_0 = \theta_0 = 15$$
$$c_1 = \dot{\theta}_0 = 0$$
$$c_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \cancel{\frac{\dot{\theta}_f + 2\dot{\theta}_0}{t_f}}^{\,0} = \frac{3}{9}\,60 = 20$$
$$c_3 = -\frac{2}{t_f^3}(\theta_f - \theta_0) + \cancel{\frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)}^{\,0} = -\frac{2}{27}\,60 = -\frac{40}{9}$$

- Polynomia

$$\theta(t) = 15 + 20t^2 - \frac{40}{9}t^3$$
$$\dot{\theta}(t) = 40t - \frac{40}{3}t^2$$
$$\ddot{\theta}(t) = 40 - \frac{80}{3}t$$

- For $t = 2$

$$\theta(2) = 15 + (20 \cdot 4) - \left(\frac{40}{9} \cdot 8\right) = \frac{535}{9} \simeq 59.4444^\circ$$
$$\dot{\theta}(2) = (40 \cdot 2) - \left(\frac{40}{3} \cdot 4\right) = \frac{80}{3} \simeq 26.6666^\circ$$
$$\ddot{\theta}(2) = 40 - \left(\frac{80}{3} \cdot 2\right) = -\frac{40}{3} \simeq -13.3333^\circ$$

Generalizating for multiple joints

- In vectorial form:

a) Position:

$$\bar{\Theta}(t) = \bar{C}_0 \bar{t}^0 + \bar{C}_1 \bar{t} + \bar{C}_2 \bar{t}^2 + \bar{C}_3 \bar{t}^3$$

$$= [\bar{t}^0 \ \bar{t} \ \bar{t}^2 \ \bar{t}^3] \cdot \begin{bmatrix} \bar{C}_0 \\ \bar{C}_1 \\ \bar{C}_2 \\ \bar{C}_3 \end{bmatrix}$$

$$\rightarrow \bar{\Theta}(t) = T_p^t \ C_p$$

b) Velocity

$$\dot{\bar{\Theta}}(t) = \bar{C}_1 \bar{t}^0 + 2\bar{C}_2 \bar{t} + 3\bar{C}_3 \bar{t}^2$$

$$= [\bar{t}^0 \ \bar{t} \ \bar{t}^2] \cdot \begin{bmatrix} \bar{C}_1 \\ 2\bar{C}_2 \\ 3\bar{C}_3 \end{bmatrix}$$

$$\rightarrow \dot{\bar{\Theta}}(t) = T_v^t \ C_v$$

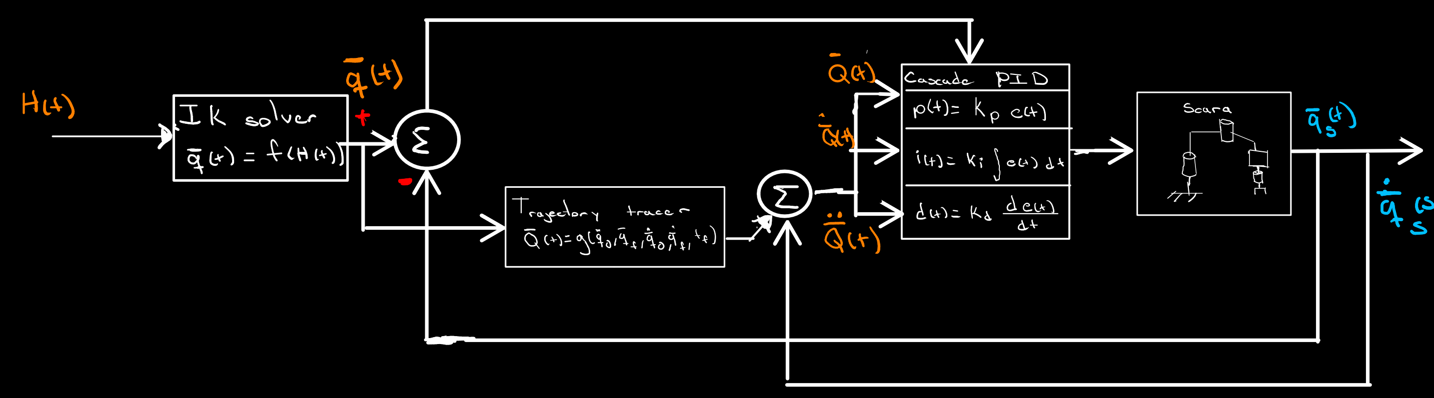
c) Acceleration

$$\ddot{\bar{\Theta}}(t) = 2\bar{C}_2 \bar{t}^0 + 6\bar{C}_3 \bar{t}$$

$$= [\bar{t}^0 \ \bar{t}] \cdot \begin{bmatrix} 2\bar{C}_2 \\ 6\bar{C}_3 \end{bmatrix}$$

$$\rightarrow \ddot{\bar{\Theta}}(t) = T_a^t \cdot C_a$$

- Scara control diagram:



- Equivalent ROS graph: