Lab 2.1 Digital Filters

Deadline: 23:59, 2023.10.31

1. Design an FIR filter with the Window method

A simple way to design FIR filter is to use window function truncate the impulse response function of an ideal filter, named the window method.

(a) Design the low-pass FIR filters, the cut-off frequency $\omega_c = 0.3\pi$, with the rectangle window, where the window length N = -30~30 and -100~100, respectively. Calculate the corresponding FRFs (Magnitudes and Phase) of the FIR filters. And then compare their impulse response functions and FRFs in terms of different window length.

(b) Design a **causal** high-pass FIR filters, the cut-off frequency $\omega_c = 0.3\pi$, with the rectangle window and Kaiser window with the window length $N = 0 \sim 59$. And then compare their impulse response functions and FRFs (amplitudes and phases) in terms of the window type.

(c) Signal $x[n] = A_1\cos(2\pi f_1 nT) + A_2\cos(2\pi f_2 nT)$, where $A_1 = 8, A_2 = 4, f_1 = 7, f_2 = 24$, n = -200:399. The sampling frequency $f_s = 1/T = 200$. In order to filter out the cosine of 7 and keep the cosine of 24 of $x[n]$, design a digital high-pass FIR filter. The filter's overshoot should be less than 5%. Show FRF of the FIR filter.

(d) Develop a sub-function y = **myfilter** (bz, az, x) by using the I/O difference equation method.

(e) Use **myfilter** to filter x with the filter you designed in (c) and plot the output signal y_1. Divide x into 3 non-overlapping blocks, 200 samples each, and use the convolution method to filter each block and piece the output blocks together to obtain and plot overall output y_2. Compare y_1 and y_2.

(f) Perform DFT on the steady state of y_1 and y_2, respectively, and compare it with the DFT result of x to verify the effectiveness of your filter. Calculate the amplitude of the remaining frequency of 24 according to the length of your filter.

2. Design an IIR filter with bilinear transform and realize filtering

a) The signal $x[n]$ is the same as that in 1.(c) . In order to filter out the cosine of 24 and keep the cosine of 7, we need to design a digital low-pass filter with the prototype filter of Butterworth by using bilinear transform, where the $f_c = 15$ and the normalized cutoff frequency $f_{cn} = \frac{f_c}{f_s}$. Develop your own bilinear transform function and design the low-pass filter mentioned above. Show ***az*** and ***bz*** of the filter and plot the frequency response function (FRF) of the filter in the normalized frequencies.

b) Develop a sub-function y = **myfilter** (bz, az, x) by using the I/O difference equation method.

c) Use **myfilter** to filter $x[n]$ in 2.(a), and produce the output signal y_1. Compare the time sequences and frequency spectra (amplitudes and phases) of x and y_1, respectively.

d) Design a high-order (at least 4-order) filter to filter x[n] in 2.(a). Write a function **filter_SOS** which can filter DT signals with SOS coefficients of filters using Direct-form II structure. Compare the filtered results with **myfilter**. (Optional)

3. Pole/Zero Designs

Pole/zero placement can be used to design simple filters, such as the notch and comb filters.

(a) In order to filter out the cosine of 24 and keep the cosine of 7 of $x[n]$ in 1(a), we need to design a notch filter with a bandwidth of 1 Hz (Try-and-error way). Show $H_1(z)$, and plot a map of poles and zeroes with the unit circle for reference. Then show FRFs of the notch filter.

(b) Filter x with the notch filter and produce the output signal y_3. Compare the time sequences and frequency spectra of y_1 in 2.c and y_3, respectively.

(c) In order to filter out the cosine of 7 and keep the cosine of 24 of $x[n]$ in 1(c), we can alternatively design a comb filter to keep only 24 of x with a bandwidth of 1 Hz. Show $H_2(z)$ and plot a map of poles and zeroes with the unit circle. Then show FRFs of the comb filter.

(d) Filter x with the comb filter and obtain the filtered signal y_4. Compare the time sequences and frequency spectra of y_1 in 1.d, and y_4, respectively.

(e) Give a discussion of advantages and disadvantages of all these filters according to the results in 2(c) and 3(b).