Lab I Sampling Theorem and Fourier Analysis

Submission Deadline: 2023.10.8 23:59

Objectives

1. Basic functions and commands of signal operations in Python.
2. Sampling and transformation of a pure harmonic signal.
3. Aliasing phenomenon in the sampling process.
4. Fourier serious and Fourier transform for digtal signals.
5. Windowing efforts on Fourier transform.

Requirements

1. The report should be written in English.
2. Include your student number in each figure title as 'No. XXXXXXX'. And include your codes in the appendix with the question numbers.
3. It is not necessary to answer the optional question(s).
4. Please submit your report in PDF format.

1. Signal operations

A continuous-time (CT) signal $x(t)$ includes three gate functions can be represented as $x(t)=\sum_{i=0}^{2} g_i$, the center gate function $g_0(t)$ is defined as

$$g_0(t)=\begin{cases} B & 0 \le t \le A \\ 0 & \text{others} \end{cases}, \tag{1}$$

and the two neighboring gate functions can be considered as a shift of $g_0(t)$, i.e. $g_1(t)=g_0(2t+D)$ and $g_2(t)=2g_0(t-D)$. The parameters of the gate function are shown in Table 1.

a) Please plot the CT signal $x(t)$ with proper parameters.

Table 1 Parameters of the gate function

Parameters	A	B	D
Values	5	3.5	10

2. Signal sampling and transformation

Consider a discrete-time signal $x(nT)$ is obtained by sampling a sinusoidal signal $x(t)=\sin(2\pi f_0 t)$ where T is the sampling interval, f_0 is the oscillation frequency of the signal, and the sampling frequency f_s equals to $1/T$.

The sampling frequency is fixed at $f_s = 5000$.

a) Assume $f_0 = 400$, plot 3 periods of $x(t)$ and $x(nT)$ in one axis, $t \geq 0$; And plot 3 periods of

$x[n]$ $n \geq 0$. Try to put these two axes in one figure.

b) Repeat a) for the sinusoidal frequencies $f_0 = \frac{1000}{2\pi}$ and indicate whether $x[n]$ is a periodic signal or not? Try to plot $x[n]$ $n = 0{:}499$, for the sinusoidal frequencies $f_0 = 20 * n$ and watch the waveform changes.

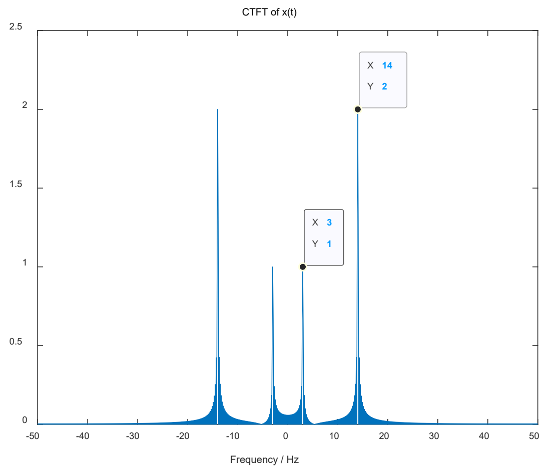
c) Assume that we sample the signal $x(t)$ created in a) and get x[n], but we do not know the analysis form of signal x[n]. However, we still want to get $y_1[n] = 2 * x[2n+1]$ and $y_2[n] = x\left[-\frac{n}{4} + 15\right]$. Please plot $y_1[n]$ and $y_2[n]$ and x[n] in one figure. (Interpolation will be used when calculating $y_2[n]$ e.g. linear, spline, cubic. Please indicate the interpolation method.)

3. Aliasing phenomenon in sampling process

A continuous time signal x(t) can be expressed as the summation of two harmonics：

$$x(t) = A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t)$$

both frequencies of the harmonics are in a narrow frequency band, i.e. from 800 Hz to 850 Hz. Sample $x(t)$with a sampling rate $f_s = 100\,Hz$ and give the samples as x_n. Calculate the parameters of the two harmonics and fill in Table 1. Assume that the integral scale is from $t = 0$ to $t = 5$s while doing CTFT.



Parameters	$i = 1$	$i = 2$
f_i		
A_i		

4. Fourier series of a CT periodic signal

Create a section of the sine signal $x(t) = A\sin(2\pi f_0 t + \phi)$ and plot it with proper format, where $f_0 = 5$, $A = 4$, $\phi = \pi/4$ over t = [-2.3, 2.3] with time interval 0.001.

a) Calculate the Continuous-time Fourier series (CTFS) coefficients $\boldsymbol{X(k)}$ of $\boldsymbol{x(t)}$ with k = -

5:5 and plot the frequency spectrum of *f*.

b) According to Parseval's formula, calculate and compare the power values of the signal in both the time and frequency domains. Are these power values the same? Why?

5. Fourier transform of a CT signal

Create a section of the gate function $y(t)$ and plot it with proper format, where $D = 7$, $H = 3$ over $t = [-15, 15]$ with time interval 0.001.

$$y(t)=\begin{cases} H & , \ -D/2 \le t \le D/2 \\ 0 & , \ t<-D/2,\ t>D/2 \end{cases} \tag{2}$$

a) Create the Continuous-time Fourier transform (CTFT) function **Yw= CTFT (*y*, *t*, *w*)**, where ω is the radian frequency.

b) Calculate CTFT of $y(t)$ and show the modules and phases of $Y(\omega)$ with ω = -10π~10π.

c) According to Parseval's formula, calculate and compare the energy values of the signal in both the time and frequency domains. Are these energy values the same? Why?

d) $y_2(t)$ is defined as $y_2(t) = y(t - D/2)$. Calculate the CTFT of $y(t)$ and $y_2(t)$. Compare the amplitude and phase of their spectra and explain the phase difference between these two signals.

6. Windowing effects on FT

Create the continuous-time harmonics $x(t)$ with $f_s = 1000$, $f_1 = 16$, $A_1 = 1.4$, $\varphi_1 = 0$, $\nabla f = 3$, $f_2 = f_1 + \nabla f$, $A_2 = 0.13$, $\varphi_2 = 0$, D = 0.8 and $0 \le t \le$ D, according to Eq. (3).

$$x(t) = A_1 \sin(2\pi f_1 t + \varphi_1) + A_2 \sin(2\pi f_2 t + \varphi_2) \tag{3}$$

a) Calculate the FT results of $x(t)$. Demonstrate the amplitudes A_1 and A_2 and harmonic frequencies f_1 and f_2 from the spectrum. Are the values equal to the ones of the original parameters?

b) Truncate the signals with D = 4 and 9, respectively, show the two windowed signals $x_1(t)$ and $x_2(t)$. Calculate and plot the FT of the signals. Indicate the amplitudes A_1 and A_2 and harmonic frequencies f_1 and f_2 and demonstrate the windowing effect on the spectra.

c) Apply the rectangle window and Hamming window with width D = 9 to truncate the signals and show the two windowed signals $x_w(t)$. Calculate and plot the modulus of FT of $x_w(t)$. Indicate the amplitudes A_1 and A_2 and harmonic frequencies f_1 and f_2.

d) Compare to the FS results of sinusoidal signals, and demonstrate the smearing and leakage effects on identification of the spectrum peaks.

7. Discrete Time Fourier Transform and Discrete Fourier Transform

7.1 DTFT and DFT

Create a triangular wave $x[n]$, see Eq.(4). Compare DTFT and N-points DFT of $x[n]$ in the Nyquist frequency range [-1/2, 1/2], where N is equal to the length of signal $x[n]$. Then indicate the computational resolution of DFT.

$$x[n]=\begin{cases} -2\times|n|+10, & |n|\le 5 \\ 0, & 6\le|n|\le 12 \end{cases} \tag{4}$$

7.2 DFT and FFT

Define a cosine signal $y[n]$, as shown in Eq. (5), where $f_0 = 1/32$, $A = 3.5$, and define a rectangle window $w[n]$, as shown in Eq. (6). The truncated cosine signal $yw[n]=y[n]\cdot w[n]$ has a length of N, where $N = 32$.

$$y[n] = A\cos(2\pi f_0 n) \tag{5}$$

$$w[n] = \begin{cases} 1, & 0 \le n \le N-1 \\ 0, & \text{others} \end{cases} \tag{6}$$

a) Plot $y[n]$ and $yw[n]$ on one figure.
b) Calculate and show DFT and *fft* (MATLAB function) in a Nyquist interval in radians $[0,\ 2\pi]$.
c) In order to investigate the computational time of DFT and *fft* with respect to the window length, use a **tic-toc** (MATLAB function) to evaluate the computational time with N = [10, 50,100, 500, 1000, 5000]. Show the curve of computational time with respect to N.
d) In order to investigate the relationship between *fft* algorithm and the power of 2 points, use a tic-toc function to evaluate the computational time with N = [1000, 2^{10}, 2000, 2^{11}]. Tell which spends less time.

(Optional question) assume that sample with frequency fs, time duration D, $f_0 = 50$ Hz, L-points in time domain and N-points in frequency domain. What conditions do the above parameters meet to make the calculated points exactly at the highest point of the main-lobe and the junction points of the side-lobes? Express it in mathematical notation.

8. Application of DFT (optional)

The ideal electricity wave is a harmonic signal with the frequency of f_0 = 50 Hz and the amplitude of $A_0 = 220\sqrt{2}$ V, written as $E_0 = A_0\sin(2\pi f_0 t)$, denoted as $e_0 = A_0@f_0$. Due to the interference of some power inverters, the practical electrical waveform is distorted and then includes the fundamental frequency $e_1 = 220\sqrt{2}@50$, the high frequency harmonics e_2 = 3.5@101, e_3 = 2@140 and inter-harmonics e_4 = 0.5@55, e_5 = 1.5@104.

$$E_n(t) = \sum_{i=1}^{n} e_i \tag{7}$$

If we want to detect the frequency components accurately in the distorted wave signal, we can simulate the detecting process as follows:

(a) Construct a uniform rectangle window with a length of 0.1 s and a sampling frequency of 1000 Hz to truncate a time sequence $x[n]$ from Eq. (7).
(b) Calculate the frequency amplitude by using DTFT at the supposed frequencies and scaled to the proper amplitudes.
(c) By choosing proper window and time section length, try use DFT or FFT identify these frequency components.
(d) Provide a set of DFT parameters to identify the <u>frequency error should be less than 1% and the amplitude error should be less than 5%</u>, show the results.