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Abstract

It is widely agreed on that most cognitive processes are *contextual* in the sense that they depend on the environment, or context, inside which they are carried on. Even concentrating on the issue of contextuality in reasoning, many different notions of context can be found in the Artificial Intelligence literature, see for instance [Giunchiglia 1991a, Giunchiglia & Weyhrauch 1988, Guha 1990, Guha & Lenat 1990, Shoham 1991, McCarthy 1990b]. Our intuition is that reasoning is usually performed on a subset of the global knowledge base; we never consider *all we know* but only a very small subset of it. The notion of context is used as a means of formalizing this idea of localization. Roughly speaking, we take a context to be the set of facts used locally to prove a given goal plus the inference routines used to reason about them (which in general are different for different sets of facts). Our perspective is similar to that proposed in [McCarthy 1990b, McCarthy 1991].

The goal of this paper is to propose an epistemologically adequate theory of reasoning with contexts. The emphasis is on motivations and intuitions, rather than on technicalities. The two basic definitions are reported in appendix A.

Ideas are described incrementally with increasing level of detail. Thus, section 2 describes why contexts are an important notion to consider as part of our ontology. This is achieved also by comparing contexts with situations, another ontologically very important concept. Section 3 then goes more into the technical details and proposes that contexts should be formalized as particular mathematical objects, namely as logical theories. Reasoning with contexts is then formalized as a set of deductions, each deduction carried out inside a context, connected by appropriate "bridge rules". Finally, section 4 describes how an important example of common sense reasoning, reasoning about reasoning, can be formalized as multicontextual reasoning.

1. Contexts

We take the explicitly known facts (axioms or explicitly derived consequences) plus the machinery used to reason about them to be all and only the state of a reasoning individual. Notice that we take a fairly restrictive notion of state. For instance our ontology does not take into consideration all those global *internal* variables, such as the state of the stack or the commitment to achieve a certain goal, which usually affect the

behaviour of a program or a human. Neither do we consider the *external* state of the outside world, as conveyed by the sensors. With our approach, given an appropriate choice of the language and of the known facts, we are able to talk and reason about sensors and internal variables but they are not part of the basic notion of state.

The basic intuition underlying the work described in this paper is that reasoning is always local to a subset of the known facts. We never consider all we know but rather a very small subset of it. This small subset is what determines the context of reasoning. We therefore take a *context* c to be *that subset of the complete state of an individual that is used for reasoning about a given goal*.

A context is a *theory* of the world which encodes an individual's subjective perspective about it. It is a *partial theory* as the individual's complete description of the world is given by the set of all the contexts. It is an *approximate theory* (as described in [McCarthy 1979]) as we never describe the world completely. A complete description of the world is in general impossible, and, even when possible, seems a very bad idea (as it would force us to consider a lot of irrelevant detail). We may have different contexts which are theories of the same phenomenon and which describe it at different levels of approximation. Contexts can be partially ordered depending on the level of approximation. The "right" level of approximation depends, among other things, on the problem to be solved.

Contexts are not situations. A comparison will help clarify things. Remember (following [McCarthy & Hayes 1969]) that a *situation* s is the complete state of the universe *at an instant of time*. (Compare this definition with that of context. Notice also that some philosophers would argue that it does not make sense to talk of the "complete state of the universe" as the universe is "incomplete". This issue is not discussed here because irrelevant to the goals of the paper. It is worth noticing, however, that a situation records what can be talked about of the universe at given instant of time, whether the universe is complete or incomplete *per se* is not relevant.)

A situation records the state of the world as it is, independently of how it is represented in the mind of the reasoner. A context, rather, is inside the reasoning individual. It is part of his state and, as such, it is responsible of his subjective view of the world. For instance a certain phenomenon can be described and reasoned about by using different predicates in different contexts, the same predicate may have different truth values in different contexts and so on. The level of approximation used in a context to describe reality is itself a subjective choice. Notice that philosophers have discussed for centuries about the distinction about object and subject, real world and mind and about their interaction, *eg.* whether the object exists independently of the subject or it is generated by it. These issues do not play any role here. We are not interested in first principles but rather in epistemologically adequate theories of common sense reasoning (using the terminology introduced in [McCarthy & Hayes 1969]).

A situation is complete in that it records *all* the state of the world. At the same time it has also a dimension of partiality along the temporal dimension as it works only for a precise instant of time. The complete (temporal) picture of the world is recorded by the sequence of situations it evolves through. A context is partial as it records only part of the state of an individual. It is at the same time complete as it records all the state which affects reasoning.

It is easy to think of axiomatizations where both situations and contexts are used. The evolution through situations will account for the evolution of the world through time, the evolution through contexts will account for the evolution of the reasoning process through different states. There are four possibilities.

The first is to take the state of the world at a given situation s and describe it as a set of contexts c_j, \dots, c_n . Having multiple contexts describing one situation corresponds to the case where the reasoner uses multiple contexts, each being a different approximate theory of the same situation. We have evolution in the contexts used in the reasoning still considering the same situation. An example, suggested by John McCarthy, where this kind of axiomatization is useful is the following. Suppose you are planning a London-Glasgow- Moscow flight. You will consider the plane connection in Glasgow without testing whether you have the ticket. You do so on the basis of some law of inertia which suggests that you should have it as you had it in London just before taking off. But now, suppose that you are in the situation $sj2$ where, in Glasgow, you are connecting flight and you realize that you have lost the ticket (an unexpected obstacle). In the context c_{jj} which records your axiomatization of the situation $sj2$ you will not be able to deal with the obstacle. Indeed it is very likely that you will not even be able to mention the formula $has\text{ticket}(John, sj2)$ as $has\text{ticket}$ will not be part of the language of c_{jj} . In order to solve this problem you will set up a new context c_{j5} for the same situation $sj2$ with $has\text{ticket}$ in the signature and with a state which axiomatizes a subset of the state variables of $sj2$. c_{j5} will have to be less approximate than c_{jj} , at least for what concerns the problem of the lost ticket. For instance one axiom of c_{j5} will say that John does not have the ticket in Glasgow, i.e. $\neg has\text{ticket}(John, sj2)$.

The second possibility is to associate contexts to situations and to describe each situation Si with a context c_i . Each situation is axiomatized with a different context; that is the time evolution of the world corresponds one-to-one to the evolution of the reasoning process through partial states. [Giunchiglia & Weyhrauch 1988, Giunchiglia 1991], for instance, take this approach. In particular in [Giunchiglia 1991] the temporal evolution of physical processes, e.g. the flowing of the water from a container to another, is modeled qualitatively as a set of situations, each situation described by a context. This one-to-one mapping of situations into contexts is very useful as it allows to keep reasoning monotonic inside each context, to maintain local consistency (inside each context) and to achieve a form of nonmonotonicity by switching between contexts which possibly contain contradictory information. This kind of nonmonotonicity has been termed inessential in [Giunchiglia & Weyhrauch 1988]. Thus, in the description of the water flowing from one container to another, we may get to a situation sis (axiomatized by context c_j) where the level of the water in the container $C1$ should be at the same time strictly higher and strictly lower than that in the container $C2$ (in formulas $HIGHER(heightwater(C1), heightwater(C2), c_j)$ and $\neg HIGHER(heightwater(C1), heightwater(C2), c_j)$). This of course cannot be and would cause c_j to be inconsistent. The solution is to switch to a new context c_g such that, we have, for instance, $HIGHER(heightwater(C1), heightwater(C2), c_g)$ and $\neg HIGHER(heightwater(C1), heightwater(C2), c_g)$ but not $HIGHER(heightwater(C1), heightwater(C2), c_g)$. The proof of $HIGHER(heightwater(C1), heightwater(C2), c_g)$ is disallowed simply by adding a new appropriate axiom to c_g . The process is similar to that of building multiple extensions in default logic.

The third possibility is to have one context correspond to many situations. These axiomatizations correspond to the case where evolution in the world is not reflected in any change in the context considered in the reasoning. Almost all the work in the literature on nonmonotonic reasoning and the situation calculus can be described as taking this approach and considering only one context (see for instance Lifschitz' solution to the Yale shooting problem [Lifschitz 1987]). Not using contexts can be described as using always the same context and by making the obvious simplification of dropping the extra context argument from all the application symbols. Differently from the two cases above, here we can make, in the same context, assertions about two distinct situations *eg.* $HIGHER(heightwater(C1), heightwater(C2), s_{15}) \wedge -, HIGHER(heightwater(C1), heightwater(C2), s_{16})$. The obvious drawback is that we are forced to consider all we know in any reasoning step.

The fourth case of many situations described by many contexts can be obtained as a combination of the cases described above.

2. Formalizing contexts

In section 1 we have taken a context to be that subset of the complete state of an individual which is used during a given reasoning process. The next step is to formalize this notion of "state" by saying more precisely what we mean by "context" and by "reasoning with contexts".

The goal is to model reasoning as deduction. The notion of deduction can be formalized only if associated with a given formal system. The usual notion of formal system, meant as the triple $\Sigma = (L, A, \Delta)$, where L is a first order language, A is a set of axioms and Δ is a set of first order inference rules, is not good enough as it does not allow us to capture, as part of the logic, the idea of localization of reasoning. We show below how contextual reasoning can be formalized as deduction in a system allowing multiple first order theories. Appendix A gives the definition of the resulting formal system, called "Multilanguage System" (ML-System) and of the notion of deduction associated with it.

2.1 Reasoning inside a context

The starting point is that the set of facts which provide the context of reasoning is in general only a subset of the knowledge base. We therefore describe the knowledge base as structured into sets of facts, which we write A_1, \dots, A_n . Taking a context to be any A_i would lead to a notion of context which is similar to the notion of a partition in partitioned data bases or to the similar notion of microtheories in CYC [Guha 1990]. Notice that these partitions would not need to be static and fixed once for all.

This does not seem a satisfactory enough solution. In fact, in general, each A_i is written using different sets of constants, predicate, and function symbols. For instance, the set of facts about arithmetic will have, as part of the signature, 0 , $+$, and $*$, while the theory of how to get on a plane will use constants like *exisisflight* and *plane*. In the

London- Glasgow- Moscow example the signature of *030* will have *hasticket* in its signature while *cjj* will not. Basic beliefs and beliefs about beliefs have again different signatures: the second set need only contain the belief predicate and the names of formulas, but not the formulas themselves (see section 3). We therefore require that each context come with its own signature.

More interestingly, we also take the notion of wellformedness to be *localized* and distinct for each A_i . (Notice that in partitioned data/knowledge bases, *eg.* CYC, the notion of wellformedness is taken to be the same everywhere.) This allows us, for instance, to have a context which is essentially a data base of atomic ground formulas and another whose facts express metalevel heuristics, all expressed in clausal form. Another interesting example (which has been implemented in GETFOL) is the following. We may have a context $C2$ containing facts which are abstractions of the facts in c ; [Giunchiglia & Walsh 1989]. $C2$ can then be used to drive search in c ? [Giunchiglia & Walsh 1989]. If we use one of the GPS abstractions, then the language of 02 allows only sets of atomic formulas [Newell et al. 1963].

We formalize the requirement that each A_i come with its own signature and wellformedness rules by associating a language L_i to each A_i . This amounts to saying that the language used to write a set of facts A_i , and not only the set of facts A_i , is part of the state used in a reasoning process.

The next step is to model reasoning. The standard solution is to have a unique inference engine (possibly consisting of a set of inference modules) which can be applied to any set of facts or, even, to combinations of them. Our proposal is to associate a distinct inference engine to each distinct set of facts A_j . This allows us to localize the form of reasoning and, for instance, to define special purpose inference engines which exploit the local form of wffs. For example, we can use PROLOG on clausal languages and set inclusion in the GPS abstract context $C2$ -

If we call ΔI the set of inference rules associated with a set of facts A_i , then we can define a context C_j to be the triple $c_j = (L_i, A_i, \Delta I)$. In other words, we take a context to be a logical theory, presented as an axiomatic formal system. This allows us to take the usual notion of deduction (see for instance [Prawitz 1965]) as the formalization of reasoning *inside* a context. The components of c_j , that is its language L_i , its set of known facts A_i , plus the explicitly derived theorem, the inference rules ΔI , are all and only the state that is used when reasoning about a given goal.

2.2 Reasoning with multiple contexts

A knowledge base contains in general a set of interacting contexts c_j, \dots, c_n . We need to capture the idea that reasoning in one context may influence reasoning in other contexts. We introduce therefore a new set of rules which allow us to derive a fact in a context because we have derived another fact in another context. We can represent these "linking" rules as follows:

$$\frac{(A_i, c_i)}{(A_j, c_j)}$$

where $c/$ and Cj are two different contexts and Ak is a formula written in the language of the context ck . We call **bridge rules** the rules of the form above as they allow us to bridge deductions in $c/$ to deductions in Cj by allowing us to derive Aj in Cj just because we have derived $A/$ in $c/$. We say also that Aj is a **justified assumption** of Cj as it is an assumption in Cj which is justified by a derivation in another context. The rules where premises and conclusion belong to the same language are called *LI* rules.

One example of L/-rule for a theory i is modus ponens

One example of bridge rule between the theories ij and the theory k is *multicontextual modus ponens*

$$\frac{(A \multimap B.j) \quad (A.j)}{(B.k)}$$

The meaning of the two rules is very different. The first allows us to derive B inside the theory i just because we have derived $A \multimap B$ and A in the same theory. Thus, for instance, if we take the theory i to represent the beliefs of an agent $a/$, this means providing the agent aj with the ability of using modus ponens. Multicontextual modus ponens allows us to derive B in the theory k just because we have derived $A \multimap B$ in the theory j and A in the theory j . If we take the theories ij,k to represent the beliefs of the agents aj, aj, afc , this means that a/c will be able to take B as known, not because he has derived it, but because this is the result of the interaction of the results derived by aj and aj . The assertion of B in k is not the result of a deduction in k but, rather the result of the "propagation" of reasoning from ij into k .

As another example, a notion similar to that captured by McCarthy's lifting axiom [McCarthy 1991] can be formalized by the following bridge rule:

$$(A.C)$$

which intuitively says that, if we can prove A in context c , then we can prove (in context c') that we can prove A in c .

2.3 Some observations

Some general observations about the role of bridge rules in the interaction among contexts are in order. Let us consider the interaction between the reasoning in two distinct contexts $cj = (Lj, Ai, Aj)$, $c2 = \{^2^2^2\}$ (the case with more than two contexts is a trivial generalization). The interaction between two contexts inside a multicontext system can be formalized provided the following conditions hold:

1. a subset of the wffs in L_j , let us call it $L_{s,j}$, can be put in relation with a subset of the wffs in L_2 , let us call it $L_{s,2}$,
2. there exist bridge rules between c_j and 02 .

In general we can formalize the relationship between L_j and $L_{s,2}$ via a general mapping function *rew* which rewrites the elements of $L_{s,j}$ into the elements $L_{s,2}$. *rew* is always total and surjective and sometimes injective. A not injective *rew* is used in abstraction to map ground formulas into abstract formulas [Giunchiglia & Walsh 1989]. For instance in GPS abstractions both $\alpha \vee \beta$ and $\alpha \wedge \beta$, where α and β are atomic wffs, are mapped into $\{(X, \beta)\}$.

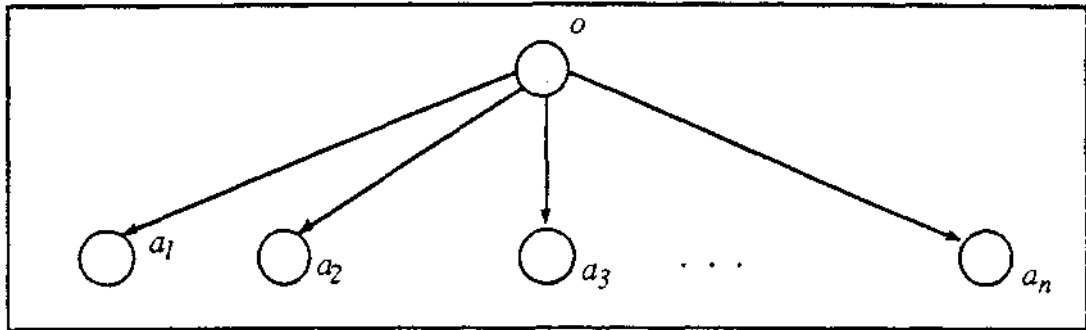
The relation between L_j and L_2 allows us to link the semantics of c_j to the semantics of $C2$. Via the relation between the languages, we can formalize the links existing between the models of c_j and those of 02 . Thus, for instance, in the London- Glasgow- Moscow-example introduced in section 2 we can say that *John* in 035 is the same *John* as that in cjg . As a more complicated example we can also say that *gate15* (used to assert *at(John, gate!5, si2)* in $C36$) corresponds to *GlasgowAirport* (used to assert *at(John, GlasgowAirport, si2)* in $c?j$). The intuition is that $cj5$ is a less approximate theory of 035 than cjg and that we need this more precise description in order to solve the problem (eg. John will buy the ticket at the gate number 15).

The relationship between the languages constrains the kind of bridge rules which exist between the two contexts c_j and $C2$ (the premises of a bridge rule must belong to L_j while its conclusion must belong to L_2). The existence of bridge rules (second condition) allows us to achieve proof theoretically the connection achieved semantically via the connection between the languages. Thus for instance, in the London- Glasgow- Moscow examples we will have a bridge rule which will allow us to assert *at(John, gate15)* in cjg from a proof of *at(John, GlasgowAirport)* in 035 and another which will allow us to assert *existsflight(Glasgow, Moscow, si2)* in $cj6$ from a proof of *existsflight(Glasgow, Moscow, S/2)* in

3. Reasoning about reasoning

In the case of reasoning about reasoning, the mapping function *rew* is such that $rew(a) = \pi("a")$; in other words *rew* maps a formula a into a predefined unary predicate " π " which takes the name of a , that is " a ", as argument. Sometimes π takes a second argument which is the name of the context a belongs to; this distinction is not relevant in this context, π can be instantiated to various predicates. For instance we can take π to be the provability predicate provable , in which case $C2$ is a metatheory of c_j (see for instance [Giunchiglia & Traverse 1991]). We can also take π to be the belief predicate believe , in which case $C2$ will be the context of the beliefs about c_j ; we will study this example in detail in the following, π can also be taken to be the defeasible belief predicate *CBB* (where *CBB* stands for "Can Be Believed") as it has been done in [Giunchiglia & Weyhrauch 1988, Giunchiglia 1991]). Finally, we can also take π to be McCarthy's *ist* predicate which takes a context c and a proposition p and is such that *ist(p, c)* means that p is true in c [McCarthy 1991].

Let us consider more in detail the case of belief in a multiagent environment. As we consider the multiagent case, not only will the computer be able to have beliefs about its own beliefs but also about the beliefs of other agents. Let us suppose that we have a situation with a (possible infinite) set $\{a_i\}_{i \in \mathbb{N}}$ of agents a_i . Each agent is associated with a context (called a_i -context) which represents his beliefs; we thus say that a_i believes a formula if that formula is provable in the a_i -context. We also have a computer observer o which is himself associated the context (called o -context) of his beliefs. Differently from the agents, o "sees" the agents' beliefs (namely all the a_i -contexts) and is able to reason about them. This is formalized by putting the o -context "on top" of the a_i -contexts, *ie.* by defining appropriate bridge rules which allow all a_i 's beliefs to propagate up into o . The following picture summarizes this idea.



Let us concentrate on the bridge rules. The computer observer o will have a belief predicate Φ_i for each agent a_i . The idea is that o believes $\Phi_i(A)$ whenever the agent a_i believes A . The ability of o to see a_i 's beliefs is formalized by defining a form of reflection up from each agent into o . For any wff A believed by a_i , o will thus be able to prove $\Phi_i(A)$. The fact that o believes only facts which correspond to the agents' beliefs is captured by defining a form of reflection down from o into each agent. This will prevent o from believing sentences of the form $\Phi_i(A)$ where A is not provable in the a_i -context. This amounts to considering the following two bridge rules:

$$\begin{array}{ccc}
 & A & \Phi_i(A, o) \\
 \text{Itup.o} & \xrightarrow{A} & \\
 & B_i(A, o) & A, a_i
 \end{array}$$

For the purpose of this paper we need to consider the case where each agent may have beliefs about its own beliefs and about the beliefs of the other agents and so on indefinitely. To formalize this situation, it is sufficient to expand the context of each agent a_i in the previous figure with the figure itself and so on with the resulting contexts. The result is a balanced tree with infinitely long branches. [Giunchiglia & Serafini 1991a] describes the technical results about this system. For instance it shows that we need the restriction that $\Phi_{j_{own}, i}$ can be applied only if $\langle A, a_i \rangle$ depends only on assumptions made in contexts above a_i . It also proves that the resulting system is equivalent to multimodal K (that is modal K with multiple modal operators) and it discusses how it relates to Konolige's logics of belief [Konolige 1984, Konolige 1985]. [Giunchiglia & Serafini 1991b] studies the proof theory of this and other related systems in detail.

This system, extended with two bridge rules which formalize common belief [Giunchiglia & Serafini 1991a] has been used to prove the three wise men problem. As an example, let us consider part of this proof. The variant of the statement of the three wise men puzzle we have considered is as follows:

A certain king wishes to test his three wise men. He arranges them in a circle so that they can see and hear each other and tells them that he will put a white or black spot on each of their foreheads but that at least one spot will be white. Infact, all three spots are white. He then repeatedly asks them, "Do you know the color of your spot?". What do they answer?

The solution is that they answer, "No", the first two times the question is asked and answer "Yes" thereafter. (See [McCarthy 1990a, Bottani & Penco 1991] for alternative descriptions of this puzzle). The following is part of the proof.

$$\begin{array}{c}
 \boxed{\rightarrow E \frac{\neg P_3 \quad \neg P_3 \rightarrow B_2(\neg P_3)}{B_2(\neg P_3)}} \quad 3 \\
 \mathcal{R}_{down.2} \overline{\hspace{10em}} \\
 \boxed{\rightarrow E \frac{\neg P_2 \quad \neg P_2 \rightarrow B_1(\neg P_2)}{B_1(\neg P_2)} \quad \rightarrow E \frac{\neg P_3 \quad \neg P_3 \rightarrow B_1(\neg P_3)}{B_1(\neg P_3)}} \quad 3, 2 \\
 \mathcal{R}_{down.1} \overline{\hspace{10em}} \quad \mathcal{R}_{down.1} \overline{\hspace{10em}} \\
 \boxed{TAUT \frac{\neg P_2, \quad P_1 \vee P_2 \vee P_3 \quad \neg P_3}{P_1}} \quad 3, 2, 1 \\
 \mathcal{R}_{up.1} \overline{\hspace{10em}} \\
 \boxed{\rightarrow E \frac{B_1(\neg P_1) \quad \neg B_1(\neg P_1)}{\perp} \quad \frac{\perp}{P_2}} \quad 3, 2 \\
 \mathcal{R}_{up.2} \overline{\hspace{10em}} \\
 \boxed{\rightarrow E \frac{B_2(\neg P_2) \quad \neg B_2(\neg P_2)}{\perp} \quad \frac{\perp}{P_3}} \quad 3 \\
 \mathcal{R}_{up.3} \overline{\hspace{10em}} \\
 \boxed{B_3(\neg P_3)} \quad c
 \end{array}$$

In the proof, P_i is the assertion that the i -th wise man has a white spot on his hat. Context c is the top context, the facts provable in this context are the computer beliefs. There are three agents named with the three natural numbers 1, 2 and 3; thus, for instance, context 3 is the third wise man, the facts provable here are his beliefs (in c 's view). At level three in the tree there are the agents' beliefs about the agents' beliefs; thus,

for instance, context 3,2 is wise man J's view of the beliefs of wise man 2 and so on. A similar argument applies to the contexts at level 4 (which is the deepest level needed in the proof), for instance to the context 3,2,1 in figure.

The part of the proof listed above ends with the theorem that *o* believes that the third agent believes that he has a white spot on his hat. The particular proof steps are not relevant. Notice on the other hand the following facts. As a notational convention, the boxes surround a reasoning session inside a context. Each box is labeled by the name of the context where the session is carried out. We may have multiple reasoning sessions inside the same context (as it happens, in the proof above, with the contexts 3 and 3,2). The box notation has been adopted to capture the intuition that reasoning inside one context is completely isolated from the reasoning done in the other contexts. The reasoning inside one context is performed as if this were the only context. In the example, the reasoning inside all the contexts is first order and monotonic and it amounts to multiple applications of modus ponens, one application of the reduction ad absurdum inference rule (in the second session in context 3) and, in context 3,2,1 to the application of a tautology decision procedure (which may not be applicable in the other contexts).

In the example, each box is linked to one or more boxes via the application of a reflection rule. In the general case a deduction inside a multicontext system can be described formally as a set of deductions inside contexts, any two or more deductions being linked by two or more applications of bridge rules (see Appendix A).

Multicontext systems have been implemented inside GETFOL [Giunchiglia 1991b], an interactive theorem prover which has been implemented on top of a re-implementation of the FOL system [Giunchiglia & Weyhrauch 1991, Weyhrauch 1980]. In particular the three wise men problem has been machine proved within GETFOL. The machine proof of the proof listed above is reported and discussed in Appendix B.

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Appendix A: Multilanguage Systems

An axiomatic formal system S is usually described as a triple consisting of a language L , a set of axioms QcL and a set of inference rules Δ , i.e. $S = (L, Q, \Delta)$. The generalization is to take many languages and many sets of axioms while keeping one set of inference rules. We thus have the following definition of multi-language system:

Definition 0.1 (Multi-Language System) Let I be a set of indices, $\{L_i\}_{i \in I}$, a family of languages and $\{\Omega_i\}_{i \in I}$, a family of sets of wffs such that $\Omega_i \subseteq L_i$. A Multi-Language Formal System (ML System) MS is a triple $(\{L_i\}_{i \in I}, \{\Omega_i\}_{i \in I}, \Delta)$ where $\{L_i\}_{i \in I}$ is the Family of Languages, $\{\Omega_i\}_{i \in I}$ is the Family of sets of Axioms and Δ is the Deductive machinery of MS .

We write (A, i) to mean A and that A is a L_i -wff. The deduction machinery Δ , is therefore defined as a set of inference rules, written as:

(i,

or as

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \wedge B} \text{ (An.in)}_s$$

Picture (2) represents a rule δ discharging the assumptions $(\Gamma_i \wedge \Delta_i), \dots, (B_{m,n}, \Delta_{m,n})$. The rules whose premises, conclusions, and, when existing, discharged assumptions belong to the same language L_i are called *Li-rules*, the others *bridge rules*. Each context c_i is defined as $C_i = (L_i \wedge A \wedge \Delta_i)$ where Δ_i is the set of L_i -rules,

Definition 0.2 (Deduction ... depending on) A formula-tree is a deduction in MS of (A, i) depending on a set of formulas according to the following rules:

- (i) if (A, i) is an axiom of MS i.e., A is an element of Ω_i , then (A, i) is a deduction in MS of (A, i) depending on the empty set;
- (ii) if (A, i) is not an axiom of MS then (A, i) is a deduction of (A, i) depending on
- (iii) if Γ_k is a deduction of (A_j, i) depending on Γ^* for any $(1 < k < n)$, then

is a deduction of (A, i) depending on Γ , provided that ι is of the form (1) and Γ is the union of all Γ^k for $1 < k < n$; or ι is of the form (2) and

$$1 = \bigcup_{IS/\text{fcSm}} \Pi \cup \bigcup$$

Definition 0.3 (Theorem) A deduction Π is a deduction in MS of (A, i) from Γ if and only if Π is a deduction in MS of (A, i) depending on Γ or any subset of Γ .

(A, i) is derivable from Γ in MS, which is denoted by $\Gamma \vdash_{\text{MS}} (A, i)$, if and only if there is a deduction of (A, i) from Γ in MS.

A deduction of (A, i) from the empty set is a proof of (A, i) . (A, i) is provable in (a theorem of) MS, in symbols $\vdash_{\text{MS}} (A, i)$, if and only if there exists a proof of (A, i) in MS.

Deductions are trees of wffs built by starting from a finite number of assumptions and axioms, possibly belonging to distinct languages, and by applying a finite number of inference rules. Any deduction can be seen as *composed of sub-deductions in distinct languages*, obtained by repeated applications of Lj-rules, any two or more subdeductions being concatenated by one or more applications of bridge rules.

Appendix B: a GETFOL proof of the three wise men problem

The following is the GETFOL listing of the (part of the) proof reported above (the listing must be read top to bottom, left to right).

```
SWITCHCONTEXT C3;          SWITCHCONTEXT C321;      SWITCHCONTEXT C3;
ROW c CB Iook2at3;         ROW C32 B1 Iookedlat2; LABEL FACT conclusions;
LABEL FACT assumptions;   ROW C32 B1 Iookedlat3; RUP C32 B2 ("P2") ;
ASSUME not P3;            ROW c CS commcnbel;    ROW c B3 tel!23;
LABEL FACT Iooked2at3;    TAUT PI BY ^1 ^2 ^3;    FALSEI ^1 conclusion3;
IMPE ^1 ^2;              NOTE ^1 assumptions;

                           SWITCHCONTEXT C32;
SWITCHCONTEXT C32;        LABEL FACT
ROW c CB Iooklat3;         conclusion32;      SWITCHCONTEXT c;
RDM C3 B2 Iooked2at3;     RUP C321 B1{"PI"};    RUP C3 B3 ("P3") ;
LABEL FACT Iookedlat3;   ROW C3 B2 tel!12;
IMPE "1 "2;              FALSEI ^1 conclusion32;
ROW c CB Iooklat2;        NOTE "1 assumption32;
LABEL FACT assumption32j
ASSUME not P2;
LABEL FACT Iookedlat2;
IMPE ^1 ^2;
```

In the above listing, each set of commands (separated from the others by an empty line) axiomatizes all the inference steps inside a box in the previous figure. The first command of each set (that is SWITCHCONTEXT) forces GETFOL to concentrate its attention to a new context. GETFOL has in fact a notion of *current context*, which is, by definition, the context where the user is currently doing the reasoning. The operation of SWITCHCONTEXT corresponds to the sequence of leaving the current context and then entering the context whose name is the argument of the command (as it has been suggested by John McCarthy [McC90]).

RUP and ROW are two of the commands implementing the reflection rules. The command "LABEL LAB" labels the result of the following proof step with LAB. This allows to refer to this proof step simply by calling it LAB. In the arguments of the commands, a number η with an arrow above it refers to the the proof line which is η proof lines above the current one.

Finally, even if this cannot be seen in the printout, GETFOL has commands which allow the user to define arbitrary multicontext systems: he can create, copy and name contexts and define arbitrary bridge rules among them. For each context the user can also define the language, the set of axioms and its internal inference rules (the default is first order ND).