Type-checking knowledge graphs

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Abstract. We first present a formal view of a knowledge graph. On this basis, the type-checking rules are developed to define correct typing relationships among the triples of a knowledge graph. We discuss the algorithms for verifying the typing relationships against the given knowledge graph. Finally, we present the experimental results of type-checking the Yago4 knowledge graph.

Keywords: type checking · knowledge graphs · RDF stores · graph databases.

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1 Introduction

Introduction to KGs ... [4, 2].
– *KGs are becoming KBs...*
– *What are the structural characteristics of KBs?*
– *What KBs can represent that (classical) data models can not.*
– *Relations between the knowledge bases and KGs.*

– *About the very (complex) KG domain.*
– *We have a specific domain, i.e., a knowledge graph including nodes and triples.*
– *The class identifiers are ordered in poset and, consequently, triples are also poset.*
– *Λ-expressions form a domain expressing derivations of λ-expressions.*
– *??*

On the type theory, using type theory, some keystones, ... [7, 3]. On intersection and union types and their use in type-checking [7, 1, 5].
– *Problem definition.*
– *Checking the types of ground triples.*
– *Errors in typing of a KG.*
– *Typing BGP queries.*
– *Typing triple patterns!*
– *Rules for TPs.*
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– *Abstract of the method.*
– *Give an abstract insight into the structure of KB. Identifiers, schema, types, ...*
– *Show the ground triples, poset of triples.types triples, schema graph, etc.*

2 Preliminaries

2.1 Knowledge graph

This section defines a knowledge graph as a RDF graph [8] using RDF-Schema [9] for the representation of the structural part of a knowledge base.

Let I be the set of URI-s, B be the set of blanks and L be the set of literals. Let us also define sets $S = I \cup B$, $P = I$, and $O = I \cup B \cup L$.

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RDF triple is a triple $(s, p, o) \in S \times P \times O$. *RDF graph* $g \subseteq S \times P \times O$ is a set of triples. Set of all graphs will be denoted as G. We suppose the existence of a set of variables V and the set of *terms* $T = O \cup V$. Term $t \in T$ is ground if $t \in O$.

We say that RDF graph g_1 is *sub-graph* of g_2, denoted $g_1 \sqsubseteq g_2$, if all triples in g_1 are also triples from g_2.

– Define major structure of KG on the basis of the sorts of data.
– ...the set I includes individual identifiers I_i, class identifiers I_c and predicate identifiers I_p.

2.2 Rule language

– Put somewhere general discussion on rules.
– What elements of rule are reasonable. Criterium: tracable elements of rule.
– What is tracable? Looping on a "selected" set. Loop may go thruogh elements of a set, all more specific/general elements, etc.
– They are extended to handle poset of identifiers that can form triples.

3 Typing identifiers

– Intro to typing idents.
– Typing idents without considering info about the triples.

The individual and class entities are represented by identifiers from the set $\mathcal{I}$. The individual identifiers $\mathcal{I}_i$ stand for literals, concrete and abstract entities. The class identifiers $\mathcal{I}_c$ represent abstract entities that have an unempty interpretation. The abstract entities include, besides the identifiers of user-defined classes, the types (classes) of literals.

A graph database includes stored definitions for typing the individual identifiers, and for representing the specialization/generalization hierarchies of classes and properties.

– Details.
– 1. First define base type of identifiers $:_1$ and stored subtyping relationship $\preceq_1$.
– 2. From the basis define the indent typing $:$ and subtyping rel $\preceq$ among identifiers.
– 3. Include the link between subtyping and typing.
– 4. Define lub type using $\wedge$ type for a given ground ident.

3.1 Typing literals

Literals are the values of an atomic type. The atomic types are in RDF provided by the RDF-Schema dictionary [9]. RDF-Schema defines a list of atomic types, such as xsd:integer, xsd:string, or xsd:boolean.

Typing of the atomic types is determined by the following rule.

$$\frac{L \in \mathcal{I}_l, T \in \mathcal{I}_c,\ \text{"L"\^{}\^{}T} \in \mathcal{L}}{L : T} \tag{1}$$

The rule states that a literal value L is of a type T if a literal "L"^^T is an element of the set of literals $\mathcal{L}$. A literal "L"^^T includes a literal value L and a literal type T referencing a type from the RDF-Schema dictionary. As an example, the literal "365"^^xsd:integer includes the literal value 365 and its type xsd:integer.

3.2 Intersection and union types

The instances of the intersection type $T_1 \wedge T_2$ are objects belonging to both T_1 and T_2. The type $T_1 \wedge T_2$ is the greatest lower bound of the types T_1 and T_2. In general, $\wedge[T_1 \ldots T_n]$ is the greatest lower bound of types $T_1 \ldots T_n$ [5, 6].

$$T_1 \wedge T_2 \preceq T_1 \tag{2}$$

$$T_1 \wedge T_2 \preceq T_2 \tag{3}$$

$$\wedge[T_1 \ldots T_n] \preceq T_i \tag{4}$$

If the type S is more specific than the types $T_1 \ldots T_n$ then S is more specific then $\wedge[T_1 \ldots T_n]$. First, we present the rule for a pair of types T_1 and T_2.

$$\frac{S \preceq T_1 \quad S \preceq T_2}{S \preceq T_1 \wedge T_2} \tag{5}$$

$$\frac{\text{forall i, } S \preceq T_i}{S \preceq \wedge[T_1 \ldots T_n]} \tag{6}$$

The intersection and union types are dual. This can be seen also from the rules that are used for each particular type.

The instances from the union type $T_1 \vee T_2$ are either the instances of T_1 or T_2, or the instances of both types. The type $T_1 \vee T_2$ is the smallest upper bound of the types T_1 and T_2. In general, $\vee[T_1 \ldots T_n]$ is the smallest upper bound of types $T_1 \ldots T_n$ [5].

$$T_1 \preceq T_1 \vee T_2 \tag{7}$$

$$T_2 \preceq T_1 \vee T_2 \tag{8}$$

$$T_i \preceq \vee[T_1 \ldots T_n] \tag{9}$$

If the type T is more general than the types $S_1 \ldots S_n$ then T is more general then $\vee[S_1 \ldots S_n]$. First, we present the rule for types T_1 and T_2.

$$\frac{S_1 \preceq T \quad S_2 \preceq T}{S_1 \vee S_2 \preceq T} \tag{10}$$

$$\frac{\text{forall i, } S_i \preceq T}{\vee[S_1 \ldots S_n] \preceq T} \tag{11}$$

3.3 Stored typing and subtyping of identifiers

Let us introduce the typing and specialization/generalization relationships formally. The expression $i :_\downarrow C$ states that a class C is a type of an individual identifier i. The expression $i_1 \preceq_\downarrow i_2$ defines the sub-class relationship between the class identifiers i_1 and i_2. The index '$\downarrow$' in relations $:_\downarrow$ and $\preceq_\downarrow$ denotes that the relationships is stored in a database. Such notation allows us to address differently the stored and the derived parts of the graph database schema.

The rule for the one-step relationship $:_\downarrow$ is defined using the predicate rdf:type.

$$\frac{I \in \mathcal{I}_i \quad I_c \in \mathcal{I}_c \quad (I, \text{rdf:type}, I_c) \in \mathcal{D}}{I :_\downarrow I_c} \tag{12}$$

The individual entity I can have more than one stored types, e.g., I_{c1} and I_{c2}. Therefore, $I :_\downarrow I_{c1}$ and $I :_\downarrow I_{c2}$ holds, and we can instead write $I :_\downarrow I_{c1} \wedge I_{c2}$. All existing stored typings of I can be gathered by Rule 23 presented later.

A one-step subtyping relationship $\preceq_\downarrow$ is defined by means of the RDF predicate rdfs:subClassOf in the following rule.

$$\frac{I_1, I_2 \in \mathcal{I}_c \quad (I_1, \text{rdfs:subClassOf}, I_2) \in \mathcal{D}}{I_1 \preceq_\downarrow I_2} \tag{13}$$

The rule for the definition of the one-step subtyping relationship $\preceq_\downarrow$ is based on the predicate rdfs:subPropertyOf.

$$\frac{I_1, I_2 \in \mathcal{I}_p \quad (I_1, \text{rdfs:subPropertyOf}, I_2) \in \mathcal{D}}{I_1 \preceq_\downarrow I_2} \tag{14}$$

3.4 Typing and subtyping identifiers

The one-step relationships $:_\downarrow$ and $\preceq_\downarrow$ are now extended with the reflexivity and transitivity to obtain the relationships : and $\preceq$. The relation $\preceq$ forms a partial ordering of class identifiers.

First, the one-step relationship $\preceq_s$ is generalized to the relationship $\preceq$ defined over class identifiers $\mathcal{I}_c$.

$$\frac{I_1, I_2 \in \mathcal{I}_c \quad I_1 \preceq_\downarrow I_2}{I_1 \preceq I_2} \tag{15}$$

Next, the subtyping relationship $\preceq$ is reflexive.

$$\frac{S \in \mathcal{I}_c}{S \preceq S} \tag{16}$$

The subtype relationship is also transitive.

$$\frac{S, U, T \in \mathcal{I}_c \quad S \preceq U \quad U \preceq T}{S \preceq T} \tag{17}$$

Finally, the subtype relationship is asymmetric which is expressed using the following rule.

$$\frac{S, U \in \mathcal{I}_c \quad S \preceq U \quad U \preceq S}{S = T} \tag{18}$$

As a consequence of the rules 16-18 the relation $\preceq$ is a poset.

Knowledge graphs include a special class $\top$ that represents the root class of the ontology. In RDF ontologies $\top$ is usually represented by the predicate owl:Thing. The following rule specifys that all class identifiers are more specific than $\top$.

$$\frac{\forall S \in \mathcal{I}_c}{S \preceq \top} \tag{19}$$

The stored typing relation $:_\downarrow$ is now extended to the typing relation $:$ that takes into account the subtyping relation $\preceq$. The following rule states that a stored type is a type.

$$\frac{I \in \mathcal{I}_i \quad C \in \mathcal{I}_c \quad I :_\downarrow C}{I : C} \tag{20}$$

The link between the typing relation and subtype relation is provided by adding a typing rule called *rule of subsumption* [7].

$$\frac{I \in \mathcal{I}_i \quad S \in \mathcal{I}_c \quad I : S \quad S \preceq T}{I : T} \tag{21}$$

– Properties have dual role: they are instances and types at the same time.
– Present the features of properties from this point of view.

4 Typing triples

– There are two basic aspects of a triple type.
– First, the type is computed bottom-up: from the stored types of triple components.
– Second, the type can be computed top-down: from the user-defined domain/range types of properties.

– About the types that are computed bottom-up.
– Ground type of a triple is computed first using $:_\downarrow$.
– Next, the lub type of a triple is derived using $:_\sqcup$.

– About the stored types that are computed as glb of valid stored types.
– From the top side of the ontology, the stored type $:_\uparrow$ is determined based on p.
– The glb types of all types obtained using $:_\uparrow$ obtaining a glb type $:_\sqcap$.
– Finally, the type $:$ of t is deterimned by summing alternative $:_\uparrow$ types.

– Unfinished!!
– Interactions between the $\wedge/\vee$ types of triple components and triples must be added.
– Analogy between the types of functions in LC and types of triples.
– Show rules relating $\wedge/\vee$ types and triple types. Example.
– E.g., $(S_1 \wedge S_2) * p * R = S_1 * p * R \wedge S_2 * p * R$.

– *Are all rules covered?*

– *Unfinished!!*

– *Predicates should be treated in the same way as the classes.*
– *They can have a rich hierarchy.*
– Note: *Where to include discussion on special role of predicates and their relations to classes?*
– *Mention Cyc as the practical KB with rich hierarchy of predicates.*

4.1 Product types

4.2 Deriving a ground type of a triple

A ground type of an individual identifier i is a class C related to i by one-step type relationship $:_\downarrow$ denoting a ground type. In terms of the concepts of a knowledge graph, C and i are related by the relationship rdf:type.

A ground type of a triple $t = (s, p, o)$ is a triple $T = T_s * p * T_o$ that includes the ground types of t's components s and o, and the property p which now has the role of a type. A ground type of a triple is defined by the following rule.

$$\frac{t \in \mathcal{T}_i,\ t = (s,p,o) \quad I_s, I_o \in \mathcal{I}_c,\ s :_\downarrow I_s \wedge o :_\downarrow I_o \quad p :_\downarrow \text{rdf:Property}}{t :_\downarrow I_s * p * I_o} \tag{22}$$

The class I_s is one of the ground types of s, and the type I_o is one of the ground types of o. The predicates are treated differently to the subject and object components of triples. The predicates have the role of classes while they are instances of rdf:Property.

There can be multiple ground types of a triple. They may be gathered into a single $\wedge$-type by using the following rule. The types T_1,...,T_n are obtained using Rule 22.

$$\frac{t \in \mathcal{T}_i \quad \forall T_i \in \mathcal{T}_t,\ t :_\downarrow T_i}{t :_\downarrow \wedge[T_i]} \tag{23}$$

– *Typing using lub types of $T_1..T_n$. Explain why this is needed?*

Let us now define the least upper bound types (abbr. *lub*) of ground types derived by Rule 23. Since a partialy ordered set is not a lattice, we can have more than one lub type for a given set of ground types.

The lub types of a given list of triple types $T_1..T_n$ are computed in two steps as before when gathering multiple ground types with conjunction. A single lub type is defined as follows.

$$\frac{t :_\downarrow \wedge[T_1..T_n] \quad T \in \mathcal{T}_t \quad \forall i,\ T_i \preceq T \quad \forall S \in \mathcal{T}_t,\ \forall i,\ T_i \preceq S \wedge T \preceq S}{\vdash t :_\sqcup T} \tag{24}$$

The above rule states that a type T is a lub type of a ground type $\wedge[T_1..T_n]$ if all ground types T_i are subtypes of T. Furthermore, the lub type T is the least (closest) supertype of all members of ground $\wedge$-type $T_1, \ldots, T_n$. The lub types can be now gathered using the following rule.

$$\frac{\forall i, T_i \in \mathcal{T}_t \quad t :_{\sqcup} T_i}{t :_{\sqcup} \wedge[T_1..T_n]} \tag{25}$$

4.3 Stored types of triples

– General comments.
– Analysis tool. Show minimality of the stored types (either enumerated or gathered with $\bigvee$).
– Reminder: *when a complete stored (user-defined) type is related to the base type of a triple, some of GLB types may be eliminated.*

– Present the complete story.
– Computing the minimal and valid stored type of a triple $t = (s, p, o) \in \mathcal{T}_i$.
– Stored types are defined by linking a predicate p to a domain and range classes.
– Only types (domains and ranges) defined for $p' \succeq p$ are valid stored types.
– There are no other valid types below, i.e., for $p' \prec p$.
– Among the valid stored types the most specific and unrelated stored types are selected.
– In other words, only glb types of valid stored types are selected.
– Finally, the minimal and complete type of t is an $\vee$-type including all previously selected glb types.

We first find stored triple types for a given triple $t = (s, p, o)$. A stored schema triple is constructed by selecting types including a predicates $p' \succeq p$ that the domain and range defined.

$$\frac{t \in \mathcal{T}_i,\, t = (s, p, o) \quad p' \in \mathcal{I}_p, p \preceq p' \quad (p', \text{domain}, T_s) \in g \quad (p', \text{range}, T_o) \in g}{t :_{\uparrow} (T_s, p, T_o)} \tag{26}$$

– Comments and description of the above rule.
– Note p is used for all types. p shoud be in most specific type -
– It makes no sense to generate types with p'.

The domain and range of a predicate p can be defined for any super-predicate, they do not need to be defined particulary for p. In addition, the domain and range of a predicates do not need to be defined for the same predicate; they can be defined for any of the super-predicates separately. The following rule captures also the last statement.

– Somewhere here, the inheritance should be noted.
– Inheritance should be treated in knowledge graphs!
– Predicates inherit in the same way as the classes.

$$\frac{t \in \mathcal{T}_i, t = (s,p,o) \quad p_1, p_2 \in \mathcal{I}_p \quad p \preceq p_1 \quad p \preceq p_2 \quad (p_1, \text{domain}, T_s) \in g \quad (p_2, \text{range}, T_o) \in g}{t :_{\uparrow} (T_s, p, T_o)} \tag{27}$$

– Explanation of the rule.
– If p inherits from multiple $p' \succeq p$, then the above rule generates multiple types. Explain.
– Note that the type is determined only if the domain and range of p or some $p' \succeq p$ is defined.
– Otherwise, the domain and range shoud be $\top$. This should be included.

The following rule is a judgment for a (user-defined) type of a concrete triple $t = (s,p,o)$. A user-defined type of t is the greatest lower bound (abbr. glb) of stored types generated by the rule 26.

– Valid stored types of t: the smallest valid glb types of all stored types.
– Justification: smallest interpretation - smallest search space for queries.
– Valid stored types are solely those defined "above" p.
– The glb types of valid stored types "above" p is selected!
– The rule generates one glb type by one.
– These (glb types) are collected in a $\vee$-type including all GLB types.

*– The meaning of $\not\sim$ is "*not related*".*
– This can be either that we have two p roots with unrelated glb schema triples (trees up).
– Or, two p-rooted but unrelated stored types through multiple inheritance.
– Therefore, we can have more than one stored GLB types.

$$\frac{t \in \mathcal{T}_i \quad T_i \in \mathcal{T}_t,\, t :_{\uparrow} T_i \quad \forall S \in \mathcal{T}_t,\, t :_{\uparrow} S \quad T_i \preceq S \vee T_i \not\sim S}{t :_{\sqcap} T_i} \tag{28}$$

The first premise says that t is a ground triple. The second premise enumerates stored types T_i of t. The third premise requires that T_i is the most specific type of all possible types S of t. In other words, there is no type S of t that is a subtype of T_i. Hence, T_i is the glb type of the stored types of t.

The implementation view of the above rule is as follows. The schema triples are obtained from the inherited values of the predicates rdfs:domain and rdfs:range. The inherited values have to be the closest when traveling from property p towards the more general properties.

The glb types are now gathered in a $\vee$-type. Hence, the resulting $\vee$-type includes all glb types of t.

$$\frac{\forall T_i \in \mathcal{T}_t,\, t :_{\sqcap} T_i}{t :_{\sqcap} \vee[T_i]} \tag{29}$$

The premise says that we identify all triple types T_i that are the individual (glb) $\sqcap$-types of t.

– What is the reason that we have multiple glb types?

Multiple different stored types of t are possible only in the case of multiple inheritance, in the case of the definition of the disjunctive domain/range types, or if predicate is defined for semantically different concepts.
– Describe each possibility in more detail.

4.4 Typing a triple

– Two ways of defining semantics.
– 1) enumeration style: stored types are enumerated as alternatives ($\vee$).
– 2) packed together: alternative types are packed in one $\bigvee$ type.
– One advantage of (1) is that individual glb types can be processed further individually.
– Advantage of (2) is the higer-level semantics without going in implementation.

– Now stored types have to be related to all lub base types to represent the correct type of a triple.
– It seems it would be easier to check the pairs one-by-one using (1) in algorithms.
– In case of using complete types in the phases, types would further have to be processed by $\wedge, \vee$ rules.

The type of a triple $t = (s, p, o)$ is computed by first deriving the base type T and the top type S of t. Then, we check if S is reachable from T through the sub-class and sub-property hierarchies, i.e., $T \preceq S$.

$$\frac{t \in \mathcal{T}_i \quad T \in \mathcal{T}_t,\, t :_{\downarrow} T \quad S \in \mathcal{T}_t,\, t :_{\uparrow} S \quad T \preceq S}{t : S} \tag{30}$$

– How to compute $T \preceq S$? Refer to position where we have a description.
– Order the possible derivations, gatherings (groupings) ... of types.

– Possible diagnoses.
– Components not related to a top type of a triple?
– Components related to sub-types of a top type?
– Above pertain to all components.

4.5 Typing a graph

– What is a type of a graph?
– A type of a graph is a graph!
– It includes a set of schema triples forming a schema graph.

– Typing a graph bottom-up?
– Checking that all the triples are of correct types.

4.6 Typing schema triples

– *What can be checked?*
– *Is a schema triple properly related to the super-classes and types of components.*
– *Consistency of the placement of a class in an ontology. What is this?*
– *A class or predicate component not related to other classes?*
– *A class or predicate component attached to "conflicting" set of classes? What can be detected?*
– *Any other examples?*
–

5 Implementation of type-checking

5.1 Computing LUB ground types

– *The ground types of a triple* $t = (s, p, o)$ *are computed first as a type* (s_t, p, o_t) *such that* $(s, rdf : type, s_t)$ *and* $(o, rdf : type, o_t)$.
– *The set of all types* s_t *(*o_t*) is stored in a set* g_s *(*g_o*).*
– *The ground type of t is then* $T_t = (\wedge g_s, p, \wedge g_o)$.
– *The lub of a type* T_t *is computed as follows.*

– *For each* $s_t \in g_s$ *(*$o_t \in g_o$*) compute a closure of a set* $\{s_t\}$ *(*$\{o_t\}$*) with respect to the relationship rdfs:superClassOf obtaining the sets* c_s *and* c_o.
– *Each step of the closure newly obtained classes are marked with the number of steps if new in the set, and the maximum of both numbers of steps otherwise.*
– *The maximum guaranties the monotonicity:* $s_t \prec_\downarrow s'_t \Rightarrow m(s_t) > m(s'_t)$.
– *Proof: Assume* $m(s_t) <= m(s'_t)$. *Since* $s_t \prec_\downarrow s'_t$ *then* $m(s'_t) + 1$ *is maximum of* s_t. *Contradiction.*

– *The lub of* T_t *is computed by intersecting all sets* c_s *(*c_o*) obtained for each* s_t *(*o_t*), yielding a lub type of* g_s *(*g_o*).*
– *The intersection of sets is computed step-by-step. Initially, intersection is the first set* c_s *(*c_o*) of some* s_t *(*o_t*).*
– *In each step, a new set* c_s *(*c_o*) for another* s_t *(*o_t*) is intersected with previous result.*
– *The classes that are in the result (intersection) are merged by selecting the maximum number of closure steps for each class in the intersection.*
– *The reason for this is to obtain the number of steps within which all ground classes reach a given lub class.*

– *Finally, the lub classes are selected from the resulted intersection of all* c_s *(*c_o*).*
– *The class with the smallest number of steps is taken. Then, it is deleted from the set together with all its super-classes.*
– *If set is not empty the previous step is repeated.*

5.2 Computing GLB stored types

– *We have a tuple* $t = (s, p, o)$.
– *The task is to compute glb of* t*'s stored types.*
– *The algorithm is using the predicates* $p' \preceq p$ *to obtain all domains and ranges of* p.

– *FUNCTION typeGlbStored* (p: *Property*, cnt: *Integer*)*: Type*
– *BEGIN*
– *if* $d_p = \{\})$ *then* $d_p = \{c_s \mid (p, \textit{rdfs:Domain}, c_s) \in G\}$
– *if* $r_p = \{\})$ *then* $r_p = \{c_o \mid (p, \textit{rdfs:Range}, c_o) \in G\}$
– *if* $d_p \neq \{\} \wedge r_p \neq \{\})$ *then*
– *RETURN* $(\vee d_p, p, \vee r_p)$
– $ts = \{\}$
– *for* p' $((p, \textit{rdfs SubPropertyOf}, p') \in G)$
– *BEGIN*
– $T_{p'} = typeGlbStored(p', cnt + 1)$
– $ts = ts \cup \{T_{p'}\}$
– *END*
– *RETURN* $\vee ts$
– *END*

5.3 Relating LUB ground and GLB stored types

6 Conclusions

References

1. V. Bono and M. Dezani-Ciancaglini. A tale of intersection types. In *Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science*, LICS '20, page 7–20, New York, NY, USA, 2020. Association for Computing Machinery.
2. L. Ehrlinger and W. Wöß. Towards a definition of knowledge graphs. In *SEMANTiCS*, 2016.
3. J. R. Hindley. *Basic simple type theory*. Cambridge University Press, USA, 1997.
4. A. Hogan, E. Blomqvist, M. Cochez, C. D'amato, G. D. Melo, C. Gutierrez, S. Kirrane, J. E. L. Gayo, R. Navigli, S. Neumaier, A.-C. N. Ngomo, A. Polleres, S. M. Rashid, A. Rula, L. Schmelzeisen, J. Sequeda, S. Staab, and A. Zimmermann. Knowledge graphs. *ACM Comput. Surv.*, 54(4), jul 2021.
5. B. C. Pierce. Programming with intersection types, union types, and polymorphism, 1991.
6. B. C. Pierce. Intersection types and bounded polymorphism, 1996.
7. B. C. Pierce. *Types and Programming Languages*. MIT Press, 1 edition, Feb. 2002.
8. Resource description framework (rdf). http://www.w3.org/RDF/, 2004.
9. Rdf schema. http://www.w3.org/TR/rdf-schema/, 2004.