Type-checking knowledge graphs

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Abstract. We first present a formal view of a knowledge graph. On this basis, the type-checking rules are developed to define correct typing relationships among the triples of a knowledge graph. We discuss the algorithms for verifying the typing relationships against the given knowledge graph. Finally, we present the experimental results of type-checking the Yago4 knowledge graph.

Keywords: RDF stores · graph databases · knowledge graphs · database statistics · statistics of graph databases.

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1 Introduction

This is intro... [1].
– Topic: KGs are becoming KBs...

– Topic: Give an abstract insight into the structure of KB. Identifiers, schema, types, ...
– Topic: Show the ground triples, poset of triples.types triples, schema graph, etc.

2 Definition of knowledge graph

This section defines a knowledge graph as a RDF graph [6] using RDF-Schema [7] for the representation of the structural part of a knowledge base.

Let I be the set of URI-s, B be the set of blanks and L be the set of literals. Let us also define sets $S = I \cup B$, $P = I$, and $O = I \cup B \cup L$.

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RDF triple is a triple $(s, p, o) \in S \times P \times O$. *RDF graph* $g \subseteq S \times P \times O$ is a set of triples. Set of all graphs will be denoted as G. We suppose the existence of a set of variables V and the set of *terms* $T = O \cup V$. Term $t \in T$ is ground if $t \in O$.

We say that RDF graph g_1 is *sub-graph* of g_2, denoted $g_1 \sqsubseteq g_2$, if all triples in g_1 are also triples from g_2.
– Define major structure of KG on the basis of the sorts of data.
– ...the set I includes individual identifiers I_i, class identifiers I_c and predicate identifiers I_p.

3 Type system used

3.1 Product types

3.2 Intersection type

The instances of the intersection type $T_1 \wedge T_2$ are objects belonging to both T_1 and T_2. The type $T_1 \wedge T_2$ is the greatest lower bound of the types T_1 and T_2. In general, $\wedge[T_1 \dots T_n]$ is the greatest lower bound of types $T_1 \dots T_n$ [3, 4].

$$T_1 \wedge T_2 \preceq T_1 \tag{1}$$

$$T_1 \wedge T_2 \preceq T_2 \tag{2}$$

$$\wedge[T_1 \dots T_n] \preceq T_i \tag{3}$$

If the type S is more specific than the types $T_1 \dots T_n$ then S is more specific then $\wedge[T_1 \dots T_n]$. First, we present the rule for a pair of types T_1 and T_2.

$$\frac{S \preceq T_1 \quad S \preceq T_2}{S \preceq T_1 \wedge T_2} \tag{4}$$

$$\frac{\text{forall i, } S \preceq T_i}{S \preceq \wedge[T_1 \dots T_n]} \tag{5}$$

3.3 Union type

The intersection and union types are dual. This can be seen also from the rules that are used for each particular type.

The instances from the union type $T_1 \vee T_2$ are either the instances of T_1 or T_2, or the instances of both types. The type $T_1 \vee T_2$ is the smallest upper bound of the types T_1 and T_2. In general, $\vee[T_1 \dots T_n]$ is the smallest upper bound of types $T_1 \dots T_n$ [2].

$$T_1 \preceq T_1 \vee T_2 \tag{6}$$

$$T_2 \preceq T_1 \vee T_2 \tag{7}$$

$$T_i \preceq \vee[T_1 \dots T_n] \tag{8}$$

If the type T is more general than the types $S_1 \dots S_n$ then T is more general then $\vee[S_1 \dots S_n]$. First, we present the rule for types T_1 and T_2.

$$\frac{S_1 \preceq T \quad S_2 \preceq T}{S_1 \vee S_2 \preceq T} \tag{9}$$

$$\frac{\text{forall i, } S_i \preceq T}{\vee[S_1 \dots S_n] \preceq T} \tag{10}$$

4 Typing identifiers

– Introduction includes the formalization of RDF, RDF-Schema as given in Angles and Peres.
– Typing idents without considering and info about the triples.

– General.
– At the end of section define the lub type using $\wedge\vee$ types.
– 1. Define lub types as the closest to base types of given ground ident.
– 2. Collect all lub types using $\wedge$ type in a single type.

– Details.
– 1. First define base type of identifiers $:_1$ and stored subtyping relationship $\preceq_1$.
– 2. From the basis define the indent typing $:$ and subtyping rel $\preceq$ among identifiers.
– 3. Include the link between subtyping and typing.
– 4. Define lub type using $\wedge$ type for a given ground ident.

4.1 Typing literals

– Literals are identifiers of atomic type!

4.2 Stored typing and subtyping of identifiers.

The individual and class entities are represented by identifiers from the set $\mathcal{I}$. The individual identifiers $\mathcal{I}_i$ stand for literals, concrete and abstract entities. The class identifiers $\mathcal{I}_c$ represent abstract entities that have an unempty interpretation. The abstract entities include, besides the identifiers of user-defined classes, the types (classes) of literals.

A graph database includes stored definitions for typing the individual identifiers, and for representing the specialization/generalization hierarchies of classes and properties.

Let us introduce the typing and specialization/generalization relationships formally. The expression $i :_{\downarrow} C$ states that a class C is a type of an individual identifier i. The expression $i_1 \preceq_{\downarrow} i_2$ defines the sub-class relationship between the class identifiers i_1 and i_2. The index '$\downarrow$' in relations $:_{\downarrow}$ and $\preceq_{\downarrow}$ denotes that the relationships is stored in a database. Such notation allows us to address differently the stored and the derived parts of the graph database schema.

The rule for the one-step relationship $:_{\downarrow}$ is defined using the predicate rdf:type.

$$\frac{I \in \mathcal{I}_i \quad I_c \in \mathcal{I}_c \quad (I, \text{rdf:type}, I_c) \in \mathcal{D}}{I :_{\downarrow} I_c} \tag{11}$$

The individual entity I can have more than one stored types, e.g., I_{c1} and I_{c2}. Therefore, $I :_{\downarrow} I_{c1}$ and $I :_{\downarrow} I_{c2}$ holds, and we can instead write $I :_{\downarrow} I_{c1} \wedge I_{c2}$. All existing stored typings of I can be gathered by Rule 22 presented later.

A one-step subtyping relationship $\preceq_{\downarrow}$ is defined by means of the RDF predicate rdfs:subClassOf in the following rule.

$$\frac{I_1, I_2 \in \mathcal{I}_c \quad (I_1, \text{rdfs:subClassOf}, I_2) \in \mathcal{D}}{I_1 \preceq_{\downarrow} I_2} \tag{12}$$

The rule for the definition of the one-step subtyping relationship $\preceq_{\downarrow}$ is based on the predicate rdfs:subPropertyOf.

$$\frac{I_1, I_2 \in \mathcal{I}_p \quad (I_1, \text{rdfs:subPropertyOf}, I_2) \in \mathcal{D}}{I_1 \preceq_{\downarrow} I_2} \tag{13}$$

4.3 Typing and subtyping identifiers.

The one-step relationships $:_{\downarrow}$ and $\preceq_{\downarrow}$ are now extended with the reflexivity and transitivity to obtain the relationships : and $\preceq$. The relation $\preceq$ forms a partial ordering of class identifiers.

First, the one-step relationship $\preceq_s$ is generalized to the relationship $\preceq$ defined over class identifiers $\mathcal{I}_c$.

$$\frac{I_1, I_2 \in \mathcal{I}_c \quad I_1 \preceq_{\downarrow} I_2}{I_1 \preceq I_2} \tag{14}$$

Next, the subtyping relationship $\preceq$ is reflexive.

$$\frac{S \in \mathcal{I}_c}{S \preceq S} \tag{15}$$

The subtype relationship is also transitive.

$$\frac{S, U, T \in \mathcal{I}_c \quad S \preceq U \quad U \preceq T}{S \preceq T} \tag{16}$$

Finally, the subtype relationship is asymmetric which is expressed using the following rule.

$$\frac{S, U \in \mathcal{I}_c \quad S \preceq U \quad U \preceq S}{S = T} \tag{17}$$

As a consequence of the rules 15-17 the relation $\preceq$ is a poset.

Knowledge graphs include a special class $\top$ that represents the root class of the ontology. In RDF ontologies $\top$ is usually represented by the predicate owl:Thing. The following rule specifys that all class identifiers are more specific than $\top$.

$$S \preceq \top \tag{18}$$

The stored typing relation $:_s$ is now extended to the typing relation : that takes into account the subtyping relation $\preceq$. The following rule states that a stored type is a type.

$$\frac{I \in \mathcal{I}_i \quad C \in \mathcal{I}_c \quad I :_s C}{I : C} \tag{19}$$

The link between the typing relation and subtype relation is provided by adding a typing rule called *rule of subsumption* [5].

$$\frac{I \in \mathcal{I}_i \quad I : S \quad S \preceq T}{I : T} \tag{20}$$

– Properties have dual role: they are instances and types at the same time.

– Present the features of properties from this point of view.

5 Typing triples

– There are two basic aspects of a triple type.
– First, the type is computed bottom-up: from the stored types of triple components.
– Second, the type can be computed top-down: from the user-defined domain/range types of properties.

– Ground type of a triple is computed first using $:_1$.
– Next, the lub type of a triple is derived using $:_l$.
– From the top side of the ontology, the stored type $:_s$ *is determined based on* p.
– Finally, the type $:$ *of* t *is deterimned by summing alternative* $:_s$ *types.*

– Interactions between the $\wedge/\vee$ *types of triple components and triples must be added.*
– Analogy between. the types of functions in lambda calculus and types of triples.
– Show rules relating $\wedge/\vee$ *types and triple types. Example.*
– E.g., $(S_1 \wedge S_2) * p * R = S_1 * p * R \wedge S_2 * p * R$.

– Predicates should be treated in the same way as the classes.
– They can have a rich hierarchy.
– Note: *Where to include discussion on special role of predicates and their relations to classes?*
– Mention Cyc as the practical KB with rich hierarchy of predicates.

5.1 Deriving a ground type of a triple.

A ground type of an individual identifier i is a class C related to i by one-step type relationship $:_\downarrow$ denoting a ground type. In terms of the concepts of a knowledge graph, C and i are related by the relationship rdf:type.

A ground type of a triple $t = (s, p, o)$ is a triple $T = T_s * p * T_o$ that includes the ground types of t's components s and o, and the property p which now has the role of a type. A ground type of a triple is defined by the following rule.

$$\frac{t \in \mathcal{T}_i,\ t = (s, p, o) \quad I_s, I_o \in \mathcal{I}_c \quad s :_\downarrow I_s \quad p :_\downarrow \text{owl:ObjectProperty} \quad o :_\downarrow I_o}{t :_\downarrow I_s * p * I_o} \tag{21}$$

The class I_s is one of the ground types of s, and the type I_o is one of the ground types of o. The predicates are treated differently to the subject and object components of triples. The predicates have the role of classes while they are instances of owl:ObjectProperty.

There can be multiple ground types of a triple. They may be gathered into a single $\wedge$-type by using the following rule. The types T_1,...,T_n are obtained using Rule 21.

$$\frac{t \in \mathcal{T}_i \quad \forall i \in [1, n], T_i \in \mathcal{T}_t \quad t :_\downarrow T_i}{t :_\downarrow \wedge[T_i]} \tag{22}$$

5.2 Deriving a lub type of a triple

– Typing using lub types of $T_1..T_n$. Explain why this is needed?

Let us now define the least upper bound types (abbr. *lub*) of ground types derived by Rule 22. Since a partialy ordered set is not a lattice, we can have more than one lub type for a given set of ground types.

The lub types of a given list of triple types $T_1..T_n$ are computed in two steps as before when gathering multiple ground types with conjunction. A single lub type is defined as follows.

$$\frac{t :_{\downarrow} \wedge[T_1..T_n] \quad T, S \in \mathcal{T}_t \quad \forall i,\ T_i \preceq T \quad \forall j, T_j \preceq S \quad T \preceq S}{\vdash t :_{\sqcup} T} \tag{23}$$

The above rule states that a lub type T is a ground type $\wedge[T_1..T_n]$ if all ground types T_i are subtypes of T. Furthermore, the lub type T is the least (closest) supertype of all members of ground $\wedge$-type $T_1, \ldots, T_n$. The lub types can be now gathered using the following rule.

$$\frac{\forall i, T_i \in \mathcal{T}_t \quad t :_{\sqcup} T_i}{t :_{\sqcup} \wedge[T_1..T_n]} \tag{24}$$

5.3 Stored types of triples.

– Make a bit more clear picture of the GLB triple types selected as the final stored type of a triple.
– Analysis tool. Show minimality of the stored types (either enumerated or gathered with $\bigvee$).
– Reminder: *when a complete stored (user-defined) type is related to the base type of a triple, some of GLB types may be eliminated.*

We first find stored triple types for a given triple $t = (s, p, o)$. A stored schema triple is constructed by selecting types including a predicate that has the domain and range defined.

$$\frac{t \in \mathcal{T}_i, t = (s, p, o) \quad p' \in \mathcal{I}_p, p \preceq p' \quad (p', \text{domain}, T_s) \in g \quad (p', \text{range}, T_o) \in g}{t :_{\uparrow} (T_s, p, T_o)} \tag{25}$$

– Comments and description of the above rule.

The domain and range of a predicate p can be defined for any super-predicate, they do not need to be defined for p. In addition, the domain and range of a predicates do not need to be defined for the same predicate; they can be defined for any of the super-predicates separately. The following rule captures also the last statement.
– Initially, this definition can be skipped!

$$\frac{t \in \mathcal{T}_i, t = (s,p,o) \quad p_1, p_2 \in \mathcal{I}_p \quad p \preceq p_1 \quad p \preceq p_2 \quad (p_1, \text{domain}, T_s) \in g \quad (p_2, \text{range}, T_o) \in g}{t :_\uparrow (T_s, p, T_o)} \tag{26}$$

– Explanation of the rule.
– If p inherits from multiple $p' \succeq p$, then the above rule generates multiple types. Explain.

– Note that the type is determined only if the domain and range of p or some $p' \succeq p$ is defined.
– Otherwise, the domain and range shoud be $\top$.

The following rule is a judgment for a (user-defined) type of a concrete triple $t = (s, p, o)$. A user-defined type of t is the greatest lower bound of stored types generated by the rule 25.

– A general explanation for choosing glb type should be given. Further expanded below. (?)
– Explanation: Among stored types of t: select the smallest GLB types that have the smallest interpretations.
– Therefore, smallest search space for queries.

– The meaning of $\not\sim$ is not related.
– This can be either that we have two p roots with unrelated glb schema triples (trees up).
– Or, two p-rooted but unrelated stored types through multiple inheritance.
– Therefore, we can have more than one stored GLB types.

$$\frac{\forall(T_i \in \mathcal{T}_t), t :_\uparrow T_i \quad \forall(t :_\uparrow S), T \preceq S \vee T \not\sim S}{t :_\sqcap \vee[T_i]} \tag{27}$$

The first premise says that t is an individual triple. The second premise determines one of the stored types T of t. The third premise requires that type T is the least general type of a (reverse) tree above the property p. Note that there may be more than one tree rooted in unrelated stored types above p.

The implementation view of the above rule is as follows. The schema triples are obtained from the inherited values of the predicates rdfs:domain and rdfs:range. The inherited values have to be the closest when traveling from property p towards the more general properties.

– Why do we have multixsple glb types? Add explanation.
– (The following paragraph is not completed! More precise!)

Multiple different schema triples for a given t are possible only in the case of multiple inheritance, in the case of the definition of the disjunctive domain/range types, or if predicate is defined for semantically different concepts. (describe each possibility in more detail.)

– Add explanation about why glb types are connected with $\vee$.
– $\vee$ is used since they represent alternative semantics of a triple.

– When type-checking a language, a context can be used to resolve disjunctions.

Now we can put together the greatest lower bounds of the top types by making the union of the glb types computed by the previous rule.

$$\frac{t \in \mathcal{T}_i \quad \forall i \in [1..n], t :_{glb} T_i}{t :_u \bigvee T_i} \tag{28}$$

– Expand on the influence/effect of using $\bigvee$ type in context and in other rules.

5.4 Typing a triple.

– Two ways of defining semantics.
– 1) enumeration style: stored types are enumerated as alternatives ($\vee$).
– 2) packed together: alternative types are packed in one $\bigvee$ type.
– One advantage of (1) is that individual glb types can be processed further individually.
– Advantage of (2) is the higer-level semantics without going in implementation.

– Now stored types have to be related to all lub base types to represent the correct type of a triple.
– It seems it would be easier to check the pairs one-by-one using (1) in algorithms.
– In case of using complete types in the phases, types would further have to be processed by $\wedge, \vee$ rules.

The type of a triple $t = (s, p, o)$ is computed by first deriving the base type T and the top type S of t. Then, we check if S is reachable from T through the sub-class and sub-property hierarchies, i.e., $T \preceq S$.

$$\frac{(s, p, o) :_1 T \quad (s, p, o) :_2 S \quad (T \preceq S)}{(s, p, o) : S} \tag{29}$$

– How to compute $T \preceq S$? Refer to position where we have a description.
– Order the possible derivations, gatherings (groupings) ... of types.

– Possible diagnoses.
– Components not related to a top type of a triple?
– Components related to sub-types of a top type?
– Above pertain to all components.

6 Typing a graph.

– What is a type of a graph?
– A type of a graph is a graph!
– It includes a set of schema triples forming a schema graph.

– Typing a graph bottom-up?
– Checking that all the triples are of correct types.

6.1 Typing a schema triple.

– What can be checked?
– Is a schema triple properly related to the super-classes and types of components.
– Consistency of the placement of a class in an ontology. What is this?
– A class or predicate component not related to other classes?
– A class or predicate component attached to "conflicting" set of classes? What can be detected?
– @kiyoshi Do you see any other examples?
–

7 Empirical analysis

8 Conclusions

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