Algorithms for Big Data

Spring Semester 2022 Exercise Set 13

Exercise 1:

Show that k-spanner of a graph is in fact k-coreset for distance queries. That is, if G' is k-spanner of G, then for any H and any u, v there is $d_{G \cup H} \leq d_{G' \cup H}(u, v) \leq k \cdot d_{G \cup H}(u, v)$.

Exercise 2:

Show that k-spanners have disjoint-merge property.

Exercise 3:

Show construction of 2t-spanner of size $\mathcal{O}(n^{1+\frac{2}{t-1}}\log W)$ for weighted graphs with integer edge weights from $\{1,\ldots,W\}$.

Exercise 4:

Show a graph-certificate for 2-edge connectivity. (G is 2-edge connected if any pair of vertices are connected by at least two edge-disjoint paths.)

Exercise 5: (No sparse directed spanners) Show that in a directed case, for some digraphs it is impossible to construct better than $\Omega(n^2)$ -size directed spanners.