

**Algorithms for Big Data**

Spring Semester 2022

## Exercise Set 7

**Exercise 1:**

Minimize the value of  $\sum_i \frac{\alpha_i^2}{p_i}$  conditioned on  $\sum_i p_i = 1$  and  $\forall_i p_i \geq 0$ , for some nonnegative  $\alpha_i$ . Find values of  $p_i$  that realize this minimum.

**Exercise 2:**

(2+1 pts)

Show existence of  $\varepsilon$ -net of  $S_{n-1}$  with size  $c = (\mathcal{O}(1/\varepsilon))^n$ . Hint: analyze greedy algorithm, and show that it induces packing of  $c$  disjoint balls of radius  $\varepsilon/2$  in ball of radius  $1 + \varepsilon/2$ . Bonus points for showing why a certain explicit geometric construction is not sufficient and produces suboptimal size.

**Exercise 3:**

(1+2+1 pts)

Show following identities:

- $\|A\|_F = \sqrt{\text{trace}(A \cdot A^T)}$
- Sub-multiplicativity:  $\|AB\|_F \leq \|A\|_F \|B\|_F$  [use Cauchy-Schwartz]
- For orthonormal  $U$ , there is  $\|UA\|_F = \|A\|_F$ .