

Algorithms for Big Data

Spring Semester 2022

Exercise Set 7

Exercise 1:

Minimize the value of $\sum_i \frac{a_i^2}{p_i}$ conditioned on $\sum_i p_i = 1$ and $\forall_i p_i \geq 0$. Find values of p_i that realize this minimum.

Exercise 2:

(2+1 pts)

Show existence of ε -net of S_{n-1} with size $c = (\mathcal{O}(1/\varepsilon))^n$. Hint: analyze greedy algorithm, and show that it induces packing of c disjoint balls of radius $\varepsilon/2$ in ball of radius $1 + \varepsilon/2$. Bonus points for showing why a certain explicit geometric construction is not sufficient and produces suboptimal size.

Exercise 3:

(1+2+1 pts)

Show following identities:

- $\|A\|_F = \sqrt{\text{trace}(A \cdot A^T)}$
- Sub-multiplicativity: $\|AB\|_F \leq \|A\|_F \|B\|_F$ [use Cauchy-Schwartz]
- For orthonormal U , there is $\|UA\|_F = \|A\|_F$.