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Algorithms for Big Data

Spring Semester 2022

Exercise Set 12

Exercise 1:
Show $\Omega(n)$ lower bound for number of vertices algorithm has to "touch" to solve 2-multiplicative approximation for estimating number of connected components.

Exercise 2:
Let $G_1, G_2, \ldots, G_w$ be unweighted graphs such that: $e \in G_i$ iff $w(e) \leq i$. Denote by K_i the number of connected components of G_i. Show that weight of MST satisfies

$$w(\mathrm{MST}) = (n-1) + \sum_{i=1}^{w-1} (K_i - 1).$$

Exercise 3:
Why does rounding weights of the input graph to nearest full power of $(1 \pm \varepsilon)$ does not provide any significant speed-up for cell-probe MST algorithm? Specifically, imagine we would use a following (up to rounding):

$$w(\mathrm{MST}) \approx (n-1) + \sum_{j=0}^{\log_{1+\varepsilon}(w)} (K_{(1+\varepsilon)^j} - 1)\Big((1+\varepsilon)^{j+1} - (1+\varepsilon)^j\Big).$$

What would be the complexity of algorithm using this formula?