

Algorithms for Big Data

Spring Semester 2022

Exercise Set 9

Recall Hadamard transform, given by a matrix H , such that $H_{i,j} = \frac{1}{\sqrt{n}} \cdot (-1)^{\text{bc}(i \& j)}$, where $i \& j$ is bit-wise AND of binary representations, and $\text{bc}(x)$ returns number of 1's in binary representation. Remember: $H = H^{-1}$, and assume n is power of two.

Exercise 1:

Recall the tests for bits of u in algorithm for $k = 1$ of Fourier transform:

$$b_i = 0 \quad \text{iff} \quad |a_r - a_{r+n/2^{i+1}}| \leq |a_r + a_{r+n/2^{i+1}}|$$

where r is randomly picked. Design analogous test for Hadamard transform.

Exercise 2:

(2 pts)

Let $(\hat{a}_0, \dots, \hat{a}_{n-1})$ be a Hadamard transform of (a_0, \dots, a_{n-1}) . Let $m \leq n$ be power of two as well. Let (b_0, \dots, b_{n-1}) be a sequence such that for any $0 \leq i < n/m$, $(b_{im}, b_{im+1}, \dots, b_{im+m-1})$ is a Hadamard transform of $(a_{im}, a_{im+1}, \dots, a_{im+m-1})$.

Show that for any $0 \leq j < m$, $(b_j, b_{m+j}, b_{2m+j}, \dots, b_{n-m+j})$ is a Hadamard transform of $(\hat{a}_j, \hat{a}_{m+j}, \hat{a}_{2m+j}, \dots, \hat{a}_{n-m+j})$. (Keep in mind those transforms are of smaller dimension.)

Exercise 3:

(2 pts)

Using previous exercise, design sparse Hadamard transform algorithm (it's almost 1-1 equivalent to the one from the lecture).