

**Algorithms for Big Data**

Spring Semester 2022

**Exercise Set 10**

Below we assume that  $k \ll m \ll n$  to avoid annoying border-cases.

**Exercise 1:**

Show that any  $k$ -disjoint set family separates  $I_1, I_2$  such that  $I_1 \neq I_2$  and  $|I_2|$  can be arbitrarily large, while  $|I_1| \leq k$ .

**Exercise 2:**

Describe a decoding procedure for  $k$ -disjoint set family: given  $\bigcup_{i \in I} F_i$ , output  $I$  if  $|I| \leq k$ , or output that it is not the case.

**Exercise 3:**

Let  $A$  be a  $k$ -disjoint matrix (columns describe  $k$ -disjoint set family). Show a decoding procedure, which for a given  $Ax$  outputs  $x$  if  $x$  is  $k$ -sparse, and otherwise outputs that it is not the case. Assume  $x \geq 0$ .

**Exercise 4:**

(2 pts)

Assume  $k$ -disjoint family which has arbitrarily slow decoding procedure. Show that there exists  $k$ -separable family with  $m' = \mathcal{O}(m \log n)$ , and decoding time  $\text{poly}(m, k, \log n)$ .