$$E_{n,l} = \hbar\omega\left(n+\frac{1}{2}\right) + \frac{\hbar^2}{2I}l(l+1) + \alpha l(l+1)\left(n+\frac{1}{2}\right)$$

Bibraziva Birakета akoplamendua

$E_{bib}(n)$ $E_{bir}(l)$ $E_{b-b}(l,n)$

$$\left[\hbar\omega \gg \frac{\hbar^2}{2I} \gg \alpha\right] \frac{1}{K_B}$$

$$\theta_{bib} \qquad \theta_{bir} \qquad \theta_{bb}$$

$$> T \quad \gg \quad \gg$$

$$\theta_{bib} > T \ggg \theta_{bir} \ggg \theta_{bb}$$

$\langle E \rangle \equiv U \leftarrow F$-tik lortu

$$\boxed{F = -(K_B T)\,\mathrm{Ln}\left[Z_N(T,V)\right]}$$

$$Z_N(T,V) = \frac{1}{N!}\left[z_1(T,V)\right]^N$$

$$z_1(T,V) = \prod_j z_1^j(T,V) \qquad j \begin{cases} \text{translazioak} \\ \text{barnekoak} \end{cases}$$

$$Z_N(T,V) = \frac{1}{N!}\left[z_1^{tran}(T,V)\cdot z_1^{barn}(T,V)\right]^N$$

$$= \underbrace{\frac{1}{N!}\left[z_1^{tra}(T,V)\right]^N}\cdot\left[z_1^{bar}(T,V)\right]^N$$

$$\boxed{Z_N(T,V) = \frac{V}{\lambda_T^3}\cdot\left[z_1^{bar}(T,V)\right]^N}$$

$$z_1^{barne}(T,V) =$$

$$Z_1(T,V) = \sum_{\text{egoera}} e^{-\epsilon/k_BT}$$

→ partikula bakoitzaren energia-adierazteko beharrak

barnekoa

$$Z_1(T,V) = \sum_{\text{egoera}} \underset{\uparrow g(\epsilon)}{} e^{-\frac{1}{k_BT}\left[E_{bib} + E_{bir} + E_{bb}\right]} \longrightarrow (n,\ell)$$

$$\boxed{Z_1^{bar}(T,V) = \sum_{n=0}^{\infty}\sum_{\ell=0}^{\infty}(2\ell+1)\, e^{-\ell(\ell+1)\frac{\Theta_{bir}}{T}}\, e^{-(n+\frac{1}{2})\frac{\Theta_{bib}}{T}}\, e^{-\ell(\ell+1)(n+\frac{1}{2})\frac{\Theta_{bb}}{T}}}$$

$$\underbrace{\sum_{\ell=0}^{\infty}(2\ell+1)\, e^{-\ell(\ell+1)\frac{\Theta_{bir}}{T}}\, e^{-\ell(\ell+1)(n+\frac{1}{2})\frac{\Theta_{bb}}{T}}}$$

+ hipotesia
Baldintza espe

$$\sum_{\ell=0}^{\infty}(2\ell+1)\, e^{-\frac{\ell(\ell+1)}{T}\left[\Theta_{bir} + (n+\frac{1}{2})\Theta_{bb}\right]} \approx \frac{T}{\left[\Theta_{bir} + (n+\frac{1}{2})\Theta_{bb}\right]}$$

$$\boxed{Z_1^{bar}(T,V) = \sum_{n=0}^{\infty}\frac{T}{\left[\Theta_{bir} + (n+\frac{1}{2})\Theta_{bb}\right]}\cdot e^{-(n+\frac{1}{2})\frac{\Theta_{bib}}{T}}}$$

$$\frac{1}{\Theta_{bir} + (n+\frac{1}{2})\Theta_{bb}} = \frac{1}{\Theta_{bir}}\,\frac{1}{\left[1 + (n+\frac{1}{2})\frac{\Theta_{bb}}{\Theta_{bir}}\right]} \approx \frac{1}{\Theta_{bir}}\left[1 - (n+\tfrac{1}{2})\frac{\Theta_{bb}}{\Theta_{bir}}\right]$$

$$\frac{1}{1+x} \underset{x\to 0}{\approx} 1 - x$$

$$\boxed{Z_1^{bar}(T,V) = \sum_{n=0}^{\infty}\frac{T}{\Theta_{bir}}\, e^{-(n+\frac{1}{2})\frac{\Theta_{bib}}{T}} - \frac{T}{\Theta_{bir}}\sum_{n=0}^{\infty}\left(n+\tfrac{1}{2}\right)\frac{\Theta_{bb}}{\Theta_{bir}}\, e^{-(n+\frac{1}{2})\frac{\Theta_{bib}}{T}}}$$

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$$-\sum_{n=0}^{\infty} \frac{T}{\theta_{bir}} \left(n+\frac{1}{2}\right) \frac{\theta_{bb}}{\theta_{bir}} e^{-\left(n+\frac{1}{2}\right)\frac{\theta_{bib}}{T}}$$

$$-\frac{T}{\theta_{bir}} \frac{\theta_{bb}}{\theta_{bir}} \sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right) e^{-\left(n+\frac{1}{2}\right)\frac{\theta_{bib}}{T}}$$

$$\sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right) \frac{\hbar\omega}{\hbar\omega} e^{-\left(n+\frac{1}{2}\right)\frac{\theta_{bib}}{T}}$$

$$-\frac{T}{\theta_{bir}} \frac{\theta_{bb}}{\theta_{bir}} \frac{1}{\hbar\omega} \sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right) \hbar\omega\, e^{-\left(n+\frac{1}{2}\right)\frac{\theta_{bib}}{T}}$$

$$\left[\qquad \right] \frac{Z_{bib}}{Z_{bib}}$$

$$-\frac{T}{\theta_{bir}} \frac{\theta_{bb}}{\theta_{bir}} \frac{1}{\hbar\omega} Z_{bib} \underbrace{\left[\frac{1}{Z_{bib}} \sum_{n=0}^{\infty} \left(n+\frac{1}{2}\right) \hbar\omega\, e^{-\left(n+\frac{1}{2}\right)\frac{\theta_{bib}}{T}} \right]}_{\bar{\epsilon}_{bib}}$$

$$\frac{1}{k_B \frac{\hbar\omega}{k_B}}$$

$$\frac{1}{k_B \theta_{bib}}$$

$$-\frac{T}{\theta_{bir}} \frac{\theta_{bb}}{\theta_{bir}} \frac{Z_{bib}}{k_B \theta_{bib}} \cdot \bar{\epsilon}_{bib}$$

$$\frac{T}{\theta_{bir}} Z_{bib} - \frac{T}{\theta_{bir}} \frac{\theta_{bb}}{\theta_{bir}} \frac{Z_{bib}}{k_B \theta_{bib}} \cdot \bar{\epsilon}_{bib}$$

$$\boxed{\frac{T}{\theta_{bir}} Z_{bib} \left[1 - \frac{\theta_{bb}}{\theta_{bir}} \frac{1}{k_B \theta_{bib}} \bar{\epsilon}_{bib} \right]}$$

$$Z_1(T,V) \approx \frac{V}{\lambda_T^3} \cdot \left[\frac{T}{\theta_{bir}} \cdot Z_{bib} \left\{ 1 - \frac{\theta_{bb}}{\theta_{bir}\theta_{bib}} \cdot \frac{\bar{\epsilon}_{bib}}{k_B} \right\} \right]$$

$$Z_N(T,V) = \frac{1}{N!} \left[\frac{V}{\lambda_T^3} \underbrace{\left[\frac{T}{\theta_{bir}} \cdot Z_{bib} \left\{ 1 - \frac{\theta_{bb}}{\theta_{bir}\theta_{bib}} \cdot \frac{\bar{\epsilon}_{bib}}{k_B} \right\} \right] \right]^N}$$

$$\left(F = -k_B T \ln Z_N(T,V) \right)^{barne}$$

$$F = -(k_B T) N \ln \left[\underbrace{\frac{T}{\theta_{bir}}}_{1} \; \underbrace{Z_{bib}}_{2} \; \underbrace{\left\{ 1 - \frac{\theta_{bb}}{\theta_{bir}\theta_{bib}} \cdot \frac{\bar{\epsilon}_{bib}}{k_B} \right\}}_{3} \right]$$

$$F^1 = -(k_B T) N \ln \left(\frac{T}{\theta_{bir}} \right) \longrightarrow \text{biraketak}$$

$$F^2 = -(k_B T) N \ln \left(Z_{bib} \right) \longrightarrow \text{bibrazioak}$$

$$F^3 = -(k_B T) N \ln \left\{ 1 - \frac{\theta_{bb}}{\theta_{bir}\theta_{bib}} \cdot \frac{\bar{\epsilon}_{bib}}{k_B} \right\}$$

$$\ln(1 - x) \approx -x$$

$$x \lll 1$$

$$+(k_B T) N \left(\frac{\theta_{bb}}{\theta_{bir}\theta_{bib}} \cdot \frac{\bar{\epsilon}_{bib}}{k_B} \right)$$