$\boxed{B09}$ (Hw 2009 6.1)

Universe, Black Body Radiation.

a) A closed system at equilibrium

$$\Delta Q = 0 = T\Delta S \rightarrow \underline{\Delta S = 0}$$

When at equilibrium? When "walls" expand more slowly than characteristic particle speed. "walls" expand as Hubble constant << speed of light.

b) $V_f = 2V$

$$S = \frac{U+PV}{T} = \frac{4}{3}\frac{U}{T} \propto VT^3 = \text{const}$$

$$\rightarrow \quad V_i T_i^3 = V_f T_f^3 \rightarrow \frac{T_f}{T_i} = \left(\frac{V_i}{V_f}\right)^{\frac{1}{3}} = \frac{1}{2^{\frac{1}{3}}}$$

c) $U_i = \frac{\pi^2 (kT_i)^4 V_i}{15\hbar^3 c^3}$, $U_f = \frac{\pi^2 (kT_f)^4 V_f}{15\hbar^3 c^3}$

$$\frac{U_f}{U_i} = \frac{V_f}{V_i}\left(\frac{T_f}{T_i}\right)^4 = 2 \cdot \frac{1}{2^{4/3}} = \frac{1}{2^{\frac{1}{3}}}$$

This process comes from

$$dU = -PdV + T\underbrace{dS}_{=0}$$

$$P = \frac{1}{3}\frac{U}{V} = \frac{1}{3}\cdot\frac{\pi^2}{15\hbar^3 c^2}(kT)^4$$

$$\Delta U = -\int_{V_i}^{V_f} PdV =$$

but $PV^{4/3} = \text{const} \rightarrow P \propto V^{-4/3} \rightarrow -\int_{V_i}^{V_f} PdV = -\int_{V_i}^{V_f} \frac{dV}{V^{4/3}}$

$VT^3 = \text{const}$

$P \propto T^4$

$$= -3\left(V_f^{-\frac{1}{3}} - V_i^{-\frac{1}{3}}\right)$$

$$\propto -3\left(2^{-1/3} - 1\right)$$