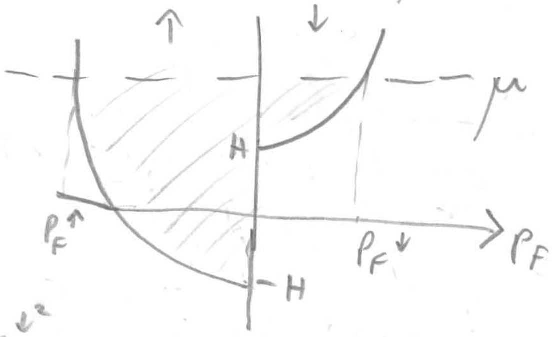
HW 6.4 (2010)

$\boxed{T=0}$

$$\epsilon_p = \frac{p^2}{2m} \pm H$$

ממלאים עד $\epsilon_p<\mu$ מומנטים



$$\frac{{p_F^\uparrow}^2}{2m} - H = \mu \qquad \frac{{p_F^\downarrow}^2}{2m} + H = \mu$$

$p_F^{\uparrow\downarrow}$ מוגדרים לפי

$$n = \alpha \int_0^{p_F^\uparrow} p^{d-1}\,dp + \alpha \int_0^{p_F^\downarrow} p^{d-1}\,dp \xleftarrow{T=0} n = \sum_p \frac{1}{e^{\beta\mu}+1}\Bigg|_{T\to 0}$$

$$\alpha = \begin{cases} \frac{4\pi}{(2\pi)^3} = \frac{1}{2\pi^2} & d=3 \\ \frac{2\pi}{(2\pi)^2} = \frac{1}{2\pi} & d=2 \\ 2 & d=1 \end{cases}$$

המקדם

$$\left(\alpha = \frac{\int d\Omega}{(2\pi)^d}\right)$$

$$n = \frac{\alpha}{d}\left[(p_F^\uparrow)^d + (p_F^\downarrow)^d\right] = \frac{\alpha}{d}(2m)^{d/2}\left[(\mu+H)^{d/2} + (\mu-H)^{d/2}\right]$$

נפתח לסדר שני ב-H (סדר ראשון מתאפס)

$$n = \frac{\alpha}{d}(2m)^{d/2}\mu^{d/2}\left[\left(1+\frac{H}{\mu}\right)^{d/2} + \left(1-\frac{H}{\mu}\right)^{d/2}\right]$$

$$\approx \frac{\alpha}{d}(2m)^{d/2}\mu^{d/2}\left[1 + \cancel{\frac{d}{2}\frac{H}{\mu}} + \frac{d}{4}\left(\frac{d}{2}-1\right)\left(\frac{H}{\mu}\right)^2 + 1 - \cancel{\frac{d}{2}\frac{H}{\mu}} + \frac{d}{4}\left(\frac{d}{2}-1\right)\left(\frac{H}{\mu}\right)^2\right]$$

$$n \approx \frac{\alpha}{d}(2m)^{d/2}\left[2\mu^{d/2} + \frac{d}{2}\left(\frac{d}{2}-1\right)H^2\cdot\mu^{\frac{d}{2}-2}\right]$$

$\underline{d=3:}$ $|A|\mu^{3/2} + |B|H^2\mu^{-\frac{1}{2}} = n$ for $H\uparrow$ we have $\mu(H)$ decreasing

$\underline{d=2:}$ $|A|\mu = n$ $\quad \mu(H) = \text{const}$

$\underline{d=1:}$ $|A|\mu^{1/2} - H^2|B|\mu^{-3/2} = n$ for $H\uparrow$ μ must increase (see (α))

$\mu_0 = \frac{n^2}{|A|^2}$ $\rightarrow$

for $\frac{dn}{dH}=0 \to \frac{d}{dH}\left[(\mu+H)^{d/2}+(\mu-H)^{d/2}\right]=0$

$\to \frac{d}{2}(\mu+H)^{d/2-1}\left(\frac{d\mu}{dH}+1\right)+\frac{d}{2}(\mu-H)^{d/2-1}\left(\frac{d\mu}{dH}-1\right)=0$

$\to \frac{d\mu}{dH}=\frac{(\mu-H)^{d/2-1}-(\mu+H)^{d/2-1}}{(\mu-H)^{d/2}+(\mu+H)^{d/2-1}}$

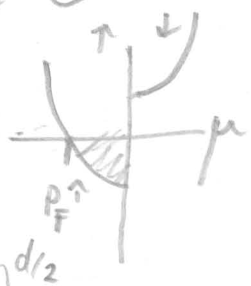
for $\underline{d=3}$: $(\mu-H)^{1/2}-(\mu+H)^{1/2}<0 \to \frac{d\mu}{dH}<0$

$\underline{d=2}$: $\frac{d\mu}{dH}=0$

$\underline{d=1}$: $\frac{1}{\sqrt{\mu-H}}-\frac{1}{\sqrt{\mu+H}}>0 \to \frac{d\mu}{dH}>0$

for $\mu>H$

b) When $\mu<H$ only $\uparrow$ states are occupied

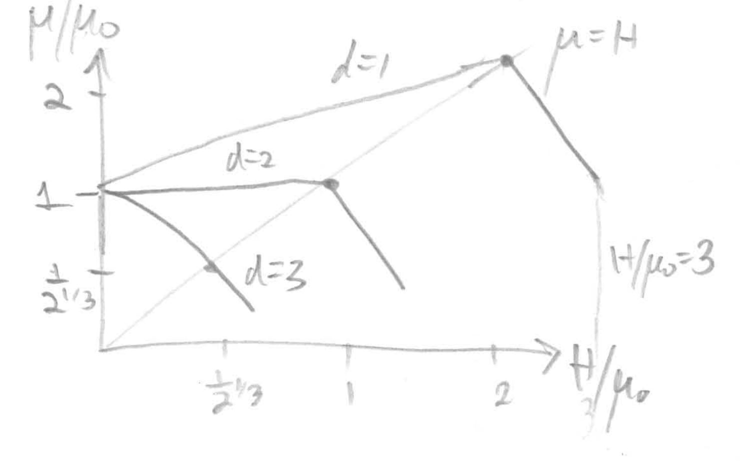


$n=\frac{a}{d}(2m)^{d/2}(\mu+H)^{d/2} \xrightarrow[\mu=H_c]{} \frac{a}{d}(2m)^{d/2}(2H_c)^{d/2}$

for $H=0$ $\quad n=\frac{a}{d}(2m)^{d/2}\underbrace{2}_{\text{degeneracy}}\mu_0^{d/2} \to \boxed{H_c=2^{\frac{2}{d}-1}\mu_0}$

For $\underline{H<H_c}$: $\quad 2\mu_0^{d/2}=(\mu+H)^{d/2}+(\mu-H)^{d/2}$

$\underline{H>H_c}$: $\quad 2\mu_0^{d/2}=(\mu+H)^{d/2} \to \boxed{\mu(H>H_c)=2^{\frac{2}{d}}\mu_0-H}$ linear decreasing



$$|A|\mu^{1/2} - H^2|B|\mu^{-3/2} = n$$

ero field: $\mu_0 = \frac{n^2}{|A|^2}$, $\mu(\delta H) = \mu_0 + \delta\mu$

$$A(\mu+\delta\mu)^{1/2} - H^2|B|(\mu_0+\delta\mu)^{-3/2} = n$$

$$|A|\mu_0^{1/2}\left(1+\frac{\delta\mu}{\mu_0}\right)^{1/2} - H^2|B|\mu_0^{-3/2}\left(1+\frac{\delta\mu}{\mu_0}\right)^{-3/2} = n$$

$$\cancel{1} + \frac{1}{2}\frac{\delta\mu}{\mu_0} - \frac{H^2|B|}{|A|\mu_0^2}\left(1 - \frac{3}{2}\frac{\delta\mu}{\mu_0}\right) = \frac{n}{|A|\mu_0^{1/2}} = \cancel{1}$$ (see here)

$$\frac{\delta\mu}{2}\left(1-\frac{3H^2|B|}{|A|\mu_0^2}\right) = \frac{H^2|B|}{|A|\mu_0}$$

נכפיל ב- μ_0 ונעביר אגפים

$$\frac{\delta\mu}{2} = \frac{1}{\frac{|A|\mu_0}{H^2|B|} - \frac{3}{\mu_0}}$$

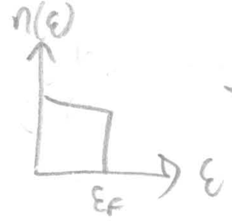
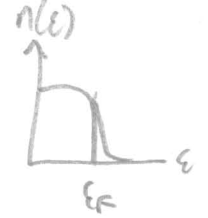
for δH as small as we want, $\delta\mu > 0$

therefore $\mu(H)$ increasing!

For $d=1,2$ $\quad \left.\frac{\partial \mu}{\partial H}\right|_{H_c} \sim \left.\frac{\partial^2 F}{\partial \mu \partial H}\right|_{H_c}$ discontinuous so transition is 2nd order

$d=3$ $\quad \left.\left(\frac{\partial \mu}{\partial H}\right)\right|_{H_c} = -1$ continuous but

$$\left.\left(\frac{\partial^2 \mu}{\partial H^2}\right)\right|_{H_c} = \begin{cases} 0 & H > H_c \\ \text{other} & H < H_c \end{cases}$$

At $T>0$,  $\rightarrow$  becomes continuous so no phase transition.

d) Flow from high μ to low μ $\quad \begin{pmatrix} B = \text{no field} \\ A = \text{field} \end{pmatrix}$

$\underline{d=3}$: $\mu(H) < \mu_0$ so flow from B to A for all H

$\underline{d=2}$ $\quad \mu(H) = \mu_0$ for $H < H_c$ no flow

$\mu(H) < \mu_0$ for $H > H_c$ flow from B to A

$\underline{d=1}$ $\quad \mu > \mu_0$ at $H < 3\mu_0$ flow from $A \to B$

$\mu < \mu_0$ at $H > 3\mu_0$ flow $B \to A$