$\boxed{BQ\,4}$ $C_V = AT^3 + Be^{-\frac{\Delta}{kT}}$ (He^4 at 3D) Deduce the excitation of the system.

for $N(\varepsilon) \sim \varepsilon^p \Rightarrow E \sim \int \frac{\varepsilon \cdot \varepsilon^p \, d\varepsilon}{e^{\beta\varepsilon} - 1}$

a) $E \sim (kT)^{p+2} \int \frac{x^{p+1} dx}{e^x - 1} \rightarrow \frac{\partial E}{\partial T} \sim (kT)^{p+1} \sim T^3$

$\rightarrow p = 2$

Therefore $N(\varepsilon) d\varepsilon = \varepsilon^2 d\varepsilon = 3D: k^2 dk$

$\rightarrow \varepsilon(k) \sim k$ like phonons

T^3 - phonon term. $e^{-\frac{\Delta}{kT}}$ means dispersion relation

$\varepsilon(k)$ has further local min such as rotons.

b) In 2D $\varepsilon^p d\varepsilon \sim k\, dk$ For a similar system

$\rightarrow \varepsilon^{p+1} \sim k^2$ but $\varepsilon(k) \sim k$

$\rightarrow p = 1$

therefore $C_V \sim AT^2 + Be^{-\frac{\Delta}{kT}}$

condensation still occurs for 2D because

$\varepsilon \sim p^s \sim p^1$ but $d = 2$

therefore $d > s$ as required.