

AO

(2009 3.4)

post AO - N 32 unsw

$$Z_1(f, T) = \int d\mathbf{r} e^{-\beta f a \cos \theta} = 2\pi \int_{-1}^1 d(\cos \theta) e^{-\beta f a \cos \theta}$$

$$= \frac{4\pi}{\beta f a} \sinh(\beta f a)$$

$$Z_N(f, T) = Z_1^N = \left(\frac{4\pi}{\beta f a} \right)^N \sinh^N(\beta f a)$$

$$\langle L \rangle = - \left(\frac{\partial \ln Z}{\partial \beta} \right)_{T, N} = +kT \left(\frac{\partial \ln Z}{\partial f} \right)_{T, N} = -kTN \frac{\partial}{\partial f} \left(-\ln f + \ln(\sinh(\beta f a)) \right)$$

$$= +kTN \left(\frac{1}{f} + \frac{\cosh(\beta f a) \cdot \beta a}{\sinh(\beta f a)} \right)$$

$$= \frac{k a N}{\beta f a} \left(1 - \coth(\beta f a) \cdot \beta f a - 1 \right)$$

$$x = \beta f a$$

mu

$$\langle L \rangle = \frac{a N}{x} \left(\coth(x) \cdot x - \frac{1}{x} \right)$$

