

B111

(HW 2008 S.4)

(HW 2009 S.4)

BEC

in potential

$$U(\vec{r}) = \frac{1}{2} m \sum_{\alpha} \omega_{\alpha}^2 x_{\alpha}^2$$

$$\hbar \omega_{\alpha} \ll kT$$

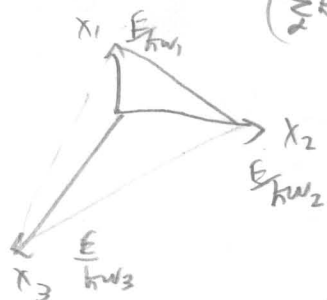
$$E' = \sum_{\alpha=1}^d \hbar \omega_{\alpha} \left( n_{\alpha} + \frac{1}{2} \right)$$

(1)

$$E = E' - \frac{1}{2} \hbar \sum_{\alpha} \omega_{\alpha} \quad \text{פרוב } n_{\alpha} \text{ מספר הקוונטים} = N(E)$$

$\sum_{\alpha} \hbar \omega_{\alpha} n_{\alpha} = E$  מספר הקוונטים  $y_{\alpha} = \hbar \omega_{\alpha} n_{\alpha}$  מספר הקוונטים  $\sum_{\alpha} y_{\alpha} = E$  מספר הקוונטים

$$N(E) = \int_{\left( \sum_{\alpha} \hbar \omega_{\alpha} n_{\alpha} = E \right)} \prod_{\alpha} d n_{\alpha} = \int_0^E \prod_{\alpha} d y_{\alpha} \cdot \left( \frac{d n_{\alpha}}{d y_{\alpha}} \right) = \frac{1}{d!} E^d \cdot \prod_{\alpha} \frac{d n_{\alpha}}{d y_{\alpha}}$$



$$0 < y_{\alpha} < E$$

$$N(E) = \frac{1}{d!} E^d \prod_{\alpha} \frac{d n_{\alpha}}{d y_{\alpha}} = \frac{1}{d!} \frac{E^d}{\prod_{\alpha} \hbar \omega_{\alpha}}$$

$$p(E) = \frac{\partial N}{\partial E} = \frac{1}{(d-1)!} \frac{E^{d-1}}{\prod_{\alpha} \hbar \omega_{\alpha}}$$

$$\langle N \rangle = \sum_{\{n_{\alpha}\}} \frac{1}{Z} e^{\beta E} = \int_0^{\infty} \frac{dE}{\frac{1}{Z} e^{\beta E}} \cdot p(E)$$

(2)

$$= \frac{1}{(d-1)! \prod_{\alpha} (\hbar \omega_{\alpha})} \int_0^{\infty} \frac{E^{d-1} dE}{\frac{1}{Z} e^{\beta E}}$$

$$= \frac{(k_B T)^d}{(d-1)!} \int_0^{\infty} \frac{x^{d-1}}{\frac{1}{Z} e^x} \cdot \frac{1}{\prod_{\alpha} (\hbar \omega_{\alpha})} = g_d(z) \prod_{\alpha=1}^d \left( \frac{k_B T}{\hbar \omega_{\alpha}} \right)$$

$$\ln$$

$$\left( n\lambda^3 = g_d(z) \right) \text{ or } z \ll 1 \quad \Leftrightarrow \text{non-degenerate gas} \quad (2)$$

$$n\lambda^3 \Big|_{T \rightarrow \infty} \sim \frac{n}{T^{3/2}} \rightarrow 0 \ll 1$$

$$g_d(z) = z + \frac{z^2}{2^{d/2}} + \frac{z^3}{3^{d/2}} + \dots \Big|_{z \ll 1} \approx z = e^{\beta\mu}$$

$$\langle N \rangle \approx e^{\beta\mu} \prod_d \left( \frac{k_B T}{\hbar \omega_d} \right)$$

$$\mu \approx k_B T \ln \frac{\langle N \rangle}{\prod_d \left( \frac{k_B T}{\hbar \omega_d} \right)}$$

$$N = g_d(1) \cdot \prod_d \frac{k_B T_c}{\hbar \omega_d} \quad \begin{array}{l} \text{for } \mu \rightarrow 0 \text{ BEC,} \\ \Leftrightarrow z=1 \end{array} \quad (3)$$

$$k_B T_c = \left[ \frac{N}{g_d(1)} \prod_d \hbar \omega_d \right]^{1/d}$$

$T \neq 0$  and BEC for  $d=1$  and  $d=2$   $g_{d=1,2}(1) \rightarrow \infty$

$T_c \neq 0$  for  $d \geq 3$   $g_{d \geq 3}(1) < \infty$