$\boxed{B28}$ β decay

(relativistic: $\mathcal{H} = cp$)

a) $$n_\nu = 2\int \frac{d^3p}{h^3} \cdot \frac{1}{\frac{1}{z}e^{\beta cp}+1} = \frac{8\pi}{h^3}\int_0^\infty \frac{p^2 dp}{\frac{1}{z}e^{\beta cp}+1} = \frac{16\pi}{(h\beta c)^3} f_3(z)$$

high $T \Leftrightarrow \frac{1}{z}e^{\beta cp} \gg 1 \rightarrow \frac{1}{\frac{1}{z}e^{\beta cp}+1} \approx z e^{-\beta cp}$, $(z \ll 1)$

$$n_{\text{high }T} \approx \frac{8\pi}{h^3} z \int_0^\infty e^{-\beta cp} p^2 dp = \frac{8\pi}{h^3} \cdot \frac{z}{(\beta c)^3} \int_0^\infty e^{-x} x^2 dx$$
$$= \frac{16\pi}{(h\beta c)^3} z$$

$$\boxed{z = \frac{n(h\beta c)^3}{16\pi} \ll 1 \Longleftrightarrow \text{high } T \text{ or low density} \quad \text{classical limit} \qquad \mu_\nu = kT \ln\left(\frac{n(h\beta c)^3}{16\pi}\right)}$$

At $T=0$ $\quad \mu(T=0) = \epsilon_F \qquad f(\epsilon) = \Theta(cp - \epsilon_F)$

בטמפ' אפס:
פונק'
מדרגה!

$$n = \frac{8\pi}{h^3}\int_0^{\epsilon_F/c} p^2 dp = \frac{8\pi}{3(hc)^3}\epsilon_F^3$$

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b) Reaction produces equal # of $p, e^-, \bar{\nu}$

$$n_p = \frac{2}{\lambda_p^3} f_{3/2}\left(e^{\beta\mu_p}\right)$$

$$n_e = n_p = \frac{2}{\lambda_e^3} f_{3/2}\left(e^{\beta\mu_e}\right)$$

$$n_{\bar\nu} = n_p = \frac{16\pi}{(h\beta c)^3} f_3\left(e^{\beta\mu_{\bar\nu}}\right)$$

$$n_n = n_0 - n_p = \frac{2}{\lambda_n^3} f_{3/2}\left(e^{\beta\mu_n}\right)$$

$$\Delta m = m_n - m_p - m_e > 0$$

הפרש המסות

$$\boxed{\mu_n = \mu_p + \mu_e + \mu_{\bar\nu}}$$

שיווי משקל

high T: $kT\ln\left(\frac{1}{2}(n_0-n_p)\lambda_n^3\right) = kT\ln\left(\frac{1}{2}n_p\lambda_p^3\right) + kT\ln\left(\frac{n_p\lambda_e^3}{2}\right)$
$+ kT\ln\left(\frac{n_p(h\beta c)^3}{16\pi}\right)$

$$(n_0-n_p)\lambda_m^3 = n_p^3\lambda_p^3\lambda_e^3\frac{(h\beta c)^3}{32\pi}$$

for $n_p \ll n_0$ $\quad n_p = \left[n_0\frac{\lambda_n^3}{\lambda_p^3\lambda_e^3}\frac{32\pi}{(h\beta c)^3}\right]^{1/3}$

at T=0: (ε_F [illegible])

$$\frac{\hbar^2}{2m_n}\left(3\pi^2(n_0-n_p)\right)^{2/3} = \frac{\hbar^2}{2m_p}\left(3\pi^2 n_p\right)^{2/3} + \frac{\hbar^2}{2m_e}\left(3\pi^2 n_p\right)^{2/3} + C\left(\frac{3h^3 n_p}{8\pi}\right)^{1/3}$$

$n_p \ll n_0 \rightarrow n_p^{1/3}$ term dominates

$$n_p = \frac{\hbar^2}{3\pi^2(c\hbar)^3}\cdot\left[\frac{\hbar^2}{2m_n}\left(3\pi^2 n_0\right)^{2/3}\right]^3$$