$\boxed{D07}$ (HW 2009, 9.3)

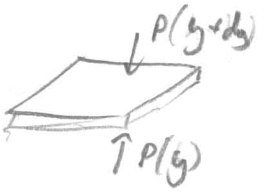
$$f_0(\vec{p},x,y,z) = \frac{n(y)}{\left(2\pi m k_B T(y)\right)^{3/2}} e^{-\frac{p^2}{2mkT(y)}}$$

a) $T(y)$ non uniform $\rightarrow$ flow of heat & particles

$\rightarrow \mu$ not uniform

Mechanical equilibrium: $P = nkT = \text{const}$

$\rightarrow n(y)\cdot T(y) = \text{const}$



i.e. $P(y+dy) = P(y)$

b) for $n(y) \sim \frac{1}{T(y)}$ we have $f_0 \sim T(y)^{-5/2} e^{-\frac{p^2}{2mk_BT(y)}}$

but $\left(\frac{\partial}{\partial t} + v_y\frac{\partial}{\partial y}\right) f_0 = \frac{p_y}{m}\left(\frac{-5}{2T(y)} + \frac{p^2}{2mk_BT^2(y)}\right)\frac{\partial T}{\partial y}\cdot f_0(p,y) \neq 0$

whereas $e^{-\frac{\beta m}{2}(v_1^2+v_2^2)} = e^{-\frac{\beta m}{2}(v_1'^2+v_2'^2)} \rightarrow f_1f_2 - f_1'f_2' = 0 \rightarrow \left(\frac{\partial f}{\partial t}\right)_{coll} = 0$

using the relaxation time approximation $\left(\frac{\partial f}{\partial t}\right)_{col} = -\frac{(f_1 - f_0)}{\tau}$

we have $f_1 = f_0\left(1 + \frac{\tau p_y}{m}\left(\frac{5}{2T(y)} - \frac{p^2}{2mkT^2(y)}\right)\frac{\partial T}{\partial y}\right)$

c) $Q = n\frac{\langle p_y p^2\rangle_1}{2m^2}$, why?

Q = Heat flow = Energy $\times$ particle flux

$= \frac{p^2}{2m}\cdot n v_y = \frac{p^2}{2m}\cdot n\frac{p_y}{m}$

$\langle Q\rangle = \frac{n}{2m^2}\langle p^2 p_y\rangle \leftarrow$ this average with the actual distribution, but f_1 approximates it since f_0 can't contribute to the flow.

$\langle p_y p^2\rangle_0 = 0 \longleftrightarrow$

$\langle Q\rangle = \frac{n}{2m^2}\frac{\tau}{m}\left\langle p_y^2p^2\cdot\frac{5}{2T(y)} - p_y^2p^4\cdot\frac{1}{2mkT^2(y)}\right\rangle_0\frac{\partial T}{\partial y}$

$= \frac{n}{2m^2}\cdot\frac{\tau}{m}\left[\frac{25m^2k_B^2T}{2} - \frac{35}{2m}k_B^2T^2\right] = \frac{-5}{2}\frac{n\tau}{m}k^2T\frac{\partial T}{\partial y}$

$Q = -\kappa \frac{\partial T}{\partial y}$ $\rightarrow$ $\underline{\kappa = \frac{5}{2m} k_B^2 n \tau T}$ = const (doesn't depend on y!)

d) Steady state $\rightarrow$ no flow (of heat or anything else)

$\rightarrow Q = \text{const}$

$\rightarrow n(y) \cdot T(y) \cdot \frac{\partial T}{\partial y} = \text{const}$

but $n(y) T(y) = \text{const}$ $\rightarrow$ $\frac{\partial T}{\partial y} = \text{const}$

$$\boxed{T = T_0 + \frac{y}{W}(T_2 - T_1)}$$ with boundary conditions

e) We are looking for $\frac{\kappa}{\sigma} \propto T$

$\sigma = \frac{J_y}{\partial_y \mu} = \frac{n v_y}{\partial_y \mu}$

but $\mu(y) = \mu_0 + \varphi(y) \rightarrow \frac{\partial \mu}{\partial y} = \frac{\partial \varphi}{\partial y} = -E$

$j = \sigma E = -\sigma \frac{\partial \mu}{\partial y}$

what we need is to replace the ΔT between the plates with electric field E and solve for Boltzmann eq. see q. D03.