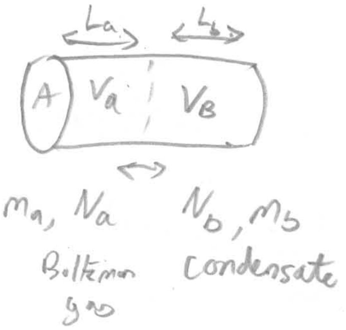
B07 (HW 2009 5.3)



$V_a + V_B = V = AL$

$L = L_a + L_b$

a) At equilibrium the pressure $P_a = P_b$

Boltzman: $P_a V_a = N_a k_B T$

BEC: (pressure doesn't depend on volume)

$$P_b = \frac{kT}{\lambda_b^3} g_{5/2}(1) = (kT)^{5/2} \cdot \left(\frac{m_b}{2\pi\hbar^2}\right)^{3/2} g_{5/2}(1)$$

By comparing $P_a = P_b$ we have

$$\frac{N_a}{V_a} = (kT)^{3/2} \left(\frac{m_b}{2\pi\hbar^2}\right)^{3/2} \cdot g_{5/2}(1)$$

$$\frac{N_a}{L_a A} = \dots$$

$$\boxed{L_a = \frac{N_a}{A} \cdot \frac{1}{g_{5/2}(1)} \cdot \left(\frac{2\pi\hbar^2}{m_b kT}\right)^{3/2} < L}$$

$L_a < L \rightarrow$

[illegible]

$$\frac{N_a}{g_{5/2}(1)} < \frac{V}{\lambda_b^3}$$

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b) Assumption (ii) requires BEC on RHS, ie.

$$\frac{\lambda_b^3}{V_b} \cdot N_b \geq g_{3/2}(1) \rightarrow \left(\frac{2\pi\hbar^2}{m_b k_B T}\right)^{3/2} \cdot \frac{N_b}{V_b} \geq g_{3/2}(1)$$

$$L_b \leq \frac{N_b}{A} \cdot \frac{1}{g_{3/2}(1)} \cdot \left(\frac{2\pi\hbar^2}{m_b k_B T}\right)^{3/2} \rightarrow L_b \leq L_a \cdot \frac{N_b}{N_a} \cdot \frac{g_{5/2}(1)}{g_{3/2}(1)}$$

$$L - L_a \leq L_a \frac{N_b}{N_a} \cdot \frac{g_{5/2}(1)}{g_{3/2}(1)}$$

$$L \leq L_a \left(\frac{N_b}{N_a} \frac{g_{5/2}}{g_{3/2}} + 1\right) = \frac{N_a}{A\, g_{5/2}(1)} \cdot \left(\frac{2\pi\hbar^2}{m_b kT}\right)^{3/2} \left(\frac{N_b}{N_a} \frac{g_{5/2}(1)}{g_{3/2}(1)} + 1\right)$$

$$\rightarrow \boxed{kT \leq \left(\frac{2\pi\hbar^2}{m_b}\right) \cdot \left(\frac{1}{AL}\right)^{2/3} \left(\frac{N_b}{g_{3/2}(1)} + \frac{N_a}{g_{5/2}(1)}\right)^{2/3}}$$

Assumption (i) requires Boltzman gas on LHS, ie.

$$\frac{\lambda_a^3}{V_a} \cdot N_a \ll 1 \rightarrow \frac{N_a}{L_a A} \cdot \left(\frac{2\pi\hbar^2}{m_a k_B T}\right)^{3/2} \ll 1$$

$$\rightarrow \boxed{kT \gg \frac{2\pi\hbar^2}{m_a} \cdot \left(\frac{N_a}{L_a A}\right)^{2/3}}$$

(again we can plug in La from sec (a) for a final result)

$$\cancel{kT} \gg \frac{\cancel{2\pi\hbar^2}}{m_a} \cdot \left(\left(\frac{m_b \cancel{kT}}{\cancel{2\pi\hbar^2}}\right) \cdot \left(g_{5/2}(1)\right)\right)^{2/3}$$

$$\frac{m_a}{m_b} \gg \left(g_{5/2}(1)\right)^{2/3}$$

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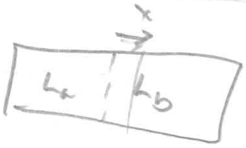
c) $\left(L_a = \frac{N_a}{A} \frac{k_B T}{P_b}\right)$



we move the partition so that $\tilde{L}_b = L_b - x$, $\tilde{L}_a = L_a + x$ with $\frac{x}{L_a} \ll 1$, now the net force is

$$F = \frac{N_a kT}{L_a + x} - \frac{N_a kT}{L_a} \approx \frac{N_a kT}{L_a}\left(1 - \frac{x}{L_a} - 1\right) = -\frac{N_a kT}{L_a^2} x$$

Comparing with $F = -\omega^2 M x$

we have

$$\omega^2 = \frac{N_a kT}{M L_a^2}$$

mass of partition.