C27 $\quad F = t^{2-\alpha} f\left(\frac{t}{h^{1/\varphi}}\right) \quad t = \frac{T-T_c}{T_c} \quad h$ - mag. field

harmonic form

a) show that α is the crit. exp. of spec. heat.

$C_V \sim t^{-\alpha}$

near T_c, $f \approx f(0) + a\frac{t}{h^{1/\varphi}}$

$$\Rightarrow F \sim t^{2-\alpha} f_0 + \frac{a t^{3-\alpha}}{h^{1/\varphi}}$$

$$\frac{\partial F}{\partial T} \sim \frac{\partial F}{\partial t} \sim t^{1-\alpha} + A t^{2-\alpha}$$

$$\frac{\partial^2 F}{\partial T^2} \sim t^{-\alpha} + B t^{1-\alpha} \overset{\text{near } T_c}{\approx} t^{-\alpha}$$

$$\Rightarrow \underline{C_V \sim t^{-\alpha}}$$

b) $m_0 \sim (T_c - T)^\beta \quad (h \to 0,\ T \lesssim T_c)$

for $h \to 0$ we need to look at harmonic expansion of $f(x)|_{x\to\infty} \approx f(\infty) + a x^{-\varphi} + b x^{-2\varphi} + \ldots$

$$\Rightarrow f\left(\frac{t}{h^{1/\varphi}}\right) \approx f(\infty) + a\left(\frac{t}{h^{1/\varphi}}\right)^{-\varphi} \approx f(\infty) + a\frac{h}{t^\varphi}$$

$$\Rightarrow F|_{h\to 0} \sim t^{2-\alpha-\varphi} \cdot h$$

$$\Rightarrow m_0 \sim t^{2-\alpha-\varphi} = t^\beta \Rightarrow \boxed{\beta = 2-\alpha-\varphi}$$

for δ we need

$m|_{T=T_c}$ as $h \to 0 \sim h^{1/\delta}$

because $T = T_c \Rightarrow t = 0$ we need $f\left(\frac{t}{h^{1/\varphi}}\right) \approx f(\infty) + \ldots + b\left(\frac{t}{h^{1/\varphi}}\right)^{\alpha-2}$

so that $F|_{t=0} \approx t^{2-\alpha} \cdot b \cdot \left(\frac{t}{h^{1/\varphi}}\right)^{\alpha-2} \sim h^{\frac{2-\alpha}{\varphi}}$

$$\Rightarrow m|_{t=0} \sim \frac{\partial F_{t=0}}{\partial h} \sim h^{\frac{2-\alpha-\varphi}{\varphi}} \sim h^{\frac{1}{\delta}} \Rightarrow \boxed{\delta = \frac{\varphi}{2-\alpha-\varphi}}$$

$$\Rightarrow \delta = \frac{2-\alpha-\beta}{\beta} \Rightarrow \boxed{2-\alpha = \delta(\beta+1)}$$