

Course: CS201

Section: 2

Assignment: 2

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### Question 1:

The first algorithm has  $O(n)$  complexity:

The first algorithm has a for loop that loops until it reaches  $n$  and its step size is 1 (meaning it loops through  $n$  elements 1 by 1). The loops executes the statements inside itself  $n$  times. The statements inside the loop have  $O(1)$  complexity on their own so they don't affect function's complexity as adding constants to  $n$  does not affect the overall complexity. The statements outside for loop also have  $O(1)$  complexities so they don't affect the complexity too. Therefore the function's complexity is for loop's complexity which is  $O(n)$ .

Second algorithm has  $O(n)$  complexity:

In this algorithm the worst case complexity is  $O(n)$  because when it cannot find the index it loops through the whole array for the mod calculation. So in the worst case algorithm loops once to find index which is  $O(n)$  and it loops again to calculate which is  $O(n)$  sum of these two complexities gives  $O(n)$  and the other statements give  $O(1)$  which does not affect function's complexity (the last loop is never entered because we didn't find the index so it more be more than  $n$ . Therefore algorithm works just like Algorithm 1). Because of this my graphs are mostly linear.

However, when the index is found the algorithm works faster and I predict this is the case when my "Algorithm 2,  $p = 101$ " graph decreases instantly. So in this case the complexity will not be  $O(n)$  as this is the best case and its complexity will be calculated by the sum of the first for loop that gives  $O(n)$  while looking for the index, the second for loop that loops until it reaches index which gives  $O(i)$  complexity ( $i$  being the index we are looking for) and the last loop that loops until  $(n) \bmod (i)$  value. But we are not looking for the worst case which is the upper bound so this only explains some differences in the graph but does not affect the  $O(n)$  complexity which is the general complexity.

Third algorithm has  $O(\log(n))$  complexity:

The recurrence relation of this algorithm is

$$T(n) = T(n/2) + \Theta(1)$$

$$T(1) = \Theta(1)$$

because every step of the recursion takes the  $n$  as  $n/2$  so the value that it must recurrence through gets smaller like this:

$$T(n) = T(n/2) + \Theta(1)$$

$$T(n) = T(n/4) + \Theta(1) + \Theta(1)$$

and eventually we react to the general case which is:

$$T(n) = T(n/2^k) + k \Theta(1)$$

$$T(n) = T(1) + \log n \Theta(1)$$

$$T(n) = \Theta(1) + \Theta(\log n)$$

$$T(n) = \Theta(\log n)$$

This is because our algorithm uses a tempMod value which makes it remember the answer of recurrence relations it calculates so it does not have to calculate two recurrence relations in every recursion. So it acts like a for loop that divides its way to half in every loop. When if we assume it executes  $x$  times,  $x$  would be our complexity and  $x$  can be found by:

$$1 = n/2^x$$

$$x = \log_2(n)$$

Because base 2 is not important this will give us  $O(\log(n))$

## Question 2:

OS: macOS Catalina

Model: MacBook Air (Retina, 13-inch, 2020)

Model Identifier: MacBookAir9,1

Processor Name: Quad-Core Intel Core i5

Processor Speed: 1,1 GHz

Number of Processors: 1

Total Number of Cores: 4

L2 Cache (per Core): 512 KB

L3 Cache: 6 MB

Hyper-Threading Technology: Enabled

Memory: 8 GB 3733 MHz LPDDR4X

Boot ROM Version: 1715.81.2.0.0 (iBridge: 19.16.10744.0.0,0)

Serial Number (system): FVFDK4DTMNHX

Hardware UUID: 97BBBA82-49EF-5052-8B01-ABF4744FD8EC

Activation Lock Status: Enabled

Graphics: Intel Iris Plus Graphics 1536 MB

Storage: (For this homework Macintosh HD is used)

Macintosh HD: 75 GB available of the total 170 GB

BOOTCAMP: 13,93 GB available of total 80,68 GB

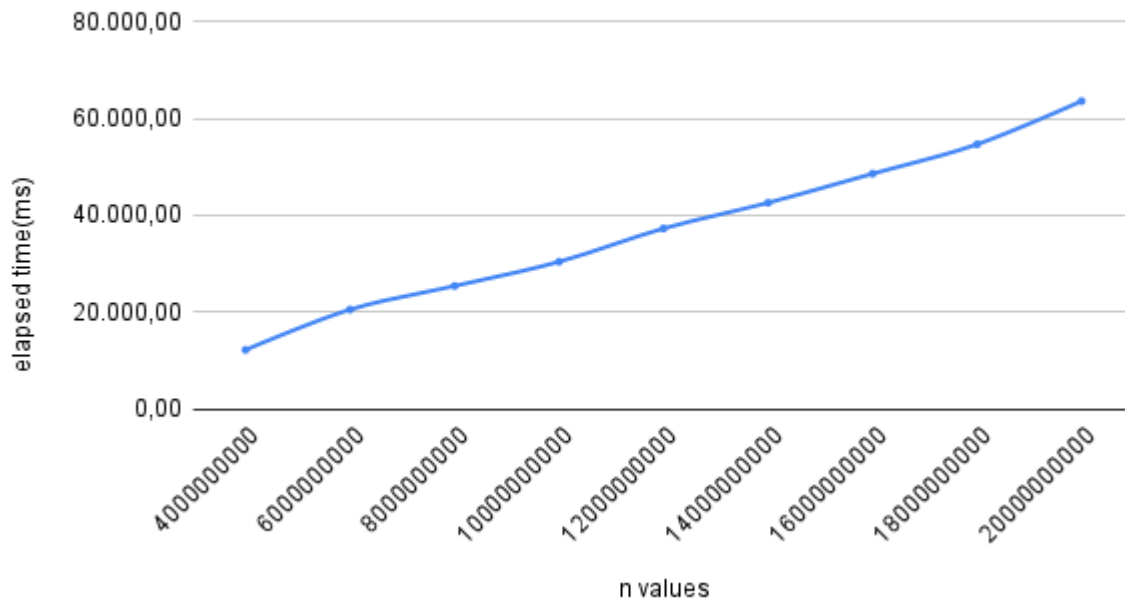
### Question 3:

In the below table, x is the value of how many times the function is called to get that time value. I only called the recursive functions more than once as you can see.

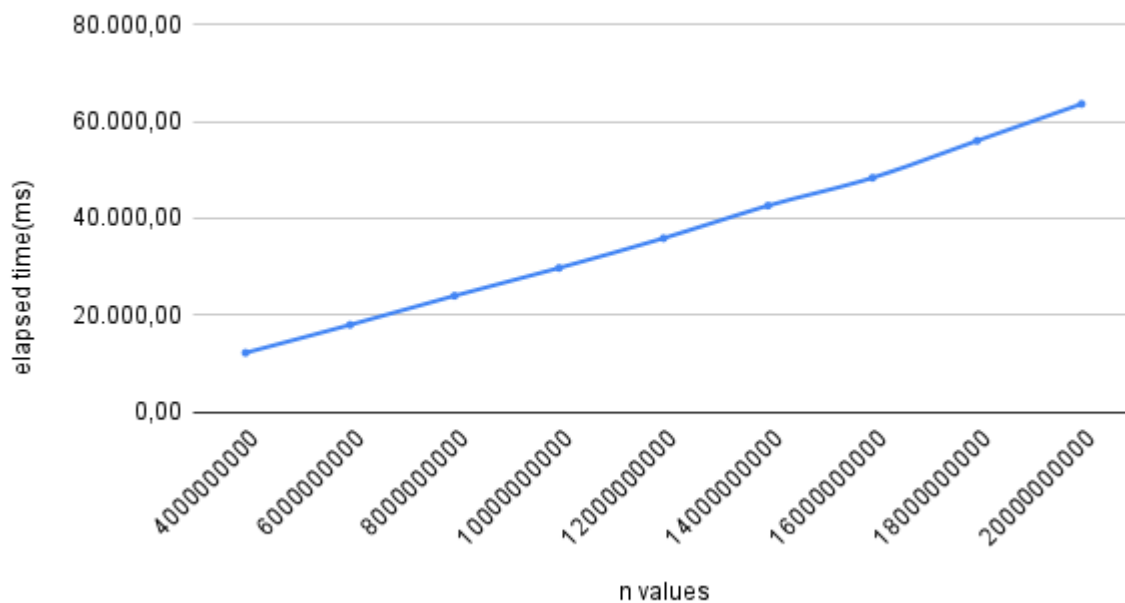
	Algorithm 1 (ms)	Algorithm 1 (ms)	Algorithm 1 (ms)	Algorithm 2 (ms)	Algorithm 2 (ms)	Algorithm 2 (ms)	Algorithm 3 (ms)	Algorithm 3 (ms)	Algorithm 3 (ms)
	p = 101	p = 1009	p = 10007	p = 101	p = 1009	p = 10007	p = 101	p = 1009	p = 10007
	(x = 1)	(x = 1)	(x = 1)	(x = 1)	(x = 1)	(x = 1)	(x = 2 * 10 <sup>8</sup> )	(x = 2 * 10 <sup>8</sup> )	(x = 2 * 10) <sup>8</sup>
$n = 2 * 10^9$	6442.7	6037.81	6095.3	13163.50	12057.20	13078.1	56348.50	60904.00	59804.1
$n = 4 * 10^9$	12171.90	12195.80	12067.70	25289.60	24545.60	25933.90	75199.20	68580.40	72607.70
$n = 6 * 10^9$	20517.70	17976.30	18204.10	39190.00	36198.30	37576.90	74494.20	78325.90	79717.70
$n = 8 * 10^9$	25395.00	23990.70	24109.20	48549.90	48803.10	49084.30	78084.90	86629.10	78070.30
$n = 10 * 10^9$	30407.00	29750.00	30493.90	66682.60	61370.10	61719.40	79717.70	87462.40	80097.30
$n = 12 * 10^9$	37293.20	35892.60	36363.70	95643.80	73726.80	74111.60	83957.80	88573.00	81777.00
$n = 14 * 10^9$	42608.00	42639.60	42762.50	84379.40	85896.40	85137.30	8274.00	91994.10	83016.70
$n = 16 * 10^9$	48618.20	48396.10	50875.30	97352.10	97545.30	97493.00	84463.40	91499.50	82001.90
$n = 18 * 10^9$	54698.20	56055.90	58406.90	108965.00	110579.00	108496.00	86878.30	93904.00	85111.50
$n = 20 * 10^9$	63633.89	63670.90	62912.00	121891.00	122040.00	121609.00	87575.70	94350.50	88671.00

#### Question 4:

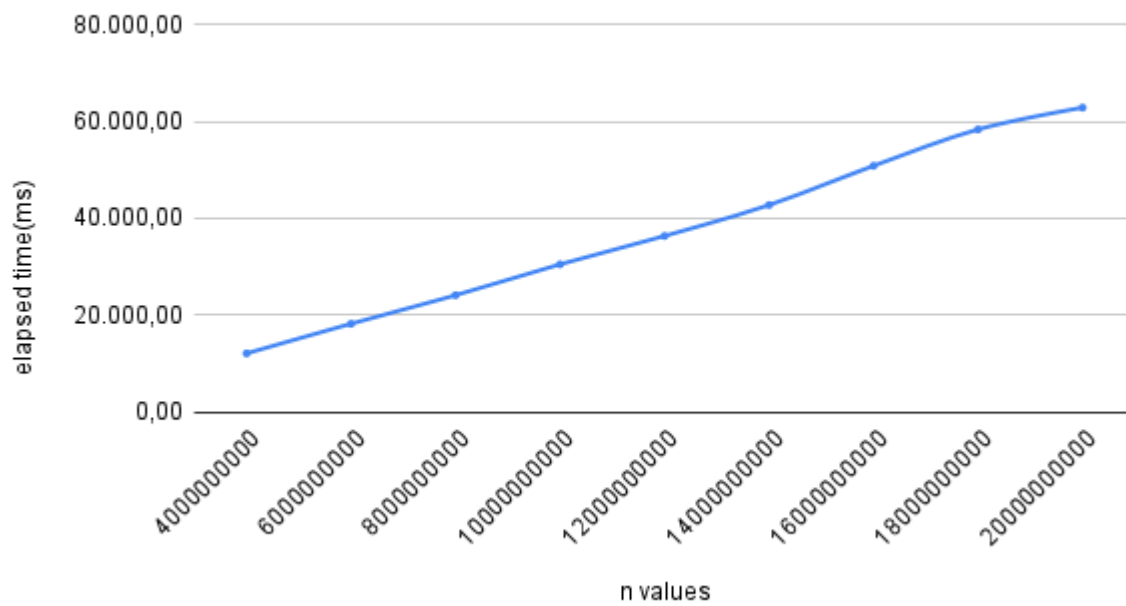
##### Algorithm 1, $p = 101$



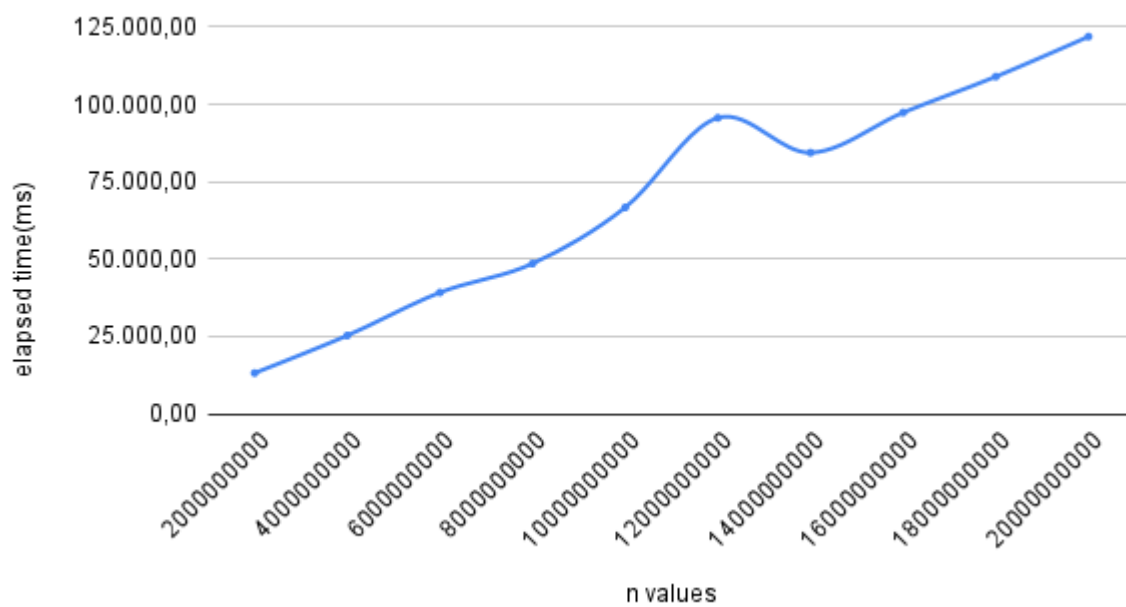
##### Algorithm 1, $p = 1009$



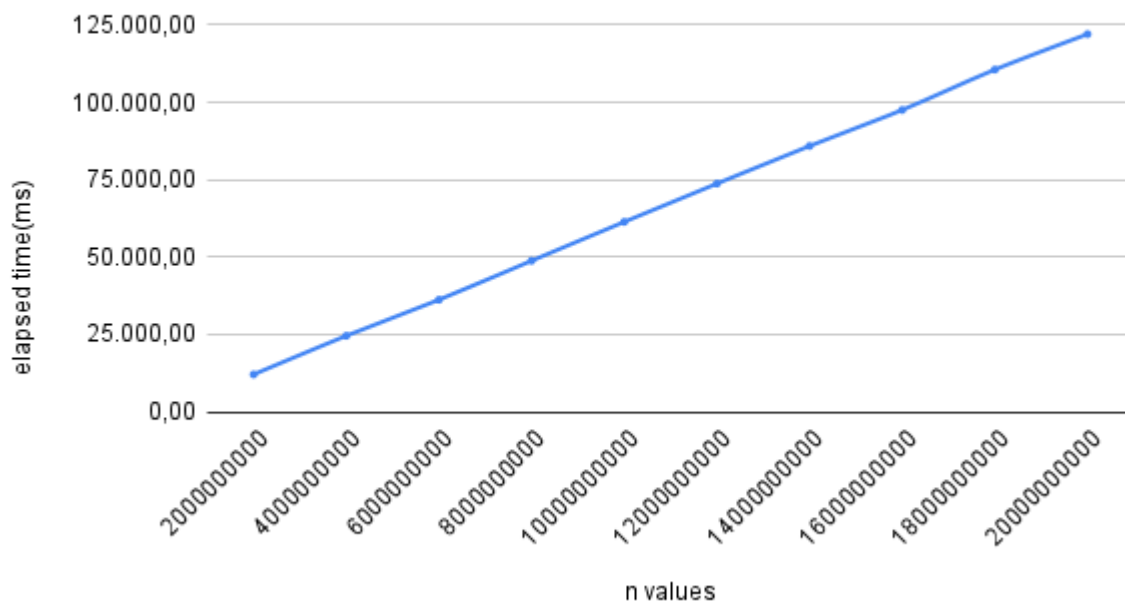
Algorithm 1,  $p = 10007$



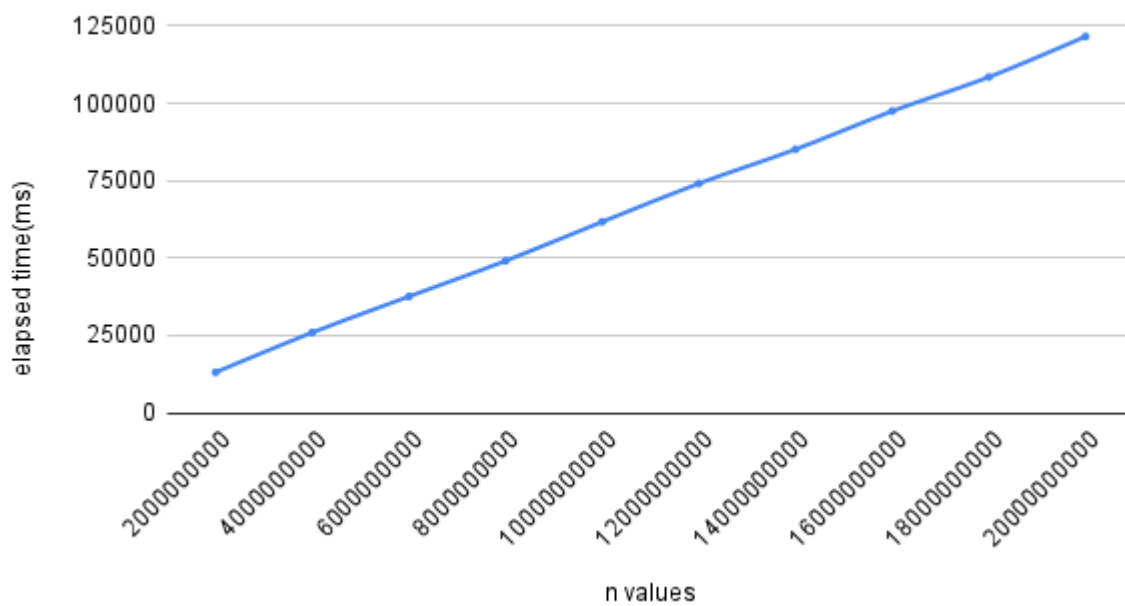
Algorithm 2,  $p = 101$



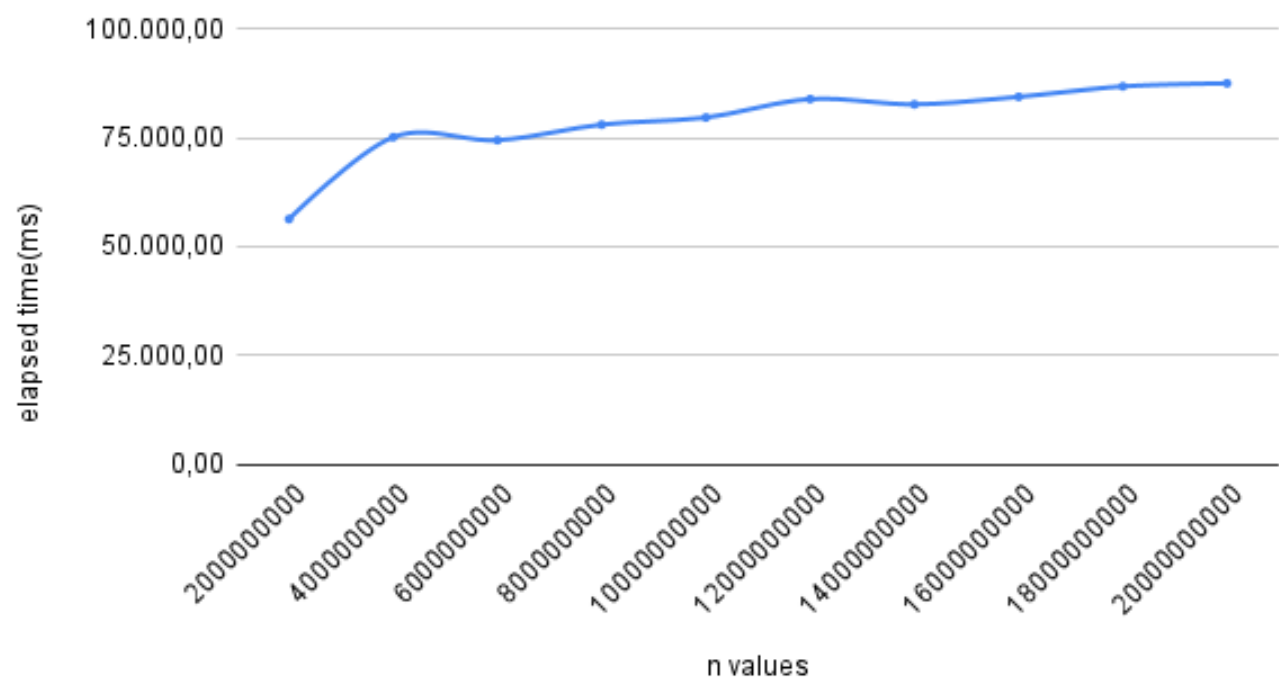
Algorithm 2, p = 1009



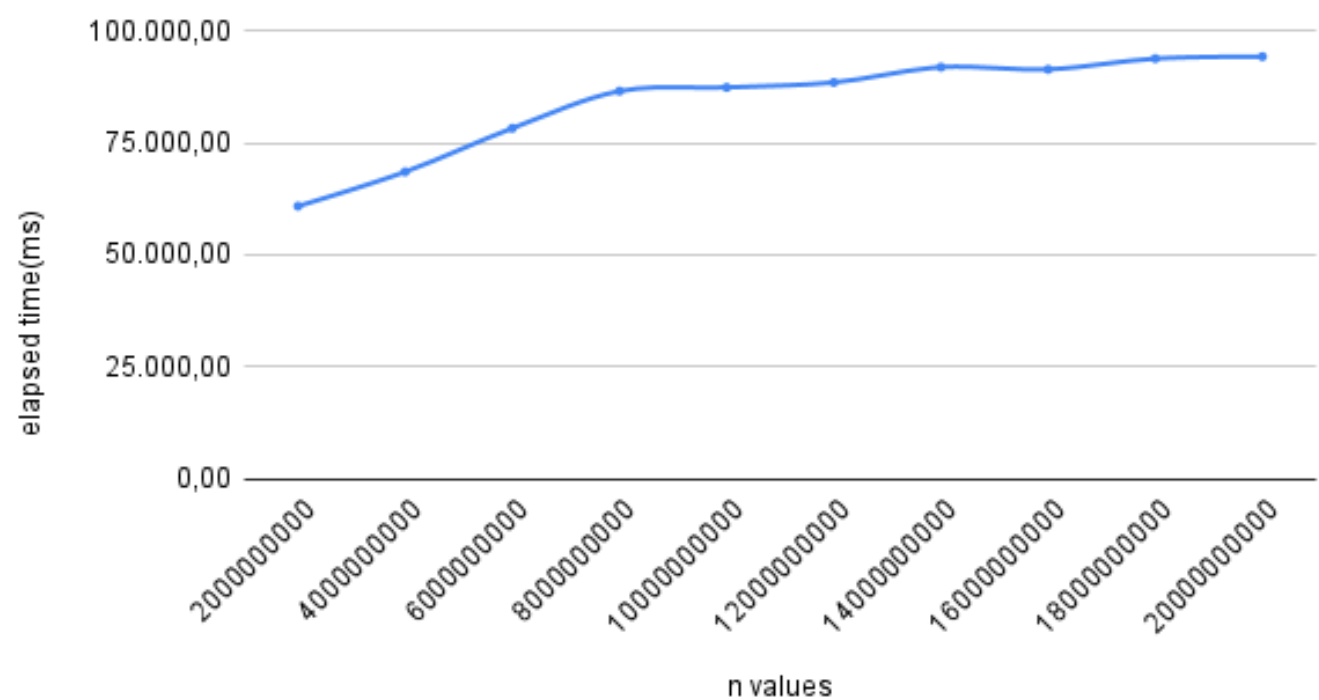
Algorithm 2, p = 10007



### Algorithm 3, $p = 101$



### Algorithm 3, $p = 1009$



### Algorithm 3, $p = 10007$

