a. Transposition of Matrix

NOTE:

The transposed matrix of A is denoted by A^T
The $n \times m$ matrix B is the transposed matrix of the $m \times n$ matrix A

Example:

A $= \begin{bmatrix} 1 & -3 & 4 \\ 2 & 10 & -7 \end{bmatrix}$ so, $A^T = \begin{bmatrix} 1 & 2 \\ -3 & 10 \\ 4 & -7 \end{bmatrix}$ (answer)

EXERCISE:
Find the transposition of Matrix A

$A = \begin{bmatrix} 7 & 1 & 9 \\ 21 & -15 & 2 \\ 8 & 7 & -1 \end{bmatrix}$	$A = \begin{bmatrix} 6 & -3 \\ 9 & 14 \end{bmatrix}$
$A = \begin{vmatrix} 6 & 7 & -8 \\ -2 & 5 & 3 \end{vmatrix}$	$A = \begin{vmatrix} 4 & 7 \\ 9 & -2 \\ -5 & -5 \end{vmatrix}$
$A = \begin{vmatrix} 2 \\ -3 \\ -1 \end{vmatrix}$	$A = \|2 \quad -6 \quad 11\|$
$A = \begin{vmatrix} -11 & 5 \\ 7 & 2 \\ 6 & 14 \end{vmatrix}$	$A = \begin{vmatrix} 4 & -3 & 2 \\ 5 & 2 & 1 \end{vmatrix}$

b. Operation of additional involving matrices.

NOTE:

- One rule that must be follow to do operation of addition which is the size of the matrix must be same.

Addition

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$ (answer)

EXERCISE:
Solve the following matrix

1. $\begin{bmatrix} 8 & 7 \\ -6 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -3 \\ 1 & 13 \end{bmatrix}$

$Ans: \begin{bmatrix} 12 & 4 \\ -5 & 18 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 4 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 7 & 3 \\ 11 & 9 \end{bmatrix}$

$Ans: \begin{bmatrix} 8 & 7 \\ 16 & 7 \end{bmatrix}$

3. $\begin{bmatrix} -3 & 4 \\ 6 & 13 \end{bmatrix} + \begin{bmatrix} 3 & -8 \\ -2 & 2 \end{bmatrix}$

$Ans: \begin{bmatrix} 0 & -4 \\ 4 & 15 \end{bmatrix}$

4. $\begin{bmatrix} -7 & 5 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} -14 & 3 \\ -3 & 1 \end{bmatrix}$

$Ans: \begin{bmatrix} -21 & 8 \\ 1 & 4 \end{bmatrix}$

5. $\begin{bmatrix} -17 & 5 \\ 22 & 8 \end{bmatrix} + \begin{bmatrix} 6 & -3 \\ 9 & 14 \end{bmatrix}$

$Ans: \begin{bmatrix} -11 & 2 \\ 31 & 22 \end{bmatrix}$

6. $\begin{bmatrix} 15 & 2 \\ -3 & 8 \end{bmatrix} + \begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix}$

$Ans: \begin{bmatrix} 22 & -1 \\ 2 & 12 \end{bmatrix}$

7. $\begin{bmatrix} 1 & -7 & 15 \\ 31 & 3 & 18 \\ 22 & 6 & 4 \end{bmatrix} + \begin{bmatrix} 13 & 17 & 5 \\ 3 & 8 & -1 \\ -9 & 2 & 12 \end{bmatrix}$

Ans: $\begin{bmatrix} 14 & 10 & 20 \\ 34 & 11 & 17 \\ 13 & 8 & 16 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 6 & 4 \\ -3 & 8 & 9 \\ -2 & 7 & -1 \end{bmatrix} + \begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix}$

Ans: $\begin{bmatrix} -1 & 10 & 3 \\ 1 & -9 & 4 \\ 4 & 8 & 7 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 6 & -2 \\ 17 & 32 & 8 \\ -12 & 9 & 5 \end{bmatrix} + \begin{bmatrix} 7 & 1 & 9 \\ 21 & -15 & 2 \\ 8 & 7 & -1 \end{bmatrix}$

Ans: $\begin{bmatrix} 10 & 7 & 7 \\ 38 & 17 & 10 \\ -4 & 16 & 4 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 20 & -1 \\ 2 & -1 & 5 \\ 6 & 7 & 8 \end{bmatrix} + \begin{bmatrix} 5 & 7 & 9 \\ -8 & -7 & 12 \\ -2 & 15 & -2 \end{bmatrix}$

Ans: $\begin{bmatrix} 6 & 27 & 8 \\ -6 & -8 & 17 \\ 4 & 22 & 6 \end{bmatrix}$

11. $\begin{bmatrix} -1 & 4 & -7 \\ 2 & -5 & 9 \\ -3 & 6 & -11 \end{bmatrix} + \begin{bmatrix} 13 & 6 & 9 \\ 10 & -4 & 2 \\ -8 & -2 & 12 \end{bmatrix}$

Ans: $\begin{bmatrix} 12 & 10 & 2 \\ 12 & -9 & 11 \\ -11 & 4 & 1 \end{bmatrix}$

12. $\begin{bmatrix} -7 & 4 & 12 \\ 8 & -16 & 5 \\ 12 & -4 & 9 \end{bmatrix} + \begin{bmatrix} 2 & -9 & 13 \\ 17 & 4 & 23 \\ 41 & 5 & -6 \end{bmatrix}$

Ans $\begin{bmatrix} -5 & -5 & 25 \\ 15 & -12 & 28 \\ 53 & 1 & 3 \end{bmatrix}$

13. $\begin{vmatrix} 2 & 4 \\ 8 & 3 \\ 1 & 5 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 9 & 2 \\ 3 & 7 \end{vmatrix}$

Ans: $\begin{vmatrix} 1 & 6 \\ 17 & 5 \\ 4 & 12 \end{vmatrix}$

14. $\begin{vmatrix} 3 & 2 & -5 \\ 2 & -7 & 5 \end{vmatrix} + \begin{vmatrix} 6 & 1 & 4 \\ 1 & 5 & 8 \end{vmatrix}$

Ans: $\begin{vmatrix} 9 & 3 & -1 \\ 3 & -2 & 13 \end{vmatrix}$

c. Operation of subtraction involving matrices.

NOTE:

- One rule that must be follow to do operation of subtraction which is the size of the matrix must be same

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

Example:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 1-5 & 2-6 \\ 3-7 & 4-8 \end{bmatrix} = \begin{bmatrix} -4 & -4 \\ -4 & -4 \end{bmatrix}$$

EXERCISE:
Solve the following matrix

1. $\begin{bmatrix} 6 & 8 \\ -14 & 33 \end{bmatrix} - \begin{bmatrix} 12 & 5 \\ -27 & -8 \end{bmatrix}$

$Ans: \begin{bmatrix} -6 & 3 \\ 13 & 41 \end{bmatrix}$

2. $\begin{bmatrix} 12 & 21 \\ -17 & 33 \end{bmatrix} - \begin{bmatrix} 5 & -8 \\ 2 & 19 \end{bmatrix}$

$Ans: \begin{bmatrix} 7 & 29 \\ -19 & 14 \end{bmatrix}$

3. $\begin{bmatrix} 12 & -15 \\ 34 & 3 \end{bmatrix} - \begin{bmatrix} 18 & 5 \\ 22 & 4 \end{bmatrix}$

$Ans: \begin{bmatrix} -6 & -20 \\ 12 & -1 \end{bmatrix}$

4. $\begin{bmatrix} -14 & 3 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} -7 & 5 \\ 4 & 3 \end{bmatrix}$

$Ans: \begin{bmatrix} -7 & 2 \\ -7 & -2 \end{bmatrix}$

5. $\begin{bmatrix} 6 & -3 \\ 9 & 14 \end{bmatrix} - \begin{bmatrix} -17 & 5 \\ 22 & 8 \end{bmatrix}$

$Ans: \begin{bmatrix} 23 & -8 \\ -13 & 6 \end{bmatrix}$

6. $\begin{bmatrix} 7 & -3 \\ 5 & 4 \end{bmatrix} - \begin{bmatrix} 15 & 2 \\ -3 & 8 \end{bmatrix}$

$Ans: \begin{bmatrix} -17 & 5 \\ 22 & 8 \end{bmatrix}$

7. $\begin{bmatrix} 3 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & -4 & 6 \end{bmatrix} - \begin{bmatrix} 11 & -8 & 1 \\ 5 & -2 & 19 \\ 4 & 12 & 3 \end{bmatrix}$

Ans: $\begin{bmatrix} -8 & 15 & 8 \\ -4 & 4 & -16 \\ 4 & -16 & 3 \end{bmatrix}$

8. $\begin{bmatrix} 13 & 6 & 9 \\ 10 & -4 & 2 \\ -8 & -2 & 12 \end{bmatrix} - \begin{bmatrix} -1 & 4 & -7 \\ 2 & -5 & 9 \\ -3 & 6 & -11 \end{bmatrix}$

Ans: $\begin{bmatrix} 14 & 2 & 16 \\ 8 & -1 & -6 \\ -5 & -8 & 23 \end{bmatrix}$

9. $\begin{bmatrix} -7 & 4 & 12 \\ 8 & -16 & 5 \\ 12 & -4 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -9 & 13 \\ 17 & 4 & 23 \\ 41 & 5 & -6 \end{bmatrix}$

Ans $\begin{bmatrix} -9 & 13 & -1 \\ -9 & -20 & -18 \\ -29 & -9 & 15 \end{bmatrix}$

10. $\begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 4 \\ -3 & 8 & 9 \\ -2 & 7 & -1 \end{bmatrix}$

Ans: $\begin{bmatrix} -3 & -2 & -5 \\ 7 & -25 & -14 \\ 8 & -6 & 9 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 6 & 4 \\ -3 & 8 & 9 \\ -2 & 7 & -1 \end{bmatrix} - \begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix}$

$\begin{bmatrix} 3 & 2 & 5 \\ -7 & 25 & 14 \\ -8 & 6 & -9 \end{bmatrix}$

12. $\begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & -4 & 6 \end{bmatrix}$

$\begin{bmatrix} -5 & -3 & -10 \\ 3 & -19 & -8 \\ -2 & 5 & 2 \end{bmatrix}$

13. $\begin{vmatrix} 3 & 2 & -5 \\ 2 & -7 & 5 \end{vmatrix} - \begin{vmatrix} 6 & 1 & 4 \\ 1 & 5 & 8 \end{vmatrix}$

Ans: $\begin{vmatrix} -3 & 1 & -9 \\ 1 & -12 & -3 \end{vmatrix}$

14. $\begin{vmatrix} 2 & 4 \\ 8 & 3 \\ 1 & 5 \end{vmatrix} - \begin{vmatrix} -1 & 2 \\ 9 & 2 \\ 3 & 7 \end{vmatrix}$

Ans: $\begin{vmatrix} 3 & 2 \\ -1 & 1 \\ 2 & -2 \end{vmatrix}$

d. Operation of Multiplication involving matrices.

NOTE:

- One rule that must be follow to do operation of multiplication which is **the number of columns of the first matrix must be same with the number of rows of the second matrix.**

Example:

$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$ $B = \begin{vmatrix} 1 \\ 2 \end{vmatrix}$, $find\ AxB$

2 x ② ② x1

$\therefore$ *we can proceed with multiplication since* number of columns of the first matrix same with the number of rows of the second matrix.

Matrix A		1 2 (Matrix B)
1	2	(1x1) + (2x2) = 1+ 4 = 5
4	6	(4x1) + (6x2) = 4 + 12 = 16

$= \begin{vmatrix} 5 \\ 16 \end{vmatrix}$

EXERCISE:
Solve the following matrix

1. $\begin{vmatrix} 10 & 11 & -5 \\ 8 & -3 & 9 \\ -2 & -3 & 6 \end{vmatrix} \times \begin{vmatrix} 2 \\ -3 \\ -1 \end{vmatrix}$

Ans: $\begin{vmatrix} -8 \\ 16 \\ -1 \end{vmatrix}$

2. $\begin{vmatrix} 6 & 7 & -8 \\ -2 & 5 & 3 \end{vmatrix} \times \begin{vmatrix} 4 & 7 \\ 9 & -2 \\ -5 & -5 \end{vmatrix}$

Ans: $\begin{vmatrix} 127 & 68 \\ 22 & -39 \end{vmatrix}$

3. $\begin{vmatrix} 3 & 7 \\ 2 & 3 \\ -5 & 20 \end{vmatrix} \times \begin{vmatrix} -3 & 4 & 6 \\ 8 & -9 & 2 \end{vmatrix}$

Ans: $\begin{vmatrix} 47 & -51 & 32 \\ 18 & -19 & 18 \\ 175 & -200 & 10 \end{vmatrix}$

4. $\begin{vmatrix} 1 & 14 \\ 3 & 5 \end{vmatrix} \times \begin{vmatrix} -2 \\ 13 \end{vmatrix}$

Ans: $\begin{vmatrix} 180 \\ 59 \end{vmatrix}$

5. $\begin{vmatrix} 3 & 3 & 2 \\ 6 & -1 & 1 \\ -2 & 5 & 2 \end{vmatrix} \times \begin{vmatrix} 6 \\ -4 \\ 7 \end{vmatrix}$

Ans: $\begin{vmatrix} 20 \\ 47 \\ -18 \end{vmatrix}$

6. $|2 \quad -6 \quad 11| \times \begin{vmatrix} 16 \\ -5 \\ 2 \end{vmatrix}$

Ans: $|84|$

7. $\begin{vmatrix} 7 & -2 \\ -3 & 5 \end{vmatrix} \times \begin{vmatrix} 2 & 2 \\ -5 & 6 \end{vmatrix}$

Ans: $\begin{vmatrix} 24 & 2 \\ -31 & 24 \end{vmatrix}$

8. $\begin{vmatrix} -11 & 5 \\ 7 & 2 \\ 6 & 14 \end{vmatrix} \times \begin{vmatrix} 4 & -3 & 2 \\ 5 & 2 & 1 \end{vmatrix}$

Ans: $\begin{vmatrix} -19 & 43 & -17 \\ 38 & -17 & 16 \\ 94 & 10 & 26 \end{vmatrix}$

9. $\begin{bmatrix} 5 & -2 \\ -2 & 11 \\ -3 & 12 \end{bmatrix} \times \begin{vmatrix} 2 & 2 \\ -5 & 6 \end{vmatrix}$

Ans: $\begin{bmatrix} -10 & -20 \\ 10 & -16 \\ 18 & 24 \end{bmatrix}$

10. $\begin{bmatrix} 6 \\ -3 \end{bmatrix} \times [-5 \quad 4]$

Ans: $\begin{bmatrix} -30 & 24 \\ 15 & -12 \end{bmatrix}$

11. Given the matrices A = $\begin{bmatrix} 1 & -3 & 4 \\ 2 & 10 & -7 \end{bmatrix}$ and B= $\begin{bmatrix} 5 & -2 \\ -2 & 11 \\ -3 & 12 \end{bmatrix}$. Calculate A x B

Ans: $\begin{vmatrix} -1 & 13 \\ 11 & 22 \end{vmatrix}$

12. Given the matrices C = $\begin{vmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{vmatrix}$ and D= $\begin{vmatrix} 12 & -1 & 2 \\ 6 & -3 & 8 \\ 7 & 1 & 5 \end{vmatrix}$ Calculate C x D

Ans: $\begin{vmatrix} -19 & 43 & -17 \\ 38 & -17 & 16 \\ 94 & 10 & 26 \end{vmatrix}$

e. Determinant of matrices.

NOTE:

- Only for square matrix → 2 x 2, 3 x 3
- Symbol for the determinant of matrix **A** is det**(A)** or **|A|**.

Example:

$\begin{vmatrix} 1 & -4 \\ 3 & 2 \end{vmatrix} = (1\ x\ 2) - (-4\ x\ 3) = 14$ (answer)

EXERCISE:
Find determinant of matrix below

1. $\begin{vmatrix} 0 & -4 \\ -6 & -2 \end{vmatrix}$

Ans: [-24]

2. $\begin{vmatrix} -6 & 0 \\ 6 & -6 \end{vmatrix}$

Ans: [36]

3. $\begin{vmatrix} -1 & 1 \\ -1 & 4 \end{vmatrix}$

Ans: [-3]

4. $\begin{vmatrix} 0 & 4 \\ 6 & 5 \end{vmatrix}$

Ans: [-24]

5. $\begin{vmatrix} 0 & -1 \\ 6 & -6 \end{vmatrix}$

Ans: [6]

6. $\begin{vmatrix} 5 & 3 \\ 6 & 6 \end{vmatrix}$

Ans: [12]

7. $\begin{vmatrix} -5 & 3 \\ 4 & 2 \end{vmatrix}$

Ans: [-24]

8. $\begin{vmatrix} -9 & -9 \\ -7 & -10 \end{vmatrix}$

Ans: [27]

NOTE:

- For matrix size 3x3, the method to find determinant is,

If $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Then $|A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

$= a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$

Example:

$A = \begin{vmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{vmatrix}$

$|A| = -4\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 6 & 5 \\ 11 & 4 \end{vmatrix} + 1\begin{vmatrix} 6 & 3 \\ 11 & 2 \end{vmatrix}$

$= -4(12 - 10) - 2(24 - 55) + 1(12 - 33)$
$= -8 + 62 - 21$
$= 33$ (answer)

EXERCISE:
Find determinant of matrix below

1. $\begin{vmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{vmatrix}$

Ans: [33]

2. $\begin{vmatrix} 12 & -1 & 2 \\ 6 & -3 & 8 \\ 7 & 1 & 5 \end{vmatrix}$

Ans: [-248]

3. $\begin{vmatrix} 3 & 3 & 2 \\ 6 & -1 & 1 \\ -2 & 5 & 2 \end{vmatrix}$

Ans: [-7]

4. $\begin{vmatrix} 5 & 3 & 3 \\ -4 & -5 & 1 \\ 5 & 3 & 0 \end{vmatrix}$

Ans: [39]

5. $\begin{bmatrix} -2 & 4 & -1 \\ 4 & -17 & -5 \\ 6 & 1 & 8 \end{bmatrix}$

Ans: [-92]

6. $\begin{bmatrix} 3 & 7 & 9 \\ 1 & 2 & 3 \\ 8 & -4 & 6 \end{bmatrix}$

Ans: [18]

7. $\begin{vmatrix} 3 & -2 & 1 \\ 3 & -1 & -2 \\ 3 & -2 & -3 \end{vmatrix}$

Ans: [-27]

8. $\begin{vmatrix} -3 & 2 & -3 \\ 0 & -1 & -1 \\ 3 & 0 & -3 \end{vmatrix}$

Ans: [-24]

9. $\begin{vmatrix} -6 & -6 & 1 \\ 3 & -5 & -2 \\ 4 & 3 & -3 \end{vmatrix}$

Ans: [-103]

10. $\begin{vmatrix} -5 & 5 & 5 \\ -8 & 9 & -3 \\ 8 & 5 & 9 \end{vmatrix}$

Ans: [-770]

11. $\begin{vmatrix} 5 & 6 & 2 \\ 1 & 4 & 0 \\ 8 & 9 & 3 \end{vmatrix}$

Ans: [-4]

12. $\begin{vmatrix} -1 & 0 & 1 \\ 3 & 4 & 5 \\ 9 & 5 & 6 \end{vmatrix}$

Ans: [-20]

f. Solve 3 simultaneous equation using inverse method

Example

Given 3 simultaneous equation below,

$-4x + 2y + z = 2$
$6x + 3y + 5z = 1$
$11x + 2y + 4z = 3$

Find the value of x, y and z.

- Step 1: transform into matrix form

$$\begin{bmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

A x b

$$A = \begin{vmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{vmatrix}$$

- Step 2: Find Determinant, |A|

$$|A| = -4\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 6 & 5 \\ 11 & 4 \end{vmatrix} + 1\begin{vmatrix} 6 & 3 \\ 11 & 2 \end{vmatrix}$$

$= -4(12 - 10) - 2(24 - 55) + 1(12 - 33)$
$= -8 + 62 - 21$
$= 33$

- Step 3: Find Minor, M

$M_{11} = 3 \times 5 \; / \; 2 \times 4$
$= 12 - 10$
$= 2$

$M_{12} = 6 \times 5 \; / \; 11 \times 4$
$= 24 - 55$
$= -31$

$M_{13} = 6 \times 3 \; / \; 11 \times 2$
$= 12 - 33$
$= -21$

$M_{21} = 2 \times 1 \; / \; 2 \times 4$
$= 8 - 2$
$= 6$

$M_{22} = -4 \times 1 \; / \; 11 \times 4$
$= -16 - 11$
$= -27$

$M_{23} = -4 \times 2 \; / \; 11 \times 2$
$= -8 - 22$
$= -30$

$M_{31} = 2 \times 1 \; / \; 3 \times 5$
$= 10 - 3$
$= 7$

$M_{32} = -4 \times 1 \; / \; 6 \times 5$
$= -20 - 6$
$= -26$

$M_{33} = -4 \times 2 \; / \; 6 \times 3$
$= -12 - 12$
$= -24$

$$\therefore M = \begin{bmatrix} 2 & -31 & -21 \\ 6 & -27 & -30 \\ 7 & -26 & -24 \end{bmatrix}$$

- Step 4:Find Cofactor, C

$$C = \begin{bmatrix} M_{11} & -M_{12} & M_{13} \\ -M_{21} & M_{22} & -M_{23} \\ M_{31} & -M_{32} & M_{33} \end{bmatrix}.$$

$$C = \begin{bmatrix} 2 & 31 & -21 \\ -6 & -27 & 30 \\ 7 & 26 & -24 \end{bmatrix}.$$

- Step 5:Find Adjoint, Adj A

$$\text{Adj A} = (\text{Cofactor})^T$$

$$\text{Adj A} = \begin{bmatrix} 2 & -6 & 7 \\ 31 & -27 & 26 \\ -21 & 30 & -24 \end{bmatrix}.$$

- Step 6:Find Inverse, A^{-1}

$$A^{-1} = \frac{1}{|A|} \times Adj\ A$$

$$A^{-1} = \frac{1}{33} \times \begin{bmatrix} 2 & -6 & 7 \\ 31 & -27 & 26 \\ -21 & 30 & -24 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{33} & \frac{-2}{11} & \frac{7}{33} \\ \frac{31}{33} & \frac{-27}{33} & \frac{26}{33} \\ \frac{-7}{11} & \frac{10}{11} & \frac{-8}{11} \end{bmatrix}$$

- Step 7: Find the value of $x, y\ and\ z$

$$x = A^{-1} \cdot b$$

$$\begin{bmatrix} \frac{2}{33} & \frac{-2}{11} & \frac{7}{33} \\ \frac{31}{33} & \frac{-27}{33} & \frac{26}{33} \\ \frac{-7}{11} & \frac{10}{11} & \frac{-8}{11} \end{bmatrix} \times \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

			$\begin{matrix}2\\1\\3\end{matrix}$
$\frac{2}{33}$	$\frac{-2}{11}$	$\frac{7}{33}$	$\frac{4}{33}-\frac{2}{11}+\frac{21}{33}=\frac{19}{33}$
$\frac{31}{33}$	$\frac{-27}{33}$	$\frac{26}{33}$	$\frac{62}{33}-\frac{27}{33}+\frac{78}{33}=-\frac{113}{33}$
$\frac{-7}{11}$	$\frac{10}{11}$	$\frac{-7}{11}$	$\frac{-14}{11}+\frac{10}{11}-\frac{24}{11}=\frac{-28}{11}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{19}{33} \\ \frac{113}{33} \\ \frac{-28}{11} \end{bmatrix}$$

$$\therefore x = \frac{19}{33}$$
$$y = \frac{113}{33}$$
$$z = \frac{-25}{11}$$

(answer)

EXERCISE:
Find the value of x,y and z by using Inverse method

1. $-4x + 6y + 8z = 40$
$-3x - 7y - 2z = -61$
$-5x + 9y - 1z = 98$

Ans: $x = -1$
$y = 10$
$z = -3$

2. $-3x - 9y + 6z = -45$
$-8x - 1y - 5z = -14$
$-4x + 7y + 2z = 107$

Ans: $x = -4$
$y = 11$
$z = 7$

3. $-9x - 3y - 2z = -72$
$-5x + 7y + 8z = 68$
$-x - 4y + 6z = 37$

Ans: $x = 5$
$y = 3$
$z = 9$

4. $-x - 9y - 3z = 120$
$2x + 6y + 8z = -124$
$-5x - 7y + 4z = 30$

Ans: $x = 3$
$y = -11$
$z = -8$

5. $3x - 2y + 5z = 6$
$9x - 8y + 1z = 46$
$-6x - 4y + 7z = 27$

Ans: $x = -1$
$y = -7$
$z = -1$

6. $-x - 8y - 5z = 143$
$-6x - 3y + 7z = -81$
$2x + 9y - 4z = -41$

Ans: $x = 5$
$y = -11$
$z = -12$

7. $5x - 3y - 7z = -50$
$-1x + 4y - 6z = -40$
$8x - 9y + 2z = -2$

$Ans: x = -8$
$y = -6$
$z = 4$

8. $-8x - 1y - 7z = 52$
$-6x + 5y - 4z = 12$
$2x - 3y + 9z = 50$

$Ans: x = -1$
$y = -7$
$z = -1$

9. $-9x + 7y - 4z = 188$
$-3x + 2y - 1z = 56$
$-6x + 5y + 8z = 44$

$Ans: x = -8$
$y = 12$
$z = -8$

10. $-5x + 7y - 3z = -95$
$-2x + 1y + 9z = -47$
$6x + 4y - 8z = 0$

$Ans: x = 4$
$y = -12$
$z = -3$

g. Cramer's rule

NOTE and Example

Given:

$-4x + 2y + z = 2$
$6x + 3y + 5z = 1$
$11x + 2y + 4z = 3$

Find the value of each value of x, y and z using the Inverse matrix.

- Step 1: Transform into matrix form

$$\begin{bmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

A x b

- Step 2: Find A, A_1, A_2, A_3

$$A = \begin{vmatrix} -4 & 2 & 1 \\ 6 & 3 & 5 \\ 11 & 2 & 4 \end{vmatrix}, \quad A_1 = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 5 \\ 3 & 2 & 4 \end{vmatrix}, \quad A_2 = \begin{vmatrix} -4 & 2 & 1 \\ 6 & 1 & 5 \\ 11 & 3 & 4 \end{vmatrix}, \quad A_3 = \begin{vmatrix} -4 & 2 & 2 \\ 6 & 3 & 1 \\ 11 & 2 & 3 \end{vmatrix}$$

- Step 3:Find determinant of matrix A, $|A|$

$$|A| = -4\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 6 & 5 \\ 11 & 4 \end{vmatrix} + 1\begin{vmatrix} 6 & 3 \\ 11 & 2 \end{vmatrix}$$

$$= -4(12 - 10) - 2(24 - 55) + 1(12 - 33)$$
$$= -8 + 62 - 21$$
$$= 33$$

- Step 4 :Find determinant of matrix A_1, $|A_1|$

$$A_1 = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 3 & 5 \\ 3 & 2 & 4 \end{vmatrix}$$

$$|A_1| = 2\begin{vmatrix} 3 & 5 \\ 2 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 5 \\ 3 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 3 & 2 \end{vmatrix}$$

$$= 2(12 - 10) - 2(4 - 15) + 1(2 - 9)$$
$$= 4 + 22 - 7$$
$$= 19$$

- Step 5:Find determinant of matrix $A_2, |A_2|$

$$A_2 = \begin{vmatrix} -4 & 2 & 1 \\ 6 & 1 & 5 \\ 11 & 3 & 4 \end{vmatrix}$$

$$|A_2| = -4\begin{vmatrix} 1 & 5 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 6 & 5 \\ 11 & 4 \end{vmatrix} + 1\begin{vmatrix} 6 & 1 \\ 11 & 3 \end{vmatrix}$$

$$= -4(4-15) - 2(24-55) + 1(18-11)$$
$$= 44 + 62 + 7$$
$$= 113$$

- Step 6:Find determinant of matrix $A_3, |A_3|$

$$|A_3| = -4\begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} - 2\begin{vmatrix} 6 & 1 \\ 11 & 3 \end{vmatrix} + 2\begin{vmatrix} 6 & 3 \\ 11 & 2 \end{vmatrix}$$

$$= -4(9-2) - 2(18-11) + 2(12-33)$$
$$= -28 - 14 - 42$$
$$= -84$$

- Step 7: Find the value of x, y and z

$$x = \frac{|A_1|}{|A|}$$
$$y = \frac{|A_2|}{|A|}$$
$$z = \frac{|A_3|}{|A|}$$

$$\therefore x = \frac{|A_1|}{|A|} = \frac{19}{33}$$

$$y = \frac{|A_2|}{|A|} = \frac{113}{33}$$

$$z = \frac{|A_3|}{|A|} = \frac{-84}{33} = \frac{-28}{11}$$

(answer)

EXERCISE:
Find the value of x,y and z by using Cramer's rule

1. $-4x + 6y + 8z = 40$
$-3x - 7y - 2z = -61$
$-5x + 9y - 1z = 98$

$Ans: x = -1$
$y = 10$
$z = -3$

2. $-3x - 9y + 6z = -45$
$-8x - 1y - 5z = -14$
$-4x + 7y + 2z = 107$

$Ans: x = -4$
$y = 11$
$z = 7$

3. $-9x - 3y - 2z = -72$
$-5x + 7y + 8z = 68$
$-x - 4y + 6z = 37$

$Ans: x = 5$
$y = 3$
$z = 9$

4. $-x - 9y - 3z = 120$
$2x + 6y + 8z = -124$
$-5x - 7y + 4z = 30$

$Ans: x = 3$
$y = -11$
$z = -8$

5. $3x - 2y + 5z = 6$
$9x - 8y + 1z = 46$
$-6x - 4y + 7z = 27$

$Ans: x = -1$
$y = -7$
$z = -1$

6. $-x - 8y - 5z = 143$
$-6x - 3y + 7z = -81$
$2x + 9y - 4z = -41$

$Ans: x = 5$
$y = -11$
$z = -12$

7. $5x - 3y - 7z = -50$
$-1x + 4y - 6z = -40$
$8x - 9y + 2z = -2$

$Ans: x = -8$
$y = -6$
$z = 4$

8. $-8x - 1y - 7z = 52$
$-6x + 5y - 4z = 12$
$2x - 3y + 9z = 50$

$Ans: x = -11$
$y = -6$
$z = 6$

9. $-9x + 7y - 4z = 188$
$-3x + 2y - 1z = 56$
$-6x + 5y + 8z = 44$

$Ans: x = -8$
$y = 12$
$z = -8$

10. $-5x + 7y - 3z = -95$
$-2x + 1y + 9z = -47$
$6x + 4y - 8z = 0$

$Ans: x = 4$
$y = -12$
$z = -3$