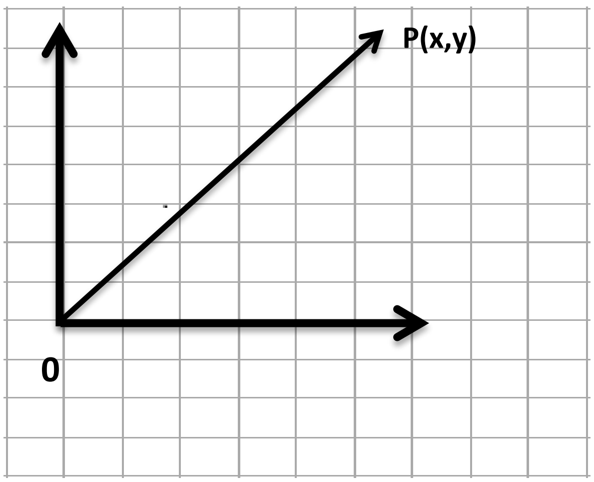
a. Magnitude Vector.

NOTE:

- The magnitude or modulus of a vector is its length and is normally denoted by $|\vec{A}|$
- If $A = xi + yj$, then the magnitude $|\vec{A}| = \sqrt{x^2 + y^2}$

Example 1:



Given $\overrightarrow{OP} = \binom{3}{4}$, find $|\overrightarrow{OP}|$

$$\overrightarrow{OP} = 3i + 4j$$

$$|\overrightarrow{OP}| = \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$= 5$ unit (answer)

EXERCISE

Solve each of the following:

1. Find the magnitude of the vector $\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j}$.

Ans : 5 unit

2. If P is the point $(13, 8)$ and O is the origin. Find the magnitude of the vector $\overrightarrow{\boldsymbol{OP}}$.

$Ans: \sqrt{233}\ unit$

3. Find $|PQ|$ if the position vector of $\boldsymbol{p} = \boldsymbol{i} - 5\boldsymbol{j}$ and position vector $\boldsymbol{q} = 7\boldsymbol{i} + 5\boldsymbol{j}$.

$Ans: \sqrt{136}\ unit$

4. If given vector $\vec{\mathrm{a}} = 2\mathrm{i} + 3\mathrm{j} + \mathrm{k}$ and $\vec{\mathrm{b}} = \mathrm{i} - \mathrm{j} + 2\mathrm{k}$, find
   a. $2\mathrm{a} + \mathrm{b}$
   b. $|2\mathrm{a} + \mathrm{b}|$

$Ans: a)\ 5i + 5j + 4k\quad b)\ \sqrt{66}$

5. Find the negative vector for OA $= 5\mathrm{i} + 3\mathrm{j} - 2\mathrm{k}$ and then unit vector OA

$Ans:\ AO = -5i - 3j + 2k, \qquad \widehat{AO} = \frac{-5i}{\sqrt{38}} - \frac{3j}{\sqrt{38}} + \frac{2k}{\sqrt{38}}$

6. Given $\vec{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\vec{B} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, find,

   a. The vector for $\vec{A} + 2\vec{B}$

   b. The magnitude for $|2\vec{A}|$

$Ans:\ a)\ \begin{pmatrix} 7 \\ 4 \end{pmatrix} \quad b)\ 4.472\ unit$

7. Given $A(2,5), B(-2,3)$ and $C(3,7)$. Find

   a. $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in form of $xi + yj$

   b. $|\overrightarrow{AC}|$

$Ans: a)\ \overrightarrow{AC} = 2i + 5j,\ \overrightarrow{CB} = -5i - 4j\ \ b)\ |\overrightarrow{AC}| = \sqrt{29}\ unit$

8. Given $\vec{A} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $\vec{B} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$, if $\vec{C} = 2\vec{A} + \vec{B}$, find.

   a. $\vec{C}$

   b. The magnitude for $|2\vec{C}|$

$Ans: a)\begin{pmatrix} 0 \\ 8 \end{pmatrix} \quad b)\ 16\ unit$

b. Unit Vector.

NOTE:

- A Vector is physical quantity which is a positif real number, the magnitude or length and a direction in space. Examples of vectors are velocity, force, displacement, etc that has a magnitude and direction.
- A Scalar is physical quantity which ia positive real number and magnitu or length without a direction in space. Examples of scalars are speed, temperature, time, etc that has only a magnitude.

$$\text{Unit Vector, } \hat{v} = \frac{\vec{A}}{|A|}$$

Example 1:

Given $\vec{A} = 3i - 4j$, Find unit vector in direction of $\vec{A}$

$$\vec{A} = 3i - 4j$$

$$|\vec{A}| = \sqrt{3^2 + -4^2}$$

$$= \sqrt{25}$$

$$= 5$$

There for, $\hat{v} = \frac{(3i-4j)}{5}$

$= \frac{3i}{5} - \frac{4j}{5}$ (answer)

Example 2:

Given $\vec{a} = 7i + 3j - 2k$ and $\vec{b} = -5i + 2j + 3k$, Find unit vector in direction of $\vec{a} + \vec{b}$

$$\vec{a} + \vec{b} = (7i + 3j - 2k) + (-5i + 2j + k)$$

$$= (2i + 5j + k)$$

$$|\vec{a} + \vec{b}| = \sqrt{2^2 + 5^2 + 1^2}$$

$$= \sqrt{30}$$

There for, $\widehat{\vec{a} + \vec{b}} = \frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$

$$= \frac{2i+5j+k}{\sqrt{30}}$$

$$= \frac{2i}{\sqrt{30}} + \frac{5j}{\sqrt{30}} + \frac{k}{\sqrt{30}}$$ (answer)

Example 3:

Solve that $\overrightarrow{OX} = 6i - 3j + k$ and $\overrightarrow{OY} = 2i + 4k - 5k$, Finf the unit vector in the direction of $\overrightarrow{XY}$

$$\overrightarrow{XY} = \overrightarrow{OY} - \overrightarrow{OX}$$

$$= \begin{pmatrix} 2 \\ 4 \\ -5 \end{pmatrix} - \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ 7 \\ -6 \end{pmatrix}$$

$$|\overrightarrow{XY}| = \sqrt{(-4)^2 + (7)^2 + (-6)^2}$$

$$= \sqrt{101}$$

There for, Unit vector $\widehat{\overrightarrow{XY}} = \frac{\overrightarrow{XY}}{|\overrightarrow{XY}|}$

$$= \frac{-4i + 7j - 6k}{\sqrt{101}}$$

$$= \frac{-4}{\sqrt{101}}i + \frac{7}{\sqrt{101}}j - \frac{6}{\sqrt{101}}k$$ (answer)

EXERCISE
Solve each of the following:

1. Find the unit vector for $\tilde{r} = 4i + 3j$.

$Ans\!: a)\ \frac{4i}{5} + \frac{3j}{5}$

2. A, B and C is a triangle with (0,1,3), (4,-1,2) and (1,3,-5) respectively. Calculate unit vector of,

   a. $\overrightarrow{AB}$
   b. $\overrightarrow{BC}$

$Ans\!: a)\ \frac{4i}{\sqrt{21}} - \frac{2j}{\sqrt{21}} - \frac{k}{\sqrt{21}}\quad b) - \frac{3i}{\sqrt{74}} + \frac{4j}{\sqrt{74}} - \frac{7k}{\sqrt{74}}$

3. Find the negative for $OA = 5i + 3j - 2k$ and than find unit vector of AO.

$Ans:\ AO = -5i - 3j + 2k,\ \ Vu = -\frac{5}{\sqrt{38}}i - \frac{3}{\sqrt{38}}j + \frac{2}{\sqrt{38}}k$

4. Given $\overrightarrow{OA} = 2i - j + 3k$ and $\overrightarrow{OB} = 3i + 2j - 4k$ and $\overrightarrow{OC} = -i + 3j - 2kk$. Determine unit vector for:

$a.\ \ \overrightarrow{AB}$

$b.\ \ \overrightarrow{BC}$

$Ans: a)\ \frac{1}{\sqrt{59}}i + \frac{3}{\sqrt{59}}j - \frac{7}{\sqrt{59}}k \ \ b) - \frac{4i}{\sqrt{21}} + \frac{1}{\sqrt{21}}j + \frac{2}{\sqrt{21}}k$

5. Given vectors $\vec{A} = 3i + 2j + k$, $\vec{B} = 2i - j + 2k$. Find the units vector for:

   a. $\vec{A}$
   b. $\vec{B}$

$Ans{:}\ a)\ \frac{3i}{\sqrt{14}} + \frac{2j}{\sqrt{14}} + \frac{k}{\sqrt{14}} \quad b)\ \frac{2i}{3} - \frac{j}{3} + \frac{2k}{3}$

6. If given vector $\vec{a} = 2i + 3j + k$ and $\vec{b} = i - j + 2k$, find unit vector of $2a + b$.

$Ans: \frac{5i}{\sqrt{66}} + \frac{5j}{\sqrt{66}} + \frac{4k}{\sqrt{66}}$

7. If given $\vec{B} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$, find the unit vector of $\vec{B}$

$Ans: \frac{i}{\sqrt{14}} + \frac{3j}{\sqrt{14}} + \frac{2k}{\sqrt{14}}$

8. Given $\vec{A} = 2i - j$ and $\vec{B} = -4i + 3j$. Find the value

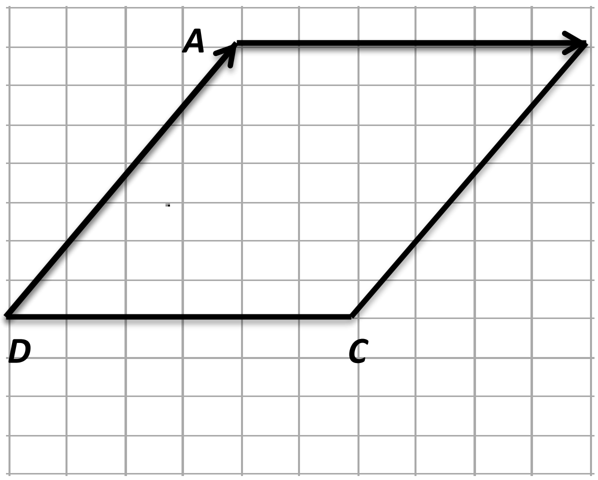
a. $\vec{R}$ if $\vec{R} = 4\vec{A} + 2\vec{B}$

b. Unit vector in the direction $\vec{R}$

$Ans: a)\ \vec{R} = 2j \quad b)\ V.u, \vec{R} = j$

c. Operations Of Vectors

Note and Example : ADDITION (parallelogram method)



ABCD is a parallelogram, sum vector in unit og vector guide

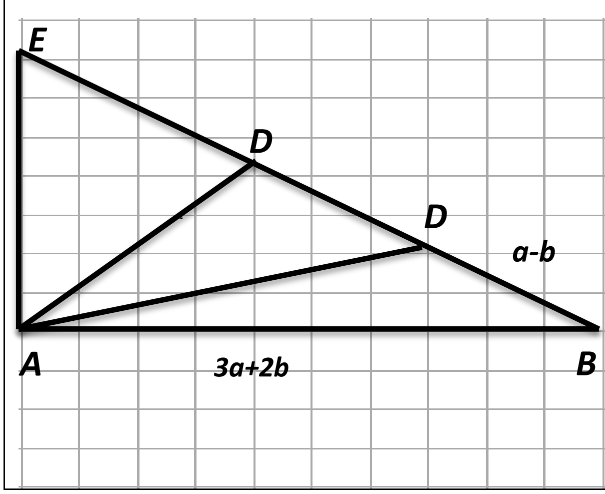
$\overrightarrow{DA} + \overrightarrow{AC} = \overrightarrow{DC}$ (answer)

$\overrightarrow{AD} + \overrightarrow{AB} = \overrightarrow{AD} + \overrightarrow{DC} = \overrightarrow{AC}$ (answer)

$\overrightarrow{AB} + \overrightarrow{CB} = \overrightarrow{DC} + \overrightarrow{CB} = \overrightarrow{DB}$ (answer)

Note and Example : ADDITION (Triangle method)

Demonstrate addition and subtraction of vectors using Triangle Constuction method.



BCDE is a straight line that $\overrightarrow{BC} = \overrightarrow{CD} = \overrightarrow{DE}$ .Find the vectors below in term of a if $\overrightarrow{AB} = 3a + 2b$ and $\overrightarrow{BC} = a - b$

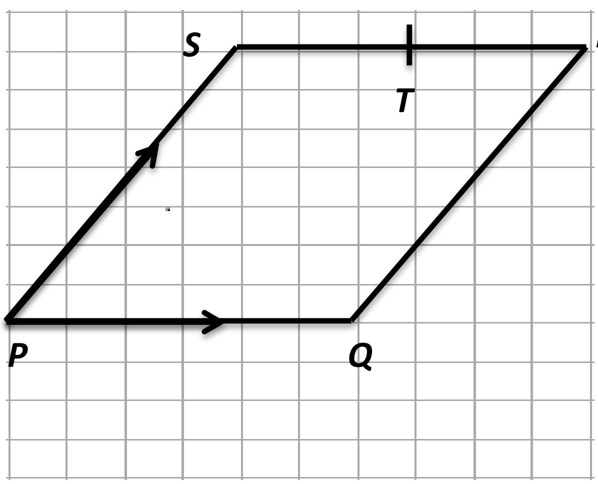
i. $\overrightarrow{ED}$
ii. $\overrightarrow{AC}$
iii. $\overrightarrow{DA}$

i. $\overrightarrow{ED} = \overrightarrow{CB} = -\overrightarrow{BC} = -(a - b) = b - a$ (answer)

ii. $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BA}$
$= 3a + 2b + a - b$
$= 4a + b$ (answer)

iii. $\overrightarrow{DA} = \overrightarrow{DB} + \overrightarrow{BA}$
$= 2\overrightarrow{CB} + (-\overrightarrow{AB})$ $\quad$ $*\overrightarrow{BC} = \overrightarrow{CD} = a - b$
$= 2(-\overrightarrow{BC}) - \overrightarrow{AB}$
$= -2(a - b) - (3a + 2b)$
$= -5a$ (answer)

Note and Example : SUBTRACTION (parallelogram method)



PQRS is parallelogram with $\overrightarrow{PQ} = m$ and $\overrightarrow{PS} = n$. T is the midpoint of SR.

$\overrightarrow{SQ} = \overrightarrow{SP} + \overrightarrow{PQ}$

$= -\overrightarrow{PS} + \overrightarrow{PQ}$

$= -n + m$ (answer)

$\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{SR}$

$= \overrightarrow{PS} + \overrightarrow{PQ}$

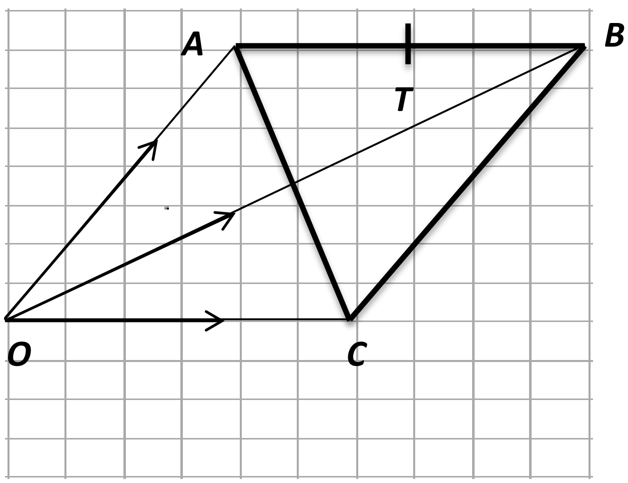
$= n + m$ (answer)

$\overrightarrow{PT} = \overrightarrow{PS} + \overrightarrow{ST}$

$= \overrightarrow{PS} + \frac{1}{2}\overrightarrow{SR}$

$= n + \frac{1}{2}m$ (answer)

Note and Example : SUBTRACTION (Triangle method)



Prove that the points of vector $\overrightarrow{OA} = 2a + 3b$, $\overrightarrow{OB} = 11a + 6b$ and $\overrightarrow{OC} = 14a + 7b$ are in the same line. Hence determine the ratio of the vectors.

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
$= 11a + 6b - 2a - 3b$
$= 9a + 3b$
$= 3(3a + b)$ (answer)

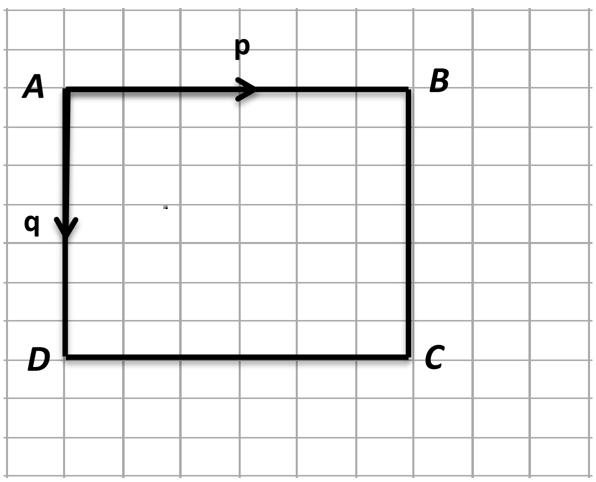
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$
$= 14a + 7b - 11a - 6b$
$= 3a + b$ (answer)

$\overrightarrow{AB} : \overrightarrow{BC}$
$3(3a+b) : (3a+b)$

There for, the ratio of the vector is $3 : 1$ (answer)

EXERCISE
Solve each of the following:

1.



Based on diagram above, $\overrightarrow{AB} = p$ and $\overrightarrow{AD} = q$. Express in terms of p and q.

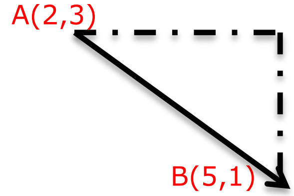
i. $\overrightarrow{AC}$
ii. $\overrightarrow{DB}$

$Ans: i)\ \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = p + q \quad ii)\ \overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB} = p - q$

2. Given A and B are the points with coordinate (2,3) and (5,1) respectively.

   a. Sketch vector $\overrightarrow{AB}$ using triangle method.
   b. Define the value of $\overrightarrow{AB}$
   c. Calculate the magnirud of vector

$Ans: a)$



$b)\ 3i - 2j \quad c)\ 3.61\ unit$

d. Scalar (dot) Product Of Two Vectors

Note

- **Scalar (dot) Product** is the vectors are expressed in terms of unit vectors i, j and k along the x, y and z directions, the scalar product can also be expressed in the form:
- Example of scalar (dot) product:

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

- Since I and j and k are all one units in length and they are all mutually perpendicular, we have:

Where,

$$\vec{A} = A_x \vec{\imath} + A_y \vec{\jmath} + A_z \vec{k}$$
$$\vec{B} = B_x \vec{\imath} + B_y \vec{\jmath} + B_z \vec{k}$$

$$i.i = j.j = k.k = 1$$
$$i.j = j.i = i.k = j.k = k.j = 0$$

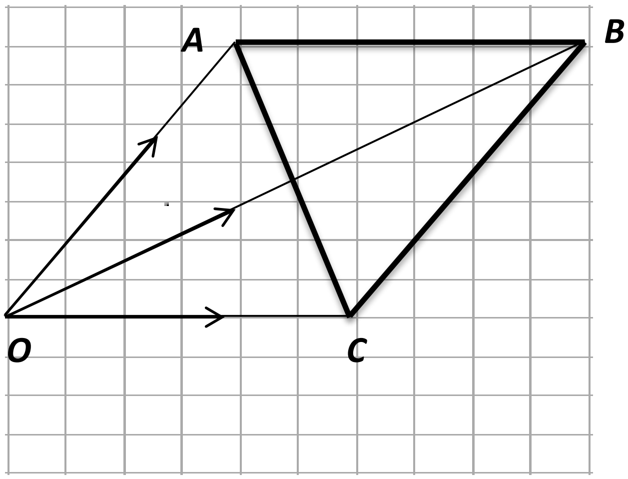
Example 1:

If $A = 2i + 3j + k$ and $B = 5i - j + 4k$, afind the $A \cdot B$.

$$\begin{aligned} A \cdot B &= (2i + 3j + k) \cdot (5i - j + 4k) \\ &= (10i^2 - 3j^2 + 4k^2 \\ &= 10 - 3 + 4 \\ &= 11 \quad \text{(answer)} \end{aligned}$$

Example 2:

Coordinates A(1,3,5), B(4,-1,2) and C(6,3,4) are the coordinates in a triangle ABC. Find the scalar (dot) product AB and BC.



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$
$= -(i + 3j + 5k) + (4i - j + 2k)$
$= 3i - 4j - 3k$

$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$
$= -(4i - j + 2k) + (6i + 3j + 4k)$
$= 2i + 4j + 2k$

$\overrightarrow{AB} \cdot \overrightarrow{BC} = (3i - 4j - 3k) \cdot (2i + 4j + 2k)$
$= 6i^2 - 16j^2 - 6k^2$
$= 6 - 16 - 6$
$= -16$ (answer)

Example 3:

Given $\overrightarrow{M} = 5i + 2j - k$ and $\overrightarrow{N} = 3i - j + 4k$, determine:

i. $\overrightarrow{M} \bullet \overrightarrow{N}$

$\overrightarrow{M} \bullet \overrightarrow{N} = (5i + 2j - k) \bullet (3i - j + 4k)$
$= 15i^2 - 2j^2 - 4k^2$
$= 15 - 2 - 4$
$= 9$ (answer)

ii. $|\overrightarrow{OM}|$

$|\overrightarrow{OM}| = \sqrt{(5)^2 + (2)^2 + (-1)^2}$

$= \sqrt{25 + 4 + 1}$

$= \sqrt{30}$ (answer)

iii. $|\overrightarrow{ON}|$

$|\overrightarrow{ON}| = \sqrt{(3)^2 + (-1)^2 + (4)^2}$

$= \sqrt{9 + 1 + 16}$

$= \sqrt{26}$ (answer)

EXERCISE

Solve each of the following:

1. If $\vec{P} = 2i + 4j + 6k$ and $\vec{Q} = 2i + 4j + 3k$. Find

   i. $\vec{P} \bullet \vec{Q}$

   ii. $(\vec{P} \bullet \vec{Q})\vec{P}$

$Ans: \ i)\ 38 \quad ii)\ 76i + 152j + 228k$

2. If $\vec{r}_1 = 2i + j$ and $\vec{r}_2 = i - 3j$, Find

   i. $\vec{r}_1 \bullet \vec{r}_2$

   ii. $\vec{r}_1 \bullet (\vec{r}_2 - \vec{r}_1)$

   iii. $(10\vec{r}_1 + \vec{r}_2) \bullet \vec{r}_1$

$Ans: \ i) -1 \quad ii) -6 \quad iii)\ 49$

3. If $\vec{r}_1 = 2i + 4j - 3k$ and $\vec{r}_2 = i + 3j + 2k$, find the dot product for this vectors.

$Ans:\ 8$

4. Given $A = 2i - 3j + 4k, B = i - 2j - 3k$ and $C = 2i + j + 2k$, Find:

   i. $A \cdot B$

   ii. $(B \cdot C)A$

$Ans:\ i) - 4\quad ii) - 12i + 18j - 24k$

i. Given vector $\vec{A} = i - 5j + 6k, \vec{B} = 2i - 3j - k$ and $\vec{C} = 2i + 3j + k$, determine:

   i. $\vec{A} \bullet \vec{B}$

   ii. $\vec{A} \bullet 2\vec{C}$

$Ans:\ i)11\quad ii) - 14$

e. Vector (cross) Product Of Two Vectors

Note

- **The Vector Cross Product** of is the vectors is abinary operation in three-dimensional space that result in athird vector that is perpendicular to the plane that contains the two input vector. The direction of the resulting vector is determined by the order of the input vector, so the vector cross product does not have a associative or commutative property. The vector cross product has extensive uses in mathematics and physics, as well as practical applications in computer graphics.
- Example of Vector (Cross) product:

$$\vec{A} = a_1 i + b_1 j + c_1 k \text{ and } \vec{B} = a_2 i + b_2 j + c_2 k$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

- Let said,

Where,

$$= (b_1 c_2 - b_1 c_1)i - (a_1 c_2 - a_2 c_1)j + (a_2 b_2 - a_2 b_2)k$$

Example 1:

If $A = 2i + 4j + 3k$ and $B = i + 5j - 2k$, Determine $A \times B.(or\ vector\ perpendicular\ A\ and\ B)$

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 2 & 4 & 3 \\ 1 & 5 & -2 \end{vmatrix}$$

$$= i\begin{vmatrix} 4 & 3 \\ 5 & -2 \end{vmatrix} - j\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} + k\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$$

$$= i(-8 - 15) - j(-4 - 3) + (10 - 4)$$

$= -23i + 7j + 6k$ (answer)

Example 2:

Find the dot product and cross product for vector $\vec{P}$ and $\vec{Q}$ respectively are $3i - 4j + 2k$ and $3i - 5j - k$.

$$\begin{aligned}\vec{P} \bullet \vec{Q} &= (3i - 4j + 2k) \bullet (2i + 5j - k)\\ &= 6i^2 - 20j^2 - 2k^2\\ &= 6(1) - 20(1) - 2(1)\\ &= -16 \quad \text{(answer)}\end{aligned}$$

$$\vec{P} \times \vec{Q} = \begin{vmatrix} i & j & k \\ 3 & -4 & 2 \\ 2 & 5 & -1 \end{vmatrix}$$

$$= i\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix}$$

$$= i(4 - 10) - j(-3 - 4) + (15 + 8)$$

$$= -6i + 7j + 23k \quad \text{(answer)}$$

Example 3:

a) Given vector $\vec{A} = 1i - 5j + 6k$, $\vec{B} = 2i - 3j - 1k$ and $\vec{C} = 2i + 3j + k$, determine:

i. $\vec{A} \times \vec{B}$

ii. $\vec{A} \bullet (\vec{B} \times \vec{C})$

iii. $\vec{A} \times (\vec{B} \times \vec{C})$

i. $$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ 1 & -5 & 6 \\ 2 & -3 & -1 \end{vmatrix}$$

$$= i\begin{vmatrix} -5 & 6 \\ -3 & -1 \end{vmatrix} - j\begin{vmatrix} 3 & 2 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} 3 & -4 \\ 2 & 5 \end{vmatrix}$$

$$= i(5 + 18) - j(-1 - 12) + k(-3 + 10)$$

$$= 23i + 13j + 7k \quad \text{(answer)}$$

ii. $\vec{A} \bullet (\vec{B} \times \vec{C})$

$$\vec{B} \times \vec{C} = \begin{vmatrix} i & j & k \\ 2 & -3 & -1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= i\begin{vmatrix} -3 & -1 \\ 3 & 1 \end{vmatrix} - j\begin{vmatrix} 2 & -1 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 2 & -3 \\ 2 & 3 \end{vmatrix}$$

$$= i(-3+4) - j(9+12) + k(6+6)$$

$$= i - 21j + 12k$$

$$\vec{A} \bullet (\vec{B} \times \vec{C}) = (i - 5j + 6k) \bullet (i - 21j + 12k)$$

$$= i^2 + 105j^2 + 72k^2$$

$$= 1 + 105 + 72$$

$= 178$ (answer)

iii. $\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} i & j & k \\ 1 & -5 & 6 \\ 1 & -21 & 12 \end{vmatrix}$

$$= i\begin{vmatrix} -5 & 6 \\ 21 & 12 \end{vmatrix} - j\begin{vmatrix} 1 & 6 \\ 1 & 12 \end{vmatrix} + k\begin{vmatrix} 1 & -5 \\ 1 & -21 \end{vmatrix}$$

$$= i(-60+126) - j(12-6) + k(-21+5)$$

$= 66i - 6j - 16k$ (answer)

EXERCISE

Solve each of the following:

1. If $\vec{P} = 2i + 2j - 2k$ and $\vec{Q} = 2i - 2j + 3k$. Find $\vec{P} \times \vec{Q}$

Ans: $2i - 10j - 8k$

2. Given vector $\vec{A} = 2i - j + 3k$, $\vec{B} = 3i - 2j - 4k$ and $\vec{C} = -i + 3j - 2k$, determine:

i. $\vec{A} \times \vec{B}$

ii. $\vec{A} \bullet (\vec{B} \times \vec{C})$

iii. $\vec{A} \times (\vec{B} \times \vec{C})$

Ans: $i) 10i + 17j - k$ $\quad ii)$ 43 $\quad iii) - 37i + 34k + 36k$

3. Given vector $\vec{A} = i - 2j + 4k$, $\vec{B} = -3i + 2j - 2k$ and $\vec{C} = i - 3j + 3k$, determine:

$i.$ $\vec{A} \times \vec{B}$

ii. $\vec{A} \bullet (\vec{B} \times \vec{C})$

iii. $\vec{A} \times (\vec{B} \times \vec{C})$

Ans: $i) - 4i - 10j + 8k$ $\quad ii)$ 14 $\quad iii) 7(-6i - j + k)$

4. Given vector $\vec{P} = 2i - 3j + 4k$, $\vec{Q} = i - 2j - 3k$ and $\vec{R} = 2i + j + 2k$, determine:

i. $\vec{P} \times \vec{Q}$

ii. $\vec{P} \bullet (\vec{Q} \times \vec{R})$

iii. $\vec{P} \times (\vec{Q} \times \vec{R})$

$Ans:\ i) 17i + 10j - k \quad ii)\ 42 \quad iii)\ 27i - 6j - 19k$

5. Given vector $\vec{M} = 5i - 2j + 3k$, $\vec{N} = 3i + j - 2k$ and $\vec{O} = i - 3j + 4k$, determine:

i. $\vec{M} \bullet (\vec{N} \times \vec{O})$

ii. $\vec{M} \times (\vec{N} \times \vec{O})$

$Ans:\ i) - 12 \quad ii)\ 62i + 46j - 74k$

f. Area Of Parallelogram

Note

- **The Area Of Parallelogram** is geometrically the magnitude of the cross product of two vectors coincides with the area of the parallelogram whose sides are formed by those vectors.
- So the area of this parallelogram is the absolute value of the determinant of formula:

$$\text{Area, A} = \left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$$

Example 1:

A parallelogram contains $A(2,-1,4)$ $B(2,1,-4)$ and $C(-2,-3,3)$, find the area of this plane.

$OA = 2i - j + 4k$

$OB = 2i + j - 4k$

$OC = -2i - 3j + 3k$

$AB = AO + OB$

$\quad = -(2\,i - j + 4k) + (2i + j - 4k)$

$\quad = 2j - 8k$

$BC = BO + OC$

$\quad = -(2i + j - 4k) + (-2i - 3j + 3k)$

$\quad = -4i - 4j + 7k$

$$AB \times BC = \begin{vmatrix} i & j & k \\ 0 & 2 & -8 \\ -4 & -4 & 7 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & -8 \\ -4 & 7 \end{vmatrix} - j\begin{vmatrix} 0 & -8 \\ -4 & 7 \end{vmatrix} + k\begin{vmatrix} 0 & 2 \\ -4 & -4 \end{vmatrix}$$

$$= (14-32)i - (0-32)j + (0+8)k$$

$$= -18i + 32j + 8k$$

$Area\ of\ parallelogram\ is = |AB \times BC|$

$= (\sqrt{(-18)^2 + (32)^2 \ + (8)^2})$

$= \ (\sqrt{324 + 1024 + 64})$

$= \ (\sqrt{1412})$

$= 18.788\ unit^2$ (answer)

Example 2:

A, B and C are points (1, -1, 3), (3, 1, 4) and (-2, 4, 3) respectively. Show that the area of parallelogram is $\sqrt{290}$ unit^2.

$OA = i - j + 3k$
$OB = 3i + j + 4k$
$OC = -2i + 4j + 3k$

$AB = AO + OB$
$= -\ (\ i - j + 3k) + (3i + j + 4k)$
$= 2i + 2j + k$

$BC = BO + OC$
$= -\ (3i + j + 4k) + (-2i + 4j + 3k)$
$= -5i + 3j - k$

$$AB \text{ x } BC = \begin{vmatrix} i & j & k \\ 2 & 2 & 1 \\ -5 & 3 & -1 \end{vmatrix}$$

$$= i\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} - j\begin{vmatrix} 2 & 1 \\ -5 & -1 \end{vmatrix} + k\begin{vmatrix} 2 & 2 \\ -5 & 3 \end{vmatrix}$$

$$= (-2-3)i - (—2—(-5j)) + (6—(-10)k$$

$$= \ -5i - 3j + 16k$$

$Area\ of\ parallelogram\ is = |\ AB \times BC|$

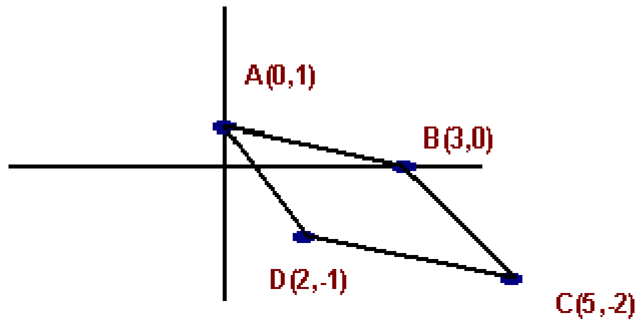
$= \ (\sqrt{(-5)^2 + (-3)^2 \ + (16)^2})$

$= \ (\sqrt{25 + 9 + 256})$

$= (\sqrt{290})\ \ unit^2$ (answer)

Example 3:

For the parallelogram with points A, B and C are (0,1), (3,0) and (2,-1)



$a = |DC| = (5{,}2) - (2,-1) = (3,-1)$

$b = |AD| = (0{,}1) - (2,-1) = (-2{,}2)$

Area A, $\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 3 & -1 & 0 \\ -2 & 2 & 0 \end{vmatrix}$

$= i\begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} - j\begin{vmatrix} 3 & 0 \\ -2 & 0 \end{vmatrix} + k\begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix}$

$= i(0 - 0) - j(0 - 0) + (6 - 2)$

$= 0i + 0j + 4k$

$Area\ of\ parallelogram\ is = |\ \vec{a} \times \vec{b}|$

$= (\sqrt{(0)^2 + (0)^2 + (4)^2})$

$= (\sqrt{16}$

$= 4\ unit^2$ (answer)

EXERCISE Solve each of the following:
1. Find the area of parallelogram spanned box by vector $\overrightarrow{AB} = -i - 5j + 2k$ and $\overrightarrow{AC} = 2i + 3k$ $Ans: \quad 19.34\ unit^2$
2. Find the area of the parallelogram which is formed by the vector $\vec{U} = (3,1,-1)$ and $\vec{V} = (2,3,4)$ $Ans: \quad \sqrt{294}\ unit^2$

3. Finf the area of the parallelogram bonded by the vector $\vec{a} = (7,3,-4)$ and $\vec{b} = (1,0,6)$

$Ans: \quad 49.5 \quad unit^2$

4. A, B and C is a points on the parallelogram bunded with (0,1,3), (4,-1,2) and (1,3,-5) respectively. Calculate the area of this parallelogram.

$Ans: \quad 37.22 \quad unit^2$

5. A, B and C are points (1, -1, 3), (3, 1, 4) and (-2, 4, 3) respectively. Show that the area of parallelogram is $\sqrt{290}$ unit^2.

Ans: $\sqrt{290}\ unit^2$

6. A parallelogram points is contains $A(2,-1,4)$ $B(2,1,-4)$ and $C(-2,-3,3)$, find the area of this parallelogram.

Ans: $37.60\ unit^2$

7. Find the area of the parallelogram if given the vertices are $S(2,0,-3)$, $B(2,1,-4)$ and $U(-2,-3,3)$.

Ans: $64.89\ unit^2$