a. Define sine, cosine, tangent, secant, cosecant and cotangent

NOTES:
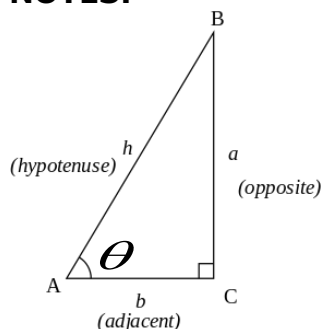
$$sin \theta = \frac{opposite}{hypotenuse} \ (SOH)$$

$$cos \theta = \frac{adjacent}{hypotenuse} \ (CAH)$$

$$tan \theta = \frac{opposite}{adjacent} \ (TOA) = \frac{sin \theta}{cos \theta}$$

$$sec \theta = \frac{1}{cos \theta}$$

$$cos \, ec\theta = \frac{1}{sin \theta}$$
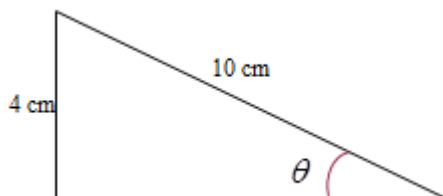
$$cot \theta = \frac{1}{tan \theta}$$

**Pythagoras Theorem
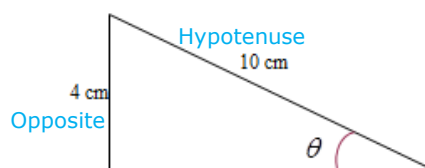Hypotenuse^2= adjacent^2 +opposite^2

Example:

Find the following:
   i. $sin \theta$
   ii. $tan \theta$
   iii. $sec \theta$

Solve:

i. $sin \theta = \dfrac{opposite}{hypotenuse} \ (SOH)$

$$sin \theta = \frac{4}{10} = \frac{2}{5} \text{ (answer)}$$

ii. $tan \theta = \dfrac{opposite}{adjacent}$

   **Pythagoras Theorem
   Hypotenuse^2= adjacent^2 +opposite^2
$$10^2 = adjacent^2 + 4^2$$
$$10^2 - 4^2 = adjacent^2$$
$$84 = adjacent^2$$
$$\sqrt{84} = 9.165 = adjacent$$
$$tan\theta = \frac{4}{\sqrt{84}} = \frac{4}{9.165} \text{ (answer)}$$

iii. $sec \theta = \dfrac{1}{cos \theta}$

$$cos \theta = \frac{adjacent}{hypotenuse} = \frac{\sqrt{84}}{10}$$

$$sec \theta = \frac{1}{\frac{\sqrt{84}}{10}} = \frac{10}{\sqrt{84}} \text{ (answer)}$$

EXERCISE

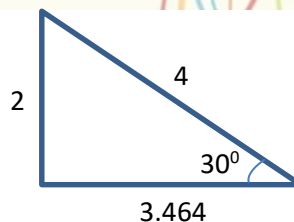**1.** Given $tan\ \theta = \dfrac{10}{24}$, evaluate :

   i. $sin\ \theta$

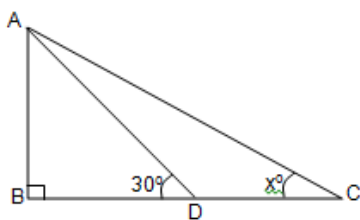   ii. $sec\ \theta$

Ans:i. $\dfrac{10}{26}$ *,ii.* $\dfrac{26}{24}$

2. Referring to figure below, find the values of: sin 30º, cos 30º and tan 30º.

Ans: $\dfrac{2}{4}$, $\dfrac{3.464}{4}$, $\dfrac{2}{3.464}$

3. From the figure below, BDC is a straight line. Given AD=4 cm and AC= 6 cm. Calculate :-

   i.   Length AB

   ii.   Sin x

Ans:i.2cm,ii.19.471⁰

4. Given sin θ =3/5.
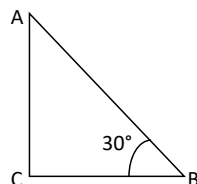
   i.   tan θ

   ii.   cos θ

   iii.   cosec θ

   iv.   cot θ

   v.   sec θ

Ans: $i.\dfrac{3}{4}$ *,ii.* $\dfrac{4}{5}$ *,iii.* $\dfrac{5}{3}$ *,iv.* $\dfrac{4}{3}$ *,v.* $\dfrac{5}{4}$

5. Figure below shows that BC=5cm, angle ABC=30°. Calculate
   - i.   The length of AB
   - ii.   The length of AC

A
30°
C   B

Ans:i.5.774cm,ii.2.887cm

6. AC=12cm and BC=10cm. Calculate
   - i.   The angle ABC
   - ii.   The hypotenuse

A
12cm
C   B
10cm
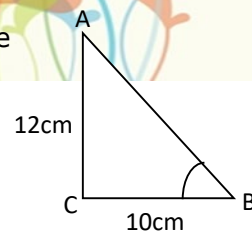
Ans:i.50.194⁰,ii.15.621cm

7. Based on triangle above, find $\cot \theta$.

θ
1 cm   2 cm
30°
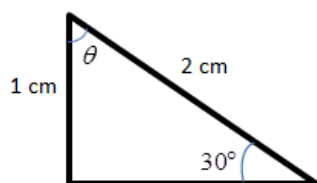
Ans:1

8. Based on diagram above, QTS is a straight line. Given that $\sin x° = \dfrac{8}{17}$ and $\tan y° = \dfrac{4}{3}$. Calculate the length of ST.

Q
y x
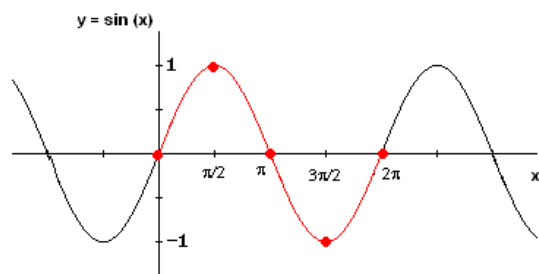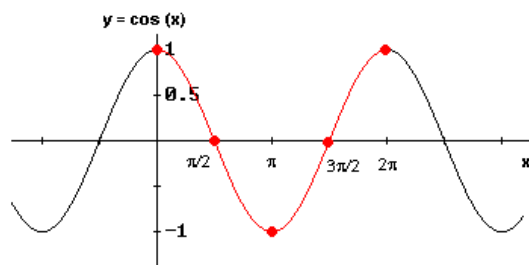6 cm
P
T
S   R
8 cm

Ans:5cm

42
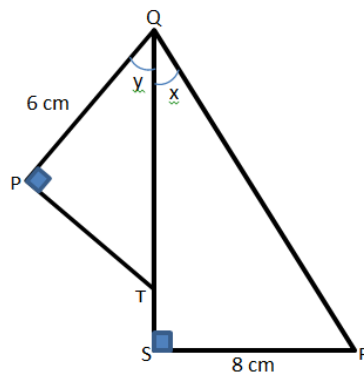
b. **Sketch the graph of sine, cosine and tangent.**
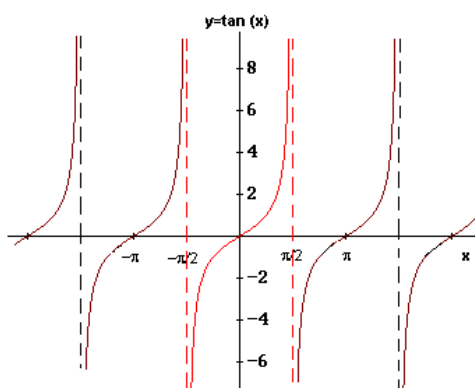
NOTES:

Graphs of sine:

Graphs of cosine:

Graphs of tan:

Example:

Sketch the graph of the function $y = 3\cos\dfrac{x}{2}$ for $0° \leq 0 \leq 360°$.

x	0^0	90^0	180^0	270^0	360^0
$y = 3\cos\dfrac{x}{2}$					

Solve:

x	0^0	90^0	180^0	270^0	360^0
$y = 3\cos\dfrac{x}{2}$	3	2.121	0	-2.121	-3

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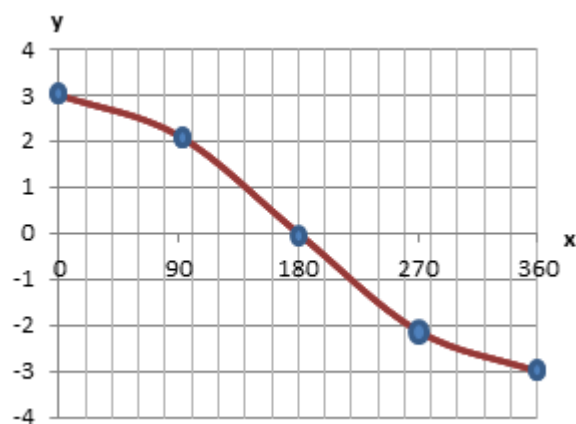
EXERCISE

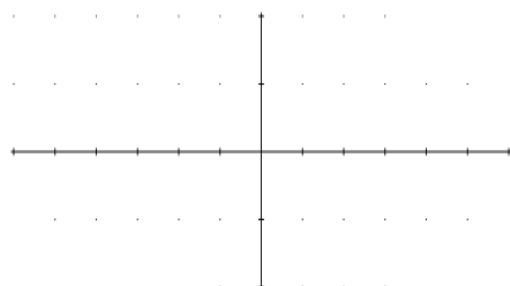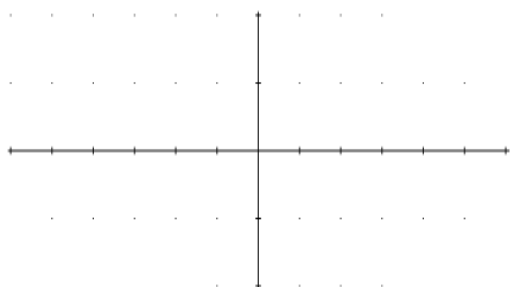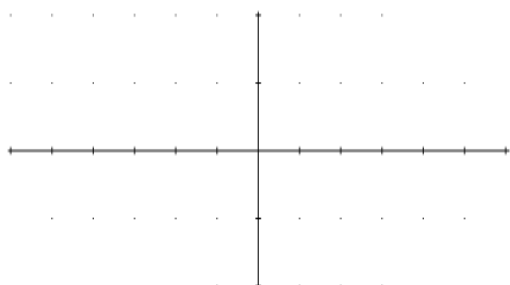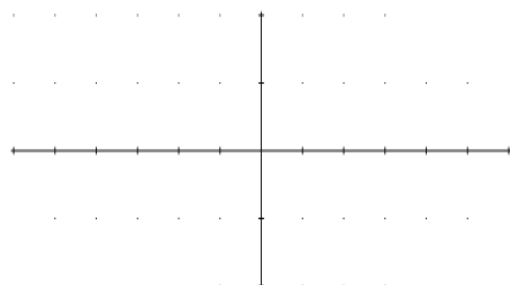| 1. Sketch the graph of the function $y = 2\sin(x - 45°)$ for $0° \leq x \leq 360°$.	2. Sketch the graph of the function $y = -2\cos(3\theta)$ for $0° \leq \theta \leq 360°$.
3. Sketch the graph of the function $y = \frac{1}{2}\cos\left(x - \frac{\pi}{2}\right)$ for $0 \leq x \leq 2\pi$.	4. Sketch the graph of the function $y = \sin\left(\frac{x}{2} + 90°\right)$ for $0° \leq x \leq 360°$.
5. Sketch the graph of the function $y = \sin x + \cos x$ for $0° \leq x \leq 360°$.	6. Sketch the graph of the function $y = \cos 2x - \cos x$ for $0° \leq x \leq 360°$.
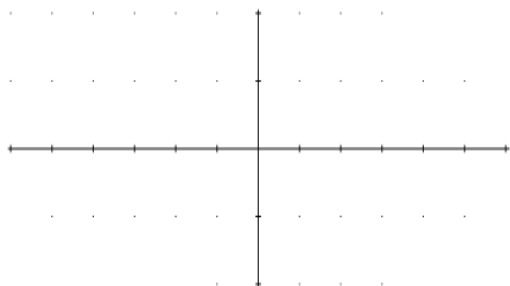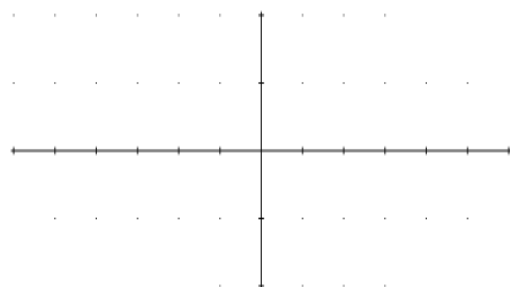
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c.   Determine the positif and negative values of trigonometric function using quadrant

NOTES:

A full circle is 360º and if we split a circle into 4 equal parts (called **quadrants**), each quadrant will take 90º.
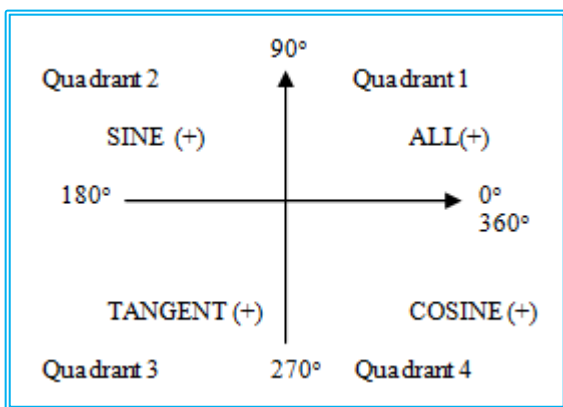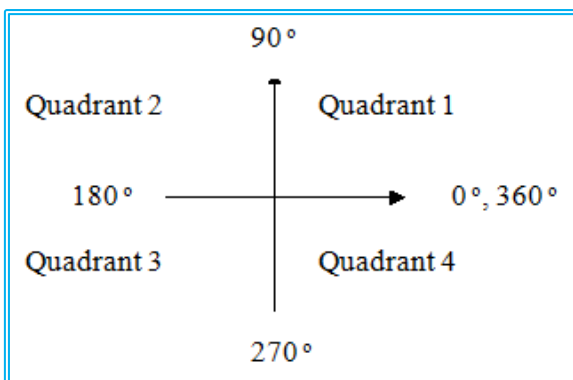
Find the **reference angle** for the angle resulting.

Example 1:

Find the value of $sin\, x = 0.6909$, for $0^o \leq x \leq 360^o$.

Solve:
$sin\, x = 0.6909$

$x = sin^{-1}\, 0.6909 = 43.701°\,(reference\ angle)$

$sin\, x$ is a +ve value.
So it is located in **quadrant I and II.**

i. $43.701°$ (answer)

ii. $180° - 43.701° = 136.299°$
(answer)

Example 2:

Find the value of $tan\,\theta = -4.011$, for $0^o \leq \theta \leq 360^o$.
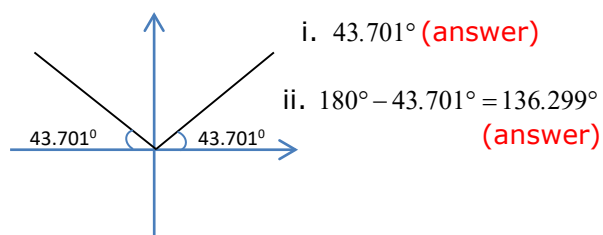
Solve:
$tan\,\theta = -4.011$

$\theta = tan^{-1}\, 4.011$

$= 76.001°\,(reference\ angle)$

$tan\,\theta$ is a −ve value.
So it is located in **quadrant II and IV.**

i. $180° - 76.001° = 103.999°$
(answer)

ii. $360° - 76.001° = 283.999°$
(answer)

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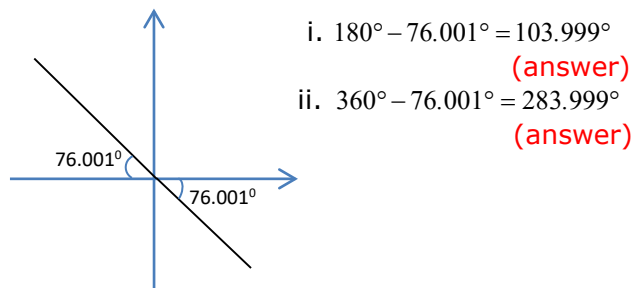| EXERCISE	

1. State the quadrant and find the reference angle for each of the following:
  i. $145°$
 	
  ii. $\dfrac{8}{5}\pi \ radian$
 	
Ans:i.Quadrant 2,35⁰ ii.Quadrant 4,72⁰
	2. Given that $cos\ y = -0.8660$ for $0° \le y \le 360°$. Find the possible values of y.
	
	Ans: 149.997°,210.003°

3. Use your scientific calculator to find the value of θ for $0^o \le \theta \le 360^o$:
  i. $cos\ ec\ \theta = -1.4226$
  ii. $cot\ \theta = -1$
  iii. $sec\ \theta = 1.5760$
 	
Ans: i.224.668°,315.332° ii.135°,315° iii.50.580°,309.42°
	4. Find the value each of the following for $0^o \le \theta \le 360^o$:
	  i. $sin\ \theta = -0.5$
	  ii. $tan\ \theta = 1.732$
	  iii. $sec\ \theta = -2.366$
	
	Ans:i. 210°,330° *,ii.* 60°,240° *, iii.* 115.005°,244.995°

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5. Given that $\sin\theta = -\sin 48°$. This means that θ is in Quadrant 3 or Quadrant 4. Find the values of θ for $0^o \le \theta \le 360^o$.

Ans: 227.988°,312.012°

6. Given that $\cos\theta = -0.3082$ and $0^o \le \theta \le 360^o$, find the values of θ.

Ans: 107.951°,252.049°

7. Given that $\tan\theta = -1.048$ and $0^o \le \theta \le 360^o$, find the values of θ.

Ans: 133.657°,313.657°

8. Given that $sin\dfrac{\theta}{2} = 0.707$ and $0^o \le \theta \le 360^o$, find the values of θ.

Ans: 89.982°,270.018°

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9. Given that $tan\,2\theta = 1.723$ and $0^o \le \theta \le 360^o$, find the values of θ.

Ans: 29.935°,119.935°,209.935°,299.935°

10. $sin\,x + cos\,42° = 0$ find the value of x for $0^o \le x \le 360^o$:

Ans: 227.988°,312.012°

11. $cos\,\theta = \dfrac{\sqrt{3}}{2}$, find the value of θ for $0^o \le \theta \le 360^o$:

Ans: 30.003°,329.997°

12. $sin\,\theta = -\dfrac{1}{\sqrt{2}}$, find the value of θ for $0^o \le \theta \le 360^o$:

Ans: 224.991°,315.009°

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d. **Solve trigonometric equations involving trigonometric basic identities, compound angel and double angle formulae.**

NOTES:

Basic Identities

$sin^2 A + cos^2 A = 1$

$tan^2 A + 1 = sec^2 A$

$cot^2 A + 1 = cosec^2 A$

Compound Angel

$sin(A \pm B) = sin A cos B \pm cos A sin B$

$cos(A \pm B) = cos A cos B \pm sin A sin B$

$tan(A \pm B) = \dfrac{tan A \pm tan B}{1 \mp tan A tan B}$

Double Angle

$sin 2A = 2 sin A cos A$

$cos 2A = cos^2 A - sin^2 A$

$\qquad = 2 cos^2 A - 1$

$\qquad = 1 - 2 sin^2 A$

$tan 2A = \dfrac{2 tan A}{1 - tan^2}$

Example 1:

$tan x \, sin^2 x = 2 tan x$

Solve:

$tan x \, sin^2 x - 2 tan x = 0$

$tan x(sin^2 x - 2) = 0$

$tan x = 0, sin^2 x - 2 = 0$

$x = tan^{-1} 0$

$x = 0°, 180°$ (answer)

$sin^2 x - 2 = 0$

$sin^2 x = 2$

$sin x = \sqrt{2}$

$x = sin^{-1} 1.414$

$no \ solution.$ (answer)

Example 2:

$sin 2t = sin t$ **#Double Angle**

Solve:

$sin 2t = 2 sin t \, cos t$

$2 sin t \, cos t = sin t$

$2 sin t \, cos t - sin t = 0$

$sin t(2 cos t - 1) = 0$

$sin t = 0$

$t = sin^{-1} 0$

$t = 0°, 180°$ (answer)

$(2 cos t - 1) = 0$

$2 cos t = 1$

$cos t = \dfrac{1}{2} @ 0.5$

$t = cos^{-1} 0.5$

$t = 60°, 300°$ (answer)

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EXERCISE

1. $2\sin^2 x - 5\sin x + 2 = 0$

Ans: $30°,150°,$ *no solution*

2. $\sin^2 x - 2\sin x - 3 = 0$

Ans: $270°,$ *no solution*

3. $3\cos 2x - 5\cos x = 1$

Ans: $120°,240°,$ *no solution*

4. $2\tan x\cos x + 2\cos x = \tan x + 1$

Ans: $60°,300°$

5. $2\sin^2 x - 1 = 0$

Ans: 44.991°,135.009°

6. $\cos x = 3\cos x - 2$

Ans: 0°,360°

7. $\tan x = \sin x$

Ans: 0°,180°,360°

8. $\cos x \sin 2x = 0$

Ans: 0°,90°,180°

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10. $4\sin^2 x - 4\sin x + 1 = 0$

Ans: 30°,150°

12. $\tan^2 x + \tan x = 0$

Ans: 0°,180°,360°,45°,135°,315°

13. $2\sin 2x = 1$

Ans: 15°,75°,195°,255°

14. $\cos 2x + \sin x = 1$

Ans: 0°,180°,360°,30°,150°

e. **Apply sine and cosine rules.**

NOTES:

a, b and c are **SIDES**.

A, B and C are **ANGLES**.
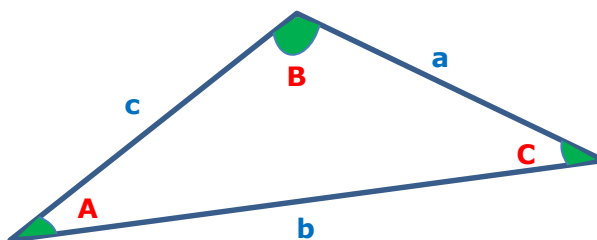
SINE RULES	**COSINE RULES**
USED to solve when given :	**USED to solve when given :**
1. **2 angles** and **1 side**	1. **3 sides**
2. **2 sides** and **1 non-included angle**	2. **2 sides** and **1 included angle**
non-included angle	included angle
3. Formula	3. Formula
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$	$$a^2 = b^2 + c^2 - 2bc \cos A$$
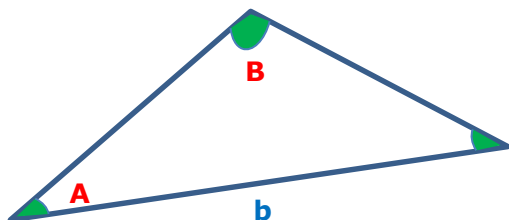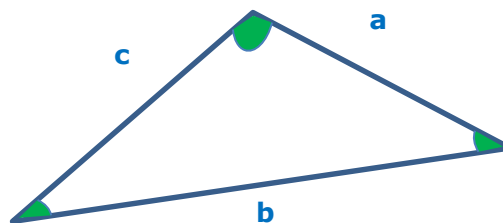
53
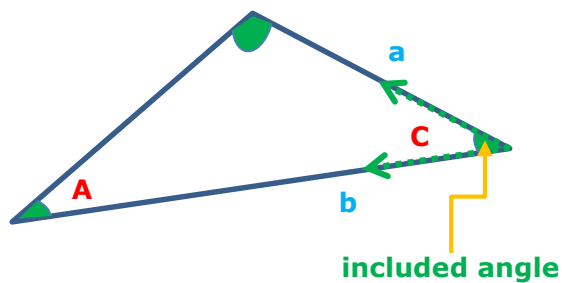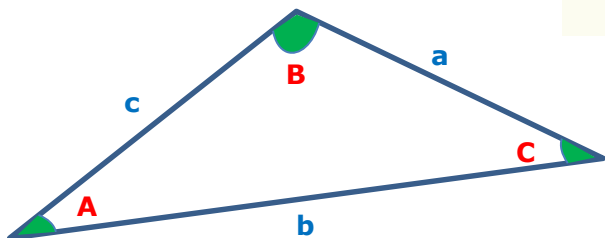
AREA OF TRIANGLE

Formula:
Area Of Triangle
$$= \frac{1}{2}ab \sin C$$

-OR-

$$= \frac{1}{2}bc \sin A$$
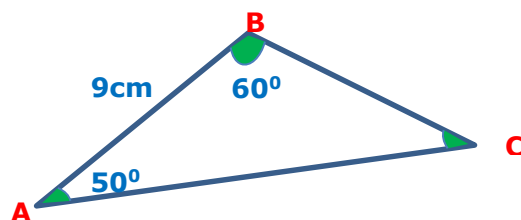
-OR-

$$= \frac{1}{2}ca \sin B$$

Example 1:
Solve the triangle ABC with side AB=9cm, A=50⁰ and B=60⁰ to find length of BC.

Solve:
i. **SKETCH THE TRIANGLE** to determine whether using formula of sine rules or cosine rules. By using all data given in question.

ii.

2 angles and **1 side = SINE RULES**

iii.   Length of:
AB=c
BC=a
AC=b

iv. $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

***This is general formula for sine rules. In reality however, you will use only two of the three fractions in one time. So the rule we will be using is
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50}{a} = \frac{\sin(180 - 50 - 60)}{9}$$

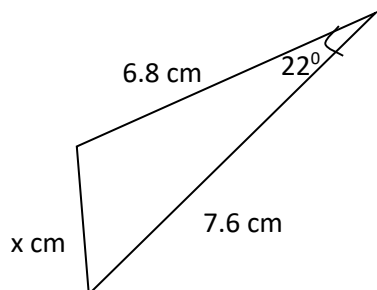$$\frac{0.766}{a} = \frac{0.940}{9}$$

$$0.766(9) = 0.940(a)$$
$$a = 7.334cm \text{ (answer)}$$
***which $a = length\ of\ BC$

Example 2:

Find the value of **x** and **the area of triangle.**

Solve:
   i.   **2 sides** and **1 included angle =**
        COSINE RULES

   ii. $a^2 = b^2 + c^2 - 2bc\,cosA$
        (you can label your own ABC)

   iii. $6.8^2 = 7.6^2 + x^2 - 2(6.8)(7.6)cos22°$

   iv. $6.8^2 = 7.6^2 + x^2 - 95.834$

   v. $6.8^2 - 7.6^2 + 95.834 = x^2$

   vi. $84.314 = x^2$

   vii. $9.182cm = x$ (answer)

the area of triangle.

Solve:

   i.   ANGLE is at C (I assume refer from
        previous question).
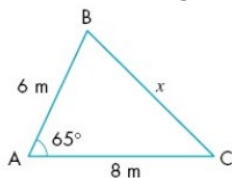
   ii. The formula is:
        $$= \frac{1}{2}ab\,sin\,C$$
        $$= \frac{1}{2}(6.8)(7.6)\,sin\,22°$$
        $$= 9.680cm^2 \text{ (answer)}$$

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EXERCISE	
1. Find the length of x	2. Find the length of x
   6 m B x 65° A 8 m C *Ans:* 7.709*m*	 B 15 cm C 102° 22 cm x A *Ans:* 29.090*cm*
3. Find the length of x	4. In triangle ABC, AB=5cm, BC=6cm and angle $\angle ABC = 55°$. Find the length of AC.
  B x C 32 cm 45 cm 37° A *Ans:* 27.367*cm*	 *Ans:* 5.156*cm*
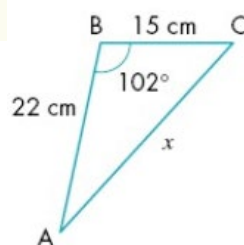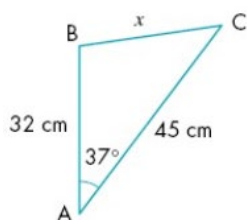
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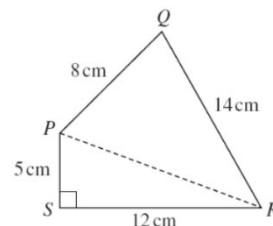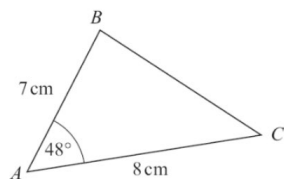
5. Find the value of x and y by using sine rule.
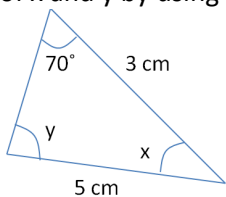
Ans: $x = 75.667°, y = 34.333°$

6. The quadrilateral PQRS has dimensions as shown. Angle PSR=90⁰. Calculate the area of triangle of PQR.

Ans: $51.160cm^2$

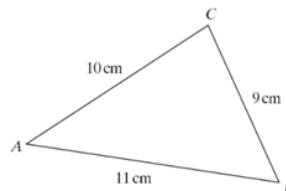7. ABC is a triangle.

i.   Calculate the length of side BC.
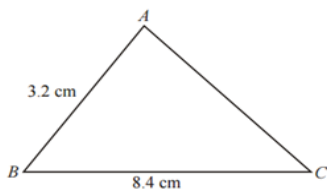ii.   Find the angle BCA.

Ans: $6.169cm$

8. In triangle ABC, AB=11cm, BC=9cm, CA=10cm. Find the area of triangle ABC.
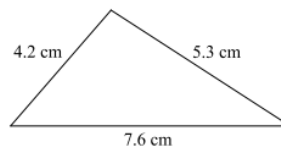

Ans: $42.443cm^2$

9.

The area of triangle ABC is 10cm².
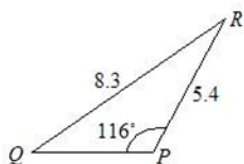Calculate the length of AC.

Ans: 6.699*cm*

10.

i. Calculate the largest angle of the triangle.
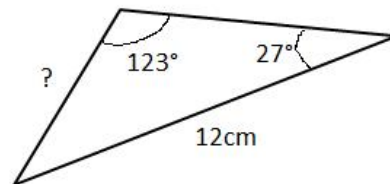ii. Calculate the area of triangle.

Ans: $i.105.704°, ii.10.715cm^2$

11. Solve $\angle PQR$ in which $\angle P = 116°, p = 8.3cm$ and $q = 5.4cm$.

Ans: 35.803°

12.

Ans: 6.493*cm*

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