a. Simplify algebraic expression.

NOTE:

- Simplifying expressions is done by cancelling out the common factor from both the numerator and denominator.
- To reduce an algebraic fraction (rational expression), follow the same process you use to reduce numeric fractions:

    a. Factor
    b. Divide out (cancel) factors common to both the numerator and denominator.
    c. Simplify

Example 1:

Simplify $\frac{x}{2x}$

$\frac{(\cancel{x})}{2(\cancel{x})} = \frac{1}{2}$ (answer)

By cancelling out the common x in the numerator and denominator.

Example 2:

Simplify $\frac{5x^2y}{15xy}$

$\frac{\cancel{5}\cancel{x}^2\cancel{y}}{\cancel{15}\cancel{x}\cancel{y}} = \frac{x}{3}$ (answer)

Cancel out common factors in algebraic fractions to make a simpler equivalent fraction.

Example 3:

$\frac{4a+2ab}{2a} = \frac{\cancel{2a}(2+b)}{\cancel{2a}} = 2+b$ (answer)

2a is a factor in common for the two terms in the sum and then cancelling.

EXERCISE Simplify the following:		
1. $\frac{3x}{15}$ Ans: $\frac{x}{5}$	2. $\frac{x^2y}{15xy}$ Ans: $\frac{x}{15}$	3. $\frac{21x^4y^7}{3xy^2}$ Ans: $7x^3y^5$

4. $\dfrac{10r^3s}{5rs^2}$ Ans: $\dfrac{2r^2}{s}$	5. $\dfrac{8x^3y^4}{2x^2y}$ Ans: $4xy^3$	6. $\dfrac{5b^3c}{15bc}$ Ans: $\dfrac{b^2}{3}$
7. $\dfrac{(2ab)^3}{4a^7b}$ Ans: $\dfrac{2b^2}{a^4}$	8. $\dfrac{2x+10}{4}$ Ans: $\dfrac{1}{2}(x+5)$	9. $\dfrac{x^2-4x}{x-4}$ Ans: x
10. $\dfrac{5x+20}{x+4}$ Ans: 5	11. $\dfrac{12x-9}{16x^2-9}$ Ans: $\dfrac{3}{4x+3}$	12. $\dfrac{6ab+2a}{2b}$ Ans: $\dfrac{3ab+a}{b}$
13. $\dfrac{9x+27}{9x+18}$ Ans: $\dfrac{x+3}{x+2}$	14. $\dfrac{16m^2n-8mn}{12m-6}$ Ans: $\dfrac{4mn}{3}$	15. $\dfrac{4mnp-8mp}{12mn}$ Ans: $\dfrac{np-2p}{3n}$
16. $\dfrac{3x^3}{12x}$ Ans: $\dfrac{x^2}{4}$	17. $\dfrac{1-x}{x-1}$ Ans: -1	18. $\dfrac{5m^2}{15m}$ Ans: $\dfrac{1}{3}m$
19. $\dfrac{15m}{-5mn}$ Ans: $\dfrac{-3}{n}$	20. $\dfrac{3x^2-9x}{x-3}$ Ans: $3x$	21. $\dfrac{16}{4x+12}$ Ans: $\dfrac{4}{x+3}$
22. $\dfrac{6x^2-6}{3x^2-6x+3}$ Ans: $\dfrac{2x+2}{x-1}$	23. $\dfrac{x^2-25}{x^2+10x+25}$ Ans: $\dfrac{x-5}{x+5}$	24. $\dfrac{k+1}{k^2-4k-5}$ Ans: $\dfrac{1}{k-5}$

25. $\dfrac{b-6}{(b+2)(6-b)}$ Ans: $\dfrac{-1}{b+2}$	26. $\dfrac{9-x^2}{x^2+8x+15}$ Ans: $\dfrac{3-x}{x+5}$	27. $\dfrac{2-x}{2x^2+3x-14}$ Ans: $\dfrac{-1}{2x+7}$
28. $\dfrac{4a^2-2a}{4a^2-1}$ Ans: $\dfrac{2a}{2a+1}$	29. $\dfrac{6x^3+12x}{x^2+2}$ Ans: $6x$	30. $\dfrac{p^2+8p+16}{8p+32}$ Ans: $\dfrac{p+4}{8}$
31. $\dfrac{x+3}{x^2-2x-15}$ Ans: $\dfrac{1}{x-5}$	32. $\dfrac{28abc^2}{49a^3c}$ Ans: $\dfrac{4bc}{7a^2}$	33. $\dfrac{25-w^2}{w^2-2w-15}$ Ans: $\dfrac{-(5+w)}{w+5}$
34. $\dfrac{2a^2-3a-2}{a^2-4}$ Ans: $\dfrac{2a+1}{a+2}$	35. $\dfrac{6a^2-a-2}{2a^2+a}$ Ans: $\dfrac{3a-2}{a}$	36. $\dfrac{8a^2+10a-3}{2a^2+5a+3}$ Ans: $\dfrac{4a-1}{a+1}$

b. Solve algebraic expression using:

i. Addition

ii. Subtraction

iii. Multiplication

iv. Division

NOTE:

- $\frac{a}{b} = \frac{numerator}{denominator}$
- To add (or subtract) two fractions that have a **common denominator**, simply add (or subtract) the numerators and retain the common denominator.
- Add and subtract rational expressions that **do not have a common denominator**, factor the denominators and find the LCD (lowest common denominator). Then rewrite each fraction in terms of the LCD.
- Note that, when either the numerator or denominators are completely cancelled, they become 1, not 0.

Steps for +/-

1. Get common denominators
2. Combine like terms in the numerator
3. Simplify expression

Example : ADDITION

common denominator:

Common rule: $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

$\frac{4a}{28} + \frac{3a}{28}$

$\frac{4a+3a}{28} = \frac{\cancel{7}a}{\cancel{28}} = \frac{a}{4}$ (answer)

do not have a common denominator:

Common rule: $\frac{a}{b} + \frac{c}{d} = \frac{ad+cd}{bd}$

$\frac{1}{2} + \frac{3}{x}$

$\frac{1}{2} + \frac{3}{x} = \frac{x+6}{2x}$ (answer)

Example : SUBTRACTION

common denominator:

Common rule: $\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}$

$\frac{4a-3a}{28}=\frac{a}{28}$ (answer)

do not have a common denominator:

Common rule: $\frac{a}{b}-\frac{c}{d}=\frac{ad-cd}{bd}$

$\frac{1}{2}-\frac{3}{x}$

$\frac{1}{2}-\frac{3}{x}=\frac{x-6}{2x}$ (answer)

EXERCISE		
1. $\frac{5}{x}+\frac{3}{x}$ Ans: $\frac{8}{x}$	2. $\frac{2a}{9b}+\frac{3}{9b}$ Ans: $\frac{2a+3}{9b}$	3. $\frac{2x-4}{2x+8}+\frac{2}{2x+8}$ Ans: $\frac{x-1}{x+4}$
4. $\frac{4x-13}{x^2-5x+6}+\frac{1}{x^2-5x+6}$ Ans: $\frac{4}{x-2}$	5. $\frac{5}{2x^2}-\frac{1}{2x^2}$ Ans: $\frac{2}{x^2}$	6. $\frac{a}{a-b}-\frac{b}{a-b}$ Ans: 1
7. $\frac{a^2+2a}{a+3}-\frac{3}{a+3}$ Ans: $a-1$	8. $\frac{8x-4}{2x+6}-\frac{4x-6}{2x+6}$ Ans: $\frac{2x+1}{x+3}$	**9.** $\frac{5}{4x}+\frac{9}{4x}-\frac{8}{4x}$ Ans: $\frac{3}{2x}$

10. $x+\frac{x}{4}$ Ans: $\frac{5x}{4}$	11. $\frac{5}{6x}-\frac{a}{y}$ Ans: $\frac{5y-6ax}{6xy}$	12. $\frac{3x}{2}+\frac{7x}{4}$ Ans: $\frac{13x}{4}$
13. $\frac{5}{a^2b}-\frac{2}{ab^2}$ Ans: $\frac{5b-2a}{a^2b^2}$	14. $\frac{4}{3d}-\frac{1}{2d^3}$ Ans: $\frac{8d^3-3}{6d^3}$	15. $\frac{x+4}{6a}-\frac{4x+7}{3a}$ Ans: $\frac{-7x-10}{6a}$
16. $\frac{7}{6x^2}+\frac{3}{4x}$ Ans: $\frac{14+9x}{12x^2}$	17. $\frac{x+4}{6a}-\frac{4x+7}{3a}$	18. $\frac{x+2}{2y}-\frac{y-4}{3x^2}$ Ans: $\frac{3x^3+6x^2-2y^2+8y}{6x^2y}$
19. $\frac{x+2}{2y}-\frac{y-4}{3x^2}$	20. $\frac{2x}{2-x}+\frac{x}{2}$ Ans: $\frac{6x-x^2}{4-2x}$	21. $\frac{10}{3x-6}+\frac{3}{2x-4}$ Ans: $\frac{29}{6x-12}$
22. $\frac{-5x}{x-4}-\frac{4x+4}{4-x}$ Ans: -1	23. $\frac{2}{x-1}-\frac{3}{x-2}$ Ans: $\frac{-x-1}{x^2-3x+2}$	24. $\frac{3x}{x^2+3x+2}+\frac{3}{x+2}$ Ans: $\frac{6x+3}{x^2+3x+2}$

MULTIPLICATION

Common rule: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a.c}{b.d}$

Example 1:

$$\frac{7x^2}{5y} \times \frac{15yz}{x} = \frac{7 \times \cancel{x} \times x \times \cancel{15} \times \cancel{y} \times z}{\cancel{5} \times \cancel{y} \times \cancel{x}}$$

$$= 7 \times x \times 3 \times z$$

$$= 21xz$$ (answer)

Example 2:

$$\frac{8}{2x+6} \cdot (x^2 + 6x + 9)$$

$$= \frac{\cancel{8}}{\cancel{2}(x+3)} \cdot \cancel{(x+3)}(x+3)$$

$$= 4 \cdot (x+3)$$

$$= 4x + 12$$

→ Put **parenthesis** around all **polynomials.**
→ Push together into one fraction.
→ **FACTOR** all polynomials.
→ **Cancel** pairs of common factors.
→ **DO NOT CANCEL SINGLE TERMS OUT OF POLYNOMIALS!!!**
→ **Multiply** back together.

DIVISION

Common rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$

Example 1:

$$\frac{x}{3} \div \boxed{\frac{2x^2}{3}}$$

Change $\div \frac{2x^2}{3} \rightarrow \times \frac{3}{2x^2}$

$$\frac{x}{3} \times \frac{3}{2x^2} = \frac{\cancel{x} \times \cancel{3}}{\cancel{3} \times 2 \times \cancel{x} \times x}$$

$$= \frac{1}{2x}$$ (answer)

Example 2:

$$\frac{m^2+m-2}{m^2-2m-8} \div \frac{3m-3}{m^2-8m+16}$$

$$= \frac{m^2+m-2}{m^2-2m-8} \times \frac{m^2-8m+16}{3m-3}$$

$$= \frac{(m+2)(m-1)}{(m-4)(m+2)} \times \frac{(m-4)(m-4)}{3(m-1)}$$

$$= \frac{\cancel{(m+2)}\cancel{(m-1)}}{\cancel{(m-4)}\cancel{(m+2)}} \times \frac{\cancel{(m-4)}(m-4)}{3\cancel{(m-1)}}$$

$$= \frac{m-4}{3}$$

→ **CHANGE** to <u>multiplication</u> of the reciprocal (keep – change – flip):

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c}$$

→ **<u>Multiply</u>** using same process as in Example 1.

EXERCISE		
1. $\dfrac{3xy}{7wy} \cdot \dfrac{14w^2}{18x}$ Ans: $\dfrac{w}{3}$	2. $\dfrac{20k^2}{39h} \div \dfrac{2k}{3h}$ Ans: $\dfrac{10k}{13}$	3. $\dfrac{x+3}{2} \times \dfrac{6}{3x+9}$ Ans: 1
4. $\dfrac{2h/8m}{13h^2m/16}$ Ans: $\dfrac{4}{13hm^2}$	5. $\dfrac{3x^2+3x}{6x^2-6x} \cdot \dfrac{7xy-7y}{5xy-10y}$ Ans: $\dfrac{7x+7}{10x-20}$	6. $\dfrac{24a^2m^3}{5m} \div \dfrac{8am}{25}$ Ans: $15am$
7. $\dfrac{2}{n^3} \div \dfrac{n-6}{n^5}$ Ans: $\dfrac{2n^2}{n-6}$	8. $\dfrac{1}{x} \div \dfrac{x-2}{2x}$ Ans: $\dfrac{2}{x-2}$	9. $\dfrac{x^2-10x+25}{x-5} \div \dfrac{x^2-25}{5x+25}$ Ans: 5

10. $\dfrac{12b^3c^2}{5ac} \bullet \dfrac{15a^2b}{3b^2c}$

Ans: $12ab^2$

11. $\dfrac{x^2+8x+15}{2} \cdot \dfrac{4}{2x+6}$

Ans: $x+5$

12. $\dfrac{3x^2+3x}{6x^2-6x} \cdot \dfrac{7xy-7y}{5xy-10y}$

13. $\dfrac{9m^2-16}{m^2-25} \cdot \dfrac{3m+15}{6m-8}$

Ans: $\dfrac{9m+12}{2m-10}$

14. $\dfrac{6m-18n}{9m+9n} \cdot \dfrac{4m-4n}{8m-24n}$

Ans: $\dfrac{m-n}{3m+3n}$

15. $\dfrac{m^2-4m+3}{m^2-1} \div \dfrac{2m-6}{m^2+2m+1}$

Ans: $\dfrac{m+1}{2}$

16. $\dfrac{\dfrac{3}{a^2}+\dfrac{5}{a^3}}{\dfrac{10}{a}+6}$

Ans: $\dfrac{1}{2a^2}$

17. $\dfrac{2-\dfrac{4}{x}}{x-6+\dfrac{8}{x}}$

Ans: $\dfrac{2}{x-4}$

18. $\dfrac{6x^2-7x-3}{2x^2-17x+21} \div \dfrac{9x^2+9x+2}{2x^2-11x-21}$

Ans: $\dfrac{2x+3}{3x+2}$

19. $\dfrac{x^2-10x+25}{x-5} \div \dfrac{x^2-25}{5x+25}$	20. $\dfrac{t^2-9}{2t^2-11t+15} \cdot \dfrac{6t^2-17t+5}{9t^2-1}$ Ans: $\dfrac{t+3}{3t+1}$	21. $\dfrac{12a^4}{a-3} \cdot \dfrac{9-3a}{2a}$ Ans: $-18a^3$
22. $\dfrac{3a^2b}{b} \cdot (3a^2+6a)$ Ans: $9a^4+18a^3$	23. $\dfrac{3a^2b-3ab^2}{4c^2} \div \dfrac{3ab}{2c}$ Ans: $\dfrac{a-b}{2c}$	24. $(3p-12) \div \dfrac{8-2p}{3}$ Ans: $\dfrac{-9}{2}$
25. $\dfrac{12x+12y}{7x-7y} \div \dfrac{18x+18y}{7x+14}$ Ans: $\dfrac{2x+4}{3x-3y}$	26. $\dfrac{x^2-2x}{x} \div \dfrac{2-x}{x^2+2x+1}$ Ans: $-x^2-2x-1$	27. $\dfrac{x^2-x-30}{2x^2-11x-6} \cdot \dfrac{4x^2-4x-3}{2x^2-11x+12}$ Ans: $\dfrac{x+5}{x-4}$

c. Solve quadratic equations by using

i. Factorization.

ii. Quadratic formula.

iii. Completing squares.

NOTE:

- "Standard" **Quadratic Equation** form: $\mathbf{ax^2 + bx + c = 0}$
- It must contain **only one unknown** and the **highest power** of the unknown is **2**.
- The letters **a**, **b** and **c** are **coefficients** (we know those values). They can have any value, except that **a** can't be 0.
- The letter "***x***" is the **variable** or unknown (we don't know it yet)
- Example: $\mathbf{5x^2 - 3x + 3 = 0}$
- 3 method to solve quadratic equation are factorization, quadratic formula and completing squares.

Example 1:

Solve $x^2 + 5x + 6 = 0$ by **factoring**.

$x^2 + 5x + 6 = (x + 2)(x + 3)$ Factor the equation

$(x + 2)(x + 3) = 0$ Set this equal to zero

$x + 2 = 0$ or $x + 3 = 0$ Solve each factor

$x = -2$ or $x = -3$
$x = -3, -2$ (answer)

Example 2:

Solve $x^2 - 6x + 2 = 0$ by using **quadratic formula**.

a = 1 , b = - 6 and c = 2

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$x_1 = \frac{6 + \sqrt{28}}{2} \qquad x_2 = \frac{6 - \sqrt{28}}{2}$$

$$= 5.65 \qquad = 0.35$$

$x = 5.65, 0.35$ (answer)

Example 3:

Solve $\boldsymbol{x^2 + 6x - 7 = 0}$ by **completing the square:**

$$\boldsymbol{ax^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \quad which \ a = 1}$$

$x^2 + 6x - 7 = 0$	
$x^2 + 6x \quad = 7$	Move the loose number over to the other side.
$x^2 + 6x + (6/2)^2 = 7 + (6/2)^2$	Take half of the $\boldsymbol{x}$ -term (divide it by two) (and don't forget the sign!), and square it.
$x^2 + 6x + (3)^2 = 7 + (3)^2$	Add this square to both sides of the equation.
$(x + 3)^2 = 16$	Convert the left-hand side to squared form. Simplify the right-hand side.
$x + 3 = \pm\sqrt{16}$ $x + 3 = \pm 4$	Square root both sides. Remember to do "±" on the right-hand side.
$x = -3 \pm 4$ $= -3 - 4, -3 + 4$ $= -7, 1$ (answer)	Solve for " $\boldsymbol{x}$ =". Remember that the "±" gives you two solutions. Simplify as necessary.

EXERCISE Solve each equation by factoring:		
1. $x^2 - 7x - 18 = 0$ (9,2)	2. $p^2 - 5p - 14 = 0$ (-2,7)	3. $m^2 - 9m + 8 = 0$ (1,8)
4. $x^2 - 16x + 63 = 0$ (9,7)	5. $7x^2 - 31x - 20 = 0$ $(5, -\frac{4}{7})$	6. $-2v^2 - v + 12 = -3v^2 + 6v$ (3 , 4)
7. $-4n^2 + 6n - 16 = -5n^2$ (2 , -8)	8. $8r^2 + 3r + 2 = 7r^2$ (-2 , -1)	9. $10n^2 - 35 = 65n$ (- ½ , 7)
10. $16b^2 - 114b = -14$ (1/8 , 7)	11. $28x^2 = -96 - 184x$ (- 4/7 , -6)	12. $42x^2 - 69x + 20 = 7x^2 - 8$ (7/5, 4/7)
13. $3k^2 + 72 = 33k$ (3, 8)	14. $k^2 = -4k - 4$ (-2)	15. $7v^2 - 42 = -35v$ (-6 , 1)

EXERCISE Solve each equation by using the quadratic formula:		
1. $5+20x-5x^2=0$ (-0.236, 4.236)	2. $2x^2+5x+3=0$ (-1 , -3/2)	3. $9-6x-3x^2=0$ (-3 , -1)
4. $k^2-5k-6=0$ (6 , -1)	5. $b^2-4b-14=-2$ (6 , -2)	6. $14m^2+1=6m^2+7m$ (-0.695 , 0.18)

EXERCISE Solve each equation by using the completing square:		
1. $b^2-4b-12=0$ (6 , -2)	2. $v^2-2v-35=0$ (7 , -5)	3. $n^2-4n+5=8$ (2 ± √ 7)

4. $x^2 - 95 = 14x$ (19 , -5)	5. $9x^2 + 5 = 18x$ $\left(\frac{5}{3}, \frac{1}{3}\right)$	6. $6k^2 = -12 + 18$ (1 , -3)
7. $9m^2 - 20m - 21 = 0$ (3 , -7/9)	8. $10x^2 - 4x - 32 = 0$ (2 , - 8/5)	9. $3x^2 + 9x + 9 = 3$ (-1, -2)

d. Define partial fractions.

- A rational function is one expressed in fractional form whose numerator and denominator are polynomials.
- A rational function is termed ***proper*** when the degree of the numerator is less than the degree of the denominator.
- It is termed ***improper*** otherwise.

$$\frac{x+1}{x^2+2},\quad \frac{x}{(x+1)(x+2)},\quad \frac{2}{x^2+3x+1}$$

are all ***proper*** functions.

$$\frac{x^3+1}{x^2+2},\quad \frac{x}{x+2},\quad \frac{x^2}{(x+1)(x+2)}$$

are all ***improper*** functions.

e. Construct partial fraction using proper fraction with:

i.Linear factor

ii.Repeated linear factors

iii.Quadratic factors

- **Obtain the partial fraction decomposition:**

1. Factor the denominator if it is not already in the factored form.
2. If the denominator contains two linear factors, break it into two partial fractions using constant A and B as follows: $\frac{-2x-20}{(x+2)(x-5)}=\frac{A}{x+2}+\frac{B}{x-2}$

   Solve for A and B
3. If the denominator contains some power of a linear factor, we break it down to partial fractions as follows:

$$\frac{-2x-20}{(x+2)^3}=\frac{A}{x+2}+\frac{B}{(x-2)^2}+\frac{C}{(x-2)^3}$$

   Solve for A , B and C
4. If the denominator contains a quadratic factor, we break it down to partial fractions as follows:

$$\frac{-2x-20}{(x+2)(x^2+1)}=\frac{A}{x+2}+\frac{Bx+C}{x^2+1}$$

   Solve for A , B and C

Example:

Obtain the partial fraction decomposition of $\frac{x-2}{x^2+4x+3}$

$$\frac{x-2}{x^2+4x+3} = \frac{x-2}{(x+3)(x+1)}$$

1.Factor the denominator

$$\frac{x-2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1}$$

2.The denominator contains two linear factors

$$\frac{x-2}{(x+3)(x+1)} = \frac{A(x+1)+B(x+3)}{(x+3)(x+1)}$$

3. Simplify right side using common denominator.

$$x-2 = A(x+1) + B(x+3)$$

4. Since the denominators of the left side and the right side are the same, the numerator on the left equal to the numerator on the right.

$$(-1)-2 = A((-1)+1) + B((-1)+3)$$
$$-3 = B(2)$$
$$B = \frac{-3}{2}$$

5. This is an identity that is true for all values of *x*
We choose a value of x that will eliminate either A or B on the right.
When $(x+1) = 0$
So x = -1
Using ***x* = -1** in the equation gives: **REFER NO 4**

$$(-3)-2 = A((-3)+1) + B((-3)+3)$$
$$-5 = A(-2)$$
$$A = \frac{5}{2}$$

When $x+3 = 0$
So x = -3
Using ***x* = -3** in the equation gives: **REFER NO 4**

$$\frac{x-2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{5/2}{x+3} + \frac{(-3/2)}{x+1} = \frac{5}{2(x+3)} - \frac{3}{2(x+1)}$$ (answer)

EXERCISE Find each **partial fraction of linear factors** decomposition:	
1. $\dfrac{2x-1}{(2x+1)(x-3)}$ Ans : $\dfrac{4}{7(2x+1)}+\dfrac{5}{7(x-3)}$	2. $\dfrac{2x}{(x-1)(2x+1)(x+2)}$ Ans: $\dfrac{2}{9(x-1)}+\dfrac{4}{9(2x+1)}-\dfrac{4}{9(x+2)}$

3. $\dfrac{2x-1}{(x-2)(x+1)(x+3)}$ Ans: $\dfrac{1}{5(x-2)}+\dfrac{1}{2(x+1)}-\dfrac{7}{10(x+3)}$	4. $\dfrac{5x+6}{(x+2)(1-x)}$ Ans: $\dfrac{-4}{3(x+2)}+\dfrac{11}{3(1-x)}$

5. $\dfrac{2x^2-5x+6}{x(x-2)(x-3)}$ Ans : $\dfrac{1}{x}-\dfrac{2}{(x-2)}+\dfrac{3}{(x-3)}$	6. $\dfrac{x^2+1}{x(x+1)(x-1)}$ Ans: $\dfrac{-1}{x}+\dfrac{1}{(x+1)}+\dfrac{1}{(x-1)}$

EXERCISE Find each **partial fraction of quadratic factors** decomposition:	
1. $\dfrac{x+2}{x^2+12x+32}$ Ans : $\dfrac{-1}{2(x+4)}+\dfrac{3}{2(x+8)}$	2. $\dfrac{2}{x^2-2x}$ Ans : $\dfrac{-1}{x}+\dfrac{1}{(x-2)}$

3. $\dfrac{2x^2+3x}{(x+1)^2(x^2-2)}$ Ans : $\dfrac{-1}{(x+1)}+\dfrac{1}{(x+1)^2}+\dfrac{x}{(x^2-2)}$	4. $\dfrac{x^2}{(x^2+2x+3)(2x+1)}$ Ans : $\dfrac{1}{9(2x-1)}+\dfrac{4x-3}{9(x^2+2x+3)}$

f. Convert improper fraction to mixed number by using long

Example:

Find the partial fraction decomposition of $\dfrac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)}$

$$(x-1)(x^2-1) = x^3 - x - x^2 + 1 = x^3 - x^2 - x + 1$$

$$\begin{array}{r} 1 \\ x^3 - x^2 - x + 1 \overline{\big) x^3 - x^2 - 3x + 5} \\ -\left(x^3 - x^2 - \quad x + 1\right) \\ \hline 0 - 0 \quad - 2x + 4 \\ \hline\hline \end{array}$$

The numerator is of degree 3; the denominator is of degree 3. So first I have to do the **long division**:

$$\frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)} = 1 + \frac{(-2x+4)}{(x-1)(x^2-1)}$$

The long division rearranges the rational expression to give:

$$\frac{-2x+4}{(x-1)(x^2-1)} = \frac{-2x+4}{(x-1)(x+1)(x-1)} = \frac{-2x+4}{(x+1)(x-1)^2}$$

$$2x + 4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

Decompose the fractional part. The denominator factors as $(x+1)(x-1)^2$

$$2x + 4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$-2(1) + 4 = A((1)-1)^2 + B((1)+1)((1)-1) + C((1)+1)$$

$$4 \qquad = A(0)^2 + B(2)(0) + C(2)$$

$$C = \frac{2}{2} = 1$$

Find all values for A,B and C

For $x = 1$

$$-2x + 4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$-2(-1) + 4 = A((-1)-1)^2 + B((-1)+1)((-1)-1) + C((-1)+1)$$

$$6 \qquad = A(-2)^2 + B(0)(-2) + C(0)$$

$$A = \frac{6}{4} = \frac{3}{2}$$

For x = -1

$$-2x+4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$
$$-2x+4 = A(x^2-2x+1) + B(x^2-1) + Cx + C$$
$$-2x+4 = x^2(A+B) + x(-2A+C) + A - B + C$$

Coefficients is use there are couple of values of x that cannot be allow to quickly get two of the three of constants

x^2 :

$$0 = A+B$$
$$B = -\frac{3}{2}$$

x :

$$-2 = -2A + C$$

x^0 :

$$4 = A - B + C$$

Coefficients:
by comparing coefficients of x^2, x and constants in the identity.

$$\frac{x^3-x^2-3x+5}{(x-1)(x^2-1)} = 1 - \frac{2x+4}{(x-1)(x^2-1)} = 1 + \frac{3}{2(x+1)} - \frac{3}{2(x-1)}$$ (answer)

EXERCISE
Find each **partial fraction of improper fraction** decomposition

1. $\dfrac{x^3}{(x+2)(x-3)}$

Ans: $x+1+\dfrac{8}{5(x+2)}+\dfrac{27}{5(x-3)}$

2. $\dfrac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)}$

Ans : $1 - \dfrac{3}{2(x-1)} + \dfrac{1}{(x-1)^2} + \dfrac{3}{2(x+1)}$

3. $\dfrac{x^2 - x + 1}{x^2 - x - 2}$

Ans: $1 - \dfrac{1}{(x+1)} + \dfrac{1}{(x-2)}$

4. $\dfrac{x^3+2x^2-10x-9}{x^2-9}$

Ans : $x+2-\dfrac{1}{(x+3)}+\dfrac{2}{(x-3)}$

EXERCISE

Find each **partial fraction** decomposition

1. $\dfrac{-2x^2+8x+6}{x^3-3x^2-9x+27}$

Ans : $\dfrac{-1}{(x+3)}+\dfrac{-1}{(x-3)}+\dfrac{2}{(x-3)^2}$

2. $\dfrac{x^2 - 2}{(x+1)(x^2+x+1)}$

Ans : $\dfrac{-1}{(x+1)} + \dfrac{2x-1}{(x^2+x+1)}$

3. $\dfrac{6x^2 - x + 1}{x^3 + x^2 + x + 1}$

Ans : $\dfrac{4}{(x+1)} + \dfrac{2x-3}{(x^2+1)}$

4. $\dfrac{2x^2-5x}{(x^2-1)(x^2-4)}$

Ans : $\dfrac{1}{2(x-1)}+\dfrac{7}{6(x+1)}-\dfrac{1}{6(x-2)}-\dfrac{3}{2(x+2)}$

5. $\dfrac{x}{(x-1)(2x^2+4x+5)}$

Ans : $\dfrac{1}{11}\left(\dfrac{1}{x-1}-\dfrac{2x-5}{2x^2+4x+5}\right)$

6. $\dfrac{2x^2+1}{x^3+2x^2+x}$

Ans : $\dfrac{1}{x}+\dfrac{1}{x+1}-\dfrac{3}{(x+1)^2}$

7. $\dfrac{9-2x}{(x-2)(x^2+1)}$

Ans : $\dfrac{1}{(x-2)} - \dfrac{(x+4)}{(x^2+1)}$

8. $\dfrac{4x^2-6}{(x-1)^3(x+2)}$

Ans : $\dfrac{10}{27(x-1)}+\dfrac{78}{27(x-1)^2}-\dfrac{2}{3(x-1)^3}-\dfrac{10}{27(x+2)}$

9. $\dfrac{4x^2+1}{x(2x-1)^2}$

Ans : $\dfrac{1}{x}+\dfrac{4}{(2x-1)^2}$

10. $\dfrac{2x^4 - x^3 - 9x^2 + x - 12}{x^3 - x^2 - 6x}$

Ans : $2x + 1 + \dfrac{2}{x} + \dfrac{3}{x-3} - \dfrac{1}{x+2}$

11. $\dfrac{x^3 + 7x^2 + 9x + 2}{x(x^2 + 3x + 2)}$

Ans : $1 + \dfrac{1}{x} + \dfrac{1}{x+1} + \dfrac{2}{x+2}$

12. $\dfrac{x^3 - 6x^2 + 5x - 3}{x^2 - 1}$

Ans : $x - 6 + \dfrac{15}{2(x+1)} - \dfrac{3}{2(x-1)}$