a. Simplify algebraic expression.

NOTE:

- **♣** Simplifying expressions is done by cancelling out the common factor from both the numerator and denominator.
- ♣ To reduce an algebraic fraction (rational expression), follow the same process you use to reduce numeric fractions:
 - a. Factor
 - b. Divide out (cancel) factors common to both the numerator and denominator.
 - c. Simplify

Example 1:

Simplify $\frac{x}{2x}$

$$\frac{(x)}{2(x)} = \frac{1}{2} \text{ (answer)}$$

By cancelling out the common x in the numerator and denominator.

Example 2:

Simplify $\frac{5x^2y}{15xy}$

$$\frac{3x^2x}{18xx} = \frac{x}{3} \text{ (answer)}$$

Cancel out common factors in algebraic fractions to make a simpler equivalent fraction.

Example 3:

$$\frac{4a+2ab}{2a} = \frac{2a(2+b)}{2a} = 2+b$$
 (answer)

2a is a factor in common for the two terms in the sum and then cancelling.

	EXERCISE
	Simplify the following
3 r	2.

1.
$$\frac{3x}{15}$$

2.
$$\frac{x^2y}{15xy}$$

3.
$$\frac{21x^4y^7}{3xy^2}$$

Ans:
$$\frac{x}{5}$$

Ans:
$$\frac{x}{15}$$

Ans:
$$7x^3y^5$$

4. $\frac{10r^3s}{5rs^2}$		5. $\frac{8x^3y^4}{2x^2y}$		$6. \frac{5b^3c}{15bc}$	7
$(2ab)^3$	Ans: $\frac{2r^2}{s}$	8. $\frac{2x+10}{4}$	Ans: $4xy^3$	x^2-4x	Ans: $\frac{b^2}{3}$
$7. \frac{(2ab)^3}{4a^7b}$	$2h^2$		1 (5)	9. $\frac{x^2 - 4x}{x - 4}$	
10. $\frac{5x+20}{x+4}$	Ans: $\frac{2b^2}{a^4}$	$11. \ \frac{12x - 9}{16x^2 - 9}$	Ans: $\frac{1}{2}(x+5)$	$12. \frac{6ab + 2a}{2b}$	Ans: x
9x + 27	Ans: 5	$16m^2n - 8mn$	Ans: $\frac{3}{4x+3}$	4mnp – 8mp	Ans: $\frac{3ab + a}{b}$
13. $\frac{9x+27}{9x+18}$	x + 3	14. $\frac{16m^2n - 8mn}{12m - 6}$	Amn	$15. \ \frac{4mnp - 8mp}{12mn}$	nn-2n
16. $\frac{3x^3}{12x}$	Ans: $\frac{x+3}{x+2}$	17. $\frac{1-x}{x-1}$	Ans: $\frac{4mn}{3}$	18. $\frac{5m^2}{15m}$	Ans: $\frac{np-2p}{3n}$
	Ans: $\frac{x^2}{4}$		Ans: -1		Ans: $\frac{1}{3}m$
$19. \ \frac{15m}{-5mn}$		$20. \ \frac{3x^2 - 9x}{x - 3}$		21. $\frac{16}{4x+12}$	
$6r^2-6$	Ans: $\frac{-3}{n}$	$x^2 - 25$	Ans: 3 <i>x</i>	k+1	Ans: $\frac{4}{x+3}$
$22. \frac{6x^2 - 6}{3x^2 - 6x + 3}$		$23. \ \frac{x^2 - 25}{x^2 + 10x + 25}$		$24. \ \frac{k+1}{k^2 - 4k - 5}$	
	Ans: $\frac{2x+2}{x-1}$		Ans: $\frac{x-5}{x+5}$		Ans: $\frac{1}{k-5}$

		1			
$25. \ \frac{b-6}{(b+2)(6-b)}$		$26. \ \frac{9 - x^2}{x^2 + 8x + 15}$		$27. \frac{2-x}{(2x^2+3x-14)}$	•
	Ans: $\frac{-1}{b+2}$		Ans: $\frac{3-x}{x+5}$		Ans: $\frac{-1}{2x+7}$
$28. \ \frac{4a^2 - 2a}{4a^2 - 1}$		$29. \ \frac{6x^3 + 12x}{x^2 + 2}$		$30. \ \frac{p^2 + 8p + 16}{8p + 32}$	
	2 <i>a</i>				
	Ans: $\frac{2a}{2a+1}$		Ans: 6 <i>x</i>		Ans: $\frac{p+4}{8}$
$31. \ \frac{x+3}{x^2 - 2x - 15}$		32. $\frac{28abc^2}{49a^3c}$		$33. \ \frac{25 - w^2}{w^2 - 2w - 15}$	
	Ans: $\frac{1}{x-5}$		Ans: $\frac{4bc}{7a^2}$		$Ans: \frac{-(5+w)}{w+5}$
$34. \ \frac{2a^2 - 3a - 2}{a^2 - 4}$		$35. \ \frac{6a^2 - a - 2}{2a^2 + a}$		$36. \ \frac{8a^2 + 10a - 3}{2a^2 + 5a + 3}$	
	Ans: $\frac{2a+1}{a+2}$		Ans: $\frac{3a-2}{a}$		Ans: $\frac{4a-1}{a+1}$
	a+2		и		a + 1

b. Solve algebraic expression using:

- i. Addition
- ii. Subtraction
- iii. Multiplication
- iv. Division

NOTE:

$$\frac{4}{b} = \frac{numerator}{deno\ min\ ator}$$

- ♣ To add (or subtract) two fractions that have a common denominator, simply add (or subtract) the numerators and retain the common denominator.
- Add and subtract rational expressions that **do not have a common denominator**, factor the denominators and find the LCD (lowest common denominator). Then rewrite each fraction in terms of the LCD.
- ♣ Note that, when either the numerator or denominators are completely cancelled, they become 1, not 0.

Steps for +/-

- 1. Get common denominators
- 2. Combine like terms in the numerator
- 3. Simplify expression

Example: ADDITION

common denominator:

Common rule:
$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

$$\frac{4a}{28} + \frac{3a}{28}$$

$$\frac{4a+3a}{28} = \frac{\sqrt{a}}{28} = \frac{a}{4} \text{ (answer)}$$

do not have a common denominator:

Common rule:
$$\frac{a}{b} + \frac{c}{d} = \frac{ad + cd}{bd}$$

$$\frac{1}{2} + \frac{3}{x}$$

$$\frac{1}{2} + \frac{3}{x} = \frac{x+6}{2x}$$
 (answer)

Example: SUBTRACTION

common denominator:

Common rule:
$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

$$\frac{4a-3a}{28} = \frac{a}{28}$$
 (answer)

do not have a common denominator:

Common rule:
$$\frac{a}{b} - \frac{c}{d} = \frac{ad - cd}{bd}$$

$$\frac{1}{2} - \frac{3}{x}$$

$$\frac{1}{2} - \frac{3}{x} = \frac{x - 6}{2x}$$
 (answer)

EXERCISE

1.
$$\frac{5}{x} + \frac{3}{x}$$

2.
$$\frac{2a}{9b} + \frac{3}{9b}$$

3.
$$\frac{2x-4}{2x+8} + \frac{2}{2x+8}$$

Ans:
$$\frac{8}{x}$$

Ans:
$$\frac{2a+3}{9b}$$

Ans:
$$\frac{x-1}{x+4}$$

4.
$$\frac{4x-13}{x^2-5x+6} + \frac{1}{x^2-5x+6}$$
 5. $\frac{5}{2x^2} - \frac{1}{2x^2}$

$$5. \ \frac{5}{2x^2} - \frac{1}{2x^2}$$

6.
$$\frac{a}{a-b} - \frac{b}{a-b}$$

Ans:
$$\frac{4}{x-2}$$

Ans:
$$\frac{2}{x^2}$$

Ans: 1

7.
$$\frac{a^2 + 2a}{a+3} - \frac{3}{a+3}$$

$$8. \ \frac{8x-4}{2x+6} - \frac{4x-6}{2x+6}$$

9.
$$\frac{5}{4x} + \frac{9}{4x} - \frac{8}{4x}$$

Ans:
$$a - 1$$

Ans:
$$\frac{2x+1}{x+3}$$

Ans:
$$\frac{3}{2x}$$

10.
$$x + \frac{x}{4}$$

11.
$$\frac{5}{6x} - \frac{a}{y}$$

12.
$$\frac{3x}{2} + \frac{7x}{4}$$

Ans:
$$\frac{5x}{4}$$

Ans:
$$\frac{5y - 6ax}{6xy}$$

Ans:
$$\frac{13x}{4}$$

13.
$$\frac{5}{a^2b} - \frac{2}{ab^2}$$

14.
$$\frac{4}{3d} - \frac{1}{2d^3}$$

15.
$$\frac{x+4}{6a} - \frac{4x+7}{3a}$$

Ans:
$$\frac{5b - 2a}{a^2b^2}$$

Ans:
$$\frac{8d^3 - 3}{6d^3}$$

Ans:
$$\frac{-7x - 10}{6a}$$

16.
$$\frac{7}{6x^2} + \frac{3}{4x}$$

17.
$$\frac{x+4}{6a} - \frac{4x+7}{3a}$$

18.
$$\frac{x+2}{2y} - \frac{y-4}{3x^2}$$

$$Ans: \frac{14 + 9x}{12x^2}$$

Ans:
$$\frac{3x^3 + 6x^2 - 2y^2 + 8y}{6x^2y}$$

19.
$$\frac{x+2}{2y} - \frac{y-4}{3x^2}$$

20.
$$\frac{2x}{2-x} + \frac{x}{2}$$

21.
$$\frac{10}{3x-6} + \frac{3}{2x-4}$$

Ans:
$$\frac{6x - x^2}{4 - 2x}$$

Ans:
$$\frac{29}{6x - 12}$$

$$22. \ \frac{-5x}{x-4} - \frac{4x+4}{4-x}$$

23.
$$\frac{2}{x-1} - \frac{3}{x-2}$$

$$\frac{3x}{x^2 + 3x + 2} + \frac{3}{x + 2}$$

Ans:
$$\frac{-x-1}{x^2-3x+2}$$

Ans:
$$\frac{6x+3}{x^2+3x+2}$$

MULTIPLICATION

Common rule: $\frac{a}{h} \cdot \frac{c}{d} = \frac{a \cdot c}{h \cdot d}$

Example 1:

$$\frac{7x^{2}}{5y} \times \frac{15yz}{x} = \frac{7 \times x \times x \times 15 \times y \times z}{5 \times y \times x}$$
$$= 7 \times x \times 3 \times z$$
$$= 21xz \text{ (answer)}$$

Example 2:

$$\frac{8}{2x+6} \cdot (x^2+6x+9)$$
⇒ Put parenthesis around all polynomials.
⇒ Push together into one fraction.
⇒ FACTOR all polynomials.
⇒ Cancel pairs of common factors.
⇒ DO NOT CANCEL SINGLE TERMS OUT OF POLYNOMIALS!!!
⇒ Multiply back together.

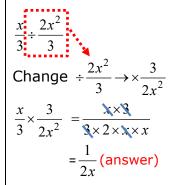
- → Put parenthesis around all polynomials.
- → Push together into one fraction.

- **POLYNOMIALS!!!**
- → Multiply back together.

DIVISION

Common rule: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$

Example 1:



Example 2:

$$\frac{m^2 + m - 2}{m^2 - 2m - 8} \div \frac{3m - 3}{m^2 - 8m + 16}$$

$$= \frac{m^2 + m - 2}{m^2 - 2m - 8} \times \frac{m^2 - 8m + 16}{3m - 3}$$

$$= \frac{(m + 2)(m - 1)}{(m - 4)(m + 2)} \times \frac{(m - 4)(m - 4)}{3(m - 1)}$$

$$= \frac{(m + 2)(m - 1)}{(m - 4)(m + 2)} \times \frac{(m - 4)(m - 4)}{3(m - 1)}$$

$$= \frac{m - 4}{3}$$

→ CHANGE to <u>multiplication</u> of the reciprocal (keep – change – flip):

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

→ Multiply using same process as in Example 1.

		EXERCISE	
$1. \ \frac{3xy}{7wy} \cdot \frac{14w^2}{18x}$		$2. \frac{20k^2}{39h} \div \frac{2k}{3h}$	$3. \ \frac{x+3}{2} \times \frac{6}{3x+9}$
	Ans: $\frac{w}{3}$	Ans: $\frac{10k}{13}$	Ans: 1
4. $\frac{2h/8m}{13h^2m/16}$		$5. \ \frac{3x^2 + 3x}{6x^2 - 6x} \cdot \frac{7xy - 7y}{5xy - 10y}$	$6. \frac{24a^2m^3}{5m} \div \frac{8am}{25}$
	Ans: $\frac{4}{13hm^2}$	Ans: $\frac{7x+7}{10x-20}$	Ans: 15 <i>am</i>
$7. \frac{2}{n^3} \div \frac{n-6}{n^5}$		$8. \ \frac{1}{x} \div \frac{x-2}{2x}$	9. $\frac{x^2 - 10x + 25}{x - 5} \div \frac{x^2 - 25}{5x + 25}$

Ans: 5

10. $\frac{12b^3c^2}{5ac} \cdot \frac{15a^2b}{3b^2c}$	11. $\frac{x^2 + 8x + 15}{2} \cdot \frac{4}{2x + 6}$	12. $\frac{3x^2 + 3x}{6x^2 - 6x} \cdot \frac{7xy - 7y}{5xy - 10y}$
Ans: 12 <i>ab</i> ²	Ans: $x + 5$	
13. $\frac{9m^2 - 16}{m^2 - 25} \cdot \frac{3m + 15}{6m - 8}$	14. $\frac{6m-18n}{9m+9n} \cdot \frac{4m-4n}{8m-24n}$	15. $\frac{m^2 - 4m + 3}{m^2 - 1} \div \frac{2m - 6}{m^2 + 2m + 1}$
Ans: $\frac{9m + 12}{2m - 10}$	Ans: $\frac{m-n}{3m+3n}$	Ans: $\frac{m+1}{2}$
16. $\frac{\frac{3}{a^2} + \frac{5}{a^3}}{\frac{10}{a} + 6}$	17. $\frac{2 - \frac{4}{x}}{x - 6 + \frac{8}{x}}$	18. $\frac{6x^2 - 7x - 3}{2x^2 - 17x + 21} \div \frac{9x^2 + 9x + 2}{2x^2 - 11x - 21}$
u	,	
1	. 2	Ans: $\frac{2x+3}{3x+2}$
Ans: $\frac{1}{2a^2}$	Ans: $\frac{2}{x-4}$	Alls. $\frac{3x+2}{3}$

$ \begin{array}{c c} 20. \\ $	$21. \frac{12a^4}{a-3} \cdot \frac{9-3a}{2a}$
Ans: $\frac{t+3}{}$	Ans: - 18a ³
3t+1	1
$23. \ \frac{3a^2b - 3ab^2}{4c^2} \div \frac{3ab}{2c}$	24. $(3p-12) \div \frac{8-2p}{3}$
Ans: $\frac{a-b}{2c}$	Ans: $\frac{-9}{2}$
26. $\frac{x^2 - 2x}{x} \div \frac{2 - x}{x^2 + 2x + 1}$	$\frac{x^2 - x - 30}{2x^2 - 11x - 6} \cdot \frac{4x^2 - 4x - 3}{2x^2 - 11x + 12}$
Ans: $-x^2 - 2x - 3$	Ans: $\frac{x+5}{x-4}$
	$\frac{t^{2}-9}{2t^{2}-11t+15} \cdot \frac{6t^{2}-17t+5}{9t^{2}-1}$ Ans: $\frac{t+3}{3t+2}$ $23. \frac{3a^{2}b-3ab^{2}}{4c^{2}} \div \frac{3ab}{2c}$ Ans: $\frac{a-b}{2c}$

c. Solve quadratic equations by using

- i. Factorization.
- ii. Quadratic formula.
- iii. Completing squares.

NOTE:

- **4** "Standard" **Quadratic Equation** form: $ax^2 + bx + c = 0$
- **↓** It must contain **only one unknown** and the **highest power** of the unknown is **2**.
- ♣ The letters a, b and c are coefficients (we know those values). They can have any value, except that a can't be 0.
- **↓** The letter "x" is the <u>variable</u> or unknown (we don't know it yet)
- **4** Example: $5x^2 3x + 3 = 0$
- **4** 3 method to solve quadratic equation are factorization, quadratic formula and completing squares.

Example 1:

Solve $x^2 + 5x + 6 = 0$ by **factoring**.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$
 Factor the equation

$$(x + 2)(x + 3) = 0$$
 Set this equal to zero

$$x + 2 = 0$$
 or $x + 3 = 0$ Solve each factor

$$x = -2$$
 or $x = -3$
 $x = -3, -2$ (answer)

Example 2:

Solve $x^2 - 6x + 2 = 0$ by using **quadratic formula**.

$$a = 1$$
, $b = -6$ and $c = 2$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{28}}{2}$$

$$x_1 = \frac{6 + \sqrt{28}}{2}$$
 $x_2 = \frac{6 - \sqrt{28}}{2}$
= 5.65 = 0.35

$$x = 5.65, 0.35$$
 (answer)

Example 3:

Solve $x^2 + 6x - 7 = 0$ by completing the square:

$$ax^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \text{ which } a = 1$$

$$x^2 + 6x - 7 = 0$$

$$x^2 + 6x = 7$$

Move the loose number over to the other side.

$$x^2 + 6x + (6/2)^2 = 7 + (6/2)^2$$

$$x^2 + 6x + (3)^2 = 7 + (3)^2$$

$$(x+3)^2 = 16$$

$$x + 3 = \pm \sqrt{16}$$

$$x + 3 = \pm 4$$

$$x = -3 \pm 4$$

= -3 - 4, -3 + 4
= -7, 1 (answer)

Take half of the \mathbf{x} -term (divide it by two) (and don't forget the sign!), and square it.

Add this square to both sides of the equation. Convert the left-hand side to squared form. Simplify the right-hand side.

Square root both sides. Remember to do " \pm " on the right-hand side.

Solve for " \mathbf{x} =". Remember that the " \pm " gives you two solutions. Simplify as necessary.

			EX	(ERC	CIS	E		
S	olve	ead	ch eq	uati	on	by	factorii	ng:
	2	2	Eas	1/		Λ	1.7	2

1.
$$x^2 - 7x - 18 = 0$$

$$2. \quad p^2 - 5p - 14 = 0$$

$$3. m^2 - 9m + 8 = 0$$

$$4. \ x^2 - 16x + 63 = 0$$
 (9,2)

$$5. 7x^2 - 31x - 20 = 0$$

$$6. -2v^2 - v + 12 = -3v^2 + 6v$$

(3,4)

$$8r^2 + 3r + 2 = 7r^2$$

9.
$$10n^2 - 35 = 65n$$

$$1 28x^2 = -96 - 184x$$

$$13.\,3k^2\,+72\,=33k$$

$$14. k^2 = -4k -$$

(3, 8)

(-2)

(-6,1)

EXERCISE

Solve each equation by using the quadratic formula: $5x^2 = 0$ | 2. $2x^2 + 5x + 3 = 0$ | 3. 9 - 6x -

1.
$$5 + 20x - 5x^2 = 0$$

$$2. \ 2x^2 + 5x + 3 = 0$$

$$3.9 - 6x - 3x^2 = 0$$

$$(-0.236, 4.236)$$

$$5. b^2 - 4b - 14 = -2$$

(-0.236, 4.236) (-1, -3/2) (-3, -1)

$$6 = 0$$
 5. $b^2 - 4b - 14 = -2$ 6. $14m^2 + 1 = 6m^2 + 7m$

EXERCISE

Solve each equation by using the completing square: 12 = 0 $2 \cdot v^2 - 2v - 35 = 0$ $3 \cdot n^2 - 4n + 5 = 8$

1.
$$b^2 - 4b - 12 = 0$$

$$2. \ v^2 - 2v - 35 = 0$$

$$3. \ n^2 - 4n + 5 = 8$$

$4. \ x^2 - 95 = 14x$	$5. \ 9x^2 + 5 = 18x$	$6. \ 6k^2 = -12 + 18$
(19,-5)	(5	(1,-3)
$7. 9m^2 - 20m - 21 = 0$	$8. \ 10x^2 - 4x - \ 32 = 0$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
(3,-7/9)	, -	- 8/5)



d. **Define partial fractions.**

- A rational function is one expressed in fractional form whose numerator and denominator are polynomials.
- 4 A rational function is termed **proper** when the degree of the numerator is less than the degree of the denominator.
- **It** is termed **improper** otherwise.

$$\frac{x+1}{x^2+2}$$
, $\frac{x}{(x+1)(x+2)}$, $\frac{2}{x^2+3x+1}$

are all **proper** functions.

$$\frac{x^3+1}{x^2+2}$$
, $\frac{x}{x+2}$, $\frac{x^2}{(x+1)(x+2)}$ are all *improper* functions.

Construct partial fraction using proper fraction with: e.

i.Linear factor

ii.Repeated linear factors

iii.Quadratic factors

4 Obtain the partial fraction decomposition:

- 1. Factor the denominator if it is not already in the factored form.
- 2. If the denominator contains two linear factors, break it into two partial fractions using constant A and B as follows: $\frac{-2x-20}{(x+2)(x-5)} = \frac{A}{x+2} + \frac{B}{x-2}$ Solve for A and B
- 3. If the denominator contains some power of a linear factor, we break it down to partial fractions as follows:

$$\frac{-2x-20}{(x+2)^3} = \frac{A}{x+2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$
 Solve for A, B and C

4. If the denominator contains a quadratic factor, we break it down to partial fractions as follows:

$$\frac{-2x-20}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$
 Solve for A, B and C

Example:

Obtain the partial fraction decomposition of $\frac{x-2}{x^2+4x+3}$

$$\frac{x-2}{x^2+4x+3} = \frac{x-2}{(x+3)(x+1)}$$

$$\frac{x-2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1}$$

$$\frac{x-2}{(x+3)(x+1)} = \frac{A(x+1) + B(x+3)}{(x+3)(x+1)}$$

$$x-2 = A(x+1) + B(x+3)$$

$$(-1)-2 = A((-1)+1)+B((-1)+3)$$
$$-3 = B(2)$$
$$B = \frac{-3}{2}$$

$$(-3)-2 = A((-3)+1)+B((-3)+3)$$
$$-5 = A(-2)$$
$$A = \frac{5}{2}$$

$$\frac{x-2}{(x+3)(x+1)} = \frac{A}{x+3} + \frac{B}{x+1} = \frac{5/2}{x+3} + \frac{(-3/2)}{x+1} = \frac{5}{2(x+3)} - \frac{3}{2(x+1)}$$
 (answer)

5. This is an identity that is true for all values of x

We choose a value of x that will eliminate either A or B on the right.

When
$$(x+1) = 0$$

So
$$x = -1$$

Using x = -1 in the equation gives: **REFER NO 4**

When
$$x+3 = 0$$

So $x = -3$

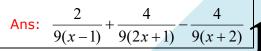
Using x = -3 in the equation gives: **REFER NO 4**

$$= \frac{5}{2(x+3)} - \frac{3}{2(x+1)}$$
 (answer)

1.
$$\frac{2x-1}{(2x+1)(x-3)}$$

$$2. \quad \frac{2x}{(x-1)(2x+1)(x+2)}$$

Ans: $\frac{4}{7(2x+1)} + \frac{5}{7(x-3)}$



3.
$$\frac{2x-1}{(x-2)(x+1)(x+3)}$$

4.
$$\frac{5x+6}{(x+2)(1-x)}$$

Ans:

$$\frac{1}{5(x-2)} + \frac{1}{2(x+1)} - \frac{7}{10(x+3)}$$

Ans: $\frac{-4}{3(x+2)} + \frac{11}{3(1-x)}$

5.
$$\frac{2x^2 - 5x + 6}{x(x-2)(x-3)}$$

6.
$$\frac{x^2 + 1}{x(x+1)(x-1)}$$

Ans:
$$\frac{1}{x} - \frac{2}{(x-2)} + \frac{3}{(x-3)}$$

1.
$$\frac{x+2}{x^2+12x+32}$$

$$2. \ \frac{2}{x^2 - 2x}$$

Ans: $\frac{-1}{2(x+4)} + \frac{3}{2(x+8)}$

$$3. \quad \frac{2x^2 + 3x}{(x+1)^2(x^2-2)}$$

4.
$$\frac{x^2}{(x^2+2x+3)(2x+1)}$$

Ans:
$$\frac{-1}{(x+1)} + \frac{1}{(x+1)^2} + \frac{x}{(x^2-2)}$$

Ans:
$$\frac{1}{9(2x-1)} + \frac{4x-3}{9(x^2+2x+3)}$$
 2 2

f. Convert improper fraction to mixed number by using long

Example:

Find the partial fraction decomposition of $\frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2 - 1)}$

$$(x-1)(x^2-1) = x^3 - x - x^2 + 1 = x^3 - x^2 - x + 1$$

$$x^{3} - x^{2} - x + 1 \overline{\smash)x^{3} - x^{2} - 3x + 5}$$

$$- (x^{3} - x^{2} - x + 1)$$

$$0 - 0 - 2x + 4$$

The numerator is of degree 3; the denominator is of degree 3. So first I have to do the **long division**:

$$\frac{x^3 - x^2 - 3x + 5}{(x - 1)(x^2 - 1)} = 1 + \frac{\left(-2x + 4\right)}{(x - 1)(x^2 - 1)}$$

The long division rearranges the rational expression to give:

$$\frac{-2x+4}{(x-1)(x^2-1)} = \frac{-2x+4}{(x-1)(x+1)(x-1)} = \frac{-2x+4}{(x+1)(x-1)^2}$$
$$2x+4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

Decompose the fractional part. The denominator factors as $(x+1)(x-1)^2$

$$2x + 4 = A(x - 1)^{2} + B(x + 1)(x - 1) + C(x + 1)$$

$$-2(1) + 4 = A((1) - 1)^{2} + B((1) + 1)((1) - 1) + C((1) + 1)$$

$$4 = A(0)^{2} + B(2)(0) + C(2)$$

$$C = \frac{2}{2} = 1$$

Find all values for A,B and C

For
$$x = 1$$

$$-2x + 4 = A(x-1)^{2} + B(x+1)(x-1) + C(x+1)$$

$$-2(-1) + 4 = A((-1)-1)^{2} + B((-1)+1)((-1)-1) + C((-1)+1)$$

$$6 = A(-2)^{2} + B(0)(-2) + C(0)$$

$$A = \frac{6}{4} = \frac{3}{2}$$
For $x = -1$

$$-2x+4 = A(x-1)^{2} + B(x+1)(x-1) + C(x+1)$$

$$-2x+4 = A(x^{2}-2x+1) + B(x^{2}-1) + Cx + C$$

$$-2x+4 = x^{2}(A+B) + x(-2A+C) + A-B+C$$

Coefficients is use there are couple of values of x that cannot be allow to quickly get two of the three of constants

$$x^2$$
: x : x^0 : Coefficients:
$$0 = A + B \qquad -2 = -2A + C \qquad 4 = A - B + C \qquad \text{by comparing coefficients of } x^2, x$$
 and constants in the identity.
$$B = -\frac{3}{2}$$

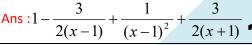
$$\frac{x^3 - x^2 - 3x + 5}{(x - 1)(x^2 - 1)} = 1 - \frac{2x + 4}{(x - 1)(x^2 - 1)} = 1 + \frac{3}{2(x + 1)} - \frac{3}{2(x - 1)}$$
 (answer)

EXERCISE

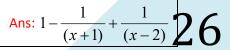
Find each partial fraction of improper fraction decomposition

1.
$$\frac{x^3}{(x+2)(x-3)}$$

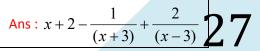
2.
$$\frac{x^3 - x^2 - 3x + 5}{(x - 1)(x^2 - 1)}$$



$$3. \quad \frac{x^2 - x + 1}{x^2 - x - 2}$$



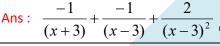
$$4. \quad \frac{x^3 + 2x^2 - 10x - 9}{x^2 - 9}$$



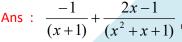
EXERCISE

Find each partial fraction decomposition

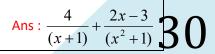
1.
$$\frac{-2x^2 + 8x + 6}{x^3 - 3x^2 - 9x + 27}$$



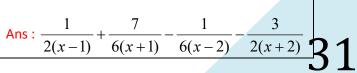
$$2. \ \frac{x^2 - 2}{(x+1)(x^2 + x + 1)}$$

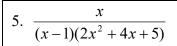


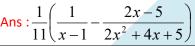
$$3. \ \frac{6x^2 - x + 1}{x^3 + x^2 + x + 1}$$



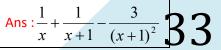
4.
$$\frac{2x^2 - 5x}{(x^2 - 1)(x^2 - 4)}$$



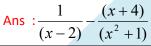




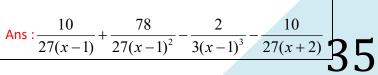
$$6. \quad \frac{2x^2 + 1}{x^3 + 2x^2 + x}$$



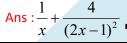
7.
$$\frac{9-2x}{(x-2)(x^2+1)}$$



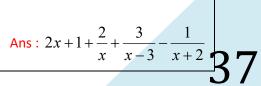
$$8. \quad \frac{4x^2 - 6}{(x - 1)^3 (x + 2)}$$



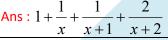
9.
$$\frac{4x^2 + 1}{x(2x - 1)^2}$$



10.
$$\frac{2x^4 - x^3 - 9x^2 + x - 12}{x^3 - x^2 - 6x}$$



11.
$$\frac{x^3 + 7x^2 + 9x + 2}{x(x^2 + 3x + 2)}$$



12.
$$\frac{x^3 - 6x^2 + 5x - 3}{x^2 - 1}$$

