Preconditioning Seismic Inverse Problems with AutoEncoders

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Ill-posed inverse problems are ubiquitous across different scientific disciplines: this is in part due to physical and economical requirements limiting the amount of data that can be acquired and subsequently used to constrain the inverse process. A powerful paradigm, originally developed in the area of medical imaging – e.g., Lustig et al. (2007) – leverages the fact that the sought models may be sparse or have a sparse representation in some transformed domain. This is also the case for seismic data, which can be explained by a few non-zero coefficients in several domains (e.g., FK, Radon, Curvelet). As a result, sparsity-promoting inversion can be employed to reconstruct seismic data when dealing with unfavourable acquisition conditions.

Motivated by the ability of deep neural networks to identify compact representations of high-dimensional vector spaces, recent work in MRI imaging (Li et al., 2020; Obmann et al., 2020) has suggested that, learned nonlinear dimensionality reduction techniques, such as AutoEncoders (AEs), may be better suited than fixed-basis sparsifying transforms for regularizing (or preconditioning) such a severely ill-posed inverse problem. Along similar lines, Ravasi (2021) proposed a two-step approach for the solution of a joint deghosting and seismic data reconstruction problem: first, an AE is trained to learn a latent representation of the input seismic data. Subsequently, its decoder is used as a nonlinear preconditioner in the solution of the physics-based problem.

Here, we extend this procedure to the problem of joint reconstruction and wavefield separation, which differs from the former application in the fact that we seek to find both the up- and down-going pressure wavefields that explain the recorded multi-component seismic data. Formally, wavefield separation can be cast as an inverse problem as follows:

$$\begin{bmatrix} \mathbf{p} \\ \mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{W}^+ & \mathbf{W}^- \end{bmatrix} \begin{bmatrix} \mathbf{p}^+ \\ \mathbf{p}^- \end{bmatrix} \rightarrow \mathbf{d} = \mathbf{G}\mathbf{p}^\pm \tag{1}$$

where $\mathbf{p}$ and $\mathbf{v_z}$ are the recorded pressure and vertical particle velocity data, $\mathbf{p}^-$ and $\mathbf{p}^+$ are the up- and down-going separated data, $\mathbf{I}$ is the identity operator, and $\mathbf{W}^\pm$ are operators that apply the obliquity factor in the frequency-wavenumber domain. A latent representation of the full pressure wavefield is obtained using a training dataset of n_s samples, $P = \{\mathbf{p}_1, \mathbf{p}_2, ..., \mathbf{p}_{n_s}\}$ by minimizing the following cost function:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \frac{1}{n_s} \sum_{i=1}^{n_s} ||\mathbf{p}_i - D_\theta(E_\theta(\mathbf{p}_i))||_p + \epsilon_\theta ||\theta||_2 \tag{2}$$

where $E_\theta : \mathbb{R}^m \rightarrow \mathbb{R}^k$ and $D_\theta : \mathbb{R}^k \rightarrow \mathbb{R}^m$ are the encoder and decoder networks, respectively. The learned decoder D_θ is then used as a nonlinear preconditioner in:

$$\hat{\mathbf{z}}^+, \hat{\mathbf{z}}^- = \underset{\mathbf{z}^+, \mathbf{z}^-}{\operatorname{argmin}} \left\| \mathbf{d} - \mathbf{G} \begin{bmatrix} D_\theta(\mathbf{z}^+) \\ D_\theta(\mathbf{z}^-) \end{bmatrix} \right\|_2 + \epsilon_Z (||\mathbf{z}^+||_2 + ||\mathbf{z}^-||_2) \tag{3}$$

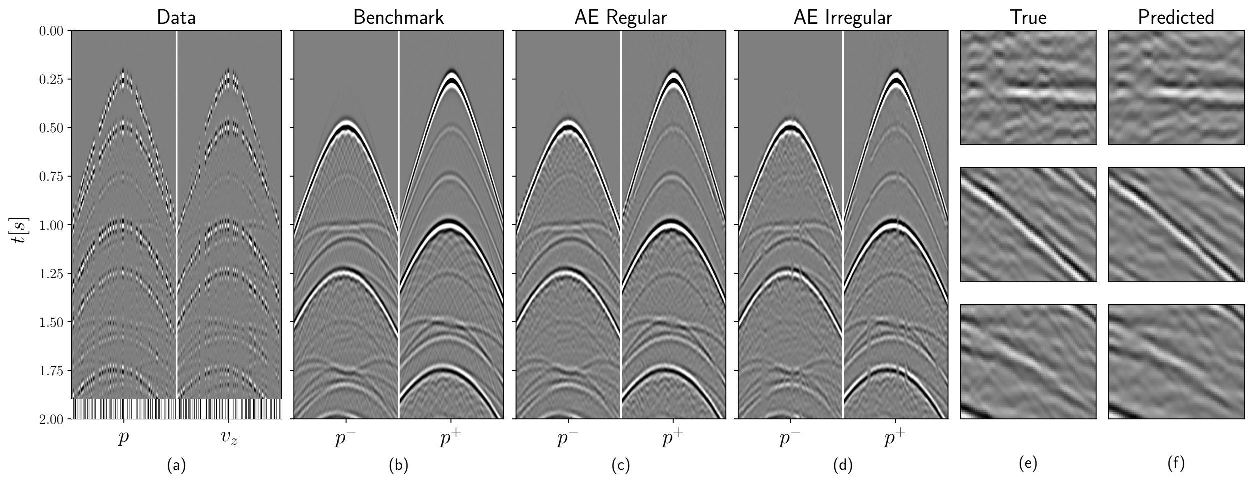

Figure 1: a) Observed multi-component data with irregular sub-sampling, b) Benchmark up- and down-going separated pressure data from finely sampled data, c-d) Up- and down-going reconstructed and separated pressure data from regularly and irregularly downsampled data, respectively, by means of. e-f) 3 seismic patches and the corresponding outputs from the AutoEncoder network.

The proposed inversion framework is applied to a synthetic dataset modelled using the same velocity model as in Ravasi (2021) and multi-component receivers in a ocean-bottom configuration. Receivers are sub-sampled in two ways: i) regularly by keeping one receiver in every 4, ii) irregularly by retaining 30% of the original array (Figure 1a). In the training phase, the pressure recordings are sorted in the common receiver gather (CRG) and patches of size 64×64 are fed to an AE with a latent space of size $k = 1000$. The network is trained for 20 epochs using the Adam optimizer. Three patches from the validation dataset and the corresponding outputs of the AE are shown in Figures 1e and f, respectively: this result shows that the information contained in the data can be comfortably compressed by a factor of 3.6 with minimal reconstruction error. The trained decoder is finally combined with the physical modelling operator to reconstruct the missing receivers and separate the up- and down-going components of the data: the estimated wavefields for regular and irregular subsampling are shown in Figure 1c and d, respectively. In both cases, the reconstructed wavefields closely resemble those obtained by performing a standard wavefield separation on the original, finely sampled data (Figure 1b).

To conclude, in this work have extended the framework proposed in Ravasi (2021) for the solution of joint wavefield separation and data interpolation of seismic data. In the presentation, we will show results from both problems and discuss similarities and differences in their training and inversion strategies.

References

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