

Accelerating seismic processing with Tile Low-Rank matrix approximations

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Co-authors



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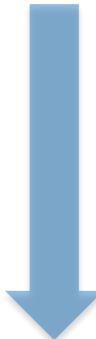
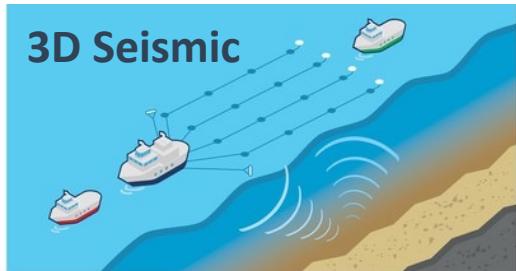
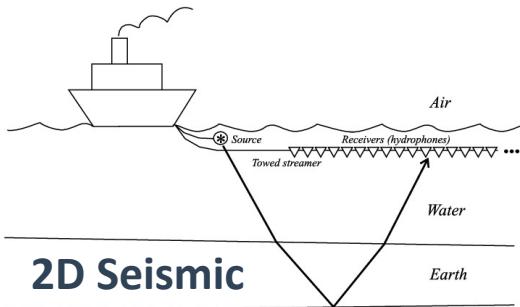


David Keyes

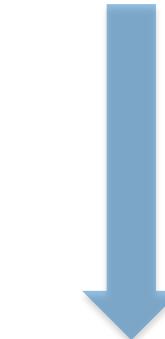


Hatem Ltaief

New challenges



More
data



More
computations

Adjoint-based
processing $m = f(d)$

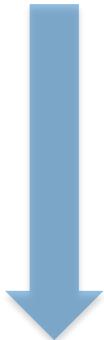
(SRME, IME)

Inversion-based
processing

$d = g(m)$
(EPSI, MDD, Mck)

New technologies

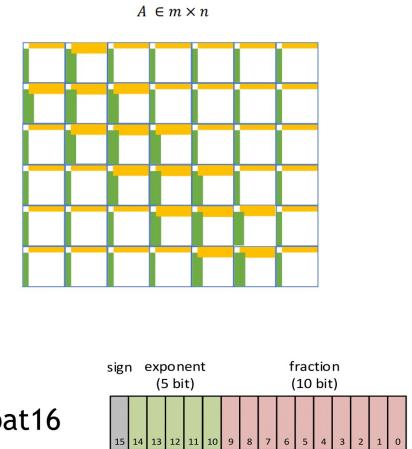
Hardware



*More memory
and compute*

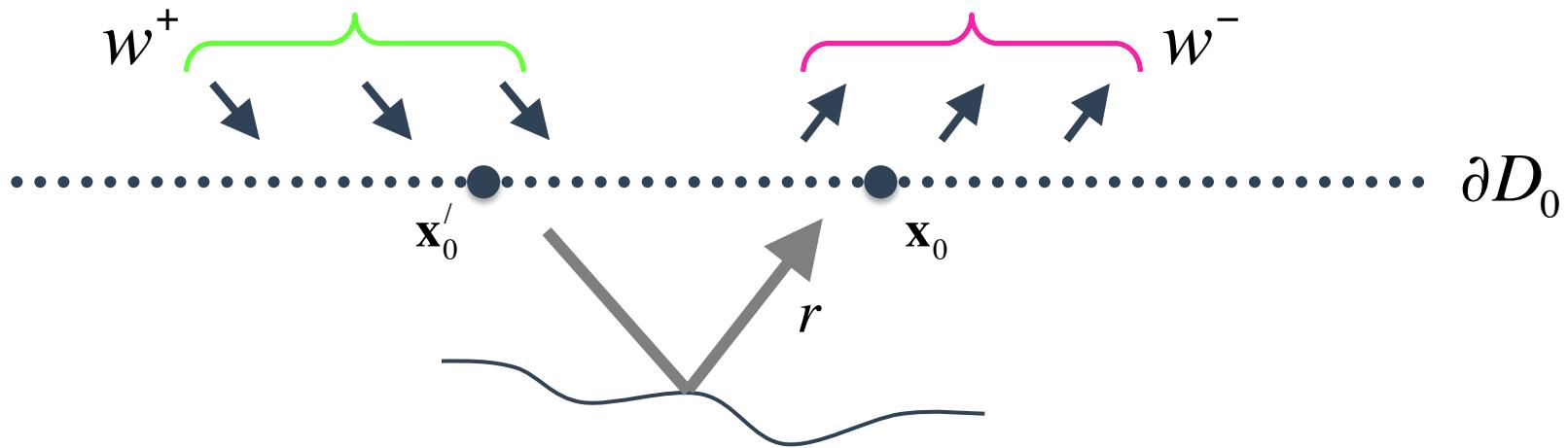


*Less memory
and compute*



Algorithms & Software

A common building block



Multidimensional
convolution (MDC)

$$\underline{w^-(\mathbf{x}_0, \mathbf{x}_S, t)} = \int \int \underline{w^+(\mathbf{x}'_0, \mathbf{x}_S, t - \tau)} \underline{r(\mathbf{x}_0, \mathbf{x}'_0, \tau)} d\tau d\mathbf{x}'_0$$

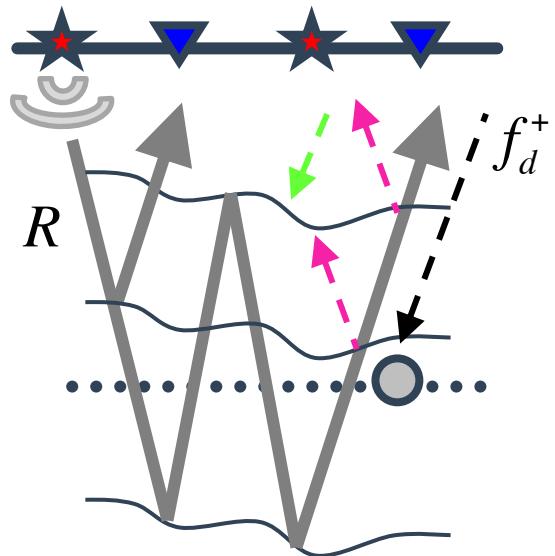
A common building block

$$\mathbf{w}^- = \mathbf{W}^+ \mathbf{r}$$

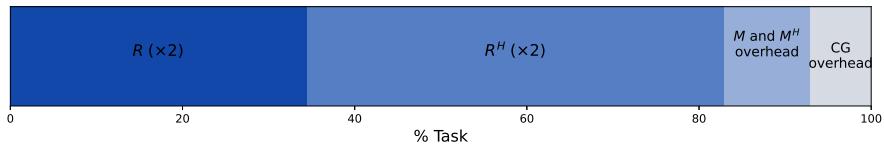
The diagram illustrates the components of the equation $\mathbf{w}^- = \mathbf{W}^+ \mathbf{r}$. A large brace groups the terms \mathbf{F}^H , $\mathbf{I}^H_{\omega_{\max}}$, $\hat{\mathbf{W}}^+$, $\mathbf{I}_{\omega_{\max}}$, and \mathbf{F} . Below this group, five arrows point upwards to their respective labels: "Real IFFT" points to \mathbf{F}^H ; "Padding" points to $\mathbf{I}^H_{\omega_{\max}}$; "Batch matrix multiplication" points to $\hat{\mathbf{W}}^+$; "Restriction" points to $\mathbf{I}_{\omega_{\max}}$; and "Real FFT (\mathbf{r} is real)" points to \mathbf{F} .

Sweet spot!!

An example application



Task % for one iteration of CG:



Marchenko equation

$$\begin{bmatrix} \Theta R f_d^+ \\ 0 \end{bmatrix} = \begin{bmatrix} I & -\Theta R \\ \Theta R^* & I \end{bmatrix} \begin{bmatrix} f^- \\ f_m^+ \end{bmatrix}$$

R, R^* : MDC operators

Forward: 2 MDC, Adjoint: 2 MDC

CGLS (10 iterations): 40 MDC

MDD



EPSI



Marchenko



Applications

Mixed-Precision Tile Low-Rank MVM (gemv) / MMM (gemm)

Algorithms



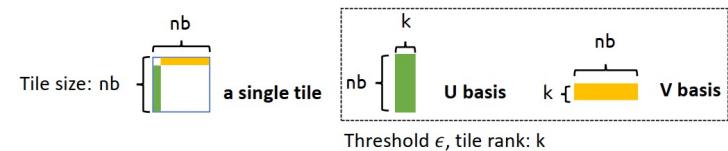
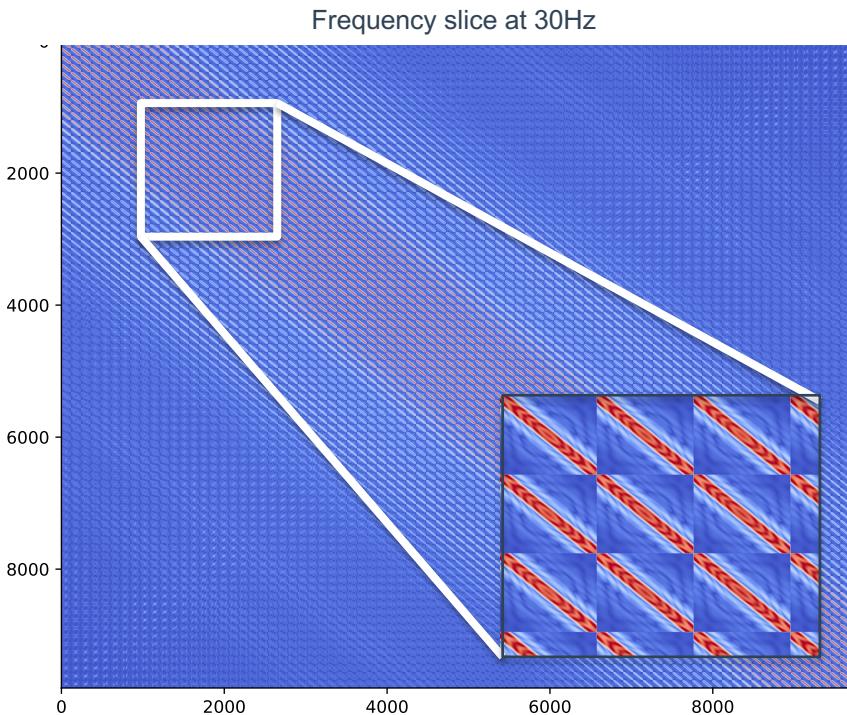
OpenMP

OPEN MPI



Libraries

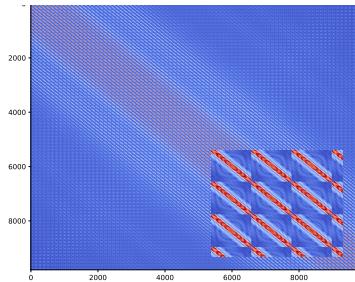
Why Tile Low-Rank compression?



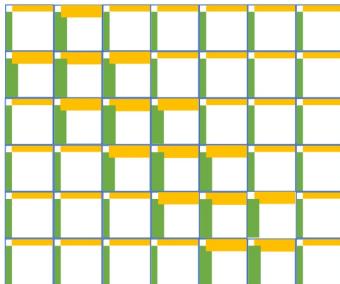
$$A \in M \times N$$

x = y

Why Tile Low-Rank compression?



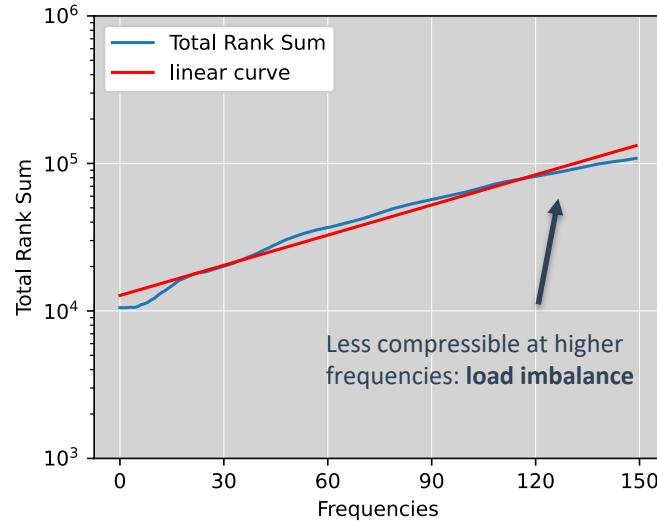
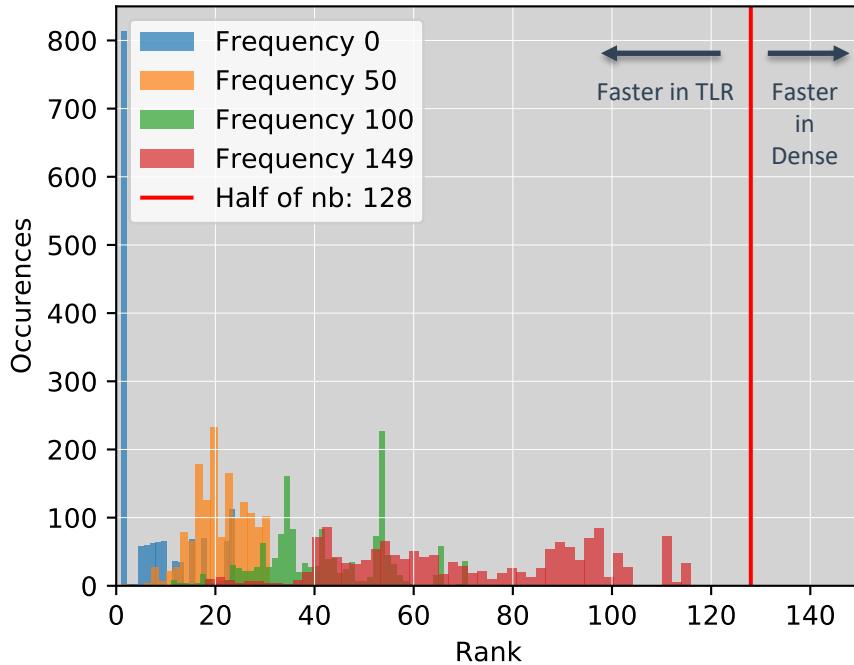
Original matrix dimension = $150 \times 9801 \times 9801 \times 8$ Bytes = 115.27 GB



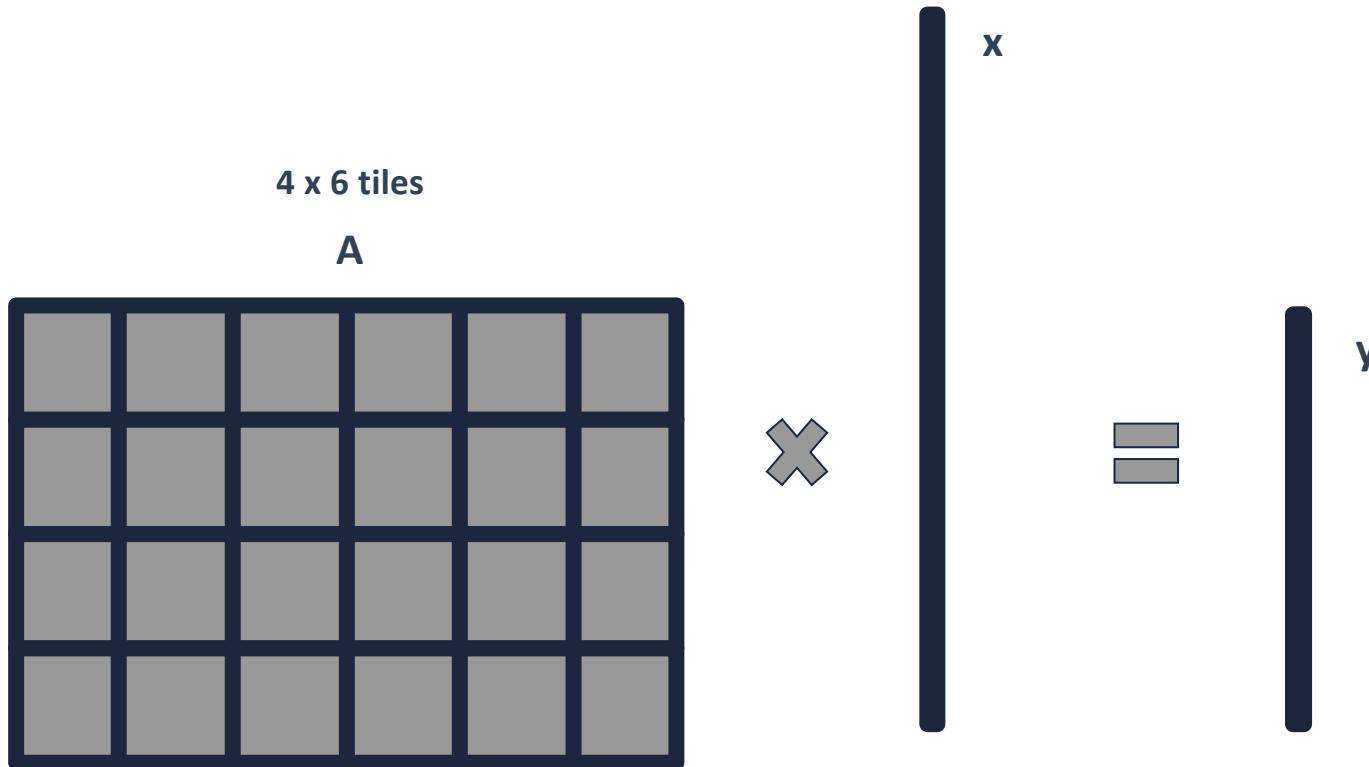
Compressed format of data (U and V for 150 frequencies) = 31 GB

Compression factor = 3.7

Why Tile Low-Rank compression?



Tile Low-Rank Matrix-Vector Multiplication



Tile Low-Rank Matrix-Vector Multiplication

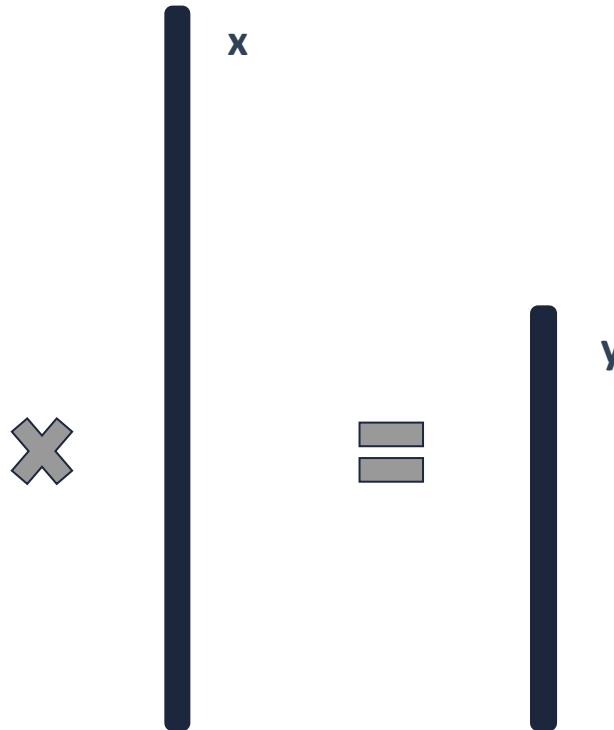
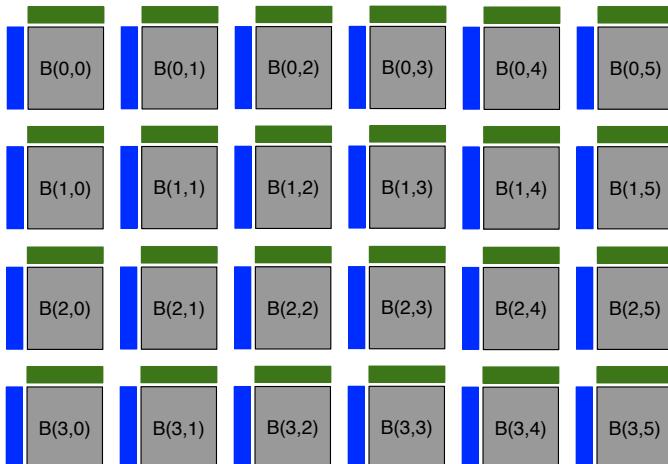
Compress (SVD-like algorithm)

Only once upfront!

4 x 6 tiles

A

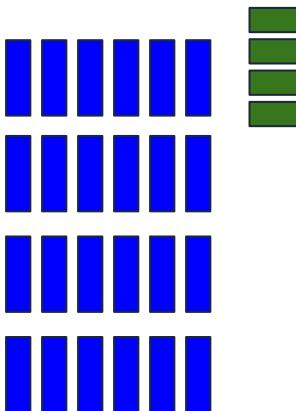
Ranks can be
different



Tile Low-Rank Matrix-Vector Multiplication

Stack U and V bases

Only once upfront!



x

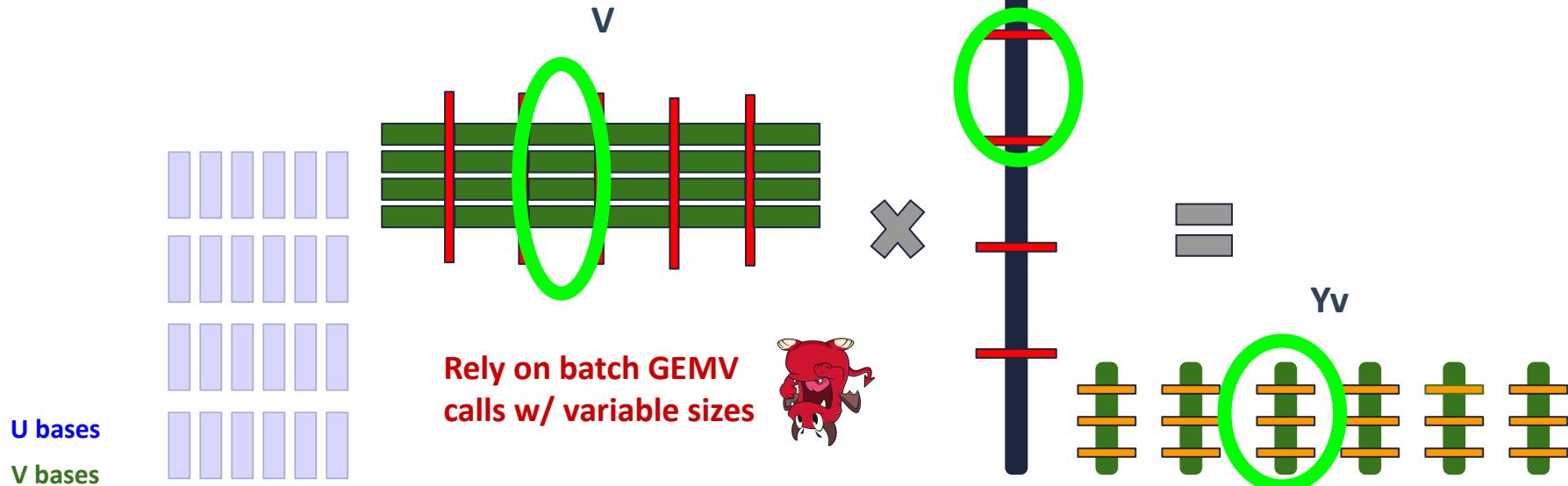


y



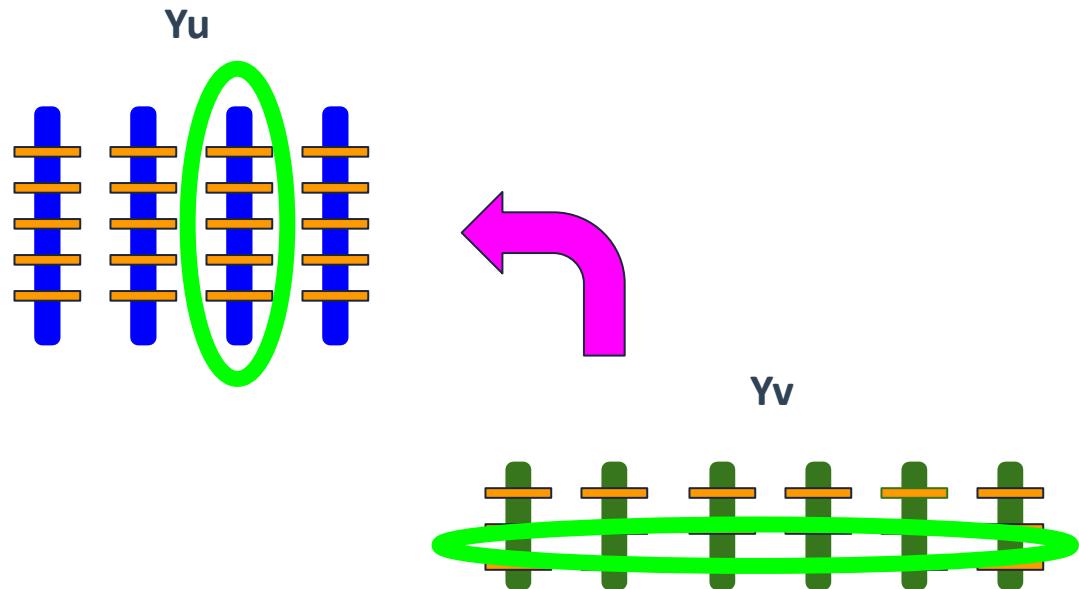
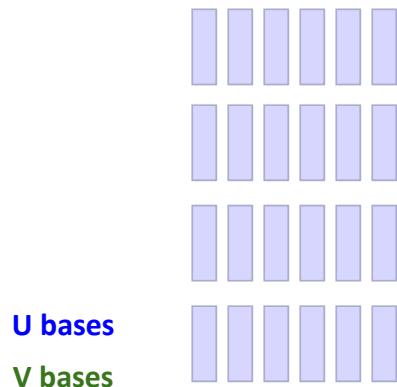
Tile Low-Rank Matrix-Vector Multiplication

PH1: Calculate (per red part): $Yv = V \cdot x$



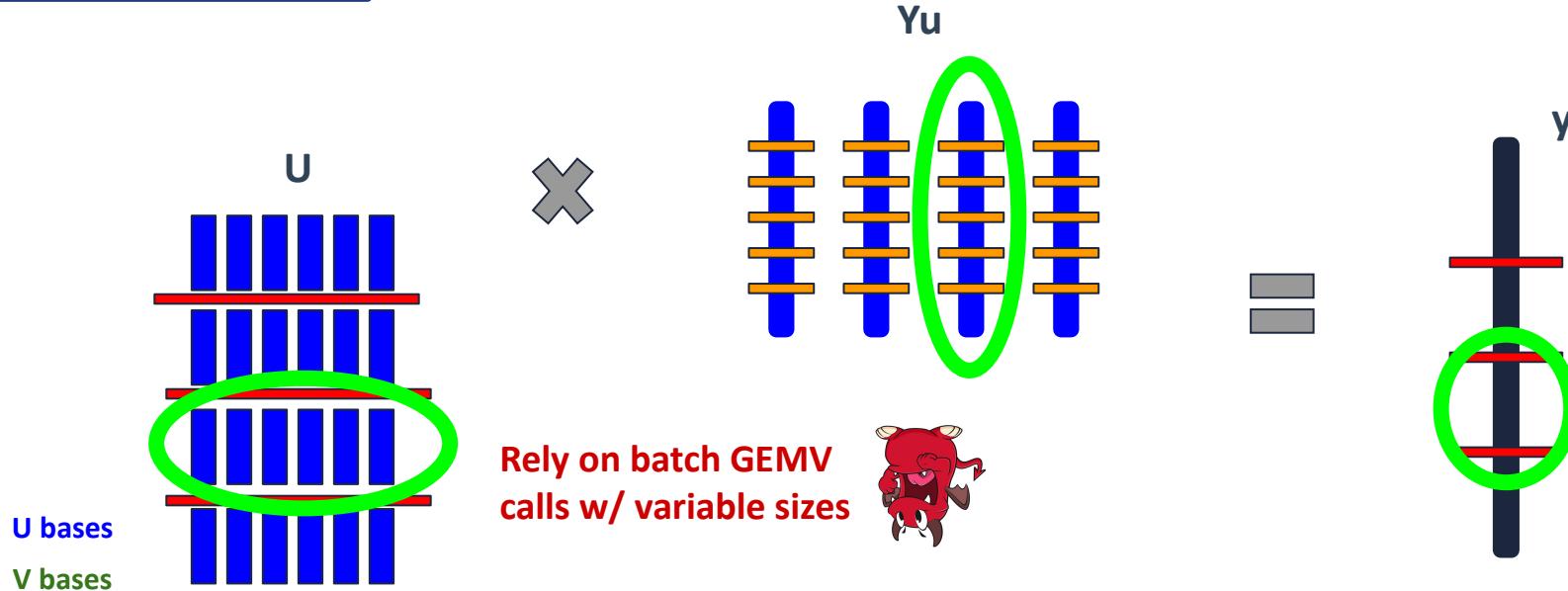
Tile Low-Rank Matrix-Vector Multiplication

PH2: Translate \mathbf{Yv} (\mathbf{V} bases) to \mathbf{Yu} (\mathbf{U} bases)

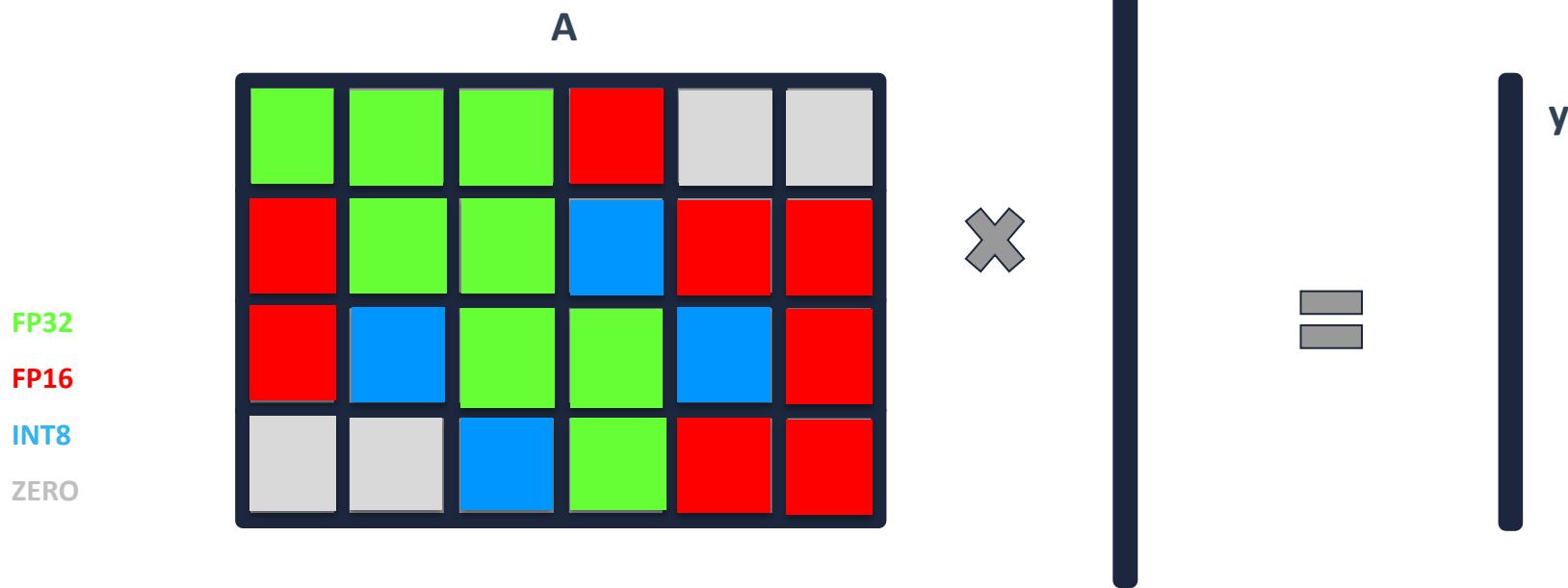


Tile Low-Rank Matrix-Vector Multiplication

PH3: Calculate $y = U \cdot Yu$



Mixed precisions

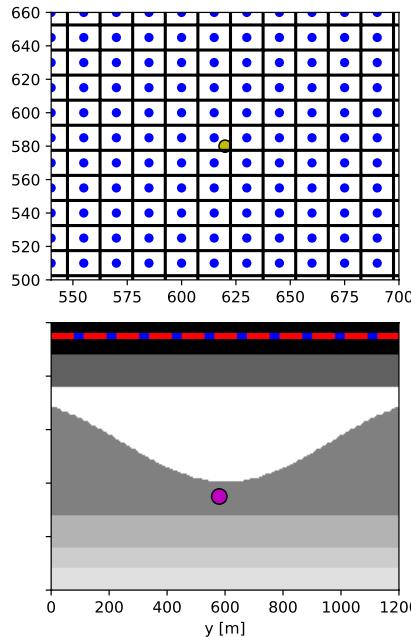
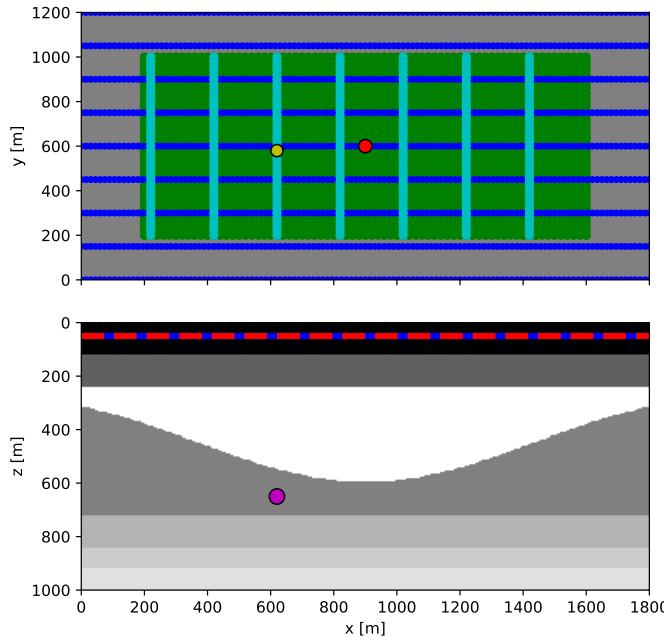


Mixed-precision hardware support

GEMV/GEMM capabilities

Hardware	FP32	FP16	INT8
intel	✓	✗	✗
AMD	✓	✓	✓
NEC	✓	✗	✗
FUJITSU	✓	✓	✓
NVIDIA	✓	✓	✓

Numerical Example



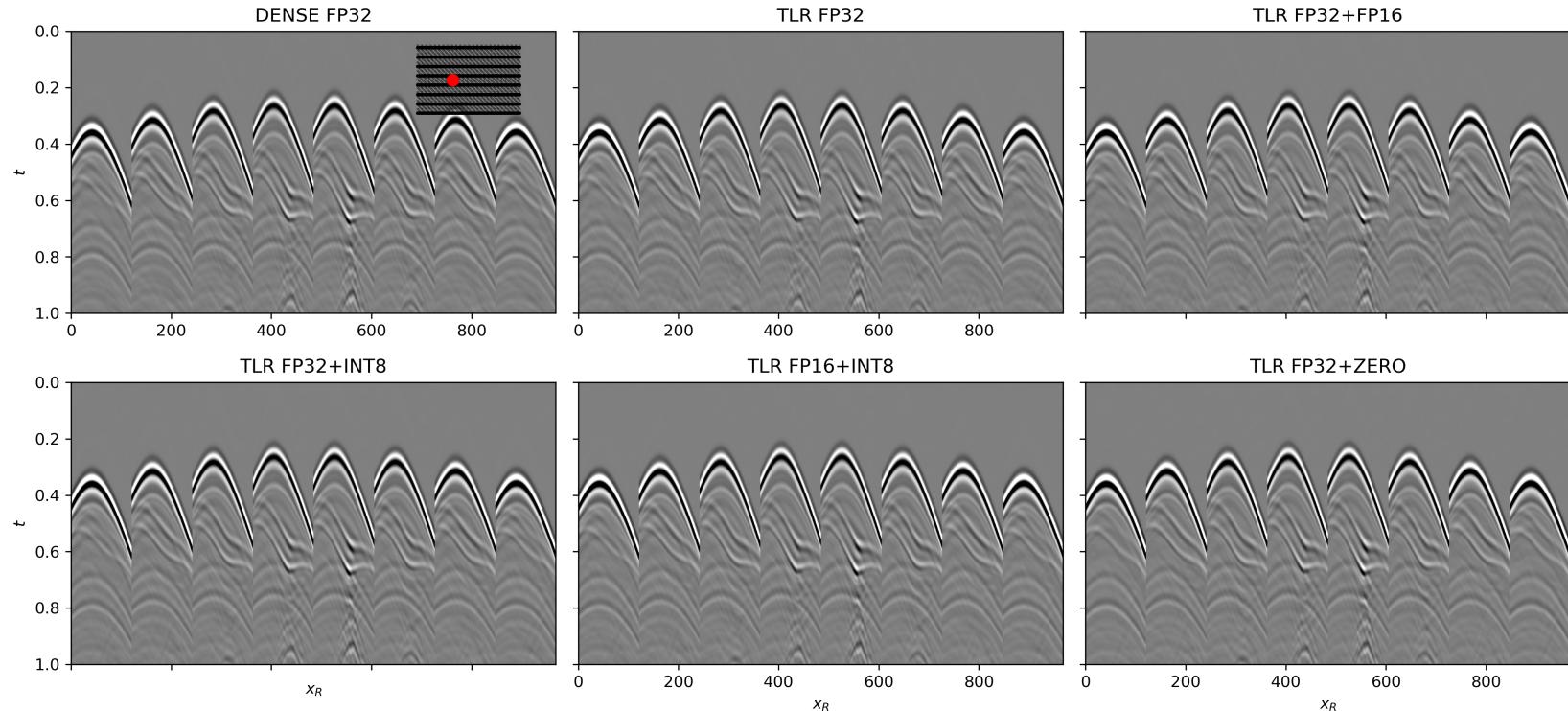
- Constant velocity, variable density model of size 1.8km x 1.2km x 1km.
- Regular carpet of sources and receivers

$$N_S = N_R = 9801, \quad dx_S = dx_R = 15\text{m}$$

Redatuming carpet @ 650m
 Marchenko lines in displays

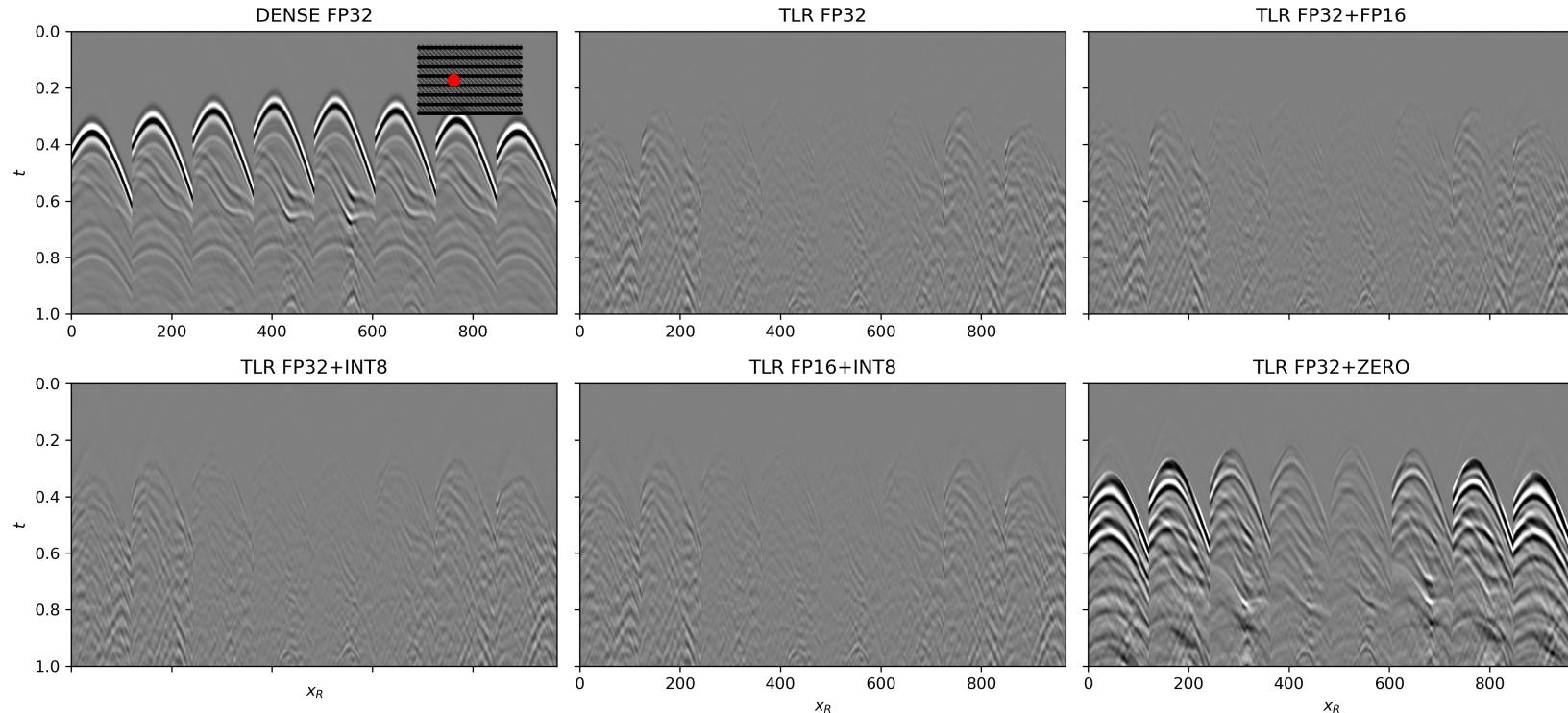
Numerical Example

Inverted Green's function HBL=20 (t^2 gain)

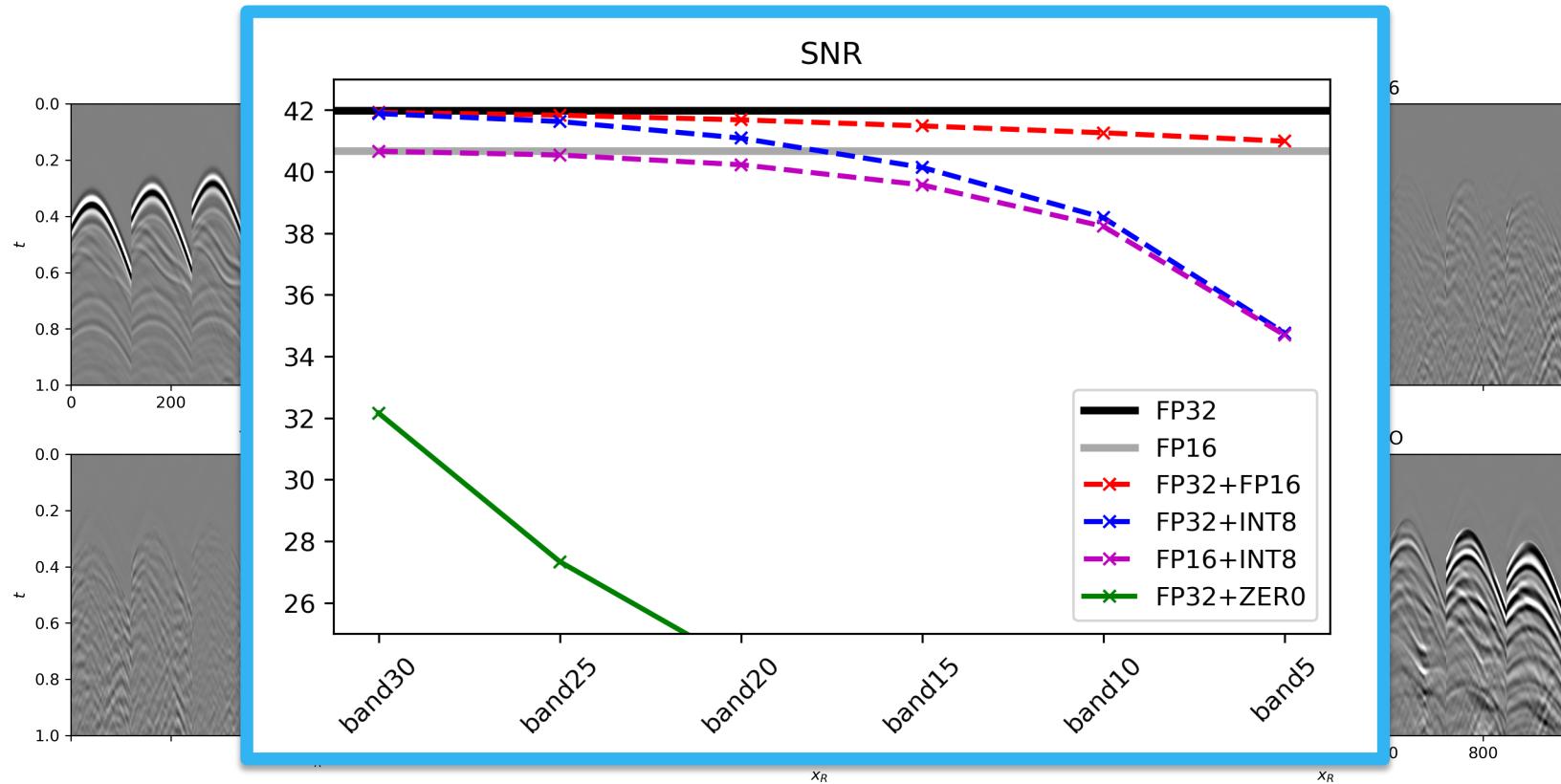


Numerical Example

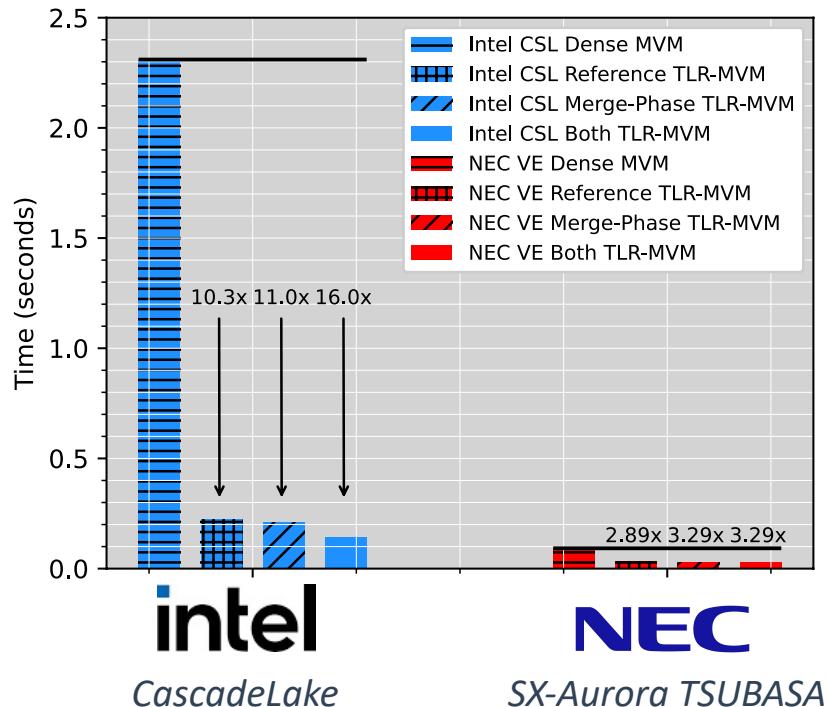
Error x10 HBL=20 (t^2 gain)



Numerical Example



TLR-MVM Performance tests



AMD

EPYC Rome

FUJITSU

A64FX

Coming
soon!

NVIDIA

A100

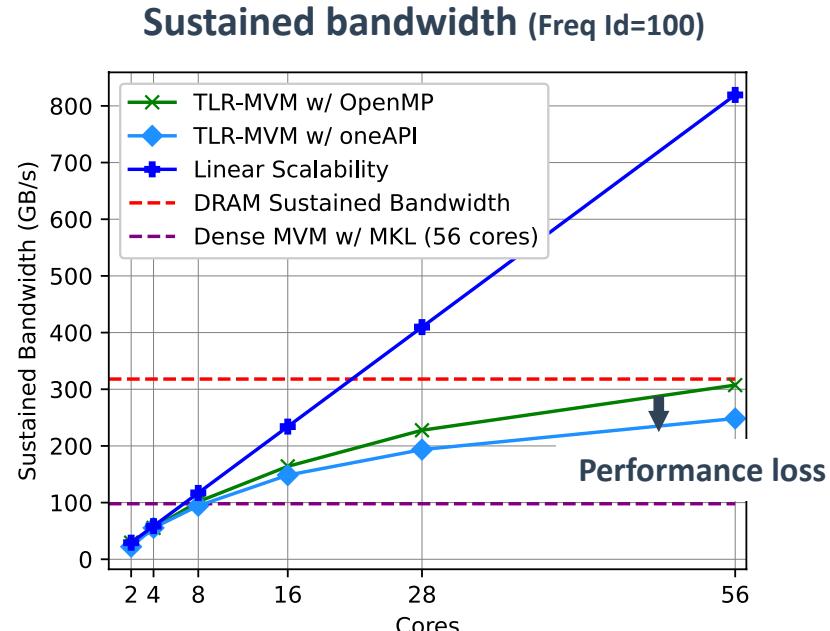
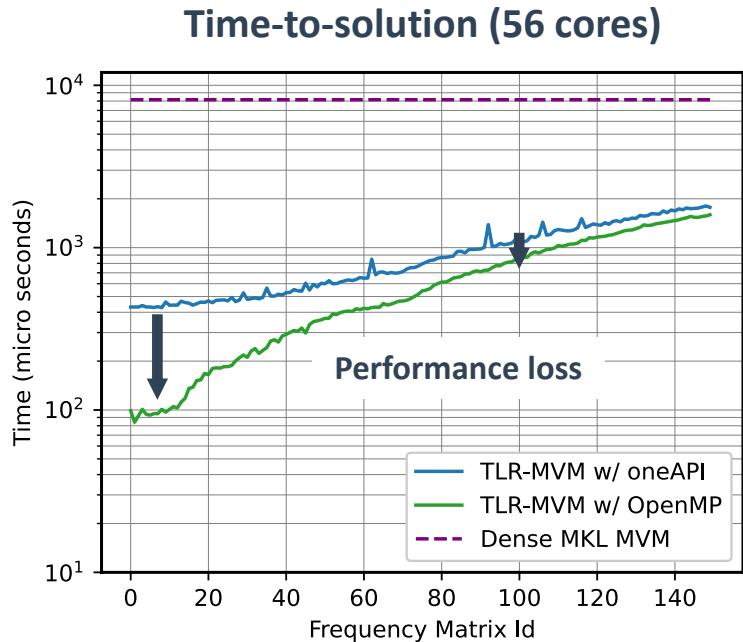
And the dream

```
// phase 1
#pragma omp parallel for           ← OpenMP
for(int i=0; i<tlrmvmptr->Ntg; i++){
    if(tlrmvmptr->colsum[i] != 0){
        cblas_cgemv(CblasColMajor, CblasNoTrans, tlrmvmptr->AvMs[i],
                      tlrmvmptr->AvKs[i], &alpha, tlrmvmptr->h_Avbp[i],
                      tlrmvmptr->AvMs[i], tlrmvmptr->h_xbp[i],
                      1, &beta, tlrmvmptr->h_yvbp[i], 1);
    }
}
```

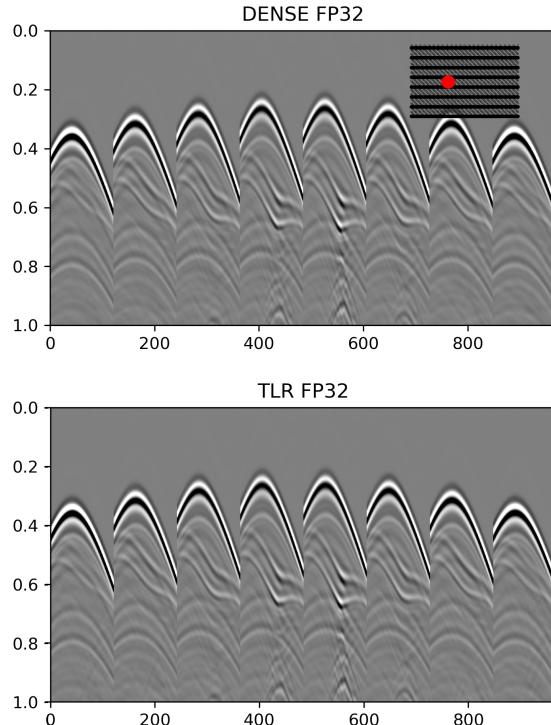
*Simplified code
for Phase 1*

```
----- ←
// phase 1
for(int i=0; i<tlrmvmptr->Ntg; i++){
    if(tlrmvmptr->colsum[i] != 0){
        oneapi::mkl::blas::gemv(tlrmvmptr->device_queue,           ← DPC++ Queue
                                transA, tlrmvmptr->AvMs[i],
                                tlrmvmptr->AvKs[i], alpha, tlrmvmptr->h_Avbp[i],
                                tlrmvmptr->AvMs[i], tlrmvmptr->h_xbp[i],
                                1, beta, tlrmvmptr->h_yvbp[i], 1);
    }
}
tlrmvmptr->device_queue.wait();
```

And the dream



Conclusion



Can you spot the difference?
... your *energy bill* will!

TLR-based MDC (NEC 128VEs)

Size reduction: x3.7

Time reduction: x3

Sustained bandwidth: **110 TB/s**

References

- M Ravasi, I Vasconcelos , An open-source framework for the implementation of large-scale integral operators with flexible, modern high-performance computing solutions: Enabling 3D Marchenko imaging by least-squares inversion, Geophysics 86 (5), WC177-WC194.
- Y Hong, H Ltaief, M Ravasi, L Gatineau, DE Keyes, Accelerating Seismic Redatuming Using Tile Low-Rank Approximations on NEC SX-Aurora TSUBASA, Supercomputing Frontiers and Innovations.

Thank you for listening!

07. 12. 2021



Deep imaging group.

