Technical Annex

1 Full Results

The complete table of results of explanation-redundancy evaluation is shown in Table 1 and the complete table of results of runtime assessment for computing AXp's are reported in Table 2. Besides, Table 3 summarizes results on path explanation redundancy for DTs included in representative bibliography on DTs, namely textbooks and surveys [14, 6, 16, 7, 8, 17, 4, 19, 3, 9, 26, 12, 1, 20, 11, 2, 22, 15, 24, 5, 25]. Table 4 shows results for DTs learned with recently proposed algorithm that specifically target the learning of optimal (and so indirectly *interpretable*) DTs, concretely [10, 13, 18] and also [23]. Table 5 shows results for DTs learned on more complex datasets (which are also publicly available).

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Dataset	IAI									ITI								
	D	**#N**	**%A**	**#P**	**%R**	**%C**	**%m**	**%M**	**%avg**	**D**	**#N**	**%A**	**#P**	**%R**	**%C**	**%m**	**%M**	**%avg**
IndiansDiabetes	6	33	67	17	35	2	20	25	21	21	67	60	34	38	22	20	33	26
adult	6	83	78	42	33	25	20	40	25	17	509	73	255	75	91	10	66	22
allhyper	6	47	95	24	25	2.5	20	33	23	14	49	96	25	64	25	12	50	35
ann-thyroid	6	61	97	31	25	30	20	50	36	48	223	93	112	65	46	11	75	22
anneal	6	29	99	15	26	16	16	33	21	9	31	100	16	25	4	12	20	16
appendicitis	2	7	68	4	0	0	—	—	—	3	7	81	4	50	3	50	66	58
australian	6	45	61	23	17	8	20	33	23	7	33	82	17	35	51	16	33	26
auto	6	33	53	17	23	1	16	33	23	10	47	75	24	33	53	14	40	26
backache	4	17	72	9	33	39	25	33	30	3	9	91	5	80	87	50	66	54
balance	6	93	81	47	61	41	25	50	27	12	105	81	53	50	62	25	50	26
bank	6	113	88	57	5	12	16	20	18	19	1467	86	734	69	64	7	63	27
banknote	3	9	67	5	20	0	66	66	66	35	71	54	36	83	2	33	50	43
biodegradation	5	19	65	10	30	1	25	50	33	8	71	76	36	50	8	14	40	21
biomed	3	9	66	5	20	1	50	50	50	8	33	64	17	29	9	25	50	33
breast-cancer	6	61	74	31	41	13	16	33	20	7	25	70	13	23	33	25	33	30
bupa	6	47	62	24	29	5	20	33	25	24	49	59	25	64	25	16	50	25
cancer	6	37	87	19	36	9	20	25	21	5	21	84	11	54	10	25	50	37
car	6	43	96	22	86	89	20	80	45	11	57	98	29	65	41	16	50	30
cleveland	6	81	31	41	24	15	16	33	20	7	65	31	33	33	71	16	40	30
coil2000	6	99	92	50	18	9	16	33	18	12	177	91	89	79	98	9	77	42
colic	6	55	81	28	46	6	16	33	20	4	17	80	9	33	27	25	25	25
compas	6	77	34	39	17	8	16	20	17	15	183	37	92	66	43	12	60	27
contraceptive	6	99	49	50	8	2	20	60	37	17	385	48	193	27	32	12	66	21
dermatology	6	33	90	17	23	3	16	33	21	7	17	95	9	22	0	14	20	17
divorce	5	15	90	8	50	19	20	33	24	2	5	96	3	33	16	50	50	50
ecoli	6	45	75	23	4	5	20	20	20	53	109	59	55	25	19	20	66	36
fars	6	75	79	38	10	86	33	66	52	60	9969	76	4985	35	90	6	50	12
german	6	25	61	13	38	10	20	40	29	10	99	72	50	46	13	12	40	22
heart-c	6	43	65	22	36	18	20	33	22	4	15	75	8	87	81	25	50	34
heart-h	6	37	59	19	31	4	20	40	24	8	25	77	13	61	60	20	50	32
heart-statlog	6	33	55	17	29	5	20	33	30	4	13	81	7	71	31	25	50	35
hepatitis	5	17	77	9	33	6	20	33	24	3	11	80	6	33	14	33	33	33
house-votes-84	6	49	91	25	68	67	16	50	27	3	9	90	5	80	75	33	50	41
hungarian	6	33	69	17	29	2	25	40	31	4	19	77	10	40	30	33	33	33
ionosphere	4	9	70	5	60	3	33	50	38	5	17	80	9	33	0	33	60	47
iris	5	23	90	12	41	25	25	33	30	10	21	63	11	36	13	50	50	50
irish	4	13	97	7	71	54	33	50	36	3	7	100	4	0	0	—	—	—
kddcup	6	91	99	46	0	0	—	—	—	29	269	99	135	22	0.9	6	20	9
kr-vs-kp	6	49	96	25	80	75	16	60	33	13	67	99	34	79	43	7	70	35
lending	6	45	73	23	73	80	16	50	25	14	507	65	254	69	80	12	75	25
letter	6	127	58	64	1	0	20	20	20	46	4857	68	2429	6	7	6	25	9
lymphography	6	61	76	31	35	25	16	33	21	6	21	86	11	9	0	16	16	16
messidor	3	7	50	4	50	1	50	66	58	22	107	52	54	88	99	10	57	28
promoters	6	17	86	9	33	19	16	33	23	3	9	81	5	20	14	33	33	33
mushroom	6	39	100	20	80	44	16	33	24	5	23	100	12	50	31	20	40	25
mux6	6	55	61	28	85	78	20	50	37	4	15	46	8	37	31	25	33	30
new-thyroid	3	11	95	6	33	4	33	33	33	14	29	79	15	26	5	20	50	30
pendigits	6	121	88	61	0	0	—	—	—	38	937	85	469	25	86	6	25	11
postoperative	6	43	50	22	59	44	16	50	25	6	15	56	8	75	25	16	50	36
primary-tumor	6	55	71	28	35	21	16	33	21	6	21	82	11	54	31	16	50	31
promoters	1	3	90	2	0	0	—	—	—	3	9	81	5	20	14	33	33	33
recidivism	6	105	61	53	28	22	16	33	18	15	611	51	306	53	38	9	44	16
schizo	6	17	55	9	55	5	50	66	58	13	51	52	26	30	64	20	25	21
segmentation	4	15	38	8	0	0	—	—	—	27	57	23	29	48	2	11	71	39
seismic_bumps	6	37	89	19	42	19	20	33	24	8	39	93	20	60	79	20	60	42
shuttle	6	63	99	32	28	7	20	33	23	23	159	99	80	33	9	14	50	30
soybean	6	63	88	32	9	5	25	25	25	16	71	89	36	22	1	9	12	10
spambase	6	63	75	32	37	12	16	33	19	15	143	91	72	76	98	7	58	25
spect	6	45	82	23	60	51	20	50	35	6	15	86	8	87	98	50	83	65
splice	3	7	50	4	0	0	—	—	—	88	177	55	89	0	0	—	—	—
student-mat	6	109	35	55	9	3	20	25	21	22	177	25	89	6	6	11	20	15
student-por	6	119	30	60	1	0	20	20	20	22	259	26	130	9	5	7	20	10
tram_2000	1	3	100	2	0	0	—	—	—	1	3	100	2	0	0	—	—	—
uci_mammo	6	53	11	27	51	43	16	40	24	9	23	38	12	66	25	25	55	39
vehicle	6	79	49	40	10	0	16	20	19	24	141	58	71	45	54	9	41	23
wdbc	2	7	87	4	0	0	—	—	—	57	115	61	58	94	10	14	73	49
yeast	6	45	49	23	4	0	25	25	25	64	493	37	247	14	0	11	25	16

Table 1: Path explanation redundancy in decision trees obtained with IAI and ITI. The table shows tree statistics for IAI and ITI, namely, tree depth **D**, number of nodes **#N**, test accuracy **%A** and number of paths **#P**. The percentage of explanation-redundant paths (XRP's) is given as **%R** while the percentage of data instances (measured for the *entire* feature space) covered by XRP's is **%C**. Focusing solely on the XRP's, the average (min. or max., resp.) percentage of explanation-redundant features per path is denoted by **%avg** (**%m** and **%M**, resp.).

Dataset	**IAI**								**ITI**							
	Traversal				**Horn**				**Traversal**				**Horn**			
	m	**M**	**avg**	**Tot**	**m**	**M**	**avg**	**Tot**	**m**	**M**	**avg**	**Tot**	**m**	**M**	**avg**	**Tot**
IndiansDiabetes	0.001	0.003	0.001	0.73	0.001	0.002	0.001	0.63	0.001	0.003	0.002	1.19	0.001	0.002	0.001	1.02
adult	0.001	0.059	0.002	3.52	0.001	0.005	0.002	2.93	0.005	0.065	0.008	0.93	0.008	0.071	0.013	1.53
allhyper	0.001	0.006	0.002	1.88	0.001	0.005	0.001	1.23	0.001	0.004	0.002	2.39	0.001	0.002	0.001	1.20
ann-thyroid	0.001	0.005	0.002	3.67	0.001	0.005	0.001	2.85	0.002	0.041	0.006	12.14	0.004	0.029	0.005	9.64
anneal	0.001	0.005	0.001	1.22	0.001	0.003	0.001	0.75	0.001	0.006	0.001	0.96	0.001	0.004	0.001	0.70
appendicitis	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.032	0.000	0.001	0.000	0.028	0.000	0.001	0.000	0.033
australian	0.001	0.003	0.001	0.93	0.001	0.002	0.001	0.71	0.001	0.004	0.001	0.94	0.001	0.002	0.001	0.61
auto	0.001	0.005	0.001	0.22	0.001	0.005	0.001	0.18	0.001	0.005	0.002	0.33	0.001	0.005	0.001	0.24
backache	0.001	0.001	0.001	0.13	0.000	0.001	0.001	0.094	0.000	0.001	0.000	0.077	0.000	0.001	0.000	0.065
balance	0.001	0.004	0.001	0.82	0.002	0.005	0.002	1.15	0.001	0.057	0.001	0.86	0.002	0.006	0.002	1.23
bank	0.002	0.062	0.003	34.45	0.002	0.008	0.002	25.53	0.013	0.090	0.027	19.64	0.025	0.092	0.033	24.21
banknote	0.000	0.001	0.000	0.12	0.000	0.001	0.000	0.14	0.001	0.002	0.001	0.47	0.001	0.003	0.001	0.54
biodegradation	0.000	0.003	0.001	0.21	0.000	0.002	0.001	0.18	0.002	0.007	0.003	0.98	0.001	0.003	0.002	0.48
biomed	0.000	0.001	0.000	0.064	0.000	0.001	0.000	0.073	0.001	0.002	0.001	0.17	0.001	0.001	0.001	0.16
breast-cancer	0.001	0.003	0.002	0.42	0.001	0.004	0.001	0.35	0.000	0.001	0.001	0.19	0.001	0.001	0.001	0.17
bupa	0.001	0.003	0.001	0.33	0.001	0.002	0.001	0.34	0.001	0.002	0.001	0.33	0.001	0.005	0.001	0.35
cancer	0.001	0.003	0.001	0.53	0.001	0.003	0.001	0.39	0.001	0.004	0.001	0.32	0.001	0.002	0.001	0.26
car	0.001	0.004	0.001	0.52	0.001	0.002	0.001	0.48	0.001	0.002	0.001	0.47	0.001	0.002	0.001	0.59
cleveland	0.001	0.005	0.001	0.25	0.001	0.004	0.002	0.31	0.001	0.002	0.001	0.23	0.001	0.002	0.001	0.25
coil2000	0.003	0.036	0.009	21.56	0.002	0.009	0.002	5.77	0.004	0.044	0.018	44.26	0.003	0.029	0.004	11.02
colic	0.001	0.005	0.002	0.71	0.001	0.002	0.001	0.43	0.000	0.001	0.001	0.24	0.000	0.001	0.000	0.18
compas	0.001	0.004	0.002	0.66	0.001	0.004	0.002	0.52	0.002	0.065	0.004	1.37	0.003	0.005	0.004	1.28
contraceptive	0.001	0.005	0.002	0.70	0.002	0.004	0.002	0.78	0.003	0.064	0.006	2.64	0.006	0.012	0.007	2.87
dermatology	0.001	0.005	0.001	0.50	0.001	0.002	0.001	0.31	0.000	0.001	0.001	0.27	0.000	0.001	0.001	0.19
divorce	0.000	0.002	0.001	0.10	0.000	0.001	0.001	0.076	0.000	0.001	0.000	0.033	0.000	0.002	0.000	0.041
ecoli	0.001	0.003	0.001	0.25	0.001	0.002	0.001	0.32	0.001	0.057	0.001	0.46	0.002	0.004	0.002	0.62
fars	0.001	0.004	0.002	3.26	0.001	0.004	0.002	2.86	0.066	0.18	0.093	172.34	0.19	0.32	0.20	374.52
german	0.001	0.004	0.001	0.76	0.001	0.002	0.001	0.65	0.002	0.007	0.003	3.19	0.002	0.003	0.002	2.03
heart-c	0.001	0.003	0.001	0.42	0.001	0.003	0.001	0.30	0.000	0.001	0.001	0.17	0.000	0.001	0.000	0.14
heart-h	0.001	0.004	0.001	0.39	0.001	0.005	0.001	0.27	0.001	0.001	0.001	0.21	0.001	0.001	0.001	0.18
heart-statlog	0.001	0.003	0.001	0.26	0.001	0.002	0.001	0.23	0.000	0.001	0.001	0.14	0.000	0.001	0.000	0.11
hepatitis	0.001	0.002	0.001	0.12	0.000	0.002	0.001	0.084	0.000	0.001	0.000	0.064	0.000	0.002	0.000	0.062
house-votes-84	0.001	0.006	0.002	0.56	0.001	0.002	0.001	0.32	0.000	0.000	0.000	0.10	0.000	0.001	0.000	0.100
hungarian	0.001	0.003	0.001	0.27	0.001	0.005	0.001	0.24	0.001	0.002	0.001	0.23	0.000	0.002	0.001	0.18
ionosphere	0.000	0.001	0.000	0.15	0.000	0.002	0.001	0.18	0.001	0.003	0.001	0.36	0.000	0.002	0.001	0.26
iris	0.000	0.003	0.001	0.094	0.001	0.001	0.001	0.094	0.000	0.001	0.000	0.050	0.000	0.001	0.001	0.082
irish	0.000	0.002	0.000	0.20	0.000	0.001	0.000	0.21	0.000	0.001	0.000	0.092	0.000	0.001	0.000	0.14
kddcup	0.002	0.032	0.003	9.54	0.002	0.007	0.002	5.69	0.003	0.045	0.011	31.00	0.005	0.010	0.006	16.17
kr-vs-kp	0.001	0.008	0.002	2.17	0.001	0.004	0.001	1.22	0.001	0.009	0.004	3.41	0.001	0.003	0.002	1.49
lending	0.001	0.003	0.001	1.99	0.001	0.003	0.001	1.51	0.004	0.065	0.007	0.73	0.008	0.069	0.011	1.12
letter	0.002	0.062	0.002	13.36	0.002	0.007	0.003	14.30	0.034	0.11	0.052	19.50	0.078	0.16	0.11	40.77
lymphography	0.001	0.007	0.002	0.32	0.001	0.003	0.001	0.20	0.000	0.002	0.001	0.13	0.001	0.001	0.001	0.090
messidor	0.000	0.001	0.000	0.096	0.000	0.001	0.000	0.11	0.001	0.006	0.003	1.06	0.002	0.004	0.002	0.74
promoters	0.000	0.001	0.001	0.063	0.000	0.001	0.000	0.053	0.000	0.002	0.000	0.046	0.000	0.001	0.000	0.045
mushroom	0.001	0.004	0.001	3.11	0.001	0.002	0.001	2.20	0.001	0.003	0.001	2.46	0.001	0.002	0.001	1.54
mux6	0.001	0.002	0.001	0.076	0.001	0.002	0.001	0.069	0.000	0.001	0.001	0.040	0.000	0.002	0.001	0.035
new-thyroid	0.000	0.002	0.000	0.078	0.000	0.001	0.000	0.086	0.000	0.001	0.001	0.17	0.001	0.001	0.001	0.15
pendigits	0.002	0.063	0.003	10.03	0.002	0.007	0.003	8.45	0.008	0.093	0.014	3.11	0.016	0.089	0.022	4.81
postoperative	0.001	0.002	0.001	0.093	0.001	0.002	0.001	0.077	0.000	0.002	0.001	0.048	0.000	0.002	0.001	0.040
primary-tumor	0.001	0.005	0.002	0.47	0.001	0.004	0.001	0.28	0.001	0.001	0.001	0.17	0.001	0.001	0.001	0.13
promoters	0.000	0.000	0.000	0.017	0.000	0.000	0.000	0.024	0.000	0.001	0.000	0.039	0.000	0.001	0.000	0.037
recidivism	0.002	0.061	0.003	3.65	0.002	0.006	0.002	2.59	0.005	0.087	0.010	11.69	0.010	0.084	0.015	18.26
schizo	0.000	0.002	0.001	0.22	0.000	0.001	0.001	0.17	0.001	0.003	0.002	0.58	0.001	0.002	0.001	0.37
segmentation	0.000	0.001	0.000	0.098	0.000	0.001	0.000	0.096	0.001	0.002	0.001	0.30	0.001	0.002	0.001	0.25
seismic_bumps	0.001	0.003	0.001	1.08	0.001	0.002	0.001	0.68	0.001	0.004	0.002	1.83	0.001	0.002	0.001	0.69
shuttle	0.001	0.006	0.001	22.65	0.001	0.005	0.001	22.35	0.002	0.061	0.002	2.85	0.003	0.060	0.003	3.56
soybean	0.001	0.058	0.002	1.17	0.001	0.005	0.001	0.82	0.001	0.005	0.003	1.65	0.001	0.003	0.002	0.95
spambase	0.001	0.008	0.003	3.43	0.001	0.003	0.001	1.81	0.002	0.069	0.009	11.20	0.003	0.062	0.003	4.18
spect	0.001	0.006	0.002	0.52	0.001	0.004	0.001	0.24	0.000	0.001	0.001	0.17	0.000	0.001	0.000	0.11
splice	0.000	0.001	0.000	0.22	0.000	0.002	0.000	0.30	0.001	0.064	0.002	1.48	0.003	0.069	0.004	3.43
student-mat	0.002	0.061	0.003	1.14	0.002	0.007	0.002	0.92	0.002	0.058	0.005	1.98	0.003	0.060	0.004	1.56
student-por	0.002	0.059	0.003	1.94	0.002	0.004	0.002	1.53	0.003	0.062	0.008	5.05	0.004	0.072	0.006	3.63
tram_2000	0.000	0.001	0.000	0.10	0.000	0.001	0.000	0.15	0.000	0.001	0.000	0.10	0.000	0.001	0.000	0.15
uci_mammo	0.001	0.003	0.002	0.23	0.001	0.002	0.001	0.15	0.001	0.002	0.001	0.12	0.001	0.001	0.001	0.079
vehicle	0.001	0.008	0.002	1.74	0.001	0.006	0.002	1.52	0.002	0.060	0.003	2.81	0.002	0.061	0.003	2.51
wdbc	0.000	0.001	0.000	0.16	0.000	0.001	0.000	0.18	0.002	0.062	0.007	3.85	0.002	0.004	0.002	1.41
yeast	0.001	0.002	0.001	0.33	0.001	0.002	0.001	0.42	0.004	0.10	0.007	3.06	0.008	0.072	0.011	5.04

Table 2: Assessing runtimes of the tree traversal algorithm and the propositional horn encoding approach for extracting one AXp. The table reports the results for DTs trained with ITI and IAI learning tools. Columns **m**, **M** and **avg** report, respectively, the minimal, maximal and average runtime (in second) to compute an AXp, while column **Tot** reports the total runtime (in second) of all tested instances in a dataset.

DT Ref	D	#N	#P	%R	%C	%m	%M	%avg
[1, Ch. 09, Fig. 9.1]	2	5	3	33	25	50	50	50
[2, Ch. 03, Fig. 3.2]	2	5	3	33	25	50	50	50
[5, Ch. 01, Fig. 1.3]	4	9	5	60	25	25	50	36
[7, Figure 1]	3	12	7	14	8	33	33	33
[3, Ch. 08, Fig. 8.2]	3	7	4	25	12	50	50	50
[6, Ch. 01, Fig. 1.1]	3	7	4	50	25	33	33	33
[8, Ch. 01, Fig. 1.2a]	2	5	3	33	25	33	33	33
[8, Ch. 01, Fig. 1.2b]	2	5	3	33	25	33	33	33
[11, Ch. 04, Fig. 4.14]	3	7	4	25	12	50	50	50
[11, Sec. 4.7, Ex. 4]	2	5	3	33	25	50	50	50
[16, Ch. 01, Fig. 1.3]	3	12	7	28	17	33	50	41
[17, Ch. 01, Fig. 1.5]	3	9	5	20	12	33	33	33
[17, Ch. 01, Fig. 1.4]	3	7	4	50	25	33	33	33
[24, Ch. 01, Fig. 1.2]	3	7	4	25	12	50	50	50
[21, Figure 4]	6	39	20	65	63	20	40	33
[9, Ch. 02, Fig. 2.1(right)]	2	5	3	33	25	50	50	50
[12, Figure 1]	3	10	6	33	11	33	33	33
[14, Figure 1]	3	9	5	80	75	33	50	41
[15, Ch. 07, Fig. 7.4]	3	7	4	50	25	33	33	33
[19, Ch. 18, Fig. 18.6]	4	12	8	25	6	25	33	29
[20, Ch. 18, Page 212]	2	5	3	33	25	50	50	50
[26, Ch. 01, Fig. 1.3]	2	5	3	33	25	33	33	33
[4, Figure 1b]	4	13	7	71	50	33	50	36
[25, Ch. 04, Fig. 4.3]	4	14	9	11	2	25	25	25

Table 3: Results on path explanation redundancy for example DTs found in the literature. Columns **D**, **#N**, **#P** and **%A** denote, resp. depth, number of nodes, number of paths and accuracy of the DT. Columns **%R** reports the percentage of explanation-redundant paths (XRP's) and **%C** shows the percentage of data instances covered by XRP's. Finally, columns **%avg** (**%m** and **%M** report, resp., the average, min. and max. percentage of explanation-redundant features (XRF's) per path.

Dataset	Tool	D	#N	%A	#P	%R	%C	%m	%M	%avg
monk1	BinOCT	3	13	91	7	28	11	66	66	66
	OSDT	5	13	100	7	57	41	33	33	33
tic-tac-toe	BinOCT	4	15	77	8	75	75	33	33	33
	OSDT	5	15	83	8	75	37	25	60	43
compas	OSDT	4	9	67	5	60	37	33	33	33
monk2	CART	6	31	69	16	62	22	20	66	33
	GOSDT	6	17	73	9	55	48	16	40	31

Table 4: Additional results on path explanation redundancy in (optimal) DTs, trained with different training tools: BinOCT [23], CART [6], OSDT [10] and GOSDT [13], that have been presented in [10, 18]. The results for CART are solely included for completeness. Columns **D**, **#N**, **#P** and **%A** denote, resp. depth, number of nodes, number of paths and accuracy of the DT. Columns **%R** reports the percentage of explanation-redundant paths (XRP's) and **%C** shows the percentage of data instances covered by XRP's. Finally, columns **%avg** (**%m** and **%M** report, resp., the average, min. and max. percentage of explanation-redundant features (XRF's) per path.

Dataset	DT			Path	AXp			
	D	**#N**	**%A**	**L**	**m**	**M**	**avg**	**n**
adult	17	509	73	16	8	8	8	2
				14	5	6	5.5	2
				16	5	5	5	1
allhyper	14	49	96	14	4	5	4.6	6
				9	4	5	4.5	8
				14	4	5	4.6	6
ann-thyroid	48	222	93	40	5	6	5.6	3
				36	5	6	5.6	3
				20	5	6	5.5	2
coil2000	12	177	91	10	2	4	3.8	39
				10	2	4	3.8	39
				10	2	4	3.8	30
fars	60	9969	76	29	11	11	11	2
				42	10	14	12.3	9
				48	9	9	9	1
kddcup	29	269	99	23	11	12	11.5	16
				23	11	12	11.5	16
				27	12	13	12.5	8

Table 5: Examples of 6 real-world datasets highlighting computed path AXp's (APXp's) in DTs learned with ITI, that require deep trees. For each dataset, the table displays 3 tested paths. Columns **D**, **#N** and **%A** denote, resp. depth, number of nodes and accuracy of the DT. Next, column **L** reports the path's length. Then, the average (min. or max., resp.) length of the computed APXp's, is denoted by **avg** (**m** and **M**, resp.). Finally, the total number of APXp's is shown in column **n**.